Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/27-1.1.3.4-e-x-^m-a+b-x^n-^p-c+d-x^n-^q

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [1081]. This is test number [27].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (1081)	0.00 (0)
Mathematica	100.00 (1081)	0.00 (0)
Maple	84.27 (911)	15.73 (170)
Fricas	75.21 (813)	24.79 (268)
Giac	52.82 (571)	47.18 (510)
Mupad	49.12 (531)	50.88 (550)
Sympy	37.56 (406)	62.44 (675)
Maxima	36.17 (391)	63.83 (690)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

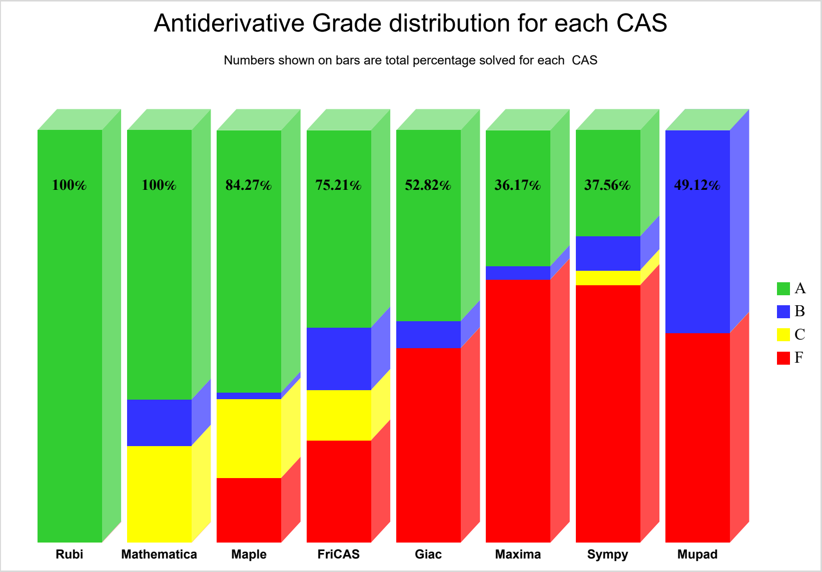
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

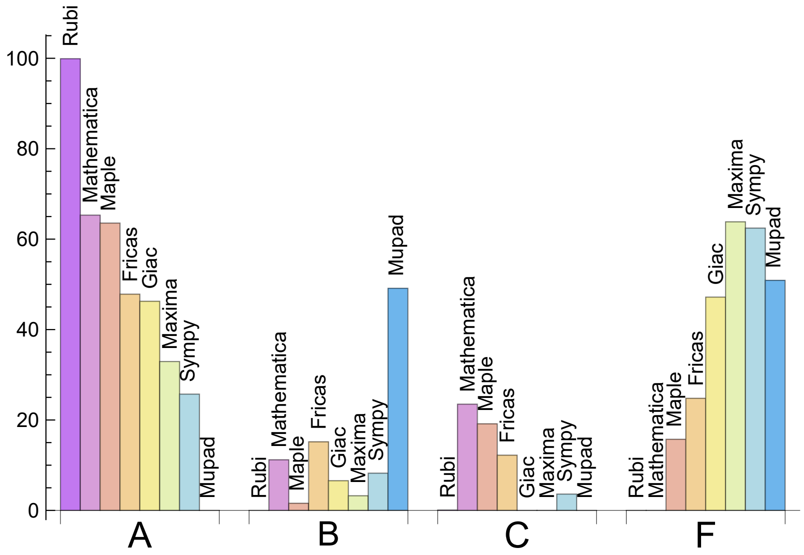
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.907	0.000	0.093	0.000
Mathematica	65.310	11.193	23.497	0.000
Maple	63.552	1.573	19.149	15.726
Fricas	47.826	15.171	12.211	24.792
Giac	46.253	6.568	0.000	47.179
Maxima	32.932	3.238	0.000	63.830
Sympy	25.717	8.233	3.608	62.442
Mupad	0.000	49.121	0.000	50.879

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	170	100.00	0.00	0.00
Fricas	268	39.93	57.46	2.61
Giac	510	95.88	0.20	3.92
Mupad	550	0.00	100.00	0.00
Sympy	675	79.70	18.22	2.07
Maxima	690	89.28	0.00	10.72

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.25
Giac	0.35
Rubi	0.37
Fricas	0.85
Mathematica	3.54
Maple	5.15
Mupad	8.09
Sympy	11.53

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	119.81	1.27	97.00	1.02
Mathematica	143.86	1.22	112.00	0.95
Giac	154.59	1.30	116.00	1.05
Sympy	204.11	1.93	114.00	1.18
Rubi	212.81	1.02	117.00	1.00
Maple	274.80	1.84	133.00	0.96
Mupad	439.68	2.41	117.00	1.12
Fricas	608.80	3.52	222.00	2.04

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

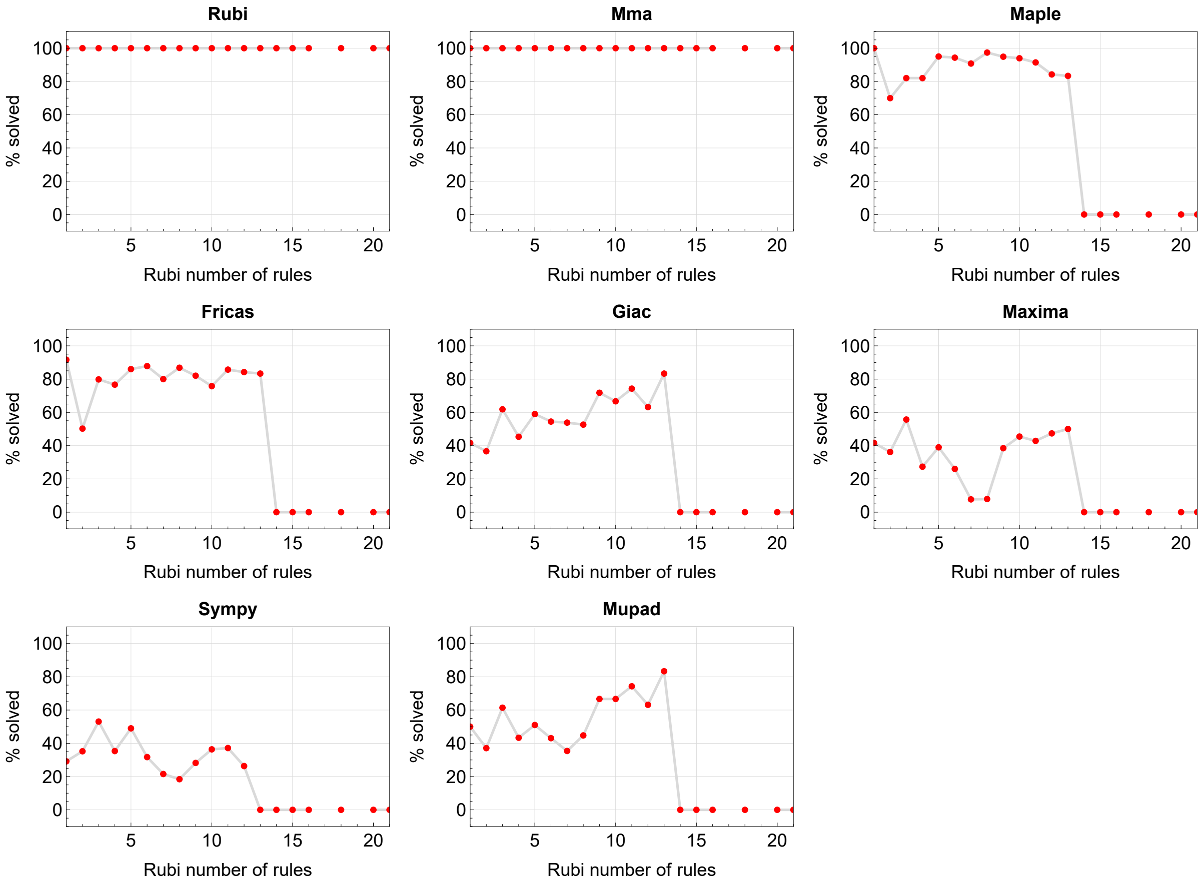


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

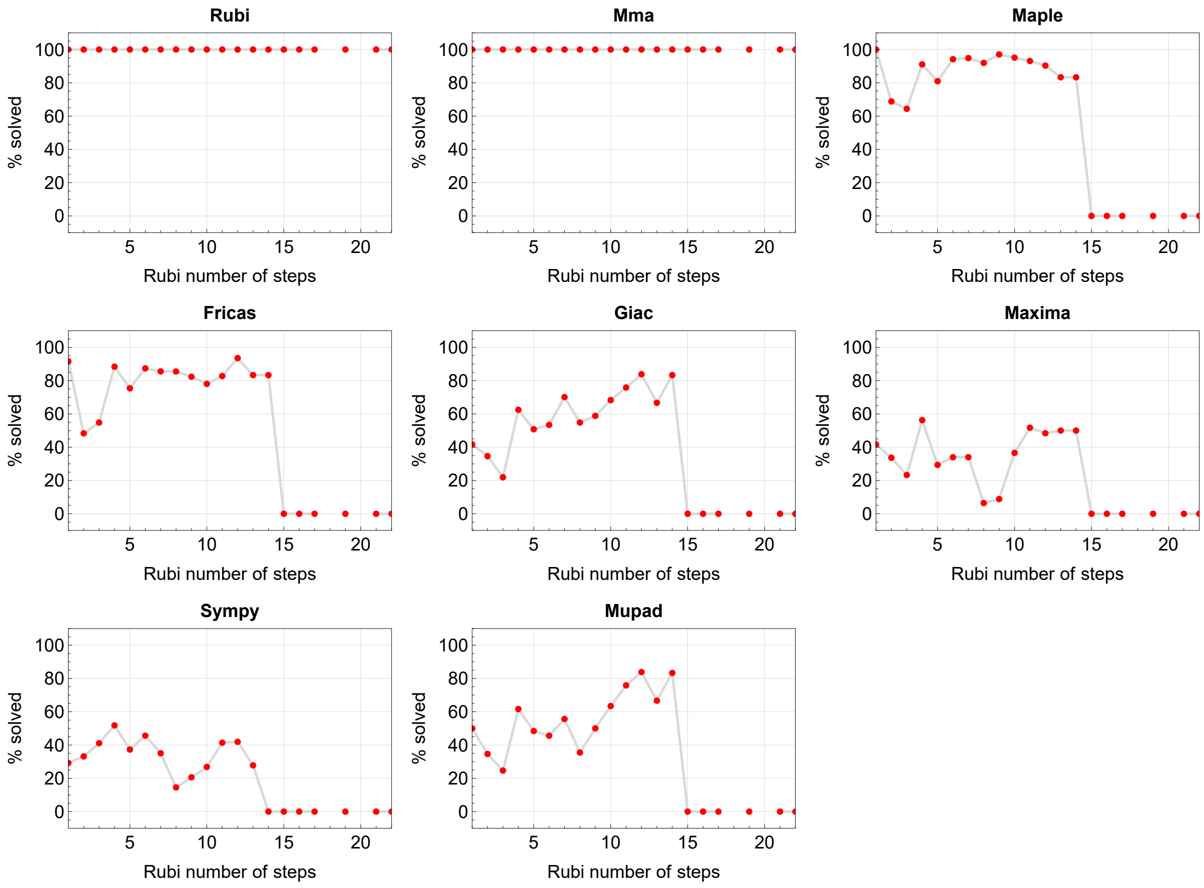


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

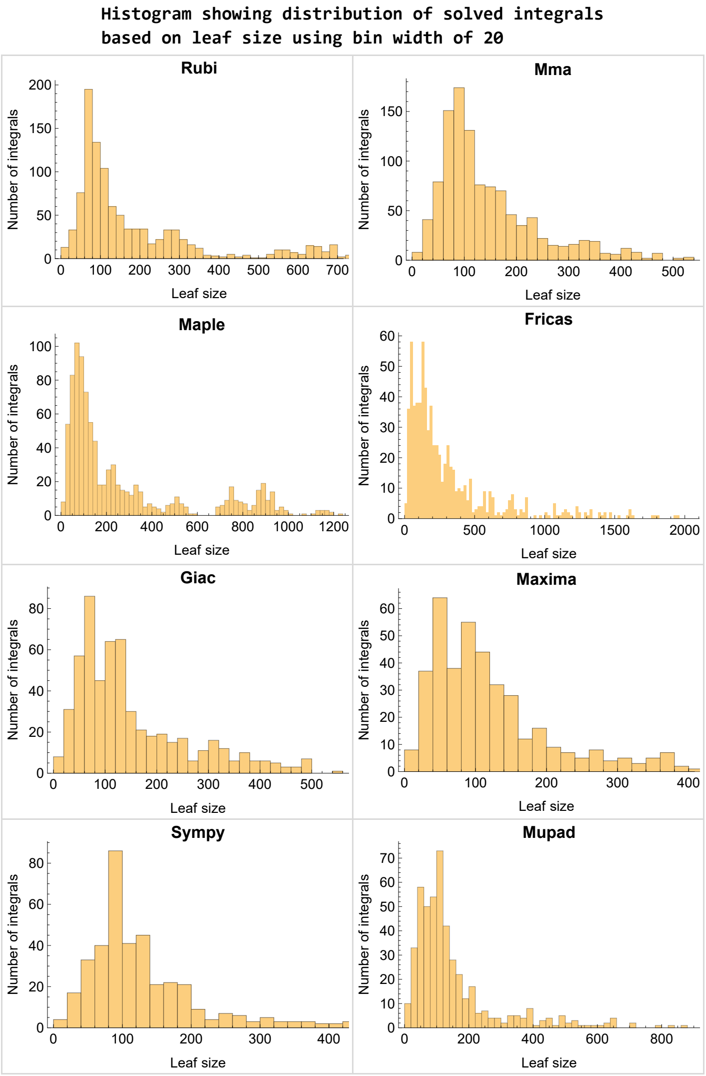


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

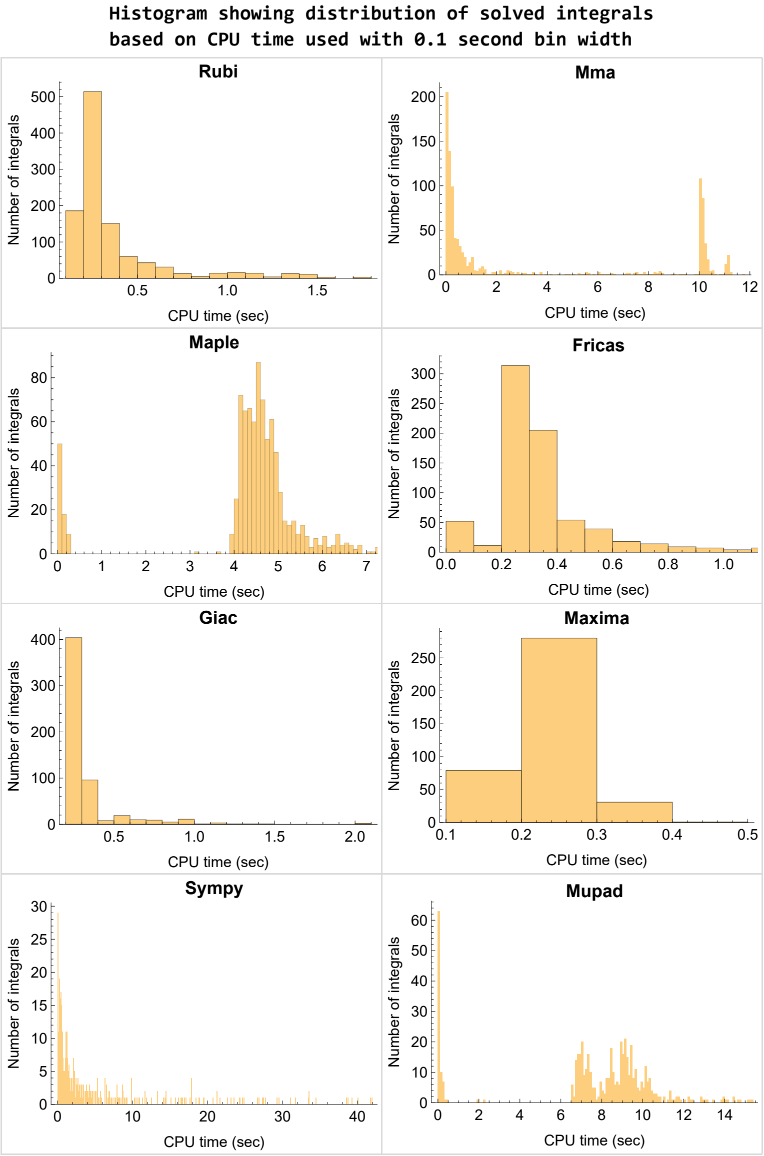


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

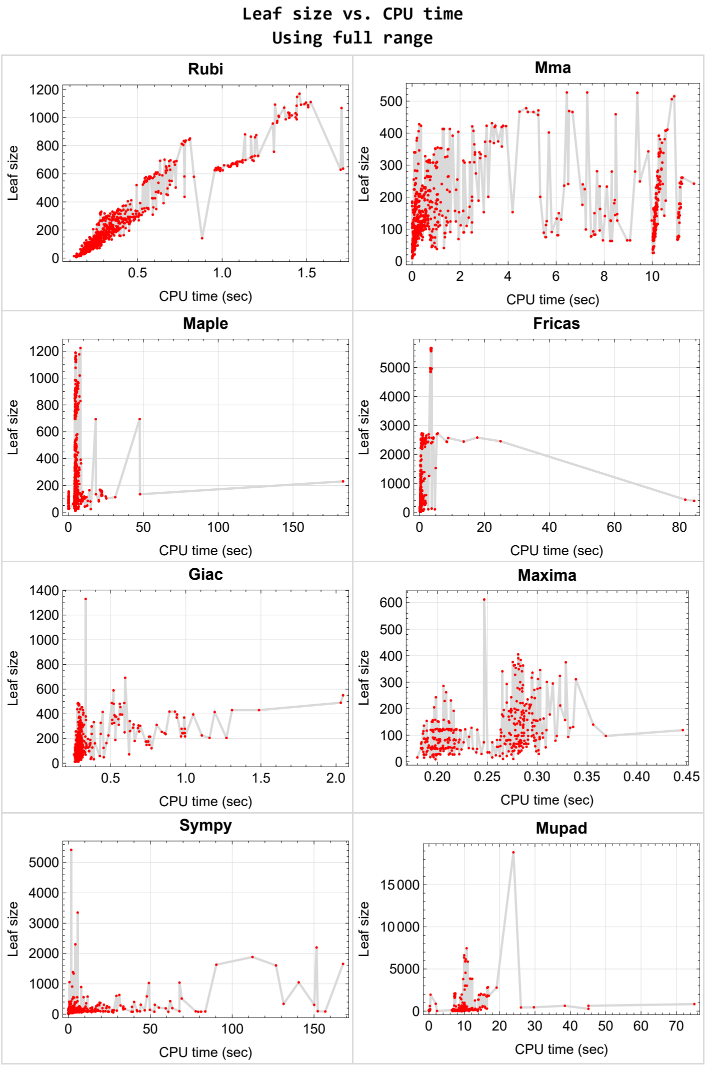


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{344, 345, 348, 349, 516, 519, 522, 525, 527, 530, 533, 535, 538, 541, 543, 546, 549, 551, 554, 559, 795, 799, 821, 834, 836, 837, 838, 839, 840, 900, 919, 920, 921, 922, 923\}$

Mathematica $\{267, 268, 269, 279, 280, 322, 323, 324, 338, 339, 340, 341, 363, 365, 367, 373, 375, 377, 385, 387, 393, 395, 397, 421, 437, 438, 439, 440, 441, 455, 456, 457, 458, 459, 465, 467, 469, 475, 477, 479, 485, 487, 489, 495, 497, 499, 513, 515, 581, 582, 584, 585, 634, 636, 637, 672, 673, 674, 675, 676, 709, 710, 711, 712, 713, 743, 745, 746, 796, 798, 800, 815, 817, 818, 834, 835, 836, 837, 866, 883, 905, 926, 994, 1002, 1003, 1011, 1012\}$

Maple {264, 265, 266, 267, 268, 269, 275, 276, 277, 278, 279, 280, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 634, 637, 795, 796, 797, 798, 799, 800, 815, 816, 817, 818, 819, 820, 821}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

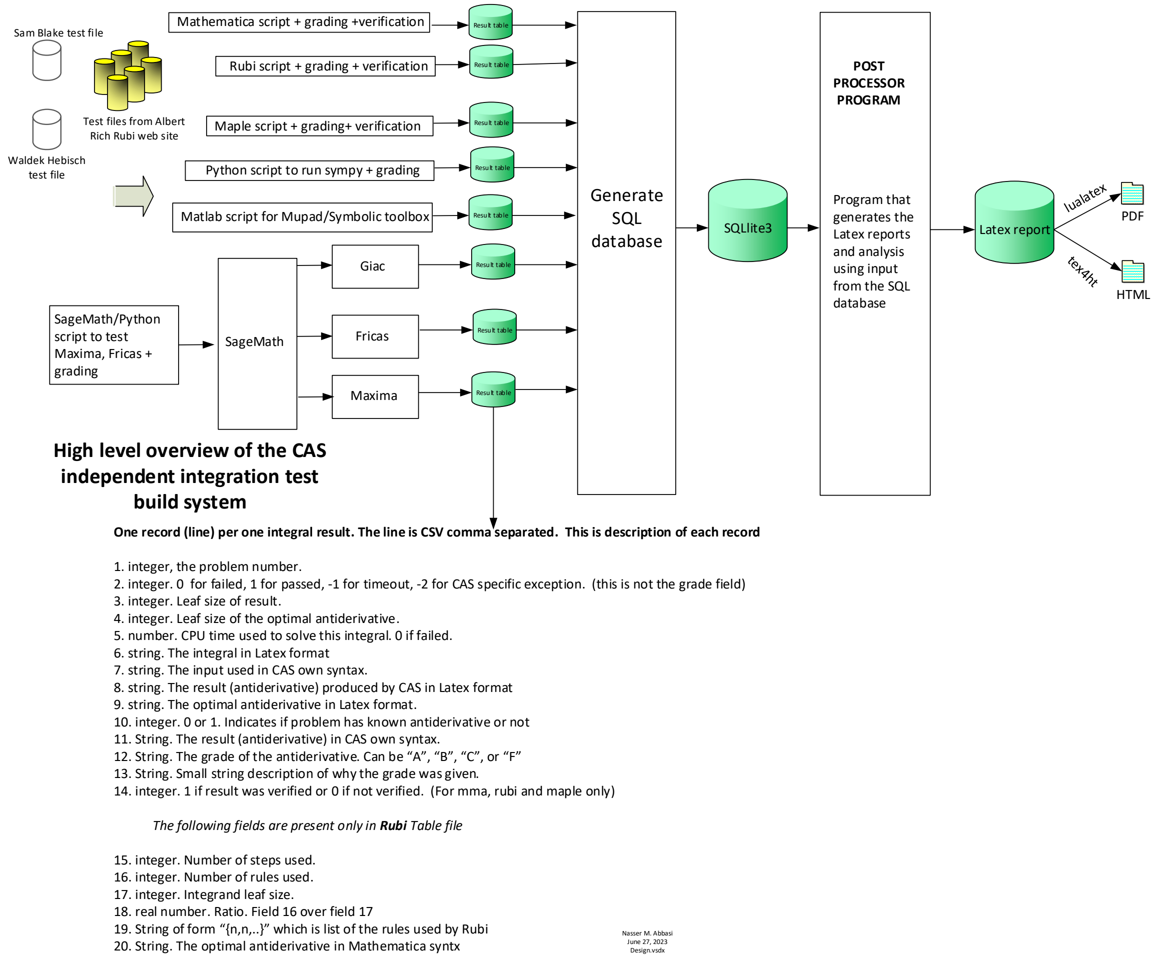
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,
28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53,
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726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745,
746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765,
766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785,
786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805,
806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825,
826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845,
846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865,
866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885,
886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905,
906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925,
926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945,
946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965,
966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985,
986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004,
1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020,
1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036,
1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052,
1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068,
1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081 }

B grade { }

C grade { 455 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 278, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 321, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 364, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 463, 464, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 516, 519, 522, 525, 527, 530, 533, 535, 538, 541, 543, 546, 549, 551, 554, 557, 559, 562, 565, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 620, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 658, 659, 660, 661, 662, 663, 664, 677, 678, 679, 680, 681, 682, 683, 693, 696, 697, 698, 699, 700, 701, 714, 715, 716, 717, 718, 719, 720, 728, 729, 732, 733, 734, 735, 736, 737, 744, 747, 748, 749, 750, 751, 752, 753, 763, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 801, 802, 803, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 841, 842, 843, 844, 846, 847, 848, 849, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 903, 904, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1008, 1009, 1010, 1011, 1012, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1054, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078 }

B grade { 30, 54, 267, 268, 269, 279, 280, 322, 323, 324, 338, 339, 340, 341, 363, 365, 366, 367, 373, 374, 375, 376, 377, 385, 386, 387, 393, 394, 395, 396, 397, 437, 438, 439, 440, 441, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 513, 514, 515, 672, 673, 674, 675, 676, 691, 692, 694, 695, 709, 710, 711, 712, 713, 727, 730, 731, 743, 745, 746, 761, 762, 764, 765, 766, 804, 845, 850, 866, 867, 868, 869, 880, 881, 882, 883, 884, 885, 886, 905, 906, 907, 924, 925, 926, 927, 928, 1004, 1005, 1006, 1007, 1013, 1014, 1015, 1016, 1051, 1052, 1053 }

C grade { 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 275, 276, 277, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 332, 333, 334, 335, 336, 337, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 449, 450, 451, 452, 453, 454, 455, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 550, 552, 553, 555, 556, 558, 560, 561, 563, 564, 566, 581, 582, 584, 585, 600, 601, 602, 603, 604, 618, 619, 621, 622, 634, 635, 636, 637, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 684, 685, 686, 687, 688, 689, 690, 702, 703, 704, 705, 706, 707, 708, 721, 722, 723, 724, 725, 726, 738, 739, 740, 741, 742, 754, 755, 756, 757, 758, 759, 760, 795, 796, 797, 798, 799, 800, 815, 816, 817, 818, 819, 820, 821, 834, 835, 836, 837, 838, 839, 840, 897, 898, 899, 900, 901, 902, 918, 919, 920, 921, 922, 923, 1024, 1055, 1079, 1080, 1081 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 57, 60, 63, 64, 65, 66, 67, 68, 69, 70, 72, 75, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 421, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 455, 460, 461, 462, 463, 464, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 516, 519, 522, 525, 527, 530, 533, 535, 538, 541, 543, 546, 549, 551, 554, 557, 559, 562, 565, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 638, 639, 640, 641, 642, 643, 646, 647, 648, 649, 650, 651, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 1002, 1003, 1004, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1029, 1036, 1039, 1040, 1041, 1042, 1043, 1044, 1046, 1047, 1048, 1049, 1050, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1078, 1079, 1080 }

B grade { 30, 54, 124, 125, 644, 645, 1005, 1014, 1024, 1045, 1051, 1052, 1053, 1054, 1055, 1065, 1081 }

C grade { 56, 58, 59, 61, 62, 71, 73, 74, 76, 77, 79, 80, 95, 96, 97, 98, 99, 100, 101, 102, 264, 265, 266, 267, 268, 269, 275, 276, 277, 278, 279, 280, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 550, 552, 553, 555, 556, 558, 560, 561, 563, 564, 566, 634, 637, 795, 796, 797, 798, 799, 800, 815, 816, 817, 818, 819, 820, 821, 834, 835, 836, 837, 838, 839, 840 }

F normal fail { 127, 128, 129, 130, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 581, 582, 583, 584, 585, 600, 601, 602, 603, 604, 618, 619, 620, 621, 622, 635, 636, 652, 653, 654, 655, 656, 657, 672, 673, 674, 675, 676, 691, 692, 693, 694, 695, 709, 710, 711, 712, 713, 727, 728, 729, 730, 731, 743, 744, 745, 746, 761, 762, 763, 764, 765, 766, 801, 802, 803, 804, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 863, 864, 865, 866, 867, 868, 869, 880, 881, 882, 883, 884, 885, 886, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 161, 163, 166, 169, 171, 174, 177, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 390, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 464, 470, 471, 472, 473, 474, 481, 483, 484, 506, 509, 516, 519, 522, 525, 527, 530, 533, 535, 538, 541, 543, 546, 549, 551, 554, 557, 559, 562, 565, 567, 568, 569, 570, 572, 573, 574, 575, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 605, 606, 607, 608, 609, 611, 612, 623, 624, 625, 626, 627, 628, 629, 630, 634, 637, 638, 639, 640, 642, 643, 644, 658, 659, 660, 661, 662, 663, 664, 665, 667, 682, 683, 696, 697, 698, 699, 700, 701, 702, 703, 714, 715, 719, 720, 721, 722, 751, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 787, 788, 789, 790, 791, 792, 793, 794, 805, 806, 807, 808, 809, 810, 811, 814, 822, 824, 826, 827, 828, 829, 833, 853, 854, 855, 856, 857, 858, 859, 862, 871, 873, 874, 875, 879, 887, 888, 889, 890, 891, 892, 893, 896, 909, 911, 912, 913, 917, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1024, 1029, 1039, 1040, 1041, 1042, 1043, 1045, 1046, 1047, 1048, 1049, 1056, 1057, 1058, 1059, 1064, 1066, 1067, 1068, 1069, 1071, 1072, 1073, 1074, 1075, 1078, 1079, 1080, 1081 }

B grade { 30, 54, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 124, 125, 126, 156, 157, 159, 160, 162, 164, 165, 167, 168, 170, 172, 173, 175, 176, 178, 267, 268, 269, 276, 278, 279, 280, 317, 321, 322, 323, 324, 338, 339, 340, 341, 350, 351, 352, 353, 354, 355, 356, 357, 388, 389, 391, 392, 437, 438, 439, 441, 456, 457, 458, 459, 463, 480, 482, 490, 491, 492, 493, 494, 507, 508, 571, 576, 577, 596, 614, 615, 616, 617, 620, 631, 632, 633, 641, 645, 646, 647, 648, 649, 650, 651, 666, 677, 678, 679, 680, 681, 684, 685, 686, 716, 717, 718, 732, 733, 734, 735, 736, 737, 738, 739, 747, 748, 749, 750, 752, 753, 754, 755, 812, 813, 823, 825, 830, 831, 832, 860, 861, 870, 872, 876, 877, 878, 894, 895, 908, 910, 914, 915, 916, 949, 975, 1014, 1036, 1044, 1050, 1051, 1052, 1053, 1054, 1055, 1060, 1061, 1062, 1063, 1065, 1070, 1076, 1077 }

C grade { 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 275, 277, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 306, 307, 315, 316, 318, 319, 320, 332, 333, 334, 335, 336, 337, 342, 343, 344, 345, 346, 347, 348, 349, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 449, 450, 451, 452, 453, 454, 455, 526, 550, 555, 556, 558, 560, 561, 563, 564, 566, 610, 613, 779, 780, 781, 782, 783, 784, 785, 786 }

F normal fail { 127, 128, 129, 130, 500, 501, 502, 503, 504, 505, 510, 511, 512, 513, 514, 515, 517, 518, 520, 521, 523, 524, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 552, 553, 618, 619, 621, 622, 635, 636, 652, 653, 654, 655, 656, 657, 796, 801, 804, 816, 819, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 867, 880, 884, 899, 900, 906, 927, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038 }

F(-1) timedout fail { 305, 363, 364, 365, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 395, 396, 397, 440, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 578, 579, 580, 581, 582, 583, 584, 585, 597, 598, 599, 600, 601, 602, 603, 604, 668, 669, 670, 671, 672, 673, 674, 675, 676, 687, 688, 689, 690, 691, 692, 693, 694, 695, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 723, 724, 725, 726, 727, 728, 729, 730, 731, 740, 741, 742, 743, 744, 745, 746, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 795, 797, 798, 799, 800, 802, 803, 815, 817, 818, 820, 821, 834, 835, 836, 837, 838, 839, 840, 864, 865, 868, 869, 881, 882, 883, 885, 886, 897, 898, 901, 902, 903, 905, 907, 918, 919, 920, 921, 922, 923, 924, 925, 926, 928 }

F(-2) exception fail { 366, 367, 393, 394, 863, 866, 904 }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 228, 229, 230, 231, 244, 245, 246, 247, 248, 259, 260, 261, 270, 271, 272, 282, 283, 284, 285, 295, 296, 297, 298, 308, 309, 310, 311, 325, 326, 327, 328, 398, 399, 400, 401, 411, 412, 413, 414, 424, 425, 426, 427, 442, 443, 444, 445, 525, 567, 568, 569, 570, 586, 587, 588, 589, 605, 606, 607, 608, 609, 623, 624, 625, 626, 638, 639, 640, 641, 642, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 964, 965, 966, 967, 968, 969, 970, 975, 976, 977, 978, 979, 980, 981, 982, 983, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1024, 1029, 1036, 1039, 1040, 1041, 1042, 1043, 1045, 1046, 1047, 1048, 1049, 1055, 1056, 1057, 1058, 1064, 1078, 1079, 1080, 1081 }

B grade { 30, 54, 184, 217, 218, 232, 233, 929, 930, 943, 944, 945, 959, 960, 961, 962, 963, 971, 972, 973, 974, 984, 985, 1014, 1044, 1050, 1051, 1052, 1053, 1054, 1059, 1060, 1061, 1062, 1063 }

C grade { }

F normal fail { 127, 128, 129, 130, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 262, 263, 264, 265, 266, 267, 268, 269, 273, 274, 275, 276, 277, 278, 279, 280, 281, 286, 287, 288, 289, 290, 291, 292, 293, 294, 299, 300, 301, 302, 303, 304, 305, 306, 307, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 361, 362, 363, 364, 365, 366, 367, 371, 372, 373, 374, 375, 376, 377, 381, 382, 383, 384, 385, 386, 387, 391, 392, 393, 394, 395, 396, 397, 402, 403, 404, 405, 406, 407, 408, 409, 410, 415, 416, 417, 418, 419, 420, 421, 422, 423, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 463, 464, 465, 466, 467, 468, 469, 473, 474, 475, 476, 477, 478, 479, 483, 484, 485, 486, 487, 488, 489, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 788, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077 }

F(-1) timedout fail { }

F(-2) exception fail { 358, 359, 360, 368, 369, 370, 378, 379, 380, 388, 389, 390, 460, 461, 462, 470, 471, 472, 480, 481, 482, 490, 491, 492, 506, 507, 508, 509, 658, 659, 660, 661, 677, 678, 679, 680, 696, 697, 698, 714, 715, 716, 717, 718, 732, 733, 734, 735, 747, 748, 749, 750, 751, 787, 789, 805, 806, 807, 822, 823, 824, 825, 853, 854, 855, 870, 871, 872, 887, 888, 889, 908, 909, 910 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 463, 464, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 506, 507, 519, 525, 543, 546, 549, 551, 554, 559, 562, 567, 568, 569, 570, 571, 572, 573, 586, 587, 588, 589, 590, 591, 592, 605, 606, 607, 608, 609, 610, 611, 623, 624, 625, 626, 627, 628, 638, 639, 640, 641, 642, 643, 644, 658, 659, 660, 661, 662, 663, 664, 677, 678, 681, 682, 683, 696, 698, 699, 700, 701, 714, 715, 716, 717, 718, 719, 720, 732, 733, 734, 735, 736, 737, 747, 748, 749, 752, 753, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 789, 791, 793, 805, 806, 807, 808, 809, 812, 813, 822, 823, 824, 825, 826, 827, 853, 854, 855, 856, 857, 860, 870, 871, 872, 873, 874, 887, 888, 889, 890, 891, 894, 895, 908, 909, 910, 911, 912, 929, 930, 931, 937, 938, 939, 940, 942, 943, 944, 945, 946, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 968, 969, 971, 972, 973, 974, 975, 976, 980, 981, 982, 984, 985, 1002, 1003, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1051, 1056, 1057, 1058, 1060, 1061, 1062, 1064, 1078 }

B grade { 27, 30, 54, 124, 125, 126, 508, 509, 516, 527, 530, 535, 538, 557, 679, 680, 697, 750, 751, 792, 794, 814, 828, 829, 830, 831, 832, 833, 858, 859, 875, 877, 896, 913, 914, 915, 916, 917, 932, 933, 934, 935, 936, 941, 947, 948, 949, 950, 951, 952, 965, 966, 967, 970, 977, 978, 979, 983, 1004, 1005, 1007, 1008, 1014, 1024, 1052, 1053, 1054, 1055, 1079, 1080, 1081 }

C grade { }

F normal fail { 127, 128, 129, 130, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 267, 268, 269, 275, 276, 277, 278, 279, 280, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 510, 511, 512, 513, 514, 515, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 550, 552, 553, 555, 556, 558, 560, 561, 563, 564, 566, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 629, 630, 631, 632, 633, 634, 635, 636, 637, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 738, 739, 740, 741, 742, 743, 744, 745, 746, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 815, 816, 817, 818, 819, 820, 821, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 861, 862, 863, 864, 865, 866, 867, 868, 869, 876, 878, 879, 880, 881, 882, 883, 884, 885, 886, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 998, 999, 1000, 1001, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1059, 1063, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077 }

F(-1) timedout fail { 1006 }

F(-2) exception fail { 350, 351, 352, 353, 354, 355, 356, 357, 522, 533, 541, 565, 788, 790, 810, 811, 892, 893, 996, 997 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 276, 281, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 317, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 463, 464, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 506, 507, 508, 509, 557, 562, 565, 567, 568, 569, 570, 571, 572, 573, 586, 587, 588, 589, 590, 591, 592, 605, 606, 607, 608, 609, 610, 611, 623, 624, 625, 626, 627, 628, 638, 639, 640, 641, 642, 643, 644, 658, 659, 660, 661, 662, 663, 664, 677, 678, 679, 680, 681, 682, 683, 696, 697, 698, 699, 700, 701, 714, 715, 716, 717, 718, 719, 720, 732, 733, 734, 735, 736, 737, 747, 748, 749, 750, 751, 752, 753, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 789, 791, 793, 805, 806, 807, 808, 809, 822, 823, 824, 825, 826, 827, 853, 854, 855, 856, 857, 870, 871, 872, 873, 874, 887, 888, 889, 890, 891, 908, 909, 910, 911, 912, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 959, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 1002, 1003, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1014, 1015, 1016, 1017, 1023, 1024, 1029, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1078, 1079, 1080, 1081 }

C grade { }

F normal fail { }

F(-1) timedout fail { 127, 128, 129, 130, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 267, 268, 269, 275, 277, 278, 279, 280, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 558, 559, 560, 561, 563, 564, 566, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 629, 630, 631, 632, 633, 634, 635, 636, 637, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 738, 739, 740, 741, 742, 743, 744, 745, 746, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 788, 790, 792, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 943, 944, 957, 958, 960, 961, 972, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1004, 1013, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 112, 114, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 182, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 199, 200, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 247, 249, 250, 251, 252, 254, 255, 256, 257, 258, 259, 260, 261, 262, 270, 271, 272, 273, 282, 283, 284, 285, 286, 295, 296, 297, 298, 299, 308, 309, 310, 311, 312, 325, 326, 327, 328, 329, 358, 359, 360, 368, 369, 370, 380, 381, 390, 391, 522, 525, 543, 549, 554, 557, 787, 789, 807, 808, 855, 856, 889, 890, 930, 931, 932, 933, 934, 935, 936, 941, 942, 943, 947, 948, 949, 950, 951, 952, 958, 962, 963, 964, 965, 966, 967, 969, 970, 971, 972, 973, 975, 976, 977, 978, 979, 982, 985, 1005, 1023, 1056, 1057, 1058, 1064 }

B grade { 27, 30, 110, 113, 124, 125, 126, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 179, 180, 181, 184, 196, 197, 198, 201, 218, 232, 244, 245, 246, 248, 361, 371, 516, 519, 527, 530, 533, 535, 538, 541, 546, 770, 791, 929, 937, 938, 939, 940, 944, 945, 946, 953, 954, 955, 956, 957, 959, 960, 961, 968, 974, 980, 981, 983, 984, 1024, 1029, 1039, 1040, 1041, 1045, 1046, 1047, 1051, 1052, 1053, 1054, 1055, 1059, 1060, 1061, 1062, 1063, 1078, 1079 }

C grade { 127, 128, 500, 501, 502, 503, 504, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 550, 553, 555, 556, 558, 841, 842, 843, 848, 849, 1081 }

F normal fail { 263, 264, 265, 266, 267, 268, 269, 274, 275, 276, 277, 278, 279, 280, 281, 287, 288, 289, 290, 291, 292, 293, 294, 300, 301, 302, 303, 304, 305, 306, 307, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 330, 331, 334, 335, 336, 337, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 362, 363, 364, 365, 366, 367, 372, 373, 374, 375, 376, 377, 378, 379, 382, 383, 384, 385, 386, 387, 388, 389, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 413, 414, 415, 419, 420, 421, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 472, 473, 475, 476, 477, 478, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 788, 790, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 844, 845, 847, 850, 853, 854, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 911, 912, 915, 916, 917, 919, 920, 922, 923, 924, 925, 926, 927, 928, 1002, 1003, 1004, 1006, 1007, 1008, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1065, 1068, 1069, 1070, 1074, 1075, 1076 }

F(-1) timedout fail { 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 107, 108, 109, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 129, 130, 163, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 253, 332, 333, 338, 404, 410, 411, 412, 416, 417, 418, 422, 423, 470, 471, 474, 479, 505, 551, 552, 559, 560, 561, 562, 563, 564, 565, 566, 708, 767, 768, 769, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 822, 823, 846, 851, 852, 870, 871, 908, 909, 910, 913, 914, 918, 921, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1009, 1010, 1066, 1067, 1071, 1072, 1073, 1077, 1080 }

F(-2) exception fail { 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1042, 1043, 1044, 1048, 1049, 1050 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	34	33	28	27	27	29	29	28
N.S.	1	1.03	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.173	0.009	0.213	0.211	0.238	0.017	0.267	0.046

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.170	0.007	0.238	0.199	0.270	0.017	0.277	6.760

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.169	0.007	0.247	0.259	0.227	0.017	0.276	0.037

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	32	29	27	28	25	27	28	26
N.S.	1	1.10	1.00	0.93	0.97	0.86	0.93	0.97	0.90
time (sec)	N/A	0.170	0.011	0.169	0.222	0.234	0.050	0.275	6.712

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	27	29	26	29	28
N.S.	1	1.00	1.00	0.97	0.87	0.94	0.84	0.94	0.90
time (sec)	N/A	0.171	0.012	0.047	0.217	0.234	0.045	0.270	0.042

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	24	24	28	24	23	24
N.S.	1	1.00	1.00	0.86	0.86	1.00	0.86	0.82	0.86
time (sec)	N/A	0.178	0.010	0.044	0.199	0.234	0.051	0.276	6.700

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	30	29	26	28	30	26	40	25
N.S.	1	1.03	1.00	0.90	0.97	1.03	0.90	1.38	0.86
time (sec)	N/A	0.184	0.013	0.051	0.200	0.224	0.108	0.267	0.045

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	32	28	29	29	31	31	29
N.S.	1	1.00	1.03	0.90	0.94	0.94	1.00	1.00	0.94
time (sec)	N/A	0.175	0.011	0.055	0.202	0.226	0.129	0.275	0.038

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	30	25	27	29	29	29	28
N.S.	1	1.00	1.07	0.89	0.96	1.04	1.04	1.04	1.00
time (sec)	N/A	0.173	0.013	0.047	0.199	0.310	0.144	0.268	6.716

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	33	31	26	30	31	29	37	29
N.S.	1	1.14	1.07	0.90	1.03	1.07	1.00	1.28	1.00
time (sec)	N/A	0.182	0.018	0.039	0.212	0.250	0.282	0.271	0.056

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	46	51	52	51	51	54	53	51
N.S.	1	1.10	1.21	1.24	1.21	1.21	1.29	1.26	1.21
time (sec)	N/A	0.205	0.017	4.142	0.194	0.257	0.027	0.262	6.784

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	54	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	0.98	0.96	0.93
time (sec)	N/A	0.200	0.008	4.156	0.205	0.349	0.022	0.264	0.049

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	51	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.02	1.00	0.96
time (sec)	N/A	0.199	0.009	4.016	0.219	0.232	0.023	0.279	0.044

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	50	51	50	52	49	53	52	49
N.S.	1	1.09	1.11	1.09	1.13	1.07	1.15	1.13	1.07
time (sec)	N/A	0.193	0.017	3.926	0.197	0.255	0.063	0.280	0.041

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	52	51	53	49	52	50
N.S.	1	1.00	1.00	0.98	0.96	1.00	0.92	0.98	0.94
time (sec)	N/A	0.195	0.021	4.227	0.206	0.237	0.060	0.263	0.049

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	53	49	48	48
N.S.	1	1.00	1.00	0.98	0.96	1.06	0.98	0.96	0.96
time (sec)	N/A	0.193	0.019	4.076	0.200	0.240	0.062	0.298	0.050

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	49	49	52	54	51	69	49
N.S.	1	1.02	0.96	0.96	1.02	1.06	1.00	1.35	0.96
time (sec)	N/A	0.199	0.026	4.187	0.215	0.246	0.150	0.294	0.046

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	50	53	53	53	54	52
N.S.	1	1.00	0.96	0.94	1.00	1.00	1.00	1.02	0.98
time (sec)	N/A	0.197	0.019	4.256	0.217	0.236	0.161	0.278	0.050

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	46	51	53	53	51	50
N.S.	1	1.00	1.00	0.92	1.02	1.06	1.06	1.02	1.00
time (sec)	N/A	0.201	0.021	3.930	0.231	0.233	0.187	0.286	6.752

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	51	46	54	55	51	70	52
N.S.	1	1.02	1.00	0.90	1.06	1.08	1.00	1.37	1.02
time (sec)	N/A	0.201	0.022	4.093	0.213	0.240	0.402	0.256	0.056

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	48	54	53	58	56	53
N.S.	1	1.00	1.02	0.91	1.02	1.00	1.09	1.06	1.00
time (sec)	N/A	0.194	0.019	4.089	0.195	0.236	0.495	0.267	0.051

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	51	53	54	53	50
N.S.	1	1.00	1.00	0.90	1.02	1.06	1.08	1.06	1.00
time (sec)	N/A	0.198	0.024	4.106	0.208	0.260	0.524	0.263	0.048

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	136	125	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.16	1.07	0.91
time (sec)	N/A	0.286	0.025	4.190	0.205	0.248	0.029	0.276	0.052

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	99	107	121	119	119	136	125	107
N.S.	1	1.04	1.13	1.27	1.25	1.25	1.43	1.32	1.13
time (sec)	N/A	0.281	0.026	4.179	0.211	0.257	0.031	0.274	6.691

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	134	125	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.15	1.07	0.91
time (sec)	N/A	0.272	0.020	4.034	0.202	0.269	0.029	0.266	0.041

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	136	125	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.16	1.07	0.91
time (sec)	N/A	0.272	0.019	4.309	0.195	0.250	0.038	0.276	0.043

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	71	107	121	119	119	138	125	107
N.S.	1	1.06	1.60	1.81	1.78	1.78	2.06	1.87	1.60
time (sec)	N/A	0.238	0.026	4.010	0.204	0.240	0.031	0.279	0.041

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	136	125	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.16	1.07	0.91
time (sec)	N/A	0.266	0.017	4.036	0.213	0.330	0.030	0.266	0.042

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	120	119	119	133	124	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.14	1.06	0.91
time (sec)	N/A	0.264	0.018	4.315	0.222	0.255	0.030	0.274	0.041

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	46	107	121	119	119	136	125	107
N.S.	1	1.10	2.55	2.88	2.83	2.83	3.24	2.98	2.55
time (sec)	N/A	0.209	0.026	4.072	0.213	0.242	0.032	0.269	0.045

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	120	119	119	134	124	106
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.15	1.06	0.91
time (sec)	N/A	0.266	0.017	4.031	0.195	0.239	0.029	0.264	0.044

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	109	116	115	115	128	120	103
N.S.	1	1.00	1.00	1.06	1.06	1.06	1.17	1.10	0.94
time (sec)	N/A	0.259	0.017	4.028	0.202	0.239	0.029	0.281	0.048

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	87	113	119	120	117	134	124	105
N.S.	1	0.99	1.28	1.35	1.36	1.33	1.52	1.41	1.19
time (sec)	N/A	0.226	0.032	3.945	0.193	0.242	0.105	0.279	0.046

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	121	118	121	129	124	106
N.S.	1	1.00	1.00	1.08	1.05	1.08	1.15	1.11	0.95
time (sec)	N/A	0.256	0.036	4.115	0.217	0.244	0.106	0.272	0.047

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	120	116	121	128	119	104
N.S.	1	1.00	1.00	1.07	1.04	1.08	1.14	1.06	0.93
time (sec)	N/A	0.257	0.035	4.191	0.202	0.239	0.110	0.283	0.043

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	115	121	120	123	133	143	105
N.S.	1	1.00	1.02	1.07	1.06	1.09	1.18	1.27	0.93
time (sec)	N/A	0.277	0.046	4.105	0.206	0.241	0.194	0.266	0.053

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	115	122	121	121	133	127	109
N.S.	1	1.00	1.02	1.08	1.07	1.07	1.18	1.12	0.96
time (sec)	N/A	0.262	0.039	4.160	0.202	0.238	0.209	0.266	0.044

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	113	119	120	121	133	124	108
N.S.	1	1.00	1.00	1.05	1.06	1.07	1.18	1.10	0.96
time (sec)	N/A	0.268	0.038	4.416	0.207	0.245	0.222	0.274	0.048

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	116	106	117	122	123	131	148	113
N.S.	1	1.02	0.93	1.03	1.07	1.08	1.15	1.30	0.99
time (sec)	N/A	0.276	0.053	4.029	0.198	0.256	0.508	0.282	0.050

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	117	121	121	129	127	113
N.S.	1	1.00	1.00	1.06	1.10	1.10	1.17	1.15	1.03
time (sec)	N/A	0.261	0.039	4.129	0.240	0.249	0.595	0.284	6.754

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	113	114	120	121	133	124	111
N.S.	1	1.00	1.00	1.01	1.06	1.07	1.18	1.10	0.98
time (sec)	N/A	0.265	0.039	4.125	0.195	0.253	0.616	0.273	0.048

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	116	106	111	123	123	129	150	118
N.S.	1	1.02	0.93	0.97	1.08	1.08	1.13	1.32	1.04
time (sec)	N/A	0.276	0.058	4.143	0.211	0.239	1.530	0.275	0.054

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	118	111	122	121	131	127	118
N.S.	1	1.00	1.03	0.97	1.06	1.05	1.14	1.10	1.03
time (sec)	N/A	0.262	0.025	4.114	0.202	0.257	8.427	0.262	6.594

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	109	108	120	121	131	124	116
N.S.	1	1.00	1.00	0.99	1.10	1.11	1.20	1.14	1.06
time (sec)	N/A	0.261	0.042	4.134	0.205	0.256	27.720	0.269	0.069

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	115	118	106	123	123	129	149	122
N.S.	1	1.01	1.04	0.93	1.08	1.08	1.13	1.31	1.07
time (sec)	N/A	0.271	0.041	4.336	0.214	0.305	46.573	0.273	0.064

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	117	107	122	121	0	128	123
N.S.	1	1.00	1.02	0.93	1.06	1.05	0.00	1.11	1.07
time (sec)	N/A	0.256	0.040	3.967	0.212	0.241	0.000	0.308	6.562

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	102	119	121	0	123	120
N.S.	1	1.00	1.00	0.93	1.08	1.10	0.00	1.12	1.09
time (sec)	N/A	0.263	0.044	4.130	0.203	0.242	0.000	0.271	6.549

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	112	116	102	123	123	0	145	121
N.S.	1	0.99	1.03	0.90	1.09	1.09	0.00	1.28	1.07
time (sec)	N/A	0.268	0.059	4.331	0.219	0.233	0.000	0.291	6.511

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	118	104	122	121	0	128	121
N.S.	1	1.00	1.03	0.90	1.06	1.05	0.00	1.11	1.05
time (sec)	N/A	0.266	0.037	4.344	0.228	0.236	0.000	0.281	6.588

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	101	119	121	0	125	119
N.S.	1	1.00	1.00	0.92	1.08	1.10	0.00	1.14	1.08
time (sec)	N/A	0.259	0.047	4.132	0.202	0.267	0.000	0.273	0.080

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	90	121	102	123	123	0	136	121
N.S.	1	0.99	1.33	1.12	1.35	1.35	0.00	1.49	1.33
time (sec)	N/A	0.228	0.040	4.117	0.209	0.234	0.000	0.288	0.096

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	119	104	121	121	0	127	119
N.S.	1	1.00	1.05	0.92	1.07	1.07	0.00	1.12	1.05
time (sec)	N/A	0.261	0.033	4.116	0.219	0.238	0.000	0.305	6.598

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	121	121	0	127	121
N.S.	1	1.00	1.03	0.89	1.03	1.03	0.00	1.09	1.03
time (sec)	N/A	0.263	0.034	3.965	0.214	0.241	0.000	0.278	6.744

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	52	118	104	121	121	0	127	122
N.S.	1	1.08	2.46	2.17	2.52	2.52	0.00	2.65	2.54
time (sec)	N/A	0.182	0.032	4.135	0.218	0.236	0.000	0.282	6.773

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	121	121	0	127	121
N.S.	1	1.00	1.00	0.89	1.03	1.03	0.00	1.09	1.03
time (sec)	N/A	0.263	0.046	4.126	0.211	0.232	0.000	0.291	0.065

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	156	171	84	182	167	114	217	164
N.S.	1	0.85	0.93	0.46	0.99	0.91	0.62	1.19	0.90
time (sec)	N/A	0.305	0.106	4.008	0.295	0.252	0.315	0.282	0.280

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	53	47	49	50	51	46	52	52
N.S.	1	0.98	0.87	0.91	0.93	0.94	0.85	0.96	0.96
time (sec)	N/A	0.214	0.026	4.220	0.206	0.239	0.252	0.289	0.079

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	157	154	65	157	162	114	207	144
N.S.	1	0.94	0.92	0.39	0.94	0.97	0.68	1.24	0.86
time (sec)	N/A	0.333	0.080	4.019	0.296	0.257	0.259	0.289	7.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	147	152	60	154	145	87	186	162
N.S.	1	0.91	0.94	0.37	0.95	0.90	0.54	1.15	1.00
time (sec)	N/A	0.328	0.081	4.277	0.300	0.367	0.274	0.292	7.055

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	33	31	32	31	30	27	32	31
N.S.	1	0.94	0.89	0.91	0.89	0.86	0.77	0.91	0.89
time (sec)	N/A	0.190	0.014	4.363	0.213	0.240	0.219	0.388	0.067

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	140	152	45	131	382	92	161	126
N.S.	1	0.93	1.01	0.30	0.87	2.55	0.61	1.07	0.84
time (sec)	N/A	0.310	0.045	4.024	0.336	0.256	0.221	0.395	7.002

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	130	129	42	128	369	71	133	123
N.S.	1	0.90	0.89	0.29	0.88	2.54	0.49	0.92	0.85
time (sec)	N/A	0.287	0.059	4.170	0.333	0.297	0.245	0.357	6.995

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	38	34	33	35	32	26	34	36
N.S.	1	1.12	1.00	0.97	1.03	0.94	0.76	1.00	1.06
time (sec)	N/A	0.192	0.015	4.155	0.239	0.251	0.606	0.367	0.115

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	139	134	114	140	372	90	155	126
N.S.	1	0.95	0.91	0.78	0.95	2.53	0.61	1.05	0.86
time (sec)	N/A	0.302	0.086	4.179	0.278	0.258	0.266	0.361	6.964

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	136	135	113	140	411	73	161	126
N.S.	1	0.91	0.91	0.76	0.94	2.76	0.49	1.08	0.85
time (sec)	N/A	0.298	0.101	4.201	0.356	0.265	0.274	0.335	0.251

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	51	49	46	48	47	41	69	46
N.S.	1	1.02	0.98	0.92	0.96	0.94	0.82	1.38	0.92
time (sec)	N/A	0.210	0.024	3.999	0.220	0.271	0.621	0.333	0.118

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	156	154	130	147	158	112	197	178
N.S.	1	0.95	0.93	0.79	0.89	0.96	0.68	1.19	1.08
time (sec)	N/A	0.330	0.113	4.203	0.295	0.251	0.288	0.348	6.803

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	153	154	130	148	176	99	176	145
N.S.	1	0.91	0.92	0.77	0.88	1.05	0.59	1.05	0.86
time (sec)	N/A	0.324	0.124	4.201	0.283	0.260	0.329	0.308	6.725

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	70	70	64	70	73	61	99	70
N.S.	1	1.01	1.01	0.93	1.01	1.06	0.88	1.43	1.01
time (sec)	N/A	0.225	0.030	4.143	0.201	0.232	0.678	0.322	0.128

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	173	173	150	178	180	139	216	161
N.S.	1	0.94	0.94	0.82	0.97	0.98	0.76	1.17	0.88
time (sec)	N/A	0.350	0.137	4.198	0.313	0.368	0.354	0.331	6.813

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	193	203	114	218	271	156	244	209
N.S.	1	0.83	0.87	0.49	0.94	1.16	0.67	1.05	0.90
time (sec)	N/A	0.322	0.134	4.171	0.289	0.328	0.575	0.289	6.798

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	79	72	76	82	121	82	106	86
N.S.	1	0.96	0.88	0.93	1.00	1.48	1.00	1.29	1.05
time (sec)	N/A	0.253	0.071	4.258	0.199	0.240	0.577	0.287	0.089

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	186	185	94	192	257	151	236	179
N.S.	1	0.87	0.86	0.44	0.89	1.20	0.70	1.10	0.83
time (sec)	N/A	0.320	0.128	4.034	0.289	0.260	0.619	0.296	6.834

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	182	181	87	187	240	126	211	193
N.S.	1	0.85	0.85	0.41	0.88	1.13	0.59	0.99	0.91
time (sec)	N/A	0.313	0.125	4.194	0.284	0.267	0.528	0.301	6.866

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	55	50	57	60	81	56	91	62
N.S.	1	0.92	0.83	0.95	1.00	1.35	0.93	1.52	1.03
time (sec)	N/A	0.215	0.039	4.145	0.197	0.248	0.520	0.305	0.085

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	183	165	71	162	578	126	189	158
N.S.	1	0.93	0.84	0.36	0.83	2.95	0.64	0.96	0.81
time (sec)	N/A	0.350	0.120	4.210	0.287	0.280	0.545	0.311	6.832

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	172	160	65	157	573	102	166	150
N.S.	1	0.91	0.84	0.34	0.83	3.02	0.54	0.87	0.79
time (sec)	N/A	0.349	0.130	4.198	0.328	0.253	0.439	0.294	6.807

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	40	41	38	40	44	36	65	37
N.S.	1	0.98	1.00	0.93	0.98	1.07	0.88	1.59	0.90
time (sec)	N/A	0.198	0.015	3.990	0.206	0.254	0.299	0.291	6.609

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	165	146	67	160	548	117	186	145
N.S.	1	0.96	0.85	0.39	0.94	3.20	0.68	1.09	0.85
time (sec)	N/A	0.329	0.096	4.205	0.286	0.265	0.373	0.269	6.770

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	158	145	65	158	537	97	160	143
N.S.	1	0.93	0.86	0.38	0.93	3.18	0.57	0.95	0.85
time (sec)	N/A	0.321	0.091	4.192	0.272	0.256	0.331	0.286	6.814

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	46	48	51	70	46	61	47
N.S.	1	1.02	0.90	0.94	1.00	1.37	0.90	1.20	0.92
time (sec)	N/A	0.205	0.031	4.008	0.204	0.230	0.292	0.277	0.148

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	181	164	139	166	570	122	180	156
N.S.	1	0.93	0.84	0.71	0.85	2.92	0.63	0.92	0.80
time (sec)	N/A	0.360	0.129	4.369	0.283	0.270	0.382	0.279	6.896

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	178	163	138	172	618	109	188	159
N.S.	1	0.91	0.83	0.70	0.88	3.15	0.56	0.96	0.81
time (sec)	N/A	0.347	0.130	4.036	0.275	0.272	0.402	0.279	0.259

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	75	64	76	76	118	70	80	78
N.S.	1	0.99	0.84	1.00	1.00	1.55	0.92	1.05	1.03
time (sec)	N/A	0.242	0.050	4.305	0.190	0.253	0.709	0.277	6.848

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	198	185	155	186	259	153	231	209
N.S.	1	0.92	0.86	0.72	0.87	1.20	0.71	1.07	0.97
time (sec)	N/A	0.384	0.147	4.443	0.274	0.259	0.424	0.278	7.034

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	195	183	154	186	277	138	206	176
N.S.	1	0.91	0.85	0.72	0.87	1.29	0.64	0.96	0.82
time (sec)	N/A	0.380	0.145	4.211	0.278	0.258	0.464	0.283	6.986

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	95	85	96	106	154	100	149	100
N.S.	1	0.98	0.88	0.99	1.09	1.59	1.03	1.54	1.03
time (sec)	N/A	0.268	0.101	4.180	0.185	0.319	0.803	0.281	0.152

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	106	94	100	115	179	112	131	117
N.S.	1	0.99	0.88	0.93	1.07	1.67	1.05	1.22	1.09
time (sec)	N/A	0.291	0.069	4.234	0.186	0.297	1.703	0.293	0.105

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	83	92	76	94	142	94	93	94
N.S.	1	0.94	1.05	0.86	1.07	1.61	1.07	1.06	1.07
time (sec)	N/A	0.250	0.041	4.016	0.232	0.329	1.404	0.286	6.831

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	65	64	57	72	89	70	61	70
N.S.	1	0.98	0.97	0.86	1.09	1.35	1.06	0.92	1.06
time (sec)	N/A	0.224	0.026	4.168	0.211	0.248	1.077	0.285	6.830

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	29	42	42	42	28	44
N.S.	1	1.00	0.94	0.91	1.31	1.31	1.31	0.88	1.38
time (sec)	N/A	0.164	0.017	3.976	0.214	0.255	0.441	0.277	6.791

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	69	59	61	77	119	75	74	71
N.S.	1	1.01	0.87	0.90	1.13	1.75	1.10	1.09	1.04
time (sec)	N/A	0.226	0.049	4.186	0.203	0.268	0.396	0.288	0.169

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	100	87	98	109	197	107	136	107
N.S.	1	0.99	0.86	0.97	1.08	1.95	1.06	1.35	1.06
time (sec)	N/A	0.271	0.059	4.175	0.216	0.264	0.836	0.289	6.905

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	108	123	136	229	133	131	130
N.S.	1	1.00	0.89	1.01	1.11	1.88	1.09	1.07	1.07
time (sec)	N/A	0.302	0.085	4.180	0.186	0.262	0.911	0.282	0.159

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	213	216	116	228	364	192	259	213
N.S.	1	0.87	0.88	0.47	0.93	1.48	0.78	1.05	0.87
time (sec)	N/A	0.350	0.191	4.372	0.279	0.252	9.878	0.295	7.081

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	209	210	109	223	347	163	234	227
N.S.	1	0.86	0.86	0.45	0.91	1.42	0.67	0.96	0.93
time (sec)	N/A	0.343	0.160	4.188	0.269	0.290	1.388	0.292	0.337

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	209	194	90	196	792	162	210	187
N.S.	1	0.94	0.87	0.41	0.88	3.57	0.73	0.95	0.84
time (sec)	N/A	0.396	0.166	4.151	0.279	0.292	7.931	0.288	6.995

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	199	188	85	191	789	141	187	183
N.S.	1	0.90	0.85	0.39	0.87	3.59	0.64	0.85	0.83
time (sec)	N/A	0.380	0.164	4.444	0.272	0.296	1.219	0.284	0.280

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	192	181	85	195	756	155	206	175
N.S.	1	0.96	0.90	0.42	0.97	3.76	0.77	1.02	0.87
time (sec)	N/A	0.366	0.176	4.040	0.267	0.299	5.366	0.276	6.965

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	185	178	83	193	743	136	187	173
N.S.	1	0.93	0.89	0.42	0.97	3.73	0.68	0.94	0.87
time (sec)	N/A	0.358	0.158	4.266	0.274	0.292	0.779	0.281	6.972

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	192	178	86	195	752	153	207	175
N.S.	1	0.96	0.89	0.43	0.97	3.74	0.76	1.03	0.87
time (sec)	N/A	0.367	0.135	4.292	0.281	0.369	0.524	0.292	0.268

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	183	175	84	192	743	133	180	173
N.S.	1	0.93	0.89	0.43	0.97	3.77	0.68	0.91	0.88
time (sec)	N/A	0.354	0.132	4.108	0.273	0.307	0.421	0.291	0.263

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	208	193	159	199	776	162	204	185
N.S.	1	0.92	0.85	0.70	0.88	3.42	0.71	0.90	0.81
time (sec)	N/A	0.390	0.163	4.261	0.274	0.294	0.482	0.307	7.014

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	205	189	158	201	812	143	209	188
N.S.	1	0.90	0.83	0.70	0.89	3.58	0.63	0.92	0.83
time (sec)	N/A	0.387	0.154	4.089	0.292	0.270	0.498	0.288	6.871

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	225	214	175	221	366	189	254	240
N.S.	1	0.91	0.87	0.71	0.90	1.49	0.77	1.03	0.98
time (sec)	N/A	0.420	0.193	4.273	0.283	0.274	0.568	0.283	6.862

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	222	210	174	221	384	173	229	207
N.S.	1	0.90	0.85	0.71	0.90	1.56	0.70	0.93	0.84
time (sec)	N/A	0.422	0.218	4.134	0.285	0.273	0.587	0.282	6.826

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	66	66	65	68	72	0	70	68
N.S.	1	0.94	0.94	0.93	0.97	1.03	0.00	1.00	0.97
time (sec)	N/A	0.230	0.036	4.154	0.217	0.368	0.000	0.288	7.044

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	316	242	228	324	273	0	311	1751
N.S.	1	1.05	0.80	0.76	1.08	0.91	0.00	1.03	5.82
time (sec)	N/A	0.490	0.153	4.212	0.323	0.545	0.000	0.287	16.266

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	271	238	225	349	228	0	308	873
N.S.	1	0.92	0.80	0.76	1.18	0.77	0.00	1.04	2.95
time (sec)	N/A	0.484	0.120	4.524	0.281	0.291	0.000	0.287	1.926

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	52	43	43	49	42	144	51	51
N.S.	1	0.98	0.81	0.81	0.92	0.79	2.72	0.96	0.96
time (sec)	N/A	0.214	0.022	4.160	0.198	0.301	5.027	0.284	7.081

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	258	224	207	289	244	0	286	1364
N.S.	1	0.90	0.78	0.72	1.00	0.85	0.00	0.99	4.74
time (sec)	N/A	0.439	0.085	4.291	0.286	0.271	0.000	0.301	14.053

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	248	224	207	317	199	342	278	1265
N.S.	1	0.86	0.78	0.72	1.10	0.69	1.19	0.97	4.39
time (sec)	N/A	0.423	0.087	4.571	0.294	0.269	131.427	0.294	13.021

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	44	31	32	41	31	138	51	602
N.S.	1	0.98	0.69	0.71	0.91	0.69	3.07	1.13	13.38
time (sec)	N/A	0.176	0.019	4.160	0.203	0.268	0.726	0.298	0.253

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	258	224	207	265	201	515	290	982
N.S.	1	0.90	0.78	0.72	0.92	0.70	1.79	1.01	3.41
time (sec)	N/A	0.419	0.099	4.168	0.285	0.280	69.176	0.290	10.072

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	248	224	207	293	254	0	278	1364
N.S.	1	0.86	0.78	0.72	1.02	0.88	0.00	0.97	4.74
time (sec)	N/A	0.400	0.103	4.238	0.269	0.315	0.000	0.305	13.944

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	63	54	55	61	54	0	71	58
N.S.	1	1.02	0.87	0.89	0.98	0.87	0.00	1.15	0.94
time (sec)	N/A	0.229	0.031	4.138	0.198	0.483	0.000	0.297	7.310

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	314	244	228	300	238	0	305	716
N.S.	1	1.05	0.82	0.76	1.00	0.80	0.00	1.02	2.39
time (sec)	N/A	0.459	0.141	4.206	0.283	0.304	0.000	0.294	8.432

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	276	259	228	328	301	0	309	1829
N.S.	1	0.92	0.86	0.76	1.09	1.00	0.00	1.03	6.08
time (sec)	N/A	0.475	0.169	4.454	0.278	0.778	0.000	0.315	16.873

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	86	88	82	87	99	0	111	87
N.S.	1	0.99	1.01	0.94	1.00	1.14	0.00	1.28	1.00
time (sec)	N/A	0.263	0.044	4.310	0.186	1.343	0.000	0.290	7.682

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	355	282	248	341	305	0	328	1734
N.S.	1	1.12	0.89	0.78	1.07	0.96	0.00	1.03	5.45
time (sec)	N/A	0.553	0.213	4.377	0.265	0.786	0.000	0.295	16.468

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	309	282	248	369	356	0	336	1860
N.S.	1	0.96	0.88	0.77	1.15	1.11	0.00	1.05	5.79
time (sec)	N/A	0.593	0.209	4.377	0.281	0.336	0.000	0.289	16.539

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	117	115	114	117	127	0	165	118
N.S.	1	0.98	0.97	0.96	0.98	1.07	0.00	1.39	0.99
time (sec)	N/A	0.300	0.060	4.175	0.189	3.729	0.000	0.286	7.641

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	391	304	279	376	332	0	377	1814
N.S.	1	1.11	0.86	0.79	1.07	0.94	0.00	1.07	5.15
time (sec)	N/A	0.648	0.261	4.365	0.275	0.404	0.000	0.290	16.927

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	137	1077	205	851	5418	1331	559
N.S.	1	1.00	0.93	7.28	1.39	5.75	36.61	8.99	3.78
time (sec)	N/A	0.319	0.558	4.737	0.197	0.274	1.651	0.333	7.625

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	261	91	215	1057	332	177
N.S.	1	1.00	0.93	3.68	1.28	3.03	14.89	4.68	2.49
time (sec)	N/A	0.220	0.146	4.073	0.195	0.261	0.602	0.283	6.906

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	53	53	92	410	143	95
N.S.	1	1.00	0.93	1.18	1.18	2.04	9.11	3.18	2.11
time (sec)	N/A	0.193	0.070	0.085	0.190	0.270	0.399	0.270	6.839

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	187	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	2.83	0.00	0.00
time (sec)	N/A	0.203	0.134	0.000	0.000	0.000	7.587	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	1049	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	11.28	0.00	0.00
time (sec)	N/A	0.217	0.224	0.000	0.000	0.000	140.649	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	0.383	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	86	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	0.363	0.000	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.175	0.033	0.271	0.198	0.240	0.976	0.301	0.055

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.171	0.032	0.281	0.189	0.243	0.630	0.274	7.001

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.172	0.028	0.253	0.203	0.243	0.434	0.266	0.045

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	30	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.77	1.18	0.74	0.79
time (sec)	N/A	0.166	0.033	0.244	0.186	0.243	0.583	0.278	0.045

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	28	27	29	44	29	31
N.S.	1	1.00	0.95	0.76	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.170	0.030	0.259	0.196	0.249	0.266	0.303	6.976

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	27	29	44	29	31
N.S.	1	1.00	0.95	0.81	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.168	0.037	0.082	0.186	0.258	0.361	0.273	7.024

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	34	30	27	28	46	29	31
N.S.	1	1.00	0.87	0.77	0.69	0.72	1.18	0.74	0.79
time (sec)	N/A	0.172	0.039	0.086	0.186	0.249	0.393	0.271	0.044

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	27	29	42	29	30
N.S.	1	1.00	0.95	0.81	0.73	0.78	1.14	0.78	0.81
time (sec)	N/A	0.168	0.037	0.084	0.215	0.266	0.479	0.270	0.043

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	52	51	56	80	53	51
N.S.	1	1.00	0.94	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.191	0.052	4.195	0.185	0.274	1.577	0.273	6.979

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.191	0.059	4.140	0.183	0.263	1.131	0.277	0.051

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.195	0.057	4.055	0.210	0.244	0.768	0.293	0.050

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	52	51	54	80	53	51
N.S.	1	1.00	0.94	0.83	0.81	0.86	1.27	0.84	0.81
time (sec)	N/A	0.193	0.059	4.169	0.196	0.245	0.853	0.263	0.055

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	52	51	53	78	53	51
N.S.	1	1.00	0.97	0.85	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.190	0.053	4.156	0.195	0.253	0.518	0.275	0.047

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	78	53	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.195	0.074	4.194	0.193	0.252	0.620	0.280	0.048

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	54	51	53	80	53	51
N.S.	1	1.00	0.94	0.86	0.81	0.84	1.27	0.84	0.81
time (sec)	N/A	0.192	0.060	4.217	0.204	0.258	0.722	0.275	0.050

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	76	53	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.25	0.87	0.84
time (sec)	N/A	0.197	0.068	4.335	0.199	0.263	0.881	0.279	0.052

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	76	73	78	114	77	69
N.S.	1	1.00	0.95	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.227	0.070	4.533	0.184	0.425	2.504	0.286	6.939

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	91	76	73	78	114	77	69
N.S.	1	1.00	1.07	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.220	0.077	4.309	0.188	0.262	1.859	0.281	0.030

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	76	73	78	114	77	69
N.S.	1	1.00	0.95	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.215	0.087	4.157	0.202	0.281	1.352	0.272	0.033

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	76	73	76	114	77	69
N.S.	1	1.00	0.94	0.89	0.86	0.89	1.34	0.91	0.81
time (sec)	N/A	0.214	0.072	4.222	0.255	0.257	1.271	0.280	0.032

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	80	76	73	75	112	77	69
N.S.	1	1.00	0.96	0.92	0.88	0.90	1.35	0.93	0.83
time (sec)	N/A	0.219	0.071	4.225	0.247	0.251	1.011	0.261	0.031

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	78	73	75	112	77	69
N.S.	1	1.00	1.00	0.94	0.88	0.90	1.35	0.93	0.83
time (sec)	N/A	0.212	0.094	4.225	0.210	0.301	1.108	0.271	0.034

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	77	78	73	75	112	77	69
N.S.	1	1.00	0.91	0.92	0.86	0.88	1.32	0.91	0.81
time (sec)	N/A	0.220	0.069	4.229	0.226	0.278	1.184	0.281	0.036

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	78	78	73	75	110	77	69
N.S.	1	1.00	0.94	0.94	0.88	0.90	1.33	0.93	0.83
time (sec)	N/A	0.218	0.088	4.242	0.221	0.266	1.409	0.275	0.035

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	70	67	55	58	143	428	64	111
N.S.	1	0.96	0.92	0.75	0.79	1.96	5.86	0.88	1.52
time (sec)	N/A	0.226	0.108	4.281	0.298	0.286	62.164	0.270	7.023

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	269	172	206	295	1289	605	289	1933
N.S.	1	0.93	0.60	0.72	1.02	4.48	2.10	1.00	6.71
time (sec)	N/A	0.505	0.297	4.361	0.316	0.296	29.283	0.300	7.334

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	263	153	190	212	1603	581	260	1640
N.S.	1	0.97	0.57	0.70	0.79	5.94	2.15	0.96	6.07
time (sec)	N/A	0.470	0.304	4.317	0.323	0.309	11.674	0.628	7.262

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	40	39	108	381	39	93
N.S.	1	1.00	1.00	0.75	0.74	2.04	7.19	0.74	1.75
time (sec)	N/A	0.193	0.084	4.447	0.299	0.283	4.678	0.272	7.051

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	251	152	190	278	1245	558	280	1915
N.S.	1	0.94	0.57	0.71	1.04	4.65	2.08	1.04	7.15
time (sec)	N/A	0.451	0.319	4.172	0.303	0.296	5.311	0.288	7.344

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	261	156	191	212	1636	561	257	1700
N.S.	1	0.97	0.58	0.71	0.79	6.10	2.09	0.96	6.34
time (sec)	N/A	0.475	0.360	4.172	0.287	0.276	9.206	0.690	7.292

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	40	39	120	371	39	102
N.S.	1	1.00	1.00	0.75	0.74	2.26	7.00	0.74	1.92
time (sec)	N/A	0.202	0.082	4.108	0.275	0.336	19.499	0.268	0.110

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	253	158	191	278	1290	586	280	2023
N.S.	1	0.94	0.59	0.71	1.03	4.78	2.17	1.04	7.49
time (sec)	N/A	0.450	0.304	4.327	0.280	0.341	47.882	0.284	7.383

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	77	65	68	222	0	68	116
N.S.	1	1.00	0.81	0.68	0.72	2.34	0.00	0.72	1.22
time (sec)	N/A	0.243	0.158	4.144	0.277	0.301	0.000	0.292	7.081

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	294	181	216	311	1426	1658	313	1884
N.S.	1	0.94	0.58	0.69	1.00	4.57	5.31	1.00	6.04
time (sec)	N/A	0.512	0.990	4.223	0.339	0.281	167.741	0.295	7.347

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	287	169	213	235	1773	1885	288	1578
N.S.	1	0.99	0.58	0.74	0.81	6.13	6.52	1.00	5.46
time (sec)	N/A	0.490	0.815	3.997	0.290	0.289	112.409	0.649	7.291

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	61	61	190	1042	63	115
N.S.	1	1.00	1.00	0.86	0.86	2.68	14.68	0.89	1.62
time (sec)	N/A	0.219	0.142	4.143	0.276	0.265	67.820	0.288	7.020

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	277	168	213	301	1417	1632	302	1922
N.S.	1	0.96	0.58	0.74	1.04	4.90	5.65	1.04	6.65
time (sec)	N/A	0.486	0.752	4.271	0.310	0.323	90.341	0.299	7.321

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	305	184	216	240	1788	2200	284	1757
N.S.	1	0.96	0.58	0.68	0.75	5.62	6.92	0.89	5.53
time (sec)	N/A	0.518	0.841	4.504	0.298	0.373	151.557	0.681	7.170

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	79	66	67	232	0	66	139
N.S.	1	1.00	0.82	0.69	0.70	2.42	0.00	0.69	1.45
time (sec)	N/A	0.239	0.161	4.042	0.287	0.429	0.000	0.277	6.797

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	297	188	217	312	1463	0	313	2080
N.S.	1	0.93	0.59	0.68	0.98	4.60	0.00	0.98	6.54
time (sec)	N/A	0.526	0.774	4.282	0.301	0.427	0.000	0.292	7.183

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	92	81	96	314	0	84	133
N.S.	1	1.00	0.88	0.78	0.92	3.02	0.00	0.81	1.28
time (sec)	N/A	0.247	0.236	4.384	0.294	0.388	0.000	0.279	7.018

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	307	192	234	341	1618	0	328	1944
N.S.	1	0.94	0.59	0.72	1.04	4.95	0.00	1.00	5.94
time (sec)	N/A	0.528	1.076	4.357	0.285	0.398	0.000	0.291	7.392

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	317	193	235	271	1959	0	314	1672
N.S.	1	0.97	0.59	0.72	0.83	5.99	0.00	0.96	5.11
time (sec)	N/A	0.523	1.048	4.186	0.297	0.429	0.000	0.652	7.347

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	92	82	96	313	0	84	136
N.S.	1	1.00	0.88	0.79	0.92	3.01	0.00	0.81	1.31
time (sec)	N/A	0.246	0.224	4.195	0.280	0.288	0.000	0.276	7.158

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	306	189	233	336	1588	0	322	1952
N.S.	1	0.95	0.59	0.73	1.05	4.95	0.00	1.00	6.08
time (sec)	N/A	0.514	1.021	4.182	0.295	0.418	0.000	0.295	0.453

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	334	208	236	273	1934	0	306	1786
N.S.	1	0.95	0.59	0.67	0.78	5.51	0.00	0.87	5.09
time (sec)	N/A	0.549	1.032	4.124	0.282	0.366	0.000	0.685	7.377

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	125	102	86	100	347	0	88	163
N.S.	1	0.97	0.79	0.67	0.78	2.69	0.00	0.68	1.26
time (sec)	N/A	0.277	0.230	4.221	0.296	0.308	0.000	0.270	7.179

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	326	209	237	346	1608	0	334	2109
N.S.	1	0.93	0.60	0.68	0.99	4.58	0.00	0.95	6.01
time (sec)	N/A	0.551	1.068	4.391	0.303	0.284	0.000	0.300	7.474

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	107	75	67	118	99	219	104	154
N.S.	1	1.04	0.73	0.65	1.15	0.96	2.13	1.01	1.50
time (sec)	N/A	0.251	0.091	4.478	0.214	0.344	0.395	0.286	7.063

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	77	56	49	84	75	168	73	114
N.S.	1	1.05	0.77	0.67	1.15	1.03	2.30	1.00	1.56
time (sec)	N/A	0.231	0.068	4.087	0.201	0.250	0.298	0.265	7.056

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	50	34	31	49	50	117	44	44
N.S.	1	1.09	0.74	0.67	1.07	1.09	2.54	0.96	0.96
time (sec)	N/A	0.195	0.045	4.110	0.198	0.255	0.213	0.271	6.976

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	65	61	50	67	125	87	61	80
N.S.	1	1.02	0.95	0.78	1.05	1.95	1.36	0.95	1.25
time (sec)	N/A	0.193	0.111	4.106	0.279	0.429	4.654	0.289	7.148

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	79	65	56	107	143	134	68	76
N.S.	1	0.94	0.77	0.67	1.27	1.70	1.60	0.81	0.90
time (sec)	N/A	0.210	0.152	4.359	0.300	0.280	11.828	0.264	7.361

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	85	78	65	158	172	160	120	93
N.S.	1	0.97	0.89	0.74	1.80	1.95	1.82	1.36	1.06
time (sec)	N/A	0.204	0.206	4.216	0.290	0.284	33.568	0.278	7.596

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	293	89	349	0	91	83	0	0
N.S.	1	0.97	0.29	1.15	0.00	0.30	0.27	0.00	0.00
time (sec)	N/A	0.345	5.486	4.421	0.000	0.104	1.170	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	264	75	325	0	67	82	0	0
N.S.	1	0.99	0.28	1.21	0.00	0.25	0.31	0.00	0.00
time (sec)	N/A	0.281	5.575	4.387	0.000	0.084	1.022	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	266	81	317	0	57	85	0	0
N.S.	1	0.99	0.30	1.18	0.00	0.21	0.32	0.00	0.00
time (sec)	N/A	0.291	6.082	4.411	0.000	0.090	1.216	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	266	80	328	0	64	94	0	0
N.S.	1	0.98	0.29	1.21	0.00	0.24	0.35	0.00	0.00
time (sec)	N/A	0.301	10.093	4.469	0.000	0.088	1.343	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	297	80	352	0	89	97	0	0
N.S.	1	0.97	0.26	1.15	0.00	0.29	0.32	0.00	0.00
time (sec)	N/A	0.313	10.093	4.557	0.000	0.083	1.532	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	577	91	503	0	102	83	0	0
N.S.	1	0.99	0.16	0.87	0.00	0.18	0.14	0.00	0.00
time (sec)	N/A	0.570	5.816	4.638	0.000	0.082	1.182	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	548	547	75	479	0	76	83	0	0
N.S.	1	1.00	0.14	0.87	0.00	0.14	0.15	0.00	0.00
time (sec)	N/A	0.562	5.555	4.607	0.000	0.081	1.103	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	545	81	469	0	64	85	0	0
N.S.	1	1.00	0.15	0.86	0.00	0.12	0.16	0.00	0.00
time (sec)	N/A	0.579	6.051	4.352	0.000	0.088	1.249	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	546	544	80	480	0	70	92	0	0
N.S.	1	1.00	0.15	0.88	0.00	0.13	0.17	0.00	0.00
time (sec)	N/A	0.546	10.090	4.249	0.000	0.092	1.366	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	575	80	504	0	97	97	0	0
N.S.	1	0.99	0.14	0.87	0.00	0.17	0.17	0.00	0.00
time (sec)	N/A	0.576	10.091	4.332	0.000	0.090	1.441	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	614	605	80	530	0	123	97	0	0
N.S.	1	0.99	0.13	0.86	0.00	0.20	0.16	0.00	0.00
time (sec)	N/A	0.610	10.097	4.489	0.000	0.078	1.651	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	107	80	68	118	124	267	104	206
N.S.	1	1.04	0.78	0.66	1.15	1.20	2.59	1.01	2.00
time (sec)	N/A	0.251	0.100	4.310	0.223	0.271	0.584	0.269	6.990

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	77	56	49	84	99	216	73	211
N.S.	1	1.05	0.77	0.67	1.15	1.36	2.96	1.00	2.89
time (sec)	N/A	0.220	0.074	4.338	0.210	0.245	0.424	0.268	6.833

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	50	34	31	49	73	165	44	150
N.S.	1	1.09	0.74	0.67	1.07	1.59	3.59	0.96	3.26
time (sec)	N/A	0.193	0.045	4.183	0.198	0.245	0.322	0.274	6.914

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	83	85	66	80	172	109	80	131
N.S.	1	1.02	1.05	0.81	0.99	2.12	1.35	0.99	1.62
time (sec)	N/A	0.203	0.147	4.204	0.284	0.261	10.221	0.267	6.906

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	98	81	79	134	169	223	103	111
N.S.	1	0.89	0.74	0.72	1.22	1.54	2.03	0.94	1.01
time (sec)	N/A	0.220	0.163	4.396	0.297	0.265	14.415	0.268	7.368

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	103	81	78	171	191	243	131	110
N.S.	1	0.90	0.70	0.68	1.49	1.66	2.11	1.14	0.96
time (sec)	N/A	0.224	0.255	4.402	0.266	0.361	38.798	0.283	7.560

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	317	93	373	0	115	172	0	0
N.S.	1	0.94	0.28	1.11	0.00	0.34	0.51	0.00	0.00
time (sec)	N/A	0.359	7.544	4.575	0.000	0.082	1.970	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	286	77	349	0	91	170	0	0
N.S.	1	0.96	0.26	1.17	0.00	0.30	0.57	0.00	0.00
time (sec)	N/A	0.308	7.453	4.601	0.000	0.082	1.700	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	288	83	334	0	80	172	0	0
N.S.	1	0.98	0.28	1.13	0.00	0.27	0.58	0.00	0.00
time (sec)	N/A	0.327	7.579	4.252	0.000	0.103	2.061	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	289	82	329	0	67	184	0	0
N.S.	1	0.97	0.28	1.11	0.00	0.23	0.62	0.00	0.00
time (sec)	N/A	0.318	10.081	4.488	0.000	0.099	2.198	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	290	82	351	0	89	196	0	0
N.S.	1	0.96	0.27	1.16	0.00	0.29	0.65	0.00	0.00
time (sec)	N/A	0.331	10.080	4.521	0.000	0.076	2.543	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	614	601	96	527	0	126	172	0	0
N.S.	1	0.98	0.16	0.86	0.00	0.21	0.28	0.00	0.00
time (sec)	N/A	0.606	7.812	4.514	0.000	0.077	2.075	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	571	78	503	0	100	172	0	0
N.S.	1	0.98	0.13	0.87	0.00	0.17	0.30	0.00	0.00
time (sec)	N/A	0.571	7.466	4.290	0.000	0.078	1.871	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	573	569	83	486	0	87	173	0	0
N.S.	1	0.99	0.14	0.85	0.00	0.15	0.30	0.00	0.00
time (sec)	N/A	0.563	7.629	4.435	0.000	0.167	2.170	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	578	569	85	482	0	77	182	0	0
N.S.	1	0.98	0.15	0.83	0.00	0.13	0.31	0.00	0.00
time (sec)	N/A	0.591	10.079	4.488	0.000	0.117	2.236	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	576	568	82	503	0	95	194	0	0
N.S.	1	0.99	0.14	0.87	0.00	0.16	0.34	0.00	0.00
time (sec)	N/A	0.580	10.086	4.502	0.000	0.113	2.440	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	608	598	82	529	0	123	199	0	0
N.S.	1	0.98	0.13	0.87	0.00	0.20	0.33	0.00	0.00
time (sec)	N/A	0.608	10.084	4.549	0.000	0.079	2.735	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	105	80	68	118	76	175	101	104
N.S.	1	1.02	0.78	0.66	1.15	0.74	1.70	0.98	1.01
time (sec)	N/A	0.251	0.082	4.234	0.201	0.242	0.397	0.266	7.119

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	75	56	49	83	52	124	70	52
N.S.	1	1.03	0.77	0.67	1.14	0.71	1.70	0.96	0.71
time (sec)	N/A	0.224	0.063	4.093	0.201	0.272	0.304	0.267	7.033

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	48	33	30	48	29	75	38	29
N.S.	1	1.04	0.72	0.65	1.04	0.63	1.63	0.83	0.63
time (sec)	N/A	0.200	0.043	4.367	0.240	0.256	0.196	0.264	7.014

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	54	105	71	40	57
N.S.	1	1.00	1.00	0.77	1.12	2.19	1.48	0.83	1.19
time (sec)	N/A	0.185	0.077	4.476	0.309	0.262	1.719	0.274	7.167

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	57	58	47	109	126	80	62	67
N.S.	1	0.98	1.00	0.81	1.88	2.17	1.38	1.07	1.16
time (sec)	N/A	0.197	0.101	4.140	0.286	0.249	5.757	0.276	7.304

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	88	78	67	178	173	163	121	95
N.S.	1	0.98	0.87	0.74	1.98	1.92	1.81	1.34	1.06
time (sec)	N/A	0.209	0.208	4.162	0.265	0.266	14.461	0.266	7.387

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	269	89	325	0	67	80	0	0
N.S.	1	1.00	0.33	1.20	0.00	0.25	0.30	0.00	0.00
time (sec)	N/A	0.314	10.085	4.197	0.000	0.090	1.232	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	74	309	0	42	78	0	0
N.S.	1	1.00	0.31	1.29	0.00	0.18	0.33	0.00	0.00
time (sec)	N/A	0.271	10.039	4.158	0.000	0.073	0.938	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	78	311	0	50	82	0	0
N.S.	1	1.00	0.32	1.28	0.00	0.21	0.34	0.00	0.00
time (sec)	N/A	0.270	10.036	4.317	0.000	0.073	1.038	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	273	78	329	0	62	90	0	0
N.S.	1	1.00	0.28	1.20	0.00	0.23	0.33	0.00	0.00
time (sec)	N/A	0.321	10.038	4.394	0.000	0.082	1.215	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	548	553	91	479	0	78	80	0	0
N.S.	1	1.01	0.17	0.87	0.00	0.14	0.15	0.00	0.00
time (sec)	N/A	0.556	10.083	4.255	0.000	0.083	1.293	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	517	523	75	463	0	51	80	0	0
N.S.	1	1.01	0.15	0.90	0.00	0.10	0.15	0.00	0.00
time (sec)	N/A	0.529	10.058	4.347	0.000	0.084	1.174	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	520	77	463	0	54	82	0	0
N.S.	1	1.02	0.15	0.91	0.00	0.11	0.16	0.00	0.00
time (sec)	N/A	0.508	10.035	4.211	0.000	0.078	0.993	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	551	78	481	0	70	88	0	0
N.S.	1	1.00	0.14	0.87	0.00	0.13	0.16	0.00	0.00
time (sec)	N/A	0.551	10.039	4.393	0.000	0.082	1.189	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	78	504	0	97	94	0	0
N.S.	1	1.00	0.13	0.87	0.00	0.17	0.16	0.00	0.00
time (sec)	N/A	0.596	10.042	4.452	0.000	0.084	1.327	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	77	68	116	88	175	114	152
N.S.	1	1.00	0.75	0.66	1.13	0.85	1.70	1.11	1.48
time (sec)	N/A	0.252	0.092	4.403	0.201	0.246	0.465	0.280	7.280

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	55	49	81	63	124	77	60
N.S.	1	1.00	0.75	0.67	1.11	0.86	1.70	1.05	0.82
time (sec)	N/A	0.234	0.074	4.335	0.208	0.253	0.339	0.276	7.243

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	33	30	47	41	75	38	33
N.S.	1	1.00	0.72	0.65	1.02	0.89	1.63	0.83	0.72
time (sec)	N/A	0.205	0.056	4.106	0.203	0.241	0.236	0.270	7.190

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	51	70	170	78	53	65
N.S.	1	1.00	1.00	0.88	1.21	2.93	1.34	0.91	1.12
time (sec)	N/A	0.196	0.113	4.266	0.282	0.255	3.864	0.278	7.310

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	83	77	74	144	233	264	99	131
N.S.	1	0.97	0.90	0.86	1.67	2.71	3.07	1.15	1.52
time (sec)	N/A	0.212	0.208	4.322	0.280	0.269	23.526	0.280	7.470

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	113	100	94	215	289	192	137	167
N.S.	1	0.96	0.85	0.80	1.82	2.45	1.63	1.16	1.42
time (sec)	N/A	0.232	0.249	4.352	0.292	0.273	59.398	0.295	7.690

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	298	103	406	0	122	80	0	0
N.S.	1	0.99	0.34	1.35	0.00	0.41	0.27	0.00	0.00
time (sec)	N/A	0.340	10.098	4.813	0.000	0.076	8.748	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	268	78	348	0	95	80	0	0
N.S.	1	1.00	0.29	1.29	0.00	0.35	0.30	0.00	0.00
time (sec)	N/A	0.301	10.075	4.679	0.000	0.085	4.060	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	73	336	0	83	78	0	0
N.S.	1	1.00	0.29	1.34	0.00	0.33	0.31	0.00	0.00
time (sec)	N/A	0.281	10.044	4.305	0.000	0.115	2.794	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	271	86	350	0	102	82	0	0
N.S.	1	1.00	0.32	1.29	0.00	0.38	0.30	0.00	0.00
time (sec)	N/A	0.303	10.041	4.674	0.000	0.076	8.289	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	302	72	386	0	119	90	0	0
N.S.	1	0.99	0.24	1.27	0.00	0.39	0.30	0.00	0.00
time (sec)	N/A	0.342	10.040	4.837	0.000	0.081	24.213	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	547	552	79	504	0	104	80	0	0
N.S.	1	1.01	0.14	0.92	0.00	0.19	0.15	0.00	0.00
time (sec)	N/A	0.555	10.077	5.056	0.000	0.080	4.844	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	524	535	71	490	0	94	80	0	0
N.S.	1	1.02	0.14	0.94	0.00	0.18	0.15	0.00	0.00
time (sec)	N/A	0.560	10.069	4.184	0.000	0.082	2.801	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	548	550	72	504	0	106	82	0	0
N.S.	1	1.00	0.13	0.92	0.00	0.19	0.15	0.00	0.00
time (sec)	N/A	0.583	10.038	4.951	0.000	0.079	6.359	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	580	580	72	540	0	127	88	0	0
N.S.	1	1.00	0.12	0.93	0.00	0.22	0.15	0.00	0.00
time (sec)	N/A	0.579	10.039	5.362	0.000	0.080	16.465	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	611	610	72	573	0	156	94	0	0
N.S.	1	1.00	0.12	0.94	0.00	0.26	0.15	0.00	0.00
time (sec)	N/A	0.640	10.039	5.225	0.000	0.092	43.675	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	73	70	116	98	338	104	145
N.S.	1	1.00	0.71	0.68	1.13	0.95	3.28	1.01	1.41
time (sec)	N/A	0.257	0.094	4.388	0.204	0.303	0.588	0.294	7.294

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	56	49	84	75	240	63	60
N.S.	1	1.00	0.77	0.67	1.15	1.03	3.29	0.86	0.82
time (sec)	N/A	0.229	0.073	4.371	0.190	0.312	0.437	0.302	7.215

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	48	33	30	49	52	144	32	33
N.S.	1	1.04	0.72	0.65	1.07	1.13	3.13	0.70	0.72
time (sec)	N/A	0.206	0.054	4.207	0.203	0.316	0.342	0.284	6.956

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	81	70	70	81	243	100	67	80
N.S.	1	1.05	0.91	0.91	1.05	3.16	1.30	0.87	1.04
time (sec)	N/A	0.223	0.157	4.234	0.273	0.295	6.744	0.302	7.033

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	106	99	94	170	351	1608	101	198
N.S.	1	0.94	0.88	0.83	1.50	3.11	14.23	0.89	1.75
time (sec)	N/A	0.240	0.198	4.268	0.271	0.333	126.770	0.307	7.232

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	297	108	397	0	153	80	0	0
N.S.	1	0.99	0.36	1.33	0.00	0.51	0.27	0.00	0.00
time (sec)	N/A	0.363	10.126	5.359	0.000	0.091	60.996	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	280	99	372	0	141	80	0	0
N.S.	1	0.99	0.35	1.31	0.00	0.50	0.28	0.00	0.00
time (sec)	N/A	0.317	10.104	4.434	0.000	0.086	38.552	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	282	103	364	0	145	78	0	0
N.S.	1	1.00	0.36	1.29	0.00	0.51	0.28	0.00	0.00
time (sec)	N/A	0.319	10.057	4.383	0.000	0.132	24.335	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	298	116	389	0	163	82	0	0
N.S.	1	0.99	0.39	1.30	0.00	0.54	0.27	0.00	0.00
time (sec)	N/A	0.320	10.070	5.541	0.000	0.085	81.003	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	331	83	425	0	178	0	0	0
N.S.	1	0.99	0.25	1.27	0.00	0.53	0.00	0.00	0.00
time (sec)	N/A	0.355	10.047	5.575	0.000	0.083	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	577	581	109	555	0	162	80	0	0
N.S.	1	1.01	0.19	0.96	0.00	0.28	0.14	0.00	0.00
time (sec)	N/A	0.606	10.109	5.561	0.000	0.081	79.235	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	559	564	92	528	0	154	80	0	0
N.S.	1	1.01	0.16	0.94	0.00	0.28	0.14	0.00	0.00
time (sec)	N/A	0.577	10.098	4.462	0.000	0.086	39.258	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	565	81	520	0	154	80	0	0
N.S.	1	1.00	0.14	0.92	0.00	0.27	0.14	0.00	0.00
time (sec)	N/A	0.554	10.077	4.276	0.000	0.108	24.604	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	578	579	86	545	0	167	82	0	0
N.S.	1	1.00	0.15	0.94	0.00	0.29	0.14	0.00	0.00
time (sec)	N/A	0.600	10.043	5.219	0.000	0.082	52.467	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	610	609	83	581	0	186	88	0	0
N.S.	1	1.00	0.14	0.95	0.00	0.30	0.14	0.00	0.00
time (sec)	N/A	0.622	10.044	5.878	0.000	0.084	156.888	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	99	78	64	69	156	95	82	109
N.S.	1	1.02	0.80	0.66	0.71	1.61	0.98	0.85	1.12
time (sec)	N/A	0.262	0.104	8.331	0.283	0.310	8.164	0.291	8.910

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	84	66	54	53	129	78	64	88
N.S.	1	1.11	0.87	0.71	0.70	1.70	1.03	0.84	1.16
time (sec)	N/A	0.203	0.086	4.535	0.275	0.290	3.407	0.294	8.534

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	59	54	44	42	110	61	44	71
N.S.	1	1.04	0.95	0.77	0.74	1.93	1.07	0.77	1.25
time (sec)	N/A	0.188	0.063	4.342	0.284	0.256	1.543	0.294	8.159

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	69	59	45	0	147	78	50	93
N.S.	1	1.06	0.91	0.69	0.00	2.26	1.20	0.77	1.43
time (sec)	N/A	0.204	0.067	4.699	0.000	0.258	2.584	0.307	9.019

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	95	88	66	0	194	0	72	113
N.S.	1	1.08	1.00	0.75	0.00	2.20	0.00	0.82	1.28
time (sec)	N/A	0.234	0.177	4.468	0.000	0.275	0.000	0.298	9.332

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	689	691	133	867	0	2442	0	0	0
N.S.	1	1.00	0.19	1.26	0.00	3.54	0.00	0.00	0.00
time (sec)	N/A	0.762	6.024	4.849	0.000	2.121	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	659	693	63	848	0	2202	0	0	0
N.S.	1	1.05	0.10	1.29	0.00	3.34	0.00	0.00	0.00
time (sec)	N/A	0.640	8.321	4.206	0.000	0.842	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	697	690	136	868	0	2253	0	0	0
N.S.	1	0.99	0.20	1.25	0.00	3.23	0.00	0.00	0.00
time (sec)	N/A	0.715	11.083	5.007	0.000	0.529	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	236	713	0	2387	0	0	0
N.S.	1	1.00	3.58	10.80	0.00	36.17	0.00	0.00	0.00
time (sec)	N/A	0.216	6.336	4.743	0.000	1.872	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	165	696	0	2240	0	0	0
N.S.	1	1.00	2.58	10.88	0.00	35.00	0.00	0.00	0.00
time (sec)	N/A	0.190	10.157	4.652	0.000	0.513	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	244	716	0	2361	0	0	0
N.S.	1	1.00	3.70	10.85	0.00	35.77	0.00	0.00	0.00
time (sec)	N/A	0.212	11.148	4.639	0.000	0.987	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	65	54	53	129	78	64	88
N.S.	1	1.00	0.83	0.69	0.68	1.65	1.00	0.82	1.13
time (sec)	N/A	0.239	0.095	4.389	0.286	0.257	8.734	0.290	9.021

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	56	44	43	112	63	49	71
N.S.	1	1.00	0.95	0.75	0.73	1.90	1.07	0.83	1.20
time (sec)	N/A	0.191	0.069	4.432	0.301	0.255	4.321	0.285	9.312

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	30	29	87	48	29	56
N.S.	1	1.00	1.00	0.75	0.72	2.18	1.20	0.72	1.40
time (sec)	N/A	0.175	0.049	4.145	0.302	0.244	2.832	0.297	8.422

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	69	59	45	0	148	80	53	94
N.S.	1	1.06	0.91	0.69	0.00	2.28	1.23	0.82	1.45
time (sec)	N/A	0.213	0.074	4.239	0.000	0.251	3.454	0.287	9.204

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	95	88	66	0	194	0	72	112
N.S.	1	1.08	1.00	0.75	0.00	2.20	0.00	0.82	1.27
time (sec)	N/A	0.242	0.138	4.455	0.000	0.264	0.000	0.282	9.496

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	667	701	67	848	0	2268	0	0	0
N.S.	1	1.05	0.10	1.27	0.00	3.40	0.00	0.00	0.00
time (sec)	N/A	0.685	10.041	4.379	0.000	1.643	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	206	206	67	416	0	2289	0	0	453
N.S.	1	1.00	0.33	2.02	0.00	11.11	0.00	0.00	2.20
time (sec)	N/A	0.235	10.040	4.350	0.000	0.673	0.000	0.000	29.695

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	697	697	136	868	0	2293	0	0	0
N.S.	1	1.00	0.20	1.25	0.00	3.29	0.00	0.00	0.00
time (sec)	N/A	0.708	11.089	4.936	0.000	0.697	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	66	66	67	696	0	2300	0	0	0
N.S.	1	1.00	1.02	10.55	0.00	34.85	0.00	0.00	0.00
time (sec)	N/A	0.220	10.039	4.362	0.000	0.712	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	165	416	0	2347	0	0	0
N.S.	1	1.00	2.58	6.50	0.00	36.67	0.00	0.00	0.00
time (sec)	N/A	0.192	10.052	4.329	0.000	0.717	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	243	716	0	2381	0	0	0
N.S.	1	1.00	3.68	10.85	0.00	36.08	0.00	0.00	0.00
time (sec)	N/A	0.214	11.180	5.039	0.000	1.416	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	28	164	0	1116	0	0	653
N.S.	1	1.00	0.22	1.29	0.00	8.79	0.00	0.00	5.14
time (sec)	N/A	0.202	0.017	13.970	0.000	0.393	0.000	0.000	7.157

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	82	70	96	169	109	100	118
N.S.	1	1.00	0.74	0.63	0.86	1.52	0.98	0.90	1.06
time (sec)	N/A	0.260	0.131	3.654	0.264	0.290	18.481	0.279	7.226

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	92	71	58	82	147	92	83	98
N.S.	1	1.02	0.79	0.64	0.91	1.63	1.02	0.92	1.09
time (sec)	N/A	0.241	0.104	4.274	0.263	0.343	8.083	0.288	7.284

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	77	59	47	66	121	75	65	78
N.S.	1	1.12	0.86	0.68	0.96	1.75	1.09	0.94	1.13
time (sec)	N/A	0.203	0.088	4.428	0.273	0.296	3.742	0.282	7.326

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	52	47	38	56	101	56	43	59
N.S.	1	1.04	0.94	0.76	1.12	2.02	1.12	0.86	1.18
time (sec)	N/A	0.187	0.064	4.341	0.260	0.295	1.895	0.274	7.291

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	62	53	38	0	138	71	48	125
N.S.	1	1.07	0.91	0.66	0.00	2.38	1.22	0.83	2.16
time (sec)	N/A	0.203	0.071	4.547	0.000	0.313	2.769	0.304	8.425

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	92	81	65	0	186	0	73	69
N.S.	1	1.14	1.00	0.80	0.00	2.30	0.00	0.90	0.85
time (sec)	N/A	0.229	0.157	4.299	0.000	0.268	0.000	0.296	7.408

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	122	95	77	0	188	0	100	83
N.S.	1	1.14	0.89	0.72	0.00	1.76	0.00	0.93	0.78
time (sec)	N/A	0.254	0.216	4.448	0.000	0.293	0.000	0.304	7.574

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	648	656	150	884	0	2442	0	0	0
N.S.	1	1.01	0.23	1.36	0.00	3.77	0.00	0.00	0.00
time (sec)	N/A	1.098	6.109	4.912	0.000	13.579	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	624	626	130	867	0	2428	0	0	0
N.S.	1	1.00	0.21	1.39	0.00	3.89	0.00	0.00	0.00
time (sec)	N/A	1.015	6.211	4.837	0.000	3.951	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	601	629	63	848	0	2194	0	0	0
N.S.	1	1.05	0.10	1.41	0.00	3.65	0.00	0.00	0.00
time (sec)	N/A	1.826	8.252	4.204	0.000	0.764	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	632	625	137	868	0	2219	0	0	0
N.S.	1	0.99	0.22	1.37	0.00	3.51	0.00	0.00	0.00
time (sec)	N/A	1.050	11.098	5.114	0.000	0.364	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	654	657	153	882	0	2401	0	0	0
N.S.	1	1.00	0.23	1.35	0.00	3.67	0.00	0.00	0.00
time (sec)	N/A	1.112	11.102	4.789	0.000	0.744	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	678	687	164	895	0	2436	0	0	0
N.S.	1	1.01	0.24	1.32	0.00	3.59	0.00	0.00	0.00
time (sec)	N/A	1.120	10.109	5.028	0.000	1.919	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	134	93	80	110	191	143	117	135
N.S.	1	1.03	0.72	0.62	0.85	1.47	1.10	0.90	1.04
time (sec)	N/A	0.286	0.146	4.398	0.264	0.272	49.715	0.292	7.461

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	113	82	69	96	169	122	100	115
N.S.	1	1.04	0.75	0.63	0.88	1.55	1.12	0.92	1.06
time (sec)	N/A	0.260	0.133	4.456	0.267	0.274	27.488	0.280	7.424

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	99	71	58	82	147	95	83	95
N.S.	1	1.12	0.81	0.66	0.93	1.67	1.08	0.94	1.08
time (sec)	N/A	0.221	0.102	4.355	0.260	0.273	13.341	0.291	7.441

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	74	59	47	68	121	71	65	75
N.S.	1	1.10	0.88	0.70	1.01	1.81	1.06	0.97	1.12
time (sec)	N/A	0.210	0.093	4.163	0.267	0.283	6.337	0.274	7.429

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	78	73	52	0	152	92	61	89
N.S.	1	1.07	1.00	0.71	0.00	2.08	1.26	0.84	1.22
time (sec)	N/A	0.227	0.081	4.406	0.000	0.273	4.392	0.282	9.971

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	86	78	59	0	186	0	64	56
N.S.	1	1.10	1.00	0.76	0.00	2.38	0.00	0.82	0.72
time (sec)	N/A	0.232	0.143	4.571	0.000	0.298	0.000	0.287	7.726

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	116	95	77	0	218	0	101	87
N.S.	1	1.12	0.91	0.74	0.00	2.10	0.00	0.97	0.84
time (sec)	N/A	0.273	0.198	4.737	0.000	0.289	0.000	0.289	7.907

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	669	680	163	895	0	2453	0	0	0
N.S.	1	1.02	0.24	1.34	0.00	3.67	0.00	0.00	0.00
time (sec)	N/A	1.188	7.829	4.752	0.000	24.896	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	645	650	150	884	0	2442	0	0	0
N.S.	1	1.01	0.23	1.37	0.00	3.79	0.00	0.00	0.00
time (sec)	N/A	1.093	7.595	4.894	0.000	8.320	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	627	626	127	864	0	2368	0	0	0
N.S.	1	1.00	0.20	1.38	0.00	3.78	0.00	0.00	0.00
time (sec)	N/A	1.004	9.980	4.846	0.000	1.810	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	626	620	137	859	0	0	0	0	0
N.S.	1	0.99	0.22	1.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.008	10.087	4.894	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	651	651	154	882	0	2369	0	0	0
N.S.	1	1.00	0.24	1.35	0.00	3.64	0.00	0.00	0.00
time (sec)	N/A	1.130	10.105	4.956	0.000	0.559	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	675	681	167	895	0	2442	0	0	0
N.S.	1	1.01	0.25	1.33	0.00	3.62	0.00	0.00	0.00
time (sec)	N/A	1.154	10.105	4.931	0.000	1.513	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	92	69	58	82	146	92	82	98
N.S.	1	1.02	0.77	0.64	0.91	1.62	1.02	0.91	1.09
time (sec)	N/A	0.244	0.114	4.500	0.271	0.308	16.603	0.311	7.472

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	73	58	47	66	121	76	65	78
N.S.	1	1.03	0.82	0.66	0.93	1.70	1.07	0.92	1.10
time (sec)	N/A	0.232	0.092	4.447	0.268	0.301	8.639	0.280	7.439

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	54	49	38	56	103	60	48	60
N.S.	1	1.04	0.94	0.73	1.08	1.98	1.15	0.92	1.15
time (sec)	N/A	0.191	0.058	4.545	0.261	0.278	4.868	0.279	7.489

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	24	42	78	42	27	45
N.S.	1	1.00	1.00	0.73	1.27	2.36	1.27	0.82	1.36
time (sec)	N/A	0.180	0.045	4.566	0.277	0.281	3.126	0.287	7.501

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	62	51	38	0	139	75	54	47
N.S.	1	1.07	0.88	0.66	0.00	2.40	1.29	0.93	0.81
time (sec)	N/A	0.207	0.065	4.286	0.000	0.310	3.294	0.292	7.309

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	92	81	64	0	184	0	73	73
N.S.	1	1.14	1.00	0.79	0.00	2.27	0.00	0.90	0.90
time (sec)	N/A	0.237	0.140	4.484	0.000	0.288	0.000	0.276	7.587

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	122	95	77	0	217	0	101	94
N.S.	1	1.14	0.89	0.72	0.00	2.03	0.00	0.94	0.88
time (sec)	N/A	0.260	0.218	4.328	0.000	0.287	0.000	0.292	7.673

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	630	632	130	867	0	2428	0	0	0
N.S.	1	1.00	0.21	1.38	0.00	3.85	0.00	0.00	0.00
time (sec)	N/A	1.002	10.094	4.873	0.000	8.429	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	601	637	67	848	0	2254	0	0	0
N.S.	1	1.06	0.11	1.41	0.00	3.75	0.00	0.00	0.00
time (sec)	N/A	1.848	10.051	4.449	0.000	1.632	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	141	141	67	416	0	2285	0	0	272
N.S.	1	1.00	0.48	2.95	0.00	16.21	0.00	0.00	1.93
time (sec)	N/A	0.917	10.039	4.231	0.000	0.540	0.000	0.000	45.124

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	632	632	137	868	0	2259	0	0	0
N.S.	1	1.00	0.22	1.37	0.00	3.57	0.00	0.00	0.00
time (sec)	N/A	1.089	11.086	4.888	0.000	0.461	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	654	657	152	882	0	2403	0	0	0
N.S.	1	1.00	0.23	1.35	0.00	3.67	0.00	0.00	0.00
time (sec)	N/A	1.090	11.111	4.932	0.000	1.108	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	678	687	167	895	0	2436	0	0	0
N.S.	1	1.01	0.25	1.32	0.00	3.59	0.00	0.00	0.00
time (sec)	N/A	1.171	10.116	5.023	0.000	2.621	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	66	66	67	696	0	2284	0	0	0
N.S.	1	1.00	1.02	10.55	0.00	34.61	0.00	0.00	0.00
time (sec)	N/A	0.218	10.045	4.361	0.000	0.562	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	166	416	0	2319	0	0	0
N.S.	1	1.00	2.59	6.50	0.00	36.23	0.00	0.00	0.00
time (sec)	N/A	0.193	10.160	4.225	0.000	0.594	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	242	716	0	2373	0	0	0
N.S.	1	1.00	3.67	10.85	0.00	35.95	0.00	0.00	0.00
time (sec)	N/A	0.216	11.236	4.899	0.000	0.991	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	261	732	0	2417	0	0	0
N.S.	1	1.00	3.95	11.09	0.00	36.62	0.00	0.00	0.00
time (sec)	N/A	0.223	11.219	4.916	0.000	2.512	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	94	70	65	82	189	90	82	95
N.S.	1	1.04	0.78	0.72	0.91	2.10	1.00	0.91	1.06
time (sec)	N/A	0.264	0.133	4.276	0.272	0.296	21.226	0.274	8.726

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	73	59	54	68	161	75	58	75
N.S.	1	1.03	0.83	0.76	0.96	2.27	1.06	0.82	1.06
time (sec)	N/A	0.231	0.105	4.474	0.277	0.288	11.697	0.428	7.924

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	56	49	48	56	149	56	47	60
N.S.	1	1.08	0.94	0.92	1.08	2.87	1.08	0.90	1.15
time (sec)	N/A	0.193	0.085	4.697	0.289	0.350	7.903	0.371	7.886

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	59	52	48	58	147	60	48	63
N.S.	1	1.07	0.95	0.87	1.05	2.67	1.09	0.87	1.15
time (sec)	N/A	0.211	0.088	4.255	0.277	0.268	6.507	0.454	7.886

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	88	69	71	0	213	97	68	68
N.S.	1	1.16	0.91	0.93	0.00	2.80	1.28	0.89	0.89
time (sec)	N/A	0.235	0.124	4.429	0.000	0.267	4.402	0.272	7.941

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	118	85	76	0	272	0	100	88
N.S.	1	1.18	0.85	0.76	0.00	2.72	0.00	1.00	0.88
time (sec)	N/A	0.268	0.210	4.512	0.000	0.280	0.000	0.276	8.061

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	148	100	88	0	303	0	118	112
N.S.	1	1.16	0.78	0.69	0.00	2.37	0.00	0.92	0.88
time (sec)	N/A	0.287	0.283	4.524	0.000	0.287	0.000	0.275	8.292

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	629	629	127	869	0	2384	0	0	0
N.S.	1	1.00	0.20	1.38	0.00	3.79	0.00	0.00	0.00
time (sec)	N/A	1.032	8.544	4.323	0.000	3.572	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	635	635	126	878	0	2525	0	0	0
N.S.	1	1.00	0.20	1.38	0.00	3.98	0.00	0.00	0.00
time (sec)	N/A	1.021	8.277	4.360	0.000	0.446	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	632	631	124	875	0	2525	0	0	0
N.S.	1	1.00	0.20	1.38	0.00	4.00	0.00	0.00	0.00
time (sec)	N/A	1.018	10.081	4.550	0.000	0.431	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	653	659	140	890	0	2379	0	0	0
N.S.	1	1.01	0.21	1.36	0.00	3.64	0.00	0.00	0.00
time (sec)	N/A	1.093	11.100	5.422	0.000	0.643	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	675	686	153	911	0	2529	0	0	0
N.S.	1	1.02	0.23	1.35	0.00	3.75	0.00	0.00	0.00
time (sec)	N/A	1.178	11.100	5.480	0.000	1.743	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	699	716	167	930	0	2564	0	0	0
N.S.	1	1.02	0.24	1.33	0.00	3.67	0.00	0.00	0.00
time (sec)	N/A	1.223	10.128	5.477	0.000	4.259	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	233	724	0	2535	0	0	0
N.S.	1	1.00	3.53	10.97	0.00	38.41	0.00	0.00	0.00
time (sec)	N/A	0.223	8.110	4.236	0.000	0.782	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	230	721	0	2483	0	0	0
N.S.	1	1.00	3.59	11.27	0.00	38.80	0.00	0.00	0.00
time (sec)	N/A	0.196	10.158	4.408	0.000	0.915	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	248	736	0	2495	0	0	0
N.S.	1	1.00	3.76	11.15	0.00	37.80	0.00	0.00	0.00
time (sec)	N/A	0.219	11.191	5.197	0.000	1.590	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	261	757	0	2543	0	0	0
N.S.	1	1.00	3.95	11.47	0.00	38.53	0.00	0.00	0.00
time (sec)	N/A	0.219	11.261	5.288	0.000	4.221	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	737	814	80	977	0	4847	0	0	0
N.S.	1	1.10	0.11	1.33	0.00	6.58	0.00	0.00	0.00
time (sec)	N/A	0.797	10.099	5.023	0.000	3.426	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	757	834	80	924	0	4855	0	0	0
N.S.	1	1.10	0.11	1.22	0.00	6.41	0.00	0.00	0.00
time (sec)	N/A	0.829	10.091	6.279	0.000	3.367	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	774	852	87	926	0	4963	0	0	0
N.S.	1	1.10	0.11	1.20	0.00	6.41	0.00	0.00	0.00
time (sec)	N/A	0.870	10.078	4.971	0.000	3.735	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	768	844	90	983	0	4981	0	0	0
N.S.	1	1.10	0.12	1.28	0.00	6.49	0.00	0.00	0.00
time (sec)	N/A	0.849	10.079	5.086	0.000	3.280	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	738	806	80	977	0	4931	0	0	0
N.S.	1	1.09	0.11	1.32	0.00	6.68	0.00	0.00	0.00
time (sec)	N/A	0.785	10.104	5.290	0.000	3.389	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	758	826	80	924	0	4953	0	0	0
N.S.	1	1.09	0.11	1.22	0.00	6.53	0.00	0.00	0.00
time (sec)	N/A	0.825	10.097	5.004	0.000	3.376	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	774	843	89	926	0	4867	0	0	0
N.S.	1	1.09	0.11	1.20	0.00	6.29	0.00	0.00	0.00
time (sec)	N/A	0.846	10.072	4.742	0.000	3.391	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	768	837	89	983	0	4875	0	0	0
N.S.	1	1.09	0.12	1.28	0.00	6.35	0.00	0.00	0.00
time (sec)	N/A	0.822	10.073	4.769	0.000	3.414	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	318	318	83	538	0	5563	0	0	0
N.S.	1	1.00	0.26	1.69	0.00	17.49	0.00	0.00	0.00
time (sec)	N/A	0.313	10.099	4.832	0.000	3.556	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	324	324	83	509	0	5587	0	0	0
N.S.	1	1.00	0.26	1.57	0.00	17.24	0.00	0.00	0.00
time (sec)	N/A	0.295	10.083	4.688	0.000	3.499	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	328	328	85	510	0	5667	0	0	0
N.S.	1	1.00	0.26	1.55	0.00	17.28	0.00	0.00	0.00
time (sec)	N/A	0.294	10.085	4.855	0.000	3.686	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	330	330	87	541	0	5679	0	0	0
N.S.	1	1.00	0.26	1.64	0.00	17.21	0.00	0.00	0.00
time (sec)	N/A	0.286	10.077	4.751	0.000	3.547	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	310	310	83	538	0	5631	0	0	0
N.S.	1	1.00	0.27	1.74	0.00	18.16	0.00	0.00	0.00
time (sec)	N/A	0.290	10.115	4.720	0.000	3.430	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	316	316	83	509	0	5667	0	0	0
N.S.	1	1.00	0.26	1.61	0.00	17.93	0.00	0.00	0.00
time (sec)	N/A	0.295	10.105	4.990	0.000	3.441	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	320	320	84	510	0	5599	0	0	0
N.S.	1	1.00	0.26	1.59	0.00	17.50	0.00	0.00	0.00
time (sec)	N/A	0.298	10.091	4.851	0.000	3.569	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	322	322	86	541	0	5599	0	0	0
N.S.	1	1.00	0.27	1.68	0.00	17.39	0.00	0.00	0.00
time (sec)	N/A	0.283	10.102	4.966	0.000	3.598	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	121	124	0	280	173	139	176
N.S.	1	1.00	0.97	0.99	0.00	2.24	1.38	1.11	1.41
time (sec)	N/A	0.296	0.390	5.612	0.000	0.277	8.957	0.288	10.527

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	98	88	81	0	195	128	96	136
N.S.	1	1.05	0.95	0.87	0.00	2.10	1.38	1.03	1.46
time (sec)	N/A	0.231	0.274	4.370	0.000	0.275	4.212	0.280	10.421

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	62	0	156	95	66	82
N.S.	1	1.00	1.00	0.89	0.00	2.23	1.36	0.94	1.17
time (sec)	N/A	0.211	0.126	4.381	0.000	0.274	2.410	0.294	10.443

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	98	0	383	165	79	114
N.S.	1	1.00	0.95	1.15	0.00	4.51	1.94	0.93	1.34
time (sec)	N/A	0.224	0.156	4.501	0.000	0.311	4.288	0.299	12.462

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	121	107	96	0	513	0	107	137
N.S.	1	1.05	0.93	0.83	0.00	4.46	0.00	0.93	1.19
time (sec)	N/A	0.275	0.427	4.582	0.000	0.301	0.000	0.269	9.444

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	241	741	0	0	0	0	0
N.S.	1	1.00	3.77	11.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	6.502	5.088	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	64	64	65	857	0	0	0	0	0
N.S.	1	1.00	1.02	13.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	9.079	4.435	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	59	59	161	705	0	0	0	0	0
N.S.	1	1.00	2.73	11.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.185	10.172	4.279	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	62	62	139	892	0	0	0	0	0
N.S.	1	1.00	2.24	14.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	10.113	5.236	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	335	740	0	0	0	0	0
N.S.	1	1.00	5.23	11.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.207	10.268	5.353	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	143	149	0	410	201	193	330
N.S.	1	1.00	0.93	0.97	0.00	2.66	1.31	1.25	2.14
time (sec)	N/A	0.310	0.472	4.396	0.000	0.276	33.412	0.296	10.386

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	129	111	113	0	297	153	151	215
N.S.	1	1.08	0.92	0.94	0.00	2.48	1.28	1.26	1.79
time (sec)	N/A	0.245	0.306	4.680	0.000	0.278	16.104	0.298	10.355

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	101	85	94	0	204	124	113	143
N.S.	1	1.05	0.89	0.98	0.00	2.12	1.29	1.18	1.49
time (sec)	N/A	0.229	0.274	4.416	0.000	0.281	7.951	0.287	10.179

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	108	106	104	0	486	189	112	155
N.S.	1	1.04	1.02	1.00	0.00	4.67	1.82	1.08	1.49
time (sec)	N/A	0.259	0.337	4.589	0.000	0.313	6.570	0.292	12.332

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	122	108	118	0	538	0	121	167
N.S.	1	1.05	0.93	1.02	0.00	4.64	0.00	1.04	1.44
time (sec)	N/A	0.283	0.395	4.429	0.000	0.305	0.000	0.297	13.457

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	65	65	280	800	0	0	0	0	0
N.S.	1	1.00	4.31	12.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	8.318	5.245	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	65	65	149	921	0	0	0	0	0
N.S.	1	1.00	2.29	14.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	10.163	5.570	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	60	60	351	769	0	0	0	0	0
N.S.	1	1.00	5.85	12.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	10.326	5.171	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	63	63	148	920	0	0	0	0	0
N.S.	1	1.00	2.35	14.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	10.133	5.699	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	65	65	343	771	0	0	0	0	0
N.S.	1	1.00	5.28	11.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	10.353	5.255	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	91	79	0	289	0	106	121
N.S.	1	1.00	0.88	0.76	0.00	2.78	0.00	1.02	1.16
time (sec)	N/A	0.259	0.309	4.337	0.000	0.351	0.000	0.266	9.690

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	75	61	0	205	0	64	86
N.S.	1	1.00	1.01	0.82	0.00	2.77	0.00	0.86	1.16
time (sec)	N/A	0.203	0.153	4.532	0.000	0.343	0.000	0.266	9.409

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	39	0	130	88	40	70
N.S.	1	1.00	1.00	0.76	0.00	2.55	1.73	0.78	1.37
time (sec)	N/A	0.187	0.072	4.673	0.000	0.292	3.930	0.281	10.230

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	82	66	0	431	119	71	114
N.S.	1	1.00	0.96	0.78	0.00	5.07	1.40	0.84	1.34
time (sec)	N/A	0.215	0.208	4.361	0.000	0.329	5.103	0.277	11.765

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	127	109	92	0	565	0	104	142
N.S.	1	1.09	0.93	0.79	0.00	4.83	0.00	0.89	1.21
time (sec)	N/A	0.262	0.399	4.606	0.000	0.347	0.000	0.269	12.828

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	64	64	65	719	0	0	0	0	0
N.S.	1	1.00	1.02	11.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	10.041	4.285	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	64	64	65	429	0	0	0	0	0
N.S.	1	1.00	1.02	6.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.191	10.036	4.262	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	59	59	161	429	0	0	0	0	0
N.S.	1	1.00	2.73	7.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.181	10.060	4.420	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	62	62	141	890	0	0	0	0	0
N.S.	1	1.00	2.27	14.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	10.115	4.929	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	339	738	0	0	0	0	0
N.S.	1	1.00	5.30	11.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	10.296	4.879	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	105	111	100	0	440	0	103	115
N.S.	1	0.98	1.04	0.93	0.00	4.11	0.00	0.96	1.07
time (sec)	N/A	0.292	0.486	4.540	0.000	0.391	0.000	0.273	10.806

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	80	88	0	326	0	78	94
N.S.	1	1.00	0.98	1.07	0.00	3.98	0.00	0.95	1.15
time (sec)	N/A	0.220	0.264	4.351	0.000	0.335	0.000	0.288	10.269

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	76	83	0	236	121	73	89
N.S.	1	1.00	0.99	1.08	0.00	3.06	1.57	0.95	1.16
time (sec)	N/A	0.214	0.168	4.202	0.000	0.349	8.689	0.283	10.220

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	136	110	103	0	790	155	111	139
N.S.	1	1.19	0.96	0.90	0.00	6.93	1.36	0.97	1.22
time (sec)	N/A	0.261	0.601	4.914	0.000	0.380	6.873	0.285	12.983

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	188	142	144	0	1120	0	173	597
N.S.	1	1.19	0.90	0.91	0.00	7.09	0.00	1.09	3.78
time (sec)	N/A	0.364	0.678	4.563	0.000	0.454	0.000	0.295	15.206

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	67	67	231	749	0	0	0	0	0
N.S.	1	1.00	3.45	11.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	10.192	4.454	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	67	67	142	907	0	0	0	0	0
N.S.	1	1.00	2.12	13.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	10.126	4.429	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	62	62	338	753	0	0	0	0	0
N.S.	1	1.00	5.45	12.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	10.323	4.261	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	65	65	193	952	0	0	0	0	0
N.S.	1	1.00	2.97	14.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	10.213	5.861	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	67	67	408	798	0	0	0	0	0
N.S.	1	1.00	6.09	11.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	10.576	6.049	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	132	91	98	107	219	0	110	127
N.S.	1	1.13	0.78	0.84	0.91	1.87	0.00	0.94	1.09
time (sec)	N/A	0.259	0.244	4.407	0.272	0.344	0.000	0.276	8.317

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	111	79	79	91	191	0	93	107
N.S.	1	1.09	0.77	0.77	0.89	1.87	0.00	0.91	1.05
time (sec)	N/A	0.230	0.163	4.669	0.297	0.305	0.000	0.285	8.279

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	90	69	62	79	165	0	69	87
N.S.	1	1.10	0.84	0.76	0.96	2.01	0.00	0.84	1.06
time (sec)	N/A	0.211	0.136	4.876	0.287	0.466	0.000	0.269	8.223

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	65	61	49	66	149	0	53	72
N.S.	1	1.02	0.95	0.77	1.03	2.33	0.00	0.83	1.12
time (sec)	N/A	0.192	0.107	4.243	0.281	0.297	0.000	0.276	8.118

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	100	83	78	0	226	0	79	76
N.S.	1	1.14	0.94	0.89	0.00	2.57	0.00	0.90	0.86
time (sec)	N/A	0.235	0.146	4.525	0.000	0.314	0.000	0.290	8.252

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	142	97	98	0	278	0	113	117
N.S.	1	1.15	0.78	0.79	0.00	2.24	0.00	0.91	0.94
time (sec)	N/A	0.265	0.291	4.629	0.000	0.333	0.000	0.287	8.433

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	184	112	116	0	310	0	105	154
N.S.	1	1.12	0.68	0.71	0.00	1.89	0.00	0.64	0.94
time (sec)	N/A	0.302	0.373	4.637	0.000	0.328	0.000	0.280	8.702

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	663	671	176	897	0	2568	0	0	0
N.S.	1	1.01	0.27	1.35	0.00	3.87	0.00	0.00	0.00
time (sec)	N/A	1.118	7.094	5.543	0.000	8.825	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	641	641	167	877	0	2397	0	0	0
N.S.	1	1.00	0.26	1.37	0.00	3.74	0.00	0.00	0.00
time (sec)	N/A	1.017	10.153	4.566	0.000	1.735	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	644	640	164	883	0	2496	0	0	0
N.S.	1	0.99	0.25	1.37	0.00	3.88	0.00	0.00	0.00
time (sec)	N/A	1.024	10.105	4.488	0.000	0.402	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	665	669	179	898	0	2390	0	0	0
N.S.	1	1.01	0.27	1.35	0.00	3.59	0.00	0.00	0.00
time (sec)	N/A	1.103	11.126	5.668	0.000	0.500	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	687	698	199	919	0	2549	0	0	0
N.S.	1	1.02	0.29	1.34	0.00	3.71	0.00	0.00	0.00
time (sec)	N/A	1.134	10.154	5.649	0.000	1.434	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	711	728	209	938	0	2582	0	0	0
N.S.	1	1.02	0.29	1.32	0.00	3.63	0.00	0.00	0.00
time (sec)	N/A	1.243	10.160	5.512	0.000	2.939	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	157	103	109	119	239	0	127	147
N.S.	1	1.17	0.77	0.81	0.89	1.78	0.00	0.95	1.10
time (sec)	N/A	0.280	0.223	4.776	0.295	0.305	0.000	0.281	8.316

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	135	93	98	107	219	0	111	127
N.S.	1	1.13	0.78	0.82	0.90	1.84	0.00	0.93	1.07
time (sec)	N/A	0.244	0.194	4.640	0.305	0.322	0.000	0.271	8.266

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	112	81	79	93	192	0	93	107
N.S.	1	1.15	0.84	0.81	0.96	1.98	0.00	0.96	1.10
time (sec)	N/A	0.225	0.167	4.414	0.331	0.306	0.000	0.277	8.045

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	84	72	63	79	162	0	69	87
N.S.	1	1.09	0.94	0.82	1.03	2.10	0.00	0.90	1.13
time (sec)	N/A	0.208	0.130	4.488	0.318	0.304	0.000	0.282	8.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	94	85	64	0	220	0	70	101
N.S.	1	1.11	1.00	0.75	0.00	2.59	0.00	0.82	1.19
time (sec)	N/A	0.232	0.133	4.531	0.000	0.315	0.000	0.272	8.754

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	136	97	99	0	280	0	114	110
N.S.	1	1.12	0.80	0.82	0.00	2.31	0.00	0.94	0.91
time (sec)	N/A	0.265	0.264	4.642	0.000	0.409	0.000	0.284	8.215

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	178	112	118	0	310	0	129	151
N.S.	1	1.11	0.70	0.73	0.00	1.93	0.00	0.80	0.94
time (sec)	N/A	0.309	0.339	4.654	0.000	0.362	0.000	0.287	8.588

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	681	687	191	921	0	2580	0	0	0
N.S.	1	1.01	0.28	1.35	0.00	3.79	0.00	0.00	0.00
time (sec)	N/A	1.148	8.442	5.508	0.000	17.752	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	657	661	176	897	0	2568	0	0	0
N.S.	1	1.01	0.27	1.37	0.00	3.91	0.00	0.00	0.00
time (sec)	N/A	1.093	10.176	5.523	0.000	3.788	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	638	631	141	874	0	2335	0	0	0
N.S.	1	0.99	0.22	1.37	0.00	3.66	0.00	0.00	0.00
time (sec)	N/A	1.027	10.194	4.489	0.000	0.994	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	522	547	242	483	0	68	0	0	0
N.S.	1	1.05	0.46	0.93	0.00	0.13	0.00	0.00	0.00
time (sec)	N/A	0.560	11.739	5.352	0.000	0.101	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	684	692	199	919	0	2549	0	0	0
N.S.	1	1.01	0.29	1.34	0.00	3.73	0.00	0.00	0.00
time (sec)	N/A	1.203	10.153	5.615	0.000	0.896	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	708	722	212	938	0	2582	0	0	0
N.S.	1	1.02	0.30	1.32	0.00	3.65	0.00	0.00	0.00
time (sec)	N/A	1.253	10.169	5.837	0.000	1.946	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	105	81	79	93	195	0	93	107
N.S.	1	1.11	0.85	0.83	0.98	2.05	0.00	0.98	1.13
time (sec)	N/A	0.229	0.186	4.575	0.302	0.280	0.000	0.269	8.046

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	93	70	63	79	167	0	69	87
N.S.	1	1.12	0.84	0.76	0.95	2.01	0.00	0.83	1.05
time (sec)	N/A	0.221	0.127	4.393	0.297	0.286	0.000	0.263	8.043

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	68	63	50	67	155	0	58	72
N.S.	1	1.06	0.98	0.78	1.05	2.42	0.00	0.91	1.12
time (sec)	N/A	0.193	0.119	4.352	0.291	0.302	0.000	0.282	7.963

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	71	64	52	72	153	0	59	75
N.S.	1	1.06	0.96	0.78	1.07	2.28	0.00	0.88	1.12
time (sec)	N/A	0.200	0.092	4.246	0.284	0.276	0.000	0.277	7.921

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	100	83	78	0	226	0	79	80
N.S.	1	1.14	0.94	0.89	0.00	2.57	0.00	0.90	0.91
time (sec)	N/A	0.235	0.144	4.512	0.000	0.294	0.000	0.280	7.999

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	142	97	99	0	280	0	114	117
N.S.	1	1.15	0.78	0.80	0.00	2.26	0.00	0.92	0.94
time (sec)	N/A	0.272	0.285	4.639	0.000	0.308	0.000	0.276	8.119

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	184	112	118	0	310	0	128	155
N.S.	1	1.12	0.68	0.72	0.00	1.89	0.00	0.78	0.95
time (sec)	N/A	0.306	0.359	4.642	0.000	0.307	0.000	0.273	8.434

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	641	641	167	877	0	2397	0	0	0
N.S.	1	1.00	0.26	1.37	0.00	3.74	0.00	0.00	0.00
time (sec)	N/A	0.997	10.167	4.691	0.000	3.665	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	647	646	166	886	0	2538	0	0	0
N.S.	1	1.00	0.26	1.37	0.00	3.92	0.00	0.00	0.00
time (sec)	N/A	1.001	10.115	4.886	0.000	0.452	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	644	641	164	883	0	2540	0	0	0
N.S.	1	1.00	0.25	1.37	0.00	3.94	0.00	0.00	0.00
time (sec)	N/A	1.023	10.105	4.318	0.000	0.567	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	665	669	180	898	0	2391	0	0	0
N.S.	1	1.01	0.27	1.35	0.00	3.60	0.00	0.00	0.00
time (sec)	N/A	1.118	11.132	5.487	0.000	0.677	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	687	698	196	919	0	2549	0	0	0
N.S.	1	1.02	0.29	1.34	0.00	3.71	0.00	0.00	0.00
time (sec)	N/A	1.148	10.163	5.596	0.000	2.066	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	711	728	212	938	0	2582	0	0	0
N.S.	1	1.02	0.30	1.32	0.00	3.63	0.00	0.00	0.00
time (sec)	N/A	1.246	10.165	5.797	0.000	4.286	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	239	723	0	2425	0	0	0
N.S.	1	1.00	3.62	10.95	0.00	36.74	0.00	0.00	0.00
time (sec)	N/A	0.218	10.327	4.505	0.000	0.718	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	237	732	0	2548	0	0	0
N.S.	1	1.00	3.59	11.09	0.00	38.61	0.00	0.00	0.00
time (sec)	N/A	0.221	10.216	4.582	0.000	0.899	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	237	729	0	2498	0	0	0
N.S.	1	1.00	3.70	11.39	0.00	39.03	0.00	0.00	0.00
time (sec)	N/A	0.193	10.199	4.481	0.000	0.874	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	266	744	0	0	0	0	0
N.S.	1	1.00	4.03	11.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	10.209	5.565	0.000	0.000	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	279	765	0	2561	0	0	0
N.S.	1	1.00	4.23	11.59	0.00	38.80	0.00	0.00	0.00
time (sec)	N/A	0.216	10.217	5.559	0.000	4.252	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	107	99	75	98	233	0	88	111
N.S.	1	1.13	1.04	0.79	1.03	2.45	0.00	0.93	1.17
time (sec)	N/A	0.227	0.184	4.582	0.286	0.313	0.000	0.294	8.308

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	95	71	61	81	223	0	67	94
N.S.	1	1.14	0.86	0.73	0.98	2.69	0.00	0.81	1.13
time (sec)	N/A	0.215	0.177	4.769	0.283	0.337	0.000	0.279	8.362

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	97	73	61	83	223	0	76	96
N.S.	1	1.14	0.86	0.72	0.98	2.62	0.00	0.89	1.13
time (sec)	N/A	0.204	0.152	4.381	0.291	0.352	0.000	0.296	8.278

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	100	73	62	85	219	0	72	97
N.S.	1	1.14	0.83	0.70	0.97	2.49	0.00	0.82	1.10
time (sec)	N/A	0.216	0.153	4.274	0.302	0.315	0.000	0.288	8.252

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	126	93	92	0	316	0	93	101
N.S.	1	1.19	0.88	0.87	0.00	2.98	0.00	0.88	0.95
time (sec)	N/A	0.266	0.202	4.554	0.000	0.353	0.000	0.289	8.347

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	168	109	113	0	368	0	129	133
N.S.	1	1.17	0.76	0.79	0.00	2.57	0.00	0.90	0.93
time (sec)	N/A	0.292	0.352	4.701	0.000	0.316	0.000	0.299	8.478

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	210	123	135	0	398	0	149	171
N.S.	1	1.14	0.66	0.73	0.00	2.15	0.00	0.81	0.92
time (sec)	N/A	0.336	0.431	4.497	0.000	0.328	0.000	0.293	8.821

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	668	669	168	910	0	2681	0	0	0
N.S.	1	1.00	0.25	1.36	0.00	4.01	0.00	0.00	0.00
time (sec)	N/A	1.083	10.125	4.591	0.000	0.596	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	671	676	169	910	0	2723	0	0	0
N.S.	1	1.01	0.25	1.36	0.00	4.06	0.00	0.00	0.00
time (sec)	N/A	1.084	10.120	4.619	0.000	0.650	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	665	669	167	904	0	2684	0	0	0
N.S.	1	1.01	0.25	1.36	0.00	4.04	0.00	0.00	0.00
time (sec)	N/A	1.093	10.119	4.528	0.000	0.762	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	686	698	180	920	0	2534	0	0	0
N.S.	1	1.02	0.26	1.34	0.00	3.69	0.00	0.00	0.00
time (sec)	N/A	1.177	11.154	6.327	0.000	0.909	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	708	727	198	943	0	2692	0	0	0
N.S.	1	1.03	0.28	1.33	0.00	3.80	0.00	0.00	0.00
time (sec)	N/A	1.255	10.163	6.309	0.000	2.796	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	732	757	210	962	0	2725	0	0	0
N.S.	1	1.03	0.29	1.31	0.00	3.72	0.00	0.00	0.00
time (sec)	N/A	1.304	10.185	6.671	0.000	5.575	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	256	66	189	339	0	89	0	0	0
N.S.	1	0.26	0.74	1.32	0.00	0.35	0.00	0.00	0.00
time (sec)	N/A	0.214	10.562	4.401	0.000	0.102	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	242	754	0	2713	0	0	0
N.S.	1	1.00	3.67	11.42	0.00	41.11	0.00	0.00	0.00
time (sec)	N/A	0.218	10.291	4.422	0.000	1.067	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	253	748	0	2640	0	0	0
N.S.	1	1.00	3.95	11.69	0.00	41.25	0.00	0.00	0.00
time (sec)	N/A	0.190	10.265	4.546	0.000	0.975	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	259	764	0	2650	0	0	0
N.S.	1	1.00	3.92	11.58	0.00	40.15	0.00	0.00	0.00
time (sec)	N/A	0.215	10.246	6.045	0.000	2.074	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	283	787	0	2698	0	0	0
N.S.	1	1.00	4.29	11.92	0.00	40.88	0.00	0.00	0.00
time (sec)	N/A	0.218	10.242	6.209	0.000	5.324	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	164	126	129	0	469	0	136	202
N.S.	1	1.02	0.78	0.80	0.00	2.91	0.00	0.84	1.25
time (sec)	N/A	0.302	0.434	4.801	0.000	0.314	0.000	0.290	10.881

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	132	98	98	0	334	0	102	152
N.S.	1	0.97	0.72	0.72	0.00	2.46	0.00	0.75	1.12
time (sec)	N/A	0.259	0.291	4.666	0.000	0.407	0.000	0.278	10.135

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	65	0	255	0	79	125
N.S.	1	1.00	1.00	0.81	0.00	3.19	0.00	0.99	1.56
time (sec)	N/A	0.210	0.229	4.457	0.000	0.351	0.000	0.273	9.669

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	126	111	120	0	856	0	114	182
N.S.	1	1.04	0.92	0.99	0.00	7.07	0.00	0.94	1.50
time (sec)	N/A	0.261	0.630	4.442	0.000	0.393	0.000	0.277	12.814

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	170	132	151	0	870	0	183	438
N.S.	1	1.06	0.82	0.94	0.00	5.40	0.00	1.14	2.72
time (sec)	N/A	0.328	0.781	4.761	0.000	0.397	0.000	0.286	14.409

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	235	748	0	0	0	0	0
N.S.	1	1.00	3.67	11.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	10.218	4.743	0.000	0.000	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	64	64	153	908	0	0	0	0	0
N.S.	1	1.00	2.39	14.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	10.109	4.557	0.000	0.000	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	59	59	232	753	0	0	0	0	0
N.S.	1	1.00	3.93	12.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.189	10.226	4.395	0.000	0.000	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	62	62	172	920	0	0	0	0	0
N.S.	1	1.00	2.77	14.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	10.151	6.387	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	338	766	0	0	0	0	0
N.S.	1	1.00	5.28	11.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.206	10.323	6.576	0.000	0.000	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	195	162	169	0	443	0	211	331
N.S.	1	1.03	0.86	0.89	0.00	2.34	0.00	1.12	1.75
time (sec)	N/A	0.324	0.562	4.651	0.000	0.384	0.000	0.287	12.037

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	125	133	0	314	0	173	229
N.S.	1	1.00	0.77	0.82	0.00	1.93	0.00	1.06	1.40
time (sec)	N/A	0.271	0.361	4.725	0.000	0.322	0.000	0.276	11.577

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	104	94	108	0	234	0	122	170
N.S.	1	1.11	1.00	1.15	0.00	2.49	0.00	1.30	1.81
time (sec)	N/A	0.231	0.311	4.259	0.000	0.315	0.000	0.274	11.373

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	140	122	140	0	686	0	155	214
N.S.	1	1.07	0.93	1.07	0.00	5.24	0.00	1.18	1.63
time (sec)	N/A	0.289	0.524	4.657	0.000	0.423	0.000	0.278	13.944

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	197	154	165	0	838	0	216	531
N.S.	1	1.16	0.91	0.97	0.00	4.93	0.00	1.27	3.12
time (sec)	N/A	0.382	0.929	4.725	0.000	0.351	0.000	0.286	15.795

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	65	65	338	808	0	0	0	0	0
N.S.	1	1.00	5.20	12.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.212	10.466	6.478	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	65	65	177	955	0	0	0	0	0
N.S.	1	1.00	2.72	14.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.207	10.199	4.767	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	60	60	339	801	0	0	0	0	0
N.S.	1	1.00	5.65	13.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.189	10.323	4.568	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	63	63	190	970	0	0	0	0	0
N.S.	1	1.00	3.02	15.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	10.253	6.316	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	65	65	370	815	0	0	0	0	0
N.S.	1	1.00	5.69	12.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	10.408	6.813	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	144	130	132	0	475	0	134	160
N.S.	1	1.17	1.06	1.07	0.00	3.86	0.00	1.09	1.30
time (sec)	N/A	0.279	0.616	4.600	0.000	0.288	0.000	0.287	11.608

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	98	100	83	0	348	0	116	111
N.S.	1	0.99	1.01	0.84	0.00	3.52	0.00	1.17	1.12
time (sec)	N/A	0.231	0.286	4.638	0.000	0.320	0.000	0.276	11.036

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	86	86	90	0	302	0	93	104
N.S.	1	0.99	0.99	1.03	0.00	3.47	0.00	1.07	1.20
time (sec)	N/A	0.217	0.260	4.440	0.000	0.269	0.000	0.279	10.663

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	155	124	146	0	862	0	139	162
N.S.	1	1.17	0.94	1.11	0.00	6.53	0.00	1.05	1.23
time (sec)	N/A	0.291	0.549	4.571	0.000	0.341	0.000	0.271	14.692

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	213	163	191	0	1236	0	257	355
N.S.	1	1.15	0.88	1.03	0.00	6.68	0.00	1.39	1.92
time (sec)	N/A	0.376	1.019	4.803	0.000	0.405	0.000	0.268	15.994

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	238	764	0	0	0	0	0
N.S.	1	1.00	3.72	11.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	10.225	4.511	0.000	0.000	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	64	64	172	923	0	0	0	0	0
N.S.	1	1.00	2.69	14.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	10.175	4.560	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	59	59	392	769	0	0	0	0	0
N.S.	1	1.00	6.64	13.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.189	10.277	4.666	0.000	0.000	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	62	62	226	963	0	0	0	0	0
N.S.	1	1.00	3.65	15.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	10.263	6.403	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	411	809	0	0	0	0	0
N.S.	1	1.00	6.42	12.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	10.634	6.312	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	162	133	146	0	746	0	195	367
N.S.	1	1.08	0.89	0.97	0.00	4.97	0.00	1.30	2.45
time (sec)	N/A	0.312	0.662	4.394	0.000	0.352	0.000	0.272	12.269

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	138	110	124	0	630	0	181	247
N.S.	1	1.03	0.82	0.93	0.00	4.70	0.00	1.35	1.84
time (sec)	N/A	0.259	0.473	4.446	0.000	0.376	0.000	0.278	11.733

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	125	101	86	0	450	0	153	199
N.S.	1	1.16	0.94	0.80	0.00	4.17	0.00	1.42	1.84
time (sec)	N/A	0.241	0.376	4.479	0.000	0.334	0.000	0.266	11.305

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	208	157	199	0	1819	0	226	288
N.S.	1	1.21	0.91	1.16	0.00	10.58	0.00	1.31	1.67
time (sec)	N/A	0.368	1.179	4.550	0.000	0.602	0.000	0.270	16.413

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	283	223	250	0	2384	0	367	18847
N.S.	1	1.17	0.93	1.04	0.00	9.89	0.00	1.52	78.20
time (sec)	N/A	0.458	1.464	4.892	0.000	0.722	0.000	0.288	23.902

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	67	67	381	787	0	0	0	0	0
N.S.	1	1.00	5.69	11.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	10.323	4.435	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	67	67	216	986	0	0	0	0	0
N.S.	1	1.00	3.22	14.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	10.283	4.383	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	62	62	381	830	0	0	0	0	0
N.S.	1	1.00	6.15	13.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	10.518	4.412	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	65	65	308	1019	0	0	0	0	0
N.S.	1	1.00	4.74	15.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	10.421	7.528	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	67	67	515	863	0	0	0	0	0
N.S.	1	1.00	7.69	12.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	10.917	7.337	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	113	0	0	0	379	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	2.83	0.00	0.00
time (sec)	N/A	0.279	0.956	0.000	0.000	0.000	18.960	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	111	0	0	0	246	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	1.86	0.00	0.00
time (sec)	N/A	0.272	0.340	0.000	0.000	0.000	6.887	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	110	0	0	0	121	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.92	0.00	0.00
time (sec)	N/A	0.275	0.228	0.000	0.000	0.000	2.224	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	110	0	0	0	117	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.89	0.00	0.00
time (sec)	N/A	0.276	0.263	0.000	0.000	0.000	1.946	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	113	0	0	0	117	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.88	0.00	0.00
time (sec)	N/A	0.271	0.496	0.000	0.000	0.000	29.539	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	113	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.998	0.000	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	87	88	0	0	256	0	104	283
N.S.	1	0.99	1.00	0.00	0.00	2.91	0.00	1.18	3.22
time (sec)	N/A	0.221	1.382	0.000	0.000	0.300	0.000	0.310	13.121

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	194	0	54	49
N.S.	1	1.00	1.00	0.00	0.00	4.04	0.00	1.12	1.02
time (sec)	N/A	0.185	0.833	0.000	0.000	0.280	0.000	0.287	8.873

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	204	0	89	136
N.S.	1	1.00	1.00	0.00	0.00	4.25	0.00	1.85	2.83
time (sec)	N/A	0.192	0.862	0.000	0.000	0.289	0.000	0.277	11.449

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	90	91	0	0	278	0	413	481
N.S.	1	0.99	1.00	0.00	0.00	3.05	0.00	4.54	5.29
time (sec)	N/A	0.222	1.594	0.000	0.000	0.317	0.000	0.288	15.120

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	2.485	0.000	0.000	0.000	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.262	2.142	0.000	0.000	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.238	2.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	170	0	0	0	0	0	0
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	2.398	0.000	0.000	0.000	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	189	0	0	0	0	0	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	2.496	0.000	0.000	0.000	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	365	0	0	0	0	0	0
N.S.	1	1.00	4.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.267	2.592	0.000	0.000	0.000	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	152	123	138	0	295	298	327	0
N.S.	1	0.94	0.76	0.86	0.00	1.83	1.85	2.03	0.00
time (sec)	N/A	0.312	1.077	5.648	0.000	0.568	18.801	0.434	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	343	112	777	0	0	97	0	0
N.S.	1	1.06	0.35	2.40	0.00	0.00	0.30	0.00	0.00
time (sec)	N/A	0.411	10.179	5.045	0.000	0.000	21.289	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	625	94	1140	0	0	97	0	0
N.S.	1	1.08	0.16	1.96	0.00	0.00	0.17	0.00	0.00
time (sec)	N/A	0.695	10.116	4.587	0.000	0.000	6.920	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	118	96	109	0	221	253	177	0
N.S.	1	0.98	0.79	0.90	0.00	1.83	2.09	1.46	0.00
time (sec)	N/A	0.265	0.408	4.769	0.000	0.578	1.552	0.324	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	308	93	744	0	0	97	0	0
N.S.	1	1.08	0.33	2.60	0.00	0.00	0.34	0.00	0.00
time (sec)	N/A	0.369	10.078	4.981	0.000	0.000	2.645	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	580	622	98	1123	0	0	100	0	0
N.S.	1	1.07	0.17	1.94	0.00	0.00	0.17	0.00	0.00
time (sec)	N/A	0.687	10.064	4.832	0.000	0.000	3.521	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	117	82	100	0	207	160	0	0
N.S.	1	0.99	0.69	0.85	0.00	1.75	1.36	0.00	0.00
time (sec)	N/A	0.267	0.387	4.599	0.000	0.567	6.392	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	311	97	743	0	0	100	0	0
N.S.	1	1.10	0.34	2.63	0.00	0.00	0.35	0.00	0.00
time (sec)	N/A	0.373	10.061	4.699	0.000	0.000	12.581	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	564	536	81	1127	0	0	97	0	0
N.S.	1	0.95	0.14	2.00	0.00	0.00	0.17	0.00	0.00
time (sec)	N/A	0.631	10.100	4.984	0.000	0.000	9.852	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	78	75	84	81	180	131	109	0
N.S.	1	0.99	0.95	1.06	1.03	2.28	1.66	1.38	0.00
time (sec)	N/A	0.230	0.449	4.460	0.299	0.362	26.869	0.298	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	265	80	745	0	76	97	0	0
N.S.	1	0.99	0.30	2.77	0.00	0.28	0.36	0.00	0.00
time (sec)	N/A	0.333	10.096	4.516	0.000	0.077	67.776	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	183	144	162	0	355	634	488	0
N.S.	1	0.91	0.72	0.81	0.00	1.77	3.15	2.43	0.00
time (sec)	N/A	0.349	0.588	4.620	0.000	0.570	31.069	0.507	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	374	116	801	0	0	199	0	0
N.S.	1	1.03	0.32	2.20	0.00	0.00	0.55	0.00	0.00
time (sec)	N/A	0.435	10.188	4.532	0.000	0.000	63.406	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	621	656	96	1164	0	0	199	0	0
N.S.	1	1.06	0.15	1.87	0.00	0.00	0.32	0.00	0.00
time (sec)	N/A	0.709	10.142	4.992	0.000	0.000	20.756	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	149	121	134	0	273	546	415	0
N.S.	1	0.93	0.75	0.83	0.00	1.70	3.39	2.58	0.00
time (sec)	N/A	0.302	0.526	4.634	0.000	0.686	3.441	0.447	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	339	96	768	0	0	199	0	0
N.S.	1	1.05	0.30	2.37	0.00	0.00	0.61	0.00	0.00
time (sec)	N/A	0.396	10.086	4.795	0.000	0.000	7.280	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	614	653	84	1140	0	0	202	0	0
N.S.	1	1.06	0.14	1.86	0.00	0.00	0.33	0.00	0.00
time (sec)	N/A	0.714	10.078	5.050	0.000	0.000	9.875	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	148	101	117	0	255	289	0	0
N.S.	1	0.97	0.66	0.77	0.00	1.68	1.90	0.00	0.00
time (sec)	N/A	0.307	0.563	4.563	0.000	0.559	17.816	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	338	85	759	0	0	202	0	0
N.S.	1	1.08	0.27	2.42	0.00	0.00	0.64	0.00	0.00
time (sec)	N/A	0.397	10.082	4.771	0.000	0.000	23.154	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	214	165	186	0	409	1028	692	0
N.S.	1	0.89	0.68	0.77	0.00	1.70	4.27	2.87	0.00
time (sec)	N/A	0.380	0.736	4.982	0.000	0.587	49.127	0.596	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	405	116	825	0	0	308	0	0
N.S.	1	1.00	0.29	2.04	0.00	0.00	0.76	0.00	0.00
time (sec)	N/A	0.473	10.232	4.886	0.000	0.000	150.121	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	661	687	99	1188	0	0	308	0	0
N.S.	1	1.04	0.15	1.80	0.00	0.00	0.47	0.00	0.00
time (sec)	N/A	0.774	10.159	4.815	0.000	0.000	52.726	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	180	141	158	0	323	896	590	0
N.S.	1	0.90	0.70	0.79	0.00	1.61	4.46	2.94	0.00
time (sec)	N/A	0.335	0.643	4.463	0.000	0.569	7.754	0.519	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	370	84	792	0	0	308	0	0
N.S.	1	1.02	0.23	2.18	0.00	0.00	0.85	0.00	0.00
time (sec)	N/A	0.433	10.090	4.844	0.000	0.000	17.853	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	650	684	87	1166	0	0	311	0	0
N.S.	1	1.05	0.13	1.79	0.00	0.00	0.48	0.00	0.00
time (sec)	N/A	0.738	10.060	4.939	0.000	0.000	21.625	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	179	127	143	0	309	403	0	0
N.S.	1	0.95	0.68	0.76	0.00	1.64	2.14	0.00	0.00
time (sec)	N/A	0.338	0.669	4.586	0.000	0.578	40.093	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	373	88	786	0	0	311	0	0
N.S.	1	1.06	0.25	2.23	0.00	0.00	0.88	0.00	0.00
time (sec)	N/A	0.429	10.055	4.862	0.000	0.000	44.170	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	100	114	0	245	194	146	0
N.S.	1	1.00	0.83	0.94	0.00	2.02	1.60	1.21	0.00
time (sec)	N/A	0.289	0.554	4.685	0.000	0.549	13.369	0.339	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	312	98	753	0	0	94	0	0
N.S.	1	1.09	0.34	2.63	0.00	0.00	0.33	0.00	0.00
time (sec)	N/A	0.372	10.130	4.900	0.000	0.000	15.700	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	594	80	1124	0	0	94	0	0
N.S.	1	1.09	0.15	2.07	0.00	0.00	0.17	0.00	0.00
time (sec)	N/A	0.663	10.101	4.809	0.000	0.000	5.781	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	84	79	94	0	184	151	94	0
N.S.	1	1.01	0.95	1.13	0.00	2.22	1.82	1.13	0.00
time (sec)	N/A	0.240	0.797	4.622	0.000	0.544	1.508	0.325	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	278	80	728	0	0	94	0	0
N.S.	1	1.12	0.32	2.92	0.00	0.00	0.38	0.00	0.00
time (sec)	N/A	0.340	10.069	4.704	0.000	0.000	2.035	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	589	83	1119	0	0	97	0	0
N.S.	1	1.09	0.15	2.06	0.00	0.00	0.18	0.00	0.00
time (sec)	N/A	0.663	10.049	4.960	0.000	0.000	2.407	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	76	66	87	0	183	60	106	0
N.S.	1	1.01	0.88	1.16	0.00	2.44	0.80	1.41	0.00
time (sec)	N/A	0.231	0.424	4.623	0.000	0.379	5.117	0.312	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	275	82	740	0	58	97	0	0
N.S.	1	1.12	0.33	3.01	0.00	0.24	0.39	0.00	0.00
time (sec)	N/A	0.338	10.053	4.939	0.000	0.080	15.173	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	93	129	0	307	0	132	0
N.S.	1	1.00	0.78	1.08	0.00	2.56	0.00	1.10	0.00
time (sec)	N/A	0.284	0.836	4.693	0.000	0.562	0.000	0.338	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	312	87	787	0	0	0	0	0
N.S.	1	1.09	0.30	2.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.381	10.131	5.912	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	607	77	1154	0	0	94	0	0
N.S.	1	1.10	0.14	2.09	0.00	0.00	0.17	0.00	0.00
time (sec)	N/A	0.690	10.134	4.591	0.000	0.000	77.731	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	86	81	92	0	234	95	93	0
N.S.	1	1.01	0.95	1.08	0.00	2.75	1.12	1.09	0.00
time (sec)	N/A	0.245	0.738	4.282	0.000	0.391	19.572	0.315	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	287	79	754	0	92	94	0	0
N.S.	1	1.11	0.31	2.92	0.00	0.36	0.36	0.00	0.00
time (sec)	N/A	0.369	10.070	4.788	0.000	0.089	22.605	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	585	629	77	1177	0	108	97	0	0
N.S.	1	1.08	0.13	2.01	0.00	0.18	0.17	0.00	0.00
time (sec)	N/A	0.697	10.061	7.205	0.000	0.107	41.997	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	44	39	0	57	90	98	70
N.S.	1	1.00	0.66	0.58	0.00	0.85	1.34	1.46	1.04
time (sec)	N/A	0.204	0.971	4.482	0.000	0.276	83.591	0.357	8.856

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	314	95	784	0	117	97	0	0
N.S.	1	1.11	0.34	2.77	0.00	0.41	0.34	0.00	0.00
time (sec)	N/A	0.371	10.069	6.015	0.000	0.086	152.274	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	119	100	157	0	345	0	126	0
N.S.	1	1.04	0.88	1.38	0.00	3.03	0.00	1.11	0.00
time (sec)	N/A	0.280	1.014	4.418	0.000	0.388	0.000	0.348	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	325	108	809	0	170	0	0	0
N.S.	1	1.09	0.36	2.71	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.395	10.164	4.603	0.000	0.088	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	596	643	86	1190	0	169	0	0	0
N.S.	1	1.08	0.14	2.00	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.714	10.131	4.627	0.000	0.086	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	44	39	0	59	0	72	73
N.S.	1	1.00	0.56	0.49	0.00	0.75	0.00	0.91	0.92
time (sec)	N/A	0.210	0.818	4.457	0.000	0.253	0.000	0.314	8.580

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	323	107	785	0	150	0	0	0
N.S.	1	1.09	0.36	2.64	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.363	10.117	4.543	0.000	0.091	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	624	665	85	1225	0	167	0	0	0
N.S.	1	1.07	0.14	1.96	0.00	0.27	0.00	0.00	0.00
time (sec)	N/A	0.733	10.066	8.026	0.000	0.093	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	67	62	0	93	0	0	115
N.S.	1	1.00	0.64	0.60	0.00	0.89	0.00	0.00	1.11
time (sec)	N/A	0.227	0.755	3.103	0.000	0.274	0.000	0.000	8.758

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	350	121	829	0	178	0	0	0
N.S.	1	1.09	0.38	2.59	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.397	10.096	7.982	0.000	0.088	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	203	202	147	183	186	0	215	240
N.S.	1	0.92	0.92	0.67	0.83	0.85	0.00	0.98	1.09
time (sec)	N/A	0.380	0.405	6.640	0.287	0.270	0.000	0.762	9.041

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	163	210	136	155	174	0	176	219
N.S.	1	0.94	1.21	0.78	0.89	1.00	0.00	1.01	1.26
time (sec)	N/A	0.324	0.277	5.047	0.287	0.256	0.000	0.738	8.983

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	171	188	125	153	157	0	174	200
N.S.	1	0.99	1.09	0.73	0.89	0.91	0.00	1.01	1.16
time (sec)	N/A	0.296	0.260	4.596	0.282	0.265	0.000	0.747	8.902

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	147	169	126	139	144	0	148	194
N.S.	1	0.98	1.13	0.84	0.93	0.96	0.00	0.99	1.29
time (sec)	N/A	0.266	0.228	4.734	0.283	0.254	0.000	0.738	8.662

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	212	235	182	0	634	0	217	345
N.S.	1	0.99	1.10	0.85	0.00	2.96	0.00	1.01	1.61
time (sec)	N/A	0.322	0.430	4.943	0.000	0.264	0.000	0.995	8.984

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	279	282	222	0	321	0	243	455
N.S.	1	1.04	1.05	0.83	0.00	1.20	0.00	0.91	1.70
time (sec)	N/A	0.376	0.622	4.858	0.000	0.281	0.000	0.984	9.291

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	267	316	254	0	345	0	279	490
N.S.	1	0.94	1.12	0.90	0.00	1.22	0.00	0.99	1.73
time (sec)	N/A	0.388	0.687	4.703	0.000	0.277	0.000	0.972	9.452

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	248	327	273	0	362	0	0	0
N.S.	1	0.93	1.22	1.02	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.428	1.469	7.070	0.000	0.269	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	219	293	243	0	338	0	0	0
N.S.	1	0.94	1.26	1.04	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.345	1.041	4.746	0.000	0.267	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	208	265	219	0	313	0	0	0
N.S.	1	1.03	1.32	1.09	0.00	1.56	0.00	0.00	0.00
time (sec)	N/A	0.287	0.703	4.680	0.000	0.265	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	153	190	150	0	395	0	0	0
N.S.	1	0.98	1.22	0.96	0.00	2.53	0.00	0.00	0.00
time (sec)	N/A	0.261	0.523	4.797	0.000	84.462	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	215	149	0	0	0	0	0
N.S.	1	1.00	1.17	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.564	4.689	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	208	206	160	0	0	0	0	0
N.S.	1	0.99	0.98	0.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.392	0.671	4.877	0.000	0.000	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	243	219	171	0	0	0	0	0
N.S.	1	1.03	0.92	0.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.467	0.761	4.932	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	521	579	234	0	0	0	0	0	0
N.S.	1	1.11	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.833	7.814	0.000	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	494	549	225	0	0	0	0	0	0
N.S.	1	1.11	0.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.749	7.192	0.000	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	416	412	431	0	0	0	0	0	0
N.S.	1	0.99	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.482	3.327	0.000	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	496	551	231	0	0	0	0	0	0
N.S.	1	1.11	0.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.726	11.163	0.000	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	523	581	243	0	0	0	0	0	0
N.S.	1	1.11	0.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.819	11.190	0.000	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	206	202	148	183	209	0	215	261
N.S.	1	0.92	0.91	0.66	0.82	0.94	0.00	0.96	1.17
time (sec)	N/A	0.356	0.464	4.637	0.292	0.390	0.000	0.755	8.721

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	166	210	136	155	197	0	176	206
N.S.	1	0.94	1.19	0.77	0.88	1.11	0.00	0.99	1.16
time (sec)	N/A	0.324	0.332	4.791	0.299	0.331	0.000	0.750	8.763

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	174	188	128	155	181	0	175	221
N.S.	1	0.99	1.07	0.73	0.89	1.03	0.00	1.00	1.26
time (sec)	N/A	0.280	0.306	5.127	0.299	0.344	0.000	0.734	8.661

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	150	170	126	140	167	0	148	186
N.S.	1	0.98	1.11	0.82	0.92	1.09	0.00	0.97	1.22
time (sec)	N/A	0.267	0.283	4.594	0.286	0.355	0.000	0.757	8.661

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	212	236	182	0	530	0	217	369
N.S.	1	0.99	1.10	0.85	0.00	2.48	0.00	1.01	1.72
time (sec)	N/A	0.310	0.431	4.674	0.000	0.365	0.000	0.968	8.975

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	283	282	221	0	612	0	243	490
N.S.	1	1.05	1.05	0.82	0.00	2.28	0.00	0.90	1.82
time (sec)	N/A	0.387	0.659	4.694	0.000	0.353	0.000	0.977	9.488

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	267	316	254	0	660	0	279	513
N.S.	1	0.94	1.11	0.89	0.00	2.32	0.00	0.98	1.81
time (sec)	N/A	0.356	0.734	4.737	0.000	0.368	0.000	0.993	9.401

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	263	325	271	0	701	0	0	0
N.S.	1	1.00	1.23	1.03	0.00	2.66	0.00	0.00	0.00
time (sec)	N/A	0.402	1.381	4.960	0.000	0.342	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	233	291	241	0	653	0	0	0
N.S.	1	1.02	1.27	1.05	0.00	2.85	0.00	0.00	0.00
time (sec)	N/A	0.332	1.012	4.885	0.000	0.303	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	205	264	219	0	611	0	0	0
N.S.	1	1.02	1.32	1.10	0.00	3.06	0.00	0.00	0.00
time (sec)	N/A	0.281	0.720	4.534	0.000	0.318	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	154	195	156	0	434	0	0	0
N.S.	1	0.98	1.24	0.99	0.00	2.76	0.00	0.00	0.00
time (sec)	N/A	0.249	0.523	4.750	0.000	81.744	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	184	216	150	0	0	0	0	0
N.S.	1	1.01	1.19	0.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.303	0.568	4.876	0.000	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	206	159	0	0	0	0	0
N.S.	1	1.00	0.99	0.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	0.667	4.828	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	244	219	174	0	0	0	0	0
N.S.	1	1.03	0.93	0.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.455	0.738	5.066	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	512	502	147	0	0	0	0	0	0
N.S.	1	0.98	0.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.758	8.541	0.000	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	485	471	127	0	0	0	0	0	0
N.S.	1	0.97	0.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.642	8.267	0.000	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	457	460	63	0	0	0	0	0	0
N.S.	1	1.01	0.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.698	10.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	483	467	136	0	0	0	0	0	0
N.S.	1	0.97	0.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.631	11.108	0.000	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	512	496	148	0	0	0	0	0	0
N.S.	1	0.97	0.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.714	11.093	0.000	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	131	145	109	119	118	0	134	133
N.S.	1	1.03	1.14	0.86	0.94	0.93	0.00	1.06	1.05
time (sec)	N/A	0.273	0.233	9.742	0.278	0.318	0.000	0.307	8.427

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	132	139	102	119	137	0	127	133
N.S.	1	1.03	1.09	0.80	0.93	1.07	0.00	0.99	1.04
time (sec)	N/A	0.259	0.205	10.106	0.446	0.308	0.000	0.294	8.362

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	101	127	109	97	106	0	98	111
N.S.	1	1.04	1.31	1.12	1.00	1.09	0.00	1.01	1.14
time (sec)	N/A	0.240	0.161	9.163	0.369	0.324	0.000	0.293	8.330

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	102	126	95	97	125	0	98	111
N.S.	1	1.04	1.29	0.97	0.99	1.28	0.00	1.00	1.13
time (sec)	N/A	0.219	0.157	9.077	0.285	0.334	0.000	0.304	8.355

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	86	104	80	86	90	0	87	100
N.S.	1	1.05	1.27	0.98	1.05	1.10	0.00	1.06	1.22
time (sec)	N/A	0.210	0.012	4.968	0.279	0.277	0.000	0.288	8.517

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	143	185	145	0	410	0	149	256
N.S.	1	1.04	1.35	1.06	0.00	2.99	0.00	1.09	1.87
time (sec)	N/A	0.278	0.272	4.933	0.000	1.002	0.000	0.314	8.408

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	169	202	211	0	187	0	163	382
N.S.	1	1.08	1.29	1.34	0.00	1.19	0.00	1.04	2.43
time (sec)	N/A	0.309	0.385	6.521	0.000	0.289	0.000	0.302	8.478

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	163	220	233	0	201	0	0	0
N.S.	1	1.06	1.43	1.51	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.273	0.682	6.395	0.000	0.259	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	205	179	0	452	0	0	0
N.S.	1	1.00	1.52	1.33	0.00	3.35	0.00	0.00	0.00
time (sec)	N/A	0.233	0.491	4.671	0.000	1.045	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	114	95	0	253	0	0	0
N.S.	1	1.00	1.30	1.08	0.00	2.88	0.00	0.00	0.00
time (sec)	N/A	0.178	0.019	4.628	0.000	1.711	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	141	126	0	307	0	0	0
N.S.	1	1.00	1.34	1.20	0.00	2.92	0.00	0.00	0.00
time (sec)	N/A	0.203	0.407	21.995	0.000	1.687	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	129	142	129	0	283	0	0	0
N.S.	1	1.04	1.15	1.04	0.00	2.28	0.00	0.00	0.00
time (sec)	N/A	0.253	0.438	22.078	0.000	1.758	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	155	154	135	0	320	0	0	0
N.S.	1	1.10	1.09	0.96	0.00	2.27	0.00	0.00	0.00
time (sec)	N/A	0.293	0.458	22.177	0.000	1.696	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	271	295	40	0	0	0	0	0	0
N.S.	1	1.09	0.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.503	10.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	254	269	26	0	0	0	0	0	0
N.S.	1	1.06	0.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.449	10.025	0.000	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	233	249	283	0	0	373	0	0	0
N.S.	1	1.07	1.21	0.00	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.409	0.124	0.000	0.000	1.702	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	270	270	67	0	0	0	0	0	0
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	11.054	0.000	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	289	288	76	0	0	0	0	0	0
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.471	11.056	0.000	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	129	137	100	119	142	0	127	135
N.S.	1	1.03	1.10	0.80	0.95	1.14	0.00	1.02	1.08
time (sec)	N/A	0.253	0.215	9.498	0.309	0.272	0.000	0.378	8.404

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	102	127	109	97	114	0	98	113
N.S.	1	1.04	1.30	1.11	0.99	1.16	0.00	1.00	1.15
time (sec)	N/A	0.238	0.178	9.509	0.279	0.270	0.000	0.357	8.435

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	99	126	95	97	130	0	98	113
N.S.	1	1.04	1.33	1.00	1.02	1.37	0.00	1.03	1.19
time (sec)	N/A	0.228	0.137	8.606	0.320	0.258	0.000	0.355	8.416

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	87	102	78	86	98	0	87	102
N.S.	1	1.05	1.23	0.94	1.04	1.18	0.00	1.05	1.23
time (sec)	N/A	0.215	0.122	4.615	0.303	0.293	0.000	0.328	8.558

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	143	185	145	0	182	0	149	344
N.S.	1	1.04	1.35	1.06	0.00	1.33	0.00	1.09	2.51
time (sec)	N/A	0.275	0.252	4.514	0.000	0.260	0.000	0.363	8.488

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	167	202	211	0	195	0	163	368
N.S.	1	1.06	1.28	1.34	0.00	1.23	0.00	1.03	2.33
time (sec)	N/A	0.307	0.330	7.280	0.000	0.278	0.000	0.365	8.557

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	162	222	234	0	232	0	0	0
N.S.	1	1.01	1.39	1.46	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.285	0.734	6.537	0.000	0.259	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	206	179	0	197	0	0	0
N.S.	1	1.00	1.48	1.29	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.245	0.487	4.668	0.000	0.267	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	114	96	0	283	0	0	0
N.S.	1	1.00	1.30	1.09	0.00	3.22	0.00	0.00	0.00
time (sec)	N/A	0.180	0.367	4.891	0.000	1.467	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	143	121	0	272	0	0	0
N.S.	1	1.00	1.39	1.17	0.00	2.64	0.00	0.00	0.00
time (sec)	N/A	0.209	0.397	22.118	0.000	1.425	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	129	141	127	0	312	0	0	0
N.S.	1	1.04	1.14	1.02	0.00	2.52	0.00	0.00	0.00
time (sec)	N/A	0.255	0.455	22.479	0.000	1.410	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	291	314	115	694	0	356	0	0	0
N.S.	1	1.08	0.40	2.38	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.416	10.129	18.211	0.000	1.723	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	294	316	26	0	0	0	0	0	0
N.S.	1	1.07	0.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.453	10.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	293	316	111	0	0	0	0	0	0
N.S.	1	1.08	0.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	10.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	294	316	120	695	0	396	0	0	0
N.S.	1	1.07	0.41	2.36	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.422	11.134	47.562	0.000	1.729	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	147	142	138	128	140	0	136	148
N.S.	1	1.04	1.01	0.98	0.91	0.99	0.00	0.96	1.05
time (sec)	N/A	0.287	0.346	8.934	0.286	0.299	0.000	0.302	8.538

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	134	137	133	119	159	0	120	139
N.S.	1	1.03	1.05	1.02	0.92	1.22	0.00	0.92	1.07
time (sec)	N/A	0.275	0.277	8.617	0.290	0.301	0.000	0.296	8.487

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	119	132	127	108	130	0	109	128
N.S.	1	1.03	1.15	1.10	0.94	1.13	0.00	0.95	1.11
time (sec)	N/A	0.257	0.261	8.368	0.272	0.329	0.000	0.299	8.475

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	107	126	122	97	148	0	98	117
N.S.	1	1.07	1.26	1.22	0.97	1.48	0.00	0.98	1.17
time (sec)	N/A	0.227	0.240	8.470	0.291	0.335	0.000	0.294	8.434

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	106	127	122	97	125	0	98	117
N.S.	1	1.06	1.27	1.22	0.97	1.25	0.00	0.98	1.17
time (sec)	N/A	0.220	0.208	8.416	0.278	0.298	0.000	0.310	8.445

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	166	198	210	0	226	0	160	253
N.S.	1	1.08	1.29	1.36	0.00	1.47	0.00	1.04	1.64
time (sec)	N/A	0.295	0.412	6.857	0.000	0.329	0.000	0.309	8.559

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	199	209	267	0	238	0	181	399
N.S.	1	1.14	1.19	1.53	0.00	1.36	0.00	1.03	2.28
time (sec)	N/A	0.323	0.515	6.549	0.000	0.347	0.000	0.288	8.603

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	186	227	292	0	271	0	0	0
N.S.	1	1.07	1.30	1.68	0.00	1.56	0.00	0.00	0.00
time (sec)	N/A	0.304	0.911	6.647	0.000	0.325	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	160	220	219	0	239	0	0	0
N.S.	1	1.05	1.44	1.43	0.00	1.56	0.00	0.00	0.00
time (sec)	N/A	0.261	0.668	5.545	0.000	0.334	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	108	139	141	0	318	0	0	0
N.S.	1	1.02	1.31	1.33	0.00	3.00	0.00	0.00	0.00
time (sec)	N/A	0.205	0.498	7.633	0.000	1.906	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	109	140	141	0	288	0	0	0
N.S.	1	1.03	1.32	1.33	0.00	2.72	0.00	0.00	0.00
time (sec)	N/A	0.196	0.481	6.833	0.000	1.775	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	126	148	156	0	340	0	0	0
N.S.	1	1.02	1.19	1.26	0.00	2.74	0.00	0.00	0.00
time (sec)	N/A	0.250	0.561	21.601	0.000	1.678	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	153	152	161	0	316	0	0	0
N.S.	1	1.06	1.06	1.12	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	0.290	0.582	22.168	0.000	1.694	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	178	157	168	0	351	0	0	0
N.S.	1	1.10	0.97	1.04	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	0.338	0.589	21.044	0.000	1.706	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	292	293	71	0	0	0	0	0	0
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.484	10.084	0.000	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	274	276	66	0	0	0	0	0	0
N.S.	1	1.01	0.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.433	10.072	0.000	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	274	277	66	0	0	0	0	0	0
N.S.	1	1.01	0.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.445	10.071	0.000	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	274	292	45	0	0	0	0	0	0
N.S.	1	1.07	0.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.489	10.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	292	293	76	0	0	0	0	0	0
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.478	11.080	0.000	0.000	0.000	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	308	308	79	0	0	0	0	0	0
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.527	11.103	0.000	0.000	0.000	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	268	308	283	0	325	0	379	442
N.S.	1	1.02	1.17	1.07	0.00	1.23	0.00	1.44	1.67
time (sec)	N/A	0.491	0.870	6.038	0.000	0.333	0.000	0.334	9.202

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	225	265	235	0	282	0	320	336
N.S.	1	1.02	1.20	1.07	0.00	1.28	0.00	1.45	1.53
time (sec)	N/A	0.400	0.626	4.731	0.000	0.395	0.000	0.299	8.894

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	207	226	205	0	222	0	276	298
N.S.	1	1.11	1.22	1.10	0.00	1.19	0.00	1.48	1.60
time (sec)	N/A	0.326	0.381	4.819	0.000	0.278	0.000	0.301	8.543

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	179	205	221	0	206	0	223	249
N.S.	1	1.13	1.29	1.39	0.00	1.30	0.00	1.40	1.57
time (sec)	N/A	0.300	0.284	4.711	0.000	0.277	0.000	0.304	8.457

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	249	312	258	0	276	0	311	1607
N.S.	1	1.01	1.27	1.05	0.00	1.12	0.00	1.26	6.53
time (sec)	N/A	0.355	0.603	4.830	0.000	0.276	0.000	0.553	8.824

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	332	351	357	0	429	0	351	1917
N.S.	1	0.98	1.03	1.05	0.00	1.26	0.00	1.03	5.64
time (sec)	N/A	0.448	0.971	5.234	0.000	0.344	0.000	0.510	14.134

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	345	413	428	0	472	0	455	2767
N.S.	1	0.93	1.12	1.16	0.00	1.28	0.00	1.23	7.48
time (sec)	N/A	0.485	1.275	5.110	0.000	1.120	0.000	0.564	16.499

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	357	527	445	0	494	0	0	0
N.S.	1	1.06	1.57	1.32	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.582	6.446	6.375	0.000	1.147	0.000	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	281	467	393	0	452	0	0	0
N.S.	1	1.02	1.69	1.42	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.420	4.475	4.861	0.000	0.335	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	240	423	338	0	330	0	0	0
N.S.	1	1.03	1.81	1.44	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	0.309	3.791	4.628	0.000	0.276	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	181	309	219	0	0	0	0	0
N.S.	1	1.08	1.84	1.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.274	2.405	4.765	0.000	0.000	0.000	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	223	333	245	0	0	0	0	0
N.S.	1	1.09	1.63	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.347	2.615	4.811	0.000	0.000	0.000	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	278	373	266	0	0	0	0	0
N.S.	1	1.08	1.45	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.451	3.043	4.839	0.000	0.000	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	352	419	306	0	0	0	0	0
N.S.	1	1.11	1.32	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.581	3.441	4.915	0.000	0.000	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	281	0	0	0	0	0	0
N.S.	1	1.00	4.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	7.703	0.000	0.000	0.000	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	240	0	0	0	0	0	0
N.S.	1	1.00	3.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	7.109	0.000	0.000	0.000	0.000	0.000	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	160	0	0	0	0	0	0
N.S.	1	1.00	2.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.193	0.040	0.000	0.000	0.000	0.000	0.000	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	327	0	0	0	0	0	0
N.S.	1	1.00	5.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	10.269	0.000	0.000	0.000	0.000	0.000	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	289	0	0	0	0	0	0
N.S.	1	1.00	4.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	10.383	0.000	0.000	0.000	0.000	0.000	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	270	310	272	0	455	0	409	490
N.S.	1	1.02	1.17	1.02	0.00	1.71	0.00	1.54	1.84
time (sec)	N/A	0.439	1.038	4.729	0.000	0.521	0.000	0.318	9.470

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	227	266	235	0	398	0	350	385
N.S.	1	1.02	1.19	1.05	0.00	1.78	0.00	1.57	1.73
time (sec)	N/A	0.392	0.623	4.756	0.000	0.486	0.000	0.335	9.303

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	209	228	206	0	353	0	306	302
N.S.	1	1.11	1.21	1.10	0.00	1.88	0.00	1.63	1.61
time (sec)	N/A	0.320	0.491	4.722	0.000	0.453	0.000	0.301	9.492

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	181	206	222	0	323	0	259	238
N.S.	1	1.12	1.27	1.37	0.00	1.99	0.00	1.60	1.47
time (sec)	N/A	0.293	0.287	4.721	0.000	0.519	0.000	0.307	9.317

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	248	310	258	0	425	0	341	1963
N.S.	1	1.01	1.27	1.05	0.00	1.73	0.00	1.39	8.01
time (sec)	N/A	0.343	0.663	4.627	0.000	0.538	0.000	0.627	9.273

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	336	352	292	0	1030	0	395	1908
N.S.	1	0.97	1.01	0.84	0.00	2.97	0.00	1.14	5.50
time (sec)	N/A	0.422	0.964	4.904	0.000	0.626	0.000	0.563	14.445

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	344	413	333	0	1151	0	488	2788
N.S.	1	0.93	1.12	0.90	0.00	3.11	0.00	1.32	7.54
time (sec)	N/A	0.472	1.326	5.029	0.000	1.356	0.000	0.590	19.141

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	527	356	0	1164	0	0	0
N.S.	1	1.00	1.58	1.07	0.00	3.49	0.00	0.00	0.00
time (sec)	N/A	0.499	7.293	5.320	0.000	1.589	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	273	466	391	0	1091	0	0	0
N.S.	1	1.00	1.71	1.44	0.00	4.01	0.00	0.00	0.00
time (sec)	N/A	0.366	5.055	4.818	0.000	0.600	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	237	423	339	0	469	0	0	0
N.S.	1	1.02	1.82	1.45	0.00	2.01	0.00	0.00	0.00
time (sec)	N/A	0.290	0.371	4.775	0.000	0.500	0.000	0.000	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	182	309	222	0	0	0	0	0
N.S.	1	1.08	1.83	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	2.591	5.075	0.000	0.000	0.000	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	225	334	245	0	0	0	0	0
N.S.	1	1.09	1.62	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.332	2.861	4.846	0.000	0.000	0.000	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	279	374	275	0	0	0	0	0
N.S.	1	1.09	1.46	1.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.444	3.252	4.855	0.000	0.000	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	354	422	307	0	0	0	0	0
N.S.	1	1.11	1.32	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.565	3.916	4.917	0.000	0.000	0.000	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	181	0	0	0	0	0	0
N.S.	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	8.420	0.000	0.000	0.000	0.000	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.212	8.028	0.000	0.000	0.000	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	10.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	138	0	0	0	0	0	0
N.S.	1	1.00	2.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	10.107	0.000	0.000	0.000	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	181	0	0	0	0	0	0
N.S.	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	10.161	0.000	0.000	0.000	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	255	308	260	0	369	0	394	477
N.S.	1	1.02	1.23	1.04	0.00	1.47	0.00	1.57	1.90
time (sec)	N/A	0.459	0.878	4.888	0.000	0.289	0.000	0.317	8.912

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	239	260	228	0	298	0	348	348
N.S.	1	1.13	1.23	1.08	0.00	1.41	0.00	1.65	1.65
time (sec)	N/A	0.343	0.545	4.895	0.000	0.289	0.000	0.300	8.912

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	211	221	203	0	246	0	297	304
N.S.	1	1.13	1.18	1.09	0.00	1.32	0.00	1.59	1.63
time (sec)	N/A	0.323	0.411	5.010	0.000	0.288	0.000	0.315	8.555

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	277	333	291	0	320	0	357	796
N.S.	1	1.06	1.28	1.11	0.00	1.23	0.00	1.37	3.05
time (sec)	N/A	0.395	0.905	4.773	0.000	0.288	0.000	0.551	9.915

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	383	353	364	0	383	0	394	2047
N.S.	1	0.96	0.88	0.91	0.00	0.96	0.00	0.99	5.13
time (sec)	N/A	0.474	1.175	5.042	0.000	0.428	0.000	0.576	14.720

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	440	392	413	454	0	503	0	481	2841
N.S.	1	0.89	0.94	1.03	0.00	1.14	0.00	1.09	6.46
time (sec)	N/A	0.530	1.573	5.024	0.000	1.166	0.000	0.561	16.695

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	349	526	472	0	550	0	0	0
N.S.	1	1.04	1.57	1.41	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	0.617	9.383	5.109	0.000	1.183	0.000	0.000	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	282	469	385	0	396	0	0	0
N.S.	1	1.02	1.69	1.39	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.442	6.538	5.190	0.000	0.406	0.000	0.000	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	261	457	334	0	0	0	0	0
N.S.	1	1.03	1.80	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.394	5.247	4.946	0.000	0.000	0.000	0.000	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	223	328	243	0	0	0	0	0
N.S.	1	1.11	1.63	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.353	3.056	5.025	0.000	0.000	0.000	0.000	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	273	369	259	0	0	0	0	0
N.S.	1	1.09	1.48	1.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.475	3.407	4.845	0.000	0.000	0.000	0.000	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	348	419	294	0	0	0	0	0
N.S.	1	1.09	1.32	0.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.598	3.731	5.048	0.000	0.000	0.000	0.000	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	436	478	336	0	0	0	0	0
N.S.	1	1.11	1.22	0.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.784	4.753	5.061	0.000	0.000	0.000	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	343	0	0	0	0	0	0
N.S.	1	1.00	5.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	9.839	0.000	0.000	0.000	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	280	0	0	0	0	0	0
N.S.	1	1.00	4.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	9.288	0.000	0.000	0.000	0.000	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	346	0	0	0	0	0	0
N.S.	1	1.00	5.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.191	0.299	0.000	0.000	0.000	0.000	0.000	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	341	0	0	0	0	0	0
N.S.	1	1.00	5.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	10.359	0.000	0.000	0.000	0.000	0.000	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	286	0	0	0	0	0	0
N.S.	1	1.00	4.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	10.384	0.000	0.000	0.000	0.000	0.000	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	294	307	275	0	1004	0	454	438
N.S.	1	1.01	1.06	0.95	0.00	3.46	0.00	1.57	1.51
time (sec)	N/A	0.467	1.008	4.803	0.000	0.298	0.000	0.307	8.955

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	248	263	234	0	873	0	371	339
N.S.	1	1.02	1.08	0.96	0.00	3.58	0.00	1.52	1.39
time (sec)	N/A	0.403	0.657	4.731	0.000	0.314	0.000	0.299	8.986

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	207	231	204	0	768	0	313	267
N.S.	1	1.02	1.14	1.00	0.00	3.78	0.00	1.54	1.32
time (sec)	N/A	0.367	0.515	5.059	0.000	0.320	0.000	0.299	9.034

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	177	203	186	0	667	0	257	219
N.S.	1	1.05	1.21	1.11	0.00	3.97	0.00	1.53	1.30
time (sec)	N/A	0.291	0.285	4.852	0.000	0.272	0.000	0.307	8.935

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	149	162	152	0	592	0	226	208
N.S.	1	1.03	1.12	1.05	0.00	4.08	0.00	1.56	1.43
time (sec)	N/A	0.261	0.172	4.500	0.000	0.265	0.000	0.286	8.849

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	241	309	248	0	628	0	326	702
N.S.	1	0.99	1.27	1.02	0.00	2.57	0.00	1.34	2.88
time (sec)	N/A	0.337	0.678	4.619	0.000	0.280	0.000	0.523	10.226

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	285	353	284	0	837	0	378	1929
N.S.	1	0.96	1.19	0.96	0.00	2.83	0.00	1.28	6.52
time (sec)	N/A	0.383	1.221	4.839	0.000	0.335	0.000	0.530	15.388

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	466	311	0	826	0	0	0
N.S.	1	1.00	1.71	1.14	0.00	3.03	0.00	0.00	0.00
time (sec)	N/A	0.370	4.821	4.969	0.000	0.342	0.000	0.000	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	229	423	286	0	761	0	0	0
N.S.	1	0.98	1.82	1.23	0.00	3.27	0.00	0.00	0.00
time (sec)	N/A	0.292	3.104	4.639	0.000	0.281	0.000	0.000	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	255	171	0	0	0	0	0
N.S.	1	1.00	1.72	1.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	0.074	4.543	0.000	0.000	0.000	0.000	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	179	314	216	0	0	0	0	0
N.S.	1	1.02	1.78	1.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.257	2.136	4.859	0.000	0.000	0.000	0.000	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	226	340	231	0	0	0	0	0
N.S.	1	1.06	1.59	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	2.622	4.986	0.000	0.000	0.000	0.000	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	280	374	270	0	0	0	0	0
N.S.	1	1.07	1.43	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.448	3.586	4.983	0.000	0.000	0.000	0.000	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	144	0	0	0	0	0	0
N.S.	1	1.00	2.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	8.461	0.000	0.000	0.000	0.000	0.000	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.212	7.949	0.000	0.000	0.000	0.000	0.000	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	10.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0	0
N.S.	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	10.107	0.000	0.000	0.000	0.000	0.000	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	183	0	0	0	0	0	0
N.S.	1	1.00	2.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	10.176	0.000	0.000	0.000	0.000	0.000	0.000

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	246	264	234	0	1322	0	372	331
N.S.	1	1.02	1.10	0.97	0.00	5.49	0.00	1.54	1.37
time (sec)	N/A	0.402	0.777	4.773	0.000	0.346	0.000	0.297	8.968

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	205	231	202	0	1156	0	312	292
N.S.	1	1.02	1.15	1.00	0.00	5.75	0.00	1.55	1.45
time (sec)	N/A	0.356	0.558	4.935	0.000	0.318	0.000	0.322	8.671

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	175	202	186	0	1060	0	253	232
N.S.	1	1.06	1.22	1.13	0.00	6.42	0.00	1.53	1.41
time (sec)	N/A	0.292	0.277	5.042	0.000	0.318	0.000	0.294	8.796

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	149	162	152	0	927	0	221	213
N.S.	1	1.03	1.12	1.05	0.00	6.39	0.00	1.52	1.47
time (sec)	N/A	0.274	0.183	4.503	0.000	0.330	0.000	0.296	9.165

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	242	308	249	0	472	0	321	1413
N.S.	1	0.99	1.26	1.02	0.00	1.93	0.00	1.31	5.77
time (sec)	N/A	0.346	0.754	4.650	0.000	0.320	0.000	0.509	9.151

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	287	355	297	0	562	0	377	1959
N.S.	1	0.96	1.19	0.99	0.00	1.88	0.00	1.26	6.55
time (sec)	N/A	0.393	1.065	4.902	0.000	0.793	0.000	0.536	15.747

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	290	471	329	0	558	0	0	0
N.S.	1	1.04	1.69	1.18	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.416	5.260	4.955	0.000	0.654	0.000	0.000	0.000

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	232	423	287	0	530	0	0	0
N.S.	1	0.99	1.81	1.23	0.00	2.26	0.00	0.00	0.00
time (sec)	N/A	0.303	3.470	4.662	0.000	0.276	0.000	0.000	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	255	169	0	0	0	0	0
N.S.	1	1.00	1.71	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	1.886	4.540	0.000	0.000	0.000	0.000	0.000

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	178	308	211	0	0	0	0	0
N.S.	1	1.03	1.78	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	2.186	4.761	0.000	0.000	0.000	0.000	0.000

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	225	340	231	0	0	0	0	0
N.S.	1	1.05	1.58	1.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.351	2.805	4.884	0.000	0.000	0.000	0.000	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	249	0	0	0	0	0	0
N.S.	1	1.00	3.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.212	9.487	0.000	0.000	0.000	0.000	0.000	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	8.959	0.000	0.000	0.000	0.000	0.000	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	338	0	0	0	0	0	0
N.S.	1	1.00	5.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	10.276	0.000	0.000	0.000	0.000	0.000	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	351	349	314	0	1300	0	431	564
N.S.	1	1.01	1.01	0.90	0.00	3.75	0.00	1.24	1.63
time (sec)	N/A	0.534	1.492	4.796	0.000	0.363	0.000	0.315	9.133

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	258	298	266	0	1141	0	372	493
N.S.	1	1.02	1.18	1.05	0.00	4.51	0.00	1.47	1.95
time (sec)	N/A	0.448	0.971	4.716	0.000	0.359	0.000	0.295	9.480

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	207	254	233	0	1004	0	325	449
N.S.	1	1.02	1.25	1.15	0.00	4.95	0.00	1.60	2.21
time (sec)	N/A	0.373	0.751	4.687	0.000	0.339	0.000	0.307	9.625

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	189	227	224	0	872	0	301	412
N.S.	1	1.09	1.30	1.29	0.00	5.01	0.00	1.73	2.37
time (sec)	N/A	0.305	0.556	4.503	0.000	0.344	0.000	0.299	9.516

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	186	220	197	0	262	0	285	389
N.S.	1	1.11	1.32	1.18	0.00	1.57	0.00	1.71	2.33
time (sec)	N/A	0.300	0.372	4.639	0.000	0.315	0.000	0.304	9.271

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	293	350	304	0	975	0	389	3804
N.S.	1	1.08	1.29	1.12	0.00	3.60	0.00	1.44	14.04
time (sec)	N/A	0.405	1.509	4.738	0.000	0.344	0.000	0.525	9.707

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	344	392	349	0	1386	0	481	5875
N.S.	1	0.96	1.10	0.98	0.00	3.88	0.00	1.35	16.46
time (sec)	N/A	0.442	1.690	4.998	0.000	0.817	0.000	0.520	11.004

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	337	506	494	0	1329	0	0	0
N.S.	1	1.05	1.57	1.53	0.00	4.13	0.00	0.00	0.00
time (sec)	N/A	0.512	10.815	5.414	0.000	0.803	0.000	0.000	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	283	466	343	0	1127	0	0	0
N.S.	1	1.09	1.79	1.32	0.00	4.33	0.00	0.00	0.00
time (sec)	N/A	0.374	6.681	4.772	0.000	0.309	0.000	0.000	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	185	322	217	0	0	0	0	0
N.S.	1	1.08	1.87	1.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	2.836	4.569	0.000	0.000	0.000	0.000	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	189	328	243	0	0	0	0	0
N.S.	1	1.06	1.83	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	0.548	4.579	0.000	0.000	0.000	0.000	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	234	358	257	0	0	0	0	0
N.S.	1	1.02	1.56	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.371	3.739	4.764	0.000	0.000	0.000	0.000	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	289	402	302	0	0	0	0	0
N.S.	1	1.01	1.40	1.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.454	5.697	4.818	0.000	0.000	0.000	0.000	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	361	459	356	0	0	0	0	0
N.S.	1	1.03	1.31	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.592	8.478	4.893	0.000	0.000	0.000	0.000	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	194	0	0	0	0	0	0
N.S.	1	1.00	2.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	10.233	0.000	0.000	0.000	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	144	0	0	0	0	0	0
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	10.129	0.000	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	129	0	0	0	0	0	0
N.S.	1	1.00	1.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	10.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	141	0	0	0	0	0	0
N.S.	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	10.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	193	0	0	0	0	0	0
N.S.	1	1.00	2.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	10.210	0.000	0.000	0.000	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	264	0	0	0	0	0	0
N.S.	1	1.00	3.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	10.317	0.000	0.000	0.000	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	87	82	78	84	100	0	88	88
N.S.	1	0.97	0.91	0.87	0.93	1.11	0.00	0.98	0.98
time (sec)	N/A	0.255	0.054	4.564	0.188	2.693	0.000	0.282	10.040

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	66	66	65	68	72	0	70	68
N.S.	1	0.94	0.94	0.93	0.97	1.03	0.00	1.00	0.97
time (sec)	N/A	0.231	0.039	4.534	0.184	1.033	0.000	0.284	9.908

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	52	43	43	49	42	0	51	51
N.S.	1	0.98	0.81	0.81	0.92	0.79	0.00	0.96	0.96
time (sec)	N/A	0.214	0.027	4.515	0.235	0.480	0.000	0.280	9.505

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	44	31	32	41	31	138	51	1012
N.S.	1	0.98	0.69	0.71	0.91	0.69	3.07	1.13	22.49
time (sec)	N/A	0.177	0.023	4.522	0.190	0.263	0.908	0.271	9.511

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	63	54	55	61	54	0	73	58
N.S.	1	1.02	0.87	0.89	0.98	0.87	0.00	1.18	0.94
time (sec)	N/A	0.225	0.036	4.531	0.191	1.153	0.000	0.275	10.198

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	86	88	82	87	99	0	112	87
N.S.	1	0.99	1.01	0.94	1.00	1.14	0.00	1.29	1.00
time (sec)	N/A	0.265	0.043	4.823	0.187	4.728	0.000	0.276	11.381

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	132	104	98	100	576	0	112	532
N.S.	1	1.18	0.93	0.88	0.89	5.14	0.00	1.00	4.75
time (sec)	N/A	0.340	0.164	4.588	0.281	1.494	0.000	0.284	10.768

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	99	82	81	80	416	0	80	518
N.S.	1	1.08	0.89	0.88	0.87	4.52	0.00	0.87	5.63
time (sec)	N/A	0.245	0.146	4.701	0.298	0.468	0.000	0.298	10.545

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	78	66	60	59	325	0	59	379
N.S.	1	0.99	0.84	0.76	0.75	4.11	0.00	0.75	4.80
time (sec)	N/A	0.206	0.042	4.801	0.263	0.303	0.000	0.298	10.433

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	78	66	60	59	325	0	59	399
N.S.	1	0.99	0.84	0.76	0.75	4.11	0.00	0.75	5.05
time (sec)	N/A	0.196	0.050	4.529	0.284	0.303	0.000	0.277	10.411

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	100	169	81	80	432	0	80	354
N.S.	1	1.09	1.84	0.88	0.87	4.70	0.00	0.87	3.85
time (sec)	N/A	0.244	0.225	4.581	0.269	0.531	0.000	0.303	10.189

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	133	193	101	101	592	0	103	535
N.S.	1	1.19	1.72	0.90	0.90	5.29	0.00	0.92	4.78
time (sec)	N/A	0.331	0.213	4.589	0.274	1.989	0.000	0.278	9.925

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	449	377	234	375	1196	0	469	6361
N.S.	1	0.98	0.82	0.51	0.82	2.62	0.00	1.03	13.92
time (sec)	N/A	0.728	0.192	4.559	0.329	0.405	0.000	0.288	9.940

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	426	340	226	363	1427	0	453	2553
N.S.	1	0.95	0.76	0.50	0.81	3.18	0.00	1.01	5.69
time (sec)	N/A	0.627	0.096	4.563	0.286	0.314	0.000	0.310	10.089

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	426	340	218	361	1067	0	437	5889
N.S.	1	0.95	0.76	0.49	0.80	2.38	0.00	0.97	13.12
time (sec)	N/A	0.618	0.098	4.546	0.283	0.276	0.000	0.297	10.268

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	426	340	218	363	1331	0	477	6633
N.S.	1	0.95	0.76	0.49	0.81	2.96	0.00	1.06	14.77
time (sec)	N/A	0.630	0.134	4.558	0.277	0.273	0.000	0.290	9.985

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	426	340	226	365	1171	0	437	6153
N.S.	1	0.95	0.76	0.50	0.81	2.61	0.00	0.97	13.70
time (sec)	N/A	0.611	0.119	4.444	0.278	0.355	0.000	0.278	10.389

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	477	385	237	384	1461	0	488	5962
N.S.	1	1.04	0.84	0.52	0.83	3.18	0.00	1.06	12.96
time (sec)	N/A	0.644	0.206	4.683	0.283	0.499	0.000	0.283	10.357

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	462	454	406	241	390	1255	0	472	7459
N.S.	1	0.98	0.88	0.52	0.84	2.72	0.00	1.02	16.15
time (sec)	N/A	0.706	0.273	4.584	0.281	2.937	0.000	0.289	10.652

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	522	428	261	405	1526	0	483	4547
N.S.	1	1.09	0.89	0.54	0.85	3.19	0.00	1.01	9.49
time (sec)	N/A	0.692	0.298	4.996	0.281	4.974	0.000	0.280	10.327

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	98	88	84	0	195	122	96	87
N.S.	1	1.05	0.95	0.90	0.00	2.10	1.31	1.03	0.94
time (sec)	N/A	0.232	0.204	6.012	0.000	0.269	5.091	0.276	9.049

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	128	142	124	0	714	0	0	0
N.S.	1	1.07	1.18	1.03	0.00	5.95	0.00	0.00	0.00
time (sec)	N/A	0.307	1.262	5.291	0.000	0.309	0.000	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	69	62	0	156	92	66	54
N.S.	1	1.00	0.99	0.89	0.00	2.23	1.31	0.94	0.77
time (sec)	N/A	0.216	0.128	4.862	0.000	0.247	2.907	0.276	9.043

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	89	107	77	0	612	0	0	0
N.S.	1	0.98	1.18	0.85	0.00	6.73	0.00	0.00	0.00
time (sec)	N/A	0.236	0.399	5.108	0.000	0.274	0.000	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	105	0	383	158	79	199
N.S.	1	1.00	0.95	1.24	0.00	4.51	1.86	0.93	2.34
time (sec)	N/A	0.227	0.170	4.867	0.000	0.273	4.431	0.269	9.236

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	96	69	0	281	0	121	0
N.S.	1	1.00	1.26	0.91	0.00	3.70	0.00	1.59	0.00
time (sec)	N/A	0.233	0.451	5.447	0.000	0.264	0.000	0.772	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	121	107	96	0	513	0	107	269
N.S.	1	1.05	0.93	0.83	0.00	4.46	0.00	0.93	2.34
time (sec)	N/A	0.269	0.416	4.960	0.000	0.266	0.000	0.263	9.582

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	118	118	92	0	329	0	225	0
N.S.	1	1.07	1.07	0.84	0.00	2.99	0.00	2.05	0.00
time (sec)	N/A	0.303	0.670	5.678	0.000	0.284	0.000	1.106	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	857	1069	141	332	0	0	0	0	0
N.S.	1	1.25	0.16	0.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.778	10.142	7.130	0.000	0.000	0.000	0.000	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	700	1002	241	303	0	0	0	0	0
N.S.	1	1.43	0.34	0.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.509	10.437	6.054	0.000	0.000	0.000	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	786	1096	65	299	0	0	0	0	0
N.S.	1	1.39	0.08	0.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.593	10.039	4.677	0.000	0.000	0.000	0.000	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	679	964	161	273	0	0	0	0	0
N.S.	1	1.42	0.24	0.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.374	10.173	4.929	0.000	0.000	0.000	0.000	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	809	1021	138	318	0	0	0	0	0
N.S.	1	1.26	0.17	0.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.504	10.098	7.401	0.000	0.000	0.000	0.000	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	703	988	333	294	0	0	0	0	0
N.S.	1	1.41	0.47	0.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.452	10.286	6.798	0.000	0.000	0.000	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	70	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	11.079	0.000	0.000	0.000	0.000	0.000	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	70	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	11.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	11.047	0.000	0.000	0.000	0.000	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	143	0	0	0	0	0	0
N.S.	1	1.00	2.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.276	11.148	0.000	0.000	0.000	0.000	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	91	91	0	289	0	106	102
N.S.	1	1.00	0.88	0.88	0.00	2.78	0.00	1.02	0.98
time (sec)	N/A	0.275	0.293	5.060	0.000	0.312	0.000	0.277	9.271

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	72	0	205	0	64	58
N.S.	1	1.00	0.99	0.97	0.00	2.77	0.00	0.86	0.78
time (sec)	N/A	0.207	0.167	4.917	0.000	0.307	0.000	0.268	9.144

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	39	0	130	87	40	40
N.S.	1	1.00	1.00	0.76	0.00	2.55	1.71	0.78	0.78
time (sec)	N/A	0.190	0.070	4.959	0.000	0.325	5.884	0.284	9.046

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	78	0	431	114	71	652
N.S.	1	1.00	0.94	0.92	0.00	5.07	1.34	0.84	7.67
time (sec)	N/A	0.223	0.191	4.896	0.000	0.329	5.308	0.282	9.198

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	127	109	92	0	565	0	104	396
N.S.	1	1.09	0.93	0.79	0.00	4.83	0.00	0.89	3.38
time (sec)	N/A	0.269	0.439	4.979	0.000	0.363	0.000	0.266	9.649

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	134	136	152	0	739	0	0	0
N.S.	1	1.09	1.11	1.24	0.00	6.01	0.00	0.00	0.00
time (sec)	N/A	0.291	0.946	5.620	0.000	0.338	0.000	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	90	108	85	0	632	0	0	0
N.S.	1	0.99	1.19	0.93	0.00	6.95	0.00	0.00	0.00
time (sec)	N/A	0.239	0.530	5.210	0.000	0.312	0.000	0.000	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	74	42	0	245	0	72	0
N.S.	1	1.00	1.37	0.78	0.00	4.54	0.00	1.33	0.00
time (sec)	N/A	0.183	0.789	5.126	0.000	0.300	0.000	0.278	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	100	80	0	332	0	116	0
N.S.	1	1.00	1.25	1.00	0.00	4.15	0.00	1.45	0.00
time (sec)	N/A	0.226	0.571	5.349	0.000	0.300	0.000	0.310	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	127	121	103	0	418	0	205	0
N.S.	1	1.10	1.05	0.90	0.00	3.63	0.00	1.78	0.00
time (sec)	N/A	0.294	1.084	6.302	0.000	0.309	0.000	1.158	0.000

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	872	1002	249	298	0	0	0	0	0
N.S.	1	1.15	0.29	0.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.444	10.346	6.114	0.000	0.000	0.000	0.000	0.000

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	638	958	65	265	0	0	0	0	0
N.S.	1	1.50	0.10	0.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.355	10.056	4.722	0.000	0.000	0.000	0.000	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	638	860	161	191	0	0	0	0	0
N.S.	1	1.35	0.25	0.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.234	10.076	4.687	0.000	0.000	0.000	0.000	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	677	988	337	288	0	0	0	0	0
N.S.	1	1.46	0.50	0.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.387	10.288	6.319	0.000	0.000	0.000	0.000	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	804	1089	65	292	0	0	0	0	0
N.S.	1	1.35	0.08	0.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.545	10.055	4.760	0.000	0.000	0.000	0.000	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	656	865	65	191	0	0	0	0	0
N.S.	1	1.32	0.10	0.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.193	10.055	4.798	0.000	0.000	0.000	0.000	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	833	999	141	310	0	0	0	0	0
N.S.	1	1.20	0.17	0.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.518	10.121	6.489	0.000	0.000	0.000	0.000	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	182	175	171	0	622	0	180	186
N.S.	1	1.04	1.00	0.98	0.00	3.55	0.00	1.03	1.06
time (sec)	N/A	0.299	0.679	5.152	0.000	0.623	0.000	0.295	9.455

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	144	130	133	0	475	0	134	144
N.S.	1	1.17	1.06	1.08	0.00	3.86	0.00	1.09	1.17
time (sec)	N/A	0.282	0.492	5.176	0.000	0.642	0.000	0.297	9.834

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	98	100	83	0	348	0	116	95
N.S.	1	0.99	1.01	0.84	0.00	3.52	0.00	1.17	0.96
time (sec)	N/A	0.237	0.283	5.224	0.000	0.911	0.000	0.277	9.690

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	86	86	90	0	302	0	93	84
N.S.	1	0.99	0.99	1.03	0.00	3.47	0.00	1.07	0.97
time (sec)	N/A	0.218	0.281	4.886	0.000	0.353	0.000	0.278	9.537

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	155	124	146	0	862	0	139	3017
N.S.	1	1.17	0.94	1.11	0.00	6.53	0.00	1.05	22.86
time (sec)	N/A	0.286	0.529	5.007	0.000	0.538	0.000	0.281	10.723

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	213	163	191	0	1236	0	257	3822
N.S.	1	1.15	0.88	1.03	0.00	6.68	0.00	1.39	20.66
time (sec)	N/A	0.366	0.975	5.134	0.000	0.543	0.000	0.308	11.896

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	213	189	164	0	1386	0	333	0
N.S.	1	1.12	0.99	0.86	0.00	7.26	0.00	1.74	0.00
time (sec)	N/A	0.456	2.768	6.404	0.000	1.448	0.000	0.388	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	166	153	117	0	1077	0	298	0
N.S.	1	1.18	1.09	0.83	0.00	7.64	0.00	2.11	0.00
time (sec)	N/A	0.319	1.957	5.705	0.000	0.522	0.000	0.373	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	97	112	81	0	426	0	244	0
N.S.	1	1.04	1.20	0.87	0.00	4.58	0.00	2.62	0.00
time (sec)	N/A	0.239	1.161	5.564	0.000	0.448	0.000	0.858	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	108	124	90	0	467	0	237	0
N.S.	1	1.04	1.19	0.87	0.00	4.49	0.00	2.28	0.00
time (sec)	N/A	0.244	1.454	5.847	0.000	0.661	0.000	0.318	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	157	157	112	0	612	0	418	0
N.S.	1	1.05	1.05	0.75	0.00	4.11	0.00	2.81	0.00
time (sec)	N/A	0.325	1.500	6.038	0.000	0.720	0.000	0.891	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	212	201	134	0	760	0	395	0
N.S.	1	1.02	0.97	0.64	0.00	3.65	0.00	1.90	0.00
time (sec)	N/A	0.415	2.497	6.751	0.000	0.737	0.000	0.942	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	996	1032	253	337	0	0	0	0	0
N.S.	1	1.04	0.25	0.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.510	10.297	5.311	0.000	0.000	0.000	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	908	1013	238	324	0	0	0	0	0
N.S.	1	1.12	0.26	0.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.503	10.229	5.204	0.000	0.000	0.000	0.000	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	983	1016	392	333	0	0	0	0	0
N.S.	1	1.03	0.40	0.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.439	10.276	4.959	0.000	0.000	0.000	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1046	1074	408	364	0	0	0	0	0
N.S.	1	1.03	0.39	0.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.587	10.549	8.392	0.000	0.000	0.000	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1146	1085	162	353	0	0	0	0	0
N.S.	1	0.95	0.14	0.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.553	10.391	5.024	0.000	0.000	0.000	0.000	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1144	1071	172	359	0	0	0	0	0
N.S.	1	0.94	0.15	0.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.426	10.175	4.980	0.000	0.000	0.000	0.000	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1225	1148	226	392	0	0	0	0	0
N.S.	1	0.94	0.18	0.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.539	10.283	8.135	0.000	0.000	0.000	0.000	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	200	195	164	0	0	0	182	0	0
N.S.	1	0.98	0.82	0.00	0.00	0.00	0.91	0.00	0.00
time (sec)	N/A	0.368	11.133	0.000	0.000	0.000	6.383	0.000	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	117	110	0	0	0	117	0	0
N.S.	1	0.95	0.89	0.00	0.00	0.00	0.95	0.00	0.00
time (sec)	N/A	0.252	2.272	0.000	0.000	0.000	2.194	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	66	0	0	0	56	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.82	0.00	0.00
time (sec)	N/A	0.188	0.785	0.000	0.000	0.000	0.601	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	125	0	0	0	0	0	0
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	5.606	0.000	0.000	0.000	0.000	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	179	0	0	0	0	0	0
N.S.	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	11.181	0.000	0.000	0.000	0.000	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	11.109	0.000	0.000	0.000	0.000	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	198	200	167	0	0	0	0	0	0
N.S.	1	1.01	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.373	11.186	0.000	0.000	0.000	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	113	0	0	0	117	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.89	0.00	0.00
time (sec)	N/A	0.253	5.577	0.000	0.000	0.000	17.871	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	69	0	0	0	56	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	0.00
time (sec)	N/A	0.190	1.934	0.000	0.000	0.000	0.734	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	169	0	0	0	0	0	0
N.S.	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	11.188	0.000	0.000	0.000	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	11.112	0.000	0.000	0.000	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	11.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	91	91	0	288	0	106	103
N.S.	1	1.00	0.88	0.88	0.00	2.77	0.00	1.02	0.99
time (sec)	N/A	0.267	0.292	7.314	0.000	0.527	0.000	0.292	9.193

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	72	0	205	0	64	58
N.S.	1	1.00	0.99	0.97	0.00	2.77	0.00	0.86	0.78
time (sec)	N/A	0.205	0.155	5.396	0.000	0.434	0.000	0.286	9.096

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	39	0	130	87	40	40
N.S.	1	1.00	1.00	0.76	0.00	2.55	1.71	0.78	0.78
time (sec)	N/A	0.189	0.073	5.112	0.000	0.320	8.628	0.298	9.120

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	78	0	431	114	71	652
N.S.	1	1.00	0.94	0.92	0.00	5.07	1.34	0.84	7.67
time (sec)	N/A	0.220	0.193	5.272	0.000	0.305	6.994	0.299	9.297

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	127	109	92	0	565	0	104	396
N.S.	1	1.09	0.93	0.79	0.00	4.83	0.00	0.89	3.38
time (sec)	N/A	0.265	0.432	5.389	0.000	0.330	0.000	0.288	9.853

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	134	136	152	0	739	0	237	0
N.S.	1	1.09	1.11	1.24	0.00	6.01	0.00	1.93	0.00
time (sec)	N/A	0.291	1.410	9.611	0.000	0.825	0.000	0.434	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	90	108	85	0	632	0	156	0
N.S.	1	0.99	1.19	0.93	0.00	6.95	0.00	1.71	0.00
time (sec)	N/A	0.231	0.697	7.217	0.000	0.608	0.000	0.303	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	74	42	0	245	0	72	0
N.S.	1	1.00	1.37	0.78	0.00	4.54	0.00	1.33	0.00
time (sec)	N/A	0.186	0.859	6.881	0.000	0.733	0.000	0.300	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	100	80	0	332	0	0	0
N.S.	1	1.00	1.25	1.00	0.00	4.15	0.00	0.00	0.00
time (sec)	N/A	0.224	0.720	8.124	0.000	0.372	0.000	0.000	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	127	121	103	0	416	0	0	0
N.S.	1	1.10	1.05	0.90	0.00	3.62	0.00	0.00	0.00
time (sec)	N/A	0.297	1.490	10.953	0.000	0.389	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	10.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	10.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	10.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	10.189	0.000	0.000	0.000	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0	0
N.S.	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	10.122	0.000	0.000	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	10.122	0.000	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	10.134	0.000	0.000	0.000	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	144	130	133	0	475	0	134	144
N.S.	1	1.17	1.06	1.08	0.00	3.86	0.00	1.09	1.17
time (sec)	N/A	0.273	0.535	5.356	0.000	0.641	0.000	0.276	9.466

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	98	100	83	0	348	0	116	95
N.S.	1	0.99	1.01	0.84	0.00	3.52	0.00	1.17	0.96
time (sec)	N/A	0.225	0.276	5.325	0.000	0.378	0.000	0.279	9.795

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	86	86	90	0	302	0	93	84
N.S.	1	0.99	0.99	1.03	0.00	3.47	0.00	1.07	0.97
time (sec)	N/A	0.210	0.251	5.159	0.000	0.411	0.000	0.273	9.241

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	155	124	146	0	862	0	139	3025
N.S.	1	1.17	0.94	1.11	0.00	6.53	0.00	1.05	22.92
time (sec)	N/A	0.281	0.521	5.352	0.000	0.480	0.000	0.281	10.306

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	213	163	189	0	1236	0	257	3860
N.S.	1	1.15	0.88	1.02	0.00	6.68	0.00	1.39	20.86
time (sec)	N/A	0.352	1.007	5.446	0.000	0.478	0.000	0.284	11.632

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	166	153	117	0	1077	0	343	0
N.S.	1	1.18	1.09	0.83	0.00	7.64	0.00	2.43	0.00
time (sec)	N/A	0.312	2.987	10.579	0.000	0.848	0.000	0.296	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	97	112	81	0	426	0	0	0
N.S.	1	1.04	1.20	0.87	0.00	4.58	0.00	0.00	0.00
time (sec)	N/A	0.230	1.433	9.337	0.000	0.361	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	108	124	90	0	467	0	237	0
N.S.	1	1.04	1.19	0.87	0.00	4.49	0.00	2.28	0.00
time (sec)	N/A	0.241	1.599	9.441	0.000	0.506	0.000	0.283	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	157	157	112	0	612	0	0	0
N.S.	1	1.05	1.05	0.75	0.00	4.11	0.00	0.00	0.00
time (sec)	N/A	0.321	1.789	12.056	0.000	0.515	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	212	201	134	0	760	0	0	0
N.S.	1	1.02	0.97	0.64	0.00	3.65	0.00	0.00	0.00
time (sec)	N/A	0.406	3.176	18.346	0.000	0.686	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	169	0	0	0	0	0	0
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.206	10.201	0.000	0.000	0.000	0.000	0.000	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	168	0	0	0	0	0	0
N.S.	1	1.00	2.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	10.193	0.000	0.000	0.000	0.000	0.000	0.000

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	172	0	0	0	0	0	0
N.S.	1	1.00	2.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	10.175	0.000	0.000	0.000	0.000	0.000	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	329	0	0	0	0	0	0
N.S.	1	1.00	5.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	10.297	0.000	0.000	0.000	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	226	0	0	0	0	0	0
N.S.	1	1.00	3.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.206	10.297	0.000	0.000	0.000	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	226	0	0	0	0	0	0
N.S.	1	1.00	3.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	10.281	0.000	0.000	0.000	0.000	0.000	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	225	0	0	0	0	0	0
N.S.	1	1.00	3.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	10.288	0.000	0.000	0.000	0.000	0.000	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	91	91	0	288	0	106	103
N.S.	1	1.00	0.88	0.88	0.00	2.77	0.00	1.02	0.99
time (sec)	N/A	0.261	0.287	8.891	0.000	0.402	0.000	0.274	9.154

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	72	0	205	0	64	58
N.S.	1	1.00	0.99	0.97	0.00	2.77	0.00	0.86	0.78
time (sec)	N/A	0.204	0.153	5.986	0.000	0.422	0.000	0.280	9.110

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	39	0	130	87	40	40
N.S.	1	1.00	1.00	0.76	0.00	2.55	1.71	0.78	0.78
time (sec)	N/A	0.187	0.072	5.898	0.000	0.452	10.988	0.274	9.144

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	78	0	431	114	71	652
N.S.	1	1.00	0.94	0.92	0.00	5.07	1.34	0.84	7.67
time (sec)	N/A	0.216	0.190	6.177	0.000	0.492	9.149	0.269	9.356

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	127	109	92	0	565	0	104	396
N.S.	1	1.09	0.93	0.79	0.00	4.83	0.00	0.89	3.38
time (sec)	N/A	0.261	0.429	6.098	0.000	0.877	0.000	0.314	10.114

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	134	136	152	0	739	0	0	0
N.S.	1	1.09	1.11	1.24	0.00	6.01	0.00	0.00	0.00
time (sec)	N/A	0.287	2.361	22.368	0.000	0.381	0.000	0.000	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	90	108	85	0	632	0	0	0
N.S.	1	0.99	1.19	0.93	0.00	6.95	0.00	0.00	0.00
time (sec)	N/A	0.235	1.093	12.914	0.000	0.395	0.000	0.000	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	74	42	0	245	0	72	0
N.S.	1	1.00	1.37	0.78	0.00	4.54	0.00	1.33	0.00
time (sec)	N/A	0.191	0.959	10.886	0.000	0.445	0.000	0.288	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	100	80	0	332	0	116	0
N.S.	1	1.00	1.25	1.00	0.00	4.15	0.00	1.45	0.00
time (sec)	N/A	0.226	0.878	14.678	0.000	0.505	0.000	0.283	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	127	121	103	0	416	0	205	0
N.S.	1	1.10	1.05	0.90	0.00	3.62	0.00	1.78	0.00
time (sec)	N/A	0.297	2.498	25.258	0.000	0.521	0.000	1.271	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	851	976	65	0	0	0	0	0	0
N.S.	1	1.15	0.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.387	10.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	754	876	65	0	0	0	0	0	0
N.S.	1	1.16	0.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.224	10.058	0.000	0.000	0.000	0.000	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	878	1006	141	0	0	0	0	0	0
N.S.	1	1.15	0.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.417	10.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1005	1111	65	0	0	0	0	0	0
N.S.	1	1.11	0.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.570	10.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	768	881	65	0	0	0	0	0	0
N.S.	1	1.15	0.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.226	10.062	0.000	0.000	0.000	0.000	0.000	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1032	1021	141	0	0	0	0	0	0
N.S.	1	0.99	0.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.489	10.125	0.000	0.000	0.000	0.000	0.000	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	10.070	0.000	0.000	0.000	0.000	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	10.062	0.000	0.000	0.000	0.000	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	10.198	0.000	0.000	0.000	0.000	0.000	0.000

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0	0
N.S.	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	10.133	0.000	0.000	0.000	0.000	0.000	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	10.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	144	130	133	0	475	0	134	144
N.S.	1	1.17	1.06	1.08	0.00	3.86	0.00	1.09	1.17
time (sec)	N/A	0.280	0.536	5.954	0.000	0.278	0.000	0.274	9.483

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	98	100	83	0	348	0	116	95
N.S.	1	0.99	1.01	0.84	0.00	3.52	0.00	1.17	0.96
time (sec)	N/A	0.235	0.273	5.837	0.000	0.324	0.000	0.273	9.294

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	86	86	90	0	302	0	93	84
N.S.	1	0.99	0.99	1.03	0.00	3.47	0.00	1.07	0.97
time (sec)	N/A	0.219	0.251	5.819	0.000	0.413	0.000	0.265	9.137

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	155	124	146	0	862	0	139	3017
N.S.	1	1.17	0.94	1.11	0.00	6.53	0.00	1.05	22.86
time (sec)	N/A	0.282	0.527	5.923	0.000	0.451	0.000	0.270	10.155

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	213	163	191	0	1236	0	257	3832
N.S.	1	1.15	0.88	1.03	0.00	6.68	0.00	1.39	20.71
time (sec)	N/A	0.361	1.023	6.152	0.000	0.522	0.000	0.270	12.251

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	166	153	117	0	1077	0	298	0
N.S.	1	1.18	1.09	0.83	0.00	7.64	0.00	2.11	0.00
time (sec)	N/A	0.323	4.186	24.895	0.000	1.194	0.000	0.374	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	97	112	81	0	426	0	244	0
N.S.	1	1.04	1.20	0.87	0.00	4.58	0.00	2.62	0.00
time (sec)	N/A	0.238	2.026	20.282	0.000	0.353	0.000	0.993	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	108	124	90	0	467	0	237	0
N.S.	1	1.04	1.19	0.87	0.00	4.49	0.00	2.28	0.00
time (sec)	N/A	0.243	1.825	20.214	0.000	0.394	0.000	0.373	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	157	157	112	0	612	0	418	0
N.S.	1	1.05	1.05	0.75	0.00	4.11	0.00	2.81	0.00
time (sec)	N/A	0.327	2.131	31.252	0.000	0.554	0.000	0.928	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	212	201	134	0	760	0	395	0
N.S.	1	1.02	0.97	0.64	0.00	3.65	0.00	1.90	0.00
time (sec)	N/A	0.405	5.337	47.789	0.000	0.661	0.000	1.049	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	924	1033	159	0	0	0	0	0	0
N.S.	1	1.12	0.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.491	10.164	0.000	0.000	0.000	0.000	0.000	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	999	1036	169	0	0	0	0	0	0
N.S.	1	1.04	0.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.433	10.178	0.000	0.000	0.000	0.000	0.000	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1060	1092	225	0	0	0	0	0	0
N.S.	1	1.03	0.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.571	10.316	0.000	0.000	0.000	0.000	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1164	1107	159	0	0	0	0	0	0
N.S.	1	0.95	0.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.550	10.171	0.000	0.000	0.000	0.000	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1162	1093	169	0	0	0	0	0	0
N.S.	1	0.94	0.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.420	10.198	0.000	0.000	0.000	0.000	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1243	1170	226	0	0	0	0	0	0
N.S.	1	0.94	0.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.535	10.310	0.000	0.000	0.000	0.000	0.000	0.000

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	170	0	0	0	0	0	0
N.S.	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.201	10.217	0.000	0.000	0.000	0.000	0.000	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	170	0	0	0	0	0	0
N.S.	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	10.186	0.000	0.000	0.000	0.000	0.000	0.000

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	328	0	0	0	0	0	0
N.S.	1	1.00	5.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.181	10.296	0.000	0.000	0.000	0.000	0.000	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	226	0	0	0	0	0	0
N.S.	1	1.00	3.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	10.330	0.000	0.000	0.000	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	226	0	0	0	0	0	0
N.S.	1	1.00	3.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	10.320	0.000	0.000	0.000	0.000	0.000	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	112	118	116	243	242	226	143	134
N.S.	1	0.91	0.96	0.94	1.98	1.97	1.84	1.16	1.09
time (sec)	N/A	0.217	0.644	0.093	0.294	0.432	24.897	0.273	10.008

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	86	97	91	159	191	144	105	93
N.S.	1	0.96	1.08	1.01	1.77	2.12	1.60	1.17	1.03
time (sec)	N/A	0.205	0.171	0.075	0.303	0.435	17.849	0.280	9.655

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	78	80	77	108	155	107	87	68
N.S.	1	0.93	0.95	0.92	1.29	1.85	1.27	1.04	0.81
time (sec)	N/A	0.200	0.261	0.083	0.287	0.453	17.144	0.328	9.611

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	66	82	84	67	166	85	163	57
N.S.	1	1.12	1.39	1.42	1.14	2.81	1.44	2.76	0.97
time (sec)	N/A	0.191	0.205	0.082	0.289	0.307	4.533	0.471	9.554

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	50	47	48	49	60	73	250	91
N.S.	1	1.09	1.02	1.04	1.07	1.30	1.59	5.43	1.98
time (sec)	N/A	0.190	0.149	0.069	0.208	0.356	1.178	0.592	9.221

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	78	69	70	84	85	109	310	126
N.S.	1	1.05	0.93	0.95	1.14	1.15	1.47	4.19	1.70
time (sec)	N/A	0.218	0.199	0.087	0.203	0.284	1.211	0.806	9.431

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	108	93	94	118	109	143	370	168
N.S.	1	1.04	0.89	0.90	1.13	1.05	1.38	3.56	1.62
time (sec)	N/A	0.242	0.225	0.104	0.202	0.326	1.293	0.989	9.704

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	138	113	118	152	133	177	430	210
N.S.	1	1.03	0.84	0.88	1.13	0.99	1.32	3.21	1.57
time (sec)	N/A	0.269	0.255	0.141	0.211	0.362	1.382	1.487	10.113

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	143	108	113	158	131	1386	175	117
N.S.	1	0.95	0.72	0.75	1.05	0.87	9.24	1.17	0.78
time (sec)	N/A	0.266	0.102	0.095	0.198	0.303	2.757	0.275	9.161

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	113	88	89	124	107	910	140	97
N.S.	1	0.97	0.75	0.76	1.06	0.91	7.78	1.20	0.83
time (sec)	N/A	0.240	0.084	0.081	0.200	0.286	2.252	0.269	8.914

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	83	64	65	90	82	422	105	77
N.S.	1	0.99	0.76	0.77	1.07	0.98	5.02	1.25	0.92
time (sec)	N/A	0.210	0.066	0.068	0.197	0.285	1.654	0.270	9.047

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	42	43	55	57	119	72	54
N.S.	1	1.00	0.79	0.81	1.04	1.08	2.25	1.36	1.02
time (sec)	N/A	0.185	0.049	0.060	0.217	0.277	1.230	0.623	8.905

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	68	68	83	75	156	107	116	80
N.S.	1	1.03	1.03	1.26	1.14	2.36	1.62	1.76	1.21
time (sec)	N/A	0.195	0.128	0.059	0.282	0.359	1.427	0.267	9.183

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	81	75	93	133	164	107	76	97
N.S.	1	0.95	0.88	1.09	1.56	1.93	1.26	0.89	1.14
time (sec)	N/A	0.197	0.200	0.095	0.295	0.426	2.024	0.285	9.635

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	88	91	102	193	194	144	130	0
N.S.	1	0.97	1.00	1.12	2.12	2.13	1.58	1.43	0.00
time (sec)	N/A	0.215	0.253	0.085	0.286	0.371	3.287	0.291	0.000

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	115	124	127	277	244	226	153	0
N.S.	1	0.93	1.01	1.03	2.25	1.98	1.84	1.24	0.00
time (sec)	N/A	0.246	0.281	0.096	0.285	0.401	7.141	0.297	0.000

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	110	123	116	240	243	253	144	130
N.S.	1	0.89	1.00	0.94	1.95	1.98	2.06	1.17	1.06
time (sec)	N/A	0.215	0.255	0.085	0.297	0.691	33.529	0.287	10.272

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	103	98	96	171	203	216	121	105
N.S.	1	0.90	0.85	0.83	1.49	1.77	1.88	1.05	0.91
time (sec)	N/A	0.217	0.486	0.089	0.287	0.311	55.222	0.332	10.183

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	97	103	99	134	195	187	225	95
N.S.	1	0.88	0.94	0.90	1.22	1.77	1.70	2.05	0.86
time (sec)	N/A	0.214	0.349	0.093	0.286	0.291	16.511	0.485	10.131

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	84	91	108	80	213	114	254	72
N.S.	1	1.11	1.20	1.42	1.05	2.80	1.50	3.34	0.95
time (sec)	N/A	0.201	0.338	0.096	0.283	0.315	9.839	0.840	10.370

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	50	49	48	49	84	189	370	122
N.S.	1	1.09	1.07	1.04	1.07	1.83	4.11	8.04	2.65
time (sec)	N/A	0.189	0.235	0.088	0.197	0.310	3.388	0.939	10.106

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	78	71	70	84	109	258	430	164
N.S.	1	1.05	0.96	0.95	1.14	1.47	3.49	5.81	2.22
time (sec)	N/A	0.218	0.291	0.111	0.217	0.300	3.547	1.307	10.253

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	108	94	94	118	134	326	490	206
N.S.	1	1.04	0.90	0.90	1.13	1.29	3.13	4.71	1.98
time (sec)	N/A	0.242	0.343	0.165	0.196	0.354	3.767	2.031	10.679

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	138	115	118	152	157	393	550	248
N.S.	1	1.03	0.86	0.88	1.13	1.17	2.93	4.10	1.85
time (sec)	N/A	0.263	0.409	0.263	0.200	0.398	3.927	2.047	11.330

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	143	110	115	158	155	3351	175	137
N.S.	1	0.95	0.73	0.77	1.05	1.03	22.34	1.17	0.91
time (sec)	N/A	0.275	0.113	0.103	0.206	0.304	5.607	0.275	9.059

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	113	89	91	124	132	2304	140	118
N.S.	1	0.97	0.76	0.78	1.06	1.13	19.69	1.20	1.01
time (sec)	N/A	0.248	0.095	0.085	0.206	0.289	4.205	0.274	9.021

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	83	66	67	90	106	1340	105	97
N.S.	1	0.99	0.79	0.80	1.07	1.26	15.95	1.25	1.15
time (sec)	N/A	0.219	0.075	0.078	0.212	0.297	3.147	0.268	8.964

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	44	45	55	80	498	72	77
N.S.	1	1.00	0.83	0.85	1.04	1.51	9.40	1.36	1.45
time (sec)	N/A	0.186	0.052	0.067	0.211	0.299	2.366	0.273	8.993

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	89	81	99	91	203	184	140	0
N.S.	1	1.03	0.94	1.15	1.06	2.36	2.14	1.63	0.00
time (sec)	N/A	0.223	0.178	0.068	0.306	0.305	2.181	0.279	0.000

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	107	105	147	163	190	202	115	0
N.S.	1	0.88	0.87	1.21	1.35	1.57	1.67	0.95	0.00
time (sec)	N/A	0.219	0.204	0.108	0.306	0.336	3.240	0.293	0.000

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	105	92	115	207	216	216	145	78
N.S.	1	0.94	0.82	1.03	1.85	1.93	1.93	1.29	0.70
time (sec)	N/A	0.216	0.298	0.102	0.293	0.323	5.378	0.299	10.150

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	112	117	127	275	246	253	173	0
N.S.	1	0.91	0.95	1.03	2.24	2.00	2.06	1.41	0.00
time (sec)	N/A	0.225	0.388	0.109	0.300	0.298	10.454	0.305	0.000

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	140	150	151	354	298	287	214	0
N.S.	1	0.88	0.94	0.95	2.23	1.87	1.81	1.35	0.00
time (sec)	N/A	0.253	0.354	0.116	0.278	0.335	31.778	0.304	0.000

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	88	105	99	178	192	150	111	99
N.S.	1	0.98	1.17	1.10	1.98	2.13	1.67	1.23	1.10
time (sec)	N/A	0.206	0.434	0.083	0.285	0.332	14.385	0.283	9.850

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	57	80	82	109	146	66	79	59
N.S.	1	0.97	1.36	1.39	1.85	2.47	1.12	1.34	1.00
time (sec)	N/A	0.189	0.101	0.073	0.285	0.304	14.104	0.273	9.557

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	48	76	69	54	130	71	66	35
N.S.	1	1.12	1.77	1.60	1.26	3.02	1.65	1.53	0.81
time (sec)	N/A	0.178	0.108	0.075	0.279	0.320	2.951	0.290	9.299

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	48	39	44	48	39	65	124	35
N.S.	1	1.12	0.91	1.02	1.12	0.91	1.51	2.88	0.81
time (sec)	N/A	0.188	0.125	0.064	0.206	0.309	1.054	0.452	9.004

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	76	60	67	83	62	100	180	58
N.S.	1	1.06	0.83	0.93	1.15	0.86	1.39	2.50	0.81
time (sec)	N/A	0.209	0.159	0.071	0.222	0.312	1.056	0.660	9.044

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	106	93	91	118	86	133	236	102
N.S.	1	1.05	0.92	0.90	1.17	0.85	1.32	2.34	1.01
time (sec)	N/A	0.236	0.194	0.080	0.194	0.302	1.181	0.866	9.091

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	81	56	62	85	59	338	99	53
N.S.	1	0.99	0.68	0.76	1.04	0.72	4.12	1.21	0.65
time (sec)	N/A	0.202	0.075	0.066	0.197	0.283	1.464	0.320	9.703

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	34	39	49	36	70	66	67
N.S.	1	1.00	0.67	0.76	0.96	0.71	1.37	1.29	1.31
time (sec)	N/A	0.171	0.049	0.058	0.197	0.287	1.077	0.280	9.422

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	71	73	58	131	39	80	65
N.S.	1	1.00	1.51	1.55	1.23	2.79	0.83	1.70	1.38
time (sec)	N/A	0.174	0.076	0.058	0.282	0.289	1.156	0.273	9.460

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	80	93	121	144	66	64	94
N.S.	1	1.00	1.31	1.52	1.98	2.36	1.08	1.05	1.54
time (sec)	N/A	0.190	0.127	0.084	0.263	0.290	2.104	0.289	10.069

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	92	101	112	200	201	150	125	0
N.S.	1	0.99	1.09	1.20	2.15	2.16	1.61	1.34	0.00
time (sec)	N/A	0.210	0.244	0.088	0.277	0.406	4.095	0.294	0.000

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	113	159	140	215	304	177	144	134
N.S.	1	0.96	1.35	1.19	1.82	2.58	1.50	1.22	1.14
time (sec)	N/A	0.220	0.548	0.114	0.294	0.430	34.409	0.290	10.336

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	82	139	115	144	249	264	105	90
N.S.	1	0.95	1.62	1.34	1.67	2.90	3.07	1.22	1.05
time (sec)	N/A	0.201	0.328	0.103	0.277	0.510	16.943	0.274	9.820

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	58	75	75	69	200	75	69	54
N.S.	1	1.12	1.44	1.44	1.33	3.85	1.44	1.33	1.04
time (sec)	N/A	0.185	0.132	0.066	0.289	0.456	4.103	0.272	9.467

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	46	36	46	46	46	68	66	46
N.S.	1	1.10	0.86	1.10	1.10	1.10	1.62	1.57	1.10
time (sec)	N/A	0.189	0.111	0.091	0.224	0.314	0.479	0.313	8.939

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	74	60	69	81	73	85	188	66
N.S.	1	1.09	0.88	1.01	1.19	1.07	1.25	2.76	0.97
time (sec)	N/A	0.210	0.167	0.095	0.200	0.288	2.250	0.531	8.977

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	104	81	94	116	98	114	303	91
N.S.	1	1.04	0.81	0.94	1.16	0.98	1.14	3.03	0.91
time (sec)	N/A	0.235	0.213	0.105	0.202	0.286	2.646	0.696	9.204

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	132	104	118	151	121	146	414	154
N.S.	1	1.05	0.83	0.94	1.20	0.96	1.16	3.29	1.22
time (sec)	N/A	0.257	0.243	0.116	0.194	0.302	3.056	1.192	9.346

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	107	80	91	128	95	561	146	79
N.S.	1	0.96	0.72	0.82	1.15	0.86	5.05	1.32	0.71
time (sec)	N/A	0.234	0.096	0.110	0.234	0.289	2.920	0.282	10.138

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	77	57	66	90	70	267	106	81
N.S.	1	0.97	0.72	0.84	1.14	0.89	3.38	1.34	1.03
time (sec)	N/A	0.199	0.076	0.091	0.190	0.307	2.613	0.272	9.695

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	33	43	53	47	65	62	38
N.S.	1	1.00	0.73	0.96	1.18	1.04	1.44	1.38	0.84
time (sec)	N/A	0.169	0.050	0.086	0.193	0.279	2.456	0.273	9.212

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	71	79	80	195	206	108	60
N.S.	1	1.00	1.20	1.34	1.36	3.31	3.49	1.83	1.02
time (sec)	N/A	0.195	0.107	0.063	0.270	0.257	3.840	0.272	9.449

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	86	91	127	162	248	262	107	0
N.S.	1	0.93	0.99	1.38	1.76	2.70	2.85	1.16	0.00
time (sec)	N/A	0.220	0.251	0.104	0.278	0.279	6.588	0.307	0.000

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	117	113	155	243	314	180	148	0
N.S.	1	0.95	0.92	1.26	1.98	2.55	1.46	1.20	0.00
time (sec)	N/A	0.240	0.269	0.119	0.274	0.279	11.330	0.309	0.000

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	101	115	0	0	0	0	0	0
N.S.	1	0.96	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.270	0.335	0.000	0.000	0.000	0.000	0.000	0.000

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	0.267	0.000	0.000	0.000	0.000	0.000	0.000

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	100	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	0.258	0.000	0.000	0.000	0.000	0.000	0.000

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	0.244	0.000	0.000	0.000	0.000	0.000	0.000

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	100	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.251	0.000	0.000	0.000	0.000	0.000	0.000

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.091	0.000	0.000	0.000	0.000	0.000	0.000

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	95	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.234	0.000	0.000	0.000	0.000	0.000	0.000

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	106	0	0	0	0	0	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	0.263	0.000	0.000	0.000	0.000	0.000	0.000

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	110	0	0	0	0	0	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.207	0.102	0.000	0.000	0.000	0.000	0.000	0.000

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	0.351	0.000	0.000	0.000	0.000	0.000	0.000

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.287	0.489	0.000	0.000	0.000	0.000	0.000	0.000

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	0.486	0.000	0.000	0.000	0.000	0.000	0.000

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	0.427	0.000	0.000	0.000	0.000	0.000	0.000

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	111	0	0	0	0	0	0
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	0.459	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	111	0	0	0	0	0	0
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	0.535	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.287	0.527	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	135	143	99	75	57	62	0	162	831
N.S.	1	1.06	0.73	0.56	0.42	0.46	0.00	1.20	6.16
time (sec)	N/A	0.259	7.236	4.579	0.195	0.309	0.000	0.288	75.080

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	104	107	87	65	47	57	0	127	632
N.S.	1	1.03	0.84	0.62	0.45	0.55	0.00	1.22	6.08
time (sec)	N/A	0.233	1.366	4.599	0.184	0.283	0.000	0.286	45.134

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	74	404	52	37	52	0	92	0
N.S.	1	1.01	5.53	0.71	0.51	0.71	0.00	1.26	0.00
time (sec)	N/A	0.194	1.927	4.592	0.189	0.277	0.000	0.286	0.000

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	264	72	26	46	61	57	41
N.S.	1	1.00	7.14	1.95	0.70	1.24	1.65	1.54	1.11
time (sec)	N/A	0.164	1.492	4.593	0.187	0.284	0.650	0.277	9.381

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	69	184	47	27	55	0	0	129
N.S.	1	1.03	2.75	0.70	0.40	0.82	0.00	0.00	1.93
time (sec)	N/A	0.193	1.192	4.574	0.274	0.274	0.000	0.000	10.549

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	421	23	10	30	0	48	31
N.S.	1	1.00	13.58	0.74	0.32	0.97	0.00	1.55	1.00
time (sec)	N/A	0.142	2.507	4.574	0.276	0.262	0.000	0.299	9.136

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	36	28	21	37	0	90	43
N.S.	1	1.00	0.57	0.44	0.33	0.59	0.00	1.43	0.68
time (sec)	N/A	0.169	10.024	4.825	0.263	0.271	0.000	0.284	9.296

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	99	41	33	31	44	0	111	55
N.S.	1	1.05	0.44	0.35	0.33	0.47	0.00	1.18	0.59
time (sec)	N/A	0.194	10.050	4.608	0.265	0.262	0.000	0.310	9.416

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	135	46	38	41	49	0	132	67
N.S.	1	1.08	0.37	0.30	0.33	0.39	0.00	1.06	0.54
time (sec)	N/A	0.222	10.078	4.696	0.281	0.250	0.000	0.319	9.363

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	104	107	88	65	47	57	0	76	632
N.S.	1	1.03	0.85	0.62	0.45	0.55	0.00	0.73	6.08
time (sec)	N/A	0.226	1.359	4.820	0.194	0.258	0.000	0.282	38.441

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	73	71	75	55	37	52	0	59	429
N.S.	1	0.97	1.03	0.75	0.51	0.71	0.00	0.81	5.88
time (sec)	N/A	0.192	1.366	4.748	0.186	0.263	0.000	0.299	26.035

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	265	41	24	46	0	39	0
N.S.	1	1.00	7.57	1.17	0.69	1.31	0.00	1.11	0.00
time (sec)	N/A	0.159	1.521	4.609	0.195	0.258	0.000	0.276	0.000

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	38	40	16	27	0	20	6
N.S.	1	1.00	4.75	5.00	2.00	3.38	0.00	2.50	0.75
time (sec)	N/A	0.138	1.034	4.587	0.180	0.259	0.000	0.273	9.438

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	146	20	10	25	0	25	19
N.S.	1	1.00	5.03	0.69	0.34	0.86	0.00	0.86	0.66
time (sec)	N/A	0.141	1.130	4.576	0.266	0.260	0.000	0.290	9.677

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	407	25	21	34	0	48	33
N.S.	1	1.00	6.46	0.40	0.33	0.54	0.00	0.76	0.52
time (sec)	N/A	0.170	2.557	4.592	0.269	0.271	0.000	0.281	9.681

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	99	41	30	31	39	0	69	43
N.S.	1	1.05	0.44	0.32	0.33	0.41	0.00	0.73	0.46
time (sec)	N/A	0.195	1.327	4.588	0.281	0.319	0.000	0.304	9.710

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	80	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	0.088	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	72	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	0.066	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	73	73	0	0	0	0	0	0
N.S.	1	1.01	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.212	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	78	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	0.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	19	15	12	15	11	10	16	11
N.S.	1	1.27	1.00	0.80	1.00	0.73	0.67	1.07	0.73
time (sec)	N/A	0.166	0.009	4.601	0.186	0.315	0.050	0.280	0.084

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	110	46	36	40	36	48	31
N.S.	1	1.00	5.00	2.09	1.64	1.82	1.64	2.18	1.41
time (sec)	N/A	0.153	0.272	12.094	0.283	0.307	2.174	0.312	9.198

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	88	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	0.295	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	78	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	0.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	78	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	0.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	58	58	69	58	348	0	162
N.S.	1	1.00	0.92	0.92	1.10	0.92	5.52	0.00	2.57
time (sec)	N/A	0.221	0.091	4.883	0.282	0.333	1.335	0.000	9.984

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	74	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.122	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	77	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	0.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	175	195	141	0	0	0	0	0	0
N.S.	1	1.11	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.447	0.437	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	160	135	0	0	0	0	0	0
N.S.	1	1.13	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.389	0.218	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	161	134	0	0	0	0	0	0
N.S.	1	1.13	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.361	0.217	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	142	108	0	0	0	0	0	0
N.S.	1	1.16	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	0.137	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	98	107	100	151	224	0	0	0
N.S.	1	0.97	1.06	0.99	1.50	2.22	0.00	0.00	0.00
time (sec)	N/A	0.284	0.216	5.128	0.201	0.303	0.000	0.000	0.000

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	158	133	0	0	0	0	0	0
N.S.	1	1.11	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.391	0.238	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	163	136	0	0	0	0	0	0
N.S.	1	1.12	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.392	0.231	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	119	134	188	231	177	347	0	0
N.S.	1	0.92	1.03	1.45	1.78	1.36	2.67	0.00	0.00
time (sec)	N/A	0.301	0.197	5.406	0.213	0.518	3.833	0.000	0.000

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	82	87	118	150	108	221	0	0
N.S.	1	0.91	0.97	1.31	1.67	1.20	2.46	0.00	0.00
time (sec)	N/A	0.261	0.137	4.932	0.195	0.446	2.132	0.000	0.000

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	55	50	65	83	56	116	0	0
N.S.	1	0.92	0.83	1.08	1.38	0.93	1.93	0.00	0.00
time (sec)	N/A	0.216	0.081	4.688	0.202	0.264	1.258	0.000	0.000

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	52	44	59	60	45	0	0	0
N.S.	1	0.96	0.81	1.09	1.11	0.83	0.00	0.00	0.00
time (sec)	N/A	0.216	0.085	5.064	0.205	0.276	0.000	0.000	0.000

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	70	75	107	121	120	0	0	0
N.S.	1	0.93	1.00	1.43	1.61	1.60	0.00	0.00	0.00
time (sec)	N/A	0.235	0.134	5.375	0.203	0.288	0.000	0.000	0.000

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	97	100	157	243	267	0	0	0
N.S.	1	0.92	0.95	1.50	2.31	2.54	0.00	0.00	0.00
time (sec)	N/A	0.261	0.191	6.238	0.198	0.308	0.000	0.000	0.000

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	144	185	342	286	230	428	0	0
N.S.	1	0.91	1.17	2.16	1.81	1.46	2.71	0.00	0.00
time (sec)	N/A	0.335	0.242	5.720	0.206	0.296	5.482	0.000	0.000

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	107	125	157	192	146	277	0	0
N.S.	1	0.91	1.06	1.33	1.63	1.24	2.35	0.00	0.00
time (sec)	N/A	0.279	0.169	4.905	0.216	0.275	3.199	0.000	0.000

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	78	76	91	112	82	150	0	0
N.S.	1	0.91	0.88	1.06	1.30	0.95	1.74	0.00	0.00
time (sec)	N/A	0.234	0.112	4.820	0.196	0.276	1.859	0.000	0.000

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	66	66	78	81	74	0	0	0
N.S.	1	0.93	0.93	1.10	1.14	1.04	0.00	0.00	0.00
time (sec)	N/A	0.230	0.119	5.027	0.216	0.283	0.000	0.000	0.000

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	90	93	125	147	166	0	0	0
N.S.	1	0.95	0.98	1.32	1.55	1.75	0.00	0.00	0.00
time (sec)	N/A	0.265	0.166	5.405	0.212	0.293	0.000	0.000	0.000

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	112	94	169	262	301	0	0	0
N.S.	1	0.93	0.78	1.41	2.18	2.51	0.00	0.00	0.00
time (sec)	N/A	0.283	0.277	6.234	0.209	0.278	0.000	0.000	0.000

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	172	155	154	154	175	13	154
N.S.	1	1.00	12.29	11.07	11.00	11.00	12.50	0.93	11.00
time (sec)	N/A	0.131	0.006	4.543	0.189	0.266	0.043	0.286	0.169

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	182	157	156	156	182	156	156
N.S.	1	1.00	11.38	9.81	9.75	9.75	11.38	9.75	9.75
time (sec)	N/A	0.142	0.007	4.741	0.195	0.270	0.044	0.285	9.152

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	186	157	156	156	185	156	156
N.S.	1	1.00	11.62	9.81	9.75	9.75	11.56	9.75	9.75
time (sec)	N/A	0.153	0.006	4.659	0.195	0.254	0.045	0.292	9.182

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	189	360	189	229
N.S.	1	1.00	1.00	10.95	10.90	9.00	17.14	9.00	10.90
time (sec)	N/A	0.154	0.061	183.423	0.207	0.267	10.799	0.364	9.550

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	107	50	16	32	39	70	25
N.S.	1	1.00	8.23	3.85	1.23	2.46	3.00	5.38	1.92
time (sec)	N/A	0.151	0.256	12.568	0.255	0.278	2.239	0.315	9.163

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	9	9	9	9	10	8	11	8
N.S.	1	1.12	1.12	1.12	1.12	1.25	1.00	1.38	1.00
time (sec)	N/A	0.148	0.007	4.615	0.198	0.247	0.051	0.266	9.027

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	17	15	14	17	13	12	18	13
N.S.	1	1.13	1.00	0.93	1.13	0.87	0.80	1.20	0.87
time (sec)	N/A	0.155	0.009	4.603	0.193	0.256	0.080	0.274	9.150

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	17	15	14	17	13	12	15	13
N.S.	1	1.13	1.00	0.93	1.13	0.87	0.80	1.00	0.87
time (sec)	N/A	0.166	0.010	4.626	0.196	0.256	0.087	0.278	9.100

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	17	19	17	47	17	29	0	15
N.S.	1	1.13	1.27	1.13	3.13	1.13	1.93	0.00	1.00
time (sec)	N/A	0.163	0.044	4.691	0.194	0.377	0.282	0.000	8.696

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	81	81	87	13	12
N.S.	1	1.00	1.00	0.93	5.79	5.79	6.21	0.93	0.86
time (sec)	N/A	0.132	0.024	4.626	0.218	0.487	0.428	0.261	11.279

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.145	0.034	4.648	0.207	0.389	0.656	0.272	2.276

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.149	0.041	4.817	0.212	0.690	0.895	0.283	13.805

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	23	612	105	129	0	105
N.S.	1	1.00	1.00	1.10	29.14	5.00	6.14	0.00	5.00
time (sec)	N/A	0.152	0.107	14.915	0.247	0.293	41.767	0.000	9.389

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	46	69	53	46	58	56	37
N.S.	1	1.00	0.88	1.33	1.02	0.88	1.12	1.08	0.71
time (sec)	N/A	0.171	0.086	5.315	0.274	0.273	27.375	0.267	9.000

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	142	143	0	757	0	0	0
N.S.	1	1.00	1.53	1.54	0.00	8.14	0.00	0.00	0.00
time (sec)	N/A	0.233	10.505	0.154	0.000	0.419	0.000	0.000	0.000

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	252	226	223	0	0	607	0	0	0
N.S.	1	0.90	0.88	0.00	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.304	1.836	0.000	0.000	0.318	0.000	0.000	0.000

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	199	181	178	0	0	469	0	0	0
N.S.	1	0.91	0.89	0.00	0.00	2.36	0.00	0.00	0.00
time (sec)	N/A	0.271	1.032	0.000	0.000	0.309	0.000	0.000	0.000

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	136	141	0	0	359	0	0	0
N.S.	1	0.93	0.97	0.00	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.240	0.568	0.000	0.000	0.304	0.000	0.000	0.000

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	87	123	0	0	281	0	0	0
N.S.	1	0.98	1.38	0.00	0.00	3.16	0.00	0.00	0.00
time (sec)	N/A	0.215	0.338	0.000	0.000	0.290	0.000	0.000	0.000

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	89	122	0	0	408	0	0	0
N.S.	1	0.98	1.34	0.00	0.00	4.48	0.00	0.00	0.00
time (sec)	N/A	0.214	1.056	0.000	0.000	0.350	0.000	0.000	0.000

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	93	57	0	0	135	0	0	0
N.S.	1	0.98	0.60	0.00	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.216	0.932	0.000	0.000	0.422	0.000	0.000	0.000

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	358	295	274	0	0	771	0	0	0
N.S.	1	0.82	0.77	0.00	0.00	2.15	0.00	0.00	0.00
time (sec)	N/A	0.365	2.657	0.000	0.000	0.366	0.000	0.000	0.000

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	251	241	0	0	607	0	0	0
N.S.	1	0.86	0.83	0.00	0.00	2.09	0.00	0.00	0.00
time (sec)	N/A	0.340	1.238	0.000	0.000	0.322	0.000	0.000	0.000

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	205	191	0	0	471	0	0	0
N.S.	1	0.93	0.86	0.00	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	0.313	0.643	0.000	0.000	0.295	0.000	0.000	0.000

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	151	157	0	0	361	0	0	0
N.S.	1	1.01	1.05	0.00	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.275	0.425	0.000	0.000	0.299	0.000	0.000	0.000

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	131	185	0	0	540	0	0	0
N.S.	1	0.98	1.39	0.00	0.00	4.06	0.00	0.00	0.00
time (sec)	N/A	0.261	0.736	0.000	0.000	0.392	0.000	0.000	0.000

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	163	217	0	0	769	0	0	0
N.S.	1	1.11	1.48	0.00	0.00	5.23	0.00	0.00	0.00
time (sec)	N/A	0.284	1.567	0.000	0.000	0.490	0.000	0.000	0.000

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	29	25	46	35	22
N.S.	1	1.00	1.00	1.05	1.45	1.25	2.30	1.75	1.10
time (sec)	N/A	0.141	0.052	4.806	0.244	0.436	0.788	0.290	9.116

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	97	26	35	32	80	52	47
N.S.	1	1.00	3.59	0.96	1.30	1.19	2.96	1.93	1.74
time (sec)	N/A	0.151	0.129	5.363	0.250	0.519	26.646	0.284	8.968

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	97	26	35	32	0	56	47
N.S.	1	1.00	3.59	0.96	1.30	1.19	0.00	2.07	1.74
time (sec)	N/A	0.155	0.130	6.586	0.252	0.274	0.000	0.279	8.916

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	101	60	39	35	116	66	54
N.S.	1	1.00	3.74	2.22	1.44	1.30	4.30	2.44	2.00
time (sec)	N/A	0.159	0.197	6.683	0.271	0.252	17.653	0.315	9.078

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [585] had the largest ratio of [.750000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.03	18	0.167
2	A	2	2	1.00	16	0.125
3	A	2	2	1.00	15	0.133
4	A	4	3	1.10	18	0.167
5	A	2	2	1.00	18	0.111
6	A	2	2	1.00	18	0.111
7	A	4	3	1.03	18	0.167
8	A	2	2	1.00	18	0.111
9	A	2	2	1.00	18	0.111
10	A	4	3	1.14	18	0.167
11	A	4	3	1.10	20	0.150
12	A	2	2	1.00	18	0.111
13	A	2	2	1.00	17	0.118
14	A	5	4	1.09	20	0.200
15	A	2	2	1.00	20	0.100
16	A	2	2	1.00	20	0.100
17	A	4	3	1.02	20	0.150
18	A	2	2	1.00	20	0.100
19	A	2	2	1.00	20	0.100
20	A	4	3	1.02	20	0.150
21	A	2	2	1.00	20	0.100
22	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	2	2	1.00	20	0.100
24	A	4	3	1.04	20	0.150
25	A	2	2	1.00	20	0.100
26	A	2	2	1.00	20	0.100
27	A	4	3	1.06	20	0.150
28	A	2	2	1.00	20	0.100
29	A	2	2	1.00	20	0.100
30	A	4	3	1.10	20	0.150
31	A	2	2	1.00	18	0.111
32	A	2	2	1.00	17	0.118
33	A	5	4	0.99	20	0.200
34	A	2	2	1.00	20	0.100
35	A	2	2	1.00	20	0.100
36	A	4	3	1.00	20	0.150
37	A	2	2	1.00	20	0.100
38	A	2	2	1.00	20	0.100
39	A	4	3	1.02	20	0.150
40	A	2	2	1.00	20	0.100
41	A	2	2	1.00	20	0.100
42	A	4	3	1.02	20	0.150
43	A	2	2	1.00	20	0.100
44	A	2	2	1.00	20	0.100
45	A	4	3	1.01	20	0.150
46	A	2	2	1.00	20	0.100
47	A	2	2	1.00	20	0.100
48	A	4	3	0.99	20	0.150
49	A	2	2	1.00	20	0.100
50	A	2	2	1.00	20	0.100
51	A	5	4	0.99	20	0.200
52	A	2	2	1.00	20	0.100
53	A	2	2	1.00	20	0.100
54	A	4	3	1.08	20	0.150
55	A	2	2	1.00	20	0.100
56	A	3	3	0.85	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	4	3	0.98	20	0.150
58	A	11	10	0.94	20	0.500
59	A	11	10	0.91	20	0.500
60	A	4	3	0.94	20	0.150
61	A	10	9	0.93	18	0.500
62	A	10	9	0.90	17	0.529
63	A	4	3	1.12	20	0.150
64	A	10	9	0.95	20	0.450
65	A	10	9	0.91	20	0.450
66	A	4	3	1.02	20	0.150
67	A	11	10	0.95	20	0.500
68	A	11	10	0.91	20	0.500
69	A	4	3	1.01	20	0.150
70	A	12	11	0.94	20	0.550
71	A	3	3	0.83	20	0.150
72	A	4	3	0.96	20	0.150
73	A	3	3	0.87	20	0.150
74	A	3	3	0.85	20	0.150
75	A	4	3	0.92	20	0.150
76	A	11	10	0.93	20	0.500
77	A	11	10	0.91	20	0.500
78	A	4	3	0.98	20	0.150
79	A	10	9	0.96	18	0.500
80	A	10	9	0.93	17	0.529
81	A	4	3	1.02	20	0.150
82	A	11	10	0.93	20	0.500
83	A	11	10	0.91	20	0.500
84	A	4	3	0.99	20	0.150
85	A	12	11	0.92	20	0.550
86	A	12	11	0.91	20	0.550
87	A	4	3	0.98	20	0.150
88	A	4	3	0.99	20	0.150
89	A	4	3	0.94	20	0.150
90	A	4	3	0.98	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	3	2	1.00	20	0.100
92	A	4	3	1.01	20	0.150
93	A	4	3	0.99	20	0.150
94	A	4	3	1.00	20	0.150
95	A	4	4	0.87	20	0.200
96	A	4	4	0.86	20	0.200
97	A	12	11	0.94	20	0.550
98	A	12	11	0.90	20	0.550
99	A	11	10	0.96	20	0.500
100	A	11	10	0.93	20	0.500
101	A	11	10	0.96	18	0.556
102	A	11	10	0.93	17	0.588
103	A	12	11	0.92	20	0.550
104	A	12	11	0.90	20	0.550
105	A	13	12	0.91	20	0.600
106	A	13	12	0.90	20	0.600
107	A	4	3	0.94	22	0.136
108	A	4	4	1.05	22	0.182
109	A	11	10	0.92	22	0.455
110	A	4	3	0.98	22	0.136
111	A	10	9	0.90	22	0.409
112	A	10	9	0.86	22	0.409
113	A	4	3	0.98	22	0.136
114	A	10	9	0.90	20	0.450
115	A	10	9	0.86	19	0.474
116	A	4	3	1.02	22	0.136
117	A	4	4	1.05	22	0.182
118	A	12	11	0.92	22	0.500
119	A	4	3	0.99	22	0.136
120	A	5	5	1.12	22	0.227
121	A	14	13	0.96	22	0.591
122	A	4	3	0.98	22	0.136
123	A	7	7	1.11	22	0.318
124	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	2	2	1.00	20	0.100
126	A	2	2	1.00	18	0.111
127	A	2	2	1.00	20	0.100
128	A	2	2	1.00	20	0.100
129	A	2	2	1.00	20	0.100
130	A	2	2	1.00	24	0.083
131	A	2	2	1.00	20	0.100
132	A	2	2	1.00	20	0.100
133	A	2	2	1.00	20	0.100
134	A	2	2	1.00	20	0.100
135	A	2	2	1.00	20	0.100
136	A	2	2	1.00	20	0.100
137	A	2	2	1.00	20	0.100
138	A	2	2	1.00	20	0.100
139	A	2	2	1.00	22	0.091
140	A	2	2	1.00	22	0.091
141	A	2	2	1.00	22	0.091
142	A	2	2	1.00	22	0.091
143	A	2	2	1.00	22	0.091
144	A	2	2	1.00	22	0.091
145	A	2	2	1.00	22	0.091
146	A	2	2	1.00	22	0.091
147	A	2	2	1.00	22	0.091
148	A	2	2	1.00	22	0.091
149	A	2	2	1.00	22	0.091
150	A	2	2	1.00	22	0.091
151	A	2	2	1.00	22	0.091
152	A	2	2	1.00	22	0.091
153	A	2	2	1.00	22	0.091
154	A	2	2	1.00	22	0.091
155	A	6	5	0.96	22	0.227
156	A	13	12	0.93	22	0.545
157	A	12	11	0.97	22	0.500
158	A	5	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	12	11	0.94	22	0.500
160	A	12	11	0.97	22	0.500
161	A	5	4	1.00	22	0.182
162	A	12	11	0.94	22	0.500
163	A	6	5	1.00	22	0.227
164	A	13	12	0.94	22	0.545
165	A	12	11	0.99	22	0.500
166	A	5	4	1.00	22	0.182
167	A	12	11	0.96	22	0.500
168	A	13	12	0.96	22	0.545
169	A	6	5	1.00	22	0.227
170	A	13	12	0.93	22	0.545
171	A	6	5	1.00	22	0.227
172	A	13	12	0.94	22	0.545
173	A	13	12	0.97	22	0.545
174	A	6	5	1.00	22	0.227
175	A	13	12	0.95	22	0.545
176	A	14	13	0.95	22	0.591
177	A	7	6	0.97	22	0.273
178	A	14	13	0.93	22	0.591
179	A	4	3	1.04	22	0.136
180	A	4	3	1.05	22	0.136
181	A	4	3	1.09	22	0.136
182	A	6	5	1.02	22	0.227
183	A	6	5	0.94	22	0.227
184	A	6	5	0.97	22	0.227
185	A	4	4	0.97	22	0.182
186	A	3	3	0.99	19	0.158
187	A	3	3	0.99	22	0.136
188	A	3	3	0.98	22	0.136
189	A	4	4	0.97	22	0.182
190	A	6	6	0.99	22	0.273
191	A	5	5	1.00	20	0.250
192	A	5	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	5	5	1.00	22	0.227
194	A	6	6	0.99	22	0.273
195	A	7	7	0.99	22	0.318
196	A	4	3	1.04	22	0.136
197	A	4	3	1.05	22	0.136
198	A	4	3	1.09	22	0.136
199	A	7	6	1.02	22	0.273
200	A	7	6	0.89	22	0.273
201	A	7	6	0.90	22	0.273
202	A	5	5	0.94	22	0.227
203	A	4	4	0.96	19	0.211
204	A	4	4	0.98	22	0.182
205	A	4	4	0.97	22	0.182
206	A	4	4	0.96	22	0.182
207	A	7	7	0.98	22	0.318
208	A	6	6	0.98	20	0.300
209	A	6	6	0.99	22	0.273
210	A	6	6	0.98	22	0.273
211	A	6	6	0.99	22	0.273
212	A	7	7	0.98	22	0.318
213	A	4	3	1.02	22	0.136
214	A	4	3	1.03	22	0.136
215	A	4	3	1.04	22	0.136
216	A	5	4	1.00	22	0.182
217	A	5	4	0.98	22	0.182
218	A	6	5	0.98	22	0.227
219	A	3	3	1.00	22	0.136
220	A	2	2	1.00	19	0.105
221	A	2	2	1.00	22	0.091
222	A	3	3	1.00	22	0.136
223	A	5	5	1.01	22	0.227
224	A	4	4	1.01	20	0.200
225	A	4	4	1.02	22	0.182
226	A	5	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	6	6	1.00	22	0.273
228	A	4	3	1.00	22	0.136
229	A	4	3	1.00	22	0.136
230	A	4	3	1.00	22	0.136
231	A	5	4	1.00	22	0.182
232	A	6	5	0.97	22	0.227
233	A	7	6	0.96	22	0.273
234	A	4	4	0.99	22	0.182
235	A	3	3	1.00	22	0.136
236	A	2	2	1.00	19	0.105
237	A	3	3	1.00	22	0.136
238	A	4	4	0.99	22	0.182
239	A	5	5	1.01	22	0.227
240	A	4	4	1.02	20	0.200
241	A	5	5	1.00	22	0.227
242	A	6	6	1.00	22	0.273
243	A	7	7	1.00	22	0.318
244	A	4	3	1.00	22	0.136
245	A	4	3	1.00	22	0.136
246	A	4	3	1.04	22	0.136
247	A	6	5	1.05	22	0.227
248	A	7	6	0.94	22	0.273
249	A	4	4	0.99	22	0.182
250	A	3	3	0.99	22	0.136
251	A	3	3	1.00	19	0.158
252	A	4	4	0.99	22	0.182
253	A	5	5	0.99	22	0.227
254	A	6	6	1.01	22	0.273
255	A	5	5	1.01	22	0.227
256	A	5	5	1.00	20	0.250
257	A	6	6	1.00	22	0.273
258	A	7	7	1.00	22	0.318
259	A	4	3	1.02	26	0.115
260	A	6	5	1.11	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	5	4	1.04	26	0.154
262	A	6	5	1.06	26	0.192
263	A	8	7	1.08	26	0.269
264	A	4	4	1.00	26	0.154
265	A	5	5	1.05	24	0.208
266	A	4	4	0.99	26	0.154
267	A	2	2	1.00	26	0.077
268	A	2	2	1.00	23	0.087
269	A	2	2	1.00	26	0.077
270	A	4	3	1.00	26	0.115
271	A	5	4	1.00	26	0.154
272	A	4	3	1.00	26	0.115
273	A	6	5	1.06	26	0.192
274	A	8	7	1.08	26	0.269
275	A	5	5	1.05	26	0.192
276	A	1	1	1.00	24	0.042
277	A	4	4	1.00	26	0.154
278	A	2	2	1.00	26	0.077
279	A	2	2	1.00	23	0.087
280	A	2	2	1.00	26	0.077
281	A	1	1	1.00	22	0.045
282	A	4	3	1.00	27	0.111
283	A	4	3	1.02	27	0.111
284	A	6	5	1.12	27	0.185
285	A	5	4	1.04	27	0.148
286	A	6	5	1.07	27	0.185
287	A	8	7	1.14	27	0.259
288	A	10	9	1.14	27	0.333
289	A	6	6	1.01	27	0.222
290	A	4	4	1.00	27	0.148
291	A	13	12	1.05	25	0.480
292	A	4	4	0.99	27	0.148
293	A	6	6	1.00	27	0.222
294	A	8	8	1.01	27	0.296

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	4	3	1.03	27	0.111
296	A	4	3	1.04	27	0.111
297	A	7	6	1.12	27	0.222
298	A	6	5	1.10	27	0.185
299	A	9	8	1.07	27	0.296
300	A	8	7	1.10	27	0.259
301	A	10	9	1.12	27	0.333
302	A	8	8	1.02	27	0.296
303	A	6	6	1.01	27	0.222
304	A	4	4	1.00	25	0.160
305	A	4	4	0.99	27	0.148
306	A	6	6	1.00	27	0.222
307	A	9	9	1.01	27	0.333
308	A	4	3	1.02	27	0.111
309	A	4	3	1.03	27	0.111
310	A	5	4	1.04	27	0.148
311	A	4	3	1.00	27	0.111
312	A	6	5	1.07	27	0.185
313	A	8	7	1.14	27	0.259
314	A	10	9	1.14	27	0.333
315	A	4	4	1.00	27	0.148
316	A	13	12	1.06	27	0.444
317	A	9	8	1.00	25	0.320
318	A	4	4	1.00	27	0.148
319	A	6	6	1.00	27	0.222
320	A	8	8	1.01	27	0.296
321	A	2	2	1.00	27	0.074
322	A	2	2	1.00	24	0.083
323	A	2	2	1.00	27	0.074
324	A	2	2	1.00	27	0.074
325	A	4	3	1.04	27	0.111
326	A	4	3	1.03	27	0.111
327	A	5	4	1.08	27	0.148
328	A	5	4	1.07	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	9	8	1.16	27	0.296
330	A	10	9	1.18	27	0.333
331	A	12	11	1.16	27	0.407
332	A	4	4	1.00	27	0.148
333	A	4	4	1.00	27	0.148
334	A	4	4	1.00	25	0.160
335	A	6	6	1.01	27	0.222
336	A	8	8	1.02	27	0.296
337	A	10	10	1.02	27	0.370
338	A	2	2	1.00	27	0.074
339	A	2	2	1.00	24	0.083
340	A	2	2	1.00	27	0.074
341	A	2	2	1.00	27	0.074
342	A	5	5	1.10	33	0.152
343	A	5	5	1.10	35	0.143
344	A	6	6	1.10	35	0.171
345	A	6	6	1.10	37	0.162
346	A	5	5	1.09	33	0.152
347	A	5	5	1.09	35	0.143
348	A	5	5	1.09	36	0.139
349	A	5	5	1.09	36	0.139
350	A	1	1	1.00	33	0.030
351	A	1	1	1.00	35	0.029
352	A	1	1	1.00	35	0.029
353	A	1	1	1.00	37	0.027
354	A	1	1	1.00	33	0.030
355	A	1	1	1.00	35	0.029
356	A	1	1	1.00	36	0.028
357	A	1	1	1.00	36	0.028
358	A	4	3	1.00	24	0.125
359	A	6	5	1.05	24	0.208
360	A	5	4	1.00	24	0.167
361	A	5	4	1.00	24	0.167
362	A	7	6	1.05	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
363	A	2	2	1.00	24	0.083
364	A	2	2	1.00	22	0.091
365	A	2	2	1.00	21	0.095
366	A	2	2	1.00	24	0.083
367	A	2	2	1.00	24	0.083
368	A	4	3	1.00	24	0.125
369	A	7	6	1.08	24	0.250
370	A	6	5	1.05	24	0.208
371	A	6	5	1.04	24	0.208
372	A	7	6	1.05	24	0.250
373	A	2	2	1.00	24	0.083
374	A	2	2	1.00	22	0.091
375	A	2	2	1.00	21	0.095
376	A	2	2	1.00	24	0.083
377	A	2	2	1.00	24	0.083
378	A	4	3	1.00	24	0.125
379	A	5	4	1.00	24	0.167
380	A	4	3	1.00	24	0.125
381	A	5	4	1.00	24	0.167
382	A	7	6	1.09	24	0.250
383	A	2	2	1.00	24	0.083
384	A	2	2	1.00	22	0.091
385	A	2	2	1.00	21	0.095
386	A	2	2	1.00	24	0.083
387	A	2	2	1.00	24	0.083
388	A	4	3	0.98	24	0.125
389	A	5	4	1.00	24	0.167
390	A	5	4	1.00	24	0.167
391	A	6	5	1.19	24	0.208
392	A	9	8	1.19	24	0.333
393	A	2	2	1.00	24	0.083
394	A	2	2	1.00	22	0.091
395	A	2	2	1.00	21	0.095
396	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
397	A	2	2	1.00	24	0.083
398	A	10	9	1.13	27	0.333
399	A	8	7	1.09	27	0.259
400	A	6	5	1.10	27	0.185
401	A	5	4	1.02	27	0.148
402	A	8	7	1.14	27	0.259
403	A	10	9	1.15	27	0.333
404	A	12	11	1.12	27	0.407
405	A	6	6	1.01	27	0.222
406	A	4	4	1.00	27	0.148
407	A	4	4	0.99	25	0.160
408	A	6	6	1.01	27	0.222
409	A	9	9	1.02	27	0.333
410	A	11	11	1.02	27	0.407
411	A	11	10	1.17	27	0.370
412	A	9	8	1.13	27	0.296
413	A	7	6	1.15	27	0.222
414	A	6	5	1.09	27	0.185
415	A	8	7	1.11	27	0.259
416	A	10	9	1.12	27	0.333
417	A	12	11	1.11	27	0.407
418	A	8	8	1.01	27	0.296
419	A	6	6	1.01	27	0.222
420	A	4	4	0.99	25	0.160
421	A	6	6	1.05	27	0.222
422	A	9	9	1.01	27	0.333
423	A	11	11	1.02	27	0.407
424	A	7	6	1.11	27	0.222
425	A	7	6	1.12	27	0.222
426	A	5	4	1.06	27	0.148
427	A	5	4	1.06	27	0.148
428	A	8	7	1.14	27	0.259
429	A	10	9	1.15	27	0.333
430	A	12	11	1.12	27	0.407

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
431	A	4	4	1.00	27	0.148
432	A	4	4	1.00	27	0.148
433	A	4	4	1.00	25	0.160
434	A	6	6	1.01	27	0.222
435	A	8	8	1.02	27	0.296
436	A	10	10	1.02	27	0.370
437	A	2	2	1.00	27	0.074
438	A	2	2	1.00	27	0.074
439	A	2	2	1.00	24	0.083
440	A	2	2	1.00	27	0.074
441	A	2	2	1.00	27	0.074
442	A	7	6	1.13	27	0.222
443	A	7	6	1.14	27	0.222
444	A	6	5	1.14	27	0.185
445	A	6	5	1.14	27	0.185
446	A	10	9	1.19	27	0.333
447	A	12	11	1.17	27	0.407
448	A	14	13	1.14	27	0.481
449	A	6	6	1.00	27	0.222
450	A	6	6	1.01	27	0.222
451	A	6	6	1.01	25	0.240
452	A	8	8	1.02	27	0.296
453	A	10	10	1.03	27	0.370
454	A	12	12	1.03	27	0.444
455	C	2	2	0.26	27	0.074
456	A	2	2	1.00	27	0.074
457	A	2	2	1.00	24	0.083
458	A	2	2	1.00	27	0.074
459	A	2	2	1.00	27	0.074
460	A	8	7	1.02	24	0.292
461	A	6	5	0.97	24	0.208
462	A	5	4	1.00	24	0.167
463	A	7	6	1.04	24	0.250
464	A	9	8	1.06	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
465	A	2	2	1.00	24	0.083
466	A	2	2	1.00	22	0.091
467	A	2	2	1.00	21	0.095
468	A	2	2	1.00	24	0.083
469	A	2	2	1.00	24	0.083
470	A	9	8	1.03	24	0.333
471	A	7	6	1.00	24	0.250
472	A	6	5	1.11	24	0.208
473	A	7	6	1.07	24	0.250
474	A	8	7	1.16	24	0.292
475	A	2	2	1.00	24	0.083
476	A	2	2	1.00	22	0.091
477	A	2	2	1.00	21	0.095
478	A	2	2	1.00	24	0.083
479	A	2	2	1.00	24	0.083
480	A	7	6	1.17	24	0.250
481	A	5	4	0.99	24	0.167
482	A	5	4	0.99	24	0.167
483	A	7	6	1.17	24	0.250
484	A	8	7	1.15	24	0.292
485	A	2	2	1.00	24	0.083
486	A	2	2	1.00	22	0.091
487	A	2	2	1.00	21	0.095
488	A	2	2	1.00	24	0.083
489	A	2	2	1.00	24	0.083
490	A	7	6	1.08	24	0.250
491	A	6	5	1.03	24	0.208
492	A	6	5	1.16	24	0.208
493	A	9	8	1.21	24	0.333
494	A	10	9	1.17	24	0.375
495	A	2	2	1.00	24	0.083
496	A	2	2	1.00	22	0.091
497	A	2	2	1.00	21	0.095
498	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
499	A	2	2	1.00	24	0.083
500	A	3	3	1.00	24	0.125
501	A	3	3	1.00	24	0.125
502	A	3	3	1.00	24	0.125
503	A	3	3	1.00	24	0.125
504	A	3	3	1.00	24	0.125
505	A	3	3	1.00	24	0.125
506	A	5	4	0.99	26	0.154
507	A	4	3	1.00	26	0.115
508	A	4	3	1.00	26	0.115
509	A	5	4	0.99	26	0.154
510	A	3	3	1.00	26	0.115
511	A	3	3	1.00	26	0.115
512	A	3	3	1.00	24	0.125
513	A	3	3	1.00	23	0.130
514	A	3	3	1.00	26	0.115
515	A	3	3	1.00	26	0.115
516	A	8	7	0.94	26	0.269
517	A	6	5	1.06	26	0.192
518	A	8	7	1.08	26	0.269
519	A	7	6	0.98	26	0.231
520	A	5	4	1.08	26	0.154
521	A	8	7	1.07	26	0.269
522	A	7	6	0.99	26	0.231
523	A	5	4	1.10	26	0.154
524	A	8	7	0.95	24	0.292
525	A	7	6	0.99	24	0.250
526	A	5	4	0.99	24	0.167
527	A	9	8	0.91	26	0.308
528	A	7	6	1.03	26	0.231
529	A	9	8	1.06	26	0.308
530	A	8	7	0.93	26	0.269
531	A	6	5	1.05	26	0.192
532	A	9	8	1.06	26	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
533	A	8	7	0.97	26	0.269
534	A	6	5	1.08	26	0.192
535	A	10	9	0.89	26	0.346
536	A	8	7	1.00	26	0.269
537	A	10	9	1.04	26	0.346
538	A	9	8	0.90	26	0.308
539	A	7	6	1.02	26	0.231
540	A	10	9	1.05	26	0.346
541	A	9	8	0.95	26	0.308
542	A	7	6	1.06	26	0.231
543	A	7	6	1.00	26	0.231
544	A	5	4	1.09	26	0.154
545	A	7	6	1.09	26	0.231
546	A	6	5	1.01	26	0.192
547	A	4	3	1.12	26	0.115
548	A	7	6	1.09	26	0.231
549	A	6	5	1.01	26	0.192
550	A	4	3	1.12	26	0.115
551	A	7	6	1.00	26	0.231
552	A	5	4	1.09	26	0.154
553	A	7	6	1.10	26	0.231
554	A	6	5	1.01	26	0.192
555	A	4	3	1.11	26	0.115
556	A	8	7	1.08	26	0.269
557	A	2	2	1.00	26	0.077
558	A	5	4	1.11	26	0.154
559	A	7	6	1.04	26	0.231
560	A	5	4	1.09	26	0.154
561	A	8	7	1.08	26	0.269
562	A	2	2	1.00	26	0.077
563	A	5	4	1.09	26	0.154
564	A	9	8	1.07	26	0.308
565	A	3	3	1.00	26	0.115
566	A	6	5	1.09	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
567	A	5	4	0.92	28	0.143
568	A	5	4	0.94	28	0.143
569	A	9	8	0.99	28	0.286
570	A	8	7	0.98	28	0.250
571	A	8	7	0.99	28	0.250
572	A	11	10	1.04	28	0.357
573	A	12	11	0.94	28	0.393
574	A	6	6	0.93	28	0.214
575	A	4	4	0.94	28	0.143
576	A	4	4	1.03	26	0.154
577	A	3	3	0.98	28	0.107
578	A	5	5	1.00	28	0.179
579	A	7	7	0.99	28	0.250
580	A	10	10	1.03	28	0.357
581	A	21	20	1.11	28	0.714
582	A	19	18	1.11	28	0.643
583	A	11	10	0.99	25	0.400
584	A	19	18	1.11	28	0.643
585	A	22	21	1.11	28	0.750
586	A	5	4	0.92	28	0.143
587	A	5	4	0.94	28	0.143
588	A	9	8	0.99	28	0.286
589	A	8	7	0.98	28	0.250
590	A	8	7	0.99	28	0.250
591	A	11	10	1.05	28	0.357
592	A	12	11	0.94	28	0.393
593	A	7	7	1.00	28	0.250
594	A	5	5	1.02	28	0.179
595	A	4	4	1.02	25	0.160
596	A	3	3	0.98	28	0.107
597	A	5	5	1.01	28	0.179
598	A	7	7	1.00	28	0.250
599	A	10	10	1.03	28	0.357
600	A	6	6	0.98	28	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
601	A	4	4	0.97	28	0.143
602	A	17	16	1.01	26	0.615
603	A	4	4	0.97	28	0.143
604	A	7	7	0.97	28	0.250
605	A	4	3	1.03	22	0.136
606	A	4	3	1.03	22	0.136
607	A	4	3	1.04	22	0.136
608	A	7	6	1.04	22	0.273
609	A	6	5	1.05	22	0.227
610	A	9	8	1.04	22	0.364
611	A	11	10	1.08	22	0.455
612	A	4	4	1.06	22	0.182
613	A	3	3	1.00	22	0.136
614	A	1	1	1.00	19	0.053
615	A	3	3	1.00	22	0.136
616	A	5	5	1.04	22	0.227
617	A	6	6	1.10	22	0.273
618	A	16	15	1.09	22	0.682
619	A	14	13	1.06	22	0.591
620	A	12	11	1.07	20	0.550
621	A	4	4	1.00	22	0.182
622	A	6	6	1.00	22	0.273
623	A	4	3	1.03	22	0.136
624	A	4	3	1.04	22	0.136
625	A	7	6	1.04	22	0.273
626	A	6	5	1.05	22	0.227
627	A	9	8	1.04	22	0.364
628	A	11	10	1.06	22	0.455
629	A	3	3	1.01	22	0.136
630	A	3	3	1.00	22	0.136
631	A	1	1	1.00	20	0.050
632	A	3	3	1.00	22	0.136
633	A	5	5	1.04	22	0.227
634	A	12	11	1.08	22	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
635	A	15	14	1.07	22	0.636
636	A	13	12	1.08	19	0.632
637	A	13	12	1.07	22	0.545
638	A	4	3	1.04	22	0.136
639	A	4	3	1.03	22	0.136
640	A	4	3	1.03	22	0.136
641	A	7	6	1.07	22	0.273
642	A	7	6	1.06	22	0.273
643	A	10	9	1.08	22	0.409
644	A	12	11	1.14	22	0.500
645	A	5	5	1.07	22	0.227
646	A	4	4	1.05	22	0.182
647	A	2	2	1.02	22	0.091
648	A	2	2	1.03	19	0.105
649	A	4	4	1.02	22	0.182
650	A	6	6	1.06	22	0.273
651	A	8	8	1.10	22	0.364
652	A	5	5	1.00	22	0.227
653	A	3	3	1.01	22	0.136
654	A	3	3	1.01	22	0.136
655	A	16	15	1.07	20	0.750
656	A	4	4	1.00	22	0.182
657	A	6	6	1.00	22	0.273
658	A	4	3	1.02	24	0.125
659	A	4	3	1.02	24	0.125
660	A	8	7	1.11	24	0.292
661	A	7	6	1.13	24	0.250
662	A	9	8	1.01	24	0.333
663	A	12	11	0.98	24	0.458
664	A	13	12	0.93	24	0.500
665	A	5	5	1.06	24	0.208
666	A	3	3	1.02	24	0.125
667	A	3	3	1.03	22	0.136
668	A	3	3	1.08	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
669	A	4	4	1.09	24	0.167
670	A	5	5	1.08	24	0.208
671	A	7	7	1.11	24	0.292
672	A	2	2	1.00	24	0.083
673	A	2	2	1.00	24	0.083
674	A	2	2	1.00	21	0.095
675	A	2	2	1.00	24	0.083
676	A	2	2	1.00	24	0.083
677	A	4	3	1.02	24	0.125
678	A	4	3	1.02	24	0.125
679	A	8	7	1.11	24	0.292
680	A	7	6	1.12	24	0.250
681	A	9	8	1.01	24	0.333
682	A	12	11	0.97	24	0.458
683	A	13	12	0.93	24	0.500
684	A	6	6	1.00	24	0.250
685	A	4	4	1.00	24	0.167
686	A	3	3	1.02	21	0.143
687	A	3	3	1.08	24	0.125
688	A	4	4	1.09	24	0.167
689	A	6	6	1.09	24	0.250
690	A	7	7	1.11	24	0.292
691	A	2	2	1.00	24	0.083
692	A	2	2	1.00	24	0.083
693	A	2	2	1.00	22	0.091
694	A	2	2	1.00	24	0.083
695	A	2	2	1.00	24	0.083
696	A	4	3	1.02	24	0.125
697	A	9	8	1.13	24	0.333
698	A	8	7	1.13	24	0.292
699	A	10	9	1.06	24	0.375
700	A	13	12	0.96	24	0.500
701	A	14	13	0.89	24	0.542
702	A	6	6	1.04	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
703	A	4	4	1.02	22	0.182
704	A	3	3	1.03	24	0.125
705	A	4	4	1.11	24	0.167
706	A	7	7	1.09	24	0.292
707	A	7	7	1.09	24	0.292
708	A	10	10	1.11	24	0.417
709	A	2	2	1.00	24	0.083
710	A	2	2	1.00	24	0.083
711	A	2	2	1.00	21	0.095
712	A	2	2	1.00	24	0.083
713	A	2	2	1.00	24	0.083
714	A	4	3	1.01	24	0.125
715	A	4	3	1.02	24	0.125
716	A	4	3	1.02	24	0.125
717	A	7	6	1.05	24	0.250
718	A	6	5	1.03	24	0.208
719	A	9	8	0.99	24	0.333
720	A	11	10	0.96	24	0.417
721	A	4	4	1.00	24	0.167
722	A	3	3	0.98	24	0.125
723	A	1	1	1.00	21	0.048
724	A	3	3	1.02	24	0.125
725	A	5	5	1.06	24	0.208
726	A	6	6	1.07	24	0.250
727	A	2	2	1.00	24	0.083
728	A	2	2	1.00	24	0.083
729	A	2	2	1.00	22	0.091
730	A	2	2	1.00	24	0.083
731	A	2	2	1.00	24	0.083
732	A	4	3	1.02	24	0.125
733	A	4	3	1.02	24	0.125
734	A	7	6	1.06	24	0.250
735	A	6	5	1.03	24	0.208
736	A	9	8	0.99	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
737	A	11	10	0.96	24	0.417
738	A	3	3	1.04	24	0.125
739	A	3	3	0.99	24	0.125
740	A	1	1	1.00	22	0.045
741	A	4	4	1.03	24	0.167
742	A	5	5	1.05	24	0.208
743	A	2	2	1.00	24	0.083
744	A	2	2	1.00	24	0.083
745	A	2	2	1.00	21	0.095
746	A	2	2	1.00	24	0.083
747	A	4	3	1.01	24	0.125
748	A	4	3	1.02	24	0.125
749	A	4	3	1.02	24	0.125
750	A	7	6	1.09	24	0.250
751	A	7	6	1.11	24	0.250
752	A	11	10	1.08	24	0.417
753	A	12	11	0.96	24	0.458
754	A	6	6	1.05	24	0.250
755	A	4	4	1.09	24	0.167
756	A	3	3	1.08	24	0.125
757	A	2	2	1.06	21	0.095
758	A	5	5	1.02	24	0.208
759	A	6	6	1.01	24	0.250
760	A	8	8	1.03	24	0.333
761	A	2	2	1.00	24	0.083
762	A	2	2	1.00	24	0.083
763	A	2	2	1.00	24	0.083
764	A	2	2	1.00	22	0.091
765	A	2	2	1.00	24	0.083
766	A	2	2	1.00	24	0.083
767	A	4	3	0.97	22	0.136
768	A	4	3	0.94	22	0.136
769	A	4	3	0.98	22	0.136
770	A	4	3	0.98	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
771	A	4	3	1.02	22	0.136
772	A	4	3	0.99	22	0.136
773	A	7	6	1.18	22	0.273
774	A	5	4	1.08	22	0.182
775	A	4	3	0.99	22	0.136
776	A	4	3	0.99	20	0.150
777	A	6	5	1.09	22	0.227
778	A	7	6	1.19	22	0.273
779	A	11	10	0.98	22	0.455
780	A	10	9	0.95	22	0.409
781	A	10	9	0.95	22	0.409
782	A	10	9	0.95	22	0.409
783	A	10	9	0.95	19	0.474
784	A	4	4	1.04	22	0.182
785	A	12	11	0.98	22	0.500
786	A	5	5	1.09	22	0.227
787	A	6	5	1.05	24	0.208
788	A	8	7	1.07	24	0.292
789	A	5	4	1.00	24	0.167
790	A	7	6	0.98	22	0.273
791	A	5	4	1.00	24	0.167
792	A	7	6	1.00	24	0.250
793	A	7	6	1.05	24	0.250
794	A	8	7	1.07	24	0.292
795	A	3	3	1.25	24	0.125
796	A	9	9	1.43	24	0.375
797	A	11	11	1.39	24	0.458
798	A	8	8	1.42	21	0.381
799	A	4	4	1.26	24	0.167
800	A	10	10	1.41	24	0.417
801	A	5	4	1.00	28	0.143
802	A	5	4	1.00	28	0.143
803	A	5	4	1.00	28	0.143
804	A	5	4	1.00	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
805	A	4	3	1.00	24	0.125
806	A	5	4	1.00	24	0.167
807	A	4	3	1.00	24	0.125
808	A	5	4	1.00	24	0.167
809	A	7	6	1.09	24	0.250
810	A	8	7	1.09	24	0.292
811	A	7	6	0.99	24	0.250
812	A	4	3	1.00	22	0.136
813	A	7	6	1.00	24	0.250
814	A	8	7	1.10	24	0.292
815	A	9	9	1.15	24	0.375
816	A	8	8	1.50	24	0.333
817	A	6	6	1.35	21	0.286
818	A	10	10	1.46	24	0.417
819	A	11	11	1.35	24	0.458
820	A	6	6	1.32	24	0.250
821	A	4	4	1.20	24	0.167
822	A	7	6	1.04	24	0.250
823	A	7	6	1.17	24	0.250
824	A	5	4	0.99	24	0.167
825	A	5	4	0.99	24	0.167
826	A	7	6	1.17	24	0.250
827	A	8	7	1.15	24	0.292
828	A	10	9	1.12	24	0.375
829	A	8	7	1.18	24	0.292
830	A	6	5	1.04	24	0.208
831	A	5	4	1.04	22	0.182
832	A	8	7	1.05	24	0.292
833	A	9	8	1.02	24	0.333
834	A	9	9	1.04	24	0.375
835	A	9	9	1.12	24	0.375
836	A	10	10	1.03	21	0.476
837	A	11	11	1.03	24	0.458
838	A	3	3	0.95	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
839	A	4	4	0.94	24	0.167
840	A	5	5	0.94	24	0.208
841	A	4	4	0.98	26	0.154
842	A	3	3	0.95	24	0.125
843	A	2	2	1.00	17	0.118
844	A	2	2	1.00	26	0.077
845	A	2	2	1.00	26	0.077
846	A	2	2	1.00	26	0.077
847	A	5	5	1.01	26	0.192
848	A	3	3	1.00	24	0.125
849	A	2	2	1.00	17	0.118
850	A	2	2	1.00	26	0.077
851	A	2	2	1.00	26	0.077
852	A	2	2	1.00	26	0.077
853	A	4	3	1.00	24	0.125
854	A	5	4	1.00	24	0.167
855	A	4	3	1.00	24	0.125
856	A	5	4	1.00	24	0.167
857	A	7	6	1.09	24	0.250
858	A	8	7	1.09	24	0.292
859	A	7	6	0.99	24	0.250
860	A	4	3	1.00	24	0.125
861	A	7	6	1.00	24	0.250
862	A	8	7	1.10	24	0.292
863	A	2	2	1.00	24	0.083
864	A	4	3	1.00	24	0.125
865	A	4	3	1.00	22	0.136
866	A	2	2	1.00	21	0.095
867	A	2	2	1.00	24	0.083
868	A	4	3	1.00	24	0.125
869	A	4	3	1.00	24	0.125
870	A	7	6	1.17	24	0.250
871	A	5	4	0.99	24	0.167
872	A	5	4	0.99	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
873	A	7	6	1.17	24	0.250
874	A	8	7	1.15	24	0.292
875	A	8	7	1.18	24	0.292
876	A	6	5	1.04	24	0.208
877	A	5	4	1.04	24	0.167
878	A	8	7	1.05	24	0.292
879	A	9	8	1.02	24	0.333
880	A	2	2	1.00	24	0.083
881	A	4	3	1.00	24	0.125
882	A	4	3	1.00	22	0.136
883	A	2	2	1.00	21	0.095
884	A	2	2	1.00	24	0.083
885	A	4	3	1.00	24	0.125
886	A	4	3	1.00	24	0.125
887	A	4	3	1.00	24	0.125
888	A	5	4	1.00	24	0.167
889	A	4	3	1.00	24	0.125
890	A	5	4	1.00	24	0.167
891	A	7	6	1.09	24	0.250
892	A	8	7	1.09	24	0.292
893	A	7	6	0.99	24	0.250
894	A	4	3	1.00	24	0.125
895	A	7	6	1.00	24	0.250
896	A	8	7	1.10	24	0.292
897	A	10	9	1.15	24	0.375
898	A	8	7	1.16	22	0.318
899	A	12	11	1.15	24	0.458
900	A	13	12	1.11	24	0.500
901	A	8	7	1.15	24	0.292
902	A	6	5	0.99	24	0.208
903	A	2	2	1.00	24	0.083
904	A	2	2	1.00	24	0.083
905	A	2	2	1.00	21	0.095
906	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
907	A	2	2	1.00	24	0.083
908	A	7	6	1.17	24	0.250
909	A	5	4	0.99	24	0.167
910	A	5	4	0.99	24	0.167
911	A	7	6	1.17	24	0.250
912	A	8	7	1.15	24	0.292
913	A	8	7	1.18	24	0.292
914	A	6	5	1.04	24	0.208
915	A	5	4	1.04	24	0.167
916	A	8	7	1.05	24	0.292
917	A	9	8	1.02	24	0.333
918	A	11	10	1.12	24	0.417
919	A	12	11	1.04	22	0.500
920	A	13	12	1.03	24	0.500
921	A	5	4	0.95	24	0.167
922	A	6	5	0.94	24	0.208
923	A	7	6	0.94	24	0.250
924	A	2	2	1.00	24	0.083
925	A	2	2	1.00	24	0.083
926	A	2	2	1.00	21	0.095
927	A	2	2	1.00	24	0.083
928	A	2	2	1.00	24	0.083
929	A	7	6	0.91	22	0.273
930	A	6	5	0.96	22	0.227
931	A	6	5	0.93	20	0.250
932	A	6	5	1.12	22	0.227
933	A	4	3	1.09	22	0.136
934	A	4	3	1.05	22	0.136
935	A	4	3	1.04	22	0.136
936	A	4	3	1.03	22	0.136
937	A	5	5	0.95	22	0.227
938	A	4	4	0.97	22	0.182
939	A	3	3	0.99	22	0.136
940	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
941	A	6	5	1.03	22	0.227
942	A	6	5	0.95	19	0.263
943	A	6	5	0.97	22	0.227
944	A	7	6	0.93	22	0.273
945	A	7	6	0.89	22	0.273
946	A	7	6	0.90	22	0.273
947	A	7	6	0.88	20	0.300
948	A	7	6	1.11	22	0.273
949	A	4	3	1.09	22	0.136
950	A	4	3	1.05	22	0.136
951	A	4	3	1.04	22	0.136
952	A	4	3	1.03	22	0.136
953	A	5	5	0.95	22	0.227
954	A	4	4	0.97	22	0.182
955	A	3	3	0.99	22	0.136
956	A	2	2	1.00	22	0.091
957	A	7	6	1.03	22	0.273
958	A	7	6	0.88	22	0.273
959	A	7	6	0.94	19	0.316
960	A	7	6	0.91	22	0.273
961	A	8	7	0.88	22	0.318
962	A	6	5	0.98	22	0.227
963	A	5	4	0.97	20	0.200
964	A	5	4	1.12	22	0.182
965	A	4	3	1.12	22	0.136
966	A	4	3	1.06	22	0.136
967	A	4	3	1.05	22	0.136
968	A	3	3	0.99	22	0.136
969	A	2	2	1.00	22	0.091
970	A	5	4	1.00	19	0.211
971	A	5	4	1.00	22	0.182
972	A	6	5	0.99	22	0.227
973	A	7	6	0.96	22	0.273
974	A	6	5	0.95	20	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
975	A	5	4	1.12	22	0.182
976	A	4	3	1.10	22	0.136
977	A	4	3	1.09	22	0.136
978	A	4	3	1.04	22	0.136
979	A	4	3	1.05	22	0.136
980	A	6	5	0.96	22	0.227
981	A	5	4	0.97	22	0.182
982	A	4	3	1.00	19	0.158
983	A	5	4	1.00	22	0.182
984	A	6	5	0.93	22	0.227
985	A	7	6	0.95	22	0.273
986	A	5	4	0.96	24	0.167
987	A	5	4	1.00	22	0.182
988	A	4	3	1.00	22	0.136
989	A	5	4	1.00	22	0.182
990	A	4	3	1.00	20	0.150
991	A	5	4	1.00	19	0.211
992	A	4	3	1.00	22	0.136
993	A	5	4	1.00	22	0.182
994	A	4	3	1.00	22	0.136
995	A	5	4	1.00	22	0.182
996	A	5	4	1.00	26	0.154
997	A	5	4	1.00	26	0.154
998	A	5	4	1.00	26	0.154
999	A	5	4	1.00	26	0.154
1000	A	5	4	1.00	26	0.154
1001	A	5	4	1.00	26	0.154
1002	A	7	6	1.06	28	0.214
1003	A	6	5	1.03	28	0.179
1004	A	5	4	1.01	28	0.143
1005	A	4	3	1.00	28	0.107
1006	A	5	4	1.03	28	0.143
1007	A	1	1	1.00	28	0.036
1008	A	2	2	1.00	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1009	A	3	3	1.05	28	0.107
1010	A	4	4	1.08	28	0.143
1011	A	6	5	1.03	28	0.179
1012	A	5	4	0.97	28	0.143
1013	A	4	3	1.00	28	0.107
1014	A	3	2	1.00	28	0.071
1015	A	1	1	1.00	28	0.036
1016	A	2	2	1.00	28	0.071
1017	A	3	3	1.05	28	0.107
1018	A	3	3	1.00	24	0.125
1019	A	3	3	1.00	22	0.136
1020	A	3	3	1.00	21	0.143
1021	A	5	4	1.01	24	0.167
1022	A	3	3	1.00	24	0.125
1023	A	4	3	1.27	18	0.167
1024	A	1	1	1.00	33	0.030
1025	A	2	2	1.00	24	0.083
1026	A	2	2	1.00	22	0.091
1027	A	2	2	1.00	20	0.100
1028	A	2	2	1.00	19	0.105
1029	A	4	3	1.00	22	0.136
1030	A	2	2	1.00	22	0.091
1031	A	2	2	1.00	22	0.091
1032	A	3	3	1.11	24	0.125
1033	A	3	3	1.13	22	0.136
1034	A	3	3	1.13	20	0.150
1035	A	3	3	1.16	19	0.158
1036	A	4	3	0.97	22	0.136
1037	A	3	3	1.11	22	0.136
1038	A	3	3	1.12	22	0.136
1039	A	4	3	0.92	26	0.115
1040	A	4	3	0.91	26	0.115
1041	A	4	3	0.92	24	0.125
1042	A	4	3	0.96	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1043	A	4	3	0.93	26	0.115
1044	A	4	3	0.92	26	0.115
1045	A	4	3	0.91	26	0.115
1046	A	4	3	0.91	26	0.115
1047	A	4	3	0.91	24	0.125
1048	A	4	3	0.93	26	0.115
1049	A	4	3	0.95	26	0.115
1050	A	4	3	0.93	26	0.115
1051	A	1	1	1.00	17	0.059
1052	A	3	2	1.00	21	0.095
1053	A	3	2	1.00	21	0.095
1054	A	3	2	1.00	25	0.080
1055	A	1	1	1.00	31	0.032
1056	A	2	2	1.12	17	0.118
1057	A	4	3	1.13	21	0.143
1058	A	4	3	1.13	21	0.143
1059	A	4	3	1.13	21	0.143
1060	A	1	1	1.00	17	0.059
1061	A	3	2	1.00	21	0.095
1062	A	3	2	1.00	21	0.095
1063	A	3	2	1.00	25	0.080
1064	A	6	5	1.00	22	0.227
1065	A	7	6	1.00	26	0.231
1066	A	8	7	0.90	30	0.233
1067	A	7	6	0.91	30	0.200
1068	A	6	5	0.93	30	0.167
1069	A	5	4	0.98	30	0.133
1070	A	5	4	0.98	30	0.133
1071	A	4	3	0.98	30	0.100
1072	A	10	9	0.82	30	0.300
1073	A	9	8	0.86	30	0.267
1074	A	8	7	0.93	30	0.233
1075	A	7	6	1.01	30	0.200
1076	A	8	7	0.98	30	0.233

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1077	A	7	6	1.11	30	0.200
1078	A	1	1	1.00	17	0.059
1079	A	1	1	1.00	27	0.037
1080	A	1	1	1.00	27	0.037
1081	A	1	1	1.00	27	0.037

CHAPTER 3

LISTING OF INTEGRALS

3.923 $\int \frac{1}{x^3\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$ 6877

3.924 $\int \frac{x^4}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$ 6885

3.925 $\int \frac{x^2}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$ 6890

3.926 $\int \frac{1}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$ 6895

3.927 $\int \frac{1}{x^2\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$ 6900

3.928 $\int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$ 6905

3.929 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx$ 6910

3.930 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx$ 6917

3.931 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx$ 6924

3.932 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx$ 6930

3.933 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx$ 6936

3.934 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx$ 6941

3.935 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx$ 6947

3.936 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$ 6953

3.937 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx$ 6959

3.938 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx$ 6966

3.939 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx$ 6973

3.940 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx$ 6979

3.941 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx$ 6984

3.942 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx$ 6990

3.943 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$ 6996

3.944 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$ 7002

3.945 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^5\,dx$ 7009

3.946 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx$ 7016

3.947 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x\,dx$ 7023

3.948 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx$ 7030

3.949 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx$ 7036

3.950 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx$ 7042

3.951 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx$ 7048

3.1 $\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx$

3.1.1 Optimal result

Integrand size = 18, antiderivative size = 33

$$\int x^2\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{6}(Ab+aB)x^6+\frac{1}{9}bBx^9$$

output
```
1/3*a*A*x^3+1/6*(A*b+B*a)*x^6+1/9*b*B*x^9
```

3.1.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{6}(Ab+aB)x^6+\frac{1}{9}bBx^9$$

input
```
Integrate[x^2*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(a*A*x^3)/3 + ((A*b + a*B)*x^6)/6 + (b*B*x^9)/9
```

3.1.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {946, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+bx^3) (A+Bx^3) \, dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3} \int (bx^3+a) (Bx^3+A) \, dx^3$$

$$\Big\downarrow \; 49$$

$$\frac{1}{3} \int (bBx^6+(Ab+aB)x^3+aA) \, dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 (aB+Ab)+aAx^3+\frac{1}{3} bBx^9\right)$$

input
```
Int[x^2*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(a*A*x^3 + ((A*b + a*B)*x^6)/2 + (b*B*x^9)/3)/3
```

3.1.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^{3}}{3}+\frac{(Ab+Ba)x^{6}}{6}+\frac{bB\,x^{9}}{9}$	28
norman	$\frac{bB\,x^{9}}{9}+\left(\frac{Ab}{6}+\frac{Ba}{6}\right)x^{6}+\frac{aA\,x^{3}}{3}$	29
gosper	$\frac{1}{9}bB\,x^{9}+\frac{1}{6}x^{6}Ab+\frac{1}{6}x^{6}Ba+\frac{1}{3}aA\,x^{3}$	30
risch	$\frac{1}{9}bB\,x^{9}+\frac{1}{6}x^{6}Ab+\frac{1}{6}x^{6}Ba+\frac{1}{3}aA\,x^{3}$	30
parallelrisch	$\frac{1}{9}bB\,x^{9}+\frac{1}{6}x^{6}Ab+\frac{1}{6}x^{6}Ba+\frac{1}{3}aA\,x^{3}$	30

input
```
int(x^2*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*A*x^3+1/6*(A*b+B*a)*x^6+1/9*b*B*x^9
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^{2}\left(a+bx^{3}\right)\left(A+Bx^{3}\right)\,dx=\frac{1}{9}\,Bbx^{9}+\frac{1}{6}\,(Ba+Ab)x^{6}+\frac{1}{3}\,Aax^{3}$$

input
```
integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/9*B*b*x^9 + 1/6*(B*a + A*b)*x^6 + 1/3*A*a*x^3
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^{2}\left(a+bx^{3}\right)\left(A+Bx^{3}\right)\,dx=\frac{Aax^{3}}{3}+\frac{Bbx^{9}}{9}+x^{6}\left(\frac{Ab}{6}+\frac{Ba}{6}\right)$$

input
```
integrate(x**2*(b*x**3+a)*(B*x**3+A),x)
```

output
```
A*a*x**3/3 + B*b*x**9/9 + x**6*(A*b/6 + B*a/6)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{9}\,Bbx^9+\frac{1}{6}\,(Ba+Ab)x^6+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/9*B*b*x^9 + 1/6*(B*a + A*b)*x^6 + 1/3*A*a*x^3
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{9}\,Bbx^9+\frac{1}{6}\,Bax^6+\frac{1}{6}\,Abx^6+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/9*B*b*x^9 + 1/6*B*a*x^6 + 1/6*A*b*x^6 + 1/3*A*a*x^3
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{B\,b\,x^9}{9}+\left(\frac{A\,b}{6}+\frac{B\,a}{6}\right)\,x^6+\frac{A\,a\,x^3}{3}$$

input
```
int(x^2*(A + B*x^3)*(a + b*x^3),x)
```

output
```
x^6*((A*b)/6 + (B*a)/6) + (A*a*x^3)/3 + (B*b*x^9)/9
```

3.2 $\int x(a+bx^3)\,(A+Bx^3)\;dx$

3.2.1 Optimal result

Integrand size = 16, antiderivative size = 33

$$\int x\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{5}(Ab+aB)x^5+\frac{1}{8}bBx^8$$

output
```
1/2*a*A*x^2+1/5*(A*b+B*a)*x^5+1/8*b*B*x^8
```

3.2.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{5}(Ab+aB)x^5+\frac{1}{8}bBx^8$$

input
```
Integrate[x*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8
```

3.2.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^4(aB+Ab)+aAx+bBx^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{8}bBx^8$$

input
```
Int[x*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8
```

3.2.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.2.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^{2}}{2}+\frac{(Ab+Ba)x^{5}}{5}+\frac{bB\,x^{8}}{8}$	28
norman	$\frac{bB\,x^{8}}{8}+\left(\frac{Ab}{5}+\frac{Ba}{5}\right)x^{5}+\frac{aA\,x^{2}}{2}$	29
gosper	$\frac{1}{8}bB\,x^{8}+\frac{1}{5}x^{5}Ab+\frac{1}{5}aB\,x^{5}+\frac{1}{2}aA\,x^{2}$	30
risch	$\frac{1}{8}bB\,x^{8}+\frac{1}{5}x^{5}Ab+\frac{1}{5}aB\,x^{5}+\frac{1}{2}aA\,x^{2}$	30
parallelrisch	$\frac{1}{8}bB\,x^{8}+\frac{1}{5}x^{5}Ab+\frac{1}{5}aB\,x^{5}+\frac{1}{2}aA\,x^{2}$	30

input
```
int(x*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*A*x^2+1/5*(A*b+B*a)*x^5+1/8*b*B*x^8
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{8}\,Bbx^8+\frac{1}{5}\,(Ba+Ab)x^5+\frac{1}{2}\,Aax^2$$

input
```
integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/8*B*b*x^8 + 1/5*(B*a + A*b)*x^5 + 1/2*A*a*x^2
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{Aax^2}{2}+\frac{Bbx^8}{8}+x^5\left(\frac{Ab}{5}+\frac{Ba}{5}\right)$$

input
```
integrate(x*(b*x**3+a)*(B*x**3+A),x)
```

output
```
A*a*x**2/2 + B*b*x**8/8 + x**5*(A*b/5 + B*a/5)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{8}\,Bbx^8+\frac{1}{5}\,(Ba+Ab)x^5+\frac{1}{2}\,Aax^2$$

input

```
integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

output

```
1/8*B*b*x^8 + 1/5*(B*a + A*b)*x^5 + 1/2*A*a*x^2
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{8}\,Bbx^8+\frac{1}{5}\,Bax^5+\frac{1}{5}\,Abx^5+\frac{1}{2}\,Aax^2$$

input

```
integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

output

```
1/8*B*b*x^8 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + 1/2*A*a*x^2
```

3.2.9 Mupad [B] (verification not implemented)

Time = 6.76 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{B\,b\,x^8}{8}+\left(\frac{A\,b}{5}+\frac{B\,a}{5}\right)\,x^5+\frac{A\,a\,x^2}{2}$$

input

```
int(x*(A + B*x^3)*(a + b*x^3),x)
```

output

```
x^5*((A*b)/5 + (B*a)/5) + (A*a*x^2)/2 + (B*b*x^8)/8
```

3.3 $\int \left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

3.3.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \left(a+bx^3\right)\left(A+Bx^3\right)\,dx = aAx+\frac{1}{4}(Ab+aB)x^4+\frac{1}{7}bBx^7$$

output
```
a*A*x+1/4*(A*b+B*a)*x^4+1/7*b*B*x^7
```

3.3.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)\left(A+Bx^3\right)\,dx = aAx+\frac{1}{4}(Ab+aB)x^4+\frac{1}{7}bBx^7$$

input
```
Integrate[(a + b*x^3)*(A + B*x^3),x]
```

output
```
a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7
```

3.3.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(x^3(aB+Ab)+aA+bBx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(aB+Ab)+aAx+\frac{1}{7}bBx^7$$

input
```
Int[(a + b*x^3)*(A + B*x^3),x]
```

output
```
a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7
```

3.3.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.3.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$aAx+\frac{(Ab+Ba)x^{4}}{4}+\frac{bB\,x^{7}}{7}$	25
norman	$\frac{bB\,x^{7}}{7}+\left(\frac{Ab}{4}+\frac{Ba}{4}\right)x^{4}+aAx$	26
gosper	$\frac{1}{7}bB\,x^{7}+\frac{1}{4}x^{4}Ab+\frac{1}{4}Ba\,x^{4}+aAx$	27
risch	$\frac{1}{7}bB\,x^{7}+\frac{1}{4}x^{4}Ab+\frac{1}{4}Ba\,x^{4}+aAx$	27
parallelrisch	$\frac{1}{7}bB\,x^{7}+\frac{1}{4}x^{4}Ab+\frac{1}{4}Ba\,x^{4}+aAx$	27

input
```
int((b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/4*(A*b+B*a)*x^4+1/7*b*B*x^7
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^3)(A+Bx^3)\,dx=\frac{1}{7}\,Bbx^7+\frac{1}{4}\,(Ba+Ab)x^4+Aax$$

input
```
integrate((b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/7*B*b*x^7 + 1/4*(B*a + A*b)*x^4 + A*a*x
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^3)(A+Bx^3)\,dx=Aax+\frac{Bbx^7}{7}+x^4\left(\frac{Ab}{4}+\frac{Ba}{4}\right)$$

input
```
integrate((b*x**3+a)*(B*x**3+A),x)
```

output
```
A*a*x + B*b*x**7/7 + x**4*(A*b/4 + B*a/4)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{7}Bbx^7+\frac{1}{4}(Ba+Ab)x^4+Aax$$

input
```
integrate((b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/7*B*b*x^7 + 1/4*(B*a + A*b)*x^4 + A*a*x
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{7}Bbx^7+\frac{1}{4}Bax^4+\frac{1}{4}Abx^4+Aax$$

input
```
integrate((b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/7*B*b*x^7 + 1/4*B*a*x^4 + 1/4*A*b*x^4 + A*a*x
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a+bx^3)\left(A+Bx^3\right)\,dx=\frac{B\,b\,x^7}{7}+\left(\frac{A\,b}{4}+\frac{B\,a}{4}\right)\,x^4+A\,a\,x$$

input
```
int((A + B*x^3)*(a + b*x^3),x)
```

output
```
x^4*((A*b)/4 + (B*a)/4) + A*a*x + (B*b*x^7)/7
```

3.4 $\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx$

3.4.1 Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx = \frac{1}{3}(Ab+aB)x^3 + \frac{1}{6}bBx^6 + aA\log(x)$$

output
```
1/3*(A*b+B*a)*x^3+1/6*b*B*x^6+a*A*ln(x)
```

3.4.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx = \frac{1}{3}(Ab+aB)x^3 + \frac{1}{6}bBx^6 + aA\log(x)$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x,x]
```

output
```
((A*b + a*B)*x^3)/3 + (b*B*x^6)/6 + a*A*Log[x]
```

3.4.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)(Bx^3+A)}{x^3}dx^3$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{3}\int \left(bBx^3+Ab+aB+\frac{aA}{x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(x^3(aB+Ab)+aA\log\left(x^3\right)+\frac{1}{2}bBx^6\right)$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x,x]
```

output
```
((A*b + a*B)*x^3 + (b*B*x^6)/2 + a*A*Log[x^3])/3
```

3.4.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.4.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result	size
norman	$\left(\frac{Ab}{3}+\frac{Ba}{3}\right)x^{3}+\frac{bB\,x^{6}}{6}+aA\ln\left(x\right)$	27
default	$\frac{bB\,x^{6}}{6}+\frac{Ab\,x^{3}}{3}+\frac{Ba\,x^{3}}{3}+aA\ln\left(x\right)$	28
parallelrisch	$\frac{bB\,x^{6}}{6}+\frac{Ab\,x^{3}}{3}+\frac{Ba\,x^{3}}{3}+aA\ln\left(x\right)$	28
risch	$\frac{bB\,x^{6}}{6}+\frac{Ab\,x^{3}}{3}+\frac{Ba\,x^{3}}{3}+\frac{b\,A^{2}}{6B}+\frac{Aa}{3}+\frac{B\,a^{2}}{6b}+aA\ln\left(x\right)$	50

input
```
int((b*x^3+a)*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/3*A*b+1/3*B*a)*x^3+1/6*b*B*x^6+a*A*ln(x)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx=\frac{1}{6}\,Bbx^{6}+\frac{1}{3}\,(Ba+Ab)x^{3}+Aa\log\left(x\right)$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="fricas")
```

output
```
1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + A*a*log(x)
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx = Aa\log{(x)} + \frac{Bbx^6}{6} + x^3\left(\frac{Ab}{3} + \frac{Ba}{3}\right)$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x,x)
```

output
```
A*a*log(x) + B*b*x**6/6 + x**3*(A*b/3 + B*a/3)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx = \frac{1}{6}\,Bbx^6 + \frac{1}{3}\,(Ba+Ab)x^3 + \frac{1}{3}\,Aa\log\left(x^3\right)$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="maxima")
```

output
```
1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + 1/3*A*a*log(x^3)
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx = \frac{1}{6}\,Bbx^6 + \frac{1}{3}\,Bax^3 + \frac{1}{3}\,Abx^3 + Aa\log\left(|x|\right)$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="giac")
```

output
```
1/6*B*b*x^6 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + A*a*log(abs(x))
```

3.4.9 Mupad [B] (verification not implemented)

Time = 6.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{x}\,dx = x^3\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)+\frac{B\,b\,x^6}{6}+A\,a\,\ln\left(x\right)$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x,x)
```

output
```
x^3*((A*b)/3 + (B*a)/3) + (B*b*x^6)/6 + A*a*log(x)
```

3.5 $\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx$

3.5.1 Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx = -\frac{aA}{x}+\frac{1}{2}(Ab+aB)x^2+\frac{1}{5}bBx^5$$

output

```
-a*A/x+1/2*(A*b+B*a)*x^2+1/5*b*B*x^5
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx = -\frac{aA}{x}+\frac{1}{2}(Ab+aB)x^2+\frac{1}{5}bBx^5$$

input

```
Integrate[((a + b*x^3)*(A + B*x^3))/x^2,x]
```

output

```
-((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5
```

3.5.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)\left(A+Bx^3\right)}{x^2}\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x(aB+Ab)+\frac{aA}{x^2}+bBx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2(aB+Ab)-\frac{aA}{x}+\frac{1}{5}bBx^5$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^2,x]
```

output
```
-((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5
```

3.5.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.5.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{bB\,x^{5}}{5}+\frac{Ab\,x^{2}}{2}+\frac{Ba\,x^{2}}{2}-\frac{aA}{x}$	30
norman	$\frac{\frac{bB\,x^{6}}{5}+\left(\frac{Ab}{2}+\frac{Ba}{2}\right)x^{3}-Aa}{x}$	30
risch	$\frac{bB\,x^{5}}{5}+\frac{Ab\,x^{2}}{2}+\frac{Ba\,x^{2}}{2}-\frac{aA}{x}$	30
gosper	$-\frac{-2bB\,x^{6}-5Ab\,x^{3}-5Ba\,x^{3}+10Aa}{10x}$	32
parallelrisch	$\frac{2bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}-10Aa}{10x}$	32

input
```
int((b*x^3+a)*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*b*B*x^5+1/2*A*b*x^2+1/2*B*a*x^2-a*A/x
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{x^2}\,dx = \frac{2\,Bbx^6+5\,(Ba+Ab)x^3-10\,Aa}{10\,x}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="fricas")
```

output
```
1/10*(2*B*b*x^6 + 5*(B*a + A*b)*x^3 - 10*A*a)/x
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{x^2}\,dx = -\frac{Aa}{x}+\frac{Bbx^5}{5}+x^2\left(\frac{Ab}{2}+\frac{Ba}{2}\right)$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**2,x)
```

output
```
-A*a/x + B*b*x**5/5 + x**2*(A*b/2 + B*a/2)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx = \frac{1}{5}\,Bbx^5 + \frac{1}{2}\,(Ba+Ab)x^2 - \frac{Aa}{x}$$

input

```
integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="maxima")
```

output

```
1/5*B*b*x^5 + 1/2*(B*a + A*b)*x^2 - A*a/x
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx = \frac{1}{5}\,Bbx^5 + \frac{1}{2}\,Bax^2 + \frac{1}{2}\,Abx^2 - \frac{Aa}{x}$$

input

```
integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="giac")
```

output

```
1/5*B*b*x^5 + 1/2*B*a*x^2 + 1/2*A*b*x^2 - A*a/x
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx = x^2\left(\frac{A\,b}{2} + \frac{B\,a}{2}\right) - \frac{A\,a}{x} + \frac{B\,b\,x^5}{5}$$

input

```
int(((A + B*x^3)*(a + b*x^3))/x^2,x)
```

output

```
x^2*((A*b)/2 + (B*a)/2) - (A*a)/x + (B*b*x^5)/5
```

3.6 $\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx$

3.6.1 Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = -\frac{aA}{2x^2} + (Ab+aB)x + \frac{1}{4}bBx^4$$

output

```
-1/2*a*A/x^2+(A*b+B*a)*x+1/4*b*B*x^4
```

3.6.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = -\frac{aA}{2x^2} + (Ab+aB)x + \frac{1}{4}bBx^4$$

input

```
Integrate[((a + b*x^3)*(A + B*x^3))/x^3,x]
```

output

```
-1/2*(a*A)/x^2 + (A*b + a*B)*x + (b*B*x^4)/4
```

3.6.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx$$

$$\Big\downarrow \; 950$$

$$\int \left(Ab\left(\frac{aB}{Ab}+1\right)+\frac{aA}{x^3}+bBx^3\right)dx$$

$$\Big\downarrow \; 2009$$

$$x(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{4}bBx^4$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(a*A)/x^2 + (A*b + a*B)*x + (b*B*x^4)/4
```

3.6.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.6.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{bB\,x^{4}}{4}+Abx+Bax-\frac{aA}{2x^{2}}$	24
risch	$\frac{bB\,x^{4}}{4}+Abx+Bax-\frac{aA}{2x^{2}}$	24
norman	$\frac{\frac{bB\,x^{6}}{4}+(Ab+Ba)x^{3}-\frac{Aa}{2}}{x^{2}}$	28
parallelrisch	$\frac{bB\,x^{6}+4Ab\,x^{3}+4Ba\,x^{3}-2Aa}{4x^{2}}$	31
gosper	$-\frac{-bB\,x^{6}-4Ab\,x^{3}-4Ba\,x^{3}+2Aa}{4x^{2}}$	32

input
```
int((b*x^3+a)*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*b*B*x^4+A*b*x+B*a*x-1/2*a*A/x^2
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{x^3}\,dx=\frac{Bbx^6+4\,(Ba+Ab)x^3-2\,Aa}{4\,x^2}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="fricas")
```

output
```
1/4*(B*b*x^6 + 4*(B*a + A*b)*x^3 - 2*A*a)/x^2
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{x^3}\,dx=-\frac{Aa}{2x^2}+\frac{Bbx^4}{4}+x(Ab+Ba)$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**3,x)
```

output
```
-A*a/(2*x**2) + B*b*x**4/4 + x*(A*b + B*a)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = \frac{1}{4}\,Bbx^4 + (Ba+Ab)x - \frac{Aa}{2\,x^2}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="maxima")
```

output
```
1/4*B*b*x^4 + (B*a + A*b)*x - 1/2*A*a/x^2
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = \frac{1}{4}\,Bbx^4 + Bax + Abx - \frac{Aa}{2\,x^2}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="giac")
```

output
```
1/4*B*b*x^4 + B*a*x + A*b*x - 1/2*A*a/x^2
```

3.6.9 Mupad [B] (verification not implemented)

Time = 6.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = x\,(A\,b + B\,a) - \frac{A\,a}{2\,x^2} + \frac{B\,b\,x^4}{4}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^3,x)
```

output
```
x*(A*b + B*a) - (A*a)/(2*x^2) + (B*b*x^4)/4
```

3.7 $\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx$

3.7.1 Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx=-\frac{aA}{3x^3}+\frac{1}{3}bBx^3+(Ab+aB)\log(x)$$

output
```
-1/3*a*A/x^3+1/3*b*B*x^3+(A*b+B*a)*ln(x)
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx=-\frac{aA}{3x^3}+\frac{1}{3}bBx^3+(Ab+aB)\log(x)$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 + (b*B*x^3)/3 + (A*b + a*B)*Log[x]
```

3.7.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)\left(A+Bx^3\right)}{x^4}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)\left(Bx^3+A\right)}{x^6}dx^3$$

$$\Big\downarrow \; 85$$

$$\frac{1}{3}\int \left(\frac{aA}{x^6}+bB+\frac{Ab+aB}{x^3}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(\log\left(x^3\right)(aB+Ab)-\frac{aA}{x^3}+bBx^3\right)$$

input

```
Int[((a + b*x^3)*(A + B*x^3))/x^4,x]
```

output

```
(-((a*A)/x^3) + b*B*x^3 + (A*b + a*B)*Log[x^3])/3
```

3.7.3.1 Defintions of rubi rules used

rule 85

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.7.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{aA}{3x^3}+\frac{bB\,x^3}{3}+(Ab+Ba)\ln(x)$	26
risch	$-\frac{aA}{3x^3}+\frac{bB\,x^3}{3}+A\ln(x)\,b+aB\ln(x)$	26
norman	$\frac{-\frac{Aa}{3}+\frac{bB\,x^6}{3}}{x^3}+(Ab+Ba)\ln(x)$	28
parallelrisch	$\frac{bB\,x^6+3A\ln(x)x^3b+3B\ln(x)x^3a-Aa}{3x^3}$	35

input
```
int((b*x^3+a)*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*A/x^3+1/3*b*B*x^3+(A*b+B*a)*ln(x)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int\frac{(a+bx^3)\,(A+Bx^3)}{x^4}\,dx=\frac{Bbx^6+3\,(Ba+Ab)x^3\log(x)-Aa}{3\,x^3}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="fricas")
```

output
```
1/3*(B*b*x^6 + 3*(B*a + A*b)*x^3*log(x) - A*a)/x^3
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx = -\frac{Aa}{3x^3} + \frac{Bbx^3}{3} + (Ab+Ba)\log(x)$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**4,x)
```

output
```
-A*a/(3*x**3) + B*b*x**3/3 + (A*b + B*a)*log(x)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx = \frac{1}{3}\,Bbx^3 + \frac{1}{3}\,(Ba+Ab)\log\left(x^3\right) - \frac{Aa}{3\,x^3}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="maxima")
```

output
```
1/3*B*b*x^3 + 1/3*(B*a + A*b)*log(x^3) - 1/3*A*a/x^3
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx = \frac{1}{3}\,Bbx^3 + (Ba+Ab)\log(|x|) - \frac{Bax^3 + Abx^3 + Aa}{3\,x^3}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="giac")
```

output
```
1/3*B*b*x^3 + (B*a + A*b)*log(abs(x)) - 1/3*(B*a*x^3 + A*b*x^3 + A*a)/x^3
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx = \ln(x)\ (A\,b+B\,a)-\frac{A\,a}{3\,x^3}+\frac{B\,b\,x^3}{3}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^4,x)
```

output
```
log(x)*(A*b + B*a) - (A*a)/(3*x^3) + (B*b*x^3)/3
```

3.8 $\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx$

3.8.1 Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx=-\frac{aA}{4x^4}-\frac{Ab+aB}{x}+\frac{1}{2}bBx^2$$

output
```
-1/4*a*A/x^4+(-A*b-B*a)/x+1/2*b*B*x^2
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx=-\frac{aA}{4x^4}+\frac{-Ab-aB}{x}+\frac{1}{2}bBx^2$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x^5,x]
```

output
```
-1/4*(a*A)/x^4 + (-(A*b) - a*B)/x + (b*B*x^2)/2
```

3.8.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \displaystyle\int \frac{(a+bx^3)(A+Bx^3)}{x^5}dx \\ \Big\downarrow \; 950 \\ \displaystyle\int \left(\frac{aB+Ab}{x^2}+\frac{aA}{x^5}+bBx\right)dx \\ \Big\downarrow \; 2009 \\ \displaystyle -\frac{aB+Ab}{x}-\frac{aA}{4x^4}+\frac{1}{2}bBx^2 \end{array}$$

input

```
Int[((a + b*x^3)*(A + B*x^3))/x^5,x]
```

output

```
-1/4*(a*A)/x^4 - (A*b + a*B)/x + (b*B*x^2)/2
```

3.8.3.1 Defintions of rubi rules used

rule 950

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.8.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{bB\,x^2}{2}-\frac{Ab+Ba}{x}-\frac{aA}{4x^4}$	28
norman	$\frac{\frac{bB\,x^6}{2}+(-Ab-Ba)x^3-\frac{Aa}{4}}{x^4}$	30
gosper	$-\frac{-2bB\,x^6+4Ab\,x^3+4Ba\,x^3+Aa}{4x^4}$	31
risch	$\frac{bB\,x^2}{2}+\frac{(-Ab-Ba)x^3-\frac{Aa}{4}}{x^4}$	31
parallelrisch	$-\frac{-2bB\,x^6+4Ab\,x^3+4Ba\,x^3+Aa}{4x^4}$	31

input
```
int((b*x^3+a)*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*B*x^2-(A*b+B*a)/x-1/4*a*A/x^4
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{x^5}\,dx = \frac{2\,Bbx^6-4\,(Ba+Ab)x^3-Aa}{4\,x^4}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="fricas")
```

output
```
1/4*(2*B*b*x^6 - 4*(B*a + A*b)*x^3 - A*a)/x^4
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{x^5}\,dx = \frac{Bbx^2}{2}+\frac{-Aa+x^3(-4Ab-4Ba)}{4x^4}$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**5,x)
```

output
```
B*b*x**2/2 + (-A*a + x**3*(-4*A*b - 4*B*a))/(4*x**4)
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx = \frac{1}{2}\,Bbx^2 - \frac{4\,(Ba+Ab)x^3+Aa}{4\,x^4}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="maxima")
```

output
```
1/2*B*b*x^2 - 1/4*(4*(B*a + A*b)*x^3 + A*a)/x^4
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx = \frac{1}{2}\,Bbx^2 - \frac{4\,Bax^3+4\,Abx^3+Aa}{4\,x^4}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="giac")
```

output
```
1/2*B*b*x^2 - 1/4*(4*B*a*x^3 + 4*A*b*x^3 + A*a)/x^4
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx = \frac{B\,b\,x^2}{2} - \frac{(A\,b+B\,a)\;x^3+\frac{A\,a}{4}}{x^4}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^5,x)
```

output
```
(B*b*x^2)/2 - ((A*a)/4 + x^3*(A*b + B*a))/x^4
```

3.9 $\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx$

3.9.1 Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = -\frac{aA}{5x^5}-\frac{Ab+aB}{2x^2}+bBx$$

output
```
-1/5*a*A/x^5+1/2*(-A*b-B*a)/x^2+b*B*x
```

3.9.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = -\frac{aA}{5x^5}+\frac{-Ab-aB}{2x^2}+bBx$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x^6,x]
```

output
```
-1/5*(a*A)/x^5 + (-(A*b) - a*B)/(2*x^2) + b*B*x
```

3.9.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{tikzcd}
\displaystyle\int \frac{(a+bx^3)(A+Bx^3)}{x^6}dx \arrow[d, "950"] \\
\displaystyle\int \left(\frac{aB+Ab}{x^3}+\frac{aA}{x^6}+bB\right)dx \arrow[d, "2009"] \\
\displaystyle -\frac{aB+Ab}{2x^2}-\frac{aA}{5x^5}+bBx
\end{tikzcd}$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^6,x]
```

output
```
-1/5*(a*A)/x^5 - (A*b + a*B)/(2*x^2) + b*B*x
```

3.9.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.9.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$bBx-\frac{Ab+Ba}{2x^2}-\frac{aA}{5x^5}$	25
risch	$bBx+\frac{\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x^3-\frac{Aa}{5}}{x^5}$	28
norman	$\frac{bB\,x^6+\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x^3-\frac{Aa}{5}}{x^5}$	29
gosper	$-\frac{-10bB\,x^6+5Ab\,x^3+5Ba\,x^3+2Aa}{10x^5}$	32
parallelrisch	$-\frac{-10bB\,x^6+5Ab\,x^3+5Ba\,x^3+2Aa}{10x^5}$	32

input

```
int((b*x^3+a)*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
```

output

```
b*B*x-1/2*(A*b+B*a)/x^2-1/5*a*A/x^5
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{x^6}\,dx = \frac{10\,Bbx^6-5\,(Ba+Ab)x^3-2\,Aa}{10\,x^5}$$

input

```
integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="fricas")
```

output

```
1/10*(10*B*b*x^6 - 5*(B*a + A*b)*x^3 - 2*A*a)/x^5
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{x^6}\,dx = Bbx+\frac{-2Aa+x^3(-5Ab-5Ba)}{10x^5}$$

input

```
integrate((b*x**3+a)*(B*x**3+A)/x**6,x)
```

output

```
B*b*x + (-2*A*a + x**3*(-5*A*b - 5*B*a))/(10*x**5)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = Bbx - \frac{5\,(Ba+Ab)x^3+2\,Aa}{10\,x^5}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="maxima")
```

output
```
B*b*x - 1/10*(5*(B*a + A*b)*x^3 + 2*A*a)/x^5
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = Bbx - \frac{5\,Bax^3+5\,Abx^3+2\,Aa}{10\,x^5}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="giac")
```

output
```
B*b*x - 1/10*(5*B*a*x^3 + 5*A*b*x^3 + 2*A*a)/x^5
```

3.9.9 Mupad [B] (verification not implemented)

Time = 6.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = B\,b\,x - \frac{\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)\,x^3+\frac{A\,a}{5}}{x^5}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^6,x)
```

output
```
B*b*x - ((A*a)/5 + x^3*((A*b)/2 + (B*a)/2))/x^5
```

3.10 $\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx$

3.10.1 Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = -\frac{aA}{6x^6} - \frac{Ab+aB}{3x^3} + bB\log(x)$$

output

```
-1/6*a*A/x^6+1/3*(-A*b-B*a)/x^3+b*B*ln(x)
```

3.10.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = -\frac{aA}{6x^6} + \frac{-Ab-aB}{3x^3} + bB\log(x)$$

input

```
Integrate[((a + b*x^3)*(A + B*x^3))/x^7,x]
```

output

```
-1/6*(a*A)/x^6 + (-(A*b) - a*B)/(3*x^3) + b*B*Log[x]
```

3.10.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{(bx^3+a)(Bx^3+A)}{x^9}dx^3$$

$$\Big\downarrow \; 85$$

$$\frac{1}{3}\int \left(\frac{aA}{x^9}+\frac{bB}{x^3}+\frac{Ab+aB}{x^6}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(-\frac{aB+Ab}{x^3}-\frac{aA}{2x^6}+bB\log\left(x^3\right)\right)$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^7,x]
```

output
```
(-1/2*(a*A)/x^6 - (A*b + a*B)/x^3 + b*B*Log[x^3])/3
```

3.10.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.10.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$bB\ln\left(x\right)-\frac{aA}{6x^{6}}-\frac{Ab+Ba}{3x^{3}}$	26
norman	$\frac{\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x^{3}-\frac{Aa}{6}}{x^{6}}+bB\ln\left(x\right)$	29
risch	$\frac{\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x^{3}-\frac{Aa}{6}}{x^{6}}+bB\ln\left(x\right)$	29
parallelrisch	$-\frac{-6Bb\ln(x)x^{6}+2Ab\,x^{3}+2Ba\,x^{3}+Aa}{6x^{6}}$	33

input
```
int((b*x^3+a)*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
b*B*ln(x)-1/6*a*A/x^6-1/3*(A*b+B*a)/x^3
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int\frac{(a+bx^3)\,(A+Bx^3)}{x^7}\,dx=\frac{6\,Bbx^6\log\left(x\right)-2\,(Ba+Ab)x^3-Aa}{6\,x^6}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="fricas")
```

output
```
1/6*(6*B*b*x^6*log(x) - 2*(B*a + A*b)*x^3 - A*a)/x^6
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = Bb\log(x) + \frac{-Aa + x^3(-2Ab-2Ba)}{6x^6}$$

input

```
integrate((b*x**3+a)*(B*x**3+A)/x**7,x)
```

output

```
B*b*log(x) + (-A*a + x**3*(-2*A*b - 2*B*a))/(6*x**6)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = \frac{1}{3}\,Bb\log\left(x^3\right) - \frac{2\,(Ba+Ab)x^3 + Aa}{6\,x^6}$$

input

```
integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="maxima")
```

output

```
1/3*B*b*log(x^3) - 1/6*(2*(B*a + A*b)*x^3 + A*a)/x^6
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = Bb\log(|x|) - \frac{3\,Bbx^6 + 2\,Bax^3 + 2\,Abx^3 + Aa}{6\,x^6}$$

input

```
integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="giac")
```

output

```
B*b*log(abs(x)) - 1/6*(3*B*b*x^6 + 2*B*a*x^3 + 2*A*b*x^3 + A*a)/x^6
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{x^7}\,dx = B\,b\,\ln\left(x\right)-\frac{\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)\,x^3+\frac{A\,a}{6}}{x^6}$$

input

```
int(((A + B*x^3)*(a + b*x^3))/x^7,x)
```

output

```
B*b*log(x) - ((A*a)/6 + x^3*((A*b)/3 + (B*a)/3))/x^6
```

3.11 $\int x^2(a+bx^3)^2 (A+Bx^3) \, dx$

3.11.1 Optimal result

Integrand size = 20, antiderivative size = 42

$$\int x^2 (a+bx^3)^2 (A+Bx^3) \, dx = \frac{(Ab-aB)(a+bx^3)^3}{9b^2} + \frac{B(a+bx^3)^4}{12b^2}$$

output
```
1/9*(A*b-B*a)*(b*x^3+a)^3/b^2+1/12*B*(b*x^3+a)^4/b^2
```

3.11.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x^2 (a+bx^3)^2 (A+Bx^3) \, dx = \frac{1}{36} x^3 (12a^2A + 6a(2Ab+aB)x^3 + 4b(Ab+2aB)x^6 + 3b^2Bx^9)$$

input
```
Integrate[x^2*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(x^3*(12*a^2*A + 6*a*(2*A*b + a*B)*x^3 + 4*b*(A*b + 2*a*B)*x^6 + 3*b^2*B*x
^9))/36
```

3.11.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {946, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3} \int \left(bx^3+a\right)^2 \left(Bx^3+A\right)dx^3$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3} \int \left(\frac{B\left(bx^3+a\right)^3}{b}+\frac{(Ab-aB)\left(bx^3+a\right)^2}{b}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(\frac{\left(a+bx^3\right)^3 (Ab-aB)}{3b^2}+\frac{B\left(a+bx^3\right)^4}{4b^2}\right)$$

input
```
Int[x^2*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(((A*b - a*B)*(a + b*x^3)^3)/(3*b^2) + (B*(a + b*x^3)^4)/(4*b^2))/3
```

3.11.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.11.4 Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{B\,b^2x^{12}}{12}+\frac{(b^2A+2abB)x^9}{9}+\frac{(2abA+a^2B)x^6}{6}+\frac{a^2A\,x^3}{3}$	52
norman	$\frac{B\,b^2x^{12}}{12}+\left(\frac{1}{9}b^2A+\frac{2}{9}abB\right)x^9+\left(\frac{1}{3}abA+\frac{1}{6}a^2B\right)x^6+\frac{a^2A\,x^3}{3}$	52
gosper	$\frac{1}{12}B\,b^2x^{12}+\frac{1}{9}x^9b^2A+\frac{2}{9}x^9abB+\frac{1}{3}x^6abA+\frac{1}{6}x^6a^2B+\frac{1}{3}a^2A\,x^3$	54
risch	$\frac{1}{12}B\,b^2x^{12}+\frac{1}{9}x^9b^2A+\frac{2}{9}x^9abB+\frac{1}{3}x^6abA+\frac{1}{6}x^6a^2B+\frac{1}{3}a^2A\,x^3$	54
parallelrisch	$\frac{1}{12}B\,b^2x^{12}+\frac{1}{9}x^9b^2A+\frac{2}{9}x^9abB+\frac{1}{3}x^6abA+\frac{1}{6}x^6a^2B+\frac{1}{3}a^2A\,x^3$	54

input
```
int(x^2*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/12*B*b^2*x^12+1/9*(A*b^2+2*B*a*b)*x^9+1/6*(2*A*a*b+B*a^2)*x^6+1/3*a^2*A*
x^3
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x^2\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{1}{12}\,Bb^2x^{12}+\frac{1}{9}\,\left(2\,Bab+Ab^2\right)x^9+\frac{1}{6}\,\left(Ba^2+2\,Aab\right)x^6+\frac{1}{3}\,Aa^2x^3$$

input
```
integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/12*B*b^2*x^12 + 1/9*(2*B*a*b + A*b^2)*x^9 + 1/6*(B*a^2 + 2*A*a*b)*x^6 +
1/3*A*a^2*x^3
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int x^2\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{Aa^2x^3}{3}+\frac{Bb^2x^{12}}{12}+x^9\left(\frac{Ab^2}{9}+\frac{2Bab}{9}\right)+x^6\left(\frac{Aab}{3}+\frac{Ba^2}{6}\right)$$

input
```
integrate(x**2*(b*x**3+a)**2*(B*x**3+A),x)
```

output
```
A*a**2*x**3/3 + B*b**2*x**12/12 + x**9*(A*b**2/9 + 2*B*a*b/9) + x**6*(A*a*
b/3 + B*a**2/6)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x^2\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{1}{12}\,Bb^2x^{12}+\frac{1}{9}\,\left(2\,Bab+Ab^2\right)x^9+\frac{1}{6}\,\left(Ba^2+2\,Aab\right)x^6+\frac{1}{3}\,Aa^2x^3$$

input
```
integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/12*B*b^2*x^12 + 1/9*(2*B*a*b + A*b^2)*x^9 + 1/6*(B*a^2 + 2*A*a*b)*x^6 +
1/3*A*a^2*x^3
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int x^2\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{1}{12}\,Bb^2x^{12}+\frac{2}{9}\,Babx^9+\frac{1}{9}\,Ab^2x^9+\frac{1}{6}\,Ba^2x^6+\frac{1}{3}\,Aabx^6+\frac{1}{3}\,Aa^2x^3$$

input
```
integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

output
```
1/12*B*b^2*x^12 + 2/9*B*a*b*x^9 + 1/9*A*b^2*x^9 + 1/6*B*a^2*x^6 + 1/3*A*a*
b*x^6 + 1/3*A*a^2*x^3
```

3.11.9 Mupad [B] (verification not implemented)

Time = 6.78 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x^2\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=x^6\left(\frac{B\,a^2}{6}+\frac{A\,b\,a}{3}\right)+x^9\left(\frac{A\,b^2}{9}+\frac{2\,B\,a\,b}{9}\right)+\frac{A\,a^2\,x^3}{3}+\frac{B\,b^2\,x^{12}}{12}$$

input

```
int(x^2*(A + B*x^3)*(a + b*x^3)^2,x)
```

output

```
x^6*((B*a^2)/6 + (A*a*b)/3) + x^9*((A*b^2)/9 + (2*B*a*b)/9) + (A*a^2*x^3)/
3 + (B*b^2*x^12)/12
```

3.12 $\int x(a+bx^3)^2 (A+Bx^3) \, dx$

3.12.1 Optimal result

Integrand size = 18, antiderivative size = 55

$$\int x(a+bx^3)^2 (A+Bx^3) \, dx = \frac{1}{2}a^2Ax^2 + \frac{1}{5}a(2Ab+aB)x^5 + \frac{1}{8}b(Ab+2aB)x^8 + \frac{1}{11}b^2Bx^{11}$$

output
```
1/2*a^2*A*x^2+1/5*a*(2*A*b+B*a)*x^5+1/8*b*(A*b+2*B*a)*x^8+1/11*b^2*B*x^11
```

3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x(a+bx^3)^2 (A+Bx^3) \, dx = \frac{1}{2}a^2Ax^2 + \frac{1}{5}a(2Ab+aB)x^5 + \frac{1}{8}b(Ab+2aB)x^8 + \frac{1}{11}b^2Bx^{11}$$

input
```
Integrate[x*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B
*x^11)/11
```

3.12.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^2\left(A+Bx^3\right)dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^2Ax+bx^7(2aB+Ab)+ax^4(aB+2Ab)+b^2Bx^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^2Ax^2+\frac{1}{8}bx^8(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{11}b^2Bx^{11}$$

input
```
Int[x*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B
*x^11)/11
```

3.12.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.12.4 Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{b^2 B\,x^{11}}{11}+\frac{(b^2A+2abB)x^8}{8}+\frac{(2abA+a^2B)x^5}{5}+\frac{a^2A\,x^2}{2}$	52
norman	$\frac{b^2 B\,x^{11}}{11}+\left(\frac{1}{8}b^2A+\frac{1}{4}abB\right)x^8+\left(\frac{2}{5}abA+\frac{1}{5}a^2B\right)x^5+\frac{a^2A\,x^2}{2}$	52
gosper	$\frac{1}{11}b^2B\,x^{11}+\frac{1}{8}x^8b^2A+\frac{1}{4}x^8abB+\frac{2}{5}x^5abA+\frac{1}{5}a^2B\,x^5+\frac{1}{2}a^2A\,x^2$	54
risch	$\frac{1}{11}b^2B\,x^{11}+\frac{1}{8}x^8b^2A+\frac{1}{4}x^8abB+\frac{2}{5}x^5abA+\frac{1}{5}a^2B\,x^5+\frac{1}{2}a^2A\,x^2$	54
parallelrisch	$\frac{1}{11}b^2B\,x^{11}+\frac{1}{8}x^8b^2A+\frac{1}{4}x^8abB+\frac{2}{5}x^5abA+\frac{1}{5}a^2B\,x^5+\frac{1}{2}a^2A\,x^2$	54

input
```
int(x*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*b^2*B*x^11+1/8*(A*b^2+2*B*a*b)*x^8+1/5*(2*A*a*b+B*a^2)*x^5+1/2*a^2*A*
x^2
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{1}{11}\,Bb^2x^{11}+\frac{1}{8}\,\left(2\,Bab+Ab^2\right)x^8+\frac{1}{5}\,\left(Ba^2+2\,Aab\right)x^5+\frac{1}{2}\,Aa^2x^2$$

input
```
integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/11*B*b^2*x^11 + 1/8*(2*B*a*b + A*b^2)*x^8 + 1/5*(B*a^2 + 2*A*a*b)*x^5 +
1/2*A*a^2*x^2
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int x\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{Aa^2x^2}{2}+\frac{Bb^2x^{11}}{11}+x^8\left(\frac{Ab^2}{8}+\frac{Bab}{4}\right)+x^5\cdot\left(\frac{2Aab}{5}+\frac{Ba^2}{5}\right)$$

input
```
integrate(x*(b*x**3+a)**2*(B*x**3+A),x)
```

output
```
A*a**2*x**2/2 + B*b**2*x**11/11 + x**8*(A*b**2/8 + B*a*b/4) + x**5*(2*A*a*
b/5 + B*a**2/5)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(a+bx^3)^2\left(A+Bx^3\right)\,dx=\frac{1}{11}\,Bb^2x^{11}+\frac{1}{8}\left(2\,Bab+Ab^2\right)x^8+\frac{1}{5}\left(Ba^2+2\,Aab\right)x^5+\frac{1}{2}\,Aa^2x^2$$

input
```
integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/11*B*b^2*x^11 + 1/8*(2*B*a*b + A*b^2)*x^8 + 1/5*(B*a^2 + 2*A*a*b)*x^5 +
1/2*A*a^2*x^2
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x(a+bx^3)^2\left(A+Bx^3\right)\,dx=\frac{1}{11}\,Bb^2x^{11}+\frac{1}{4}\,Babx^8+\frac{1}{8}\,Ab^2x^8+\frac{1}{5}\,Ba^2x^5+\frac{2}{5}\,Aabx^5+\frac{1}{2}\,Aa^2x^2$$

input
```
integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

output
```
1/11*B*b^2*x^11 + 1/4*B*a*b*x^8 + 1/8*A*b^2*x^8 + 1/5*B*a^2*x^5 + 2/5*A*a*
b*x^5 + 1/2*A*a^2*x^2
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(a+bx^3)^2\left(A+Bx^3\right)\,dx=x^5\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+x^8\left(\frac{A\,b^2}{8}+\frac{B\,a\,b}{4}\right)+\frac{A\,a^2\,x^2}{2}+\frac{B\,b^2\,x^{11}}{11}$$

input
```
int(x*(A + B*x^3)*(a + b*x^3)^2,x)
```

output
```
x^5*((B*a^2)/5 + (2*A*a*b)/5) + x^8*((A*b^2)/8 + (B*a*b)/4) + (A*a^2*x^2)/
2 + (B*b^2*x^11)/11
```

3.13 $\int \left(a+b x^3\right)^2 \left(A+B x^3\right) \, dx$

3.13.1 Optimal result

Integrand size = 17, antiderivative size = 50

$$\int \left(a+b x^3\right)^2 \left(A+B x^3\right) \, dx = a^2 A x+\frac{1}{4} a (2 A b+a B) x^4+\frac{1}{7} b (A b+2 a B) x^7+\frac{1}{10} b^2 B x^{10}$$

output
```
a^2*A*x+1/4*a*(2*A*b+B*a)*x^4+1/7*b*(A*b+2*B*a)*x^7+1/10*b^2*B*x^10
```

3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+b x^3\right)^2 \left(A+B x^3\right) \, dx = a^2 A x+\frac{1}{4} a (2 A b+a B) x^4+\frac{1}{7} b (A b+2 a B) x^7+\frac{1}{10} b^2 B x^{10}$$

input
```
Integrate[(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)
/10
```

3.13.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(a^2A+bx^6(2aB+Ab)+ax^3(aB+2Ab)+b^2Bx^9\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{4}ax^4(aB+2Ab)+\frac{1}{10}b^2Bx^{10}$$

input
```
Int[(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)
/10
```

3.13.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.13.4 Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 B\,x^{10}}{10}+\frac{(b^2 A+2abB)x^7}{7}+\frac{(2abA+a^2 B)x^4}{4}+a^2 A x$	49
norman	$\frac{b^2 B\,x^{10}}{10}+\left(\frac{1}{7}b^2 A+\frac{2}{7}abB\right)x^7+\left(\frac{1}{2}abA+\frac{1}{4}a^2 B\right)x^4+a^2 A x$	49
gosper	$\frac{1}{10}b^2 B\,x^{10}+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+\frac{1}{2}x^4 abA+\frac{1}{4}a^2 B\,x^4+a^2 A x$	51
risch	$\frac{1}{10}b^2 B\,x^{10}+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+\frac{1}{2}x^4 abA+\frac{1}{4}a^2 B\,x^4+a^2 A x$	51
parallelrisch	$\frac{1}{10}b^2 B\,x^{10}+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+\frac{1}{2}x^4 abA+\frac{1}{4}a^2 B\,x^4+a^2 A x$	51

input
```
int((b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/10*b^2*B*x^10+1/7*(A*b^2+2*B*a*b)*x^7+1/4*(2*A*a*b+B*a^2)*x^4+a^2*A*x
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)^2 \left(A+Bx^3\right)\,dx=\frac{1}{10}\,Bb^2x^{10}+\frac{1}{7}\,(2\,Bab+Ab^2)x^7+\frac{1}{4}\,(Ba^2+2\,Aab)x^4+Aa^2x$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/10*B*b^2*x^10 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/4*(B*a^2 + 2*A*a*b)*x^4 +
A*a^2*x
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)^2 \left(A+Bx^3\right)\,dx=Aa^2x+\frac{Bb^2x^{10}}{10}+x^7\left(\frac{Ab^2}{7}+\frac{2Bab}{7}\right)+x^4\left(\frac{Aab}{2}+\frac{Ba^2}{4}\right)$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A),x)
```

output
```
A*a**2*x + B*b**2*x**10/10 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**4*(A*a*b/2 +
 B*a**2/4)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)^2 \left(A+Bx^3\right)\, dx = \frac{1}{10}\,Bb^2x^{10} + \frac{1}{7}\left(2\,Bab + Ab^2\right)x^7 + \frac{1}{4}\left(Ba^2 + 2\,Aab\right)x^4 + Aa^2x$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/10*B*b^2*x^10 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/4*(B*a^2 + 2*A*a*b)*x^4 +
A*a^2*x
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)^2 \left(A+Bx^3\right)\, dx = \frac{1}{10}\,Bb^2x^{10} + \frac{2}{7}\,Babx^7 + \frac{1}{7}\,Ab^2x^7 + \frac{1}{4}\,Ba^2x^4 + \frac{1}{2}\,Aabx^4 + Aa^2x$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

output
```
1/10*B*b^2*x^10 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/4*B*a^2*x^4 + 1/2*A*a*
b*x^4 + A*a^2*x
```

3.13.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)^2 \left(A+Bx^3\right)\, dx = x^4\left(\frac{B\,a^2}{4} + \frac{A\,b\,a}{2}\right) + x^7\left(\frac{A\,b^2}{7} + \frac{2\,B\,a\,b}{7}\right) + \frac{B\,b^2\,x^{10}}{10} + A\,a^2\,x$$

input
```
int((A + B*x^3)*(a + b*x^3)^2,x)
```

output
```
x^4*((B*a^2)/4 + (A*a*b)/2) + x^7*((A*b^2)/7 + (2*B*a*b)/7) + (B*b^2*x^10)
/10 + A*a^2*x
```

3.14 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x}\,dx$

3.14.1 Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x}\,dx=\frac{2}{3}aAbx^3+\frac{1}{6}Ab^2x^6+\frac{B(a+bx^3)^3}{9b}+a^2A\log(x)$$

output
```
2/3*a*A*b*x^3+1/6*A*b^2*x^6+1/9*B*(b*x^3+a)^3/b+a^2*A*ln(x)
```

3.14.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x}\,dx=\frac{1}{3}a(2Ab+aB)x^3+\frac{1}{6}b(Ab+2aB)x^6+\frac{1}{9}b^2Bx^9+a^2A\log(x)$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x,x]
```

output
```
(a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^6)/6 + (b^2*B*x^9)/9 + a^2*A*
Log[x]
```

3.14.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {948, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)^2\left(Bx^3+A\right)}{x^3}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(A\int \frac{\left(bx^3+a\right)^2}{x^3}dx^3+\frac{B\left(a+bx^3\right)^3}{3b}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\left(A\int \left(b^2x^3+2ab+\frac{a^2}{x^3}\right)dx^3+\frac{B\left(a+bx^3\right)^3}{3b}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(A\left(a^2\log\left(x^3\right)+2abx^3+\frac{b^2x^6}{2}\right)+\frac{B\left(a+bx^3\right)^3}{3b}\right)$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x,x]
```

output
```
((B*(a + b*x^3)^3)/(3*b) + A*(2*a*b*x^3 + (b^2*x^6)/2 + a^2*Log[x^3]))/3
```

3.14.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.14.4 Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

method	result	size
norman	$\left(\frac{1}{6}b^{2}A+\frac{1}{3}abB\right)x^{6}+\left(\frac{2}{3}abA+\frac{1}{3}a^{2}B\right)x^{3}+\frac{b^{2}B\,x^{9}}{9}+a^{2}A\ln\left(x\right)$	50
default	$\frac{b^{2}B\,x^{9}}{9}+\frac{A\,b^{2}x^{6}}{6}+\frac{Bab\,x^{6}}{3}+\frac{2aAb\,x^{3}}{3}+\frac{a^{2}B\,x^{3}}{3}+a^{2}A\ln\left(x\right)$	52
risch	$\frac{b^{2}B\,x^{9}}{9}+\frac{A\,b^{2}x^{6}}{6}+\frac{Bab\,x^{6}}{3}+\frac{2aAb\,x^{3}}{3}+\frac{a^{2}B\,x^{3}}{3}+a^{2}A\ln\left(x\right)$	52
parallelrisch	$\frac{b^{2}B\,x^{9}}{9}+\frac{A\,b^{2}x^{6}}{6}+\frac{Bab\,x^{6}}{3}+\frac{2aAb\,x^{3}}{3}+\frac{a^{2}B\,x^{3}}{3}+a^{2}A\ln\left(x\right)$	52

input
```
int((b*x^3+a)^2*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/6*b^2*A+1/3*a*b*B)*x^6+(2/3*a*b*A+1/3*a^2*B)*x^3+1/9*b^2*B*x^9+a^2*A*ln
(x)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x}\,dx = \frac{1}{9}Bb^2x^9+\frac{1}{6}\left(2\,Bab+Ab^2\right)x^6+\frac{1}{3}\left(Ba^2+2\,Aab\right)x^3+Aa^2\log\left(x\right)$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="fricas")
```

output
```
1/9*B*b^2*x^9 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/3*(B*a^2 + 2*A*a*b)*x^3 + A*
a^2*log(x)
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x}\,dx = Aa^2\log\left(x\right)+\frac{Bb^2x^9}{9}+x^6\left(\frac{Ab^2}{6}+\frac{Bab}{3}\right)+x^3\cdot\left(\frac{2Aab}{3}+\frac{Ba^2}{3}\right)$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x,x)
```

output
```
A*a**2*log(x) + B*b**2*x**9/9 + x**6*(A*b**2/6 + B*a*b/3) + x**3*(2*A*a*b/
3 + B*a**2/3)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{(a+bx^3)^2 (A+Bx^3)}{x}\,dx &= \frac{1}{9}Bb^2x^9+\frac{1}{6}\left(2\,Bab+Ab^2\right)x^6\\ &\quad+\frac{1}{3}\left(Ba^2+2\,Aab\right)x^3+\frac{1}{3}Aa^2\log\left(x^3\right)\end{aligned}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="maxima")
```

output
```
1/9*B*b^2*x^9 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/3*(B*a^2 + 2*A*a*b)*x^3 + 1/
3*A*a^2*log(x^3)
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x}\,dx = \frac{1}{9}\,Bb^2x^9+\frac{1}{3}\,Babx^6+\frac{1}{6}\,Ab^2x^6 + \frac{1}{3}\,Ba^2x^3+\frac{2}{3}\,Aabx^3+Aa^2\log\left(|x|\right)$$

input

```
integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="giac")
```

output

```
1/9*B*b^2*x^9 + 1/3*B*a*b*x^6 + 1/6*A*b^2*x^6 + 1/3*B*a^2*x^3 + 2/3*A*a*b*
x^3 + A*a^2*log(abs(x))
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x}\,dx = x^3\left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^6\left(\frac{A\,b^2}{6}+\frac{B\,a\,b}{3}\right)+\frac{B\,b^2\,x^9}{9}+A\,a^2\,\ln\left(x\right)$$

input

```
int(((A + B*x^3)*(a + b*x^3)^2)/x,x)
```

output

```
x^3*((B*a^2)/3 + (2*A*a*b)/3) + x^6*((A*b^2)/6 + (B*a*b)/3) + (B*b^2*x^9)/
9 + A*a^2*log(x)
```

3.15 $\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^2}\,dx$

3.15.1 Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^2}\,dx=-\frac{a^2A}{x}+\frac{1}{2}a(2Ab+aB)x^2+\frac{1}{5}b(Ab+2aB)x^5+\frac{1}{8}b^2Bx^8$$

output
```
-a^2*A/x+1/2*a*(2*A*b+B*a)*x^2+1/5*b*(A*b+2*B*a)*x^5+1/8*b^2*B*x^8
```

3.15.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^2}\,dx=-\frac{a^2A}{x}+\frac{1}{2}a(2Ab+aB)x^2+\frac{1}{5}b(Ab+2aB)x^5+\frac{1}{8}b^2Bx^8$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^2,x]
```

output
```
-((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*
x^8)/8
```

3.15.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^2} dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2A}{x^2} + bx^4(2aB+Ab) + ax(aB+2Ab) + b^2Bx^7\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{x} + \frac{1}{5}bx^5(2aB+Ab) + \frac{1}{2}ax^2(aB+2Ab) + \frac{1}{8}b^2Bx^8$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^2,x]
```

output
```
-((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*
x^8)/8
```

3.15.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.15.4 Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result	size
norman	$\frac{\frac{b^2 B x^9}{8}+\left(\frac{1}{5} b^2 A+\frac{2}{5} a b B\right) x^6+\left(a b A+\frac{1}{2} a^2 B\right) x^3-a^2 A}{x}$	52
default	$\frac{b^2 B x^8}{8}+\frac{A b^2 x^5}{5}+\frac{2 B a b x^5}{5}+a A b x^2+\frac{a^2 B x^2}{2}-\frac{a^2 A}{x}$	53
risch	$\frac{b^2 B x^8}{8}+\frac{A b^2 x^5}{5}+\frac{2 B a b x^5}{5}+a A b x^2+\frac{a^2 B x^2}{2}-\frac{a^2 A}{x}$	53
gosper	$-\frac{-5 b^2 B x^9-8 A b^2 x^6-16 B a b x^6-40 a A b x^3-20 a^2 B x^3+40 a^2 A}{40 x}$	56
parallelrisch	$\frac{5 b^2 B x^9+8 A b^2 x^6+16 B a b x^6+40 a A b x^3+20 a^2 B x^3-40 a^2 A}{40 x}$	56

input

```
int((b*x^3+a)^2*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
```

output

```
1/x*(1/8*b^2*B*x^9+(1/5*b^2*A+2/5*a*b*B)*x^6+(a*b*A+1/2*a^2*B)*x^3-a^2*A)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2\,(A+Bx^3)}{x^2}\,dx=\frac{5\,Bb^2x^9+8\,(2\,Bab+Ab^2)x^6+20\,(Ba^2+2\,Aab)x^3-40\,Aa^2}{40\,x}$$

input

```
integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="fricas")
```

output

```
1/40*(5*B*b^2*x^9 + 8*(2*B*a*b + A*b^2)*x^6 + 20*(B*a^2 + 2*A*a*b)*x^3 - 4
0*A*a^2)/x
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^2\,(A+Bx^3)}{x^2}\,dx=-\frac{Aa^2}{x}+\frac{Bb^2x^8}{8}+x^5\left(\frac{Ab^2}{5}+\frac{2Bab}{5}\right)+x^2\left(Aab+\frac{Ba^2}{2}\right)$$

input

```
integrate((b*x**3+a)**2*(B*x**3+A)/x**2,x)
```

output
```
-A*a**2/x + B*b**2*x**8/8 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**2*(A*a*b + B*
a**2/2)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^2}\,dx = \frac{1}{8}\,Bb^2x^8 + \frac{1}{5}\,(2\,Bab + Ab^2)x^5 + \frac{1}{2}\,(Ba^2 + 2\,Aab)x^2 - \frac{Aa^2}{x}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="maxima")
```

output
```
1/8*B*b^2*x^8 + 1/5*(2*B*a*b + A*b^2)*x^5 + 1/2*(B*a^2 + 2*A*a*b)*x^2 - A*
a^2/x
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^2}\,dx = \frac{1}{8}\,Bb^2x^8 + \frac{2}{5}\,Babx^5 + \frac{1}{5}\,Ab^2x^5 + \frac{1}{2}\,Ba^2x^2 + Aabx^2 - \frac{Aa^2}{x}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="giac")
```

output
```
1/8*B*b^2*x^8 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/2*B*a^2*x^2 + A*a*b*x^2
- A*a^2/x
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^2}\,dx = x^2\left(\frac{B\,a^2}{2} + A\,b\,a\right) + x^5\left(\frac{A\,b^2}{5} + \frac{2\,B\,a\,b}{5}\right) - \frac{A\,a^2}{x} + \frac{B\,b^2\,x^8}{8}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^2,x)
```

output
```
x^2*((B*a^2)/2 + A*a*b) + x^5*((A*b^2)/5 + (2*B*a*b)/5) - (A*a^2)/x + (B*b
^2*x^8)/8
```

3.16 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3}\,dx$

3.16.1 Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3}\,dx = -\frac{a^2A}{2x^2}+a(2Ab+aB)x+\frac{1}{4}b(Ab+2aB)x^4+\frac{1}{7}b^2Bx^7$$

output
```
-1/2*a^2*A/x^2+a*(2*A*b+B*a)*x+1/4*b*(A*b+2*B*a)*x^4+1/7*b^2*B*x^7
```

3.16.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3}\,dx = -\frac{a^2A}{2x^2}+a(2Ab+aB)x+\frac{1}{4}b(Ab+2aB)x^4+\frac{1}{7}b^2Bx^7$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(a^2*A)/x^2 + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^
7)/7
```

3.16.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^3}\,dx$$

$$\Big\downarrow \; 950$$

$$\int \left(\frac{a^2A}{x^3}+bx^3(2aB+Ab)+a(aB+2Ab)+b^2Bx^6\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2A}{2x^2}+\frac{1}{4}bx^4(2aB+Ab)+ax(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

input

```
Int[((a + b*x^3)^2*(A + B*x^3))/x^3,x]
```

output

```
-1/2*(a^2*A)/x^2 + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^
7)/7
```

3.16.3.1 Defintions of rubi rules used

rule 950

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.16.4 Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^{2} B \,x^{7}}{7}+\frac{A \,b^{2} x^{4}}{4}+\frac{B a b \,x^{4}}{2}+2 a A b x +a^{2} B x -\frac{a^{2} A}{2 x^{2}}$	49
risch	$\frac{b^{2} B \,x^{7}}{7}+\frac{A \,b^{2} x^{4}}{4}+\frac{B a b \,x^{4}}{2}+2 a A b x +a^{2} B x -\frac{a^{2} A}{2 x^{2}}$	49
norman	$\frac{\frac{b^{2} B \,x^{9}}{7}+\left(\frac{1}{4} b^{2} A +\frac{1}{2} a b B \right) x^{6}+\left(2 a b A +a^{2} B \right) x^{3}-\frac{a^{2} A}{2}}{x^{2}}$	52
gosper	$-\frac{-4 b^{2} B \,x^{9}-7 A \,b^{2} x^{6}-14 B a b \,x^{6}-56 a A b \,x^{3}-28 a^{2} B \,x^{3}+14 a^{2} A}{28 x^{2}}$	56
parallelrisch	$\frac{4 b^{2} B \,x^{9}+7 A \,b^{2} x^{6}+14 B a b \,x^{6}+56 a A b \,x^{3}+28 a^{2} B \,x^{3}-14 a^{2} A}{28 x^{2}}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^2*B*x^7+1/4*A*b^2*x^4+1/2*B*a*b*x^4+2*a*A*b*x+a^2*B*x-1/2*a^2*A/x^2
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^3}\,dx = \frac{4\,Bb^2x^9 + 7\,(2\,Bab + Ab^2)x^6 + 28\,(Ba^2 + 2\,Aab)x^3 - 14\,Aa^2}{28\,x^2}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="fricas")
```

output
```
1/28*(4*B*b^2*x^9 + 7*(2*B*a*b + A*b^2)*x^6 + 28*(B*a^2 + 2*A*a*b)*x^3 - 1
4*A*a^2)/x^2
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^3}\,dx = -\frac{Aa^2}{2x^2} + \frac{Bb^2x^7}{7} + x^4\left(\frac{Ab^2}{4} + \frac{Bab}{2}\right) + x\left(2Aab + Ba^2\right)$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**3,x)
```

output
```
-A*a**2/(2*x**2) + B*b**2*x**7/7 + x**4*(A*b**2/4 + B*a*b/2) + x*(2*A*a*b
+ B*a**2)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3}\,dx = \frac{1}{7}Bb^2x^7 + \frac{1}{4}\left(2\,Bab + Ab^2\right)x^4 + \left(Ba^2 + 2\,Aab\right)x - \frac{Aa^2}{2\,x^2}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="maxima")
```

output
```
1/7*B*b^2*x^7 + 1/4*(2*B*a*b + A*b^2)*x^4 + (B*a^2 + 2*A*a*b)*x - 1/2*A*a^
2/x^2
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3}\,dx = \frac{1}{7}Bb^2x^7 + \frac{1}{2}Babx^4 + \frac{1}{4}Ab^2x^4 + Ba^2x + 2\,Aabx - \frac{Aa^2}{2\,x^2}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="giac")
```

output
```
1/7*B*b^2*x^7 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + B*a^2*x + 2*A*a*b*x - 1/2*
A*a^2/x^2
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3}\,dx = x^4\left(\frac{A\,b^2}{4} + \frac{B\,a\,b}{2}\right) + x\left(B\,a^2 + 2\,A\,b\,a\right) - \frac{A\,a^2}{2\,x^2} + \frac{B\,b^2\,x^7}{7}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^3,x)
```

output
```
x^4*((A*b^2)/4 + (B*a*b)/2) + x*(B*a^2 + 2*A*a*b) - (A*a^2)/(2*x^2) + (B*b
^2*x^7)/7
```

3.17 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}\,dx$

3.17.1 Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}\,dx = -\frac{a^2A}{3x^3}+\frac{1}{3}b(Ab+2aB)x^3+\frac{1}{6}b^2Bx^6+a(2Ab+aB)\log(x)$$

output
```
-1/3*a^2*A/x^3+1/3*b*(A*b+2*B*a)*x^3+1/6*b^2*B*x^6+a*(2*A*b+B*a)*ln(x)
```

3.17.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}\,dx = \frac{1}{6}\left(-\frac{2a^2A}{x^3}+2b(Ab+2aB)x^3+b^2Bx^6+6a(2Ab+aB)\log(x)\right)$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^4,x]
```

output
```
((-2*a^2*A)/x^3 + 2*b*(A*b + 2*a*B)*x^3 + b^2*B*x^6 + 6*a*(2*A*b + a*B)*Lo
g[x])/6
```

3.17.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^2(Bx^3+A)}{x^6}dx^3$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{3}\int \left(b^2Bx^3+b(Ab+2aB)+\frac{a(2Ab+aB)}{x^3}+\frac{a^2A}{x^6}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^2A}{x^3}+bx^3(2aB+Ab)+a\log\left(x^3\right)(aB+2Ab)+\frac{1}{2}b^2Bx^6\right)$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^4,x]
```

output
```
(-((a^2*A)/x^3) + b*(A*b + 2*a*B)*x^3 + (b^2*B*x^6)/2 + a*(2*A*b + a*B)*Lo
g[x^3])/3
```

3.17.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.17.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{b^2 B x^6}{6}+\frac{A b^2 x^3}{3}+\frac{2 B a b x^3}{3}+a(2Ab+Ba)\ln(x)-\frac{a^2 A}{3x^3}$	49
norman	$\frac{\left(\frac{1}{3}b^2 A+\frac{2}{3}abB\right)x^6-\frac{a^2 A}{3}+\frac{b^2 B x^9}{6}}{x^3}+(2abA+a^2 B)\ln(x)$	52
parallelrisch	$\frac{b^2 B x^9+2A b^2 x^6+4Bab x^6+12A\ln(x)x^3 ab+6B\ln(x)x^3 a^2-2a^2 A}{6x^3}$	59
risch	$\frac{b^2 B x^6}{6}+\frac{A b^2 x^3}{3}+\frac{2B ab x^3}{3}+\frac{A^2 b^2}{6B}+\frac{2abA}{3}+\frac{2a^2 B}{3}-\frac{a^2 A}{3x^3}+2A\ln(x)ab+a^2 B\ln(x)$	73

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^2*B*x^6+1/3*A*b^2*x^3+2/3*B*a*b*x^3+a*(2*A*b+B*a)*ln(x)-1/3*a^2*A/x^
3
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}\,dx = \frac{Bb^2x^9+2\,(2\,Bab+Ab^2)x^6+6\,(Ba^2+2\,Aab)x^3\log(x)-2\,Aa^2}{6\,x^3}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="fricas")
```

output
```
1/6*(B*b^2*x^9 + 2*(2*B*a*b + A*b^2)*x^6 + 6*(B*a^2 + 2*A*a*b)*x^3*log(x)
- 2*A*a^2)/x^3
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^4} \, dx = -\frac{Aa^2}{3x^3} + \frac{Bb^2x^6}{6} + a(2Ab+Ba)\log(x) + x^3\left(\frac{Ab^2}{3} + \frac{2Bab}{3}\right)$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**4,x)
```

output
```
-A*a**2/(3*x**3) + B*b**2*x**6/6 + a*(2*A*b + B*a)*log(x) + x**3*(A*b**2/3
 + 2*B*a*b/3)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^4} \, dx = \frac{1}{6} Bb^2x^6 + \frac{1}{3} (2Bab + Ab^2)x^3 + \frac{1}{3} (Ba^2 + 2Aab) \log(x^3) - \frac{Aa^2}{3x^3}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="maxima")
```

output
```
1/6*B*b^2*x^6 + 1/3*(2*B*a*b + A*b^2)*x^3 + 1/3*(B*a^2 + 2*A*a*b)*log(x^3)
 - 1/3*A*a^2/x^3
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^4} \, dx = \frac{1}{6} Bb^2x^6 + \frac{2}{3} Babx^3 + \frac{1}{3} Ab^2x^3 + (Ba^2 + 2Aab) \log(|x|) - \frac{Ba^2x^3 + 2Aabx^3 + Aa^2}{3x^3}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="giac")
```

output
```
1/6*B*b^2*x^6 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + (B*a^2 + 2*A*a*b)*log(abs(
x)) - 1/3*(B*a^2*x^3 + 2*A*a*b*x^3 + A*a^2)/x^3
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2\,(A+Bx^3)}{x^4}\,dx = x^3\left(\frac{A\,b^2}{3}+\frac{2\,B\,a\,b}{3}\right)+\ln(x)\,\left(B\,a^2+2\,A\,b\,a\right)-\frac{A\,a^2}{3\,x^3}+\frac{B\,b^2\,x^6}{6}$$

input

```
int(((A + B*x^3)*(a + b*x^3)^2)/x^4,x)
```

output

```
x^3*((A*b^2)/3 + (2*B*a*b)/3) + log(x)*(B*a^2 + 2*A*a*b) - (A*a^2)/(3*x^3)
 + (B*b^2*x^6)/6
```

3.18 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5}\,dx$

3.18.1 Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5}\,dx = -\frac{a^2A}{4x^4} - \frac{a(2Ab+aB)}{x} + \frac{1}{2}b(Ab+2aB)x^2 + \frac{1}{5}b^2Bx^5$$

output
```
-1/4*a^2*A/x^4-a*(2*A*b+B*a)/x+1/2*b*(A*b+2*B*a)*x^2+1/5*b^2*B*x^5
```

3.18.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5}\,dx = \frac{-5a^2A - 20a(2Ab+aB)x^3 + 10b(Ab+2aB)x^6 + 4b^2Bx^9}{20x^4}$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^5,x]
```

output
```
(-5*a^2*A - 20*a*(2*A*b + a*B)*x^3 + 10*b*(A*b + 2*a*B)*x^6 + 4*b^2*B*x^9)
/(20*x^4)
```

3.18.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^2 \left(A+Bx^3\right)}{x^5}dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2A}{x^5}+\frac{a(aB+2Ab)}{x^2}+bx(2aB+Ab)+b^2Bx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{4x^4}+\frac{1}{2}bx^2(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{5}b^2Bx^5$$

input

```
Int[((a + b*x^3)^2*(A + B*x^3))/x^5,x]
```

output

```
-1/4*(a^2*A)/x^4 - (a*(2*A*b + a*B))/x + (b*(A*b + 2*a*B)*x^2)/2 + (b^2*B*
x^5)/5
```

3.18.3.1 Defintions of rubi rules used

rule 950

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.18.4 Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{b^{2} B \,x^{5}}{5}+\frac{A \,b^{2} x^{2}}{2}+B a b \,x^{2}-\frac{a \left(2 A b +B a \right)}{x}-\frac{a^{2} A}{4 x^{4}}$	50
norman	$\frac{\frac{b^{2} B \,x^{9}}{5}+\left(\frac{1}{2} b^{2} A +a b B \right) x^{6}+\left(-2 a b A -a^{2} B \right) x^{3}-\frac{a^{2} A}{4}}{x^{4}}$	52
risch	$\frac{b^{2} B \,x^{5}}{5}+\frac{A \,b^{2} x^{2}}{2}+B a b \,x^{2}+\frac{\left(-2 a b A -a^{2} B \right) x^{3}-\frac{a^{2} A}{4}}{x^{4}}$	54
gosper	$-\frac{-4 b^{2} B \,x^{9}-10 A \,b^{2} x^{6}-20 B a b \,x^{6}+40 a A b \,x^{3}+20 a^{2} B \,x^{3}+5 a^{2} A}{20 x^{4}}$	56
parallelrisch	$\frac{4 b^{2} B \,x^{9}+10 A \,b^{2} x^{6}+20 B a b \,x^{6}-40 a A b \,x^{3}-20 a^{2} B \,x^{3}-5 a^{2} A}{20 x^{4}}$	56

input

```
int((b*x^3+a)^2*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
```

output

```
1/5*b^2*B*x^5+1/2*A*b^2*x^2+B*a*b*x^2-a*(2*A*b+B*a)/x-1/4*a^2*A/x^4
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^5}\,dx=\frac{4\,Bb^{2}x^{9}+10\,{\left(2\,Bab+Ab^{2}\right)}x^{6}-20\,{\left(Ba^{2}+2\,Aab\right)}x^{3}-5\,Aa^{2}}{20\,x^{4}}$$

input

```
integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="fricas")
```

output

```
1/20*(4*B*b^2*x^9 + 10*(2*B*a*b + A*b^2)*x^6 - 20*(B*a^2 + 2*A*a*b)*x^3 -
5*A*a^2)/x^4
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^5}\,dx=\frac{Bb^{2}x^{5}}{5}+x^{2}\left(\frac{Ab^{2}}{2}+Bab\right)+\frac{-Aa^{2}+x^{3}\left(-8Aab-4Ba^{2}\right)}{4x^{4}}$$

input

```
integrate((b*x**3+a)**2*(B*x**3+A)/x**5,x)
```

output

```
B*b**2*x**5/5 + x**2*(A*b**2/2 + B*a*b) + (-A*a**2 + x**3*(-8*A*a*b - 4*B*
a**2))/(4*x**4)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^5}\,dx = \frac{1}{5}\,Bb^2x^5 + \frac{1}{2}\,(2\,Bab + Ab^2)x^2 - \frac{4\,(Ba^2 + 2\,Aab)x^3 + Aa^2}{4\,x^4}$$

input

```
integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="maxima")
```

output

```
1/5*B*b^2*x^5 + 1/2*(2*B*a*b + A*b^2)*x^2 - 1/4*(4*(B*a^2 + 2*A*a*b)*x^3 +
 A*a^2)/x^4
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^5}\,dx = \frac{1}{5}\,Bb^2x^5 + Babx^2 + \frac{1}{2}\,Ab^2x^2 - \frac{4\,Ba^2x^3 + 8\,Aabx^3 + Aa^2}{4\,x^4}$$

input

```
integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="giac")
```

output

```
1/5*B*b^2*x^5 + B*a*b*x^2 + 1/2*A*b^2*x^2 - 1/4*(4*B*a^2*x^3 + 8*A*a*b*x^3
 + A*a^2)/x^4
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^5}\,dx = x^2\left(\frac{A\,b^2}{2} + B\,a\,b\right) - \frac{x^3\,(B\,a^2 + 2\,A\,b\,a) + \frac{A\,a^2}{4}}{x^4} + \frac{B\,b^2\,x^5}{5}$$

input

```
int(((A + B*x^3)*(a + b*x^3)^2)/x^5,x)
```

output

```
x^2*((A*b^2)/2 + B*a*b) - (x^3*(B*a^2 + 2*A*a*b) + (A*a^2)/4)/x^4 + (B*b^2
*x^5)/5
```

3.19 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6} \, dx$

3.19.1 Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6} \, dx = -\frac{a^2A}{5x^5} - \frac{a(2Ab+aB)}{2x^2} + b(Ab+2aB)x + \frac{1}{4}b^2Bx^4$$

output
```
-1/5*a^2*A/x^5-1/2*a*(2*A*b+B*a)/x^2+b*(A*b+2*B*a)*x+1/4*b^2*B*x^4
```

3.19.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6} \, dx = -\frac{a^2A}{5x^5} - \frac{a(2Ab+aB)}{2x^2} + b(Ab+2aB)x + \frac{1}{4}b^2Bx^4$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^6,x]
```

output
```
-1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*
x^4)/4
```

3.19.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^6}\,dx$$

$$\Big\downarrow \; 950$$

$$\int \left(\frac{a^2A}{x^6}+\frac{a(aB+2Ab)}{x^3}+b(2aB+Ab)+b^2Bx^3\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{2x^2}+bx(2aB+Ab)+\frac{1}{4}b^2Bx^4$$

input

```
Int[((a + b*x^3)^2*(A + B*x^3))/x^6,x]
```

output

```
-1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*
x^4)/4
```

3.19.3.1 Defintions of rubi rules used

rule 950

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.19.4 Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{b^2 B\,x^4}{4}+A\,b^2 x+2Babx-\frac{a(2Ab+Ba)}{2x^2}-\frac{a^2 A}{5x^5}$	46
risch	$\frac{b^2 B\,x^4}{4}+A\,b^2 x+2Babx+\frac{\left(-abA-\frac{1}{2}a^2 B\right)x^3-\frac{a^2 A}{5}}{x^5}$	50
norman	$\frac{\frac{b^2 B\,x^9}{4}+\left(b^2 A+2abB\right)x^6+\left(-abA-\frac{1}{2}a^2 B\right)x^3-\frac{a^2 A}{5}}{x^5}$	52
gosper	$-\frac{-5b^2 B\,x^9-20A\,b^2 x^6-40Bab\,x^6+20aAb\,x^3+10a^2 B\,x^3+4a^2 A}{20x^5}$	56
parallelrisch	$\frac{5b^2 B\,x^9+20A\,b^2 x^6+40Bab\,x^6-20aAb\,x^3-10a^2 B\,x^3-4a^2 A}{20x^5}$	56

input

```
int((b*x^3+a)^2*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
```

output

```
1/4*b^2*B*x^4+A*b^2*x+2*B*a*b*x-1/2*a*(2*A*b+B*a)/x^2-1/5*a^2*A/x^5
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^6}\,dx=\frac{5\,Bb^2x^9+20\,(2\,Bab+Ab^2)x^6-10\,(Ba^2+2\,Aab)x^3-4\,Aa^2}{20\,x^5}$$

input

```
integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="fricas")
```

output

```
1/20*(5*B*b^2*x^9 + 20*(2*B*a*b + A*b^2)*x^6 - 10*(B*a^2 + 2*A*a*b)*x^3 -
4*A*a^2)/x^5
```

3.19.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^6}\,dx=\frac{Bb^2x^4}{4}+x\left(Ab^2+2Bab\right)+\frac{-2Aa^2+x^3(-10Aab-5Ba^2)}{10x^5}$$

input

```
integrate((b*x**3+a)**2*(B*x**3+A)/x**6,x)
```

output
```
B*b**2*x**4/4 + x*(A*b**2 + 2*B*a*b) + (-2*A*a**2 + x**3*(-10*A*a*b - 5*B*
a**2))/(10*x**5)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6}\,dx = \frac{1}{4}\,Bb^2x^4 + \left(2\,Bab + Ab^2\right)x - \frac{5\,(Ba^2 + 2\,Aab)x^3 + 2\,Aa^2}{10\,x^5}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="maxima")
```

output
```
1/4*B*b^2*x^4 + (2*B*a*b + A*b^2)*x - 1/10*(5*(B*a^2 + 2*A*a*b)*x^3 + 2*A*
a^2)/x^5
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6}\,dx = \frac{1}{4}\,Bb^2x^4 + 2\,Babx + Ab^2x - \frac{5\,Ba^2x^3 + 10\,Aabx^3 + 2\,Aa^2}{10\,x^5}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="giac")
```

output
```
1/4*B*b^2*x^4 + 2*B*a*b*x + A*b^2*x - 1/10*(5*B*a^2*x^3 + 10*A*a*b*x^3 + 2
*A*a^2)/x^5
```

3.19.9 Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6}\,dx = x\left(A\,b^2 + 2\,B\,a\,b\right) - \frac{x^3\left(\frac{B\,a^2}{2} + A\,b\,a\right) + \frac{A\,a^2}{5}}{x^5} + \frac{B\,b^2\,x^4}{4}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^6,x)
```

output
```
x*(A*b^2 + 2*B*a*b) - (x^3*((B*a^2)/2 + A*a*b) + (A*a^2)/5)/x^5 + (B*b^2*x
^4)/4
```

3.19. $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6}\,dx$

3.20 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx$

3.20.1 Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx=-\frac{a^2A}{6x^6}-\frac{a(2Ab+aB)}{3x^3}+\frac{1}{3}b^2Bx^3+b(Ab+2aB)\log(x)$$

output
```
-1/6*a^2*A/x^6-1/3*a*(2*A*b+B*a)/x^3+1/3*b^2*B*x^3+b*(A*b+2*B*a)*ln(x)
```

3.20.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx=\frac{1}{6}\left(-\frac{4aAb}{x^3}+2b^2Bx^3-\frac{a^2(A+2Bx^3)}{x^6}+6b(Ab+2aB)\log(x)\right)$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^7,x]
```

output
```
((-4*a*A*b)/x^3 + 2*b^2*B*x^3 - (a^2*(A + 2*B*x^3))/x^6 + 6*b*(A*b + 2*a*B
)*Log[x])/6
```

3.20.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^2(Bx^3+A)}{x^9}dx^3$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{3}\int \left(\frac{Aa^2}{x^9}+\frac{(2Ab+aB)a}{x^6}+b^2B+\frac{b(Ab+2aB)}{x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^2A}{2x^6}-\frac{a(aB+2Ab)}{x^3}+b\log\left(x^3\right)(2aB+Ab)+b^2Bx^3\right)$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^7,x]
```

output
```
(-1/2*(a^2*A)/x^6 - (a*(2*A*b + a*B))/x^3 + b^2*B*x^3 + b*(A*b + 2*a*B)*Lo
g[x^3])/3
```

3.20.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.20.4 Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^{2} A}{6 x^{6}}-\frac{a \left(2 A b+B a\right)}{3 x^{3}}+\frac{b^{2} B \,x^{3}}{3}+b \left(A b+2 B a\right) \ln \left(x\right)$	46
norman	$\frac{\left(-\frac{2}{3} a b A-\frac{1}{3} a^{2} B\right) x^{3}-\frac{a^{2} A}{6}+\frac{b^{2} B \,x^{9}}{3}}{x^{6}}+\left(b^{2} A+2 a b B\right) \ln \left(x\right)$	52
risch	$\frac{b^{2} B \,x^{3}}{3}+\frac{\left(-\frac{2}{3} a b A-\frac{1}{3} a^{2} B\right) x^{3}-\frac{a^{2} A}{6}}{x^{6}}+A \ln \left(x\right) b^{2}+2 B \ln \left(x\right) a b$	52
parallelrisch	$\frac{2 b^{2} B \,x^{9}+6 A \ln \left(x\right) x^{6} b^{2}+12 B \ln \left(x\right) x^{6} a b-4 a A b \,x^{3}-2 a^{2} B \,x^{3}-a^{2} A}{6 x^{6}}$	60

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^2*A/x^6-1/3*a*(2*A*b+B*a)/x^3+1/3*b^2*B*x^3+b*(A*b+2*B*a)*ln(x)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^7}\,dx = \frac{2\,Bb^{2}x^{9}+6\,(2\,Bab+Ab^{2})x^{6}\log\left(x\right)-2\,(Ba^{2}+2\,Aab)x^{3}-Aa^{2}}{6\,x^{6}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="fricas")
```

output
```
1/6*(2*B*b^2*x^9 + 6*(2*B*a*b + A*b^2)*x^6*log(x) - 2*(B*a^2 + 2*A*a*b)*x^
3 - A*a^2)/x^6
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx = \frac{Bb^2x^3}{3} + b(Ab+2Ba)\log(x) + \frac{-Aa^2+x^3(-4Aab-2Ba^2)}{6x^6}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**7,x)
```

output
```
B*b**2*x**3/3 + b*(A*b + 2*B*a)*log(x) + (-A*a**2 + x**3*(-4*A*a*b - 2*B*a
**2))/(6*x**6)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx = \frac{1}{3}\,Bb^2x^3 + \frac{1}{3}\left(2\,Bab+Ab^2\right)\log\left(x^3\right) - \frac{2\,(Ba^2+2\,Aab)x^3+Aa^2}{6\,x^6}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="maxima")
```

output
```
1/3*B*b^2*x^3 + 1/3*(2*B*a*b + A*b^2)*log(x^3) - 1/6*(2*(B*a^2 + 2*A*a*b)*
x^3 + A*a^2)/x^6
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx = {} & \frac{1}{3}\,Bb^2x^3 + \left(2\,Bab+Ab^2\right)\log\left(|x|\right) \\ & - \frac{6\,Babx^6+3\,Ab^2x^6+2\,Ba^2x^3+4\,Aabx^3+Aa^2}{6\,x^6}\end{aligned}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="giac")
```

output
```
1/3*B*b^2*x^3 + (2*B*a*b + A*b^2)*log(abs(x)) - 1/6*(6*B*a*b*x^6 + 3*A*b^2
*x^6 + 2*B*a^2*x^3 + 4*A*a*b*x^3 + A*a^2)/x^6
```

3.20.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^7} \, dx = \ln(x) \, \left(A\,b^2+2\,B\,a\,b\right)-\frac{x^3\,\left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+\frac{A\,a^2}{6}}{x^6}+\frac{B\,b^2\,x^3}{3}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^7,x)
```

output
```
log(x)*(A*b^2 + 2*B*a*b) - (x^3*((B*a^2)/3 + (2*A*a*b)/3) + (A*a^2)/6)/x^6
 + (B*b^2*x^3)/3
```

3.21 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8}\,dx$

3.21.1 Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8}\,dx = -\frac{a^2A}{7x^7}-\frac{a(2Ab+aB)}{4x^4}-\frac{b(Ab+2aB)}{x}+\frac{1}{2}b^2Bx^2$$

output
```
-1/7*a^2*A/x^7-1/4*a*(2*A*b+B*a)/x^4-b*(A*b+2*B*a)/x+1/2*b^2*B*x^2
```

3.21.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8}\,dx = -\frac{-14b^2x^6(-2A+Bx^3)+14abx^3(A+4Bx^3)+a^2(4A+7Bx^3)}{28x^7}$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^8,x]
```

output
```
-1/28*(-14*b^2*x^6*(-2*A + B*x^3) + 14*a*b*x^3*(A + 4*B*x^3) + a^2*(4*A +
7*B*x^3))/x^7
```

3.21.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^8} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2 A}{x^8}+\frac{a(aB+2Ab)}{x^5}+\frac{b(2aB+Ab)}{x^2}+b^2 Bx\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2 A}{7x^7}-\frac{a(aB+2Ab)}{4x^4}-\frac{b(2aB+Ab)}{x}+\frac{1}{2}b^2 Bx^2$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^8,x]
```

output
```
-1/7*(a^2*A)/x^7 - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/x + (b^2*
B*x^2)/2
```

3.21.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.21.4 Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{a^2A}{7x^7}-\frac{a(2Ab+Ba)}{4x^4}-\frac{b(Ab+2Ba)}{x}+\frac{b^2B\,x^2}{2}$	48
norman	$\frac{\frac{b^2B\,x^9}{2}+(-b^2A-2abB)x^6+(-\frac{1}{2}abA-\frac{1}{4}a^2B)x^3-\frac{a^2A}{7}}{x^7}$	53
risch	$\frac{b^2B\,x^2}{2}+\frac{(-b^2A-2abB)x^6+(-\frac{1}{2}abA-\frac{1}{4}a^2B)x^3-\frac{a^2A}{7}}{x^7}$	54
gosper	$-\frac{-14b^2B\,x^9+28A\,b^2x^6+56Bab\,x^6+14aAb\,x^3+7a^2B\,x^3+4a^2A}{28x^7}$	56
parallelrisch	$-\frac{-14b^2B\,x^9+28A\,b^2x^6+56Bab\,x^6+14aAb\,x^3+7a^2B\,x^3+4a^2A}{28x^7}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^2*A/x^7-1/4*a*(2*A*b+B*a)/x^4-b*(A*b+2*B*a)/x+1/2*b^2*B*x^2
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2\,(A+Bx^3)}{x^8}\,dx = \frac{14\,Bb^2x^9-28\,(2\,Bab+Ab^2)x^6-7\,(Ba^2+2\,Aab)x^3-4\,Aa^2}{28\,x^7}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="fricas")
```

output
```
1/28*(14*B*b^2*x^9 - 28*(2*B*a*b + A*b^2)*x^6 - 7*(B*a^2 + 2*A*a*b)*x^3 -
4*A*a^2)/x^7
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^2\,(A+Bx^3)}{x^8}\,dx$$
$$=\frac{Bb^2x^2}{2}+\frac{-4Aa^2+x^6(-28Ab^2-56Bab)+x^3(-14Aab-7Ba^2)}{28x^7}$$

input

```
integrate((b*x**3+a)**2*(B*x**3+A)/x**8,x)
```

output

```
B*b**2*x**2/2 + (-4*A*a**2 + x**6*(-28*A*b**2 - 56*B*a*b) + x**3*(-14*A*a*
b - 7*B*a**2))/(28*x**7)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^8}\,dx = \frac{1}{2}\,Bb^2x^2 - \frac{28\,(2\,Bab+Ab^2)x^6+7\,(Ba^2+2\,Aab)x^3+4\,Aa^2}{28\,x^7}$$

input

```
integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="maxima")
```

output

```
1/2*B*b^2*x^2 - 1/28*(28*(2*B*a*b + A*b^2)*x^6 + 7*(B*a^2 + 2*A*a*b)*x^3 +
 4*A*a^2)/x^7
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^8}\,dx = \frac{1}{2}\,Bb^2x^2 - \frac{56\,Babx^6+28\,Ab^2x^6+7\,Ba^2x^3+14\,Aabx^3+4\,Aa^2}{28\,x^7}$$

input

```
integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="giac")
```

output

```
1/2*B*b^2*x^2 - 1/28*(56*B*a*b*x^6 + 28*A*b^2*x^6 + 7*B*a^2*x^3 + 14*A*a*b
*x^3 + 4*A*a^2)/x^7
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^8}\,dx = \frac{B\,b^2\,x^2}{2}-\frac{x^3\,\left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^6\,\left(A\,b^2+2\,B\,a\,b\right)+\frac{A\,a^2}{7}}{x^7}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^8,x)
```

output
```
(B*b^2*x^2)/2 - (x^3*((B*a^2)/4 + (A*a*b)/2) + x^6*(A*b^2 + 2*B*a*b) + (A*
a^2)/7)/x^7
```

3.22 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx$

3.22.1 Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx = -\frac{a^2A}{8x^8} - \frac{a(2Ab+aB)}{5x^5} - \frac{b(Ab+2aB)}{2x^2} + b^2Bx$$

output
```
-1/8*a^2*A/x^8-1/5*a*(2*A*b+B*a)/x^5-1/2*b*(A*b+2*B*a)/x^2+b^2*B*x
```

3.22.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx = -\frac{a^2A}{8x^8} - \frac{a(2Ab+aB)}{5x^5} - \frac{b(Ab+2aB)}{2x^2} + b^2Bx$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^9,x]
```

output
```
-1/8*(a^2*A)/x^8 - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) +
 b^2*B*x
```

3.22.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^2 \left(A+Bx^3\right)}{x^9} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2 A}{x^9}+\frac{a(aB+2Ab)}{x^6}+\frac{b(2aB+Ab)}{x^3}+b^2 B\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2 A}{8x^8}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{2x^2}+b^2 Bx$$

input

```
Int[((a + b*x^3)^2*(A + B*x^3))/x^9,x]
```

output

```
-1/8*(a^2*A)/x^8 - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) +
 b^2*B*x
```

3.22.3.1 Defintions of rubi rules used

rule 950

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.22.4 Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^{2} A}{8 x^{8}}-\frac{a(2 A b+B a)}{5 x^{5}}-\frac{b(A b+2 B a)}{2 x^{2}}+b^{2} B x$	45
risch	$b^{2} B x+\frac{\left(-\frac{1}{2} b^{2} A-a b B\right) x^{6}+\left(-\frac{2}{5} a b A-\frac{1}{5} a^{2} B\right) x^{3}-\frac{a^{2} A}{8}}{x^{8}}$	51
norman	$\frac{b^{2} B x^{9}+\left(-\frac{1}{2} b^{2} A-a b B\right) x^{6}+\left(-\frac{2}{5} a b A-\frac{1}{5} a^{2} B\right) x^{3}-\frac{a^{2} A}{8}}{x^{8}}$	52
gosper	$-\frac{-40 b^{2} B x^{9}+20 A b^{2} x^{6}+40 B a b x^{6}+16 a A b x^{3}+8 a^{2} B x^{3}+5 a^{2} A}{40 x^{8}}$	56
parallelrisch	$-\frac{-40 b^{2} B x^{9}+20 A b^{2} x^{6}+40 B a b x^{6}+16 a A b x^{3}+8 a^{2} B x^{3}+5 a^{2} A}{40 x^{8}}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^2*A/x^8-1/5*a*(2*A*b+B*a)/x^5-1/2*b*(A*b+2*B*a)/x^2+b^2*B*x
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^9}\,dx=\frac{40\,Bb^2x^9-20\,(2\,Bab+Ab^2)x^6-8\,(Ba^2+2\,Aab)x^3-5\,Aa^2}{40\,x^8}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="fricas")
```

output
```
1/40*(40*B*b^2*x^9 - 20*(2*B*a*b + A*b^2)*x^6 - 8*(B*a^2 + 2*A*a*b)*x^3 -
5*A*a^2)/x^8
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^9}\,dx=Bb^2x+\frac{-5Aa^2+x^6(-20Ab^2-40Bab)+x^3(-16Aab-8Ba^2)}{40x^8}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**9,x)
```

output
```
B*b**2*x + (-5*A*a**2 + x**6*(-20*A*b**2 - 40*B*a*b) + x**3*(-16*A*a*b - 8
*B*a**2))/(40*x**8)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^9}\,dx = Bb^2x - \frac{20\,(2\,Bab+Ab^2)x^6 + 8\,(Ba^2+2\,Aab)x^3 + 5\,Aa^2}{40\,x^8}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="maxima")
```

output
```
B*b^2*x - 1/40*(20*(2*B*a*b + A*b^2)*x^6 + 8*(B*a^2 + 2*A*a*b)*x^3 + 5*A*a
^2)/x^8
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^9}\,dx = Bb^2x - \frac{40\,Babx^6 + 20\,Ab^2x^6 + 8\,Ba^2x^3 + 16\,Aabx^3 + 5\,Aa^2}{40\,x^8}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="giac")
```

output
```
B*b^2*x - 1/40*(40*B*a*b*x^6 + 20*A*b^2*x^6 + 8*B*a^2*x^3 + 16*A*a*b*x^3 +
 5*A*a^2)/x^8
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^9}\,dx = B\,b^2\,x - \frac{x^3\,\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+x^6\,\left(\frac{A\,b^2}{2}+B\,a\,b\right)+\frac{A\,a^2}{8}}{x^8}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^9,x)
```

output
```
B*b^2*x - (x^3*((B*a^2)/5 + (2*A*a*b)/5) + x^6*((A*b^2)/2 + B*a*b) + (A*a^
2)/8)/x^8
```

3.22 $\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^9}\,dx$

3.23 $\int x^9(a+bx^3)^5 (A+Bx^3)\, dx$

3.23.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\begin{aligned}\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=&\ \frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4(5Ab+aB)x^{13}+\frac{5}{16}a^3b(2Ab+aB)x^{16}\\&+\frac{10}{19}a^2b^2(Ab+aB)x^{19}+\frac{5}{22}ab^3(Ab+2aB)x^{22}\\&+\frac{1}{25}b^4(Ab+5aB)x^{25}+\frac{1}{28}b^5Bx^{28}\end{aligned}$$

output
```
1/10*a^5*A*x^10+1/13*a^4*(5*A*b+B*a)*x^13+5/16*a^3*b*(2*A*b+B*a)*x^16+10/1
9*a^2*b^2*(A*b+B*a)*x^19+5/22*a*b^3*(A*b+2*B*a)*x^22+1/25*b^4*(A*b+5*B*a)*
x^25+1/28*b^5*B*x^28
```

3.23.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=&\ \frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4(5Ab+aB)x^{13}+\frac{5}{16}a^3b(2Ab+aB)x^{16}\\&+\frac{10}{19}a^2b^2(Ab+aB)x^{19}+\frac{5}{22}ab^3(Ab+2aB)x^{22}\\&+\frac{1}{25}b^4(Ab+5aB)x^{25}+\frac{1}{28}b^5Bx^{28}\end{aligned}$$

input
```
Integrate[x^9*(a + b*x^3)^5*(A + B*x^3),x]
```

output

```
(a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^1
6)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22
 + (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28
```

3.23.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \; 950$$

$$\int \left(a^5Ax^9+a^4x^{12}(aB+5Ab)+5a^3bx^{15}(aB+2Ab)+10a^2b^2x^{18}(aB+Ab)+b^4x^{24}(5aB+Ab)+5ab^3x^{21}(2aB+\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4x^{13}(aB+5Ab)+\frac{5}{16}a^3bx^{16}(aB+2Ab)+\frac{10}{19}a^2b^2x^{19}(aB+Ab)+\frac{1}{25}b^4x^{25}(5aB+Ab)+\frac{5}{22}ab^3x^{22}(2aB+Ab)+\frac{1}{28}b^5Bx^{28}$$

input

```
Int[x^9*(a + b*x^3)^5*(A + B*x^3),x]
```

output

```
(a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^1
6)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22
 + (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28
```

3.23.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.23.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^{10}}{10}+\left(\frac{5}{13}a^4bA+\frac{1}{13}a^5B\right)x^{13}+\left(\frac{5}{8}a^3b^2A+\frac{5}{16}a^4bB\right)x^{16}+\left(\frac{10}{19}a^2b^3A+\frac{10}{19}a^3b^2B\right)x^{19}+$
default	$\frac{b^5 B x^{28}}{28}+\frac{(b^5A+5a\,b^4B)x^{25}}{25}+\frac{(5a\,b^4A+10a^2b^3B)x^{22}}{22}+\frac{(10a^2b^3A+10a^3b^2B)x^{19}}{19}+\frac{(10a^3b^2A+5a^4bB)x^{16}}{16}+$
gosper	$\frac{1}{10}a^5Ax^{10}+\frac{5}{13}x^{13}a^4bA+\frac{1}{13}x^{13}a^5B+\frac{5}{8}x^{16}a^3b^2A+\frac{5}{16}x^{16}a^4bB+\frac{10}{19}x^{19}a^2b^3A+\frac{10}{19}x^{19}a^3b^2B$
risch	$\frac{1}{10}a^5Ax^{10}+\frac{5}{13}x^{13}a^4bA+\frac{1}{13}x^{13}a^5B+\frac{5}{8}x^{16}a^3b^2A+\frac{5}{16}x^{16}a^4bB+\frac{10}{19}x^{19}a^2b^3A+\frac{10}{19}x^{19}a^3b^2B$
parallelrisch	$\frac{1}{10}a^5Ax^{10}+\frac{5}{13}x^{13}a^4bA+\frac{1}{13}x^{13}a^5B+\frac{5}{8}x^{16}a^3b^2A+\frac{5}{16}x^{16}a^4bB+\frac{10}{19}x^{19}a^2b^3A+\frac{10}{19}x^{19}a^3b^2B$

input
```
int(x^9*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/10*a^5*A*x^10+(5/13*a^4*b*A+1/13*a^5*B)*x^13+(5/8*a^3*b^2*A+5/16*a^4*b*B
)*x^16+(10/19*a^2*b^3*A+10/19*a^3*b^2*B)*x^19+(5/22*a*b^4*A+5/11*a^2*b^3*B
)*x^22+(1/25*b^5*A+1/5*a*b^4*B)*x^25+1/28*b^5*B*x^28
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{28}\,Bb^5x^{28}+\frac{1}{25}\left(5\,Bab^4+Ab^5\right)x^{25}+\frac{5}{22}\left(2\,Ba^2b^3+Aab^4\right)x^{22}+\frac{10}{19}\left(Ba^3b^2+Aa^2b^3\right)x^{19}+\frac{5}{16}\left(Ba^4b+2\,Aa^3b^2\right)x^{16}+\frac{1}{10}\,Aa^5x^{10}+\frac{1}{13}\left(Ba^5+5\,Aa^4b\right)x^{13}$$

input
```
integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/28*B*b^5*x^28 + 1/25*(5*B*a*b^4 + A*b^5)*x^25 + 5/22*(2*B*a^2*b^3 + A*a*
b^4)*x^22 + 10/19*(B*a^3*b^2 + A*a^2*b^3)*x^19 + 5/16*(B*a^4*b + 2*A*a^3*b
^2)*x^16 + 1/10*A*a^5*x^10 + 1/13*(B*a^5 + 5*A*a^4*b)*x^13
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16

$$\int x^9(a+bx^3)^5(A+Bx^3)\,dx = \frac{Aa^5x^{10}}{10}+\frac{Bb^5x^{28}}{28}+x^{25}\left(\frac{Ab^5}{25}+\frac{Bab^4}{5}\right)+x^{22}\cdot\left(\frac{5Aab^4}{22}+\frac{5Ba^2b^3}{11}\right)+x^{19}\cdot\left(\frac{10Aa^2b^3}{19}+\frac{10Ba^3b^2}{19}\right)+x^{16}\cdot\left(\frac{5Aa^3b^2}{8}+\frac{5Ba^4b}{16}\right)+x^{13}\cdot\left(\frac{5Aa^4b}{13}+\frac{Ba^5}{13}\right)$$

input
```
integrate(x**9*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**10/10 + B*b**5*x**28/28 + x**25*(A*b**5/25 + B*a*b**4/5) + x**22
*(5*A*a*b**4/22 + 5*B*a**2*b**3/11) + x**19*(10*A*a**2*b**3/19 + 10*B*a**3
*b**2/19) + x**16*(5*A*a**3*b**2/8 + 5*B*a**4*b/16) + x**13*(5*A*a**4*b/13
 + B*a**5/13)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^9(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{28}Bb^5x^{28}+\frac{1}{25}\left(5Bab^4+Ab^5\right)x^{25}+\frac{5}{22}\left(2Ba^2b^3+Aab^4\right)x^{22}+\frac{10}{19}\left(Ba^3b^2+Aa^2b^3\right)x^{19}+\frac{5}{16}\left(Ba^4b+2Aa^3b^2\right)x^{16}+\frac{1}{10}Aa^5x^{10}+\frac{1}{13}\left(Ba^5+5Aa^4b\right)x^{13}$$

input
```
integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/28*B*b^5*x^28 + 1/25*(5*B*a*b^4 + A*b^5)*x^25 + 5/22*(2*B*a^2*b^3 + A*a*
b^4)*x^22 + 10/19*(B*a^3*b^2 + A*a^2*b^3)*x^19 + 5/16*(B*a^4*b + 2*A*a^3*b
^2)*x^16 + 1/10*A*a^5*x^10 + 1/13*(B*a^5 + 5*A*a^4*b)*x^13
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{28}Bb^5x^{28}+\frac{1}{5}Bab^4x^{25}+\frac{1}{25}Ab^5x^{25}+\frac{5}{11}Ba^2b^3x^{22}+\frac{5}{22}Aab^4x^{22}+\frac{10}{19}Ba^3b^2x^{19}+\frac{10}{19}Aa^2b^3x^{19}+\frac{5}{16}Ba^4bx^{16}+\frac{5}{8}Aa^3b^2x^{16}+\frac{1}{13}Ba^5x^{13}+\frac{5}{13}Aa^4bx^{13}+\frac{1}{10}Aa^5x^{10}$$

input
```
integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/28*B*b^5*x^28 + 1/5*B*a*b^4*x^25 + 1/25*A*b^5*x^25 + 5/11*B*a^2*b^3*x^22
 + 5/22*A*a*b^4*x^22 + 10/19*B*a^3*b^2*x^19 + 10/19*A*a^2*b^3*x^19 + 5/16*
B*a^4*b*x^16 + 5/8*A*a^3*b^2*x^16 + 1/13*B*a^5*x^13 + 5/13*A*a^4*b*x^13 +
1/10*A*a^5*x^10
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=x^{13}\left(\frac{B\,a^5}{13}+\frac{5\,A\,b\,a^4}{13}\right)+x^{25}\left(\frac{A\,b^5}{25}+\frac{B\,a\,b^4}{5}\right)+\frac{A\,a^5\,x^{10}}{10}+\frac{B\,b^5\,x^{28}}{28}+\frac{10\,a^2\,b^2\,x^{19}\,(A\,b+B\,a)}{19}+\frac{5\,a^3\,b\,x^{16}\,(2\,A\,b+B\,a)}{16}+\frac{5\,a\,b^3\,x^{22}\,(A\,b+2\,B\,a)}{22}$$

input
```
int(x^9*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^13*((B*a^5)/13 + (5*A*a^4*b)/13) + x^25*((A*b^5)/25 + (B*a*b^4)/5) + (A*
a^5*x^10)/10 + (B*b^5*x^28)/28 + (10*a^2*b^2*x^19*(A*b + B*a))/19 + (5*a^3
*b*x^16*(2*A*b + B*a))/16 + (5*a*b^3*x^22*(A*b + 2*B*a))/22
```

3.24 $\int x^8(a+bx^3)^5(A+Bx^3)\,dx$

3.24.1 Optimal result

Integrand size = 20, antiderivative size = 95

$$\int x^8\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{a^2(Ab-aB)\left(a+bx^3\right)^6}{18b^4}-\frac{a(2Ab-3aB)\left(a+bx^3\right)^7}{21b^4}+\frac{(Ab-3aB)\left(a+bx^3\right)^8}{24b^4}+\frac{B\left(a+bx^3\right)^9}{27b^4}$$

output

```
1/18*a^2*(A*b-B*a)*(b*x^3+a)^6/b^4-1/21*a*(2*A*b-3*B*a)*(b*x^3+a)^7/b^4+1/
24*(A*b-3*B*a)*(b*x^3+a)^8/b^4+1/27*B*(b*x^3+a)^9/b^4
```

3.24.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int x^8\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx$$
$$=\frac{x^9(168a^5A+126a^4(5Ab+aB)x^3+504a^3b(2Ab+aB)x^6+840a^2b^2(Ab+aB)x^9+360ab^3(Ab+2aB)x^{12}}{1512}$$

input

```
Integrate[x^8*(a + b*x^3)^5*(A + B*x^3),x]
```

output

```
(x^9*(168*a^5*A + 126*a^4*(5*A*b + a*B)*x^3 + 504*a^3*b*(2*A*b + a*B)*x^6
+ 840*a^2*b^2*(A*b + a*B)*x^9 + 360*a*b^3*(A*b + 2*a*B)*x^12 + 63*b^4*(A*b
 + 5*a*B)*x^15 + 56*b^5*B*x^18))/1512
```

3.24.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int x^6 \left(bx^3+a\right)^5 \left(Bx^3+A\right) dx^3$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{3} \int \left(\frac{B\left(bx^3+a\right)^8}{b^3}+\frac{(Ab-3aB)\left(bx^3+a\right)^7}{b^3}+\frac{a(3aB-2Ab)\left(bx^3+a\right)^6}{b^3}-\frac{a^2(aB-Ab)\left(bx^3+a\right)^5}{b^3}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{a^2\left(a+bx^3\right)^6 (Ab-aB)}{6b^4}+\frac{\left(a+bx^3\right)^8 (Ab-3aB)}{8b^4}-\frac{a\left(a+bx^3\right)^7 (2Ab-3aB)}{7b^4}+\frac{B\left(a+bx^3\right)^9}{9b^4}\right)$$

input
```
Int[x^8*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
((a^2*(A*b - a*B)*(a + b*x^3)^6)/(6*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^3)^
7)/(7*b^4) + ((A*b - 3*a*B)*(a + b*x^3)^8)/(8*b^4) + (B*(a + b*x^3)^9)/(9*
b^4))/3
```

3.24.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.24.4 Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.27

method	result
norman	$\frac{b^5 B x^{27}}{27}+\frac{a^5 A x^9}{9}+\left(\frac{5}{12}a^4bA+\frac{1}{12}a^5B\right)x^{12}+\left(\frac{2}{3}a^3b^2A+\frac{1}{3}a^4bB\right)x^{15}+\left(\frac{5}{9}a^2b^3A+\frac{5}{9}a^3b^2B\right)x^{18}$
default	$\frac{b^5 B x^{27}}{27}+\frac{(b^5A+5ab^4B)x^{24}}{24}+\frac{(5ab^4A+10a^2b^3B)x^{21}}{21}+\frac{(10a^2b^3A+10a^3b^2B)x^{18}}{18}+\frac{(10a^3b^2A+5a^4bB)x^{15}}{15}+$
gosper	$\frac{1}{27}b^5Bx^{27}+\frac{1}{9}a^5Ax^9+\frac{5}{12}x^{12}a^4bA+\frac{1}{12}x^{12}a^5B+\frac{2}{3}x^{15}a^3b^2A+\frac{1}{3}x^{15}a^4bB+\frac{5}{9}x^{18}a^2b^3A+\frac{5}{9}x$
risch	$\frac{1}{27}b^5Bx^{27}+\frac{1}{9}a^5Ax^9+\frac{5}{12}x^{12}a^4bA+\frac{1}{12}x^{12}a^5B+\frac{2}{3}x^{15}a^3b^2A+\frac{1}{3}x^{15}a^4bB+\frac{5}{9}x^{18}a^2b^3A+\frac{5}{9}x$
parallelrisch	$\frac{1}{27}b^5Bx^{27}+\frac{1}{9}a^5Ax^9+\frac{5}{12}x^{12}a^4bA+\frac{1}{12}x^{12}a^5B+\frac{2}{3}x^{15}a^3b^2A+\frac{1}{3}x^{15}a^4bB+\frac{5}{9}x^{18}a^2b^3A+\frac{5}{9}x$

input
```
int(x^8*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/27*b^5*B*x^27+1/9*a^5*A*x^9+(5/12*a^4*b*A+1/12*a^5*B)*x^12+(2/3*a^3*b^2*
A+1/3*a^4*b*B)*x^15+(5/9*a^2*b^3*A+5/9*a^3*b^2*B)*x^18+(5/21*a*b^4*A+10/21
*a^2*b^3*B)*x^21+(1/24*b^5*A+5/24*a*b^4*B)*x^24
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$\begin{aligned}\int x^8(a+bx^3)^5(A+Bx^3)\,dx &= \frac{1}{27}Bb^5x^{27}+\frac{1}{24}(5Bab^4+Ab^5)x^{24}+\frac{5}{21}(2Ba^2b^3+Aab^4)x^{21}\\ &\quad+\frac{5}{9}(Ba^3b^2+Aa^2b^3)x^{18}+\frac{1}{3}(Ba^4b+2Aa^3b^2)x^{15}\\ &\quad+\frac{1}{9}Aa^5x^9+\frac{1}{12}(Ba^5+5Aa^4b)x^{12}\end{aligned}$$

input
```
integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/27*B*b^5*x^27 + 1/24*(5*B*a*b^4 + A*b^5)*x^24 + 5/21*(2*B*a^2*b^3 + A*a*
b^4)*x^21 + 5/9*(B*a^3*b^2 + A*a^2*b^3)*x^18 + 1/3*(B*a^4*b + 2*A*a^3*b^2)
*x^15 + 1/9*A*a^5*x^9 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

$$\int x^8(a+bx^3)^5(A+Bx^3)\,dx = \frac{Aa^5x^9}{9}+\frac{Bb^5x^{27}}{27}+x^{24}\left(\frac{Ab^5}{24}+\frac{5Bab^4}{24}\right)+x^{21}\cdot\left(\frac{5Aab^4}{21}+\frac{10Ba^2b^3}{21}\right)+x^{18}\cdot\left(\frac{5Aa^2b^3}{9}+\frac{5Ba^3b^2}{9}\right)+x^{15}\cdot\left(\frac{2Aa^3b^2}{3}+\frac{Ba^4b}{3}\right)+x^{12}\cdot\left(\frac{5Aa^4b}{12}+\frac{Ba^5}{12}\right)$$

input
```
integrate(x**8*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**9/9 + B*b**5*x**27/27 + x**24*(A*b**5/24 + 5*B*a*b**4/24) + x**2
1*(5*A*a*b**4/21 + 10*B*a**2*b**3/21) + x**18*(5*A*a**2*b**3/9 + 5*B*a**3*
b**2/9) + x**15*(2*A*a**3*b**2/3 + B*a**4*b/3) + x**12*(5*A*a**4*b/12 + B*
a**5/12)
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$\int x^8(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{27}\,Bb^5x^{27}+\frac{1}{24}\,(5\,Bab^4+Ab^5)x^{24}+\frac{5}{21}\,(2\,Ba^2b^3+Aab^4)x^{21}+\frac{5}{9}\,(Ba^3b^2+Aa^2b^3)x^{18}+\frac{1}{3}\,(Ba^4b+2\,Aa^3b^2)x^{15}+\frac{1}{9}\,Aa^5x^9+\frac{1}{12}\,(Ba^5+5\,Aa^4b)x^{12}$$

input
```
integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/27*B*b^5*x^27 + 1/24*(5*B*a*b^4 + A*b^5)*x^24 + 5/21*(2*B*a^2*b^3 + A*a*
b^4)*x^21 + 5/9*(B*a^3*b^2 + A*a^2*b^3)*x^18 + 1/3*(B*a^4*b + 2*A*a^3*b^2)
*x^15 + 1/9*A*a^5*x^9 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.32

$$\int x^8 (a+bx^3)^5 \left(A+Bx^3\right) \,dx = \frac{1}{27}\,Bb^5x^{27} + \frac{5}{24}\,Bab^4x^{24} + \frac{1}{24}\,Ab^5x^{24} + \frac{10}{21}\,Ba^2b^3x^{21} + \frac{5}{21}\,Aab^4x^{21} + \frac{5}{9}\,Ba^3b^2x^{18} + \frac{5}{9}\,Aa^2b^3x^{18} + \frac{1}{3}\,Ba^4bx^{15} + \frac{2}{3}\,Aa^3b^2x^{15} + \frac{1}{12}\,Ba^5x^{12} + \frac{5}{12}\,Aa^4bx^{12} + \frac{1}{9}\,Aa^5x^9$$

input
```
integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/27*B*b^5*x^27 + 5/24*B*a*b^4*x^24 + 1/24*A*b^5*x^24 + 10/21*B*a^2*b^3*x^
21 + 5/21*A*a*b^4*x^21 + 5/9*B*a^3*b^2*x^18 + 5/9*A*a^2*b^3*x^18 + 1/3*B*a
^4*b*x^15 + 2/3*A*a^3*b^2*x^15 + 1/12*B*a^5*x^12 + 5/12*A*a^4*b*x^12 + 1/9
*A*a^5*x^9
```

3.24.9 Mupad [B] (verification not implemented)

Time = 6.69 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int x^8 (a+bx^3)^5 \left(A+Bx^3\right) \,dx = x^{12}\left(\frac{B\,a^5}{12}+\frac{5\,A\,b\,a^4}{12}\right)+x^{24}\left(\frac{A\,b^5}{24}+\frac{5\,B\,a\,b^4}{24}\right) + \frac{A\,a^5\,x^9}{9}+\frac{B\,b^5\,x^{27}}{27}+\frac{5\,a^2\,b^2\,x^{18}\,(A\,b+B\,a)}{9} + \frac{a^3\,b\,x^{15}\,(2\,A\,b+B\,a)}{3}+\frac{5\,a\,b^3\,x^{21}\,(A\,b+2\,B\,a)}{21}$$

input
```
int(x^8*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^12*((B*a^5)/12 + (5*A*a^4*b)/12) + x^24*((A*b^5)/24 + (5*B*a*b^4)/24) +
(A*a^5*x^9)/9 + (B*b^5*x^27)/27 + (5*a^2*b^2*x^18*(A*b + B*a))/9 + (a^3*b*
x^15*(2*A*b + B*a))/3 + (5*a*b^3*x^21*(A*b + 2*B*a))/21
```

3.25 $\int x^7(a+bx^3)^5\left(A+Bx^3\right)\,dx$

3.25.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\begin{aligned}\int x^7\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\ &\frac{1}{8}a^5Ax^8+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{14}a^3b(2Ab+aB)x^{14}\\&+\frac{10}{17}a^2b^2(Ab+aB)x^{17}+\frac{1}{4}ab^3(Ab+2aB)x^{20}\\&+\frac{1}{23}b^4(Ab+5aB)x^{23}+\frac{1}{26}b^5Bx^{26}\end{aligned}$$

output
```
1/8*a^5*A*x^8+1/11*a^4*(5*A*b+B*a)*x^11+5/14*a^3*b*(2*A*b+B*a)*x^14+10/17*
a^2*b^2*(A*b+B*a)*x^17+1/4*a*b^3*(A*b+2*B*a)*x^20+1/23*b^4*(A*b+5*B*a)*x^2
3+1/26*b^5*B*x^26
```

3.25.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^7\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\ &\frac{1}{8}a^5Ax^8+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{14}a^3b(2Ab+aB)x^{14}\\&+\frac{10}{17}a^2b^2(Ab+aB)x^{17}+\frac{1}{4}ab^3(Ab+2aB)x^{20}\\&+\frac{1}{23}b^4(Ab+5aB)x^{23}+\frac{1}{26}b^5Bx^{26}\end{aligned}$$

input
```
Integrate[x^7*(a + b*x^3)^5*(A + B*x^3),x]
```

output

```
(a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)
/14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b
^4*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26
```

3.25.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\left(a+bx^3\right)^5\left(A+Bx^3\right)dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^5Ax^7+a^4x^{10}(aB+5Ab)+5a^3bx^{13}(aB+2Ab)+10a^2b^2x^{16}(aB+Ab)+b^4x^{22}(5aB+Ab)+5ab^3x^{19}(2aB+$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}a^5Ax^8+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{10}{17}a^2b^2x^{17}(aB+Ab)+\frac{1}{23}b^4x^{23}(5aB+Ab)+\frac{1}{4}ab^3x^{20}(2aB+Ab)+\frac{1}{26}b^5Bx^{26}$$

input

```
Int[x^7*(a + b*x^3)^5*(A + B*x^3),x]
```

output

```
(a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)
/14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b
^4*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26
```

3.25.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.25.4 Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^8}{8}+\left(\frac{5}{11}a^4bA+\frac{1}{11}a^5B\right)x^{11}+\left(\frac{5}{7}a^3b^2A+\frac{5}{14}a^4bB\right)x^{14}+\left(\frac{10}{17}a^2b^3A+\frac{10}{17}a^3b^2B\right)x^{17}+ (\frac{1}{4}a$
default	$\frac{b^5 B x^{26}}{26}+\frac{(b^5A+5a b^4B)x^{23}}{23}+\frac{(5a b^4A+10a^2b^3B)x^{20}}{20}+\frac{(10a^2b^3A+10a^3b^2B)x^{17}}{17}+\frac{(10a^3b^2A+5a^4bB)x^{14}}{14}+$
gosper	$\frac{1}{8}a^5A x^8+\frac{5}{11}x^{11}a^4bA+\frac{1}{11}x^{11}a^5B+\frac{5}{7}x^{14}a^3b^2A+\frac{5}{14}x^{14}a^4bB+\frac{10}{17}x^{17}a^2b^3A+\frac{10}{17}x^{17}a^3b^2B+$
risch	$\frac{1}{8}a^5A x^8+\frac{5}{11}x^{11}a^4bA+\frac{1}{11}x^{11}a^5B+\frac{5}{7}x^{14}a^3b^2A+\frac{5}{14}x^{14}a^4bB+\frac{10}{17}x^{17}a^2b^3A+\frac{10}{17}x^{17}a^3b^2B+$
parallelrisch	$\frac{1}{8}a^5A x^8+\frac{5}{11}x^{11}a^4bA+\frac{1}{11}x^{11}a^5B+\frac{5}{7}x^{14}a^3b^2A+\frac{5}{14}x^{14}a^4bB+\frac{10}{17}x^{17}a^2b^3A+\frac{10}{17}x^{17}a^3b^2B+$

input
```
int(x^7*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*a^5*A*x^8+(5/11*a^4*b*A+1/11*a^5*B)*x^11+(5/7*a^3*b^2*A+5/14*a^4*b*B)*
x^14+(10/17*a^2*b^3*A+10/17*a^3*b^2*B)*x^17+(1/4*a*b^4*A+1/2*a^2*b^3*B)*x^
20+(1/23*b^5*A+5/23*a*b^4*B)*x^23+1/26*b^5*B*x^26
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\begin{aligned}\int x^7(a+bx^3)^5\left(A+Bx^3\right)\,dx &= \frac{1}{26}\,Bb^5x^{26}+\frac{1}{23}\,(5\,Bab^4+Ab^5)x^{23}+\frac{1}{4}\,(2\,Ba^2b^3+Aab^4)x^{20}\\ &\quad+\frac{10}{17}\,(Ba^3b^2+Aa^2b^3)x^{17}+\frac{5}{14}\,(Ba^4b+2\,Aa^3b^2)x^{14}\\ &\quad+\frac{1}{8}\,Aa^5x^8+\frac{1}{11}\,(Ba^5+5\,Aa^4b)x^{11}\end{aligned}$$

input
```
integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/26*B*b^5*x^26 + 1/23*(5*B*a*b^4 + A*b^5)*x^23 + 1/4*(2*B*a^2*b^3 + A*a*b
^4)*x^20 + 10/17*(B*a^3*b^2 + A*a^2*b^3)*x^17 + 5/14*(B*a^4*b + 2*A*a^3*b^
2)*x^14 + 1/8*A*a^5*x^8 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.15

$$\begin{aligned}\int x^7(a+bx^3)^5\left(A+Bx^3\right)\,dx=&\ \frac{Aa^5x^8}{8}+\frac{Bb^5x^{26}}{26}+x^{23}\left(\frac{Ab^5}{23}+\frac{5Bab^4}{23}\right)\\&+x^{20}\left(\frac{Aab^4}{4}+\frac{Ba^2b^3}{2}\right)+x^{17}\cdot\left(\frac{10Aa^2b^3}{17}+\frac{10Ba^3b^2}{17}\right)\\&+x^{14}\cdot\left(\frac{5Aa^3b^2}{7}+\frac{5Ba^4b}{14}\right)+x^{11}\cdot\left(\frac{5Aa^4b}{11}+\frac{Ba^5}{11}\right)\end{aligned}$$

input
```
integrate(x**7*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**8/8 + B*b**5*x**26/26 + x**23*(A*b**5/23 + 5*B*a*b**4/23) + x**2
0*(A*a*b**4/4 + B*a**2*b**3/2) + x**17*(10*A*a**2*b**3/17 + 10*B*a**3*b**2
/17) + x**14*(5*A*a**3*b**2/7 + 5*B*a**4*b/14) + x**11*(5*A*a**4*b/11 + B*
a**5/11)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\begin{aligned}\int x^7(a+bx^3)^5\left(A+Bx^3\right)\,dx=&\ \frac{1}{26}\,Bb^5x^{26}+\frac{1}{23}\,(5\,Bab^4+Ab^5)x^{23}+\frac{1}{4}\,(2\,Ba^2b^3+Aab^4)x^{20}\\&+\frac{10}{17}\,(Ba^3b^2+Aa^2b^3)x^{17}+\frac{5}{14}\,(Ba^4b+2\,Aa^3b^2)x^{14}\\&+\frac{1}{8}\,Aa^5x^8+\frac{1}{11}\,(Ba^5+5\,Aa^4b)x^{11}\end{aligned}$$

input
```
integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/26*B*b^5*x^26 + 1/23*(5*B*a*b^4 + A*b^5)*x^23 + 1/4*(2*B*a^2*b^3 + A*a*b
^4)*x^20 + 10/17*(B*a^3*b^2 + A*a^2*b^3)*x^17 + 5/14*(B*a^4*b + 2*A*a^3*b^
2)*x^14 + 1/8*A*a^5*x^8 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^7\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{26}Bb^5x^{26}+\frac{5}{23}Bab^4x^{23}+\frac{1}{23}Ab^5x^{23}+\frac{1}{2}Ba^2b^3x^{20}+\frac{1}{4}Aab^4x^{20}+\frac{10}{17}Ba^3b^2x^{17}+\frac{10}{17}Aa^2b^3x^{17}+\frac{5}{14}Ba^4bx^{14}+\frac{5}{7}Aa^3b^2x^{14}+\frac{1}{11}Ba^5x^{11}+\frac{5}{11}Aa^4bx^{11}+\frac{1}{8}Aa^5x^8$$

input
```
integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/26*B*b^5*x^26 + 5/23*B*a*b^4*x^23 + 1/23*A*b^5*x^23 + 1/2*B*a^2*b^3*x^20
 + 1/4*A*a*b^4*x^20 + 10/17*B*a^3*b^2*x^17 + 10/17*A*a^2*b^3*x^17 + 5/14*B
*a^4*b*x^14 + 5/7*A*a^3*b^2*x^14 + 1/11*B*a^5*x^11 + 5/11*A*a^4*b*x^11 + 1
/8*A*a^5*x^8
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^7\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=x^{11}\left(\frac{B\,a^5}{11}+\frac{5\,A\,b\,a^4}{11}\right)+x^{23}\left(\frac{A\,b^5}{23}+\frac{5\,B\,a\,b^4}{23}\right)+\frac{A\,a^5\,x^8}{8}+\frac{B\,b^5\,x^{26}}{26}+\frac{10\,a^2\,b^2\,x^{17}\,\left(A\,b+B\,a\right)}{17}+\frac{5\,a^3\,b\,x^{14}\,\left(2\,A\,b+B\,a\right)}{14}+\frac{a\,b^3\,x^{20}\,\left(A\,b+2\,B\,a\right)}{4}$$

input
```
int(x^7*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^11*((B*a^5)/11 + (5*A*a^4*b)/11) + x^23*((A*b^5)/23 + (5*B*a*b^4)/23) +
(A*a^5*x^8)/8 + (B*b^5*x^26)/26 + (10*a^2*b^2*x^17*(A*b + B*a))/17 + (5*a^
3*b*x^14*(2*A*b + B*a))/14 + (a*b^3*x^20*(A*b + 2*B*a))/4
```

3.26 $\int x^6(a+bx^3)^5 (A+Bx^3)\,dx$

3.26.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\begin{aligned}\int x^6(a+bx^3)^5 (A+Bx^3)\,dx = {} & \frac{1}{7}a^5Ax^7+\frac{1}{10}a^4(5Ab+aB)x^{10}+\frac{5}{13}a^3b(2Ab+aB)x^{13}\\ & +\frac{5}{8}a^2b^2(Ab+aB)x^{16}+\frac{5}{19}ab^3(Ab+2aB)x^{19}\\ & +\frac{1}{22}b^4(Ab+5aB)x^{22}+\frac{1}{25}b^5Bx^{25}\end{aligned}$$

output
```
1/7*a^5*A*x^7+1/10*a^4*(5*A*b+B*a)*x^10+5/13*a^3*b*(2*A*b+B*a)*x^13+5/8*a^
2*b^2*(A*b+B*a)*x^16+5/19*a*b^3*(A*b+2*B*a)*x^19+1/22*b^4*(A*b+5*B*a)*x^22
+1/25*b^5*B*x^25
```

3.26.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^6(a+bx^3)^5 (A+Bx^3)\,dx = {} & \frac{1}{7}a^5Ax^7+\frac{1}{10}a^4(5Ab+aB)x^{10}+\frac{5}{13}a^3b(2Ab+aB)x^{13}\\ & +\frac{5}{8}a^2b^2(Ab+aB)x^{16}+\frac{5}{19}ab^3(Ab+2aB)x^{19}\\ & +\frac{1}{22}b^4(Ab+5aB)x^{22}+\frac{1}{25}b^5Bx^{25}\end{aligned}$$

input
```
Integrate[x^6*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)
/13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (
b^4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25
```

3.26.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^5 A x^6 + a^4 x^9 (aB+5Ab) + 5a^3 b x^{12} (aB+2Ab) + 10a^2 b^2 x^{15} (aB+Ab) + b^4 x^{21} (5aB+Ab) + 5ab^3 x^{18} (2aB+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}a^5 A x^7 + \frac{1}{10}a^4 x^{10}(aB+5Ab) + \frac{5}{13}a^3 b x^{13}(aB+2Ab) + \frac{5}{8}a^2 b^2 x^{16}(aB+Ab) + \frac{1}{22}b^4 x^{22}(5aB+ Ab) + \frac{5}{19}ab^3 x^{19}(2aB+Ab) + \frac{1}{25}b^5 B x^{25}$$

input
```
Int[x^6*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)
/13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (
b^4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25
```

3.26.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.26.4 Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^7}{7}+\left(\frac{1}{2}a^4 b A+\frac{1}{10}a^5 B\right)x^{10}+\left(\frac{10}{13}a^3 b^2 A+\frac{5}{13}a^4 b B\right)x^{13}+\left(\frac{5}{8}a^2 b^3 A+\frac{5}{8}a^3 b^2 B\right)x^{16}+$
default	$\frac{b^5 B x^{25}}{25}+\frac{\left(b^5 A+5a b^4 B\right)x^{22}}{22}+\frac{\left(5a b^4 A+10a^2 b^3 B\right)x^{19}}{19}+\frac{\left(10a^2 b^3 A+10a^3 b^2 B\right)x^{16}}{16}+\frac{\left(10a^3 b^2 A+5a^4 b B\right)x^{13}}{13}+$
gosper	$\frac{1}{7}a^5 A x^7+\frac{1}{2}x^{10}a^4 b A+\frac{1}{10}x^{10}a^5 B+\frac{10}{13}x^{13}a^3 b^2 A+\frac{5}{13}x^{13}a^4 b B+\frac{5}{8}x^{16}a^2 b^3 A+\frac{5}{8}x^{16}a^3 b^2 B+$
risch	$\frac{1}{7}a^5 A x^7+\frac{1}{2}x^{10}a^4 b A+\frac{1}{10}x^{10}a^5 B+\frac{10}{13}x^{13}a^3 b^2 A+\frac{5}{13}x^{13}a^4 b B+\frac{5}{8}x^{16}a^2 b^3 A+\frac{5}{8}x^{16}a^3 b^2 B+$
parallelrisch	$\frac{1}{7}a^5 A x^7+\frac{1}{2}x^{10}a^4 b A+\frac{1}{10}x^{10}a^5 B+\frac{10}{13}x^{13}a^3 b^2 A+\frac{5}{13}x^{13}a^4 b B+\frac{5}{8}x^{16}a^2 b^3 A+\frac{5}{8}x^{16}a^3 b^2 B+$

input
```
int(x^6*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*a^5*A*x^7+(1/2*a^4*b*A+1/10*a^5*B)*x^10+(10/13*a^3*b^2*A+5/13*a^4*b*B)
*x^13+(5/8*a^2*b^3*A+5/8*a^3*b^2*B)*x^16+(5/19*a*b^4*A+10/19*a^2*b^3*B)*x^
19+(1/22*b^5*A+5/22*a*b^4*B)*x^22+1/25*b^5*B*x^25
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^6\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{25}\,Bb^5x^{25}+\frac{1}{22}\,\left(5\,Bab^4+Ab^5\right)x^{22}+\frac{5}{19}\,\left(2\,Ba^2b^3+Aab^4\right)x^{19}+\frac{5}{8}\,\left(Ba^3b^2+Aa^2b^3\right)x^{16}+\frac{5}{13}\,\left(Ba^4b+2\,Aa^3b^2\right)x^{13}+\frac{1}{7}\,Aa^5x^7+\frac{1}{10}\,\left(Ba^5+5\,Aa^4b\right)x^{10}$$

input
```
integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/25*B*b^5*x^25 + 1/22*(5*B*a*b^4 + A*b^5)*x^22 + 5/19*(2*B*a^2*b^3 + A*a*
b^4)*x^19 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 5/13*(B*a^4*b + 2*A*a^3*b^2
)*x^13 + 1/7*A*a^5*x^7 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16

$$\int x^6(a+bx^3)^5\left(A+Bx^3\right)\,dx=\frac{Aa^5x^7}{7}+\frac{Bb^5x^{25}}{25}+x^{22}\left(\frac{Ab^5}{22}+\frac{5Bab^4}{22}\right)+x^{19}\cdot\left(\frac{5Aab^4}{19}+\frac{10Ba^2b^3}{19}\right)+x^{16}\cdot\left(\frac{5Aa^2b^3}{8}+\frac{5Ba^3b^2}{8}\right)+x^{13}\cdot\left(\frac{10Aa^3b^2}{13}+\frac{5Ba^4b}{13}\right)+x^{10}\left(\frac{Aa^4b}{2}+\frac{Ba^5}{10}\right)$$

input
```
integrate(x**6*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**7/7 + B*b**5*x**25/25 + x**22*(A*b**5/22 + 5*B*a*b**4/22) + x**1
9*(5*A*a*b**4/19 + 10*B*a**2*b**3/19) + x**16*(5*A*a**2*b**3/8 + 5*B*a**3*
b**2/8) + x**13*(10*A*a**3*b**2/13 + 5*B*a**4*b/13) + x**10*(A*a**4*b/2 +
B*a**5/10)
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^6(a+bx^3)^5\left(A+Bx^3\right)\,dx=\frac{1}{25}\,Bb^5x^{25}+\frac{1}{22}\,\left(5\,Bab^4+Ab^5\right)x^{22}+\frac{5}{19}\,\left(2\,Ba^2b^3+Aab^4\right)x^{19}+\frac{5}{8}\,\left(Ba^3b^2+Aa^2b^3\right)x^{16}+\frac{5}{13}\,\left(Ba^4b+2\,Aa^3b^2\right)x^{13}+\frac{1}{7}\,Aa^5x^7+\frac{1}{10}\,\left(Ba^5+5\,Aa^4b\right)x^{10}$$

input
```
integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/25*B*b^5*x^25 + 1/22*(5*B*a*b^4 + A*b^5)*x^22 + 5/19*(2*B*a^2*b^3 + A*a*
b^4)*x^19 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 5/13*(B*a^4*b + 2*A*a^3*b^2
)*x^13 + 1/7*A*a^5*x^7 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^6 (a+bx^3)^5 (A+Bx^3)\, dx = \frac{1}{25}Bb^5x^{25} + \frac{5}{22}Bab^4x^{22} + \frac{1}{22}Ab^5x^{22} + \frac{10}{19}Ba^2b^3x^{19} + \frac{5}{19}Aab^4x^{19} + \frac{5}{8}Ba^3b^2x^{16} + \frac{5}{8}Aa^2b^3x^{16} + \frac{5}{13}Ba^4bx^{13} + \frac{10}{13}Aa^3b^2x^{13} + \frac{1}{10}Ba^5x^{10} + \frac{1}{2}Aa^4bx^{10} + \frac{1}{7}Aa^5x^7$$

input
```
integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/25*B*b^5*x^25 + 5/22*B*a*b^4*x^22 + 1/22*A*b^5*x^22 + 10/19*B*a^2*b^3*x^
19 + 5/19*A*a*b^4*x^19 + 5/8*B*a^3*b^2*x^16 + 5/8*A*a^2*b^3*x^16 + 5/13*B*
a^4*b*x^13 + 10/13*A*a^3*b^2*x^13 + 1/10*B*a^5*x^10 + 1/2*A*a^4*b*x^10 + 1
/7*A*a^5*x^7
```

3.26.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^6 (a+bx^3)^5 (A+Bx^3)\, dx = x^{10}\left(\frac{B\,a^5}{10} + \frac{A\,b\,a^4}{2}\right) + x^{22}\left(\frac{A\,b^5}{22} + \frac{5\,B\,a\,b^4}{22}\right) + \frac{A\,a^5\,x^7}{7} + \frac{B\,b^5\,x^{25}}{25} + \frac{5\,a^2\,b^2\,x^{16}\,(A\,b+B\,a)}{8} + \frac{5\,a^3\,b\,x^{13}\,(2\,A\,b+B\,a)}{13} + \frac{5\,a\,b^3\,x^{19}\,(A\,b+2\,B\,a)}{19}$$

input
```
int(x^6*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^10*((B*a^5)/10 + (A*a^4*b)/2) + x^22*((A*b^5)/22 + (5*B*a*b^4)/22) + (A*
a^5*x^7)/7 + (B*b^5*x^25)/25 + (5*a^2*b^2*x^16*(A*b + B*a))/8 + (5*a^3*b*x
^13*(2*A*b + B*a))/13 + (5*a*b^3*x^19*(A*b + 2*B*a))/19
```

3.27 $\int x^5(a+bx^3)^5 (A+Bx^3)\,dx$

3.27.1 Optimal result

Integrand size = 20, antiderivative size = 67

$$\int x^5(a+bx^3)^5 (A+Bx^3)\,dx = -\frac{a(Ab-aB)(a+bx^3)^6}{18b^3} + \frac{(Ab-2aB)(a+bx^3)^7}{21b^3} + \frac{B(a+bx^3)^8}{24b^3}$$

output
```
-1/18*a*(A*b-B*a)*(b*x^3+a)^6/b^3+1/21*(A*b-2*B*a)*(b*x^3+a)^7/b^3+1/24*B*
(b*x^3+a)^8/b^3
```

3.27.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.60

$$\int x^5(a+bx^3)^5 (A+Bx^3)\,dx = \frac{1}{504}x^6\big(84a^5A+56a^4(5Ab+aB)x^3+210a^3b(2Ab+aB)x^6 + 336a^2b^2(Ab+aB)x^9+140ab^3(Ab+2aB)x^{12} + 24b^4(Ab+5aB)x^{15}+21b^5Bx^{18}\big)$$

input
```
Integrate[x^5*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(x^6*(84*a^5*A + 56*a^4*(5*A*b + a*B)*x^3 + 210*a^3*b*(2*A*b + a*B)*x^6 +
336*a^2*b^2*(A*b + a*B)*x^9 + 140*a*b^3*(A*b + 2*a*B)*x^12 + 24*b^4*(A*b +
 5*a*B)*x^15 + 21*b^5*B*x^18))/504
```

3.27.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3} \int x^3 \left(bx^3+a\right)^5 \left(Bx^3+A\right) dx^3$$

$$\Big\downarrow \ 85$$

$$\frac{1}{3} \int \left(\frac{B\left(bx^3+a\right)^7}{b^2}+\frac{(Ab-2aB)\left(bx^3+a\right)^6}{b^2}+\frac{a(aB-Ab)\left(bx^3+a\right)^5}{b^2}\right) dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3} \left(\frac{\left(a+bx^3\right)^7 (Ab-2aB)}{7b^3}-\frac{a\left(a+bx^3\right)^6 (Ab-aB)}{6b^3}+\frac{B\left(a+bx^3\right)^8}{8b^3}\right)$$

input
```
Int[x^5*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(-1/6*(a*(A*b - a*B)*(a + b*x^3)^6)/b^3 + ((A*b - 2*a*B)*(a + b*x^3)^7)/(7
*b^3) + (B*(a + b*x^3)^8)/(8*b^3))/3
```

3.27.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.27.4 Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.81

method	result
norman	$\frac{a^5 A x^6}{6} + \left(\frac{5}{9}a^4bA + \frac{1}{9}a^5B\right)x^9 + \left(\frac{5}{6}a^3b^2A + \frac{5}{12}a^4bB\right)x^{12} + \left(\frac{2}{3}a^2b^3A + \frac{2}{3}a^3b^2B\right)x^{15} + \left(\frac{5}{18}a\,b^4$
default	$\frac{b^5 B x^{24}}{24} + \frac{\left(b^5A+5a\,b^4B\right)x^{21}}{21} + \frac{\left(5a\,b^4A+10a^2b^3B\right)x^{18}}{18} + \frac{\left(10a^2b^3A+10a^3b^2B\right)x^{15}}{15} + \frac{\left(10a^3b^2A+5a^4bB\right)x^{12}}{12} + (5a^4$
gosper	$\frac{1}{6}a^5 A x^6 + \frac{5}{9}x^9a^4bA + \frac{1}{9}x^9a^5B + \frac{5}{6}x^{12}a^3b^2A + \frac{5}{12}x^{12}a^4bB + \frac{2}{3}x^{15}a^2b^3A + \frac{2}{3}x^{15}a^3b^2B + \frac{5}{18}x$
risch	$\frac{1}{6}a^5 A x^6 + \frac{5}{9}x^9a^4bA + \frac{1}{9}x^9a^5B + \frac{5}{6}x^{12}a^3b^2A + \frac{5}{12}x^{12}a^4bB + \frac{2}{3}x^{15}a^2b^3A + \frac{2}{3}x^{15}a^3b^2B + \frac{5}{18}x$
parallelrisch	$\frac{1}{6}a^5 A x^6 + \frac{5}{9}x^9a^4bA + \frac{1}{9}x^9a^5B + \frac{5}{6}x^{12}a^3b^2A + \frac{5}{12}x^{12}a^4bB + \frac{2}{3}x^{15}a^2b^3A + \frac{2}{3}x^{15}a^3b^2B + \frac{5}{18}x$

input
```
int(x^5*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/6*a^5*A*x^6+(5/9*a^4*b*A+1/9*a^5*B)*x^9+(5/6*a^3*b^2*A+5/12*a^4*b*B)*x^1
2+(2/3*a^2*b^3*A+2/3*a^3*b^2*B)*x^15+(5/18*a*b^4*A+5/9*a^2*b^3*B)*x^18+(1/
21*b^5*A+5/21*a*b^4*B)*x^21+1/24*b^5*B*x^24
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.78

$$\begin{aligned}\int x^5\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx = {} & \frac{1}{24}\,Bb^5x^{24} + \frac{1}{21}\left(5\,Bab^4 + Ab^5\right)x^{21} + \frac{5}{18}\left(2\,Ba^2b^3 + Aab^4\right)x^{18} \\ & + \frac{2}{3}\left(Ba^3b^2 + Aa^2b^3\right)x^{15} + \frac{5}{12}\left(Ba^4b + 2\,Aa^3b^2\right)x^{12} \\ & + \frac{1}{6}\,Aa^5x^6 + \frac{1}{9}\left(Ba^5 + 5\,Aa^4b\right)x^9\end{aligned}$$

input
```
integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/24*B*b^5*x^24 + 1/21*(5*B*a*b^4 + A*b^5)*x^21 + 5/18*(2*B*a^2*b^3 + A*a*
b^4)*x^18 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 5/12*(B*a^4*b + 2*A*a^3*b^2
)*x^12 + 1/6*A*a^5*x^6 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

3.27.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(60) = 120$.

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.06

$$\int x^5(a+bx^3)^5(A+Bx^3)\,dx = \frac{Aa^5x^6}{6} + \frac{Bb^5x^{24}}{24} + x^{21}\left(\frac{Ab^5}{21} + \frac{5Bab^4}{21}\right) + x^{18} \cdot \left(\frac{5Aab^4}{18} + \frac{5Ba^2b^3}{9}\right) + x^{15} \cdot \left(\frac{2Aa^2b^3}{3} + \frac{2Ba^3b^2}{3}\right) + x^{12} \cdot \left(\frac{5Aa^3b^2}{6} + \frac{5Ba^4b}{12}\right) + x^9 \cdot \left(\frac{5Aa^4b}{9} + \frac{Ba^5}{9}\right)$$

input
```
integrate(x**5*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**6/6 + B*b**5*x**24/24 + x**21*(A*b**5/21 + 5*B*a*b**4/21) + x**1
8*(5*A*a*b**4/18 + 5*B*a**2*b**3/9) + x**15*(2*A*a**2*b**3/3 + 2*B*a**3*b*
*2/3) + x**12*(5*A*a**3*b**2/6 + 5*B*a**4*b/12) + x**9*(5*A*a**4*b/9 + B*a
**5/9)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.78

$$\int x^5(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{24}Bb^5x^{24} + \frac{1}{21}(5Bab^4 + Ab^5)x^{21} + \frac{5}{18}(2Ba^2b^3 + Aab^4)x^{18} + \frac{2}{3}(Ba^3b^2 + Aa^2b^3)x^{15} + \frac{5}{12}(Ba^4b + 2Aa^3b^2)x^{12} + \frac{1}{6}Aa^5x^6 + \frac{1}{9}(Ba^5 + 5Aa^4b)x^9$$

input
```
integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/24*B*b^5*x^24 + 1/21*(5*B*a*b^4 + A*b^5)*x^21 + 5/18*(2*B*a^2*b^3 + A*a*
b^4)*x^18 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 5/12*(B*a^4*b + 2*A*a^3*b^2
)*x^12 + 1/6*A*a^5*x^6 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

3.27.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(62) = 124$.

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.87

$$\int x^5\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx = \frac{1}{24}Bb^5x^{24}+\frac{5}{21}Bab^4x^{21}+\frac{1}{21}Ab^5x^{21}+\frac{5}{9}Ba^2b^3x^{18}+\frac{5}{18}Aab^4x^{18}+\frac{2}{3}Ba^3b^2x^{15}+\frac{2}{3}Aa^2b^3x^{15}+\frac{5}{12}Ba^4bx^{12}+\frac{5}{6}Aa^3b^2x^{12}+\frac{1}{9}Ba^5x^9+\frac{5}{9}Aa^4bx^9+\frac{1}{6}Aa^5x^6$$

input
```
integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/24*B*b^5*x^24 + 5/21*B*a*b^4*x^21 + 1/21*A*b^5*x^21 + 5/9*B*a^2*b^3*x^18
 + 5/18*A*a*b^4*x^18 + 2/3*B*a^3*b^2*x^15 + 2/3*A*a^2*b^3*x^15 + 5/12*B*a^
4*b*x^12 + 5/6*A*a^3*b^2*x^12 + 1/9*B*a^5*x^9 + 5/9*A*a^4*b*x^9 + 1/6*A*a^
5*x^6
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.60

$$\int x^5\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx = x^9\left(\frac{B\,a^5}{9}+\frac{5\,A\,b\,a^4}{9}\right)+x^{21}\left(\frac{A\,b^5}{21}+\frac{5\,B\,a\,b^4}{21}\right)+\frac{A\,a^5\,x^6}{6}+\frac{B\,b^5\,x^{24}}{24}+\frac{2\,a^2\,b^2\,x^{15}\,(A\,b+B\,a)}{3}+\frac{5\,a^3\,b\,x^{12}\,(2\,A\,b+B\,a)}{12}+\frac{5\,a\,b^3\,x^{18}\,(A\,b+2\,B\,a)}{18}$$

input
```
int(x^5*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^9*((B*a^5)/9 + (5*A*a^4*b)/9) + x^21*((A*b^5)/21 + (5*B*a*b^4)/21) + (A*
a^5*x^6)/6 + (B*b^5*x^24)/24 + (2*a^2*b^2*x^15*(A*b + B*a))/3 + (5*a^3*b*x
^12*(2*A*b + B*a))/12 + (5*a*b^3*x^18*(A*b + 2*B*a))/18
```

3.28 $\int x^4(a+bx^3)^5\left(A+Bx^3\right)\,dx$

3.28.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\begin{aligned}\int x^4(a+bx^3)^5\left(A+Bx^3\right)\,dx=&\ \frac{1}{5}a^5Ax^5+\frac{1}{8}a^4(5Ab+aB)x^8+\frac{5}{11}a^3b(2Ab+aB)x^{11}\\&+\frac{5}{7}a^2b^2(Ab+aB)x^{14}+\frac{5}{17}ab^3(Ab+2aB)x^{17}\\&+\frac{1}{20}b^4(Ab+5aB)x^{20}+\frac{1}{23}b^5Bx^{23}\end{aligned}$$

output
```
1/5*a^5*A*x^5+1/8*a^4*(5*A*b+B*a)*x^8+5/11*a^3*b*(2*A*b+B*a)*x^11+5/7*a^2*
b^2*(A*b+B*a)*x^14+5/17*a*b^3*(A*b+2*B*a)*x^17+1/20*b^4*(A*b+5*B*a)*x^20+1
/23*b^5*B*x^23
```

3.28.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^4(a+bx^3)^5\left(A+Bx^3\right)\,dx=&\ \frac{1}{5}a^5Ax^5+\frac{1}{8}a^4(5Ab+aB)x^8+\frac{5}{11}a^3b(2Ab+aB)x^{11}\\&+\frac{5}{7}a^2b^2(Ab+aB)x^{14}+\frac{5}{17}ab^3(Ab+2aB)x^{17}\\&+\frac{1}{20}b^4(Ab+5aB)x^{20}+\frac{1}{23}b^5Bx^{23}\end{aligned}$$

input
```
Integrate[x^4*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^8)/8 + (5*a^3*b*(2*A*b + a*B)*x^11)/1
1 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^
4*(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^23)/23
```

3.28.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^5Ax^4+a^4x^7(aB+5Ab)+5a^3bx^{10}(aB+2Ab)+10a^2b^2x^{13}(aB+Ab)+b^4x^{19}(5aB+Ab)+5ab^3x^{16}(2aB+$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^5Ax^5+\frac{1}{8}a^4x^8(aB+5Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{5}{7}a^2b^2x^{14}(aB+Ab)+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)+\frac{1}{23}b^5Bx^{23}$$

input
```
Int[x^4*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^8)/8 + (5*a^3*b*(2*A*b + a*B)*x^11)/1
1 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^
4*(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^23)/23
```

3.28.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.28.4 Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^5}{5} + \left(\frac{5}{8} a^4 b A + \frac{1}{8} a^5 B\right) x^8 + \left(\frac{10}{11} a^3 b^2 A + \frac{5}{11} a^4 b B\right) x^{11} + \left(\frac{5}{7} a^2 b^3 A + \frac{5}{7} a^3 b^2 B\right) x^{14} + (\frac{5}{17} a b^4$
default	$\frac{b^5 B x^{23}}{23} + \frac{(b^5 A + 5 a b^4 B) x^{20}}{20} + \frac{(5 a b^4 A + 10 a^2 b^3 B) x^{17}}{17} + \frac{(10 a^2 b^3 A + 10 a^3 b^2 B) x^{14}}{14} + \frac{(10 a^3 b^2 A + 5 a^4 b B) x^{11}}{11} +$
gosper	$\frac{1}{5} a^5 A x^5 + \frac{5}{8} x^8 a^4 b A + \frac{1}{8} x^8 a^5 B + \frac{10}{11} x^{11} a^3 b^2 A + \frac{5}{11} x^{11} a^4 b B + \frac{5}{7} x^{14} a^2 b^3 A + \frac{5}{7} x^{14} a^3 b^2 B + \frac{5}{17} x$
risch	$\frac{1}{5} a^5 A x^5 + \frac{5}{8} x^8 a^4 b A + \frac{1}{8} x^8 a^5 B + \frac{10}{11} x^{11} a^3 b^2 A + \frac{5}{11} x^{11} a^4 b B + \frac{5}{7} x^{14} a^2 b^3 A + \frac{5}{7} x^{14} a^3 b^2 B + \frac{5}{17} x$
parallelrisch	$\frac{1}{5} a^5 A x^5 + \frac{5}{8} x^8 a^4 b A + \frac{1}{8} x^8 a^5 B + \frac{10}{11} x^{11} a^3 b^2 A + \frac{5}{11} x^{11} a^4 b B + \frac{5}{7} x^{14} a^2 b^3 A + \frac{5}{7} x^{14} a^3 b^2 B + \frac{5}{17} x$

input
```
int(x^4*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/5*a^5*A*x^5+(5/8*a^4*b*A+1/8*a^5*B)*x^8+(10/11*a^3*b^2*A+5/11*a^4*b*B)*x
^11+(5/7*a^2*b^3*A+5/7*a^3*b^2*B)*x^14+(5/17*a*b^4*A+10/17*a^2*b^3*B)*x^17
+(1/20*b^5*A+1/4*a*b^4*B)*x^20+1/23*b^5*B*x^23
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\begin{aligned}\int x^4 (a + b x^3)^5 \left(A + B x^3\right) \, dx = & \frac{1}{23} B b^5 x^{23} + \frac{1}{20} \left(5 B a b^4 + A b^5\right) x^{20} \\ & + \frac{5}{17} \left(2 B a^2 b^3 + A a b^4\right) x^{17} + \frac{5}{7} \left(B a^3 b^2 + A a^2 b^3\right) x^{14} \\ & + \frac{5}{11} \left(B a^4 b + 2 A a^3 b^2\right) x^{11} + \frac{1}{5} A a^5 x^5 + \frac{1}{8} \left(B a^5 + 5 A a^4 b\right) x^8\end{aligned}$$

input
```
integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/23*B*b^5*x^23 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/17*(2*B*a^2*b^3 + A*a*
b^4)*x^17 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 5/11*(B*a^4*b + 2*A*a^3*b^2
)*x^11 + 1/5*A*a^5*x^5 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

3.28.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16

$$\int x^4(a+bx^3)^5\left(A+Bx^3\right)\,dx=\frac{Aa^5x^5}{5}+\frac{Bb^5x^{23}}{23}+x^{20}\left(\frac{Ab^5}{20}+\frac{Bab^4}{4}\right)+x^{17}\cdot\left(\frac{5Aab^4}{17}+\frac{10Ba^2b^3}{17}\right)+x^{14}\cdot\left(\frac{5Aa^2b^3}{7}+\frac{5Ba^3b^2}{7}\right)+x^{11}\cdot\left(\frac{10Aa^3b^2}{11}+\frac{5Ba^4b}{11}\right)+x^8\cdot\left(\frac{5Aa^4b}{8}+\frac{Ba^5}{8}\right)$$

input
```
integrate(x**4*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**5/5 + B*b**5*x**23/23 + x**20*(A*b**5/20 + B*a*b**4/4) + x**17*(
5*A*a*b**4/17 + 10*B*a**2*b**3/17) + x**14*(5*A*a**2*b**3/7 + 5*B*a**3*b**
2/7) + x**11*(10*A*a**3*b**2/11 + 5*B*a**4*b/11) + x**8*(5*A*a**4*b/8 + B*
a**5/8)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^4(a+bx^3)^5\left(A+Bx^3\right)\,dx=\frac{1}{23}\,Bb^5x^{23}+\frac{1}{20}\left(5\,Bab^4+Ab^5\right)x^{20}+\frac{5}{17}\left(2\,Ba^2b^3+Aab^4\right)x^{17}+\frac{5}{7}\left(Ba^3b^2+Aa^2b^3\right)x^{14}+\frac{5}{11}\left(Ba^4b+2\,Aa^3b^2\right)x^{11}+\frac{1}{5}\,Aa^5x^5+\frac{1}{8}\left(Ba^5+5\,Aa^4b\right)x^8$$

input
```
integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/23*B*b^5*x^23 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/17*(2*B*a^2*b^3 + A*a*
b^4)*x^17 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 5/11*(B*a^4*b + 2*A*a^3*b^2
)*x^11 + 1/5*A*a^5*x^5 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\begin{aligned}\int x^4(a+bx^3)^5\left(A+Bx^3\right)\,dx &= \frac{1}{23}Bb^5x^{23}+\frac{1}{4}Bab^4x^{20}+\frac{1}{20}Ab^5x^{20}+\frac{10}{17}Ba^2b^3x^{17}\\ &\quad+\frac{5}{17}Aab^4x^{17}+\frac{5}{7}Ba^3b^2x^{14}+\frac{5}{7}Aa^2b^3x^{14}+\frac{5}{11}Ba^4bx^{11}\\ &\quad+\frac{10}{11}Aa^3b^2x^{11}+\frac{1}{8}Ba^5x^8+\frac{5}{8}Aa^4bx^8+\frac{1}{5}Aa^5x^5\end{aligned}$$

input
```
integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/23*B*b^5*x^23 + 1/4*B*a*b^4*x^20 + 1/20*A*b^5*x^20 + 10/17*B*a^2*b^3*x^1
7 + 5/17*A*a*b^4*x^17 + 5/7*B*a^3*b^2*x^14 + 5/7*A*a^2*b^3*x^14 + 5/11*B*a
^4*b*x^11 + 10/11*A*a^3*b^2*x^11 + 1/8*B*a^5*x^8 + 5/8*A*a^4*b*x^8 + 1/5*A
*a^5*x^5
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\begin{aligned}\int x^4(a+bx^3)^5\left(A+Bx^3\right)\,dx &= x^8\left(\frac{B\,a^5}{8}+\frac{5\,A\,b\,a^4}{8}\right)+x^{20}\left(\frac{A\,b^5}{20}+\frac{B\,a\,b^4}{4}\right)\\ &\quad+\frac{A\,a^5\,x^5}{5}+\frac{B\,b^5\,x^{23}}{23}+\frac{5\,a^2\,b^2\,x^{14}\,(A\,b+B\,a)}{7}\\ &\quad+\frac{5\,a^3\,b\,x^{11}\,(2\,A\,b+B\,a)}{11}+\frac{5\,a\,b^3\,x^{17}\,(A\,b+2\,B\,a)}{17}\end{aligned}$$

input
```
int(x^4*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^8*((B*a^5)/8 + (5*A*a^4*b)/8) + x^20*((A*b^5)/20 + (B*a*b^4)/4) + (A*a^5
*x^5)/5 + (B*b^5*x^23)/23 + (5*a^2*b^2*x^14*(A*b + B*a))/7 + (5*a^3*b*x^11
*(2*A*b + B*a))/11 + (5*a*b^3*x^17*(A*b + 2*B*a))/17
```

3.29 $\int x^3(a+bx^3)^5\left(A+Bx^3\right)\,dx$

3.29.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\begin{aligned}\int x^3\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \frac{1}{4}a^5Ax^4+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{1}{2}a^3b(2Ab+aB)x^{10}\\ &\quad+\frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{5}{16}ab^3(Ab+2aB)x^{16}\\ &\quad+\frac{1}{19}b^4(Ab+5aB)x^{19}+\frac{1}{22}b^5Bx^{22}\end{aligned}$$

output
```
1/4*a^5*A*x^4+1/7*a^4*(5*A*b+B*a)*x^7+1/2*a^3*b*(2*A*b+B*a)*x^10+10/13*a^2
*b^2*(A*b+B*a)*x^13+5/16*a*b^3*(A*b+2*B*a)*x^16+1/19*b^4*(A*b+5*B*a)*x^19+
1/22*b^5*B*x^22
```

3.29.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^3\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \frac{1}{4}a^5Ax^4+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{1}{2}a^3b(2Ab+aB)x^{10}\\ &\quad+\frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{5}{16}ab^3(Ab+2aB)x^{16}\\ &\quad+\frac{1}{19}b^4(Ab+5aB)x^{19}+\frac{1}{22}b^5Bx^{22}\end{aligned}$$

input
```
Integrate[x^3*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 +
 (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4
*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22
```

3.29.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \; 950$$

$$\int \left(a^5Ax^3+a^4x^6(aB+5Ab)+5a^3bx^9(aB+2Ab)+10a^2b^2x^{12}(aB+Ab)+b^4x^{18}(5aB+Ab)+5ab^3x^{15}(2aB+$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}a^5Ax^4+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{2}a^3bx^{10}(aB+2Ab)+\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{16}ab^3x^{16}(2aB+Ab)+\frac{1}{22}b^5Bx^{22}$$

input
```
Int[x^3*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 +
 (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4
*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22
```

3.29.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.29.4 Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^4}{4}+\left(\frac{5}{7}a^4 bA+\frac{1}{7}a^5 B\right)x^7+\left(a^3 b^2 A+\frac{1}{2}a^4 bB\right)x^{10}+\left(\frac{10}{13}a^2 b^3 A+\frac{10}{13}a^3 b^2 B\right)x^{13}+\big(\frac{5}{16}a b^4 A$
default	$\frac{b^5 B x^{22}}{22}+\frac{(b^5 A+5a b^4 B)x^{19}}{19}+\frac{(5a b^4 A+10a^2 b^3 B)x^{16}}{16}+\frac{(10a^2 b^3 A+10a^3 b^2 B)x^{13}}{13}+\frac{(10a^3 b^2 A+5a^4 bB)x^{10}}{10}+$
gosper	$\frac{1}{4}a^5 A x^4+\frac{5}{7}x^7 a^4 bA+\frac{1}{7}x^7 a^5 B+x^{10}a^3 b^2 A+\frac{1}{2}x^{10}a^4 bB+\frac{10}{13}x^{13}a^2 b^3 A+\frac{10}{13}x^{13}a^3 b^2 B+\frac{5}{16}x^{16}$
risch	$\frac{1}{4}a^5 A x^4+\frac{5}{7}x^7 a^4 bA+\frac{1}{7}x^7 a^5 B+x^{10}a^3 b^2 A+\frac{1}{2}x^{10}a^4 bB+\frac{10}{13}x^{13}a^2 b^3 A+\frac{10}{13}x^{13}a^3 b^2 B+\frac{5}{16}x^{16}$
parallelrisch	$\frac{1}{4}a^5 A x^4+\frac{5}{7}x^7 a^4 bA+\frac{1}{7}x^7 a^5 B+x^{10}a^3 b^2 A+\frac{1}{2}x^{10}a^4 bB+\frac{10}{13}x^{13}a^2 b^3 A+\frac{10}{13}x^{13}a^3 b^2 B+\frac{5}{16}x^{16}$

input
```
int(x^3*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^5*A*x^4+(5/7*a^4*b*A+1/7*a^5*B)*x^7+(a^3*b^2*A+1/2*a^4*b*B)*x^10+(10
/13*a^2*b^3*A+10/13*a^3*b^2*B)*x^13+(5/16*a*b^4*A+5/8*a^2*b^3*B)*x^16+(1/1
9*b^5*A+5/19*a*b^4*B)*x^19+1/22*b^5*B*x^22
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\begin{aligned}\int x^3(a+bx^3)^5\left(A+Bx^3\right)\,dx=&\ \frac{1}{22}\,Bb^5x^{22}+\frac{1}{19}\left(5\,Bab^4+Ab^5\right)x^{19}\\&+\frac{5}{16}\left(2\,Ba^2b^3+Aab^4\right)x^{16}+\frac{10}{13}\left(Ba^3b^2+Aa^2b^3\right)x^{13}\\&+\frac{1}{2}\left(Ba^4b+2\,Aa^3b^2\right)x^{10}+\frac{1}{4}\,Aa^5x^4+\frac{1}{7}\left(Ba^5+5\,Aa^4b\right)x^7\end{aligned}$$

input
```
integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/22*B*b^5*x^22 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/16*(2*B*a^2*b^3 + A*a*
b^4)*x^16 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/2*(B*a^4*b + 2*A*a^3*b^
2)*x^10 + 1/4*A*a^5*x^4 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14

$$\int x^3(a+bx^3)^5(A+Bx^3)\,dx = \frac{Aa^5x^4}{4}+\frac{Bb^5x^{22}}{22}+x^{19}\left(\frac{Ab^5}{19}+\frac{5Bab^4}{19}\right)+x^{16}\cdot\left(\frac{5Aab^4}{16}+\frac{5Ba^2b^3}{8}\right)+x^{13}\cdot\left(\frac{10Aa^2b^3}{13}+\frac{10Ba^3b^2}{13}\right)+x^{10}\left(Aa^3b^2+\frac{Ba^4b}{2}\right)+x^7\cdot\left(\frac{5Aa^4b}{7}+\frac{Ba^5}{7}\right)$$

input
```
integrate(x**3*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**4/4 + B*b**5*x**22/22 + x**19*(A*b**5/19 + 5*B*a*b**4/19) + x**1
6*(5*A*a*b**4/16 + 5*B*a**2*b**3/8) + x**13*(10*A*a**2*b**3/13 + 10*B*a**3
*b**2/13) + x**10*(A*a**3*b**2 + B*a**4*b/2) + x**7*(5*A*a**4*b/7 + B*a**5
/7)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^3(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{22}\,Bb^5x^{22}+\frac{1}{19}\,(5\,Bab^4+Ab^5)x^{19}+\frac{5}{16}\,(2\,Ba^2b^3+Aab^4)x^{16}+\frac{10}{13}\,(Ba^3b^2+Aa^2b^3)x^{13}+\frac{1}{2}\,(Ba^4b+2\,Aa^3b^2)x^{10}+\frac{1}{4}\,Aa^5x^4+\frac{1}{7}\,(Ba^5+5\,Aa^4b)x^7$$

input
```
integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/22*B*b^5*x^22 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/16*(2*B*a^2*b^3 + A*a*
b^4)*x^16 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/2*(B*a^4*b + 2*A*a^3*b^
2)*x^10 + 1/4*A*a^5*x^4 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\begin{aligned}\int x^3\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=&\ \frac{1}{22}Bb^5x^{22}+\frac{5}{19}Bab^4x^{19}+\frac{1}{19}Ab^5x^{19}+\frac{5}{8}Ba^2b^3x^{16}\\&+\frac{5}{16}Aab^4x^{16}+\frac{10}{13}Ba^3b^2x^{13}+\frac{10}{13}Aa^2b^3x^{13}+\frac{1}{2}Ba^4bx^{10}\\&+Aa^3b^2x^{10}+\frac{1}{7}Ba^5x^7+\frac{5}{7}Aa^4bx^7+\frac{1}{4}Aa^5x^4\end{aligned}$$

input
```
integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/22*B*b^5*x^22 + 5/19*B*a*b^4*x^19 + 1/19*A*b^5*x^19 + 5/8*B*a^2*b^3*x^16
 + 5/16*A*a*b^4*x^16 + 10/13*B*a^3*b^2*x^13 + 10/13*A*a^2*b^3*x^13 + 1/2*B
*a^4*b*x^10 + A*a^3*b^2*x^10 + 1/7*B*a^5*x^7 + 5/7*A*a^4*b*x^7 + 1/4*A*a^5
*x^4
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\begin{aligned}\int x^3\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=&\ x^7\left(\frac{B\,a^5}{7}+\frac{5\,A\,b\,a^4}{7}\right)+x^{19}\left(\frac{A\,b^5}{19}+\frac{5\,B\,a\,b^4}{19}\right)\\&+\frac{A\,a^5\,x^4}{4}+\frac{B\,b^5\,x^{22}}{22}+\frac{10\,a^2\,b^2\,x^{13}\,\left(A\,b+B\,a\right)}{13}\\&+\frac{a^3\,b\,x^{10}\,\left(2\,A\,b+B\,a\right)}{2}+\frac{5\,a\,b^3\,x^{16}\,\left(A\,b+2\,B\,a\right)}{16}\end{aligned}$$

input
```
int(x^3*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^7*((B*a^5)/7 + (5*A*a^4*b)/7) + x^19*((A*b^5)/19 + (5*B*a*b^4)/19) + (A*
a^5*x^4)/4 + (B*b^5*x^22)/22 + (10*a^2*b^2*x^13*(A*b + B*a))/13 + (a^3*b*x
^10*(2*A*b + B*a))/2 + (5*a*b^3*x^16*(A*b + 2*B*a))/16
```

3.30 $\int x^2(a+bx^3)^5 (A+Bx^3) \, dx$

3.30.1 Optimal result

Integrand size = 20, antiderivative size = 42

$$\int x^2 (a+bx^3)^5 (A+Bx^3) \, dx = \frac{(Ab-aB)(a+bx^3)^6}{18b^2} + \frac{B(a+bx^3)^7}{21b^2}$$

output
```
1/18*(A*b-B*a)*(b*x^3+a)^6/b^2+1/21*B*(b*x^3+a)^7/b^2
```

3.30.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 107 vs. 2(42) = 84.

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.55

$$\begin{aligned}\int x^2 (a+bx^3)^5 (A+Bx^3) \, dx = \frac{1}{126}x^3\big(42a^5A + 21a^4(5Ab+aB)x^3 + 70a^3b(2Ab+aB)x^6 \\ + 105a^2b^2(Ab+aB)x^9 + 42ab^3(Ab+2aB)x^{12} \\ + 7b^4(Ab+5aB)x^{15} + 6b^5Bx^{18}\big)\end{aligned}$$

input
```
Integrate[x^2*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(x^3*(42*a^5*A + 21*a^4*(5*A*b + a*B)*x^3 + 70*a^3*b*(2*A*b + a*B)*x^6 + 1
05*a^2*b^2*(A*b + a*B)*x^9 + 42*a*b^3*(A*b + 2*a*B)*x^12 + 7*b^4*(A*b + 5*
a*B)*x^15 + 6*b^5*B*x^18))/126
```

3.30.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {946, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3} \int \left(bx^3+a\right)^5 \left(Bx^3+A\right)dx^3$$

$$\Big\downarrow \; 49$$

$$\frac{1}{3} \int \left(\frac{B\left(bx^3+a\right)^6}{b}+\frac{(Ab-aB)\left(bx^3+a\right)^5}{b}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3} \left(\frac{\left(a+bx^3\right)^6 (Ab-aB)}{6b^2}+\frac{B\left(a+bx^3\right)^7}{7b^2}\right)$$

input

```
Int[x^2*(a + b*x^3)^5*(A + B*x^3),x]
```

output

```
(((A*b - a*B)*(a + b*x^3)^6)/(6*b^2) + (B*(a + b*x^3)^7)/(7*b^2))/3
```

3.30.3.1 Defintions of rubi rules used

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.30.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(38) = 76.

Time = 4.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.88

method	result
norman	$\frac{a^5 A x^3}{3}+\left(\frac{5}{6}a^4 bA+\frac{1}{6}a^5 B\right)x^6+\left(\frac{10}{9}a^3 b^2 A+\frac{5}{9}a^4 bB\right)x^9+\left(\frac{5}{6}a^2 b^3 A+\frac{5}{6}a^3 b^2 B\right)x^{12}+\left(\frac{1}{3}a\,b^4 A\right.$
default	$\frac{b^5 B\,x^{21}}{21}+\frac{\left(b^5 A+5a\,b^4 B\right)x^{18}}{18}+\frac{\left(5a\,b^4 A+10a^2 b^3 B\right)x^{15}}{15}+\frac{\left(10a^2 b^3 A+10a^3 b^2 B\right)x^{12}}{12}+\frac{\left(10a^3 b^2 A+5a^4 bB\right)x^9}{9}+$
gosper	$\frac{1}{3}a^5 A\,x^3+\frac{5}{6}x^6 a^4 bA+\frac{1}{6}x^6 a^5 B+\frac{10}{9}x^9 a^3 b^2 A+\frac{5}{9}x^9 a^4 bB+\frac{5}{6}x^{12}a^2 b^3 A+\frac{5}{6}x^{12}a^3 b^2 B+\frac{1}{3}x^{15}a$
risch	$\frac{1}{3}a^5 A\,x^3+\frac{5}{6}x^6 a^4 bA+\frac{1}{6}x^6 a^5 B+\frac{10}{9}x^9 a^3 b^2 A+\frac{5}{9}x^9 a^4 bB+\frac{5}{6}x^{12}a^2 b^3 A+\frac{5}{6}x^{12}a^3 b^2 B+\frac{1}{3}x^{15}a$
parallelrisch	$\frac{1}{3}a^5 A\,x^3+\frac{5}{6}x^6 a^4 bA+\frac{1}{6}x^6 a^5 B+\frac{10}{9}x^9 a^3 b^2 A+\frac{5}{9}x^9 a^4 bB+\frac{5}{6}x^{12}a^2 b^3 A+\frac{5}{6}x^{12}a^3 b^2 B+\frac{1}{3}x^{15}a$

input
```
int(x^2*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^5*A*x^3+(5/6*a^4*b*A+1/6*a^5*B)*x^6+(10/9*a^3*b^2*A+5/9*a^4*b*B)*x^9
+(5/6*a^2*b^3*A+5/6*a^3*b^2*B)*x^12+(1/3*a*b^4*A+2/3*a^2*b^3*B)*x^15+(1/18
*b^5*A+5/18*a*b^4*B)*x^18+1/21*b^5*B*x^21
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(38) = 76.

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.83

$$\begin{aligned}\int x^2\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=&\ \frac{1}{21}\,Bb^5x^{21}+\frac{1}{18}\,\left(5\,Bab^4+Ab^5\right)x^{18}\\&+\frac{1}{3}\,\left(2\,Ba^2b^3+Aab^4\right)x^{15}+\frac{5}{6}\,\left(Ba^3b^2+Aa^2b^3\right)x^{12}\\&+\frac{5}{9}\,\left(Ba^4b+2\,Aa^3b^2\right)x^9+\frac{1}{3}\,Aa^5x^3+\frac{1}{6}\,\left(Ba^5+5\,Aa^4b\right)x^6\end{aligned}$$

input
```
integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/21*B*b^5*x^21 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 1/3*(2*B*a^2*b^3 + A*a*b
^4)*x^15 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 5/9*(B*a^4*b + 2*A*a^3*b^2)*
x^9 + 1/3*A*a^5*x^3 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

3.30.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(36) = 72$.

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.24

$$\int x^2(a+bx^3)^5(A+Bx^3)\,dx = \frac{Aa^5x^3}{3} + \frac{Bb^5x^{21}}{21} + x^{18}\left(\frac{Ab^5}{18} + \frac{5Bab^4}{18}\right) + x^{15}\left(\frac{Aab^4}{3} + \frac{2Ba^2b^3}{3}\right) + x^{12}\cdot\left(\frac{5Aa^2b^3}{6} + \frac{5Ba^3b^2}{6}\right) + x^9\cdot\left(\frac{10Aa^3b^2}{9} + \frac{5Ba^4b}{9}\right) + x^6\cdot\left(\frac{5Aa^4b}{6} + \frac{Ba^5}{6}\right)$$

input
```
integrate(x**2*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**3/3 + B*b**5*x**21/21 + x**18*(A*b**5/18 + 5*B*a*b**4/18) + x**1
5*(A*a*b**4/3 + 2*B*a**2*b**3/3) + x**12*(5*A*a**2*b**3/6 + 5*B*a**3*b**2/
6) + x**9*(10*A*a**3*b**2/9 + 5*B*a**4*b/9) + x**6*(5*A*a**4*b/6 + B*a**5/
6)
```

3.30.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(38) = 76$.

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.83

$$\int x^2(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{21}Bb^5x^{21} + \frac{1}{18}(5\,Bab^4 + Ab^5)x^{18} + \frac{1}{3}(2\,Ba^2b^3 + Aab^4)x^{15} + \frac{5}{6}(Ba^3b^2 + Aa^2b^3)x^{12} + \frac{5}{9}(Ba^4b + 2\,Aa^3b^2)x^9 + \frac{1}{3}Aa^5x^3 + \frac{1}{6}(Ba^5 + 5\,Aa^4b)x^6$$

input
```
integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/21*B*b^5*x^21 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 1/3*(2*B*a^2*b^3 + A*a*b
^4)*x^15 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 5/9*(B*a^4*b + 2*A*a^3*b^2)*
x^9 + 1/3*A*a^5*x^3 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

3.30.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(38) = 76$.

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.98

$$\int x^2(a+bx^3)^5\left(A+Bx^3\right)\,dx = \frac{1}{21}\,Bb^5x^{21}+\frac{5}{18}\,Bab^4x^{18}+\frac{1}{18}\,Ab^5x^{18}+\frac{2}{3}\,Ba^2b^3x^{15}+\frac{1}{3}\,Aab^4x^{15}+\frac{5}{6}\,Ba^3b^2x^{12}+\frac{5}{6}\,Aa^2b^3x^{12}+\frac{5}{9}\,Ba^4bx^9+\frac{10}{9}\,Aa^3b^2x^9+\frac{1}{6}\,Ba^5x^6+\frac{5}{6}\,Aa^4bx^6+\frac{1}{3}\,Aa^5x^3$$

input
```
integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/21*B*b^5*x^21 + 5/18*B*a*b^4*x^18 + 1/18*A*b^5*x^18 + 2/3*B*a^2*b^3*x^15
 + 1/3*A*a*b^4*x^15 + 5/6*B*a^3*b^2*x^12 + 5/6*A*a^2*b^3*x^12 + 5/9*B*a^4*
b*x^9 + 10/9*A*a^3*b^2*x^9 + 1/6*B*a^5*x^6 + 5/6*A*a^4*b*x^6 + 1/3*A*a^5*x
^3
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.55

$$\int x^2(a+bx^3)^5\left(A+Bx^3\right)\,dx = x^6\left(\frac{B\,a^5}{6}+\frac{5\,A\,b\,a^4}{6}\right)+x^{18}\left(\frac{A\,b^5}{18}+\frac{5\,B\,a\,b^4}{18}\right)+\frac{A\,a^5\,x^3}{3}+\frac{B\,b^5\,x^{21}}{21}+\frac{5\,a^2\,b^2\,x^{12}\,(A\,b+B\,a)}{6}+\frac{5\,a^3\,b\,x^9\,(2\,A\,b+B\,a)}{9}+\frac{a\,b^3\,x^{15}\,(A\,b+2\,B\,a)}{3}$$

input
```
int(x^2*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^6*((B*a^5)/6 + (5*A*a^4*b)/6) + x^18*((A*b^5)/18 + (5*B*a*b^4)/18) + (A*
a^5*x^3)/3 + (B*b^5*x^21)/21 + (5*a^2*b^2*x^12*(A*b + B*a))/6 + (5*a^3*b*x
^9*(2*A*b + B*a))/9 + (a*b^3*x^15*(A*b + 2*B*a))/3
```

3.31 $\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx$

3.31.1 Optimal result

Integrand size = 18, antiderivative size = 117

$$\begin{aligned}\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=&\ \frac{1}{2}a^5Ax^2+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{8}a^3b(2Ab+aB)x^8\\&+\frac{10}{11}a^2b^2(Ab+aB)x^{11}+\frac{5}{14}ab^3(Ab+2aB)x^{14}\\&+\frac{1}{17}b^4(Ab+5aB)x^{17}+\frac{1}{20}b^5Bx^{20}\end{aligned}$$

output
```
1/2*a^5*A*x^2+1/5*a^4*(5*A*b+B*a)*x^5+5/8*a^3*b*(2*A*b+B*a)*x^8+10/11*a^2*
b^2*(A*b+B*a)*x^11+5/14*a*b^3*(A*b+2*B*a)*x^14+1/17*b^4*(A*b+5*B*a)*x^17+1
/20*b^5*B*x^20
```

3.31.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=&\ \frac{1}{2}a^5Ax^2+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{8}a^3b(2Ab+aB)x^8\\&+\frac{10}{11}a^2b^2(Ab+aB)x^{11}+\frac{5}{14}ab^3(Ab+2aB)x^{14}\\&+\frac{1}{17}b^4(Ab+5aB)x^{17}+\frac{1}{20}b^5Bx^{20}\end{aligned}$$

input
```
Integrate[x*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8
+ (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^
4*(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20
```

3.31.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^5Ax+a^4x^4(aB+5Ab)+5a^3bx^7(aB+2Ab)+10a^2b^2x^{10}(aB+Ab)+b^4x^{16}(5aB+Ab)+5ab^3x^{13}(2aB+A$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^5Ax^2+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{5}{14}ab^3x^{14}(2aB+Ab)+\frac{1}{20}b^5Bx^{20}$$

input
```
Int[x*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8
+ (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^
4*(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20
```

3.31.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.31.4 Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^2}{2} + \left(a^4 b A + \frac{1}{5} a^5 B\right) x^5 + \left(\frac{5}{4} a^3 b^2 A + \frac{5}{8} a^4 b B\right) x^8 + \left(\frac{10}{11} a^2 b^3 A + \frac{10}{11} a^3 b^2 B\right) x^{11} + (\frac{5}{14} a b^4 A$
default	$\frac{b^5 B x^{20}}{20} + \frac{(b^5 A + 5 a b^4 B) x^{17}}{17} + \frac{(5 a b^4 A + 10 a^2 b^3 B) x^{14}}{14} + \frac{(10 a^2 b^3 A + 10 a^3 b^2 B) x^{11}}{11} + \frac{(10 a^3 b^2 A + 5 a^4 b B) x^8}{8} +$
gosper	$\frac{1}{2} a^5 A x^2 + x^5 a^4 b A + \frac{1}{5} x^5 a^5 B + \frac{5}{4} x^8 a^3 b^2 A + \frac{5}{8} x^8 a^4 b B + \frac{10}{11} x^{11} a^2 b^3 A + \frac{10}{11} x^{11} a^3 b^2 B + \frac{5}{14} x^{14} a$
risch	$\frac{1}{2} a^5 A x^2 + x^5 a^4 b A + \frac{1}{5} x^5 a^5 B + \frac{5}{4} x^8 a^3 b^2 A + \frac{5}{8} x^8 a^4 b B + \frac{10}{11} x^{11} a^2 b^3 A + \frac{10}{11} x^{11} a^3 b^2 B + \frac{5}{14} x^{14} a$
parallelrisch	$\frac{1}{2} a^5 A x^2 + x^5 a^4 b A + \frac{1}{5} x^5 a^5 B + \frac{5}{4} x^8 a^3 b^2 A + \frac{5}{8} x^8 a^4 b B + \frac{10}{11} x^{11} a^2 b^3 A + \frac{10}{11} x^{11} a^3 b^2 B + \frac{5}{14} x^{14} a$

input
```
int(x*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^5*A*x^2+(a^4*b*A+1/5*a^5*B)*x^5+(5/4*a^3*b^2*A+5/8*a^4*b*B)*x^8+(10/
11*a^2*b^3*A+10/11*a^3*b^2*B)*x^11+(5/14*a*b^4*A+5/7*a^2*b^3*B)*x^14+(1/17
*b^5*A+5/17*a*b^4*B)*x^17+1/20*b^5*B*x^20
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\begin{aligned}\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \frac{1}{20}\,Bb^5x^{20} + \frac{1}{17}\left(5\,Bab^4 + Ab^5\right)x^{17} \\ &\quad + \frac{5}{14}\left(2\,Ba^2b^3 + Aab^4\right)x^{14} + \frac{10}{11}\left(Ba^3b^2 + Aa^2b^3\right)x^{11} \\ &\quad + \frac{5}{8}\left(Ba^4b + 2\,Aa^3b^2\right)x^8 + \frac{1}{2}\,Aa^5x^2 + \frac{1}{5}\left(Ba^5 + 5\,Aa^4b\right)x^5\end{aligned}$$

input
```
integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/20*B*b^5*x^20 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 5/14*(2*B*a^2*b^3 + A*a*
b^4)*x^14 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 5/8*(B*a^4*b + 2*A*a^3*b^
2)*x^8 + 1/2*A*a^5*x^2 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

3.31.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.15

$$\int x(a+bx^3)^5\left(A+Bx^3\right)\,dx=\frac{Aa^5x^2}{2}+\frac{Bb^5x^{20}}{20}+x^{17}\left(\frac{Ab^5}{17}+\frac{5Bab^4}{17}\right)+x^{14}\cdot\left(\frac{5Aab^4}{14}+\frac{5Ba^2b^3}{7}\right)+x^{11}\cdot\left(\frac{10Aa^2b^3}{11}+\frac{10Ba^3b^2}{11}\right)+x^8\cdot\left(\frac{5Aa^3b^2}{4}+\frac{5Ba^4b}{8}\right)+x^5\left(Aa^4b+\frac{Ba^5}{5}\right)$$

input
```
integrate(x*(b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x**2/2 + B*b**5*x**20/20 + x**17*(A*b**5/17 + 5*B*a*b**4/17) + x**1
4*(5*A*a*b**4/14 + 5*B*a**2*b**3/7) + x**11*(10*A*a**2*b**3/11 + 10*B*a**3
*b**2/11) + x**8*(5*A*a**3*b**2/4 + 5*B*a**4*b/8) + x**5*(A*a**4*b + B*a**
5/5)
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x(a+bx^3)^5\left(A+Bx^3\right)\,dx=\frac{1}{20}\,Bb^5x^{20}+\frac{1}{17}\,(5\,Bab^4+Ab^5)x^{17}+\frac{5}{14}\,(2\,Ba^2b^3+Aab^4)x^{14}+\frac{10}{11}\,(Ba^3b^2+Aa^2b^3)x^{11}+\frac{5}{8}\,(Ba^4b+2\,Aa^3b^2)x^8+\frac{1}{2}\,Aa^5x^2+\frac{1}{5}\,(Ba^5+5\,Aa^4b)x^5$$

input
```
integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/20*B*b^5*x^20 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 5/14*(2*B*a^2*b^3 + A*a*
b^4)*x^14 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 5/8*(B*a^4*b + 2*A*a^3*b^
2)*x^8 + 1/2*A*a^5*x^2 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int x(a+bx^3)^5 (A+Bx^3)\,dx = \frac{1}{20}Bb^5x^{20} + \frac{5}{17}Bab^4x^{17} + \frac{1}{17}Ab^5x^{17} + \frac{5}{7}Ba^2b^3x^{14} + \frac{5}{14}Aab^4x^{14} + \frac{10}{11}Ba^3b^2x^{11} + \frac{10}{11}Aa^2b^3x^{11} + \frac{5}{8}Ba^4bx^8 + \frac{5}{4}Aa^3b^2x^8 + \frac{1}{5}Ba^5x^5 + Aa^4bx^5 + \frac{1}{2}Aa^5x^2$$

input
```
integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/20*B*b^5*x^20 + 5/17*B*a*b^4*x^17 + 1/17*A*b^5*x^17 + 5/7*B*a^2*b^3*x^14
 + 5/14*A*a*b^4*x^14 + 10/11*B*a^3*b^2*x^11 + 10/11*A*a^2*b^3*x^11 + 5/8*B
*a^4*b*x^8 + 5/4*A*a^3*b^2*x^8 + 1/5*B*a^5*x^5 + A*a^4*b*x^5 + 1/2*A*a^5*x
^2
```

3.31.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int x(a+bx^3)^5 (A+Bx^3)\,dx = x^5\left(\frac{B\,a^5}{5} + A\,b\,a^4\right) + x^{17}\left(\frac{A\,b^5}{17} + \frac{5\,B\,a\,b^4}{17}\right) + \frac{A\,a^5\,x^2}{2} + \frac{B\,b^5\,x^{20}}{20} + \frac{10\,a^2\,b^2\,x^{11}\,(A\,b+B\,a)}{11} + \frac{5\,a^3\,b\,x^8\,(2\,A\,b+B\,a)}{8} + \frac{5\,a\,b^3\,x^{14}\,(A\,b+2\,B\,a)}{14}$$

input
```
int(x*(A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^5*((B*a^5)/5 + A*a^4*b) + x^17*((A*b^5)/17 + (5*B*a*b^4)/17) + (A*a^5*x^
2)/2 + (B*b^5*x^20)/20 + (10*a^2*b^2*x^11*(A*b + B*a))/11 + (5*a^3*b*x^8*(
2*A*b + B*a))/8 + (5*a*b^3*x^14*(A*b + 2*B*a))/14
```

3.32 $\int (a+bx^3)^5 (A+Bx^3)\, dx$

3.32.1 Optimal result

Integrand size = 17, antiderivative size = 109

$$\int (a+bx^3)^5 (A+Bx^3)\, dx = a^5Ax + \frac{1}{4}a^4(5Ab+aB)x^4 + \frac{5}{7}a^3b(2Ab+aB)x^7 + a^2b^2(Ab + aB)x^{10} + \frac{5}{13}ab^3(Ab+2aB)x^{13} + \frac{1}{16}b^4(Ab+5aB)x^{16} + \frac{1}{19}b^5Bx^{19}$$

output
```
a^5*A*x+1/4*a^4*(5*A*b+B*a)*x^4+5/7*a^3*b*(2*A*b+B*a)*x^7+a^2*b^2*(A*b+B*a
)*x^10+5/13*a*b^3*(A*b+2*B*a)*x^13+1/16*b^4*(A*b+5*B*a)*x^16+1/19*b^5*B*x^
19
```

3.32.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^5 (A+Bx^3)\, dx = a^5Ax + \frac{1}{4}a^4(5Ab+aB)x^4 + \frac{5}{7}a^3b(2Ab+aB)x^7 + a^2b^2(Ab + aB)x^{10} + \frac{5}{13}ab^3(Ab+2aB)x^{13} + \frac{1}{16}b^4(Ab+5aB)x^{16} + \frac{1}{19}b^5Bx^{19}$$

input
```
Integrate[(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*
b^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B
)*x^16)/16 + (b^5*B*x^19)/19
```

3.32.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \textcolor{blue}{897}$$

$$\int \left(a^5A+a^4x^3(aB+5Ab)+5a^3bx^6(aB+2Ab)+10a^2b^2x^9(aB+Ab)+b^4x^{15}(5aB+Ab)+5ab^3x^{12}(2aB+Ab)\right.$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$a^5Ax+\frac{1}{4}a^4x^4(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+a^2b^2x^{10}(aB+Ab)+\frac{1}{16}b^4x^{16}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{19}b^5Bx^{19}$$

input
```
Int[(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*
b^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B
)*x^16)/16 + (b^5*B*x^19)/19
```

3.32.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.32.4 Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06

method	result
norman	$a^5Ax+\left(\frac{5}{4}a^4bA+\frac{1}{4}a^5B\right)x^4+\left(\frac{10}{7}a^3b^2A+\frac{5}{7}a^4bB\right)x^7+\left(a^2b^3A+a^3b^2B\right)x^{10}+\left(\frac{5}{13}a\,b^4A+\right.$
gosper	$a^5Ax+\frac{5}{4}x^4a^4bA+\frac{1}{4}x^4a^5B+\frac{10}{7}x^7a^3b^2A+\frac{5}{7}x^7a^4bB+A\,a^2b^3x^{10}+B\,a^3b^2x^{10}+\frac{5}{13}x^{13}a\,b^4A$
default	$\frac{b^5B\,x^{19}}{19}+\frac{\left(b^5A+5a\,b^4B\right)x^{16}}{16}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{13}}{13}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{10}}{10}+\frac{\left(10a^3b^2A+5a^4bB\right)x^7}{7}+$
risch	$a^5Ax+\frac{5}{4}x^4a^4bA+\frac{1}{4}x^4a^5B+\frac{10}{7}x^7a^3b^2A+\frac{5}{7}x^7a^4bB+A\,a^2b^3x^{10}+B\,a^3b^2x^{10}+\frac{5}{13}x^{13}a\,b^4A$
parallelrisch	$a^5Ax+\frac{5}{4}x^4a^4bA+\frac{1}{4}x^4a^5B+\frac{10}{7}x^7a^3b^2A+\frac{5}{7}x^7a^4bB+A\,a^2b^3x^{10}+B\,a^3b^2x^{10}+\frac{5}{13}x^{13}a\,b^4A$

input
```
int((b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
a^5*A*x+(5/4*a^4*b*A+1/4*a^5*B)*x^4+(10/7*a^3*b^2*A+5/7*a^4*b*B)*x^7+(A*a^
2*b^3+B*a^3*b^2)*x^10+(5/13*a*b^4*A+10/13*a^2*b^3*B)*x^13+(1/16*b^5*A+5/16
*a*b^4*B)*x^16+1/19*b^5*B*x^19
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \frac{1}{19}\,Bb^5x^{19}+\frac{1}{16}\,\left(5\,Bab^4+Ab^5\right)x^{16} \\ &\quad +\frac{5}{13}\,\left(2\,Ba^2b^3+Aab^4\right)x^{13}+\left(Ba^3b^2+Aa^2b^3\right)x^{10} \\ &\quad +\frac{5}{7}\,\left(Ba^4b+2\,Aa^3b^2\right)x^7+Aa^5x+\frac{1}{4}\,\left(Ba^5+5\,Aa^4b\right)x^4\end{aligned}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output
```
1/19*B*b^5*x^19 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/13*(2*B*a^2*b^3 + A*a*
b^4)*x^13 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 5/7*(B*a^4*b + 2*A*a^3*b^2)*x^7
 + A*a^5*x + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

3.32.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.17

$$\int (a+bx^3)^5 (A+Bx^3)\,dx = Aa^5x + \frac{Bb^5x^{19}}{19} + x^{16}\left(\frac{Ab^5}{16} + \frac{5Bab^4}{16}\right) + x^{13} \cdot \left(\frac{5Aab^4}{13} + \frac{10Ba^2b^3}{13}\right) + x^{10}(Aa^2b^3 + Ba^3b^2) + x^7 \cdot \left(\frac{10Aa^3b^2}{7} + \frac{5Ba^4b}{7}\right) + x^4 \cdot \left(\frac{5Aa^4b}{4} + \frac{Ba^5}{4}\right)$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A),x)
```

output
```
A*a**5*x + B*b**5*x**19/19 + x**16*(A*b**5/16 + 5*B*a*b**4/16) + x**13*(5*
A*a*b**4/13 + 10*B*a**2*b**3/13) + x**10*(A*a**2*b**3 + B*a**3*b**2) + x**
7*(10*A*a**3*b**2/7 + 5*B*a**4*b/7) + x**4*(5*A*a**4*b/4 + B*a**5/4)
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int (a+bx^3)^5 (A+Bx^3)\,dx = \frac{1}{19}Bb^5x^{19} + \frac{1}{16}(5\,Bab^4 + Ab^5)x^{16} + \frac{5}{13}(2\,Ba^2b^3 + Aab^4)x^{13} + (Ba^3b^2 + Aa^2b^3)x^{10} + \frac{5}{7}(Ba^4b + 2\,Aa^3b^2)x^7 + Aa^5x + \frac{1}{4}(Ba^5 + 5\,Aa^4b)x^4$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
1/19*B*b^5*x^19 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/13*(2*B*a^2*b^3 + A*a*
b^4)*x^13 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 5/7*(B*a^4*b + 2*A*a^3*b^2)*x^7
 + A*a^5*x + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.10

$$\begin{aligned}\int \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx &= \frac{1}{19}Bb^5x^{19}+\frac{5}{16}Bab^4x^{16}+\frac{1}{16}Ab^5x^{16}+\frac{10}{13}Ba^2b^3x^{13}\\ &\quad+\frac{5}{13}Aab^4x^{13}+Ba^3b^2x^{10}+Aa^2b^3x^{10}+\frac{5}{7}Ba^4bx^7\\ &\quad+\frac{10}{7}Aa^3b^2x^7+\frac{1}{4}Ba^5x^4+\frac{5}{4}Aa^4bx^4+Aa^5x\end{aligned}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
1/19*B*b^5*x^19 + 5/16*B*a*b^4*x^16 + 1/16*A*b^5*x^16 + 10/13*B*a^2*b^3*x^
13 + 5/13*A*a*b^4*x^13 + B*a^3*b^2*x^10 + A*a^2*b^3*x^10 + 5/7*B*a^4*b*x^7
 + 10/7*A*a^3*b^2*x^7 + 1/4*B*a^5*x^4 + 5/4*A*a^4*b*x^4 + A*a^5*x
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx &= x^4\left(\frac{B\,a^5}{4}+\frac{5\,A\,b\,a^4}{4}\right)+x^{16}\left(\frac{A\,b^5}{16}+\frac{5\,B\,a\,b^4}{16}\right)\\ &\quad+\frac{B\,b^5\,x^{19}}{19}+A\,a^5\,x+a^2\,b^2\,x^{10}\,(A\,b+B\,a)\\ &\quad+\frac{5\,a^3\,b\,x^7\,(2\,A\,b+B\,a)}{7}+\frac{5\,a\,b^3\,x^{13}\,(A\,b+2\,B\,a)}{13}\end{aligned}$$

input
```
int((A + B*x^3)*(a + b*x^3)^5,x)
```

output
```
x^4*((B*a^5)/4 + (5*A*a^4*b)/4) + x^16*((A*b^5)/16 + (5*B*a*b^4)/16) + (B*
b^5*x^19)/19 + A*a^5*x + a^2*b^2*x^10*(A*b + B*a) + (5*a^3*b*x^7*(2*A*b +
B*a))/7 + (5*a*b^3*x^13*(A*b + 2*B*a))/13
```

3.33 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x}\,dx$

3.33.1 Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x}\,dx = \frac{5}{3}a^4Abx^3+\frac{5}{3}a^3Ab^2x^6+\frac{10}{9}a^2Ab^3x^9+\frac{5}{12}aAb^4x^{12}+\frac{1}{15}Ab^5x^{15}+\frac{B(a+bx^3)^6}{18b}+a^5A\log(x)$$

output
```
5/3*a^4*A*b*x^3+5/3*a^3*A*b^2*x^6+10/9*a^2*A*b^3*x^9+5/12*a*A*b^4*x^12+1/1
5*A*b^5*x^15+1/18*B*(b*x^3+a)^6/b+a^5*A*ln(x)
```

3.33.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x}\,dx = \frac{1}{3}a^4(5Ab+aB)x^3+\frac{5}{6}a^3b(2Ab+aB)x^6+\frac{10}{9}a^2b^2(Ab+aB)x^9+\frac{5}{12}ab^3(Ab+2aB)x^{12}+\frac{1}{15}b^4(Ab+5aB)x^{15}+\frac{1}{18}b^5Bx^{18}+a^5A\log(x)$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x,x]
```

output
```
(a^4*(5*A*b + a*B)*x^3)/3 + (5*a^3*b*(2*A*b + a*B)*x^6)/6 + (10*a^2*b^2*(A
*b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^12)/12 + (b^4*(A*b + 5*a*B)*x^
15)/15 + (b^5*B*x^18)/18 + a^5*A*Log[x]
```

3.33.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {948, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3} \int \frac{(bx^3+a)^5 (Bx^3+A)}{x^3} dx^3$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{3} \left(A \int \frac{(bx^3+a)^5}{x^3} dx^3 + \frac{B(a+bx^3)^6}{6b} \right)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3} \left(A \int \left(b^5x^{12} + 5ab^4x^9 + 10a^2b^3x^6 + 10a^3b^2x^3 + 5a^4b + \frac{a^5}{x^3} \right) dx^3 + \frac{B(a+bx^3)^6}{6b} \right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(A \left(a^5 \log\left(x^3\right) + 5a^4bx^3 + 5a^3b^2x^6 + \frac{10}{3}a^2b^3x^9 + \frac{5}{4}ab^4x^{12} + \frac{b^5x^{15}}{5} \right) + \frac{B(a+bx^3)^6}{6b} \right)$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x,x]
```

output
```
((B*(a + b*x^3)^6)/(6*b) + A*(5*a^4*b*x^3 + 5*a^3*b^2*x^6 + (10*a^2*b^3*x^
9)/3 + (5*a*b^4*x^12)/4 + (b^5*x^15)/5 + a^5*Log[x^3]))/3
```

3.33.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.33.4 Maple [A] (verified)

Time = 3.94 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

method	result
norman	$\left(\frac{1}{15} b^5 A+\frac{1}{3} a\, b^4 B\right) x^{15}+\left(\frac{5}{12} a\, b^4 A+\frac{5}{6} a^2 b^3 B\right) x^{12}+\left(\frac{10}{9} a^2 b^3 A+\frac{10}{9} a^3 b^2 B\right) x^9+\left(\frac{5}{3} a^3 b^2 A+\frac{5}{6}\right.$
default	$\frac{b^5 B\, x^{18}}{18}+\frac{A\, b^5 x^{15}}{15}+\frac{B a\, b^4 x^{15}}{3}+\frac{5 a A\, b^4 x^{12}}{12}+\frac{5 B\, a^2 b^3 x^{12}}{6}+\frac{10 a^2 A\, b^3 x^9}{9}+\frac{10 B\, a^3 b^2 x^9}{9}+\frac{5 a^3 A\, b^2 x^6}{3}+\frac{5 B\, a^4 b}{6}$
risch	$\frac{b^5 B\, x^{18}}{18}+\frac{A\, b^5 x^{15}}{15}+\frac{B a\, b^4 x^{15}}{3}+\frac{5 a A\, b^4 x^{12}}{12}+\frac{5 B\, a^2 b^3 x^{12}}{6}+\frac{10 a^2 A\, b^3 x^9}{9}+\frac{10 B\, a^3 b^2 x^9}{9}+\frac{5 a^3 A\, b^2 x^6}{3}+\frac{5 B\, a^4 b}{6}$
parallelrisch	$\frac{b^5 B\, x^{18}}{18}+\frac{A\, b^5 x^{15}}{15}+\frac{B a\, b^4 x^{15}}{3}+\frac{5 a A\, b^4 x^{12}}{12}+\frac{5 B\, a^2 b^3 x^{12}}{6}+\frac{10 a^2 A\, b^3 x^9}{9}+\frac{10 B\, a^3 b^2 x^9}{9}+\frac{5 a^3 A\, b^2 x^6}{3}+\frac{5 B\, a^4 b}{6}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/15*b^5*A+1/3*a*b^4*B)*x^15+(5/12*a*b^4*A+5/6*a^2*b^3*B)*x^12+(10/9*a^2*
b^3*A+10/9*a^3*b^2*B)*x^9+(5/3*a^3*b^2*A+5/6*a^4*b*B)*x^6+(5/3*a^4*b*A+1/3
*a^5*B)*x^3+1/18*b^5*B*x^18+a^5*A*ln(x)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x}\,dx = \frac{1}{18}\,Bb^5x^{18} + \frac{1}{15}\left(5\,Bab^4 + Ab^5\right)x^{15} + \frac{5}{12}\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + \frac{10}{9}\left(Ba^3b^2 + Aa^2b^3\right)x^9 + \frac{5}{6}\left(Ba^4b + 2\,Aa^3b^2\right)x^6 + Aa^5\log\left(x\right) + \frac{1}{3}\left(Ba^5 + 5\,Aa^4b\right)x^3$$

input

```
integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="fricas")
```

output

```
1/18*B*b^5*x^18 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/12*(2*B*a^2*b^3 + A*a*
b^4)*x^12 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/6*(B*a^4*b + 2*A*a^3*b^2)
*x^6 + A*a^5*log(x) + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x}\,dx = Aa^5\log\left(x\right) + \frac{Bb^5x^{18}}{18} + x^{15}\left(\frac{Ab^5}{15} + \frac{Bab^4}{3}\right) + x^{12} \cdot \left(\frac{5Aab^4}{12} + \frac{5Ba^2b^3}{6}\right) + x^9 \cdot \left(\frac{10Aa^2b^3}{9} + \frac{10Ba^3b^2}{9}\right) + x^6 \cdot \left(\frac{5Aa^3b^2}{3} + \frac{5Ba^4b}{6}\right) + x^3 \cdot \left(\frac{5Aa^4b}{3} + \frac{Ba^5}{3}\right)$$

input

```
integrate((b*x**3+a)**5*(B*x**3+A)/x,x)
```

output

```
A*a**5*log(x) + B*b**5*x**18/18 + x**15*(A*b**5/15 + B*a*b**4/3) + x**12*(
5*A*a*b**4/12 + 5*B*a**2*b**3/6) + x**9*(10*A*a**2*b**3/9 + 10*B*a**3*b**2
/9) + x**6*(5*A*a**3*b**2/3 + 5*B*a**4*b/6) + x**3*(5*A*a**4*b/3 + B*a**5/
3)
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x}\,dx = \frac{1}{18}Bb^5x^{18} + \frac{1}{15}\left(5\,Bab^4 + Ab^5\right)x^{15} + \frac{5}{12}\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + \frac{10}{9}\left(Ba^3b^2 + Aa^2b^3\right)x^9 + \frac{5}{6}\left(Ba^4b + 2\,Aa^3b^2\right)x^6 + \frac{1}{3}Aa^5\log\left(x^3\right) + \frac{1}{3}\left(Ba^5 + 5\,Aa^4b\right)x^3$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="maxima")
```

output
```
1/18*B*b^5*x^18 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/12*(2*B*a^2*b^3 + A*a*
b^4)*x^12 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/6*(B*a^4*b + 2*A*a^3*b^2)
*x^6 + 1/3*A*a^5*log(x^3) + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.41

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x}\,dx = \frac{1}{18}Bb^5x^{18} + \frac{1}{3}Bab^4x^{15} + \frac{1}{15}Ab^5x^{15} + \frac{5}{6}Ba^2b^3x^{12} + \frac{5}{12}Aab^4x^{12} + \frac{10}{9}Ba^3b^2x^9 + \frac{10}{9}Aa^2b^3x^9 + \frac{5}{6}Ba^4bx^6 + \frac{5}{3}Aa^3b^2x^6 + \frac{1}{3}Ba^5x^3 + \frac{5}{3}Aa^4bx^3 + Aa^5\log\left(|x|\right)$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="giac")
```

output
```
1/18*B*b^5*x^18 + 1/3*B*a*b^4*x^15 + 1/15*A*b^5*x^15 + 5/6*B*a^2*b^3*x^12
+ 5/12*A*a*b^4*x^12 + 10/9*B*a^3*b^2*x^9 + 10/9*A*a^2*b^3*x^9 + 5/6*B*a^4*
b*x^6 + 5/3*A*a^3*b^2*x^6 + 1/3*B*a^5*x^3 + 5/3*A*a^4*b*x^3 + A*a^5*log(ab
s(x))
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x}\,dx = x^3\left(\frac{B\,a^5}{3}+\frac{5\,A\,b\,a^4}{3}\right)+x^{15}\left(\frac{A\,b^5}{15}+\frac{B\,a\,b^4}{3}\right) + \frac{B\,b^5\,x^{18}}{18}+A\,a^5\,\ln\left(x\right)+\frac{10\,a^2\,b^2\,x^9\,\left(A\,b+B\,a\right)}{9} + \frac{5\,a^3\,b\,x^6\,\left(2\,A\,b+B\,a\right)}{6}+\frac{5\,a\,b^3\,x^{12}\,\left(A\,b+2\,B\,a\right)}{12}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x,x)
```

output
```
x^3*((B*a^5)/3 + (5*A*a^4*b)/3) + x^15*((A*b^5)/15 + (B*a*b^4)/3) + (B*b^5
*x^18)/18 + A*a^5*log(x) + (10*a^2*b^2*x^9*(A*b + B*a))/9 + (5*a^3*b*x^6*(
2*A*b + B*a))/6 + (5*a*b^3*x^12*(A*b + 2*B*a))/12
```

3.34 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2}\,dx$

3.34.1 Optimal result

Integrand size = 20, antiderivative size = 112

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2}\,dx = &-\frac{a^5A}{x}+\frac{1}{2}a^4(5Ab+aB)x^2+a^3b(2Ab+aB)x^5\\&+\frac{5}{4}a^2b^2(Ab+aB)x^8+\frac{5}{11}ab^3(Ab+2aB)x^{11}\\&+\frac{1}{14}b^4(Ab+5aB)x^{14}+\frac{1}{17}b^5Bx^{17}\end{aligned}$$

output

```
-a^5*A/x+1/2*a^4*(5*A*b+B*a)*x^2+a^3*b*(2*A*b+B*a)*x^5+5/4*a^2*b^2*(A*b+B*
a)*x^8+5/11*a*b^3*(A*b+2*B*a)*x^11+1/14*b^4*(A*b+5*B*a)*x^14+1/17*b^5*B*x^
17
```

3.34.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2}\,dx = &-\frac{a^5A}{x}+\frac{1}{2}a^4(5Ab+aB)x^2+a^3b(2Ab+aB)x^5\\&+\frac{5}{4}a^2b^2(Ab+aB)x^8+\frac{5}{11}ab^3(Ab+2aB)x^{11}\\&+\frac{1}{14}b^4(Ab+5aB)x^{14}+\frac{1}{17}b^5Bx^{17}\end{aligned}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^2,x]
```

output
```
-((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^
2*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5
*a*B)*x^14)/14 + (b^5*B*x^17)/17
```

3.34.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^2} \, dx$$

$$\Big\downarrow \; 950$$

$$\int \Bigg(\frac{a^5 A}{x^2} + a^4 x (aB+5Ab) + 5a^3 b x^4 (aB+2Ab) + 10a^2 b^2 x^7 (aB+Ab) + b^4 x^{13} (5aB+Ab) + 5ab^3 x^{10} (2aB+Ab)$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^5 A}{x} + \frac{1}{2} a^4 x^2 (aB+5Ab) + a^3 b x^5 (aB+2Ab) + \frac{5}{4} a^2 b^2 x^8 (aB+Ab) + \frac{1}{14} b^4 x^{14} (5aB+Ab) + \frac{5}{11} ab^3 x^{11} (2aB+Ab) + \frac{1}{17} b^5 B x^{17}$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^2,x]
```

output
```
-((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^
2*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5
*a*B)*x^14)/14 + (b^5*B*x^17)/17
```

3.34.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.34.4 Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

method	result
norman	$\frac{-a^5A+\left(\frac{5}{2}a^4bA+\frac{1}{2}a^5B\right)x^3+\left(2a^3b^2A+a^4bB\right)x^6+\left(\frac{5}{4}a^2b^3A+\frac{5}{4}a^3b^2B\right)x^9+\left(\frac{5}{11}a\,b^4A+\frac{10}{11}a^2b^3B\right)x^{12}+\left(\frac{1}{14}b^5A+\frac{5}{14}a\,b^4B\right)x^{15}+}{x}$
default	$\frac{b^5B\,x^{17}}{17}+\frac{A\,b^5x^{14}}{14}+\frac{5Ba\,b^4x^{14}}{14}+\frac{5Aa\,b^4x^{11}}{11}+\frac{10B\,a^2b^3x^{11}}{11}+\frac{5A\,a^2b^3x^8}{4}+\frac{5B\,a^3b^2x^8}{4}+2a^3A\,b^2x^5+B\,a^4$
risch	$\frac{b^5B\,x^{17}}{17}+\frac{A\,b^5x^{14}}{14}+\frac{5Ba\,b^4x^{14}}{14}+\frac{5Aa\,b^4x^{11}}{11}+\frac{10B\,a^2b^3x^{11}}{11}+\frac{5A\,a^2b^3x^8}{4}+\frac{5B\,a^3b^2x^8}{4}+2a^3A\,b^2x^5+B\,a^4$
gosper	$-\frac{-308b^5B\,x^{18}-374A\,b^5x^{15}-1870Ba\,b^4x^{15}-2380aA\,b^4x^{12}-4760B\,a^2b^3x^{12}-6545a^2A\,b^3x^9-6545B\,a^3b^2x^9-10472a^3A\,b^2x^6}{5236x}$
parallelrisch	$\frac{308b^5B\,x^{18}+374A\,b^5x^{15}+1870Ba\,b^4x^{15}+2380aA\,b^4x^{12}+4760B\,a^2b^3x^{12}+6545a^2A\,b^3x^9+6545B\,a^3b^2x^9+10472a^3A\,b^2x^6+52}{5236x}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x*(-a^5*A+(5/2*a^4*b*A+1/2*a^5*B)*x^3+(2*A*a^3*b^2+B*a^4*b)*x^6+(5/4*a^2
*b^3*A+5/4*a^3*b^2*B)*x^9+(5/11*a*b^4*A+10/11*a^2*b^3*B)*x^12+(1/14*b^5*A+
5/14*a*b^4*B)*x^15+1/17*b^5*B*x^18)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^2}\,dx$$
$$=\frac{308\,Bb^5x^{18}+374\,(5\,Bab^4+Ab^5)x^{15}+2380\,(2\,Ba^2b^3+Aab^4)x^{12}+6545\,(Ba^3b^2+Aa^2b^3)x^9+5236\,(Ba^4}{5236\,x}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="fricas")
```

output
```
1/5236*(308*B*b^5*x^18 + 374*(5*B*a*b^4 + A*b^5)*x^15 + 2380*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5236*(B*a^4*b + 2*A*a
^3*b^2)*x^6 - 5236*A*a^5 + 2618*(B*a^5 + 5*A*a^4*b)*x^3)/x
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^2}\,dx = -\frac{Aa^5}{x} + \frac{Bb^5x^{17}}{17} + x^{14}\left(\frac{Ab^5}{14} + \frac{5Bab^4}{14}\right) + x^{11} \cdot \left(\frac{5Aab^4}{11} + \frac{10Ba^2b^3}{11}\right) + x^8 \cdot \left(\frac{5Aa^2b^3}{4} + \frac{5Ba^3b^2}{4}\right) + x^5 \cdot \left(2Aa^3b^2 + Ba^4b\right) + x^2 \cdot \left(\frac{5Aa^4b}{2} + \frac{Ba^5}{2}\right)$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**2,x)
```

output
```
-A*a**5/x + B*b**5*x**17/17 + x**14*(A*b**5/14 + 5*B*a*b**4/14) + x**11*(5
*A*a*b**4/11 + 10*B*a**2*b**3/11) + x**8*(5*A*a**2*b**3/4 + 5*B*a**3*b**2/
4) + x**5*(2*A*a**3*b**2 + B*a**4*b) + x**2*(5*A*a**4*b/2 + B*a**5/2)
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^2}\,dx = \frac{1}{17}\,Bb^5x^{17} + \frac{1}{14}\,(5\,Bab^4 + Ab^5)x^{14} + \frac{5}{11}\,(2\,Ba^2b^3 + Aab^4)x^{11} + \frac{5}{4}\,(Ba^3b^2 + Aa^2b^3)x^8 + (Ba^4b + 2\,Aa^3b^2)x^5 - \frac{Aa^5}{x} + \frac{1}{2}\,(Ba^5 + 5\,Aa^4b)x^2$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="maxima")
```

output
```
1/17*B*b^5*x^17 + 1/14*(5*B*a*b^4 + A*b^5)*x^14 + 5/11*(2*B*a^2*b^3 + A*a*
b^4)*x^11 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + (B*a^4*b + 2*A*a^3*b^2)*x^5
- A*a^5/x + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2}\,dx = \frac{1}{17}\,Bb^5x^{17} + \frac{5}{14}\,Bab^4x^{14} + \frac{1}{14}\,Ab^5x^{14} + \frac{10}{11}\,Ba^2b^3x^{11} + \frac{5}{11}\,Aab^4x^{11} + \frac{5}{4}\,Ba^3b^2x^8 + \frac{5}{4}\,Aa^2b^3x^8 + Ba^4bx^5 + 2\,Aa^3b^2x^5 + \frac{1}{2}\,Ba^5x^2 + \frac{5}{2}\,Aa^4bx^2 - \frac{Aa^5}{x}$$

input

```
integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="giac")
```

output

```
1/17*B*b^5*x^17 + 5/14*B*a*b^4*x^14 + 1/14*A*b^5*x^14 + 10/11*B*a^2*b^3*x^
11 + 5/11*A*a*b^4*x^11 + 5/4*B*a^3*b^2*x^8 + 5/4*A*a^2*b^3*x^8 + B*a^4*b*x
^5 + 2*A*a^3*b^2*x^5 + 1/2*B*a^5*x^2 + 5/2*A*a^4*b*x^2 - A*a^5/x
```

3.34.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2}\,dx = x^2\left(\frac{B\,a^5}{2} + \frac{5\,A\,b\,a^4}{2}\right) + x^{14}\left(\frac{A\,b^5}{14} + \frac{5\,B\,a\,b^4}{14}\right) - \frac{A\,a^5}{x} + \frac{B\,b^5\,x^{17}}{17} + \frac{5\,a^2\,b^2\,x^8\,(A\,b + B\,a)}{4} + a^3\,b\,x^5\,(2\,A\,b + B\,a) + \frac{5\,a\,b^3\,x^{11}\,(A\,b + 2\,B\,a)}{11}$$

input

```
int(((A + B*x^3)*(a + b*x^3)^5)/x^2,x)
```

output

```
x^2*((B*a^5)/2 + (5*A*a^4*b)/2) + x^14*((A*b^5)/14 + (5*B*a*b^4)/14) - (A*
a^5)/x + (B*b^5*x^17)/17 + (5*a^2*b^2*x^8*(A*b + B*a))/4 + a^3*b*x^5*(2*A*
b + B*a) + (5*a*b^3*x^11*(A*b + 2*B*a))/11
```

3.35 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^3}\,dx$

3.35.1 Optimal result

Integrand size = 20, antiderivative size = 112

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^3}\,dx = &-\frac{a^5A}{2x^2}+a^4(5Ab+aB)x+\frac{5}{4}a^3b(2Ab+aB)x^4\\&+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{1}{2}ab^3(Ab+2aB)x^{10}\\&+\frac{1}{13}b^4(Ab+5aB)x^{13}+\frac{1}{16}b^5Bx^{16}\end{aligned}$$

output
```
-1/2*a^5*A/x^2+a^4*(5*A*b+B*a)*x+5/4*a^3*b*(2*A*b+B*a)*x^4+10/7*a^2*b^2*(A
*b+B*a)*x^7+1/2*a*b^3*(A*b+2*B*a)*x^10+1/13*b^4*(A*b+5*B*a)*x^13+1/16*b^5*
B*x^16
```

3.35.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^3}\,dx = &-\frac{a^5A}{2x^2}+a^4(5Ab+aB)x+\frac{5}{4}a^3b(2Ab+aB)x^4\\&+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{1}{2}ab^3(Ab+2aB)x^{10}\\&+\frac{1}{13}b^4(Ab+5aB)x^{13}+\frac{1}{16}b^5Bx^{16}\end{aligned}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(a^5*A)/x^2 + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (
10*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b +
 5*a*B)*x^13)/13 + (b^5*B*x^16)/16
```

3.35.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5 A}{x^3} + a^4(aB+5Ab) + 5a^3bx^3(aB+2Ab) + 10a^2b^2x^6(aB+Ab) + b^4x^{12}(5aB+Ab) + 5ab^3x^9(2aB+Ab) + \right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{2x^2} + a^4x(aB+5Ab) + \frac{5}{4}a^3bx^4(aB+2Ab) + \frac{10}{7}a^2b^2x^7(aB+Ab) + \frac{1}{13}b^4x^{13}(5aB+Ab) + \frac{1}{2}ab^3x^{10}(2aB+Ab) + \frac{1}{16}b^5Bx^{16}$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(a^5*A)/x^2 + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (
10*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b +
 5*a*B)*x^13)/13 + (b^5*B*x^16)/16
```

3.35.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.35.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

method	result
default	$\frac{b^5 B\,x^{16}}{16}+\frac{A\,b^5 x^{13}}{13}+\frac{5Ba\,b^4 x^{13}}{13}+\frac{Aa\,b^4 x^{10}}{2}+B\,a^2 b^3 x^{10}+\frac{10a^2 A\,b^3 x^7}{7}+\frac{10B\,a^3 b^2 x^7}{7}+\frac{5a^3 A\,b^2 x^4}{2}+\frac{5B\,a^4 b}{4}$
norman	$\frac{-\frac{a^5 A}{2}+\left(5a^4 bA+a^5 B\right)x^3+\left(\frac{5}{2}a^3 b^2 A+\frac{5}{4}a^4 bB\right)x^6+\left(\frac{10}{7}a^2 b^3 A+\frac{10}{7}a^3 b^2 B\right)x^9+\left(\frac{1}{2}a\,b^4 A+a^2 b^3 B\right)x^{12}+\left(\frac{1}{13}b^5 A+\frac{5}{13}a\,b^4 B\right)x^{15}+b}{x^2}$
risch	$\frac{b^5 B\,x^{16}}{16}+\frac{A\,b^5 x^{13}}{13}+\frac{5Ba\,b^4 x^{13}}{13}+\frac{Aa\,b^4 x^{10}}{2}+B\,a^2 b^3 x^{10}+\frac{10a^2 A\,b^3 x^7}{7}+\frac{10B\,a^3 b^2 x^7}{7}+\frac{5a^3 A\,b^2 x^4}{2}+\frac{5B\,a^4 b}{4}$
gosper	$-\frac{-91b^5 B\,x^{18}-112A\,b^5 x^{15}-560Ba\,b^4 x^{15}-728aA\,b^4 x^{12}-1456B\,a^2 b^3 x^{12}-2080a^2 A\,b^3 x^9-2080B\,a^3 b^2 x^9-3640a^3 A\,b^2 x^6-18}{1456x^2}$
parallelrisch	$\frac{91b^5 B\,x^{18}+112A\,b^5 x^{15}+560Ba\,b^4 x^{15}+728aA\,b^4 x^{12}+1456B\,a^2 b^3 x^{12}+2080a^2 A\,b^3 x^9+2080B\,a^3 b^2 x^9+3640a^3 A\,b^2 x^6+1820B}{1456x^2}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*b^5*B*x^16+1/13*A*b^5*x^13+5/13*B*a*b^4*x^13+1/2*A*a*b^4*x^10+B*a^2*b
^3*x^10+10/7*a^2*A*b^3*x^7+10/7*B*a^3*b^2*x^7+5/2*a^3*A*b^2*x^4+5/4*B*a^4*
b*x^4+5*a^4*A*b*x+a^5*B*x-1/2*a^5*A/x^2
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^3}\,dx = \frac{91\,Bb^5x^{18}+112\,(5\,Bab^4+Ab^5)x^{15}+728\,(2\,Ba^2b^3+Aab^4)x^{12}+2080\,(Ba^3b^2+Aa^2b^3)x^9+1820\,(Ba^4b+}{1456\,x^2}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="fricas")
```

output
```
1/1456*(91*B*b^5*x^18 + 112*(5*B*a*b^4 + A*b^5)*x^15 + 728*(2*B*a^2*b^3 +
A*a*b^4)*x^12 + 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1820*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 728*A*a^5 + 1456*(B*a^5 + 5*A*a^4*b)*x^3)/x^2
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3}\,dx = &-\frac{Aa^5}{2x^2}+\frac{Bb^5x^{16}}{16}+x^{13}\left(\frac{Ab^5}{13}+\frac{5Bab^4}{13}\right)\\ &+x^{10}\left(\frac{Aab^4}{2}+Ba^2b^3\right)+x^7\cdot\left(\frac{10Aa^2b^3}{7}+\frac{10Ba^3b^2}{7}\right)\\ &+x^4\cdot\left(\frac{5Aa^3b^2}{2}+\frac{5Ba^4b}{4}\right)+x\left(5Aa^4b+Ba^5\right)\end{aligned}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**3,x)
```

output
```
-A*a**5/(2*x**2) + B*b**5*x**16/16 + x**13*(A*b**5/13 + 5*B*a*b**4/13) + x
**10*(A*a*b**4/2 + B*a**2*b**3) + x**7*(10*A*a**2*b**3/7 + 10*B*a**3*b**2/
7) + x**4*(5*A*a**3*b**2/2 + 5*B*a**4*b/4) + x*(5*A*a**4*b + B*a**5)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3}\,dx = &\frac{1}{16}Bb^5x^{16}+\frac{1}{13}\left(5\,Bab^4+Ab^5\right)x^{13}\\ &+\frac{1}{2}\left(2\,Ba^2b^3+Aab^4\right)x^{10}+\frac{10}{7}\left(Ba^3b^2+Aa^2b^3\right)x^7\\ &+\frac{5}{4}\left(Ba^4b+2\,Aa^3b^2\right)x^4-\frac{Aa^5}{2\,x^2}+\left(Ba^5+5\,Aa^4b\right)x\end{aligned}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="maxima")
```

output
```
1/16*B*b^5*x^16 + 1/13*(5*B*a*b^4 + A*b^5)*x^13 + 1/2*(2*B*a^2*b^3 + A*a*b
^4)*x^10 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + 5/4*(B*a^4*b + 2*A*a^3*b^2)*
x^4 - 1/2*A*a^5/x^2 + (B*a^5 + 5*A*a^4*b)*x
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} \, dx = \frac{1}{16} Bb^5x^{16} + \frac{5}{13} Bab^4x^{13} + \frac{1}{13} Ab^5x^{13} + Ba^2b^3x^{10} + \frac{1}{2} Aab^4x^{10} + \frac{10}{7} Ba^3b^2x^7 + \frac{10}{7} Aa^2b^3x^7 + \frac{5}{4} Ba^4bx^4 + \frac{5}{2} Aa^3b^2x^4 + Ba^5x + 5\, Aa^4bx - \frac{Aa^5}{2\, x^2}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="giac")
```

output
```
1/16*B*b^5*x^16 + 5/13*B*a*b^4*x^13 + 1/13*A*b^5*x^13 + B*a^2*b^3*x^10 + 1
/2*A*a*b^4*x^10 + 10/7*B*a^3*b^2*x^7 + 10/7*A*a^2*b^3*x^7 + 5/4*B*a^4*b*x^
4 + 5/2*A*a^3*b^2*x^4 + B*a^5*x + 5*A*a^4*b*x - 1/2*A*a^5/x^2
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} \, dx = x \left(B\, a^5 + 5\, A\, b\, a^4\right) + x^{13} \left(\frac{A\, b^5}{13} + \frac{5\, B\, a\, b^4}{13}\right) - \frac{A\, a^5}{2\, x^2} + \frac{B\, b^5\, x^{16}}{16} + \frac{10\, a^2\, b^2\, x^7\, (A\, b + B\, a)}{7} + \frac{5\, a^3\, b\, x^4\, (2\, A\, b + B\, a)}{4} + \frac{a\, b^3\, x^{10}\, (A\, b + 2\, B\, a)}{2}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^3,x)
```

output
```
x*(B*a^5 + 5*A*a^4*b) + x^13*((A*b^5)/13 + (5*B*a*b^4)/13) - (A*a^5)/(2*x^
2) + (B*b^5*x^16)/16 + (10*a^2*b^2*x^7*(A*b + B*a))/7 + (5*a^3*b*x^4*(2*A*
b + B*a))/4 + (a*b^3*x^10*(A*b + 2*B*a))/2
```

3.36 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^4}\,dx$

3.36.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^4}\,dx = -\frac{a^5A}{3x^3}+\frac{5}{3}a^3b(2Ab+aB)x^3+\frac{5}{3}a^2b^2(Ab+aB)x^6+\frac{5}{9}ab^3(Ab+2aB)x^9+\frac{1}{12}b^4(Ab+5aB)x^{12}+\frac{1}{15}b^5Bx^{15}+a^4(5Ab+aB)\log(x)$$

output
```
-1/3*a^5*A/x^3+5/3*a^3*b*(2*A*b+B*a)*x^3+5/3*a^2*b^2*(A*b+B*a)*x^6+5/9*a*b
^3*(A*b+2*B*a)*x^9+1/12*b^4*(A*b+5*B*a)*x^12+1/15*b^5*B*x^15+a^4*(5*A*b+B*
a)*ln(x)
```

3.36.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^4}\,dx = -\frac{a^5A}{3x^3}+\frac{5}{3}a^3b(2Ab+aB)x^3+\frac{5}{3}a^2b^2(Ab+aB)x^6+\frac{5}{9}ab^3(Ab+2aB)x^9+\frac{1}{12}b^4(Ab+5aB)x^{12}+\frac{1}{15}b^5Bx^{15}+\left(5a^4Ab+a^5B\right)\log(x)$$

input

```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^4,x]
```

output

```
-1/3*(a^5*A)/x^3 + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a^2*b^2*(A*b + a*B)*
x^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^
5*B*x^15)/15 + (5*a^4*A*b + a^5*B)*Log[x]
```

3.36.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^4} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{\left(bx^3+a\right)^5 \left(Bx^3+A\right)}{x^6}dx^3$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{3} \int \left(b^5 B x^{12}+b^4 (Ab+5aB) x^9+5 a b^3 (Ab+2aB) x^6+10 a^2 b^2 (Ab+aB) x^3+5 a^3 b (2Ab+aB)+\frac{a^4 (5Ab+aB)}{x^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{a^5 A}{x^3}+a^4 \log \left(x^3\right) (aB+5Ab)+5 a^3 b x^3 (aB+2Ab)+5 a^2 b^2 x^6 (aB+Ab)+\frac{1}{4} b^4 x^{12} (5aB+Ab)+\frac{5}{3} a b^3 x^9 (2a\right.$$

input

```
Int[((a + b*x^3)^5*(A + B*x^3))/x^4,x]
```

output

```
(-((a^5*A)/x^3) + 5*a^3*b*(2*A*b + a*B)*x^3 + 5*a^2*b^2*(A*b + a*B)*x^6 +
(5*a*b^3*(A*b + 2*a*B)*x^9)/3 + (b^4*(A*b + 5*a*B)*x^12)/4 + (b^5*B*x^15)/
5 + a^4*(5*A*b + a*B)*Log[x^3])/3
```

3.36.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.36.4 Maple [A] (verified)

Time = 4.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

method	result
default	$\frac{b^{5} B\,x^{15}}{15}+\frac{A\,b^{5} x^{12}}{12}+\frac{5 B a\,b^{4} x^{12}}{12}+\frac{5 A a\,b^{4} x^{9}}{9}+\frac{10 B\,a^{2} b^{3} x^{9}}{9}+\frac{5 a^{2} A\,b^{3} x^{6}}{3}+\frac{5 B\,a^{3} b^{2} x^{6}}{3}+\frac{10 a^{3} A\,b^{2} x^{3}}{3}+\frac{5 B\,a^{4} b\,x}{3}$
norman	$\frac{\left(\frac{1}{12} b^{5} A+\frac{5}{12} a\,b^{4} B\right) x^{15}+\left(\frac{5}{9} a\,b^{4} A+\frac{10}{9} a^{2} b^{3} B\right) x^{12}+\left(\frac{5}{3} a^{2} b^{3} A+\frac{5}{3} a^{3} b^{2} B\right) x^{9}+\left(\frac{10}{3} a^{3} b^{2} A+\frac{5}{3} a^{4} b B\right) x^{6}-\frac{a^{5} A}{3}+\frac{b^{5} B\,x^{18}}{15}}{x^{3}}+(5a^{4}b$
risch	$\frac{b^{5} B\,x^{15}}{15}+\frac{A\,b^{5} x^{12}}{12}+\frac{5 B a\,b^{4} x^{12}}{12}+\frac{5 A a\,b^{4} x^{9}}{9}+\frac{10 B\,a^{2} b^{3} x^{9}}{9}+\frac{5 a^{2} A\,b^{3} x^{6}}{3}+\frac{5 B\,a^{3} b^{2} x^{6}}{3}+\frac{10 a^{3} A\,b^{2} x^{3}}{3}+\frac{5 B\,a^{4} b\,x}{3}$
parallelrisch	$\frac{12 b^{5} B\,x^{18}+15 A\,b^{5} x^{15}+75 B a\,b^{4} x^{15}+100 a A\,b^{4} x^{12}+200 B\,a^{2} b^{3} x^{12}+300 a^{2} A\,b^{3} x^{9}+300 B\,a^{3} b^{2} x^{9}+600 a^{3} A\,b^{2} x^{6}+300 B\,a^{4} b\,x^{6}+}{180 x^{3}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/15*b^5*B*x^15+1/12*A*b^5*x^12+5/12*B*a*b^4*x^12+5/9*A*a*b^4*x^9+10/9*B*a
^2*b^3*x^9+5/3*a^2*A*b^3*x^6+5/3*B*a^3*b^2*x^6+10/3*a^3*A*b^2*x^3+5/3*B*a^
4*b*x^3+a^4*(5*A*b+B*a)*ln(x)-1/3*a^5*A/x^3
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^4}\,dx$$
$$= \frac{12\,Bb^5x^{18}+15\,(5\,Bab^4+Ab^5)x^{15}+100\,(2\,Ba^2b^3+Aab^4)x^{12}+300\,(Ba^3b^2+Aa^2b^3)x^9+300\,(Ba^4b+2}{180\,x^3}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="fricas")
```

output
```
1/180*(12*B*b^5*x^18 + 15*(5*B*a*b^4 + A*b^5)*x^15 + 100*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 300*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 60*A*a^5 + 180*(B*a^5 + 5*A*a^4*b)*x^3*log(x))/x^3
```

3.36.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^4}\,dx = {} & -\frac{Aa^5}{3x^3}+\frac{Bb^5x^{15}}{15}+a^4\cdot(5Ab+Ba)\log{(x)} \\ & +x^{12}\left(\frac{Ab^5}{12}+\frac{5Bab^4}{12}\right)+x^9\cdot\left(\frac{5Aab^4}{9}+\frac{10Ba^2b^3}{9}\right)+x^6 \\ & \cdot\left(\frac{5Aa^2b^3}{3}+\frac{5Ba^3b^2}{3}\right)+x^3\cdot\left(\frac{10Aa^3b^2}{3}+\frac{5Ba^4b}{3}\right)\end{aligned}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**4,x)
```

output
```
-A*a**5/(3*x**3) + B*b**5*x**15/15 + a**4*(5*A*b + B*a)*log(x) + x**12*(A*
b**5/12 + 5*B*a*b**4/12) + x**9*(5*A*a*b**4/9 + 10*B*a**2*b**3/9) + x**6*(
5*A*a**2*b**3/3 + 5*B*a**3*b**2/3) + x**3*(10*A*a**3*b**2/3 + 5*B*a**4*b/3
)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^4}\,dx = \frac{1}{15}\,Bb^5x^{15} + \frac{1}{12}\left(5\,Bab^4 + Ab^5\right)x^{12} + \frac{5}{9}\left(2\,Ba^2b^3 + Aab^4\right)x^9 + \frac{5}{3}\left(Ba^3b^2 + Aa^2b^3\right)x^6 + \frac{5}{3}\left(Ba^4b + 2\,Aa^3b^2\right)x^3 - \frac{Aa^5}{3\,x^3} + \frac{1}{3}\left(Ba^5 + 5\,Aa^4b\right)\log\left(x^3\right)$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="maxima")
```

output
```
1/15*B*b^5*x^15 + 1/12*(5*B*a*b^4 + A*b^5)*x^12 + 5/9*(2*B*a^2*b^3 + A*a*b
^4)*x^9 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*x^
3 - 1/3*A*a^5/x^3 + 1/3*(B*a^5 + 5*A*a^4*b)*log(x^3)
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^4}\,dx = \frac{1}{15}\,Bb^5x^{15} + \frac{5}{12}\,Bab^4x^{12} + \frac{1}{12}\,Ab^5x^{12} + \frac{10}{9}\,Ba^2b^3x^9 + \frac{5}{9}\,Aab^4x^9 + \frac{5}{3}\,Ba^3b^2x^6 + \frac{5}{3}\,Aa^2b^3x^6 + \frac{5}{3}\,Ba^4bx^3 + \frac{10}{3}\,Aa^3b^2x^3 + \left(Ba^5 + 5\,Aa^4b\right)\log\left(|x|\right) - \frac{Ba^5x^3 + 5\,Aa^4bx^3 + Aa^5}{3\,x^3}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="giac")
```

output
```
1/15*B*b^5*x^15 + 5/12*B*a*b^4*x^12 + 1/12*A*b^5*x^12 + 10/9*B*a^2*b^3*x^9
 + 5/9*A*a*b^4*x^9 + 5/3*B*a^3*b^2*x^6 + 5/3*A*a^2*b^3*x^6 + 5/3*B*a^4*b*x
^3 + 10/3*A*a^3*b^2*x^3 + (B*a^5 + 5*A*a^4*b)*log(abs(x)) - 1/3*(B*a^5*x^3
 + 5*A*a^4*b*x^3 + A*a^5)/x^3
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^4}\,dx = x^{12}\left(\frac{A\,b^5}{12}+\frac{5\,B\,a\,b^4}{12}\right)+\ln\left(x\right)\,\left(B\,a^5+5\,A\,b\,a^4\right) - \frac{A\,a^5}{3\,x^3}+\frac{B\,b^5\,x^{15}}{15}+\frac{5\,a^2\,b^2\,x^6\,\left(A\,b+B\,a\right)}{3} + \frac{5\,a^3\,b\,x^3\,\left(2\,A\,b+B\,a\right)}{3}+\frac{5\,a\,b^3\,x^9\,\left(A\,b+2\,B\,a\right)}{9}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^4,x)
```

output
```
x^12*((A*b^5)/12 + (5*B*a*b^4)/12) + log(x)*(B*a^5 + 5*A*a^4*b) - (A*a^5)/
(3*x^3) + (B*b^5*x^15)/15 + (5*a^2*b^2*x^6*(A*b + B*a))/3 + (5*a^3*b*x^3*(
2*A*b + B*a))/3 + (5*a*b^3*x^9*(A*b + 2*B*a))/9
```

3.37 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5}\,dx$

3.37.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5}\,dx = -\frac{a^5A}{4x^4} - \frac{a^4(5Ab+aB)}{x} + \frac{5}{2}a^3b(2Ab+aB)x^2 + 2a^2b^2(Ab+aB)x^5 + \frac{5}{8}ab^3(Ab+2aB)x^8 + \frac{1}{11}b^4(Ab+5aB)x^{11} + \frac{1}{14}b^5Bx^{14}$$

output
```
-1/4*a^5*A/x^4-a^4*(5*A*b+B*a)/x+5/2*a^3*b*(2*A*b+B*a)*x^2+2*a^2*b^2*(A*b+
B*a)*x^5+5/8*a*b^3*(A*b+2*B*a)*x^8+1/11*b^4*(A*b+5*B*a)*x^11+1/14*b^5*B*x^
14
```

3.37.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5}\,dx = -\frac{a^5A}{4x^4} + \frac{-5a^4Ab - a^5B}{x} + \frac{5}{2}a^3b(2Ab+aB)x^2 + 2a^2b^2(Ab+aB)x^5 + \frac{5}{8}ab^3(Ab+2aB)x^8 + \frac{1}{11}b^4(Ab+5aB)x^{11} + \frac{1}{14}b^5Bx^{14}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^5,x]
```

output
```
-1/4*(a^5*A)/x^4 + (-5*a^4*A*b - a^5*B)/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2
+ 2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b +
5*a*B)*x^11)/11 + (b^5*B*x^14)/14
```

3.37.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^5} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5A}{x^5}+\frac{a^4(aB+5Ab)}{x^2}+5a^3bx(aB+2Ab)+10a^2b^2x^4(aB+Ab)+b^4x^{10}(5aB+Ab)+5ab^3x^7(2aB+Ab)+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{4x^4}-\frac{a^4(aB+5Ab)}{x}+\frac{5}{2}a^3bx^2(aB+2Ab)+2a^2b^2x^5(aB+Ab)+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{8}ab^3x^8(2aB+Ab)+\frac{1}{14}b^5Bx^{14}$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^5,x]
```

output
```
-1/4*(a^5*A)/x^4 - (a^4*(5*A*b + a*B))/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 +
 2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5
*a*B)*x^11)/11 + (b^5*B*x^14)/14
```

3.37.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.37.4 Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.08

method	result
norman	$\frac{-\frac{a^5 A}{4}+(-5a^4bA-a^5B)x^3+(5a^3b^2A+\frac{5}{2}a^4bB)x^6+(2a^2b^3A+2a^3b^2B)x^9+(\frac{5}{8}ab^4A+\frac{5}{4}a^2b^3B)x^{12}+(\frac{1}{11}b^5A+\frac{5}{11}ab^4B)x^{15}+}{x^4}$
default	$\frac{b^5Bx^{14}}{14}+\frac{Ab^5x^{11}}{11}+\frac{5Bab^4x^{11}}{11}+\frac{5aAb^4x^8}{8}+\frac{5Ba^2b^3x^8}{4}+2Aa^2b^3x^5+2Ba^3b^2x^5+5a^3Ab^2x^2+$
risch	$\frac{b^5Bx^{14}}{14}+\frac{Ab^5x^{11}}{11}+\frac{5Bab^4x^{11}}{11}+\frac{5aAb^4x^8}{8}+\frac{5Ba^2b^3x^8}{4}+2Aa^2b^3x^5+2Ba^3b^2x^5+5a^3Ab^2x^2+$
gosper	$-\frac{-44b^5Bx^{18}-56Ab^5x^{15}-280Bab^4x^{15}-385aAb^4x^{12}-770Ba^2b^3x^{12}-1232a^2Ab^3x^9-1232Ba^3b^2x^9-3080a^3Ab^2x^6-1540}{616x^4}$
parallelrisch	$\frac{44b^5Bx^{18}+56Ab^5x^{15}+280Bab^4x^{15}+385aAb^4x^{12}+770Ba^2b^3x^{12}+1232a^2Ab^3x^9+1232Ba^3b^2x^9+3080a^3Ab^2x^6+1540Ba^4}{616x^4}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/x^4*(-1/4*a^5*A+(-5*A*a^4*b-B*a^5)*x^3+(5*a^3*b^2*A+5/2*a^4*b*B)*x^6+(2*
A*a^2*b^3+2*B*a^3*b^2)*x^9+(5/8*a*b^4*A+5/4*a^2*b^3*B)*x^12+(1/11*b^5*A+5/
11*a*b^4*B)*x^15+1/14*b^5*B*x^18)
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5}\,dx = \frac{44\,Bb^5x^{18}+56\,(5\,Bab^4+Ab^5)x^{15}+385\,(2\,Ba^2b^3+Aab^4)x^{12}+1232\,(Ba^3b^2+Aa^2b^3)x^9+1540\,(Ba^4b+}{616\,x^4}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="fricas")
```

output
```
1/616*(44*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 385*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1540*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 154*A*a^5 - 616*(B*a^5 + 5*A*a^4*b)*x^3)/x^4
```

3.37.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5}\,dx = \frac{Bb^5x^{14}}{14} + x^{11}\left(\frac{Ab^5}{11} + \frac{5Bab^4}{11}\right) + x^8 \cdot \left(\frac{5Aab^4}{8} + \frac{5Ba^2b^3}{4}\right) + x^5 \cdot \left(2Aa^2b^3 + 2Ba^3b^2\right) + x^2 \cdot \left(5Aa^3b^2 + \frac{5Ba^4b}{2}\right) + \frac{-Aa^5 + x^3(-20Aa^4b - 4Ba^5)}{4x^4}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**5,x)
```

output
```
B*b**5*x**14/14 + x**11*(A*b**5/11 + 5*B*a*b**4/11) + x**8*(5*A*a*b**4/8 +
 5*B*a**2*b**3/4) + x**5*(2*A*a**2*b**3 + 2*B*a**3*b**2) + x**2*(5*A*a**3*
b**2 + 5*B*a**4*b/2) + (-A*a**5 + x**3*(-20*A*a**4*b - 4*B*a**5))/(4*x**4)
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5}\,dx = \frac{1}{14}Bb^5x^{14} + \frac{1}{11}\left(5\,Bab^4 + Ab^5\right)x^{11} + \frac{5}{8}\left(2\,Ba^2b^3 + Aab^4\right)x^8 + 2\left(Ba^3b^2 + Aa^2b^3\right)x^5 + \frac{5}{2}\left(Ba^4b + 2\,Aa^3b^2\right)x^2 - \frac{Aa^5 + 4\left(Ba^5 + 5\,Aa^4b\right)x^3}{4\,x^4}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="maxima")
```

output
```
1/14*B*b^5*x^14 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 5/8*(2*B*a^2*b^3 + A*a*b
^4)*x^8 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 + 5/2*(B*a^4*b + 2*A*a^3*b^2)*x^2
- 1/4*(A*a^5 + 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^4
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5}\,dx = \frac{1}{14}\,Bb^5x^{14} + \frac{5}{11}\,Bab^4x^{11} + \frac{1}{11}\,Ab^5x^{11} + \frac{5}{4}\,Ba^2b^3x^8 + \frac{5}{8}\,Aab^4x^8 + 2\,Ba^3b^2x^5 + 2\,Aa^2b^3x^5 + \frac{5}{2}\,Ba^4bx^2 + 5\,Aa^3b^2x^2 - \frac{4\,Ba^5x^3 + 20\,Aa^4bx^3 + Aa^5}{4\,x^4}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="giac")
```

output
```
1/14*B*b^5*x^14 + 5/11*B*a*b^4*x^11 + 1/11*A*b^5*x^11 + 5/4*B*a^2*b^3*x^8
+ 5/8*A*a*b^4*x^8 + 2*B*a^3*b^2*x^5 + 2*A*a^2*b^3*x^5 + 5/2*B*a^4*b*x^2 +
5*A*a^3*b^2*x^2 - 1/4*(4*B*a^5*x^3 + 20*A*a^4*b*x^3 + A*a^5)/x^4
```

3.37.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5}\,dx = x^{11}\left(\frac{A\,b^5}{11} + \frac{5\,B\,a\,b^4}{11}\right) - \frac{\frac{A\,a^5}{4} + x^3\,(B\,a^5 + 5\,A\,b\,a^4)}{x^4} + \frac{B\,b^5\,x^{14}}{14} + 2\,a^2\,b^2\,x^5\,(A\,b + B\,a) + \frac{5\,a^3\,b\,x^2\,(2\,A\,b + B\,a)}{2} + \frac{5\,a\,b^3\,x^8\,(A\,b + 2\,B\,a)}{8}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^5,x)
```

output
```
x^11*((A*b^5)/11 + (5*B*a*b^4)/11) - ((A*a^5)/4 + x^3*(B*a^5 + 5*A*a^4*b))
/x^4 + (B*b^5*x^14)/14 + 2*a^2*b^2*x^5*(A*b + B*a) + (5*a^3*b*x^2*(2*A*b +
 B*a))/2 + (5*a*b^3*x^8*(A*b + 2*B*a))/8
```

3.38 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^6}\,dx$

3.38.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^6}\,dx = -\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{2x^2}+5a^3b(2Ab+aB)x+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{7}ab^3(Ab+2aB)x^7+\frac{1}{10}b^4(Ab+5aB)x^{10}+\frac{1}{13}b^5Bx^{13}$$

output
```
-1/5*a^5*A/x^5-1/2*a^4*(5*A*b+B*a)/x^2+5*a^3*b*(2*A*b+B*a)*x+5/2*a^2*b^2*(
A*b+B*a)*x^4+5/7*a*b^3*(A*b+2*B*a)*x^7+1/10*b^4*(A*b+5*B*a)*x^10+1/13*b^5*
B*x^13
```

3.38.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^6}\,dx = -\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{2x^2}+5a^3b(2Ab+aB)x+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{7}ab^3(Ab+2aB)x^7+\frac{1}{10}b^4(Ab+5aB)x^{10}+\frac{1}{13}b^5Bx^{13}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^6,x]
```

output
```
-1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x +
 (5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b
 + 5*a*B)*x^10)/10 + (b^5*B*x^13)/13
```

3.38.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5 A}{x^6}+\frac{a^4 (aB+5Ab)}{x^3}+5a^3 b(aB+2Ab)+10a^2 b^2 x^3 (aB+Ab)+b^4 x^9 (5aB+Ab)+5ab^3 x^6 (2aB+Ab)+b^5$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5 A}{5x^5}-\frac{a^4 (aB+5Ab)}{2x^2}+5a^3 bx(aB+2Ab)+\frac{5}{2}a^2 b^2 x^4 (aB+Ab)+\frac{1}{10}b^4 x^{10} (5aB+Ab)+ \frac{5}{7}ab^3 x^7 (2aB+Ab)+\frac{1}{13}b^5 Bx^{13}$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^6,x]
```

output
```
-1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x +
 (5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b
 + 5*a*B)*x^10)/10 + (b^5*B*x^13)/13
```

3.38.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.38.4 Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

method	result
default	$\frac{b^5 B x^{13}}{13}+\frac{A b^5 x^{10}}{10}+\frac{B a b^4 x^{10}}{2}+\frac{5 A a b^4 x^7}{7}+\frac{10 B a^2 b^3 x^7}{7}+\frac{5 a^2 A b^3 x^4}{2}+\frac{5 B a^3 b^2 x^4}{2}+10 a^3 b^2 A x+5 a^4 b B$
norman	$\frac{-\frac{a^5 A}{5}+\left(-\frac{5}{2} a^4 b A-\frac{1}{2} a^5 B\right) x^3+\left(10 a^3 b^2 A+5 a^4 b B\right) x^6+\left(\frac{5}{2} a^2 b^3 A+\frac{5}{2} a^3 b^2 B\right) x^9+\left(\frac{5}{7} a b^4 A+\frac{10}{7} a^2 b^3 B\right) x^{12}+\left(\frac{1}{10} b^5 A+\frac{1}{2} a b^4 B\right) x^{1}}{x^5}$
risch	$\frac{b^5 B x^{13}}{13}+\frac{A b^5 x^{10}}{10}+\frac{B a b^4 x^{10}}{2}+\frac{5 A a b^4 x^7}{7}+\frac{10 B a^2 b^3 x^7}{7}+\frac{5 a^2 A b^3 x^4}{2}+\frac{5 B a^3 b^2 x^4}{2}+10 a^3 b^2 A x+5 a^4 b B$
gosper	$-\frac{-70 b^5 B x^{18}-91 A b^5 x^{15}-455 B a b^4 x^{15}-650 a A b^4 x^{12}-1300 B a^2 b^3 x^{12}-2275 a^2 A b^3 x^9-2275 B a^3 b^2 x^9-9100 a^3 A b^2 x^6-455}{910 x^5}$
parallelrisch	$\frac{70 b^5 B x^{18}+91 A b^5 x^{15}+455 B a b^4 x^{15}+650 a A b^4 x^{12}+1300 B a^2 b^3 x^{12}+2275 a^2 A b^3 x^9+2275 B a^3 b^2 x^9+9100 a^3 A b^2 x^6+4550 B a}{910 x^5}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/13*b^5*B*x^13+1/10*A*b^5*x^10+1/2*B*a*b^4*x^10+5/7*A*a*b^4*x^7+10/7*B*a^
2*b^3*x^7+5/2*a^2*A*b^3*x^4+5/2*B*a^3*b^2*x^4+10*a^3*b^2*A*x+5*a^4*b*B*x-1
/2*a^4*(5*A*b+B*a)/x^2-1/5*a^5*A/x^5
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+b x^3\right)^5 \left(A+B x^3\right)}{x^6} \, dx$$
$$=\frac{70 B b^5 x^{18}+91 (5 B a b^4+A b^5) x^{15}+650 (2 B a^2 b^3+A a b^4) x^{12}+2275 (B a^3 b^2+A a^2 b^3) x^9+4550 (B a^4 b+}{910 x^5}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="fricas")
```

output
```
1/910*(70*B*b^5*x^18 + 91*(5*B*a*b^4 + A*b^5)*x^15 + 650*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 4550*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 182*A*a^5 - 455*(B*a^5 + 5*A*a^4*b)*x^3)/x^5
```

3.38.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^6}\,dx = \frac{Bb^5x^{13}}{13} + x^{10}\left(\frac{Ab^5}{10} + \frac{Bab^4}{2}\right) + x^7\cdot\left(\frac{5Aab^4}{7} + \frac{10Ba^2b^3}{7}\right) + x^4\cdot\left(\frac{5Aa^2b^3}{2} + \frac{5Ba^3b^2}{2}\right) + x\left(10Aa^3b^2 + 5Ba^4b\right) + \frac{-2Aa^5 + x^3(-25Aa^4b - 5Ba^5)}{10x^5}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**6,x)
```

output
```
B*b**5*x**13/13 + x**10*(A*b**5/10 + B*a*b**4/2) + x**7*(5*A*a*b**4/7 + 10
*B*a**2*b**3/7) + x**4*(5*A*a**2*b**3/2 + 5*B*a**3*b**2/2) + x*(10*A*a**3*
b**2 + 5*B*a**4*b) + (-2*A*a**5 + x**3*(-25*A*a**4*b - 5*B*a**5))/(10*x**5
)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^6}\,dx = \frac{1}{13}\,Bb^5x^{13} + \frac{1}{10}\left(5\,Bab^4 + Ab^5\right)x^{10} + \frac{5}{7}\left(2\,Ba^2b^3 + Aab^4\right)x^7 + \frac{5}{2}\left(Ba^3b^2 + Aa^2b^3\right)x^4 + 5\left(Ba^4b + 2\,Aa^3b^2\right)x - \frac{2\,Aa^5 + 5\,(Ba^5 + 5\,Aa^4b)x^3}{10\,x^5}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="maxima")
```

output
```
1/13*B*b^5*x^13 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/7*(2*B*a^2*b^3 + A*a*b
^4)*x^7 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 + 5*(B*a^4*b + 2*A*a^3*b^2)*x -
1/10*(2*A*a^5 + 5*(B*a^5 + 5*A*a^4*b)*x^3)/x^5
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^6}\,dx = \frac{1}{13}Bb^5x^{13} + \frac{1}{2}Bab^4x^{10} + \frac{1}{10}Ab^5x^{10} + \frac{10}{7}Ba^2b^3x^7 + \frac{5}{7}Aab^4x^7 + \frac{5}{2}Ba^3b^2x^4 + \frac{5}{2}Aa^2b^3x^4 + 5\,Ba^4bx + 10\,Aa^3b^2x - \frac{5\,Ba^5x^3 + 25\,Aa^4bx^3 + 2\,Aa^5}{10\,x^5}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="giac")
```

output
```
1/13*B*b^5*x^13 + 1/2*B*a*b^4*x^10 + 1/10*A*b^5*x^10 + 10/7*B*a^2*b^3*x^7
+ 5/7*A*a*b^4*x^7 + 5/2*B*a^3*b^2*x^4 + 5/2*A*a^2*b^3*x^4 + 5*B*a^4*b*x +
10*A*a^3*b^2*x - 1/10*(5*B*a^5*x^3 + 25*A*a^4*b*x^3 + 2*A*a^5)/x^5
```

3.38.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^6}\,dx = x^{10}\left(\frac{A\,b^5}{10} + \frac{B\,a\,b^4}{2}\right) - \frac{\frac{A\,a^5}{5} + x^3\left(\frac{B\,a^5}{2} + \frac{5\,A\,b\,a^4}{2}\right)}{x^5} + \frac{B\,b^5\,x^{13}}{13} + \frac{5\,a^2\,b^2\,x^4\,(A\,b + B\,a)}{2} + 5\,a^3\,b\,x\,(2\,A\,b + B\,a) + \frac{5\,a\,b^3\,x^7\,(A\,b + 2\,B\,a)}{7}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^6,x)
```

output
```
x^10*((A*b^5)/10 + (B*a*b^4)/2) - ((A*a^5)/5 + x^3*((B*a^5)/2 + (5*A*a^4*b
)/2))/x^5 + (B*b^5*x^13)/13 + (5*a^2*b^2*x^4*(A*b + B*a))/2 + 5*a^3*b*x*(2
*A*b + B*a) + (5*a*b^3*x^7*(A*b + 2*B*a))/7
```

3.39 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^7}\,dx$

3.39.1 Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^7}\,dx = -\frac{a^5A}{6x^6}-\frac{a^4(5Ab+aB)}{3x^3}+\frac{10}{3}a^2b^2(Ab+aB)x^3 + \frac{5}{6}ab^3(Ab+2aB)x^6+\frac{1}{9}b^4(Ab+5aB)x^9 + \frac{1}{12}b^5Bx^{12}+5a^3b(2Ab+aB)\log(x)$$

output
```
-1/6*a^5*A/x^6-1/3*a^4*(5*A*b+B*a)/x^3+10/3*a^2*b^2*(A*b+B*a)*x^3+5/6*a*b^
3*(A*b+2*B*a)*x^6+1/9*b^4*(A*b+5*B*a)*x^9+1/12*b^5*B*x^12+5*a^3*b*(2*A*b+B
*a)*ln(x)
```

3.39.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^7}\,dx = \frac{1}{36}\left(-\frac{6a^5A}{x^6}-\frac{12a^4(5Ab+aB)}{x^3}+120a^2b^2(Ab+aB)x^3 + 30ab^3(Ab+2aB)x^6+4b^4(Ab+5aB)x^9+3b^5Bx^{12} + 180a^3b(2Ab+aB)\log(x)\right)$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^7,x]
```

output
```
((-6*a^5*A)/x^6 - (12*a^4*(5*A*b + a*B))/x^3 + 120*a^2*b^2*(A*b + a*B)*x^3
 + 30*a*b^3*(A*b + 2*a*B)*x^6 + 4*b^4*(A*b + 5*a*B)*x^9 + 3*b^5*B*x^12 + 1
80*a^3*b*(2*A*b + a*B)*Log[x])/36
```

3.39.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^7} \, dx$$

$$\Big\downarrow \textcolor{blue}{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5 (Bx^3+A)}{x^9}dx^3$$

$$\Big\downarrow \textcolor{blue}{85}$$

$$\frac{1}{3}\int \Bigg(b^5Bx^9+b^4(Ab+5aB)x^6+5ab^3(Ab+2aB)x^3+10a^2b^2(Ab+aB)+\frac{5a^3b(2Ab+aB)}{x^3}+\frac{a^4(5Ab+aB)}{x^6}+$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{1}{3}\Bigg(-\frac{a^5A}{2x^6}-\frac{a^4(aB+5Ab)}{x^3}+5a^3b\log\left(x^3\right)(aB+2Ab)+10a^2b^2x^3(aB+Ab)+\frac{1}{3}b^4x^9(5aB+Ab)+\frac{5}{2}ab^3x^6(2aB$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^7,x]
```

output
```
(-1/2*(a^5*A)/x^6 - (a^4*(5*A*b + a*B))/x^3 + 10*a^2*b^2*(A*b + a*B)*x^3 +
 (5*a*b^3*(A*b + 2*a*B)*x^6)/2 + (b^4*(A*b + 5*a*B)*x^9)/3 + (b^5*B*x^12)/
4 + 5*a^3*b*(2*A*b + a*B)*Log[x^3])/3
```

3.39.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.39.4 Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

method	result
default	$\frac{b^5 B\, x^{12}}{12}+\frac{A\, b^5 x^9}{9}+\frac{5 B a\, b^4 x^9}{9}+\frac{5 A a\, b^4 x^6}{6}+\frac{5 B\, a^2 b^3 x^6}{3}+\frac{10 a^2 A\, b^3 x^3}{3}+\frac{10 B\, a^3 b^2 x^3}{3}+5 a^3 b (2 A b+B a) \ln$
norman	$\frac{\left(\frac{1}{9} b^5 A+\frac{5}{9} a\, b^4 B\right) x^{15}+\left(\frac{5}{6} a\, b^4 A+\frac{5}{3} a^2 b^3 B\right) x^{12}+\left(\frac{10}{3} a^2 b^3 A+\frac{10}{3} a^3 b^2 B\right) x^9+\left(-\frac{5}{3} a^4 b A-\frac{1}{3} a^5 B\right) x^3-\frac{a^5 A}{6}+\frac{b^5 B\, x^{18}}{12}}{x^6}+(10 a^3 b^2$
risch	$\frac{b^5 B\, x^{12}}{12}+\frac{A\, b^5 x^9}{9}+\frac{5 B a\, b^4 x^9}{9}+\frac{5 A a\, b^4 x^6}{6}+\frac{5 B\, a^2 b^3 x^6}{3}+\frac{10 a^2 A\, b^3 x^3}{3}+\frac{10 B\, a^3 b^2 x^3}{3}+\frac{\left(-\frac{5}{3} a^4 b A-\frac{1}{3} a^5 B\right) x^3-\frac{a^5}{6}}{x^6}$
parallelrisch	$\frac{3 b^5 B\, x^{18}+4 A\, b^5 x^{15}+20 B a\, b^4 x^{15}+30 a A\, b^4 x^{12}+60 B\, a^2 b^3 x^{12}+120 a^2 A\, b^3 x^9+120 B\, a^3 b^2 x^9+360 A \ln(x) x^6 a^3 b^2+180 B \ln(x) x^6}{36 x^6}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/12*b^5*B*x^12+1/9*A*b^5*x^9+5/9*B*a*b^4*x^9+5/6*A*a*b^4*x^6+5/3*B*a^2*b^
3*x^6+10/3*a^2*A*b^3*x^3+10/3*B*a^3*b^2*x^3+5*a^3*b*(2*A*b+B*a)*ln(x)-1/6*
a^5*A/x^6-1/3*a^4*(5*A*b+B*a)/x^3
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^7}\,dx$$
$$= \frac{3\,Bb^5x^{18} + 4\,(5\,Bab^4 + Ab^5)x^{15} + 30\,(2\,Ba^2b^3 + Aab^4)x^{12} + 120\,(Ba^3b^2 + Aa^2b^3)x^9 + 180\,(Ba^4b + 2\,Aa^3}{36\,x^6}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="fricas")
```

output
```
1/36*(3*B*b^5*x^18 + 4*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^
4)*x^12 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 180*(B*a^4*b + 2*A*a^3*b^2)*x^
6*log(x) - 6*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^6
```

3.39.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^7}\,dx = \frac{Bb^5x^{12}}{12} + 5a^3b(2Ab+Ba)\log{(x)} + x^9\left(\frac{Ab^5}{9} + \frac{5Bab^4}{9}\right) + x^6\cdot\left(\frac{5Aab^4}{6} + \frac{5Ba^2b^3}{3}\right) + x^3\cdot\left(\frac{10Aa^2b^3}{3} + \frac{10Ba^3b^2}{3}\right) + \frac{-Aa^5 + x^3(-10Aa^4b - 2Ba^5)}{6x^6}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**7,x)
```

output
```
B*b**5*x**12/12 + 5*a**3*b*(2*A*b + B*a)*log(x) + x**9*(A*b**5/9 + 5*B*a*b
**4/9) + x**6*(5*A*a*b**4/6 + 5*B*a**2*b**3/3) + x**3*(10*A*a**2*b**3/3 +
10*B*a**3*b**2/3) + (-A*a**5 + x**3*(-10*A*a**4*b - 2*B*a**5))/(6*x**6)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^7}\,dx = \frac{1}{12}\,Bb^5x^{12} + \frac{1}{9}\left(5\,Bab^4 + Ab^5\right)x^9 + \frac{5}{6}\left(2\,Ba^2b^3 + Aab^4\right)x^6 + \frac{10}{3}\left(Ba^3b^2 + Aa^2b^3\right)x^3 + \frac{5}{3}\left(Ba^4b + 2\,Aa^3b^2\right)\log\left(x^3\right) - \frac{Aa^5 + 2\,(Ba^5 + 5\,Aa^4b)x^3}{6\,x^6}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="maxima")
```

output
```
1/12*B*b^5*x^12 + 1/9*(5*B*a*b^4 + A*b^5)*x^9 + 5/6*(2*B*a^2*b^3 + A*a*b^4
)*x^6 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*log
(x^3) - 1/6*(A*a^5 + 2*(B*a^5 + 5*A*a^4*b)*x^3)/x^6
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^7}\,dx = \frac{1}{12}\,Bb^5x^{12} + \frac{5}{9}\,Bab^4x^9 + \frac{1}{9}\,Ab^5x^9 + \frac{5}{3}\,Ba^2b^3x^6 + \frac{5}{6}\,Aab^4x^6 + \frac{10}{3}\,Ba^3b^2x^3 + \frac{10}{3}\,Aa^2b^3x^3 + 5\left(Ba^4b + 2\,Aa^3b^2\right)\log\left(|x|\right) - \frac{15\,Ba^4bx^6 + 30\,Aa^3b^2x^6 + 2\,Ba^5x^3 + 10\,Aa^4bx^3 + Aa^5}{6\,x^6}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="giac")
```

output
```
1/12*B*b^5*x^12 + 5/9*B*a*b^4*x^9 + 1/9*A*b^5*x^9 + 5/3*B*a^2*b^3*x^6 + 5/
6*A*a*b^4*x^6 + 10/3*B*a^3*b^2*x^3 + 10/3*A*a^2*b^3*x^3 + 5*(B*a^4*b + 2*A
*a^3*b^2)*log(abs(x)) - 1/6*(15*B*a^4*b*x^6 + 30*A*a^3*b^2*x^6 + 2*B*a^5*x
^3 + 10*A*a^4*b*x^3 + A*a^5)/x^6
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^7}\,dx = \ln(x)\,\left(5\,B\,a^4\,b+10\,A\,a^3\,b^2\right)-\frac{\frac{A\,a^5}{6}+x^3\,\left(\frac{B\,a^5}{3}+\frac{5\,A\,b\,a^4}{3}\right)}{x^6}+x^9\,\left(\frac{A\,b^5}{9}+\frac{5\,B\,a\,b^4}{9}\right)+\frac{B\,b^5\,x^{12}}{12}+\frac{10\,a^2\,b^2\,x^3\,(A\,b+B\,a)}{3}+\frac{5\,a\,b^3\,x^6\,(A\,b+2\,B\,a)}{6}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^7,x)
```

output
```
log(x)*(10*A*a^3*b^2 + 5*B*a^4*b) - ((A*a^5)/6 + x^3*((B*a^5)/3 + (5*A*a^4
*b)/3))/x^6 + x^9*((A*b^5)/9 + (5*B*a*b^4)/9) + (B*b^5*x^12)/12 + (10*a^2*
b^2*x^3*(A*b + B*a))/3 + (5*a*b^3*x^6*(A*b + 2*B*a))/6
```

3.40 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^8}\,dx$

3.40.1 Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^8}\,dx = -\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{x}+5a^2b^2(Ab+aB)x^2+ab^3(Ab+2aB)x^5+\frac{1}{8}b^4(Ab+5aB)x^8+\frac{1}{11}b^5Bx^{11}$$

output
```
-1/7*a^5*A/x^7-1/4*a^4*(5*A*b+B*a)/x^4-5*a^3*b*(2*A*b+B*a)/x+5*a^2*b^2*(A*
b+B*a)*x^2+a*b^3*(A*b+2*B*a)*x^5+1/8*b^4*(A*b+5*B*a)*x^8+1/11*b^5*B*x^11
```

3.40.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^8}\,dx = -\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{x}+5a^2b^2(Ab+aB)x^2+ab^3(Ab+2aB)x^5+\frac{1}{8}b^4(Ab+5aB)x^8+\frac{1}{11}b^5Bx^{11}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^8,x]
```

output
```
-1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x
 + 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B
)*x^8)/8 + (b^5*B*x^11)/11
```

3.40.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} \, dx$$

$$\Big\downarrow \; 950$$

$$\int \left(\frac{a^5 A}{x^8}+\frac{a^4(aB+5Ab)}{x^5}+\frac{5a^3b(aB+2Ab)}{x^2}+10a^2b^2x(aB+Ab)+b^4x^7(5aB+Ab)+5ab^3x^4(2aB+Ab)+b^5$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^5 A}{7x^7}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{x}+5a^2b^2x^2(aB+Ab)+\frac{1}{8}b^4x^8(5aB+Ab)+ ab^3x^5(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^8,x]
```

output
```
-1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x
 + 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B
)*x^8)/8 + (b^5*B*x^11)/11
```

3.40.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.40.4 Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06

method	result
default	$\frac{b^5 B x^{11}}{11}+\frac{A b^5 x^8}{8}+\frac{5 B a b^4 x^8}{8}+A a b^4 x^5+2 B a^2 b^3 x^5+5 A a^2 b^3 x^2+5 B a^3 b^2 x^2-\frac{a^5 A}{7 x^7}-\frac{5 a^3 b(2 A b+}{x}$
norman	$\frac{-\frac{a^5 A}{7}+(-\frac{5}{4} a^4 b A-\frac{1}{4} a^5 B) x^3+(-10 a^3 b^2 A-5 a^4 b B) x^6+(5 a^2 b^3 A+5 a^3 b^2 B) x^9+(a b^4 A+2 a^2 b^3 B) x^{12}+(\frac{1}{8} b^5 A+\frac{5}{8} a b^4 B) x^{15}+}{x^7}$
risch	$\frac{b^5 B x^{11}}{11}+\frac{A b^5 x^8}{8}+\frac{5 B a b^4 x^8}{8}+A a b^4 x^5+2 B a^2 b^3 x^5+5 A a^2 b^3 x^2+5 B a^3 b^2 x^2+\frac{(-10 a^3 b^2 A-5 a^4 b B}{}$
gosper	$-\frac{-56 b^5 B x^{18}-77 A b^5 x^{15}-385 B a b^4 x^{15}-616 a A b^4 x^{12}-1232 B a^2 b^3 x^{12}-3080 a^2 A b^3 x^9-3080 B a^3 b^2 x^9+6160 a^3 A b^2 x^6+3080}{616 x^7}$
parallelrisch	$\frac{56 b^5 B x^{18}+77 A b^5 x^{15}+385 B a b^4 x^{15}+616 a A b^4 x^{12}+1232 B a^2 b^3 x^{12}+3080 a^2 A b^3 x^9+3080 B a^3 b^2 x^9-6160 a^3 A b^2 x^6-3080 B a}{616 x^7}$

input

```
int((b*x^3+a)^5*(B*x^3+A)/x^8,x,method=_RETURNVERBOSE)
```

output

```
1/11*b^5*B*x^11+1/8*A*b^5*x^8+5/8*B*a*b^4*x^8+A*a*b^4*x^5+2*B*a^2*b^3*x^5+
5*A*a^2*b^3*x^2+5*B*a^3*b^2*x^2-1/7*a^5*A/x^7-5*a^3*b*(2*A*b+B*a)/x-1/4*a^
4*(5*A*b+B*a)/x^4
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^8}\,dx$$
$$=\frac{56\,Bb^5x^{18}+77\,(5\,Bab^4+Ab^5)x^{15}+616\,(2\,Ba^2b^3+Aab^4)x^{12}+3080\,(Ba^3b^2+Aa^2b^3)x^9-3080\,(Ba^4b+}{616\,x^7}$$

input

```
integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="fricas")
```

output

```
1/616*(56*B*b^5*x^18 + 77*(5*B*a*b^4 + A*b^5)*x^15 + 616*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 3080*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 88*A*a^5 - 154*(B*a^5 + 5*A*a^4*b)*x^3)/x^7
```

3.40.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8}\,dx = \frac{Bb^5x^{11}}{11} + x^8\left(\frac{Ab^5}{8}+\frac{5Bab^4}{8}\right) + x^5\left(Aab^4+2Ba^2b^3\right) + x^2\cdot\left(5Aa^2b^3+5Ba^3b^2\right) + \frac{-4Aa^5+x^6(-280Aa^3b^2-140Ba^4b)+x^3(-35Aa^4b-7Ba^5)}{28x^7}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**8,x)
```

output
```
B*b**5*x**11/11 + x**8*(A*b**5/8 + 5*B*a*b**4/8) + x**5*(A*a*b**4 + 2*B*a*
*2*b**3) + x**2*(5*A*a**2*b**3 + 5*B*a**3*b**2) + (-4*A*a**5 + x**6*(-280*
A*a**3*b**2 - 140*B*a**4*b) + x**3*(-35*A*a**4*b - 7*B*a**5))/(28*x**7)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8}\,dx = \frac{1}{11}\,Bb^5x^{11} + \frac{1}{8}\left(5\,Bab^4+Ab^5\right)x^8 + \left(2\,Ba^2b^3+Aab^4\right)x^5 + 5\left(Ba^3b^2+Aa^2b^3\right)x^2 - \frac{140\,(Ba^4b+2\,Aa^3b^2)x^6+4\,Aa^5+7\,(Ba^5+5\,Aa^4b)x^3}{28\,x^7}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="maxima")
```

output
```
1/11*B*b^5*x^11 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + (2*B*a^2*b^3 + A*a*b^4)*x^
5 + 5*(B*a^3*b^2 + A*a^2*b^3)*x^2 - 1/28*(140*(B*a^4*b + 2*A*a^3*b^2)*x^6
+ 4*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^7
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8}\,dx = \frac{1}{11}Bb^5x^{11} + \frac{5}{8}Bab^4x^8 + \frac{1}{8}Ab^5x^8 + 2\,Ba^2b^3x^5 + Aab^4x^5 + 5\,Ba^3b^2x^2 + 5\,Aa^2b^3x^2 - \frac{140\,Ba^4bx^6 + 280\,Aa^3b^2x^6 + 7\,Ba^5x^3 + 35\,Aa^4bx^3 + 4\,Aa^5}{28\,x^7}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="giac")
```

output
```
1/11*B*b^5*x^11 + 5/8*B*a*b^4*x^8 + 1/8*A*b^5*x^8 + 2*B*a^2*b^3*x^5 + A*a*
b^4*x^5 + 5*B*a^3*b^2*x^2 + 5*A*a^2*b^3*x^2 - 1/28*(140*B*a^4*b*x^6 + 280*
A*a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 4*A*a^5)/x^7
```

3.40.9 Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8}\,dx = x^8\left(\frac{A\,b^5}{8} + \frac{5\,B\,a\,b^4}{8}\right) - \frac{\frac{A\,a^5}{7} + x^6\left(5\,B\,a^4\,b + 10\,A\,a^3\,b^2\right) + x^3\left(\frac{B\,a^5}{4} + \frac{5\,A\,b\,a^4}{4}\right)}{x^7} + \frac{B\,b^5\,x^{11}}{11} + 5\,a^2\,b^2\,x^2\,(A\,b + B\,a) + a\,b^3\,x^5\,(A\,b + 2\,B\,a)$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^8,x)
```

output
```
x^8*((A*b^5)/8 + (5*B*a*b^4)/8) - ((A*a^5)/7 + x^6*(10*A*a^3*b^2 + 5*B*a^4
*b) + x^3*((B*a^5)/4 + (5*A*a^4*b)/4))/x^7 + (B*b^5*x^11)/11 + 5*a^2*b^2*x
^2*(A*b + B*a) + a*b^3*x^5*(A*b + 2*B*a)
```

3.41 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^9}\,dx$

3.41.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^9}\,dx = -\frac{a^5A}{8x^8}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{2x^2}+10a^2b^2(Ab+aB)x+\frac{5}{4}ab^3(Ab+2aB)x^4+\frac{1}{7}b^4(Ab+5aB)x^7+\frac{1}{10}b^5Bx^{10}$$

output

```
-1/8*a^5*A/x^8-1/5*a^4*(5*A*b+B*a)/x^5-5/2*a^3*b*(2*A*b+B*a)/x^2+10*a^2*b^
2*(A*b+B*a)*x+5/4*a*b^3*(A*b+2*B*a)*x^4+1/7*b^4*(A*b+5*B*a)*x^7+1/10*b^5*B
*x^10
```

3.41.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^9}\,dx = -\frac{a^5A}{8x^8}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{2x^2}+10a^2b^2(Ab+aB)x+\frac{5}{4}ab^3(Ab+2aB)x^4+\frac{1}{7}b^4(Ab+5aB)x^7+\frac{1}{10}b^5Bx^{10}$$

input

```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^9,x]
```

output

```
-1/8*(a^5*A)/x^8 - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(
2*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(
A*b + 5*a*B)*x^7)/7 + (b^5*B*x^10)/10
```

3.41.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^9}\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5A}{x^9}+\frac{a^4(aB+5Ab)}{x^6}+\frac{5a^3b(aB+2Ab)}{x^3}+10a^2b^2(aB+Ab)+b^4x^6(5aB+Ab)+5ab^3x^3(2aB+Ab)+b^5B$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{8x^8}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{2x^2}+10a^2b^2x(aB+Ab)+\frac{1}{7}b^4x^7(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

input

```
Int[((a + b*x^3)^5*(A + B*x^3))/x^9,x]
```

output

```
-1/8*(a^5*A)/x^8 - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(
2*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(
A*b + 5*a*B)*x^7)/7 + (b^5*B*x^10)/10
```

3.41.3.1 Defintions of rubi rules used

rule 950

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.41.4 Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

method	result
default	$\frac{b^5 B x^{10}}{10}+\frac{A b^5 x^7}{7}+\frac{5 B a b^4 x^7}{7}+\frac{5 a A b^4 x^4}{4}+\frac{5 B a^2 b^3 x^4}{2}+10 A a^2 b^3 x+10 B a^3 b^2 x-\frac{a^5 A}{8 x^8}-\frac{5 a^3 b (2 A b+B}{2 x^2}$
norman	$\frac{-\frac{a^5 A}{8}+\left(-a^4 b A-\frac{1}{5} a^5 B\right) x^3+\left(-5 a^3 b^2 A-\frac{5}{2} a^4 b B\right) x^6+\left(10 a^2 b^3 A+10 a^3 b^2 B\right) x^9+\left(\frac{5}{4} a b^4 A+\frac{5}{2} a^2 b^3 B\right) x^{12}+\left(\frac{1}{7} b^5 A+\frac{5}{7} a b^4 B\right) x^{15}}{x^8}$
risch	$\frac{b^5 B x^{10}}{10}+\frac{A b^5 x^7}{7}+\frac{5 B a b^4 x^7}{7}+\frac{5 a A b^4 x^4}{4}+\frac{5 B a^2 b^3 x^4}{2}+10 A a^2 b^3 x+10 B a^3 b^2 x+\frac{\left(-5 a^3 b^2 A-\frac{5}{2} a^4 b B\right) x^6}{}$
gosper	$-\frac{-28 b^5 B x^{18}-40 A b^5 x^{15}-200 B a b^4 x^{15}-350 a A b^4 x^{12}-700 B a^2 b^3 x^{12}-2800 a^2 A b^3 x^9-2800 B a^3 b^2 x^9+1400 a^3 A b^2 x^6+700 B}{280 x^8}$
parallelrisch	$\frac{28 b^5 B x^{18}+40 A b^5 x^{15}+200 B a b^4 x^{15}+350 a A b^4 x^{12}+700 B a^2 b^3 x^{12}+2800 a^2 A b^3 x^9+2800 B a^3 b^2 x^9-1400 a^3 A b^2 x^6-700 B a^4 b}{280 x^8}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/10*b^5*B*x^10+1/7*A*b^5*x^7+5/7*B*a*b^4*x^7+5/4*a*A*b^4*x^4+5/2*B*a^2*b^
3*x^4+10*A*a^2*b^3*x+10*B*a^3*b^2*x-1/8*a^5*A/x^8-5/2*a^3*b*(2*A*b+B*a)/x^
2-1/5*a^4*(5*A*b+B*a)/x^5
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+b x^3\right)^5\left(A+B x^3\right)}{x^9} dx$$
$$=\frac{28 B b^5 x^{18}+40\left(5 B a b^4+A b^5\right) x^{15}+350\left(2 B a^2 b^3+A a b^4\right) x^{12}+2800\left(B a^3 b^2+A a^2 b^3\right) x^9-700\left(B a^4 b+2\right.}{280 x^8}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="fricas")
```

output
```
1/280*(28*B*b^5*x^18 + 40*(5*B*a*b^4 + A*b^5)*x^15 + 350*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 2800*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 700*(B*a^4*b + 2*A*a^3*b^
2)*x^6 - 35*A*a^5 - 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^8
```

3.41.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^9}\,dx = \frac{Bb^5x^{10}}{10} + x^7\left(\frac{Ab^5}{7} + \frac{5Bab^4}{7}\right) + x^4\cdot\left(\frac{5Aab^4}{4} + \frac{5Ba^2b^3}{2}\right) + x\left(10Aa^2b^3 + 10Ba^3b^2\right) + \frac{-5Aa^5 + x^6(-200Aa^3b^2 - 100Ba^4b) + x^3(-40Aa^4b - 8Ba^5)}{40x^8}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**9,x)
```

output
```
B*b**5*x**10/10 + x**7*(A*b**5/7 + 5*B*a*b**4/7) + x**4*(5*A*a*b**4/4 + 5*
B*a**2*b**3/2) + x*(10*A*a**2*b**3 + 10*B*a**3*b**2) + (-5*A*a**5 + x**6*(
-200*A*a**3*b**2 - 100*B*a**4*b) + x**3*(-40*A*a**4*b - 8*B*a**5))/(40*x**
8)
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^9}\,dx = \frac{1}{10}\,Bb^5x^{10} + \frac{1}{7}\left(5\,Bab^4 + Ab^5\right)x^7 + \frac{5}{4}\left(2\,Ba^2b^3 + Aab^4\right)x^4 + 10\left(Ba^3b^2 + Aa^2b^3\right)x - \frac{100\,(Ba^4b + 2\,Aa^3b^2)x^6 + 5\,Aa^5 + 8\,(Ba^5 + 5\,Aa^4b)x^3}{40\,x^8}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="maxima")
```

output
```
1/10*B*b^5*x^10 + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + 5/4*(2*B*a^2*b^3 + A*a*b^4
)*x^4 + 10*(B*a^3*b^2 + A*a^2*b^3)*x - 1/40*(100*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 5*A*a^5 + 8*(B*a^5 + 5*A*a^4*b)*x^3)/x^8
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^9}\,dx = \frac{1}{10}\,Bb^5x^{10}+\frac{5}{7}\,Bab^4x^7+\frac{1}{7}\,Ab^5x^7+\frac{5}{2}\,Ba^2b^3x^4+\frac{5}{4}\,Aab^4x^4+10\,Ba^3b^2x+10\,Aa^2b^3x-\frac{100\,Ba^4bx^6+200\,Aa^3b^2x^6+8\,Ba^5x^3+40\,Aa^4bx^3+5\,Aa^5}{40\,x^8}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="giac")
```

output
```
1/10*B*b^5*x^10 + 5/7*B*a*b^4*x^7 + 1/7*A*b^5*x^7 + 5/2*B*a^2*b^3*x^4 + 5/
4*A*a*b^4*x^4 + 10*B*a^3*b^2*x + 10*A*a^2*b^3*x - 1/40*(100*B*a^4*b*x^6 +
200*A*a^3*b^2*x^6 + 8*B*a^5*x^3 + 40*A*a^4*b*x^3 + 5*A*a^5)/x^8
```

3.41.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^9}\,dx = x^7\left(\frac{A\,b^5}{7}+\frac{5\,B\,a\,b^4}{7}\right)-\frac{\frac{A\,a^5}{8}+x^6\left(\frac{5\,B\,a^4\,b}{2}+5\,A\,a^3\,b^2\right)+x^3\left(\frac{B\,a^5}{5}+A\,b\,a^4\right)}{x^8}+\frac{B\,b^5\,x^{10}}{10}+10\,a^2\,b^2\,x\,(A\,b+B\,a)+\frac{5\,a\,b^3\,x^4\,(A\,b+2\,B\,a)}{4}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^9,x)
```

output
```
x^7*((A*b^5)/7 + (5*B*a*b^4)/7) - ((A*a^5)/8 + x^6*(5*A*a^3*b^2 + (5*B*a^4
*b)/2) + x^3*((B*a^5)/5 + A*a^4*b))/x^8 + (B*b^5*x^10)/10 + 10*a^2*b^2*x*(
A*b + B*a) + (5*a*b^3*x^4*(A*b + 2*B*a))/4
```

3.42 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{10}}\,dx$

3.42.1 Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{10}}\,dx = -\frac{a^5A}{9x^9}-\frac{a^4(5Ab+aB)}{6x^6}-\frac{5a^3b(2Ab+aB)}{3x^3}+\frac{5}{3}ab^3(Ab+2aB)x^3 + \frac{1}{6}b^4(Ab+5aB)x^6+\frac{1}{9}b^5Bx^9+10a^2b^2(Ab+aB)\log(x)$$

output
```
-1/9*a^5*A/x^9-1/6*a^4*(5*A*b+B*a)/x^6-5/3*a^3*b*(2*A*b+B*a)/x^3+5/3*a*b^3
*(A*b+2*B*a)*x^3+1/6*b^4*(A*b+5*B*a)*x^6+1/9*b^5*B*x^9+10*a^2*b^2*(A*b+B*a
)*ln(x)
```

3.42.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{10}}\,dx = \frac{1}{18}\left(-\frac{2a^5A}{x^9}-\frac{3a^4(5Ab+aB)}{x^6}-\frac{30a^3b(2Ab+aB)}{x^3} + 30ab^3(Ab+2aB)x^3+3b^4(Ab+5aB)x^6+2b^5Bx^9 + 180a^2b^2(Ab+aB)\log(x)\right)$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^10,x]
```

output
```
((-2*a^5*A)/x^9 - (3*a^4*(5*A*b + a*B))/x^6 - (30*a^3*b*(2*A*b + a*B))/x^3
 + 30*a*b^3*(A*b + 2*a*B)*x^3 + 3*b^4*(A*b + 5*a*B)*x^6 + 2*b^5*B*x^9 + 18
0*a^2*b^2*(A*b + a*B)*Log[x])/18
```

3.42.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{10}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)^5\left(Bx^3+A\right)}{x^{12}}dx^3$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{3}\int \Bigg(b^5Bx^6+b^4(Ab+5aB)x^3+5ab^3(Ab+2aB)+\frac{10a^2b^2(Ab+aB)}{x^3}+\frac{5a^3b(2Ab+aB)}{x^6}+\frac{a^4(5Ab+aB)}{x^9}+$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\Bigg(-\frac{a^5A}{3x^9}-\frac{a^4(aB+5Ab)}{2x^6}-\frac{5a^3b(aB+2Ab)}{x^3}+10a^2b^2\log\left(x^3\right)(aB+Ab)+\frac{1}{2}b^4x^6(5aB+Ab)+5ab^3x^3(2aB+$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^10,x]
```

output
```
(-1/3*(a^5*A)/x^9 - (a^4*(5*A*b + a*B))/(2*x^6) - (5*a^3*b*(2*A*b + a*B))/
x^3 + 5*a*b^3*(A*b + 2*a*B)*x^3 + (b^4*(A*b + 5*a*B)*x^6)/2 + (b^5*B*x^9)/
3 + 10*a^2*b^2*(A*b + a*B)*Log[x^3])/3
```

3.42.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.42.4 Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^5 B x^9}{9}+\frac{A b^5 x^6}{6}+\frac{5 B a b^4 x^6}{6}+\frac{5 A a b^4 x^3}{3}+\frac{10 B a^2 b^3 x^3}{3}+10 a^2 b^2 (A b+B a) \ln (x)-\frac{a^4 (5 A b+B a)}{6 x^6}-$
norman	$\frac{\left(\frac{1}{6} b^5 A+\frac{5}{6} a b^4 B\right) x^{15}+\left(\frac{5}{3} a b^4 A+\frac{10}{3} a^2 b^3 B\right) x^{12}+\left(-\frac{10}{3} a^3 b^2 A-\frac{5}{3} a^4 b B\right) x^6+\left(-\frac{5}{6} a^4 b A-\frac{1}{6} a^5 B\right) x^3-\frac{a^5 A}{9}+\frac{b^5 B x^{18}}{9}}{x^9}+$
risch	$\frac{b^5 B x^9}{9}+\frac{A b^5 x^6}{6}+\frac{5 B a b^4 x^6}{6}+\frac{5 A a b^4 x^3}{3}+\frac{10 B a^2 b^3 x^3}{3}+\frac{\left(-\frac{10}{3} a^3 b^2 A-\frac{5}{3} a^4 b B\right) x^6+\left(-\frac{5}{6} a^4 b A-\frac{1}{6} a^5 B\right) x^3-\frac{a^5 A}{9}}{x^9}+$
parallelrisch	$\frac{2 b^5 B x^{18}+3 A b^5 x^{15}+15 B a b^4 x^{15}+30 a A b^4 x^{12}+60 B a^2 b^3 x^{12}+180 A \ln (x) x^9 a^2 b^3+180 B \ln (x) x^9 a^3 b^2-60 a^3 A b^2 x^6-30 B a^4 b}{18 x^9}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/9*b^5*B*x^9+1/6*A*b^5*x^6+5/6*B*a*b^4*x^6+5/3*A*a*b^4*x^3+10/3*B*a^2*b^3
*x^3+10*a^2*b^2*(A*b+B*a)*ln(x)-1/6*a^4*(5*A*b+B*a)/x^6-5/3*a^3*b*(2*A*b+B
*a)/x^3-1/9*a^5*A/x^9
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}}\,dx$$
$$= \frac{2\,Bb^5x^{18} + 3\,(5\,Bab^4 + Ab^5)x^{15} + 30\,(2\,Ba^2b^3 + Aab^4)x^{12} + 180\,(Ba^3b^2 + Aa^2b^3)x^9\log(x) - 30\,(Ba^4b +}{18\,x^9}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="fricas")
```

output
```
1/18*(2*B*b^5*x^18 + 3*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^
4)*x^12 + 180*(B*a^3*b^2 + A*a^2*b^3)*x^9*log(x) - 30*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 2*A*a^5 - 3*(B*a^5 + 5*A*a^4*b)*x^3)/x^9
```

3.42.6 Sympy [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}}\,dx = {} & \frac{Bb^5x^9}{9} + 10a^2b^2(Ab+Ba)\log(x) \\ & + x^6\left(\frac{Ab^5}{6} + \frac{5Bab^4}{6}\right) + x^3\cdot\left(\frac{5Aab^4}{3} + \frac{10Ba^2b^3}{3}\right) \\ & + \frac{-2Aa^5 + x^6(-60Aa^3b^2 - 30Ba^4b) + x^3(-15Aa^4b - 3Ba^5)}{18x^9}\end{aligned}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**10,x)
```

output
```
B*b**5*x**9/9 + 10*a**2*b**2*(A*b + B*a)*log(x) + x**6*(A*b**5/6 + 5*B*a*b
**4/6) + x**3*(5*A*a*b**4/3 + 10*B*a**2*b**3/3) + (-2*A*a**5 + x**6*(-60*A
*a**3*b**2 - 30*B*a**4*b) + x**3*(-15*A*a**4*b - 3*B*a**5))/(18*x**9)
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}}\,dx = \frac{1}{9}\,Bb^5x^9 + \frac{1}{6}\left(5\,Bab^4 + Ab^5\right)x^6 + \frac{5}{3}\left(2\,Ba^2b^3 + Aab^4\right)x^3 + \frac{10}{3}\left(Ba^3b^2 + Aa^2b^3\right)\log\left(x^3\right) - \frac{30\,(Ba^4b + 2\,Aa^3b^2)x^6 + 2\,Aa^5 + 3\,(Ba^5 + 5\,Aa^4b)x^3}{18\,x^9}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="maxima")
```

output
```
1/9*B*b^5*x^9 + 1/6*(5*B*a*b^4 + A*b^5)*x^6 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*
x^3 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*log(x^3) - 1/18*(30*(B*a^4*b + 2*A*a^3*
b^2)*x^6 + 2*A*a^5 + 3*(B*a^5 + 5*A*a^4*b)*x^3)/x^9
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}}\,dx = \frac{1}{9}\,Bb^5x^9 + \frac{5}{6}\,Bab^4x^6 + \frac{1}{6}\,Ab^5x^6 + \frac{10}{3}\,Ba^2b^3x^3 + \frac{5}{3}\,Aab^4x^3 + 10\left(Ba^3b^2 + Aa^2b^3\right)\log\left(|x|\right) - \frac{110\,Ba^3b^2x^9 + 110\,Aa^2b^3x^9 + 30\,Ba^4bx^6 + 60\,Aa^3b^2x^6 + 3\,Ba^5x^3 + 15\,Aa^4bx^3 + 2\,Aa^5}{18\,x^9}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="giac")
```

output
```
1/9*B*b^5*x^9 + 5/6*B*a*b^4*x^6 + 1/6*A*b^5*x^6 + 10/3*B*a^2*b^3*x^3 + 5/3
*A*a*b^4*x^3 + 10*(B*a^3*b^2 + A*a^2*b^3)*log(abs(x)) - 1/18*(110*B*a^3*b^
2*x^9 + 110*A*a^2*b^3*x^9 + 30*B*a^4*b*x^6 + 60*A*a^3*b^2*x^6 + 3*B*a^5*x^
3 + 15*A*a^4*b*x^3 + 2*A*a^5)/x^9
```

3.42.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}}\,dx = x^6 \left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right) - \frac{\frac{A\,a^5}{9}+x^6\left(\frac{5\,B\,a^4\,b}{3}+\frac{10\,A\,a^3\,b^2}{3}\right)+x^3\left(\frac{B\,a^5}{6}+\frac{5\,A\,b\,a^4}{6}\right)}{x^9} + \ln(x)\,\left(10\,B\,a^3\,b^2+10\,A\,a^2\,b^3\right) + \frac{B\,b^5\,x^9}{9}+\frac{5\,a\,b^3\,x^3\,(A\,b+2\,B\,a)}{3}$$

input

```
int(((A + B*x^3)*(a + b*x^3)^5)/x^10,x)
```

output

```
x^6*((A*b^5)/6 + (5*B*a*b^4)/6) - ((A*a^5)/9 + x^6*((10*A*a^3*b^2)/3 + (5*
B*a^4*b)/3) + x^3*((B*a^5)/6 + (5*A*a^4*b)/6))/x^9 + log(x)*(10*A*a^2*b^3
+ 10*B*a^3*b^2) + (B*b^5*x^9)/9 + (5*a*b^3*x^3*(A*b + 2*B*a))/3
```

3.43 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx$

3.43.1 Optimal result

Integrand size = 20, antiderivative size = 115

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx = &-\frac{a^5A}{10x^{10}}-\frac{a^4(5Ab+aB)}{7x^7}-\frac{5a^3b(2Ab+aB)}{4x^4}\\ &-\frac{10a^2b^2(Ab+aB)}{x}+\frac{5}{2}ab^3(Ab+2aB)x^2\\ &+\frac{1}{5}b^4(Ab+5aB)x^5+\frac{1}{8}b^5Bx^8\end{aligned}$$

output
```
-1/10*a^5*A/x^10-1/7*a^4*(5*A*b+B*a)/x^7-5/4*a^3*b*(2*A*b+B*a)/x^4-10*a^2*
b^2*(A*b+B*a)/x+5/2*a*b^3*(A*b+2*B*a)*x^2+1/5*b^4*(A*b+5*B*a)*x^5+1/8*b^5*
B*x^8
```

3.43.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx$$
$$=\frac{1400a^2b^3x^9(-2A+Bx^3)+140ab^4x^{12}(5A+2Bx^3)-700a^3b^2x^6(A+4Bx^3)+7b^5x^{15}(8A+5Bx^3)-50a^4}{280x^{10}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^11,x]
```

output
```
(1400*a^2*b^3*x^9*(-2*A + B*x^3) + 140*a*b^4*x^12*(5*A + 2*B*x^3) - 700*a^
3*b^2*x^6*(A + 4*B*x^3) + 7*b^5*x^15*(8*A + 5*B*x^3) - 50*a^4*b*x^3*(4*A +
 7*B*x^3) - 4*a^5*(7*A + 10*B*x^3))/(280*x^10)
```

3.43.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^{11}} \, dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(\frac{a^5 A}{x^{11}}+\frac{a^4(aB+5Ab)}{x^8}+\frac{5a^3b(aB+2Ab)}{x^5}+\frac{10a^2b^2(aB+Ab)}{x^2}+b^4x^4(5aB+Ab)+5ab^3x(2aB+Ab)+b^5B$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5A}{10x^{10}}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{10a^2b^2(aB+Ab)}{x}+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^11,x]
```

output
```
-1/10*(a^5*A)/x^10 - (a^4*(5*A*b + a*B))/(7*x^7) - (5*a^3*b*(2*A*b + a*B))
/(4*x^4) - (10*a^2*b^2*(A*b + a*B))/x + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b
^4*(A*b + 5*a*B)*x^5)/5 + (b^5*B*x^8)/8
```

3.43.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.43.4 Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^5 B x^8}{8}+\frac{A b^5 x^5}{5}+B a b^4 x^5+\frac{5 A a b^4 x^2}{2}+5 B a^2 b^3 x^2-\frac{a^4 (5 A b+B a)}{7 x^7}-\frac{a^5 A}{10 x^{10}}-\frac{10 a^2 b^2 (A b+B a)}{x}-$
norman	$\frac{-\frac{a^5 A}{10}+\left(-\frac{5}{7} a^4 b A-\frac{1}{7} a^5 B\right) x^3+\left(-\frac{5}{2} a^3 b^2 A-\frac{5}{4} a^4 b B\right) x^6+\left(-10 a^2 b^3 A-10 a^3 b^2 B\right) x^9+\left(\frac{5}{2} a b^4 A+5 a^2 b^3 B\right) x^{12}+\left(\frac{1}{5} b^5 A+a b^4 B\right)}{x^{10}}$
risch	$\frac{b^5 B x^8}{8}+\frac{A b^5 x^5}{5}+B a b^4 x^5+\frac{5 A a b^4 x^2}{2}+5 B a^2 b^3 x^2+\frac{\left(-10 a^2 b^3 A-10 a^3 b^2 B\right) x^9+\left(-\frac{5}{2} a^3 b^2 A-\frac{5}{4} a^4 b B\right) x^6+}{x^{10}}$
gosper	$-\frac{-35 b^5 B x^{18}-56 A b^5 x^{15}-280 B a b^4 x^{15}-700 a A b^4 x^{12}-1400 B a^2 b^3 x^{12}+2800 a^2 A b^3 x^9+2800 B a^3 b^2 x^9+700 a^3 A b^2 x^6+350}{280 x^{10}}$
parallelrisch	$\frac{35 b^5 B x^{18}+56 A b^5 x^{15}+280 B a b^4 x^{15}+700 a A b^4 x^{12}+1400 B a^2 b^3 x^{12}-2800 a^2 A b^3 x^9-2800 B a^3 b^2 x^9-700 a^3 A b^2 x^6-350 B a^4}{280 x^{10}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^11,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^5*B*x^8+1/5*A*b^5*x^5+B*a*b^4*x^5+5/2*A*a*b^4*x^2+5*B*a^2*b^3*x^2-1/
7*a^4*(5*A*b+B*a)/x^7-1/10*a^5*A/x^10-10*a^2*b^2*(A*b+B*a)/x-5/4*a^3*b*(2*
A*b+B*a)/x^4
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b x^3\right)^5 \left(A+B x^3\right)}{x^{11}}\,dx$$
$$=\frac{35\,B b^5 x^{18}+56\,(5\,B a b^4+A b^5) x^{15}+700\,(2\,B a^2 b^3+A a b^4) x^{12}-2800\,(B a^3 b^2+A a^2 b^3) x^9-350\,(B a^4 b+2}{280\,x^{10}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="fricas")
```

output
```
1/280*(35*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 700*(2*B*a^2*b^3 + A*
a*b^4)*x^12 - 2800*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 350*(B*a^4*b + 2*A*a^3*b^
2)*x^6 - 28*A*a^5 - 40*(B*a^5 + 5*A*a^4*b)*x^3)/x^10
```

3.43.6 Sympy [A] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}} \, dx = \frac{Bb^5x^8}{8} + x^5\left(\frac{Ab^5}{5} + Bab^4\right) + x^2 \cdot \left(\frac{5Aab^4}{2} + 5Ba^2b^3\right) + \frac{-14Aa^5 + x^9(-1400Aa^2b^3 - 1400Ba^3b^2) + x^6(-350Aa^3b^2 - 175Ba^4b) + x^3(-100Aa^4b - 20Ba^5)}{140x^{10}}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**11,x)
```

output
```
B*b**5*x**8/8 + x**5*(A*b**5/5 + B*a*b**4) + x**2*(5*A*a*b**4/2 + 5*B*a**2
*b**3) + (-14*A*a**5 + x**9*(-1400*A*a**2*b**3 - 1400*B*a**3*b**2) + x**6*
(-350*A*a**3*b**2 - 175*B*a**4*b) + x**3*(-100*A*a**4*b - 20*B*a**5))/(140
*x**10)
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}} \, dx = \frac{1}{8}\,Bb^5x^8 + \frac{1}{5}\left(5\,Bab^4 + Ab^5\right)x^5 + \frac{5}{2}\left(2\,Ba^2b^3 + Aab^4\right)x^2 - \frac{1400\,(Ba^3b^2 + Aa^2b^3)x^9 + 175\,(Ba^4b + 2\,Aa^3b^2)x^6 + 14\,Aa^5 + 20\,(Ba^5 + 5\,Aa^4b)x^3}{140\,x^{10}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="maxima")
```

output
```
1/8*B*b^5*x^8 + 1/5*(5*B*a*b^4 + A*b^5)*x^5 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*
x^2 - 1/140*(1400*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 175*(B*a^4*b + 2*A*a^3*b^2
)*x^6 + 14*A*a^5 + 20*(B*a^5 + 5*A*a^4*b)*x^3)/x^10
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx = \frac{1}{8}\,Bb^5x^8 + Bab^4x^5 + \frac{1}{5}\,Ab^5x^5 + 5\,Ba^2b^3x^2 + \frac{5}{2}\,Aab^4x^2 - \frac{1400\,Ba^3b^2x^9 + 1400\,Aa^2b^3x^9 + 175\,Ba^4bx^6 + 350\,Aa^3b^2x^6 + 20\,Ba^5x^3 + 100\,Aa^4bx^3 + 14\,Aa^5}{140\,x^{10}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="giac")
```

output
```
1/8*B*b^5*x^8 + B*a*b^4*x^5 + 1/5*A*b^5*x^5 + 5*B*a^2*b^3*x^2 + 5/2*A*a*b^
4*x^2 - 1/140*(1400*B*a^3*b^2*x^9 + 1400*A*a^2*b^3*x^9 + 175*B*a^4*b*x^6 +
 350*A*a^3*b^2*x^6 + 20*B*a^5*x^3 + 100*A*a^4*b*x^3 + 14*A*a^5)/x^10
```

3.43.9 Mupad [B] (verification not implemented)

Time = 6.59 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\begin{aligned}
&\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx \\
&= x^5\left(\frac{A\,b^5}{5} + B\,a\,b^4\right) \\
&\quad - \frac{\frac{A\,a^5}{10} + x^6\left(\frac{5\,B\,a^4\,b}{4} + \frac{5\,A\,a^3\,b^2}{2}\right) + x^3\left(\frac{B\,a^5}{7} + \frac{5\,A\,b\,a^4}{7}\right) + x^9\left(10\,B\,a^3\,b^2 + 10\,A\,a^2\,b^3\right)}{x^{10}} \\
&\quad + \frac{B\,b^5\,x^8}{8} + \frac{5\,a\,b^3\,x^2\,(A\,b + 2\,B\,a)}{2}
\end{aligned}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^11,x)
```

output
```
x^5*((A*b^5)/5 + B*a*b^4) - ((A*a^5)/10 + x^6*((5*A*a^3*b^2)/2 + (5*B*a^4*
b)/4) + x^3*((B*a^5)/7 + (5*A*a^4*b)/7) + x^9*(10*A*a^2*b^3 + 10*B*a^3*b^2
))/x^10 + (B*b^5*x^8)/8 + (5*a*b^3*x^2*(A*b + 2*B*a))/2
```

3.44 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx$

3.44.1 Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx = -\frac{a^5A}{11x^{11}} - \frac{a^4(5Ab+aB)}{8x^8} - \frac{a^3b(2Ab+aB)}{x^5} - \frac{5a^2b^2(Ab+aB)}{x^2} + 5ab^3(Ab+2aB)x + \frac{1}{4}b^4(Ab+5aB)x^4 + \frac{1}{7}b^5Bx^7$$

output
```
-1/11*a^5*A/x^11-1/8*a^4*(5*A*b+B*a)/x^8-a^3*b*(2*A*b+B*a)/x^5-5*a^2*b^2*(
A*b+B*a)/x^2+5*a*b^3*(A*b+2*B*a)*x+1/4*b^4*(A*b+5*B*a)*x^4+1/7*b^5*B*x^7
```

3.44.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx = -\frac{a^5A}{11x^{11}} - \frac{a^4(5Ab+aB)}{8x^8} - \frac{a^3b(2Ab+aB)}{x^5} - \frac{5a^2b^2(Ab+aB)}{x^2} + 5ab^3(Ab+2aB)x + \frac{1}{4}b^4(Ab+5aB)x^4 + \frac{1}{7}b^5Bx^7$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^12,x]
```

output
```
-1/11*(a^5*A)/x^11 - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x
^5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5
*a*B)*x^4)/4 + (b^5*B*x^7)/7
```

3.44.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{12}}\,dx$$

$$\Big\downarrow \ 950$$

$$\int \left(\frac{a^5A}{x^{12}}+\frac{a^4(aB+5Ab)}{x^9}+\frac{5a^3b(aB+2Ab)}{x^6}+\frac{10a^2b^2(aB+Ab)}{x^3}+b^4x^3(5aB+Ab)+5ab^3(2aB+Ab)+b^5Bx^6$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{5a^2b^2(aB+Ab)}{x^2}+\frac{1}{4}b^4x^4(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^12,x]
```

output
```
-1/11*(a^5*A)/x^11 - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x
^5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5
*a*B)*x^4)/4 + (b^5*B*x^7)/7
```

3.44.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.44.4 Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

method	result
default	$\frac{b^5 B x^7}{7}+\frac{A b^5 x^4}{4}+\frac{5 B a b^4 x^4}{4}+5 A a b^4 x+10 B a^2 b^3 x-\frac{a^4 (5 A b+B a)}{8 x^8}-\frac{a^5 A}{11 x^{11}}-\frac{5 a^2 b^2 (A b+B a)}{x^2}-\frac{a^3 b(2A}{x}$
risch	$\frac{b^5 B x^7}{7}+\frac{A b^5 x^4}{4}+\frac{5 B a b^4 x^4}{4}+5 A a b^4 x+10 B a^2 b^3 x+\frac{(-5 a^2 b^3 A-5 a^3 b^2 B) x^9+(-2 a^3 b^2 A-a^4 b B) x^6+(-\frac{5}{8} a^4}{x^{11}}$
norman	$\frac{-\frac{a^5 A}{11}+(-\frac{5}{8} a^4 b A-\frac{1}{8} a^5 B) x^3+(-2 a^3 b^2 A-a^4 b B) x^6+(-5 a^2 b^3 A-5 a^3 b^2 B) x^9+(5 a b^4 A+10 a^2 b^3 B) x^{12}+(\frac{1}{4} b^5 A+\frac{5}{4} a b^4 B) x^{15}+}{x^{11}}$
gosper	$-\frac{-88 b^5 B x^{18}-154 A b^5 x^{15}-770 B a b^4 x^{15}-3080 a A b^4 x^{12}-6160 B a^2 b^3 x^{12}+3080 a^2 A b^3 x^9+3080 B a^3 b^2 x^9+1232 a^3 A b^2 x^6+6}{616 x^{11}}$
parallelrisch	$\frac{88 b^5 B x^{18}+154 A b^5 x^{15}+770 B a b^4 x^{15}+3080 a A b^4 x^{12}+6160 B a^2 b^3 x^{12}-3080 a^2 A b^3 x^9-3080 B a^3 b^2 x^9-1232 a^3 A b^2 x^6-616 B}{616 x^{11}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^12,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^5*B*x^7+1/4*A*b^5*x^4+5/4*B*a*b^4*x^4+5*A*a*b^4*x+10*B*a^2*b^3*x-1/8
*a^4*(5*A*b+B*a)/x^8-1/11*a^5*A/x^11-5*a^2*b^2*(A*b+B*a)/x^2-a^3*b*(2*A*b+
B*a)/x^5
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}} dx$$
$$= \frac{88 B b^5 x^{18} + 154 (5 B a b^4 + A b^5) x^{15} + 3080 (2 B a^2 b^3 + A a b^4) x^{12} - 3080 (B a^3 b^2 + A a^2 b^3) x^9 - 616 (B a^4 b +}{616 x^{11}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="fricas")
```

output
```
1/616*(88*B*b^5*x^18 + 154*(5*B*a*b^4 + A*b^5)*x^15 + 3080*(2*B*a^2*b^3 +
A*a*b^4)*x^12 - 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 616*(B*a^4*b + 2*A*a^3*
b^2)*x^6 - 56*A*a^5 - 77*(B*a^5 + 5*A*a^4*b)*x^3)/x^11
```

3.44.6 Sympy [A] (verification not implemented)

Time = 27.72 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}}\,dx = \frac{Bb^5x^7}{7} + x^4\left(\frac{Ab^5}{4}+\frac{5Bab^4}{4}\right) + x\left(5Aab^4+10Ba^2b^3\right) + \frac{-8Aa^5 + x^9(-440Aa^2b^3-440Ba^3b^2) + x^6(-176Aa^3b^2-88Ba^4b) + x^3(-55Aa^4b-11Ba^5)}{88x^{11}}$$

input

```
integrate((b*x**3+a)**5*(B*x**3+A)/x**12,x)
```

output

```
B*b**5*x**7/7 + x**4*(A*b**5/4 + 5*B*a*b**4/4) + x*(5*A*a*b**4 + 10*B*a**2
*b**3) + (-8*A*a**5 + x**9*(-440*A*a**2*b**3 - 440*B*a**3*b**2) + x**6*(-1
76*A*a**3*b**2 - 88*B*a**4*b) + x**3*(-55*A*a**4*b - 11*B*a**5))/(88*x**11
)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}}\,dx = \frac{1}{7}\,Bb^5x^7 + \frac{1}{4}\,(5\,Bab^4+Ab^5)x^4 + 5\,(2\,Ba^2b^3+Aab^4)x - \frac{440\,(Ba^3b^2+Aa^2b^3)x^9 + 88\,(Ba^4b+2\,Aa^3b^2)x^6 + 8\,Aa^5 + 11\,(Ba^5+5\,Aa^4b)x^3}{88\,x^{11}}$$

input

```
integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="maxima")
```

output

```
1/7*B*b^5*x^7 + 1/4*(5*B*a*b^4 + A*b^5)*x^4 + 5*(2*B*a^2*b^3 + A*a*b^4)*x
- 1/88*(440*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 88*(B*a^4*b + 2*A*a^3*b^2)*x^6 +
 8*A*a^5 + 11*(B*a^5 + 5*A*a^4*b)*x^3)/x^11
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}}\,dx = \frac{1}{7}\,Bb^5x^7 + \frac{5}{4}\,Bab^4x^4 + \frac{1}{4}\,Ab^5x^4 + 10\,Ba^2b^3x + 5\,Aab^4x - \frac{440\,Ba^3b^2x^9 + 440\,Aa^2b^3x^9 + 88\,Ba^4bx^6 + 176\,Aa^3b^2x^6 + 11\,Ba^5x^3 + 55\,Aa^4bx^3 + 8\,Aa^5}{88\,x^{11}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="giac")
```

output
```
1/7*B*b^5*x^7 + 5/4*B*a*b^4*x^4 + 1/4*A*b^5*x^4 + 10*B*a^2*b^3*x + 5*A*a*b
^4*x - 1/88*(440*B*a^3*b^2*x^9 + 440*A*a^2*b^3*x^9 + 88*B*a^4*b*x^6 + 176*
A*a^3*b^2*x^6 + 11*B*a^5*x^3 + 55*A*a^4*b*x^3 + 8*A*a^5)/x^11
```

3.44.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06

$$\begin{aligned}
&\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}}\,dx \\
&= x^4 \left(\frac{A\,b^5}{4} + \frac{5\,B\,a\,b^4}{4}\right) \\
&\quad - \frac{\frac{A\,a^5}{11} + x^6\,(B\,a^4\,b + 2\,A\,a^3\,b^2) + x^3\left(\frac{B\,a^5}{8} + \frac{5\,A\,b\,a^4}{8}\right) + x^9\,(5\,B\,a^3\,b^2 + 5\,A\,a^2\,b^3)}{x^{11}} \\
&\quad + \frac{B\,b^5\,x^7}{7} + 5\,a\,b^3\,x\,(A\,b + 2\,B\,a)
\end{aligned}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^12,x)
```

output
```
x^4*((A*b^5)/4 + (5*B*a*b^4)/4) - ((A*a^5)/11 + x^6*(2*A*a^3*b^2 + B*a^4*b
) + x^3*((B*a^5)/8 + (5*A*a^4*b)/8) + x^9*(5*A*a^2*b^3 + 5*B*a^3*b^2))/x^1
1 + (B*b^5*x^7)/7 + 5*a*b^3*x*(A*b + 2*B*a)
```

3.45 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{13}}\,dx$

3.45.1 Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{13}}\,dx = -\frac{a^5A}{12x^{12}} - \frac{a^4(5Ab+aB)}{9x^9} - \frac{5a^3b(2Ab+aB)}{6x^6} - \frac{10a^2b^2(Ab+aB)}{3x^3} + \frac{1}{3}b^4(Ab+5aB)x^3 + \frac{1}{6}b^5Bx^6 + 5ab^3(Ab + 2aB)\log(x)$$

output
```
-1/12*a^5*A/x^12-1/9*a^4*(5*A*b+B*a)/x^9-5/6*a^3*b*(2*A*b+B*a)/x^6-10/3*a^
2*b^2*(A*b+B*a)/x^3+1/3*b^4*(A*b+5*B*a)*x^3+1/6*b^5*B*x^6+5*a*b^3*(A*b+2*B
*a)*ln(x)
```

3.45.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{13}}\,dx = -\frac{120a^2Ab^3x^9 - 60ab^4Bx^{15} - 6b^5x^{15}(2A+Bx^3) + 60a^3b^2x^6(A+2Bx^3) + 10a^4bx^3(2A+3Bx^3) + a^5(3A}{36x^{12}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^13,x]
```

output
```
-1/36*(120*a^2*A*b^3*x^9 - 60*a*b^4*B*x^15 - 6*b^5*x^15*(2*A + B*x^3) + 60
*a^3*b^2*x^6*(A + 2*B*x^3) + 10*a^4*b*x^3*(2*A + 3*B*x^3) + a^5*(3*A + 4*B
*x^3) - 180*a*b^3*(A*b + 2*a*B)*x^12*Log[x])/x^12
```

3.45.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^{13}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{\left(bx^3+a\right)^5 \left(Bx^3+A\right)}{x^{15}}dx^3$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{3} \int \Bigg(\frac{Aa^5}{x^{15}}+\frac{(5Ab+aB)a^4}{x^{12}}+\frac{5b(2Ab+aB)a^3}{x^9}+\frac{10b^2(Ab+aB)a^2}{x^6}+\frac{5b^3(Ab+2aB)a}{x^3}+b^5Bx^3+b^4(Ab+5aB$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\Bigg(-\frac{a^5A}{4x^{12}}-\frac{a^4(aB+5Ab)}{3x^9}-\frac{5a^3b(aB+2Ab)}{2x^6}-\frac{10a^2b^2(aB+Ab)}{x^3}+b^4x^3(5aB+Ab)+5ab^3\log\left(x^3\right)(2aB+A$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^13,x]
```

output
```
(-1/4*(a^5*A)/x^12 - (a^4*(5*A*b + a*B))/(3*x^9) - (5*a^3*b*(2*A*b + a*B))
/(2*x^6) - (10*a^2*b^2*(A*b + a*B))/x^3 + b^4*(A*b + 5*a*B)*x^3 + (b^5*B*x
^6)/2 + 5*a*b^3*(A*b + 2*a*B)*Log[x^3])/3
```

3.45.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.45.4 Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

method	result
default	$\frac{b^{5} B \,x^{6}}{6}+\frac{A \,b^{5} x^{3}}{3}+\frac{5 B a \,b^{4} x^{3}}{3}+5 a \,b^{3} \left(A b +2 B a \right) \ln \left(x \right)-\frac{5 a^{3} b \left(2 A b +B a \right)}{6 x^{6}}-\frac{a^{5} A}{12 x^{12}}-\frac{10 a^{2} b^{2} \left(A b +B a \right)}{3 x^{3}}-a$
norman	$\frac{\left(\frac{1}{3} b^{5} A +\frac{5}{3} a \,b^{4} B \right) x^{15}+\left(-\frac{10}{3} a^{2} b^{3} A -\frac{10}{3} a^{3} b^{2} B \right) x^{9}+\left(-\frac{5}{3} a^{3} b^{2} A -\frac{5}{6} a^{4} b B \right) x^{6}+\left(-\frac{5}{9} a^{4} b A -\frac{1}{9} a^{5} B \right) x^{3}-\frac{a^{5} A}{12}+\frac{b^{5} B \,x^{18}}{6}}{x^{12}}+\left(5 a \,b^{4}\right.$
parallelrisch	$\frac{6 b^{5} B \,x^{18}+12 A \,b^{5} x^{15}+60 B a \,b^{4} x^{15}+180 A \ln \left(x \right) x^{12} a \,b^{4}+360 B \ln \left(x \right) x^{12} a^{2} b^{3}-120 a^{2} A \,b^{3} x^{9}-120 B \,a^{3} b^{2} x^{9}-60 a^{3} A \,b^{2} x^{6}-30 B a}{36 x^{12}}$
risch	$\frac{b^{5} B \,x^{6}}{6}+\frac{A \,b^{5} x^{3}}{3}+\frac{5 B a \,b^{4} x^{3}}{3}+\frac{b^{5} A^{2}}{6 B}+\frac{5 a \,b^{4} A}{3}+\frac{25 a^{2} b^{3} B}{6}+\frac{\left(-\frac{10}{3} a^{2} b^{3} A -\frac{10}{3} a^{3} b^{2} B \right) x^{9}+\left(-\frac{5}{3} a^{3} b^{2} A -\frac{5}{6} a^{4} b B \right) x^{6}+}{x^{12}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^13,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^5*B*x^6+1/3*A*b^5*x^3+5/3*B*a*b^4*x^3+5*a*b^3*(A*b+2*B*a)*ln(x)-5/6*
a^3*b*(2*A*b+B*a)/x^6-1/12*a^5*A/x^12-10/3*a^2*b^2*(A*b+B*a)/x^3-1/9*a^4*(
5*A*b+B*a)/x^9
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}}\,dx$$
$$= \frac{6\,Bb^5x^{18} + 12\,(5\,Bab^4 + Ab^5)x^{15} + 180\,(2\,Ba^2b^3 + Aab^4)x^{12}\log{(x)} - 120\,(Ba^3b^2 + Aa^2b^3)x^9 - 30\,(Ba^4b}{36\,x^{12}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="fricas")
```

output
```
1/36*(6*B*b^5*x^18 + 12*(5*B*a*b^4 + A*b^5)*x^15 + 180*(2*B*a^2*b^3 + A*a*
b^4)*x^12*log(x) - 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 30*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 3*A*a^5 - 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^12
```

3.45.6 Sympy [A] (verification not implemented)

Time = 46.57 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}}\,dx = \frac{Bb^5x^6}{6} + 5ab^3(Ab+2Ba)\log{(x)} + x^3\left(\frac{Ab^5}{3} + \frac{5Bab^4}{3}\right)$$
$$+ \frac{-3Aa^5 + x^9(-120Aa^2b^3 - 120Ba^3b^2) + x^6(-60Aa^3b^2 - 30Ba^4b) + x^3(-20Aa^4b - 4Ba^5)}{36x^{12}}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**13,x)
```

output
```
B*b**5*x**6/6 + 5*a*b**3*(A*b + 2*B*a)*log(x) + x**3*(A*b**5/3 + 5*B*a*b**
4/3) + (-3*A*a**5 + x**9*(-120*A*a**2*b**3 - 120*B*a**3*b**2) + x**6*(-60*
A*a**3*b**2 - 30*B*a**4*b) + x**3*(-20*A*a**4*b - 4*B*a**5))/(36*x**12)
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}}\,dx$$
$$= \frac{1}{6}\,Bb^5x^6 + \frac{1}{3}\,(5\,Bab^4 + Ab^5)x^3 + \frac{5}{3}\,(2\,Ba^2b^3 + Aab^4)\log{(x^3)}$$
$$- \frac{120\,(Ba^3b^2 + Aa^2b^3)x^9 + 30\,(Ba^4b + 2\,Aa^3b^2)x^6 + 3\,Aa^5 + 4\,(Ba^5 + 5\,Aa^4b)x^3}{36\,x^{12}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="maxima")
```

output
```
1/6*B*b^5*x^6 + 1/3*(5*B*a*b^4 + A*b^5)*x^3 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*
log(x^3) - 1/36*(120*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 30*(B*a^4*b + 2*A*a^3*b
^2)*x^6 + 3*A*a^5 + 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^12
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}}\,dx = \frac{1}{6}\,Bb^5x^6 + \frac{5}{3}\,Bab^4x^3 + \frac{1}{3}\,Ab^5x^3 + 5\left(2\,Ba^2b^3 + Aab^4\right)\log\left(|x|\right) - \frac{250\,Ba^2b^3x^{12} + 125\,Aab^4x^{12} + 120\,Ba^3b^2x^9 + 120\,Aa^2b^3x^9 + 30\,Ba^4bx^6 + 60\,Aa^3b^2x^6 + 4\,Ba^5x^3 + 20}{36\,x^{12}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="giac")
```

output
```
1/6*B*b^5*x^6 + 5/3*B*a*b^4*x^3 + 1/3*A*b^5*x^3 + 5*(2*B*a^2*b^3 + A*a*b^4
)*log(abs(x)) - 1/36*(250*B*a^2*b^3*x^12 + 125*A*a*b^4*x^12 + 120*B*a^3*b^
2*x^9 + 120*A*a^2*b^3*x^9 + 30*B*a^4*b*x^6 + 60*A*a^3*b^2*x^6 + 4*B*a^5*x^
3 + 20*A*a^4*b*x^3 + 3*A*a^5)/x^12
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07

$$\begin{aligned}
&\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}}\,dx \\
&= \ln(x)\,\left(10\,B\,a^2\,b^3 + 5\,A\,a\,b^4\right) \\
&\quad - \frac{\frac{A\,a^5}{12} + x^6\left(\frac{5\,B\,a^4\,b}{6} + \frac{5\,A\,a^3\,b^2}{3}\right) + x^3\left(\frac{B\,a^5}{9} + \frac{5\,A\,b\,a^4}{9}\right) + x^9\left(\frac{10\,B\,a^3\,b^2}{3} + \frac{10\,A\,a^2\,b^3}{3}\right)}{x^{12}} \\
&\quad + x^3\left(\frac{A\,b^5}{3} + \frac{5\,B\,a\,b^4}{3}\right) + \frac{B\,b^5\,x^6}{6}
\end{aligned}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^13,x)
```

output
```
log(x)*(10*B*a^2*b^3 + 5*A*a*b^4) - ((A*a^5)/12 + x^6*((5*A*a^3*b^2)/3 + (
5*B*a^4*b)/6) + x^3*((B*a^5)/9 + (5*A*a^4*b)/9) + x^9*((10*A*a^2*b^3)/3 +
(10*B*a^3*b^2)/3))/x^12 + x^3*((A*b^5)/3 + (5*B*a*b^4)/3) + (B*b^5*x^6)/6
```

3.45. $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{13}}\,dx$

3.46 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}}\,dx$

3.46.1 Optimal result

Integrand size = 20, antiderivative size = 115

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}}\,dx = -\frac{a^5A}{13x^{13}} - \frac{a^4(5Ab+aB)}{10x^{10}} - \frac{5a^3b(2Ab+aB)}{7x^7} - \frac{5a^2b^2(Ab+aB)}{2x^4} - \frac{5ab^3(Ab+2aB)}{x} + \frac{1}{2}b^4(Ab+5aB)x^2 + \frac{1}{5}b^5Bx^5$$

output
```
-1/13*a^5*A/x^13-1/10*a^4*(5*A*b+B*a)/x^10-5/7*a^3*b*(2*A*b+B*a)/x^7-5/2*a
^2*b^2*(A*b+B*a)/x^4-5*a*b^3*(A*b+2*B*a)/x+1/2*b^4*(A*b+5*B*a)*x^2+1/5*b^5
*B*x^5
```

3.46.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}}\,dx = -\frac{-2275ab^4x^{12}(-2A+Bx^3) - 91b^5x^{15}(5A+2Bx^3) + 2275a^2b^3x^9(A+4Bx^3) + 325a^3b^2x^6(4A+7Bx^3)}{910x^{13}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^14,x]
```

output
```
-1/910*(-2275*a*b^4*x^12*(-2*A + B*x^3) - 91*b^5*x^15*(5*A + 2*B*x^3) + 22
75*a^2*b^3*x^9*(A + 4*B*x^3) + 325*a^3*b^2*x^6*(4*A + 7*B*x^3) + 65*a^4*b*
x^3*(7*A + 10*B*x^3) + a^5*(70*A + 91*B*x^3))/x^13
```

3.46.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} \, dx$$

$$\Big\downarrow \; 950$$

$$\int \left(\frac{a^5 A}{x^{14}}+\frac{a^4(aB+5Ab)}{x^{11}}+\frac{5a^3b(aB+2Ab)}{x^8}+\frac{10a^2b^2(aB+Ab)}{x^5}+b^4x(5aB+Ab)+\frac{5ab^3(2aB+Ab)}{x^2}+b^5Bx^4\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^5A}{13x^{13}}-\frac{a^4(aB+5Ab)}{10x^{10}}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{5a^2b^2(aB+Ab)}{2x^4}+\frac{1}{2}b^4x^2(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+\frac{1}{5}b^5Bx^5$$

input

```
Int[((a + b*x^3)^5*(A + B*x^3))/x^14,x]
```

output

```
-1/13*(a^5*A)/x^13 - (a^4*(5*A*b + a*B))/(10*x^10) - (5*a^3*b*(2*A*b + a*B
))/(7*x^7) - (5*a^2*b^2*(A*b + a*B))/(2*x^4) - (5*a*b^3*(A*b + 2*a*B))/x +
 (b^4*(A*b + 5*a*B)*x^2)/2 + (b^5*B*x^5)/5
```

3.46.3.1 Defintions of rubi rules used

rule 950

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.46.4 Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

method	result
default	$\frac{b^5Bx^5}{5}+\frac{Ab^5x^2}{2}+\frac{5Bab^4x^2}{2}-\frac{5a^3b(2Ab+Ba)}{7x^7}-\frac{a^4(5Ab+Ba)}{10x^{10}}-\frac{a^5A}{13x^{13}}-\frac{5ab^3(Ab+2Ba)}{x}-\frac{5a^2b^2(Ab+Ba)}{2x^4}$
norman	$\frac{-\frac{a^5A}{13}+(-\frac{1}{2}a^4bA-\frac{1}{10}a^5B)x^3+(-\frac{10}{7}a^3b^2A-\frac{5}{7}a^4bB)x^6+(-\frac{5}{2}a^2b^3A-\frac{5}{2}a^3b^2B)x^9+(-5ab^4A-10a^2b^3B)x^{12}+(\frac{1}{2}b^5A+\frac{5}{2}ab^4}{x^{13}}$
risch	$\frac{b^5Bx^5}{5}+\frac{Ab^5x^2}{2}+\frac{5Bab^4x^2}{2}+\frac{(-5ab^4A-10a^2b^3B)x^{12}+(-\frac{5}{2}a^2b^3A-\frac{5}{2}a^3b^2B)x^9+(-\frac{10}{7}a^3b^2A-\frac{5}{7}a^4bB)x^6+(-\frac{1}{2}a^4bA}{x^{13}}$
gosper	$-\frac{-182b^5Bx^{18}-455Ab^5x^{15}-2275Bab^4x^{15}+4550aAb^4x^{12}+9100Ba^2b^3x^{12}+2275a^2Ab^3x^9+2275Ba^3b^2x^9+1300a^3Ab^2x^6+}{910x^{13}}$
parallelrisch	$\frac{182b^5Bx^{18}+455Ab^5x^{15}+2275Bab^4x^{15}-4550aAb^4x^{12}-9100Ba^2b^3x^{12}-2275a^2Ab^3x^9-2275Ba^3b^2x^9-1300a^3Ab^2x^6-650}{910x^{13}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^14,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^5*B*x^5+1/2*A*b^5*x^2+5/2*B*a*b^4*x^2-5/7*a^3*b*(2*A*b+B*a)/x^7-1/10
*a^4*(5*A*b+B*a)/x^10-1/13*a^5*A/x^13-5*a*b^3*(A*b+2*B*a)/x-5/2*a^2*b^2*(A
*b+B*a)/x^4
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}}\,dx$$
$$=\frac{182\,Bb^5x^{18}+455\,(5\,Bab^4+Ab^5)x^{15}-4550\,(2\,Ba^2b^3+Aab^4)x^{12}-2275\,(Ba^3b^2+Aa^2b^3)x^9-650\,(Ba^4b}{910\,x^{13}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="fricas")
```

output
```
1/910*(182*B*b^5*x^18 + 455*(5*B*a*b^4 + A*b^5)*x^15 - 4550*(2*B*a^2*b^3 +
 A*a*b^4)*x^12 - 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 650*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 70*A*a^5 - 91*(B*a^5 + 5*A*a^4*b)*x^3)/x^13
```

3.46.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{14}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**14,x)
```

output
```
Timed out
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{14}}\,dx = \frac{1}{5}\,Bb^5x^5 + \frac{1}{2}\,(5\,Bab^4 + Ab^5)x^2 - \frac{4550\,(2\,Ba^2b^3 + Aab^4)x^{12} + 2275\,(Ba^3b^2 + Aa^2b^3)x^9 + 650\,(Ba^4b + 2\,Aa^3b^2)x^6 + 70\,Aa^5 + 91\,(Ba^5 +}{910\,x^{13}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="maxima")
```

output
```
1/5*B*b^5*x^5 + 1/2*(5*B*a*b^4 + A*b^5)*x^2 - 1/910*(4550*(2*B*a^2*b^3 + A
*a*b^4)*x^12 + 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 650*(B*a^4*b + 2*A*a^3*b
^2)*x^6 + 70*A*a^5 + 91*(B*a^5 + 5*A*a^4*b)*x^3)/x^13
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{14}}\,dx = \frac{1}{5}\,Bb^5x^5 + \frac{5}{2}\,Bab^4x^2 + \frac{1}{2}\,Ab^5x^2 - \frac{9100\,Ba^2b^3x^{12} + 4550\,Aab^4x^{12} + 2275\,Ba^3b^2x^9 + 2275\,Aa^2b^3x^9 + 650\,Ba^4bx^6 + 1300\,Aa^3b^2x^6 + 91\,Ba}{910\,x^{13}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="giac")
```

output
```
1/5*B*b^5*x^5 + 5/2*B*a*b^4*x^2 + 1/2*A*b^5*x^2 - 1/910*(9100*B*a^2*b^3*x^
12 + 4550*A*a*b^4*x^12 + 2275*B*a^3*b^2*x^9 + 2275*A*a^2*b^3*x^9 + 650*B*a
^4*b*x^6 + 1300*A*a^3*b^2*x^6 + 91*B*a^5*x^3 + 455*A*a^4*b*x^3 + 70*A*a^5)
/x^13
```

3.46.9 Mupad [B] (verification not implemented)

Time = 6.56 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}}\,dx = x^2\left(\frac{A\,b^5}{2}+\frac{5\,B\,a\,b^4}{2}\right) - \frac{\frac{A\,a^5}{13}+x^{12}\left(10\,B\,a^2\,b^3+5\,A\,a\,b^4\right)+x^6\left(\frac{5\,B\,a^4\,b}{7}+\frac{10\,A\,a^3\,b^2}{7}\right)+x^3\left(\frac{B\,a^5}{10}+\frac{A\,b\,a^4}{2}\right)+x^9\left(\frac{5\,B\,a^3\,b^2}{2}+\frac{5\,A\,a^2\,b^3}{2}\right)}{x^{13}} + \frac{B\,b^5\,x^5}{5}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^14,x)
```

output
```
x^2*((A*b^5)/2 + (5*B*a*b^4)/2) - ((A*a^5)/13 + x^12*(10*B*a^2*b^3 + 5*A*a
*b^4) + x^6*((10*A*a^3*b^2)/7 + (5*B*a^4*b)/7) + x^3*((B*a^5)/10 + (A*a^4*
b)/2) + x^9*((5*A*a^2*b^3)/2 + (5*B*a^3*b^2)/2))/x^13 + (B*b^5*x^5)/5
```

3.47 $\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{15}}\,dx$

3.47.1 Optimal result

Integrand size = 20, antiderivative size = 110

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{15}}\,dx &= -\frac{a^5A}{14x^{14}}-\frac{a^4(5Ab+aB)}{11x^{11}}-\frac{5a^3b(2Ab+aB)}{8x^8}-\frac{2a^2b^2(Ab+aB)}{x^5}\\&\quad-\frac{5ab^3(Ab+2aB)}{2x^2}+b^4(Ab+5aB)x+\frac{1}{4}b^5Bx^4\end{aligned}$$

output
```
-1/14*a^5*A/x^14-1/11*a^4*(5*A*b+B*a)/x^11-5/8*a^3*b*(2*A*b+B*a)/x^8-2*a^2
*b^2*(A*b+B*a)/x^5-5/2*a*b^3*(A*b+2*B*a)/x^2+b^4*(A*b+5*B*a)*x+1/4*b^5*B*x
^4
```

3.47.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{15}}\,dx &= -\frac{a^5A}{14x^{14}}-\frac{a^4(5Ab+aB)}{11x^{11}}-\frac{5a^3b(2Ab+aB)}{8x^8}-\frac{2a^2b^2(Ab+aB)}{x^5}\\&\quad-\frac{5ab^3(Ab+2aB)}{2x^2}+b^4(Ab+5aB)x+\frac{1}{4}b^5Bx^4\end{aligned}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^15,x]
```

output
```
-1/14*(a^5*A)/x^14 - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B
))/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2)
 + b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4
```

3.47.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5 A}{x^{15}}+\frac{a^4 (aB+5Ab)}{x^{12}}+\frac{5a^3 b (aB+2Ab)}{x^9}+\frac{10a^2 b^2 (aB+Ab)}{x^6}+b^4 (5aB+Ab)+\frac{5ab^3 (2aB+Ab)}{x^3}+b^5 B x^3\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5 A}{14x^{14}}-\frac{a^4 (aB+5Ab)}{11x^{11}}-\frac{5a^3 b (aB+2Ab)}{8x^8}-\frac{2a^2 b^2 (aB+Ab)}{x^5}+b^4 x (5aB+Ab)-\frac{5ab^3 (2aB+Ab)}{2x^2}+\frac{1}{4} b^5 B x^4$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^15,x]
```

output
```
-1/14*(a^5*A)/x^14 - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B
))/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2)
 + b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4
```

3.47.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.47.4 Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

method	result
default	$\frac{b^5 B x^4}{4}+A b^5 x+5 B a b^4 x-\frac{a^5 A}{14 x^{14}}-\frac{5 a^3 b (2 A b+B a)}{8 x^8}-\frac{a^4 (5 A b+B a)}{11 x^{11}}-\frac{5 a b^3 (A b+2 B a)}{2 x^2}-\frac{2 a^2 b^2 (A b+B a)}{x^5}$
risch	$\frac{b^5 B x^4}{4}+A b^5 x+5 B a b^4 x+\frac{\left(-\frac{5}{2} a b^4 A-5 a^2 b^3 B\right) x^{12}+\left(-2 a^2 b^3 A-2 a^3 b^2 B\right) x^9+\left(-\frac{5}{4} a^3 b^2 A-\frac{5}{8} a^4 b B\right) x^6+\left(-\frac{5}{11} a^4 b A\right.}{x^{14}}$
norman	$\frac{-\frac{a^5 A}{14}+\left(-\frac{5}{11} a^4 b A-\frac{1}{11} a^5 B\right) x^3+\left(-\frac{5}{4} a^3 b^2 A-\frac{5}{8} a^4 b B\right) x^6+\left(-2 a^2 b^3 A-2 a^3 b^2 B\right) x^9+\left(-\frac{5}{2} a b^4 A-5 a^2 b^3 B\right) x^{12}+\left(b^5 A+5 a b^4 B\right) x}{x^{14}}$
gosper	$-\frac{-154 b^5 B x^{18}-616 A b^5 x^{15}-3080 B a b^4 x^{15}+1540 a A b^4 x^{12}+3080 B a^2 b^3 x^{12}+1232 a^2 A b^3 x^9+1232 B a^3 b^2 x^9+770 a^3 A b^2 x^6+}{616 x^{14}}$
parallelrisch	$\frac{154 b^5 B x^{18}+616 A b^5 x^{15}+3080 B a b^4 x^{15}-1540 a A b^4 x^{12}-3080 B a^2 b^3 x^{12}-1232 a^2 A b^3 x^9-1232 B a^3 b^2 x^9-770 a^3 A b^2 x^6-385}{616 x^{14}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^15,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^5*B*x^4+A*b^5*x+5*B*a*b^4*x-1/14*a^5*A/x^14-5/8*a^3*b*(2*A*b+B*a)/x^
8-1/11*a^4*(5*A*b+B*a)/x^11-5/2*a*b^3*(A*b+2*B*a)/x^2-2*a^2*b^2*(A*b+B*a)/
x^5
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx = \frac{154\,Bb^5x^{18}+616\,(5\,Bab^4+Ab^5)x^{15}-1540\,(2\,Ba^2b^3+Aab^4)x^{12}-1232\,(Ba^3b^2+Aa^2b^3)x^9-385\,(Ba^4b}{616\,x^{14}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="fricas")
```

output
```
1/616*(154*B*b^5*x^18 + 616*(5*B*a*b^4 + A*b^5)*x^15 - 1540*(2*B*a^2*b^3 +
 A*a*b^4)*x^12 - 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 385*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 44*A*a^5 - 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^14
```

3.47.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**15,x)
```

output
```
Timed out
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx = \frac{1}{4}\,Bb^5x^4 + \left(5\,Bab^4 + Ab^5\right)x - \frac{1540\,(2\,Ba^2b^3 + Aab^4)x^{12} + 1232\,(Ba^3b^2 + Aa^2b^3)x^9 + 385\,(Ba^4b + 2\,Aa^3b^2)x^6 + 44\,Aa^5 + 56\,(Ba^5 +}{616\,x^{14}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="maxima")
```

output
```
1/4*B*b^5*x^4 + (5*B*a*b^4 + A*b^5)*x - 1/616*(1540*(2*B*a^2*b^3 + A*a*b^4
)*x^12 + 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 385*(B*a^4*b + 2*A*a^3*b^2)*x^
6 + 44*A*a^5 + 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^14
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx = \frac{1}{4}\,Bb^5x^4 + 5\,Bab^4x + Ab^5x - \frac{3080\,Ba^2b^3x^{12} + 1540\,Aab^4x^{12} + 1232\,Ba^3b^2x^9 + 1232\,Aa^2b^3x^9 + 385\,Ba^4bx^6 + 770\,Aa^3b^2x^6 + 56\,Ba^5}{616\,x^{14}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="giac")
```

output
```
1/4*B*b^5*x^4 + 5*B*a*b^4*x + A*b^5*x - 1/616*(3080*B*a^2*b^3*x^12 + 1540*
A*a*b^4*x^12 + 1232*B*a^3*b^2*x^9 + 1232*A*a^2*b^3*x^9 + 385*B*a^4*b*x^6 +
 770*A*a^3*b^2*x^6 + 56*B*a^5*x^3 + 280*A*a^4*b*x^3 + 44*A*a^5)/x^14
```

3.47.9 Mupad [B] (verification not implemented)

Time = 6.55 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{15}}\,dx = x\left(A\,b^5+5\,B\,a\,b^4\right) - \frac{\frac{A\,a^5}{14}+x^{12}\left(5\,B\,a^2\,b^3+\frac{5\,A\,a\,b^4}{2}\right)+x^6\left(\frac{5\,B\,a^4\,b}{8}+\frac{5\,A\,a^3\,b^2}{4}\right)+x^3\left(\frac{B\,a^5}{11}+\frac{5\,A\,b\,a^4}{11}\right)+x^9\left(2\,B\,a^3\,b^2+2\,A\,a^2\,b^3\right)}{x^{14}} + \frac{B\,b^5\,x^4}{4}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^15,x)
```

output
```
x*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/14 + x^12*(5*B*a^2*b^3 + (5*A*a*b^4)/2) +
 x^6*((5*A*a^3*b^2)/4 + (5*B*a^4*b)/8) + x^3*((B*a^5)/11 + (5*A*a^4*b)/11)
 + x^9*(2*A*a^2*b^3 + 2*B*a^3*b^2))/x^14 + (B*b^5*x^4)/4
```

3.48 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx$

3.48.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx = -\frac{a^5A}{15x^{15}}-\frac{a^4(5Ab+aB)}{12x^{12}}-\frac{5a^3b(2Ab+aB)}{9x^9}-\frac{5a^2b^2(Ab+aB)}{3x^6}-\frac{5ab^3(Ab+2aB)}{3x^3}+\frac{1}{3}b^5Bx^3+b^4(Ab+5aB)\log(x)$$

output
```
-1/15*a^5*A/x^15-1/12*a^4*(5*A*b+B*a)/x^12-5/9*a^3*b*(2*A*b+B*a)/x^9-5/3*a
^2*b^2*(A*b+B*a)/x^6-5/3*a*b^3*(A*b+2*B*a)/x^3+1/3*b^5*B*x^3+b^4*(A*b+5*B*
a)*ln(x)
```

3.48.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx = -\frac{300aAb^4x^{12}-60b^5Bx^{18}+300a^2b^3x^9(A+2Bx^3)+100a^3b^2x^6(2A+3Bx^3)+25a^4bx^3(3A+4Bx^3)+3}{180x^{15}}+b^4(Ab+5aB)\log(x)$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^16,x]
```

output
```
-1/180*(300*a*A*b^4*x^12 - 60*b^5*B*x^18 + 300*a^2*b^3*x^9*(A + 2*B*x^3) +
 100*a^3*b^2*x^6*(2*A + 3*B*x^3) + 25*a^4*b*x^3*(3*A + 4*B*x^3) + 3*a^5*(4
*A + 5*B*x^3))/x^15 + b^4*(A*b + 5*a*B)*Log[x]
```

3.48.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{16}}dx$$

$$\Big\downarrow 948$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)^5\left(Bx^3+A\right)}{x^{18}}dx^3$$

$$\Big\downarrow 85$$

$$\frac{1}{3}\int \left(\frac{Aa^5}{x^{18}}+\frac{(5Ab+aB)a^4}{x^{15}}+\frac{5b(2Ab+aB)a^3}{x^{12}}+\frac{10b^2(Ab+aB)a^2}{x^9}+\frac{5b^3(Ab+2aB)a}{x^6}+b^5B+\frac{b^4(Ab+5aB)}{x^3}\right)$$

$$\Big\downarrow 2009$$

$$\frac{1}{3}\left(-\frac{a^5A}{5x^{15}}-\frac{a^4(aB+5Ab)}{4x^{12}}-\frac{5a^3b(aB+2Ab)}{3x^9}-\frac{5a^2b^2(aB+Ab)}{x^6}+b^4\log\left(x^3\right)(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x^3}+\right.$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^16,x]
```

output
```
(-1/5*(a^5*A)/x^15 - (a^4*(5*A*b + a*B))/(4*x^12) - (5*a^3*b*(2*A*b + a*B)
)/(3*x^9) - (5*a^2*b^2*(A*b + a*B))/x^6 - (5*a*b^3*(A*b + 2*a*B))/x^3 + b^
5*B*x^3 + b^4*(A*b + 5*a*B)*Log[x^3])/3
```

3.48.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.48.4 Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a^{5} A}{15 x^{15}}-\frac{a^{4} \left(5 A b +B a \right)}{12 x^{12}}-\frac{5 a^{3} b \left(2 A b +B a \right)}{9 x^{9}}-\frac{5 a^{2} b^{2} \left(A b +B a \right)}{3 x^{6}}-\frac{5 a \,b^{3} \left(A b +2 B a \right)}{3 x^{3}}+\frac{b^{5} B \,x^{3}}{3}+b^{4} \left(A b +5 B a \right) \ln$
norman	$\frac{\left(-\frac{5}{3} a \,b^{4} A -\frac{10}{3} a^{2} b^{3} B \right) x^{12}+\left(-\frac{5}{3} a^{2} b^{3} A -\frac{5}{3} a^{3} b^{2} B \right) x^{9}+\left(-\frac{10}{9} a^{3} b^{2} A -\frac{5}{9} a^{4} b B \right) x^{6}+\left(-\frac{5}{12} a^{4} b A -\frac{1}{12} a^{5} B \right) x^{3}-\frac{a^{5} A}{15}+\frac{b^{5} B \,x^{18}}{3}}{x^{15}}+($
risch	$\frac{b^{5} B \,x^{3}}{3}+\frac{-\frac{a^{5} A}{15}+\left(-\frac{5}{12} a^{4} b A -\frac{1}{12} a^{5} B \right) x^{3}+\left(-\frac{10}{9} a^{3} b^{2} A -\frac{5}{9} a^{4} b B \right) x^{6}+\left(-\frac{5}{3} a^{2} b^{3} A -\frac{5}{3} a^{3} b^{2} B \right) x^{9}+\left(-\frac{5}{3} a \,b^{4} A -\frac{10}{3} a^{2} b^{3} B \right) x^{12}}{x^{15}}+$
parallelrisch	$\frac{60 b^{5} B \,x^{18}+180 A \ln \left(x \right) x^{15} b^{5}+900 B \ln \left(x \right) x^{15} a \,b^{4}-300 a A \,b^{4} x^{12}-600 B \,a^{2} b^{3} x^{12}-300 a^{2} A \,b^{3} x^{9}-300 B \,a^{3} b^{2} x^{9}-200 a^{3} A \,b^{2} x^{6}-}{180 x^{15}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/15*a^5*A/x^15-1/12*a^4*(5*A*b+B*a)/x^12-5/9*a^3*b*(2*A*b+B*a)/x^9-5/3*a
^2*b^2*(A*b+B*a)/x^6-5/3*a*b^3*(A*b+2*B*a)/x^3+1/3*b^5*B*x^3+b^4*(A*b+5*B*
a)*ln(x)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}}\,dx$$
$$= \frac{60\,Bb^5x^{18} + 180\,(5\,Bab^4 + Ab^5)x^{15}\log(x) - 300\,(2\,Ba^2b^3 + Aab^4)x^{12} - 300\,(Ba^3b^2 + Aa^2b^3)x^9 - 100\,(B}{180\,x^{15}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="fricas")
```

output
```
1/180*(60*B*b^5*x^18 + 180*(5*B*a*b^4 + A*b^5)*x^15*log(x) - 300*(2*B*a^2*
b^3 + A*a*b^4)*x^12 - 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 100*(B*a^4*b + 2*A
*a^3*b^2)*x^6 - 12*A*a^5 - 15*(B*a^5 + 5*A*a^4*b)*x^3)/x^15
```

3.48.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**16,x)
```

output
```
Timed out
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}}\,dx = \frac{1}{3}\,Bb^5x^3 + \frac{1}{3}\left(5\,Bab^4 + Ab^5\right)\log\left(x^3\right)$$
$$- \frac{300\,(2\,Ba^2b^3 + Aab^4)x^{12} + 300\,(Ba^3b^2 + Aa^2b^3)x^9 + 100\,(Ba^4b + 2\,Aa^3b^2)x^6 + 12\,Aa^5 + 15\,(Ba^5 + 5\,A}{180\,x^{15}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="maxima")
```

output
```
1/3*B*b^5*x^3 + 1/3*(5*B*a*b^4 + A*b^5)*log(x^3) - 1/180*(300*(2*B*a^2*b^3
 + A*a*b^4)*x^12 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 100*(B*a^4*b + 2*A*a^
3*b^2)*x^6 + 12*A*a^5 + 15*(B*a^5 + 5*A*a^4*b)*x^3)/x^15
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}}\,dx = \frac{1}{3}Bb^5x^3 + \left(5\,Bab^4 + Ab^5\right)\log\left(|x|\right) - \frac{685\,Bab^4x^{15} + 137\,Ab^5x^{15} + 600\,Ba^2b^3x^{12} + 300\,Aab^4x^{12} + 300\,Ba^3b^2x^9 + 300\,Aa^2b^3x^9 + 100\,Ba^4bx^6}{180\,x^{15}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="giac")
```

output
```
1/3*B*b^5*x^3 + (5*B*a*b^4 + A*b^5)*log(abs(x)) - 1/180*(685*B*a*b^4*x^15
+ 137*A*b^5*x^15 + 600*B*a^2*b^3*x^12 + 300*A*a*b^4*x^12 + 300*B*a^3*b^2*x
^9 + 300*A*a^2*b^3*x^9 + 100*B*a^4*b*x^6 + 200*A*a^3*b^2*x^6 + 15*B*a^5*x^
3 + 75*A*a^4*b*x^3 + 12*A*a^5)/x^15
```

3.48.9 Mupad [B] (verification not implemented)

Time = 6.51 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}}\,dx = \ln(x)\ \left(A\,b^5 + 5\,B\,a\,b^4\right) - \frac{\frac{A\,a^5}{15} + x^{12}\left(\frac{10\,B\,a^2\,b^3}{3} + \frac{5\,A\,a\,b^4}{3}\right) + x^6\left(\frac{5\,B\,a^4\,b}{9} + \frac{10\,A\,a^3\,b^2}{9}\right) + x^3\left(\frac{B\,a^5}{12} + \frac{5\,A\,b\,a^4}{12}\right) + x^9\left(\frac{5\,B\,a^3\,b^2}{3} + \frac{5\,A\,a^2\,b^3}{3}\right)}{x^{15}} + \frac{B\,b^5\,x^3}{3}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^16,x)
```

output
```
log(x)*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/15 + x^12*((10*B*a^2*b^3)/3 + (5*A*a
*b^4)/3) + x^6*((10*A*a^3*b^2)/9 + (5*B*a^4*b)/9) + x^3*((B*a^5)/12 + (5*A
*a^4*b)/12) + x^9*((5*A*a^2*b^3)/3 + (5*B*a^3*b^2)/3))/x^15 + (B*b^5*x^3)/
3
```

3.49 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx$

3.49.1 Optimal result

Integrand size = 20, antiderivative size = 115

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx = -\frac{a^5A}{16x^{16}} - \frac{a^4(5Ab+aB)}{13x^{13}} - \frac{a^3b(2Ab+aB)}{2x^{10}} - \frac{10a^2b^2(Ab+aB)}{7x^7} - \frac{5ab^3(Ab+2aB)}{4x^4} - \frac{b^4(Ab+5aB)}{x} + \frac{1}{2}b^5Bx^2$$

output
```
-1/16*a^5*A/x^16-1/13*a^4*(5*A*b+B*a)/x^13-1/2*a^3*b*(2*A*b+B*a)/x^10-10/7
*a^2*b^2*(A*b+B*a)/x^7-5/4*a*b^3*(A*b+2*B*a)/x^4-b^4*(A*b+5*B*a)/x+1/2*b^5
*B*x^2
```

3.49.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx = -\frac{-728b^5x^{15}(-2A+Bx^3)+1820ab^4x^{12}(A+4Bx^3)+520a^2b^3x^9(4A+7Bx^3)+208a^3b^2x^6(7A+10Bx^3}{1456x^{16}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^17,x]
```

output
```
-1/1456*(-728*b^5*x^15*(-2*A + B*x^3) + 1820*a*b^4*x^12*(A + 4*B*x^3) + 52
0*a^2*b^3*x^9*(4*A + 7*B*x^3) + 208*a^3*b^2*x^6*(7*A + 10*B*x^3) + 56*a^4*
b*x^3*(10*A + 13*B*x^3) + 7*a^5*(13*A + 16*B*x^3))/x^16
```

3.49.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^{17}} \, dx$$

$$\Big\downarrow \ \color{blue}{950}$$

$$\int \left(\frac{a^5 A}{x^{17}}+\frac{a^4 (aB+5Ab)}{x^{14}}+\frac{5a^3 b (aB+2Ab)}{x^{11}}+\frac{10a^2 b^2 (aB+Ab)}{x^8}+\frac{b^4 (5aB+Ab)}{x^2}+\frac{5ab^3 (2aB+Ab)}{x^5}+b^5 Bx\right)$$

$$\Big\downarrow \ \color{blue}{2009}$$

$$-\frac{a^5 A}{16x^{16}}-\frac{a^4 (aB+5Ab)}{13x^{13}}-\frac{a^3 b (aB+2Ab)}{2x^{10}}-\frac{10a^2 b^2 (aB+Ab)}{7x^7}-\frac{b^4 (5aB+Ab)}{x}-\frac{5ab^3 (2aB+Ab)}{4x^4}+\frac{1}{2}b^5 Bx^2$$

input

```
Int[((a + b*x^3)^5*(A + B*x^3))/x^17,x]
```

output

```
-1/16*(a^5*A)/x^16 - (a^4*(5*A*b + a*B))/(13*x^13) - (a^3*b*(2*A*b + a*B))
/(2*x^10) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (5*a*b^3*(A*b + 2*a*B))/(4*
x^4) - (b^4*(A*b + 5*a*B))/x + (b^5*B*x^2)/2
```

3.49.3.1 Defintions of rubi rules used

rule 950

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.49.4 Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a^{5} A}{16 x^{16}}-\frac{a^{4} (5 A b+B a)}{13 x^{13}}-\frac{a^{3} b (2 A b+B a)}{2 x^{10}}-\frac{10 a^{2} b^{2} (A b+B a)}{7 x^{7}}-\frac{5 a\, b^{3} (A b+2 B a)}{4 x^{4}}-\frac{b^{4} (A b+5 B a)}{x}+\frac{b^{5} B\, x^{2}}{2}$
norman	$\frac{-\frac{a^{5} A}{16}+(-\frac{5}{13} a^{4} b A-\frac{1}{13} a^{5} B) x^{3}+(-a^{3} b^{2} A-\frac{1}{2} a^{4} b B) x^{6}+(-\frac{10}{7} a^{2} b^{3} A-\frac{10}{7} a^{3} b^{2} B) x^{9}+(-\frac{5}{4} a\, b^{4} A-\frac{5}{2} a^{2} b^{3} B) x^{12}+(-b^{5} A-5 a\, b^{4}}{x^{16}}$
risch	$\frac{b^{5} B\, x^{2}}{2}+\frac{-\frac{a^{5} A}{16}+(-\frac{5}{13} a^{4} b A-\frac{1}{13} a^{5} B) x^{3}+(-a^{3} b^{2} A-\frac{1}{2} a^{4} b B) x^{6}+(-\frac{10}{7} a^{2} b^{3} A-\frac{10}{7} a^{3} b^{2} B) x^{9}+(-\frac{5}{4} a\, b^{4} A-\frac{5}{2} a^{2} b^{3} B) x^{12}+(-}{x^{16}}$
gosper	$-\frac{-728 b^{5} B\, x^{18}+1456 A\, b^{5} x^{15}+7280 B a\, b^{4} x^{15}+1820 a A\, b^{4} x^{12}+3640 B\, a^{2} b^{3} x^{12}+2080 a^{2} A\, b^{3} x^{9}+2080 B\, a^{3} b^{2} x^{9}+1456 a^{3} A\, b^{2} x^{6}}{1456 x^{16}}$
parallelrisch	$-\frac{-728 b^{5} B\, x^{18}+1456 A\, b^{5} x^{15}+7280 B a\, b^{4} x^{15}+1820 a A\, b^{4} x^{12}+3640 B\, a^{2} b^{3} x^{12}+2080 a^{2} A\, b^{3} x^{9}+2080 B\, a^{3} b^{2} x^{9}+1456 a^{3} A\, b^{2} x^{6}}{1456 x^{16}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^17,x,method=_RETURNVERBOSE)
```

output
```
-1/16*a^5*A/x^16-1/13*a^4*(5*A*b+B*a)/x^13-1/2*a^3*b*(2*A*b+B*a)/x^10-10/7
*a^2*b^2*(A*b+B*a)/x^7-5/4*a*b^3*(A*b+2*B*a)/x^4-b^4*(A*b+5*B*a)/x+1/2*b^5
*B*x^2
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}}\,dx$$
$$=\frac{728\,Bb^5x^{18}-1456\,(5\,Bab^4+Ab^5)x^{15}-1820\,(2\,Ba^2b^3+Aab^4)x^{12}-2080\,(Ba^3b^2+Aa^2b^3)x^9-728\,(Ba^4}{1456\,x^{16}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="fricas")
```

output
```
1/1456*(728*B*b^5*x^18 - 1456*(5*B*a*b^4 + A*b^5)*x^15 - 1820*(2*B*a^2*b^3
 + A*a*b^4)*x^12 - 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 728*(B*a^4*b + 2*A*a
^3*b^2)*x^6 - 91*A*a^5 - 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^16
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**17,x)
```

output
```
Timed out
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx = \frac{1}{2}\,Bb^5x^2 - \frac{1456\,(5\,Bab^4+Ab^5)x^{15}+1820\,(2\,Ba^2b^3+Aab^4)x^{12}+2080\,(Ba^3b^2+Aa^2b^3)x^9+728\,(Ba^4b+2\,Aa^3b^2)}{1456\,x^{16}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="maxima")
```

output
```
1/2*B*b^5*x^2 - 1/1456*(1456*(5*B*a*b^4 + A*b^5)*x^15 + 1820*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 728*(B*a^4*b + 2*A*a^
3*b^2)*x^6 + 91*A*a^5 + 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^16
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx = \frac{1}{2}\,Bb^5x^2 - \frac{7280\,Bab^4x^{15}+1456\,Ab^5x^{15}+3640\,Ba^2b^3x^{12}+1820\,Aab^4x^{12}+2080\,Ba^3b^2x^9+2080\,Aa^2b^3x^9+728\,B}{1456\,x^{16}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="giac")
```

output
```
1/2*B*b^5*x^2 - 1/1456*(7280*B*a*b^4*x^15 + 1456*A*b^5*x^15 + 3640*B*a^2*b
^3*x^12 + 1820*A*a*b^4*x^12 + 2080*B*a^3*b^2*x^9 + 2080*A*a^2*b^3*x^9 + 72
8*B*a^4*b*x^6 + 1456*A*a^3*b^2*x^6 + 112*B*a^5*x^3 + 560*A*a^4*b*x^3 + 91*
A*a^5)/x^16
```

3.49.9 Mupad [B] (verification not implemented)

Time = 6.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}}\,dx = \frac{B\,b^5\,x^2}{2} - \frac{\frac{A\,a^5}{16} + x^6\left(\frac{B\,a^4\,b}{2} + A\,a^3\,b^2\right) + x^{12}\left(\frac{5\,B\,a^2\,b^3}{2} + \frac{5\,A\,a\,b^4}{4}\right) + x^3\left(\frac{B\,a^5}{13} + \frac{5\,A\,b\,a^4}{13}\right) + x^{15}\left(A\,b^5 + 5\,B\,a\,b^4\right) + x^9\left(\right.}{x^{16}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^17,x)
```

output
```
(B*b^5*x^2)/2 - ((A*a^5)/16 + x^6*(A*a^3*b^2 + (B*a^4*b)/2) + x^12*((5*B*a
^2*b^3)/2 + (5*A*a*b^4)/4) + x^3*((B*a^5)/13 + (5*A*a^4*b)/13) + x^15*(A*b
^5 + 5*B*a*b^4) + x^9*((10*A*a^2*b^3)/7 + (10*B*a^3*b^2)/7))/x^16
```

3.50 $\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{18}}\,dx$

3.50.1 Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{18}}\,dx=-\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{14x^{14}}-\frac{5a^3b(2Ab+aB)}{11x^{11}}-\frac{5a^2b^2(Ab+aB)}{4x^8}-\frac{ab^3(Ab+2aB)}{x^5}-\frac{b^4(Ab+5aB)}{2x^2}+b^5Bx$$

output
```
-1/17*a^5*A/x^17-1/14*a^4*(5*A*b+B*a)/x^14-5/11*a^3*b*(2*A*b+B*a)/x^11-5/4
*a^2*b^2*(A*b+B*a)/x^8-a*b^3*(A*b+2*B*a)/x^5-1/2*b^4*(A*b+5*B*a)/x^2+b^5*B
*x
```

3.50.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{18}}\,dx=-\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{14x^{14}}-\frac{5a^3b(2Ab+aB)}{11x^{11}}-\frac{5a^2b^2(Ab+aB)}{4x^8}-\frac{ab^3(Ab+2aB)}{x^5}-\frac{b^4(Ab+5aB)}{2x^2}+b^5Bx$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^18,x]
```

output
```
-1/17*(a^5*A)/x^17 - (a^4*(5*A*b + a*B))/(14*x^14) - (5*a^3*b*(2*A*b + a*B
))/(11*x^11) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5
 - (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x
```

3.50.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^{18}}dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5A}{x^{18}}+\frac{a^4(aB+5Ab)}{x^{15}}+\frac{5a^3b(aB+2Ab)}{x^{12}}+\frac{10a^2b^2(aB+Ab)}{x^9}+\frac{b^4(5aB+Ab)}{x^3}+\frac{5ab^3(2aB+Ab)}{x^6}+b^5B\right)d$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(aB+5Ab)}{14x^{14}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{5a^2b^2(aB+Ab)}{4x^8}-\frac{b^4(5aB+Ab)}{2x^2}-\frac{ab^3(2aB+Ab)}{x^5}+b^5Bx$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^18,x]
```

output
```
-1/17*(a^5*A)/x^17 - (a^4*(5*A*b + a*B))/(14*x^14) - (5*a^3*b*(2*A*b + a*B
))/(11*x^11) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5
 - (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x
```

3.50.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.50.4 Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^{5} A}{17 x^{17}}-\frac{a^{4} (5 A b+B a)}{14 x^{14}}-\frac{5 a^{3} b (2 A b+B a)}{11 x^{11}}-\frac{5 a^{2} b^{2} (A b+B a)}{4 x^{8}}-\frac{a\, b^{3} (A b+2 B a)}{x^{5}}-\frac{b^{4} (A b+5 B a)}{2 x^{2}}+b^{5} B x$
risch	$b^{5} B x+\frac{-\frac{a^{5} A}{17}+(-\frac{5}{14} a^{4} b A-\frac{1}{14} a^{5} B) x^{3}+(-\frac{10}{11} a^{3} b^{2} A-\frac{5}{11} a^{4} b B) x^{6}+(-\frac{5}{4} a^{2} b^{3} A-\frac{5}{4} a^{3} b^{2} B) x^{9}+(-a\, b^{4} A-2 a^{2} b^{3} B) x^{12}+(-}{x^{17}}$
norman	$\frac{-\frac{a^{5} A}{17}+(-\frac{5}{14} a^{4} b A-\frac{1}{14} a^{5} B) x^{3}+(-\frac{10}{11} a^{3} b^{2} A-\frac{5}{11} a^{4} b B) x^{6}+(-\frac{5}{4} a^{2} b^{3} A-\frac{5}{4} a^{3} b^{2} B) x^{9}+(-a\, b^{4} A-2 a^{2} b^{3} B) x^{12}+(-\frac{1}{2} b^{5} A-\frac{5}{2} a\, b}{x^{17}}$
gosper	$-\frac{-5236 b^{5} B\, x^{18}+2618 A\, b^{5} x^{15}+13090 B a\, b^{4} x^{15}+5236 a A\, b^{4} x^{12}+10472 B\, a^{2} b^{3} x^{12}+6545 a^{2} A\, b^{3} x^{9}+6545 B\, a^{3} b^{2} x^{9}+4760 a^{3} A\, b}{5236 x^{17}}$
parallelrisch	$-\frac{-5236 b^{5} B\, x^{18}+2618 A\, b^{5} x^{15}+13090 B a\, b^{4} x^{15}+5236 a A\, b^{4} x^{12}+10472 B\, a^{2} b^{3} x^{12}+6545 a^{2} A\, b^{3} x^{9}+6545 B\, a^{3} b^{2} x^{9}+4760 a^{3} A\, b}{5236 x^{17}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^18,x,method=_RETURNVERBOSE)
```

output
```
-1/17*a^5*A/x^17-1/14*a^4*(5*A*b+B*a)/x^14-5/11*a^3*b*(2*A*b+B*a)/x^11-5/4
*a^2*b^2*(A*b+B*a)/x^8-a*b^3*(A*b+2*B*a)/x^5-1/2*b^4*(A*b+5*B*a)/x^2+b^5*B
*x
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}}\, dx = \frac{5236\, Bb^5x^{18} - 2618\, (5\, Bab^4 + Ab^5)x^{15} - 5236\, (2\, Ba^2b^3 + Aab^4)x^{12} - 6545\, (Ba^3b^2 + Aa^2b^3)x^9 - 2380\, (B}{5236\, x^{17}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="fricas")
```

output
```
1/5236*(5236*B*b^5*x^18 - 2618*(5*B*a*b^4 + A*b^5)*x^15 - 5236*(2*B*a^2*b^
3 + A*a*b^4)*x^12 - 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 2380*(B*a^4*b + 2*A
*a^3*b^2)*x^6 - 308*A*a^5 - 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^17
```

3.50.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**18,x)
```

output
```
Timed out
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}}\,dx = Bb^5x \\ - \frac{2618\,(5\,Bab^4 + Ab^5)x^{15} + 5236\,(2\,Ba^2b^3 + Aab^4)x^{12} + 6545\,(Ba^3b^2 + Aa^2b^3)x^9 + 2380\,(Ba^4b + 2\,Aa^3b}{5236\,x^{17}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="maxima")
```

output
```
B*b^5*x - 1/5236*(2618*(5*B*a*b^4 + A*b^5)*x^15 + 5236*(2*B*a^2*b^3 + A*a*
b^4)*x^12 + 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 2380*(B*a^4*b + 2*A*a^3*b^2
)*x^6 + 308*A*a^5 + 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^17
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}}\,dx = Bb^5x \\ - \frac{13090\,Bab^4x^{15} + 2618\,Ab^5x^{15} + 10472\,Ba^2b^3x^{12} + 5236\,Aab^4x^{12} + 6545\,Ba^3b^2x^9 + 6545\,Aa^2b^3x^9 + 238}{5236\,x^{17}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="giac")
```

output
```
B*b^5*x - 1/5236*(13090*B*a*b^4*x^15 + 2618*A*b^5*x^15 + 10472*B*a^2*b^3*x
^12 + 5236*A*a*b^4*x^12 + 6545*B*a^3*b^2*x^9 + 6545*A*a^2*b^3*x^9 + 2380*B
*a^4*b*x^6 + 4760*A*a^3*b^2*x^6 + 374*B*a^5*x^3 + 1870*A*a^4*b*x^3 + 308*A
*a^5)/x^17
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}}\,dx = B\,b^5\,x$$
$$-\frac{\frac{A\,a^5}{17}+x^{12}\left(2\,B\,a^2\,b^3+A\,a\,b^4\right)+x^6\left(\frac{5\,B\,a^4\,b}{11}+\frac{10\,A\,a^3\,b^2}{11}\right)+x^3\left(\frac{B\,a^5}{14}+\frac{5\,A\,b\,a^4}{14}\right)+x^{15}\left(\frac{A\,b^5}{2}+\frac{5\,B\,a\,b^4}{2}\right)+x^9}{x^{17}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^18,x)
```

output
```
B*b^5*x - ((A*a^5)/17 + x^12*(2*B*a^2*b^3 + A*a*b^4) + x^6*((10*A*a^3*b^2)
/11 + (5*B*a^4*b)/11) + x^3*((B*a^5)/14 + (5*A*a^4*b)/14) + x^15*((A*b^5)/
2 + (5*B*a*b^4)/2) + x^9*((5*A*a^2*b^3)/4 + (5*B*a^3*b^2)/4))/x^17
```

3.51 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{19}}\,dx$

3.51.1 Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{19}}\,dx = -\frac{a^5B}{15x^{15}} - \frac{5a^4bB}{12x^{12}} - \frac{10a^3b^2B}{9x^9} - \frac{5a^2b^3B}{3x^6} - \frac{5ab^4B}{3x^3} - \frac{A(a+bx^3)^6}{18ax^{18}} + b^5B\log(x)$$

output
```
-1/15*a^5*B/x^15-5/12*a^4*b*B/x^12-10/9*a^3*b^2*B/x^9-5/3*a^2*b^3*B/x^6-5/
3*a*b^4*B/x^3-1/18*A*(b*x^3+a)^6/a/x^18+b^5*B*ln(x)
```

3.51.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{19}}\,dx = -\frac{60Ab^5x^{15} + 150ab^4x^{12}(A+2Bx^3) + 100a^2b^3x^9(2A+3Bx^3) + 50a^3b^2x^6(3A+4Bx^3) + 15a^4bx^3(4A+}{180x^{18}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^19,x]
```

output
```
-1/180*(60*A*b^5*x^15 + 150*a*b^4*x^12*(A + 2*B*x^3) + 100*a^2*b^3*x^9*(2*
A + 3*B*x^3) + 50*a^3*b^2*x^6*(3*A + 4*B*x^3) + 15*a^4*b*x^3*(4*A + 5*B*x^
3) + 2*a^5*(5*A + 6*B*x^3) - 180*b^5*B*x^18*Log[x])/x^18
```

3.51.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {948, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5 (Bx^3+A)}{x^{21}}dx^3$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{3}\left(B\int \frac{(bx^3+a)^5}{x^{18}}dx^3-\frac{A(a+bx^3)^6}{6ax^{18}}\right)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3}\left(B\int \left(\frac{a^5}{x^{18}}+\frac{5ba^4}{x^{15}}+\frac{10b^2a^3}{x^{12}}+\frac{10b^3a^2}{x^9}+\frac{5b^4a}{x^6}+\frac{b^5}{x^3}\right)dx^3-\frac{A(a+bx^3)^6}{6ax^{18}}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(B\left(-\frac{a^5}{5x^{15}}-\frac{5a^4b}{4x^{12}}-\frac{10a^3b^2}{3x^9}-\frac{5a^2b^3}{x^6}-\frac{5ab^4}{x^3}+b^5\log\left(x^3\right)\right)-\frac{A(a+bx^3)^6}{6ax^{18}}\right)$$

input

```
Int[((a + b*x^3)^5*(A + B*x^3))/x^19,x]
```

output

```
(-1/6*(A*(a + b*x^3)^6)/(a*x^18) + B*(-1/5*a^5/x^15 - (5*a^4*b)/(4*x^12) -
 (10*a^3*b^2)/(3*x^9) - (5*a^2*b^3)/x^6 - (5*a*b^4)/x^3 + b^5*Log[x^3]))/3
```

3.51.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.51.4 Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

method	result
default	$b^5 B \ln(x) - \frac{5a\,b^3(Ab+2Ba)}{6x^6} - \frac{5a^3b(2Ab+Ba)}{12x^{12}} - \frac{a^4(5Ab+Ba)}{15x^{15}} - \frac{a^5A}{18x^{18}} - \frac{b^4(Ab+5Ba)}{3x^3} - \frac{10a^2b^2(Ab+Ba)}{9x^9}$
norman	$\frac{(-\frac{1}{3}b^5A-\frac{5}{3}a\,b^4B)x^{15}+(-\frac{5}{6}a\,b^4A-\frac{5}{3}a^2b^3B)x^{12}+(-\frac{10}{9}a^2b^3A-\frac{10}{9}a^3b^2B)x^9+(-\frac{5}{6}a^3b^2A-\frac{5}{12}a^4bB)x^6+(-\frac{1}{3}a^4bA-\frac{1}{15}a^5B)}{x^{18}}$
risch	$\frac{(-\frac{1}{3}b^5A-\frac{5}{3}a\,b^4B)x^{15}+(-\frac{5}{6}a\,b^4A-\frac{5}{3}a^2b^3B)x^{12}+(-\frac{10}{9}a^2b^3A-\frac{10}{9}a^3b^2B)x^9+(-\frac{5}{6}a^3b^2A-\frac{5}{12}a^4bB)x^6+(-\frac{1}{3}a^4bA-\frac{1}{15}a^5B)}{x^{18}}$
parallelrisch	$-\frac{-180b^5B\ln(x)x^{18}+60A\,b^5x^{15}+300Ba\,b^4x^{15}+150aA\,b^4x^{12}+300B\,a^2b^3x^{12}+200a^2A\,b^3x^9+200B\,a^3b^2x^9+150a^3A\,b^2x^6+75}{180x^{18}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^19,x,method=_RETURNVERBOSE)
```

output
```
b^5*B*ln(x)-5/6*a*b^3*(A*b+2*B*a)/x^6-5/12*a^3*b*(2*A*b+B*a)/x^12-1/15*a^4
*(5*A*b+B*a)/x^15-1/18*a^5*A/x^18-1/3*b^4*(A*b+5*B*a)/x^3-10/9*a^2*b^2*(A*
b+B*a)/x^9
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{19}}\,dx$$
$$= \frac{180\,Bb^5x^{18}\log\left(x\right)-60\,(5\,Bab^4+Ab^5)x^{15}-150\,(2\,Ba^2b^3+Aab^4)x^{12}-200\,(Ba^3b^2+Aa^2b^3)x^9-75\,(Ba}{180\,x^{18}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="fricas")
```

output
```
1/180*(180*B*b^5*x^18*log(x) - 60*(5*B*a*b^4 + A*b^5)*x^15 - 150*(2*B*a^2*
b^3 + A*a*b^4)*x^12 - 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 75*(B*a^4*b + 2*A*
a^3*b^2)*x^6 - 10*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18
```

3.51.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{19}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**19,x)
```

output
```
Timed out
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{19}}\,dx = \frac{1}{3}\,Bb^5\log\left(x^3\right)$$
$$-\frac{60\,(5\,Bab^4+Ab^5)x^{15}+150\,(2\,Ba^2b^3+Aab^4)x^{12}+200\,(Ba^3b^2+Aa^2b^3)x^9+75\,(Ba^4b+2\,Aa^3b^2)x^6+}{180\,x^{18}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="maxima")
```

output
```
1/3*B*b^5*log(x^3) - 1/180*(60*(5*B*a*b^4 + A*b^5)*x^15 + 150*(2*B*a^2*b^3
 + A*a*b^4)*x^12 + 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 75*(B*a^4*b + 2*A*a^3
*b^2)*x^6 + 10*A*a^5 + 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{19}}\,dx = Bb^5\log\left(|x|\right)$$
$$-\frac{147\,Bb^5x^{18}+300\,Bab^4x^{15}+60\,Ab^5x^{15}+300\,Ba^2b^3x^{12}+150\,Aab^4x^{12}+200\,Ba^3b^2x^9+200\,Aa^2b^3x^9+}{180\,x^{18}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="giac")
```

output
```
B*b^5*log(abs(x)) - 1/180*(147*B*b^5*x^18 + 300*B*a*b^4*x^15 + 60*A*b^5*x^
15 + 300*B*a^2*b^3*x^12 + 150*A*a*b^4*x^12 + 200*B*a^3*b^2*x^9 + 200*A*a^2
*b^3*x^9 + 75*B*a^4*b*x^6 + 150*A*a^3*b^2*x^6 + 12*B*a^5*x^3 + 60*A*a^4*b*
x^3 + 10*A*a^5)/x^18
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}}\,dx = B\,b^5\,\ln(x)$$
$$-\frac{\frac{A\,a^5}{18}+x^{12}\left(\frac{5\,B\,a^2\,b^3}{3}+\frac{5\,A\,a\,b^4}{6}\right)+x^6\left(\frac{5\,B\,a^4\,b}{12}+\frac{5\,A\,a^3\,b^2}{6}\right)+x^3\left(\frac{B\,a^5}{15}+\frac{A\,b\,a^4}{3}\right)+x^{15}\left(\frac{A\,b^5}{3}+\frac{5\,B\,a\,b^4}{3}\right)+x^9\Bigg(}{x^{18}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^19,x)
```

output
```
B*b^5*log(x) - ((A*a^5)/18 + x^12*((5*B*a^2*b^3)/3 + (5*A*a*b^4)/6) + x^6*
((5*A*a^3*b^2)/6 + (5*B*a^4*b)/12) + x^3*((B*a^5)/15 + (A*a^4*b)/3) + x^15
*((A*b^5)/3 + (5*B*a*b^4)/3) + x^9*((10*A*a^2*b^3)/9 + (10*B*a^3*b^2)/9))/
x^18
```

3.52 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{20}}\,dx$

3.52.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{20}}\,dx = -\frac{a^5A}{19x^{19}} - \frac{a^4(5Ab+aB)}{16x^{16}} - \frac{5a^3b(2Ab+aB)}{13x^{13}} - \frac{a^2b^2(Ab+aB)}{x^{10}} - \frac{5ab^3(Ab+2aB)}{7x^7} - \frac{b^4(Ab+5aB)}{4x^4} - \frac{b^5B}{x}$$

output
```
-1/19*a^5*A/x^19-1/16*a^4*(5*A*b+B*a)/x^16-5/13*a^3*b*(2*A*b+B*a)/x^13-a^2
*b^2*(A*b+B*a)/x^10-5/7*a*b^3*(A*b+2*B*a)/x^7-1/4*b^4*(A*b+5*B*a)/x^4-b^5*
B/x
```

3.52.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{20}}\,dx = -\frac{6916b^5x^{15}(A+4Bx^3)+4940ab^4x^{12}(4A+7Bx^3)+3952a^2b^3x^9(7A+10Bx^3)+2128a^3b^2x^6(10A+13B}{27664x^{19}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^20,x]
```

output
```
-1/27664*(6916*b^5*x^15*(A + 4*B*x^3) + 4940*a*b^4*x^12*(4*A + 7*B*x^3) +
3952*a^2*b^3*x^9*(7*A + 10*B*x^3) + 2128*a^3*b^2*x^6*(10*A + 13*B*x^3) + 6
65*a^4*b*x^3*(13*A + 16*B*x^3) + 91*a^5*(16*A + 19*B*x^3))/x^19
```

3.52.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^{20}}dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5A}{x^{20}}+\frac{a^4(aB+5Ab)}{x^{17}}+\frac{5a^3b(aB+2Ab)}{x^{14}}+\frac{10a^2b^2(aB+Ab)}{x^{11}}+\frac{b^4(5aB+Ab)}{x^5}+\frac{5ab^3(2aB+Ab)}{x^8}+\frac{b^5B}{x^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{19x^{19}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{a^2b^2(aB+Ab)}{x^{10}}-\frac{b^4(5aB+Ab)}{4x^4}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^5B}{x}$$

input

```
Int[((a + b*x^3)^5*(A + B*x^3))/x^20,x]
```

output

```
-1/19*(a^5*A)/x^19 - (a^4*(5*A*b + a*B))/(16*x^16) - (5*a^3*b*(2*A*b + a*B
))/(13*x^13) - (a^2*b^2*(A*b + a*B))/x^10 - (5*a*b^3*(A*b + 2*a*B))/(7*x^7
) - (b^4*(A*b + 5*a*B))/(4*x^4) - (b^5*B)/x
```

3.52.3.1 Defintions of rubi rules used

rule 950

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.52.4 Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^5 A}{19x^{19}}-\frac{a^4(5Ab+Ba)}{16x^{16}}-\frac{5a^3b(2Ab+Ba)}{13x^{13}}-\frac{a^2b^2(Ab+Ba)}{x^{10}}-\frac{5a\,b^3(Ab+2Ba)}{7x^7}-\frac{b^4(Ab+5Ba)}{4x^4}-\frac{b^5B}{x}$
norman	$\frac{-\frac{a^5A}{19}+(-\frac{5}{16}a^4bA-\frac{1}{16}a^5B)x^3+(-\frac{10}{13}a^3b^2A-\frac{5}{13}a^4bB)x^6+(-a^2b^3A-a^3b^2B)x^9+(-\frac{5}{7}a\,b^4A-\frac{10}{7}a^2b^3B)x^{12}+(-\frac{1}{4}b^5A-\frac{5}{4}a\,b}{x^{19}}$
risch	$\frac{-\frac{a^5A}{19}+(-\frac{5}{16}a^4bA-\frac{1}{16}a^5B)x^3+(-\frac{10}{13}a^3b^2A-\frac{5}{13}a^4bB)x^6+(-a^2b^3A-a^3b^2B)x^9+(-\frac{5}{7}a\,b^4A-\frac{10}{7}a^2b^3B)x^{12}+(-\frac{1}{4}b^5A-\frac{5}{4}a\,b}{x^{19}}$
gosper	$-\frac{27664b^5B\,x^{18}+6916A\,b^5x^{15}+34580Ba\,b^4x^{15}+19760aA\,b^4x^{12}+39520B\,a^2b^3x^{12}+27664a^2A\,b^3x^9+27664B\,a^3b^2x^9+21280a}{27664x^{19}}$
parallelrisch	$-\frac{27664b^5B\,x^{18}+6916A\,b^5x^{15}+34580Ba\,b^4x^{15}+19760aA\,b^4x^{12}+39520B\,a^2b^3x^{12}+27664a^2A\,b^3x^9+27664B\,a^3b^2x^9+21280a}{27664x^{19}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^20,x,method=_RETURNVERBOSE)
```

output
```
-1/19*a^5*A/x^19-1/16*a^4*(5*A*b+B*a)/x^16-5/13*a^3*b*(2*A*b+B*a)/x^13-a^2
*b^2*(A*b+B*a)/x^10-5/7*a*b^3*(A*b+2*B*a)/x^7-1/4*b^4*(A*b+5*B*a)/x^4-b^5*
B/x
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{20}}\,dx =$$
$$-\frac{27664\,Bb^5x^{18}+6916\,(5\,Bab^4+Ab^5)x^{15}+19760\,(2\,Ba^2b^3+Aab^4)x^{12}+27664\,(Ba^3b^2+Aa^2b^3)x^9+10}{27664\,x^{19}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="fricas")
```

output
```
-1/27664*(27664*B*b^5*x^18 + 6916*(5*B*a*b^4 + A*b^5)*x^15 + 19760*(2*B*a^
2*b^3 + A*a*b^4)*x^12 + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b
 + 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^19
```

3.52.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**20,x)
```

output
```
Timed out
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}}\,dx = -\frac{27664\,Bb^5x^{18} + 6916\,(5\,Bab^4 + Ab^5)x^{15} + 19760\,(2\,Ba^2b^3 + Aab^4)x^{12} + 27664\,(Ba^3b^2 + Aa^2b^3)x^9 + 10}{27664\,x^{19}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="maxima")
```

output
```
-1/27664*(27664*B*b^5*x^18 + 6916*(5*B*a*b^4 + A*b^5)*x^15 + 19760*(2*B*a^
2*b^3 + A*a*b^4)*x^12 + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b
 + 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^19
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}}\,dx = -\frac{27664\,Bb^5x^{18} + 34580\,Bab^4x^{15} + 6916\,Ab^5x^{15} + 39520\,Ba^2b^3x^{12} + 19760\,Aab^4x^{12} + 27664\,Ba^3b^2x^9 + 2}{27664\,x^{19}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="giac")
```

output
```
-1/27664*(27664*B*b^5*x^18 + 34580*B*a*b^4*x^15 + 6916*A*b^5*x^15 + 39520*
B*a^2*b^3*x^12 + 19760*A*a*b^4*x^12 + 27664*B*a^3*b^2*x^9 + 27664*A*a^2*b^
3*x^9 + 10640*B*a^4*b*x^6 + 21280*A*a^3*b^2*x^6 + 1729*B*a^5*x^3 + 8645*A*
a^4*b*x^3 + 1456*A*a^5)/x^19
```

3.52.9 Mupad [B] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}}\,dx =$$

$$-\frac{\frac{A\,a^5}{19}+x^{12}\left(\frac{10\,B\,a^2\,b^3}{7}+\frac{5\,A\,a\,b^4}{7}\right)+x^6\left(\frac{5\,B\,a^4\,b}{13}+\frac{10\,A\,a^3\,b^2}{13}\right)+x^3\left(\frac{B\,a^5}{16}+\frac{5\,A\,b\,a^4}{16}\right)+x^{15}\left(\frac{A\,b^5}{4}+\frac{5\,B\,a\,b^4}{4}\right)+x^9}{x^{19}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^20,x)
```

output
```
-((A*a^5)/19 + x^12*((10*B*a^2*b^3)/7 + (5*A*a*b^4)/7) + x^6*((10*A*a^3*b^
2)/13 + (5*B*a^4*b)/13) + x^3*((B*a^5)/16 + (5*A*a^4*b)/16) + x^15*((A*b^5
)/4 + (5*B*a*b^4)/4) + x^9*(A*a^2*b^3 + B*a^3*b^2) + B*b^5*x^18)/x^19
```

3.53 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{21}}\,dx$

3.53.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{21}}\,dx = -\frac{a^5A}{20x^{20}} - \frac{a^4(5Ab+aB)}{17x^{17}} - \frac{5a^3b(2Ab+aB)}{14x^{14}} - \frac{10a^2b^2(Ab+aB)}{11x^{11}} - \frac{5ab^3(Ab+2aB)}{8x^8} - \frac{b^4(Ab+5aB)}{5x^5} - \frac{b^5B}{2x^2}$$

output
```
-1/20*a^5*A/x^20-1/17*a^4*(5*A*b+B*a)/x^17-5/14*a^3*b*(2*A*b+B*a)/x^14-10/
11*a^2*b^2*(A*b+B*a)/x^11-5/8*a*b^3*(A*b+2*B*a)/x^8-1/5*b^4*(A*b+5*B*a)/x^
5-1/2*b^5*B/x^2
```

3.53.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{21}}\,dx = -\frac{5236b^5x^{15}(2A+5Bx^3)+6545ab^4x^{12}(5A+8Bx^3)+5950a^2b^3x^9(8A+11Bx^3)+3400a^3b^2x^6(11A+14}{52360x^{20}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^21,x]
```

output
```
-1/52360*(5236*b^5*x^15*(2*A + 5*B*x^3) + 6545*a*b^4*x^12*(5*A + 8*B*x^3)
+ 5950*a^2*b^3*x^9*(8*A + 11*B*x^3) + 3400*a^3*b^2*x^6*(11*A + 14*B*x^3) +
 1100*a^4*b*x^3*(14*A + 17*B*x^3) + 154*a^5*(17*A + 20*B*x^3))/x^20
```

3.53.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{21}}\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^5A}{x^{21}}+\frac{a^4(aB+5Ab)}{x^{18}}+\frac{5a^3b(aB+2Ab)}{x^{15}}+\frac{10a^2b^2(aB+Ab)}{x^{12}}+\frac{b^4(5aB+Ab)}{x^6}+\frac{5ab^3(2aB+Ab)}{x^9}+\frac{b^5B}{x^3}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{20x^{20}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{5ab^3(2aB+Ab)}{8x^8}-\frac{b^5B}{2x^2}$$

input

```
Int[((a + b*x^3)^5*(A + B*x^3))/x^21,x]
```

output

```
-1/20*(a^5*A)/x^20 - (a^4*(5*A*b + a*B))/(17*x^17) - (5*a^3*b*(2*A*b + a*B
))/(14*x^14) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B)
)/(8*x^8) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(2*x^2)
```

3.53.3.1 Defintions of rubi rules used

rule 950

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.53.4 Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^{5}A}{20x^{20}}-\frac{a^{4}(5Ab+Ba)}{17x^{17}}-\frac{5a^{3}b(2Ab+Ba)}{14x^{14}}-\frac{10a^{2}b^{2}(Ab+Ba)}{11x^{11}}-\frac{5a\,b^{3}(Ab+2Ba)}{8x^{8}}-\frac{b^{4}(Ab+5Ba)}{5x^{5}}-\frac{b^{5}B}{2x^{2}}$
norman	$\frac{-\frac{a^{5}A}{20}+\left(-\frac{5}{17}a^{4}bA-\frac{1}{17}a^{5}B\right)x^{3}+\left(-\frac{5}{7}a^{3}b^{2}A-\frac{5}{14}a^{4}bB\right)x^{6}+\left(-\frac{10}{11}a^{2}b^{3}A-\frac{10}{11}a^{3}b^{2}B\right)x^{9}+\left(-\frac{5}{8}a\,b^{4}A-\frac{5}{4}a^{2}b^{3}B\right)x^{12}+\left(-\frac{1}{5}b^{5}A-a\right.}{x^{20}}$
risch	$\frac{-\frac{a^{5}A}{20}+\left(-\frac{5}{17}a^{4}bA-\frac{1}{17}a^{5}B\right)x^{3}+\left(-\frac{5}{7}a^{3}b^{2}A-\frac{5}{14}a^{4}bB\right)x^{6}+\left(-\frac{10}{11}a^{2}b^{3}A-\frac{10}{11}a^{3}b^{2}B\right)x^{9}+\left(-\frac{5}{8}a\,b^{4}A-\frac{5}{4}a^{2}b^{3}B\right)x^{12}+\left(-\frac{1}{5}b^{5}A-a\right.}{x^{20}}$
gosper	$-\frac{26180b^{5}B\,x^{18}+10472A\,b^{5}x^{15}+52360Ba\,b^{4}x^{15}+32725aA\,b^{4}x^{12}+65450B\,a^{2}b^{3}x^{12}+47600a^{2}A\,b^{3}x^{9}+47600B\,a^{3}b^{2}x^{9}+37400}{52360x^{20}}$
parallelrisch	$-\frac{26180b^{5}B\,x^{18}+10472A\,b^{5}x^{15}+52360Ba\,b^{4}x^{15}+32725aA\,b^{4}x^{12}+65450B\,a^{2}b^{3}x^{12}+47600a^{2}A\,b^{3}x^{9}+47600B\,a^{3}b^{2}x^{9}+37400}{52360x^{20}}$

input

```
int((b*x^3+a)^5*(B*x^3+A)/x^21,x,method=_RETURNVERBOSE)
```

output

```
-1/20*a^5*A/x^20-1/17*a^4*(5*A*b+B*a)/x^17-5/14*a^3*b*(2*A*b+B*a)/x^14-10/
11*a^2*b^2*(A*b+B*a)/x^11-5/8*a*b^3*(A*b+2*B*a)/x^8-1/5*b^4*(A*b+5*B*a)/x^
5-1/2*b^5*B/x^2
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{21}}\,dx =$$

$$-\frac{26180\,Bb^{5}x^{18}+10472\,(5\,Bab^{4}+Ab^{5})x^{15}+32725\,(2\,Ba^{2}b^{3}+Aab^{4})x^{12}+47600\,(Ba^{3}b^{2}+Aa^{2}b^{3})x^{9}+1}{52360\,x^{20}}$$

input

```
integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="fricas")
```

output

```
-1/52360*(26180*B*b^5*x^18 + 10472*(5*B*a*b^4 + A*b^5)*x^15 + 32725*(2*B*a
^2*b^3 + A*a*b^4)*x^12 + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*
b + 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^20
```

3.53.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**21,x)
```

output
```
Timed out
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}}\,dx = -\frac{26180\,Bb^5x^{18} + 10472\,(5\,Bab^4 + Ab^5)x^{15} + 32725\,(2\,Ba^2b^3 + Aab^4)x^{12} + 47600\,(Ba^3b^2 + Aa^2b^3)x^9 + 1}{52360\,x^{20}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="maxima")
```

output
```
-1/52360*(26180*B*b^5*x^18 + 10472*(5*B*a*b^4 + A*b^5)*x^15 + 32725*(2*B*a
^2*b^3 + A*a*b^4)*x^12 + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*
b + 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^20
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}}\,dx = -\frac{26180\,Bb^5x^{18} + 52360\,Bab^4x^{15} + 10472\,Ab^5x^{15} + 65450\,Ba^2b^3x^{12} + 32725\,Aab^4x^{12} + 47600\,Ba^3b^2x^9 +}{52360\,x^{20}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="giac")
```

output
```
-1/52360*(26180*B*b^5*x^18 + 52360*B*a*b^4*x^15 + 10472*A*b^5*x^15 + 65450
*B*a^2*b^3*x^12 + 32725*A*a*b^4*x^12 + 47600*B*a^3*b^2*x^9 + 47600*A*a^2*b
^3*x^9 + 18700*B*a^4*b*x^6 + 37400*A*a^3*b^2*x^6 + 3080*B*a^5*x^3 + 15400*
A*a^4*b*x^3 + 2618*A*a^5)/x^20
```

3.53.9 Mupad [B] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}} \, dx =$$

$$-\frac{\frac{A\,a^5}{20}+x^{12}\left(\frac{5\,B\,a^2\,b^3}{4}+\frac{5\,A\,a\,b^4}{8}\right)+x^6\left(\frac{5\,B\,a^4\,b}{14}+\frac{5\,A\,a^3\,b^2}{7}\right)+x^3\left(\frac{B\,a^5}{17}+\frac{5\,A\,b\,a^4}{17}\right)+x^{15}\left(\frac{A\,b^5}{5}+B\,a\,b^4\right)+x^9\,(}{x^{20}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^21,x)
```

output
```
-((A*a^5)/20 + x^12*((5*B*a^2*b^3)/4 + (5*A*a*b^4)/8) + x^6*((5*A*a^3*b^2)
/7 + (5*B*a^4*b)/14) + x^3*((B*a^5)/17 + (5*A*a^4*b)/17) + x^15*((A*b^5)/5
 + B*a*b^4) + x^9*((10*A*a^2*b^3)/11 + (10*B*a^3*b^2)/11) + (B*b^5*x^18)/2
)/x^20
```

3.54 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}}\,dx$

3.54.1 Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}}\,dx = -\frac{A(a+bx^3)^6}{21ax^{21}} + \frac{(Ab-7aB)(a+bx^3)^6}{126a^2x^{18}}$$

output
```
-1/21*A*(b*x^3+a)^6/a/x^21+1/126*(A*b-7*B*a)*(b*x^3+a)^6/a^2/x^18
```

3.54.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 118 vs. 2(48) = 96.

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.46

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}}\,dx = -\frac{21b^5x^{15}(A+2Bx^3)+35ab^4x^{12}(2A+3Bx^3)+35a^2b^3x^9(3A+4Bx^3)+21a^3b^2x^6(4A+5Bx^3)+7a^4bx}{126x^{21}}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^22,x]
```

output
```
-1/126*(21*b^5*x^15*(A + 2*B*x^3) + 35*a*b^4*x^12*(2*A + 3*B*x^3) + 35*a^2
*b^3*x^9*(3*A + 4*B*x^3) + 21*a^3*b^2*x^6*(4*A + 5*B*x^3) + 7*a^4*b*x^3*(5
*A + 6*B*x^3) + a^5*(6*A + 7*B*x^3))/x^21
```

3.54.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{22}} \, dx$$

$$\Big\downarrow \color{blue}{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5 (Bx^3+A)}{x^{24}}dx^3$$

$$\Big\downarrow \color{blue}{87}$$

$$\frac{1}{3}\left(-\frac{(Ab-7aB)\int \frac{(bx^3+a)^5}{x^{21}}dx^3}{7a}-\frac{A(a+bx^3)^6}{7ax^{21}}\right)$$

$$\Big\downarrow \color{blue}{48}$$

$$\frac{1}{3}\left(\frac{(a+bx^3)^6 (Ab-7aB)}{42a^2x^{18}}-\frac{A(a+bx^3)^6}{7ax^{21}}\right)$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^22,x]
```

output
```
(-1/7*(A*(a + b*x^3)^6)/(a*x^21) + ((A*b - 7*a*B)*(a + b*x^3)^6)/(42*a^2*x
^18))/3
```

3.54.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.54.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(44) = 88$.

Time = 4.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.17

method	result
default	$-\frac{b^4(Ab+5Ba)}{6x^6}-\frac{5a^2b^2(Ab+Ba)}{6x^{12}}-\frac{a^5A}{21x^{21}}-\frac{a^3b(2Ab+Ba)}{3x^{15}}-\frac{a^4(5Ab+Ba)}{18x^{18}}-\frac{b^5B}{3x^3}-\frac{5a\,b^3(Ab+2Ba)}{9x^9}$
norman	$\frac{-\frac{a^5A}{21}+(-\frac{5}{18}a^4bA-\frac{1}{18}a^5B)x^3+(-\frac{2}{3}a^3b^2A-\frac{1}{3}a^4bB)x^6+(-\frac{5}{6}a^2b^3A-\frac{5}{6}a^3b^2B)x^9+(-\frac{5}{9}a\,b^4A-\frac{10}{9}a^2b^3B)x^{12}+(-\frac{1}{6}b^5A-\frac{5}{6}a}{x^{21}}$
risch	$\frac{-\frac{a^5A}{21}+(-\frac{5}{18}a^4bA-\frac{1}{18}a^5B)x^3+(-\frac{2}{3}a^3b^2A-\frac{1}{3}a^4bB)x^6+(-\frac{5}{6}a^2b^3A-\frac{5}{6}a^3b^2B)x^9+(-\frac{5}{9}a\,b^4A-\frac{10}{9}a^2b^3B)x^{12}+(-\frac{1}{6}b^5A-\frac{5}{6}a}{x^{21}}$
gosper	$-\frac{42b^5B\,x^{18}+21A\,b^5x^{15}+105Ba\,b^4x^{15}+70aA\,b^4x^{12}+140B\,a^2b^3x^{12}+105a^2A\,b^3x^9+105B\,a^3b^2x^9+84a^3A\,b^2x^6+42B\,a^4b\,x^6+}{126x^{21}}$
parallelrisch	$-\frac{42b^5B\,x^{18}+21A\,b^5x^{15}+105Ba\,b^4x^{15}+70aA\,b^4x^{12}+140B\,a^2b^3x^{12}+105a^2A\,b^3x^9+105B\,a^3b^2x^9+84a^3A\,b^2x^6+42B\,a^4b\,x^6+}{126x^{21}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^22,x,method=_RETURNVERBOSE)
```

output
```
-1/6*b^4*(A*b+5*B*a)/x^6-5/6*a^2*b^2*(A*b+B*a)/x^12-1/21*a^5*A/x^21-1/3*a^
3*b*(2*A*b+B*a)/x^15-1/18*a^4*(5*A*b+B*a)/x^18-1/3*b^5*B/x^3-5/9*a*b^3*(A*
b+2*B*a)/x^9
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(45) = 90$.

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.52

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{22}}\,dx = -\frac{42\,Bb^5x^{18} + 21\,(5\,Bab^4 + Ab^5)x^{15} + 70\,(2\,Ba^2b^3 + Aab^4)x^{12} + 105\,(Ba^3b^2 + Aa^2b^3)x^9 + 42\,(Ba^4b + 2}{126\,x^{21}}$$

input

```
integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="fricas")
```

output

```
-1/126*(42*B*b^5*x^18 + 21*(5*B*a*b^4 + A*b^5)*x^15 + 70*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)
*x^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^21
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{22}}\,dx = \text{Timed out}$$

input

```
integrate((b*x**3+a)**5*(B*x**3+A)/x**22,x)
```

output

```
Timed out
```

3.54.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(45) = 90$.

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.52

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{22}}\,dx = -\frac{42\,Bb^5x^{18} + 21\,(5\,Bab^4 + Ab^5)x^{15} + 70\,(2\,Ba^2b^3 + Aab^4)x^{12} + 105\,(Ba^3b^2 + Aa^2b^3)x^9 + 42\,(Ba^4b + 2}{126\,x^{21}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="maxima")
```

output
```
-1/126*(42*B*b^5*x^18 + 21*(5*B*a*b^4 + A*b^5)*x^15 + 70*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)
*x^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^21
```

3.54.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(45) = 90$.

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.65

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{22}}\,dx =$$
$$-\frac{42\,Bb^5x^{18}+105\,Bab^4x^{15}+21\,Ab^5x^{15}+140\,Ba^2b^3x^{12}+70\,Aab^4x^{12}+105\,Ba^3b^2x^9+105\,Aa^2b^3x^9+42}{126\,x^{21}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="giac")
```

output
```
-1/126*(42*B*b^5*x^18 + 105*B*a*b^4*x^15 + 21*A*b^5*x^15 + 140*B*a^2*b^3*x
^12 + 70*A*a*b^4*x^12 + 105*B*a^3*b^2*x^9 + 105*A*a^2*b^3*x^9 + 42*B*a^4*b
*x^6 + 84*A*a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 6*A*a^5)/x^21
```

3.54.9 Mupad [B] (verification not implemented)

Time = 6.77 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{22}}\,dx =$$
$$-\frac{\frac{A\,a^5}{21}+x^6\left(\frac{B\,a^4\,b}{3}+\frac{2\,A\,a^3\,b^2}{3}\right)+x^{12}\left(\frac{10\,B\,a^2\,b^3}{9}+\frac{5\,A\,a\,b^4}{9}\right)+x^3\left(\frac{B\,a^5}{18}+\frac{5\,A\,b\,a^4}{18}\right)+x^{15}\left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right)+x^9\,\Bigg(}{x^{21}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^22,x)
```

output
```
-((A*a^5)/21 + x^6*((2*A*a^3*b^2)/3 + (B*a^4*b)/3) + x^12*((10*B*a^2*b^3)/
9 + (5*A*a*b^4)/9) + x^3*((B*a^5)/18 + (5*A*a^4*b)/18) + x^15*((A*b^5)/6 +
 (5*B*a*b^4)/6) + x^9*((5*A*a^2*b^3)/6 + (5*B*a^3*b^2)/6) + (B*b^5*x^18)/3
)/x^21
```

3.55 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}}\,dx$

3.55.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}}\,dx = -\frac{a^5A}{22x^{22}}-\frac{a^4(5Ab+aB)}{19x^{19}}-\frac{5a^3b(2Ab+aB)}{16x^{16}} -\frac{10a^2b^2(Ab+aB)}{13x^{13}}-\frac{ab^3(Ab+2aB)}{2x^{10}}-\frac{b^4(Ab+5aB)}{7x^7}-\frac{b^5B}{4x^4}$$

output
```
-1/22*a^5*A/x^22-1/19*a^4*(5*A*b+B*a)/x^19-5/16*a^3*b*(2*A*b+B*a)/x^16-10/
13*a^2*b^2*(A*b+B*a)/x^13-1/2*a*b^3*(A*b+2*B*a)/x^10-1/7*b^4*(A*b+5*B*a)/x
^7-1/4*b^5*B/x^4
```

3.55.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}}\,dx = -\frac{a^5A}{22x^{22}}-\frac{a^4(5Ab+aB)}{19x^{19}}-\frac{5a^3b(2Ab+aB)}{16x^{16}} -\frac{10a^2b^2(Ab+aB)}{13x^{13}}-\frac{ab^3(Ab+2aB)}{2x^{10}}-\frac{b^4(Ab+5aB)}{7x^7}-\frac{b^5B}{4x^4}$$

input
```
Integrate[((a + b*x^3)^5*(A + B*x^3))/x^23,x]
```

output
```
-1/22*(a^5*A)/x^22 - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B
))/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/
(2*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)
```

3.55.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^{23}} \, dx$$

$$\Big\downarrow \; 950$$

$$\int \left(\frac{a^5 A}{x^{23}}+\frac{a^4 (aB+5Ab)}{x^{20}}+\frac{5a^3 b(aB+2Ab)}{x^{17}}+\frac{10a^2 b^2 (aB+Ab)}{x^{14}}+\frac{b^4 (5aB+Ab)}{x^8}+\frac{5ab^3 (2aB+Ab)}{x^{11}}+\frac{b^5 B}{x^5}\right)$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^5 A}{22x^{22}}-\frac{a^4 (aB+5Ab)}{19x^{19}}-\frac{5a^3 b(aB+2Ab)}{16x^{16}}-\frac{10a^2 b^2 (aB+Ab)}{13x^{13}}-\frac{b^4 (5aB+Ab)}{7x^7}-\frac{ab^3 (2aB+Ab)}{2x^{10}}-\frac{b^5 B}{4x^4}$$

input
```
Int[((a + b*x^3)^5*(A + B*x^3))/x^23,x]
```

output
```
-1/22*(a^5*A)/x^22 - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B
))/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/
(2*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)
```

3.55.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.55.4 Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^{5} A}{22 x^{22}}-\frac{a^{4} (5 A b+B a)}{19 x^{19}}-\frac{5 a^{3} b (2 A b+B a)}{16 x^{16}}-\frac{10 a^{2} b^{2} (A b+B a)}{13 x^{13}}-\frac{a\,b^{3} (A b+2 B a)}{2 x^{10}}-\frac{b^{4} (A b+5 B a)}{7 x^{7}}-\frac{b^{5} B}{4 x^{4}}$
norman	$\frac{-\frac{a^{5} A}{22}+\left(-\frac{5}{19} a^{4} b A-\frac{1}{19} a^{5} B\right) x^{3}+\left(-\frac{5}{8} a^{3} b^{2} A-\frac{5}{16} a^{4} b B\right) x^{6}+\left(-\frac{10}{13} a^{2} b^{3} A-\frac{10}{13} a^{3} b^{2} B\right) x^{9}+\left(-\frac{1}{2} a\,b^{4} A-a^{2} b^{3} B\right) x^{12}+\left(-\frac{1}{7} b^{5} A-\frac{5}{7} a\right.}{x^{22}}$
risch	$\frac{-\frac{a^{5} A}{22}+\left(-\frac{5}{19} a^{4} b A-\frac{1}{19} a^{5} B\right) x^{3}+\left(-\frac{5}{8} a^{3} b^{2} A-\frac{5}{16} a^{4} b B\right) x^{6}+\left(-\frac{10}{13} a^{2} b^{3} A-\frac{10}{13} a^{3} b^{2} B\right) x^{9}+\left(-\frac{1}{2} a\,b^{4} A-a^{2} b^{3} B\right) x^{12}+\left(-\frac{1}{7} b^{5} A-\frac{5}{7} a\right.}{x^{22}}$
gosper	$-\frac{76076 b^{5} B\,x^{18}+43472 A\,b^{5} x^{15}+217360 B a\,b^{4} x^{15}+152152 a A\,b^{4} x^{12}+304304 B\,a^{2} b^{3} x^{12}+234080 a^{2} A\,b^{3} x^{9}+234080 B\,a^{3} b^{2} x^{9}+}{304304 x^{22}}$
parallelrisch	$-\frac{76076 b^{5} B\,x^{18}+43472 A\,b^{5} x^{15}+217360 B a\,b^{4} x^{15}+152152 a A\,b^{4} x^{12}+304304 B\,a^{2} b^{3} x^{12}+234080 a^{2} A\,b^{3} x^{9}+234080 B\,a^{3} b^{2} x^{9}+}{304304 x^{22}}$

input
```
int((b*x^3+a)^5*(B*x^3+A)/x^23,x,method=_RETURNVERBOSE)
```

output
```
-1/22*a^5*A/x^22-1/19*a^4*(5*A*b+B*a)/x^19-5/16*a^3*b*(2*A*b+B*a)/x^16-10/
13*a^2*b^2*(A*b+B*a)/x^13-1/2*a*b^3*(A*b+2*B*a)/x^10-1/7*b^4*(A*b+5*B*a)/x
^7-1/4*b^5*B/x^4
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+b x^3\right)^5 \left(A+B x^3\right)}{x^{23}}\,dx = -\frac{76076\,B b^{5} x^{18} + 43472\,(5\,B a b^{4} + A b^{5}) x^{15} + 152152\,(2\,B a^{2} b^{3} + A a b^{4}) x^{12} + 234080\,(B a^{3} b^{2} + A a^{2} b^{3}) x^{9} +}{304304\,x^{22}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="fricas")
```

output
```
-1/304304*(76076*B*b^5*x^18 + 43472*(5*B*a*b^4 + A*b^5)*x^15 + 152152*(2*B
*a^2*b^3 + A*a*b^4)*x^12 + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a
^4*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^2
2
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**5*(B*x**3+A)/x**23,x)
```

output
```
Timed out
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}}\,dx =$$
$$-\frac{76076\,Bb^5x^{18} + 43472\,(5\,Bab^4 + Ab^5)x^{15} + 152152\,(2\,Ba^2b^3 + Aab^4)x^{12} + 234080\,(Ba^3b^2 + Aa^2b^3)x^9}{304304\,x^{22}}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="maxima")
```

output
```
-1/304304*(76076*B*b^5*x^18 + 43472*(5*B*a*b^4 + A*b^5)*x^15 + 152152*(2*B
*a^2*b^3 + A*a*b^4)*x^12 + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a
^4*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^2
2
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}}\,dx =$$
$$-\frac{76076\,Bb^5x^{18} + 217360\,Bab^4x^{15} + 43472\,Ab^5x^{15} + 304304\,Ba^2b^3x^{12} + 152152\,Aab^4x^{12} + 234080\,Ba^3b^2}{30430}$$

input
```
integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="giac")
```

output
```
-1/304304*(76076*B*b^5*x^18 + 217360*B*a*b^4*x^15 + 43472*A*b^5*x^15 + 304
304*B*a^2*b^3*x^12 + 152152*A*a*b^4*x^12 + 234080*B*a^3*b^2*x^9 + 234080*A
*a^2*b^3*x^9 + 95095*B*a^4*b*x^6 + 190190*A*a^3*b^2*x^6 + 16016*B*a^5*x^3
+ 80080*A*a^4*b*x^3 + 13832*A*a^5)/x^22
```

3.55.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}}\,dx =$$
$$-\frac{\frac{A\,a^5}{22} + x^{12}\left(B\,a^2\,b^3 + \frac{A\,a\,b^4}{2}\right) + x^6\left(\frac{5\,B\,a^4\,b}{16} + \frac{5\,A\,a^3\,b^2}{8}\right) + x^3\left(\frac{B\,a^5}{19} + \frac{5\,A\,b\,a^4}{19}\right) + x^{15}\left(\frac{A\,b^5}{7} + \frac{5\,B\,a\,b^4}{7}\right) + x^9\Bigg(}{x^{22}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^5)/x^23,x)
```

output
```
-((A*a^5)/22 + x^12*(B*a^2*b^3 + (A*a*b^4)/2) + x^6*((5*A*a^3*b^2)/8 + (5*
B*a^4*b)/16) + x^3*((B*a^5)/19 + (5*A*a^4*b)/19) + x^15*((A*b^5)/7 + (5*B*
a*b^4)/7) + x^9*((10*A*a^2*b^3)/13 + (10*B*a^3*b^2)/13) + (B*b^5*x^18)/4)/
x^22
```

3.56 $\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx$

3.56.1 Optimal result

Integrand size = 20, antiderivative size = 183

$$\begin{aligned}\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = & -\frac{a(Ab-aB)x}{b^3}+\frac{(Ab-aB)x^4}{4b^2}+\frac{Bx^7}{7b} \\ & -\frac{a^{4/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}} \\ & +\frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}} \\ & -\frac{a^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}}\end{aligned}$$

output
```
-a*(A*b-B*a)*x/b^3+1/4*(A*b-B*a)*x^4/b^2+1/7*B*x^7/b+1/3*a^(4/3)*(A*b-B*a)
*ln(a^(1/3)+b^(1/3)*x)/b^(10/3)-1/6*a^(4/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/b^(10/3)-1/3*a^(4/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(10/3)*3^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.93

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx$$

$$= \frac{84a\sqrt[3]{b}(-Ab+aB)x+21b^{4/3}(Ab-aB)x^4+12b^{7/3}Bx^7+28\sqrt{3}a^{4/3}(-Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-28}{84b^{10/3}}$$

input
```
Integrate[(x^6*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(84*a*b^(1/3)*(-(A*b) + a*B)*x + 21*b^(4/3)*(A*b - a*B)*x^4 + 12*b^(7/3)*B
*x^7 + 28*Sqrt[3]*a^(4/3)*(-(A*b) + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]] - 28*a^(4/3)*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/
3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(84*b^(1
0/3))
```

3.56.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {959, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(Ab-aB)\int \frac{x^6}{bx^3+a}dx}{b}+\frac{Bx^7}{7b}$$

$$\Big\downarrow \; 831$$

$$\frac{(Ab-aB)\int \left(\frac{x^3}{b}+\frac{a^2}{b^2(bx^3+a)}-\frac{a}{b^2}\right)dx}{b}+\frac{Bx^7}{7b}$$

$$\Big\downarrow \; 2009$$

$$\frac{(Ab-aB)\left(-\frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}}-\frac{ax}{b^2}+\frac{x^4}{4b}\right)}{b}+\frac{Bx^7}{7b}$$

input

```
Int[(x^6*(A + B*x^3))/(a + b*x^3),x]
```

output

```
(B*x^7)/(7*b) + ((A*b - a*B)*(-((a*x)/b^2) + x^4/(4*b) - (a^(4/3)*ArcTan[(
a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) + (a^(4/3)*Lo
g[a^(1/3) + b^(1/3)*x])/(3*b^(7/3)) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(6*b^(7/3))))/b
```

3.56.3.1 Defintions of rubi rules used

rule 831

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 959

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.56.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.46

method	result
risch	$\frac{B\,x^{7}}{7b}+\frac{A\,x^{4}}{4b}-\frac{Ba\,x^{4}}{4b^{2}}-\frac{aAx}{b^{2}}+\frac{a^{2}Bx}{b^{3}}+\frac{a^{2}\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{(Ab-Ba)\ln\left(x-\_R\right)}{\_R^{2}}\right)}{3b^{4}}$
default	$-\frac{-\frac{1}{7}b^{2}B\,x^{7}-\frac{1}{4}A\,b^{2}x^{4}+\frac{1}{4}Bab\,x^{4}+aAbx-a^{2}Bx}{b^{3}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a^{2}(}{b^{3}}$

input
```
int(x^6*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/7*B*x^7/b+1/4/b*A*x^4-1/4/b^2*B*a*x^4-1/b^2*a*A*x+1/b^3*a^2*B*x+1/3/b^4*
a^2*sum((A*b-B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int\frac{x^{6}(A+Bx^{3})}{a+bx^{3}}\,dx$$
$$=\frac{12\,Bb^{2}x^{7}-21\,(Bab-Ab^{2})x^{4}-28\,\sqrt{3}(Ba^{2}-Aab)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)+14\,(Ba^{2}-Aab)\left(\frac{a}{b}\right)^{\frac{1}{3}}}{84\,b^{3}}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/84*(12*B*b^2*x^7 - 21*(B*a*b - A*b^2)*x^4 - 28*sqrt(3)*(B*a^2 - A*a*b)*(
a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 14*(B*a
^2 - A*a*b)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(B*a^2
 - A*a*b)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(B*a^2 - A*a*b)*x)/b^3
```

3.56.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.62

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^7}{7b}+x^4\left(\frac{A}{4b}-\frac{Ba}{4b^2}\right)+x\left(-\frac{Aa}{b^2}+\frac{Ba^2}{b^3}\right) + \operatorname{RootSum}\left(27t^3b^{10}-A^3a^4b^3+3A^2Ba^5b^2-3AB^2a^6b+B^3a^7,\left(t\mapsto t\log\left(-\frac{3tb^3}{-Aab+Ba^2}+x\right)\right)\right)$$

input
```
integrate(x**6*(B*x**3+A)/(b*x**3+a),x)
```

output
```
B*x**7/(7*b) + x**4*(A/(4*b) - B*a/(4*b**2)) + x*(-A*a/b**2 + B*a**2/b**3)
 + RootSum(27*_t**3*b**10 - A**3*a**4*b**3 + 3*A**2*B*a**5*b**2 - 3*A*B**2
*a**6*b + B**3*a**7, Lambda(_t, _t*log(-3*_t*b**3/(-A*a*b + B*a**2) + x)))
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx &= \frac{4\,Bb^2x^7-7\,(Bab-Ab^2)x^4+28\,(Ba^2-Aab)x}{28\,b^3} \\ &\quad - \frac{\sqrt{3}(Ba^3-Aa^2b)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &\quad + \frac{(Ba^3-Aa^2b)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &\quad - \frac{(Ba^3-Aa^2b)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/28*(4*B*b^2*x^7 - 7*(B*a*b - A*b^2)*x^4 + 28*(B*a^2 - A*a*b)*x)/b^3 - 1/
3*sqrt(3)*(B*a^3 - A*a^2*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/(b^4*(a/b)^(2/3)) + 1/6*(B*a^3 - A*a^2*b)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(b^4*(a/b)^(2/3)) - 1/3*(B*a^3 - A*a^2*b)*log(x + (a/b)^(1/3)
)/(b^4*(a/b)^(2/3))
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.19

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = -\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba^2-\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba^2-\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4} + \frac{\left(Ba^3b^4-Aa^2b^5\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^7} + \frac{4\,Bb^6x^7-7\,Bab^5x^4+7\,Ab^6x^4+28\,Ba^2b^4x-28\,Aab^5x}{28\,b^7}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a^2 - (-a*b^2)^(1/3)*A*a*b)*arctan(1/3*sqrt
(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)*B*a^2 - (
-a*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/3*(B
*a^3*b^4 - A*a^2*b^5)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/
28*(4*B*b^6*x^7 - 7*B*a*b^5*x^4 + 7*A*b^6*x^4 + 28*B*a^2*b^4*x - 28*A*a*b^
5*x)/b^7
```

3.56.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.90

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = x^4\left(\frac{A}{4\,b}-\frac{B\,a}{4\,b^2}\right)+\frac{B\,x^7}{7\,b} + \frac{a^{4/3}\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,b^{10/3}} - \frac{a\,x\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{b} - \frac{a^{4/3}\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,b^{10/3}} + \frac{a^{4/3}\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,b^{10/3}}$$

input
```
int((x^6*(A + B*x^3))/(a + b*x^3),x)
```

output
```
x^4*(A/(4*b) - (B*a)/(4*b^2)) + (B*x^7)/(7*b) + (a^(4/3)*log(b^(1/3)*x + a
^(1/3))*(A*b - B*a))/(3*b^(10/3)) - (a*x*(A/b - (B*a)/b^2))/b - (a^(4/3)*l
og(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b
 - B*a))/(3*b^(10/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^
(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(10/3))
```

3.56. $\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx$

3.57 $\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx$

3.57.1 Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx=\frac{(Ab-aB)x^3}{3b^2}+\frac{Bx^6}{6b}-\frac{a(Ab-aB)\log(a+bx^3)}{3b^3}$$

output
```
1/3*(A*b-B*a)*x^3/b^2+1/6*B*x^6/b-1/3*a*(A*b-B*a)*ln(b*x^3+a)/b^3
```

3.57.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx=\frac{bx^3(2Ab-2aB+bBx^3)+2a(-Ab+aB)\log(a+bx^3)}{6b^3}$$

input
```
Integrate[(x^5*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(b*x^3*(2*A*b - 2*a*B + b*B*x^3) + 2*a*(-(A*b) + a*B)*Log[a + b*x^3])/(6*b
^3)
```

3.57.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(A+Bx^3\right)}{a+bx^3} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3} \int \frac{x^3 \left(Bx^3+A\right)}{bx^3+a} dx^3$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{3} \int \left(\frac{Bx^3}{b}+\frac{Ab-aB}{b^2}+\frac{a(aB-Ab)}{b^2 \left(bx^3+a\right)}\right) dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(-\frac{a(Ab-aB) \log \left(a+bx^3\right)}{b^3}+\frac{x^3(Ab-aB)}{b^2}+\frac{Bx^6}{2b}\right)$$

input
```
Int[(x^5*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(((A*b - a*B)*x^3)/b^2 + (B*x^6)/(2*b) - (a*(A*b - a*B)*Log[a + b*x^3])/b^
3)/3
```

3.57.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.57.4 Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result	size
norman	$\frac{(Ab-Ba)x^3}{3b^2}+\frac{B\,x^6}{6b}-\frac{a(Ab-Ba)\ln\left(b\,x^3+a\right)}{3b^3}$	49
default	$\frac{\frac{1}{2}bB\,x^6+Ab\,x^3-Ba\,x^3}{3b^2}-\frac{a(Ab-Ba)\ln\left(b\,x^3+a\right)}{3b^3}$	50
parallelrisch	$-\frac{-b^2B\,x^6-2A\,b^2x^3+2Bab\,x^3+2A\ln\left(b\,x^3+a\right)ab-2B\ln\left(b\,x^3+a\right)a^2}{6b^3}$	60
risch	$\frac{B\,x^6}{6b}+\frac{A\,x^3}{3b}-\frac{Ba\,x^3}{3b^2}+\frac{A^2}{6bB}-\frac{aA}{3b^2}+\frac{a^2B}{6b^3}-\frac{a\ln\left(b\,x^3+a\right)A}{3b^2}+\frac{a^2\ln\left(b\,x^3+a\right)B}{3b^3}$	89

input
```
int(x^5*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*(A*b-B*a)*x^3/b^2+1/6*B*x^6/b-1/3*a*(A*b-B*a)*ln(b*x^3+a)/b^3
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx=\frac{Bb^2x^6-2\,(Bab-Ab^2)x^3+2\,(Ba^2-Aab)\log\left(bx^3+a\right)}{6\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(B*b^2*x^6 - 2*(B*a*b - A*b^2)*x^3 + 2*(B*a^2 - A*a*b)*log(b*x^3 + a))
/b^3
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^6}{6b} + \frac{a(-Ab+Ba)\log(a+bx^3)}{3b^3} + x^3\left(\frac{A}{3b} - \frac{Ba}{3b^2}\right)$$

input
```
integrate(x**5*(B*x**3+A)/(b*x**3+a),x)
```

output
```
B*x**6/(6*b) + a*(-A*b + B*a)*log(a + b*x**3)/(3*b**3) + x**3*(A/(3*b) - B
*a/(3*b**2))
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = \frac{Bbx^6 - 2\,(Ba-Ab)x^3}{6\,b^2} + \frac{(Ba^2-Aab)\log(bx^3+a)}{3\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/6*(B*b*x^6 - 2*(B*a - A*b)*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(b*x^3 + a)
/b^3
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = \frac{Bbx^6 - 2\,Bax^3 + 2\,Abx^3}{6\,b^2} + \frac{(Ba^2-Aab)\log(|bx^3+a|)}{3\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/6*(B*b*x^6 - 2*B*a*x^3 + 2*A*b*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(abs(b*
x^3 + a))/b^3
```

3.57.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = x^3\left(\frac{A}{3\,b}-\frac{B\,a}{3\,b^2}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(B\,a^2-A\,a\,b\right)}{3\,b^3}+\frac{B\,x^6}{6\,b}$$

input
```
int((x^5*(A + B*x^3))/(a + b*x^3),x)
```

output
```
x^3*(A/(3*b) - (B*a)/(3*b^2)) + (log(a + b*x^3)*(B*a^2 - A*a*b))/(3*b^3) +
 (B*x^6)/(6*b)
```

3.58 $\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx$

3.58.1 Optimal result

Integrand size = 20, antiderivative size = 167

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = \frac{(Ab-aB)x^2}{2b^2}+\frac{Bx^5}{5b}+\frac{a^{2/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}}+\frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}}-\frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}$$

output
```
1/2*(A*b-B*a)*x^2/b^2+1/5*B*x^5/b+1/3*a^(2/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)
*x)/b^(8/3)-1/6*a^(2/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/b^(8/3)+1/3*a^(2/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3
^(1/2))/b^(8/3)*3^(1/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.92

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx$$

$$= \frac{15b^{2/3}(Ab-aB)x^2+6b^{5/3}Bx^5-10\sqrt{3}a^{2/3}(-Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-10a^{2/3}(-Ab+aB)\log\left(\sqrt[3]{}\right.}{30b^{8/3}}$$

input
```
Integrate[(x^4*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(15*b^(2/3)*(A*b - a*B)*x^2 + 6*b^(5/3)*B*x^5 - 10*Sqrt[3]*a^(2/3)*(-(A*b)
 + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-(A*b) +
 a*B)*Log[a^(1/3) + b^(1/3)*x] + 5*a^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(30*b^(8/3))
```

3.58.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {959, 843, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(A+Bx^3\right)}{a+bx^3} \, dx$$

$$\Big\downarrow \ 959$$

$$\frac{(Ab-aB) \int \frac{x^4}{bx^3+a}dx}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \ 843$$

$$\frac{(Ab-aB) \left(\frac{x^2}{2b}-\frac{a \int \frac{x}{bx^3+a}dx}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \ 821$$

$$\frac{(Ab-aB) \left(\frac{x^2}{2b}-\frac{a \left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \ 16$$

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \; 1142$$

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \; 25$$

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \; 27$$

3.58. $\int \frac{x^4\left(A+Bx^3\right)}{a+bx^3}\,dx$

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \ 1082$$

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \ 217$$

3.58. $\int \frac{x^4\left(A+Bx^3\right)}{a+bx^3}\,dx$

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{(Ab-aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

input
```
Int[(x^4*(A + B*x^3))/(a + b*x^3),x]
```

3.58. $\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx$

output
```
(B*x^5)/(5*b) + ((A*b - a*B)*(x^2/(2*b) - (a*(-1/3*Log[a^(1/3) + b^(1/3)*x
]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3))
)/(3*a^(1/3)*b^(1/3))))/b))/b
```

3.58.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.58.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{B\,x^{5}}{5b}+\frac{A\,x^{2}}{2b}-\frac{Ba\,x^{2}}{2b^{2}}+\frac{a\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-Ab+Ba\right)\ln\left(x-\_R\right)}{\_R}\right)}{3b^{3}}$	65
default	$\frac{\frac{bB\,x^{5}}{5}+\frac{\left(Ab-Ba\right)x^{2}}{2}}{b^{2}}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a\left(Ab-Ba\right)}{b^{2}}$	131

input
```
int(x^4*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*B*x^5/b+1/2/b*A*x^2-1/2/b^2*B*a*x^2+1/3/b^3*a*sum((-A*b+B*a)/_R*ln(x-_
R),_R=RootOf(_Z^3*b+a))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.97

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx$$

$$= \frac{6\,Bbx^5 - 15\,(Ba-Ab)x^2 + 10\,\sqrt{3}(Ba-Ab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right) + 5\,(Ba-Ab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\Bigg(}{30\,b^2}$$

input

```
integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output

```
1/30*(6*B*b*x^5 - 15*(B*a - A*b)*x^2 + 10*sqrt(3)*(B*a - A*b)*(a^2/b^2)^(1
/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*(B*a - A
*b)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) -
 10*(B*a - A*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b^2
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.68

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^5}{5b} + x^2\left(\frac{A}{2b} - \frac{Ba}{2b^2}\right)$$

$$+\mathrm{RootSum}\Bigg(27t^3b^8 - A^3a^2b^3 + 3A^2Ba^3b^2 - 3AB^2a^4b + B^3a^5, \Bigg(t \mapsto t\log\Bigg(\frac{9t^2b^5}{A^2ab^2 - 2ABa^2b + B^2a^3} +$$

input

```
integrate(x**4*(B*x**3+A)/(b*x**3+a),x)
```

output

```
B*x**5/(5*b) + x**2*(A/(2*b) - B*a/(2*b**2)) + RootSum(27*_t**3*b**8 - A**
3*a**2*b**3 + 3*A**2*B*a**3*b**2 - 3*A*B**2*a**4*b + B**3*a**5, Lambda(_t,
 _t*log(9*_t**2*b**5/(A**2*a*b**2 - 2*A*B*a**2*b + B**2*a**3) + x)))
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.94

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = \frac{\sqrt{3}(Ba^2-Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2\,Bbx^5-5\,(Ba-Ab)x^2}{10\,b^2} + \frac{(Ba^2-Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba^2-Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(B*a^2 - A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/(b^3*(a/b)^(1/3)) + 1/10*(2*B*b*x^5 - 5*(B*a - A*b)*x^2)/b^2 + 1/6*(
B*a^2 - A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) -
1/3*(B*a^2 - A*a*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.24

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = -\frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}\,Ba-(-ab^2)^{\frac{2}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4} + \frac{\left((-ab^2)^{\frac{2}{3}}\,Ba-(-ab^2)^{\frac{2}{3}}\,Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4} - \frac{\left(Ba^2b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Aab^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^5} + \frac{2\,Bb^4x^5-5\,Bab^3x^2+5\,Ab^4x^2}{10\,b^5}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output

```
-1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/6*((-a*b^2)^(2/3)*B*a - (-a*b^2
)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 - 1/3*(B*a^2*b^3
*(-a/b)^(1/3) - A*a*b^4*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3
)))/(a*b^5) + 1/10*(2*B*b^4*x^5 - 5*B*a*b^3*x^2 + 5*A*b^4*x^2)/b^5
```

3.58.9 Mupad [B] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = {} & x^2\left(\frac{A}{2\,b}-\frac{B\,a}{2\,b^2}\right)+\frac{B\,x^5}{5\,b}+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,b^{8/3}}\\ & +\frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,b^{8/3}}\\ & -\frac{a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,b^{8/3}}\end{aligned}$$

input

```
int((x^4*(A + B*x^3))/(a + b*x^3),x)
```

output

```
x^2*(A/(2*b) - (B*a)/(2*b^2)) + (B*x^5)/(5*b) + (a^(2/3)*log(b^(1/3)*x + a
^(1/3))*(A*b - B*a))/(3*b^(8/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(
1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(8/3)) - (a^(2/
3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(A*b - B*a))/(3*b^(8/3))
```

3.59 $\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx$

3.59.1 Optimal result

Integrand size = 20, antiderivative size = 162

$$\begin{aligned}\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx = {}&\frac{(Ab-aB)x}{b^2}+\frac{Bx^4}{4b}+\frac{\sqrt[3]{a}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}}\\ &-\frac{\sqrt[3]{a}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}}\\ &+\frac{\sqrt[3]{a}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}}\end{aligned}$$

output
```
(A*b-B*a)*x/b^2+1/4*B*x^4/b-1/3*a^(1/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/b^
(7/3)+1/6*a^(1/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(7
/3)+1/3*a^(1/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2)
)/b^(7/3)*3^(1/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.94

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx$$

$$= \frac{12\sqrt[3]{b}(Ab-aB)x+3b^{4/3}Bx^4-4\sqrt{3}\sqrt[3]{a}(-Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{a}(-Ab+aB)\log\left(\sqrt[3]{a}+\right.}{12b^{7/3}}$$

input
```
Integrate[(x^3*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(12*b^(1/3)*(A*b - a*B)*x + 3*b^(4/3)*B*x^4 - 4*Sqrt[3]*a^(1/3)*(-(A*b) +
a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-(A*b) + a*B
)*Log[a^(1/3) + b^(1/3)*x] - 2*a^(1/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(12*b^(7/3))
```

3.59.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {959, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(A+Bx^3\right)}{a+bx^3} \, dx$$

$$\Big\downarrow \; 959$$

$$\frac{(Ab-aB)\int \frac{x^3}{bx^3+a}dx}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \; 843$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^3+a}dx}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \; 750$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \; 16$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \text{ 27}$$

3.59. $\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \text{217}$$

3.59. $\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

$$\Big\downarrow \; 1103$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}+\frac{Bx^4}{4b}$$

input

```
Int[(x^3*(A + B*x^3))/(a + b*x^3),x]
```

3.59. $\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx$

output
```
(B*x^4)/(4*b) + ((A*b - a*B)*(x/b - (a*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3
)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1
/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2
/3))))/b))/b
```

3.59.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.59.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.37

method	result	size
risch	$\frac{B\,x^{4}}{4b}+\frac{Ax}{b}-\frac{Bax}{b^{2}}+\frac{a\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-Ab+Ba\right)\ln\left(x-\_R\right)}{\_R^{2}}\right)}{3b^{3}}$	60
default	$\frac{\frac{1}{4}bB\,x^{4}+Abx-Bax}{b^{2}}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a\left(Ab-Ba\right)}{b^{2}}$	127

input
```
int(x^3*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*B*x^4/b+1/b*A*x-1/b^2*B*a*x+1/3/b^3*a*sum((-A*b+B*a)/_R^2*ln(x-_R),_R=
RootOf(_Z^3*b+a))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx = \frac{3\,Bbx^4 - 4\sqrt{3}(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right) + 2\,(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+(-\right.}{12\,b^2}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/12*(3*B*b*x^4 - 4*sqrt(3)*(B*a - A*b)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)
*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 2*(B*a - A*b)*(-a/b)^(1/3)*log(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3)) - 4*(B*a - A*b)*(-a/b)^(1/3)*log(x - (-a/b)^
(1/3)) - 12*(B*a - A*b)*x)/b^2
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.54

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^4}{4b} + x\left(\frac{A}{b}-\frac{Ba}{b^2}\right) + \operatorname{RootSum}\left(27t^3b^7 + A^3ab^3 - 3A^2Ba^2b^2 + 3AB^2a^3b - B^3a^4, \left(t \mapsto t\log\left(\frac{3tb^2}{-Ab+Ba}+x\right)\right)\right)$$

input
```
integrate(x**3*(B*x**3+A)/(b*x**3+a),x)
```

output
```
B*x**4/(4*b) + x*(A/b - B*a/b**2) + RootSum(27*_t**3*b**7 + A**3*a*b**3 -
3*A**2*B*a**2*b**2 + 3*A*B**2*a**3*b - B**3*a**4, Lambda(_t, _t*log(3*_t*b
**2/(-A*b + B*a) + x)))
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.95

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx = \frac{Bbx^4-4\,(Ba-Ab)x}{4\,b^2} + \frac{\sqrt{3}(Ba^2-Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba^2-Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba^2-Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input

```
integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output

```
1/4*(B*b*x^4 - 4*(B*a - A*b)*x)/b^2 + 1/3*sqrt(3)*(B*a^2 - A*a*b)*arctan(1
/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) - 1/6*(B*a^2
 - A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(
B*a^2 - A*a*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.15

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx = \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3} - \frac{(Ba^2b^2-Aab^3)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^4} + \frac{Bb^3x^4-4\,Bab^2x+4\,Ab^3x}{4\,b^4}$$

input

```
integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)
^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 - 1/3*(B*a^2*b^2
- A*a*b^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) + 1/4*(B*b^3*x^
4 - 4*B*a*b^2*x + 4*A*b^3*x)/b^4
```

3.59.9 Mupad [B] (verification not implemented)

Time = 7.05 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx &= x\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)+\frac{B\,x^4}{4\,b}+\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)\,(A\,b-B\,a)}{3\,b^{7/3}}\\&\quad-\frac{(-a)^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}-\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a)}{3\,b^{7/3}}\\&\quad+\frac{(-a)^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}+\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a)}{3\,b^{7/3}}\end{aligned}$$

input
```
int((x^3*(A + B*x^3))/(a + b*x^3),x)
```

output
```
x*(A/b - (B*a)/b^2) + (B*x^4)/(4*b) + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/
3)*x)*(A*b - B*a))/(3*b^(7/3)) - ((-a)^(1/3)*log(2*a*b^(1/3)*x - 3^(1/2)*(
-a)^(4/3)*1i - (-a)^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*b^(7/3))
 + ((-a)^(1/3)*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3
^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(7/3))
```

3.60 $\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx$

3.60.1 Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^3}{3b} + \frac{(Ab-aB)\log(a+bx^3)}{3b^2}$$

output
```
1/3*B*x^3/b+1/3*(A*b-B*a)*ln(b*x^3+a)/b^2
```

3.60.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{bBx^3 + (Ab-aB)\log(a+bx^3)}{3b^2}$$

input
```
Integrate[(x^2*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(b*B*x^3 + (A*b - a*B)*Log[a + b*x^3])/(3*b^2)
```

3.60.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {946, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx$$
$$\Big\downarrow \text{946}$$
$$\frac{1}{3}\int \frac{Bx^3+A}{bx^3+a}dx^3$$
$$\Big\downarrow \text{49}$$
$$\frac{1}{3}\int \left(\frac{B}{b}+\frac{Ab-aB}{b(bx^3+a)}\right)dx^3$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}\left(\frac{(Ab-aB)\log(a+bx^3)}{b^2}+\frac{Bx^3}{b}\right)$$

input
```
Int[(x^2*(A + B*x^3))/(a + b*x^3),x]
```

output
```
((B*x^3)/b + ((A*b - a*B)*Log[a + b*x^3])/b^2)/3
```

3.60.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.60.4 Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{B\,x^{3}}{3b}+\frac{(Ab-Ba)\ln\left(b\,x^{3}+a\right)}{3b^{2}}$	32
norman	$\frac{B\,x^{3}}{3b}+\frac{(Ab-Ba)\ln\left(b\,x^{3}+a\right)}{3b^{2}}$	32
parallelrisch	$\frac{bB\,x^{3}+A\ln\left(b\,x^{3}+a\right)b-B\ln\left(b\,x^{3}+a\right)a}{3b^{2}}$	36
risch	$\frac{B\,x^{3}}{3b}+\frac{\ln\left(b\,x^{3}+a\right)A}{3b}-\frac{\ln\left(b\,x^{3}+a\right)Ba}{3b^{2}}$	40

input
```
int(x^2*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*B*x^3/b+1/3*(A*b-B*a)*ln(b*x^3+a)/b^2
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx=\frac{Bbx^3-(Ba-Ab)\log\left(bx^3+a\right)}{3\,b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/3*(B*b*x^3 - (B*a - A*b)*log(b*x^3 + a))/b^2
```

3.60.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^3}{3b} - \frac{(-Ab+Ba)\log{(a+bx^3)}}{3b^2}$$

input
```
integrate(x**2*(B*x**3+A)/(b*x**3+a),x)
```

output
```
B*x**3/(3*b) - (-A*b + B*a)*log(a + b*x**3)/(3*b**2)
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^3}{3\,b} - \frac{(Ba-Ab)\log{(bx^3+a)}}{3\,b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*B*x^3/b - 1/3*(B*a - A*b)*log(b*x^3 + a)/b^2
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^3}{3\,b} - \frac{(Ba-Ab)\log{(|bx^3+a|)}}{3\,b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*B*x^3/b - 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/b^2
```

3.60.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx=\frac{B\,x^3}{3\,b}+\frac{\ln\left(b\,x^3+a\right)\,\left(A\,b-B\,a\right)}{3\,b^2}$$

input
```
int((x^2*(A + B*x^3))/(a + b*x^3),x)
```

output
```
(B*x^3)/(3*b) + (log(a + b*x^3)*(A*b - B*a))/(3*b^2)
```

3.61 $\int \frac{x(A+Bx^3)}{a+bx^3}\,dx$

3.61.1 Optimal result

Integrand size = 18, antiderivative size = 150

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2b} - \frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}}$$

output
```
1/2*B*x^2/b-1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(5/3)+1/6*(A*b-B
*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(5/3)-1/3*(A*b-B*a
)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(5/3)*3^(1/2
)
```

3.61.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2b} - \frac{(-Ab+aB)\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}} + \frac{(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}} - \frac{(-Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}}$$

input

```
Integrate[(x*(A + B*x^3))/(a + b*x^3),x]
```

output

```
(B*x^2)/(2*b) - ((-(A*b) + a*B)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a
^(1/3))])/(Sqrt[3]*a^(1/3)*b^(5/3)) + ((-(A*b) + a*B)*Log[a^(1/3) + b^(1/3
)*x])/(3*a^(1/3)*b^(5/3)) - ((-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(6*a^(1/3)*b^(5/3))
```

3.61.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {959, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^3)}{a+bx^3}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{x}{bx^3+a}dx}{b}+\frac{Bx^2}{2b}$$

$$\Big\downarrow \text{821}$$

$$\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}+\frac{Bx^2}{2b}$$

$$\Big\downarrow \text{16}$$

$$\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}+\frac{Bx^2}{2b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}+\frac{Bx^2}{2b}$$

$$\Big\downarrow \ 25$$

$$\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}+\frac{Bx^2}{2b}$$

$$\Big\downarrow \ 27$$

$$\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}+\frac{Bx^2}{2b}$$

$$\Big\downarrow \ 1082$$

$$\frac{(Ab-aB)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}+\frac{Bx^2}{2b}$$

$$\Big\downarrow \ 217$$

3.61. $\int \frac{x(A+Bx^3)}{a+bx^3}\,dx$

$$\frac{(Ab-aB)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}+\frac{Bx^2}{2b}$$

$$\Big\downarrow \text{1103}$$

$$\frac{(Ab-aB)\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}+\frac{Bx^2}{2b}$$

input
```
Int[(x*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(B*x^2)/(2*b) + ((A*b - a*B)*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/
3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) +
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(
1/3))))/b
```

3.61.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.61.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.30

method	result	size
risch	$\frac{B\,x^2}{2b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^3+a\right)}\frac{(Ab-Ba)\ln\left(x-\_R\right)}{\_R}}{3b^2}$	45
default	$\frac{B\,x^2}{2b}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)(Ab-Ba)}{b}$	113

input
```
int(x*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x^2/b+1/3/b^2*sum((A*b-B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.55

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx$$

$$= \Bigg[\frac{3\,Bab^2x^2-3\,\sqrt{\frac{1}{3}}(Ba^2b-Aab^2)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\Bigg(\frac{2\,b^2x^3-ab+3\,\sqrt{\frac{1}{3}}\left(abx+2\,(-ab^2)^{\frac{2}{3}}x^2+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}}{bx^3+a}}{6}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output

```
[1/6*(3*B*a*b^2*x^2 - 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt((-a*b^2)^(1/3)/
a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*
b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) -
(-a*b^2)^(2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/
3)) + 2*(-a*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3), 1/6
*(3*B*a*b^2*x^2 - 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt(-(-a*b^2)^(1/3)/a)*
arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (-a
*b^2)^(2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3))
 + 2*(-a*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3)]
```

3.61.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.61

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2b} + \operatorname{RootSum}\left(27t^3ab^5 + A^3b^3 - 3A^2Bab^2 + 3AB^2a^2b - B^3a^3, \left(t \mapsto t\log\left(\frac{9t^2ab^3}{A^2b^2 - 2ABab + B^2a^2} + x\right)\right)\right)$$

input

```
integrate(x*(B*x**3+A)/(b*x**3+a),x)
```

output

```
B*x**2/(2*b) + RootSum(27*_t**3*a*b**5 + A**3*b**3 - 3*A**2*B*a*b**2 + 3*A
*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(9*_t**2*a*b**3/(A**2*b**2 - 2*
A*B*a*b + B**2*a**2) + x)))
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2\,b} - \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba-Ab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input

```
integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output

```
1/2*B*x^2/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3
))/(a/b)^(1/3))/(b^2*(a/b)^(1/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(b^2*(a/b)^(1/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/
(b^2*(a/b)^(1/3))
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2\,b} - \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{1}{3}}b} + \frac{(Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{1}{3}}b} + \frac{\left(Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}$$

input

```
integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output

```
1/2*B*x^2/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b) + 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^
(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b) + 1/3*(B*a*b*(-a/b)^(1/3) - A*b^2
*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{B\,x^2}{2\,b} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(A\,b-B\,a)}{3\,a^{1/3}\,b^{5/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(A\,b-B\,a)}{3\,a^{1/3}\,b^{5/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(A\,b-B\,a)}{3\,a^{1/3}\,b^{5/3}}$$

input
```
int((x*(A + B*x^3))/(a + b*x^3),x)
```

output
```
(B*x^2)/(2*b) - (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(1/3)*b^(5/3))
 - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)
*(A*b - B*a))/(3*a^(1/3)*b^(5/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
- a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(1/3)*b^(5/3))
```

3.61. $\int \frac{x(A+Bx^3)}{a+bx^3}\,dx$

3.62 $\int \frac{A+Bx^3}{a+bx^3}\,dx$

3.62.1 Optimal result

Integrand size = 17, antiderivative size = 145

$$\int \frac{A+Bx^3}{a+bx^3}\,dx=\frac{Bx}{b}-\frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}$$

output
```
B*x/b+1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(4/3)-1/6*(A*b-B*a)*ln
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(4/3)-1/3*(A*b-B*a)*arct
an(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(4/3)*3^(1/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{a+bx^3}\,dx$$

$$=\frac{6a^{2/3}\sqrt[3]{b}Bx-2\sqrt{3}(Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-(Ab-aB)\log\Big(a^{2/3}}{6a^{2/3}b^{4/3}}$$

input
```
Integrate[(A + B*x^3)/(a + b*x^3),x]
```

output
```
(6*a^(2/3)*b^(1/3)*B*x - 2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

3.62.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {913, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{a+bx^3} \, dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(Ab-aB)\int \frac{1}{bx^3+a}dx}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{750}$$

$$\frac{(Ab-aB)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{(Ab-aB)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \; 1082$$

$$\frac{(Ab-aB)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \; 217$$

3.62. $\int \frac{A+Bx^3}{a+bx^3}\,dx$

$$\frac{(Ab-aB)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{1103}$$

$$\frac{(Ab-aB)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{Bx}{b}$$

input

```
Int[(A + B*x^3)/(a + b*x^3),x]
```

output

```
(B*x)/b + ((A*b - a*B)*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-(
(Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/b
```

3.62.3.1 Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.62. $\int \frac{A+Bx^3}{a+bx^3}\,dx$

3.62.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{Bx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(Ab-Ba)\ln\left(x-\_R\right)}{\_R^{2}}}{3b^{2}}$	42
default	$\frac{Bx}{b}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(Ab-Ba)}{b}$	110

input
```
int((B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
B*x/b+1/3/b^2*sum((A*b-B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.54

$$\int\frac{A+Bx^{3}}{a+bx^{3}}\,dx$$

$$=\left[\frac{6\,Ba^{2}bx-3\,\sqrt{\frac{1}{3}}(Ba^{2}b-Aab^{2})\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}-3\,(a^{2}b)^{\frac{1}{3}}\,ax-a^{2}+3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2}+(a^{2}b)^{\frac{2}{3}}\,x-(a^{2}b)^{\frac{1}{3}}\,a\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)}{6\,a^{2}b^{2}}\right.$$

input
```
integrate((B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output

```
[1/6*(6*B*a^2*b*x - 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt(-(a^2*b)^(1/3)/b)
*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^
2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + (a^
2*b)^(2/3)*(B*a - A*b)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) -
2*(a^2*b)^(2/3)*(B*a - A*b)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^2), 1/6*(6*
B*a^2*b*x - 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt((a^2*b)^(1/3)/b)*arctan(s
qrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2)
+ (a^2*b)^(2/3)*(B*a - A*b)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*
a) - 2*(a^2*b)^(2/3)*(B*a - A*b)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^2)]
```

3.62.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{Bx}{b} + \operatorname{RootSum}\left(27t^3a^2b^4 - A^3b^3 + 3A^2Bab^2 - 3AB^2a^2b + B^3a^3, \left(t \mapsto t\log\left(-\frac{3tab}{-Ab+Ba}+x\right)\right)\right)$$

input

```
integrate((B*x**3+A)/(b*x**3+a),x)
```

output

```
B*x/b + RootSum(27*_t**3*a**2*b**4 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B**
2*a**2*b + B**3*a**3, Lambda(_t, _t*log(-3*_t*a*b/(-A*b + B*a) + x)))
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{Bx}{b} - \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input

```
integrate((B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
B*x/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/
b)^(1/3))/(b^2*(a/b)^(2/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a
/b)^(2/3))/(b^2*(a/b)^(2/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(b^2*(
a/b)^(2/3))
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}} + \frac{(Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}} + \frac{Bx}{b} + \frac{(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

input
```
integrate((B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/(-a*b^2)^(2/3) + 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/
3))/(-a*b^2)^(2/3) + B*x/b + 1/3*(B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/(a*b)
```

3.62.9 Mupad [B] (verification not implemented)

Time = 6.99 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{B\,x}{b} + \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}}$$

input
```
int((A + B*x^3)/(a + b*x^3),x)
```

output
```
(B*x)/b + (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(2/3)*b^(4/3)) - (lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b
- B*a))/(3*a^(2/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1
/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*a^(2/3)*b^(4/3))
```

3.62. $\int \frac{A+Bx^3}{a+bx^3}\,dx$

3.63 $\int \frac{A+Bx^3}{x(a+bx^3)}\,dx$

3.63.1 Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)}\,dx=\frac{A\log(x)}{a}-\frac{(Ab-aB)\log\left(a+bx^3\right)}{3ab}$$

output

```
A*ln(x)/a-1/3*(A*b-B*a)*ln(b*x^3+a)/a/b
```

3.63.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)}\,dx=\frac{A\log(x)}{a}+\frac{(-Ab+aB)\log\left(a+bx^3\right)}{3ab}$$

input

```
Integrate[(A + B*x^3)/(x*(a + b*x^3)),x]
```

output

```
(A*Log[x])/a + ((-(A*b) + a*B)*Log[a + b*x^3])/(3*a*b)
```

3.63.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^3\left(bx^3+a\right)}dx^3$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{3}\int \left(\frac{A}{ax^3}+\frac{aB-Ab}{a\left(bx^3+a\right)}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{A\log\left(x^3\right)}{a}-\frac{(Ab-aB)\log\left(a+bx^3\right)}{ab}\right)$$

input
```
Int[(A + B*x^3)/(x*(a + b*x^3)),x]
```

output
```
((A*Log[x^3])/a - ((A*b - a*B)*Log[a + b*x^3])/(a*b))/3
```

3.63.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.63.4 Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{A\ln(x)}{a}-\frac{(Ab-Ba)\ln(b\,x^3+a)}{3ab}$	33
norman	$\frac{A\ln(x)}{a}-\frac{(Ab-Ba)\ln(b\,x^3+a)}{3ab}$	33
risch	$\frac{A\ln(x)}{a}-\frac{\ln(b\,x^3+a)A}{3a}+\frac{\ln(b\,x^3+a)B}{3b}$	37
parallelrisch	$\frac{3A\ln(x)b-A\ln(b\,x^3+a)b+B\ln(b\,x^3+a)a}{3ab}$	39

input
```
int((B*x^3+A)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a-1/3*(A*b-B*a)*ln(b*x^3+a)/a/b
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^3}{x(a+bx^3)}\,dx=\frac{3\,Ab\log(x)+(Ba-Ab)\log(bx^3+a)}{3\,ab}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/3*(3*A*b*log(x) + (B*a - A*b)*log(b*x^3 + a))/(a*b)
```

3.63.6 Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^3}{x\,(a+bx^3)}\,dx = \frac{A\log{(x)}}{a} + \frac{(-Ab+Ba)\log{\left(\frac{a}{b}+x^3\right)}}{3ab}$$

input
```
integrate((B*x**3+A)/x/(b*x**3+a),x)
```

output
```
A*log(x)/a + (-A*b + B*a)*log(a/b + x**3)/(3*a*b)
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x\,(a+bx^3)}\,dx = \frac{A\log{(x^3)}}{3\,a} + \frac{(Ba-Ab)\log{(bx^3+a)}}{3\,ab}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*A*log(x^3)/a + 1/3*(B*a - A*b)*log(b*x^3 + a)/(a*b)
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x\,(a+bx^3)}\,dx = \frac{A\log{(|x|)}}{a} + \frac{(Ba-Ab)\log{(|bx^3+a|)}}{3\,ab}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="giac")
```

output
```
A*log(abs(x))/a + 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/(a*b)
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)}\,dx = \frac{B\,\ln\left(b\,x^3+a\right)}{3\,b}-\frac{A\,\ln\left(b\,x^3+a\right)}{3\,a}+\frac{A\,\ln\left(x\right)}{a}$$

input
```
int((A + B*x^3)/(x*(a + b*x^3)),x)
```

output
```
(B*log(a + b*x^3))/(3*b) - (A*log(a + b*x^3))/(3*a) + (A*log(x))/a
```

3.64 $\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx$

3.64.1 Optimal result

Integrand size = 20, antiderivative size = 147

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx=-\frac{A}{ax}+\frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}}-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}}$$

output
```
-A/a/x+1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(2/3)-1/6*(A*b-B*a)*l
n(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(2/3)+1/3*(A*b-B*a)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(2/3)*3^(1/2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx$$

$$=\frac{-6\sqrt[3]{a}Ab^{2/3}+2\sqrt{3}(Ab-aB)x\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2(Ab-aB)x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-(Ab-aB)x\log\Big(}{6a^{4/3}b^{2/3}x}$$

input
```
Integrate[(A + B*x^3)/(x^2*(a + b*x^3)),x]
```

output
```
(-6*a^(1/3)*A*b^(2/3) + 2*Sqrt[3]*(A*b - a*B)*x*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]] + 2*(A*b - a*B)*x*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)
*x*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(2/3)*x)
```

3.64.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {955, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(Ab-aB)\int \frac{x}{bx^3+a}dx}{a}-\frac{A}{ax}$$

$$\Big\downarrow \; 821$$

$$-\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \; 16$$

$$-\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \; 1142$$

$$-\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \ 25$$

$$-\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \ 27$$

$$-\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \ 1082$$

$$-\frac{(Ab-aB)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \ 217$$

3.64. $\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx$

$$-\frac{(Ab-aB)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{(Ab-aB)\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{A}{ax}$$

input
```
Int[(A + B*x^3)/(x^2*(a + b*x^3)),x]
```

output
```
-(A/(a*x)) - ((A*b - a*B)*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3))
 + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3
))))/a
```

3.64.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.64. $\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx$

3.64.4 Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)(Ab-Ba)}{a}-\frac{A}{ax}$
risch	$-\frac{A}{ax}+\frac{\Bigl(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{3}b^{2}a^{4}-A^{3}b^{3}+3A^{2}Bab^{2}-3AB^{2}a^{2}b+B^{3}a^{3}\right)}\_R\ln\bigl(\bigl(-4\_R^{3}a^{4}b^{2}+3A^{3}b^{3}-9A^{2}Bab^{2}+9AB^{2}a^{2}b-3B^{3}a^{3}}{3}$

input

```
int((B*x^3+A)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output

```
-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*x-1)))*(A*b-B*a)/a-A/a/x
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.53

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx$$

$$=\left[-\frac{6\,Aab^2+3\,\sqrt{\frac{1}{3}}(Ba^2b-Aab^2)x\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab-3\,\sqrt{\frac{1}{3}}\left(abx+2\,(ab^2)^{\frac{2}{3}}x^2-(ab^2)^{\frac{1}{3}}a\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}-3\,(ab^2)^{\frac{2}{3}}x}{bx^3+a}\right)}{6\,a^2b^2}\right.$$

$$-\frac{6\,Aab^2+6\,\sqrt{\frac{1}{3}}(Ba^2b-Aab^2)x\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2\,bx-(ab^2)^{\frac{1}{3}}\right)\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}}{b}\right)-(ab^2)^{\frac{2}{3}}\,(Ba-Ab)x\log\left(\right.}{6\,a^2b^2x}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/6*(6*A*a*b^2 + 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x*sqrt(-(a*b^2)^(1/3)/a
)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2
)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) - (a*b
^2)^(2/3)*(B*a - A*b)*x*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) +
 2*(a*b^2)^(2/3)*(B*a - A*b)*x*log(b*x + (a*b^2)^(1/3)))/(a^2*b^2*x), -1/6
*(6*A*a*b^2 + 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x*sqrt((a*b^2)^(1/3)/a)*arct
an(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - (a*b^2)^(
2/3)*(B*a - A*b)*x*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*(a
*b^2)^(2/3)*(B*a - A*b)*x*log(b*x + (a*b^2)^(1/3)))/(a^2*b^2*x)]
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx = -\frac{A}{ax} + \operatorname{RootSum}\left(27t^3a^4b^2 - A^3b^3 + 3A^2Bab^2 - 3AB^2a^2b + B^3a^3, \left(t \mapsto t\log\left(\frac{9t^2a^3b}{A^2b^2 - 2ABab + B^2a^2} + x\right)\right.\right.$$

input
```
integrate((B*x**3+A)/x**2/(b*x**3+a),x)
```

output
```
-A/(a*x) + RootSum(27*_t**3*a**4*b**2 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*
B**2*a**2*b + B**3*a**3, Lambda(_t, _t*log(9*_t**2*a**3*b/(A**2*b**2 - 2*A
*B*a*b + B**2*a**2) + x)))
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx = \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba-Ab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{A}{ax}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(a*b*(a/b)^(1/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
))/(a*b*(a/b)^(1/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(1/
3)) - A/(a*x)
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx = \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{1}{3}}a} - \frac{(Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{1}{3}}a} - \frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2} - \frac{A}{ax}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/((-a*b^2)^(1/3)*a) - 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)
^(2/3))/((-a*b^2)^(1/3)*a) - 1/3*(B*a*(-a/b)^(1/3) - A*b*(-a/b)^(1/3))*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - A/(a*x)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 6.96 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{4/3}\,b^{2/3}} - \frac{A}{a\,x} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{4/3}\,b^{2/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{4/3}\,b^{2/3}}$$

input
```
int((A + B*x^3)/(x^2*(a + b*x^3)),x)
```

output
```
(log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(4/3)*b^(2/3)) - A/(a*x) + (lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b
- B*a))/(3*a^(4/3)*b^(2/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1
/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(4/3)*b^(2/3))
```

3.65 $\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx$

3.65.1 Optimal result

Integrand size = 20, antiderivative size = 149

$$\begin{aligned}\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx = {} & -\frac{A}{2ax^2}+\frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}} \\ & -\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}} \\ & +\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}}\end{aligned}$$

output
```
-1/2*A/a/x^2-1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(1/3)+1/6*(A*b-
B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(1/3)+1/3*(A*b-B*
a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(1/3)*3^(1/
2)
```

3.65.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx = \frac{-\frac{3a^{2/3}A}{x^2}+\frac{2\sqrt{3}(Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{2(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}}{6a^{5/3}}$$

input
```
Integrate[(A + B*x^3)/(x^3*(a + b*x^3)),x]
```

output
```
((-3*a^(2/3)*A)/x^2 + (2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]])/b^(1/3) + (2*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1
/3) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3)
)/(6*a^(5/3))
```

3.65.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {955, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(Ab-aB)\int \frac{1}{bx^3+a}dx}{a}-\frac{A}{2ax^2}$$

$$\Big\downarrow \text{750}$$

$$-\frac{(Ab-aB)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{A}{2ax^2}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{(Ab-aB)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{A}{2ax^2}$$

$$\Big\downarrow \text{ 1142}$$

$$-\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{A}{2ax^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{A}{2ax^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{(Ab-aB)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{A}{2ax^2}$$

$$\Big\downarrow \text{ 1082}$$

$$-\frac{(Ab-aB)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{A}{2ax^2}$$

$$\Big\downarrow \text{ 217}$$

3.65. $\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx$

$$-\frac{(Ab-aB)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{A}{2ax^2}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{(Ab-aB)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{A}{2ax^2}$$

input
```
Int[(A + B*x^3)/(x^3*(a + b*x^3)),x]
```

output
```
-1/2*A/(a*x^2) - ((A*b - a*B)*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)
) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/a
```

3.65.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.65. $\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx$

3.65.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.76

method	result
default	$\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(-Ab+Ba)}{a}-\frac{A}{2a\,x^{2}}$
risch	$-\frac{A}{2a\,x^{2}}+\frac{\Bigg(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{3}b\,a^{5}+A^{3}b^{3}-3A^{2}Ba\,b^{2}+3A\,B^{2}a^{2}b-B^{3}a^{3}\right)}\_R\ln\Big(\Big(-4\_R^{3}a^{5}b-3A^{3}b^{3}+9A^{2}Ba\,b^{2}-9A\,B^{2}a^{2}b+3B^{3}a}{3}$

input
```
int((B*x^3+A)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1)))*(-A*b+B*a)/a-1/2*A/a/x^2
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.76

$$\int\frac{A+Bx^{3}}{x^{3}\left(a+bx^{3}\right)}\,dx$$

$$=\Bigg[-\frac{3\sqrt{\frac{1}{3}}(Ba^{2}b-Aab^{2})x^{2}\sqrt{\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}+3\left(-a^{2}b\right)^{\frac{1}{3}}ax-a^{2}-3\sqrt{\frac{1}{3}}\left(2\,abx^{2}+\left(-a^{2}b\right)^{\frac{2}{3}}x+\left(-a^{2}b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-a^{2}b\right)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)+(}{}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/6*(3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x^2*sqrt((-a^2*b)^(1/3)/b)*log((2*a
*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2
/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) + (-a^2*b)^
(2/3)*(B*a - A*b)*x^2*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) -
 2*(-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x + (-a^2*b)^(2/3)) + 3*A*a^2*b)
/(a^3*b*x^2), 1/6*(6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x^2*sqrt(-(-a^2*b)^(1/3
)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*
b)^(1/3)/b)/a^2) - (-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x^2 - (-a^2*b)^(
2/3)*x - (-a^2*b)^(1/3)*a) + 2*(-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x +
(-a^2*b)^(2/3)) - 3*A*a^2*b)/(a^3*b*x^2)]
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx = -\frac{A}{2ax^2} + \operatorname{RootSum}\left(27t^3a^5b + A^3b^3 - 3A^2Bab^2 + 3AB^2a^2b - B^3a^3, \left(t \mapsto t\log\left(\frac{3ta^2}{-Ab+Ba} + x\right)\right)\right)$$

input
```
integrate((B*x**3+A)/x**3/(b*x**3+a),x)
```

output
```
-A/(2*a*x**2) + RootSum(27*_t**3*a**5*b + A**3*b**3 - 3*A**2*B*a*b**2 + 3*
A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(3*_t*a**2/(-A*b + B*a) + x)))
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx = \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba-Ab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{A}{2\,ax^2}$$

input

```
integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="maxima")
```

output

```
1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(a*b*(a/b)^(2/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
))/(a*b*(a/b)^(2/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/
3)) - 1/2*A/(a*x^2)
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx = -\frac{(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b} - \frac{A}{2\,ax^2}$$

input

```
integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="giac")
```

output

```
-1/3*(B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*sqrt(3)
*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/(a^2*b) + 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*
A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 1/2*A/(a*x^2)
```

3.65.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx = -\frac{A}{2\,a\,x^2} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{5/3}\,b^{1/3}} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{5/3}\,b^{1/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{5/3}\,b^{1/3}}$$

input

```
int((A + B*x^3)/(x^3*(a + b*x^3)),x)
```

output

```
(log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A
*b - B*a))/(3*a^(5/3)*b^(1/3)) - (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3
*a^(5/3)*b^(1/3)) - A/(2*a*x^2) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x -
a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*a^(5/3)*b^(1/3))
```

3.65. $\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx$

3.66 $\int \frac{A+Bx^3}{x^4(a+bx^3)}\,dx$

3.66.1 Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx=-\frac{A}{3ax^3}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}$$

output
```
-1/3*A/a/x^3-(A*b-B*a)*ln(x)/a^2+1/3*(A*b-B*a)*ln(b*x^3+a)/a^2
```

3.66.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx=-\frac{A}{3ax^3}+\frac{(-Ab+aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}$$

input
```
Integrate[(A + B*x^3)/(x^4*(a + b*x^3)),x]
```

output
```
-1/3*A/(a*x^3) + ((-(A*b) + a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^3]
)/(3*a^2)
```

3.66.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^4\,(a+bx^3)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^6\,(bx^3+a)}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{A}{ax^6}-\frac{b(aB-Ab)}{a^2\,(bx^3+a)}+\frac{aB-Ab}{a^2x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\log\left(x^3\right)(Ab-aB)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{a^2}-\frac{A}{ax^3}\right)$$

input
```
Int[(A + B*x^3)/(x^4*(a + b*x^3)),x]
```

output
```
(-(A/(a*x^3)) - ((A*b - a*B)*Log[x^3])/a^2 + ((A*b - a*B)*Log[a + b*x^3])/
a^2)/3
```

3.66.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.66.4 Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{A}{3a\,x^{3}}+\frac{(-Ab+Ba)\ln(x)}{a^{2}}+\frac{(Ab-Ba)\ln(b\,x^{3}+a)}{3a^{2}}$	46
norman	$-\frac{A}{3a\,x^{3}}-\frac{(Ab-Ba)\ln(x)}{a^{2}}+\frac{(Ab-Ba)\ln(b\,x^{3}+a)}{3a^{2}}$	47
parallelrisch	$-\frac{3A\ln(x)x^{3}b-A\ln(b\,x^{3}+a)x^{3}b-3B\ln(x)x^{3}a+B\ln(b\,x^{3}+a)x^{3}a+Aa}{3x^{3}a^{2}}$	60
risch	$-\frac{A}{3a\,x^{3}}-\frac{\ln(x)Ab}{a^{2}}+\frac{B\ln(x)}{a}+\frac{\ln(-b\,x^{3}-a)Ab}{3a^{2}}-\frac{\ln(-b\,x^{3}-a)B}{3a}$	62

input
```
int((B*x^3+A)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/a/x^3+1/a^2*(-A*b+B*a)*ln(x)+1/3*(A*b-B*a)*ln(b*x^3+a)/a^2
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx=-\frac{(Ba-Ab)x^3\log\left(bx^3+a\right)-3\,(Ba-Ab)x^3\log\left(x\right)+Aa}{3\,a^2x^3}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/3*((B*a - A*b)*x^3*log(b*x^3 + a) - 3*(B*a - A*b)*x^3*log(x) + A*a)/(a^
2*x^3)
```

3.66.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx = -\frac{A}{3ax^3}+\frac{(-Ab+Ba)\log\left(x\right)}{a^2}-\frac{(-Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^2}$$

input

```
integrate((B*x**3+A)/x**4/(b*x**3+a),x)
```

output

```
-A/(3*a*x**3) + (-A*b + B*a)*log(x)/a**2 - (-A*b + B*a)*log(a/b + x**3)/(3
*a**2)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx = -\frac{(Ba-Ab)\log\left(bx^3+a\right)}{3\,a^2}+\frac{(Ba-Ab)\log\left(x^3\right)}{3\,a^2}-\frac{A}{3\,ax^3}$$

input

```
integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="maxima")
```

output

```
-1/3*(B*a - A*b)*log(b*x^3 + a)/a^2 + 1/3*(B*a - A*b)*log(x^3)/a^2 - 1/3*A
/(a*x^3)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx = \frac{(Ba-Ab)\log\left(|x|\right)}{a^2}-\frac{\left(Bab-Ab^2\right)\log\left(|bx^3+a|\right)}{3\,a^2b}-\frac{Bax^3-Abx^3+Aa}{3\,a^2x^3}$$

input

```
integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="giac")
```

output

```
(B*a - A*b)*log(abs(x))/a^2 - 1/3*(B*a*b - A*b^2)*log(abs(b*x^3 + a))/(a^2
*b) - 1/3*(B*a*x^3 - A*b*x^3 + A*a)/(a^2*x^3)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx=\frac{\ln\left(b\,x^3+a\right)\,\left(A\,b-B\,a\right)}{3\,a^2}-\frac{A}{3\,a\,x^3}-\frac{\ln\left(x\right)\,\left(A\,b-B\,a\right)}{a^2}$$

input
```
int((A + B*x^3)/(x^4*(a + b*x^3)),x)
```

output
```
(log(a + b*x^3)*(A*b - B*a))/(3*a^2) - A/(3*a*x^3) - (log(x)*(A*b - B*a))/
a^2
```

3.67 $\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx$

3.67.1 Optimal result

Integrand size = 20, antiderivative size = 165

$$\begin{aligned}\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)}\,dx=&-\frac{A}{4ax^4}+\frac{Ab-aB}{a^2x}-\frac{\sqrt[3]{b}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}}\\&-\frac{\sqrt[3]{b}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}}\\&+\frac{\sqrt[3]{b}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}}\end{aligned}$$

output

```
-1/4*A/a/x^4+(A*b-B*a)/a^2/x-1/3*b^(1/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a
^(7/3)+1/6*b^(1/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(
7/3)-1/3*b^(1/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2
))/a^(7/3)*3^(1/2)
```

3.67.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)}\,dx = \frac{-\frac{3a^{4/3}A}{x^4}+\frac{12\sqrt[3]{a}(Ab-aB)}{x}-4\sqrt{3}\sqrt[3]{b}(Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{b}(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2}{12a^{7/3}}$$

input

```
Integrate[(A + B*x^3)/(x^5*(a + b*x^3)),x]
```

output

```
((-3*a^(4/3)*A)/x^4 + (12*a^(1/3)*(A*b - a*B))/x - 4*Sqrt[3]*b^(1/3)*(A*b
- a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*(-(A*b) + a
*B)*Log[a^(1/3) + b^(1/3)*x] + 2*b^(1/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/(12*a^(7/3))
```

3.67.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {955, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)}\,dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(Ab-aB)\int \frac{1}{x^2\left(bx^3+a\right)}dx}{a}-\frac{A}{4ax^4}$$

$$\Big\downarrow \; 847$$

$$-\frac{(Ab-aB)\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$$\Big\downarrow \; 821$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$$\Big\downarrow \ 16$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$$\Big\downarrow \ 1142$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$$\Big\downarrow \ 25$$

3.67. $\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$$\Big\downarrow 27$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$$\Big\downarrow 1082$$

3.67. $\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$$\Big\downarrow 217$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

$$\Big\downarrow 1103$$

3.67. $\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{4ax^4}$$

input
```
Int[(A + B*x^3)/(x^5*(a + b*x^3)),x]
```

output
```
-1/4*A/(a*x^4) - ((A*b - a*B)*(-(1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(1/3)
*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3
)))/(3*a^(1/3)*b^(1/3))))/a))/a
```

3.67.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.67. $\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx$

3.67.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.79

method	result
default	$-\frac{A}{4a\,x^4}-\frac{-Ab+Ba}{x\,a^2}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b(Ab-Ba)}{a^2}$
risch	$\frac{\frac{(Ab-Ba)x^3}{a^2}-\frac{A}{4a}}{x^4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^7\_Z^3+A^3b^4-3A^2Ba\,b^3+3A\,B^2a^2b^2-B^3a^3b\right)}\_R\ln\left(\left(-4a^7\_R^3-3A^3b^4+9A^2Ba\,b^3-9A\,B^2a^2\right.\right.\right.}{3}$

input
```
int((B*x^3+A)/x^5/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/a/x^4-(-A*b+B*a)/x/a^2+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/
(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b*(A*b-B*a)/a^2
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96

$$\int\frac{A+Bx^3}{x^5\left(a+bx^3\right)}\,dx=$$
$$-\frac{4\sqrt{3}(Ba-Ab)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-2\left(Ba-Ab\right)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-\right.}{12\,a^2x^4}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/12*(4*sqrt(3)*(B*a - A*b)*x^4*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^
(1/3) + 1/3*sqrt(3)) - 2*(B*a - A*b)*x^4*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/
a)^(2/3) - a*(-b/a)^(1/3)) + 4*(B*a - A*b)*x^4*(-b/a)^(1/3)*log(b*x + a*(-
b/a)^(2/3)) + 12*(B*a - A*b)*x^3 + 3*A*a)/(a^2*x^4)
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^3}{x^5\,(a+bx^3)}\,dx$$
$$=\operatorname{RootSum}\Bigg(27t^3a^7+A^3b^4-3A^2Bab^3+3AB^2a^2b^2-B^3a^3b,\ \Bigg(t\mapsto t\log\Bigg(\frac{9t^2a^5}{A^2b^3-2ABab^2+B^2a^2b}+x\Bigg)$$
$$+\frac{-Aa+x^3\cdot(4Ab-4Ba)}{4a^2x^4}$$

input
```
integrate((B*x**3+A)/x**5/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**7 + A**3*b**4 - 3*A**2*B*a*b**3 + 3*A*B**2*a**2*b**2 -
 B**3*a**3*b, Lambda(_t, _t*log(9*_t**2*a**5/(A**2*b**3 - 2*A*B*a*b**2 + B
**2*a**2*b) + x))) + (-A*a + x**3*(4*A*b - 4*B*a))/(4*a**2*x**4)
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{x^5\,(a+bx^3)}\,dx = -\frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{4\,(Ba-Ab)x^3+Aa}{4\,a^2x^4}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(a^2*(a/b)^(1/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/
3))/(a^2*(a/b)^(1/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a^2*(a/b)^(1
/3)) - 1/4*(4*(B*a - A*b)*x^3 + A*a)/(a^2*x^4)
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)}\,dx = {} & \frac{\left(Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3} \\ & + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba - \left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b} \\ & - \frac{\left(\left(-ab^2\right)^{\frac{2}{3}}Ba - \left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^3b} \\ & - \frac{4\,Bax^3-4\,Abx^3+Aa}{4\,a^2x^4}\end{aligned}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*(B*a*b*(-a/b)^(1/3) - A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/a^3 + 1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*ar
ctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) - 1/6*((-a*b^2
)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/(a^3*b) - 1/4*(4*B*a*x^3 - 4*A*b*x^3 + A*a)/(a^2*x^4)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.08

$$\begin{aligned}& \int \frac{A+Bx^3}{x^5\left(a+bx^3\right)}\,dx \\ & = \frac{\left(-b\right)^{1/3}\ln\left(a^{1/3}\left(-b\right)^{8/3}+b^3\,x\right)\,\left(A\,b-B\,a\right)}{3\,a^{7/3}} - \frac{\frac{A}{4\,a}-\frac{x^3\,\left(A\,b-B\,a\right)}{a^2}}{x^4} \\ & + \frac{\left(-b\right)^{1/3}\ln\left(a^{1/3}\left(-b\right)^{8/3}-2\,b^3\,x+\sqrt{3}\,a^{1/3}\left(-b\right)^{8/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{7/3}} \\ & - \frac{\left(-b\right)^{1/3}\ln\left(2\,b^3\,x-a^{1/3}\left(-b\right)^{8/3}+\sqrt{3}\,a^{1/3}\left(-b\right)^{8/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{7/3}}\end{aligned}$$

input
```
int((A + B*x^3)/(x^5*(a + b*x^3)),x)
```

output

```
((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(A*b - B*a))/(3*a^(7/3)) - (A/
(4*a) - (x^3*(A*b - B*a))/a^2)/x^4 + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) -
2*b^3*x + 3^(1/2)*a^(1/3)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a
))/(3*a^(7/3)) - ((-b)^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^
(1/3)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(7/3))
```

3.67. $\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx$

3.68 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx$

3.68.1 Optimal result

Integrand size = 20, antiderivative size = 168

$$\begin{aligned}\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx=&-\frac{A}{5ax^5}+\frac{Ab-aB}{2a^2x^2}-\frac{b^{2/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}}\\&+\frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}}\\&-\frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}}\end{aligned}$$

output
```
-1/5*A/a/x^5+1/2*(A*b-B*a)/a^2/x^2+1/3*b^(2/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3
)*x)/a^(8/3)-1/6*b^(2/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/a^(8/3)-1/3*b^(2/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*
3^(1/2))/a^(8/3)*3^(1/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx$$

$$=\frac{-\frac{6a^{5/3}A}{x^5}+\frac{15a^{2/3}(Ab-aB)}{x^2}-10\sqrt{3}b^{2/3}(Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+}{30a^{8/3}}$$

input
```
Integrate[(A + B*x^3)/(x^6*(a + b*x^3)),x]
```

output
```
((-6*a^(5/3)*A)/x^5 + (15*a^(2/3)*(A*b - a*B))/x^2 - 10*Sqrt[3]*b^(2/3)*(A
*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*b^(2/3)*(A*b -
a*B)*Log[a^(1/3) + b^(1/3)*x] + 5*b^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(30*a^(8/3))
```

3.68.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {955, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(Ab-aB)\int \frac{1}{x^3\left(bx^3+a\right)}dx}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(Ab-aB)\left(-\frac{b\int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \text{750}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \ 16$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \ 1142$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \ 25$$

3.68. $\int \frac{A+Bx^3}{x^6(a+bx^3)}\,dx$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \; 27$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \; 1082$$

3.68. $\int \frac{A+Bx^3}{x^6(a+bx^3)}\,dx$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \; 217$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \; 1103$$

3.68. $\int \frac{A+Bx^3}{x^6(a+bx^3)}\,dx$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{A}{5ax^5}$$

input

```
Int[(A + B*x^3)/(x^6*(a + b*x^3)),x]
```

output

```
-1/5*A/(a*x^5) - ((A*b - a*B)*(-1/2*1/(a*x^2) - (b*(Log[a^(1/3) + b^(1/3)*
x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/
3)))/(3*a^(2/3))))/a))/a
```

3.68.3.1 Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.68. $\int \frac{A+Bx^3}{x^6(a+bx^3)}\,dx$

3.68.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.77

method	result
default	$-\frac{A}{5a\,x^{5}}-\frac{-Ab+Ba}{2x^{2}a^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b(Ab-Ba)}{a^{2}}$
risch	$\frac{\frac{(Ab-Ba)x^{3}}{2a^{2}}-\frac{A}{5a}}{x^{5}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{8}\_Z^{3}-A^{3}b^{5}+3A^{2}Ba\,b^{4}-3A\,B^{2}a^{2}b^{3}+B^{3}a^{3}b^{2}\right)}\_R\ln\left(\left(-4\_R^{3}a^{8}+3A^{3}b^{5}-9A^{2}Ba\,b^{4}+9A\,B^{2}a\right.\right.\right.}{3}$

input
```
int((B*x^3+A)/x^6/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/5*A/a/x^5-1/2*(-A*b+B*a)/x^2/a^2+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1
/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b*(A*b-B*a)/a^2
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^{3}}{x^{6}\left(a+bx^{3}\right)}\,dx =$$

$$-\frac{10\sqrt{3}(Ba-Ab)x^{5}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-5\,(Ba-Ab)x^{5}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}-abx\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+\right.}{30\,a^{2}x^{5}}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/30*(10*sqrt(3)*(B*a - A*b)*x^5*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*
x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*(B*a - A*b)*x^5*(b^2/a^2)^(1/3)*log(
b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*(B*a - A*b)*x^
5*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 15*(B*a - A*b)*x^3 + 6*A*
a)/(a^2*x^5)
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^3}{x^6\,(a+bx^3)}\,dx$$
$$=\operatorname{RootSum}\left(27t^3a^8-A^3b^5+3A^2Bab^4-3AB^2a^2b^3+B^3a^3b^2,\left(t\mapsto t\log\left(-\frac{3ta^3}{-Ab^2+Bab}+x\right)\right)\right)$$
$$+\frac{-2Aa+x^3\cdot(5Ab-5Ba)}{10a^2x^5}$$

input
```
integrate((B*x**3+A)/x**6/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**8 - A**3*b**5 + 3*A**2*B*a*b**4 - 3*A*B**2*a**2*b**3 +
 B**3*a**3*b**2, Lambda(_t, _t*log(-3*_t*a**3/(-A*b**2 + B*a*b) + x))) + (
-2*A*a + x**3*(5*A*b - 5*B*a))/(10*a**2*x**5)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^6\,(a+bx^3)}\,dx=-\frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,(Ba-Ab)x^3+2\,Aa}{10\,a^2x^5}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(a^2*(a/b)^(2/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/
3))/(a^2*(a/b)^(2/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a^2*(a/b)^(2
/3)) - 1/10*(5*(B*a - A*b)*x^3 + 2*A*a)/(a^2*x^5)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx = &-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3}\\ &+\frac{\left(Bab-Ab^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3}\\ &-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^3}\\ &-\frac{5\,Bax^3-5\,Abx^3+2\,Aa}{10\,a^2x^5}\end{aligned}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^3 + 1/3*(B*a*b - A*b^2)*(-a/b)^(1/3)*
log(abs(x - (-a/b)^(1/3)))/a^3 - 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*
A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^3 - 1/10*(5*B*a*x^3 - 5*A*
b*x^3 + 2*A*a)/(a^2*x^5)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 6.73 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx &= \frac{b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{8/3}}-\frac{\frac{A}{5\,a}-\frac{x^3\,\left(A\,b-B\,a\right)}{2\,a^2}}{x^5}\\&\quad-\frac{b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{8/3}}\\&\quad+\frac{b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{8/3}}\end{aligned}$$

input
```
int((A + B*x^3)/(x^6*(a + b*x^3)),x)
```

output
```
(b^(2/3)*log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(8/3)) - (A/(5*a) - (x
^3*(A*b - B*a))/(2*a^2))/x^5 - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)
*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(8/3)) + (b^(2/3)*l
og(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b
 - B*a))/(3*a^(8/3))
```

3.69 $\int \frac{A+Bx^3}{x^7(a+bx^3)}\,dx$

3.69.1 Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx=-\frac{A}{6ax^6}+\frac{Ab-aB}{3a^2x^3}+\frac{b(Ab-aB)\log(x)}{a^3}-\frac{b(Ab-aB)\log\left(a+bx^3\right)}{3a^3}$$

output
```
-1/6*A/a/x^6+1/3*(A*b-B*a)/a^2/x^3+b*(A*b-B*a)*ln(x)/a^3-1/3*b*(A*b-B*a)*l
n(b*x^3+a)/a^3
```

3.69.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx =\frac{-a(aA-2Abx^3+2aBx^3)+6b(Ab-aB)x^6\log(x)+2b(-Ab+aB)x^6\log\left(a+bx^3\right)}{6a^3x^6}$$

input
```
Integrate[(A + B*x^3)/(x^7*(a + b*x^3)),x]
```

output
```
(-(a*(a*A - 2*A*b*x^3 + 2*a*B*x^3)) + 6*b*(A*b - a*B)*x^6*Log[x] + 2*b*(-(
A*b) + a*B)*x^6*Log[a + b*x^3])/(6*a^3*x^6)
```

3.69.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^9\left(bx^3+a\right)}dx^3$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{3}\int \left(\frac{(aB-Ab)b^2}{a^3\left(bx^3+a\right)}-\frac{(aB-Ab)b}{a^3x^3}+\frac{aB-Ab}{a^2x^6}+\frac{A}{ax^9}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{b\log\left(x^3\right)(Ab-aB)}{a^3}-\frac{b(Ab-aB)\log\left(a+bx^3\right)}{a^3}+\frac{Ab-aB}{a^2x^3}-\frac{A}{2ax^6}\right)$$

input
```
Int[(A + B*x^3)/(x^7*(a + b*x^3)),x]
```

output
```
(-1/2*A/(a*x^6) + (A*b - a*B)/(a^2*x^3) + (b*(A*b - a*B)*Log[x^3])/a^3 - (
b*(A*b - a*B)*Log[a + b*x^3])/a^3)/3
```

3.69.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.69.4 Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{A}{6a\,x^{6}}-\frac{-Ab+Ba}{3x^{3}a^{2}}+\frac{b(Ab-Ba)\ln(x)}{a^{3}}-\frac{b(Ab-Ba)\ln(b\,x^{3}+a)}{3a^{3}}$	64
norman	$\frac{-\frac{A}{6a}+\frac{(Ab-Ba)x^{3}}{3a^{2}}}{x^{6}}+\frac{b(Ab-Ba)\ln(x)}{a^{3}}-\frac{b(Ab-Ba)\ln(b\,x^{3}+a)}{3a^{3}}$	66
risch	$\frac{-\frac{A}{6a}+\frac{(Ab-Ba)x^{3}}{3a^{2}}}{x^{6}}+\frac{b^{2}\ln(x)A}{a^{3}}-\frac{b\ln(x)B}{a^{2}}-\frac{b^{2}\ln(b\,x^{3}+a)A}{3a^{3}}+\frac{b\ln(b\,x^{3}+a)B}{3a^{2}}$	80
parallelrisch	$\frac{6A\ln(x)x^{6}b^{2}-2A\ln(b\,x^{3}+a)x^{6}b^{2}-6B\ln(x)x^{6}ab+2B\ln(b\,x^{3}+a)x^{6}ab+2aAb\,x^{3}-2a^{2}B\,x^{3}-a^{2}A}{6a^{3}x^{6}}$	87

input
```
int((B*x^3+A)/x^7/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*A/a/x^6-1/3*(-A*b+B*a)/x^3/a^2+b*(A*b-B*a)*ln(x)/a^3-1/3*b*(A*b-B*a)*
ln(b*x^3+a)/a^3
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx = \frac{2\left(Bab-Ab^2\right)x^6\log\left(bx^3+a\right)-6\left(Bab-Ab^2\right)x^6\log\left(x\right)-2\left(Ba^2-Aab\right)x^3-Aa^2}{6\,a^3x^6}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(2*(B*a*b - A*b^2)*x^6*log(b*x^3 + a) - 6*(B*a*b - A*b^2)*x^6*log(x) -
 2*(B*a^2 - A*a*b)*x^3 - A*a^2)/(a^3*x^6)
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx = \frac{-Aa+x^3\cdot(2Ab-2Ba)}{6a^2x^6} - \frac{b(-Ab+Ba)\log(x)}{a^3} + \frac{b(-Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^3}$$

input
```
integrate((B*x**3+A)/x**7/(b*x**3+a),x)
```

output
```
(-A*a + x**3*(2*A*b - 2*B*a))/(6*a**2*x**6) - b*(-A*b + B*a)*log(x)/a**3 +
 b*(-A*b + B*a)*log(a/b + x**3)/(3*a**3)
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx = \frac{(Bab-Ab^2)\log\left(bx^3+a\right)}{3\,a^3} - \frac{(Bab-Ab^2)\log\left(x^3\right)}{3\,a^3} - \frac{2\,(Ba-Ab)x^3+Aa}{6\,a^2x^6}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*(B*a*b - A*b^2)*log(b*x^3 + a)/a^3 - 1/3*(B*a*b - A*b^2)*log(x^3)/a^3
- 1/6*(2*(B*a - A*b)*x^3 + A*a)/(a^2*x^6)
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx = -\frac{(Bab-Ab^2)\log\left(|x|\right)}{a^3} + \frac{(Bab^2-Ab^3)\log\left(|bx^3+a|\right)}{3\,a^3b} + \frac{3\,Babx^6-3\,Ab^2x^6-2\,Ba^2x^3+2\,Aabx^3-Aa^2}{6\,a^3x^6}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="giac")
```

output
```
-(B*a*b - A*b^2)*log(abs(x))/a^3 + 1/3*(B*a*b^2 - A*b^3)*log(abs(b*x^3 + a
))/(a^3*b) + 1/6*(3*B*a*b*x^6 - 3*A*b^2*x^6 - 2*B*a^2*x^3 + 2*A*a*b*x^3 -
A*a^2)/(a^3*x^6)
```

3.69.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx=\frac{\ln\left(x\right)\,\left(A\,b^2-B\,a\,b\right)}{a^3}-\frac{\ln\left(b\,x^3+a\right)\,\left(A\,b^2-B\,a\,b\right)}{3\,a^3}-\frac{\frac{A}{6\,a}-\frac{x^3\,\left(A\,b-B\,a\right)}{3\,a^2}}{x^6}$$

input
```
int((A + B*x^3)/(x^7*(a + b*x^3)),x)
```

output
```
(log(x)*(A*b^2 - B*a*b))/a^3 - (log(a + b*x^3)*(A*b^2 - B*a*b))/(3*a^3) -
(A/(6*a) - (x^3*(A*b - B*a))/(3*a^2))/x^6
```

3.70 $\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)}\,dx$

3.70.1 Optimal result

Integrand size = 20, antiderivative size = 184

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)}\,dx = -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{b^{4/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}}$$
$$+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}$$
$$-\frac{b^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}}$$

output

```
-1/7*A/a/x^7+1/4*(A*b-B*a)/a^2/x^4-b*(A*b-B*a)/a^3/x+1/3*b^(4/3)*(A*b-B*a)
*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)-1/6*b^(4/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/a^(10/3)+1/3*b^(4/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)*3^(1/2)
```

3.70.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)}\,dx = \frac{-\frac{12a^{7/3}A}{x^7}+\frac{21a^{4/3}(Ab-aB)}{x^4}+\frac{84\sqrt[3]{a}b(-Ab+aB)}{x}+28\sqrt{3}b^{4/3}(Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+28b^{4/3}(Ab-aB)\mathrm{lo}}{84a^{10/3}}$$

input
```
Integrate[(A + B*x^3)/(x^8*(a + b*x^3)),x]
```

output
```
((-12*a^(7/3)*A)/x^7 + (21*a^(4/3)*(A*b - a*B))/x^4 + (84*a^(1/3)*b*(-(A*b
) + a*B))/x + 28*Sqrt[3]*b^(4/3)*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]] + 28*b^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*b^(4
/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(84*a^(
10/3))
```

3.70.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {955, 847, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(Ab-aB)\int \frac{1}{x^5\left(bx^3+a\right)}dx}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(Ab-aB)\left(-\frac{b\int \frac{1}{x^2\left(bx^3+a\right)}dx}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow \text{ 821}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow \text{ 1142}$$

3.70. $\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow 25$$

3.70. $\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow \textcolor{blue}{27}$$

3.70. $\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow 1082$$

3.70. $\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow \text{217}$$

3.70. $\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow 1103$$

3.70. $\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{a}-\frac{A}{7ax^7}$$

input
```
Int[(A + B*x^3)/(x^8*(a + b*x^3)),x]
```

3.70. $\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx$

output
```
-1/7*A/(a*x^7) - ((A*b - a*B)*(-1/4*1/(a*x^4) - (b*(-(1/(a*x)) - (b*(-1/3*
Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a))/a))/a
```

3.70.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.70.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.82

method	result
default	$-\frac{A}{7a\,x^7}-\frac{-Ab+Ba}{4x^4a^2}-\frac{b(Ab-Ba)}{a^3x}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2(Ab-Ba)}{a^3}$
risch	$\frac{-\frac{b(Ab-Ba)x^6}{a^3}+\frac{(Ab-Ba)x^3}{4a^2}-\frac{A}{7a}}{x^7}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}\_Z^3-A^3b^7+3A^2Ba\,b^6-3A\,B^2a^2b^5+B^3a^3b^4\right)}\_R\ln\left(\left(-4a^{10}\_R^3+3A^3b^7-9\right.\right.}{3}$

input
```
int((B*x^3+A)/x^8/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/7*A/a/x^7-1/4*(-A*b+B*a)/x^4/a^2-b*(A*b-B*a)/a^3/x-(-1/3/b/(a/b)^(1/3)*
ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*
3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b^2*(A*b-B*
a)/a^3
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)}\,dx$$
$$=\frac{28\sqrt{3}(Bab-Ab^2)x^7\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+14\,(Bab-Ab^2)x^7\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\Bigg(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\Bigg(}{84\,a^3x}$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/84*(28*sqrt(3)*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a
)^(1/3) - 1/3*sqrt(3)) + 14*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*log(b*x^2 - a*
x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 28*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*log(b*
x + a*(b/a)^(2/3)) + 84*(B*a*b - A*b^2)*x^6 - 21*(B*a^2 - A*a*b)*x^3 - 12*
A*a^2)/(a^3*x^7)
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)}\,dx$$
$$=\operatorname{RootSum}\Bigg(27t^3a^{10}-A^3b^7+3A^2Bab^6-3AB^2a^2b^5+B^3a^3b^4,\Bigg(t\mapsto t\log\Bigg(\frac{9t^2a^7}{A^2b^5-2ABab^4+B^2a^2b^3}+x$$
$$+\frac{-4Aa^2+x^6(-28Ab^2+28Bab)+x^3\cdot(7Aab-7Ba^2)}{28a^3x^7}$$

input
```
integrate((B*x**3+A)/x**8/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**10 - A**3*b**7 + 3*A**2*B*a*b**6 - 3*A*B**2*a**2*b**5
+ B**3*a**3*b**4, Lambda(_t, _t*log(9*_t**2*a**7/(A**2*b**5 - 2*A*B*a*b**4
 + B**2*a**2*b**3) + x))) + (-4*A*a**2 + x**6*(-28*A*b**2 + 28*B*a*b) + x*
*3*(7*A*a*b - 7*B*a**2))/(28*a**3*x**7)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)}\,dx = \frac{\sqrt{3}(Bab-Ab^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Bab-Ab^2)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Bab-Ab^2)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{28\,(Bab-Ab^2)x^6-7\,(Ba^2-Aab)x^3-4\,Aa^2}{28\,a^3x^7}$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(B*a*b - A*b^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/(a^3*(a/b)^(1/3)) + 1/6*(B*a*b - A*b^2)*log(x^2 - x*(a/b)^(1/3) + (a
/b)^(2/3))/(a^3*(a/b)^(1/3)) - 1/3*(B*a*b - A*b^2)*log(x + (a/b)^(1/3))/(a
^3*(a/b)^(1/3)) + 1/28*(28*(B*a*b - A*b^2)*x^6 - 7*(B*a^2 - A*a*b)*x^3 - 4
*A*a^2)/(a^3*x^7)
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx = -\frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}Ba-(-ab^2)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^4} - \frac{\left(Bab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^4} + \frac{\left((-ab^2)^{\frac{2}{3}}Ba-(-ab^2)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^4} + \frac{28\,Babx^6-28\,Ab^2x^6-7\,Ba^2x^3+7\,Aabx^3-4\,Aa^2}{28\,a^3x^7}$$

input

```
integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="giac")
```

output

```
-1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4 - 1/3*(B*a*b^2*(-a/b)^(1/3) - A*b^3
*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 1/6*((-a*b^2)
^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
a^4 + 1/28*(28*B*a*b*x^6 - 28*A*b^2*x^6 - 7*B*a^2*x^3 + 7*A*a*b*x^3 - 4*A*
a^2)/(a^3*x^7)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 6.81 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx = \frac{b^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{10/3}} - \frac{\frac{A}{7\,a}-\frac{x^3\,\left(A\,b-B\,a\right)}{4\,a^2}+\frac{b\,x^6\,\left(A\,b-B\,a\right)}{a^3}}{x^7} + \frac{b^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{10/3}} - \frac{b^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{10/3}}$$

input

```
int((A + B*x^3)/(x^8*(a + b*x^3)),x)
```

output

```
(b^(4/3)*log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(10/3)) - (A/(7*a) - (
x^3*(A*b - B*a))/(4*a^2) + (b*x^6*(A*b - B*a))/a^3)/x^7 + (b^(4/3)*log(3^(
1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a
))/(3*a^(10/3)) - (b^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))
*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(10/3))
```

3.70. $\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx$

3.71 $\int \frac{x^9\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

3.71.1 Optimal result

Integrand size = 20, antiderivative size = 233

$$\begin{aligned}\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx = &-\frac{a(7Ab-10aB)x}{3b^4}+\frac{(7Ab-10aB)x^4}{12b^3}-\frac{(7Ab-10aB)x^7}{21ab^2}\\&+\frac{(Ab-aB)x^{10}}{3ab\,(a+bx^3)}-\frac{a^{4/3}(7Ab-10aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{13/3}}\\&+\frac{a^{4/3}(7Ab-10aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{13/3}}\\&-\frac{a^{4/3}(7Ab-10aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{13/3}}\end{aligned}$$

output
```
-1/3*a*(7*A*b-10*B*a)*x/b^4+1/12*(7*A*b-10*B*a)*x^4/b^3-1/21*(7*A*b-10*B*a
)*x^7/a/b^2+1/3*(A*b-B*a)*x^10/a/b/(b*x^3+a)+1/9*a^(4/3)*(7*A*b-10*B*a)*ln
(a^(1/3)+b^(1/3)*x)/b^(13/3)-1/18*a^(4/3)*(7*A*b-10*B*a)*ln(a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/b^(13/3)-1/9*a^(4/3)*(7*A*b-10*B*a)*arctan(1/3*(a
^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(13/3)*3^(1/2)
```

3.71.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.87

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{252a\sqrt[3]{b}(-2Ab+3aB)x+63b^{4/3}(Ab-2aB)x^4+36b^{7/3}Bx^7+\frac{84a^2\sqrt[3]{b}(-Ab+aB)x}{a+bx^3}+28\sqrt{3}a^{4/3}(-7Ab+10a}{}$$

input
```
Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(252*a*b^(1/3)*(-2*A*b + 3*a*B)*x + 63*b^(4/3)*(A*b - 2*a*B)*x^4 + 36*b^(7
/3)*B*x^7 + (84*a^2*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + 28*Sqrt[3]*a^(
4/3)*(-7*A*b + 10*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 28*a^
(4/3)*(-7*A*b + 10*a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/3)*(-7*A*b + 10
*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(252*b^(13/3))
```

3.71.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.83, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {957, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{10}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(7Ab-10aB)\int \frac{x^9}{bx^3+a}dx}{3ab}$$

$$\Big\downarrow \text{831}$$

$$\frac{x^{10}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(7Ab-10aB)\int \left(\frac{x^6}{b}-\frac{ax^3}{b^2}-\frac{a^3}{b^3\left(bx^3+a\right)}+\frac{a^2}{b^3}\right)dx}{3ab}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{10}(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(7Ab-10aB)\left(\frac{a^{7/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}}+\frac{a^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}}-\frac{a^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}}+\frac{a^2x}{b^3}-\frac{ax^4}{4b^2}+\frac{x^7}{7b}\right)}{3ab}$$

input
```
Int[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^10)/(3*a*b*(a + b*x^3)) - ((7*A*b - 10*a*B)*((a^2*x)/b^3 -
(a*x^4)/(4*b^2) + x^7/(7*b) + (a^(7/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*b^(10/3)) - (a^(7/3)*Log[a^(1/3) + b^(1/3)*x])/(3
*b^(10/3)) + (a^(7/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b
^(10/3))))/(3*a*b)
```

3.71.3.1 Defintions of rubi rules used

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.71.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.49

method	result
risch	$\frac{B\,x^{7}}{7 b^{2}}+\frac{A\,x^{4}}{4 b^{2}}-\frac{B a\,x^{4}}{2 b^{3}}-\frac{2 a A x}{b^{3}}+\frac{3 a^{2} B x}{b^{4}}+\frac{\left(-\frac{1}{3} a^{2} b A+\frac{1}{3} a^{3} B\right) x}{b^{4} \left(b\,x^{3}+a\right)}+\frac{a^{2} \left(\sum_{\_R=\operatorname{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\left(7 A b-10 B a\right) \ln\left(x-\_R\right)}{\_R^{2}}\right)}{9 b^{5}}$
default	$-\frac{-\frac{1}{7} b^{2} B\,x^{7}-\frac{1}{4} A\,b^{2} x^{4}+\frac{1}{2} B a b\,x^{4}+2 a A b x-3 a^{2} B x}{b^{4}}+\frac{a^{2} \left(\frac{\left(-\frac{A b}{3}+\frac{B a}{3}\right) x}{b\,x^{3}+a}+\frac{\left(7 A b-10 B a\right) \left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}+\sqrt{}\right)}{3}\right)}{b^{4}}$

input
```
int(x^9*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*B*x^7/b^2+1/4/b^2*A*x^4-1/2/b^3*B*a*x^4-2/b^3*a*A*x+3/b^4*a^2*B*x+(-1/
3*a^2*b*A+1/3*a^3*B)*x/b^4/(b*x^3+a)+1/9/b^5*a^2*sum((7*A*b-10*B*a)/_R^2*l
n(x-_R),_R=RootOf(_Z^3*b+a))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.16

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{36\,Bb^3x^{10} - 9\,(10\,Bab^2 - 7\,Ab^3)x^7 + 63\,(10\,Ba^2b - 7\,Aab^2)x^4 - 28\,\sqrt{3}(10\,Ba^3 - 7\,Aa^2b + (10\,Ba^2b - 7}{}$$

input
```
integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/252*(36*B*b^3*x^10 - 9*(10*B*a*b^2 - 7*A*b^3)*x^7 + 63*(10*B*a^2*b - 7*A
*a*b^2)*x^4 - 28*sqrt(3)*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*
x^3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 1
4*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*x^3)*(a/b)^(1/3)*log(x^
2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b
- 7*A*a*b^2)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(10*B*a^3 - 7*A*a^
2*b)*x)/(b^5*x^3 + a*b^4)
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.67

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^7}{7b^2} + x^4\left(\frac{A}{4b^2} - \frac{Ba}{2b^3}\right) + x\left(-\frac{2Aa}{b^3} + \frac{3Ba^2}{b^4}\right) + \frac{x(-Aa^2b + Ba^3)}{3ab^4 + 3b^5x^3}$$

$$+\mathrm{RootSum}\left(729t^3b^{13} - 343A^3a^4b^3 + 1470A^2Ba^5b^2 - 2100AB^2a^6b + 1000B^3a^7, \left(t \mapsto t\log\left(-\frac{9tb}{-7Aab+}\right.\right.\right.$$

input
```
integrate(x**9*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
B*x**7/(7*b**2) + x**4*(A/(4*b**2) - B*a/(2*b**3)) + x*(-2*A*a/b**3 + 3*B*
a**2/b**4) + x*(-A*a**2*b + B*a**3)/(3*a*b**4 + 3*b**5*x**3) + RootSum(729
*_t**3*b**13 - 343*A**3*a**4*b**3 + 1470*A**2*B*a**5*b**2 - 2100*A*B**2*a*
*6*b + 1000*B**3*a**7, Lambda(_t, _t*log(-9*_t*b**4/(-7*A*a*b + 10*B*a**2)
 + x)))
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.94

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba^3-Aa^2b)x}{3\,(b^5x^3+ab^4)} + \frac{4\,Bb^2x^7-7\,(2\,Bab-Ab^2)x^4+28\,(3\,Ba^2-2\,Aab)x}{28\,b^4} - \frac{\sqrt{3}(10\,Ba^3-7\,Aa^2b)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(10\,Ba^3-7\,Aa^2b)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(10\,Ba^3-7\,Aa^2b)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(B*a^3 - A*a^2*b)*x/(b^5*x^3 + a*b^4) + 1/28*(4*B*b^2*x^7 - 7*(2*B*a*b
 - A*b^2)*x^4 + 28*(3*B*a^2 - 2*A*a*b)*x)/b^4 - 1/9*sqrt(3)*(10*B*a^3 - 7*
A*a^2*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2
/3)) + 1/18*(10*B*a^3 - 7*A*a^2*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/
(b^5*(a/b)^(2/3)) - 1/9*(10*B*a^3 - 7*A*a^2*b)*log(x + (a/b)^(1/3))/(b^5*(
a/b)^(2/3))
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx = &-\frac{\sqrt{3}\left(10\left(-ab^2\right)^{\frac{1}{3}}Ba^2-7\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5}\\ &+\frac{\left(10\,Ba^3-7\,Aa^2b\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4}\\ &-\frac{\left(10\left(-ab^2\right)^{\frac{1}{3}}Ba^2-7\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5}\\ &+\frac{Ba^3x-Aa^2bx}{3\left(bx^3+a\right)b^4}\\ &+\frac{4\,Bb^{12}x^7-14\,Bab^{11}x^4+7\,Ab^{12}x^4+84\,Ba^2b^{10}x-56\,Aab^{11}x}{28\,b^{14}}\end{aligned}$$

input

```
integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output

```
-1/9*sqrt(3)*(10*(-a*b^2)^(1/3)*B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*arctan(1/3
*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 + 1/9*(10*B*a^3 - 7*A*a^2*
b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 1/18*(10*(-a*b^2)^(1/
3)*B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)
)/b^5 + 1/3*(B*a^3*x - A*a^2*b*x)/((b*x^3 + a)*b^4) + 1/28*(4*B*b^12*x^7 -
 14*B*a*b^11*x^4 + 7*A*b^12*x^4 + 84*B*a^2*b^10*x - 56*A*a*b^11*x)/b^14
```

3.71.9 Mupad [B] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx = &\,x^4\left(\frac{A}{4\,b^2}-\frac{B\,a}{2\,b^3}\right)-x\left(\frac{2\,a\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)+\frac{B\,x^7}{7\,b^2}\\ &+\frac{x\left(\frac{B\,a^3}{3}-\frac{A\,a^2\,b}{3}\right)}{b^5\,x^3+a\,b^4}+\frac{a^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(7\,A\,b-10\,B\,a\right)}{9\,b^{13/3}}\\ &-\frac{a^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(7\,A\,b-10\,B\,a\right)}{9\,b^{13/3}}\\ &+\frac{a^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(7\,A\,b-10\,B\,a\right)}{9\,b^{13/3}}\end{aligned}$$

input

```
int((x^9*(A + B*x^3))/(a + b*x^3)^2,x)
```

output

```
x^4*(A/(4*b^2) - (B*a)/(2*b^3)) - x*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^
2)/b^4) + (B*x^7)/(7*b^2) + (x*((B*a^3)/3 - (A*a^2*b)/3))/(a*b^4 + b^5*x^3
) + (a^(4/3)*log(b^(1/3)*x + a^(1/3))*(7*A*b - 10*B*a))/(9*b^(13/3)) - (a^
(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/
2)*(7*A*b - 10*B*a))/(9*b^(13/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^
(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - 10*B*a))/(9*b^(13/3))
```

3.71. $\int \frac{x^9\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

3.72 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx$

3.72.1 Optimal result

Integrand size = 20, antiderivative size = 82

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx=\frac{(Ab-2aB)x^3}{3b^3}+\frac{Bx^6}{6b^2}-\frac{a^2(Ab-aB)}{3b^4(a+bx^3)}-\frac{a(2Ab-3aB)\log(a+bx^3)}{3b^4}$$

output

```
1/3*(A*b-2*B*a)*x^3/b^3+1/6*B*x^6/b^2-1/3*a^2*(A*b-B*a)/b^4/(b*x^3+a)-1/3*
a*(2*A*b-3*B*a)*ln(b*x^3+a)/b^4
```

3.72.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$=\frac{2b(Ab-2aB)x^3+b^2Bx^6+\frac{2a^2(-Ab+aB)}{a+bx^3}+2a(-2Ab+3aB)\log(a+bx^3)}{6b^4}$$

input

```
Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]
```

output

```
(2*b*(A*b - 2*a*B)*x^3 + b^2*B*x^6 + (2*a^2*(-(A*b) + a*B))/(a + b*x^3) +
2*a*(-2*A*b + 3*a*B)*Log[a + b*x^3])/(6*b^4)
```

3.72.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (A+Bx^3)}{(a+bx^3)^2} \, dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^6 (Bx^3+A)}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \ 86$$

$$\frac{1}{3}\int \left(\frac{Bx^3}{b^2}+\frac{Ab-2aB}{b^3}+\frac{a(3aB-2Ab)}{b^3 (bx^3+a)}-\frac{a^2(aB-Ab)}{b^3 (bx^3+a)^2}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(-\frac{a^2(Ab-aB)}{b^4 (a+bx^3)}-\frac{a(2Ab-3aB)\log (a+bx^3)}{b^4}+\frac{x^3(Ab-2aB)}{b^3}+\frac{Bx^6}{2b^2}\right)$$

input
```
Int[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(((A*b - 2*a*B)*x^3)/b^3 + (B*x^6)/(2*b^2) - (a^2*(A*b - a*B))/(b^4*(a + b
*x^3)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^3])/b^4)/3
```

3.72.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.72.4 Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

method	result
default	$\frac{\left(bB\,x^{3}+Ab-2Ba\right)^{2}}{6b^{4}B}-\frac{a\left(\frac{\left(2Ab-3Ba\right)\ln\left(b\,x^{3}+a\right)}{b}+\frac{a\left(Ab-Ba\right)}{b\left(b\,x^{3}+a\right)}\right)}{3b^{3}}$
norman	$\frac{\frac{B\,x^{9}}{6b}-\frac{a\left(2abA-3a^{2}B\right)}{3b^{4}}+\frac{\left(2Ab-3Ba\right)x^{6}}{6b^{2}}}{b\,x^{3}+a}-\frac{a\left(2Ab-3Ba\right)\ln\left(b\,x^{3}+a\right)}{3b^{4}}$
parallelrisch	$-\frac{-b^{3}B\,x^{9}-2x^{6}b^{3}A+3B\,x^{6}a\,b^{2}+4A\ln\left(b\,x^{3}+a\right)x^{3}a\,b^{2}-6B\ln\left(b\,x^{3}+a\right)x^{3}a^{2}b+4A\ln\left(b\,x^{3}+a\right)a^{2}b-6B\ln\left(b\,x^{3}+a\right)a^{3}+4a^{2}bA-}{6b^{4}\left(b\,x^{3}+a\right)}$
risch	$\frac{B\,x^{6}}{6b^{2}}+\frac{A\,x^{3}}{3b^{2}}-\frac{2Ba\,x^{3}}{3b^{3}}+\frac{A^{2}}{6b^{2}B}-\frac{2aA}{3b^{3}}+\frac{2a^{2}B}{3b^{4}}-\frac{a^{2}A}{3b^{3}\left(b\,x^{3}+a\right)}+\frac{a^{3}B}{3b^{4}\left(b\,x^{3}+a\right)}-\frac{2a\ln\left(b\,x^{3}+a\right)A}{3b^{3}}+\frac{a^{2}\ln\left(b\,x^{3}+\right.}{b^{4}}$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(B*b*x^3+A*b-2*B*a)^2/b^4/B-1/3*a/b^3*((2*A*b-3*B*a)/b*ln(b*x^3+a)+a*(
A*b-B*a)/b/(b*x^3+a))
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.48

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$=\frac{Bb^3x^9-(3\,Bab^2-2\,Ab^3)x^6+2\,Ba^3-2\,Aa^2b-2\,(2\,Ba^2b-Aab^2)x^3+2\,\left(3\,Ba^3-2\,Aa^2b+\left(3\,Ba^2b-2\right.\right.}{6\,(b^5x^3+ab^4)}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/6*(B*b^3*x^9 - (3*B*a*b^2 - 2*A*b^3)*x^6 + 2*B*a^3 - 2*A*a^2*b - 2*(2*B*
a^2*b - A*a*b^2)*x^3 + 2*(3*B*a^3 - 2*A*a^2*b + (3*B*a^2*b - 2*A*a*b^2)*x^
3)*log(b*x^3 + a))/(b^5*x^3 + a*b^4)
```

3.72.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^6}{6b^2} + \frac{a(-2Ab+3Ba)\log{(a+bx^3)}}{3b^4} + x^3\left(\frac{A}{3b^2} - \frac{2Ba}{3b^3}\right) + \frac{-Aa^2b+Ba^3}{3ab^4+3b^5x^3}$$

input
```
integrate(x**8*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
B*x**6/(6*b**2) + a*(-2*A*b + 3*B*a)*log(a + b*x**3)/(3*b**4) + x**3*(A/(3
*b**2) - 2*B*a/(3*b**3)) + (-A*a**2*b + B*a**3)/(3*a*b**4 + 3*b**5*x**3)
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Ba^3 - Aa^2b}{3\,(b^5x^3+ab^4)} + \frac{Bbx^6 - 2\,(2\,Ba - Ab)x^3}{6\,b^3} + \frac{(3\,Ba^2 - 2\,Aab)\log{(bx^3+a)}}{3\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(B*a^3 - A*a^2*b)/(b^5*x^3 + a*b^4) + 1/6*(B*b*x^6 - 2*(2*B*a - A*b)*x
^3)/b^3 + 1/3*(3*B*a^2 - 2*A*a*b)*log(b*x^3 + a)/b^4
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.29

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(3\,Ba^2-2\,Aab)\log\left(|bx^3+a|\right)}{3\,b^4} + \frac{Bb^2x^6-4\,Babx^3+2\,Ab^2x^3}{6\,b^4} - \frac{3\,Ba^2bx^3-2\,Aab^2x^3+2\,Ba^3-Aa^2b}{3\,(bx^3+a)b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*(3*B*a^2 - 2*A*a*b)*log(abs(b*x^3 + a))/b^4 + 1/6*(B*b^2*x^6 - 4*B*a*b
*x^3 + 2*A*b^2*x^3)/b^4 - 1/3*(3*B*a^2*b*x^3 - 2*A*a*b^2*x^3 + 2*B*a^3 - A
*a^2*b)/((b*x^3 + a)*b^4)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = x^3\left(\frac{A}{3\,b^2}-\frac{2\,B\,a}{3\,b^3}\right) + \frac{\ln\left(b\,x^3+a\right)\,\left(3\,B\,a^2-2\,A\,a\,b\right)}{3\,b^4} + \frac{B\,x^6}{6\,b^2} + \frac{B\,a^3-A\,a^2\,b}{3\,b\,\left(b^4\,x^3+a\,b^3\right)}$$

input
```
int((x^8*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
x^3*(A/(3*b^2) - (2*B*a)/(3*b^3)) + (log(a + b*x^3)*(3*B*a^2 - 2*A*a*b))/(
3*b^4) + (B*x^6)/(6*b^2) + (B*a^3 - A*a^2*b)/(3*b*(a*b^3 + b^4*x^3))
```

3.73 $\int \frac{x^7 (A+Bx^3)}{(a+bx^3)^2} \, dx$

3.73.1 Optimal result

Integrand size = 20, antiderivative size = 215

$$\begin{aligned}\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2} \, dx = {} & \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab\,(a+bx^3)} \\ & + \frac{a^{2/3}(5Ab-8aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{11/3}} \\ & + \frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{11/3}} \\ & - \frac{a^{2/3}(5Ab-8aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{11/3}}\end{aligned}$$

output
```
1/6*(5*A*b-8*B*a)*x^2/b^3-1/15*(5*A*b-8*B*a)*x^5/a/b^2+1/3*(A*b-B*a)*x^8/a
/b/(b*x^3+a)+1/9*a^(2/3)*(5*A*b-8*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(11/3)-1/18
*a^(2/3)*(5*A*b-8*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(11/3)+
1/9*a^(2/3)*(5*A*b-8*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2)
)/b^(11/3)*3^(1/2)
```

3.73.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.86

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{45b^{2/3}(Ab-2aB)x^2+18b^{5/3}Bx^5+\frac{30ab^{2/3}(Ab-aB)x^2}{a+bx^3}-10\sqrt{3}a^{2/3}(-5Ab+8aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-10a}{90b^{11/3}}$$

input
```
Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(45*b^(2/3)*(A*b - 2*a*B)*x^2 + 18*b^(5/3)*B*x^5 + (30*a*b^(2/3)*(A*b - a*
B)*x^2)/(a + b*x^3) - 10*Sqrt[3]*a^(2/3)*(-5*A*b + 8*a*B)*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(1/3) + b^
(1/3)*x] + 5*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(90*b^(11/3))
```

3.73.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.87, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {957, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^8(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(5Ab-8aB)\int \frac{x^7}{bx^3+a}dx}{3ab}$$

$$\Big\downarrow \text{831}$$

$$\frac{x^8(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(5Ab-8aB)\int \left(\frac{x^4}{b}+\frac{a^2x}{b^2(bx^3+a)}-\frac{ax}{b^2}\right)dx}{3ab}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^8(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(5Ab-8aB)\left(-\frac{a^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}}+\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}}-\frac{ax^2}{2b^2}+\frac{x^5}{5b}\right)}{3ab}$$

input
```
Int[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^8)/(3*a*b*(a + b*x^3)) - ((5*A*b - 8*a*B)*(-1/2*(a*x^2)/b^2
 + x^5/(5*b) - (a^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/(Sqrt[3]*b^(8/3)) - (a^(5/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(8/3)) + (a^(
5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(8/3))))/(3*a*b)
```

3.73.3.1 Defintions of rubi rules used

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.73.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.44

method	result
risch	$\frac{B\,x^{5}}{5b^{2}}+\frac{A\,x^{2}}{2b^{2}}-\frac{Ba\,x^{2}}{b^{3}}+\frac{\left(\frac{1}{3}abA-\frac{1}{3}a^{2}B\right)x^{2}}{b^{3}\left(b\,x^{3}+a\right)}+\frac{a\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-5Ab+8Ba\right)\ln\left(x-\_R\right)}{\_R}\right)}{9b^{4}}$
default	$\frac{\frac{bB\,x^{5}}{5}+\frac{x^{2}\left(Ab-2Ba\right)}{2}}{b^{3}}-\frac{a\left(\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x^{2}}{b\,x^{3}+a}+\left(\frac{5Ab}{3}-\frac{8Ba}{3}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.\right.}{b^{3}}$

input
```
int(x^7*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*B*x^5/b^2+1/2/b^2*A*x^2-1/b^3*B*a*x^2+(1/3*a*b*A-1/3*a^2*B)*x^2/b^3/(b
*x^3+a)+1/9/b^4*a*sum((-5*A*b+8*B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.20

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$=\frac{18\,Bb^{2}x^{8}-9\,(8\,Bab-5\,Ab^{2})x^{5}-15\,(8\,Ba^{2}-5\,Aab)x^{2}+10\,\sqrt{3}((8\,Bab-5\,Ab^{2})x^{3}+8\,Ba^{2}-5\,Aab)\left(\frac{a^{2}}{b^{2}}\right.}{}$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/90*(18*B*b^2*x^8 - 9*(8*B*a*b - 5*A*b^2)*x^5 - 15*(8*B*a^2 - 5*A*a*b)*x^
2 + 10*sqrt(3)*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/
3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*((8*B*a*b
 - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/
b^2)^(2/3) + a*(a^2/b^2)^(1/3)) - 10*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 -
5*A*a*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/(b^4*x^3 + a*b^3)
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.70

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^5}{5b^2} + x^2\left(\frac{A}{2b^2} - \frac{Ba}{b^3}\right) + \frac{x^2(Aab - Ba^2)}{3ab^3 + 3b^4x^3}$$

$$+\text{RootSum}\left(729t^3b^{11} - 125A^3a^2b^3 + 600A^2Ba^3b^2 - 960AB^2a^4b + 512B^3a^5, \left(t \mapsto t\log\left(\frac{81}{25A^2ab^2 - 80A}\right.\right.\right.$$

input
```
integrate(x**7*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
B*x**5/(5*b**2) + x**2*(A/(2*b**2) - B*a/b**3) + x**2*(A*a*b - B*a**2)/(3*
a*b**3 + 3*b**4*x**3) + RootSum(729*_t**3*b**11 - 125*A**3*a**2*b**3 + 600
*A**2*B*a**3*b**2 - 960*A*B**2*a**4*b + 512*B**3*a**5, Lambda(_t, _t*log(8
1*_t**2*b**7/(25*A**2*a*b**2 - 80*A*B*a**2*b + 64*B**2*a**3) + x)))
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx = &-\frac{(Ba^2 - Aab)x^2}{3\,(b^4x^3 + ab^3)} + \frac{\sqrt{3}(8\,Ba^2 - 5\,Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &+ \frac{2\,Bbx^5 - 5\,(2\,Ba - Ab)x^2}{10\,b^3} \\ &+ \frac{(8\,Ba^2 - 5\,Aab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &- \frac{(8\,Ba^2 - 5\,Aab)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}\end{aligned}$$

input

```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output

```
-1/3*(B*a^2 - A*a*b)*x^2/(b^4*x^3 + a*b^3) + 1/9*sqrt(3)*(8*B*a^2 - 5*A*a*
b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) +
 1/10*(2*B*b*x^5 - 5*(2*B*a - A*b)*x^2)/b^3 + 1/18*(8*B*a^2 - 5*A*a*b)*log
(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3)) - 1/9*(8*B*a^2 - 5*A
*a*b)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(1/3))
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.10

$$
\begin{aligned}
\int \frac{x^7(A+Bx^3)}{\left(a+bx^3\right)^2}\,dx = & -\frac{\left(8\,Ba^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,Aab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^3} \\
& -\frac{\sqrt{3}\left(8\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5} \\
& -\frac{Ba^2x^2-Aabx^2}{3\left(bx^3+a\right)b^3} \\
& +\frac{\left(8\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5} \\
& +\frac{2\,Bb^8x^5-10\,Bab^7x^2+5\,Ab^8x^2}{10\,b^{10}}
\end{aligned}
$$

input

```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output

```
-1/9*(8*B*a^2*(-a/b)^(1/3) - 5*A*a*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a*b^3) - 1/9*sqrt(3)*(8*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^
(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 1/3
*(B*a^2*x^2 - A*a*b*x^2)/((b*x^3 + a)*b^3) + 1/18*(8*(-a*b^2)^(2/3)*B*a -
5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 + 1/10*
(2*B*b^8*x^5 - 10*B*a*b^7*x^2 + 5*A*b^8*x^2)/b^10
```

3.73.9 Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx &= x^2\left(\frac{A}{2\,b^2}-\frac{B\,a}{b^3}\right)+\frac{B\,x^5}{5\,b^2}-\frac{x^2\left(\frac{B\,a^2}{3}-\frac{A\,a\,b}{3}\right)}{b^4\,x^3+a\,b^3}\\&+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(5\,A\,b-8\,B\,a\right)}{9\,b^{11/3}}\\&+\frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,A\,b-8\,B\,a\right)}{9\,b^{11/3}}\\&-\frac{a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,A\,b-8\,B\,a\right)}{9\,b^{11/3}}\end{aligned}$$

input
```
int((x^7*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
x^2*(A/(2*b^2) - (B*a)/b^3) + (B*x^5)/(5*b^2) - (x^2*((B*a^2)/3 - (A*a*b)/
3))/(a*b^3 + b^4*x^3) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*A*b - 8*B*a))
/(9*b^(11/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*(
(3^(1/2)*1i)/2 - 1/2)*(5*A*b - 8*B*a))/(9*b^(11/3)) - (a^(2/3)*log(3^(1/2)
*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b - 8*B*a
))/(9*b^(11/3))
```

3.74 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx$

3.74.1 Optimal result

Integrand size = 20, antiderivative size = 213

$$\begin{aligned}\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\,(a+bx^3)} \\ &\quad + \frac{\sqrt[3]{a}(4Ab-7aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{10/3}} \\ &\quad - \frac{\sqrt[3]{a}(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{10/3}} \\ &\quad + \frac{\sqrt[3]{a}(4Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{10/3}}\end{aligned}$$

output
```
1/3*(4*A*b-7*B*a)*x/b^3-1/12*(4*A*b-7*B*a)*x^4/a/b^2+1/3*(A*b-B*a)*x^7/a/b
/(b*x^3+a)-1/9*a^(1/3)*(4*A*b-7*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(10/3)+1/18*a
^(1/3)*(4*A*b-7*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(10/3)+1/
9*a^(1/3)*(4*A*b-7*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/
b^(10/3)*3^(1/2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.85

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$= \frac{36\sqrt[3]{b}(Ab-2aB)x+9b^{4/3}Bx^4+\frac{12a\sqrt[3]{b}(Ab-aB)x}{a+bx^3}-4\sqrt{3}\sqrt[3]{a}(-4Ab+7aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{a}(-4}{36b^{10/3}}$$

input
```
Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(36*b^(1/3)*(A*b - 2*a*B)*x + 9*b^(4/3)*B*x^4 + (12*a*b^(1/3)*(A*b - a*B)*
x)/(a + b*x^3) - 4*Sqrt[3]*a^(1/3)*(-4*A*b + 7*a*B)*ArcTan[(1 - (2*b^(1/3)
*x)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(1/3) + b^(1/3)*x
] - 2*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(36*b^(10/3))
```

3.74.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {957, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}dx$$
$$\Big\downarrow \ 957$$
$$\frac{x^7(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(4Ab-7aB)\int \frac{x^6}{bx^3+a}dx}{3ab}$$
$$\Big\downarrow \ 831$$
$$\frac{x^7(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(4Ab-7aB)\int \left(\frac{x^3}{b}+\frac{a^2}{b^2\left(bx^3+a\right)}-\frac{a}{b^2}\right)dx}{3ab}$$
$$\Big\downarrow \ 2009$$

$$\frac{x^7(Ab-aB)}{3ab\,(a+bx^3)} - \frac{(4Ab-7aB)\left(-\frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}} - \frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}} - \frac{ax}{b^2} + \frac{x^4}{4b}\right)}{3ab}$$

input
```
Int[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^7)/(3*a*b*(a + b*x^3)) - ((4*A*b - 7*a*B)*(-((a*x)/b^2) + x
^4/(4*b) - (a^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^(7/3)) + (a^(4/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(7/3)) - (a^(4/3)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(7/3))))/(3*a*b)
```

3.74.3.1 Defintions of rubi rules used

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.74.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.41

method	result
risch	$\frac{Bx^4}{4b^2}+\frac{Ax}{b^2}-\frac{2Bax}{b^3}+\frac{\left(\frac{1}{3}abA-\frac{1}{3}a^2B\right)x}{b^3(bx^3+a)}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(-4Ab+7Ba)\ln\left(x-\_R\right)}{\_R^2}\right)}{9b^4}$
default	$\frac{\frac{1}{4}bBx^4+Abx-2Bax}{b^3}-\frac{a\left(\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x}{bx^3+a}+\frac{(4Ab-7Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}\right)}{b^3}$

input
```
int(x^6*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*B*x^4/b^2+1/b^2*A*x-2/b^3*B*a*x+(1/3*a*b*A-1/3*a^2*B)*x/b^3/(b*x^3+a)+
1/9/b^4*a*sum((-4*A*b+7*B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.13

$$\int\frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$=\frac{9\,Bb^2x^7-9\,(7\,Bab-4\,Ab^2)x^4-4\,\sqrt{3}((7\,Bab-4\,Ab^2)x^3+7\,Ba^2-4\,Aab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{}}{3\,a}\right.}{}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*(9*B*b^2*x^7 - 9*(7*B*a*b - 4*A*b^2)*x^4 - 4*sqrt(3)*((7*B*a*b - 4*A*
b^2)*x^3 + 7*B*a^2 - 4*A*a*b)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b
)^(2/3) - sqrt(3)*a)/a) + 2*((7*B*a*b - 4*A*b^2)*x^3 + 7*B*a^2 - 4*A*a*b)*
(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 4*((7*B*a*b - 4*A*
b^2)*x^3 + 7*B*a^2 - 4*A*a*b)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 12*(7*B
*a^2 - 4*A*a*b)*x)/(b^4*x^3 + a*b^3)
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.59

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^4}{4b^2} + x\left(\frac{A}{b^2} - \frac{2Ba}{b^3}\right) + \frac{x(Aab-Ba^2)}{3ab^3+3b^4x^3}$$

$$+\operatorname{RootSum}\left(729t^3b^{10} + 64A^3ab^3 - 336A^2Ba^2b^2 + 588AB^2a^3b - 343B^3a^4, \left(t \mapsto t\log\left(\frac{9tb^3}{-4Ab+7Ba} +\right.\right.\right.$$

input
```
integrate(x**6*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
B*x**4/(4*b**2) + x*(A/b**2 - 2*B*a/b**3) + x*(A*a*b - B*a**2)/(3*a*b**3 +
 3*b**4*x**3) + RootSum(729*_t**3*b**10 + 64*A**3*a*b**3 - 336*A**2*B*a**2
*b**2 + 588*A*B**2*a**3*b - 343*B**3*a**4, Lambda(_t, _t*log(9*_t*b**3/(-4
*A*b + 7*B*a) + x)))
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx = &-\frac{(Ba^2-Aab)x}{3\,(b^4x^3+ab^3)} + \frac{Bbx^4-4\,(2\,Ba-Ab)x}{4\,b^3}\\ &+ \frac{\sqrt{3}(7\,Ba^2-4\,Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\ &- \frac{(7\,Ba^2-4\,Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\ &+ \frac{(7\,Ba^2-4\,Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

input

```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output

```
-1/3*(B*a^2 - A*a*b)*x/(b^4*x^3 + a*b^3) + 1/4*(B*b*x^4 - 4*(2*B*a - A*b)*
x)/b^3 + 1/9*sqrt(3)*(7*B*a^2 - 4*A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(
1/3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) - 1/18*(7*B*a^2 - 4*A*a*b)*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/9*(7*B*a^2 - 4*A*a*b)*l
og(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx = &\frac{\sqrt{3}\left(7\left(-ab^2\right)^{\frac{1}{3}}Ba-4\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4}\\ &- \frac{(7\,Ba^2-4\,Aab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^3}\\ &+ \frac{\left(7\left(-ab^2\right)^{\frac{1}{3}}Ba-4\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4}\\ &- \frac{Ba^2x-Aabx}{3\,(bx^3+a)b^3} + \frac{Bb^6x^4-8\,Bab^5x+4\,Ab^6x}{4\,b^8}\end{aligned}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/9*(7*B*a^2 - 4*A*a*b)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/18*(7*(-a*b^2)^(1/3)*B*a - 4
*(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 - 1/3*(B
*a^2*x - A*a*b*x)/((b*x^3 + a)*b^3) + 1/4*(B*b^6*x^4 - 8*B*a*b^5*x + 4*A*b
^6*x)/b^8
```

3.74.9 Mupad [B] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.91

$$\begin{aligned}
&\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx \\
&= x\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)-\frac{x\left(\frac{B\,a^2}{3}-\frac{A\,a\,b}{3}\right)}{b^4\,x^3+a\,b^3}+\frac{B\,x^4}{4\,b^2} \\
&\quad+\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)\,(4\,A\,b-7\,B\,a)}{9\,b^{10/3}} \\
&\quad-\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}-2\,a\,b^{1/3}\,x+\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(4\,A\,b-7\,B\,a)}{9\,b^{10/3}} \\
&\quad+\frac{(-a)^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}+\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(4\,A\,b-7\,B\,a)}{9\,b^{10/3}}
\end{aligned}$$

input
```
int((x^6*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
x*(A/b^2 - (2*B*a)/b^3) - (x*((B*a^2)/3 - (A*a*b)/3))/(a*b^3 + b^4*x^3) +
(B*x^4)/(4*b^2) + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x)*(4*A*b - 7*B*a
))/(9*b^(10/3)) - ((-a)^(1/3)*log((-a)^(4/3) + 3^(1/2)*(-a)^(4/3)*1i - 2*a
*b^(1/3)*x)*((3^(1/2)*1i)/2 + 1/2)*(4*A*b - 7*B*a))/(9*b^(10/3)) + ((-a)^(
1/3)*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)
/2 - 1/2)*(4*A*b - 7*B*a))/(9*b^(10/3))
```

3.75 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx$

3.75.1 Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^3}{3b^2}+\frac{a(Ab-aB)}{3b^3(a+bx^3)}+\frac{(Ab-2aB)\log(a+bx^3)}{3b^3}$$

output
```
1/3*B*x^3/b^2+1/3*a*(A*b-B*a)/b^3/(b*x^3+a)+1/3*(A*b-2*B*a)*ln(b*x^3+a)/b^
3
```

3.75.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{bBx^3+\frac{a(Ab-aB)}{a+bx^3}+(Ab-2aB)\log(a+bx^3)}{3b^3}$$

input
```
Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(b*B*x^3 + (a*(A*b - a*B))/(a + b*x^3) + (A*b - 2*a*B)*Log[a + b*x^3])/(3*
b^3)
```

3.75.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3 (Bx^3+A)}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{B}{b^2}+\frac{Ab-2aB}{b^2 (bx^3+a)}+\frac{a(aB-Ab)}{b^2 (bx^3+a)^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{a(Ab-aB)}{b^3 (a+bx^3)}+\frac{(Ab-2aB)\log (a+bx^3)}{b^3}+\frac{Bx^3}{b^2}\right)$$

input
```
Int[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((B*x^3)/b^2 + (a*(A*b - a*B))/(b^3*(a + b*x^3)) + ((A*b - 2*a*B)*Log[a +
b*x^3])/b^3)/3
```

3.75.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.75.4 Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

method	result	size
norman	$\frac{\frac{B\,x^{6}}{3b}+\frac{a(Ab-2Ba)}{3b^{3}}}{b\,x^{3}+a}+\frac{(Ab-2Ba)\ln\left(b\,x^{3}+a\right)}{3b^{3}}$	57
default	$\frac{B\,x^{3}}{3b^{2}}+\frac{\frac{(Ab-2Ba)\ln\left(b\,x^{3}+a\right)}{b}+\frac{a(Ab-Ba)}{b\left(b\,x^{3}+a\right)}}{3b^{2}}$	59
risch	$\frac{B\,x^{3}}{3b^{2}}+\frac{aA}{3b^{2}(b\,x^{3}+a)}-\frac{a^{2}B}{3b^{3}(b\,x^{3}+a)}+\frac{\ln\left(b\,x^{3}+a\right)A}{3b^{2}}-\frac{2\ln\left(b\,x^{3}+a\right)Ba}{3b^{3}}$	74
parallelrisch	$\frac{b^{2}B\,x^{6}+A\ln\left(b\,x^{3}+a\right)x^{3}b^{2}-2B\ln\left(b\,x^{3}+a\right)x^{3}ab+A\ln\left(b\,x^{3}+a\right)ab-2B\ln\left(b\,x^{3}+a\right)a^{2}+abA-2a^{2}B}{3b^{3}\left(b\,x^{3}+a\right)}$	92

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/3*B*x^6/b+1/3*a*(A*b-2*B*a)/b^3)/(b*x^3+a)+1/3*(A*b-2*B*a)*ln(b*x^3+a)/
b^3
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.35

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bb^2x^6+Babx^3-Ba^2+Aab-((2\,Bab-Ab^2)x^3+2\,Ba^2-Aab)\log\left(bx^3+a\right)}{3\left(b^4x^3+ab^3\right)}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output

```
1/3*(B*b^2*x^6 + B*a*b*x^3 - B*a^2 + A*a*b - ((2*B*a*b - A*b^2)*x^3 + 2*B*
a^2 - A*a*b)*log(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^3}{3b^2} + \frac{Aab - Ba^2}{3ab^3 + 3b^4x^3} - \frac{(-Ab+2Ba)\log(a+bx^3)}{3b^3}$$

input

```
integrate(x**5*(B*x**3+A)/(b*x**3+a)**2,x)
```

output

```
B*x**3/(3*b**2) + (A*a*b - B*a**2)/(3*a*b**3 + 3*b**4*x**3) - (-A*b + 2*B*
a)*log(a + b*x**3)/(3*b**3)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^3}{3\,b^2} - \frac{Ba^2 - Aab}{3\,(b^4x^3 + ab^3)} - \frac{(2\,Ba - Ab)\log(bx^3+a)}{3\,b^3}$$

input

```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output

```
1/3*B*x^3/b^2 - 1/3*(B*a^2 - A*a*b)/(b^4*x^3 + a*b^3) - 1/3*(2*B*a - A*b)*
log(b*x^3 + a)/b^3
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.52

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\frac{(bx^3+a)B}{b^2} + \frac{(2\,Ba-Ab)\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^2} - \frac{\frac{Ba^2b}{bx^3+a} - \frac{Aab^2}{bx^3+a}}{b^3}}{3\,b}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*((b*x^3 + a)*B/b^2 + (2*B*a - A*b)*log(abs(b*x^3 + a)/((b*x^3 + a)^2*a
bs(b)))/b^2 - (B*a^2*b/(b*x^3 + a) - A*a*b^2/(b*x^3 + a))/b^3)/b
```

3.75.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{B\,x^3}{3\,b^2} + \frac{\ln\left(b\,x^3+a\right)\,\left(A\,b-2\,B\,a\right)}{3\,b^3} - \frac{B\,a^2-A\,a\,b}{3\,b\,\left(b^3\,x^3+a\,b^2\right)}$$

input
```
int((x^5*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(B*x^3)/(3*b^2) + (log(a + b*x^3)*(A*b - 2*B*a))/(3*b^3) - (B*a^2 - A*a*b)
/(3*b*(a*b^2 + b^3*x^3))
```

3.76 $\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

3.76.1 Optimal result

Integrand size = 20, antiderivative size = 196

$$\int \frac{x^4(A+Bx^3)}{\left(a+bx^3\right)^2}\,dx = -\frac{(2Ab-5aB)x^2}{6ab^2}+\frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)} - \frac{(2Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{8/3}} - \frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{8/3}} + \frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{8/3}}$$

output
```
-1/6*(2*A*b-5*B*a)*x^2/a/b^2+1/3*(A*b-B*a)*x^5/a/b/(b*x^3+a)-1/9*(2*A*b-5*
B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(8/3)+1/18*(2*A*b-5*B*a)*ln(a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(8/3)-1/9*(2*A*b-5*B*a)*arctan(1/3
*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(8/3)*3^(1/2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.84

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$= \frac{9b^{2/3}Bx^2 - \frac{6b^{2/3}(Ab-aB)x^2}{a+bx^3} + \frac{2\sqrt{3}(-2Ab+5aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{2(-2Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} + \frac{(2Ab-5aB)\log\left(a^{2/3}-\right.}{\sqrt[3]{}}}{18b^{8/3}}$$

input
```
Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(9*b^(2/3)*B*x^2 - (6*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3) + (2*Sqrt[3]*(-
2*A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (2*(
-2*A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + ((2*A*b - 5*a*B)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(18*b^(8/3))
```

3.76.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {957, 843, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$\Big\downarrow \text{957}$$
$$\frac{x^5(Ab-aB)}{3ab(a+bx^3)} - \frac{(2Ab-5aB)\int \frac{x^4}{bx^3+a}dx}{3ab}$$
$$\Big\downarrow \text{843}$$
$$\frac{x^5(Ab-aB)}{3ab(a+bx^3)} - \frac{(2Ab-5aB)\left(\frac{x^2}{2b} - \frac{a\int \frac{x}{bx^3+a}dx}{b}\right)}{3ab}$$

$$\Big\downarrow \ 821$$

$$\frac{x^5(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \ 16$$

$$\frac{x^5(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \ 1142$$

$$\frac{x^5(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \ 25$$

3.76. $\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

$$\frac{\frac{x^5(Ab-aB)}{3ab\left(a+bx^3\right)}-(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{x^5(Ab-aB)}{3ab\left(a+bx^3\right)}-(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \text{1082}$$

3.76. $\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

$$\frac{x^5(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \; 217$$

$$\frac{x^5(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \; 1103$$

3.76. $\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}dx$

$$\frac{x^5(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(2Ab-5aB)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3ab}$$

input
```
Int[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^5)/(3*a*b*(a + b*x^3)) - ((2*A*b - 5*a*B)*(x^2/(2*b) - (a*(
-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/b))/(3*a*b)
```

3.76.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.76. $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx$

3.76.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{B\,x^{2}}{2b^{2}}+\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x^{2}}{b^{2}\left(b\,x^{3}+a\right)}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(2Ab-5Ba\right)\ln\left(x-\_R\right)}{\_R}}{9b^{3}}$	71
default	$\frac{B\,x^{2}}{2b^{2}}+\frac{\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x^{2}}{b\,x^{3}+a}+\left(-\frac{5Ba}{3}+\frac{2Ab}{3}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}}$	138

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x^2/b^2+(-1/3*A*b+1/3*B*a)*x^2/b^2/(b*x^3+a)+1/9/b^3*sum((2*A*b-5*B*
a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 578, normalized size of antiderivative = 2.95

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \left[9\,Bab^3x^5 + 3\,(5\,Ba^2b^2 - 2\,Aab^3)x^2 - 3\,\sqrt{\tfrac{1}{3}}(5\,Ba^3b - 2\,Aa^2b^2 + (5\,Ba^2b^2 - 2\,Aab^3)x^3)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\right.\right.$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/18*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 3*sqrt(1/3)*(5*B*
a^3*b - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b
^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - (
(5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 +
(-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^
2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^2*b^
4), 1/18*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 6*sqrt(1/3)*(5
*B*a^3*b - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt(-(-a*b^2)^(1/
3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b)
 - ((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^
2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*
B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^
2*b^4)]
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.64

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^2}{2b^2} + \frac{x^2(-Ab+Ba)}{3ab^2+3b^3x^3}$$
$$+\mathrm{RootSum}\left(729t^3ab^8 + 8A^3b^3 - 60A^2Bab^2 + 150AB^2a^2b - 125B^3a^3, \left(t \mapsto t\log\left(\frac{81t^2ab^5}{4A^2b^2 - 20ABab +}\right.\right.\right.$$

input
```
integrate(x**4*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
B*x**2/(2*b**2) + x**2*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729
*_t**3*a*b**8 + 8*A**3*b**3 - 60*A**2*B*a*b**2 + 150*A*B**2*a**2*b - 125*B
**3*a**3, Lambda(_t, _t*log(81*_t**2*a*b**5/(4*A**2*b**2 - 20*A*B*a*b + 25
*B**2*a**2) + x)))
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{(Ba-Ab)x^2}{3(b^3x^3+ab^2)} + \frac{Bx^2}{2b^2} - \frac{\sqrt{3}(5\,Ba-2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\\ &\quad - \frac{(5\,Ba-2\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\\ &\quad + \frac{(5\,Ba-2\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\end{aligned}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(B*a - A*b)*x^2/(b^3*x^3 + a*b^2) + 1/2*B*x^2/b^2 - 1/9*sqrt(3)*(5*B*a
 - 2*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(
1/3)) - 1/18*(5*B*a - 2*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(
a/b)^(1/3)) + 1/9*(5*B*a - 2*A*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{Bx^2}{2\,b^2} - \frac{\sqrt{3}(5\,Ba-2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{1}{3}}\,b^2}\\ &\quad + \frac{(5\,Ba-2\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{1}{3}}\,b^2}\\ &\quad + \frac{\left(5\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2}\\ &\quad + \frac{Bax^2-Abx^2}{3\,(bx^3+a)b^2}\end{aligned}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/2*B*x^2/b^2 - 1/9*sqrt(3)*(5*B*a - 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^2) + 1/18*(5*B*a - 2*A*b)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^2) + 1/9*(5*B*a*(-a/b
)^(1/3) - 2*A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b
^2) + 1/3*(B*a*x^2 - A*b*x^2)/((b*x^3 + a)*b^2)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.81

$$
\begin{aligned}
\int \frac{x^4(A+Bx^3)}{\left(a+bx^3\right)^2}\,dx &= \frac{B\,x^2}{2\,b^2}-\frac{x^2\,\left(\frac{A\,b}{3}-\frac{B\,a}{3}\right)}{b^3\,x^3+a\,b^2}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(2\,A\,b-5\,B\,a\right)}{9\,a^{1/3}\,b^{8/3}} \\
&\quad -\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,A\,b-5\,B\,a\right)}{9\,a^{1/3}\,b^{8/3}} \\
&\quad +\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,A\,b-5\,B\,a\right)}{9\,a^{1/3}\,b^{8/3}}
\end{aligned}
$$

input
```
int((x^4*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(B*x^2)/(2*b^2) - (x^2*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) - (log(b^(1/
3)*x + a^(1/3))*(2*A*b - 5*B*a))/(9*a^(1/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3
)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*A*b - 5*B*a))/(9*a
^(1/3)*b^(8/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/
2)*1i)/2 + 1/2)*(2*A*b - 5*B*a))/(9*a^(1/3)*b^(8/3))
```

3.77 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx$

3.77.1 Optimal result

Integrand size = 20, antiderivative size = 190

$$\begin{aligned}\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx = &-\frac{(Ab-4aB)x}{3ab^2}+\frac{(Ab-aB)x^4}{3ab\,(a+bx^3)}-\frac{(Ab-4aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}}\\&+\frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}}\\&-\frac{(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{7/3}}\end{aligned}$$

output
```
-1/3*(A*b-4*B*a)*x/a/b^2+1/3*(A*b-B*a)*x^4/a/b/(b*x^3+a)+1/9*(A*b-4*B*a)*l
n(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(7/3)-1/18*(A*b-4*B*a)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(7/3)-1/9*(A*b-4*B*a)*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(7/3)*3^(1/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.84

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$=\frac{18\sqrt[3]{b}Bx-\frac{6\sqrt[3]{b}(Ab-aB)x}{a+bx^3}+\frac{2\sqrt{3}(-Ab+4aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{2(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}+\frac{(-Ab+4aB)\log\left(a^{2/3}-\sqrt[3]{}\right.}{a^{2/3}}}{18b^{7/3}}$$

input
```
Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(18*b^(1/3)*B*x - (6*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3) + (2*Sqrt[3]*(-(A*
b) + 4*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(A*b
 - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + ((-(A*b) + 4*a*B)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(18*b^(7/3))
```

3.77.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {957, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\int \frac{x^3}{bx^3+a}dx}{3ab}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^3+a}dx}{b}\right)}{3ab}$$

$$\Big\downarrow \text{750}$$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \text{16}$$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \text{1142}$$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \text{25}$$

3.77. $\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \; 27$$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \; 1082$$

3.77. $\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \ 217$$

$$\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

$$\Big\downarrow \ 1103$$

3.77. $\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

$$\frac{x^4(Ab-aB)}{3ab\,(a+bx^3)}-\frac{(Ab-4aB)\left(\frac{x}{b}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3ab}$$

input
```
Int[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^4)/(3*a*b*(a + b*x^3)) - ((A*b - 4*a*B)*(x/b - (a*(Log[a^(1
/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/b))/(3*a*b)
```

3.77.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.77. $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx$

3.77.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{Bx}{b^2}+\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x}{b^2(b\,x^3+a)}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab-4Ba)\ln\left(x-\_R\right)}{\_R^2}}{9b^3}$	65
default	$\frac{Bx}{b^2}+\frac{\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x}{b\,x^3+a}+\frac{(Ab-4Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}}{b^2}$	133

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
B*x/b^2+(-1/3*A*b+1/3*B*a)*x/b^2/(b*x^3+a)+1/9/b^3*sum((A*b-4*B*a)/_R^2*ln
(x-_R),_R=RootOf(_Z^3*b+a))
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.02

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \Bigg[\frac{18\,Ba^2b^2x^4-3\sqrt{\tfrac{1}{3}}(4\,Ba^3b-Aa^2b^2+(4\,Ba^2b^2-Aab^3)x^3)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\Bigg(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\Big(2\,abx}{bx^3+}}{}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output

```
[1/18*(18*B*a^2*b^2*x^4 - 3*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^
2 - A*a*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*
a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sq
rt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*
a*b)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((
4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^
(2/3)) + 6*(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3), 1/18*(18*B*
a^2*b^2*x^4 - 6*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^2 - A*a*b^3)
*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^
(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A
*a*b)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*(
(4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)
^(2/3)) + 6*(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3)]
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.54

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx}{b^2} + \frac{x(-Ab+Ba)}{3ab^2+3b^3x^3} + \operatorname{RootSum}\left(729t^3a^2b^7 - A^3b^3 + 12A^2Bab^2 - 48AB^2a^2b + 64B^3a^3, \left(t \mapsto t\log\left(-\frac{9tab^2}{-Ab+4Ba}+x\right)\right)\right)$$

input

```
integrate(x**3*(B*x**3+A)/(b*x**3+a)**2,x)
```

output

```
B*x/b**2 + x*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729*_t**3*a**
2*b**7 - A**3*b**3 + 12*A**2*B*a*b**2 - 48*A*B**2*a**2*b + 64*B**3*a**3, L
ambda(_t, _t*log(-9*_t*a*b**2/(-A*b + 4*B*a) + x)))
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.83

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba-Ab)x}{3\,(b^3x^3+ab^2)} + \frac{Bx}{b^2} - \frac{\sqrt{3}(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(4\,Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(4\,Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(B*a - A*b)*x/(b^3*x^3 + a*b^2) + B*x/b^2 - 1/9*sqrt(3)*(4*B*a - A*b)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 1/
18*(4*B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3))
- 1/9*(4*B*a - A*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.87

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\sqrt{3}(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,b} + \frac{(4\,Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,b} + \frac{Bx}{b^2} + \frac{(4\,Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2} + \frac{Bax-Abx}{3\,(bx^3+a)b^2}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*(4*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(2/3)*b) + 1/18*(4*B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/((-a*b^2)^(2/3)*b) + B*x/b^2 + 1/9*(4*B*a - A*b)*(-a/b)^(1/3)
*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*b^2
)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 6.81 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{B\,x}{b^2}-\frac{x\,\left(\frac{A\,b}{3}-\frac{B\,a}{3}\right)}{b^3\,x^3+a\,b^2}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}}\\ &\quad-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}}\\ &\quad+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}}\end{aligned}$$

input
```
int((x^3*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(B*x)/b^2 - (x*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) + (log(b^(1/3)*x + a
^(1/3))*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b
^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/
3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1
/2)*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3))
```

3.78 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx$

3.78.1 Optimal result

Integrand size = 20, antiderivative size = 41

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{-Ab+aB}{3b^2(a+bx^3)} + \frac{B\log(a+bx^3)}{3b^2}$$

output
```
1/3*(-A*b+B*a)/b^2/(b*x^3+a)+1/3*B*ln(b*x^3+a)/b^2
```

3.78.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{-Ab+aB}{3b^2(a+bx^3)} + \frac{B\log(a+bx^3)}{3b^2}$$

input
```
Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(-(A*b) + a*B)/(3*b^2*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^2)
```

3.78.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {946, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{B}{b(bx^3+a)}+\frac{Ab-aB}{b(bx^3+a)^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{B\log(a+bx^3)}{b^2}-\frac{Ab-aB}{b^2(a+bx^3)}\right)$$

input
```
Int[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(-((A*b - a*B)/(b^2*(a + b*x^3))) + (B*Log[a + b*x^3])/b^2)/3
```

3.78.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.78.4 Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{B\ln(b\,x^3+a)}{3b^2}-\frac{Ab-Ba}{3b^2(b\,x^3+a)}$	38
norman	$\frac{B\ln(b\,x^3+a)}{3b^2}-\frac{Ab-Ba}{3b^2(b\,x^3+a)}$	38
risch	$\frac{B\ln(b\,x^3+a)}{3b^2}-\frac{A}{3b(b\,x^3+a)}+\frac{Ba}{3b^2(b\,x^3+a)}$	47
parallelrisch	$-\frac{-B\ln(b\,x^3+a)x^3b-B\ln(b\,x^3+a)a+Ab-Ba}{3b^2(b\,x^3+a)}$	50

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*B*ln(b*x^3+a)/b^2-1/3/b^2*(A*b-B*a)/(b*x^3+a)
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx=\frac{Ba-Ab+(Bbx^3+Ba)\log(bx^3+a)}{3\,(b^3x^3+ab^2)}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/3*(B*a - A*b + (B*b*x^3 + B*a)*log(b*x^3 + a))/(b^3*x^3 + a*b^2)
```

3.78.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{B\log(a+bx^3)}{3b^2} + \frac{-Ab+Ba}{3ab^2+3b^3x^3}$$

input
```
integrate(x**2*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
B*log(a + b*x**3)/(3*b**2) + (-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3)
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Ba-Ab}{3\,(b^3x^3+ab^2)} + \frac{B\log(bx^3+a)}{3\,b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(B*a - A*b)/(b^3*x^3 + a*b^2) + 1/3*B*log(b*x^3 + a)/b^2
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{B\left(\frac{\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b} - \frac{a}{(bx^3+a)b}\right)}{3\,b} - \frac{A}{3\,(bx^3+a)b}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*B*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 + a)*b))/
b - 1/3*A/((b*x^3 + a)*b)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 6.61 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{x^2(A+Bx^3)}{\left(a+bx^3\right)^2}\,dx = \frac{B\,\ln\left(b\,x^3+a\right)}{3\,b^2}-\frac{A\,b-B\,a}{3\,b^2\,\left(b\,x^3+a\right)}$$

input

```
int((x^2*(A + B*x^3))/(a + b*x^3)^2,x)
```

output

```
(B*log(a + b*x^3))/(3*b^2) - (A*b - B*a)/(3*b^2*(a + b*x^3))
```

3.79 $\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx$

3.79.1 Optimal result

Integrand size = 18, antiderivative size = 171

$$\begin{aligned}\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = {} & \frac{(Ab-aB)x^2}{3ab\,(a+bx^3)} - \frac{(Ab+2aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{5/3}} \\ & - \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{5/3}} \\ & + \frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{5/3}}\end{aligned}$$

output
```
1/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)-1/9*(A*b+2*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(4
/3)/b^(5/3)+1/18*(A*b+2*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(
4/3)/b^(5/3)-1/9*(A*b+2*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1
/2))/a^(4/3)/b^(5/3)*3^(1/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.85

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{6\sqrt[3]{a}b^{2/3}(-Ab+aB)x^2}{a+bx^3} - 2\sqrt{3}(Ab+2aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) + (Ab+2a}{18a^{4/3}b^{5/3}}$$

input
```
Integrate[(x*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((-6*a^(1/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) - 2*Sqrt[3]*(A*b + 2*
a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(A*b + 2*a*B)*Log[a^(
1/3) + b^(1/3)*x] + (A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*a^(4/3)*b^(5/3))
```

3.79.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {957, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(2aB+Ab)\int \frac{x}{bx^3+a}dx}{3ab}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{821}$$

$$\frac{(2aB+Ab)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 16$$

$$\frac{(2aB+Ab)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{(2aB+Ab)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{(2aB+Ab)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(2aB+Ab)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1082$$

3.79. $\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

$$\frac{(2aB+Ab)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{(2aB+Ab)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{(2aB+Ab)\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3ab}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

input

```
Int[(x*(A + B*x^3))/(a + b*x^3)^2,x]
```

output

```
((A*b - a*B)*x^2)/(3*a*b*(a + b*x^3)) + ((A*b + 2*a*B)*(-1/3*Log[a^(1/3) +
 b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(
2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/(3*a*b)
```

3.79. $\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

3.79.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.79. $\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.79.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{(Ab-Ba)x^2}{3ab(b\,x^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab+2Ba)\ln\left(x-\_R\right)}{\_R}}{9a\,b^2}$	67
default	$\frac{(Ab-Ba)x^2}{3ab(b\,x^3+a)}+\frac{(Ab+2Ba)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab}$	136

input
```
int(x*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)+1/9/a/b^2*sum((A*b+2*B*a)/_R*ln(x-_R),_R=R
ootOf(_Z^3*b+a))
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.20

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \left[-\frac{6\,(Ba^2b^2-Aab^3)x^2-3\,\sqrt{\tfrac{1}{3}}(2\,Ba^3b+Aa^2b^2+(2\,Ba^2b^2+Aab^3)x^3)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\,\sqrt{\frac{1}{3}}\left(abx+\right.}{}\right.}{}\right.$$

$$-\frac{6\,(Ba^2b^2-Aab^3)x^2-6\,\sqrt{\tfrac{1}{3}}(2\,Ba^3b+Aa^2b^2+(2\,Ba^2b^2+Aab^3)x^3)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx+(-ab^2)\right.}{b}\right.}{}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/18*(6*(B*a^2*b^2 - A*a*b^3)*x^2 - 3*sqrt(1/3)*(2*B*a^3*b + A*a^2*b^2 +
 (2*B*a^2*b^2 + A*a*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b
+ 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b
^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((2*B*a*b + A*b^2)*x^3 +
 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^
2)^(2/3)) + 2*((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log
(b*x - (-a*b^2)^(1/3)))/(a^2*b^4*x^3 + a^3*b^3), -1/18*(6*(B*a^2*b^2 - A*a
*b^3)*x^2 - 6*sqrt(1/3)*(2*B*a^3*b + A*a^2*b^2 + (2*B*a^2*b^2 + A*a*b^3)*x
^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt
(-(-a*b^2)^(1/3)/a)/b) - ((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2
)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((2*B*a*b +
 A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(
a^2*b^4*x^3 + a^3*b^3)]
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.68

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{x^2(Ab-Ba)}{3a^2b+3ab^2x^3} + \mathrm{RootSum}\Bigg(729t^3a^4b^5 + A^3b^3 + 6A^2Bab^2 + 12AB^2a^2b + 8B^3a^3, \Bigg(t \mapsto t\log\Bigg(\frac{81t^2a^3b^3}{A^2b^2+4ABab+4B^2a^2} +$$

input
```
integrate(x*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
x**2*(A*b - B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**4*b**5
+ A**3*b**3 + 6*A**2*B*a*b**2 + 12*A*B**2*a**2*b + 8*B**3*a**3, Lambda(_t,
 _t*log(81*_t**2*a**3*b**3/(A**2*b**2 + 4*A*B*a*b + 4*B**2*a**2) + x)))
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ba-Ab)x^2}{3\,(ab^2x^3+a^2b)} + \frac{\sqrt{3}(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(2\,Ba+Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(2\,Ba+Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(B*a - A*b)*x^2/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(2*B*a + A*b)*arcta
n(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(1/3)) + 1/18*
(2*B*a + A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(1/3)) -
 1/9*(2*B*a + A*b)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\sqrt{3}(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{1}{3}}\,ab} - \frac{(2\,Ba+Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{1}{3}}\,ab} - \frac{\left(2\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b} - \frac{Bax^2-Abx^2}{3\,(bx^3+a)ab}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(1/3)*a*b) - 1/18*(2*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/((-a*b^2)^(1/3)*a*b) - 1/9*(2*B*a*(-a/b)^(1/3) + A*b*(-a/b)
^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) - 1/3*(B*a*x^2 - A
*b*x^2)/((b*x^3 + a)*a*b)
```

3.79.9 Mupad [B] (verification not implemented)

Time = 6.77 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{x^2\,(A\,b-B\,a)}{3\,a\,b\,\left(b\,x^3+a\right)} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b+2\,B\,a\right)}{9\,a^{4/3}\,b^{5/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b+2\,B\,a\right)}{9\,a^{4/3}\,b^{5/3}} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b+2\,B\,a\right)}{9\,a^{4/3}\,b^{5/3}}$$

input
```
int((x*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A
*b + 2*B*a))/(9*a^(4/3)*b^(5/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x +
 a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b + 2*B*a))/(9*a^(4/3)*b^(5/3)) - (log
(b^(1/3)*x + a^(1/3))*(A*b + 2*B*a))/(9*a^(4/3)*b^(5/3)) + (x^2*(A*b - B*a
))/(3*a*b*(a + b*x^3))
```

3.79. $\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx$

3.80 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^2}\,dx$

3.80.1 Optimal result

Integrand size = 17, antiderivative size = 169

$$\begin{aligned}\int \frac{A+Bx^3}{\left(a+bx^3\right)^2}\,dx &= \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} - \frac{(2Ab+aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} \\ &\quad + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} \\ &\quad - \frac{(2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}\end{aligned}$$

output

```
1/3*(A*b-B*a)*x/a/b/(b*x^3+a)+1/9*(2*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3
)/b^(4/3)-1/18*(2*A*b+B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/
3)/b^(4/3)-1/9*(2*A*b+B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2
))/a^(5/3)/b^(4/3)*3^(1/2)
```

3.80.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{6a^{2/3}\sqrt[3]{b}(-Ab+aB)x}{a+bx^3} - 2\sqrt{3}(2Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - (2Ab+aB}{18a^{5/3}b^{4/3}}$$

input
```
Integrate[(A + B*x^3)/(a + b*x^3)^2,x]
```

output
```
((-6*a^(2/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) - 2*Sqrt[3]*(2*A*b + a*
B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*A*b + a*B)*Log[a^(1/
3) + b^(1/3)*x] - (2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(18*a^(5/3)*b^(4/3))
```

3.80.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {910, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(aB+2Ab)\int \frac{1}{bx^3+a}dx}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{750}$$

$$\frac{(aB+2Ab)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \ 16$$

$$\frac{(aB+2Ab)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \ 1142$$

$$\frac{(aB+2Ab)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{(aB+2Ab)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{(aB+2Ab)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \ 1082$$

3.80. $\int \frac{A+Bx^3}{\left(a+bx^3\right)^2}\,dx$

$$\frac{(aB+2Ab)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow 217$$

$$\frac{(aB+2Ab)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow 1103$$

$$\frac{(aB+2Ab)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x)/(3*a*b*(a + b*x^3)) + ((2*A*b + a*B)*(Log[a^(1/3) + b^(1/3
)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(
1/3)))/(3*a^(2/3))))/(3*a*b)
```

3.80. $\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx$

3.80.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.80.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.38

method	result	size
risch	$\frac{(Ab-Ba)x}{3ab(b\,x^3+a)}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{(2Ab+Ba)\ln\left(x-\_R\right)}{\_R^2}}{9a\,b^2}$	65
default	$\frac{(Ab-Ba)x}{3ab(b\,x^3+a)}+\frac{(2Ab+Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3ab}$	134

input
```
int((B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(A*b-B*a)*x/a/b/(b*x^3+a)+1/9/a/b^2*sum((2*A*b+B*a)/_R^2*ln(x-_R),_R=R
ootOf(_Z^3*b+a))
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.18

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(Ba^3b+2\,Aa^2b^2+(Ba^2b^2+2\,Aab^3)x^3)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}\,ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}\,x-(a^2b)^{\frac{1}{3}}\right.}{bx^3+a}\right.}{}\right.$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*(B*a^3*b + 2*A*a^2*b^2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*s
qrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(
1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b
))/(b*x^3 + a)) - ((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*
log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^
3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b
 - A*a^2*b^2)*x)/(a^3*b^3*x^3 + a^4*b^2), 1/18*(6*sqrt(1/3)*(B*a^3*b + 2*A
*a^2*b^2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(
1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((
B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b
)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)
*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b - A*a^2*b^2)*x)/(a^
3*b^3*x^3 + a^4*b^2)]
```

3.80.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = \frac{x(Ab-Ba)}{3a^2b+3ab^2x^3} + \operatorname{RootSum}\left(729t^3a^5b^4-8A^3b^3-12A^2Bab^2-6AB^2a^2b-B^3a^3, \left(t \mapsto t\log\left(\frac{9ta^2b}{2Ab+Ba}+x\right)\right)\right)$$

input
```
integrate((B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
x*(A*b - B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**5*b**4 - 8
*A**3*b**3 - 12*A**2*B*a*b**2 - 6*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t
*log(9*_t*a**2*b/(2*A*b + B*a) + x)))
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = -\frac{(Ba-Ab)x}{3\,(ab^2x^3+a^2b)} + \frac{\sqrt{3}(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba+2\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba+2\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(B*a - A*b)*x/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(B*a + 2*A*b)*arctan(
1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/18*(B
*a + 2*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1
/9*(B*a + 2*A*b)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{(Ba+2\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{(Ba+2\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b} - \frac{Bax-Abx}{3\,(bx^3+a)ab}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(B*a + 2*A*b)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(B*a + 2*A*b)*(-a/b)^(1/3)*log(abs(
x - (-a/b)^(1/3)))/(a^2*b) - 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*a*b)
```

3.80.9 Mupad [B] (verification not implemented)

Time = 6.81 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(2\,A\,b+B\,a)}{9\,a^{5/3}\,b^{4/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{9\,a^{5/3}\,b^{4/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{9\,a^{5/3}\,b^{4/3}} + \frac{x\,(A\,b-B\,a)}{3\,a\,b\,\left(b\,x^3+a\right)}$$

input
```
int((A + B*x^3)/(a + b*x^3)^2,x)
```

output
```
(log(b^(1/3)*x + a^(1/3))*(2*A*b + B*a))/(9*a^(5/3)*b^(4/3)) - (log(3^(1/2
)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*A*b + B*a)
)/(9*a^(5/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*(
(3^(1/2)*1i)/2 - 1/2)*(2*A*b + B*a))/(9*a^(5/3)*b^(4/3)) + (x*(A*b - B*a))
/(3*a*b*(a + b*x^3))
```

3.81 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx$

3.81.1 Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx=\frac{Ab-aB}{3ab\left(a+bx^3\right)}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^3\right)}{3a^2}$$

output
```
1/3*(A*b-B*a)/a/b/(b*x^3+a)+A*ln(x)/a^2-1/3*A*ln(b*x^3+a)/a^2
```

3.81.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx=\frac{\frac{a(Ab-aB)}{b(a+bx^3)}+3A\log(x)-A\log\left(a+bx^3\right)}{3a^2}$$

input
```
Integrate[(A + B*x^3)/(x*(a + b*x^3)^2),x]
```

output
```
((a*(A*b - a*B))/(b*(a + b*x^3)) + 3*A*Log[x] - A*Log[a + b*x^3])/(3*a^2)
```

3.81.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^3\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \ 86$$

$$\frac{1}{3}\int \left(-\frac{bA}{a^2\left(bx^3+a\right)}+\frac{A}{a^2x^3}+\frac{aB-Ab}{a\left(bx^3+a\right)^2}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(-\frac{A\log\left(a+bx^3\right)}{a^2}+\frac{A\log\left(x^3\right)}{a^2}+\frac{Ab-aB}{ab\left(a+bx^3\right)}\right)$$

input
```
Int[(A + B*x^3)/(x*(a + b*x^3)^2),x]
```

output
```
((A*b - a*B)/(a*b*(a + b*x^3)) + (A*Log[x^3])/a^2 - (A*Log[a + b*x^3])/a^2
)/3
```

3.81.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.81.4 Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{A\ln(x)}{a^2}-\frac{A\ln\left(b\,x^3+a\right)-\frac{a(Ab-Ba)}{b\left(b\,x^3+a\right)}}{3a^2}$	48
norman	$-\frac{(Ab-Ba)x^3}{3a^2(b\,x^3+a)}+\frac{A\ln(x)}{a^2}-\frac{A\ln\left(b\,x^3+a\right)}{3a^2}$	48
risch	$\frac{A}{3a(b\,x^3+a)}-\frac{B}{3b(b\,x^3+a)}+\frac{A\ln(x)}{a^2}-\frac{A\ln\left(b\,x^3+a\right)}{3a^2}$	53
parallelrisch	$\frac{3A\ln(x)x^3b-A\ln\left(b\,x^3+a\right)x^3b-Ab\,x^3+Ba\,x^3+3aA\ln(x)-A\ln\left(b\,x^3+a\right)a}{3a^2(b\,x^3+a)}$	71

input
```
int((B*x^3+A)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a^2-1/3/a^2*(A*ln(b*x^3+a)-a*(A*b-B*a)/b/(b*x^3+a))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx=-\frac{Ba^2-Aab+\left(Ab^2x^3+Aab\right)\log\left(bx^3+a\right)-3\left(Ab^2x^3+Aab\right)\log\left(x\right)}{3\left(a^2b^2x^3+a^3b\right)}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/3*(B*a^2 - A*a*b + (A*b^2*x^3 + A*a*b)*log(b*x^3 + a) - 3*(A*b^2*x^3 +
A*a*b)*log(x))/(a^2*b^2*x^3 + a^3*b)
```

3.81.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx=\frac{A\log\left(x\right)}{a^2}-\frac{A\log\left(\frac{a}{b}+x^3\right)}{3a^2}+\frac{Ab-Ba}{3a^2b+3ab^2x^3}$$

input
```
integrate((B*x**3+A)/x/(b*x**3+a)**2,x)
```

output
```
A*log(x)/a**2 - A*log(a/b + x**3)/(3*a**2) + (A*b - B*a)/(3*a**2*b + 3*a*b
**2*x**3)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx=-\frac{Ba-Ab}{3\left(ab^2x^3+a^2b\right)}-\frac{A\log\left(bx^3+a\right)}{3\,a^2}+\frac{A\log\left(x^3\right)}{3\,a^2}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(B*a - A*b)/(a*b^2*x^3 + a^2*b) - 1/3*A*log(b*x^3 + a)/a^2 + 1/3*A*lo
g(x^3)/a^2
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx=-\frac{A\log\left(|bx^3+a|\right)}{3\,a^2}+\frac{A\log\left(|x|\right)}{a^2}+\frac{Ab^2x^3-Ba^2+2\,Aab}{3\,(bx^3+a)a^2b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*A*log(abs(b*x^3 + a))/a^2 + A*log(abs(x))/a^2 + 1/3*(A*b^2*x^3 - B*a^
2 + 2*A*a*b)/((b*x^3 + a)*a^2*b)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx=\frac{A\,\ln\left(x\right)}{a^2}-\frac{A\,\ln\left(b\,x^3+a\right)}{3\,a^2}+\frac{A\,b-B\,a}{3\,a\,b\,\left(b\,x^3+a\right)}$$

input

```
int((A + B*x^3)/(x*(a + b*x^3)^2),x)
```

output

```
(A*log(x))/a^2 - (A*log(a + b*x^3))/(3*a^2) + (A*b - B*a)/(3*a*b*(a + b*x^
3))
```

3.82 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx$

3.82.1 Optimal result

Integrand size = 20, antiderivative size = 195

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx=\frac{-4Ab+aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}}-\frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{2/3}}$$

output
```
1/3*(-4*A*b+B*a)/a^2/b/x+1/3*(A*b-B*a)/a/b/x/(b*x^3+a)+1/9*(4*A*b-B*a)*ln(
a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(2/3)-1/18*(4*A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(2/3)+1/9*(4*A*b-B*a)*arctan(1/3*(a^(1/3)-2*
b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(2/3)*3^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx$$
$$=\frac{-\frac{18\sqrt[3]{a}A}{x}+\frac{6\sqrt[3]{a}(-Ab+aB)x^2}{a+bx^3}+\frac{2\sqrt{3}(4Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{2(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{(-4Ab+aB)\log\left(a^{2/3}-\right.}{b^{2/3}}}{18a^{7/3}}$$

input
```
Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^2),x]
```

output
```
((-18*a^(1/3)*A)/x + (6*a^(1/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) + (2*Sqrt[
3]*(4*A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2
*(4*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-4*A*b + a*B)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(18*a^(7/3))
```

3.82.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {957, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx$$
$$\Big\downarrow \text{957}$$
$$\frac{(4Ab-aB)\int \frac{1}{x^2\left(bx^3+a\right)}dx}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$
$$\Big\downarrow \text{847}$$
$$\frac{(4Ab-aB)\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$
$$\Big\downarrow \text{821}$$

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

$$\Big\downarrow 16$$

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

$$\Big\downarrow 1142$$

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

$$\Big\downarrow 25$$

3.82. $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx$

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

$$\Big\downarrow 27$$

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

$$\Big\downarrow 1082$$

3.82. $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx$

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

$$\Big\downarrow \text{1103}$$

3.82. $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx$

$$\frac{(4Ab-aB)\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3ab}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(x^2*(a + b*x^3)^2),x]
```

output
```
(A*b - a*B)/(3*a*b*x*(a + b*x^3)) + ((4*A*b - a*B)*(-(1/(a*x)) - (b*(-1/3*
Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a))/(3*a*b)
```

3.82.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.82. $\int \frac{A+Bx^3}{x^2(a+bx^3)^2}\,dx$

3.82.4 Maple [A] (verified)

Time = 4.37 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.71

method	result
default	$-\frac{A}{a^{2} x}-\frac{\frac{\left(\frac{A b}{3}-\frac{B a}{3}\right) x^{2}}{b\,x^{3}+a}+\left(\frac{4 A b}{3}-\frac{B a}{3}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}}$
risch	$\frac{-\frac{(4 A b-B a) x^{3}}{3 a^{2}}-\frac{A}{a}}{x (b\,x^{3}+a)}+\frac{\Bigl(\sum_{\_R=\operatorname{RootOf}\left(a^{7} b^{2} \_Z^{3}-64 A^{3} b^{3}+48 A^{2} B a\,b^{2}-12 A\,B^{2} a^{2} b+B^{3} a^{3}\right)}\_R \ln\Bigl(\Bigl(-4 \_R^{3} a^{7} b^{2}+192 A^{3} b^{3}-144 A^{2} B a}{9}$

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/a^2/x-1/a^2*((1/3*A*b-1/3*B*a)*x^2/(b*x^3+a)+(4/3*A*b-1/3*B*a)*(-1/3/b/
(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))
)
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.92

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx$$

$$= \Bigg[-\frac{18\,Aa^2b^2 - 6\,(Ba^2b^2-4\,Aab^3)x^3 + 3\,\sqrt{\frac{1}{3}}((Ba^2b^2-4\,Aab^3)x^4+(Ba^3b-4\,Aa^2b^2)x)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\Bigg(}{}$$

$$-\frac{18\,Aa^2b^2 - 6\,(Ba^2b^2-4\,Aab^3)x^3 + 6\,\sqrt{\frac{1}{3}}((Ba^2b^2-4\,Aab^3)x^4+(Ba^3b-4\,Aa^2b^2)x)\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}\arctan\Bigg(}{}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/18*(18*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 3*sqrt(1/3)*((B*a^2
*b^2 - 4*A*a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(-(a*b^2)^(1/3)/a)*
log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^
(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((B*a*
b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2
)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b
)*x)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^3*x^4 + a^4*b^2*x), -1
/18*(18*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 6*sqrt(1/3)*((B*a^2*b^
2 - 4*A*a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*sqrt((a*b^2)^(1/3)/a)*arct
an(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - ((B*a*b -
 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(
1/3)*b*x + (a*b^2)^(2/3)) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x
)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^3*x^4 + a^4*b^2*x)]
```

3.82.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx=\frac{-3Aa+x^3(-4Ab+Ba)}{3a^3x+3a^2bx^4}$$
$$+\operatorname{RootSum}\left(729t^3a^7b^2-64A^3b^3+48A^2Bab^2-12AB^2a^2b+B^3a^3,\left(t\mapsto t\log\left(\frac{81t^2a^5b}{16A^2b^2-8ABab+B^2}\right.\right.\right.$$

input
```
integrate((B*x**3+A)/x**2/(b*x**3+a)**2,x)
```

output
```
(-3*A*a + x**3*(-4*A*b + B*a))/(3*a**3*x + 3*a**2*b*x**4) + RootSum(729*_t
**3*a**7*b**2 - 64*A**3*b**3 + 48*A**2*B*a*b**2 - 12*A*B**2*a**2*b + B**3*
a**3, Lambda(_t, _t*log(81*_t**2*a**5*b/(16*A**2*b**2 - 8*A*B*a*b + B**2*a
**2) + x)))
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx=\frac{(Ba-4\,Ab)x^3-3\,Aa}{3\left(a^2bx^4+a^3x\right)}+\frac{\sqrt{3}(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+\frac{(Ba-4\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$-\frac{(Ba-4\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*((B*a - 4*A*b)*x^3 - 3*A*a)/(a^2*b*x^4 + a^3*x) + 1/9*sqrt(3)*(B*a - 4
*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(1/
3)) + 1/18*(B*a - 4*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/
b)^(1/3)) - 1/9*(B*a - 4*A*b)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx = \frac{\sqrt{3}(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}a^2} - \frac{(Ba-4\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}a^2} - \frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} + \frac{Bax^3-4\,Abx^3-3\,Aa}{3\left(bx^4+ax\right)a^2}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*(B*a - 4*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(1/3)*a^2) - 1/18*(B*a - 4*A*b)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2) - 1/9*(B*a*(-a/b)^(1/3) - 4*A*b*(-a/b)
^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/3*(B*a*x^3 - 4*A*b
*x^3 - 3*A*a)/((b*x^4 + a*x)*a^2)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 6.90 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(4\,A\,b-B\,a\right)}{9\,a^{7/3}\,b^{2/3}} - \frac{\frac{A}{a}+\frac{x^3\,\left(4\,A\,b-B\,a\right)}{3\,a^2}}{b\,x^4+a\,x} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(4\,A\,b-B\,a\right)}{9\,a^{7/3}\,b^{2/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(4\,A\,b-B\,a\right)}{9\,a^{7/3}\,b^{2/3}}$$

input
```
int((A + B*x^3)/(x^2*(a + b*x^3)^2),x)
```

output

```
(log(b^(1/3)*x + a^(1/3))*(4*A*b - B*a))/(9*a^(7/3)*b^(2/3)) - (A/a + (x^3
*(4*A*b - B*a))/(3*a^2))/(a*x + b*x^4) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/
3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*A*b - B*a))/(9*a^(7/3)*b^(2/3))
- (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(4*A*b - B*a))/(9*a^(7/3)*b^(2/3))
```

3.82. $\int \frac{A+Bx^3}{x^2(a+bx^3)^2}\,dx$

3.83 $\int \frac{A+Bx^3}{x^3(a+bx^3)^2}\,dx$

3.83.1 Optimal result

Integrand size = 20, antiderivative size = 196

$$\begin{aligned}\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx &= \frac{-5Ab+2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}+\frac{(5Ab-2aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}}\\ &\quad-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}}\\ &\quad+\frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}}\end{aligned}$$

output

```
1/6*(-5*A*b+2*B*a)/a^2/b/x^2+1/3*(A*b-B*a)/a/b/x^2/(b*x^3+a)-1/9*(5*A*b-2*
B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(1/3)+1/18*(5*A*b-2*B*a)*ln(a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(1/3)+1/9*(5*A*b-2*B*a)*arctan(1/3
*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(1/3)*3^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{9a^{2/3}A}{x^2}+\frac{6a^{2/3}(-Ab+aB)x}{a+bx^3}+\frac{2\sqrt{3}(5Ab-2aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{2(-5Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{(5Ab-2aB)\log\left(a^{2/3}-\right.}{\sqrt[3]{b}}}{18a^{8/3}}$$

input
```
Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^2),x]
```

output
```
((-9*a^(2/3)*A)/x^2 + (6*a^(2/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + (2*Sqrt[3
]*(5*A*b - 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (
2*(-5*A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((5*A*b - 2*a*B)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(18*a^(8/3))
```

3.83.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {957, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(5Ab-2aB)\int \frac{1}{x^3\left(bx^3+a\right)}dx}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

$$\Big\downarrow \text{847}$$

$$\frac{(5Ab-2aB)\left(-\frac{b\int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

$$\Big\downarrow \text{750}$$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 16$$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 25$$

3.83. $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

$$\Big\downarrow \text{1082}$$

3.83. $\int\frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1103$$

3.83. $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx$

$$\frac{(5Ab-2aB)\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3ab}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(x^3*(a + b*x^3)^2),x]
```

output
```
(A*b - a*B)/(3*a*b*x^2*(a + b*x^3)) + ((5*A*b - 2*a*B)*(-1/2*1/(a*x^2) - (
b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/a))/(3*a*b)
```

3.83.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.83. $\int \frac{A+Bx^3}{x^3(a+bx^3)^2}\,dx$

3.83.4 Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.70

method	result
default	$-\frac{A}{2a^{2}x^{2}}-\frac{\frac{\left(\frac{Ab}{3}-\frac{Ba}{3}\right)x}{b\,x^{3}+a}+\frac{(5Ab-2Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}}{a^{2}}$
risch	$\frac{-\frac{(5Ab-2Ba)x^{3}}{6a^{2}}-\frac{A}{2a}}{x^{2}(b\,x^{3}+a)}+\frac{\Bigg(\sum_{\_R=\operatorname{RootOf}\left(a^{8}b\_Z^{3}+125A^{3}b^{3}-150A^{2}Ba\,b^{2}+60A\,B^{2}a^{2}b-8B^{3}a^{3}\right)}\_R\ln\Big(\Big(-4\_R^{3}a^{8}b-375A^{3}b^{3}+450A^{2}}{9}$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a^2/x^2-1/a^2*((1/3*A*b-1/3*B*a)*x/(b*x^3+a)+1/3*(5*A*b-2*B*a)*(1/3
/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))))
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 618, normalized size of antiderivative = 3.15

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx$$

$$= \left[-\frac{9\,Aa^3b - 3\,(2\,Ba^3b - 5\,Aa^2b^2)x^3 + 3\,\sqrt{\frac{1}{3}}((2\,Ba^2b^2 - 5\,Aab^3)x^5 + (2\,Ba^3b - 5\,Aa^2b^2)x^2)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}\log}{} \right.$$

$$-\frac{9\,Aa^3b - 3\,(2\,Ba^3b - 5\,Aa^2b^2)x^3 - 6\,\sqrt{\frac{1}{3}}((2\,Ba^2b^2 - 5\,Aab^3)x^5 + (2\,Ba^3b - 5\,Aa^2b^2)x^2)\sqrt{-\frac{(-a^2b)^{\frac{1}{3}}}{b}}\,\mathrm{arc}}{}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*x^3 + 3*sqrt(1/3)*((2*B*a^
2*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt((-a^2*b)^(1/3
)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2
+ (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)
) + ((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log
(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((2*B*a*b - 5*A*b^2)*x
^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/
(a^4*b^2*x^5 + a^5*b*x^2), -1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*
x^3 - 6*sqrt(1/3)*((2*B*a^2*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^
2)*x^2)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a
^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) + ((2*B*a*b - 5*A*b^2)*x^5 + (
2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-
a^2*b)^(1/3)*a) - 2*((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-
a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^2*x^5 + a^5*b*x^2)]
```

3.83.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.56

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx = \frac{-3Aa+x^3(-5Ab+2Ba)}{6a^3x^2+6a^2bx^5}$$
$$+\mathrm{RootSum}\left(729t^3a^8b+125A^3b^3-150A^2Bab^2+60AB^2a^2b-8B^3a^3,\left(t\mapsto t\log\left(\frac{9ta^3}{-5Ab+2Ba}+x\right)\right)\right)$$

input

```
integrate((B*x**3+A)/x**3/(b*x**3+a)**2,x)
```

output

```
(-3*A*a + x**3*(-5*A*b + 2*B*a))/(6*a**3*x**2 + 6*a**2*b*x**5) + RootSum(7
29*_t**3*a**8*b + 125*A**3*b**3 - 150*A**2*B*a*b**2 + 60*A*B**2*a**2*b - 8
*B**3*a**3, Lambda(_t, _t*log(9*_t*a**3/(-5*A*b + 2*B*a) + x)))
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx = \frac{(2\,Ba-5\,Ab)x^3-3\,Aa}{6\,(a^2bx^5+a^3x^2)}+\frac{\sqrt{3}(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(2\,Ba-5\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(2\,Ba-5\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input

```
integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

output

```
1/6*((2*B*a - 5*A*b)*x^3 - 3*A*a)/(a^2*b*x^5 + a^3*x^2) + 1/9*sqrt(3)*(2*B
*a - 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/
b)^(2/3)) - 1/18*(2*B*a - 5*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
^2*b*(a/b)^(2/3)) + 1/9*(2*B*a - 5*A*b)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^
(2/3))
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{\left(2\,Ba-5\,Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} + \frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}Ba-5\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} + \frac{Bax-Abx}{3\left(bx^3+a\right)a^2} + \frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}Ba-5\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b} - \frac{A}{2\,a^2x^2}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*(2*B*a - 5*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/9*sqr
t(3)*(2*(-a*b^2)^(1/3)*B*a - 5*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x
 + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) + 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*
a^2) + 1/18*(2*(-a*b^2)^(1/3)*B*a - 5*(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a^3*b) - 1/2*A/(a^2*x^2)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{\frac{A}{2\,a}+\frac{x^3\,\left(5\,A\,b-2\,B\,a\right)}{6\,a^2}}{b\,x^5+a\,x^2} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(5\,A\,b-2\,B\,a\right)}{9\,a^{8/3}\,b^{1/3}} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,A\,b-2\,B\,a\right)}{9\,a^{8/3}\,b^{1/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,A\,b-2\,B\,a\right)}{9\,a^{8/3}\,b^{1/3}}$$

input
```
int((A + B*x^3)/(x^3*(a + b*x^3)^2),x)
```

output
```
(log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5
*A*b - 2*B*a))/(9*a^(8/3)*b^(1/3)) - (log(b^(1/3)*x + a^(1/3))*(5*A*b - 2*
B*a))/(9*a^(8/3)*b^(1/3)) - (A/(2*a) + (x^3*(5*A*b - 2*B*a))/(6*a^2))/(a*x
^2 + b*x^5) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1
i)/2 - 1/2)*(5*A*b - 2*B*a))/(9*a^(8/3)*b^(1/3))
```

3.83. $\int \frac{A+Bx^3}{x^3(a+bx^3)^2}\,dx$

3.84 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx$

3.84.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\begin{aligned}&\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx\\&=-\frac{A}{3a^2x^3}-\frac{Ab-aB}{3a^2\left(a+bx^3\right)}-\frac{(2Ab-aB)\log(x)}{a^3}+\frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}\end{aligned}$$

output
```
-1/3*A/a^2/x^3+1/3*(-A*b+B*a)/a^2/(b*x^3+a)-(2*A*b-B*a)*ln(x)/a^3+1/3*(2*A
*b-B*a)*ln(b*x^3+a)/a^3
```

3.84.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx=\frac{-\frac{aA}{x^3}+\frac{a(-Ab+aB)}{a+bx^3}+3(-2Ab+aB)\log(x)+(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}$$

input
```
Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^2),x]
```

output
```
(-((a*A)/x^3) + (a*(-(A*b) + a*B))/(a + b*x^3) + 3*(-2*A*b + a*B)*Log[x] +
 (2*A*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

3.84.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^6\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \; 86$$

$$\frac{1}{3}\int \left(\frac{A}{a^2x^6}-\frac{b(aB-2Ab)}{a^3\left(bx^3+a\right)}-\frac{b(aB-Ab)}{a^2\left(bx^3+a\right)^2}+\frac{aB-2Ab}{a^3x^3}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(-\frac{\log\left(x^3\right)(2Ab-aB)}{a^3}+\frac{(2Ab-aB)\log\left(a+bx^3\right)}{a^3}-\frac{Ab-aB}{a^2\left(a+bx^3\right)}-\frac{A}{a^2x^3}\right)$$

input
```
Int[(A + B*x^3)/(x^4*(a + b*x^3)^2),x]
```

output
```
(-(A/(a^2*x^3)) - (A*b - a*B)/(a^2*(a + b*x^3)) - ((2*A*b - a*B)*Log[x^3])
/a^3 + ((2*A*b - a*B)*Log[a + b*x^3])/a^3)/3
```

3.84.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.84.4 Maple [A] (verified)

Time = 4.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

method	result
default	$-\frac{A}{3a^2x^3}+\frac{(-2Ab+Ba)\ln(x)}{a^3}+\frac{b\left(\frac{(2Ab-Ba)\ln\left(bx^3+a\right)}{b}-\frac{a(Ab-Ba)}{b\left(bx^3+a\right)}\right)}{3a^3}$
norman	$\frac{-\frac{A}{3a}+\frac{b(2Ab-Ba)x^6}{3a^3}}{x^3(bx^3+a)}-\frac{(2Ab-Ba)\ln(x)}{a^3}+\frac{(2Ab-Ba)\ln\left(bx^3+a\right)}{3a^3}$
risch	$\frac{-\frac{(2Ab-Ba)x^3}{3a^2}-\frac{A}{3a}}{x^3(bx^3+a)}-\frac{2\ln(x)Ab}{a^3}+\frac{B\ln(x)}{a^2}+\frac{2\ln(-bx^3-a)Ab}{3a^3}-\frac{\ln(-bx^3-a)B}{3a^2}$
parallelrisch	$-\frac{6A\ln(x)x^6b^2-2A\ln(bx^3+a)x^6b^2-3B\ln(x)x^6ab+B\ln(bx^3+a)x^6ab-2Ab^2x^6+Bx^6ab+6A\ln(x)x^3ab-2A\ln(bx^3+a)x^3}{3a^3x^3(bx^3+a)}$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/a^2/x^3+(-2*A*b+B*a)/a^3*ln(x)+1/3/a^3*b*((2*A*b-B*a)/b*ln(b*x^3+a)
-a*(A*b-B*a)/b/(b*x^3+a))
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.55

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx$$
$$=\frac{(Ba^2-2\,Aab)x^3-Aa^2-((Bab-2\,Ab^2)x^6+(Ba^2-2\,Aab)x^3)\log\left(bx^3+a\right)+3\,((Bab-2\,Ab^2)x^6+(}{3\left(a^3bx^6+a^4x^3\right)}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/3*((B*a^2 - 2*A*a*b)*x^3 - A*a^2 - ((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A
*a*b)*x^3)*log(b*x^3 + a) + 3*((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A*a*b)*x
^3)*log(x))/(a^3*b*x^6 + a^4*x^3)
```

3.84.6 Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx = \frac{-Aa+x^3(-2Ab+Ba)}{3a^3x^3+3a^2bx^6}+\frac{(-2Ab+Ba)\log\left(x\right)}{a^3} - \frac{(-2Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^3}$$

input
```
integrate((B*x**3+A)/x**4/(b*x**3+a)**2,x)
```

output
```
(-A*a + x**3*(-2*A*b + B*a))/(3*a**3*x**3 + 3*a**2*b*x**6) + (-2*A*b + B*a
)*log(x)/a**3 - (-2*A*b + B*a)*log(a/b + x**3)/(3*a**3)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx = \frac{(Ba-2\,Ab)x^3-Aa}{3\left(a^2bx^6+a^3x^3\right)}-\frac{(Ba-2\,Ab)\log\left(bx^3+a\right)}{3\,a^3}+\frac{(Ba-2\,Ab)\log\left(x^3\right)}{3\,a^3}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*((B*a - 2*A*b)*x^3 - A*a)/(a^2*b*x^6 + a^3*x^3) - 1/3*(B*a - 2*A*b)*lo
g(b*x^3 + a)/a^3 + 1/3*(B*a - 2*A*b)*log(x^3)/a^3
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx = \frac{(Ba-2\,Ab)\log\left(|x|\right)}{a^3} + \frac{Bax^3-2\,Abx^3-Aa}{3\,(bx^6+ax^3)a^2} - \frac{(Bab-2\,Ab^2)\log\left(|bx^3+a|\right)}{3\,a^3b}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
(B*a - 2*A*b)*log(abs(x))/a^3 + 1/3*(B*a*x^3 - 2*A*b*x^3 - A*a)/((b*x^6 +
a*x^3)*a^2) - 1/3*(B*a*b - 2*A*b^2)*log(abs(b*x^3 + a))/(a^3*b)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 6.85 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx = \frac{\ln\left(b\,x^3+a\right)\,\left(2\,A\,b-B\,a\right)}{3\,a^3} - \frac{\frac{A}{3\,a}+\frac{x^3\,(2\,A\,b-B\,a)}{3\,a^2}}{b\,x^6+a\,x^3} - \frac{\ln\left(x\right)\,\left(2\,A\,b-B\,a\right)}{a^3}$$

input
```
int((A + B*x^3)/(x^4*(a + b*x^3)^2),x)
```

output
```
(log(a + b*x^3)*(2*A*b - B*a))/(3*a^3) - (A/(3*a) + (x^3*(2*A*b - B*a))/(3
*a^2))/(a*x^3 + b*x^6) - (log(x)*(2*A*b - B*a))/a^3
```

3.85 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$

3.85.1 Optimal result

Integrand size = 20, antiderivative size = 215

$$\begin{aligned}\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx=&\frac{-7Ab+4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}\\&-\frac{\sqrt[3]{b}(7Ab-4aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}\\&-\frac{\sqrt[3]{b}(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}}\\&+\frac{\sqrt[3]{b}(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{10/3}}\end{aligned}$$

output
```
1/12*(-7*A*b+4*B*a)/a^2/b/x^4+1/3*(7*A*b-4*B*a)/a^3/x+1/3*(A*b-B*a)/a/b/x^
4/(b*x^3+a)-1/9*b^(1/3)*(7*A*b-4*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)+1/18*
b^(1/3)*(7*A*b-4*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)-1
/9*b^(1/3)*(7*A*b-4*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))
/a^(10/3)*3^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{9a^{4/3}A}{x^4}-\frac{36\sqrt[3]{a}(-2Ab+aB)}{x}-\frac{12\sqrt[3]{a}b(-Ab+aB)x^2}{a+bx^3}-4\sqrt{3}\sqrt[3]{b}(7Ab-4aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{b}(-7Ab+}{36a^{10/3}}$$

input
```
Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^2),x]
```

output
```
((-9*a^(4/3)*A)/x^4 - (36*a^(1/3)*(-2*A*b + a*B))/x - (12*a^(1/3)*b*(-(A*b
) + a*B)*x^2)/(a + b*x^3) - 4*Sqrt[3]*b^(1/3)*(7*A*b - 4*a*B)*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*(-7*A*b + 4*a*B)*Log[a^(1/3) +
 b^(1/3)*x] + 2*b^(1/3)*(7*A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(36*a^(10/3))
```

3.85.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {957, 847, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{(7Ab-4aB)\int \frac{1}{x^5\left(bx^3+a\right)}dx}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 847$$

$$\frac{(7Ab-4aB)\left(-\frac{b\int \frac{1}{x^2\left(bx^3+a\right)}dx}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 847$$

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 821$$

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 16$$

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1142$$

3.85. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$$\Big\downarrow 25$$

3.85. $\int\frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$$\Big\downarrow \textcolor{blue}{27}$$

3.85. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$$\Big\downarrow \text{1082}$$

3.85. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 217$$

3.85. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1103$$

3.85. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$

$$\frac{(7Ab-4aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3ab}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(x^5*(a + b*x^3)^2),x]
```

3.85. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$

output
```
(A*b - a*B)/(3*a*b*x^4*(a + b*x^3)) + ((7*A*b - 4*a*B)*(-1/4*1/(a*x^4) - (
b*(-(1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((
Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a)
)/a))/(3*a*b)
```

3.85.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.85.4 Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.72

method	result
default	$-\frac{A}{4a^2x^4}-\frac{-2Ab+Ba}{x\,a^3}+\frac{b\left(\frac{\left(\frac{Ab}{3}-\frac{Ba}{3}\right)x^2}{b\,x^3+a}+\left(\frac{7Ab}{3}-\frac{4Ba}{3}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\ \mathrm{arctan}\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.\right.}{a^3}$
risch	$\frac{\frac{b(7Ab-4Ba)x^6}{3a^3}+\frac{(7Ab-4Ba)x^3}{4a^2}-\frac{A}{4a}}{x^4(b\,x^3+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}\_Z^3+343A^3b^4-588A^2Ba\,b^3+336A\,B^2a^2b^2-64B^3a^3b\right)}\_R\ln\left(\left(-4a^{10}\_R^3-\right.\right.\right.}{9}$

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/a^2/x^4-(-2*A*b+B*a)/x/a^3+1/a^3*b*((1/3*A*b-1/3*B*a)*x^2/(b*x^3+a)
+(7/3*A*b-4/3*B*a)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx =$$

$$-\frac{12\,(4\,Bab-7\,Ab^2)x^6+9\,(4\,Ba^2-7\,Aab)x^3+9\,Aa^2+4\,\sqrt{3}((4\,Bab-7\,Ab^2)x^7+(4\,Ba^2-7\,Aab)x^4)(}{}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/36*(12*(4*B*a*b - 7*A*b^2)*x^6 + 9*(4*B*a^2 - 7*A*a*b)*x^3 + 9*A*a^2 +
4*sqrt(3)*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^(1/3)
*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 2*((4*B*a*b - 7*A*b^2)
*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3)
- a*(-b/a)^(1/3)) + 4*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*
(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^3*b*x^7 + a^4*x^4)
```

3.85.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$$

$$= \operatorname{RootSum}\left(729t^3a^{10}+343A^3b^4-588A^2Bab^3+336AB^2a^2b^2-64B^3a^3b,\ \left(t\mapsto t\log\left(\frac{81t^2a^7}{49A^2b^3-56ABab^2}\right.\right.\right.$$

$$+\frac{-3Aa^2+x^6\cdot(28Ab^2-16Bab)+x^3\cdot(21Aab-12Ba^2)}{12a^4x^4+12a^3bx^7}$$

input
```
integrate((B*x**3+A)/x**5/(b*x**3+a)**2,x)
```

output
```
RootSum(729*_t**3*a**10 + 343*A**3*b**4 - 588*A**2*B*a*b**3 + 336*A*B**2*a
**2*b**2 - 64*B**3*a**3*b, Lambda(_t, _t*log(81*_t**2*a**7/(49*A**2*b**3 -
 56*A*B*a*b**2 + 16*B**2*a**2*b) + x))) + (-3*A*a**2 + x**6*(28*A*b**2 - 1
6*B*a*b) + x**3*(21*A*a*b - 12*B*a**2))/(12*a**4*x**4 + 12*a**3*b*x**7)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx = -\frac{4\,(4\,Bab-7\,Ab^2)x^6+3\,(4\,Ba^2-7\,Aab)x^3+3\,Aa^2}{12\,(a^3bx^7+a^4x^4)} - \frac{\sqrt{3}(4\,Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(4\,Ba-7\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(4\,Ba-7\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/12*(4*(4*B*a*b - 7*A*b^2)*x^6 + 3*(4*B*a^2 - 7*A*a*b)*x^3 + 3*A*a^2)/(a
^3*b*x^7 + a^4*x^4) - 1/9*sqrt(3)*(4*B*a - 7*A*b)*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(a^3*(a/b)^(1/3)) - 1/18*(4*B*a - 7*A*b)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(1/3)) + 1/9*(4*B*a - 7*A*b)*
log(x + (a/b)^(1/3))/(a^3*(a/b)^(1/3))
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx &= \frac{\left(4\,Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7\,Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4}\\ &\quad+\frac{\sqrt{3}\left(4\,(-ab^2)^{\frac{2}{3}}\,Ba-7\,(-ab^2)^{\frac{2}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4b}\\ &\quad-\frac{Babx^2-Ab^2x^2}{3\,(bx^3+a)a^3}\\ &\quad-\frac{\left(4\,(-ab^2)^{\frac{2}{3}}\,Ba-7\,(-ab^2)^{\frac{2}{3}}\,Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^4b}\\ &\quad-\frac{4\,Bax^3-8\,Abx^3+Aa}{4\,a^3x^4}\end{aligned}$$

input

```
integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="giac")
```

output

```
1/9*(4*B*a*b*(-a/b)^(1/3) - 7*A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/a^4 + 1/9*sqrt(3)*(4*(-a*b^2)^(2/3)*B*a - 7*(-a*b^2)^(2/3)
*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) - 1/3*
(B*a*b*x^2 - A*b^2*x^2)/((b*x^3 + a)*a^3) - 1/18*(4*(-a*b^2)^(2/3)*B*a - 7
*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/
4*(4*B*a*x^3 - 8*A*b*x^3 + A*a)/(a^3*x^4)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 7.03 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.97

$$\begin{aligned}&\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx\\ &= \frac{\frac{x^3\,(7\,A\,b-4\,B\,a)}{4\,a^2}-\frac{A}{4\,a}+\frac{b\,x^6\,(7\,A\,b-4\,B\,a)}{3\,a^3}}{b\,x^7+a\,x^4}+\frac{(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}+b^3\,x\right)\,(7\,A\,b-4\,B\,a)}{9\,a^{10/3}}\\ &\quad+\frac{(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}-2\,b^3\,x+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,(7\,A\,b-4\,B\,a)}{9\,a^{10/3}}\\ &\quad-\frac{(-b)^{1/3}\,\ln\left(2\,b^3\,x-a^{1/3}\,(-b)^{8/3}+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,(7\,A\,b-4\,B\,a)}{9\,a^{10/3}}\end{aligned}$$

input
```
int((A + B*x^3)/(x^5*(a + b*x^3)^2),x)
```

output
```
((x^3*(7*A*b - 4*B*a))/(4*a^2) - A/(4*a) + (b*x^6*(7*A*b - 4*B*a))/(3*a^3)
)/(a*x^4 + b*x^7) + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(7*A*b - 4
*B*a))/(9*a^(10/3)) + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*b^3*x + 3^(1/
2)*a^(1/3)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - 4*B*a))/(9*a^(10
/3)) - ((-b)^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3)*(-b)
^(8/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - 4*B*a))/(9*a^(10/3))
```

3.85. $\int \frac{A+Bx^3}{x^5(a+bx^3)^2}\,dx$

3.86 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx$

3.86.1 Optimal result

Integrand size = 20, antiderivative size = 215

$$\begin{aligned}\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx = {} & \frac{-8Ab+5aB}{15a^2bx^5}+\frac{8Ab-5aB}{6a^3x^2}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)} \\ & -\frac{b^{2/3}(8Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}} \\ & +\frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}} \\ & -\frac{b^{2/3}(8Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{11/3}}\end{aligned}$$

output

```
1/15*(-8*A*b+5*B*a)/a^2/b/x^5+1/6*(8*A*b-5*B*a)/a^3/x^2+1/3*(A*b-B*a)/a/b/
x^5/(b*x^3+a)+1/9*b^(2/3)*(8*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)-1/1
8*b^(2/3)*(8*A*b-5*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)
-1/9*b^(2/3)*(8*A*b-5*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2
))/a^(11/3)*3^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{18a^{5/3}A}{x^5}-\frac{45a^{2/3}(-2Ab+aB)}{x^2}-\frac{30a^{2/3}b(-Ab+aB)x}{a+bx^3}-10\sqrt{3}b^{2/3}(8Ab-5aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10b^{2/3}(8Ab-}{90a^{11/3}}$$

input
```
Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^2),x]
```

output
```
((-18*a^(5/3)*A)/x^5 - (45*a^(2/3)*(-2*A*b + a*B))/x^2 - (30*a^(2/3)*b*(-(
A*b) + a*B)*x)/(a + b*x^3) - 10*Sqrt[3]*b^(2/3)*(8*A*b - 5*a*B)*ArcTan[(1
- (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*b^(2/3)*(8*A*b - 5*a*B)*Log[a^(1/3)
 + b^(1/3)*x] + 5*b^(2/3)*(-8*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(90*a^(11/3))
```

3.86.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {957, 847, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(8Ab-5aB)\int \frac{1}{x^6\left(bx^3+a\right)}dx}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{847}$$

$$\frac{(8Ab-5aB)\left(-\frac{b\int \frac{1}{x^3\left(bx^3+a\right)}dx}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$$\Big\downarrow 847$$

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$$\Big\downarrow 750$$

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$$\Big\downarrow 16$$

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$$\Big\downarrow 1142$$

3.86. $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx$

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$$\Big\downarrow \; 25$$

3.86. $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx$

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$$\Big\downarrow \; 27$$

3.86. $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx$

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{1082}$$

3.86. $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx$

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$$\Big\downarrow 217$$

3.86. $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx$

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1103$$

3.86. $\int \frac{A+Bx^3}{x^6(a+bx^3)^2}\,dx$

$$\frac{(8Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3ab}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(x^6*(a + b*x^3)^2),x]
```

3.86. $\int \frac{A+Bx^3}{x^6(a+bx^3)^2}\,dx$

output
```
(A*b - a*B)/(3*a*b*x^5*(a + b*x^3)) + ((8*A*b - 5*a*B)*(-1/5*1/(a*x^5) - (
b*(-1/2*1/(a*x^2) - (b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-(
(Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/a))/a))/(
3*a*b)
```

3.86.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.86.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.72

method	result
default	$-\frac{A}{5a^{2}x^{5}}-\frac{-2Ab+Ba}{2x^{2}a^{3}}+\frac{b\left(\frac{\left(\frac{Ab}{3}-\frac{Ba}{3}\right)x}{b\,x^{3}+a}+\frac{(8Ab-5Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}\right)}{a^{3}}$
risch	$\frac{\frac{b(8Ab-5Ba)x^{6}}{6a^{3}}+\frac{(8Ab-5Ba)x^{3}}{10a^{2}}-\frac{A}{5a}}{x^{5}(b\,x^{3}+a)}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{11}\_Z^{3}-512A^{3}b^{5}+960A^{2}Ba\,b^{4}-600A\,B^{2}a^{2}b^{3}+125B^{3}a^{3}b^{2}\right)}\_R\ln\left(\left(-4\_R^{3}a^{11}\right.\right.\right.}{}$

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*A/a^2/x^5-1/2*(-2*A*b+B*a)/x^2/a^3+1/a^3*b*((1/3*A*b-1/3*B*a)*x/(b*x^
3+a)+1/3*(8*A*b-5*B*a)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.29

$$\int\frac{A+Bx^{3}}{x^{6}\left(a+bx^{3}\right)^{2}}\,dx=$$

$$-\frac{15\,(5\,Bab-8\,Ab^{2})x^{6}+9\,(5\,Ba^{2}-8\,Aab)x^{3}+18\,Aa^{2}+10\,\sqrt{3}((5\,Bab-8\,Ab^{2})x^{8}+(5\,Ba^{2}-8\,Aab)x^{5}}{}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/90*(15*(5*B*a*b - 8*A*b^2)*x^6 + 9*(5*B*a^2 - 8*A*a*b)*x^3 + 18*A*a^2 +
 10*sqrt(3)*((5*B*a*b - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^
(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*((5*B*
a*b - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1/3)*log(b^2*x^2
- a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*((5*B*a*b - 8*A*b^2)*x
^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3))
)/(a^3*b*x^8 + a^4*x^5)
```

3.86.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx^3}{x^6\,(a+bx^3)^2}\,dx$$

$$=\mathrm{RootSum}\left(729t^3a^{11}-512A^3b^5+960A^2Bab^4-600AB^2a^2b^3+125B^3a^3b^2,\left(t\mapsto t\log\left(-\frac{9ta^4}{-8Ab^2+5Bab}\right.\right.\right.$$

$$+\frac{-6Aa^2+x^6\cdot(40Ab^2-25Bab)+x^3\cdot(24Aab-15Ba^2)}{30a^4x^5+30a^3bx^8}$$

input
```
integrate((B*x**3+A)/x**6/(b*x**3+a)**2,x)
```

output
```
RootSum(729*_t**3*a**11 - 512*A**3*b**5 + 960*A**2*B*a*b**4 - 600*A*B**2*a
**2*b**3 + 125*B**3*a**3*b**2, Lambda(_t, _t*log(-9*_t*a**4/(-8*A*b**2 + 5
*B*a*b) + x))) + (-6*A*a**2 + x**6*(40*A*b**2 - 25*B*a*b) + x**3*(24*A*a*b
 - 15*B*a**2))/(30*a**4*x**5 + 30*a**3*b*x**8)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx = -\frac{5\,(5\,Bab-8\,Ab^2)x^6+3\,(5\,Ba^2-8\,Aab)x^3+6\,Aa^2}{30\,(a^3bx^8+a^4x^5)} - \frac{\sqrt{3}(5\,Ba-8\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(5\,Ba-8\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(5\,Ba-8\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input

```
integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="maxima")
```

output

```
-1/30*(5*(5*B*a*b - 8*A*b^2)*x^6 + 3*(5*B*a^2 - 8*A*a*b)*x^3 + 6*A*a^2)/(a
^3*b*x^8 + a^4*x^5) - 1/9*sqrt(3)*(5*B*a - 8*A*b)*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(a^3*(a/b)^(2/3)) + 1/18*(5*B*a - 8*A*b)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(2/3)) - 1/9*(5*B*a - 8*A*b)*
log(x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx = -\frac{\sqrt{3}\left(5\,(-ab^2)^{\frac{1}{3}}\,Ba-8\,(-ab^2)^{\frac{1}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4} + \frac{(5\,Bab-8\,Ab^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4} - \frac{\left(5\,(-ab^2)^{\frac{1}{3}}\,Ba-8\,(-ab^2)^{\frac{1}{3}}\,Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^4} - \frac{Babx-Ab^2x}{3\,(bx^3+a)a^3} - \frac{5\,Bax^3-10\,Abx^3+2\,Aa}{10\,a^3x^5}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 8*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt
(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4 + 1/9*(5*B*a*b - 8*A*b^2)*(-a/b
)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - 1/18*(5*(-a*b^2)^(1/3)*B*a - 8*(-
a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^4 - 1/3*(B*a*
b*x - A*b^2*x)/((b*x^3 + a)*a^3) - 1/10*(5*B*a*x^3 - 10*A*b*x^3 + 2*A*a)/(
a^3*x^5)
```

3.86.9 Mupad [B] (verification not implemented)

Time = 6.99 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx = \frac{\frac{x^3\,\left(8\,A\,b-5\,B\,a\right)}{10\,a^2}-\frac{A}{5\,a}+\frac{b\,x^6\,\left(8\,A\,b-5\,B\,a\right)}{6\,a^3}}{b\,x^8+a\,x^5} + \frac{b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(8\,A\,b-5\,B\,a\right)}{9\,a^{11/3}} - \frac{b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(8\,A\,b-5\,B\,a\right)}{9\,a^{11/3}} + \frac{b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(8\,A\,b-5\,B\,a\right)}{9\,a^{11/3}}$$

input
```
int((A + B*x^3)/(x^6*(a + b*x^3)^2),x)
```

output
```
((x^3*(8*A*b - 5*B*a))/(10*a^2) - A/(5*a) + (b*x^6*(8*A*b - 5*B*a))/(6*a^3
))/(a*x^5 + b*x^8) + (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(8*A*b - 5*B*a))/(9
*a^(11/3)) - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^
(1/2)*1i)/2 + 1/2)*(8*A*b - 5*B*a))/(9*a^(11/3)) + (b^(2/3)*log(3^(1/2)*a^
(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(8*A*b - 5*B*a))/
(9*a^(11/3))
```

3.87 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx$

3.87.1 Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx=-\frac{A}{6a^2x^6}+\frac{2Ab-aB}{3a^3x^3}+\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)}+\frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log\left(a+bx^3\right)}{3a^4}$$

output
```
-1/6*A/a^2/x^6+1/3*(2*A*b-B*a)/a^3/x^3+1/3*b*(A*b-B*a)/a^3/(b*x^3+a)+b*(3*
A*b-2*B*a)*ln(x)/a^4-1/3*b*(3*A*b-2*B*a)*ln(b*x^3+a)/a^4
```

3.87.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx=-\frac{\frac{a^2A}{x^6}+\frac{2a(-2Ab+aB)}{x^3}+\frac{2ab(-Ab+aB)}{a+bx^3}-6b(3Ab-2aB)\log(x)+2b(3Ab-2aB)\log\left(a+bx^3\right)}{6a^4}$$

input
```
Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^2),x]
```

output
```
-1/6*((a^2*A)/x^6 + (2*a*(-2*A*b + a*B))/x^3 + (2*a*b*(-(A*b) + a*B))/(a +
 b*x^3) - 6*b*(3*A*b - 2*a*B)*Log[x] + 2*b*(3*A*b - 2*a*B)*Log[a + b*x^3])
/a^4
```

3.87.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^9\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{3}\int \left(\frac{(2aB-3Ab)b^2}{a^4\left(bx^3+a\right)}+\frac{(aB-Ab)b^2}{a^3\left(bx^3+a\right)^2}-\frac{(2aB-3Ab)b}{a^4x^3}+\frac{aB-2Ab}{a^3x^6}+\frac{A}{a^2x^9}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{b\log\left(x^3\right)(3Ab-2aB)}{a^4}-\frac{b(3Ab-2aB)\log\left(a+bx^3\right)}{a^4}+\frac{b(Ab-aB)}{a^3\left(a+bx^3\right)}+\frac{2Ab-aB}{a^3x^3}-\frac{A}{2a^2x^6}\right)$$

input
```
Int[(A + B*x^3)/(x^7*(a + b*x^3)^2),x]
```

output
```
(-1/2*A/(a^2*x^6) + (2*A*b - a*B)/(a^3*x^3) + (b*(A*b - a*B))/(a^3*(a + b*
x^3)) + (b*(3*A*b - 2*a*B)*Log[x^3])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x^
3])/a^4)/3
```

3.87.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.87.4 Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

method	result
default	$-\frac{A}{6a^{2}x^{6}}-\frac{-2Ab+Ba}{3x^{3}a^{3}}+\frac{b(3Ab-2Ba)\ln(x)}{a^{4}}-\frac{b^{2}\left(\frac{(3Ab-2Ba)\ln\left(b\,x^{3}+a\right)}{b}-\frac{a(Ab-Ba)}{b\left(b\,x^{3}+a\right)}\right)}{3a^{4}}$
norman	$\frac{-\frac{A}{6a}+\frac{(3Ab-2Ba)x^{3}}{6a^{2}}-\frac{b\left(3b^{2}A-2abB\right)x^{9}}{3a^{4}}}{x^{6}(b\,x^{3}+a)}+\frac{b(3Ab-2Ba)\ln(x)}{a^{4}}-\frac{b(3Ab-2Ba)\ln\left(b\,x^{3}+a\right)}{3a^{4}}$
risch	$\frac{\frac{b(3Ab-2Ba)x^{6}}{3a^{3}}+\frac{(3Ab-2Ba)x^{3}}{6a^{2}}-\frac{A}{6a}}{x^{6}(b\,x^{3}+a)}+\frac{3b^{2}\ln(x)A}{a^{4}}-\frac{2b\ln(x)B}{a^{3}}-\frac{b^{2}\ln\left(b\,x^{3}+a\right)A}{a^{4}}+\frac{2b\ln\left(b\,x^{3}+a\right)B}{3a^{3}}$
parallelrisch	$\frac{18A\ln(x)x^{9}b^{3}-6A\ln\left(b\,x^{3}+a\right)x^{9}b^{3}-12B\ln(x)x^{9}a\,b^{2}+4B\ln\left(b\,x^{3}+a\right)x^{9}a\,b^{2}-6A\,x^{9}b^{3}+4B\,x^{9}a\,b^{2}+18A\ln(x)x^{6}a\,b^{2}-6A\ln(b\,x}{6a^{4}x^{6}(b\,x^{3}+a)}$

input
```
int((B*x^3+A)/x^7/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*A/a^2/x^6-1/3*(-2*A*b+B*a)/x^3/a^3+b*(3*A*b-2*B*a)*ln(x)/a^4-1/3/a^4*
b^2*((3*A*b-2*B*a)/b*ln(b*x^3+a)-a*(A*b-B*a)/b/(b*x^3+a))
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx = -\frac{2\,(2\,Ba^2b-3\,Aab^2)x^6+Aa^3+(2\,Ba^3-3\,Aa^2b)x^3-2\,((2\,Bab^2-3\,Ab^3)x^9+(2\,Ba^2b-3\,Aab^2)x^6)\log}{6\,(a^4bx^9+a^5x^6)}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(2*B*a^2*b - 3*A*a*b^2)*x^6 + A*a^3 + (2*B*a^3 - 3*A*a^2*b)*x^3 -
2*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*log(b*x^3 + a)
 + 6*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*log(x))/(a^
4*b*x^9 + a^5*x^6)
```

3.87.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx = \frac{-Aa^2+x^6\cdot(6Ab^2-4Bab)+x^3\cdot(3Aab-2Ba^2)}{6a^4x^6+6a^3bx^9} - \frac{b(-3Ab+2Ba)\log(x)}{a^4} + \frac{b(-3Ab+2Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^4}$$

input
```
integrate((B*x**3+A)/x**7/(b*x**3+a)**2,x)
```

output
```
(-A*a**2 + x**6*(6*A*b**2 - 4*B*a*b) + x**3*(3*A*a*b - 2*B*a**2))/(6*a**4*
x**6 + 6*a**3*b*x**9) - b*(-3*A*b + 2*B*a)*log(x)/a**4 + b*(-3*A*b + 2*B*a
)*log(a/b + x**3)/(3*a**4)
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx = -\frac{2\,(2\,Bab-3\,Ab^2)x^6+(2\,Ba^2-3\,Aab)x^3+Aa^2}{6\,(a^3bx^9+a^4x^6)} + \frac{(2\,Bab-3\,Ab^2)\log(bx^3+a)}{3\,a^4} - \frac{(2\,Bab-3\,Ab^2)\log(x^3)}{3\,a^4}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/6*(2*(2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3 + A*a^2)/(a^3*b*
x^9 + a^4*x^6) + 1/3*(2*B*a*b - 3*A*b^2)*log(b*x^3 + a)/a^4 - 1/3*(2*B*a*b
 - 3*A*b^2)*log(x^3)/a^4
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.54

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx = -\frac{(2\,Bab-3\,Ab^2)\log\left(|x|\right)}{a^4}+\frac{(2\,Bab^2-3\,Ab^3)\log\left(|bx^3+a|\right)}{3\,a^4b} - \frac{2\,Bab^2x^3-3\,Ab^3x^3+3\,Ba^2b-4\,Aab^2}{3\,(bx^3+a)a^4} + \frac{6\,Babx^6-9\,Ab^2x^6-2\,Ba^2x^3+4\,Aabx^3-Aa^2}{6\,a^4x^6}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-(2*B*a*b - 3*A*b^2)*log(abs(x))/a^4 + 1/3*(2*B*a*b^2 - 3*A*b^3)*log(abs(b
*x^3 + a))/(a^4*b) - 1/3*(2*B*a*b^2*x^3 - 3*A*b^3*x^3 + 3*B*a^2*b - 4*A*a*
b^2)/((b*x^3 + a)*a^4) + 1/6*(6*B*a*b*x^6 - 9*A*b^2*x^6 - 2*B*a^2*x^3 + 4*
A*a*b*x^3 - A*a^2)/(a^4*x^6)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx = \frac{\frac{x^3\,(3\,A\,b-2\,B\,a)}{6\,a^2}-\frac{A}{6\,a}+\frac{b\,x^6\,(3\,A\,b-2\,B\,a)}{3\,a^3}}{b\,x^9+a\,x^6} - \frac{\ln\left(b\,x^3+a\right)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{3\,a^4}+\frac{\ln\left(x\right)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{a^4}$$

input
```
int((A + B*x^3)/(x^7*(a + b*x^3)^2),x)
```

output
```
((x^3*(3*A*b - 2*B*a))/(6*a^2) - A/(6*a) + (b*x^6*(3*A*b - 2*B*a))/(3*a^3)
)/(a*x^6 + b*x^9) - (log(a + b*x^3)*(3*A*b^2 - 2*B*a*b))/(3*a^4) + (log(x)
*(3*A*b^2 - 2*B*a*b))/a^4
```

3.88 $\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.88.1 Optimal result

Integrand size = 20, antiderivative size = 107

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-3aB)x^3}{3b^4} + \frac{Bx^6}{6b^3} + \frac{a^3(Ab-aB)}{6b^5(a+bx^3)^2} - \frac{a^2(3Ab-4aB)}{3b^5(a+bx^3)} - \frac{a(Ab-2aB)\log(a+bx^3)}{b^5}$$

output

```
1/3*(A*b-3*B*a)*x^3/b^4+1/6*B*x^6/b^3+1/6*a^3*(A*b-B*a)/b^5/(b*x^3+a)^2-1/
3*a^2*(3*A*b-4*B*a)/b^5/(b*x^3+a)-a*(A*b-2*B*a)*ln(b*x^3+a)/b^5
```

3.88.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.88

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2b(Ab-3aB)x^3 + b^2Bx^6 + \frac{a^3(Ab-aB)}{(a+bx^3)^2} + \frac{2a^2(-3Ab+4aB)}{a+bx^3} + 6a(-Ab+2aB)\log(a+bx^3)}{6b^5}$$

input

```
Integrate[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]
```

output

```
(2*b*(A*b - 3*a*B)*x^3 + b^2*B*x^6 + (a^3*(A*b - a*B))/(a + b*x^3)^2 + (2*
a^2*(-3*A*b + 4*a*B))/(a + b*x^3) + 6*a*(-(A*b) + 2*a*B)*Log[a + b*x^3])/(
6*b^5)
```

3.88.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}dx$$
$$\Big\downarrow \text{948}$$
$$\frac{1}{3}\int \frac{x^9(Bx^3+A)}{(bx^3+a)^3}dx^3$$
$$\Big\downarrow \text{86}$$
$$\frac{1}{3}\int \left(\frac{(aB-Ab)a^3}{b^4(bx^3+a)^3}-\frac{(4aB-3Ab)a^2}{b^4(bx^3+a)^2}+\frac{3(2aB-Ab)a}{b^4(bx^3+a)}+\frac{Bx^3}{b^3}+\frac{Ab-3aB}{b^4}\right)dx^3$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}\left(\frac{a^3(Ab-aB)}{2b^5(a+bx^3)^2}-\frac{a^2(3Ab-4aB)}{b^5(a+bx^3)}-\frac{3a(Ab-2aB)\log(a+bx^3)}{b^5}+\frac{x^3(Ab-3aB)}{b^4}+\frac{Bx^6}{2b^3}\right)$$

input
```
Int[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(((A*b - 3*a*B)*x^3)/b^4 + (B*x^6)/(2*b^3) + (a^3*(A*b - a*B))/(2*b^5*(a +
 b*x^3)^2) - (a^2*(3*A*b - 4*a*B))/(b^5*(a + b*x^3)) - (3*a*(A*b - 2*a*B)*
Log[a + b*x^3])/b^5)/3
```

3.88.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.88.4 Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

method	result
norman	$\frac{\frac{B\,x^{12}}{6b}-\frac{a^{2}\left(3abA-6a^{2}B\right)}{2b^{5}}+\frac{(Ab-2Ba)x^{9}}{3b^{2}}-\frac{2a\left(abA-2a^{2}B\right)x^{3}}{b^{4}}}{(b\,x^{3}+a)^{2}}-\frac{a(Ab-2Ba)\ln\left(b\,x^{3}+a\right)}{b^{5}}$
default	$\frac{\left(bB\,x^{3}+Ab-3Ba\right)^{2}}{6b^{5}B}-\frac{a\left(\frac{(3Ab-6Ba)\ln\left(b\,x^{3}+a\right)}{b}-\frac{a^{2}(Ab-Ba)}{2b\left(b\,x^{3}+a\right)^{2}}+\frac{a(3Ab-4Ba)}{b\left(b\,x^{3}+a\right)}\right)}{3b^{4}}$
risch	$\frac{B\,x^{6}}{6b^{3}}+\frac{A\,x^{3}}{3b^{3}}-\frac{Ba\,x^{3}}{b^{4}}+\frac{A^{2}}{6b^{3}B}-\frac{Aa}{b^{4}}+\frac{3B\,a^{2}}{2b^{5}}+\frac{(-a^{2}bA+\frac{4}{3}a^{3}B)x^{3}-\frac{a^{3}(5Ab-7Ba)}{6b}}{b^{4}(b\,x^{3}+a)^{2}}-\frac{a\ln(b\,x^{3}+a)A}{b^{4}}+\frac{2a^{2}\ln(b\,x}{b^{5}}$
parallelrisch	$-\frac{-B\,x^{12}b^{4}-2A\,x^{9}b^{4}+4B\,x^{9}a\,b^{3}+6A\ln(b\,x^{3}+a)x^{6}a\,b^{3}-12B\ln(b\,x^{3}+a)x^{6}a^{2}b^{2}+12A\ln(b\,x^{3}+a)x^{3}a^{2}b^{2}-24B\ln(b\,x^{3}+a)x^{3}}{6b^{5}(b\,x^{3}+a)^{2}}$

input
```
int(x^11*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/6*B/b*x^12-1/2*a^2*(3*A*a*b-6*B*a^2)/b^5+1/3*(A*b-2*B*a)/b^2*x^9-2*a*(A
*a*b-2*B*a^2)/b^4*x^3)/(b*x^3+a)^2-a*(A*b-2*B*a)*ln(b*x^3+a)/b^5
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.67

$$\int \frac{x^{11}(A+Bx^{3})}{\left(a+bx^{3}\right)^{3}}\,dx$$
$$=\frac{Bb^{4}x^{12}-2\,(2\,Bab^{3}-Ab^{4})x^{9}-(11\,Ba^{2}b^{2}-4\,Aab^{3})x^{6}+7\,Ba^{4}-5\,Aa^{3}b+2\,(Ba^{3}b-2\,Aa^{2}b^{2})x^{3}+6\,((2}{6\,(b^{7}x^{6}+2\,ab^{6}x^{3}+a^{2}b^{5})}$$

input
```
integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(B*b^4*x^12 - 2*(2*B*a*b^3 - A*b^4)*x^9 - (11*B*a^2*b^2 - 4*A*a*b^3)*x
^6 + 7*B*a^4 - 5*A*a^3*b + 2*(B*a^3*b - 2*A*a^2*b^2)*x^3 + 6*((2*B*a^2*b^2
 - A*a*b^3)*x^6 + 2*B*a^4 - A*a^3*b + 2*(2*B*a^3*b - A*a^2*b^2)*x^3)*log(b
*x^3 + a))/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5)
```

3.88.6 Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^6}{6b^3} + \frac{a(-Ab+2Ba)\log(a+bx^3)}{b^5} + x^3\left(\frac{A}{3b^3} - \frac{Ba}{b^4}\right) + \frac{-5Aa^3b+7Ba^4+x^3(-6Aa^2b^2+8Ba^3b)}{6a^2b^5+12ab^6x^3+6b^7x^6}$$

input
```
integrate(x**11*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
B*x**6/(6*b**3) + a*(-A*b + 2*B*a)*log(a + b*x**3)/b**5 + x**3*(A/(3*b**3)
 - B*a/b**4) + (-5*A*a**3*b + 7*B*a**4 + x**3*(-6*A*a**2*b**2 + 8*B*a**3*b
))/(6*a**2*b**5 + 12*a*b**6*x**3 + 6*b**7*x**6)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{7\,Ba^4-5\,Aa^3b+2\,(4\,Ba^3b-3\,Aa^2b^2)x^3}{6\,(b^7x^6+2\,ab^6x^3+a^2b^5)} + \frac{Bbx^6-2\,(3\,Ba-Ab)x^3}{6\,b^4} + \frac{(2\,Ba^2-Aab)\log(bx^3+a)}{b^5}$$

input
```
integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/6*(7*B*a^4 - 5*A*a^3*b + 2*(4*B*a^3*b - 3*A*a^2*b^2)*x^3)/(b^7*x^6 + 2*a
*b^6*x^3 + a^2*b^5) + 1/6*(B*b*x^6 - 2*(3*B*a - A*b)*x^3)/b^4 + (2*B*a^2 -
 A*a*b)*log(b*x^3 + a)/b^5
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{(2\,Ba^2-Aab)\log\left(|bx^3+a|\right)}{b^5}+\frac{Bb^3x^6-6\,Bab^2x^3+2\,Ab^3x^3}{6\,b^6}\\ &\quad-\frac{18\,Ba^2b^2x^6-9\,Aab^3x^6+28\,Ba^3bx^3-12\,Aa^2b^2x^3+11\,Ba^4-4\,Aa^3b}{6\,(bx^3+a)^2b^5}\end{aligned}$$

input

```
integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output

```
(2*B*a^2 - A*a*b)*log(abs(b*x^3 + a))/b^5 + 1/6*(B*b^3*x^6 - 6*B*a*b^2*x^3
 + 2*A*b^3*x^3)/b^6 - 1/6*(18*B*a^2*b^2*x^6 - 9*A*a*b^3*x^6 + 28*B*a^3*b*x
^3 - 12*A*a^2*b^2*x^3 + 11*B*a^4 - 4*A*a^3*b)/((b*x^3 + a)^2*b^5)
```

3.88.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{\frac{7\,B\,a^4-5\,A\,a^3\,b}{6\,b}+x^3\left(\frac{4\,B\,a^3}{3}-A\,a^2\,b\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x^3\left(\frac{A}{3\,b^3}-\frac{B\,a}{b^4}\right)\\ &\quad+\frac{\ln\left(b\,x^3+a\right)\,\left(2\,B\,a^2-A\,a\,b\right)}{b^5}+\frac{B\,x^6}{6\,b^3}\end{aligned}$$

input

```
int((x^11*(A + B*x^3))/(a + b*x^3)^3,x)
```

output

```
((7*B*a^4 - 5*A*a^3*b)/(6*b) + x^3*((4*B*a^3)/3 - A*a^2*b))/(a^2*b^4 + b^6
*x^6 + 2*a*b^5*x^3) + x^3*(A/(3*b^3) - (B*a)/b^4) + (log(a + b*x^3)*(2*B*a
^2 - A*a*b))/b^5 + (B*x^6)/(6*b^3)
```

3.89 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.89.1 Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx=\frac{Bx^3}{3b^3}-\frac{a^2(Ab-aB)}{6b^4(a+bx^3)^2}+\frac{a(2Ab-3aB)}{3b^4(a+bx^3)}+\frac{(Ab-3aB)\log(a+bx^3)}{3b^4}$$

output
```
1/3*B*x^3/b^3-1/6*a^2*(A*b-B*a)/b^4/(b*x^3+a)^2+1/3*a*(2*A*b-3*B*a)/b^4/(b
*x^3+a)+1/3*(A*b-3*B*a)*ln(b*x^3+a)/b^4
```

3.89.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx=\frac{Bx^3}{3b^3}+\frac{-a^2Ab+a^3B}{6b^4(a+bx^3)^2}+\frac{2aAb-3a^2B}{3b^4(a+bx^3)}+\frac{(Ab-3aB)\log(a+bx^3)}{3b^4}$$

input
```
Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(B*x^3)/(3*b^3) + (-(a^2*A*b) + a^3*B)/(6*b^4*(a + b*x^3)^2) + (2*a*A*b -
3*a^2*B)/(3*b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

3.89.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 \left(A+Bx^3\right)}{\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^6 \left(Bx^3+A\right)}{\left(bx^3+a\right)^3}dx^3$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{3}\int \left(-\frac{(aB-Ab)a^2}{b^3 \left(bx^3+a\right)^3}+\frac{(3aB-2Ab)a}{b^3 \left(bx^3+a\right)^2}+\frac{B}{b^3}+\frac{Ab-3aB}{b^3 \left(bx^3+a\right)}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(-\frac{a^2 (Ab-aB)}{2 b^4 \left(a+bx^3\right)^2}+\frac{a (2 Ab-3 aB)}{b^4 \left(a+bx^3\right)}+\frac{(Ab-3 aB) \log \left(a+bx^3\right)}{b^4}+\frac{Bx^3}{b^3}\right)$$

input

```
Int[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]
```

output

```
((B*x^3)/b^3 - (a^2*(A*b - a*B))/(2*b^4*(a + b*x^3)^2) + (a*(2*A*b - 3*a*B
))/(b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/b^4)/3
```

3.89.3.1 Defintions of rubi rules used

rule 86

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.89.4 Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

method	result
norman	$\frac{\frac{B\,x^{9}}{3b}+\frac{a^{2}(Ab-3Ba)}{2b^{4}}+\frac{2a(Ab-3Ba)x^{3}}{3b^{3}}}{(b\,x^{3}+a)^{2}}+\frac{(Ab-3Ba)\ln(b\,x^{3}+a)}{3b^{4}}$
default	$\frac{B\,x^{3}}{3b^{3}}+\frac{\frac{(Ab-3Ba)\ln\left(b\,x^{3}+a\right)}{b}-\frac{a^{2}(Ab-Ba)}{2b\left(b\,x^{3}+a\right)^{2}}+\frac{a(2Ab-3Ba)}{b\left(b\,x^{3}+a\right)}}{3b^{3}}$
risch	$\frac{B\,x^{3}}{3b^{3}}+\frac{\left(\frac{2}{3}abA-a^{2}B\right)x^{3}+\frac{a^{2}(3Ab-5Ba)}{6b}}{b^{3}(b\,x^{3}+a)^{2}}+\frac{\ln(b\,x^{3}+a)A}{3b^{3}}-\frac{\ln(b\,x^{3}+a)Ba}{b^{4}}$
parallelrisch	$\frac{2b^{3}B\,x^{9}+2A\ln(b\,x^{3}+a)x^{6}b^{3}-6B\ln(b\,x^{3}+a)x^{6}a\,b^{2}+4A\ln(b\,x^{3}+a)x^{3}a\,b^{2}-12B\ln(b\,x^{3}+a)x^{3}a^{2}b+4aA\,b^{2}x^{3}-12B\,a^{2}b\,x^{3}+2A}{6b^{4}(b\,x^{3}+a)^{2}}$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/3*B/b*x^9+1/2*a^2*(A*b-3*B*a)/b^4+2/3*a*(A*b-3*B*a)/b^3*x^3)/(b*x^3+a)^
2+1/3*(A*b-3*B*a)*ln(b*x^3+a)/b^4
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.61

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$=\frac{2\,Bb^3x^9+4\,Bab^2x^6-5\,Ba^3+3\,Aa^2b-4\,(Ba^2b-Aab^2)x^3-2\,((3\,Bab^2-Ab^3)x^6+3\,Ba^3-Aa^2b+2\,(}{6\,(b^6x^6+2\,ab^5x^3+a^2b^4)}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(2*B*b^3*x^9 + 4*B*a*b^2*x^6 - 5*B*a^3 + 3*A*a^2*b - 4*(B*a^2*b - A*a*
b^2)*x^3 - 2*((3*B*a*b^2 - A*b^3)*x^6 + 3*B*a^3 - A*a^2*b + 2*(3*B*a^2*b -
 A*a*b^2)*x^3)*log(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

3.89.6 Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^3}{3b^3} + \frac{3Aa^2b - 5Ba^3 + x^3\cdot(4Aab^2 - 6Ba^2b)}{6a^2b^4 + 12ab^5x^3 + 6b^6x^6} - \frac{(-Ab+3Ba)\log(a+bx^3)}{3b^4}$$

input
```
integrate(x**8*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
B*x**3/(3*b**3) + (3*A*a**2*b - 5*B*a**3 + x**3*(4*A*a*b**2 - 6*B*a**2*b))
/(6*a**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) - (-A*b + 3*B*a)*log(a + b*x
**3)/(3*b**4)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^3}{3\,b^3} - \frac{5\,Ba^3 - 3\,Aa^2b + 2\,(3\,Ba^2b - 2\,Aab^2)x^3}{6\,(b^6x^6 + 2\,ab^5x^3 + a^2b^4)} - \frac{(3\,Ba - Ab)\log(bx^3+a)}{3\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/3*B*x^3/b^3 - 1/6*(5*B*a^3 - 3*A*a^2*b + 2*(3*B*a^2*b - 2*A*a*b^2)*x^3)/
(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) - 1/3*(3*B*a - A*b)*log(b*x^3 + a)/b^4
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^3}{3\,b^3} - \frac{(3\,Ba - Ab)\log\left(|bx^3+a|\right)}{3\,b^4} + \frac{9\,Bab^2x^6 - 3\,Ab^3x^6 + 12\,Ba^2bx^3 - 2\,Aab^2x^3 + 4\,Ba^3}{6\,(bx^3+a)^2b^4}$$

input

```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output

```
1/3*B*x^3/b^3 - 1/3*(3*B*a - A*b)*log(abs(b*x^3 + a))/b^4 + 1/6*(9*B*a*b^2
*x^6 - 3*A*b^3*x^6 + 12*B*a^2*b*x^3 - 2*A*a*b^2*x^3 + 4*B*a^3)/((b*x^3 + a
)^2*b^4)
```

3.89.9 Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{B\,x^3}{3\,b^3} - \frac{x^3\left(B\,a^2 - \frac{2\,A\,a\,b}{3}\right) + \frac{5\,B\,a^3 - 3\,A\,a^2\,b}{6\,b}}{a^2\,b^3 + 2\,a\,b^4\,x^3 + b^5\,x^6} + \frac{\ln\left(b\,x^3+a\right)\,\left(A\,b - 3\,B\,a\right)}{3\,b^4}$$

input

```
int((x^8*(A + B*x^3))/(a + b*x^3)^3,x)
```

output

```
(B*x^3)/(3*b^3) - (x^3*(B*a^2 - (2*A*a*b)/3) + (5*B*a^3 - 3*A*a^2*b)/(6*b)
)/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (log(a + b*x^3)*(A*b - 3*B*a))/(3*b^
4)
```

3.90 $\int \frac{x^5\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

3.90.1 Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx=\frac{a(Ab-aB)}{6b^3\left(a+bx^3\right)^2}-\frac{Ab-2aB}{3b^3\left(a+bx^3\right)}+\frac{B\log\left(a+bx^3\right)}{3b^3}$$

output
```
1/6*a*(A*b-B*a)/b^3/(b*x^3+a)^2+1/3*(-A*b+2*B*a)/b^3/(b*x^3+a)+1/3*B*ln(b*
x^3+a)/b^3
```

3.90.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx=\frac{3a^2B-2Ab^2x^3-ab(A-4Bx^3)+2B(a+bx^3)^2\log\left(a+bx^3\right)}{6b^3\left(a+bx^3\right)^2}$$

input
```
Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(3*a^2*B - 2*A*b^2*x^3 - a*b*(A - 4*B*x^3) + 2*B*(a + b*x^3)^2*Log[a + b*x
^3])/(6*b^3*(a + b*x^3)^2)
```

3.90.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^3)}{(a+bx^3)^3} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^3 (Bx^3+A)}{(bx^3+a)^3} dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3} \int \left(\frac{B}{b^2 (bx^3+a)} + \frac{Ab-2aB}{b^2 (bx^3+a)^2} + \frac{a(aB-Ab)}{b^2 (bx^3+a)^3}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{Ab-2aB}{b^3 (a+bx^3)} + \frac{a(Ab-aB)}{2b^3 (a+bx^3)^2} + \frac{B \log (a+bx^3)}{b^3}\right)$$

input
```
Int[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((a*(A*b - a*B))/(2*b^3*(a + b*x^3)^2) - (A*b - 2*a*B)/(b^3*(a + b*x^3)) +
 (B*Log[a + b*x^3])/b^3)/3
```

3.90.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.90.4 Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result	size
norman	$\frac{-\frac{a(Ab-3Ba)}{6b^3}-\frac{(Ab-2Ba)x^3}{3b^2}}{(b\,x^3+a)^2}+\frac{B\ln(b\,x^3+a)}{3b^3}$	57
risch	$\frac{-\frac{a(Ab-3Ba)}{6b^3}-\frac{(Ab-2Ba)x^3}{3b^2}}{(b\,x^3+a)^2}+\frac{B\ln(b\,x^3+a)}{3b^3}$	57
default	$\frac{B\ln(b\,x^3+a)}{3b^3}+\frac{a(Ab-Ba)}{6b^3(b\,x^3+a)^2}-\frac{Ab-2Ba}{3b^3(b\,x^3+a)}$	61
parallelrisch	$-\frac{-2B\ln(b\,x^3+a)x^6b^2-4B\ln(b\,x^3+a)x^3ab+2A\,b^2x^3-4Bab\,x^3-2B\ln(b\,x^3+a)a^2+abA-3a^2B}{6b^3(b\,x^3+a)^2}$	90

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/6*a*(A*b-3*B*a)/b^3-1/3*(A*b-2*B*a)/b^2*x^3)/(b*x^3+a)^2+1/3*B*ln(b*x^
3+a)/b^3
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.35

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$=\frac{2\,(2\,Bab-Ab^2)x^3+3\,Ba^2-Aab+2\,(Bb^2x^6+2\,Babx^3+Ba^2)\log\left(bx^3+a\right)}{6\,(b^5x^6+2\,ab^4x^3+a^2b^3)}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b + 2*(B*b^2*x^6 + 2*B*a*b*x^
3 + B*a^2)*log(b*x^3 + a))/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

3.90.6 Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{B\log(a+bx^3)}{3b^3} + \frac{-Aab+3Ba^2+x^3(-2Ab^2+4Bab)}{6a^2b^3+12ab^4x^3+6b^5x^6}$$

input
```
integrate(x**5*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
B*log(a + b*x**3)/(3*b**3) + (-A*a*b + 3*B*a**2 + x**3*(-2*A*b**2 + 4*B*a*
b))/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\,(2\,Bab-Ab^2)x^3+3\,Ba^2-Aab}{6\,(b^5x^6+2\,ab^4x^3+a^2b^3)} + \frac{B\log(bx^3+a)}{3\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/6*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)/(b^5*x^6 + 2*a*b^4*x^3 + a
^2*b^3) + 1/3*B*log(b*x^3 + a)/b^3
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{B\log(|bx^3+a|)}{3\,b^3} + \frac{2\,(2\,Ba-Ab)x^3+\frac{3\,Ba^2-Aab}{b}}{6\,(bx^3+a)^2b^2}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/3*B*log(abs(b*x^3 + a))/b^3 + 1/6*(2*(2*B*a - A*b)*x^3 + (3*B*a^2 - A*a*
b)/b)/((b*x^3 + a)^2*b^2)
```

3.90.9 Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx=\frac{\frac{3\,B\,a^2-A\,a\,b}{6\,b^3}-\frac{x^3\,(A\,b-2\,B\,a)}{3\,b^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{B\,\ln\left(b\,x^3+a\right)}{3\,b^3}$$

input

```
int((x^5*(A + B*x^3))/(a + b*x^3)^3,x)
```

output

```
((3*B*a^2 - A*a*b)/(6*b^3) - (x^3*(A*b - 2*B*a))/(3*b^2))/(a^2 + b^2*x^6 +
 2*a*b*x^3) + (B*log(a + b*x^3))/(3*b^3)
```

3.91 $\int \frac{x^2\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

3.91.1 Optimal result

Integrand size = 20, antiderivative size = 32

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx=-\frac{(A+Bx^3)^2}{6(Ab-aB)\,(a+bx^3)^2}$$

output
```
-1/6*(B*x^3+A)^2/(A*b-B*a)/(b*x^3+a)^2
```

3.91.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx=-\frac{Ab+B(a+2bx^3)}{6b^2\,(a+bx^3)^2}$$

input
```
Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
-1/6*(A*b + B*(a + 2*b*x^3))/(b^2*(a + b*x^3)^2)
```

3.91.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {946, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}dx$$
$$\Big\downarrow \text{946}$$
$$\frac{1}{3}\int \frac{Bx^3+A}{(bx^3+a)^3}dx^3$$
$$\Big\downarrow \text{48}$$
$$-\frac{(A+Bx^3)^2}{6(a+bx^3)^2(Ab-aB)}$$

input
```
Int[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
-1/6*(A + B*x^3)^2/((A*b - a*B)*(a + b*x^3)^2)
```

3.91.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.91.4 Maple [A] (verified)

Time = 3.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
gosper	$-\frac{2bB\,x^3+Ab+Ba}{6(b\,x^3+a)^2 b^2}$	29
parallelrisch	$-\frac{2bB\,x^3+Ab+Ba}{6(b\,x^3+a)^2 b^2}$	29
norman	$\frac{-\frac{B\,x^3}{3b}-\frac{Ab+Ba}{6b^2}}{(b\,x^3+a)^2}$	33
risch	$\frac{-\frac{B\,x^3}{3b}-\frac{Ab+Ba}{6b^2}}{(b\,x^3+a)^2}$	33
default	$-\frac{Ab-Ba}{6b^2(b\,x^3+a)^2}-\frac{B}{3b^2(b\,x^3+a)}$	39

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*B*b*x^3+A*b+B*a)/(b*x^3+a)^2/b^2
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{2\,Bbx^3+Ba+Ab}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

3.91.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{-Ab-Ba-2Bbx^3}{6a^2b^2+12ab^3x^3+6b^4x^6}$$

input
```
integrate(x**2*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
(-A*b - B*a - 2*B*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{2\,Bbx^3+Ba+Ab}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{2\,Bbx^3+Ba+Ab}{6\,(bx^3+a)^2b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/6*(2*B*b*x^3 + B*a + A*b)/((b*x^3 + a)^2*b^2)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 6.79 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{\frac{A\,b+B\,a}{6\,b^2}+\frac{B\,x^3}{3\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

input
```
int((x^2*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
-((A*b + B*a)/(6*b^2) + (B*x^3)/(3*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

3.92 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx$

3.92.1 Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx=\frac{Ab-aB}{6ab\left(a+bx^3\right)^2}+\frac{A}{3a^2\left(a+bx^3\right)}+\frac{A\log(x)}{a^3}-\frac{A\log\left(a+bx^3\right)}{3a^3}$$

output
```
1/6*(A*b-B*a)/a/b/(b*x^3+a)^2+1/3*A/a^2/(b*x^3+a)+A*ln(x)/a^3-1/3*A*ln(b*x
^3+a)/a^3
```

3.92.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx=\frac{\frac{a\left(3aAb-a^2B+2Ab^2x^3\right)}{b\left(a+bx^3\right)^2}+6A\log(x)-2A\log\left(a+bx^3\right)}{6a^3}$$

input
```
Integrate[(A + B*x^3)/(x*(a + b*x^3)^3),x]
```

output
```
((a*(3*a*A*b - a^2*B + 2*A*b^2*x^3))/(b*(a + b*x^3)^2) + 6*A*Log[x] - 2*A*
Log[a + b*x^3])/(6*a^3)
```

3.92.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \textbf{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^3\left(bx^3+a\right)^3}dx^3$$

$$\Big\downarrow \textbf{86}$$

$$\frac{1}{3}\int \left(-\frac{bA}{a^3\left(bx^3+a\right)}-\frac{bA}{a^2\left(bx^3+a\right)^2}+\frac{A}{a^3x^3}+\frac{aB-Ab}{a\left(bx^3+a\right)^3}\right)dx^3$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{1}{3}\left(-\frac{A\log\left(a+bx^3\right)}{a^3}+\frac{A\log\left(x^3\right)}{a^3}+\frac{A}{a^2\left(a+bx^3\right)}+\frac{Ab-aB}{2ab\left(a+bx^3\right)^2}\right)$$

input
```
Int[(A + B*x^3)/(x*(a + b*x^3)^3),x]
```

output
```
((A*b - a*B)/(2*a*b*(a + b*x^3)^2) + A/(a^2*(a + b*x^3)) + (A*Log[x^3])/a^
3 - (A*Log[a + b*x^3])/a^3)/3
```

3.92.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.92.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

method	result
risch	$\frac{\frac{A b\,x^{3}}{3 a^{2}}+\frac{3 A b -B a}{6 a b}}{\left(b\,x^{3}+a \right)^{2}}+\frac{A \ln \left(x \right)}{a^{3}}-\frac{A \ln \left(b\,x^{3}+a \right)}{3 a^{3}}$
default	$\frac{A \ln \left(x \right)}{a^{3}}-\frac{A \ln \left(b\,x^{3}+a \right)-\frac{a^{2} \left(A b -B a \right)}{2 b \left(b\,x^{3}+a \right)^{2}}-\frac{A a}{b\,x^{3}+a}}{3 a^{3}}$
norman	$\frac{-\frac{\left(2 A b -B a \right) x^{3}}{3 a^{2}}-\frac{b \left(3 A b -B a \right) x^{6}}{6 a^{3}}}{\left(b\,x^{3}+a \right)^{2}}+\frac{A \ln \left(x \right)}{a^{3}}-\frac{A \ln \left(b\,x^{3}+a \right)}{3 a^{3}}$
parallelrisch	$\frac{6 A \ln \left(x \right) x^{6} b^{2}-2 A \ln \left(b\,x^{3}+a \right) x^{6} b^{2}-3 A\,b^{2} x^{6}+B\,x^{6} a b +12 A \ln \left(x \right) x^{3} a b -4 A \ln \left(b\,x^{3}+a \right) x^{3} a b -4 a A b\,x^{3}+2 a^{2} B\,x^{3}+6 a^{2} A \ln \left(x \right)-2}{6 a^{3} \left(b\,x^{3}+a \right)^{2}}$

input
```
int((B*x^3+A)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/3/a^2*A*b*x^3+1/6*(3*A*b-B*a)/a/b)/(b*x^3+a)^2+A*ln(x)/a^3-1/3*A*ln(b*x
^3+a)/a^3
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx$$
$$=\frac{2\,Aab^2x^3-Ba^3+3\,Aa^2b-2\left(Ab^3x^6+2\,Aab^2x^3+Aa^2b\right)\log\left(bx^3+a\right)+6\left(Ab^3x^6+2\,Aab^2x^3+Aa^2b\right)\log}{6\left(a^3b^3x^6+2\,a^4b^2x^3+a^5b\right)}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(2*A*a*b^2*x^3 - B*a^3 + 3*A*a^2*b - 2*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*
a^2*b)*log(b*x^3 + a) + 6*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*log(x))/(a
^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b)
```

3.92.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx=\frac{A\log\left(x\right)}{a^3}-\frac{A\log\left(\frac{a}{b}+x^3\right)}{3a^3}+\frac{3Aab+2Ab^2x^3-Ba^2}{6a^4b+12a^3b^2x^3+6a^2b^3x^6}$$

input
```
integrate((B*x**3+A)/x/(b*x**3+a)**3,x)
```

output
```
A*log(x)/a**3 - A*log(a/b + x**3)/(3*a**3) + (3*A*a*b + 2*A*b**2*x**3 - B*
a**2)/(6*a**4*b + 12*a**3*b**2*x**3 + 6*a**2*b**3*x**6)
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx=\frac{2\,Ab^2x^3-Ba^2+3\,Aab}{6\,\left(a^2b^3x^6+2\,a^3b^2x^3+a^4b\right)}-\frac{A\log\left(bx^3+a\right)}{3\,a^3}+\frac{A\log\left(x^3\right)}{3\,a^3}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/6*(2*A*b^2*x^3 - B*a^2 + 3*A*a*b)/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
- 1/3*A*log(b*x^3 + a)/a^3 + 1/3*A*log(x^3)/a^3
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx=-\frac{A\log\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{A\log\left(\left|x\right|\right)}{a^3}+\frac{3\,Ab^3x^6+8\,Aab^2x^3-Ba^3+6\,Aa^2b}{6\,\left(bx^3+a\right)^2a^3b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/3*A*log(abs(b*x^3 + a))/a^3 + A*log(abs(x))/a^3 + 1/6*(3*A*b^3*x^6 + 8*
A*a*b^2*x^3 - B*a^3 + 6*A*a^2*b)/((b*x^3 + a)^2*a^3*b)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx=\frac{\frac{3\,A\,b-B\,a}{6\,a\,b}+\frac{A\,b\,x^3}{3\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}-\frac{A\,\ln\left(b\,x^3+a\right)}{3\,a^3}+\frac{A\,\ln\left(x\right)}{a^3}$$

input
```
int((A + B*x^3)/(x*(a + b*x^3)^3),x)
```

output
```
((3*A*b - B*a)/(6*a*b) + (A*b*x^3)/(3*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) -
(A*log(a + b*x^3))/(3*a^3) + (A*log(x))/a^3
```

3.93 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx$

3.93.1 Optimal result

Integrand size = 20, antiderivative size = 101

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx=-\frac{A}{3a^3x^3}-\frac{Ab-aB}{6a^2\left(a+bx^3\right)^2}-\frac{2Ab-aB}{3a^3\left(a+bx^3\right)}-\frac{(3Ab-aB)\log(x)}{a^4}+\frac{(3Ab-aB)\log\left(a+bx^3\right)}{3a^4}$$

output

```
-1/3*A/a^3/x^3+1/6*(-A*b+B*a)/a^2/(b*x^3+a)^2+1/3*(-2*A*b+B*a)/a^3/(b*x^3+
a)-(3*A*b-B*a)*ln(x)/a^4+1/3*(3*A*b-B*a)*ln(b*x^3+a)/a^4
```

3.93.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx=\frac{-\frac{2aA}{x^3}+\frac{a^2(-Ab+aB)}{\left(a+bx^3\right)^2}+\frac{2a(-2Ab+aB)}{a+bx^3}+6(-3Ab+aB)\log(x)+2(3Ab-aB)\log\left(a+bx^3\right)}{6a^4}$$

input

```
Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^3),x]
```

output

```
((-2*a*A)/x^3 + (a^2*(-(A*b) + a*B))/(a + b*x^3)^2 + (2*a*(-2*A*b + a*B))/
(a + b*x^3) + 6*(-3*A*b + a*B)*Log[x] + 2*(3*A*b - a*B)*Log[a + b*x^3])/(6
*a^4)
```

3.93.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^6\left(bx^3+a\right)^3}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{A}{a^3x^6}-\frac{b(aB-3Ab)}{a^4\left(bx^3+a\right)}-\frac{b(aB-2Ab)}{a^3\left(bx^3+a\right)^2}+\frac{aB-3Ab}{a^4x^3}-\frac{b(aB-Ab)}{a^2\left(bx^3+a\right)^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\log\left(x^3\right)(3Ab-aB)}{a^4}+\frac{(3Ab-aB)\log\left(a+bx^3\right)}{a^4}-\frac{2Ab-aB}{a^3\left(a+bx^3\right)}-\frac{A}{a^3x^3}-\frac{Ab-aB}{2a^2\left(a+bx^3\right)^2}\right)$$

input
```
Int[(A + B*x^3)/(x^4*(a + b*x^3)^3),x]
```

output
```
(-(A/(a^3*x^3)) - (A*b - a*B)/(2*a^2*(a + b*x^3)^2) - (2*A*b - a*B)/(a^3*(
a + b*x^3)) - ((3*A*b - a*B)*Log[x^3])/a^4 + ((3*A*b - a*B)*Log[a + b*x^3]
)/a^4)/3
```

3.93.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.93.4 Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

method	result
norman	$\frac{-\frac{A}{3a}+\frac{2b(3Ab-Ba)x^6}{3a^3}+\frac{b^2(3Ab-Ba)x^9}{2a^4}}{x^3(b\,x^3+a)^2}-\frac{(3Ab-Ba)\ln(x)}{a^4}+\frac{(3Ab-Ba)\ln\left(b\,x^3+a\right)}{3a^4}$
default	$-\frac{A}{3a^3x^3}+\frac{(-3Ab+Ba)\ln(x)}{a^4}+\frac{b\left(\frac{(3Ab-Ba)\ln\left(b\,x^3+a\right)}{b}-\frac{a^2(Ab-Ba)}{2b\left(b\,x^3+a\right)^2}-\frac{a(2Ab-Ba)}{b\left(b\,x^3+a\right)}\right)}{3a^4}$
risch	$\frac{-\frac{b(3Ab-Ba)x^6}{3a^3}-\frac{(3Ab-Ba)x^3}{2a^2}-\frac{A}{3a}}{x^3(b\,x^3+a)^2}-\frac{3\ln(x)Ab}{a^4}+\frac{B\ln(x)}{a^3}+\frac{\ln\left(-b\,x^3-a\right)Ab}{a^4}-\frac{\ln\left(-b\,x^3-a\right)B}{3a^3}$
parallelrisch	$-\frac{18A\ln(x)x^9b^3-6A\ln\left(b\,x^3+a\right)x^9b^3-6B\ln(x)x^9a\,b^2+2B\ln\left(b\,x^3+a\right)x^9a\,b^2-9A\,x^9b^3+3B\,x^9a\,b^2+36A\ln(x)x^6a\,b^2-12A\ln(}{}$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*A/a+2/3*b*(3*A*b-B*a)/a^3*x^6+1/2*b^2*(3*A*b-B*a)/a^4*x^9)/x^3/(b*x^
3+a)^2-(3*A*b-B*a)*ln(x)/a^4+1/3*(3*A*b-B*a)*ln(b*x^3+a)/a^4
```

3.93.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(89) = 178.

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.95

$$\int\frac{A+Bx^3}{x^4\,(a+bx^3)^3}\,dx$$
$$=\frac{2\,(Ba^2b-3\,Aab^2)x^6-2\,Aa^3+3\,(Ba^3-3\,Aa^2b)x^3-2\,((Bab^2-3\,Ab^3)x^9+2\,(Ba^2b-3\,Aab^2)x^6+(Ba^3}{6\,(a^4b^2x^9+2\,a^5}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(2*(B*a^2*b - 3*A*a*b^2)*x^6 - 2*A*a^3 + 3*(B*a^3 - 3*A*a^2*b)*x^3 - 2
*((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*b^2)*x^6 + (B*a^3 - 3*A*a^2
*b)*x^3)*log(b*x^3 + a) + 6*((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*
b^2)*x^6 + (B*a^3 - 3*A*a^2*b)*x^3)*log(x))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a
^6*x^3)
```

3.93.6 Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx = \frac{-2Aa^2+x^6(-6Ab^2+2Bab)+x^3(-9Aab+3Ba^2)}{6a^5x^3+12a^4bx^6+6a^3b^2x^9} + \frac{(-3Ab+Ba)\log{(x)}}{a^4} - \frac{(-3Ab+Ba)\log{\left(\frac{a}{b}+x^3\right)}}{3a^4}$$

input
```
integrate((B*x**3+A)/x**4/(b*x**3+a)**3,x)
```

output
```
(-2*A*a**2 + x**6*(-6*A*b**2 + 2*B*a*b) + x**3*(-9*A*a*b + 3*B*a**2))/(6*a
**5*x**3 + 12*a**4*b*x**6 + 6*a**3*b**2*x**9) + (-3*A*b + B*a)*log(x)/a**4
 - (-3*A*b + B*a)*log(a/b + x**3)/(3*a**4)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx = \frac{2\,(Bab-3\,Ab^2)x^6+3\,(Ba^2-3\,Aab)x^3-2\,Aa^2}{6\,(a^3b^2x^9+2\,a^4bx^6+a^5x^3)} - \frac{(Ba-3\,Ab)\log{(bx^3+a)}}{3\,a^4} + \frac{(Ba-3\,Ab)\log{(x^3)}}{3\,a^4}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/6*(2*(B*a*b - 3*A*b^2)*x^6 + 3*(B*a^2 - 3*A*a*b)*x^3 - 2*A*a^2)/(a^3*b^2
*x^9 + 2*a^4*b*x^6 + a^5*x^3) - 1/3*(B*a - 3*A*b)*log(b*x^3 + a)/a^4 + 1/3
*(B*a - 3*A*b)*log(x^3)/a^4
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx = \frac{(Ba-3\,Ab)\log\left(|x|\right)}{a^4} - \frac{\left(Bab-3\,Ab^2\right)\log\left(|bx^3+a|\right)}{3\,a^4b} + \frac{3\,Bab^2x^6-9\,Ab^3x^6+8\,Ba^2bx^3-22\,Aab^2x^3+6\,Ba^3-14\,Aa^2b}{6\left(bx^3+a\right)^2a^4} - \frac{Bax^3-3\,Abx^3+Aa}{3\,a^4x^3}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
(B*a - 3*A*b)*log(abs(x))/a^4 - 1/3*(B*a*b - 3*A*b^2)*log(abs(b*x^3 + a))/
(a^4*b) + 1/6*(3*B*a*b^2*x^6 - 9*A*b^3*x^6 + 8*B*a^2*b*x^3 - 22*A*a*b^2*x^
3 + 6*B*a^3 - 14*A*a^2*b)/((b*x^3 + a)^2*a^4) - 1/3*(B*a*x^3 - 3*A*b*x^3 +
 A*a)/(a^4*x^3)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 6.90 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx = \frac{\ln\left(b\,x^3+a\right)\,\left(3\,A\,b-B\,a\right)}{3\,a^4} - \frac{\frac{A}{3\,a}+\frac{x^3\,\left(3\,A\,b-B\,a\right)}{2\,a^2}+\frac{b\,x^6\,\left(3\,A\,b-B\,a\right)}{3\,a^3}}{a^2\,x^3+2\,a\,b\,x^6+b^2\,x^9} - \frac{\ln\left(x\right)\,\left(3\,A\,b-B\,a\right)}{a^4}$$

input
```
int((A + B*x^3)/(x^4*(a + b*x^3)^3),x)
```

output
```
(log(a + b*x^3)*(3*A*b - B*a))/(3*a^4) - (A/(3*a) + (x^3*(3*A*b - B*a))/(2
*a^2) + (b*x^6*(3*A*b - B*a))/(3*a^3))/(a^2*x^3 + b^2*x^9 + 2*a*b*x^6) - (
log(x)*(3*A*b - B*a))/a^4
```

3.94 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx$

3.94.1 Optimal result

Integrand size = 20, antiderivative size = 122

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx=-\frac{A}{6a^3x^6}+\frac{3Ab-aB}{3a^4x^3}+\frac{b(Ab-aB)}{6a^3\left(a+bx^3\right)^2}+\frac{b(3Ab-2aB)}{3a^4\left(a+bx^3\right)}+\frac{3b(2Ab-aB)\log(x)}{a^5}-\frac{b(2Ab-aB)\log\left(a+bx^3\right)}{a^5}$$

output
```
-1/6*A/a^3/x^6+1/3*(3*A*b-B*a)/a^4/x^3+1/6*b*(A*b-B*a)/a^3/(b*x^3+a)^2+1/3
*b*(3*A*b-2*B*a)/a^4/(b*x^3+a)+3*b*(2*A*b-B*a)*ln(x)/a^5-b*(2*A*b-B*a)*ln(
b*x^3+a)/a^5
```

3.94.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx=\frac{-\frac{a^2A}{x^6}-\frac{2a(-3Ab+aB)}{x^3}+\frac{a^2b(Ab-aB)}{\left(a+bx^3\right)^2}+\frac{2ab(3Ab-2aB)}{a+bx^3}+18b(2Ab-aB)\log(x)+6b(-2Ab+aB)\log\left(a+bx^3\right)}{6a^5}$$

input
```
Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^3),x]
```

output
```
(-((a^2*A)/x^6) - (2*a*(-3*A*b + a*B))/x^3 + (a^2*b*(A*b - a*B))/(a + b*x^
3)^2 + (2*a*b*(3*A*b - 2*a*B))/(a + b*x^3) + 18*b*(2*A*b - a*B)*Log[x] + 6
*b*(-2*A*b + a*B)*Log[a + b*x^3])/(6*a^5)
```

3.94.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^9\left(bx^3+a\right)^3}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{3(aB-2Ab)b^2}{a^5\left(bx^3+a\right)}+\frac{(2aB-3Ab)b^2}{a^4\left(bx^3+a\right)^2}+\frac{(aB-Ab)b^2}{a^3\left(bx^3+a\right)^3}-\frac{3(aB-2Ab)b}{a^5x^3}+\frac{aB-3Ab}{a^4x^6}+\frac{A}{a^3x^9}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3b\log\left(x^3\right)(2Ab-aB)}{a^5}-\frac{3b(2Ab-aB)\log\left(a+bx^3\right)}{a^5}+\frac{b(3Ab-2aB)}{a^4\left(a+bx^3\right)}+\frac{3Ab-aB}{a^4x^3}+\frac{b(Ab-aB)}{2a^3\left(a+bx^3\right)^2}-\frac{A}{2a^3x}\right.$$

input
```
Int[(A + B*x^3)/(x^7*(a + b*x^3)^3),x]
```

output
```
(-1/2*A/(a^3*x^6) + (3*A*b - a*B)/(a^4*x^3) + (b*(A*b - a*B))/(2*a^3*(a +
b*x^3)^2) + (b*(3*A*b - 2*a*B))/(a^4*(a + b*x^3)) + (3*b*(2*A*b - a*B)*Log
[x^3])/a^5 - (3*b*(2*A*b - a*B)*Log[a + b*x^3])/a^5)/3
```

3.94.3.1 Defintions of rubi rules used

rule 86

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.94.4 Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01

method	result
default	$-\frac{A}{6a^3x^6}-\frac{-3Ab+Ba}{3a^4x^3}+\frac{3b(2Ab-Ba)\ln(x)}{a^5}-\frac{b^2\left(\frac{(6Ab-3Ba)\ln\left(b\,x^3+a\right)}{b}-\frac{a^2(Ab-Ba)}{2b\left(b\,x^3+a\right)^2}-\frac{a(3Ab-2Ba)}{b\left(b\,x^3+a\right)}\right)}{3a^5}$
norman	$\frac{-\frac{A}{6a}+\frac{(2Ab-Ba)x^3}{3a^2}-\frac{2b\left(2b^2A-abB\right)x^9}{a^4}-\frac{b^2\left(6b^2A-3abB\right)x^{12}}{2a^5}}{x^6(b\,x^3+a)^2}+\frac{3b(2Ab-Ba)\ln(x)}{a^5}-\frac{b(2Ab-Ba)\ln(b\,x^3+a)}{a^5}$
risch	$\frac{\frac{b^2(2Ab-Ba)x^9}{a^4}+\frac{3b(2Ab-Ba)x^6}{2a^3}+\frac{(2Ab-Ba)x^3}{3a^2}-\frac{A}{6a}}{x^6(b\,x^3+a)^2}+\frac{6b^2\ln(x)A}{a^5}-\frac{3b\ln(x)B}{a^4}-\frac{2b^2\ln(b\,x^3+a)A}{a^5}+\frac{b\ln(b\,x^3+a)B}{a^4}$
parallelrisch	$\frac{36A\ln(x)x^{12}b^4-12A\ln(b\,x^3+a)x^{12}b^4-18B\ln(x)x^{12}a\,b^3+6B\ln(b\,x^3+a)x^{12}a\,b^3-18A\,x^{12}b^4+9B\,x^{12}a\,b^3+72A\ln(x)x^9a\,b^3-}{}$

input

```
int((B*x^3+A)/x^7/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/6*A/a^3/x^6-1/3*(-3*A*b+B*a)/a^4/x^3+3*b*(2*A*b-B*a)*ln(x)/a^5-1/3/a^5*
b^2*((6*A*b-3*B*a)/b*ln(b*x^3+a)-1/2*a^2*(A*b-B*a)/b/(b*x^3+a)^2-a*(3*A*b-
2*B*a)/b/(b*x^3+a))
```

3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(110) = 220$.

Time = 0.26 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx = -\frac{6\left(Ba^2b^2-2\,Aab^3\right)x^9+9\left(Ba^3b-2\,Aa^2b^2\right)x^6+Aa^4+2\left(Ba^4-2\,Aa^3b\right)x^3-6\left(\left(Bab^3-2\,Ab^4\right)x^{12}+2\right.}{}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/6*(6*(B*a^2*b^2 - 2*A*a*b^3)*x^9 + 9*(B*a^3*b - 2*A*a^2*b^2)*x^6 + A*a^
4 + 2*(B*a^4 - 2*A*a^3*b)*x^3 - 6*((B*a*b^3 - 2*A*b^4)*x^12 + 2*(B*a^2*b^2
 - 2*A*a*b^3)*x^9 + (B*a^3*b - 2*A*a^2*b^2)*x^6)*log(b*x^3 + a) + 18*((B*a
*b^3 - 2*A*b^4)*x^12 + 2*(B*a^2*b^2 - 2*A*a*b^3)*x^9 + (B*a^3*b - 2*A*a^2*
b^2)*x^6)*log(x))/(a^5*b^2*x^12 + 2*a^6*b*x^9 + a^7*x^6)
```

3.94.6 Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx &= \frac{-Aa^3+x^9\cdot\left(12Ab^3-6Bab^2\right)+x^6\cdot\left(18Aab^2-9Ba^2b\right)+x^3\cdot\left(4Aa^2b-2Ba^3\right)}{6a^6x^6+12a^5bx^9+6a^4b^2x^{12}}\\ &\quad-\frac{3b(-2Ab+Ba)\log{(x)}}{a^5}+\frac{b(-2Ab+Ba)\log{\left(\frac{a}{b}+x^3\right)}}{a^5}\end{aligned}$$

input
```
integrate((B*x**3+A)/x**7/(b*x**3+a)**3,x)
```

output
```
(-A*a**3 + x**9*(12*A*b**3 - 6*B*a*b**2) + x**6*(18*A*a*b**2 - 9*B*a**2*b)
 + x**3*(4*A*a**2*b - 2*B*a**3))/(6*a**6*x**6 + 12*a**5*b*x**9 + 6*a**4*b*
*2*x**12) - 3*b*(-2*A*b + B*a)*log(x)/a**5 + b*(-2*A*b + B*a)*log(a/b + x*
*3)/a**5
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx = -\frac{6\,(Bab^2-2\,Ab^3)x^9+9\,(Ba^2b-2\,Aab^2)x^6+Aa^3+2\,(Ba^3-2\,Aa^2b)x^3}{6\,(a^4b^2x^{12}+2\,a^5bx^9+a^6x^6)} + \frac{(Bab-2\,Ab^2)\log\left(bx^3+a\right)}{a^5} - \frac{(Bab-2\,Ab^2)\log\left(x^3\right)}{a^5}$$

input

```
integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="maxima")
```

output

```
-1/6*(6*(B*a*b^2 - 2*A*b^3)*x^9 + 9*(B*a^2*b - 2*A*a*b^2)*x^6 + A*a^3 + 2*
(B*a^3 - 2*A*a^2*b)*x^3)/(a^4*b^2*x^12 + 2*a^5*b*x^9 + a^6*x^6) + (B*a*b -
 2*A*b^2)*log(b*x^3 + a)/a^5 - (B*a*b - 2*A*b^2)*log(x^3)/a^5
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx = -\frac{3\,(Bab-2\,Ab^2)\log\left(|x|\right)}{a^5} + \frac{(Bab^2-2\,Ab^3)\log\left(|bx^3+a|\right)}{a^5b} - \frac{6\,Bab^2x^9-12\,Ab^3x^9+9\,Ba^2bx^6-18\,Aab^2x^6+2\,Ba^3x^3-4\,Aa^2bx^3+Aa^3}{6\,\left(bx^6+ax^3\right)^2a^4}$$

input

```
integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="giac")
```

output

```
-3*(B*a*b - 2*A*b^2)*log(abs(x))/a^5 + (B*a*b^2 - 2*A*b^3)*log(abs(b*x^3 +
 a))/(a^5*b) - 1/6*(6*B*a*b^2*x^9 - 12*A*b^3*x^9 + 9*B*a^2*b*x^6 - 18*A*a*
b^2*x^6 + 2*B*a^3*x^3 - 4*A*a^2*b*x^3 + A*a^3)/((b*x^6 + a*x^3)^2*a^4)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx = \frac{\frac{x^3\,(2\,A\,b-B\,a)}{3\,a^2}-\frac{A}{6\,a}+\frac{b^2\,x^9\,(2\,A\,b-B\,a)}{a^4}+\frac{3\,b\,x^6\,(2\,A\,b-B\,a)}{2\,a^3}}{a^2\,x^6+2\,a\,b\,x^9+b^2\,x^{12}} - \frac{\ln\left(b\,x^3+a\right)\,\left(2\,A\,b^2-B\,a\,b\right)}{a^5}+\frac{\ln\left(x\right)\,\left(6\,A\,b^2-3\,B\,a\,b\right)}{a^5}$$

input

```
int((A + B*x^3)/(x^7*(a + b*x^3)^3),x)
```

output

```
((x^3*(2*A*b - B*a))/(3*a^2) - A/(6*a) + (b^2*x^9*(2*A*b - B*a))/a^4 + (3*
b*x^6*(2*A*b - B*a))/(2*a^3))/(a^2*x^6 + b^2*x^12 + 2*a*b*x^9) - (log(a +
b*x^3)*(2*A*b^2 - B*a*b))/a^5 + (log(x)*(6*A*b^2 - 3*B*a*b))/a^5
```

3.95 $\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.95.1 Optimal result

Integrand size = 20, antiderivative size = 246

$$\begin{aligned}\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{2(5Ab-11aB)x^2}{9b^4}-\frac{4(5Ab-11aB)x^5}{45ab^3}+\frac{(Ab-aB)x^{11}}{6ab\,(a+bx^3)^2}\\ &\quad+\frac{(5Ab-11aB)x^8}{18ab^2\,(a+bx^3)}+\frac{4a^{2/3}(5Ab-11aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{14/3}}\\ &\quad+\frac{4a^{2/3}(5Ab-11aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{14/3}}\\ &\quad-\frac{2a^{2/3}(5Ab-11aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27b^{14/3}}\end{aligned}$$

output
```
2/9*(5*A*b-11*B*a)*x^2/b^4-4/45*(5*A*b-11*B*a)*x^5/a/b^3+1/6*(A*b-B*a)*x^1
1/a/b/(b*x^3+a)^2+1/18*(5*A*b-11*B*a)*x^8/a/b^2/(b*x^3+a)+4/27*a^(2/3)*(5*
A*b-11*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(14/3)-2/27*a^(2/3)*(5*A*b-11*B*a)*ln(
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(14/3)+4/27*a^(2/3)*(5*A*b-11*B*a
)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(14/3)*3^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.88

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{135b^{2/3}(Ab-3aB)x^2+54b^{5/3}Bx^5+\frac{45a^2b^{2/3}(-Ab+aB)x^2}{(a+bx^3)^2}+\frac{30ab^{2/3}(7Ab-10aB)x^2}{a+bx^3}-40\sqrt{3}a^{2/3}(-5Ab+11aB)\,\mathrm{a}}{}$$

input
```
Integrate[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(135*b^(2/3)*(A*b - 3*a*B)*x^2 + 54*b^(5/3)*B*x^5 + (45*a^2*b^(2/3)*(-(A*b
) + a*B)*x^2)/(a + b*x^3)^2 + (30*a*b^(2/3)*(7*A*b - 10*a*B)*x^2)/(a + b*x
^3) - 40*Sqrt[3]*a^(2/3)*(-5*A*b + 11*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] - 40*a^(2/3)*(-5*A*b + 11*a*B)*Log[a^(1/3) + b^(1/3)*x] + 20*
a^(2/3)*(-5*A*b + 11*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(270*b^(14/3))
```

3.95.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {957, 817, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{11}(Ab-aB)}{6ab\,(a+bx^3)^2}-\frac{(5Ab-11aB)\int \frac{x^{10}}{(bx^3+a)^2}dx}{6ab}$$

$$\Big\downarrow \text{817}$$

$$\frac{x^{11}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(5Ab-11aB)\left(\frac{8\int \frac{x^7}{bx^3+a}dx}{3b}-\frac{x^8}{3b(a+bx^3)}\right)}{6ab}$$

$$\Big\downarrow \text{831}$$

$$\frac{x^{11}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(5Ab-11aB)\left(\frac{8\int \left(\frac{x^4}{b}+\frac{a^2x}{b^2\left(bx^3+a\right)}-\frac{ax}{b^2}\right)dx}{3b}-\frac{x^8}{3b(a+bx^3)}\right)}{6ab}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{11}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(5Ab-11aB)\left(\frac{8\left(-\frac{a^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}}+\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}}-\frac{ax^2}{2b^2}+\frac{x^5}{5b}\right)}{3b}-\frac{x^8}{3b(a+bx^3)}\right)}{6ab}$$

input
```
Int[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^11)/(6*a*b*(a + b*x^3)^2) - ((5*A*b - 11*a*B)*(-1/3*x^8/(b*
(a + b*x^3)) + (8*(-1/2*(a*x^2)/b^2 + x^5/(5*b) - (a^(5/3)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) - (a^(5/3)*Log[a^(1/
3) + b^(1/3)*x])/(3*b^(8/3)) + (a^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(6*b^(8/3))))/(3*b)))/(6*a*b)
```

3.95.3.1 Defintions of rubi rules used

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.95.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.47

method	result
risch	$\frac{B\,x^{5}}{5b^{3}}+\frac{A\,x^{2}}{2b^{3}}-\frac{3Ba\,x^{2}}{2b^{4}}+\frac{\left(\frac{7}{9}a\,b^{2}A-\frac{10}{9}a^{2}bB\right)x^{5}+\frac{a^{2}(11Ab-17Ba)x^{2}}{18}}{b^{4}(b\,x^{3}+a)^{2}}-\frac{4a\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{(5Ab-11Ba)\ln\left(x-\_R\right)}{\_R}\right)}{27b^{5}}$
default	$\frac{\frac{bB\,x^{5}}{5}+\frac{(Ab-3Ba)x^{2}}{2}}{b^{4}}-\frac{a\left(\frac{\left(-\frac{7}{9}b^{2}A+\frac{10}{9}abB\right)x^{5}-\frac{a(11Ab-17Ba)x^{2}}{18}}{\left(b\,x^{3}+a\right)^{2}}+\left(\frac{20Ab}{9}-\frac{44Ba}{9}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\sqrt{3}\right.\right.}{b^{4}}$

input
```
int(x^10*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*B*x^5/b^3+1/2/b^3*A*x^2-3/2/b^4*B*a*x^2+((7/9*a*b^2*A-10/9*a^2*b*B)*x^
5+1/18*a^2*(11*A*b-17*B*a)*x^2)/b^4/(b*x^3+a)^2-4/27/b^5*a*sum((5*A*b-11*B
*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.48

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{54\,Bb^3x^{11} - 27\,(11\,Bab^2 - 5\,Ab^3)x^8 - 96\,(11\,Ba^2b - 5\,Aab^2)x^5 - 60\,(11\,Ba^3 - 5\,Aa^2b)x^2 + 40\,\sqrt{3}((11\,B}{}$$

input
```
integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/270*(54*B*b^3*x^11 - 27*(11*B*a*b^2 - 5*A*b^3)*x^8 - 96*(11*B*a^2*b - 5*
A*a*b^2)*x^5 - 60*(11*B*a^3 - 5*A*a^2*b)*x^2 + 40*sqrt(3)*((11*B*a*b^2 - 5
*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a*b^2)*x^3)*(a^2/
b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 20*
((11*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a
*b^2)*x^3)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(
1/3)) - 40*((11*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^
2*b - 5*A*a*b^2)*x^3)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/(b^6*x
^6 + 2*a*b^5*x^3 + a^2*b^4)
```

3.95.6 Sympy [A] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.78

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{Bx^5}{5b^3} + x^2\left(\frac{A}{2b^3} - \frac{3Ba}{2b^4}\right) + \frac{x^5\cdot(14Aab^2 - 20Ba^2b) + x^2\cdot(11Aa^2b - 17Ba^3)}{18a^2b^4 + 36ab^5x^3 + 18b^6x^6}$$

$$+\mathrm{RootSum}\left(19683t^3b^{14} - 8000A^3a^2b^3 + 52800A^2Ba^3b^2 - 116160AB^2a^4b + 85184B^3a^5, \left(t \mapsto t\log\left(\frac{}{40}\right.\right.\right.$$

input

```
integrate(x**10*(B*x**3+A)/(b*x**3+a)**3,x)
```

output

```
B*x**5/(5*b**3) + x**2*(A/(2*b**3) - 3*B*a/(2*b**4)) + (x**5*(14*A*a*b**2
- 20*B*a**2*b) + x**2*(11*A*a**2*b - 17*B*a**3))/(18*a**2*b**4 + 36*a*b**5
*x**3 + 18*b**6*x**6) + RootSum(19683*_t**3*b**14 - 8000*A**3*a**2*b**3 +
52800*A**2*B*a**3*b**2 - 116160*A*B**2*a**4*b + 85184*B**3*a**5, Lambda(_t
, _t*log(729*_t**2*b**9/(400*A**2*a*b**2 - 1760*A*B*a**2*b + 1936*B**2*a**
3) + x)))
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.93

$$\int \frac{x^{10}(A+Bx^3)}{\left(a+bx^3\right)^3}\,dx = -\frac{2\,(10\,Ba^2b-7\,Aab^2)x^5+(17\,Ba^3-11\,Aa^2b)x^2}{18\,(b^6x^6+2\,ab^5x^3+a^2b^4)} + \frac{4\,\sqrt{3}(11\,Ba^2-5\,Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^5\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2\,Bbx^5-5\,(3\,Ba-Ab)x^2}{10\,b^4} + \frac{2\,(11\,Ba^2-5\,Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,b^5\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{4\,(11\,Ba^2-5\,Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^5\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input

```
integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output

```
-1/18*(2*(10*B*a^2*b - 7*A*a*b^2)*x^5 + (17*B*a^3 - 11*A*a^2*b)*x^2)/(b^6*
x^6 + 2*a*b^5*x^3 + a^2*b^4) + 4/27*sqrt(3)*(11*B*a^2 - 5*A*a*b)*arctan(1/
3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(1/3)) + 1/10*(2*B*b
*x^5 - 5*(3*B*a - A*b)*x^2)/b^4 + 2/27*(11*B*a^2 - 5*A*a*b)*log(x^2 - x*(a
/b)^(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(1/3)) - 4/27*(11*B*a^2 - 5*A*a*b)*log
(x + (a/b)^(1/3))/(b^5*(a/b)^(1/3))
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.05

$$
\begin{aligned}
\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx = &-\frac{4\left(11\,Ba^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,Aab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^4} \\
&-\frac{4\sqrt{3}\left(11\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^6} \\
&+\frac{2\left(11\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,b^6} \\
&-\frac{20\,Ba^2bx^5-14\,Aab^2x^5+17\,Ba^3x^2-11\,Aa^2bx^2}{18\left(bx^3+a\right)^2b^4} \\
&+\frac{2\,Bb^{12}x^5-15\,Bab^{11}x^2+5\,Ab^{12}x^2}{10\,b^{15}}
\end{aligned}
$$

input
```
integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-4/27*(11*B*a^2*(-a/b)^(1/3) - 5*A*a*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(
x - (-a/b)^(1/3)))/(a*b^4) - 4/27*sqrt(3)*(11*(-a*b^2)^(2/3)*B*a - 5*(-a*b
^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^6 +
 2/27*(11*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/b^6 - 1/18*(20*B*a^2*b*x^5 - 14*A*a*b^2*x^5 + 17*B*a^3
*x^2 - 11*A*a^2*b*x^2)/((b*x^3 + a)^2*b^4) + 1/10*(2*B*b^12*x^5 - 15*B*a*b
^11*x^2 + 5*A*b^12*x^2)/b^15
```

3.95.9 Mupad [B] (verification not implemented)

Time = 7.08 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.87

$$\begin{aligned}
&\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx \\
&= \frac{x^5\left(\frac{7\,A\,a\,b^2}{9}-\frac{10\,B\,a^2\,b}{9}\right)-x^2\left(\frac{17\,B\,a^3}{18}-\frac{11\,A\,a^2\,b}{18}\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x^2\left(\frac{A}{2\,b^3}-\frac{3\,B\,a}{2\,b^4}\right) \\
&\quad+\frac{B\,x^5}{5\,b^3}+\frac{4\,a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(5\,A\,b-11\,B\,a\right)}{27\,b^{14/3}} \\
&\quad+\frac{4\,a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,A\,b-11\,B\,a\right)}{27\,b^{14/3}} \\
&\quad-\frac{4\,a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,A\,b-11\,B\,a\right)}{27\,b^{14/3}}
\end{aligned}$$

input
```
int((x^10*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(x^5*((7*A*a*b^2)/9 - (10*B*a^2*b)/9) - x^2*((17*B*a^3)/18 - (11*A*a^2*b)/
18))/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + x^2*(A/(2*b^3) - (3*B*a)/(2*b^4))
 + (B*x^5)/(5*b^3) + (4*a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*A*b - 11*B*a))
/(27*b^(14/3)) + (4*a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(5*A*b - 11*B*a))/(27*b^(14/3)) - (4*a^(2/3)*log(
3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b
- 11*B*a))/(27*b^(14/3))
```

3.96 $\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.96.1 Optimal result

Integrand size = 20, antiderivative size = 244

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)}+\frac{7\sqrt[3]{a}(2Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{13/3}}-\frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{13/3}}+\frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{13/3}}$$

output
```
7/9*(2*A*b-5*B*a)*x/b^4-7/36*(2*A*b-5*B*a)*x^4/a/b^3+1/6*(A*b-B*a)*x^10/a/
b/(b*x^3+a)^2+1/9*(2*A*b-5*B*a)*x^7/a/b^2/(b*x^3+a)-7/27*a^(1/3)*(2*A*b-5*
B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(13/3)+7/54*a^(1/3)*(2*A*b-5*B*a)*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(13/3)+7/27*a^(1/3)*(2*A*b-5*B*a)*arctan(
1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(13/3)*3^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.86

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{108\sqrt[3]{b}(Ab-3aB)x+27b^{4/3}Bx^4+\frac{18a^2\sqrt[3]{b}(-Ab+aB)x}{(a+bx^3)^2}+\frac{6a\sqrt[3]{b}(13Ab-19aB)x}{a+bx^3}-28\sqrt{3}\sqrt[3]{a}(-2Ab+5aB)\,\mathrm{arcta}}{10}$$

input
```
Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(108*b^(1/3)*(A*b - 3*a*B)*x + 27*b^(4/3)*B*x^4 + (18*a^2*b^(1/3)*(-(A*b)
+ a*B)*x)/(a + b*x^3)^2 + (6*a*b^(1/3)*(13*A*b - 19*a*B)*x)/(a + b*x^3) -
28*Sqrt[3]*a^(1/3)*(-2*A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] + 28*a^(1/3)*(-2*A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x] - 14*a^(1/3)*
(-2*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(108*b^(1
3/3))
```

3.96.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {957, 817, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{10}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(2Ab-5aB)\int \frac{x^9}{\left(bx^3+a\right)^2}dx}{3ab}$$

$$\Big\downarrow \text{817}$$

$$\frac{x^{10}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(2Ab-5aB)\left(\frac{7\int\frac{x^6}{bx^3+a}dx}{3b}-\frac{x^7}{3b(a+bx^3)}\right)}{3ab}$$

$$\Big\downarrow \text{831}$$

$$\frac{x^{10}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(2Ab-5aB)\left(\frac{7\int\left(\frac{x^3}{b}+\frac{a^2}{b^2\left(bx^3+a\right)}-\frac{a}{b^2}\right)dx}{3b}-\frac{x^7}{3b(a+bx^3)}\right)}{3ab}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{10}(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(2Ab-5aB)\left(\frac{7\left(-\frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}}-\frac{ax}{b^2}+\frac{x^4}{4b}\right)}{3b}-\frac{x^7}{3b(a+bx^3)}\right)}{3ab}$$

input
```
Int[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^10)/(6*a*b*(a + b*x^3)^2) - ((2*A*b - 5*a*B)*(-1/3*x^7/(b*(
a + b*x^3)) + (7*(-((a*x)/b^2) + x^4/(4*b) - (a^(4/3)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) + (a^(4/3)*Log[a^(1/3) +
b^(1/3)*x])/(3*b^(7/3)) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(6*b^(7/3))))/(3*b)))/(3*a*b)
```

3.96.3.1 Defintions of rubi rules used

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.96.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.45

method	result
risch	$\frac{Bx^4}{4b^3}+\frac{Ax}{b^3}-\frac{3Bax}{b^4}+\frac{\left(\frac{13}{18}ab^2A-\frac{19}{18}a^2bB\right)x^4+\frac{a^2(5Ab-8Ba)x}{9}}{b^4(bx^3+a)^2}-\frac{7a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(2Ab-5Ba)\ln\left(x-\_R\right)}{\_R^2}\right)}{27b^5}$
default	$\frac{\frac{1}{4}bBx^4+Abx-3Bax}{b^4}-\frac{a\left(\frac{\left(-\frac{13}{18}b^2A+\frac{19}{18}abB\right)x^4-\frac{a(5Ab-8Ba)x}{9}}{\left(bx^3+a\right)^2}+\frac{7(2Ab-5Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan}{}\right.}{9}\right.}{b^4}$

input
```
int(x^9*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*B*x^4/b^3+1/b^3*A*x-3/b^4*B*a*x+((13/18*a*b^2*A-19/18*a^2*b*B)*x^4+1/9
*a^2*(5*A*b-8*B*a)*x)/b^4/(b*x^3+a)^2-7/27/b^5*a*sum((2*A*b-5*B*a)/_R^2*ln
(x-_R),_R=RootOf(_Z^3*b+a))
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.42

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$=\frac{27\,Bb^3x^{10}-54\,(5\,Bab^2-2\,Ab^3)x^7-147\,(5\,Ba^2b-2\,Aab^2)x^4-28\,\sqrt{3}((5\,Bab^2-2\,Ab^3)x^6+5\,Ba^3-2\,A}{}$$

input
```
integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output

```
1/108*(27*B*b^3*x^10 - 54*(5*B*a*b^2 - 2*A*b^3)*x^7 - 147*(5*B*a^2*b - 2*A
*a*b^2)*x^4 - 28*sqrt(3)*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b
+ 2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-
a/b)^(2/3) - sqrt(3)*a)/a) + 14*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A
*a^2*b + 2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3)) - 28*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b
+ 2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 84*(
5*B*a^3 - 2*A*a^2*b)*x)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

3.96.6 Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.67

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^4}{4b^3}+x\left(\frac{A}{b^3}-\frac{3Ba}{b^4}\right)+\frac{x^4\cdot(13Aab^2-19Ba^2b)+x(10Aa^2b-16Ba^3)}{18a^2b^4+36ab^5x^3+18b^6x^6}$$

$$+\operatorname{RootSum}\left(19683t^3b^{13}+2744A^3ab^3-20580A^2Ba^2b^2+51450AB^2a^3b-42875B^3a^4,\left(t\mapsto t\log\left(\frac{}{-14}\right.\right.\right.$$

input

```
integrate(x**9*(B*x**3+A)/(b*x**3+a)**3,x)
```

output

```
B*x**4/(4*b**3) + x*(A/b**3 - 3*B*a/b**4) + (x**4*(13*A*a*b**2 - 19*B*a**2
*b) + x*(10*A*a**2*b - 16*B*a**3))/(18*a**2*b**4 + 36*a*b**5*x**3 + 18*b**
6*x**6) + RootSum(19683*_t**3*b**13 + 2744*A**3*a*b**3 - 20580*A**2*B*a**2
*b**2 + 51450*A*B**2*a**3*b - 42875*B**3*a**4, Lambda(_t, _t*log(27*_t*b**
4/(-14*A*b + 35*B*a) + x)))
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.91

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(19\,Ba^2b-13\,Aab^2)x^4+2\,(8\,Ba^3-5\,Aa^2b)x}{18\,(b^6x^6+2\,ab^5x^3+a^2b^4)} + \frac{Bbx^4-4\,(3\,Ba-Ab)x}{4\,b^4} + \frac{7\,\sqrt{3}(5\,Ba^2-2\,Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{7\,(5\,Ba^2-2\,Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{7\,(5\,Ba^2-2\,Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*((19*B*a^2*b - 13*A*a*b^2)*x^4 + 2*(8*B*a^3 - 5*A*a^2*b)*x)/(b^6*x^6
 + 2*a*b^5*x^3 + a^2*b^4) + 1/4*(B*b*x^4 - 4*(3*B*a - A*b)*x)/b^4 + 7/27*s
qrt(3)*(5*B*a^2 - 2*A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/(b^5*(a/b)^(2/3)) - 7/54*(5*B*a^2 - 2*A*a*b)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b^5*(a/b)^(2/3)) + 7/27*(5*B*a^2 - 2*A*a*b)*log(x + (a/b)^
(1/3))/(b^5*(a/b)^(2/3))
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.96

$$
\begin{aligned}
\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{7\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-2\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^5} \\
&\quad - \frac{7\left(5\,Ba^2-2\,Aab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^4} \\
&\quad + \frac{7\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-2\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^5} \\
&\quad - \frac{19\,Ba^2bx^4-13\,Aab^2x^4+16\,Ba^3x-10\,Aa^2bx}{18\left(bx^3+a\right)^2b^4} \\
&\quad + \frac{Bb^9x^4-12\,Bab^8x+4\,Ab^9x}{4\,b^{12}}
\end{aligned}
$$

input

```
integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output

```
7/27*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 2*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt
(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 7/27*(5*B*a^2 - 2*A*a*b)*(-a/
b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) + 7/54*(5*(-a*b^2)^(1/3)*B*a -
 2*(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 - 1/18
*(19*B*a^2*b*x^4 - 13*A*a*b^2*x^4 + 16*B*a^3*x - 10*A*a^2*b*x)/((b*x^3 + a
)^2*b^4) + 1/4*(B*b^9*x^4 - 12*B*a*b^8*x + 4*A*b^9*x)/b^12
```

3.96.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.93

$$
\begin{aligned}
\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx
&= \frac{x^4\left(\frac{13\,A\,a\,b^2}{18}-\frac{19\,B\,a^2\,b}{18}\right)-x\left(\frac{8\,B\,a^3}{9}-\frac{5\,A\,a^2\,b}{9}\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)\\
&\quad+\frac{B\,x^4}{4\,b^3}+\frac{7\,(-a)^{1/3}\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)\,(2\,A\,b-5\,B\,a)}{27\,b^{13/3}}\\
&\quad-\frac{7\,(-a)^{1/3}\,\ln\left((-a)^{4/3}-2\,a\,b^{1/3}\,x+\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b-5\,B\,a)}{27\,b^{13/3}}\\
&\quad+\frac{7\,(-a)^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}+\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b-5\,B\,a)}{27\,b^{13/3}}
\end{aligned}
$$

input

```
int((x^9*(A + B*x^3))/(a + b*x^3)^3,x)
```

output

```
(x^4*((13*A*a*b^2)/18 - (19*B*a^2*b)/18) - x*((8*B*a^3)/9 - (5*A*a^2*b)/9)
)/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + x*(A/b^3 - (3*B*a)/b^4) + (B*x^4)/(4
*b^3) + (7*(-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x)*(2*A*b - 5*B*a))/(27*b
^(13/3)) - (7*(-a)^(1/3)*log((-a)^(4/3) + 3^(1/2)*(-a)^(4/3)*1i - 2*a*b^(1
/3)*x)*((3^(1/2)*1i)/2 + 1/2)*(2*A*b - 5*B*a))/(27*b^(13/3)) + (7*(-a)^(1/
3)*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2
 - 1/2)*(2*A*b - 5*B*a))/(27*b^(13/3))
```

3.97 $\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

3.97.1 Optimal result

Integrand size = 20, antiderivative size = 222

$$\begin{aligned}\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx &= -\frac{5(Ab-4aB)x^2}{18ab^3}+\frac{(Ab-aB)x^8}{6ab\,(a+bx^3)^2}+\frac{(Ab-4aB)x^5}{9ab^2\,(a+bx^3)} \\ &\quad -\frac{5(Ab-4aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a}b^{11/3}}-\frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{11/3}} \\ &\quad +\frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54\sqrt[3]{a}b^{11/3}}\end{aligned}$$

output

```
-5/18*(A*b-4*B*a)*x^2/a/b^3+1/6*(A*b-B*a)*x^8/a/b/(b*x^3+a)^2+1/9*(A*b-4*B
*a)*x^5/a/b^2/(b*x^3+a)-5/27*(A*b-4*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(
11/3)+5/54*(A*b-4*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b
^(11/3)-5/27*(A*b-4*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))
/a^(1/3)/b^(11/3)*3^(1/2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.87

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{27b^{2/3}Bx^2 + \frac{9ab^{2/3}(Ab-aB)x^2}{(a+bx^3)^2} - \frac{6b^{2/3}(4Ab-7aB)x^2}{a+bx^3} + \frac{10\sqrt{3}(-Ab+4aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{10(-Ab+4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}}{54b^{11/3}}$$

input
```
Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(27*b^(2/3)*B*x^2 + (9*a*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 - (6*b^(2/
3)*(4*A*b - 7*a*B)*x^2)/(a + b*x^3) + (10*Sqrt[3]*(-(A*b) + 4*a*B)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (10*(-(A*b) + 4*a*B)*Log[a
^(1/3) + b^(1/3)*x])/a^(1/3) + (5*(A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/a^(1/3))/(54*b^(11/3))
```

3.97.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {957, 817, 843, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^8(Ab-aB)}{6ab\,(a+bx^3)^2} - \frac{(Ab-4aB)\int \frac{x^7}{(bx^3+a)^2}dx}{3ab}$$

$$\Big\downarrow \text{817}$$

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\int\frac{x^4}{bx^3+a}dx}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\int\frac{x}{bx^3+a}dx}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$$\Big\downarrow \text{ 821}$$

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$$\Big\downarrow \text{ 16}$$

3.97. $\int\frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$$\Big\downarrow \text{1142}$$

3.97. $\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$$\Big\downarrow \; 25$$

3.97. $\int\frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$$\Big\downarrow \, 27$$

3.97. $\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^5}{3b\left(a+bx^3\right)}\right)}{3ab}$$

$$\Big\downarrow \color{blue}{1082}$$

3.97. $\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$$\Big\downarrow 217$$

3.97. $\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

$$\Big\downarrow 1103$$

3.97. $\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-4aB)\left(\frac{5\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^5}{3b(a+bx^3)}\right)}{3ab}$$

input
```
Int[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]
```

3.97. $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx$

output
```
((A*b - a*B)*x^8)/(6*a*b*(a + b*x^3)^2) - ((A*b - 4*a*B)*(-1/3*x^5/(b*(a +
 b*x^3)) + (5*(x^2/(2*b) - (a*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2
/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) +
 Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^
(1/3))))/b))/(3*b)))/(3*a*b)
```

3.97.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.97. $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.97.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.41

method	result
risch	$\frac{B\,x^{2}}{2b^{3}}+\frac{\left(-\frac{4}{9}b^{2}A+\frac{7}{9}abB\right)x^{5}-\frac{a(5Ab-11Ba)x^{2}}{18}}{b^{3}\left(b\,x^{3}+a\right)^{2}}+\frac{5\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{(Ab-4Ba)\ln\left(x-\_R\right)}{\_R}\right)}{27b^{4}}$
default	$\frac{B\,x^{2}}{2b^{3}}+\frac{\frac{\left(-\frac{4}{9}b^{2}A+\frac{7}{9}abB\right)x^{5}-\frac{a(5Ab-11Ba)x^{2}}{18}}{\left(b\,x^{3}+a\right)^{2}}+\left(\frac{5Ab}{9}-\frac{20Ba}{9}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{b^{3}}$

input
```
int(x^7*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x^2/b^3+((-4/9*b^2*A+7/9*a*b*B)*x^5-1/18*a*(5*A*b-11*B*a)*x^2)/b^3/(
b*x^3+a)^2+5/27/b^4*sum((A*b-4*B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(184) = 368.

Time = 0.29 (sec) , antiderivative size = 792, normalized size of antiderivative = 3.57

$$\int \frac{x^{7}(A+Bx^{3})}{(a+bx^{3})^{3}}\,dx$$

$$=\left[\frac{27\,Bab^{4}x^{8}+24\,(4\,Ba^{2}b^{3}-Aab^{4})x^{5}+15\,(4\,Ba^{3}b^{2}-Aa^{2}b^{3})x^{2}-15\,\sqrt{\frac{1}{3}}((4\,Ba^{2}b^{3}-Aab^{4})x^{6}+4\,Ba^{4}b-}{}\right.$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(27*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 -
 A*a^2*b^3)*x^2 - 15*sqrt(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b -
A*a^3*b^2 + 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2
*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3
)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 5*((4*B*a
*b^2 - A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b
^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*((4*B*a*
b^2 - A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^
2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5),
 1/54*(27*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 -
 A*a^2*b^3)*x^2 - 30*sqrt(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b -
A*a^3*b^2 + 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arcta
n(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 5*((4*B*
a*b^2 - A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*
b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*((4*B*a
*b^2 - A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b
^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5)
]
```

3.97.6 Sympy [A] (verification not implemented)

Time = 7.93 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.73

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^2}{2b^3} + \frac{x^5(-8Ab^2+14Bab)+x^2(-5Aab+11Ba^2)}{18a^2b^3+36ab^4x^3+18b^5x^6}$$
$$+\mathrm{RootSum}\left(19683t^3ab^{11}+125A^3b^3-1500A^2Bab^2+6000AB^2a^2b-8000B^3a^3,\left(t\mapsto t\log\left(\frac{}{25A^2b^2-}\right.\right.\right.$$

input
```
integrate(x**7*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
B*x**2/(2*b**3) + (x**5*(-8*A*b**2 + 14*B*a*b) + x**2*(-5*A*a*b + 11*B*a**
2))/(18*a**2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + RootSum(19683*_t**3*a
*b**11 + 125*A**3*b**3 - 1500*A**2*B*a*b**2 + 6000*A*B**2*a**2*b - 8000*B*
*3*a**3, Lambda(_t, _t*log(729*_t**2*a*b**7/(25*A**2*b**2 - 200*A*B*a*b +
400*B**2*a**2) + x)))
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\,(7\,Bab-4\,Ab^2)x^5+(11\,Ba^2-5\,Aab)x^2}{18\,(b^5x^6+2\,ab^4x^3+a^2b^3)} + \frac{Bx^2}{2\,b^3} - \frac{5\,\sqrt{3}(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{5\,(4\,Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{5\,(4\,Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(2*(7*B*a*b - 4*A*b^2)*x^5 + (11*B*a^2 - 5*A*a*b)*x^2)/(b^5*x^6 + 2*a
*b^4*x^3 + a^2*b^3) + 1/2*B*x^2/b^3 - 5/27*sqrt(3)*(4*B*a - A*b)*arctan(1/
3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) - 5/54*(4*B*a
 - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3)) + 5/27*(4
*B*a - A*b)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(1/3))
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.95

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^2}{2\,b^3} - \frac{5\,\sqrt{3}(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}\,b^3} + \frac{5\,(4\,Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}\,b^3} + \frac{5\,\left(4\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^3} + \frac{14\,Babx^5-8\,Ab^2x^5+11\,Ba^2x^2-5\,Aabx^2}{18\,\left(bx^3+a\right)^2b^3}$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/2*B*x^2/b^3 - 5/27*sqrt(3)*(4*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^3) + 5/54*(4*B*a - A*b)*log(x^2 +
 x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^3) + 5/27*(4*B*a*(-a/b)^
(1/3) - A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3)
+ 1/18*(14*B*a*b*x^5 - 8*A*b^2*x^5 + 11*B*a^2*x^2 - 5*A*a*b*x^2)/((b*x^3 +
 a)^2*b^3)
```

3.97.9 Mupad [B] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{x^2\left(\frac{11\,B\,a^2}{18}-\frac{5\,A\,a\,b}{18}\right)-x^5\left(\frac{4\,A\,b^2}{9}-\frac{7\,B\,a\,b}{9}\right)}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}\\ &\quad + \frac{B\,x^2}{2\,b^3} - \frac{5\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(A\,b-4\,B\,a)}{27\,a^{1/3}\,b^{11/3}}\\ &\quad - \frac{5\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-4\,B\,a)}{27\,a^{1/3}\,b^{11/3}}\\ &\quad + \frac{5\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-4\,B\,a)}{27\,a^{1/3}\,b^{11/3}}\end{aligned}$$

input
```
int((x^7*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(x^2*((11*B*a^2)/18 - (5*A*a*b)/18) - x^5*((4*A*b^2)/9 - (7*B*a*b)/9))/(a^
2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (B*x^2)/(2*b^3) - (5*log(b^(1/3)*x + a^(1
/3))*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3)) - (5*log(3^(1/2)*a^(1/3)*1i - 2*
b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - 4*B*a))/(27*a^(1/3)*b^(
11/3)) + (5*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/
2 + 1/2)*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3))
```

3.98 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.98.1 Optimal result

Integrand size = 20, antiderivative size = 220

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{2(Ab-7aB)x}{9ab^3}+\frac{(Ab-aB)x^7}{6ab(a+bx^3)^2}+\frac{(Ab-7aB)x^4}{18ab^2(a+bx^3)} - \frac{2(Ab-7aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}}+\frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}} - \frac{(Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{2/3}b^{10/3}}$$

output

```
-2/9*(A*b-7*B*a)*x/a/b^3+1/6*(A*b-B*a)*x^7/a/b/(b*x^3+a)^2+1/18*(A*b-7*B*a
)*x^4/a/b^2/(b*x^3+a)+2/27*(A*b-7*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(10
/3)-1/27*(A*b-7*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(
10/3)-2/27*(A*b-7*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a
^(2/3)/b^(10/3)*3^(1/2)
```

3.98.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.85

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$= \frac{54\sqrt[3]{b}Bx + \frac{9a\sqrt[3]{b}(Ab-aB)x}{(a+bx^3)^2} - \frac{3\sqrt[3]{b}(7Ab-13aB)x}{a+bx^3} + \frac{4\sqrt{3}(-Ab+7aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{4(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} +}{54b^{10/3}}$$

input
```
Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(54*b^(1/3)*B*x + (9*a*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 - (3*b^(1/3)*(
7*A*b - 13*a*B)*x)/(a + b*x^3) + (4*Sqrt[3]*(-(A*b) + 7*a*B)*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*(A*b - 7*a*B)*Log[a^(1/3) + b
^(1/3)*x])/a^(2/3) + (2*(-(A*b) + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/a^(2/3))/(54*b^(10/3))
```

3.98.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {957, 817, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}dx$$
$$\Big\downarrow \text{957}$$
$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\int \frac{x^6}{\left(bx^3+a\right)^2}dx}{6ab}$$
$$\Big\downarrow \text{817}$$
$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\int \frac{x^3}{bx^3+a}dx}{3b}-\frac{x^4}{3b\left(a+bx^3\right)}\right)}{6ab}$$

$$\Big\downarrow \ 843$$

$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\int\frac{1}{bx^3+a}dx}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$$\Big\downarrow \ 750$$

$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$$\Big\downarrow \ 16$$

$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$$\Big\downarrow \ 1142$$

3.98. $\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$$\Big\downarrow \; 25$$

3.98. $\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$$\Big\downarrow 27$$

3.98. $\int\frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$$\Big\downarrow \color{blue}{1082}$$

3.98. $\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$$\Big\downarrow 217$$

3.98. $\int\frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}$$

$$\Big\downarrow \; 1103$$

3.98. $\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}-\frac{(Ab-7aB)\left(\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}\right)}{6ab}}{}$$

input
```
Int[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]
```

3.98. $\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

output
```
((A*b - a*B)*x^7)/(6*a*b*(a + b*x^3)^2) - ((A*b - 7*a*B)*(-1/3*x^4/(b*(a +
 b*x^3)) + (4*(x/b - (a*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-
((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/b))/(3*b
)))/(6*a*b)
```

3.98.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

3.98. $\int \frac{x^6 (A+Bx^3)}{(a+bx^3)^3} \, dx$

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.98. $\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

3.98.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.44 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.39

method	result
risch	$\frac{Bx}{b^3}+\frac{\left(-\frac{7}{18}b^2A+\frac{13}{18}abB\right)x^4-\frac{a(2Ab-5Ba)x}{9}}{b^3\left(b\,x^3+a\right)^2}+\frac{2\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab-7Ba)\ln\left(x-\_R\right)}{\_R^2}\right)}{27b^4}$
default	$\frac{Bx}{b^3}+\frac{\frac{\left(-\frac{7}{18}b^2A+\frac{13}{18}abB\right)x^4-\frac{a(2Ab-5Ba)x}{9}}{\left(b\,x^3+a\right)^2}+\frac{2(Ab-7Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{9}}{b^3}$

input
```
int(x^6*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
B*x/b^3+((-7/18*b^2*A+13/18*a*b*B)*x^4-1/9*a*(2*A*b-5*B*a)*x)/b^3/(b*x^3+a
)^2+2/27/b^4*sum((A*b-7*B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(182) = 364.

Time = 0.30 (sec) , antiderivative size = 789, normalized size of antiderivative = 3.59

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \left[\frac{54\,Ba^2b^3x^7+21\,(7\,Ba^3b^2-Aa^2b^3)x^4-6\,\sqrt{\frac{1}{3}}((7\,Ba^2b^3-Aab^4)x^6+7\,Ba^4b-Aa^3b^2+2\,(7\,Ba^3b^2-Aa}{}\right.$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(54*B*a^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 6*sqrt(1/3)*(
(7*B*a^2*b^3 - A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a
^2*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x -
 a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(
a^2*b)^(1/3)/b))/(b*x^3 + a)) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a
^2*b + 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2
/3)*x + (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b
+ 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) +
12*(7*B*a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), 1/5
4*(54*B*a^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 12*sqrt(1/3)*((7*
B*a^2*b^3 - A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a^2*
b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2
*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B
*a^3 - A*a^2*b + 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 -
(a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3
- A*a^2*b + 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)
^(2/3)) + 12*(7*B*a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4
*b^4)]
```

3.98.6 Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.64

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx}{b^3} + \frac{x^4(-7Ab^2+13Bab)+x(-4Aab+10Ba^2)}{18a^2b^3+36ab^4x^3+18b^5x^6}$$
$$+\mathrm{RootSum}\left(19683t^3a^2b^{10}-8A^3b^3+168A^2Bab^2-1176AB^2a^2b+2744B^3a^3,\left(t\mapsto t\log\left(-\frac{27tab^3}{-2Ab+14}\right.\right.\right.$$

input
```
integrate(x**6*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
B*x/b**3 + (x**4*(-7*A*b**2 + 13*B*a*b) + x*(-4*A*a*b + 10*B*a**2))/(18*a*
*2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + RootSum(19683*_t**3*a**2*b**10
- 8*A**3*b**3 + 168*A**2*B*a*b**2 - 1176*A*B**2*a**2*b + 2744*B**3*a**3, L
ambda(_t, _t*log(-27*_t*a*b**3/(-2*A*b + 14*B*a) + x)))
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.87

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(13\,Bab-7\,Ab^2)x^4+2\,(5\,Ba^2-2\,Aab)x}{18\,(b^5x^6+2\,ab^4x^3+a^2b^3)}+\frac{Bx}{b^3} - \frac{2\,\sqrt{3}(7\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(7\,Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{2\,(7\,Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*((13*B*a*b - 7*A*b^2)*x^4 + 2*(5*B*a^2 - 2*A*a*b)*x)/(b^5*x^6 + 2*a*b
^4*x^3 + a^2*b^3) + B*x/b^3 - 2/27*sqrt(3)*(7*B*a - A*b)*arctan(1/3*sqrt(3
)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) + 1/27*(7*B*a - A*b)*
log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) - 2/27*(7*B*a - A
*b)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.85

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\,\sqrt{3}(7\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}b^2} + \frac{(7\,Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}b^2} + \frac{Bx}{b^3} + \frac{2\,(7\,Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^3} + \frac{13\,Babx^4-7\,Ab^2x^4+10\,Ba^2x-4\,Aabx}{18\,(bx^3+a)^2b^3}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
2/27*sqrt(3)*(7*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(2/3)*b^2) + 1/27*(7*B*a - A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) + B*x/b^3 + 2/27*(7*B*a - A*b)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/18*(13*B*a*b*x^4 - 7*A*b^2*x
^4 + 10*B*a^2*x - 4*A*a*b*x)/((b*x^3 + a)^2*b^3)
```

3.98.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.83

$$
\begin{aligned}
\int \frac{x^6(A+Bx^3)}{\left(a+bx^3\right)^3}\,dx &= \frac{B\,x}{b^3}-\frac{x^4\,\left(\frac{7\,A\,b^2}{18}-\frac{13\,B\,a\,b}{18}\right)-x\,\left(\frac{5\,B\,a^2}{9}-\frac{2\,A\,a\,b}{9}\right)}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6} \\
&\quad+\frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-7\,B\,a\right)}{27\,a^{2/3}\,b^{10/3}} \\
&\quad-\frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-7\,B\,a\right)}{27\,a^{2/3}\,b^{10/3}} \\
&\quad+\frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-7\,B\,a\right)}{27\,a^{2/3}\,b^{10/3}}
\end{aligned}
$$

input
```
int((x^6*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(B*x)/b^3 - (x^4*((7*A*b^2)/18 - (13*B*a*b)/18) - x*((5*B*a^2)/9 - (2*A*a*
b)/9))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (2*log(b^(1/3)*x + a^(1/3))*(A*
b - 7*B*a))/(27*a^(2/3)*b^(10/3)) - (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*
x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - 7*B*a))/(27*a^(2/3)*b^(10/3)) +
 (2*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)
*(A*b - 7*B*a))/(27*a^(2/3)*b^(10/3))
```

3.99 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.99.1 Optimal result

Integrand size = 20, antiderivative size = 201

$$\begin{aligned}\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{(Ab-aB)x^5}{6ab\,(a+bx^3)^2}-\frac{(Ab+5aB)x^2}{18ab^2\,(a+bx^3)}\\ &\quad-\frac{(Ab+5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{8/3}}-\frac{(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}}\\ &\quad+\frac{(Ab+5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{8/3}}\end{aligned}$$

output
```
1/6*(A*b-B*a)*x^5/a/b/(b*x^3+a)^2-1/18*(A*b+5*B*a)*x^2/a/b^2/(b*x^3+a)-1/2
7*(A*b+5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(8/3)+1/54*(A*b+5*B*a)*ln(a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(8/3)-1/27*(A*b+5*B*a)*arct
an(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(8/3)*3^(1/2)
```

3.99.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.90

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$=\frac{-\frac{9b^{2/3}(Ab-aB)x^2}{(a+bx^3)^2}+\frac{6b^{2/3}(Ab-4aB)x^2}{a(a+bx^3)}-\frac{2\sqrt{3}(Ab+5aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}-\frac{2(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}}+(Ab+5aB)\log\Big(a^{2/}}{54b^{8/3}}$$

input
```
Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((-9*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 + (6*b^(2/3)*(A*b - 4*a*B)*x^2
)/(a*(a + b*x^3)) - (2*Sqrt[3]*(A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]])/a^(4/3) - (2*(A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(4/
3) + ((A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3
))/(54*b^(8/3))
```

3.99.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {957, 817, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$$
$$\Big\downarrow \text{957}$$
$$\frac{(5aB+Ab)\int \frac{x^4}{\left(bx^3+a\right)^2}dx}{6ab}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$
$$\Big\downarrow \text{817}$$
$$\frac{(5aB+Ab)\left(\frac{2\int \frac{x}{bx^3+a}dx}{3b}-\frac{x^2}{3b\left(a+bx^3\right)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 821$$

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 16$$

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

3.99. $\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1082$$

3.99. $\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1103$$

3.99. $\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{(5aB+Ab)\left(\frac{2\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6ab}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^5)/(6*a*b*(a + b*x^3)^2) + ((A*b + 5*a*B)*(-1/3*x^2/(b*(a +
 b*x^3)) + (2*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[
3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/(3*b)))
/(6*a*b)
```

3.99.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.99. $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.99.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.42

method	result	size
risch	$\frac{\frac{(Ab-4Ba)x^5}{9ab}-\frac{(Ab+5Ba)x^2}{18b^2}}{(bx^3+a)^2}+\frac{\sum_{\_R=\operatorname{RootOf}(b\_Z^3+a)}\frac{(Ab+5Ba)\ln(x-\_R)}{\_R}}{27a\,b^3}$	85
default	$\frac{\frac{(Ab-4Ba)x^5}{9ab}-\frac{(Ab+5Ba)x^2}{18b^2}}{(bx^3+a)^2}+\frac{(Ab+5Ba)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^2a}$	154

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/9*(A*b-4*B*a)/a/b*x^5-1/18*(A*b+5*B*a)/b^2*x^2)/(b*x^3+a)^2+1/27/a/b^3*
sum((A*b+5*B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(160) = 320.

Time = 0.30 (sec) , antiderivative size = 756, normalized size of antiderivative = 3.76

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \left[-\frac{6\,(4\,Ba^2b^3-Aab^4)x^5+3\,(5\,Ba^3b^2+Aa^2b^3)x^2-3\,\sqrt{\frac{1}{3}}((5\,Ba^2b^3+Aab^4)x^6+5\,Ba^4b+Aa^3b^2+2\,(5\,}{}\right.$$

$$-\frac{6\,(4\,Ba^2b^3-Aab^4)x^5+3\,(5\,Ba^3b^2+Aa^2b^3)x^2-6\,\sqrt{\frac{1}{3}}((5\,Ba^2b^3+Aab^4)x^6+5\,Ba^4b+Aa^3b^2+2\,(5\,B}{}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[-1/54*(6*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 -
3*sqrt(1/3)*((5*B*a^2*b^3 + A*a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*
a^3*b^2 + A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*
sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^
(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((5*B*a*b^2 + A*b^3)*x^6 + 5
*B*a^3 + A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2
 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B
*a^3 + A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a
*b^2)^(1/3)))/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), -1/54*(6*(4*B*a^2*b
^3 - A*a*b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 - 6*sqrt(1/3)*((5*B*a^
2*b^3 + A*a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*a^3*b^2 + A*a^2*b^3)
*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sq
rt(-(-a*b^2)^(1/3)/a)/b) - ((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b +
2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b
*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*
(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*
b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4)]
```

3.99. $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.99.6 Sympy [A] (verification not implemented)

Time = 5.37 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.77

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^5\cdot(2Ab^2-8Bab)+x^2(-Aab-5Ba^2)}{18a^3b^2+36a^2b^3x^3+18ab^4x^6}$$
$$+\mathrm{RootSum}\Bigg(19683t^3a^4b^8+A^3b^3+15A^2Bab^2+75AB^2a^2b+125B^3a^3, \Bigg(t\mapsto t\log\Bigg(\frac{729t^2a^3b^5}{A^2b^2+10ABab+2}$$

input
```
integrate(x**4*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
(x**5*(2*A*b**2 - 8*B*a*b) + x**2*(-A*a*b - 5*B*a**2))/(18*a**3*b**2 + 36*
a**2*b**3*x**3 + 18*a*b**4*x**6) + RootSum(19683*_t**3*a**4*b**8 + A**3*b*
*3 + 15*A**2*B*a*b**2 + 75*A*B**2*a**2*b + 125*B**3*a**3, Lambda(_t, _t*lo
g(729*_t**2*a**3*b**5/(A**2*b**2 + 10*A*B*a*b + 25*B**2*a**2) + x)))
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = &-\frac{2\,(4\,Bab-Ab^2)x^5+(5\,Ba^2+Aab)x^2}{18\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)}\\ &+\frac{\sqrt{3}(5\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\\ &+\frac{(5\,Ba+Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\\ &-\frac{(5\,Ba+Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\end{aligned}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*(2*(4*B*a*b - A*b^2)*x^5 + (5*B*a^2 + A*a*b)*x^2)/(a*b^4*x^6 + 2*a^2
*b^3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(5*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x -
 (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(1/3)) + 1/54*(5*B*a + A*b)*log(x^
2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(1/3)) - 1/27*(5*B*a + A*b)*
log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(1/3))
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.02

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\sqrt{3}(5\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}ab^2} - \frac{(5\,Ba+Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}ab^2} - \frac{\left(5\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^2} - \frac{8\,Babx^5-2\,Ab^2x^5+5\,Ba^2x^2+Aabx^2}{18\,\left(bx^3+a\right)^2ab^2}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/27*sqrt(3)*(5*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(1/3)*a*b^2) - 1/54*(5*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a*b^2) - 1/27*(5*B*a*(-a/b)^(1/3) + A*b*
(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^2) - 1/18*(8*
B*a*b*x^5 - 2*A*b^2*x^5 + 5*B*a^2*x^2 + A*a*b*x^2)/((b*x^3 + a)^2*a*b^2)
```

3.99.9 Mupad [B] (verification not implemented)

Time = 6.97 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{\frac{x^2\,(A\,b+5\,B\,a)}{18\,b^2}-\frac{x^5\,(A\,b-4\,B\,a)}{9\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b+5\,B\,a\right)}{27\,a^{4/3}\,b^{8/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b+5\,B\,a\right)}{27\,a^{4/3}\,b^{8/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b+5\,B\,a\right)}{27\,a^{4/3}\,b^{8/3}}$$

input
```
int((x^4*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A
*b + 5*B*a))/(27*a^(4/3)*b^(8/3)) - (log(b^(1/3)*x + a^(1/3))*(A*b + 5*B*a
))/(27*a^(4/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))
*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a))/(27*a^(4/3)*b^(8/3)) - ((x^2*(A*b +
 5*B*a))/(18*b^2) - (x^5*(A*b - 4*B*a))/(9*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^
3)
```

3.99. $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.100 $\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

3.100.1 Optimal result

Integrand size = 20, antiderivative size = 199

$$\begin{aligned}\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = {} & \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)}-\frac{(Ab+2aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}}\\ & +\frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}}\\ & -\frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{7/3}}\end{aligned}$$

output
```
1/6*(A*b-B*a)*x^4/a/b/(b*x^3+a)^2-1/9*(A*b+2*B*a)*x/a/b^2/(b*x^3+a)+1/27*(
A*b+2*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(7/3)-1/54*(A*b+2*B*a)*ln(a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(7/3)-1/27*(A*b+2*B*a)*arctan(
1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(7/3)*3^(1/2)
```

3.100.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.89

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{9\sqrt[3]{b}(Ab-aB)x}{(a+bx^3)^2}+\frac{3\sqrt[3]{b}(Ab-7aB)x}{a(a+bx^3)}-\frac{2\sqrt{3}(Ab+2aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{2(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}-\frac{(Ab+2aB)\log\left(a^{2/3}\right.}{a}}{54b^{7/3}}$$

input
```
Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((-9*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 + (3*b^(1/3)*(A*b - 7*a*B)*x)/(a
*(a + b*x^3)) - (2*Sqrt[3]*(A*b + 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]])/a^(5/3) + (2*(A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) -
 ((A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(
54*b^(7/3))
```

3.100.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {957, 817, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(2aB+Ab)\int \frac{x^3}{(bx^3+a)^2}dx}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{817}$$

$$\frac{(2aB+Ab)\left(\frac{\int \frac{1}{bx^3+a}dx}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow 750$$

$$\frac{(2aB+Ab)\left(\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow 16$$

$$\frac{(2aB+Ab)\left(\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow 1142$$

$$\frac{(2aB+Ab)\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow 25$$

$$\frac{(2aB+Ab)\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

3.100. $\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\Big\downarrow \text{27}$$

$$\frac{(2aB+Ab)\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(2aB+Ab)\left(\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{(2aB+Ab)\left(\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1103}$$

3.100. $\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{(2aB+Ab)\left(\frac{\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3ab}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^4)/(6*a*b*(a + b*x^3)^2) + ((A*b + 2*a*B)*(-1/3*x/(b*(a + b
*x^3)) + (Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3)))/(3*b)))/(3*a*b)
```

3.100.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.100. $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.100.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.42

method	result	size
risch	$\frac{\frac{(Ab-7Ba)x^4}{18ab}-\frac{(Ab+2Ba)x}{9b^2}}{(b\,x^3+a)^2}+\frac{\sum\limits_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab+2Ba)\ln\left(x-\_R\right)}{\_R^2}}{27a\,b^3}$	83
default	$\frac{\frac{(Ab-7Ba)x^4}{18ab}-\frac{(Ab+2Ba)x}{9b^2}}{(b\,x^3+a)^2}+\frac{(Ab+2Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{9b^2a}$	152

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/18*(A*b-7*B*a)/a/b*x^4-1/9*(A*b+2*B*a)/b^2*x)/(b*x^3+a)^2+1/27/a/b^3*su
m((A*b+2*B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(158) = 316.

Time = 0.29 (sec) , antiderivative size = 743, normalized size of antiderivative = 3.73

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \Bigg[-\frac{3\,(7\,Ba^3b^2 - Aa^2b^3)x^4 - 3\,\sqrt{\tfrac{1}{3}}((2\,Ba^2b^3 + Aab^4)x^6 + 2\,Ba^4b + Aa^3b^2 + 2\,(2\,Ba^3b^2 + Aa^2b^3)x^3)\sqrt{-\tfrac{(a}{}}}{}$$

$$-\frac{3\,(7\,Ba^3b^2 - Aa^2b^3)x^4 - 6\,\sqrt{\tfrac{1}{3}}((2\,Ba^2b^3 + Aab^4)x^6 + 2\,Ba^4b + Aa^3b^2 + 2\,(2\,Ba^3b^2 + Aa^2b^3)x^3)\sqrt{\tfrac{(a^2b)}{b}}}{}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[-1/54*(3*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 3*sqrt(1/3)*((2*B*a^2*b^3 + A*a*
b^4)*x^6 + 2*B*a^4*b + A*a^3*b^2 + 2*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt(-
(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*
(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b
*x^3 + a)) + ((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b +
 A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)
*a) - 2*((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*
b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(2*B*a^4*b + A*a^3*
b^2)*x)/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), -1/54*(3*(7*B*a^3*b^2 - A
*a^2*b^3)*x^4 - 6*sqrt(1/3)*((2*B*a^2*b^3 + A*a*b^4)*x^6 + 2*B*a^4*b + A*a
^3*b^2 + 2*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqr
t(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) +
((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3
)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((2*B
*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^
2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(2*B*a^4*b + A*a^3*b^2)*x)/(a^3*
b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3)]
```

3.100. $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.100.6 Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.68

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^4(Ab^2-7Bab)+x(-2Aab-4Ba^2)}{18a^3b^2+36a^2b^3x^3+18ab^4x^6} + \operatorname{RootSum}\left(19683t^3a^5b^7-A^3b^3-6A^2Bab^2-12AB^2a^2b-8B^3a^3, \left(t\mapsto t\log\left(\frac{27ta^2b^2}{Ab+2Ba}+x\right)\right)\right)$$

input
```
integrate(x**3*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
(x**4*(A*b**2 - 7*B*a*b) + x*(-2*A*a*b - 4*B*a**2))/(18*a**3*b**2 + 36*a**
2*b**3*x**3 + 18*a*b**4*x**6) + RootSum(19683*_t**3*a**5*b**7 - A**3*b**3
- 6*A**2*B*a*b**2 - 12*A*B**2*a**2*b - 8*B**3*a**3, Lambda(_t, _t*log(27*_
t*a**2*b**2/(A*b + 2*B*a) + x)))
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = &-\frac{(7\,Bab-Ab^2)x^4+2\,(2\,Ba^2+Aab)x}{18\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)} \\ &+\frac{\sqrt{3}(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &-\frac{(2\,Ba+Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &+\frac{(2\,Ba+Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*((7*B*a*b - A*b^2)*x^4 + 2*(2*B*a^2 + A*a*b)*x)/(a*b^4*x^6 + 2*a^2*b
^3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(2/3)) - 1/54*(2*B*a + A*b)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 1/27*(2*B*a + A*b)*lo
g(x + (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.94

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}ab} - \frac{(2\,Ba+Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{2}{3}}ab} - \frac{(2\,Ba+Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^2} - \frac{7\,Babx^4-Ab^2x^4+4\,Ba^2x+2\,Aabx}{18\,\left(bx^3+a\right)^2ab^2}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/54*(2*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b) - 1/27*(2*B*a + A*b)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a^2*b^2) - 1/18*(7*B*a*b*x^4 - A*b^2*x^4 + 4*B*a
^2*x + 2*A*a*b*x)/((b*x^3 + a)^2*a*b^2)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 6.97 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.87

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}} - \frac{\frac{x\,\left(A\,b+2\,B\,a\right)}{9\,b^2}-\frac{x^4\,\left(A\,b-7\,B\,a\right)}{18\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}}$$

input
```
int((x^3*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(log(b^(1/3)*x + a^(1/3))*(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3)) - ((x*(A*b +
 2*B*a))/(9*b^2) - (x^4*(A*b - 7*B*a))/(18*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^
3) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/
2)*(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/
3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3))
```

3.100. $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.101 $\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

3.101.1 Optimal result

Integrand size = 18, antiderivative size = 201

$$\begin{aligned}\int \frac{x(A+Bx^3)}{\left(a+bx^3\right)^3}\,dx &= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)}-\frac{(2Ab+aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{5/3}}\\&\quad-\frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}}\\&\quad+\frac{(2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{5/3}}\end{aligned}$$

output

```
1/6*(A*b-B*a)*x^2/a/b/(b*x^3+a)^2+1/9*(2*A*b+B*a)*x^2/a^2/b/(b*x^3+a)-1/27
*(2*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(5/3)+1/54*(2*A*b+B*a)*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(5/3)-1/27*(2*A*b+B*a)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(5/3)*3^(1/2)
```

3.101.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.89

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{9a^{4/3}b^{2/3}(-Ab+aB)x^2}{(a+bx^3)^2}+\frac{6\sqrt[3]{a}b^{2/3}(2Ab+aB)x^2}{a+bx^3}-2\sqrt{3}(2Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2(2Ab+aB)\log\left(\sqrt[3]{a}+\right.}{54a^{7/3}b^{5/3}}$$

input
```
Integrate[(x*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((-9*a^(4/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(1/3)*b^(2/3
)*(2*A*b + a*B)*x^2)/(a + b*x^3) - 2*Sqrt[3]*(2*A*b + a*B)*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] +
(2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*
b^(5/3))
```

3.101.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {957, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \ 957$$

$$\frac{(aB+2Ab)\int \frac{x}{(bx^3+a)^2}dx}{3ab}+\frac{x^2(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \ 819$$

$$\frac{(aB+2Ab)\left(\frac{\int \frac{x}{bx^3+a}dx}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow 821$$

$$\frac{(aB+2Ab)\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow 16$$

$$\frac{(aB+2Ab)\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow 1142$$

$$\frac{(aB+2Ab)\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow 25$$

$$\frac{(aB+2Ab)\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

3.101. $\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\Big\downarrow \text{27}$$

$$\frac{(aB+2Ab)\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(aB+2Ab)\left(\frac{\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{(aB+2Ab)\left(\frac{\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1103}$$

3.101. $\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{(aB+2Ab)\left(\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3ab}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input

```
Int[(x*(A + B*x^3))/(a + b*x^3)^3,x]
```

output

```
((A*b - a*B)*x^2)/(6*a*b*(a + b*x^3)^2) + ((2*A*b + a*B)*(x^2/(3*a*(a + b*
x^3)) + (-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*Arc
Tan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3)))/(3*a)))/(3*a*b
)
```

3.101.3.1 Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.101. $\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.101.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\frac{(2Ab+Ba)x^5}{9a^2}+\frac{(7Ab-Ba)x^2}{18ab}}{(b x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(2Ab+Ba)\ln\left(x-\_R\right)}{\_R}}{27a^2b^2}$	86
default	$\frac{\frac{(2Ab+Ba)x^5}{9a^2}+\frac{(7Ab-Ba)x^2}{18ab}}{(b x^3+a)^2}+\frac{(2Ab+Ba)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b}$	155

input
```
int(x*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/9*(2*A*b+B*a)/a^2*x^5+1/18*(7*A*b-B*a)/a/b*x^2)/(b*x^3+a)^2+1/27/a^2/b^
2*sum((2*A*b+B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(160) = 320.

Time = 0.37 (sec) , antiderivative size = 752, normalized size of antiderivative = 3.74

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \left[\frac{6\,(Ba^2b^3+2\,Aab^4)x^5-3\,(Ba^3b^2-7\,Aa^2b^3)x^2+3\,\sqrt{\frac{1}{3}}((Ba^2b^3+2\,Aab^4)x^6+Ba^4b+2\,Aa^3b^2+2\,(Ba^3}{}\right.$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(6*(B*a^2*b^3 + 2*A*a*b^4)*x^5 - 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 3
*sqrt(1/3)*((B*a^2*b^3 + 2*A*a*b^4)*x^6 + B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3
*b^2 + 2*A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*s
qrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(
1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((B*a*b^2 + 2*A*b^3)*x^6 + B*
a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2
+ (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^
3 + 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*
b^2)^(1/3)))/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), 1/54*(6*(B*a^2*b^3 +
 2*A*a*b^4)*x^5 - 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 6*sqrt(1/3)*((B*a^2*b^
3 + 2*A*a*b^4)*x^6 + B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3*b^2 + 2*A*a^2*b^3)*x
^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt
(-(-a*b^2)^(1/3)/a)/b) + ((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*
(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x
 + (-a*b^2)^(2/3)) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*(B
*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^
5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3)]
```

3.101.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.76

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^5\cdot(4Ab^2+2Bab)+x^2\cdot(7Aab-Ba^2)}{18a^4b+36a^3b^2x^3+18a^2b^3x^6}$$

$$+\mathrm{RootSum}\Bigg(19683t^3a^7b^5+8A^3b^3+12A^2Bab^2+6AB^2a^2b+B^3a^3, \Bigg(t\mapsto t\log\Bigg(\frac{729t^2a^5b^3}{4A^2b^2+4ABab+B^2a^2}$$

input
```
integrate(x*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
(x**5*(4*A*b**2 + 2*B*a*b) + x**2*(7*A*a*b - B*a**2))/(18*a**4*b + 36*a**3
*b**2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**7*b**5 + 8*A**3*b
**3 + 12*A**2*B*a*b**2 + 6*A*B**2*a**2*b + B**3*a**3, Lambda(_t, _t*log(72
9*_t**2*a**5*b**3/(4*A**2*b**2 + 4*A*B*a*b + B**2*a**2) + x)))
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\,(Bab+2\,Ab^2)x^5-(Ba^2-7\,Aab)x^2}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)} + \frac{\sqrt{3}(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba+2\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba+2\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(2*(B*a*b + 2*A*b^2)*x^5 - (B*a^2 - 7*A*a*b)*x^2)/(a^2*b^3*x^6 + 2*a^
3*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3)) + 1/54*(B*a + 2*A*b)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(1/3)) - 1/27*(B*a + 2*A*
b)*log(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3))
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.03

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\sqrt{3}(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}\,a^2b} - \frac{(Ba+2\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}\,a^2b} - \frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b} + \frac{2\,Babx^5+4\,Ab^2x^5-Ba^2x^2+7\,Aabx^2}{18\,(bx^3+a)^2a^2b}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/27*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(1/3)*a^2*b) - 1/54*(B*a + 2*A*b)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*b) - 1/27*(B*a*(-a/b)^(1/3) + 2*A*b*
(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/18*(2*B*
a*b*x^5 + 4*A*b^2*x^5 - B*a^2*x^2 + 7*A*a*b*x^2)/((b*x^3 + a)^2*a^2*b)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\frac{x^5\,(2\,A\,b+B\,a)}{9\,a^2}+\frac{x^2\,(7\,A\,b-B\,a)}{18\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}}$$

input
```
int((x*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
((x^5*(2*A*b + B*a))/(9*a^2) + (x^2*(7*A*b - B*a))/(18*a*b))/(a^2 + b^2*x^
6 + 2*a*b*x^3) - (log(b^(1/3)*x + a^(1/3))*(2*A*b + B*a))/(27*a^(7/3)*b^(5
/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 -
1/2)*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(
1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3
))
```

3.102 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^3}\,dx$

3.102.1 Optimal result

Integrand size = 17, antiderivative size = 197

$$\begin{aligned}\int \frac{A+Bx^3}{\left(a+bx^3\right)^3}\,dx &= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2}+\frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)} \\ &\quad -\frac{(5Ab+aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{4/3}}+\frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}} \\ &\quad -\frac{(5Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}}\end{aligned}$$

output
```
1/6*(A*b-B*a)*x/a/b/(b*x^3+a)^2+1/18*(5*A*b+B*a)*x/a^2/b/(b*x^3+a)+1/27*(5
*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(4/3)-1/54*(5*A*b+B*a)*ln(a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(4/3)-1/27*(5*A*b+B*a)*arctan(1
/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(4/3)*3^(1/2)
```

3.102.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^3}\,dx$$

$$=\frac{-\frac{9a^{5/3}\sqrt[3]{b}(-Ab+aB)x}{\left(a+bx^3\right)^2}+\frac{3a^{2/3}\sqrt[3]{b}(5Ab+aB)x}{a+bx^3}-2\sqrt{3}(5Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2(5Ab+aB)\log\Big(\sqrt[3]{a}+}{54a^{8/3}b^{4/3}}$$

input
```
Integrate[(A + B*x^3)/(a + b*x^3)^3,x]
```

output
```
((-9*a^(5/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(2/3)*b^(1/3)*
(5*A*b + a*B)*x)/(a + b*x^3) - 2*Sqrt[3]*(5*A*b + a*B)*ArcTan[(1 - (2*b^(1
/3)*x)/a^(1/3))/Sqrt[3]] + 2*(5*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] - (5*A
*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(4
/3))
```

3.102.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {910, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(aB+5Ab)\int \frac{1}{\left(bx^3+a\right)^2}dx}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{749}$$

$$\frac{(aB+5Ab)\left(\frac{2\int \frac{1}{bx^3+a}dx}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{750}$$

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 1142$$

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 25$$

3.102. $\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx$

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \textbf{1082}$$

3.102. $\int \frac{A+Bx^3}{\left(a+bx^3\right)^3}\,dx$

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1103}$$

3.102. $\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx$

$$\frac{(aB+5Ab)\left(\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(A + B*x^3)/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x)/(6*a*b*(a + b*x^3)^2) + ((5*A*b + a*B)*(x/(3*a*(a + b*x^3)
) + (2*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(3*a)))/(6*a*b)
```

3.102.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.102. $\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx$

3.102.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\frac{(5Ab+Ba)x^4}{18a^2}+\frac{(4Ab-Ba)x}{9ab}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(5Ab+Ba)\ln\left(x-\_R\right)}{\_R^2}}{27b^2a^2}$	84
default	$\frac{\frac{(5Ab+Ba)x^4}{18a^2}+\frac{(4Ab-Ba)x}{9ab}}{(b\,x^3+a)^2}+\frac{(5Ab+Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{9b\,a^2}$	153

input
```
int((B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/18*(5*A*b+B*a)/a^2*x^4+1/9*(4*A*b-B*a)/a/b*x)/(b*x^3+a)^2+1/27/b^2/a^2*
sum((5*A*b+B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(156) = 312.

Time = 0.31 (sec) , antiderivative size = 743, normalized size of antiderivative = 3.77

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^3}\,dx$$

$$= \left[\frac{3\left(Ba^3b^2+5\,Aa^2b^3\right)x^4+3\,\sqrt{\frac{1}{3}}\left(\left(Ba^2b^3+5\,Aab^4\right)x^6+Ba^4b+5\,Aa^3b^2+2\left(Ba^3b^2+5\,Aa^2b^3\right)x^3\right)\sqrt{-\frac{(a^2b}{b}}}{}\right.$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(3*(B*a^3*b^2 + 5*A*a^2*b^3)*x^4 + 3*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b
^4)*x^6 + B*a^4*b + 5*A*a^3*b^2 + 2*(B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt(-(
a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(
2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*
x^3 + a)) - ((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*
A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*
a) + 2*((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b
^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b
^2)*x)/(a^4*b^4*x^6 + 2*a^5*b^3*x^3 + a^6*b^2), 1/54*(3*(B*a^3*b^2 + 5*A*a
^2*b^3)*x^4 + 6*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b^4)*x^6 + B*a^4*b + 5*A*a^3
*b^2 + 2*(B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(
1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((
B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*
(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b
^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*
b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b^2)*x)/(a^4*b^
4*x^6 + 2*a^5*b^3*x^3 + a^6*b^2)]
```

3.102.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx = \frac{x^4\cdot(5Ab^2+Bab)+x(8Aab-2Ba^2)}{18a^4b+36a^3b^2x^3+18a^2b^3x^6}$$
$$+\operatorname{RootSum}\left(19683t^3a^8b^4-125A^3b^3-75A^2Bab^2-15AB^2a^2b-B^3a^3,\left(t\mapsto t\log\left(\frac{27ta^3b}{5Ab+Ba}+x\right)\right)\right)$$

input
```
integrate((B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
(x**4*(5*A*b**2 + B*a*b) + x*(8*A*a*b - 2*B*a**2))/(18*a**4*b + 36*a**3*b*
*2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**8*b**4 - 125*A**3*b*
*3 - 75*A**2*B*a*b**2 - 15*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(27
*_t*a**3*b/(5*A*b + B*a) + x)))
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx = \frac{(Bab+5\,Ab^2)x^4 - 2\,(Ba^2-4\,Aab)x}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)} + \frac{\sqrt{3}(Ba+5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba+5\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba+5\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*((B*a*b + 5*A*b^2)*x^4 - 2*(B*a^2 - 4*A*a*b)*x)/(a^2*b^3*x^6 + 2*a^3*
b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(B*a + 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a
/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) - 1/54*(B*a + 5*A*b)*log(x^2
 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2/3)) + 1/27*(B*a + 5*A*b)
*log(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}(Ba+5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{(Ba+5\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{(Ba+5\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b} + \frac{Babx^4+5\,Ab^2x^4-2\,Ba^2x+8\,Aabx}{18\,(bx^3+a)^2a^2b}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output

```
-1/27*sqrt(3)*(B*a + 5*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(B*a + 5*A*b)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(B*a + 5*A*b)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/18*(B*a*b*x^4 + 5*A*b^2*x^4 - 2*B*a^2
*x + 8*A*a*b*x)/((b*x^3 + a)^2*a^2*b)
```

3.102.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.88

$$
\begin{aligned}
\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx &= \frac{\frac{x^4\,(5\,A\,b+B\,a)}{18\,a^2}+\frac{x\,(4\,A\,b-B\,a)}{9\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}} \\
&\quad -\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}} \\
&\quad +\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}}
\end{aligned}
$$

input

```
int((A + B*x^3)/(a + b*x^3)^3,x)
```

output

```
((x^4*(5*A*b + B*a))/(18*a^2) + (x*(4*A*b - B*a))/(9*a*b))/(a^2 + b^2*x^6
+ 2*a*b*x^3) + (log(b^(1/3)*x + a^(1/3))*(5*A*b + B*a))/(27*a^(8/3)*b^(4/3
)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/
2)*(5*A*b + B*a))/(27*a^(8/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/
3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a))/(27*a^(8/3)*b^(4/3))
```

3.103 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx$

3.103.1 Optimal result

Integrand size = 20, antiderivative size = 227

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx = -\frac{2(7Ab-aB)}{9a^3bx}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)} + \frac{2(7Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}} - \frac{(7Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}b^{2/3}}$$

output
```
-2/9*(7*A*b-B*a)/a^3/b/x+1/6*(A*b-B*a)/a/b/x/(b*x^3+a)^2+1/18*(7*A*b-B*a)/
a^2/b/x/(b*x^3+a)+2/27*(7*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(2/3)-
1/27*(7*A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(2/3
)+2/27*(7*A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10
/3)/b^(2/3)*3^(1/2)
```

3.103.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx$$
$$=\frac{-\frac{54\sqrt[3]{a}A}{x}+\frac{9a^{4/3}(-Ab+aB)x^2}{(a+bx^3)^2}+\frac{6\sqrt[3]{a}(-5Ab+2aB)x^2}{a+bx^3}+\frac{4\sqrt{3}(7Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{4(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+}{54a^{10/3}}$$

input
```
Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^3),x]
```

output
```
((-54*a^(1/3)*A)/x + (9*a^(4/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(
1/3)*(-5*A*b + 2*a*B)*x^2)/(a + b*x^3) + (4*Sqrt[3]*(7*A*b - a*B)*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (4*(7*A*b - a*B)*Log[a^(1/3
) + b^(1/3)*x])/b^(2/3) + (2*(-7*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/b^(2/3))/(54*a^(10/3))
```

3.103.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {957, 819, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx$$
$$\Big\downarrow \text{957}$$
$$\frac{(7Ab-aB)\int \frac{1}{x^2\left(bx^3+a\right)^2}dx}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$
$$\Big\downarrow \text{819}$$
$$\frac{(7Ab-aB)\left(\frac{4\int \frac{1}{x^2\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{847}$$

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\int\frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{821}$$

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1142}$$

3.103. $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx$

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

3.103. $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx$

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

3.103. $\int \frac{A+Bx^3}{x^2(a+bx^3)^3}\,dx$

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; {\color{blue}1082}$$

3.103. $\int \frac{A+Bx^3}{x^2(a+bx^3)^3}\,dx$

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$$\Big\downarrow 217$$

3.103. $\int \frac{A+Bx^3}{x^2(a+bx^3)^3}\,dx$

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1103}$$

3.103. $\int \frac{A+Bx^3}{x^2(a+bx^3)^3}\,dx$

$$\frac{(7Ab-aB)\left(\frac{4\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

input
```
Int[(A + B*x^3)/(x^2*(a + b*x^3)^3),x]
```

3.103. $\int \frac{A+Bx^3}{x^2(a+bx^3)^3}\,dx$

output
```
(A*b - a*B)/(6*a*b*x*(a + b*x^3)^2) + ((7*A*b - a*B)*(1/(3*a*x*(a + b*x^3)
) + (4*(-(1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) +
 (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))
))/a))/(3*a)))/(6*a*b)
```

3.103.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.103. $\int \frac{A+Bx^3}{x^2(a+bx^3)^3}\,dx$

3.103.4 Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.70

method	result
default	$-\frac{A}{a^{3}x}-\frac{\frac{\left(\frac{5}{9}b^{2}A-\frac{2}{9}abB\right)x^{5}+\frac{a(13Ab-7Ba)x^{2}}{18}}{\left(bx^{3}+a\right)^{2}}+\left(\frac{14Ab}{9}-\frac{2Ba}{9}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{a^{3}}$
risch	$\frac{-\frac{2b(7Ab-Ba)x^{6}}{9a^{3}}-\frac{7(7Ab-Ba)x^{3}}{18a^{2}}-\frac{A}{a}}{x\left(bx^{3}+a\right)^{2}}+\frac{2\left(\sum_{\_R=\operatorname{RootOf}\left(a^{10}b^{2}\_Z^{3}-343A^{3}b^{3}+147A^{2}Bab^{2}-21AB^{2}a^{2}b+B^{3}a^{3}\right)}\_R\ln\left(\left(-4\_R^{3}a^{10}b^{2}\right.\right.\right.}{27}$

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-A/a^3/x-1/a^3*(((5/9*b^2*A-2/9*a*b*B)*x^5+1/18*a*(13*A*b-7*B*a)*x^2)/(b*x
^3+a)^2+(14/9*A*b-2/9*B*a)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/
b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))
```

3.103.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(179) = 358.

Time = 0.29 (sec) , antiderivative size = 776, normalized size of antiderivative = 3.42

$$\int\frac{A+Bx^{3}}{x^{2}\left(a+bx^{3}\right)^{3}}\,dx$$

$$=\left[\frac{12\,(Ba^{2}b^{3}-7\,Aab^{4})x^{6}-54\,Aa^{3}b^{2}+21\,(Ba^{3}b^{2}-7\,Aa^{2}b^{3})x^{3}-6\,\sqrt{\frac{1}{3}}((Ba^{2}b^{3}-7\,Aab^{4})x^{7}+2\,(Ba^{3}b^{2}-}{}\right.$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 - 7*A
*a^2*b^3)*x^3 - 6*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b^2 -
7*A*a^2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt(-(a*b^2)^(1/3)/a)*log((
2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)
*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*((B*a*b^2
 - 7*A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*
b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a*b^2
- 7*A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b
^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^6*b^2
*x), 1/54*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 -
 7*A*a^2*b^3)*x^3 - 12*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b
^2 - 7*A*a^2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt((a*b^2)^(1/3)/a)*a
rctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + 2*((B*
a*b^2 - 7*A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x
)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a
*b^2 - 7*A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)
*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^
6*b^2*x)]
```

3.103.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx = \frac{-18Aa^2+x^6(-28Ab^2+4Bab)+x^3(-49Aab+7Ba^2)}{18a^5x+36a^4bx^4+18a^3b^2x^7}$$

$$+\mathrm{RootSum}\Bigg(19683t^3a^{10}b^2-2744A^3b^3+1176A^2Bab^2-168AB^2a^2b+8B^3a^3,\ \Bigg(t\mapsto t\log\Bigg(\frac{729}{196A^2b^2-56}$$

input
```
integrate((B*x**3+A)/x**2/(b*x**3+a)**3,x)
```

output
```
(-18*A*a**2 + x**6*(-28*A*b**2 + 4*B*a*b) + x**3*(-49*A*a*b + 7*B*a**2))/(
18*a**5*x + 36*a**4*b*x**4 + 18*a**3*b**2*x**7) + RootSum(19683*_t**3*a**1
0*b**2 - 2744*A**3*b**3 + 1176*A**2*B*a*b**2 - 168*A*B**2*a**2*b + 8*B**3*
a**3, Lambda(_t, _t*log(729*_t**2*a**7*b/(196*A**2*b**2 - 56*A*B*a*b + 4*B
**2*a**2) + x)))
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx &= \frac{4\left(Bab-7\,Ab^2\right)x^6+7\left(Ba^2-7\,Aab\right)x^3-18\,Aa^2}{18\left(a^3b^2x^7+2\,a^4bx^4+a^5x\right)} \\ &\quad+\frac{2\sqrt{3}(Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &\quad+\frac{(Ba-7\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &\quad-\frac{2\left(Ba-7\,Ab\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\end{aligned}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(4*(B*a*b - 7*A*b^2)*x^6 + 7*(B*a^2 - 7*A*a*b)*x^3 - 18*A*a^2)/(a^3*b
^2*x^7 + 2*a^4*b*x^4 + a^5*x) + 2/27*sqrt(3)*(B*a - 7*A*b)*arctan(1/3*sqrt
(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(1/3)) + 1/27*(B*a - 7*A
*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(1/3)) - 2/27*(B*a
 - 7*A*b)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(1/3))
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx &= \frac{2\sqrt{3}(Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^3} \\ &\quad-\frac{(Ba-7\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^3} \\ &\quad-\frac{2\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4} \\ &\quad-\frac{A}{a^3x}+\frac{4\,Babx^5-10\,Ab^2x^5+7\,Ba^2x^2-13\,Aabx^2}{18\left(bx^3+a\right)^2a^3}\end{aligned}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
2/27*sqrt(3)*(B*a - 7*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(1/3)*a^3) - 1/27*(B*a - 7*A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^3) - 2/27*(B*a*(-a/b)^(1/3) - 7*A*b*(-a/
b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - A/(a^3*x) + 1/18*(
4*B*a*b*x^5 - 10*A*b^2*x^5 + 7*B*a^2*x^2 - 13*A*a*b*x^2)/((b*x^3 + a)^2*a^
3)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 7.01 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx = \frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(7\,A\,b-B\,a\right)}{27\,a^{10/3}\,b^{2/3}} - \frac{\frac{A}{a}+\frac{7\,x^3\,\left(7\,A\,b-B\,a\right)}{18\,a^2}+\frac{2\,b\,x^6\,\left(7\,A\,b-B\,a\right)}{9\,a^3}}{a^2\,x+2\,a\,b\,x^4+b^2\,x^7} + \frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(7\,A\,b-B\,a\right)}{27\,a^{10/3}\,b^{2/3}} - \frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(7\,A\,b-B\,a\right)}{27\,a^{10/3}\,b^{2/3}}$$

input
```
int((A + B*x^3)/(x^2*(a + b*x^3)^3),x)
```

output
```
(2*log(b^(1/3)*x + a^(1/3))*(7*A*b - B*a))/(27*a^(10/3)*b^(2/3)) - (A/a +
(7*x^3*(7*A*b - B*a))/(18*a^2) + (2*b*x^6*(7*A*b - B*a))/(9*a^3))/(a^2*x +
 b^2*x^7 + 2*a*b*x^4) + (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))
*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - B*a))/(27*a^(10/3)*b^(2/3)) - (2*log(3^(1
/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - B*
a))/(27*a^(10/3)*b^(2/3))
```

3.104 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx$

3.104.1 Optimal result

Integrand size = 20, antiderivative size = 227

$$\begin{aligned}\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx =& -\frac{5(4Ab-aB)}{18a^3bx^2}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}+\frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)}\\ &+\frac{5(4Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt[3]{b}}-\frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}}\\ &+\frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}}\end{aligned}$$

output
```
-5/18*(4*A*b-B*a)/a^3/b/x^2+1/6*(A*b-B*a)/a/b/x^2/(b*x^3+a)^2+1/9*(4*A*b-B
*a)/a^2/b/x^2/(b*x^3+a)-5/27*(4*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^
(1/3)+5/54*(4*A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/
b^(1/3)+5/27*(4*A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))
/a^(11/3)/b^(1/3)*3^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx$$

$$=\frac{-\frac{27a^{2/3}A}{x^2}+\frac{9a^{5/3}(-Ab+aB)x}{(a+bx^3)^2}+\frac{3a^{2/3}(-11Ab+5aB)x}{a+bx^3}+\frac{10\sqrt{3}(4Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{10(-4Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}}{54a^{11/3}}$$

input
```
Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^3),x]
```

output
```
((-27*a^(2/3)*A)/x^2 + (9*a^(5/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(
2/3)*(-11*A*b + 5*a*B)*x)/(a + b*x^3) + (10*Sqrt[3]*(4*A*b - a*B)*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (10*(-4*A*b + a*B)*Log[a^(1
/3) + b^(1/3)*x])/b^(1/3) + (5*(4*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/b^(1/3))/(54*a^(11/3))
```

3.104.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {957, 819, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(4Ab-aB)\int \frac{1}{x^3\left(bx^3+a\right)^2}dx}{3ab}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{(4Ab-aB)\left(\frac{5\int \frac{1}{x^3\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 847$$

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 750$$

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 16$$

3.104. $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx$

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 1142$$

3.104. $\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx$

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}$$

$$\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{25}$$

3.104. $\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx$

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}+$$

$$\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow 27$$

3.104. $\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx$

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}$$

$$\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1082$$

3.104. $\int\frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx$

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}$$

$$\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{217}$$

3.104. $\int\frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx$

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 1103}$$

3.104. $\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx$

$$\frac{(4Ab-aB)\left(\frac{5\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

input

```
Int[(A + B*x^3)/(x^3*(a + b*x^3)^3),x]
```

3.104. $\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx$

output
```
(A*b - a*B)/(6*a*b*x^2*(a + b*x^3)^2) + ((4*A*b - a*B)*(1/(3*a*x^2*(a + b*
x^3)) + (5*(-1/2*1/(a*x^2) - (b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/
3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) -
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/
a))/(3*a)))/(3*a*b)
```

3.104.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.104. $\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx$

3.104.4 Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.70

method	result
default	$-\frac{A}{2a^{3}x^{2}}-\frac{\frac{\left(\frac{11}{18}b^{2}A-\frac{5}{18}abB\right)x^{4}+\frac{a(7Ab-4Ba)x}{9}}{\left(bx^{3}+a\right)^{2}}+\frac{5(4Ab-Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{9}}{a^{3}}$
risch	$\frac{-\frac{5b(4Ab-Ba)x^{6}}{18a^{3}}-\frac{4(4Ab-Ba)x^{3}}{9a^{2}}-\frac{A}{2a}}{x^{2}\left(bx^{3}+a\right)^{2}}+\frac{5\left(\sum_{\_R=\operatorname{RootOf}\left(a^{11}b\_Z^{3}+64A^{3}b^{3}-48A^{2}Bab^{2}+12AB^{2}a^{2}b-B^{3}a^{3}\right)}\_R\ln\left(\left(-4\_R^{3}a^{11}b-1\right.\right.\right.}{27}$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a^3/x^2-1/a^3*(((11/18*b^2*A-5/18*a*b*B)*x^4+1/9*a*(7*A*b-4*B*a)*x)
/(b*x^3+a)^2+5/9*(4*A*b-B*a)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a
/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(179) = 358.

Time = 0.27 (sec) , antiderivative size = 812, normalized size of antiderivative = 3.58

$$\int \frac{A+Bx^{3}}{x^{3}\left(a+bx^{3}\right)^{3}}\,dx$$

$$=\left[\frac{15\left(Ba^{3}b^{2}-4\,Aa^{2}b^{3}\right)x^{6}-27\,Aa^{4}b+24\left(Ba^{4}b-4\,Aa^{3}b^{2}\right)x^{3}-15\sqrt{\frac{1}{3}}\left(\left(Ba^{2}b^{3}-4\,Aab^{4}\right)x^{8}+2\left(Ba^{3}b^{2}-\right.\right.}{}\right.$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b + 24*(B*a^4*b - 4*A*a
^3*b^2)*x^3 - 15*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*x^8 + 2*(B*a^3*b^2 - 4
*A*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt((-a^2*b)^(1/3)/b)*log(
(2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b
)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*((B
*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*
x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 1
0*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^
2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^5*b^3*x^8 + 2*a^6
*b^2*x^5 + a^7*b*x^2), 1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b
 + 24*(B*a^4*b - 4*A*a^3*b^2)*x^3 + 30*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*
x^8 + 2*(B*a^3*b^2 - 4*A*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt(
-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a
)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 5*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b -
 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-
a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^
2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x +
 (-a^2*b)^(2/3)))/(a^5*b^3*x^8 + 2*a^6*b^2*x^5 + a^7*b*x^2)]
```

3.104.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx = \frac{-9Aa^2+x^6(-20Ab^2+5Bab)+x^3(-32Aab+8Ba^2)}{18a^5x^2+36a^4bx^5+18a^3b^2x^8}$$
$$+\mathrm{RootSum}\left(19683t^3a^{11}b+8000A^3b^3-6000A^2Bab^2+1500AB^2a^2b-125B^3a^3,\left(t\mapsto t\log\left(\frac{27ta^4}{-20Ab+5}\right.\right.\right.$$

input
```
integrate((B*x**3+A)/x**3/(b*x**3+a)**3,x)
```

output
```
(-9*A*a**2 + x**6*(-20*A*b**2 + 5*B*a*b) + x**3*(-32*A*a*b + 8*B*a**2))/(1
8*a**5*x**2 + 36*a**4*b*x**5 + 18*a**3*b**2*x**8) + RootSum(19683*_t**3*a*
*11*b + 8000*A**3*b**3 - 6000*A**2*B*a*b**2 + 1500*A*B**2*a**2*b - 125*B**
3*a**3, Lambda(_t, _t*log(27*_t*a**4/(-20*A*b + 5*B*a) + x)))
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx = \frac{5\,(Bab-4\,Ab^2)x^6+8\,(Ba^2-4\,Aab)x^3-9\,Aa^2}{18\,(a^3b^2x^8+2\,a^4bx^5+a^5x^2)} + \frac{5\,\sqrt{3}(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\,(Ba-4\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\,(Ba-4\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(5*(B*a*b - 4*A*b^2)*x^6 + 8*(B*a^2 - 4*A*a*b)*x^3 - 9*A*a^2)/(a^3*b^
2*x^8 + 2*a^4*b*x^5 + a^5*x^2) + 5/27*sqrt(3)*(B*a - 4*A*b)*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) - 5/54*(B*a - 4*
A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) + 5/27*(B*
a - 4*A*b)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx = -\frac{5\,(Ba-4\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4} + \frac{5\,\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-4\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b} + \frac{5\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-4\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b} + \frac{5\,Babx^6-20\,Ab^2x^6+8\,Ba^2x^3-32\,Aabx^3-9\,Aa^2}{18\left(bx^4+ax\right)^2a^3}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-5/27*(B*a - 4*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 5/27*sqr
t(3)*((-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) + 5/54*((-a*b^2)^(1/3)*B*a - 4*(-a*b^
2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) + 1/18*(5*B
*a*b*x^6 - 20*A*b^2*x^6 + 8*B*a^2*x^3 - 32*A*a*b*x^3 - 9*A*a^2)/((b*x^4 +
a*x)^2*a^3)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx=-\frac{\frac{A}{2\,a}+\frac{4\,x^3\,\left(4\,A\,b-B\,a\right)}{9\,a^2}+\frac{5\,b\,x^6\,\left(4\,A\,b-B\,a\right)}{18\,a^3}}{a^2\,x^2+2\,a\,b\,x^5+b^2\,x^8}-\frac{5\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(4\,A\,b-B\,a\right)}{27\,a^{11/3}\,b^{1/3}}+\frac{5\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(4\,A\,b-B\,a\right)}{27\,a^{11/3}\,b^{1/3}}-\frac{5\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(4\,A\,b-B\,a\right)}{27\,a^{11/3}\,b^{1/3}}$$

input
```
int((A + B*x^3)/(x^3*(a + b*x^3)^3),x)
```

output
```
(5*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(4*A*b - B*a))/(27*a^(11/3)*b^(1/3)) - (5*log(b^(1/3)*x + a^(1/3))*(4*A*b
- B*a))/(27*a^(11/3)*b^(1/3)) - (A/(2*a) + (4*x^3*(4*A*b - B*a))/(9*a^2) +
 (5*b*x^6*(4*A*b - B*a))/(18*a^3))/(a^2*x^2 + b^2*x^8 + 2*a*b*x^5) - (5*lo
g(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*A*
b - B*a))/(27*a^(11/3)*b^(1/3))
```

3.105 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$

3.105.1 Optimal result

Integrand size = 20, antiderivative size = 246

$$\begin{aligned}\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx =& -\frac{7(5Ab-2aB)}{36a^3bx^4}+\frac{7(5Ab-2aB)}{9a^4x}+\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}\\ &+\frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)}-\frac{7\sqrt[3]{b}(5Ab-2aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{13/3}}\\ &-\frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{13/3}}\\ &+\frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{13/3}}\end{aligned}$$

output
```
-7/36*(5*A*b-2*B*a)/a^3/b/x^4+7/9*(5*A*b-2*B*a)/a^4/x+1/6*(A*b-B*a)/a/b/x^
4/(b*x^3+a)^2+1/9*(5*A*b-2*B*a)/a^2/b/x^4/(b*x^3+a)-7/27*b^(1/3)*(5*A*b-2*
B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)+7/54*b^(1/3)*(5*A*b-2*B*a)*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(13/3)-7/27*b^(1/3)*(5*A*b-2*B*a)*arctan(
1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)*3^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx = \frac{-\frac{27a^{4/3}A}{x^4}-\frac{108\sqrt[3]{a}(-3Ab+aB)}{x}-\frac{18a^{4/3}b(-Ab+aB)x^2}{\left(a+bx^3\right)^2}-\frac{12\sqrt[3]{a}b(-8Ab+5aB)x^2}{a+bx^3}-28\sqrt{3}\sqrt[3]{b}(5Ab-2aB)\arctan\left(\frac{1-2}{}\right.}{108a^{13/}}$$

input
```
Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^3),x]
```

output
```
((-27*a^(4/3)*A)/x^4 - (108*a^(1/3)*(-3*A*b + a*B))/x - (18*a^(4/3)*b*(-(A
*b) + a*B)*x^2)/(a + b*x^3)^2 - (12*a^(1/3)*b*(-8*A*b + 5*a*B)*x^2)/(a + b
*x^3) - 28*Sqrt[3]*b^(1/3)*(5*A*b - 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] + 28*b^(1/3)*(-5*A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*b
^(1/3)*(5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(10
8*a^(13/3))
```

3.105.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.91, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {957, 819, 847, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(5Ab-2aB)\int \frac{1}{x^5\left(bx^3+a\right)^2}dx}{3ab}+\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{(5Ab-2aB)\left(\frac{7\int \frac{1}{x^5\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax^4(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{847}$$

$$\frac{(5Ab-2aB)\left(\frac{7\left(-\frac{b\int \frac{1}{x^2\left(bx^3+a\right)}dx}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{1}{3ax^4(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{847}$$

$$\frac{(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{1}{3ax^4(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{821}$$

3.105. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$

$$\frac{(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{1}{3ax^4(a+bx^3)}\right)}{3ab}+$$

$$\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 16$$

3.105. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$

$$\frac{(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{1}{3ax^4(a+bx^3)}\right)}{3ab}+\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1142}$$

3.105. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$

$$(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{}{3ax}\right.$$

3.105. $\int\frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$

$$\Big\downarrow \ 25$$

3.105. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$

$$(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{}{3ax^4(}\right.$$

3.105. $\int\frac{A+Bx^3}{x^5(a+bx^3)^3}\,dx$

$$\Big\downarrow 27$$

$$\frac{(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{}{3ax^4}\right)}{3ab}$$

$$\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow 1082$$

3.105. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$

$$(5Ab-2aB)\,\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}\,d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{}{3a}$$

3.105. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$

$$\Big\downarrow \; 217$$

3.105. $\int \frac{A+Bx^3}{x^5(a+bx^3)^3}\,dx$

$$(5Ab-2aB)\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{1}{3ax^4(a+bx^3)}$$

3.105. $\int\frac{A+Bx^3}{x^5(a+bx^3)^3}\,dx$

$$\downarrow\ 1103$$

3.105. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$

$$(5Ab-2aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{3a}+\frac{1}{3ax^4(a+bx^3)}\right)$$

3.105. $\int \frac{A+Bx^3}{x^5(a+bx^3)^3}\,dx$

input
```
Int[(A + B*x^3)/(x^5*(a + b*x^3)^3),x]
```

output
```
(A*b - a*B)/(6*a*b*x^4*(a + b*x^3)^2) + ((5*A*b - 2*a*B)*(1/(3*a*x^4*(a +
b*x^3)) + (7*(-1/4*1/(a*x^4) - (b*(-(1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(
1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^
(1/3)))/(3*a^(1/3)*b^(1/3))))/a))/a))/(3*a)))/(3*a*b)
```

3.105.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.105. $\int \frac{A+Bx^3}{x^5(a+bx^3)^3}\,dx$

3.105.4 Maple [A] (verified)

Time = 4.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.71

method	result
default	$-\frac{A}{4a^3x^4}-\frac{-3Ab+Ba}{a^4x}+\frac{b\left(\frac{\left(\frac{8}{9}b^2A-\frac{5}{9}abB\right)x^5+\frac{a(19Ab-13Ba)x^2}{18}}{\left(b\,x^3+a\right)^2}+\left(\frac{35Ab}{9}-\frac{14Ba}{9}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+-\right.\right.}{a^4}$
risch	$\frac{\frac{7b^2(5Ab-2Ba)x^9}{9a^4}+\frac{49b(5Ab-2Ba)x^6}{36a^3}+\frac{(5Ab-2Ba)x^3}{2a^2}-\frac{A}{4a}}{x^4\left(b\,x^3+a\right)^2}+\frac{7\left(\sum_{\_R=\operatorname{RootOf}\left(a^{13}\_Z^3+125A^3b^4-150A^2Ba\,b^3+60A\,B^2a^2b^2-8B^3a^3b\right)}\_R\right.}{}$

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/a^3/x^4-(-3*A*b+B*a)/a^4/x+1/a^4*b*(((8/9*b^2*A-5/9*a*b*B)*x^5+1/18
*a*(19*A*b-13*B*a)*x^2)/(b*x^3+a)^2+(35/9*A*b-14/9*B*a)*(-1/3/b/(a/b)^(1/3
)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/
3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx =$$

$$-\frac{84\,(2\,Bab^2-5\,Ab^3)x^9+147\,(2\,Ba^2b-5\,Aab^2)x^6+27\,Aa^3+54\,(2\,Ba^3-5\,Aa^2b)x^3+28\,\sqrt{3}((2\,Bab^2-}{}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/108*(84*(2*B*a*b^2 - 5*A*b^3)*x^9 + 147*(2*B*a^2*b - 5*A*a*b^2)*x^6 + 2
7*A*a^3 + 54*(2*B*a^3 - 5*A*a^2*b)*x^3 + 28*sqrt(3)*((2*B*a*b^2 - 5*A*b^3)
*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*(-b/a)^
(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 14*((2*B*a*b^2 -
5*A*b^3)*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)
*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) + 28*((2*B*a*
b^2 - 5*A*b^3)*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b
)*x^4)*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^4*b^2*x^10 + 2*a^5*b*x^7
 + a^6*x^4)
```

3.105.6 Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$$

$$= \operatorname{RootSum}\Bigg(19683t^3a^{13} + 42875A^3b^4 - 51450A^2Bab^3 + 20580AB^2a^2b^2 - 2744B^3a^3b, \Bigg(t \mapsto t\log\Bigg(\frac{}{1225A}$$

$$+ \frac{-9Aa^3 + x^9\cdot\left(140Ab^3 - 56Bab^2\right) + x^6\cdot\left(245Aab^2 - 98Ba^2b\right) + x^3\cdot\left(90Aa^2b - 36Ba^3\right)}{36a^6x^4 + 72a^5bx^7 + 36a^4b^2x^{10}}$$

input
```
integrate((B*x**3+A)/x**5/(b*x**3+a)**3,x)
```

output
```
RootSum(19683*_t**3*a**13 + 42875*A**3*b**4 - 51450*A**2*B*a*b**3 + 20580*
A*B**2*a**2*b**2 - 2744*B**3*a**3*b, Lambda(_t, _t*log(729*_t**2*a**9/(122
5*A**2*b**3 - 980*A*B*a*b**2 + 196*B**2*a**2*b) + x))) + (-9*A*a**3 + x**9
*(140*A*b**3 - 56*B*a*b**2) + x**6*(245*A*a*b**2 - 98*B*a**2*b) + x**3*(90
*A*a**2*b - 36*B*a**3))/(36*a**6*x**4 + 72*a**5*b*x**7 + 36*a**4*b**2*x**1
0)
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$$
$$= -\frac{28\,(2\,Bab^2-5\,Ab^3)x^9+49\,(2\,Ba^2b-5\,Aab^2)x^6+9\,Aa^3+18\,(2\,Ba^3-5\,Aa^2b)x^3}{36\,(a^4b^2x^{10}+2\,a^5bx^7+a^6x^4)}$$
$$-\frac{7\sqrt{3}(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$-\frac{7\,(2\,Ba-5\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7\,(2\,Ba-5\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input

```
integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="maxima")
```

output

```
-1/36*(28*(2*B*a*b^2 - 5*A*b^3)*x^9 + 49*(2*B*a^2*b - 5*A*a*b^2)*x^6 + 9*A
*a^3 + 18*(2*B*a^3 - 5*A*a^2*b)*x^3)/(a^4*b^2*x^10 + 2*a^5*b*x^7 + a^6*x^4
) - 7/27*sqrt(3)*(2*B*a - 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a
/b)^(1/3))/(a^4*(a/b)^(1/3)) - 7/54*(2*B*a - 5*A*b)*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(a^4*(a/b)^(1/3)) + 7/27*(2*B*a - 5*A*b)*log(x + (a/b)^(1
/3))/(a^4*(a/b)^(1/3))
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx = \frac{7\left(2\,Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5}$$
$$+\frac{7\sqrt{3}\left(2\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5b}$$
$$-\frac{7\left(2\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^5b}$$
$$-\frac{10\,Bab^2x^5-16\,Ab^3x^5+13\,Ba^2bx^2-19\,Aab^2x^2}{18\,(bx^3+a)^2a^4}$$
$$-\frac{4\,Bax^3-12\,Abx^3+Aa}{4\,a^4x^4}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
7/27*(2*B*a*b*(-a/b)^(1/3) - 5*A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/a^5 + 7/27*sqrt(3)*(2*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/
3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b) - 7/
54*(2*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/(a^5*b) - 1/18*(10*B*a*b^2*x^5 - 16*A*b^3*x^5 + 13*B*a^2*b
*x^2 - 19*A*a*b^2*x^2)/((b*x^3 + a)^2*a^4) - 1/4*(4*B*a*x^3 - 12*A*b*x^3 +
 A*a)/(a^4*x^4)
```

3.105.9 Mupad [B] (verification not implemented)

Time = 6.86 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.98

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx
&= \frac{\frac{x^3\,(5\,A\,b-2\,B\,a)}{2\,a^2}-\frac{A}{4\,a}+\frac{7\,b^2\,x^9\,(5\,A\,b-2\,B\,a)}{9\,a^4}+\frac{49\,b\,x^6\,(5\,A\,b-2\,B\,a)}{36\,a^3}}{a^2\,x^4+2\,a\,b\,x^7+b^2\,x^{10}} \\
&\quad+\frac{7\,{(-b)}^{1/3}\,\ln\left(a^{1/3}\,{(-b)}^{8/3}+b^3\,x\right)\,(5\,A\,b-2\,B\,a)}{27\,a^{13/3}} \\
&\quad+\frac{7\,{(-b)}^{1/3}\,\ln\left(a^{1/3}\,{(-b)}^{8/3}-2\,b^3\,x+\sqrt{3}\,a^{1/3}\,{(-b)}^{8/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-2\,B\,a)}{27\,a^{13/3}} \\
&\quad-\frac{7\,{(-b)}^{1/3}\,\ln\left(2\,b^3\,x-a^{1/3}\,{(-b)}^{8/3}+\sqrt{3}\,a^{1/3}\,{(-b)}^{8/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-2\,B\,a)}{27\,a^{13/3}}
\end{aligned}
$$

input
```
int((A + B*x^3)/(x^5*(a + b*x^3)^3),x)
```

output
```
((x^3*(5*A*b - 2*B*a))/(2*a^2) - A/(4*a) + (7*b^2*x^9*(5*A*b - 2*B*a))/(9*
a^4) + (49*b*x^6*(5*A*b - 2*B*a))/(36*a^3))/(a^2*x^4 + b^2*x^10 + 2*a*b*x^
7) + (7*(-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(5*A*b - 2*B*a))/(27*a^
(13/3)) + (7*(-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*b^3*x + 3^(1/2)*a^(1/3)
*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(5*A*b - 2*B*a))/(27*a^(13/3)) - (7
*(-b)^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3)*(-b)^(8/3)*
1i)*((3^(1/2)*1i)/2 + 1/2)*(5*A*b - 2*B*a))/(27*a^(13/3))
```

3.106 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$

3.106.1 Optimal result

Integrand size = 20, antiderivative size = 246

$$\begin{aligned}\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx = &-\frac{4(11Ab-5aB)}{45a^3bx^5}+\frac{2(11Ab-5aB)}{9a^4x^2}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}\\ &+\frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)}-\frac{4b^{2/3}(11Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{14/3}}\\ &+\frac{4b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}}\\ &-\frac{2b^{2/3}(11Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{14/3}}\end{aligned}$$

output

```
-4/45*(11*A*b-5*B*a)/a^3/b/x^5+2/9*(11*A*b-5*B*a)/a^4/x^2+1/6*(A*b-B*a)/a/
b/x^5/(b*x^3+a)^2+1/18*(11*A*b-5*B*a)/a^2/b/x^5/(b*x^3+a)+4/27*b^(2/3)*(11
*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)-2/27*b^(2/3)*(11*A*b-5*B*a)*ln(
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)-4/27*b^(2/3)*(11*A*b-5*B*a
)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)*3^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$$

$$=\frac{-\frac{54a^{5/3}A}{x^5}-\frac{135a^{2/3}(-3Ab+aB)}{x^2}-\frac{45a^{5/3}b(-Ab+aB)x}{(a+bx^3)^2}-\frac{15a^{2/3}b(-17Ab+11aB)x}{a+bx^3}-40\sqrt{3}b^{2/3}(11Ab-5aB)\arctan\left(\frac{1-}{}\right.}{270a}$$

input
```
Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^3),x]
```

output
```
((-54*a^(5/3)*A)/x^5 - (135*a^(2/3)*(-3*A*b + a*B))/x^2 - (45*a^(5/3)*b*(-
(A*b) + a*B)*x)/(a + b*x^3)^2 - (15*a^(2/3)*b*(-17*A*b + 11*a*B)*x)/(a + b
*x^3) - 40*Sqrt[3]*b^(2/3)*(11*A*b - 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]] + 40*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x] + 20*
b^(2/3)*(-11*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(270*a^(14/3))
```

3.106.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.90, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {957, 819, 847, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(11Ab-5aB)\int \frac{1}{x^6\left(bx^3+a\right)^2}dx}{6ab}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{(11Ab-5aB)\left(\frac{8\int \frac{1}{x^6\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax^5(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 847$$

$$\frac{(11Ab-5aB)\left(\frac{8\left(-\frac{b\int \frac{1}{x^3\left(bx^3+a\right)}dx}{a}-\frac{1}{5ax^5}\right)}{3a}+\frac{1}{3ax^5(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 847$$

$$\frac{(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+\frac{1}{3ax^5(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 750$$

3.106. $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$

$$\frac{(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+\frac{1}{3ax^5(a+bx^3)}\right)}{6ab}+$$

$$\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{16}$$

3.106. $\int \frac{A+Bx^3}{x^6(a+bx^3)^3}\,dx$

$$\frac{(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+\frac{1}{3ax^5(a+bx^3)}\right)}{6ab}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1142$$

3.106. $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$

$$(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+$$

3.106. $\int \frac{A+Bx^3}{x^6(a+bx^3)^3}\,dx$

$$\Big\downarrow \ 25$$

3.106. $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$

$$(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+$$

3.106. $\int \frac{A+Bx^3}{x^6(a+bx^3)^3}\,dx$

$$\Big\downarrow \; 27$$

$$\frac{(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+\right.}{6ab}$$

$$\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1082$$

3.106. $\int \frac{A+Bx^3}{x^6(a+bx^3)^3}\,dx$

$$(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+\right.$$

3.106. $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$

$$\downarrow 217$$

3.106. $\int \frac{A+Bx^3}{x^6(a+bx^3)^3}\,dx$

$$(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}\right)+\frac{1}{3ax^5(a+bx^3}$$

3.106. $\int\frac{A+Bx^3}{x^6(a+bx^3)^3}\,dx$

$\downarrow$ 1103

3.106. $\int \frac{A+Bx^3}{x^6(a+bx^3)^3}\,dx$

$$(11Ab-5aB)\left(\frac{8\left(-\frac{b\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{a}-\frac{1}{5ax^5}\right)}{3a}+\frac{1}{3ax^5(a+bx}\right.$$

3.106. $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$

input
```
Int[(A + B*x^3)/(x^6*(a + b*x^3)^3),x]
```

output
```
(A*b - a*B)/(6*a*b*x^5*(a + b*x^3)^2) + ((11*A*b - 5*a*B)*(1/(3*a*x^5*(a +
 b*x^3)) + (8*(-1/5*1/(a*x^5) - (b*(-1/2*1/(a*x^2) - (b*(Log[a^(1/3) + b^(
1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*
b^(1/3)))/(3*a^(2/3))))/a))/a))/(3*a)))/(6*a*b)
```

3.106.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.106. $\int \frac{A+Bx^3}{x^6(a+bx^3)^3}\,dx$

3.106.4 Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.71

method	result
default	$-\frac{A}{5a^3x^5}-\frac{-3Ab+Ba}{2x^2a^4}+\frac{b\left(\frac{\left(\frac{17}{18}b^2A-\frac{11}{18}abB\right)x^4+\frac{a(10Ab-7Ba)x}{9}}{\left(bx^3+a\right)^2}+\frac{4(11Ab-5Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\ \mathrm{arct}}{}\right.}{9}\right.}{a^4}$
risch	$\frac{\frac{2b^2(11Ab-5Ba)x^9}{9a^4}+\frac{16b(11Ab-5Ba)x^6}{45a^3}+\frac{(11Ab-5Ba)x^3}{10a^2}-\frac{A}{5a}}{x^5\left(bx^3+a\right)^2}+\frac{4\left(\sum_{\_R=\mathrm{RootOf}\left(a^{14}\_Z^3-1331A^3b^5+1815A^2Bab^4-825AB^2a^2b^3+125B^3a\right.}\right.}{}$

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5*A/a^3/x^5-1/2*(-3*A*b+B*a)/x^2/a^4+1/a^4*b*(((17/18*b^2*A-11/18*a*b*B
)*x^4+1/9*a*(10*A*b-7*B*a)*x)/(b*x^3+a)^2+4/9*(11*A*b-5*B*a)*(1/3/b/(a/b)^
(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.56

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx =$$

$$-\frac{60\,(5\,Bab^2-11\,Ab^3)x^9+96\,(5\,Ba^2b-11\,Aab^2)x^6+54\,Aa^3+27\,(5\,Ba^3-11\,Aa^2b)x^3+40\,\sqrt{3}((5\,Bab}{}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/270*(60*(5*B*a*b^2 - 11*A*b^3)*x^9 + 96*(5*B*a^2*b - 11*A*a*b^2)*x^6 +
54*A*a^3 + 27*(5*B*a^3 - 11*A*a^2*b)*x^3 + 40*sqrt(3)*((5*B*a*b^2 - 11*A*b
^3)*x^11 + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b
^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) -
20*((5*B*a*b^2 - 11*A*b^3)*x^11 + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^
3 - 11*A*a^2*b)*x^5)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) +
 a^2*(b^2/a^2)^(2/3)) + 40*((5*B*a*b^2 - 11*A*b^3)*x^11 + 2*(5*B*a^2*b - 1
1*A*a*b^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b^2/a^2)^(1/3)*log(b*x + a*(
b^2/a^2)^(1/3)))/(a^4*b^2*x^11 + 2*a^5*b*x^8 + a^6*x^5)
```

3.106.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$$

$$=\operatorname{RootSum}\left(19683t^3a^{14}-85184A^3b^5+116160A^2Bab^4-52800AB^2a^2b^3+8000B^3a^3b^2,\left(t\mapsto t\log\left(-\frac{}{-4}\right.\right.\right.$$

$$+\frac{-18Aa^3+x^9\cdot(220Ab^3-100Bab^2)+x^6\cdot(352Aab^2-160Ba^2b)+x^3\cdot(99Aa^2b-45Ba^3)}{90a^6x^5+180a^5bx^8+90a^4b^2x^{11}}$$

input
```
integrate((B*x**3+A)/x**6/(b*x**3+a)**3,x)
```

output
```
RootSum(19683*_t**3*a**14 - 85184*A**3*b**5 + 116160*A**2*B*a*b**4 - 52800
*A*B**2*a**2*b**3 + 8000*B**3*a**3*b**2, Lambda(_t, _t*log(-27*_t*a**5/(-4
4*A*b**2 + 20*B*a*b) + x))) + (-18*A*a**3 + x**9*(220*A*b**3 - 100*B*a*b**
2) + x**6*(352*A*a*b**2 - 160*B*a**2*b) + x**3*(99*A*a**2*b - 45*B*a**3))/
(90*a**6*x**5 + 180*a**5*b*x**8 + 90*a**4*b**2*x**11)
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx &= -\frac{20\,(5\,Bab^2-11\,Ab^3)x^9+32\,(5\,Ba^2b-11\,Aab^2)x^6+18\,Aa^3+9\,(5\,Ba^3-11\,Aa^2b)x^3}{90\,(a^4b^2x^{11}+2\,a^5bx^8+a^6x^5)} \\ &\quad - \frac{4\,\sqrt{3}(5\,Ba-11\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &\quad + \frac{2\,(5\,Ba-11\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{4\,(5\,Ba-11\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/90*(20*(5*B*a*b^2 - 11*A*b^3)*x^9 + 32*(5*B*a^2*b - 11*A*a*b^2)*x^6 + 1
8*A*a^3 + 9*(5*B*a^3 - 11*A*a^2*b)*x^3)/(a^4*b^2*x^11 + 2*a^5*b*x^8 + a^6*
x^5) - 4/27*sqrt(3)*(5*B*a - 11*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/(a^4*(a/b)^(2/3)) + 2/27*(5*B*a - 11*A*b)*log(x^2 - x*(a/b)
^(1/3) + (a/b)^(2/3))/(a^4*(a/b)^(2/3)) - 4/27*(5*B*a - 11*A*b)*log(x + (a
/b)^(1/3))/(a^4*(a/b)^(2/3))
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx &= -\frac{4\,\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-11\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5} \\ &\quad + \frac{4\,(5\,Bab-11\,Ab^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5} \\ &\quad - \frac{2\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-11\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^5} \\ &\quad - \frac{11\,Bab^2x^4-17\,Ab^3x^4+14\,Ba^2bx-20\,Aab^2x}{18\,\left(bx^3+a\right)^2a^4} \\ &\quad - \frac{5\,Bax^3-15\,Abx^3+2\,Aa}{10\,a^4x^5}\end{aligned}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-4/27*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 11*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sq
rt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^5 + 4/27*(5*B*a*b - 11*A*b^2)*(
-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^5 - 2/27*(5*(-a*b^2)^(1/3)*B*a -
11*(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^5 - 1/18
*(11*B*a*b^2*x^4 - 17*A*b^3*x^4 + 14*B*a^2*b*x - 20*A*a*b^2*x)/((b*x^3 + a
)^2*a^4) - 1/10*(5*B*a*x^3 - 15*A*b*x^3 + 2*A*a)/(a^4*x^5)
```

3.106.9 Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.84

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx &= \frac{\frac{x^3\,\left(11\,A\,b-5\,B\,a\right)}{10\,a^2}-\frac{A}{5\,a}+\frac{2\,b^2\,x^9\,\left(11\,A\,b-5\,B\,a\right)}{9\,a^4}+\frac{16\,b\,x^6\,\left(11\,A\,b-5\,B\,a\right)}{45\,a^3}}{a^2\,x^5+2\,a\,b\,x^8+b^2\,x^{11}} \\
&\quad + \frac{4\,b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(11\,A\,b-5\,B\,a\right)}{27\,a^{14/3}} \\
&\quad - \frac{4\,b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(11\,A\,b-5\,B\,a\right)}{27\,a^{14/3}} \\
&\quad + \frac{4\,b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(11\,A\,b-5\,B\,a\right)}{27\,a^{14/3}}
\end{aligned}
$$

input
```
int((A + B*x^3)/(x^6*(a + b*x^3)^3),x)
```

output
```
((x^3*(11*A*b - 5*B*a))/(10*a^2) - A/(5*a) + (2*b^2*x^9*(11*A*b - 5*B*a))/
(9*a^4) + (16*b*x^6*(11*A*b - 5*B*a))/(45*a^3))/(a^2*x^5 + b^2*x^11 + 2*a*
b*x^8) + (4*b^(2/3)*log(b^(1/3)*x + a^(1/3))*(11*A*b - 5*B*a))/(27*a^(14/3
)) - (4*b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*
1i)/2 + 1/2)*(11*A*b - 5*B*a))/(27*a^(14/3)) + (4*b^(2/3)*log(3^(1/2)*a^(1
/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a))/(
27*a^(14/3))
```

3.107 $\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx$

3.107.1 Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx=\frac{x^3}{3bd}+\frac{a^2\log(a+bx^3)}{3b^2(bc-ad)}-\frac{c^2\log(c+dx^3)}{3d^2(bc-ad)}$$

output
```
1/3*x^3/b/d+1/3*a^2*ln(b*x^3+a)/b^2/(-a*d+b*c)-1/3*c^2*ln(d*x^3+c)/d^2/(-a
*d+b*c)
```

3.107.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx=\frac{a^2d^2\log(a+bx^3)-b(d(-bc+ad)x^3+bc^2\log(c+dx^3))}{3b^2d^2(bc-ad)}$$

input
```
Integrate[x^8/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(a^2*d^2*Log[a + b*x^3] - b*(d*(-(b*c) + a*d)*x^3 + b*c^2*Log[c + d*x^3]))
/(3*b^2*d^2*(b*c - a*d))
```

3.107.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^3+a)(dx^3+c)}dx^3$$

$$\Big\downarrow \text{93}$$

$$\frac{1}{3}\int \left(\frac{a^2}{b(bc-ad)(bx^3+a)}+\frac{1}{bd}+\frac{c^2}{d(ad-bc)(dx^3+c)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{a^2\log\left(a+bx^3\right)}{b^2(bc-ad)}-\frac{c^2\log\left(c+dx^3\right)}{d^2(bc-ad)}+\frac{x^3}{bd}\right)$$

input
```
Int[x^8/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(x^3/(b*d) + (a^2*Log[a + b*x^3])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x^3])
/(d^2*(b*c - a*d)))/3
```

3.107.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.107.4 Maple [A] (verified)

Time = 4.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{x^{3}}{3bd}+\frac{c^{2}\ln\left(d\,x^{3}+c\right)}{3(ad-bc)d^{2}}-\frac{a^{2}\ln\left(b\,x^{3}+a\right)}{3(ad-bc)b^{2}}$	65
norman	$\frac{x^{3}}{3bd}+\frac{c^{2}\ln\left(d\,x^{3}+c\right)}{3(ad-bc)d^{2}}-\frac{a^{2}\ln\left(b\,x^{3}+a\right)}{3(ad-bc)b^{2}}$	65
risch	$\frac{x^{3}}{3bd}-\frac{a^{2}\ln\left(-b\,x^{3}-a\right)}{3b^{2}(ad-bc)}+\frac{c^{2}\ln\left(d\,x^{3}+c\right)}{3(ad-bc)d^{2}}$	68
parallelrisch	$-\frac{-x^{3}ab\,d^{2}+x^{3}b^{2}cd+a^{2}\ln\left(b\,x^{3}+a\right)d^{2}-c^{2}\ln\left(d\,x^{3}+c\right)b^{2}}{3b^{2}d^{2}(ad-bc)}$	70

input
```
int(x^8/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/b/d+1/3*c^2/(a*d-b*c)/d^2*ln(d*x^3+c)-1/3*a^2/(a*d-b*c)/b^2*ln(b*x
^3+a)
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx=\frac{a^2d^2\log\left(bx^3+a\right)-b^2c^2\log\left(dx^3+c\right)+(b^2cd-abd^2)x^3}{3\left(b^3cd^2-ab^2d^3\right)}$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/3*(a^2*d^2*log(b*x^3 + a) - b^2*c^2*log(d*x^3 + c) + (b^2*c*d - a*b*d^2)
*x^3)/(b^3*c*d^2 - a*b^2*d^3)
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x**8/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx=\frac{a^2\log(bx^3+a)}{3(b^3c-ab^2d)}-\frac{c^2\log(dx^3+c)}{3(bcd^2-ad^3)}+\frac{x^3}{3\,bd}$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*a^2*log(b*x^3 + a)/(b^3*c - a*b^2*d) - 1/3*c^2*log(d*x^3 + c)/(b*c*d^2
 - a*d^3) + 1/3*x^3/(b*d)
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx=\frac{a^2\log(|bx^3+a|)}{3(b^3c-ab^2d)}-\frac{c^2\log(|dx^3+c|)}{3(bcd^2-ad^3)}+\frac{x^3}{3\,bd}$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*a^2*log(abs(b*x^3 + a))/(b^3*c - a*b^2*d) - 1/3*c^2*log(abs(d*x^3 + c)
)/(b*c*d^2 - a*d^3) + 1/3*x^3/(b*d)
```

3.107.9 Mupad [B] (verification not implemented)

Time = 7.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx=\frac{a^2\,\ln\left(b\,x^3+a\right)}{3\,b^3\,c-3\,a\,b^2\,d}+\frac{c^2\,\ln\left(d\,x^3+c\right)}{3\,a\,d^3-3\,b\,c\,d^2}+\frac{x^3}{3\,b\,d}$$

input
```
int(x^8/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
(a^2*log(a + b*x^3))/(3*b^3*c - 3*a*b^2*d) + (c^2*log(c + d*x^3))/(3*a*d^3
 - 3*b*c*d^2) + x^3/(3*b*d)
```

3.108 $\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx$

3.108.1 Optimal result

Integrand size = 22, antiderivative size = 301

$$\begin{aligned}\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = {} & \frac{x^2}{2bd}-\frac{a^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}(bc-ad)}+\frac{c^{5/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{5/3}(bc-ad)} \\ & -\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)}+\frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{5/3}(bc-ad)} \\ & +\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}(bc-ad)} \\ & -\frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{5/3}(bc-ad)}\end{aligned}$$

output
```
1/2*x^2/b/d-1/3*a^(5/3)*ln(a^(1/3)+b^(1/3)*x)/b^(5/3)/(-a*d+b*c)+1/3*c^(5/
3)*ln(c^(1/3)+d^(1/3)*x)/d^(5/3)/(-a*d+b*c)+1/6*a^(5/3)*ln(a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)/(-a*d+b*c)-1/6*c^(5/3)*ln(c^(2/3)-c^(1/3)*
d^(1/3)*x+d^(2/3)*x^2)/d^(5/3)/(-a*d+b*c)-1/3*a^(5/3)*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)/(-a*d+b*c)*3^(1/2)+1/3*c^(5/3)*arcta
n(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(5/3)/(-a*d+b*c)*3^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.80

$$\int \frac{x^7}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$
$$=\frac{-\frac{3ax^2}{b}+\frac{3cx^2}{d}-\frac{2\sqrt{3}a^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}}+\frac{2\sqrt{3}c^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{5/3}}-\frac{2a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{5/3}}+\frac{2c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{d^{5/3}}}{6bc-6ad}$$

input
```
Integrate[x^7/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
((-3*a*x^2)/b + (3*c*x^2)/d - (2*Sqrt[3]*a^(5/3)*ArcTan[(1 - (2*b^(1/3)*x)
/a^(1/3))/Sqrt[3]])/b^(5/3) + (2*Sqrt[3]*c^(5/3)*ArcTan[(1 - (2*d^(1/3)*x)
/c^(1/3))/Sqrt[3]])/d^(5/3) - (2*a^(5/3)*Log[a^(1/3) + b^(1/3)*x])/b^(5/3)
 + (2*c^(5/3)*Log[c^(1/3) + d^(1/3)*x])/d^(5/3) + (a^(5/3)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - (c^(5/3)*Log[c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2])/d^(5/3))/(6*b*c - 6*a*d)
```

3.108.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {979, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\left(a+bx^3\right)\left(c+dx^3\right)}dx$$
$$\Big\downarrow \text{979}$$
$$\frac{x^2}{2bd}-\frac{\int \frac{2x\left(\left(bc+ad\right)x^3+ac\right)}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{2bd}$$
$$\Big\downarrow \text{27}$$
$$\frac{x^2}{2bd}-\frac{\int \frac{x\left(\left(bc+ad\right)x^3+ac\right)}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{bd}$$

$$\Big\downarrow \text{1054}$$

$$\frac{x^2}{2bd}-\frac{\int \left(\frac{dxa^2}{(ad-bc)(bx^3+a)}+\frac{bc^2x}{(bc-ad)(dx^3+c)}\right)dx}{bd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2bd}-\frac{\frac{a^{5/3}d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc-ad)}-\frac{a^{5/3}d\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)}+\frac{a^{5/3}d\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)}-\frac{bc^{5/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{2/3}(bc-ad)}+\frac{bc^{5/3}\,\mathrm{lo}}{}}{bd}$$

input
```
Int[x^7/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
x^2/(2*b*d) - ((a^(5/3)*d*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*b^(2/3)*(b*c - a*d)) - (b*c^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*
x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*d^(2/3)*(b*c - a*d)) + (a^(5/3)*d*Log[a^(1
/3) + b^(1/3)*x])/(3*b^(2/3)*(b*c - a*d)) - (b*c^(5/3)*Log[c^(1/3) + d^(1/
3)*x])/(3*d^(2/3)*(b*c - a*d)) - (a^(5/3)*d*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(6*b^(2/3)*(b*c - a*d)) + (b*c^(5/3)*Log[c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2])/(6*d^(2/3)*(b*c - a*d)))/(b*d)
```

3.108.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.108.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.76

method	result
default	$\frac{x^2}{2bd}+\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)c^2}{(ad-bc)d}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{(ad-}$
risch	$\frac{x^2}{2bd}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\left(a^3d^5-3a^2bc\,d^4+3a\,b^2c^2d^3-b^3c^3d^2\right)\_Z^3+b^3c^5\right)}\_R\ln\left(\left(\left(-a^5b^2c\,d^6+2a^4b^3c^2d^5-2a^3b^4c^3d^4+2a^2b^5c^4d^3-a\,b^6c^5\right.\right.\right.}{}$

input
```
int(x^7/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/b/d+(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2
-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(
2/(c/d)^(1/3)*x-1)))*c^2/(a*d-b*c)/d-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))
+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a^2/(a*d-b*c)/b
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.91

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx$$

$$= 2\sqrt{3}ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)-2\sqrt{3}bc\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}c}{3c}\right)+ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(a\right.$$

input
```
integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*a*d*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/
3) - sqrt(3)*a)/a) - 2*sqrt(3)*b*c*(-c^2/d^2)^(1/3)*arctan(1/3*(2*sqrt(3)*
d*x*(-c^2/d^2)^(1/3) + sqrt(3)*c)/c) + a*d*(a^2/b^2)^(1/3)*log(a*x^2 - b*x
*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) + b*c*(-c^2/d^2)^(1/3)*log(c*x^2 - d
*x*(-c^2/d^2)^(2/3) - c*(-c^2/d^2)^(1/3)) - 2*a*d*(a^2/b^2)^(1/3)*log(a*x
+ b*(a^2/b^2)^(2/3)) - 2*b*c*(-c^2/d^2)^(1/3)*log(c*x + d*(-c^2/d^2)^(2/3)
) + 3*(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2)
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(x**7/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.08

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^3c-ab^2d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}c^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd^2-ad^3\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{a^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{c^2\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} - \frac{a^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{c^2\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{x^2}{2\,bd}$$

input
```
integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^3*
c - a*b^2*d)*(a/b)^(1/3)) - 1/3*sqrt(3)*c^2*arctan(1/3*sqrt(3)*(2*x - (c/d
)^(1/3))/(c/d)^(1/3))/((b*c*d^2 - a*d^3)*(c/d)^(1/3)) + 1/6*a^2*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*c*(a/b)^(1/3) - a*b^2*d*(a/b)^(1/3)) - 1
/6*c^2*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*d^2*(c/d)^(1/3) - a*d^3
*(c/d)^(1/3)) - 1/3*a^2*log(x + (a/b)^(1/3))/(b^3*c*(a/b)^(1/3) - a*b^2*d*
(a/b)^(1/3)) + 1/3*c^2*log(x + (c/d)^(1/3))/(b*c*d^2*(c/d)^(1/3) - a*d^3*(
c/d)^(1/3)) + 1/2*x^2/(b*d)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.03

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(ab^2c-a^2bd)} + \frac{c^2\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2d-acd^2)} - \frac{(-ab^2)^{\frac{2}{3}}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^4c-\sqrt{3}ab^3d} + \frac{(-cd^2)^{\frac{2}{3}}\,c\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^3-\sqrt{3}ad^4} + \frac{(-ab^2)^{\frac{2}{3}}\,a\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(b^4c-ab^3d)} - \frac{(-cd^2)^{\frac{2}{3}}\,c\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^3-ad^4)} + \frac{x^2}{2\,bd}$$

input
```
integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*a^2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c - a^2*b*d) + 1/3
*c^2*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2*d - a*c*d^2) - (-a*b^2
)^(2/3)*a*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b
^4*c - sqrt(3)*a*b^3*d) + (-c*d^2)^(2/3)*c*arctan(1/3*sqrt(3)*(2*x + (-c/d
)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) + 1/6*(-a*b^2)^(2
/3)*a*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^4*c - a*b^3*d) - 1/6*(-c
*d^2)^(2/3)*c*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^3 - a*d^4) +
 1/2*x^2/(b*d)
```

3.108.9 Mupad [B] (verification not implemented)

Time = 16.27 (sec) , antiderivative size = 1751, normalized size of antiderivative = 5.82

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int(x^7/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(((((27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + 27*a*b^3*c*d^
3*(a*d + b*c)*(a*d - b*c)^4*(a^5/(b^5*(a*d - b*c)^3))^(2/3))*(a^5/(b^5*(a*
d - b*c)^3))^(1/3))/3 - (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*
c^2*d^6))/(b^2*d^2))*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/9 - (a^4*c^4*x*(a^2*
d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-a^5/(27*b^8*c^3 - 27*a^3*b^5*d^3 +
81*a^2*b^6*c*d^2 - 81*a*b^7*c^2*d))^(1/3) + log(((((27*a^2*b*c^2*d*x*(a^2*
d^2 + b^2*c^2)*(a*d - b*c)^2 + 27*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(-
c^5/(d^5*(a*d - b*c)^3))^(2/3))*(-c^5/(d^5*(a*d - b*c)^3))^(1/3))/3 - (9*(
a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(-c^5/(
d^5*(a*d - b*c)^3))^(2/3))/9 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(
b^2*d^2))*(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d^5 + 81*a*b^2*c^2*d^6 - 81*a^2*b
*c*d^7))^(1/3) - (log(((3^(1/2)*1i + 1)^2*(((3^(1/2)*1i + 1)*(27*a^2*b*c^2
*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1)^
2*(a*d + b*c)*(a*d - b*c)^4*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/4)*(a^5/(b^5*
(a*d - b*c)^3))^(1/3))/6 + (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7
*b*c^2*d^6))/(b^2*d^2))*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/36 + (a^4*c^4*x*(
a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-a^5/(27*b^8*c^3 - 27*a^3*b^5*d^
3 + 81*a^2*b^6*c*d^2 - 81*a*b^7*c^2*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log((
(3^(1/2)*1i - 1)^2*(((3^(1/2)*1i - 1)*(27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2
)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d -...
```

3.108. $\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx$

3.109 $\int \frac{x^6}{(a+bx^3)\left(c+dx^3\right)}\,dx$

3.109.1 Optimal result

Integrand size = 22, antiderivative size = 296

$$
\begin{aligned}
\int \frac{x^6}{(a+bx^3)\left(c+dx^3\right)}\,dx &= \frac{x}{bd} - \frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}(bc-ad)} + \frac{c^{4/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{4/3}(bc-ad)} \\
&\quad + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{4/3}(bc-ad)} \\
&\quad - \frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}(bc-ad)} \\
&\quad + \frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{4/3}(bc-ad)}
\end{aligned}
$$

output
```
x/b/d+1/3*a^(4/3)*ln(a^(1/3)+b^(1/3)*x)/b^(4/3)/(-a*d+b*c)-1/3*c^(4/3)*ln(
c^(1/3)+d^(1/3)*x)/d^(4/3)/(-a*d+b*c)-1/6*a^(4/3)*ln(a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/b^(4/3)/(-a*d+b*c)+1/6*c^(4/3)*ln(c^(2/3)-c^(1/3)*d^(1/3
)*x+d^(2/3)*x^2)/d^(4/3)/(-a*d+b*c)-1/3*a^(4/3)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)/a^(1/3)*3^(1/2))/b^(4/3)/(-a*d+b*c)*3^(1/2)+1/3*c^(4/3)*arctan(1/3*
(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(4/3)/(-a*d+b*c)*3^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.80

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx$$
$$=\frac{-\frac{6ax}{b}+\frac{6cx}{d}-\frac{2\sqrt{3}a^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}}+\frac{2\sqrt{3}c^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{4/3}}+\frac{2a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{4/3}}-\frac{2c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{4/3}}}{6bc-6ad}$$

input
```
Integrate[x^6/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
((-6*a*x)/b + (6*c*x)/d - (2*Sqrt[3]*a^(4/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]])/b^(4/3) + (2*Sqrt[3]*c^(4/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(
1/3))/Sqrt[3]])/d^(4/3) + (2*a^(4/3)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) - (
2*c^(4/3)*Log[c^(1/3) + d^(1/3)*x])/d^(4/3) - (a^(4/3)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3) + (c^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2])/d^(4/3))/(6*b*c - 6*a*d)
```

3.109.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {979, 1020, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx$$
$$\Big\downarrow \text{979}$$
$$\frac{x}{bd}-\frac{\int \frac{(bc+ad)x^3+ac}{(bx^3+a)(dx^3+c)}dx}{bd}$$
$$\Big\downarrow \text{1020}$$
$$\frac{x}{bd}-\frac{\frac{bc^2\int \frac{1}{dx^3+c}dx}{bc-ad}-\frac{a^2d\int \frac{1}{bx^3+a}dx}{bc-ad}}{bd}$$

$$\Big\downarrow \text{750}$$

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}}{bd}$$

$$\Big\downarrow \text{16}$$

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}}{bd}$$

$$\Big\downarrow \text{1142}$$

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\int -}{3a^{2/3}}\right.}{bc}}{bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\int}{3a^{2/3}}\right.}{bc-a}}{bd}$$

$$\Big\downarrow \text{27}$$

3.109. $\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int}{3a^{2/3}}\right.}{bc-}}{bd}$$

$$\Big\downarrow \; 1082$$

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int}{}}{3a^{2/3}}\right.}{}}{bd}$$

$$\Big\downarrow \; 217$$

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}\right.}{bc-ad}}{bd}$$

$$\Big\downarrow \; 1103$$

3.109. $\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{\frac{x}{bd}-\frac{bc^2\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a^2d\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}\right.}{bc-ad}}{bd}$$

input
```
Int[x^6/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
x/(b*d) - (-((a^2*d*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sq
rt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(b*c - a*d))
 + (b*c^2*(Log[c^(1/3) + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcT
an[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1/3)))/(3*c^(2/3))))/(b*c - a*d))/(b*d)
```

3.109.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 979

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1020

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.109. $\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx$

3.109.4 Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.76

method	result
default	$\frac{x}{bd}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)c^{2}}{d(ad-bc)}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\right.}{b(ad-bc)}$
risch	$\frac{x}{bd}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\left(a^{3}bd^{3}-3cd^{2}a^{2}b^{2}+3c^{2}dab^{3}-b^{4}c^{3}\right)\_Z^{3}+a^{4}d^{3}\right)}\_R\ln\left(\left(-a^{5}bcd^{5}-ab^{5}c^{5}d\right)x+\left(-a^{5}bd^{6}+3d^{5}cb^{2}a^{4}-2d^{4}c^{2}b^{3}a^{3}-\right.\right.}{3bd}$

input
```
int(x^6/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
x/b/d+(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^
(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)
^(1/3)*x-1)))/d*c^2/(a*d-b*c)-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/b*a^2/(a*d-b*c)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.77

$$\int\frac{x^{6}}{(a+bx^{3})\,(c+dx^{3})}\,dx=$$
$$-\frac{2\sqrt{3}ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+2\sqrt{3}bc\left(\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)-ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{2}+\right.}{}$$

input
```
integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*a*d*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) -
sqrt(3)*a)/a) + 2*sqrt(3)*b*c*(c/d)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(c/d)^
(2/3) - sqrt(3)*c)/c) - a*d*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)
^(2/3)) - b*c*(c/d)^(1/3)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3)) + 2*a*d*(
-a/b)^(1/3)*log(x - (-a/b)^(1/3)) + 2*b*c*(c/d)^(1/3)*log(x + (c/d)^(1/3))
 - 6*(b*c - a*d)*x)/(b^2*c*d - a*b*d^2)
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(x**6/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx ={}& \frac{\sqrt{3}a^2 \arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}c^2 \arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}} - ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} \\ &- \frac{a^2\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c\left(\frac{a}{b}\right)^{\frac{2}{3}} - ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{c^2\log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^2\left(\frac{c}{d}\right)^{\frac{2}{3}} - ad^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} \\ &+ \frac{a^2\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{2}{3}} - ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{c^2\log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{2}{3}} - ad^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{x}{bd}\end{aligned}$$

input
```
integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^3*
c*(a/b)^(1/3) - a*b^2*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*c^2*arctan
(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*d^2*(c/d)^(1/3) - a*d^
3*(c/d)^(1/3))*(c/d)^(1/3)) - 1/6*a^2*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
))/(b^3*c*(a/b)^(2/3) - a*b^2*d*(a/b)^(2/3)) + 1/6*c^2*log(x^2 - x*(c/d)^(
1/3) + (c/d)^(2/3))/(b*c*d^2*(c/d)^(2/3) - a*d^3*(c/d)^(2/3)) + 1/3*a^2*lo
g(x + (a/b)^(1/3))/(b^3*c*(a/b)^(2/3) - a*b^2*d*(a/b)^(2/3)) - 1/3*c^2*log
(x + (c/d)^(1/3))/(b*c*d^2*(c/d)^(2/3) - a*d^3*(c/d)^(2/3)) + x/(b*d)
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.04

$$
\begin{aligned}
\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = {} & -\frac{a^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(ab^2c-a^2bd)} + \frac{c^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2d-acd^2)} \\
& + \frac{\left(-ab^2\right)^{\frac{1}{3}} a\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^3c-\sqrt{3}ab^2d} \\
& - \frac{\left(-cd^2\right)^{\frac{1}{3}} c\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3} \\
& + \frac{\left(-ab^2\right)^{\frac{1}{3}} a\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(b^3c-ab^2d)} \\
& - \frac{\left(-cd^2\right)^{\frac{1}{3}} c\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^2-ad^3)} + \frac{x}{bd}
\end{aligned}
$$

input
```
integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*a^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c - a^2*b*d) + 1/3
*c^2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2*d - a*c*d^2) + (-a*b^2
)^(1/3)*a*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b
^3*c - sqrt(3)*a*b^2*d) - (-c*d^2)^(1/3)*c*arctan(1/3*sqrt(3)*(2*x + (-c/d
)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3) + 1/6*(-a*b^2)^(1
/3)*a*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c - a*b^2*d) - 1/6*(-c
*d^2)^(1/3)*c*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^2 - a*d^3) +
 x/(b*d)
```

3.109.9 Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 873, normalized size of antiderivative = 2.95

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = \ln\left(a\,x + b^2\,c\left(-\frac{a^4}{b^4\,(a\,d-b\,c)^3}\right)^{1/3} - a\,b\,d\left(-\frac{a^4}{b^4\,(a\,d-b\,c)^3}\right)^{1/3}\right)\left(\frac{a^4}{-27\,a^3\,b^4\,d^3+81\,a^2\,b^5\,c\,d^2-81\,a\,b^6\,c^2\,d+27\,b^7\,c^3}\right)^{1/3}$$

$$+\ln\left(c\,x+a\,d^2\left(\frac{c^4}{d^4\,(a\,d-b\,c)^3}\right)^{1/3}-b\,c\,d\left(\frac{c^4}{d^4\,(a\,d-b\,c)^3}\right)^{1/3}\right)\Bigg(\frac{c^4}{27\,a^3\,d^7-81\,a^2\,b\,c\,d^6+81\,a\,b^2\,c^2\,d^5-}$$

input
```
int(x^6/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(a*x + b^2*c*(-a^4/(b^4*(a*d - b*c)^3))^(1/3) - a*b*d*(-a^4/(b^4*(a*d -
 b*c)^3))^(1/3))*(a^4/(27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^2 - 81
*a*b^6*c^2*d))^(1/3) + log(c*x + a*d^2*(c^4/(d^4*(a*d - b*c)^3))^(1/3) - b
*c*d*(c^4/(d^4*(a*d - b*c)^3))^(1/3))*(c^4/(27*a^3*d^7 - 27*b^3*c^3*d^4 +
81*a*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^(1/3) + x/(b*d) + (log((3*x*(a^2*b^4*c
^6 + a^6*c^2*d^4))/(b*d) - (3*a*c^2*(3^(1/2)*1i - 1)*(-a^4/(b^4*(a*d - b*c
)^3))^(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d - a^4*b*c*d^4))/(2*d))*(a^4/(
27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^2 - 81*a*b^6*c^2*d))^(1/3)*(3
^(1/2)*1i - 1))/2 - (log((3*x*(a^2*b^4*c^6 + a^6*c^2*d^4))/(b*d) + (3*a*c^
2*(3^(1/2)*1i + 1)*(-a^4/(b^4*(a*d - b*c)^3))^(1/3)*(a^5*d^5 - b^5*c^5 + a
*b^4*c^4*d - a^4*b*c*d^4))/(2*d))*(a^4/(27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a
^2*b^5*c*d^2 - 81*a*b^6*c^2*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log((3*x*(a^2
*b^4*c^6 + a^6*c^2*d^4))/(b*d) + (3*a^2*c*(3^(1/2)*1i - 1)*(c^4/(d^4*(a*d
- b*c)^3))^(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d - a^4*b*c*d^4))/(2*b))*(
c^4/(27*a^3*d^7 - 27*b^3*c^3*d^4 + 81*a*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^(1/
3)*(3^(1/2)*1i - 1))/2 - (log((3*x*(a^2*b^4*c^6 + a^6*c^2*d^4))/(b*d) - (3
*a^2*c*(3^(1/2)*1i + 1)*(c^4/(d^4*(a*d - b*c)^3))^(1/3)*(a^5*d^5 - b^5*c^5
 + a*b^4*c^4*d - a^4*b*c*d^4))/(2*b))*(c^4/(27*a^3*d^7 - 27*b^3*c^3*d^4 +
81*a*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^(1/3)*(3^(1/2)*1i + 1))/2
```

3.110 $\int \frac{x^5}{(a+bx^3)(c+dx^3)} \, dx$

3.110.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)} \, dx = -\frac{a\log(a+bx^3)}{3b(bc-ad)} + \frac{c\log(c+dx^3)}{3d(bc-ad)}$$

output
```
-1/3*a*ln(b*x^3+a)/b/(-a*d+b*c)+1/3*c*ln(d*x^3+c)/d/(-a*d+b*c)
```

3.110.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)} \, dx = -\frac{ad\log(a+bx^3) - bc\log(c+dx^3)}{3b^2cd - 3abd^2}$$

input
```
Integrate[x^5/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
-((a*d*Log[a + b*x^3] - b*c*Log[c + d*x^3])/(3*b^2*c*d - 3*a*b*d^2))
```

3.110.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(bx^3+a)(dx^3+c)}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{c}{(bc-ad)(dx^3+c)}-\frac{a}{(bc-ad)(bx^3+a)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{c\log(c+dx^3)}{d(bc-ad)}-\frac{a\log(a+bx^3)}{b(bc-ad)}\right)$$

input
```
Int[x^5/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(-((a*Log[a + b*x^3])/(b*(b*c - a*d))) + (c*Log[c + d*x^3])/(d*(b*c - a*d)
))/3
```

3.110.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.110.4 Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$\frac{a\ln\left(b\,x^{3}+a\right)d-c\ln\left(d\,x^{3}+c\right)b}{3\left(ad-bc\right)bd}$	43
default	$-\frac{c\ln\left(d\,x^{3}+c\right)}{3\left(ad-bc\right)d}+\frac{a\ln\left(b\,x^{3}+a\right)}{3\left(ad-bc\right)b}$	50
norman	$-\frac{c\ln\left(d\,x^{3}+c\right)}{3\left(ad-bc\right)d}+\frac{a\ln\left(b\,x^{3}+a\right)}{3\left(ad-bc\right)b}$	50
risch	$-\frac{c\ln\left(-d\,x^{3}-c\right)}{3\left(ad-bc\right)d}+\frac{a\ln\left(b\,x^{3}+a\right)}{3\left(ad-bc\right)b}$	53

input
```
int(x^5/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*ln(b*x^3+a)*d-c*ln(d*x^3+c)*b)/(a*d-b*c)/b/d
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = -\frac{ad\log\left(bx^3+a\right)-bc\log\left(dx^3+c\right)}{3\left(b^2cd-abd^2\right)}$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/3*(a*d*log(b*x^3 + a) - b*c*log(d*x^3 + c))/(b^2*c*d - a*b*d^2)
```

3.110.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(39) = 78.

Time = 5.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.72

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = \frac{a\log\left(x^3+\frac{\frac{a^3d^2}{b(ad-bc)}-\frac{2a^2cd}{ad-bc}+\frac{abc^2}{ad-bc}+2ac}{ad+bc}\right)}{3b\,(ad-bc)} - \frac{c\log\left(x^3+\frac{-\frac{a^2cd}{ad-bc}+\frac{2abc^2}{ad-bc}+2ac-\frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{3d\,(ad-bc)}$$

input
```
integrate(x**5/(b*x**3+a)/(d*x**3+c),x)
```

output
```
a*log(x**3 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**
2/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(3*b*(a*d - b*c)) - c*log(x**3 + (-a**
2*c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b
*c)))/(a*d + b*c))/(3*d*(a*d - b*c))
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a\log(bx^3+a)}{3\,(b^2c-abd)} + \frac{c\log(dx^3+c)}{3\,(bcd-ad^2)}$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*a*log(b*x^3 + a)/(b^2*c - a*b*d) + 1/3*c*log(d*x^3 + c)/(b*c*d - a*d^
2)
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a\log(|bx^3+a|)}{3(b^2c-abd)}+\frac{c\log(|dx^3+c|)}{3(bcd-ad^2)}$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*a*log(abs(b*x^3 + a))/(b^2*c - a*b*d) + 1/3*c*log(abs(d*x^3 + c))/(b*
c*d - a*d^2)
```

3.110.9 Mupad [B] (verification not implemented)

Time = 7.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a\ln(b\,x^3+a)}{3\,b^2\,c-3\,a\,b\,d}-\frac{c\ln(d\,x^3+c)}{3\,a\,d^2-3\,b\,c\,d}$$

input
```
int(x^5/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
- (a*log(a + b*x^3))/(3*b^2*c - 3*a*b*d) - (c*log(c + d*x^3))/(3*a*d^2 - 3
*b*c*d)
```

3.111 $\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$

3.111.1 Optimal result

Integrand size = 22, antiderivative size = 288

$$\begin{aligned}\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx &= \frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc-ad)} - \frac{c^{2/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{2/3}(bc-ad)} \\ &\quad + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{2/3}(bc-ad)} \\ &\quad - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)} \\ &\quad + \frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{2/3}(bc-ad)}\end{aligned}$$

output
```
1/3*a^(2/3)*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/(-a*d+b*c)-1/3*c^(2/3)*ln(c^(1/3
)+d^(1/3)*x)/d^(2/3)/(-a*d+b*c)-1/6*a^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/b^(2/3)/(-a*d+b*c)+1/6*c^(2/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^
(2/3)*x^2)/d^(2/3)/(-a*d+b*c)+1/3*a^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)
/a^(1/3)*3^(1/2))/b^(2/3)/(-a*d+b*c)*3^(1/2)-1/3*c^(2/3)*arctan(1/3*(c^(1/
3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(2/3)/(-a*d+b*c)*3^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$$

$$= \frac{\frac{2\sqrt{3}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} - \frac{2\sqrt{3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{2/3}} + \frac{2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} - \frac{2c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{2/3}} - \frac{a^{2/3}\log\left(a^{2/3}\right.}{}}{6bc-6ad}$$

input
```
Integrate[x^4/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
((2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) -
 (2*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(2/3) +
 (2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (2*c^(2/3)*Log[c^(1/3) + d
^(1/3)*x])/d^(2/3) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/b^(2/3) + (c^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(
2/3))/(6*b*c - 6*a*d)
```

3.111.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {981, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$$

$$\Big\downarrow \text{ 981}$$

$$\frac{c\int \frac{x}{dx^3+c}dx}{bc-ad} - \frac{a\int \frac{x}{bx^3+a}dx}{bc-ad}$$

$$\Big\downarrow \text{ 821}$$

$$\frac{c\left(\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{bc-ad}$$

$$\big\downarrow \; 16$$

$$\frac{c\left(\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}$$

$$\big\downarrow \; 1142$$

$$\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}$$

$$\big\downarrow \; 25$$

3.111. $\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow 27$$

$$\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow 1082$$

3.111. $\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{c\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}-\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \; 217$$

$$\frac{c\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1103$$

3.111. $\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{c\left(\frac{\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}$$

input
```
Int[x^4/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
-((a*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan
[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/(b*c - a*d)) + (
c*(-1/3*Log[c^(1/3) + d^(1/3)*x]/(c^(1/3)*d^(2/3)) + (-((Sqrt[3]*ArcTan[(1
 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3)) + Log[c^(2/3) - c^(1/3)*d^(1/
3)*x + d^(2/3)*x^2]/(2*d^(1/3)))/(3*c^(1/3)*d^(1/3))))/(b*c - a*d)
```

3.111.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 981
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[(-a)*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(a + b*x^n),
x], x] + Simp[c*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(c + d*x^n), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
 m, 2*n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.111. $\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$

3.111.4 Maple [A] (verified)

Time = 4.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

method	result
default	$-\frac{\Bigg(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\Bigg)c}{ad-bc}+\frac{\Bigg(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+}{ad-bc}$
risch	$\frac{\Bigg(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{3}b^{2}d^{3}-3a^{2}b^{3}c\,d^{2}+3a\,b^{4}c^{2}d-b^{5}c^{3}\right)\_Z^{3}+a^{2}\right)}\_R\ln\Big(\Big(\left(-2a^{3}b^{2}c\,d^{4}+4a^{2}b^{3}c^{2}d^{3}-2a\,b^{4}c^{3}d^{2}\right)\_R^{3}-a^{2}cd-b\,c^{2}a\Big)x+(-}{3}$

input
```
int(x^4/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3
)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/
3)*x-1)))*c/(a*d-b*c)+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a/(a*d-b*c)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.85

$$\int\frac{x^{4}}{\left(a+bx^{3}\right)\left(c+dx^{3}\right)}\,dx = -\frac{2\sqrt{3}\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}+\sqrt{3}a}{3\,a}\right)-2\sqrt{3}\left(\frac{c^{2}}{d^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(\frac{c^{2}}{d^{2}}\right)^{\frac{1}{3}}-\sqrt{3}c}{3\,c}\right)-\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\Big(ax^{2}}{}$$

input
```
integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3
) + sqrt(3)*a)/a) - 2*sqrt(3)*(c^2/d^2)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(c
^2/d^2)^(1/3) - sqrt(3)*c)/c) - (-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2
)^(2/3) - a*(-a^2/b^2)^(1/3)) - (c^2/d^2)^(1/3)*log(c*x^2 - d*x*(c^2/d^2)^
(2/3) + c*(c^2/d^2)^(1/3)) + 2*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/
3)) + 2*(c^2/d^2)^(1/3)*log(c*x + d*(c^2/d^2)^(2/3)))/(b*c - a*d)
```

3.111.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^3)\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+bx^3)\,(c+dx^3)}\,dx = -\frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(b^2c-abd)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}c\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,(bcd-ad^2)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{c\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{c\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}$$

input
```
integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c
 - a*b*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*c*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/
3))/(c/d)^(1/3))/((b*c*d - a*d^2)*(c/d)^(1/3)) - 1/6*a*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(b^2*c*(a/b)^(1/3) - a*b*d*(a/b)^(1/3)) + 1/6*c*log(x^
2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1/3)) +
 1/3*a*log(x + (a/b)^(1/3))/(b^2*c*(a/b)^(1/3) - a*b*d*(a/b)^(1/3)) - 1/3*
c*log(x + (c/d)^(1/3))/(b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1/3))
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.99

$$
\begin{aligned}
\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx &= \frac{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)} - \frac{c\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} \\
&\quad + \frac{(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^3c-\sqrt{3}ab^2d} \\
&\quad - \frac{(-cd^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3} \\
&\quad - \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(b^3c-ab^2d)} \\
&\quad + \frac{(-cd^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^2-ad^3)}
\end{aligned}
$$

input
```
integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*a*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) - 1/3*c*(-c/
d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(2/3)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^3*c - sqrt(3)*
a*b^2*d) - (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(
1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3) - 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c - a*b^2*d) + 1/6*(-c*d^2)^(2/3)*log(x^2
 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^2 - a*d^3)
```

3.111.9 Mupad [B] (verification not implemented)

Time = 14.05 (sec) , antiderivative size = 1364, normalized size of antiderivative = 4.74

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int(x^4/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(a*x + b^3*c^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3) + a^2*b*d^2*(-a^2/(b^2*
(a*d - b*c)^3))^(2/3) - 2*a*b^2*c*d*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))*(a^2
/(27*b^5*c^3 - 27*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3)
+ log(c*x + a^2*d^3*(c^2/(d^2*(a*d - b*c)^3))^(2/3) + b^2*c^2*d*(c^2/(d^2*
(a*d - b*c)^3))^(2/3) - 2*a*b*c*d^2*(c^2/(d^2*(a*d - b*c)^3))^(2/3))*(c^2/
(27*a^3*d^5 - 27*b^3*c^3*d^2 + 81*a*b^2*c^2*d^3 - 81*a^2*b*c*d^4))^(1/3) +
 (log(((3^(1/2)*1i - 1)^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i -
 1)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)
^2*(a*d + b*c)*(a*d - b*c)^4*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))/4)*(-a^2/(b
^2*(a*d - b*c)^3))^(1/3))/6 - 9*a^2*b^4*c^4*d^2 - 9*a^4*b^2*c^2*d^4 + 9*a*
b^5*c^5*d + 9*a^5*b*c*d^5))/36 + a^2*b*c^2*d*x*(a*d + b*c))*(a^2/(27*b^5*c
^3 - 27*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3)*(3^(1/2)*1
i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3)*(((3
^(1/2)*1i + 1)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1
/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))/
4)*(-a^2/(b^2*(a*d - b*c)^3))^(1/3))/6 + 9*a^2*b^4*c^4*d^2 + 9*a^4*b^2*c^2
*d^4 - 9*a*b^5*c^5*d - 9*a^5*b*c*d^5))/36 - a^2*b*c^2*d*x*(a*d + b*c))*(a^
2/(27*b^5*c^3 - 27*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3)
*(3^(1/2)*1i + 1))/2 + (log(((3^(1/2)*1i - 1)^2*(c^2/(d^2*(a*d - b*c)^3))^
(2/3)*(((3^(1/2)*1i - 1)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^...
```

3.112 $\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$

3.112.1 Optimal result

Integrand size = 22, antiderivative size = 288

$$\begin{aligned}\int \frac{x^3}{(a+bx^3)\,(c+dx^3)}\,dx = {} & \frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}(bc-ad)} - \frac{\sqrt[3]{c}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)} \\ & - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)} \\ & + \frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bc-ad)} \\ & - \frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{d}(bc-ad)}\end{aligned}$$

output
```
-1/3*a^(1/3)*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)/(-a*d+b*c)+1/3*c^(1/3)*ln(c^(1/
3)+d^(1/3)*x)/d^(1/3)/(-a*d+b*c)+1/6*a^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/b^(1/3)/(-a*d+b*c)-1/6*c^(1/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d
^(2/3)*x^2)/d^(1/3)/(-a*d+b*c)+1/3*a^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x
)/a^(1/3)*3^(1/2))/b^(1/3)/(-a*d+b*c)*3^(1/2)-1/3*c^(1/3)*arctan(1/3*(c^(1
/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(1/3)/(-a*d+b*c)*3^(1/2)
```

3.112.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \frac{\frac{2\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} - \frac{2\sqrt{3}\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}} - \frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \frac{2\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt[3]{d}} + \sqrt[3]{a}\log\left(a^{2/}\right.}{6bc-6ad}$$

input
```
Integrate[x^3/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
((2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) -
 (2*Sqrt[3]*c^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3) -
 (2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (2*c^(1/3)*Log[c^(1/3) + d
^(1/3)*x])/d^(1/3) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/b^(1/3) - (c^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(
1/3))/(6*b*c - 6*a*d)
```

3.112.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {981, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$$
$$\Big\downarrow \text{981}$$
$$\frac{c\int \frac{1}{dx^3+c}dx}{bc-ad} - \frac{a\int \frac{1}{bx^3+a}dx}{bc-ad}$$
$$\Big\downarrow \text{750}$$

$$\frac{c\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \; 16$$

$$\frac{c\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1142$$

$$\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}$$

$$\Big\downarrow \; 25$$

3.112. $\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1082}$$

3.112. $\int\frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{c\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}$$

$$\Big\downarrow \ 217$$

$$\frac{c\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}$$

$$\Big\downarrow \ 1103$$

3.112. $\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{c\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}$$

input
```
Int[x^3/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
-((a*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(b*c - a*d)) + (c*(Log[c^(1
/3) + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*
x)/c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2]/(2*d^(1/3)))/(3*c^(2/3))))/(b*c - a*d)
```

3.112.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 981
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[(-a)*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(a + b*x^n),
x], x] + Simp[c*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(c + d*x^n), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
 m, 2*n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.112. $\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$

3.112.4 Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

method	result
default	$-\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)c}{ad-bc}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\sqrt{3}\ \mathrm{ar}\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{3}b\,d^{3}-3c\,d^{2}a^{2}b^{2}+3c^{2}da\,b^{3}-b^{4}c^{3}\right)\_Z^{3}-a\right)}\_R\ln\left(\left(\left(a^{4}b\,d^{5}-4a^{3}b^{2}c\,d^{4}+6a^{2}b^{3}c^{2}d^{3}-4a\,b^{4}c^{3}d^{2}+b^{5}c^{4}d\right)\_R^{3}-a^{2}d^{2}-\right.\right.\right.}{3}$

input
```
int(x^3/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)
*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3
)*x-1)))*c/(a*d-b*c)+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1)))*a/(a*d-b*c)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx = -\frac{2\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+2\sqrt{3}\left(-\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(-\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\right.}{6\,(bc-}$$

input
```
integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3
)*a)/a) + 2*sqrt(3)*(-c/d)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(-c/d)^(2/3) -
sqrt(3)*c)/c) - (a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - (-c/d
)^(1/3)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3)) + 2*(a/b)^(1/3)*log(x + (
a/b)^(1/3)) + 2*(-c/d)^(1/3)*log(x - (-c/d)^(1/3)))/(b*c - a*d)
```

3.112.6 Sympy [A] (verification not implemented)

Time = 131.43 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$$

$$= \operatorname{RootSum}\left(t^3\cdot\left(27a^3d^4-81a^2bcd^3+81ab^2c^2d^2-27b^3c^3d\right)+c,\left(t\mapsto t\log\left(x+\frac{162t^4a^4bd^5-648t^4a^3b^2}{}\right.\right.\right.$$

$$+\operatorname{RootSum}\left(t^3\cdot\left(27a^3bd^3-81a^2b^2cd^2+81ab^3c^2d-27b^4c^3\right)-a,\left(t\mapsto t\log\left(x+\frac{162t^4a^4bd^5-648t^4a^3}{}\right.\right.\right.$$

input
```
integrate(x**3/(b*x**3+a)/(d*x**3+c),x)
```

output
```
RootSum(_t**3*(27*a**3*d**4 - 81*a**2*b*c*d**3 + 81*a*b**2*c**2*d**2 - 27*
b**3*c**3*d) + c, Lambda(_t, _t*log(x + (162*_t**4*a**4*b*d**5 - 648*_t**4
*a**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 - 648*_t**4*a*b**4*c**3*
d**2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t*a*b*c*d - 3*_t*b**2*c
**2)/(a*d + b*c)))) + RootSum(_t**3*(27*a**3*b*d**3 - 81*a**2*b**2*c*d**2
+ 81*a*b**3*c**2*d - 27*b**4*c**3) - a, Lambda(_t, _t*log(x + (162*_t**4*a
**4*b*d**5 - 648*_t**4*a**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 -
648*_t**4*a*b**4*c**3*d**2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t
*a*b*c*d - 3*_t*b**2*c**2)/(a*d + b*c))))
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.10

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx = -\frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}c\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}$$

$$+\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{c\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

$$-\frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{c\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

input

```
integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output

```
-1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c
*(a/b)^(1/3) - a*b*d*(a/b)^(1/3))*(a/b)^(1/3)) + 1/3*sqrt(3)*c*arctan(1/3*
sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*d*(c/d)^(1/3) - a*d^2*(c/d)
^(1/3))*(c/d)^(1/3)) + 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c
*(a/b)^(2/3) - a*b*d*(a/b)^(2/3)) - 1/6*c*log(x^2 - x*(c/d)^(1/3) + (c/d)^
(2/3))/(b*c*d*(c/d)^(2/3) - a*d^2*(c/d)^(2/3)) - 1/3*a*log(x + (a/b)^(1/3)
)/(b^2*c*(a/b)^(2/3) - a*b*d*(a/b)^(2/3)) + 1/3*c*log(x + (c/d)^(1/3))/(b*
c*d*(c/d)^(2/3) - a*d^2*(c/d)^(2/3))
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx = \frac{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)} - \frac{c\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} - \frac{(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c-\sqrt{3}abd} + \frac{(-cd^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd-\sqrt{3}ad^2} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(b^2c-abd)} + \frac{(-cd^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd-ad^2)}$$

input

```
integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output

```
1/3*a*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) - 1/3*c*(-c/
d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) - (-a*b^2)^(1/3)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^2*c - sqrt(3)*
a*b*d) + (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/
3))/(sqrt(3)*b*c*d - sqrt(3)*a*d^2) - 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/(b^2*c - a*b*d) + 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(
-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d - a*d^2)
```

3.112.9 Mupad [B] (verification not implemented)

Time = 13.02 (sec) , antiderivative size = 1265, normalized size of antiderivative = 4.39

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

input

```
int(x^3/((a + b*x^3)*(c + d*x^3)),x)
```

output

```
log(x + a*d*(a/(b*(a*d - b*c)^3))^(1/3) - b*c*(a/(b*(a*d - b*c)^3))^(1/3))
*(-a/(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^(1/3
) + log(x - a*d*(-c/(d*(a*d - b*c)^3))^(1/3) + b*c*(-c/(d*(a*d - b*c)^3))^
(1/3))*(-c/(27*a^3*d^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3)
)^(1/3) + (log(((3^(1/2)*1i - 1)*(a/(b*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i
 - 1)^2*(81*a*b^3*c*d^3*x*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i - 1)
*(a*d + b*c)*(a*d - b*c)^4*(a/(b*(a*d - b*c)^3))^(1/3))/2)*(a/(b*(a*d - b*
c)^3))^(2/3))/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 -
 9*a^3*b^3*c^2*d^4))/6 - 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i
- 1)*(-a/(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^
(1/3))/2 - (log(((3^(1/2)*1i + 1)*(a/(b*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1
i + 1)^2*(81*a*b^3*c*d^3*x*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(1/2)*1i + 1
)*(a*d + b*c)*(a*d - b*c)^4*(a/(b*(a*d - b*c)^3))^(1/3))/2)*(a/(b*(a*d - b
*c)^3))^(2/3))/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3
- 9*a^3*b^3*c^2*d^4))/6 + 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i
 + 1)*(-a/(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))
^(1/3))/2 + (log(((3^(1/2)*1i - 1)*(-c/(d*(a*d - b*c)^3))^(1/3)*(((3^(1/2)
*1i - 1)^2*(81*a*b^3*c*d^3*x*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i -
 1)*(a*d + b*c)*(a*d - b*c)^4*(-c/(d*(a*d - b*c)^3))^(1/3))/2)*(-c/(d*(a*d
 - b*c)^3))^(2/3))/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c...
```

3.112. $\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$

3.113 $\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx$

3.113.1 Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx=\frac{\log(a+bx^3)}{3(bc-ad)}-\frac{\log(c+dx^3)}{3(bc-ad)}$$

output
```
1/3*ln(b*x^3+a)/(-a*d+b*c)-1/3*ln(d*x^3+c)/(-a*d+b*c)
```

3.113.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx=\frac{\log(a+bx^3)-\log(c+dx^3)}{3bc-3ad}$$

input
```
Integrate[x^2/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(Log[a + b*x^3] - Log[c + d*x^3])/(3*b*c - 3*a*d)
```

3.113.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3}\int \frac{1}{(bx^3+a)(dx^3+c)}dx^3$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{3}\left(\frac{b\int \frac{1}{bx^3+a}dx^3}{bc-ad}-\frac{d\int \frac{1}{dx^3+c}dx^3}{bc-ad}\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\left(\frac{\log\left(a+bx^3\right)}{bc-ad}-\frac{\log\left(c+dx^3\right)}{bc-ad}\right)$$

input
```
Int[x^2/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(Log[a + b*x^3]/(b*c - a*d) - Log[c + d*x^3]/(b*c - a*d))/3
```

3.113.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.113.4 Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

method	result	size
parallelrisch	$-\frac{\ln(b x^3+a)-\ln(d x^3+c)}{3(ad-bc)}$	32
default	$\frac{\ln(d x^3+c)}{3ad-3bc}-\frac{\ln(b x^3+a)}{3(ad-bc)}$	42
norman	$\frac{\ln(d x^3+c)}{3ad-3bc}-\frac{\ln(b x^3+a)}{3(ad-bc)}$	42
risch	$\frac{\ln(d x^3+c)}{3ad-3bc}-\frac{\ln(-b x^3-a)}{3(ad-bc)}$	45

input
```
int(x^2/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(ln(b*x^3+a)-ln(d*x^3+c))/(a*d-b*c)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx=\frac{\log(bx^3+a)-\log(dx^3+c)}{3(bc-ad)}$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/3*(log(b*x^3 + a) - log(d*x^3 + c))/(b*c - a*d)
```

3.113.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(36) = 72$.

Time = 0.73 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.07

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{\log\left(x^3 + \frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{3(ad-bc)} - \frac{\log\left(x^3 + \frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{3(ad-bc)}$$

input
```
integrate(x**2/(b*x**3+a)/(d*x**3+c),x)
```

output
```
log(x**3 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c*
*2/(a*d - b*c) + b*c)/(2*b*d))/(3*(a*d - b*c)) - log(x**3 + (a**2*d**2/(a*
d - b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b
*d))/(3*(a*d - b*c))
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{\log(bx^3+a)}{3(bc-ad)} - \frac{\log(dx^3+c)}{3(bc-ad)}$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*log(b*x^3 + a)/(b*c - a*d) - 1/3*log(d*x^3 + c)/(b*c - a*d)
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{b\log\left(\left|bx^3+a\right|\right)}{3\left(b^2c-abd\right)} - \frac{d\log\left(\left|dx^3+c\right|\right)}{3\left(bcd-ad^2\right)}$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b*log(abs(b*x^3 + a))/(b^2*c - a*b*d) - 1/3*d*log(abs(d*x^3 + c))/(b*c
*d - a*d^2)
```

3.113.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 602, normalized size of antiderivative = 13.38

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = -\frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{x^3\left(36\,c\,b^4\,d^3+36\,a\,b^3\,d^4\right)+\frac{x^3\left(54\,a^2\,b^3\,d^5+108\,a\,b^4\,c\,d^4+54\,b^5\,c^2\,d^3\right)+108\,a\,b^4\,c^2\,d^3+108\,a^2\,b^3\,c\,d^4}{3\,a\,d-3\,b\,c}+36\,a\,b^3\,c\,d^3}{3\,a\,d-3\,b\,c}+6\,b^3\,d^3\,x^3\right)1\mathrm{i}}{3\,a\,d-3\,b\,c}-{}}{\frac{\frac{x^3\left(36\,c\,b^4\,d^3+36\,a\,b^3\,d^4\right)+\frac{x^3\left(54\,a^2\,b^3\,d^5+108\,a\,b^4\,c\,d^4+54\,b^5\,c^2\,d^3\right)+108\,a\,b^4\,c^2\,d^3+108\,a^2\,b^3\,c\,d^4}{3\,a\,d-3\,b\,c}+36\,a\,b^3\,c\,d^3}{3\,a\,d-3\,b\,c}+6\,b^3\,d^3\,x^3}{3\,a\,d-3\,b\,c}+{}}\right)}{3\,a\,d-3\,b\,c}$$

input
```
int(x^2/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
-(atan(((((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) + (x^3*(54*a^2*b^3*d^5 + 54*b
^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*
a*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) + 6*b^3*d^3*x^3)*1i)/(3*a*d
 - 3*b*c) - (((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) - (x^3*(54*a^2*b^3*d^5 +
54*b^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)
/(3*a*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) - 6*b^3*d^3*x^3)*1i)/(3
*a*d - 3*b*c))/(((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) + (x^3*(54*a^2*b^3*d^5
 + 54*b^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d
^4)/(3*a*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) + 6*b^3*d^3*x^3)/(3*
a*d - 3*b*c) + ((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) - (x^3*(54*a^2*b^3*d^5
+ 54*b^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^
4)/(3*a*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) - 6*b^3*d^3*x^3)/(3*a
*d - 3*b*c)))*2i)/(3*a*d - 3*b*c)
```

3.114 $\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$

3.114.1 Optimal result

Integrand size = 20, antiderivative size = 288

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = -\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}(bc-ad)}+\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)} + \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)} - \frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)}$$

output
```
-1/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/(-a*d+b*c)+1/3*d^(1/3)*ln(c^(1/
3)+d^(1/3)*x)/c^(1/3)/(-a*d+b*c)+1/6*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/a^(1/3)/(-a*d+b*c)-1/6*d^(1/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d
^(2/3)*x^2)/c^(1/3)/(-a*d+b*c)-1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x
)/a^(1/3)*3^(1/2))/a^(1/3)/(-a*d+b*c)*3^(1/2)+1/3*d^(1/3)*arctan(1/3*(c^(1
/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(1/3)/(-a*d+b*c)*3^(1/2)
```

3.114.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = \frac{\frac{2\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} - \frac{2\sqrt{3}\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{c}} + \frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{2\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt[3]{c}} - \sqrt[3]{b}\log\left(a^{2/}\right.}{-6bc+6ad}$$

input
```
Integrate[x/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
((2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) -
 (2*Sqrt[3]*d^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(1/3) +
 (2*b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) - (2*d^(1/3)*Log[c^(1/3) + d
^(1/3)*x])/c^(1/3) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/a^(1/3) + (d^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(
1/3))/(-6*b*c + 6*a*d)
```

3.114.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$$

$$\Big\downarrow \; 982$$

$$\frac{b\int \frac{x}{bx^3+a}dx}{bc-ad} - \frac{d\int \frac{x}{dx^3+c}dx}{bc-ad}$$

$$\Big\downarrow \; 821$$

$$\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \; 16$$

$$\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1142$$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \; 25$$

3.114. $\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1082}$$

3.114. $\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}-\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \; 217$$

$$\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1103$$

3.114. $\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3\sqrt[3]{c}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{bc-ad}$$

input
```
Int[x/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(b*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/(b*c - a*d) - (d*(
-1/3*Log[c^(1/3) + d^(1/3)*x]/(c^(1/3)*d^(2/3)) + (-((Sqrt[3]*ArcTan[(1 -
(2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3)) + Log[c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2]/(2*d^(1/3)))/(3*c^(1/3)*d^(1/3))))/(b*c - a*d)
```

3.114.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 982

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.114. $\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$

3.114.4 Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

method	result
default	$\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)d}{ad-bc}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{3}c\,d^{3}-3a^{2}b\,c^{2}d^{2}+3a\,b^{2}c^{3}d-c^{4}b^{3}\right)\_Z^{3}+d\right)}\_R\ln\left(\left(\left(-a^{4}d^{4}+2a^{3}bc\,d^{3}-2a^{2}b^{2}c^{2}d^{2}+2a\,b^{3}c^{3}d-b^{4}c^{4}\right)\_R^{3}+bd\right)x+(-\right.\right.}{3}$

input
```
int(x/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)
*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3
)*x-1)))*d/(a*d-b*c)-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b/(a*d-b*c)
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.70

$$\int\frac{x}{\left(a+bx^{3}\right)\left(c+dx^{3}\right)}\,dx$$
$$=\frac{2\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-2\sqrt{3}\left(-\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-a\right.}{}$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3))
 - 2*sqrt(3)*(-d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(-d/c)^(1/3) + 1/3*sqrt(3))
 + (b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) + (-d/c)^(1/3)
*log(d*x^2 - c*x*(-d/c)^(2/3) - c*(-d/c)^(1/3)) - 2*(b/a)^(1/3)*log(b*x +
a*(b/a)^(2/3)) - 2*(-d/c)^(1/3)*log(d*x + c*(-d/c)^(2/3)))/(b*c - a*d)
```

3.114.6 Sympy [A] (verification not implemented)

Time = 69.18 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.79

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \operatorname{RootSum}\left(t^3\cdot\left(27a^4d^3-81a^3bcd^2+81a^2b^2c^2d-27ab^3c^3\right)-b,\left(t\mapsto t\log\left(x+\frac{243t^5a^7cd^6-1458t^5a^6b}{}\right.\right.\right.$$
$$+\operatorname{RootSum}\left(t^3\cdot\left(27a^3cd^3-81a^2bc^2d^2+81ab^2c^3d-27b^3c^4\right)+d,\left(t\mapsto t\log\left(x+\frac{243t^5a^7cd^6-1458t^5a}{}\right.\right.\right.$$

input
```
integrate(x/(b*x**3+a)/(d*x**3+c),x)
```

output
```
RootSum(_t**3*(27*a**4*d**3 - 81*a**3*b*c*d**2 + 81*a**2*b**2*c**2*d - 27*
a*b**3*c**3) - b, Lambda(_t, _t*log(x + (243*_t**5*a**7*c*d**6 - 1458*_t**
5*a**6*b*c**2*d**5 + 3645*_t**5*a**5*b**2*c**3*d**4 - 4860*_t**5*a**4*b**3
*c**4*d**3 + 3645*_t**5*a**3*b**4*c**5*d**2 - 1458*_t**5*a**2*b**5*c**6*d
+ 243*_t**5*a*b**6*c**7 + 9*_t**2*a**4*d**4 - 18*_t**2*a**3*b*c*d**3 + 18*
_t**2*a**2*b**2*c**2*d**2 - 18*_t**2*a*b**3*c**3*d + 9*_t**2*b**4*c**4)/(a
*b*d**2 + b**2*c*d))) + RootSum(_t**3*(27*a**3*c*d**3 - 81*a**2*b*c**2*d*
*2 + 81*a*b**2*c**3*d - 27*b**3*c**4) + d, Lambda(_t, _t*log(x + (243*_t**
5*a**7*c*d**6 - 1458*_t**5*a**6*b*c**2*d**5 + 3645*_t**5*a**5*b**2*c**3*d*
*4 - 4860*_t**5*a**4*b**3*c**4*d**3 + 3645*_t**5*a**3*b**4*c**5*d**2 - 145
8*_t**5*a**2*b**5*c**6*d + 243*_t**5*a*b**6*c**7 + 9*_t**2*a**4*d**4 - 18*
_t**2*a**3*b*c*d**3 + 18*_t**2*a**2*b**2*c**2*d**2 - 18*_t**2*a*b**3*c**3*
d + 9*_t**2*b**4*c**4)/(a*b*d**2 + b**2*c*d)))
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.92

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(bc-ad)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3(bc-ad)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} - \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c - a*
d)*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)
^(1/3))/((b*c - a*d)*(c/d)^(1/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2
/3))/(b*c*(a/b)^(1/3) - a*d*(a/b)^(1/3)) - 1/6*log(x^2 - x*(c/d)^(1/3) + (
c/d)^(2/3))/(b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3)) - 1/3*log(x + (a/b)^(1/3))
/(b*c*(a/b)^(1/3) - a*d*(a/b)^(1/3)) + 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)
^(1/3) - a*d*(c/d)^(1/3))
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{x}{(a+bx^3)\,(c+dx^3)}\,dx = &-\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)}\\ &-\frac{(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c-\sqrt{3}a^2bd}\\ &+\frac{(-cd^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2d-\sqrt{3}acd^2}\\ &+\frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(ab^2c-a^2bd)}\\ &-\frac{(-cd^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2d-acd^2)}\end{aligned}$$

input

```
integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output

```
-1/3*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c
/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) - (-a*b^2)^(2/3)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b^2*c - sqrt(
3)*a^2*b*d) + (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d
)^(1/3))/(sqrt(3)*b*c^2*d - sqrt(3)*a*c*d^2) + 1/6*(-a*b^2)^(2/3)*log(x^2
+ x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2*c - a^2*b*d) - 1/6*(-c*d^2)^(2/3)*
log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^2*d - a*c*d^2)
```

3.114.9 Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 982, normalized size of antiderivative = 3.41

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = \ln\left(b\,x + a^3\,d^2\left(\frac{b}{a\,(a\,d-b\,c)^3}\right)^{2/3} + a\,b^2\,c^2\left(\frac{b}{a\,(a\,d-b\,c)^3}\right)^{2/3}\right.$$

$$\left. -2\,a^2\,b\,c\,d\left(\frac{b}{a\,(a\,d-b\,c)^3}\right)^{2/3}\right)\left(\frac{b}{27\,a^4\,d^3-81\,a^3\,b\,c\,d^2+81\,a^2\,b^2\,c^2\,d-27\,a\,b^3\,c^3}\right)^{1/3}+\ln\left(d\,x+b^2\,c^3\left(\right.\right.$$

input
```
int(x/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(b*x + a^3*d^2*(b/(a*(a*d - b*c)^3))^(2/3) + a*b^2*c^2*(b/(a*(a*d - b*c
)^3))^(2/3) - 2*a^2*b*c*d*(b/(a*(a*d - b*c)^3))^(2/3))*(b/(27*a^4*d^3 - 27
*a*b^3*c^3 + 81*a^2*b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3) + log(d*x + b^2*c^3
*(-d/(c*(a*d - b*c)^3))^(2/3) + a^2*c*d^2*(-d/(c*(a*d - b*c)^3))^(2/3) - 2
*a*b*c^2*d*(-d/(c*(a*d - b*c)^3))^(2/3))*(d/(27*b^3*c^4 - 27*a^3*c*d^3 + 8
1*a^2*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3) + (log(b^4*d^4*x - (b*(3^(1/2)*1i
 - 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*
(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b/(a*(a*d - b*c)^3))^(2/3))/
4))/(216*a*(a*d - b*c)^3))*(3^(1/2)*1i - 1)*(b/(27*a^4*d^3 - 27*a*b^3*c^3
+ 81*a^2*b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3))/2 - (log(b^4*d^4*x + (b*(3^(1
/2)*1i + 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*
c*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b/(a*(a*d - b*c)^3))^(
2/3))/4))/(216*a*(a*d - b*c)^3))*(3^(1/2)*1i + 1)*(b/(27*a^4*d^3 - 27*a*b^
3*c^3 + 81*a^2*b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3))/2 + (log(b^4*d^4*x + (d
*(3^(1/2)*1i - 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*
a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d/(c*(a*d - b*c
)^3))^(2/3))/4))/(216*c*(a*d - b*c)^3))*(3^(1/2)*1i - 1)*(d/(27*b^3*c^4 -
27*a^3*c*d^3 + 81*a^2*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3))/2 - (log(b^4*d^4
*x - (d*(3^(1/2)*1i + 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2
 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d/(c*...
```

3.115 $\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$

3.115.1 Optimal result

Integrand size = 19, antiderivative size = 288

$$\begin{aligned}\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = &-\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)}+\frac{d^{2/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)}\\ &+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)}\\ &-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}\\ &+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}\end{aligned}$$

output

```
1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/(-a*d+b*c)-1/3*d^(2/3)*ln(c^(1/3
)+d^(1/3)*x)/c^(2/3)/(-a*d+b*c)-1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/a^(2/3)/(-a*d+b*c)+1/6*d^(2/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^
(2/3)*x^2)/c^(2/3)/(-a*d+b*c)-1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)
/a^(1/3)*3^(1/2))/a^(2/3)/(-a*d+b*c)*3^(1/2)+1/3*d^(2/3)*arctan(1/3*(c^(1/
3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/(-a*d+b*c)*3^(1/2)
```

3.115.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \frac{\frac{2\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{2\sqrt{3}d^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{2/3}} - \frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{2/3}} + \frac{b^{2/3}\log\left(a^{2/3}-\right.}{}}{-6bc+6ad}$$

input
```
Integrate[1/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) -
 (2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) -
 (2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d
^(1/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(
2/3))/(-6*b*c + 6*a*d)
```

3.115.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {917, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$$
$$\Big\downarrow \; 917$$
$$\frac{b\int \frac{1}{bx^3+a}dx}{bc-ad} - \frac{d\int \frac{1}{dx^3+c}dx}{bc-ad}$$
$$\Big\downarrow \; 750$$

$$\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \ 16$$

$$\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \ 1142$$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \ 25$$

3.115. $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1082$$

3.115. $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \; 217$$

$$\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1103$$

3.115. $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

input

```
Int[1/((a + b*x^3)*(c + d*x^3)),x]
```

output

```
(b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(b*c - a*d) - (d*(Log[c^(1/3)
 + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/
c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^
2]/(2*d^(1/3)))/(3*c^(2/3))))/(b*c - a*d)
```

3.115.3.1 Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 917
```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^n), x], x] - Simp[d/(b*c - a*d)   Int[1/(c
 + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.115. $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

3.115.4 Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

method	result
default	$\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)d}{ad-bc}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\mathrm{arcta}}{}\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^{5}d^{3}-3a^{4}bc\,d^{2}+3a^{3}b^{2}c^{2}d-a^{2}b^{3}c^{3}\right)\_Z^{3}+b^{2}\right)}\_R\ln\left(\left(\left(-a^{5}d^{5}+3a^{4}bc\,d^{4}-2a^{3}b^{2}c^{2}d^{3}-2a^{2}b^{3}c^{3}d^{2}+3a\,b^{4}c^{4}d-b^{5}c^{5}\right)\_R\right.\right.\right.}{3}$

input
```
int(1/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*
x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)
*x-1)))*d/(a*d-b*c)-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1)))*b/(a*d-b*c)
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)}\,dx = -\frac{2\sqrt{3}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)+2\sqrt{3}\left(\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^{2}}{c^{2}}\right)^{\frac{2}{3}}-\sqrt{3}d}{3\,d}\right)-\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x\right.}{}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3
) - sqrt(3)*b)/b) + 2*sqrt(3)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d
^2/c^2)^(2/3) - sqrt(3)*d)/d) - (-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2
/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - (d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*
(d^2/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*(-b^2/a^2)^(1/3)*log(b*x - a*(-
b^2/a^2)^(1/3)) + 2*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)))/(b*c - a
*d)
```

3.115.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = {} & \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} \\ & - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} \\ & + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}\end{aligned}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c*(a/b
)^(1/3) - a*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(
2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3))*(c/d)
^(1/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*c*(a/b)^(2/3) - a*
d*(a/b)^(2/3)) + 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*(c/d)^(2/
3) - a*d*(c/d)^(2/3)) + 1/3*log(x + (a/b)^(1/3))/(b*c*(a/b)^(2/3) - a*d*(a
/b)^(2/3)) - 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(2/3) - a*d*(c/d)^(2/3))
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.97

$$
\begin{aligned}
\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = &-\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} \\
&+\frac{(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}abc-\sqrt{3}a^2d} \\
&-\frac{(-cd^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2-\sqrt{3}acd} \\
&+\frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(abc-a^2d)} \\
&-\frac{(-cd^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2-acd)}
\end{aligned}
$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c
/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(1/3)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b*c - sqrt(3)
*a^2*d) - (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1
/3))/(sqrt(3)*b*c^2 - sqrt(3)*a*c*d) + 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a*b*c - a^2*d) - 1/6*(-c*d^2)^(1/3)*log(x^2 + x*
(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^2 - a*c*d)
```

3.115.9 Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 1364, normalized size of antiderivative = 4.74

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(((-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*
a*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))*(a*d +
b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-b
^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3
) + log(((d^2/(c^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 -
18*a*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(d^2/(c^2*(a*d - b*c)^3))^(1/3))*(a*d
+ b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-
d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/
3) + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)
*(((3^(1/2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*a*b^3*
c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3)
)^(1/3))/2)*(-b^2/(a^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c
^2*d^4 + 18*a*b^5*c*d^5))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b
^2*c^2*d - 81*a^4*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x -
 ((3^(1/2)*1i + 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(
81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*
(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)*(-b^2/(a^2*
(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5
))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d
^2))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(...
```

3.116 $\int \frac{1}{x(a+bx^3)(c+dx^3)}\,dx$

3.116.1 Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx=\frac{\log(x)}{ac}-\frac{b\log\left(a+bx^3\right)}{3a(bc-ad)}+\frac{d\log\left(c+dx^3\right)}{3c(bc-ad)}$$

output
```
ln(x)/a/c-1/3*b*ln(b*x^3+a)/a/(-a*d+b*c)+1/3*d*ln(d*x^3+c)/c/(-a*d+b*c)
```

3.116.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx=\frac{3bc\log(x)-3ad\log(x)-bc\log\left(a+bx^3\right)+ad\log\left(c+dx^3\right)}{3abc^2-3a^2cd}$$

input
```
Integrate[1/(x*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
(3*b*c*Log[x] - 3*a*d*Log[x] - b*c*Log[a + b*x^3] + a*d*Log[c + d*x^3])/(3
*a*b*c^2 - 3*a^2*c*d)
```

3.116.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^3)(c+dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3(bx^3+a)(dx^3+c)}dx^3$$

$$\Big\downarrow \text{93}$$

$$\frac{1}{3}\int \left(\frac{b^2}{a(ad-bc)(bx^3+a)}+\frac{d^2}{c(bc-ad)(dx^3+c)}+\frac{1}{acx^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{b\log(a+bx^3)}{a(bc-ad)}+\frac{d\log(c+dx^3)}{c(bc-ad)}+\frac{\log(x^3)}{ac}\right)$$

input
```
Int[1/(x*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
(Log[x^3]/(a*c) - (b*Log[a + b*x^3])/(a*(b*c - a*d)) + (d*Log[c + d*x^3])/
(c*(b*c - a*d)))/3
```

3.116.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.116.4 Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{3\ln(x)ad-3\ln(x)bc+b\ln(b\,x^3+a)c-d\ln(d\,x^3+c)a}{3ac(ad-bc)}$	55
default	$\frac{\ln(x)}{ac}-\frac{d\ln(d\,x^3+c)}{3(ad-bc)c}+\frac{b\ln(b\,x^3+a)}{3(ad-bc)a}$	59
norman	$\frac{\ln(x)}{ac}-\frac{d\ln(d\,x^3+c)}{3(ad-bc)c}+\frac{b\ln(b\,x^3+a)}{3(ad-bc)a}$	59
risch	$\frac{\ln(x)}{ac}-\frac{d\ln(-d\,x^3-c)}{3c(ad-bc)}+\frac{b\ln(-b\,x^3-a)}{3(ad-bc)a}$	65

input
```
int(1/x/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*ln(x)*a*d-3*ln(x)*b*c+b*ln(b*x^3+a)*c-d*ln(d*x^3+c)*a)/a/c/(a*d-b*c
)
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(a+bx^3)(c+dx^3)}\,dx = -\frac{bc\log(bx^3+a)-ad\log(dx^3+c)-3(bc-ad)\log(x)}{3(abc^2-a^2cd)}$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/3*(b*c*log(b*x^3 + a) - a*d*log(d*x^3 + c) - 3*(b*c - a*d)*log(x))/(a*b
*c^2 - a^2*c*d)
```

3.116.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{b\log\left(bx^3+a\right)}{3\left(abc-a^2d\right)}+\frac{d\log\left(dx^3+c\right)}{3\left(bc^2-acd\right)}+\frac{\log\left(x^3\right)}{3\,ac}$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*b*log(b*x^3 + a)/(a*b*c - a^2*d) + 1/3*d*log(d*x^3 + c)/(b*c^2 - a*c*
d) + 1/3*log(x^3)/(a*c)
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{b^2\log\left(\left|bx^3+a\right|\right)}{3\left(ab^2c-a^2bd\right)}+\frac{d^2\log\left(\left|dx^3+c\right|\right)}{3\left(bc^2d-acd^2\right)}+\frac{\log\left(\left|x\right|\right)}{ac}$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*b^2*log(abs(b*x^3 + a))/(a*b^2*c - a^2*b*d) + 1/3*d^2*log(abs(d*x^3 +
 c))/(b*c^2*d - a*c*d^2) + log(abs(x))/(a*c)
```

3.116.9 Mupad [B] (verification not implemented)

Time = 7.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx=\frac{b\,\ln\left(b\,x^3+a\right)}{3\,a^2\,d-3\,a\,b\,c}+\frac{d\,\ln\left(d\,x^3+c\right)}{3\,b\,c^2-3\,a\,c\,d}+\frac{\ln\left(x\right)}{a\,c}$$

input
```
int(1/(x*(a + b*x^3)*(c + d*x^3)),x)
```

output
```
(b*log(a + b*x^3))/(3*a^2*d - 3*a*b*c) + (d*log(c + d*x^3))/(3*b*c^2 - 3*a
*c*d) + log(x)/(a*c)
```

3.117 $\int \frac{1}{x^2(a+bx^3)(c+dx^3)}\,dx$

3.117.1 Optimal result

Integrand size $= 22$, antiderivative size $= 299$

$$\begin{aligned}\int \frac{1}{x^2(a+bx^3)(c+dx^3)}\,dx = {} & -\frac{1}{acx}+\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}(bc-ad)}-\frac{d^{4/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{4/3}(bc-ad)} \\ & +\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)}-\frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{4/3}(bc-ad)} \\ & -\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}(bc-ad)} \\ & +\frac{d^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{4/3}(bc-ad)}\end{aligned}$$

output

```
-1/a/c/x+1/3*b^(4/3)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/(-a*d+b*c)-1/3*d^(4/3)*
ln(c^(1/3)+d^(1/3)*x)/c^(4/3)/(-a*d+b*c)-1/6*b^(4/3)*ln(a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/a^(4/3)/(-a*d+b*c)+1/6*d^(4/3)*ln(c^(2/3)-c^(1/3)*d^(
1/3)*x+d^(2/3)*x^2)/c^(4/3)/(-a*d+b*c)+1/3*b^(4/3)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/(-a*d+b*c)*3^(1/2)-1/3*d^(4/3)*arctan(1
/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(4/3)/(-a*d+b*c)*3^(1/2)
```

3.117.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$
$$=\frac{\frac{6b}{a}-\frac{6d}{c}-\frac{2\sqrt{3}b^{4/3}x\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}+\frac{2\sqrt{3}d^{4/3}x\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{4/3}}-\frac{2b^{4/3}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}}+\frac{2d^{4/3}x\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{4/3}}}{-6bcx+6adx}$$

input
```
Integrate[1/(x^2*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
((6*b)/a - (6*d)/c - (2*Sqrt[3]*b^(4/3)*x*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]])/a^(4/3) + (2*Sqrt[3]*d^(4/3)*x*ArcTan[(1 - (2*d^(1/3)*x)/c^(1
/3))/Sqrt[3]])/c^(4/3) - (2*b^(4/3)*x*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) +
(2*d^(4/3)*x*Log[c^(1/3) + d^(1/3)*x])/c^(4/3) + (b^(4/3)*x*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3) - (d^(4/3)*x*Log[c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2])/c^(4/3))/(-6*b*c*x + 6*a*d*x)
```

3.117.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {980, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$
$$\Big\downarrow \text{980}$$
$$\frac{\int -\frac{x\left(bdx^3+bc+ad\right)}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{ac}-\frac{1}{acx}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int \frac{x\left(bdx^3+bc+ad\right)}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{ac}-\frac{1}{acx}$$

$$\Big\downarrow \text{1054}$$

$$-\frac{\int \left(\frac{cxb^2}{(bc-ad)(bx^3+a)}+\frac{ad^2x}{(ad-bc)(dx^3+c)}\right)dx}{ac}-\frac{1}{acx}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{b^{4/3}c\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)}-\frac{b^{4/3}c\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}(bc-ad)}+\frac{ad^{4/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)}-\frac{b^{4/3}c\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)}-ad^{4/3}}{ac}$$

$$\frac{1}{acx}$$

input
```
Int[1/(x^2*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
-(1/(a*c*x)) - (-((b^(4/3)*c*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(Sqrt[3]*a^(1/3)*(b*c - a*d))) + (a*d^(4/3)*ArcTan[(c^(1/3) - 2*d^(1
/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(1/3)*(b*c - a*d)) - (b^(4/3)*c*Log[
a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*(b*c - a*d)) + (a*d^(4/3)*Log[c^(1/3) + d
^(1/3)*x])/(3*c^(1/3)*(b*c - a*d)) + (b^(4/3)*c*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*(b*c - a*d)) - (a*d^(4/3)*Log[c^(2/3) - c
^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(1/3)*(b*c - a*d)))/(a*c)
```

3.117.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.117.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)d^2}{(ad-bc)c}+\frac{(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+}{(ad-bc)a}$
risch	$-\frac{1}{acx}+\frac{(\sum_{\_R=\mathrm{RootOf}\left(\left(d^3a^7-3a^6bc\,d^2+3a^5b^2c^2d-a^4b^3c^3\right)\_Z^3+b^4\right)}\_R\ln(((-4a^{10}c^4d^6+22a^9b\,c^5d^5-52a^8b^2c^6d^4+68a^7b^3c^7d^3-}{}$

input
```
int(1/x^2/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3
)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/
3)*x-1)))*d^2/(a*d-b*c)/c+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b
)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b^2/(a*d-b*c)/a-1/a/c/x
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{2\sqrt{3}bcx\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-2\sqrt{3}adx\left(\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{d}{c}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-bcx\left(-\frac{b}{a}\right.}{}$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*b*c*x*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3
*sqrt(3)) - 2*sqrt(3)*a*d*x*(d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(d/c)^(1/3) -
 1/3*sqrt(3)) - b*c*x*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)
^(1/3)) - a*d*x*(d/c)^(1/3)*log(d*x^2 - c*x*(d/c)^(2/3) + c*(d/c)^(1/3)) +
 2*b*c*x*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) + 2*a*d*x*(d/c)^(1/3)*log(
d*x + c*(d/c)^(2/3)) + 6*b*c - 6*a*d)/((a*b*c^2 - a^2*c*d)*x)
```

3.117.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a+bx^3) (c+dx^3)} \, dx = -\frac{\sqrt{3} b \arctan\left(\frac{\sqrt{3}\left(2 x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 \left(abc-a^2 d\right) \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3} d \arctan\left(\frac{\sqrt{3}\left(2 x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3 \left(bc^2-acd\right) \left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{b \log\left(x^2 - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(abc \left(\frac{a}{b}\right)^{\frac{1}{3}} - a^2 d \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{d \log\left(x^2 - x \left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6 \left(bc^2 \left(\frac{c}{d}\right)^{\frac{1}{3}} - acd \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{b \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(abc \left(\frac{a}{b}\right)^{\frac{1}{3}} - a^2 d \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{d \log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3 \left(bc^2 \left(\frac{c}{d}\right)^{\frac{1}{3}} - acd \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} - \frac{1}{acx}$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a*b*c
 - a^2*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/
3))/(c/d)^(1/3))/((b*c^2 - a*c*d)*(c/d)^(1/3)) - 1/6*b*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(a*b*c*(a/b)^(1/3) - a^2*d*(a/b)^(1/3)) + 1/6*d*log(x^
2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1/3)) +
 1/3*b*log(x + (a/b)^(1/3))/(a*b*c*(a/b)^(1/3) - a^2*d*(a/b)^(1/3)) - 1/3*
d*log(x + (c/d)^(1/3))/(b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1/3)) - 1/(a*c*x)
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{b^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^2bc-a^3d\right)} - \frac{d^2\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)} + \frac{\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^2bc-\sqrt{3}a^3d} - \frac{\left(-cd^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^3-\sqrt{3}ac^2d} - \frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2bc-a^3d\right)} + \frac{\left(-cd^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^3-ac^2d\right)} - \frac{1}{acx}$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b^2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*c - a^3*d) - 1/3*d^
2*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^3 - a*c^2*d) + (-a*b^2)^(2/
3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b*c
- sqrt(3)*a^3*d) - (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/
(-c/d)^(1/3))/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) - 1/6*(-a*b^2)^(2/3)*log(x
^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)^(2/3)
*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^3 - a*c^2*d) - 1/(a*c*x)
```

3.117.9 Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 716, normalized size of antiderivative = 2.39

$$\int \frac{1}{x^2\,(a+bx^3)\,(c+dx^3)}\,dx = \ln\left(b - a^2\,d\,x\left(-\frac{b^4}{a^4\,(a\,d-b\,c)^3}\right)^{1/3}\right.$$

$$\left.+\,a\,b\,c\,x\left(-\frac{b^4}{a^4\,(a\,d-b\,c)^3}\right)^{1/3}\right)\left(-\frac{b^4}{27\,a^7\,d^3-81\,a^6\,b\,c\,d^2+81\,a^5\,b^2\,c^2\,d-27\,a^4\,b^3\,c^3}\right)^{1/3}$$

$$+\ln\left(d - b\,c^2\,x\left(\frac{d^4}{c^4\,(a\,d-b\,c)^3}\right)^{1/3} + a\,c\,d\,x\left(\frac{d^4}{c^4\,(a\,d-b\,c)^3}\right)^{1/3}\right)\left(-\frac{d^4}{-27\,a^3\,c^4\,d^3+81\,a^2\,b\,c^5\,d^2-81\,a\,b\right.$$

input
```
int(1/(x^2*(a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(b - a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) + a*b*c*x*(-b^4/(a^4*(a*d
 - b*c)^3))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d -
 81*a^6*b*c*d^2))^(1/3) + log(d - b*c^2*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3)
+ a*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(-d^4/(27*b^3*c^7 - 27*a^3*c^4*
d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1/3) - 1/(a*c*x) - (log(b - 3^(
1/2)*b*1i + 2*a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) - 2*a*b*c*x*(-b^4/(
a^4*(a*d - b*c)^3))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2
*c^2*d - 81*a^6*b*c*d^2))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(b + 3^(1/2)*b*1
i + 2*a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) - 2*a*b*c*x*(-b^4/(a^4*(a*d
 - b*c)^3))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d -
 81*a^6*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(d - 3^(1/2)*d*1i + 2*b*
c^2*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3) - 2*a*c*d*x*(d^4/(c^4*(a*d - b*c)^3)
)^(1/3))*(-d^4/(27*b^3*c^7 - 27*a^3*c^4*d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*
c^6*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(d + 3^(1/2)*d*1i + 2*b*c^2*x*(d^4
/(c^4*(a*d - b*c)^3))^(1/3) - 2*a*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(
-d^4/(27*b^3*c^7 - 27*a^3*c^4*d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1
/3)*(3^(1/2)*1i - 1))/2
```

3.118 $\int \frac{1}{x^3\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$

3.118.1 Optimal result

Integrand size = 22, antiderivative size = 301

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = {} & -\frac{1}{2acx^2}+\frac{b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}(bc-ad)} \\ & -\frac{d^{5/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{5/3}(bc-ad)}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)} \\ & +\frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{5/3}(bc-ad)}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}(bc-ad)} \\ & -\frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{5/3}(bc-ad)}\end{aligned}$$

output
```
-1/2/a/c/x^2-1/3*b^(5/3)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/(-a*d+b*c)+1/3*d^(5
/3)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*d+b*c)+1/6*b^(5/3)*ln(a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/(-a*d+b*c)-1/6*d^(5/3)*ln(c^(2/3)-c^(1/3)
*d^(1/3)*x+d^(2/3)*x^2)/c^(5/3)/(-a*d+b*c)+1/3*b^(5/3)*arctan(1/3*(a^(1/3)
-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/(-a*d+b*c)*3^(1/2)-1/3*d^(5/3)*arct
an(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/(-a*d+b*c)*3^(1/2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)}\,dx$$
$$=\frac{\frac{3b}{a}-\frac{3d}{c}-\frac{2\sqrt{3}b^{5/3}x^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{2\sqrt{3}d^{5/3}x^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}}+\frac{2b^{5/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}-\frac{2d^{5/3}x^2\log\left(\sqrt[3]{c}+\sqrt[3]{}\right.}{c^{5/3}}}{6(-bc+ad)x^2}$$

input
```
Integrate[1/(x^3*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
((3*b)/a - (3*d)/c - (2*Sqrt[3]*b^(5/3)*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/a^(5/3) + (2*Sqrt[3]*d^(5/3)*x^2*ArcTan[(1 - (2*d^(1/3)*x)/
c^(1/3))/Sqrt[3]])/c^(5/3) + (2*b^(5/3)*x^2*Log[a^(1/3) + b^(1/3)*x])/a^(5
/3) - (2*d^(5/3)*x^2*Log[c^(1/3) + d^(1/3)*x])/c^(5/3) - (b^(5/3)*x^2*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + (d^(5/3)*x^2*Log[c^(
2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(5/3))/(6*(-(b*c) + a*d)*x^2)
```

3.118.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {980, 27, 1020, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)}\,dx$$
$$\Big\downarrow \; 980$$
$$\frac{\int -\frac{2(bdx^3+bc+ad)}{(bx^3+a)(dx^3+c)}dx}{2ac}-\frac{1}{2acx^2}$$
$$\Big\downarrow \; 27$$
$$-\frac{\int \frac{bdx^3+bc+ad}{(bx^3+a)(dx^3+c)}dx}{ac}-\frac{1}{2acx^2}$$

$$\Big\downarrow 1020$$

$$-\frac{\frac{b^2c\int\frac{1}{bx^3+a}dx}{bc-ad}-\frac{ad^2\int\frac{1}{dx^3+c}dx}{bc-ad}}{ac}-\frac{1}{2acx^2}$$

$$\Big\downarrow 750$$

$$-\frac{\frac{b^2c\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}}{ac}-\frac{1}{2acx^2}$$

$$\Big\downarrow 16$$

$$-\frac{\frac{b^2c\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}}{ac}-\frac{1}{2acx^2}$$

$$\Big\downarrow 1142$$

$$-\frac{\frac{b^2c\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\quad\int}{3c^{2/3}}\right.}{}}{ac}$$

$$\frac{1}{2acx^2}$$

$$\Big\downarrow 25$$

3.118. $\int\frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx$

$$-\frac{\frac{b^2c\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int - \quad d}{}}{3c^{2/3}}\right)}{bc}}{ac}$$

$$\frac{1}{2acx^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{b^2c\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}}{3c^{2/3}}\right)}{}}{ac}$$

$$\frac{1}{2acx^2}$$

$$\Big\downarrow \; 1082$$

$$-\frac{\frac{b^2c\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int -}{-}}{3c^{2/3}}\right)}{}}{ac}$$

$$\frac{1}{2acx^2}$$

$$\Big\downarrow \; 217$$

3.118. $\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx$

$$-\frac{\frac{b^2c\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan}{\sqrt[3]{d}}}{3c^{2/3}}\right.}{bc-ad}}{ac}$$

$$\frac{1}{2acx^2}$$

$$\Big\downarrow 1103$$

$$-\frac{\frac{b^2c\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{ad^2\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+\right.}{2\sqrt[3]{d}}}{3c^{2/3}}\right.}{bc-ad}}{ac}$$

$$\frac{1}{2acx^2}$$

input
```
Int[1/(x^3*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
-1/2*1/(a*c*x^2) - ((b^2*c*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) +
 (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(b*c
- a*d) - (a*d^2*(Log[c^(1/3) + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3
]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^
(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1/3)))/(3*c^(2/3))))/(b*c - a*d))/(a*
c)
```

3.118. $\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx$

3.118.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.118.4 Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)d^{2}}{c(ad-bc)}+\frac{\Bigg(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\sqrt{3}}{a(ad-bc)}$
risch	$-\frac{1}{2ac\,x^{2}}+\frac{\Bigg(\sum_{\_R=\mathrm{RootOf}\left(\left(d^{3}a^{8}-3c\,d^{2}a^{7}b+3c^{2}d\,a^{6}b^{2}-a^{5}b^{3}c^{3}\right)\_Z^{3}-b^{5}\right)}\_R\ln\Big(\Big(\big(-4a^{11}c^{5}d^{6}+22a^{10}b\,c^{6}d^{5}-52a^{9}b^{2}c^{7}d^{4}+68a^{8}b^{3}c^{8}}{}$

input
```
int(1/x^3/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)
*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3
)*x-1)))/c*d^2/(a*d-b*c)+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^
(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/a*b^2/(a*d-b*c)-1/2/a/c/x^2
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx =$$

$$-\frac{2\sqrt{3}bcx^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+2\sqrt{3}adx^2\left(-\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(-\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-bcx^2\left(\frac{b^2}{a^2}\right.}{}$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*b*c*x^2*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2
)^(2/3) - sqrt(3)*b)/b) + 2*sqrt(3)*a*d*x^2*(-d^2/c^2)^(1/3)*arctan(1/3*(2
*sqrt(3)*c*x*(-d^2/c^2)^(2/3) - sqrt(3)*d)/d) - b*c*x^2*(b^2/a^2)^(1/3)*lo
g(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) - a*d*x^2*(-d^2/c
^2)^(1/3)*log(d^2*x^2 + c*d*x*(-d^2/c^2)^(1/3) + c^2*(-d^2/c^2)^(2/3)) + 2
*b*c*x^2*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 2*a*d*x^2*(-d^2/c^
2)^(1/3)*log(d*x - c*(-d^2/c^2)^(1/3)) + 3*b*c - 3*a*d)/((a*b*c^2 - a^2*c*
d)*x^2)
```

3.118.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)}\,dx = -\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}d\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{b\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(abc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{d\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-acd\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} - \frac{b\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{d\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-acd\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} - \frac{1}{2\,acx^2}$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a*b*c
*(a/b)^(1/3) - a^2*d*(a/b)^(1/3))*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*
sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c^2*(c/d)^(1/3) - a*c*d*(c/d)
^(1/3))*(c/d)^(1/3)) + 1/6*b*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*c
*(a/b)^(2/3) - a^2*d*(a/b)^(2/3)) - 1/6*d*log(x^2 - x*(c/d)^(1/3) + (c/d)^
(2/3))/(b*c^2*(c/d)^(2/3) - a*c*d*(c/d)^(2/3)) - 1/3*b*log(x + (a/b)^(1/3)
)/(a*b*c*(a/b)^(2/3) - a^2*d*(a/b)^(2/3)) + 1/3*d*log(x + (c/d)^(1/3))/(b*
c^2*(c/d)^(2/3) - a*c*d*(c/d)^(2/3)) - 1/2/(a*c*x^2)
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx = {} & \frac{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^2bc-a^3d\right)} - \frac{d^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)} \\ & - \frac{\left(-ab^2\right)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^2bc-\sqrt{3}a^3d} \\ & + \frac{\left(-cd^2\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^3-\sqrt{3}ac^2d} \\ & - \frac{\left(-ab^2\right)^{\frac{1}{3}}b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2bc-a^3d\right)} \\ & + \frac{\left(-cd^2\right)^{\frac{1}{3}}d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^3-ac^2d\right)} - \frac{1}{2\,acx^2}\end{aligned}$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*c - a^3*d) - 1/3*d^
2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^3 - a*c^2*d) - (-a*b^2)^(1/
3)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b*
c - sqrt(3)*a^3*d) + (-c*d^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/
3))/(-c/d)^(1/3))/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) - 1/6*(-a*b^2)^(1/3)*b
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)
^(1/3)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^3 - a*c^2*d) - 1/2/
(a*c*x^2)
```

3.118.9 Mupad [B] (verification not implemented)

Time = 16.87 (sec) , antiderivative size = 1829, normalized size of antiderivative = 6.08

$$\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*x^3)*(c + d*x^3)),x)
```

output

```
log(((b^5/(a^5*(a*d - b*c)^3))^(1/3)*(((81*a^10*b^3*c^10*d^3*(a*d + b*c)*(
a*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3) - 81*a^8*b^3*c^8*d^3*x*(a*d -
 b*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d))*(b^5/(a^5*(a*d - b*c)^3))^(2/3))/9
+ 9*a^6*b^9*c^11*d^4 - 9*a^7*b^8*c^10*d^5 - 9*a^10*b^5*c^7*d^8 + 9*a^11*b^
4*c^6*d^9))/3 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(b^5/(27*a^8*d^3
- 27*a^5*b^3*c^3 + 81*a^6*b^2*c^2*d - 81*a^7*b*c*d^2))^(1/3) + log(((-d^5/
(c^5*(a*d - b*c)^3))^(1/3)*(((81*a^10*b^3*c^10*d^3*(a*d + b*c)*(a*d - b*c)
^4*(-d^5/(c^5*(a*d - b*c)^3))^(1/3) - 81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(
a^2*d^2 + b^2*c^2 + a*b*c*d))*(-d^5/(c^5*(a*d - b*c)^3))^(2/3))/9 + 9*a^6*
b^9*c^11*d^4 - 9*a^7*b^8*c^10*d^5 - 9*a^10*b^5*c^7*d^8 + 9*a^11*b^4*c^6*d^
9))/3 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(d^5/(27*b^3*c^8 - 27*a^3
*c^5*d^3 + 81*a^2*b*c^6*d^2 - 81*a*b^2*c^7*d))^(1/3) + (log(((3^(1/2)*1i -
 1)*(b^5/(a^5*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a^8*b^3*c^8*d
^3*x*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d) - (81*a^10*b^3*c^10*d^3*(
3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3))
/2)*(b^5/(a^5*(a*d - b*c)^3))^(2/3))/36 - 9*a^6*b^9*c^11*d^4 + 9*a^7*b^8*c
^10*d^5 + 9*a^10*b^5*c^7*d^8 - 9*a^11*b^4*c^6*d^9))/6 - 3*a^6*b^6*c^6*d^6*
x*(a^2*d^2 + b^2*c^2))*(b^5/(27*a^8*d^3 - 27*a^5*b^3*c^3 + 81*a^6*b^2*c^2*
d - 81*a^7*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)*(b
^5/(a^5*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a^8*b^3*c^8*d^3*...
```

3.118. $\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx$

3.119 $\int \frac{1}{x^4(a+bx^3)(c+dx^3)}\,dx$

3.119.1 Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)}\,dx=-\frac{1}{3acx^3}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^3\right)}{3a^2(bc-ad)}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}$$

output
```
-1/3/a/c/x^3-(a*d+b*c)*ln(x)/a^2/c^2+1/3*b^2*ln(b*x^3+a)/a^2/(-a*d+b*c)-1/
3*d^2*ln(d*x^3+c)/c^2/(-a*d+b*c)
```

3.119.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)}\,dx=-\frac{1}{3acx^3}+\frac{(-bc-ad)\log(x)}{a^2c^2}-\frac{b^2\log\left(a+bx^3\right)}{3a^2(-bc+ad)}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}$$

input
```
Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
-1/3*1/(a*c*x^3) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^3]
)/(3*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^3])/(3*c^2*(b*c - a*d))
```

3.119.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)} \, dx$$

$$\Big\downarrow \textbf{948}$$

$$\frac{1}{3} \int \frac{1}{x^6 (bx^3+a) (dx^3+c)} dx^3$$

$$\Big\downarrow \textbf{93}$$

$$\frac{1}{3} \int \left(-\frac{b^3}{a^2(ad-bc) (bx^3+a)} - \frac{d^3}{c^2(bc-ad) (dx^3+c)} + \frac{-bc-ad}{a^2c^2x^3} + \frac{1}{acx^6}\right) dx^3$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{1}{3} \left(\frac{b^2 \log (a+bx^3)}{a^2(bc-ad)} - \frac{\log (x^3) (ad+bc)}{a^2c^2} - \frac{d^2 \log (c+dx^3)}{c^2(bc-ad)} - \frac{1}{acx^3}\right)$$

input
```
Int[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
(-(1/(a*c*x^3)) - ((b*c + a*d)*Log[x^3])/(a^2*c^2) + (b^2*Log[a + b*x^3])/
(a^2*(b*c - a*d)) - (d^2*Log[c + d*x^3])/(c^2*(b*c - a*d)))/3
```

3.119.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.119.4 Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

method	result	size
norman	$-\frac{1}{3ac\,x^{3}}-\frac{b^{2}\ln\left(b\,x^{3}+a\right)}{3a^{2}\left(ad-bc\right)}+\frac{d^{2}\ln\left(d\,x^{3}+c\right)}{3c^{2}\left(ad-bc\right)}-\frac{\left(ad+bc\right)\ln\left(x\right)}{a^{2}c^{2}}$	82
default	$-\frac{1}{3ac\,x^{3}}+\frac{\left(-ad-bc\right)\ln\left(x\right)}{a^{2}c^{2}}+\frac{d^{2}\ln\left(d\,x^{3}+c\right)}{3c^{2}\left(ad-bc\right)}-\frac{b^{2}\ln\left(b\,x^{3}+a\right)}{3a^{2}\left(ad-bc\right)}$	83
risch	$-\frac{1}{3ac\,x^{3}}-\frac{\ln\left(x\right)d}{a\,c^{2}}-\frac{\ln\left(x\right)b}{a^{2}c}-\frac{b^{2}\ln\left(-b\,x^{3}-a\right)}{3\left(ad-bc\right)a^{2}}+\frac{d^{2}\ln\left(d\,x^{3}+c\right)}{3c^{2}\left(ad-bc\right)}$	90
parallelrisch	$-\frac{3\ln\left(x\right)x^{3}a^{2}d^{2}-3\ln\left(x\right)x^{3}b^{2}c^{2}+b^{2}\ln\left(b\,x^{3}+a\right)c^{2}x^{3}-d^{2}\ln\left(d\,x^{3}+c\right)a^{2}x^{3}+a^{2}cd-b\,c^{2}a}{3a^{2}c^{2}x^{3}\left(ad-bc\right)}$	99

input
```
int(1/x^4/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/c/x^3-1/3*b^2/a^2/(a*d-b*c)*ln(b*x^3+a)+1/3*d^2/c^2/(a*d-b*c)*ln(d*
x^3+c)-(a*d+b*c)*ln(x)/a^2/c^2
```

3.119.5 Fricas [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{b^2c^2x^3\log\left(bx^3+a\right)-a^2d^2x^3\log\left(dx^3+c\right)-3\left(b^2c^2-a^2d^2\right)x^3\log\left(x\right)-abc^2+a^2cd}{3\left(a^2bc^3-a^3c^2d\right)x^3}$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/3*(b^2*c^2*x^3*log(b*x^3 + a) - a^2*d^2*x^3*log(d*x^3 + c) - 3*(b^2*c^2
- a^2*d^2)*x^3*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^3)
```

3.119.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**4/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)}\,dx = \frac{b^2 \log (bx^3+a)}{3 (a^2bc-a^3d)} - \frac{d^2 \log (dx^3+c)}{3 (bc^3-ac^2d)} - \frac{(bc+ad) \log (x^3)}{3\,a^2c^2} - \frac{1}{3\,acx^3}$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*b^2*log(b*x^3 + a)/(a^2*b*c - a^3*d) - 1/3*d^2*log(d*x^3 + c)/(b*c^3 -
 a*c^2*d) - 1/3*(b*c + a*d)*log(x^3)/(a^2*c^2) - 1/3/(a*c*x^3)
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \frac{1}{x^4 (a+bx^3) (c+dx^3)}\,dx = {} & \frac{b^3 \log (|bx^3+a|)}{3 (a^2b^2c-a^3bd)} - \frac{d^3 \log (|dx^3+c|)}{3 (bc^3d-ac^2d^2)} \\ & - \frac{(bc+ad) \log (|x|)}{a^2c^2} + \frac{bcx^3+adx^3-ac}{3\,a^2c^2x^3}\end{aligned}$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b^3*log(abs(b*x^3 + a))/(a^2*b^2*c - a^3*b*d) - 1/3*d^3*log(abs(d*x^3
+ c))/(b*c^3*d - a*c^2*d^2) - (b*c + a*d)*log(abs(x))/(a^2*c^2) + 1/3*(b*c
*x^3 + a*d*x^3 - a*c)/(a^2*c^2*x^3)
```

3.119.9 Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{b^2\,\ln\left(b\,x^3+a\right)}{3\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{d^2\,\ln\left(d\,x^3+c\right)}{3\,\left(b\,c^3-a\,c^2\,d\right)} -\frac{1}{3\,a\,c\,x^3}-\frac{\ln\left(x\right)\,\left(a\,d+b\,c\right)}{a^2\,c^2}$$

input

```
int(1/(x^4*(a + b*x^3)*(c + d*x^3)),x)
```

output

```
- (b^2*log(a + b*x^3))/(3*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^3))/(3*(b*
c^3 - a*c^2*d)) - 1/(3*a*c*x^3) - (log(x)*(a*d + b*c))/(a^2*c^2)
```

3.120 $\int \frac{1}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$

3.120.1 Optimal result

Integrand size = 22, antiderivative size = 318

$$\begin{aligned}\int \frac{1}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = & -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}(bc-ad)} \\ & +\frac{d^{7/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{7/3}(bc-ad)}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}(bc-ad)} \\ & +\frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{7/3}(bc-ad)}+\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}(bc-ad)} \\ & -\frac{d^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{7/3}(bc-ad)}\end{aligned}$$

output
```
-1/4/a/c/x^4+(a*d+b*c)/a^2/c^2/x-1/3*b^(7/3)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)
/(-a*d+b*c)+1/3*d^(7/3)*ln(c^(1/3)+d^(1/3)*x)/c^(7/3)/(-a*d+b*c)+1/6*b^(7/
3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/(-a*d+b*c)-1/6*d^(7/3
)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(7/3)/(-a*d+b*c)-1/3*b^(7/3)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/(-a*d+b*c)*3^(1
/2)+1/3*d^(7/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(7/3)/
(-a*d+b*c)*3^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$
$$=\frac{\frac{3b}{a}-\frac{3d}{c}-\frac{12b^2x^3}{a^2}+\frac{12d^2x^3}{c^2}+\frac{4\sqrt{3}b^{7/3}x^4\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}}-\frac{4\sqrt{3}d^{7/3}x^4\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{7/3}}+\frac{4b^{7/3}x^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{7/3}}-}{12(-bc+ad)x^4}$$

input
```
Integrate[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
((3*b)/a - (3*d)/c - (12*b^2*x^3)/a^2 + (12*d^2*x^3)/c^2 + (4*Sqrt[3]*b^(7
/3)*x^4*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(7/3) - (4*Sqrt[3]*
d^(7/3)*x^4*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(7/3) + (4*b^(7
/3)*x^4*Log[a^(1/3) + b^(1/3)*x])/a^(7/3) - (4*d^(7/3)*x^4*Log[c^(1/3) + d
^(1/3)*x])/c^(7/3) - (2*b^(7/3)*x^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/a^(7/3) + (2*d^(7/3)*x^4*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2])/c^(7/3))/(12*(-(b*c) + a*d)*x^4)
```

3.120.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {980, 27, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}dx$$
$$\Big\downarrow \ 980$$
$$\frac{\int -\frac{4\left(bdx^3+bc+ad\right)}{x^2\left(bx^3+a\right)\left(dx^3+c\right)}dx}{4ac}-\frac{1}{4acx^4}$$
$$\Big\downarrow \ 27$$
$$-\frac{\int \frac{bdx^3+bc+ad}{x^2\left(bx^3+a\right)\left(dx^3+c\right)}dx}{ac}-\frac{1}{4acx^4}$$

$$\Big\downarrow \text{1053}$$

$$-\frac{-\frac{\int \frac{x\left(bd(bc+ad)x^3+b^2c^2+a^2d^2+abcd\right)}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{ac}-\frac{ad+bc}{acx}}{ac}-\frac{1}{4acx^4}$$

$$\Big\downarrow \text{1054}$$

$$-\frac{-\frac{\int \left(\frac{c^2xb^3}{(bc-ad)\left(bx^3+a\right)}+\frac{a^2d^3x}{(ad-bc)\left(dx^3+c\right)}\right)dx}{ac}-\frac{ad+bc}{acx}}{ac}-\frac{1}{4acx^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{\frac{b^{7/3}c^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)}+\frac{a^2d^{7/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)}-\frac{a^2d^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)}+\frac{a^2d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)}-b^{7/3}c}{ac}}{ac}$$

$$\frac{1}{4acx^4}$$

input
```
Int[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
-1/4*1/(a*c*x^4) - (-((b*c + a*d)/(a*c*x)) - (-((b^(7/3)*c^2*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*(b*c - a*d))) + (a^
2*d^(7/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(1
/3)*(b*c - a*d)) - (b^(7/3)*c^2*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*(b*c
- a*d)) + (a^2*d^(7/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(1/3)*(b*c - a*d)) +
 (b^(7/3)*c^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*(
b*c - a*d)) - (a^2*d^(7/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])
/(6*c^(1/3)*(b*c - a*d)))/(a*c))/(a*c)
```

3.120. $\int \frac{1}{x^5(a+bx^3)(c+dx^3)}\,dx$

3.120.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.120.4 Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.78

method	result
default	$-\frac{1}{4acx^4}-\frac{-ad-bc}{a^2c^2x}+\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)d^3}{c^2(ad-bc)}-\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\ln\right.$
risch	$\frac{\frac{(ad+bc)x^3}{a^2c^2}-\frac{1}{4ac}}{x^4}+\left(\sum_{\_R=\operatorname{RootOf}\left(\left(d^3c^7a^3-3d^2c^8a^2b+3dc^9ab^2-b^3c^{10}\right)\_Z^3+d^7\right)}\_R\ln\left(\left(\left(-4a^{13}c^7d^6+22a^{12}bc^8d^5-52a^{11}b^2c^9d^4+\right.\right.\right.\right.$

input
```
int(1/x^5/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/c/x^4-1/a^2/c^2*(-a*d-b*c)/x+(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+
1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1
/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))*d^3/c^2/(a*d-b*c)-(-1/3/b/(a/
b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b^
3/a^2/(a*d-b*c)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.96

$$\int\frac{1}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$

$$=4\sqrt{3}b^2c^2x^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-4\sqrt{3}a^2d^2x^4\left(-\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+2b^2c$$

input
```
integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/12*(4*sqrt(3)*b^2*c^2*x^4*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) -
 1/3*sqrt(3)) - 4*sqrt(3)*a^2*d^2*x^4*(-d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(-
d/c)^(1/3) + 1/3*sqrt(3)) + 2*b^2*c^2*x^4*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a
)^(2/3) + a*(b/a)^(1/3)) + 2*a^2*d^2*x^4*(-d/c)^(1/3)*log(d*x^2 - c*x*(-d/
c)^(2/3) - c*(-d/c)^(1/3)) - 4*b^2*c^2*x^4*(b/a)^(1/3)*log(b*x + a*(b/a)^(
2/3)) - 4*a^2*d^2*x^4*(-d/c)^(1/3)*log(d*x + c*(-d/c)^(2/3)) - 3*a*b*c^2 +
 3*a^2*c*d + 12*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*d)*x^4)
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**5/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{1}{x^5 (a+bx^3)(c+dx^3)}\,dx = {} & \frac{\sqrt{3}b^2 \arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(a^2bc-a^3d)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}d^2 \arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,(bc^3-ac^2d)\left(\frac{c}{d}\right)^{\frac{1}{3}}} \\ & + \frac{b^2 \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2bc\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^3d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} \\ & - \frac{d^2 \log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^3\left(\frac{c}{d}\right)^{\frac{1}{3}} - ac^2d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} - \frac{b^2 \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a^2bc\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^3d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} \\ & + \frac{d^2 \log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^3\left(\frac{c}{d}\right)^{\frac{1}{3}} - ac^2d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{4\,(bc+ad)x^3 - ac}{4\,a^2c^2x^4}\end{aligned}$$

input
```
integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^2*
b*c - a^3*d)*(a/b)^(1/3)) - 1/3*sqrt(3)*d^2*arctan(1/3*sqrt(3)*(2*x - (c/d
)^(1/3))/(c/d)^(1/3))/((b*c^3 - a*c^2*d)*(c/d)^(1/3)) + 1/6*b^2*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*c*(a/b)^(1/3) - a^3*d*(a/b)^(1/3)) - 1
/6*d^2*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^3*(c/d)^(1/3) - a*c^2*d
*(c/d)^(1/3)) - 1/3*b^2*log(x + (a/b)^(1/3))/(a^2*b*c*(a/b)^(1/3) - a^3*d*
(a/b)^(1/3)) + 1/3*d^2*log(x + (c/d)^(1/3))/(b*c^3*(c/d)^(1/3) - a*c^2*d*(
c/d)^(1/3)) + 1/4*(4*(b*c + a*d)*x^3 - a*c)/(a^2*c^2*x^4)
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{1}{x^5(a+bx^3)(c+dx^3)}\,dx = & -\frac{b^3\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(a^3bc-a^4d)}+\frac{d^3\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^4-ac^3d)} \\ & -\frac{\left(-ab^2\right)^{\frac{2}{3}}b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^3bc-\sqrt{3}a^4d} \\ & +\frac{\left(-cd^2\right)^{\frac{2}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^4-\sqrt{3}ac^3d} \\ & +\frac{\left(-ab^2\right)^{\frac{2}{3}}b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(a^3bc-a^4d)} \\ & -\frac{\left(-cd^2\right)^{\frac{2}{3}}d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^4-ac^3d)} \\ & +\frac{4\,bcx^3+4\,adx^3-ac}{4\,a^2c^2x^4}\end{aligned}$$

input
```
integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output

```
-1/3*b^3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b*c - a^4*d) + 1/3*d
^3*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^4 - a*c^3*d) - (-a*b^2)^(2
/3)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^3*b
*c - sqrt(3)*a^4*d) + (-c*d^2)^(2/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c^4 - sqrt(3)*a*c^3*d) + 1/6*(-a*b^2)^(2/3)*
b*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b*c - a^4*d) - 1/6*(-c*d^2
)^(2/3)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^4 - a*c^3*d) + 1/4
*(4*b*c*x^3 + 4*a*d*x^3 - a*c)/(a^2*c^2*x^4)
```

3.120.9 Mupad [B] (verification not implemented)

Time = 16.47 (sec) , antiderivative size = 1734, normalized size of antiderivative = 5.45

$$\int \frac{1}{x^5 (a + bx^3) (c + dx^3)} \, dx = \text{Too large to display}$$

input

```
int(1/(x^5*(a + b*x^3)*(c + d*x^3)),x)
```

output

```
log(((b^7/(a^7*(a*d - b*c)^3))^(2/3)*(((27*a^14*b^3*c^14*d^3*x*(a^6*d^6 +
b^6*c^6)*(a*d - b*c)^2 + 27*a^19*b^3*c^19*d^3*(a*d + b*c)*(a*d - b*c)^4*(b
^7/(a^7*(a*d - b*c)^3))^(2/3))*(b^7/(a^7*(a*d - b*c)^3))^(1/3))/3 + 9*a^13
*b^11*c^20*d^4 - 9*a^14*b^10*c^19*d^5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*
c^13*d^11))/9 + a^13*b^9*c^13*d^9*x)*(b^7/(27*a^10*d^3 - 27*a^7*b^3*c^3 +
81*a^8*b^2*c^2*d - 81*a^9*b*c*d^2))^(1/3) + log(((-d^7/(c^7*(a*d - b*c)^3)
)^(2/3)*(((27*a^14*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + 27*a
^19*b^3*c^19*d^3*(a*d + b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^(2/3
))*(-d^7/(c^7*(a*d - b*c)^3))^(1/3))/3 + 9*a^13*b^11*c^20*d^4 - 9*a^14*b^1
0*c^19*d^5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/9 + a^13*b^9*c^
13*d^9*x)*(d^7/(27*b^3*c^10 - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*a*b^2
*c^9*d))^(1/3) - (1/(4*a*c) - (x^3*(a*d + b*c))/(a^2*c^2))/x^4 + (log(((3^
(1/2)*1i - 1)^2*(b^7/(a^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(27*a^1
4*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^19*b^3*c^19*d^3
*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b^7/(a^7*(a*d - b*c)^3))^(2
/3))/4)*(b^7/(a^7*(a*d - b*c)^3))^(1/3))/6 + 9*a^13*b^11*c^20*d^4 - 9*a^14
*b^10*c^19*d^5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/36 + a^13*b
^9*c^13*d^9*x)*(b^7/(27*a^10*d^3 - 27*a^7*b^3*c^3 + 81*a^8*b^2*c^2*d - 81*
a^9*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(b^7/(a
^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i + 1)*(27*a^14*b^3*c^14*d^3*x*(a^...
```

3.121 $\int \frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx$

3.121.1 Optimal result

Integrand size = 22, antiderivative size = 321

$$\begin{aligned}\int \frac{1}{x^6\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = {} & -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}-\frac{b^{8/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}(bc-ad)} \\ & +\frac{d^{8/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{8/3}(bc-ad)}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)} \\ & -\frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{8/3}(bc-ad)}-\frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}(bc-ad)} \\ & +\frac{d^{8/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{8/3}(bc-ad)}\end{aligned}$$

output
```
-1/5/a/c/x^5+1/2*(a*d+b*c)/a^2/c^2/x^2+1/3*b^(8/3)*ln(a^(1/3)+b^(1/3)*x)/a
^(8/3)/(-a*d+b*c)-1/3*d^(8/3)*ln(c^(1/3)+d^(1/3)*x)/c^(8/3)/(-a*d+b*c)-1/6
*b^(8/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/(-a*d+b*c)+1/6*
d^(8/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(8/3)/(-a*d+b*c)-1/3*b
^(8/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/(-a*d+b*c
)*3^(1/2)+1/3*d^(8/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^
(8/3)/(-a*d+b*c)*3^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^6\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{\frac{6b}{a}-\frac{6d}{c}-\frac{15b^2x^3}{a^2}+\frac{15d^2x^3}{c^2}+\frac{10\sqrt{3}b^{8/3}x^5\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}}-\frac{10\sqrt{3}d^{8/3}x^5\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{8/3}}-\frac{10b^{8/3}x^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{8/3}}}{30(-bc+ad)x^5}$$

input
```
Integrate[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
((6*b)/a - (6*d)/c - (15*b^2*x^3)/a^2 + (15*d^2*x^3)/c^2 + (10*Sqrt[3]*b^(
8/3)*x^5*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(8/3) - (10*Sqrt[3
]*d^(8/3)*x^5*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(8/3) - (10*b
^(8/3)*x^5*Log[a^(1/3) + b^(1/3)*x]])/a^(8/3) + (10*d^(8/3)*x^5*Log[c^(1/3)
 + d^(1/3)*x]])/c^(8/3) + (5*b^(8/3)*x^5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2]])/a^(8/3) - (5*d^(8/3)*x^5*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2]])/c^(8/3))/(30*(-(b*c) + a*d)*x^5)
```

3.121.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {980, 27, 1053, 27, 1020, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$

$$\Big\downarrow \text{980}$$

$$\frac{\int -\frac{5\left(bdx^3+bc+ad\right)}{x^3\left(bx^3+a\right)\left(dx^3+c\right)}dx}{5ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{bdx^3+bc+ad}{x^3\left(bx^3+a\right)\left(dx^3+c\right)}dx}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{1053}$$

$$-\frac{-\frac{\int \frac{2\left(bd(bc+ad)x^3+b^2c^2+a^2d^2+abcd\right)}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{2ac}-\frac{ad+bc}{2acx^2}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{bd(bc+ad)x^3+b^2c^2+a^2d^2+abcd}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{ac}-\frac{ad+bc}{2acx^2}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{1020}$$

$$-\frac{-\frac{\frac{b^3c^2\int \frac{1}{bx^3+a}dx}{bc-ad}-\frac{a^2d^3\int \frac{1}{dx^3+c}dx}{bc-ad}}{ac}-\frac{ad+bc}{2acx^2}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{750}$$

$$-\frac{-\frac{\frac{b^3c^2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}-\frac{a^2d^3\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}}{ac}-\frac{ad+bc}{2acx^2}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{16}$$

$$-\frac{-\frac{\frac{b^3c^2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{a^2d^3\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}}{ac}-\frac{ad+bc}{2acx^2}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{1142}$$

3.121. $\int \frac{1}{x^6\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$

$$-\frac{-\frac{b^3c^2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-a^2d^3\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-}{3c^{2/3}}\right.}{ac}}{ac}$$

$$\frac{1}{5acx^5}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{b^3c^2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-a^2d^3\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+}{3c^{2/3}}\right.}{ac}}{ac}$$

$$\frac{1}{5acx^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{b^3c^2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-a^2d^3\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+}{3c^{2/3}}\right.}{ac}}{ac}$$

$$\frac{1}{5acx^5}$$

$$\Big\downarrow \text{1082}$$

3.121. $\int\frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx$

$$-\frac{-\frac{\frac{b^3c^2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-a^2d^3\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int}{}}{3c^{2/3}}\right.}{ac}}{ac}$$

$$\frac{1}{5acx^5}$$

$$\Big\downarrow \; 217$$

$$-\frac{-\frac{\frac{b^3c^2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{a^2d^3\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}\right.}{bc-ad}}{ac}}{ac}$$

$$\frac{1}{5acx^5}$$

$$\Big\downarrow \; 1103$$

$$-\frac{-\frac{\frac{b^3c^2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{a^2d^3\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+\right.}{2\sqrt[3]{d}}}{3c^{2/3}}\right.}{bc-ad}}{ac}}{ac}$$

$$\frac{1}{5acx^5}$$

input
```
Int[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]
```

3.121. $\int \frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx$

output

```
-1/5*1/(a*c*x^5) - (-1/2*(b*c + a*d)/(a*c*x^2) - ((b^3*c^2*(Log[a^(1/3) +
b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/
(2*b^(1/3)))/(3*a^(2/3))))/(b*c - a*d) - (a^2*d^3*(Log[c^(1/3) + d^(1/3)*x
]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqr
t[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1/3
)))/(3*c^(2/3))))/(b*c - a*d))/(a*c))/(a*c)
```

3.121.3.1 Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 980

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.121. $\int \frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx$

3.121.4 Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.77

method	result
default	$-\frac{1}{5ac\,x^5}-\frac{-ad-bc}{2x^2a^2c^2}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)d^3}{c^2(ad-bc)}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2\right.}{}\right.}{}$
risch	$\frac{\frac{(ad+bc)x^3}{2a^2c^2}-\frac{1}{5ac}}{x^5}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(d^3c^8a^3-3a^2b\,c^9d^2+3a\,b^2c^{10}d-b^3c^{11}\right)\_Z^3-d^8\right)}\_R\ln\left(\left(\left(-4a^{14}c^8d^6+22a^{13}b\,c^9d^5-52a^{12}b^2c^{10}d^4\right.\right.\right.\right.}{}$

input
```
int(1/x^6/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/5/a/c/x^5-1/2*(-a*d-b*c)/x^2/a^2/c^2+(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3
))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))/c^2*d^3/(a*d-b*c)-(1/3/b/(
a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/
a^2*b^3/(a*d-b*c)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^6\left(a+bx^3\right)\left(c+dx^3\right)}\,dx =$$

$$-\frac{10\sqrt{3}b^2c^2x^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+10\sqrt{3}a^2d^2x^5\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-5b}{}$$

input
```
integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output

```
-1/30*(10*sqrt(3)*b^2*c^2*x^5*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(
-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 10*sqrt(3)*a^2*d^2*x^5*(d^2/c^2)^(1/3)*a
rctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3) - sqrt(3)*d)/d) - 5*b^2*c^2*x^5*(
-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3
)) - 5*a^2*d^2*x^5*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c
^2*(d^2/c^2)^(2/3)) + 10*b^2*c^2*x^5*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^
2)^(1/3)) + 10*a^2*d^2*x^5*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) +
6*a*b*c^2 - 6*a^2*c*d - 15*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*
d)*x^5)
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^3) (c+dx^3)} \, dx = \text{Timed out}$$

input

```
integrate(1/x**6/(b*x**3+a)/(d*x**3+c),x)
```

output

```
Timed out
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a^2bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}d^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^3\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^2d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{b^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{d^2\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^3\left(\frac{c}{d}\right)^{\frac{2}{3}}-ac^2d\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{b^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a^2bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{d^2\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^3\left(\frac{c}{d}\right)^{\frac{2}{3}}-ac^2d\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{5\,(bc+ad)x^3-2\,ac}{10\,a^2c^2x^5}$$

input
```
integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^2*
b*c*(a/b)^(1/3) - a^3*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*d^2*arctan
(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c^3*(c/d)^(1/3) - a*c^2*
d*(c/d)^(1/3))*(c/d)^(1/3)) - 1/6*b^2*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
))/(a^2*b*c*(a/b)^(2/3) - a^3*d*(a/b)^(2/3)) + 1/6*d^2*log(x^2 - x*(c/d)^(
1/3) + (c/d)^(2/3))/(b*c^3*(c/d)^(2/3) - a*c^2*d*(c/d)^(2/3)) + 1/3*b^2*lo
g(x + (a/b)^(1/3))/(a^2*b*c*(a/b)^(2/3) - a^3*d*(a/b)^(2/3)) - 1/3*d^2*log
(x + (c/d)^(1/3))/(b*c^3*(c/d)^(2/3) - a*c^2*d*(c/d)^(2/3)) + 1/10*(5*(b*c
 + a*d)*x^3 - 2*a*c)/(a^2*c^2*x^5)
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx = {} & -\frac{b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3(a^3bc-a^4d)}+\frac{d^3\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3(bc^4-ac^3d)} \\ & +\frac{(-ab^2)^{\frac{1}{3}}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^3bc-\sqrt{3}a^4d} \\ & -\frac{(-cd^2)^{\frac{1}{3}}\,d^2\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^4-\sqrt{3}ac^3d} \\ & +\frac{(-ab^2)^{\frac{1}{3}}\,b^2\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(a^3bc-a^4d)} \\ & -\frac{(-cd^2)^{\frac{1}{3}}\,d^2\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(bc^4-ac^3d)} \\ & +\frac{5\,bcx^3+5\,adx^3-2\,ac}{10\,a^2c^2x^5}\end{aligned}$$

input

```
integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output

```
-1/3*b^3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b*c - a^4*d) + 1/3*d
^3*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^4 - a*c^3*d) + (-a*b^2)^(1
/3)*b^2*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^3
*b*c - sqrt(3)*a^4*d) - (-c*d^2)^(1/3)*d^2*arctan(1/3*sqrt(3)*(2*x + (-c/d
)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b*c^4 - sqrt(3)*a*c^3*d) + 1/6*(-a*b^2)^(1
/3)*b^2*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b*c - a^4*d) - 1/6*(
-c*d^2)^(1/3)*d^2*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^4 - a*c^3*
d) + 1/10*(5*b*c*x^3 + 5*a*d*x^3 - 2*a*c)/(a^2*c^2*x^5)
```

3.121.9 Mupad [B] (verification not implemented)

Time = 16.54 (sec) , antiderivative size = 1860, normalized size of antiderivative = 5.79

$$\int \frac{1}{x^6 (a+bx^3) (c+dx^3)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^6*(a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(((-b^8/(a^8*(a*d - b*c)^3))^(1/3)*(9*a^13*b^11*c^19*d^5 - 9*a^12*b^12*
c^20*d^4 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12 + 9*a^16*b^3*c^16*d
^3*(a*d + b*c)*(a*d - b*c)^4*(a^3*c^3*(-b^8/(a^8*(a*d - b*c)^3))^(1/3) + a
^2*d^2*x + b^2*c^2*x)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3)))/3 + 3*a^12*b^7*c^
12*d^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^3 - 27*a^8*b^3*c^3 + 81*a^9
*b^2*c^2*d - 81*a^10*b*c*d^2))^(1/3) - (1/(5*a*c) - (x^3*(a*d + b*c))/(2*a
^2*c^2))/x^5 + log(((d^8/(c^8*(a*d - b*c)^3))^(1/3)*(9*a^13*b^11*c^19*d^5
- 9*a^12*b^12*c^20*d^4 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12 + 9*a
^16*b^3*c^16*d^3*(a*d + b*c)*(a*d - b*c)^4*(a^3*c^3*(d^8/(c^8*(a*d - b*c)^
3))^(1/3) + a^2*d^2*x + b^2*c^2*x)*(d^8/(c^8*(a*d - b*c)^3))^(2/3)))/3 + 3
*a^12*b^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-d^8/(27*b^3*c^11 - 27*a^3*c^8*
d^3 + 81*a^2*b*c^9*d^2 - 81*a*b^2*c^10*d))^(1/3) + (log(((3^(1/2)*1i - 1)*
(-b^8/(a^8*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a^16*b^3*c^16*d^
3*x*(a*d - b*c)^4*(a^3*d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2) + (81*a^
19*b^3*c^19*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^8/(a^8*(a*d
 - b*c)^3))^(1/3))/2)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3))/36 - 9*a^12*b^12*c
^20*d^4 + 9*a^13*b^11*c^19*d^5 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^
12))/6 + 3*a^12*b^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^3 - 2
7*a^8*b^3*c^3 + 81*a^9*b^2*c^2*d - 81*a^10*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1
))/2 - (log(((3^(1/2)*1i + 1)*(-b^8/(a^8*(a*d - b*c)^3))^(1/3)*(((3^(1/...
```

3.122 $\int \frac{1}{x^7(a+bx^3)(c+dx^3)}\,dx$

3.122.1 Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{1}{x^7\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{6acx^6}+\frac{bc+ad}{3a^2c^2x^3}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log(x)}{a^3c^3} - \frac{b^3\log\left(a+bx^3\right)}{3a^3(bc-ad)}+\frac{d^3\log\left(c+dx^3\right)}{3c^3(bc-ad)}$$

output
```
-1/6/a/c/x^6+1/3*(a*d+b*c)/a^2/c^2/x^3+(a^2*d^2+a*b*c*d+b^2*c^2)*ln(x)/a^3
/c^3-1/3*b^3*ln(b*x^3+a)/a^3/(-a*d+b*c)+1/3*d^3*ln(d*x^3+c)/c^3/(-a*d+b*c)
```

3.122.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^7\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{-ac(-bc+ad)\left(-2bcx^3+a(c-2dx^3)\right)+6\left(-b^3c^3+a^3d^3\right)x^6\log(x)+2b^3c^3x^6\log\left(a+bx^3\right)-2a^3d^3x^6\log}{6a^3c^3(-bc+ad)x^6}$$

input
```
Integrate[1/(x^7*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
(-(a*c*(-(b*c) + a*d)*(-2*b*c*x^3 + a*(c - 2*d*x^3))) + 6*(-(b^3*c^3) + a^
3*d^3)*x^6*Log[x] + 2*b^3*c^3*x^6*Log[a + b*x^3] - 2*a^3*d^3*x^6*Log[c + d
*x^3])/(6*a^3*c^3*(-(b*c) + a*d)*x^6)
```

3.122.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^3) (c+dx^3)} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^9 (bx^3+a) (dx^3+c)} dx^3$$

$$\Big\downarrow \text{93}$$

$$\frac{1}{3} \int \left(\frac{b^4}{a^3 (ad-bc) (bx^3+a)}+\frac{d^4}{c^3 (bc-ad) (dx^3+c)}+\frac{b^2 c^2+abdc+a^2 d^2}{a^3 c^3 x^3}+\frac{-bc-ad}{a^2 c^2 x^6}+\frac{1}{acx^9}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{b^3 \log \left(a+bx^3\right)}{a^3 (bc-ad)}+\frac{ad+bc}{a^2 c^2 x^3}+\frac{\log \left(x^3\right) \left(a^2 d^2+abcd+b^2 c^2\right)}{a^3 c^3}+\frac{d^3 \log \left(c+dx^3\right)}{c^3 (bc-ad)}-\frac{1}{2acx^6}\right)$$

input

```
Int[1/(x^7*(a + b*x^3)*(c + d*x^3)),x]
```

output

```
(-1/2*1/(a*c*x^6) + (b*c + a*d)/(a^2*c^2*x^3) + ((b^2*c^2 + a*b*c*d + a^2*
d^2)*Log[x^3])/(a^3*c^3) - (b^3*Log[a + b*x^3])/(a^3*(b*c - a*d)) + (d^3*L
og[c + d*x^3])/(c^3*(b*c - a*d)))/3
```

3.122.3.1 Defintions of rubi rules used

rule 93

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.122.4 Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{1}{6ac\,x^{6}}-\frac{-ad-bc}{3x^{3}a^{2}c^{2}}+\frac{\left(a^{2}d^{2}+abcd+b^{2}c^{2}\right)\ln \left(x \right)}{a^{3}c^{3}}-\frac{d^{3}\ln \left(d\,x^{3}+c \right)}{3c^{3}\left(ad-bc \right)}+\frac{b^{3}\ln \left(b\,x^{3}+a \right)}{3a^{3}\left(ad-bc \right)}$	114
norman	$\frac{-\frac{1}{6ac}+\frac{\left(ad+bc \right)x^{3}}{3a^{2}c^{2}}}{x^{6}}+\frac{\left(a^{2}d^{2}+abcd+b^{2}c^{2}\right)\ln \left(x \right)}{a^{3}c^{3}}+\frac{b^{3}\ln \left(b\,x^{3}+a \right)}{3a^{3}\left(ad-bc \right)}-\frac{d^{3}\ln \left(d\,x^{3}+c \right)}{3c^{3}\left(ad-bc \right)}$	114
risch	$\frac{-\frac{1}{6ac}+\frac{\left(ad+bc \right)x^{3}}{3a^{2}c^{2}}}{x^{6}}+\frac{\ln \left(x \right)d^{2}}{a\,c^{3}}+\frac{\ln \left(x \right)bd}{a^{2}c^{2}}+\frac{\ln \left(x \right)b^{2}}{a^{3}c}-\frac{d^{3}\ln \left(d\,x^{3}+c \right)}{3c^{3}\left(ad-bc \right)}+\frac{b^{3}\ln \left(-b\,x^{3}-a \right)}{3\left(ad-bc \right)a^{3}}$	123
parallelrisch	$\frac{6\ln \left(x \right)x^{6}a^{3}d^{3}-6\ln \left(x \right)x^{6}b^{3}c^{3}+2b^{3}\ln \left(b\,x^{3}+a \right)c^{3}x^{6}-2d^{3}\ln \left(d\,x^{3}+c \right)a^{3}x^{6}+2d^{2}a^{3}c\,x^{3}-2a\,b^{2}c^{3}x^{3}-a^{3}c^{2}d+a^{2}b\,c^{3}}{6a^{3}c^{3}x^{6}\left(ad-bc \right)}$	128

input

```
int(1/x^7/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output

```
-1/6/a/c/x^6-1/3*(-a*d-b*c)/x^3/a^2/c^2+(a^2*d^2+a*b*c*d+b^2*c^2)*ln(x)/a^
3/c^3-1/3*d^3/c^3/(a*d-b*c)*ln(d*x^3+c)+1/3*b^3/a^3/(a*d-b*c)*ln(b*x^3+a)
```

3.122.5 Fricas [A] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^7\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{2\,b^3c^3x^6\log\left(bx^3+a\right)-2\,a^3d^3x^6\log\left(dx^3+c\right)-6\,(b^3c^3-a^3d^3)x^6\log\left(x\right)+a^2bc^3-a^3c^2d-2\,(ab^2c^3-}{6\,(a^3bc^4-a^4c^3d)x^6}$$

input

```
integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*b^3*c^3*x^6*log(b*x^3 + a) - 2*a^3*d^3*x^6*log(d*x^3 + c) - 6*(b^3
*c^3 - a^3*d^3)*x^6*log(x) + a^2*b*c^3 - a^3*c^2*d - 2*(a*b^2*c^3 - a^3*c*
d^2)*x^3)/((a^3*b*c^4 - a^4*c^3*d)*x^6)
```

3.122.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^7\left(a+bx^3\right)\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x**7/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^7\left(a+bx^3\right)\left(c+dx^3\right)}\,dx=-\frac{b^3\log\left(bx^3+a\right)}{3\left(a^3bc-a^4d\right)}+\frac{d^3\log\left(dx^3+c\right)}{3\left(bc^4-ac^3d\right)}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log\left(x^3\right)}{3\,a^3c^3}+\frac{2\left(bc+ad\right)x^3-ac}{6\,a^2c^2x^6}$$

input
```
integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*b^3*log(b*x^3 + a)/(a^3*b*c - a^4*d) + 1/3*d^3*log(d*x^3 + c)/(b*c^4
- a*c^3*d) + 1/3*(b^2*c^2 + a*b*c*d + a^2*d^2)*log(x^3)/(a^3*c^3) + 1/6*(2
*(b*c + a*d)*x^3 - a*c)/(a^2*c^2*x^6)
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^7 (a+bx^3) (c+dx^3)} \, dx = -\frac{b^4 \log (|bx^3+a|)}{3 (a^3b^2c-a^4bd)} + \frac{d^4 \log (|dx^3+c|)}{3 (bc^4d-ac^3d^2)} + \frac{(b^2c^2+abcd+a^2d^2) \log (|x|)}{a^3c^3} - \frac{3 b^2c^2x^6+3 abcdx^6+3 a^2d^2x^6-2 abc^2x^3-2 a^2cdx^3+a^2c^2}{6 a^3c^3x^6}$$

input
```
integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*b^4*log(abs(b*x^3 + a))/(a^3*b^2*c - a^4*b*d) + 1/3*d^4*log(abs(d*x^3
 + c))/(b*c^4*d - a*c^3*d^2) + (b^2*c^2 + a*b*c*d + a^2*d^2)*log(abs(x))/(
a^3*c^3) - 1/6*(3*b^2*c^2*x^6 + 3*a*b*c*d*x^6 + 3*a^2*d^2*x^6 - 2*a*b*c^2*
x^3 - 2*a^2*c*d*x^3 + a^2*c^2)/(a^3*c^3*x^6)
```

3.122.9 Mupad [B] (verification not implemented)

Time = 7.64 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^7 (a+bx^3) (c+dx^3)} \, dx = \frac{b^3 \ln (b\,x^3+a)}{3\,a^4\,d-3\,a^3\,b\,c} - \frac{\frac{1}{6\,a\,c}-\frac{x^3\,(a\,d+b\,c)}{3\,a^2\,c^2}}{x^6} + \frac{d^3 \ln (d\,x^3+c)}{3\,b\,c^4-3\,a\,c^3\,d} + \frac{\ln (x)\ (a^2\,d^2+a\,b\,c\,d+b^2\,c^2)}{a^3\,c^3}$$

input
```
int(1/(x^7*(a + b*x^3)*(c + d*x^3)),x)
```

output
```
(b^3*log(a + b*x^3))/(3*a^4*d - 3*a^3*b*c) - (1/(6*a*c) - (x^3*(a*d + b*c)
)/(3*a^2*c^2))/x^6 + (d^3*log(c + d*x^3))/(3*b*c^4 - 3*a*c^3*d) + (log(x)*
(a^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3)
```

3.123 $\int \frac{1}{x^8\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$

3.123.1 Optimal result

Integrand size = 22, antiderivative size = 352

$$\begin{aligned}\int \frac{1}{x^8\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = &-\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} \\ &+\frac{b^{10/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}(bc-ad)}-\frac{d^{10/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{10/3}(bc-ad)} \\ &+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)}-\frac{d^{10/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{10/3}(bc-ad)} \\ &-\frac{b^{10/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}(bc-ad)} \\ &+\frac{d^{10/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{10/3}(bc-ad)}\end{aligned}$$

output

```
-1/7/a/c/x^7+1/4*(a*d+b*c)/a^2/c^2/x^4+(-a^2*d^2-a*b*c*d-b^2*c^2)/a^3/c^3/
x+1/3*b^(10/3)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/(-a*d+b*c)-1/3*d^(10/3)*ln(c
^(1/3)+d^(1/3)*x)/c^(10/3)/(-a*d+b*c)-1/6*b^(10/3)*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/a^(10/3)/(-a*d+b*c)+1/6*d^(10/3)*ln(c^(2/3)-c^(1/3)*d^(
1/3)*x+d^(2/3)*x^2)/c^(10/3)/(-a*d+b*c)+1/3*b^(10/3)*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/(-a*d+b*c)*3^(1/2)-1/3*d^(10/3)*arct
an(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(10/3)/(-a*d+b*c)*3^(1/2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^8\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$
$$=\frac{\frac{12b}{a}-\frac{12d}{c}-\frac{21b^2x^3}{a^2}+\frac{21d^2x^3}{c^2}+\frac{84b^3x^6}{a^3}-\frac{84d^3x^6}{c^3}-\frac{28\sqrt{3}b^{10/3}x^7\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{10/3}}+\frac{28\sqrt{3}d^{10/3}x^7\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{10/3}}-}{84(}$$

input
```
Integrate[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]
```

output
```
((12*b)/a - (12*d)/c - (21*b^2*x^3)/a^2 + (21*d^2*x^3)/c^2 + (84*b^3*x^6)/
a^3 - (84*d^3*x^6)/c^3 - (28*Sqrt[3]*b^(10/3)*x^7*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]])/a^(10/3) + (28*Sqrt[3]*d^(10/3)*x^7*ArcTan[(1 - (2*d^
(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(10/3) - (28*b^(10/3)*x^7*Log[a^(1/3) + b^(1
/3)*x])/a^(10/3) + (28*d^(10/3)*x^7*Log[c^(1/3) + d^(1/3)*x])/c^(10/3) + (
14*b^(10/3)*x^7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(10/3) -
 (14*d^(10/3)*x^7*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(10/3)
)/(84*(-(b*c) + a*d)*x^7)
```

3.123.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {980, 27, 1053, 27, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$
$$\Big\downarrow \; 980$$
$$\frac{\int -\frac{7\left(bdx^3+bc+ad\right)}{x^5\left(bx^3+a\right)\left(dx^3+c\right)}dx}{7ac}-\frac{1}{7acx^7}$$
$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{bdx^3+bc+ad}{x^5(bx^3+a)(dx^3+c)}dx}{ac}-\frac{1}{7acx^7}$$

$$\Big\downarrow \; 1053$$

$$-\frac{-\frac{\int \frac{4\left(bd(bc+ad)x^3+b^2c^2+a^2d^2+abcd\right)}{x^2\left(bx^3+a\right)\left(dx^3+c\right)}dx}{4ac}-\frac{ad+bc}{4acx^4}}{ac}-\frac{1}{7acx^7}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{bd(bc+ad)x^3+b^2c^2+a^2d^2+abcd}{x^2\left(bx^3+a\right)\left(dx^3+c\right)}dx}{ac}-\frac{ad+bc}{4acx^4}}{ac}-\frac{1}{7acx^7}$$

$$\Big\downarrow \; 1053$$

$$-\frac{-\frac{-\frac{\int \frac{x\left(bd\left(b^2c^2+abdc+a^2d^2\right)x^3+(bc+ad)\left(b^2c^2+a^2d^2\right)\right)}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{ac}-\frac{\frac{b^2c}{a}+\frac{ad^2}{c}+bd}{x}}{ac}-\frac{ad+bc}{4acx^4}}{ac}-\frac{1}{7acx^7}$$

$$\Big\downarrow \; 1054$$

$$-\frac{-\frac{-\frac{\int \left(\frac{c^3xb^4}{(bc-ad)\left(bx^3+a\right)}+\frac{a^3d^4x}{(ad-bc)\left(dx^3+c\right)}\right)dx}{ac}-\frac{\frac{b^2c}{a}+\frac{ad^2}{c}+bd}{x}}{ac}-\frac{ad+bc}{4acx^4}}{ac}-\frac{1}{7acx^7}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-\frac{-\frac{\frac{b^{10/3}c^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)}+\frac{a^3d^{10/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)}-\frac{a^3d^{10/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)}+\frac{a^3d^{10/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)}-}{ac}}{ac}}{ac}$$

$$\frac{1}{7acx^7}$$

input

```
Int[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]
```

3.123. $\int \frac{1}{x^8(a+bx^3)(c+dx^3)}\,dx$

output

```
-1/7*1/(a*c*x^7) - (-1/4*(b*c + a*d)/(a*c*x^4) - (-(((b^2*c)/a + b*d + (a*
d^2)/c)/x) - (-((b^(10/3)*c^3*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(1/3)*(b*c - a*d))) + (a^3*d^(10/3)*ArcTan[(c^(1/3) - 2*
d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(1/3)*(b*c - a*d)) - (b^(10/3)*c
^3*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*(b*c - a*d)) + (a^3*d^(10/3)*Log[c
^(1/3) + d^(1/3)*x])/(3*c^(1/3)*(b*c - a*d)) + (b^(10/3)*c^3*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*(b*c - a*d)) - (a^3*d^(10/3)
*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(1/3)*(b*c - a*d)))/
(a*c))/(a*c))/(a*c)
```

3.123.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.123.4 Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.79

method	result
default	$-\frac{1}{7ac\,x^{7}}-\frac{-ad-bc}{4x^{4}a^{2}c^{2}}-\frac{a^{2}d^{2}+abcd+b^{2}c^{2}}{c^{3}a^{3}x}-\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)d^{4}}{c^{3}\left(ad-bc\right)}+$
risch	$\frac{-\frac{\left(a^{2}d^{2}+abcd+b^{2}c^{2}\right)x^{6}}{c^{3}a^{3}}+\frac{\left(ad+bc\right)x^{3}}{4a^{2}c^{2}}-\frac{1}{7ac}}{x^{7}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^{3}c^{10}a^{3}-3a^{2}b\,c^{11}d^{2}+3a\,b^{2}c^{12}d-b^{3}c^{13}\right)\_Z^{3}-d^{10}\right)}\_R\ln\left(\left(\left(-4a^{16}c^{10}d^{6}\right.\right.\right.\right.}{}$

input
```
int(1/x^8/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/7/a/c/x^7-1/4*(-a*d-b*c)/x^4/a^2/c^2-(a^2*d^2+a*b*c*d+b^2*c^2)/c^3/a^3/
x-(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/
3)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1
/3)*x-1)))*d^4/c^3/(a*d-b*c)+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(
a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b^4/a^3/(a*d-b*c)
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^{8}\left(a+bx^{3}\right)\left(c+dx^{3}\right)}\,dx =$$
$$-\frac{28\sqrt{3}b^{3}c^{3}x^{7}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-28\sqrt{3}a^{3}d^{3}x^{7}\left(\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{d}{c}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-}{}$$

input
```
integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/84*(28*sqrt(3)*b^3*c^3*x^7*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/
3) + 1/3*sqrt(3)) - 28*sqrt(3)*a^3*d^3*x^7*(d/c)^(1/3)*arctan(2/3*sqrt(3)*
x*(d/c)^(1/3) - 1/3*sqrt(3)) - 14*b^3*c^3*x^7*(-b/a)^(1/3)*log(b*x^2 - a*x
*(-b/a)^(2/3) - a*(-b/a)^(1/3)) - 14*a^3*d^3*x^7*(d/c)^(1/3)*log(d*x^2 - c
*x*(d/c)^(2/3) + c*(d/c)^(1/3)) + 28*b^3*c^3*x^7*(-b/a)^(1/3)*log(b*x + a*
(-b/a)^(2/3)) + 28*a^3*d^3*x^7*(d/c)^(1/3)*log(d*x + c*(d/c)^(2/3)) + 84*(
b^3*c^3 - a^3*d^3)*x^6 + 12*a^2*b*c^3 - 12*a^3*c^2*d - 21*(a*b^2*c^3 - a^3
*c*d^2)*x^3)/((a^3*b*c^4 - a^4*c^3*d)*x^7)
```

3.123.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^8\left(a+bx^3\right)\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x**8/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{1}{x^8\left(a+bx^3\right)\left(c+dx^3\right)}\,dx &= -\frac{\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a^3bc-a^4d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &\quad+\frac{\sqrt{3}d^3\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^4-ac^3d\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} \\ &\quad-\frac{b^3\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^3bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^4d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{d^3\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^3d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} \\ &\quad+\frac{b^3\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a^3bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^4d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{d^3\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^3d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} \\ &\quad-\frac{28\left(b^2c^2+abcd+a^2d^2\right)x^6+4\,a^2c^2-7\left(abc^2+a^2cd\right)x^3}{28\,a^3c^3x^7}\end{aligned}$$

input
```
integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^3
*b*c - a^4*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*d^3*arctan(1/3*sqrt(3)*(2*x - (c/
d)^(1/3))/(c/d)^(1/3))/((b*c^4 - a*c^3*d)*(c/d)^(1/3)) - 1/6*b^3*log(x^2 -
 x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*c*(a/b)^(1/3) - a^4*d*(a/b)^(1/3)) +
1/6*d^3*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^4*(c/d)^(1/3) - a*c^3*
d*(c/d)^(1/3)) + 1/3*b^3*log(x + (a/b)^(1/3))/(a^3*b*c*(a/b)^(1/3) - a^4*d
*(a/b)^(1/3)) - 1/3*d^3*log(x + (c/d)^(1/3))/(b*c^4*(c/d)^(1/3) - a*c^3*d*
(c/d)^(1/3)) - 1/28*(28*(b^2*c^2 + a*b*c*d + a^2*d^2)*x^6 + 4*a^2*c^2 - 7*
(a*b*c^2 + a^2*c*d)*x^3)/(a^3*c^3*x^7)
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.07

$$
\begin{aligned}
\int \frac{1}{x^8\left(a+bx^3\right)\left(c+dx^3\right)}\,dx
&= \frac{b^4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^4bc-a^5d\right)} - \frac{d^4\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^5-ac^4d\right)} \\
&\quad + \frac{\left(-ab^2\right)^{\frac{2}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^4bc-\sqrt{3}a^5d} - \frac{\left(-cd^2\right)^{\frac{2}{3}}d^2\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^5-\sqrt{3}ac^4d} \\
&\quad - \frac{\left(-ab^2\right)^{\frac{2}{3}}b^2\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^4bc-a^5d\right)} + \frac{\left(-cd^2\right)^{\frac{2}{3}}d^2\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^5-ac^4d\right)} \\
&\quad - \frac{28\,b^2c^2x^6+28\,abcdx^6+28\,a^2d^2x^6-7\,abc^2x^3-7\,a^2cdx^3+4\,a^2c^2}{28\,a^3c^3x^7}
\end{aligned}
$$

input
```
integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b^4*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b*c - a^5*d) - 1/3*d^
4*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^5 - a*c^4*d) + (-a*b^2)^(2/
3)*b^2*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^4*
b*c - sqrt(3)*a^5*d) - (-c*d^2)^(2/3)*d^2*arctan(1/3*sqrt(3)*(2*x + (-c/d)
^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b*c^5 - sqrt(3)*a*c^4*d) - 1/6*(-a*b^2)^(2/
3)*b^2*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b*c - a^5*d) + 1/6*(-
c*d^2)^(2/3)*d^2*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^5 - a*c^4*d
) - 1/28*(28*b^2*c^2*x^6 + 28*a*b*c*d*x^6 + 28*a^2*d^2*x^6 - 7*a*b*c^2*x^3
 - 7*a^2*c*d*x^3 + 4*a^2*c^2)/(a^3*c^3*x^7)
```

3.123.9 Mupad [B] (verification not implemented)

Time = 16.93 (sec) , antiderivative size = 1814, normalized size of antiderivative = 5.15

$$\int \frac{1}{x^8 (a+bx^3) (c+dx^3)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^8*(a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(((-b^10/(a^10*(a*d - b*c)^3))^(2/3)*(((27*a^21*b^3*c^21*d^3*x*(a^8*d^8
 + b^8*c^8)*(a*d - b*c)^2 + 27*a^28*b^3*c^28*d^3*(a*d + b*c)*(a*d - b*c)^4
*(-b^10/(a^10*(a*d - b*c)^3))^(2/3))*(-b^10/(a^10*(a*d - b*c)^3))^(1/3))/3
 - 9*a^19*b^14*c^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*
a^29*b^4*c^19*d^14))/9 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27*a^1
3*d^3 - 27*a^10*b^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/3) + lo
g(((d^10/(c^10*(a*d - b*c)^3))^(2/3)*(((27*a^21*b^3*c^21*d^3*x*(a^8*d^8 +
b^8*c^8)*(a*d - b*c)^2 + 27*a^28*b^3*c^28*d^3*(a*d + b*c)*(a*d - b*c)^4*(d
^10/(c^10*(a*d - b*c)^3))^(2/3))*(d^10/(c^10*(a*d - b*c)^3))^(1/3))/3 - 9*
a^19*b^14*c^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29*
b^4*c^19*d^14))/9 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-d^10/(27*b^3*c^13
 - 27*a^3*c^10*d^3 + 81*a^2*b*c^11*d^2 - 81*a*b^2*c^12*d))^(1/3) - (1/(7*a
*c) - (x^3*(a*d + b*c))/(4*a^2*c^2) + (x^6*(a^2*d^2 + b^2*c^2 + a*b*c*d))/
(a^3*c^3))/x^7 - (log(((3^(1/2)*1i + 1)^2*(-b^10/(a^10*(a*d - b*c)^3))^(2/
3)*(((3^(1/2)*1i + 1)*(27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b
*c)^2 + (27*a^28*b^3*c^28*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4
*(-b^10/(a^10*(a*d - b*c)^3))^(2/3))/4)*(-b^10/(a^10*(a*d - b*c)^3))^(1/3)
)/6 + 9*a^19*b^14*c^29*d^4 - 9*a^20*b^13*c^28*d^5 - 9*a^28*b^5*c^20*d^13 +
 9*a^29*b^4*c^19*d^14))/36 + a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27
*a^13*d^3 - 27*a^10*b^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/...
```

3.124 $\int x^m(a+bx^3)^5 (A+Bx^3)\,dx$

3.124.1 Optimal result

Integrand size = 20, antiderivative size = 148

$$\int x^m(a+bx^3)^5 (A+Bx^3)\,dx = \frac{a^5Ax^{1+m}}{1+m}+\frac{a^4(5Ab+aB)x^{4+m}}{4+m}+\frac{5a^3b(2Ab+aB)x^{7+m}}{7+m}+\frac{10a^2b^2(Ab+aB)x^{10+m}}{10+m}+\frac{5ab^3(Ab+2aB)x^{13+m}}{13+m}+\frac{b^4(Ab+5aB)x^{16+m}}{16+m}+\frac{b^5Bx^{19+m}}{19+m}$$

output
```
a^5*A*x^(1+m)/(1+m)+a^4*(5*A*b+B*a)*x^(4+m)/(4+m)+5*a^3*b*(2*A*b+B*a)*x^(7
+m)/(7+m)+10*a^2*b^2*(A*b+B*a)*x^(10+m)/(10+m)+5*a*b^3*(A*b+2*B*a)*x^(13+m
)/(13+m)+b^4*(A*b+5*B*a)*x^(16+m)/(16+m)+b^5*B*x^(19+m)/(19+m)
```

3.124.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\int x^m(a+bx^3)^5 (A+Bx^3)\,dx = x^{1+m}\left(\frac{a^5A}{1+m}+\frac{a^4(5Ab+aB)x^3}{4+m}+\frac{5a^3b(2Ab+aB)x^6}{7+m}+\frac{10a^2b^2(Ab+aB)x^9}{10+m}+\frac{5ab^3(Ab+2aB)x^{12}}{13+m}+\frac{b^4(Ab+5aB)x^{15}}{16+m}+\frac{b^5Bx^{18}}{19+m}\right)$$

input
```
Integrate[x^m*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
x^(1 + m)*((a^5*A)/(1 + m) + (a^4*(5*A*b + a*B)*x^3)/(4 + m) + (5*a^3*b*(2
*A*b + a*B)*x^6)/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^9)/(10 + m) + (5*a*b^
3*(A*b + 2*a*B)*x^12)/(13 + m) + (b^4*(A*b + 5*a*B)*x^15)/(16 + m) + (b^5*
B*x^18)/(19 + m))
```

3.124.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \ 950$$

$$\int \left(a^5Ax^m+a^4x^{m+3}(aB+5Ab)+5a^3bx^{m+6}(aB+2Ab)+10a^2b^2x^{m+9}(aB+Ab)+b^4x^{m+15}(5aB+Ab)+5ab^3$$

$$\Big\downarrow \ 2009$$

$$\frac{a^5Ax^{m+1}}{m+1}+\frac{a^4x^{m+4}(aB+5Ab)}{m+4}+\frac{5a^3bx^{m+7}(aB+2Ab)}{m+7}+\frac{10a^2b^2x^{m+10}(aB+Ab)}{m+10}+\frac{b^4x^{m+16}(5aB+Ab)}{m+16}+\frac{5ab^3x^{m+13}(2aB+Ab)}{m+13}+\frac{b^5Bx^{m+19}}{m+19}$$

input
```
Int[x^m*(a + b*x^3)^5*(A + B*x^3),x]
```

output
```
(a^5*A*x^(1 + m))/(1 + m) + (a^4*(5*A*b + a*B)*x^(4 + m))/(4 + m) + (5*a^3
*b*(2*A*b + a*B)*x^(7 + m))/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^(10 + m))/
(10 + m) + (5*a*b^3*(A*b + 2*a*B)*x^(13 + m))/(13 + m) + (b^4*(A*b + 5*a*B
)*x^(16 + m))/(16 + m) + (b^5*B*x^(19 + m))/(19 + m)
```

3.124.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.124.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. 2(148) = 296.

Time = 4.74 (sec) , antiderivative size = 1077, normalized size of antiderivative = 7.28

method	result	size
risch	Expression too large to display	1077
gosper	Expression too large to display	1078
parallelrisch	Expression too large to display	1332

input
```
int(x^m*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
x*(B*b^5*m^6*x^18+51*B*b^5*m^5*x^18+1005*B*b^5*m^4*x^18+A*b^5*m^6*x^15+5*B
*a*b^4*m^6*x^15+9605*B*b^5*m^3*x^18+54*A*b^5*m^5*x^15+270*B*a*b^4*m^5*x^15
+45474*B*b^5*m^2*x^18+1110*A*b^5*m^4*x^15+5550*B*a*b^4*m^4*x^15+95064*B*b^
5*m*x^18+5*A*a*b^4*m^6*x^12+10940*A*b^5*m^3*x^15+10*B*a^2*b^3*m^6*x^12+547
00*B*a*b^4*m^3*x^15+58240*B*b^5*x^18+285*A*a*b^4*m^5*x^12+52929*A*b^5*m^2*
x^15+570*B*a^2*b^3*m^5*x^12+264645*B*a*b^4*m^2*x^15+6165*A*a*b^4*m^4*x^12+
112206*A*b^5*m*x^15+12330*B*a^2*b^3*m^4*x^12+561030*B*a*b^4*m*x^15+10*A*a^
2*b^3*m^6*x^9+63355*A*a*b^4*m^3*x^12+69160*A*b^5*x^15+10*B*a^3*b^2*m^6*x^9
+126710*B*a^2*b^3*m^3*x^12+345800*B*a*b^4*x^15+600*A*a^2*b^3*m^5*x^9+31623
0*A*a*b^4*m^2*x^12+600*B*a^3*b^2*m^5*x^9+632460*B*a^2*b^3*m^2*x^12+13740*A
*a^2*b^3*m^4*x^9+684360*A*a*b^4*m*x^12+13740*B*a^3*b^2*m^4*x^9+1368720*B*a
^2*b^3*m*x^12+10*A*a^3*b^2*m^6*x^6+149600*A*a^2*b^3*m^3*x^9+425600*A*a*b^4
*x^12+5*B*a^4*b*m^6*x^6+149600*B*a^3*b^2*m^3*x^9+851200*B*a^2*b^3*x^12+630
*A*a^3*b^2*m^5*x^6+783690*A*a^2*b^3*m^2*x^9+315*B*a^4*b*m^5*x^6+783690*B*a
^3*b^2*m^2*x^9+15330*A*a^3*b^2*m^4*x^6+1753800*A*a^2*b^3*m*x^9+7665*B*a^4*
b*m^4*x^6+1753800*B*a^3*b^2*m*x^9+5*A*a^4*b*m^6*x^3+179690*A*a^3*b^2*m^3*x
^6+1106560*A*a^2*b^3*x^9+B*a^5*m^6*x^3+89845*B*a^4*b*m^3*x^6+1106560*B*a^3
*b^2*x^9+330*A*a^4*b*m^5*x^3+1021860*A*a^3*b^2*m^2*x^6+66*B*a^5*m^5*x^3+51
0930*B*a^4*b*m^2*x^6+8550*A*a^4*b*m^4*x^3+2437680*A*a^3*b^2*m*x^6+1710*B*a
^5*m^4*x^3+1218840*B*a^4*b*m*x^6+A*a^5*m^6+109300*A*a^4*b*m^3*x^3+15808...
```

3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(148) = 296$.

Time = 0.27 (sec) , antiderivative size = 851, normalized size of antiderivative = 5.75

$$\int x^m \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx$$
$$= \frac{((Bb^5m^6 + 51\,Bb^5m^5 + 1005\,Bb^5m^4 + 9605\,Bb^5m^3 + 45474\,Bb^5m^2 + 95064\,Bb^5m + 58240\,Bb^5)x^{19} + ((}{}$$

input
```
integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

output

```
((B*b^5*m^6 + 51*B*b^5*m^5 + 1005*B*b^5*m^4 + 9605*B*b^5*m^3 + 45474*B*b^5
*m^2 + 95064*B*b^5*m + 58240*B*b^5)*x^19 + ((5*B*a*b^4 + A*b^5)*m^6 + 3458
00*B*a*b^4 + 69160*A*b^5 + 54*(5*B*a*b^4 + A*b^5)*m^5 + 1110*(5*B*a*b^4 +
A*b^5)*m^4 + 10940*(5*B*a*b^4 + A*b^5)*m^3 + 52929*(5*B*a*b^4 + A*b^5)*m^2
 + 112206*(5*B*a*b^4 + A*b^5)*m)*x^16 + 5*((2*B*a^2*b^3 + A*a*b^4)*m^6 + 1
70240*B*a^2*b^3 + 85120*A*a*b^4 + 57*(2*B*a^2*b^3 + A*a*b^4)*m^5 + 1233*(2
*B*a^2*b^3 + A*a*b^4)*m^4 + 12671*(2*B*a^2*b^3 + A*a*b^4)*m^3 + 63246*(2*B
*a^2*b^3 + A*a*b^4)*m^2 + 136872*(2*B*a^2*b^3 + A*a*b^4)*m)*x^13 + 10*((B*
a^3*b^2 + A*a^2*b^3)*m^6 + 110656*B*a^3*b^2 + 110656*A*a^2*b^3 + 60*(B*a^3
*b^2 + A*a^2*b^3)*m^5 + 1374*(B*a^3*b^2 + A*a^2*b^3)*m^4 + 14960*(B*a^3*b^
2 + A*a^2*b^3)*m^3 + 78369*(B*a^3*b^2 + A*a^2*b^3)*m^2 + 175380*(B*a^3*b^2
 + A*a^2*b^3)*m)*x^10 + 5*((B*a^4*b + 2*A*a^3*b^2)*m^6 + 158080*B*a^4*b +
316160*A*a^3*b^2 + 63*(B*a^4*b + 2*A*a^3*b^2)*m^5 + 1533*(B*a^4*b + 2*A*a^
3*b^2)*m^4 + 17969*(B*a^4*b + 2*A*a^3*b^2)*m^3 + 102186*(B*a^4*b + 2*A*a^3
*b^2)*m^2 + 243768*(B*a^4*b + 2*A*a^3*b^2)*m)*x^7 + ((B*a^5 + 5*A*a^4*b)*m
^6 + 276640*B*a^5 + 1383200*A*a^4*b + 66*(B*a^5 + 5*A*a^4*b)*m^5 + 1710*(B
*a^5 + 5*A*a^4*b)*m^4 + 21860*(B*a^5 + 5*A*a^4*b)*m^3 + 140529*(B*a^5 + 5*
A*a^4*b)*m^2 + 396954*(B*a^5 + 5*A*a^4*b)*m)*x^4 + (A*a^5*m^6 + 69*A*a^5*m
^5 + 1905*A*a^5*m^4 + 26795*A*a^5*m^3 + 201174*A*a^5*m^2 + 757896*A*a^5*m
+ 1106560*A*a^5)*x)*x^m/(m^7 + 70*m^6 + 1974*m^5 + 28700*m^4 + 227969*m...
```

3.124.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5418 vs. $2(138) = 276$.

Time = 1.65 (sec) , antiderivative size = 5418, normalized size of antiderivative = 36.61

$$\int x^m \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx = \text{Too large to display}$$

input

```
integrate(x**m*(b*x**3+a)**5*(B*x**3+A),x)
```

output

```
Piecewise((-A*a**5/(18*x**18) - A*a**4*b/(3*x**15) - 5*A*a**3*b**2/(6*x**1
2) - 10*A*a**2*b**3/(9*x**9) - 5*A*a*b**4/(6*x**6) - A*b**5/(3*x**3) - B*a
**5/(15*x**15) - 5*B*a**4*b/(12*x**12) - 10*B*a**3*b**2/(9*x**9) - 5*B*a**
2*b**3/(3*x**6) - 5*B*a*b**4/(3*x**3) + B*b**5*log(x), Eq(m, -19)), (-A*a*
*5/(15*x**15) - 5*A*a**4*b/(12*x**12) - 10*A*a**3*b**2/(9*x**9) - 5*A*a**2
*b**3/(3*x**6) - 5*A*a*b**4/(3*x**3) + A*b**5*log(x) - B*a**5/(12*x**12) -
 5*B*a**4*b/(9*x**9) - 5*B*a**3*b**2/(3*x**6) - 10*B*a**2*b**3/(3*x**3) +
5*B*a*b**4*log(x) + B*b**5*x**3/3, Eq(m, -16)), (-A*a**5/(12*x**12) - 5*A*
a**4*b/(9*x**9) - 5*A*a**3*b**2/(3*x**6) - 10*A*a**2*b**3/(3*x**3) + 5*A*a
*b**4*log(x) + A*b**5*x**3/3 - B*a**5/(9*x**9) - 5*B*a**4*b/(6*x**6) - 10*
B*a**3*b**2/(3*x**3) + 10*B*a**2*b**3*log(x) + 5*B*a*b**4*x**3/3 + B*b**5*
x**6/6, Eq(m, -13)), (-A*a**5/(9*x**9) - 5*A*a**4*b/(6*x**6) - 10*A*a**3*b
**2/(3*x**3) + 10*A*a**2*b**3*log(x) + 5*A*a*b**4*x**3/3 + A*b**5*x**6/6 -
 B*a**5/(6*x**6) - 5*B*a**4*b/(3*x**3) + 10*B*a**3*b**2*log(x) + 10*B*a**2
*b**3*x**3/3 + 5*B*a*b**4*x**6/6 + B*b**5*x**9/9, Eq(m, -10)), (-A*a**5/(6
*x**6) - 5*A*a**4*b/(3*x**3) + 10*A*a**3*b**2*log(x) + 10*A*a**2*b**3*x**3
/3 + 5*A*a*b**4*x**6/6 + A*b**5*x**9/9 - B*a**5/(3*x**3) + 5*B*a**4*b*log(
x) + 10*B*a**3*b**2*x**3/3 + 5*B*a**2*b**3*x**6/3 + 5*B*a*b**4*x**9/9 + B*
b**5*x**12/12, Eq(m, -7)), (-A*a**5/(3*x**3) + 5*A*a**4*b*log(x) + 10*A*a*
*3*b**2*x**3/3 + 5*A*a**2*b**3*x**6/3 + 5*A*a*b**4*x**9/9 + A*b**5*x**1...
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.39

$$\int x^m\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{Bb^5x^{m+19}}{m+19}+\frac{5\,Bab^4x^{m+16}}{m+16}+\frac{Ab^5x^{m+16}}{m+16}+\frac{10\,Ba^2b^3x^{m+13}}{m+13}+\frac{5\,Aab^4x^{m+13}}{m+13}+\frac{10\,Ba^3b^2x^{m+10}}{m+10}+\frac{10\,Aa^2b^3x^{m+10}}{m+10}+\frac{5\,Ba^4bx^{m+7}}{m+7}+\frac{10\,Aa^3b^2x^{m+7}}{m+7}+\frac{Ba^5x^{m+4}}{m+4}+\frac{5\,Aa^4bx^{m+4}}{m+4}+\frac{Aa^5x^{m+1}}{m+1}$$

input

```
integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

output
```
B*b^5*x^(m + 19)/(m + 19) + 5*B*a*b^4*x^(m + 16)/(m + 16) + A*b^5*x^(m + 1
6)/(m + 16) + 10*B*a^2*b^3*x^(m + 13)/(m + 13) + 5*A*a*b^4*x^(m + 13)/(m +
 13) + 10*B*a^3*b^2*x^(m + 10)/(m + 10) + 10*A*a^2*b^3*x^(m + 10)/(m + 10)
 + 5*B*a^4*b*x^(m + 7)/(m + 7) + 10*A*a^3*b^2*x^(m + 7)/(m + 7) + B*a^5*x^
(m + 4)/(m + 4) + 5*A*a^4*b*x^(m + 4)/(m + 4) + A*a^5*x^(m + 1)/(m + 1)
```

3.124.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1331 vs. 2(148) = 296.

Time = 0.33 (sec) , antiderivative size = 1331, normalized size of antiderivative = 8.99

$$\int x^m \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

output
```
(B*b^5*m^6*x^19*x^m + 51*B*b^5*m^5*x^19*x^m + 1005*B*b^5*m^4*x^19*x^m + 5*
B*a*b^4*m^6*x^16*x^m + A*b^5*m^6*x^16*x^m + 9605*B*b^5*m^3*x^19*x^m + 270*
B*a*b^4*m^5*x^16*x^m + 54*A*b^5*m^5*x^16*x^m + 45474*B*b^5*m^2*x^19*x^m +
5550*B*a*b^4*m^4*x^16*x^m + 1110*A*b^5*m^4*x^16*x^m + 95064*B*b^5*m*x^19*x
^m + 10*B*a^2*b^3*m^6*x^13*x^m + 5*A*a*b^4*m^6*x^13*x^m + 54700*B*a*b^4*m^
3*x^16*x^m + 10940*A*b^5*m^3*x^16*x^m + 58240*B*b^5*x^19*x^m + 570*B*a^2*b
^3*m^5*x^13*x^m + 285*A*a*b^4*m^5*x^13*x^m + 264645*B*a*b^4*m^2*x^16*x^m +
 52929*A*b^5*m^2*x^16*x^m + 12330*B*a^2*b^3*m^4*x^13*x^m + 6165*A*a*b^4*m^
4*x^13*x^m + 561030*B*a*b^4*m*x^16*x^m + 112206*A*b^5*m*x^16*x^m + 10*B*a^
3*b^2*m^6*x^10*x^m + 10*A*a^2*b^3*m^6*x^10*x^m + 126710*B*a^2*b^3*m^3*x^13
*x^m + 63355*A*a*b^4*m^3*x^13*x^m + 345800*B*a*b^4*x^16*x^m + 69160*A*b^5*
x^16*x^m + 600*B*a^3*b^2*m^5*x^10*x^m + 600*A*a^2*b^3*m^5*x^10*x^m + 63246
0*B*a^2*b^3*m^2*x^13*x^m + 316230*A*a*b^4*m^2*x^13*x^m + 13740*B*a^3*b^2*m
^4*x^10*x^m + 13740*A*a^2*b^3*m^4*x^10*x^m + 1368720*B*a^2*b^3*m*x^13*x^m
+ 684360*A*a*b^4*m*x^13*x^m + 5*B*a^4*b*m^6*x^7*x^m + 10*A*a^3*b^2*m^6*x^7
*x^m + 149600*B*a^3*b^2*m^3*x^10*x^m + 149600*A*a^2*b^3*m^3*x^10*x^m + 851
200*B*a^2*b^3*x^13*x^m + 425600*A*a*b^4*x^13*x^m + 315*B*a^4*b*m^5*x^7*x^m
 + 630*A*a^3*b^2*m^5*x^7*x^m + 783690*B*a^3*b^2*m^2*x^10*x^m + 783690*A*a^
2*b^3*m^2*x^10*x^m + 7665*B*a^4*b*m^4*x^7*x^m + 15330*A*a^3*b^2*m^4*x^7*x^
m + 1753800*B*a^3*b^2*m*x^10*x^m + 1753800*A*a^2*b^3*m*x^10*x^m + B*a^5...
```

3.124.9 Mupad [B] (verification not implemented)

Time = 7.63 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.78

$$
\begin{aligned}
&\int x^m \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx \\
&= \frac{B\,b^5\,x^m\,x^{19}\,\left(m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560} \\
&\quad+ \frac{a^4\,x^m\,x^4\,\left(5\,A\,b+B\,a\right)\,\left(m^6+66\,m^5+1710\,m^4+21860\,m^3+140529\,m^2+396954\,m+276640\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560} \\
&\quad+ \frac{b^4\,x^m\,x^{16}\,\left(A\,b+5\,B\,a\right)\,\left(m^6+54\,m^5+1110\,m^4+10940\,m^3+52929\,m^2+112206\,m+69160\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560} \\
&\quad+ \frac{A\,a^5\,x\,x^m\,\left(m^6+69\,m^5+1905\,m^4+26795\,m^3+201174\,m^2+757896\,m+1106560\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560} \\
&\quad+ \frac{10\,a^2\,b^2\,x^m\,x^{10}\,\left(A\,b+B\,a\right)\,\left(m^6+60\,m^5+1374\,m^4+14960\,m^3+78369\,m^2+175380\,m+110656\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560} \\
&\quad+ \frac{5\,a\,b^3\,x^m\,x^{13}\,\left(A\,b+2\,B\,a\right)\,\left(m^6+57\,m^5+1233\,m^4+12671\,m^3+63246\,m^2+136872\,m+85120\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560} \\
&\quad+ \frac{5\,a^3\,b\,x^m\,x^7\,\left(2\,A\,b+B\,a\right)\,\left(m^6+63\,m^5+1533\,m^4+17969\,m^3+102186\,m^2+243768\,m+158080\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}
\end{aligned}
$$

input

```
int(x^m*(A + B*x^3)*(a + b*x^3)^5,x)
```

output

```
(B*b^5*x^m*x^19*(95064*m + 45474*m^2 + 9605*m^3 + 1005*m^4 + 51*m^5 + m^6
+ 58240))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70
*m^6 + m^7 + 1106560) + (a^4*x^m*x^4*(5*A*b + B*a)*(396954*m + 140529*m^2
+ 21860*m^3 + 1710*m^4 + 66*m^5 + m^6 + 276640))/(1864456*m + 959070*m^2 +
 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (b^4*x^m*x^
16*(A*b + 5*B*a)*(112206*m + 52929*m^2 + 10940*m^3 + 1110*m^4 + 54*m^5 + m
^6 + 69160))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 +
 70*m^6 + m^7 + 1106560) + (A*a^5*x*x^m*(757896*m + 201174*m^2 + 26795*m^3
 + 1905*m^4 + 69*m^5 + m^6 + 1106560))/(1864456*m + 959070*m^2 + 227969*m^
3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (10*a^2*b^2*x^m*x^10*
(A*b + B*a)*(175380*m + 78369*m^2 + 14960*m^3 + 1374*m^4 + 60*m^5 + m^6 +
110656))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*
m^6 + m^7 + 1106560) + (5*a*b^3*x^m*x^13*(A*b + 2*B*a)*(136872*m + 63246*m
^2 + 12671*m^3 + 1233*m^4 + 57*m^5 + m^6 + 85120))/(1864456*m + 959070*m^2
 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (5*a^3*b*
x^m*x^7*(2*A*b + B*a)*(243768*m + 102186*m^2 + 17969*m^3 + 1533*m^4 + 63*m
^5 + m^6 + 158080))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 197
4*m^5 + 70*m^6 + m^7 + 1106560)
```

3.125 $\int x^m(a+bx^3)^2(A+Bx^3)\,dx$

3.125.1 Optimal result

Integrand size = 20, antiderivative size = 71

$$\int x^m(a+bx^3)^2(A+Bx^3)\,dx=\frac{a^2Ax^{1+m}}{1+m}+\frac{a(2Ab+aB)x^{4+m}}{4+m}+\frac{b(Ab+2aB)x^{7+m}}{7+m}+\frac{b^2Bx^{10+m}}{10+m}$$

output
```
a^2*A*x^(1+m)/(1+m)+a*(2*A*b+B*a)*x^(4+m)/(4+m)+b*(A*b+2*B*a)*x^(7+m)/(7+m
)+b^2*B*x^(10+m)/(10+m)
```

3.125.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int x^m(a+bx^3)^2(A+Bx^3)\,dx=x^{1+m}\left(\frac{a^2A}{1+m}+\frac{a(2Ab+aB)x^3}{4+m}+\frac{b(Ab+2aB)x^6}{7+m}+\frac{b^2Bx^9}{10+m}\right)$$

input
```
Integrate[x^m*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
x^(1 + m)*((a^2*A)/(1 + m) + (a*(2*A*b + a*B)*x^3)/(4 + m) + (b*(A*b + 2*a
*B)*x^6)/(7 + m) + (b^2*B*x^9)/(10 + m))
```

3.125.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^2Ax^m+ax^{m+3}(aB+2Ab)+bx^{m+6}(2aB+Ab)+b^2Bx^{m+9}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+4}(aB+2Ab)}{m+4}+\frac{bx^{m+7}(2aB+Ab)}{m+7}+\frac{b^2Bx^{m+10}}{m+10}$$

input
```
Int[x^m*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(4 + m))/(4 + m) + (b*(A*b
+ 2*a*B)*x^(7 + m))/(7 + m) + (b^2*B*x^(10 + m))/(10 + m)
```

3.125.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.125.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(71) = 142$.

Time = 4.07 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.68

method	result
risch	$\frac{x\left(B\,b^2m^3x^9+12B\,b^2m^2x^9+39m\,x^9B\,b^2+A\,b^2m^3x^6+2Bab\,m^3x^6+28b^2B\,x^9+15A\,b^2m^2x^6+30Bab\,m^2x^6+54A\,b^2x^6m+108B}{}$
gosper	$\frac{x^{1+m}\left(B\,b^2m^3x^9+12B\,b^2m^2x^9+39m\,x^9B\,b^2+A\,b^2m^3x^6+2Bab\,m^3x^6+28b^2B\,x^9+15A\,b^2m^2x^6+30Bab\,m^2x^6+54A\,b^2x^6m+1}{}$
parallelrisch	$\frac{174A\,x^4x^m abm+B\,x^{10}x^m b^2m^3+12B\,x^{10}x^m b^2m^2+2B\,x^7x^m ab\,m^3+30B\,x^7x^m ab\,m^2+108B\,x^7x^m abm+2A\,x^4x^m ab\,m^3+36A}{}$

input
```
int(x^m*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
x*(B*b^2*m^3*x^9+12*B*b^2*m^2*x^9+39*B*b^2*m*x^9+A*b^2*m^3*x^6+2*B*a*b*m^3
*x^6+28*B*b^2*x^9+15*A*b^2*m^2*x^6+30*B*a*b*m^2*x^6+54*A*b^2*m*x^6+108*B*a
*b*m*x^6+2*A*a*b*m^3*x^3+40*A*b^2*x^6+B*a^2*m^3*x^3+80*B*a*b*x^6+36*A*a*b*
m^2*x^3+18*B*a^2*m^2*x^3+174*A*a*b*m*x^3+87*B*a^2*m*x^3+A*a^2*m^3+140*A*a*
b*x^3+70*B*a^2*x^3+21*A*a^2*m^2+138*A*a^2*m+280*A*a^2)*x^m/(10+m)/(7+m)/(4
+m)/(1+m)
```

3.125.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(71) = 142$.

Time = 0.26 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.03

$$\int x^m\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx$$
$$=\frac{((Bb^2m^3+12\,Bb^2m^2+39\,Bb^2m+28\,Bb^2)x^{10}+((2\,Bab+Ab^2)m^3+80\,Bab+40\,Ab^2+15\,(2\,Bab+Ab}{}$$

input
```
integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

output
```
((B*b^2*m^3 + 12*B*b^2*m^2 + 39*B*b^2*m + 28*B*b^2)*x^10 + ((2*B*a*b + A*b
^2)*m^3 + 80*B*a*b + 40*A*b^2 + 15*(2*B*a*b + A*b^2)*m^2 + 54*(2*B*a*b + A
*b^2)*m)*x^7 + ((B*a^2 + 2*A*a*b)*m^3 + 70*B*a^2 + 140*A*a*b + 18*(B*a^2 +
 2*A*a*b)*m^2 + 87*(B*a^2 + 2*A*a*b)*m)*x^4 + (A*a^2*m^3 + 21*A*a^2*m^2 +
138*A*a^2*m + 280*A*a^2)*x)*x^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

3.125.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1057 vs. $2(63) = 126$.

Time = 0.60 (sec) , antiderivative size = 1057, normalized size of antiderivative = 14.89

$$\int x^m \left(a+bx^3\right)^2 \left(A+Bx^3\right)\,dx$$
$$= \begin{cases} -\frac{Aa^2}{9x^9} - \frac{Aab}{3x^6} - \frac{Ab^2}{3x^3} - \frac{Ba^2}{6x^6} - \frac{2Bab}{3x^3} + Bb^2\log\left(x\right) \\ -\frac{Aa^2}{6x^6} - \frac{2Aab}{3x^3} + Ab^2\log\left(x\right) - \frac{Ba^2}{3x^3} + 2Bab\log\left(x\right) + \frac{Bb^2x^3}{3} \\ -\frac{Aa^2}{3x^3} + 2Aab\log\left(x\right) + \frac{Ab^2x^3}{3} + Ba^2\log\left(x\right) + \frac{2Babx^3}{3} + \frac{Bb^2x^6}{6} \\ Aa^2\log\left(x\right) + \frac{2Aabx^3}{3} + \frac{Ab^2x^6}{6} + \frac{Ba^2x^3}{3} + \frac{Babx^6}{3} + \frac{Bb^2x^9}{9} \\ \frac{Aa^2m^3xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{21Aa^2m^2xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{138Aa^2mxx^m}{m^4+22m^3+159m^2+418m+280} + \frac{280Aa^2xx^m}{m^4+22m^3+159m^2+418m+280} \end{cases}$$

input
```
integrate(x**m*(b*x**3+a)**2*(B*x**3+A),x)
```

output
```
Piecewise((-A*a**2/(9*x**9) - A*a*b/(3*x**6) - A*b**2/(3*x**3) - B*a**2/(6
*x**6) - 2*B*a*b/(3*x**3) + B*b**2*log(x), Eq(m, -10)), (-A*a**2/(6*x**6)
- 2*A*a*b/(3*x**3) + A*b**2*log(x) - B*a**2/(3*x**3) + 2*B*a*b*log(x) + B*
b**2*x**3/3, Eq(m, -7)), (-A*a**2/(3*x**3) + 2*A*a*b*log(x) + A*b**2*x**3/
3 + B*a**2*log(x) + 2*B*a*b*x**3/3 + B*b**2*x**6/6, Eq(m, -4)), (A*a**2*lo
g(x) + 2*A*a*b*x**3/3 + A*b**2*x**6/6 + B*a**2*x**3/3 + B*a*b*x**6/3 + B*b
**2*x**9/9, Eq(m, -1)), (A*a**2*m**3*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 4
18*m + 280) + 21*A*a**2*m**2*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 2
80) + 138*A*a**2*m*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 280*
A*a**2*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 2*A*a*b*m**3*x**
4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 36*A*a*b*m**2*x**4*x**m
/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 174*A*a*b*m*x**4*x**m/(m**4 +
 22*m**3 + 159*m**2 + 418*m + 280) + 140*A*a*b*x**4*x**m/(m**4 + 22*m**3 +
 159*m**2 + 418*m + 280) + A*b**2*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**
2 + 418*m + 280) + 15*A*b**2*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 4
18*m + 280) + 54*A*b**2*m*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 2
80) + 40*A*b**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + B*a*
*2*m**3*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 18*B*a**2*m*
*2*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 87*B*a**2*m*x**4*
x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 70*B*a**2*x**4*x**m/(m...
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.28

$$\int x^m \left(a+bx^3\right)^2 \left(A+Bx^3\right)\,dx = \frac{Bb^2x^{m+10}}{m+10} + \frac{2\,Babx^{m+7}}{m+7} + \frac{Ab^2x^{m+7}}{m+7} + \frac{Ba^2x^{m+4}}{m+4} + \frac{2\,Aabx^{m+4}}{m+4} + \frac{Aa^2x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

output
```
B*b^2*x^(m + 10)/(m + 10) + 2*B*a*b*x^(m + 7)/(m + 7) + A*b^2*x^(m + 7)/(m
 + 7) + B*a^2*x^(m + 4)/(m + 4) + 2*A*a*b*x^(m + 4)/(m + 4) + A*a^2*x^(m +
 1)/(m + 1)
```

3.125.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(71) = 142.

Time = 0.28 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.68

$$\int x^m \left(a+bx^3\right)^2 \left(A+Bx^3\right)\,dx = \frac{Bb^2m^3x^{10}x^m + 12\,Bb^2m^2x^{10}x^m + 39\,Bb^2mx^{10}x^m + 2\,Babm^3x^7x^m + Ab^2m^3x^7x^m + 28\,Bb^2x^{10}x^m + 30\,Ba}{}$$

input
```
integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

output
```
(B*b^2*m^3*x^10*x^m + 12*B*b^2*m^2*x^10*x^m + 39*B*b^2*m*x^10*x^m + 2*B*a*
b*m^3*x^7*x^m + A*b^2*m^3*x^7*x^m + 28*B*b^2*x^10*x^m + 30*B*a*b*m^2*x^7*x
^m + 15*A*b^2*m^2*x^7*x^m + 108*B*a*b*m*x^7*x^m + 54*A*b^2*m*x^7*x^m + B*a
^2*m^3*x^4*x^m + 2*A*a*b*m^3*x^4*x^m + 80*B*a*b*x^7*x^m + 40*A*b^2*x^7*x^m
 + 18*B*a^2*m^2*x^4*x^m + 36*A*a*b*m^2*x^4*x^m + 87*B*a^2*m*x^4*x^m + 174*
A*a*b*m*x^4*x^m + A*a^2*m^3*x*x^m + 70*B*a^2*x^4*x^m + 140*A*a*b*x^4*x^m +
 21*A*a^2*m^2*x*x^m + 138*A*a^2*m*x*x^m + 280*A*a^2*x*x^m)/(m^4 + 22*m^3 +
 159*m^2 + 418*m + 280)
```

3.125.9 Mupad [B] (verification not implemented)

Time = 6.91 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.49

$$\int x^m \left(a+bx^3\right)^2 \left(A+Bx^3\right)\,dx = x^m \left(\frac{B\,b^2\,x^{10}\,\left(m^3+12\,m^2+39\,m+28\right)}{m^4+22\,m^3+159\,m^2+418\,m+280} + \frac{A\,a^2\,x\,\left(m^3+21\,m^2+138\,m+280\right)}{m^4+22\,m^3+159\,m^2+418\,m+280} + \frac{a\,x^4\,\left(2\,A\,b+B\,a\right)\,\left(m^3+18\,m^2+87\,m+70\right)}{m^4+22\,m^3+159\,m^2+418\,m+280} + \frac{b\,x^7\,\left(A\,b+2\,B\,a\right)\,\left(m^3+15\,m^2+54\,m+40\right)}{m^4+22\,m^3+159\,m^2+418\,m+280}\right)$$

input

```
int(x^m*(A + B*x^3)*(a + b*x^3)^2,x)
```

output

```
x^m*((B*b^2*x^10*(39*m + 12*m^2 + m^3 + 28))/(418*m + 159*m^2 + 22*m^3 + m
^4 + 280) + (A*a^2*x*(138*m + 21*m^2 + m^3 + 280))/(418*m + 159*m^2 + 22*m
^3 + m^4 + 280) + (a*x^4*(2*A*b + B*a)*(87*m + 18*m^2 + m^3 + 70))/(418*m
+ 159*m^2 + 22*m^3 + m^4 + 280) + (b*x^7*(A*b + 2*B*a)*(54*m + 15*m^2 + m^
3 + 40))/(418*m + 159*m^2 + 22*m^3 + m^4 + 280))
```

3.126 $\int x^m(a+bx^3)\left(A+Bx^3\right)\,dx$

3.126.1 Optimal result

Integrand size = 18, antiderivative size = 45

$$\int x^m\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{aAx^{1+m}}{1+m}+\frac{(Ab+aB)x^{4+m}}{4+m}+\frac{bBx^{7+m}}{7+m}$$

output
```
a*A*x^(1+m)/(1+m)+(A*b+B*a)*x^(4+m)/(4+m)+b*B*x^(7+m)/(7+m)
```

3.126.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x^m\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=x^{1+m}\left(\frac{aA}{1+m}+\frac{(Ab+aB)x^3}{4+m}+\frac{bBx^6}{7+m}\right)$$

input
```
Integrate[x^m*(a + b*x^3)*(A + B*x^3),x]
```

output
```
x^(1 + m)*((a*A)/(1 + m) + ((A*b + a*B)*x^3)/(4 + m) + (b*B*x^6)/(7 + m))
```

3.126.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (a+bx^3) (A+Bx^3) \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{m+3}(aB+Ab)+aAx^m+bBx^{m+6}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{m+4}(aB+Ab)}{m+4}+\frac{aAx^{m+1}}{m+1}+\frac{bBx^{m+7}}{m+7}$$

input
```
Int[x^m*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(4 + m))/(4 + m) + (b*B*x^(7 + m)
)/(7 + m)
```

3.126.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.126.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

method	result
norman	$\frac{(Ab+Ba)x^4 \mathrm{e}^{m\ln(x)}}{4+m}+\frac{Aax\,\mathrm{e}^{m\ln(x)}}{1+m}+\frac{Bb\,x^7 \mathrm{e}^{m\ln(x)}}{7+m}$
risch	$\frac{x\left(Bb\,m^2x^6+5Bbm\,x^6+4bB\,x^6+Ab\,m^2x^3+Ba\,m^2x^3+8Abm\,x^3+8Bam\,x^3+7Ab\,x^3+7Ba\,x^3+Aa\,m^2+11Aam+28Aa\right)x^m}{(7+m)(4+m)(1+m)}$
gosper	$\frac{x^{1+m}\left(Bb\,m^2x^6+5Bbm\,x^6+4bB\,x^6+Ab\,m^2x^3+Ba\,m^2x^3+8Abm\,x^3+8Bam\,x^3+7Ab\,x^3+7Ba\,x^3+Aa\,m^2+11Aam+28Aa\right)}{(1+m)(4+m)(7+m)}$
parallelrisch	$\frac{B\,x^7x^mb\,m^2+5B\,x^7x^mbm+4B\,x^7x^mb+A\,x^4x^mb\,m^2+B\,x^4x^ma\,m^2+8A\,x^4x^mbm+8B\,x^4x^mam+7A\,x^4x^mb+7B\,x^4x^ma+Ax}{(7+m)(4+m)(1+m)}$

input
```
int(x^m*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
(A*b+B*a)/(4+m)*x^4*exp(m*ln(x))+A*a/(1+m)*x*exp(m*ln(x))+B*b/(7+m)*x^7*ex
p(m*ln(x))
```

3.126.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(45) = 90.

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.04

$$\int x^m\left(a+bx^3\right)\left(A+Bx^3\right)\,dx = \frac{((Bbm^2+5\,Bbm+4\,Bb)x^7+((Ba+Ab)m^2+7\,Ba+7\,Ab+8\,(Ba+Ab)m)x^4+(Aam^2+11\,Aam+2}{m^3+12\,m^2+39\,m+28}$$

input
```
integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

output
```
((B*b*m^2 + 5*B*b*m + 4*B*b)*x^7 + ((B*a + A*b)*m^2 + 7*B*a + 7*A*b + 8*(B
*a + A*b)*m)*x^4 + (A*a*m^2 + 11*A*a*m + 28*A*a)*x)*x^m/(m^3 + 12*m^2 + 39
*m + 28)
```

3.126.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(37) = 74$.

Time = 0.40 (sec) , antiderivative size = 410, normalized size of antiderivative = 9.11

$$\int x^m(a+bx^3)\left(A+Bx^3\right)\,dx$$
$$= \begin{cases} -\frac{Aa}{6x^6} - \frac{Ab}{3x^3} - \frac{Ba}{3x^3} + Bb\log(x) \\ -\frac{Aa}{3x^3} + Ab\log(x) + Ba\log(x) + \frac{Bbx^3}{3} \\ Aa\log(x) + \frac{Abx^3}{3} + \frac{Bax^3}{3} + \frac{Bbx^6}{6} \\ \frac{Aam^2xx^m}{m^3+12m^2+39m+28} + \frac{11Aamxx^m}{m^3+12m^2+39m+28} + \frac{28Aaxx^m}{m^3+12m^2+39m+28} + \frac{Abm^2x^4x^m}{m^3+12m^2+39m+28} + \frac{8Abmx^4x^m}{m^3+12m^2+39m+28} + \frac{7Abx^4}{m^3+12m^2+} \end{cases}$$

input
```
integrate(x**m*(b*x**3+a)*(B*x**3+A),x)
```

output
```
Piecewise((-A*a/(6*x**6) - A*b/(3*x**3) - B*a/(3*x**3) + B*b*log(x), Eq(m,
 -7)), (-A*a/(3*x**3) + A*b*log(x) + B*a*log(x) + B*b*x**3/3, Eq(m, -4)),
(A*a*log(x) + A*b*x**3/3 + B*a*x**3/3 + B*b*x**6/6, Eq(m, -1)), (A*a*m**2*
x*x**m/(m**3 + 12*m**2 + 39*m + 28) + 11*A*a*m*x*x**m/(m**3 + 12*m**2 + 39
*m + 28) + 28*A*a*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + A*b*m**2*x**4*x**m
/(m**3 + 12*m**2 + 39*m + 28) + 8*A*b*m*x**4*x**m/(m**3 + 12*m**2 + 39*m +
 28) + 7*A*b*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*a*m**2*x**4*x**m/(
m**3 + 12*m**2 + 39*m + 28) + 8*B*a*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 2
8) + 7*B*a*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*b*m**2*x**7*x**m/(m*
*3 + 12*m**2 + 39*m + 28) + 5*B*b*m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28)
 + 4*B*b*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28), True))
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int x^m(a+bx^3)\left(A+Bx^3\right)\,dx = \frac{Bbx^{m+7}}{m+7} + \frac{Bax^{m+4}}{m+4} + \frac{Abx^{m+4}}{m+4} + \frac{Aax^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

output
```
B*b*x^(m + 7)/(m + 7) + B*a*x^(m + 4)/(m + 4) + A*b*x^(m + 4)/(m + 4) + A*
a*x^(m + 1)/(m + 1)
```

3.126.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.18

$$\int x^m(a+bx^3)\left(A+Bx^3\right)\,dx$$
$$= \frac{Bbm^2x^7x^m + 5\,Bbmx^7x^m + 4\,Bbx^7x^m + Bam^2x^4x^m + Abm^2x^4x^m + 8\,Bamx^4x^m + 8\,Abmx^4x^m + 7\,Bax}{m^3 + 12\,m^2 + 39\,m + 28}$$

input
```
integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

output
```
(B*b*m^2*x^7*x^m + 5*B*b*m*x^7*x^m + 4*B*b*x^7*x^m + B*a*m^2*x^4*x^m + A*b
*m^2*x^4*x^m + 8*B*a*m*x^4*x^m + 8*A*b*m*x^4*x^m + 7*B*a*x^4*x^m + 7*A*b*x
^4*x^m + A*a*m^2*x*x^m + 11*A*a*m*x*x^m + 28*A*a*x*x^m)/(m^3 + 12*m^2 + 39
*m + 28)
```

3.126.9 Mupad [B] (verification not implemented)

Time = 6.84 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.11

$$\int x^m(a+bx^3)\left(A+Bx^3\right)\,dx = x^m\left(\frac{x^4\,(A\,b+B\,a)\,\left(m^2+8\,m+7\right)}{m^3+12\,m^2+39\,m+28} + \frac{B\,b\,x^7\,\left(m^2+5\,m+4\right)}{m^3+12\,m^2+39\,m+28} + \frac{A\,a\,x\,\left(m^2+11\,m+28\right)}{m^3+12\,m^2+39\,m+28}\right)$$

input
```
int(x^m*(A + B*x^3)*(a + b*x^3),x)
```

output
```
x^m*((x^4*(A*b + B*a)*(8*m + m^2 + 7))/(39*m + 12*m^2 + m^3 + 28) + (B*b*x
^7*(5*m + m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (A*a*x*(11*m + m^2 + 28))
/(39*m + 12*m^2 + m^3 + 28))
```

3.127 $\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx$

3.127.1 Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx=\frac{Bx^{1+m}}{b(1+m)}+\frac{(Ab-aB)x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{ab(1+m)}$$

output
```
B*x^(1+m)/b/(1+m)+(A*b-B*a)*x^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-
b*x^3/a)/a/b/(1+m)
```

3.127.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx=\frac{x^{1+m}\left(aB+(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)\right)}{ab(1+m)}$$

input
```
Integrate[(x^m*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(x^(1 + m)*(a*B + (A*b - a*B)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -
((b*x^3)/a)]))/(a*b*(1 + m))
```

3.127.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (A+Bx^3)}{a+bx^3} \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{x^m}{bx^3+a}dx}{b}+\frac{Bx^{m+1}}{b(m+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1}(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{ab(m+1)}+\frac{Bx^{m+1}}{b(m+1)}$$

input
```
Int[(x^m*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(B*x^(1 + m))/(b*(1 + m)) + ((A*b - a*B)*x^(1 + m)*Hypergeometric2F1[1, (1
 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(a*b*(1 + m))
```

3.127.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))    Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.127.4 Maple [F]

$$\int \frac{x^m(x^3B+A)}{b\,x^3+a}dx$$

input
```
int(x^m*(B*x^3+A)/(b*x^3+a),x)
```

output
```
int(x^m*(B*x^3+A)/(b*x^3+a),x)
```

3.127.5 Fricas [F]

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \int \frac{(Bx^3+A)x^m}{bx^3+a}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

3.127.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.59 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.83

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \frac{Amx^{m+1}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{Ax^{m+1}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{Bmx^{m+4}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{4Bx^{m+4}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

input
```
integrate(x**m*(B*x**3+A)/(b*x**3+a),x)
```

output
```
A*m*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3
+ 1/3)/(9*a*gamma(m/3 + 4/3)) + A*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*p
i)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(9*a*gamma(m/3 + 4/3)) + B*m*x**(m +
4)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 + 4/3)/(9*a*
gamma(m/3 + 7/3)) + 4*B*x**(m + 4)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m
/3 + 4/3)*gamma(m/3 + 4/3)/(9*a*gamma(m/3 + 7/3))
```

3.127.7 Maxima [F]

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \int \frac{(Bx^3+A)x^m}{bx^3+a}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

3.127.8 Giac [F]

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \int \frac{(Bx^3+A)x^m}{bx^3+a}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \int \frac{x^m\,(B\,x^3+A)}{b\,x^3+a}\,dx$$

input
```
int((x^m*(A + B*x^3))/(a + b*x^3),x)
```

output
```
int((x^m*(A + B*x^3))/(a + b*x^3), x)
```

3.128 $\int \frac{x^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

3.128.1 Optimal result

Integrand size = 20, antiderivative size = 93

$$\begin{aligned}\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{(Ab-aB)x^{1+m}}{3ab\,(a+bx^3)}\\ &\quad+\frac{(Ab(2-m)+aB(1+m))x^{1+m}\operatorname{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{3a^2b(1+m)}\end{aligned}$$

output
```
1/3*(A*b-B*a)*x^(1+m)/a/b/(b*x^3+a)+1/3*(A*b*(-m+2)+a*B*(1+m))*x^(1+m)*hyp
ergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^2/b/(1+m)
```

3.128.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{x^{1+m}\Big(aB\operatorname{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+(Ab-aB)\operatorname{Hypergeometric2F1}\Big(2,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}}{a^2b(1+m)}$$

input
```
Integrate[(x^m*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(x^(1 + m)*(a*B*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)] +
 (A*b - a*B)*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)]))/(a
^2*b*(1 + m))
```

3.128.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (A+Bx^3)}{(a+bx^3)^2} \, dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB(m+1)+Ab(2-m))\int \frac{x^m}{bx^3+a}dx}{3ab}+\frac{x^{m+1}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1}(aB(m+1)+Ab(2-m))\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{3a^2b(m+1)}+\frac{x^{m+1}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

input
```
Int[(x^m*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^(1 + m))/(3*a*b*(a + b*x^3)) + ((A*b*(2 - m) + a*B*(1 + m))
*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(3*a^
2*b*(1 + m))
```

3.128.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.128.4 Maple [F]

$$\int \frac{x^m(x^3B+A)}{(b\,x^3+a)^2}dx$$

input
```
int(x^m*(B*x^3+A)/(b*x^3+a)^2,x)
```

output
```
int(x^m*(B*x^3+A)/(b*x^3+a)^2,x)
```

3.128.5 Fricas [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \int \frac{(Bx^3+A)x^m}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*x^m/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

3.128.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 140.65 (sec) , antiderivative size = 1049, normalized size of antiderivative = 11.28

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**m*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
A*(-a*m**2*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gam
ma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3))
 + a*m*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m
/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + 3
*a*m*x**(m + 1)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**
3*gamma(m/3 + 4/3)) + 2*a*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1,
 m/3 + 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*ga
mma(m/3 + 4/3)) + 3*a*x**(m + 1)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3
) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) - b*m**2*x**3*x**(m + 1)*lerchphi(b*x
**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 +
 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + b*m*x**3*x**(m + 1)*lerchphi(b*
x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3
+ 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + 2*b*x**3*x**(m + 1)*lerchphi(b
*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3
 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3))) + B*(-a*m**2*x**(m + 4)*lerchp
hi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 + 4/3)/(27*a**3*gamma
(m/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7/3)) - 5*a*m*x**(m + 4)*lerchphi
(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 + 4/3)/(27*a**3*gamma(m
/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7/3)) + 3*a*m*x**(m + 4)*gamma(m/3
+ 4/3)/(27*a**3*gamma(m/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7/3)) - 4...
```

3.128.7 Maxima [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \int \frac{(Bx^3+A)x^m}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^m/(b*x^3 + a)^2, x)
```

3.128.8 Giac [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \int \frac{(Bx^3+A)x^m}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^m/(b*x^3 + a)^2, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \int \frac{x^m\,(B\,x^3+A)}{(b\,x^3+a)^2}\,dx$$

input
```
int((x^m*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
int((x^m*(A + B*x^3))/(a + b*x^3)^2, x)
```

3.129 $\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.129.1 Optimal result

Integrand size = 20, antiderivative size = 93

$$\begin{aligned}\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{(Ab-aB)x^{1+m}}{6ab\,(a+bx^3)^2} \\ &\quad+ \frac{(Ab(5-m)+aB(1+m))x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{6a^3b(1+m)}\end{aligned}$$

output
```
1/6*(A*b-B*a)*x^(1+m)/a/b/(b*x^3+a)^2+1/6*(A*b*(5-m)+a*B*(1+m))*x^(1+m)*hy
pergeom([2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^3/b/(1+m)
```

3.129.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^{1+m}\left(aB\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+(Ab-aB)\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right.\right.}{a^3b(1+m)}$$

input
```
Integrate[(x^m*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
(x^(1 + m)*(a*B*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)] +
 (A*b - a*B)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)]))/(a
^3*b*(1 + m))
```

3.129.3 Rubi [A] (verified)

Time $= 0.22$ (sec) , antiderivative size $= 93$, normalized size of antiderivative $= 1.00$, number of steps used $= 2$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used $= \{957, 888\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB(m+1)+Ab(5-m))\int \frac{x^m}{\left(bx^3+a\right)^2}dx}{6ab}+\frac{x^{m+1}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1}(aB(m+1)+Ab(5-m))\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{6a^3b(m+1)}+\frac{x^{m+1}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(x^m*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^(1 + m))/(6*a*b*(a + b*x^3)^2) + ((A*b*(5 - m) + a*B*(1 + m
))*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(6*
a^3*b*(1 + m))
```

3.129.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.129.4 Maple [F]

$$\int \frac{x^{m} \left(x^{3} B +A \right)}{\left(b \,x^{3}+a \right)^{3}}d x$$

input
```
int(x^m*(B*x^3+A)/(b*x^3+a)^3,x)
```

output
```
int(x^m*(B*x^3+A)/(b*x^3+a)^3,x)
```

3.129.5 Fricas [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx=\int \frac{(Bx^3+A)x^m}{(bx^3+a)^3}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*x^m/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3 + a^3), x)
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.129.7 Maxima [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \int \frac{(Bx^3+A)x^m}{(bx^3+a)^3}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^m/(b*x^3 + a)^3, x)
```

3.129.8 Giac [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \int \frac{(Bx^3+A)x^m}{(bx^3+a)^3}\,dx$$

input
```
integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^m/(b*x^3 + a)^3, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^3)}{\left(a+bx^3\right)^3}\,dx=\int \frac{x^m\,\left(B\,x^3+A\right)}{{\left(b\,x^3+a\right)}^3}\,dx$$

input
```
int((x^m*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
int((x^m*(A + B*x^3))/(a + b*x^3)^3, x)
```

3.130 $\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx$

3.130.1 Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx = \frac{b(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a(bc-ad)e(1+m)} - \frac{d(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{dx^3}{c}\right)}{c(bc-ad)e(1+m)}$$

output
```
b*(e*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a/(-a*d+b*c)/
e/(1+m)-d*(e*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-d*x^3/c)/c/(-a
*d+b*c)/e/(1+m)
```

3.130.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx = \frac{x(ex)^m\left(-bc\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+ad\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{dx^3}{c}\right)\right)}{ac(-bc+ad)(1+m)}$$

input
```
Integrate[(e*x)^m/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(x*(e*x)^m*(-(b*c*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)]
) + a*d*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((d*x^3)/c)]))/(a*c*(-
(b*c) + a*d)*(1 + m))
```

3.130.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {982, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}dx$$

$$\Big\downarrow \text{982}$$

$$\frac{b\int \frac{(ex)^m}{bx^3+a}dx}{bc-ad}-\frac{d\int \frac{(ex)^m}{dx^3+c}dx}{bc-ad}$$

$$\Big\downarrow \text{888}$$

$$\frac{b(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{ae(m+1)(bc-ad)}-\frac{d(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{dx^3}{c}\right)}{ce(m+1)(bc-ad)}$$

input
```
Int[(e*x)^m/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(b*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])
/(a*(b*c - a*d)*e*(1 + m)) - (d*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)
/3, (4 + m)/3, -((d*x^3)/c)])/(c*(b*c - a*d)*e*(1 + m))
```

3.130.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

3.130.4 Maple [F]

$$\int \frac{(ex)^m}{(b\,x^3+a)\,(d\,x^3+c)}dx$$

input
```
int((e*x)^m/(b*x^3+a)/(d*x^3+c),x)
```

output
```
int((e*x)^m/(b*x^3+a)/(d*x^3+c),x)
```

3.130.5 Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^3)\,(c+dx^3)}\,dx = \int \frac{(ex)^m}{(bx^3+a)(dx^3+c)}\,dx$$

input
```
integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((e*x)^m/(b*d*x^6 + (b*c + a*d)*x^3 + a*c), x)
```

3.130.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.130.7 Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx = \int \frac{(ex)^m}{(bx^3+a)(dx^3+c)}\,dx$$

input
```
integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^3 + a)*(d*x^3 + c)), x)
```

3.130.8 Giac [F]

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx = \int \frac{(ex)^m}{(bx^3+a)(dx^3+c)}\,dx$$

input
```
integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^3 + a)*(d*x^3 + c)), x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx = \int \frac{(e\,x)^m}{(b\,x^3+a)\,(d\,x^3+c)}\,dx$$

input

```
int((e*x)^m/((a + b*x^3)*(c + d*x^3)),x)
```

output

```
int((e*x)^m/((a + b*x^3)*(c + d*x^3)), x)
```

3.131 $\int x^{7/2}(a+bx^3)\left(A+Bx^3\right)\,dx$

3.131.1 Optimal result

Integrand size = 20, antiderivative size = 39

$$\int x^{7/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{9}aAx^{9/2}+\frac{2}{15}(Ab+aB)x^{15/2}+\frac{2}{21}bBx^{21/2}$$

output
```
2/9*a*A*x^(9/2)+2/15*(A*b+B*a)*x^(15/2)+2/21*b*B*x^(21/2)
```

3.131.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int x^{7/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{315}x^{9/2}\left(35aA+21Abx^3+21aBx^3+15bBx^6\right)$$

input
```
Integrate[x^(7/2)*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(2*x^(9/2)*(35*a*A + 21*A*b*x^3 + 21*a*B*x^3 + 15*b*B*x^6))/315
```

3.131.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}(a+bx^3)\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(x^{13/2}(aB+Ab)+aAx^{7/2}+bBx^{19/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{15}x^{15/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{21}bBx^{21/2}$$

input
```
Int[x^(7/2)*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(15/2))/15 + (2*b*B*x^(21/2))/21
```

3.131.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.131.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{9}{2}}}{9}+\frac{2(Ab+Ba)x^{\frac{15}{2}}}{15}+\frac{2bB\,x^{\frac{21}{2}}}{21}$	28
default	$\frac{2aA\,x^{\frac{9}{2}}}{9}+\frac{2(Ab+Ba)x^{\frac{15}{2}}}{15}+\frac{2bB\,x^{\frac{21}{2}}}{21}$	28
gosper	$\frac{2x^{\frac{9}{2}}\left(15bB\,x^{6}+21Ab\,x^{3}+21Ba\,x^{3}+35Aa\right)}{315}$	32
trager	$\frac{2x^{\frac{9}{2}}\left(15bB\,x^{6}+21Ab\,x^{3}+21Ba\,x^{3}+35Aa\right)}{315}$	32
risch	$\frac{2x^{\frac{9}{2}}\left(15bB\,x^{6}+21Ab\,x^{3}+21Ba\,x^{3}+35Aa\right)}{315}$	32

input
```
int(x^(7/2)*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/9*a*A*x^(9/2)+2/15*(A*b+B*a)*x^(15/2)+2/21*b*B*x^(21/2)
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{315}\left(15\,Bbx^{10}+21\,(Ba+Ab)x^7+35\,Aax^4\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/315*(15*B*b*x^10 + 21*(B*a + A*b)*x^7 + 35*A*a*x^4)*sqrt(x)
```

3.131.6 Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2Aax^{\frac{9}{2}}}{9}+\frac{2Abx^{\frac{15}{2}}}{15}+\frac{2Bax^{\frac{15}{2}}}{15}+\frac{2Bbx^{\frac{21}{2}}}{21}$$

input
```
integrate(x**(7/2)*(b*x**3+a)*(B*x**3+A),x)
```

output
```
2*A*a*x**(9/2)/9 + 2*A*b*x**(15/2)/15 + 2*B*a*x**(15/2)/15 + 2*B*b*x**(21/
2)/21
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{21}\,Bbx^{\frac{21}{2}}+\frac{2}{15}\,(Ba+Ab)x^{\frac{15}{2}}+\frac{2}{9}\,Aax^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/21*B*b*x^(21/2) + 2/15*(B*a + A*b)*x^(15/2) + 2/9*A*a*x^(9/2)
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{21}\,Bbx^{\frac{21}{2}}+\frac{2}{15}\,Bax^{\frac{15}{2}}+\frac{2}{15}\,Abx^{\frac{15}{2}}+\frac{2}{9}\,Aax^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

output
```
2/21*B*b*x^(21/2) + 2/15*B*a*x^(15/2) + 2/15*A*b*x^(15/2) + 2/9*A*a*x^(9/2
)
```

3.131.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2\,x^{9/2}\,\left(35\,A\,a+21\,A\,b\,x^3+21\,B\,a\,x^3+15\,B\,b\,x^6\right)}{315}$$

input
```
int(x^(7/2)*(A + B*x^3)*(a + b*x^3),x)
```

output
```
(2*x^(9/2)*(35*A*a + 21*A*b*x^3 + 21*B*a*x^3 + 15*B*b*x^6))/315
```

3.132 $\int x^{5/2}(a+bx^3)\left(A+Bx^3\right)\,dx$

3.132.1 Optimal result

Integrand size = 20, antiderivative size = 39

$$\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{7}aAx^{7/2}+\frac{2}{13}(Ab+aB)x^{13/2}+\frac{2}{19}bBx^{19/2}$$

output
```
2/7*a*A*x^(7/2)+2/13*(A*b+B*a)*x^(13/2)+2/19*b*B*x^(19/2)
```

3.132.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2x^{7/2}(247aA+133Abx^3+133aBx^3+91bBx^6)}{1729}$$

input
```
Integrate[x^(5/2)*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(2*x^(7/2)*(247*a*A + 133*A*b*x^3 + 133*a*B*x^3 + 91*b*B*x^6))/1729
```

3.132.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \; 950$$

$$\int \left(x^{11/2}(aB+Ab)+aAx^{5/2}+bBx^{17/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{19}bBx^{19/2}$$

input
```
Int[x^(5/2)*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(19/2))/19
```

3.132.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.132.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{13}{2}}}{13}+\frac{2bB\,x^{\frac{19}{2}}}{19}$	28
default	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{13}{2}}}{13}+\frac{2bB\,x^{\frac{19}{2}}}{19}$	28
gosper	$\frac{2x^{\frac{7}{2}}\left(91bB\,x^{6}+133Ab\,x^{3}+133Ba\,x^{3}+247Aa\right)}{1729}$	32
trager	$\frac{2x^{\frac{7}{2}}\left(91bB\,x^{6}+133Ab\,x^{3}+133Ba\,x^{3}+247Aa\right)}{1729}$	32
risch	$\frac{2x^{\frac{7}{2}}\left(91bB\,x^{6}+133Ab\,x^{3}+133Ba\,x^{3}+247Aa\right)}{1729}$	32

input
```
int(x^(5/2)*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/7*a*A*x^(7/2)+2/13*(A*b+B*a)*x^(13/2)+2/19*b*B*x^(19/2)
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{1729}\left(91\,Bbx^9+133\,(Ba+Ab)x^6+247\,Aax^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/1729*(91*B*b*x^9 + 133*(B*a + A*b)*x^6 + 247*A*a*x^3)*sqrt(x)
```

3.132.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2Aax^{\frac{7}{2}}}{7}+\frac{2Abx^{\frac{13}{2}}}{13}+\frac{2Bax^{\frac{13}{2}}}{13}+\frac{2Bbx^{\frac{19}{2}}}{19}$$

input
```
integrate(x**(5/2)*(b*x**3+a)*(B*x**3+A),x)
```

output
```
2*A*a*x**(7/2)/7 + 2*A*b*x**(13/2)/13 + 2*B*a*x**(13/2)/13 + 2*B*b*x**(19/
2)/19
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{5/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{19}\,Bbx^{\frac{19}{2}}+\frac{2}{13}\,(Ba+Ab)x^{\frac{13}{2}}+\frac{2}{7}\,Aax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/19*B*b*x^(19/2) + 2/13*(B*a + A*b)*x^(13/2) + 2/7*A*a*x^(7/2)
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{5/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{19}\,Bbx^{\frac{19}{2}}+\frac{2}{13}\,Bax^{\frac{13}{2}}+\frac{2}{13}\,Abx^{\frac{13}{2}}+\frac{2}{7}\,Aax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

output
```
2/19*B*b*x^(19/2) + 2/13*B*a*x^(13/2) + 2/13*A*b*x^(13/2) + 2/7*A*a*x^(7/2
)
```

3.132.9 Mupad [B] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{5/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2\,x^{7/2}\,\left(247\,A\,a+133\,A\,b\,x^3+133\,B\,a\,x^3+91\,B\,b\,x^6\right)}{1729}$$

input
```
int(x^(5/2)*(A + B*x^3)*(a + b*x^3),x)
```

output
```
(2*x^(7/2)*(247*A*a + 133*A*b*x^3 + 133*B*a*x^3 + 91*B*b*x^6))/1729
```

3.133 $\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx$

3.133.1 Optimal result

Integrand size = 20, antiderivative size = 39

$$\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{5}aAx^{5/2}+\frac{2}{11}(Ab+aB)x^{11/2}+\frac{2}{17}bBx^{17/2}$$

output
```
2/5*a*A*x^(5/2)+2/11*(A*b+B*a)*x^(11/2)+2/17*b*B*x^(17/2)
```

3.133.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{935}x^{5/2}\left(187aA+85Abx^3+85aBx^3+55bBx^6\right)$$

input
```
Integrate[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(2*x^(5/2)*(187*a*A + 85*A*b*x^3 + 85*a*B*x^3 + 55*b*B*x^6))/935
```

3.133.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2} \left(a+b x^3\right) \left(A+B x^3\right) \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{9/2} (a B+A b)+a A x^{3/2}+b B x^{15/2}\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{11} x^{11/2} (a B+A b)+\frac{2}{5} a A x^{5/2}+\frac{2}{17} b B x^{17/2}$$

input
```
Int[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(17/2))/17
```

3.133.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.133.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2bB\,x^{\frac{17}{2}}}{17}$	28
default	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2bB\,x^{\frac{17}{2}}}{17}$	28
gosper	$\frac{2x^{\frac{5}{2}}\left(55bB\,x^{6}+85Ab\,x^{3}+85Ba\,x^{3}+187Aa\right)}{935}$	32
trager	$\frac{2x^{\frac{5}{2}}\left(55bB\,x^{6}+85Ab\,x^{3}+85Ba\,x^{3}+187Aa\right)}{935}$	32
risch	$\frac{2x^{\frac{5}{2}}\left(55bB\,x^{6}+85Ab\,x^{3}+85Ba\,x^{3}+187Aa\right)}{935}$	32

input
```
int(x^(3/2)*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/5*a*A*x^(5/2)+2/11*(A*b+B*a)*x^(11/2)+2/17*b*B*x^(17/2)
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{935}\left(55\,Bbx^8+85\,(Ba+Ab)x^5+187\,Aax^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/935*(55*B*b*x^8 + 85*(B*a + A*b)*x^5 + 187*A*a*x^2)*sqrt(x)
```

3.133.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2Aax^{\frac{5}{2}}}{5}+\frac{2Abx^{\frac{11}{2}}}{11}+\frac{2Bax^{\frac{11}{2}}}{11}+\frac{2Bbx^{\frac{17}{2}}}{17}$$

input
```
integrate(x**(3/2)*(b*x**3+a)*(B*x**3+A),x)
```

output
```
2*A*a*x**(5/2)/5 + 2*A*b*x**(11/2)/11 + 2*B*a*x**(11/2)/11 + 2*B*b*x**(17/
2)/17
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{11}\,(Ba+Ab)x^{\frac{11}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/17*B*b*x^(17/2) + 2/11*(B*a + A*b)*x^(11/2) + 2/5*A*a*x^(5/2)
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{11}\,Bax^{\frac{11}{2}}+\frac{2}{11}\,Abx^{\frac{11}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

output
```
2/17*B*b*x^(17/2) + 2/11*B*a*x^(11/2) + 2/11*A*b*x^(11/2) + 2/5*A*a*x^(5/2
)
```

3.133.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2\,x^{5/2}\,\left(187\,A\,a+85\,A\,b\,x^3+85\,B\,a\,x^3+55\,B\,b\,x^6\right)}{935}$$

input
```
int(x^(3/2)*(A + B*x^3)*(a + b*x^3),x)
```

output
```
(2*x^(5/2)*(187*A*a + 85*A*b*x^3 + 85*B*a*x^3 + 55*B*b*x^6))/935
```

3.134 $\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right) dx$

3.134.1 Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right) dx = \frac{2}{3}aAx^{3/2}+\frac{2}{9}(Ab+aB)x^{9/2}+\frac{2}{15}bBx^{15/2}$$

output
```
2/3*a*A*x^(3/2)+2/9*(A*b+B*a)*x^(9/2)+2/15*b*B*x^(15/2)
```

3.134.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right) dx = \frac{2}{45}x^{3/2}\left(15aA+5Abx^3+5aBx^3+3bBx^6\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(2*x^(3/2)*(15*a*A + 5*A*b*x^3 + 5*a*B*x^3 + 3*b*B*x^6))/45
```

3.134.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{7/2}(aB+Ab)+aA\sqrt{x}+bBx^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{15}bBx^{15/2}$$

input
```
Int[Sqrt[x]*(a + b*x^3)*(A + B*x^3),x]
```

output
```
(2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(15/2))/15
```

3.134.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.134.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2bB\,x^{\frac{15}{2}}}{15}$	28
default	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2bB\,x^{\frac{15}{2}}}{15}$	28
gosper	$\frac{2x^{\frac{3}{2}}\left(3bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+15Aa\right)}{45}$	32
trager	$\frac{2x^{\frac{3}{2}}\left(3bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+15Aa\right)}{45}$	32
risch	$\frac{2x^{\frac{3}{2}}\left(3bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+15Aa\right)}{45}$	32

input
```
int((b*x^3+a)*(B*x^3+A)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*a*A*x^(3/2)+2/9*(A*b+B*a)*x^(9/2)+2/15*b*B*x^(15/2)
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{45}\left(3\,Bbx^7+5\,(Ba+Ab)x^4+15\,Aax\right)\sqrt{x}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*B*b*x^7 + 5*(B*a + A*b)*x^4 + 15*A*a*x)*sqrt(x)
```

3.134.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2Aax^{\frac{3}{2}}}{3}+\frac{2Abx^{\frac{9}{2}}}{9}+\frac{2Bax^{\frac{9}{2}}}{9}+\frac{2Bbx^{\frac{15}{2}}}{15}$$

input
```
integrate((b*x**3+a)*(B*x**3+A)*x**(1/2),x)
```

output
```
2*A*a*x**(3/2)/3 + 2*A*b*x**(9/2)/9 + 2*B*a*x**(9/2)/9 + 2*B*b*x**(15/2)/1
5
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{9}\,(Ba+Ab)x^{\frac{9}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
```

output
```
2/15*B*b*x^(15/2) + 2/9*(B*a + A*b)*x^(9/2) + 2/3*A*a*x^(3/2)
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{9}\,Bax^{\frac{9}{2}}+\frac{2}{9}\,Abx^{\frac{9}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

output
```
2/15*B*b*x^(15/2) + 2/9*B*a*x^(9/2) + 2/9*A*b*x^(9/2) + 2/3*A*a*x^(3/2)
```

3.134.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2\,x^{3/2}\,\left(15\,A\,a+5\,A\,b\,x^3+5\,B\,a\,x^3+3\,B\,b\,x^6\right)}{45}$$

input
```
int(x^(1/2)*(A + B*x^3)*(a + b*x^3),x)
```

output
```
(2*x^(3/2)*(15*A*a + 5*A*b*x^3 + 5*B*a*x^3 + 3*B*b*x^6))/45
```

3.135 $\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx$

3.135.1 Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = 2aA\sqrt{x}+\frac{2}{7}(Ab+aB)x^{7/2}+\frac{2}{13}bBx^{13/2}$$

output
```
2/7*(A*b+B*a)*x^(7/2)+2/13*b*B*x^(13/2)+2*a*A*x^(1/2)
```

3.135.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{91}\sqrt{x}\left(91aA+13Abx^3+13aBx^3+7bBx^6\right)$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(91*a*A + 13*A*b*x^3 + 13*a*B*x^3 + 7*b*B*x^6))/91
```

3.135.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{5/2}(aB+Ab)+\frac{aA}{\sqrt{x}}+bBx^{11/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}x^{7/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{13}bBx^{13/2}$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/Sqrt[x],x]
```

output
```
2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(13/2))/13
```

3.135.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.135.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2bB\,x^{\frac{13}{2}}}{13}+2aA\sqrt{x}$	28
default	$\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2bB\,x^{\frac{13}{2}}}{13}+2aA\sqrt{x}$	28
trager	$\left(\frac{2}{13}bB\,x^{6}+\frac{2}{7}Ab\,x^{3}+\frac{2}{7}Ba\,x^{3}+2Aa\right)\sqrt{x}$	31
gosper	$\frac{2\sqrt{x}\,(7bB\,x^{6}+13Ab\,x^{3}+13Ba\,x^{3}+91Aa)}{91}$	32
risch	$\frac{2\sqrt{x}\,(7bB\,x^{6}+13Ab\,x^{3}+13Ba\,x^{3}+91Aa)}{91}$	32

input
```
int((b*x^3+a)*(B*x^3+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*(A*b+B*a)*x^(7/2)+2/13*b*B*x^(13/2)+2*a*A*x^(1/2)
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx=\frac{2}{91}\left(7\,Bbx^6+13\,(Ba+Ab)x^3+91\,Aa\right)\sqrt{x}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

output
```
2/91*(7*B*b*x^6 + 13*(B*a + A*b)*x^3 + 91*A*a)*sqrt(x)
```

3.135.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx=2Aa\sqrt{x}+\frac{2Abx^{\frac{7}{2}}}{7}+\frac{2Bax^{\frac{7}{2}}}{7}+\frac{2Bbx^{\frac{13}{2}}}{13}$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**(1/2),x)
```

output
```
2*A*a*sqrt(x) + 2*A*b*x**(7/2)/7 + 2*B*a*x**(7/2)/7 + 2*B*b*x**(13/2)/13
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{7}\,(Ba+Ab)x^{\frac{7}{2}} + 2\,Aa\sqrt{x}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

output
```
2/13*B*b*x^(13/2) + 2/7*(B*a + A*b)*x^(7/2) + 2*A*a*sqrt(x)
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{7}\,Bax^{\frac{7}{2}} + \frac{2}{7}\,Abx^{\frac{7}{2}} + 2\,Aa\sqrt{x}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

output
```
2/13*B*b*x^(13/2) + 2/7*B*a*x^(7/2) + 2/7*A*b*x^(7/2) + 2*A*a*sqrt(x)
```

3.135.9 Mupad [B] (verification not implemented)

Time = 6.98 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2\,\sqrt{x}\,(91\,A\,a + 13\,A\,b\,x^3 + 13\,B\,a\,x^3 + 7\,B\,b\,x^6)}{91}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^(1/2),x)
```

output
```
(2*x^(1/2)*(91*A*a + 13*A*b*x^3 + 13*B*a*x^3 + 7*B*b*x^6))/91
```

3.136 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx$

3.136.1 Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2aA}{\sqrt{x}} + \frac{2}{5}(Ab+aB)x^{5/2} + \frac{2}{11}bBx^{11/2}$$

output
```
2/5*(A*b+B*a)*x^(5/2)+2/11*b*B*x^(11/2)-2*a*A/x^(1/2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2(55aA-11Abx^3-11aBx^3-5bBx^6)}{55\sqrt{x}}$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x^(3/2),x]
```

output
```
(-2*(55*a*A - 11*A*b*x^3 - 11*a*B*x^3 - 5*b*B*x^6))/(55*Sqrt[x])
```

3.136.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx$$

$$\Big\downarrow \; 950$$

$$\int \left(x^{3/2}(aB+Ab)+\frac{aA}{x^{3/2}}+bBx^{9/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{5}x^{5/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{11}bBx^{11/2}$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^(3/2),x]
```

output
```
(-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(11/2))/11
```

3.136.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.136.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{11}{2}}}{11}+\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2Ba\,x^{\frac{5}{2}}}{5}-\frac{2aA}{\sqrt{x}}$	30
default	$\frac{2bB\,x^{\frac{11}{2}}}{11}+\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2Ba\,x^{\frac{5}{2}}}{5}-\frac{2aA}{\sqrt{x}}$	30
gosper	$-\frac{2(-5bB\,x^{6}-11Ab\,x^{3}-11Ba\,x^{3}+55Aa)}{55\sqrt{x}}$	32
trager	$-\frac{2(-5bB\,x^{6}-11Ab\,x^{3}-11Ba\,x^{3}+55Aa)}{55\sqrt{x}}$	32
risch	$-\frac{2(-5bB\,x^{6}-11Ab\,x^{3}-11Ba\,x^{3}+55Aa)}{55\sqrt{x}}$	32

input
```
int((b*x^3+a)*(B*x^3+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*b*B*x^(11/2)+2/5*A*b*x^(5/2)+2/5*B*a*x^(5/2)-2*a*A/x^(1/2)
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx=\frac{2\left(5\,Bbx^6+11\,(Ba+Ab)x^3-55\,Aa\right)}{55\,\sqrt{x}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

output
```
2/55*(5*B*b*x^6 + 11*(B*a + A*b)*x^3 - 55*A*a)/sqrt(x)
```

3.136.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx=-\frac{2Aa}{\sqrt{x}}+\frac{2Abx^{\frac{5}{2}}}{5}+\frac{2Bax^{\frac{5}{2}}}{5}+\frac{2Bbx^{\frac{11}{2}}}{11}$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**(3/2),x)
```

output
```
-2*A*a/sqrt(x) + 2*A*b*x**(5/2)/5 + 2*B*a*x**(5/2)/5 + 2*B*b*x**(11/2)/11
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx=\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{5}\,(Ba+Ab)x^{\frac{5}{2}}-\frac{2\,Aa}{\sqrt{x}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
```

output
```
2/11*B*b*x^(11/2) + 2/5*(B*a + A*b)*x^(5/2) - 2*A*a/sqrt(x)
```

3.136.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx=\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{5}\,Bax^{\frac{5}{2}}+\frac{2}{5}\,Abx^{\frac{5}{2}}-\frac{2\,Aa}{\sqrt{x}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

output
```
2/11*B*b*x^(11/2) + 2/5*B*a*x^(5/2) + 2/5*A*b*x^(5/2) - 2*A*a/sqrt(x)
```

3.136.9 Mupad [B] (verification not implemented)

Time = 7.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx=\frac{22\,A\,b\,x^3-110\,A\,a+22\,B\,a\,x^3+10\,B\,b\,x^6}{55\,\sqrt{x}}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^(3/2),x)
```

output
```
(22*A*b*x^3 - 110*A*a + 22*B*a*x^3 + 10*B*b*x^6)/(55*x^(1/2))
```

3.137 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}} \, dx$

3.137.1 Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}} \, dx = -\frac{2aA}{3x^{3/2}} + \frac{2}{3}(Ab+aB)x^{3/2} + \frac{2}{9}bBx^{9/2}$$

output
```
-2/3*a*A/x^(3/2)+2/3*(A*b+B*a)*x^(3/2)+2/9*b*B*x^(9/2)
```

3.137.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}} \, dx = \frac{2(-3aA + 3Abx^3 + 3aBx^3 + bBx^6)}{9x^{3/2}}$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x^(5/2),x]
```

output
```
(2*(-3*a*A + 3*A*b*x^3 + 3*a*B*x^3 + b*B*x^6))/(9*x^(3/2))
```

3.137.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\sqrt{x}(aB+Ab)+\frac{aA}{x^{5/2}}+bBx^{7/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{9}bBx^{9/2}$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^(5/2),x]
```

output
```
(-2*a*A)/(3*x^(3/2)) + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(9/2))/9
```

3.137.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.137.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2Ba\,x^{\frac{3}{2}}}{3}-\frac{2aA}{3x^{\frac{3}{2}}}$	30
default	$\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2Ba\,x^{\frac{3}{2}}}{3}-\frac{2aA}{3x^{\frac{3}{2}}}$	30
gosper	$-\frac{2(-bB\,x^{6}-3Ab\,x^{3}-3Ba\,x^{3}+3Aa)}{9x^{\frac{3}{2}}}$	32
trager	$-\frac{2(-bB\,x^{6}-3Ab\,x^{3}-3Ba\,x^{3}+3Aa)}{9x^{\frac{3}{2}}}$	32
risch	$-\frac{2(-bB\,x^{6}-3Ab\,x^{3}-3Ba\,x^{3}+3Aa)}{9x^{\frac{3}{2}}}$	32

input
```
int((b*x^3+a)*(B*x^3+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*b*B*x^(9/2)+2/3*A*b*x^(3/2)+2/3*B*a*x^(3/2)-2/3*a*A/x^(3/2)
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx=\frac{2\,(Bbx^6+3\,(Ba+Ab)x^3-3\,Aa)}{9\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

output
```
2/9*(B*b*x^6 + 3*(B*a + A*b)*x^3 - 3*A*a)/x^(3/2)
```

3.137.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx=-\frac{2Aa}{3x^{\frac{3}{2}}}+\frac{2Abx^{\frac{3}{2}}}{3}+\frac{2Bax^{\frac{3}{2}}}{3}+\frac{2Bbx^{\frac{9}{2}}}{9}$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**(5/2),x)
```

output
```
-2*A*a/(3*x**(3/2)) + 2*A*b*x**(3/2)/3 + 2*B*a*x**(3/2)/3 + 2*B*b*x**(9/2)
/9
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx=\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{3}\,(Ba+Ab)x^{\frac{3}{2}}-\frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

output
```
2/9*B*b*x^(9/2) + 2/3*(B*a + A*b)*x^(3/2) - 2/3*A*a/x^(3/2)
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx=\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{3}\,Bax^{\frac{3}{2}}+\frac{2}{3}\,Abx^{\frac{3}{2}}-\frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

output
```
2/9*B*b*x^(9/2) + 2/3*B*a*x^(3/2) + 2/3*A*b*x^(3/2) - 2/3*A*a/x^(3/2)
```

3.137.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx=\frac{6\,A\,b\,x^3-6\,A\,a+6\,B\,a\,x^3+2\,B\,b\,x^6}{9\,x^{3/2}}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^(5/2),x)
```

output
```
(6*A*b*x^3 - 6*A*a + 6*B*a*x^3 + 2*B*b*x^6)/(9*x^(3/2))
```

3.138 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx$

3.138.1 Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = -\frac{2aA}{5x^{5/2}} + 2(Ab+aB)\sqrt{x} + \frac{2}{7}bBx^{7/2}$$

output
```
-2/5*a*A/x^(5/2)+2/7*b*B*x^(7/2)+2*(A*b+B*a)*x^(1/2)
```

3.138.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = -\frac{2(7aA-35Abx^3-35aBx^3-5bBx^6)}{35x^{5/2}}$$

input
```
Integrate[((a + b*x^3)*(A + B*x^3))/x^(7/2),x]
```

output
```
(-2*(7*a*A - 35*A*b*x^3 - 35*a*B*x^3 - 5*b*B*x^6))/(35*x^(5/2))
```

3.138.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{aB+Ab}{\sqrt{x}}+\frac{aA}{x^{7/2}}+bBx^{5/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\sqrt{x}(aB+Ab)-\frac{2aA}{5x^{5/2}}+\frac{2}{7}bBx^{7/2}$$

input
```
Int[((a + b*x^3)*(A + B*x^3))/x^(7/2),x]
```

output
```
(-2*a*A)/(5*x^(5/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(7/2))/7
```

3.138.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.138.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{5x^{\frac{5}{2}}}$	30
default	$\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{5x^{\frac{5}{2}}}$	30
gosper	$-\frac{2\left(-5bB\,x^{6}-35Ab\,x^{3}-35Ba\,x^{3}+7Aa\right)}{35x^{\frac{5}{2}}}$	32
trager	$-\frac{2\left(-5bB\,x^{6}-35Ab\,x^{3}-35Ba\,x^{3}+7Aa\right)}{35x^{\frac{5}{2}}}$	32
risch	$-\frac{2\left(-5bB\,x^{6}-35Ab\,x^{3}-35Ba\,x^{3}+7Aa\right)}{35x^{\frac{5}{2}}}$	32

input
```
int((b*x^3+a)*(B*x^3+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*b*B*x^(7/2)+2*A*b*x^(1/2)+2*B*a*x^(1/2)-2/5*a*A/x^(5/2)
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{x^{7/2}}\,dx=\frac{2\,(5\,Bbx^6+35\,(Ba+Ab)x^3-7\,Aa)}{35\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

output
```
2/35*(5*B*b*x^6 + 35*(B*a + A*b)*x^3 - 7*A*a)/x^(5/2)
```

3.138.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)\,(A+Bx^3)}{x^{7/2}}\,dx=-\frac{2Aa}{5x^{\frac{5}{2}}}+2Ab\sqrt{x}+2Ba\sqrt{x}+\frac{2Bbx^{\frac{7}{2}}}{7}$$

input
```
integrate((b*x**3+a)*(B*x**3+A)/x**(7/2),x)
```

output
```
-2*A*a/(5*x**(5/2)) + 2*A*b*sqrt(x) + 2*B*a*sqrt(x) + 2*B*b*x**(7/2)/7
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{7}\,Bbx^{\frac{7}{2}} + 2\,(Ba+Ab)\sqrt{x} - \frac{2\,Aa}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

output
```
2/7*B*b*x^(7/2) + 2*(B*a + A*b)*sqrt(x) - 2/5*A*a/x^(5/2)
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{7}\,Bbx^{\frac{7}{2}} + 2\,Ba\sqrt{x} + 2\,Ab\sqrt{x} - \frac{2\,Aa}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

output
```
2/7*B*b*x^(7/2) + 2*B*a*sqrt(x) + 2*A*b*sqrt(x) - 2/5*A*a/x^(5/2)
```

3.138.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = \frac{2\,A\,b\,x^3 - \frac{2\,A\,a}{5} + 2\,B\,a\,x^3 + \frac{2\,B\,b\,x^6}{7}}{x^{5/2}}$$

input
```
int(((A + B*x^3)*(a + b*x^3))/x^(7/2),x)
```

output
```
(2*A*b*x^3 - (2*A*a)/5 + 2*B*a*x^3 + (2*B*b*x^6)/7)/x^(5/2)
```

3.139 $\int x^{7/2}(a+bx^3)^2\left(A+Bx^3\right)\,dx$

3.139.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{7/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{9}a^2Ax^{9/2}+\frac{2}{15}a(2Ab+aB)x^{15/2}+\frac{2}{21}b(Ab+2aB)x^{21/2}+\frac{2}{27}b^2Bx^{27/2}$$

output
```
2/9*a^2*A*x^(9/2)+2/15*a*(2*A*b+B*a)*x^(15/2)+2/21*b*(A*b+2*B*a)*x^(21/2)+
2/27*b^2*B*x^(27/2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int x^{7/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{945}x^{9/2}\left(105a^2A+126aAbx^3+63a^2Bx^3+45Ab^2x^6+90abBx^6+35b^2Bx^9\right)$$

input
```
Integrate[x^(7/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(2*x^(9/2)*(105*a^2*A + 126*a*A*b*x^3 + 63*a^2*B*x^3 + 45*A*b^2*x^6 + 90*a
*b*B*x^6 + 35*b^2*B*x^9))/945
```

3.139.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^2Ax^{7/2}+bx^{19/2}(2aB+Ab)+ax^{13/2}(aB+2Ab)+b^2Bx^{25/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{21}bx^{21/2}(2aB+Ab)+\frac{2}{15}ax^{15/2}(aB+2Ab)+\frac{2}{27}b^2Bx^{27/2}$$

input
```
Int[x^(7/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(15/2))/15 + (2*b*(A*b + 2*a*B)
*x^(21/2))/21 + (2*b^2*B*x^(27/2))/27
```

3.139.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.139.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2 B x^{\frac{27}{2}}}{27}+\frac{2(b^2 A+2abB)x^{\frac{21}{2}}}{21}+\frac{2(2abA+a^2 B)x^{\frac{15}{2}}}{15}+\frac{2a^2 A x^{\frac{9}{2}}}{9}$	52
default	$\frac{2b^2 B x^{\frac{27}{2}}}{27}+\frac{2(b^2 A+2abB)x^{\frac{21}{2}}}{21}+\frac{2(2abA+a^2 B)x^{\frac{15}{2}}}{15}+\frac{2a^2 A x^{\frac{9}{2}}}{9}$	52
gosper	$\frac{2x^{\frac{9}{2}}(35b^2 B x^9+45A b^2 x^6+90B x^6 ab+126aAb x^3+63a^2 B x^3+105a^2 A)}{945}$	56
trager	$\frac{2x^{\frac{9}{2}}(35b^2 B x^9+45A b^2 x^6+90B x^6 ab+126aAb x^3+63a^2 B x^3+105a^2 A)}{945}$	56
risch	$\frac{2x^{\frac{9}{2}}(35b^2 B x^9+45A b^2 x^6+90B x^6 ab+126aAb x^3+63a^2 B x^3+105a^2 A)}{945}$	56

input
```
int(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/27*b^2*B*x^(27/2)+2/21*(A*b^2+2*B*a*b)*x^(21/2)+2/15*(2*A*a*b+B*a^2)*x^(
15/2)+2/9*a^2*A*x^(9/2)
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{7/2}(a+bx^3)^2\left(A+Bx^3\right)\,dx=\frac{2}{945}\left(35\,Bb^2x^{13}+45\left(2\,Bab+Ab^2\right)x^{10}+63\left(Ba^2+2\,Aab\right)x^7+105\,Aa^2x^4\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/945*(35*B*b^2*x^13 + 45*(2*B*a*b + A*b^2)*x^10 + 63*(B*a^2 + 2*A*a*b)*x^
7 + 105*A*a^2*x^4)*sqrt(x)
```

3.139.6 Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{7/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2Aa^2x^{\frac{9}{2}}}{9}+\frac{4Aabx^{\frac{15}{2}}}{15}+\frac{2Ab^2x^{\frac{21}{2}}}{21}+\frac{2Ba^2x^{\frac{15}{2}}}{15}+\frac{4Babx^{\frac{21}{2}}}{21}+\frac{2Bb^2x^{\frac{27}{2}}}{27}$$

input
```
integrate(x**(7/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

output
```
2*A*a**2*x**(9/2)/9 + 4*A*a*b*x**(15/2)/15 + 2*A*b**2*x**(21/2)/21 + 2*B*a
**2*x**(15/2)/15 + 4*B*a*b*x**(21/2)/21 + 2*B*b**2*x**(27/2)/27
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{27}\,Bb^2x^{\frac{27}{2}}+\frac{2}{21}\,\left(2\,Bab+Ab^2\right)x^{\frac{21}{2}}+\frac{2}{15}\,\left(Ba^2+2\,Aab\right)x^{\frac{15}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/27*B*b^2*x^(27/2) + 2/21*(2*B*a*b + A*b^2)*x^(21/2) + 2/15*(B*a^2 + 2*A*
a*b)*x^(15/2) + 2/9*A*a^2*x^(9/2)
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{7/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{27}\,Bb^2x^{\frac{27}{2}}+\frac{4}{21}\,Babx^{\frac{21}{2}}+\frac{2}{21}\,Ab^2x^{\frac{21}{2}}+\frac{2}{15}\,Ba^2x^{\frac{15}{2}}+\frac{4}{15}\,Aabx^{\frac{15}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

output
```
2/27*B*b^2*x^(27/2) + 4/21*B*a*b*x^(21/2) + 2/21*A*b^2*x^(21/2) + 2/15*B*a
^2*x^(15/2) + 4/15*A*a*b*x^(15/2) + 2/9*A*a^2*x^(9/2)
```

3.139.9 Mupad [B] (verification not implemented)

Time = 6.98 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}(a+bx^3)^2\,(A+Bx^3)\ dx = x^{15/2}\left(\frac{2\,B\,a^2}{15}+\frac{4\,A\,b\,a}{15}\right) + x^{21/2}\left(\frac{2\,A\,b^2}{21}+\frac{4\,B\,a\,b}{21}\right)+\frac{2\,A\,a^2\,x^{9/2}}{9}+\frac{2\,B\,b^2\,x^{27/2}}{27}$$

input
```
int(x^(7/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

output
```
x^(15/2)*((2*B*a^2)/15 + (4*A*a*b)/15) + x^(21/2)*((2*A*b^2)/21 + (4*B*a*b
)/21) + (2*A*a^2*x^(9/2))/9 + (2*B*b^2*x^(27/2))/27
```

3.140 $\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx$

3.140.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{7}a^2Ax^{7/2}+\frac{2}{13}a(2Ab+aB)x^{13/2}+\frac{2}{19}b(Ab+2aB)x^{19/2}+\frac{2}{25}b^2Bx^{25/2}$$

output
```
2/7*a^2*A*x^(7/2)+2/13*a*(2*A*b+B*a)*x^(13/2)+2/19*b*(A*b+2*B*a)*x^(19/2)+
2/25*b^2*B*x^(25/2)
```

3.140.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2x^{7/2}\left(475a^2\left(13A+7Bx^3\right)+350abx^3\left(19A+13Bx^3\right)+91b^2x^6\left(25A+19Bx^3\right)\right)}{43225}$$

input
```
Integrate[x^(5/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(2*x^(7/2)*(475*a^2*(13*A + 7*B*x^3) + 350*a*b*x^3*(19*A + 13*B*x^3) + 91*
b^2*x^6*(25*A + 19*B*x^3)))/43225
```

3.140.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^2Ax^{5/2}+bx^{17/2}(2aB+Ab)+ax^{11/2}(aB+2Ab)+b^2Bx^{23/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{19}bx^{19/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{25}b^2Bx^{25/2}$$

input
```
Int[x^(5/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)
*x^(19/2))/19 + (2*b^2*B*x^(25/2))/25
```

3.140.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.140.4 Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^{2}B\,x^{\frac{25}{2}}}{25}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{19}{2}}}{19}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{13}{2}}}{13}+\frac{2a^{2}A\,x^{\frac{7}{2}}}{7}$	52
default	$\frac{2b^{2}B\,x^{\frac{25}{2}}}{25}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{19}{2}}}{19}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{13}{2}}}{13}+\frac{2a^{2}A\,x^{\frac{7}{2}}}{7}$	52
gosper	$\frac{2x^{\frac{7}{2}}\left(1729b^{2}B\,x^{9}+2275A\,b^{2}x^{6}+4550B\,x^{6}ab+6650aAb\,x^{3}+3325a^{2}B\,x^{3}+6175a^{2}A\right)}{43225}$	56
trager	$\frac{2x^{\frac{7}{2}}\left(1729b^{2}B\,x^{9}+2275A\,b^{2}x^{6}+4550B\,x^{6}ab+6650aAb\,x^{3}+3325a^{2}B\,x^{3}+6175a^{2}A\right)}{43225}$	56
risch	$\frac{2x^{\frac{7}{2}}\left(1729b^{2}B\,x^{9}+2275A\,b^{2}x^{6}+4550B\,x^{6}ab+6650aAb\,x^{3}+3325a^{2}B\,x^{3}+6175a^{2}A\right)}{43225}$	56

input
```
int(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/25*b^2*B*x^(25/2)+2/19*(A*b^2+2*B*a*b)*x^(19/2)+2/13*(2*A*a*b+B*a^2)*x^(
13/2)+2/7*a^2*A*x^(7/2)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{5/2}\left(a+bx^{3}\right)^{2}\left(A+Bx^{3}\right)\,dx=\frac{2}{43225}\left(1729\,Bb^{2}x^{12}+2275\left(2\,Bab+Ab^{2}\right)x^{9}+3325\left(Ba^{2}+2\,Aab\right)x^{6}+6175\,Aa^{2}x^{3}\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/43225*(1729*B*b^2*x^12 + 2275*(2*B*a*b + A*b^2)*x^9 + 3325*(B*a^2 + 2*A*
a*b)*x^6 + 6175*A*a^2*x^3)*sqrt(x)
```

3.140.6 Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{5/2}(a+bx^3)^2\left(A+Bx^3\right)\,dx=\frac{2Aa^2x^{\frac{7}{2}}}{7}+\frac{4Aabx^{\frac{13}{2}}}{13}+\frac{2Ab^2x^{\frac{19}{2}}}{19}+\frac{2Ba^2x^{\frac{13}{2}}}{13}+\frac{4Babx^{\frac{19}{2}}}{19}+\frac{2Bb^2x^{\frac{25}{2}}}{25}$$

input
```
integrate(x**(5/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

output
```
2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(13/2)/13 + 2*A*b**2*x**(19/2)/19 + 2*B*a
**2*x**(13/2)/13 + 4*B*a*b*x**(19/2)/19 + 2*B*b**2*x**(25/2)/25
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}(a+bx^3)^2\left(A+Bx^3\right)\,dx=\frac{2}{25}\,Bb^2x^{\frac{25}{2}}+\frac{2}{19}\left(2\,Bab+Ab^2\right)x^{\frac{19}{2}}+\frac{2}{13}\left(Ba^2+2\,Aab\right)x^{\frac{13}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/25*B*b^2*x^(25/2) + 2/19*(2*B*a*b + A*b^2)*x^(19/2) + 2/13*(B*a^2 + 2*A*
a*b)*x^(13/2) + 2/7*A*a^2*x^(7/2)
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{5/2}(a+bx^3)^2\left(A+Bx^3\right)\,dx=\frac{2}{25}\,Bb^2x^{\frac{25}{2}}+\frac{4}{19}\,Babx^{\frac{19}{2}}+\frac{2}{19}\,Ab^2x^{\frac{19}{2}}+\frac{2}{13}\,Ba^2x^{\frac{13}{2}}+\frac{4}{13}\,Aabx^{\frac{13}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

output
```
2/25*B*b^2*x^(25/2) + 4/19*B*a*b*x^(19/2) + 2/19*A*b^2*x^(19/2) + 2/13*B*a
^2*x^(13/2) + 4/13*A*a*b*x^(13/2) + 2/7*A*a^2*x^(7/2)
```

3.140.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=x^{13/2}\left(\frac{2\,B\,a^2}{13}+\frac{4\,A\,b\,a}{13}\right)+x^{19/2}\left(\frac{2\,A\,b^2}{19}+\frac{4\,B\,a\,b}{19}\right)+\frac{2\,A\,a^2\,x^{7/2}}{7}+\frac{2\,B\,b^2\,x^{25/2}}{25}$$

input
```
int(x^(5/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

output
```
x^(13/2)*((2*B*a^2)/13 + (4*A*a*b)/13) + x^(19/2)*((2*A*b^2)/19 + (4*B*a*b
)/19) + (2*A*a^2*x^(7/2))/7 + (2*B*b^2*x^(25/2))/25
```

3.141 $\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx$

3.141.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{5}a^2Ax^{5/2}+\frac{2}{11}a(2Ab+aB)x^{11/2}+\frac{2}{17}b(Ab+2aB)x^{17/2}+\frac{2}{23}b^2Bx^{23/2}$$

output
```
2/5*a^2*A*x^(5/2)+2/11*a*(2*A*b+B*a)*x^(11/2)+2/17*b*(A*b+2*B*a)*x^(17/2)+
2/23*b^2*B*x^(23/2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2x^{5/2}\left(391a^2\left(11A+5Bx^3\right)+230abx^3\left(17A+11Bx^3\right)+55b^2x^6\left(23A+17Bx^3\right)\right)}{21505}$$

input
```
Integrate[x^(3/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(2*x^(5/2)*(391*a^2*(11*A + 5*B*x^3) + 230*a*b*x^3*(17*A + 11*B*x^3) + 55*
b^2*x^6*(23*A + 17*B*x^3)))/21505
```

3.141.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^2Ax^{3/2}+bx^{15/2}(2aB+Ab)+ax^{9/2}(aB+2Ab)+b^2Bx^{21/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{17}bx^{17/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{23}b^2Bx^{23/2}$$

input
```
Int[x^(3/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)
*x^(17/2))/17 + (2*b^2*B*x^(23/2))/23
```

3.141.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.141.4 Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{23}{2}}}{23}+\frac{2(b^2A+2abB)x^{\frac{17}{2}}}{17}+\frac{2(2abA+a^2B)x^{\frac{11}{2}}}{11}+\frac{2a^2Ax^{\frac{5}{2}}}{5}$	52
default	$\frac{2b^2Bx^{\frac{23}{2}}}{23}+\frac{2(b^2A+2abB)x^{\frac{17}{2}}}{17}+\frac{2(2abA+a^2B)x^{\frac{11}{2}}}{11}+\frac{2a^2Ax^{\frac{5}{2}}}{5}$	52
gosper	$\frac{2x^{\frac{5}{2}}(935b^2Bx^9+1265Ab^2x^6+2530Bx^6ab+3910aAbx^3+1955a^2Bx^3+4301a^2A)}{21505}$	56
trager	$\frac{2x^{\frac{5}{2}}(935b^2Bx^9+1265Ab^2x^6+2530Bx^6ab+3910aAbx^3+1955a^2Bx^3+4301a^2A)}{21505}$	56
risch	$\frac{2x^{\frac{5}{2}}(935b^2Bx^9+1265Ab^2x^6+2530Bx^6ab+3910aAbx^3+1955a^2Bx^3+4301a^2A)}{21505}$	56

input
```
int(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/23*b^2*B*x^(23/2)+2/17*(A*b^2+2*B*a*b)*x^(17/2)+2/11*(2*A*a*b+B*a^2)*x^(
11/2)+2/5*a^2*A*x^(5/2)
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{21505}\left(935\,Bb^2x^{11}+1265\left(2\,Bab+Ab^2\right)x^8+1955\left(Ba^2+2\,Aab\right)x^5+4301\,Aa^2x^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/21505*(935*B*b^2*x^11 + 1265*(2*B*a*b + A*b^2)*x^8 + 1955*(B*a^2 + 2*A*a
*b)*x^5 + 4301*A*a^2*x^2)*sqrt(x)
```

3.141.6 Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2Aa^2x^{\frac{5}{2}}}{5}+\frac{4Aabx^{\frac{11}{2}}}{11}+\frac{2Ab^2x^{\frac{17}{2}}}{17}+\frac{2Ba^2x^{\frac{11}{2}}}{11}+\frac{4Babx^{\frac{17}{2}}}{17}+\frac{2Bb^2x^{\frac{23}{2}}}{23}$$

input
```
integrate(x**(3/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

output
```
2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(11/2)/11 + 2*A*b**2*x**(17/2)/17 + 2*B*a
**2*x**(11/2)/11 + 4*B*a*b*x**(17/2)/17 + 2*B*b**2*x**(23/2)/23
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{23}\,Bb^2x^{\frac{23}{2}}+\frac{2}{17}\,\left(2\,Bab+Ab^2\right)x^{\frac{17}{2}}+\frac{2}{11}\,\left(Ba^2+2\,Aab\right)x^{\frac{11}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/23*B*b^2*x^(23/2) + 2/17*(2*B*a*b + A*b^2)*x^(17/2) + 2/11*(B*a^2 + 2*A*
a*b)*x^(11/2) + 2/5*A*a^2*x^(5/2)
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{23}\,Bb^2x^{\frac{23}{2}}+\frac{4}{17}\,Babx^{\frac{17}{2}}+\frac{2}{17}\,Ab^2x^{\frac{17}{2}}+\frac{2}{11}\,Ba^2x^{\frac{11}{2}}+\frac{4}{11}\,Aabx^{\frac{11}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

output
```
2/23*B*b^2*x^(23/2) + 4/17*B*a*b*x^(17/2) + 2/17*A*b^2*x^(17/2) + 2/11*B*a
^2*x^(11/2) + 4/11*A*a*b*x^(11/2) + 2/5*A*a^2*x^(5/2)
```

3.141.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=x^{11/2}\left(\frac{2\,B\,a^2}{11}+\frac{4\,A\,b\,a}{11}\right)+x^{17/2}\left(\frac{2\,A\,b^2}{17}+\frac{4\,B\,a\,b}{17}\right)+\frac{2\,A\,a^2\,x^{5/2}}{5}+\frac{2\,B\,b^2\,x^{23/2}}{23}$$

input
```
int(x^(3/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

output
```
x^(11/2)*((2*B*a^2)/11 + (4*A*a*b)/11) + x^(17/2)*((2*A*b^2)/17 + (4*B*a*b
)/17) + (2*A*a^2*x^(5/2))/5 + (2*B*b^2*x^(23/2))/23
```

3.142 $\int \sqrt{x}(a+bx^3)^2 (A+Bx^3)\,dx$

3.142.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \sqrt{x}(a+bx^3)^2 (A+Bx^3)\,dx = \frac{2}{3}a^2Ax^{3/2} + \frac{2}{9}a(2Ab+aB)x^{9/2} + \frac{2}{15}b(Ab+2aB)x^{15/2} + \frac{2}{21}b^2Bx^{21/2}$$

output
```
2/3*a^2*A*x^(3/2)+2/9*a*(2*A*b+B*a)*x^(9/2)+2/15*b*(A*b+2*B*a)*x^(15/2)+2/
21*b^2*B*x^(21/2)
```

3.142.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \sqrt{x}(a+bx^3)^2 (A+Bx^3)\,dx = \frac{2}{315}x^{3/2}\left(35a^2(3A+Bx^3) + 14abx^3(5A+3Bx^3) + 3b^2x^6(7A+5Bx^3)\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(2*x^(3/2)*(35*a^2*(3*A + B*x^3) + 14*a*b*x^3*(5*A + 3*B*x^3) + 3*b^2*x^6*
(7*A + 5*B*x^3)))/315
```

3.142.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^2A\sqrt{x}+bx^{13/2}(2aB+Ab)+ax^{7/2}(aB+2Ab)+b^2Bx^{19/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{21}b^2Bx^{21/2}$$

input
```
Int[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3),x]
```

output
```
(2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x
^(15/2))/15 + (2*b^2*B*x^(21/2))/21
```

3.142.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.142.4 Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2 b^{2} B \,x^{\frac{21}{2}}}{21}+\frac{2 \left(b^{2} A +2 a b B \right) x^{\frac{15}{2}}}{15}+\frac{2 \left(2 a b A +a^{2} B \right) x^{\frac{9}{2}}}{9}+\frac{2 a^{2} A \,x^{\frac{3}{2}}}{3}$	52
default	$\frac{2 b^{2} B \,x^{\frac{21}{2}}}{21}+\frac{2 \left(b^{2} A +2 a b B \right) x^{\frac{15}{2}}}{15}+\frac{2 \left(2 a b A +a^{2} B \right) x^{\frac{9}{2}}}{9}+\frac{2 a^{2} A \,x^{\frac{3}{2}}}{3}$	52
gosper	$\frac{2 x^{\frac{3}{2}} \left(15 b^{2} B \,x^{9}+21 A \,b^{2} x^{6}+42 B \,x^{6} a b +70 a A b \,x^{3}+35 a^{2} B \,x^{3}+105 a^{2} A \right)}{315}$	56
trager	$\frac{2 x^{\frac{3}{2}} \left(15 b^{2} B \,x^{9}+21 A \,b^{2} x^{6}+42 B \,x^{6} a b +70 a A b \,x^{3}+35 a^{2} B \,x^{3}+105 a^{2} A \right)}{315}$	56
risch	$\frac{2 x^{\frac{3}{2}} \left(15 b^{2} B \,x^{9}+21 A \,b^{2} x^{6}+42 B \,x^{6} a b +70 a A b \,x^{3}+35 a^{2} B \,x^{3}+105 a^{2} A \right)}{315}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*b^2*B*x^(21/2)+2/15*(A*b^2+2*B*a*b)*x^(15/2)+2/9*(2*A*a*b+B*a^2)*x^(9
/2)+2/3*a^2*A*x^(3/2)
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(a+bx^3)^2\left(A+Bx^3\right)\,dx$$
$$=\frac{2}{315}\left(15\,Bb^2x^{10}+21\left(2\,Bab+Ab^2\right)x^7+35\left(Ba^2+2\,Aab\right)x^4+105\,Aa^2x\right)\sqrt{x}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
```

output
```
2/315*(15*B*b^2*x^10 + 21*(2*B*a*b + A*b^2)*x^7 + 35*(B*a^2 + 2*A*a*b)*x^4
 + 105*A*a^2*x)*sqrt(x)
```

3.142.6 Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2Aa^2x^{\frac{3}{2}}}{3}+\frac{4Aabx^{\frac{9}{2}}}{9}+\frac{2Ab^2x^{\frac{15}{2}}}{15}+\frac{2Ba^2x^{\frac{9}{2}}}{9}+\frac{4Babx^{\frac{15}{2}}}{15}+\frac{2Bb^2x^{\frac{21}{2}}}{21}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)*x**(1/2),x)
```

output
```
2*A*a**2*x**(3/2)/3 + 4*A*a*b*x**(9/2)/9 + 2*A*b**2*x**(15/2)/15 + 2*B*a**
2*x**(9/2)/9 + 4*B*a*b*x**(15/2)/15 + 2*B*b**2*x**(21/2)/21
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{21}\,Bb^2x^{\frac{21}{2}}+\frac{2}{15}\,\left(2\,Bab+Ab^2\right)x^{\frac{15}{2}}+\frac{2}{9}\,\left(Ba^2+2\,Aab\right)x^{\frac{9}{2}}+\frac{2}{3}\,Aa^2x^{\frac{3}{2}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
```

output
```
2/21*B*b^2*x^(21/2) + 2/15*(2*B*a*b + A*b^2)*x^(15/2) + 2/9*(B*a^2 + 2*A*a
*b)*x^(9/2) + 2/3*A*a^2*x^(3/2)
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{21}\,Bb^2x^{\frac{21}{2}}+\frac{4}{15}\,Babx^{\frac{15}{2}}+\frac{2}{15}\,Ab^2x^{\frac{15}{2}}+\frac{2}{9}\,Ba^2x^{\frac{9}{2}}+\frac{4}{9}\,Aabx^{\frac{9}{2}}+\frac{2}{3}\,Aa^2x^{\frac{3}{2}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

output
```
2/21*B*b^2*x^(21/2) + 4/15*B*a*b*x^(15/2) + 2/15*A*b^2*x^(15/2) + 2/9*B*a^
2*x^(9/2) + 4/9*A*a*b*x^(9/2) + 2/3*A*a^2*x^(3/2)
```

3.142.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(a+bx^3)^2\left(A+Bx^3\right)\,dx = x^{9/2}\left(\frac{2\,B\,a^2}{9}+\frac{4\,A\,b\,a}{9}\right) + x^{15/2}\left(\frac{2\,A\,b^2}{15}+\frac{4\,B\,a\,b}{15}\right)+\frac{2\,A\,a^2\,x^{3/2}}{3}+\frac{2\,B\,b^2\,x^{21/2}}{21}$$

input
```
int(x^(1/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

output
```
x^(9/2)*((2*B*a^2)/9 + (4*A*a*b)/9) + x^(15/2)*((2*A*b^2)/15 + (4*B*a*b)/1
5) + (2*A*a^2*x^(3/2))/3 + (2*B*b^2*x^(21/2))/21
```

3.143 $\int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}}\,dx$

3.143.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}}\,dx = 2a^2A\sqrt{x}+\frac{2}{7}a(2Ab+aB)x^{7/2} + \frac{2}{13}b(Ab+2aB)x^{13/2}+\frac{2}{19}b^2Bx^{19/2}$$

output
```
2/7*a*(2*A*b+B*a)*x^(7/2)+2/13*b*(A*b+2*B*a)*x^(13/2)+2/19*b^2*B*x^(19/2)+
2*a^2*A*x^(1/2)
```

3.143.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}(247a^2(7A+Bx^3)+38abx^3(13A+7Bx^3)+7b^2x^6(19A+13Bx^3))}{1729}$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(247*a^2*(7*A + B*x^3) + 38*a*b*x^3*(13*A + 7*B*x^3) + 7*b^2*x^
6*(19*A + 13*B*x^3)))/1729
```

3.143.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}}dx$$

$$\Big\downarrow \textcolor{blue}{950}$$

$$\int \left(\frac{a^2A}{\sqrt{x}}+bx^{11/2}(2aB+Ab)+ax^{5/2}(aB+2Ab)+b^2Bx^{17/2}\right)dx$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$2a^2A\sqrt{x}+\frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x],x]
```

output
```
2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b + 2*a*B)*x^(13
/2))/13 + (2*b^2*B*x^(19/2))/19
```

3.143.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.143.4 Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2b^2 B\, x^{\frac{19}{2}}}{19}+\frac{2\left(b^2 A+2abB\right) x^{\frac{13}{2}}}{13}+\frac{2\left(2abA+a^2 B\right) x^{\frac{7}{2}}}{7}+2a^2 A\sqrt{x}$	52
default	$\frac{2b^2 B\, x^{\frac{19}{2}}}{19}+\frac{2\left(b^2 A+2abB\right) x^{\frac{13}{2}}}{13}+\frac{2\left(2abA+a^2 B\right) x^{\frac{7}{2}}}{7}+2a^2 A\sqrt{x}$	52
trager	$\left(\frac{2}{19}b^2 B\, x^9+\frac{2}{13}A\, b^2 x^6+\frac{4}{13}B\, x^6 ab+\frac{4}{7}aAb\, x^3+\frac{2}{7}a^2 B\, x^3+2a^2 A\right)\sqrt{x}$	55
gosper	$\frac{2\sqrt{x}\,\left(91b^2 B\, x^9+133A\, b^2 x^6+266B\, x^6 ab+494aAb\, x^3+247a^2 B\, x^3+1729a^2 A\right)}{1729}$	56
risch	$\frac{2\sqrt{x}\,\left(91b^2 B\, x^9+133A\, b^2 x^6+266B\, x^6 ab+494aAb\, x^3+247a^2 B\, x^3+1729a^2 A\right)}{1729}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/19*b^2*B*x^(19/2)+2/13*(A*b^2+2*B*a*b)*x^(13/2)+2/7*(2*A*a*b+B*a^2)*x^(7
/2)+2*a^2*A*x^(1/2)
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{\sqrt{x}}\,dx$$
$$=\frac{2}{1729}\left(91\,Bb^2x^9+133\left(2\,Bab+Ab^2\right)x^6+247\left(Ba^2+2\,Aab\right)x^3+1729\,Aa^2\right)\sqrt{x}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

output
```
2/1729*(91*B*b^2*x^9 + 133*(2*B*a*b + A*b^2)*x^6 + 247*(B*a^2 + 2*A*a*b)*x
^3 + 1729*A*a^2)*sqrt(x)
```

3.143.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}}\,dx = 2Aa^2\sqrt{x} + \frac{4Aabx^{\frac{7}{2}}}{7} + \frac{2Ab^2x^{\frac{13}{2}}}{13} + \frac{2Ba^2x^{\frac{7}{2}}}{7} + \frac{4Babx^{\frac{13}{2}}}{13} + \frac{2Bb^2x^{\frac{19}{2}}}{19}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**(1/2),x)
```

output
```
2*A*a**2*sqrt(x) + 4*A*a*b*x**(7/2)/7 + 2*A*b**2*x**(13/2)/13 + 2*B*a**2*x
**(7/2)/7 + 4*B*a*b*x**(13/2)/13 + 2*B*b**2*x**(19/2)/19
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{19}\,Bb^2x^{\frac{19}{2}} + \frac{2}{13}\,(2\,Bab + Ab^2)x^{\frac{13}{2}} + \frac{2}{7}\,(Ba^2 + 2\,Aab)x^{\frac{7}{2}} + 2\,Aa^2\sqrt{x}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

output
```
2/19*B*b^2*x^(19/2) + 2/13*(2*B*a*b + A*b^2)*x^(13/2) + 2/7*(B*a^2 + 2*A*a
*b)*x^(7/2) + 2*A*a^2*sqrt(x)
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{19}\,Bb^2x^{\frac{19}{2}} + \frac{4}{13}\,Babx^{\frac{13}{2}} + \frac{2}{13}\,Ab^2x^{\frac{13}{2}} + \frac{2}{7}\,Ba^2x^{\frac{7}{2}} + \frac{4}{7}\,Aabx^{\frac{7}{2}} + 2\,Aa^2\sqrt{x}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

output
```
2/19*B*b^2*x^(19/2) + 4/13*B*a*b*x^(13/2) + 2/13*A*b^2*x^(13/2) + 2/7*B*a^
2*x^(7/2) + 4/7*A*a*b*x^(7/2) + 2*A*a^2*sqrt(x)
```

3.143.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}}\,dx = x^{7/2}\left(\frac{2\,B\,a^2}{7}+\frac{4\,A\,b\,a}{7}\right) + x^{13/2}\left(\frac{2\,A\,b^2}{13}+\frac{4\,B\,a\,b}{13}\right) + 2\,A\,a^2\,\sqrt{x} + \frac{2\,B\,b^2\,x^{19/2}}{19}$$

input

```
int(((A + B*x^3)*(a + b*x^3)^2)/x^(1/2),x)
```

output

```
x^(7/2)*((2*B*a^2)/7 + (4*A*a*b)/7) + x^(13/2)*((2*A*b^2)/13 + (4*B*a*b)/1
3) + 2*A*a^2*x^(1/2) + (2*B*b^2*x^(19/2))/19
```

3.144 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx$

3.144.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2a^2A}{\sqrt{x}} + \frac{2}{5}a(2Ab+aB)x^{5/2} + \frac{2}{11}b(Ab+2aB)x^{11/2} + \frac{2}{17}b^2Bx^{17/2}$$

output
```
2/5*a*(2*A*b+B*a)*x^(5/2)+2/11*b*(A*b+2*B*a)*x^(11/2)+2/17*b^2*B*x^(17/2)-
2*a^2*A/x^(1/2)
```

3.144.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2(935a^2A-374aAbx^3-187a^2Bx^3-85Ab^2x^6-170abBx^6-55b^2Bx^9)}{935\sqrt{x}}$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(3/2),x]
```

output
```
(-2*(935*a^2*A - 374*a*A*b*x^3 - 187*a^2*B*x^3 - 85*A*b^2*x^6 - 170*a*b*B*
x^6 - 55*b^2*B*x^9))/(935*Sqrt[x])
```

3.144.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^2 \left(A+Bx^3\right)}{x^{3/2}}\,dx$$

$$\Big\downarrow \ 950$$

$$\int \left(\frac{a^2A}{x^{3/2}}+bx^{9/2}(2aB+Ab)+ax^{3/2}(aB+2Ab)+b^2Bx^{15/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

input

```
Int[((a + b*x^3)^2*(A + B*x^3))/x^(3/2),x]
```

output

```
(-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x^
(11/2))/11 + (2*b^2*B*x^(17/2))/17
```

3.144.3.1 Defintions of rubi rules used

rule 950

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.144.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2b^{2}B\,x^{\frac{17}{2}}}{17}+\frac{2A\,b^{2}x^{\frac{11}{2}}}{11}+\frac{4Bab\,x^{\frac{11}{2}}}{11}+\frac{4Aab\,x^{\frac{5}{2}}}{5}+\frac{2B\,a^{2}x^{\frac{5}{2}}}{5}-\frac{2a^{2}A}{\sqrt{x}}$	54
default	$\frac{2b^{2}B\,x^{\frac{17}{2}}}{17}+\frac{2A\,b^{2}x^{\frac{11}{2}}}{11}+\frac{4Bab\,x^{\frac{11}{2}}}{11}+\frac{4Aab\,x^{\frac{5}{2}}}{5}+\frac{2B\,a^{2}x^{\frac{5}{2}}}{5}-\frac{2a^{2}A}{\sqrt{x}}$	54
gosper	$-\frac{2\left(-55b^{2}B\,x^{9}-85A\,b^{2}x^{6}-170B\,x^{6}ab-374aAb\,x^{3}-187a^{2}B\,x^{3}+935a^{2}A\right)}{935\sqrt{x}}$	56
trager	$-\frac{2\left(-55b^{2}B\,x^{9}-85A\,b^{2}x^{6}-170B\,x^{6}ab-374aAb\,x^{3}-187a^{2}B\,x^{3}+935a^{2}A\right)}{935\sqrt{x}}$	56
risch	$-\frac{2\left(-55b^{2}B\,x^{9}-85A\,b^{2}x^{6}-170B\,x^{6}ab-374aAb\,x^{3}-187a^{2}B\,x^{3}+935a^{2}A\right)}{935\sqrt{x}}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/17*b^2*B*x^(17/2)+2/11*A*b^2*x^(11/2)+4/11*B*a*b*x^(11/2)+4/5*A*a*b*x^(5
/2)+2/5*B*a^2*x^(5/2)-2*a^2*A/x^(1/2)
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{3/2}}\,dx = \frac{2\left(55\,Bb^2x^9+85\,(2\,Bab+Ab^2)x^6+187\,(Ba^2+2\,Aab)x^3-935\,Aa^2\right)}{935\,\sqrt{x}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

output
```
2/935*(55*B*b^2*x^9 + 85*(2*B*a*b + A*b^2)*x^6 + 187*(B*a^2 + 2*A*a*b)*x^3
 - 935*A*a^2)/sqrt(x)
```

3.144.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2Aa^2}{\sqrt{x}} + \frac{4Aabx^{\frac{5}{2}}}{5} + \frac{2Ab^2x^{\frac{11}{2}}}{11} + \frac{2Ba^2x^{\frac{5}{2}}}{5} + \frac{4Babx^{\frac{11}{2}}}{11} + \frac{2Bb^2x^{\frac{17}{2}}}{17}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**(3/2),x)
```

output
```
-2*A*a**2/sqrt(x) + 4*A*a*b*x**(5/2)/5 + 2*A*b**2*x**(11/2)/11 + 2*B*a**2*
x**(5/2)/5 + 4*B*a*b*x**(11/2)/11 + 2*B*b**2*x**(17/2)/17
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx = \frac{2}{17}Bb^2x^{\frac{17}{2}} + \frac{2}{11}\left(2\,Bab + Ab^2\right)x^{\frac{11}{2}} + \frac{2}{5}\left(Ba^2 + 2\,Aab\right)x^{\frac{5}{2}} - \frac{2\,Aa^2}{\sqrt{x}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
```

output
```
2/17*B*b^2*x^(17/2) + 2/11*(2*B*a*b + A*b^2)*x^(11/2) + 2/5*(B*a^2 + 2*A*a
*b)*x^(5/2) - 2*A*a^2/sqrt(x)
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx = \frac{2}{17}Bb^2x^{\frac{17}{2}} + \frac{4}{11}Babx^{\frac{11}{2}} + \frac{2}{11}Ab^2x^{\frac{11}{2}} + \frac{2}{5}Ba^2x^{\frac{5}{2}} + \frac{4}{5}Aabx^{\frac{5}{2}} - \frac{2\,Aa^2}{\sqrt{x}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

output
```
2/17*B*b^2*x^(17/2) + 4/11*B*a*b*x^(11/2) + 2/11*A*b^2*x^(11/2) + 2/5*B*a^
2*x^(5/2) + 4/5*A*a*b*x^(5/2) - 2*A*a^2/sqrt(x)
```

3.144.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx = x^{5/2}\left(\frac{2\,B\,a^2}{5}+\frac{4\,A\,b\,a}{5}\right) + x^{11/2}\left(\frac{2\,A\,b^2}{11}+\frac{4\,B\,a\,b}{11}\right) - \frac{2\,A\,a^2}{\sqrt{x}} + \frac{2\,B\,b^2\,x^{17/2}}{17}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^(3/2),x)
```

output
```
x^(5/2)*((2*B*a^2)/5 + (4*A*a*b)/5) + x^(11/2)*((2*A*b^2)/11 + (4*B*a*b)/1
1) - (2*A*a^2)/x^(1/2) + (2*B*b^2*x^(17/2))/17
```

3.145 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}}\,dx$

3.145.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}}\,dx = -\frac{2a^2A}{3x^{3/2}}+\frac{2}{3}a(2Ab+aB)x^{3/2}+\frac{2}{9}b(Ab+2aB)x^{9/2}+\frac{2}{15}b^2Bx^{15/2}$$

output
```
-2/3*a^2*A/x^(3/2)+2/3*a*(2*A*b+B*a)*x^(3/2)+2/9*b*(A*b+2*B*a)*x^(9/2)+2/1
5*b^2*B*x^(15/2)
```

3.145.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}}\,dx = \frac{2(-15a^2A+30aAbx^3+15a^2Bx^3+5Ab^2x^6+10abBx^6+3b^2Bx^9)}{45x^{3/2}}$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]
```

output
```
(2*(-15*a^2*A + 30*a*A*b*x^3 + 15*a^2*B*x^3 + 5*A*b^2*x^6 + 10*a*b*B*x^6 +
 3*b^2*B*x^9))/(45*x^(3/2))
```

3.145.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2 A}{x^{5/2}}+bx^{7/2}(2aB+Ab)+a\sqrt{x}(aB+2Ab)+b^2Bx^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]
```

output
```
(-2*a^2*A)/(3*x^(3/2)) + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B
)*x^(9/2))/9 + (2*b^2*B*x^(15/2))/15
```

3.145.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.145.4 Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2b^{2}B\,x^{\frac{15}{2}}}{15}+\frac{2A\,b^{2}x^{\frac{9}{2}}}{9}+\frac{4Bab\,x^{\frac{9}{2}}}{9}+\frac{4Aab\,x^{\frac{3}{2}}}{3}+\frac{2B\,a^{2}x^{\frac{3}{2}}}{3}-\frac{2a^{2}A}{3x^{\frac{3}{2}}}$	54
default	$\frac{2b^{2}B\,x^{\frac{15}{2}}}{15}+\frac{2A\,b^{2}x^{\frac{9}{2}}}{9}+\frac{4Bab\,x^{\frac{9}{2}}}{9}+\frac{4Aab\,x^{\frac{3}{2}}}{3}+\frac{2B\,a^{2}x^{\frac{3}{2}}}{3}-\frac{2a^{2}A}{3x^{\frac{3}{2}}}$	54
gosper	$-\frac{2\left(-3b^{2}B\,x^{9}-5A\,b^{2}x^{6}-10B\,x^{6}ab-30aAb\,x^{3}-15a^{2}B\,x^{3}+15a^{2}A\right)}{45x^{\frac{3}{2}}}$	56
trager	$-\frac{2\left(-3b^{2}B\,x^{9}-5A\,b^{2}x^{6}-10B\,x^{6}ab-30aAb\,x^{3}-15a^{2}B\,x^{3}+15a^{2}A\right)}{45x^{\frac{3}{2}}}$	56
risch	$-\frac{2\left(-3b^{2}B\,x^{9}-5A\,b^{2}x^{6}-10B\,x^{6}ab-30aAb\,x^{3}-15a^{2}B\,x^{3}+15a^{2}A\right)}{45x^{\frac{3}{2}}}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*b^2*B*x^(15/2)+2/9*A*b^2*x^(9/2)+4/9*B*a*b*x^(9/2)+4/3*A*a*b*x^(3/2)+
2/3*B*a^2*x^(3/2)-2/3*a^2*A/x^(3/2)
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2\,(A+Bx^3)}{x^{5/2}}\,dx = \frac{2\,(3\,Bb^2x^9+5\,(2\,Bab+Ab^2)x^6+15\,(Ba^2+2\,Aab)x^3-15\,Aa^2)}{45\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

output
```
2/45*(3*B*b^2*x^9 + 5*(2*B*a*b + A*b^2)*x^6 + 15*(B*a^2 + 2*A*a*b)*x^3 - 1
5*A*a^2)/x^(3/2)
```

3.145.6 Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}}\,dx = -\frac{2Aa^2}{3x^{\frac{3}{2}}} + \frac{4Aabx^{\frac{3}{2}}}{3} + \frac{2Ab^2x^{\frac{9}{2}}}{9} + \frac{2Ba^2x^{\frac{3}{2}}}{3} + \frac{4Babx^{\frac{9}{2}}}{9} + \frac{2Bb^2x^{\frac{15}{2}}}{15}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**(5/2),x)
```

output
```
-2*A*a**2/(3*x**(3/2)) + 4*A*a*b*x**(3/2)/3 + 2*A*b**2*x**(9/2)/9 + 2*B*a*
*2*x**(3/2)/3 + 4*B*a*b*x**(9/2)/9 + 2*B*b**2*x**(15/2)/15
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}}\,dx = \frac{2}{15}\,Bb^2x^{\frac{15}{2}} + \frac{2}{9}\,(2\,Bab + Ab^2)x^{\frac{9}{2}} + \frac{2}{3}\,(Ba^2 + 2\,Aab)x^{\frac{3}{2}} - \frac{2\,Aa^2}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

output
```
2/15*B*b^2*x^(15/2) + 2/9*(2*B*a*b + A*b^2)*x^(9/2) + 2/3*(B*a^2 + 2*A*a*b
)*x^(3/2) - 2/3*A*a^2/x^(3/2)
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}}\,dx = \frac{2}{15}\,Bb^2x^{\frac{15}{2}} + \frac{4}{9}\,Babx^{\frac{9}{2}} + \frac{2}{9}\,Ab^2x^{\frac{9}{2}} + \frac{2}{3}\,Ba^2x^{\frac{3}{2}} + \frac{4}{3}\,Aabx^{\frac{3}{2}} - \frac{2\,Aa^2}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

output
```
2/15*B*b^2*x^(15/2) + 4/9*B*a*b*x^(9/2) + 2/9*A*b^2*x^(9/2) + 2/3*B*a^2*x^
(3/2) + 4/3*A*a*b*x^(3/2) - 2/3*A*a^2/x^(3/2)
```

3.145.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^2\,(A+Bx^3)}{x^{5/2}}\,dx = x^{3/2}\left(\frac{2\,B\,a^2}{3}+\frac{4\,A\,b\,a}{3}\right) + x^{9/2}\left(\frac{2\,A\,b^2}{9}+\frac{4\,B\,a\,b}{9}\right) - \frac{2\,A\,a^2}{3\,x^{3/2}} + \frac{2\,B\,b^2\,x^{15/2}}{15}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^(5/2),x)
```

output
```
x^(3/2)*((2*B*a^2)/3 + (4*A*a*b)/3) + x^(9/2)*((2*A*b^2)/9 + (4*B*a*b)/9)
- (2*A*a^2)/(3*x^(3/2)) + (2*B*b^2*x^(15/2))/15
```

3.146 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}}\,dx$

3.146.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}}\,dx=-\frac{2a^2A}{5x^{5/2}}+2a(2Ab+aB)\sqrt{x}+\frac{2}{7}b(Ab+2aB)x^{7/2}+\frac{2}{13}b^2Bx^{13/2}$$

output
```
-2/5*a^2*A/x^(5/2)+2/7*b*(A*b+2*B*a)*x^(7/2)+2/13*b^2*B*x^(13/2)+2*a*(2*A*
b+B*a)*x^(1/2)
```

3.146.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}}\,dx = -\frac{2(91a^2A-910aAbx^3-455a^2Bx^3-65Ab^2x^6-130abBx^6-35b^2Bx^9)}{455x^{5/2}}$$

input
```
Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(7/2),x]
```

output
```
(-2*(91*a^2*A - 910*a*A*b*x^3 - 455*a^2*B*x^3 - 65*A*b^2*x^6 - 130*a*b*B*x
^6 - 35*b^2*B*x^9))/(455*x^(5/2))
```

3.146.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}} \, dx$$

$$\Big\downarrow \; 950$$

$$\int \left(\frac{a^2 A}{x^{7/2}}+bx^{5/2}(2aB+Ab)+\frac{a(aB+2Ab)}{\sqrt{x}}+b^2 Bx^{11/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+2a\sqrt{x}(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

input
```
Int[((a + b*x^3)^2*(A + B*x^3))/x^(7/2),x]
```

output
```
(-2*a^2*A)/(5*x^(5/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^
(7/2))/7 + (2*b^2*B*x^(13/2))/13
```

3.146.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.146.4 Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2b^{2}B\,x^{\frac{13}{2}}}{13}+\frac{2A\,b^{2}x^{\frac{7}{2}}}{7}+\frac{4Bab\,x^{\frac{7}{2}}}{7}+4Aab\sqrt{x}+2B\,a^{2}\sqrt{x}-\frac{2a^{2}A}{5x^{\frac{5}{2}}}$	54
default	$\frac{2b^{2}B\,x^{\frac{13}{2}}}{13}+\frac{2A\,b^{2}x^{\frac{7}{2}}}{7}+\frac{4Bab\,x^{\frac{7}{2}}}{7}+4Aab\sqrt{x}+2B\,a^{2}\sqrt{x}-\frac{2a^{2}A}{5x^{\frac{5}{2}}}$	54
gosper	$-\frac{2\left(-35b^{2}B\,x^{9}-65A\,b^{2}x^{6}-130B\,x^{6}ab-910aAb\,x^{3}-455a^{2}B\,x^{3}+91a^{2}A\right)}{455x^{\frac{5}{2}}}$	56
trager	$-\frac{2\left(-35b^{2}B\,x^{9}-65A\,b^{2}x^{6}-130B\,x^{6}ab-910aAb\,x^{3}-455a^{2}B\,x^{3}+91a^{2}A\right)}{455x^{\frac{5}{2}}}$	56
risch	$-\frac{2\left(-35b^{2}B\,x^{9}-65A\,b^{2}x^{6}-130B\,x^{6}ab-910aAb\,x^{3}-455a^{2}B\,x^{3}+91a^{2}A\right)}{455x^{\frac{5}{2}}}$	56

input
```
int((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/13*b^2*B*x^(13/2)+2/7*A*b^2*x^(7/2)+4/7*B*a*b*x^(7/2)+4*A*a*b*x^(1/2)+2*
B*a^2*x^(1/2)-2/5*a^2*A/x^(5/2)
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2\,(A+Bx^3)}{x^{7/2}}\,dx=\frac{2\,(35\,Bb^2x^9+65\,(2\,Bab+Ab^2)x^6+455\,(Ba^2+2\,Aab)x^3-91\,Aa^2)}{455\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

output
```
2/455*(35*B*b^2*x^9 + 65*(2*B*a*b + A*b^2)*x^6 + 455*(B*a^2 + 2*A*a*b)*x^3
 - 91*A*a^2)/x^(5/2)
```

3.146.6 Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}}\,dx = -\frac{2Aa^2}{5x^{\frac{5}{2}}}+4Aab\sqrt{x}+\frac{2Ab^2x^{\frac{7}{2}}}{7}+2Ba^2\sqrt{x}+\frac{4Babx^{\frac{7}{2}}}{7}+\frac{2Bb^2x^{\frac{13}{2}}}{13}$$

input
```
integrate((b*x**3+a)**2*(B*x**3+A)/x**(7/2),x)
```

output
```
-2*A*a**2/(5*x**(5/2)) + 4*A*a*b*sqrt(x) + 2*A*b**2*x**(7/2)/7 + 2*B*a**2*
sqrt(x) + 4*B*a*b*x**(7/2)/7 + 2*B*b**2*x**(13/2)/13
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{13}\,Bb^2x^{\frac{13}{2}}+\frac{2}{7}\left(2\,Bab+Ab^2\right)x^{\frac{7}{2}}+2\left(Ba^2+2\,Aab\right)\sqrt{x}-\frac{2\,Aa^2}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

output
```
2/13*B*b^2*x^(13/2) + 2/7*(2*B*a*b + A*b^2)*x^(7/2) + 2*(B*a^2 + 2*A*a*b)*
sqrt(x) - 2/5*A*a^2/x^(5/2)
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{13}\,Bb^2x^{\frac{13}{2}}+\frac{4}{7}\,Babx^{\frac{7}{2}}+\frac{2}{7}\,Ab^2x^{\frac{7}{2}}+2\,Ba^2\sqrt{x}+4\,Aab\sqrt{x}-\frac{2\,Aa^2}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

output
```
2/13*B*b^2*x^(13/2) + 4/7*B*a*b*x^(7/2) + 2/7*A*b^2*x^(7/2) + 2*B*a^2*sqrt
(x) + 4*A*a*b*sqrt(x) - 2/5*A*a^2/x^(5/2)
```

3.146.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}}\,dx = \sqrt{x}\,\left(2\,B\,a^2+4\,A\,b\,a\right) + x^{7/2}\left(\frac{2\,A\,b^2}{7}+\frac{4\,B\,a\,b}{7}\right)-\frac{2\,A\,a^2}{5\,x^{5/2}}+\frac{2\,B\,b^2\,x^{13/2}}{13}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^2)/x^(7/2),x)
```

output
```
x^(1/2)*(2*B*a^2 + 4*A*a*b) + x^(7/2)*((2*A*b^2)/7 + (4*B*a*b)/7) - (2*A*a
^2)/(5*x^(5/2)) + (2*B*b^2*x^(13/2))/13
```

3.147 $\int x^{7/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx$

3.147.1 Optimal result

Integrand size = 22, antiderivative size = 85

$$\int x^{7/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx=\frac{2}{9}a^3Ax^{9/2}+\frac{2}{15}a^2(3Ab+aB)x^{15/2}+\frac{2}{7}ab(Ab+aB)x^{21/2}+\frac{2}{27}b^2(Ab+3aB)x^{27/2}+\frac{2}{33}b^3Bx^{33/2}$$

output
```
2/9*a^3*A*x^(9/2)+2/15*a^2*(3*A*b+B*a)*x^(15/2)+2/7*a*b*(A*b+B*a)*x^(21/2)
+2/27*b^2*(A*b+3*B*a)*x^(27/2)+2/33*b^3*B*x^(33/2)
```

3.147.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int x^{7/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx=\frac{2x^{9/2}(231a^3(5A+3Bx^3)+297a^2bx^3(7A+5Bx^3)+165ab^2x^6(9A+7Bx^3)+35b^3x^9(11A+9}{10395}$$

input
```
Integrate[x^(7/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

output
```
(2*x^(9/2)*(231*a^3*(5*A + 3*B*x^3) + 297*a^2*b*x^3*(7*A + 5*B*x^3) + 165*
a*b^2*x^6*(9*A + 7*B*x^3) + 35*b^3*x^9*(11*A + 9*B*x^3)))/10395
```

3.147.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^3Ax^{7/2}+a^2x^{13/2}(aB+3Ab)+b^2x^{25/2}(3aB+Ab)+3abx^{19/2}(aB+Ab)+b^3Bx^{31/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2}{15}a^2x^{15/2}(aB+3Ab)+\frac{2}{27}b^2x^{27/2}(3aB+Ab)+\frac{2}{7}abx^{21/2}(aB+Ab)+\frac{2}{33}b^3Bx^{33/2}$$

input
```
Int[x^(7/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

output
```
(2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(15/2))/15 + (2*a*b*(A*b + a*
B)*x^(21/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(27/2))/27 + (2*b^3*B*x^(33/2))/33
```

3.147.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.147.4 Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result	si
derivativedivides	$\frac{2b^{3}B\,x^{\frac{33}{2}}}{33}+\frac{2(b^{3}A+3a\,b^{2}B)x^{\frac{27}{2}}}{27}+\frac{2(3a\,b^{2}A+3a^{2}bB)x^{\frac{21}{2}}}{21}+\frac{2(3a^{2}bA+a^{3}B)x^{\frac{15}{2}}}{15}+\frac{2a^{3}A\,x^{\frac{9}{2}}}{9}$	76
default	$\frac{2b^{3}B\,x^{\frac{33}{2}}}{33}+\frac{2(b^{3}A+3a\,b^{2}B)x^{\frac{27}{2}}}{27}+\frac{2(3a\,b^{2}A+3a^{2}bB)x^{\frac{21}{2}}}{21}+\frac{2(3a^{2}bA+a^{3}B)x^{\frac{15}{2}}}{15}+\frac{2a^{3}A\,x^{\frac{9}{2}}}{9}$	76
gosper	$\frac{2x^{\frac{9}{2}}(315B\,b^{3}x^{12}+385A\,x^{9}b^{3}+1155B\,x^{9}a\,b^{2}+1485A\,x^{6}a\,b^{2}+1485B\,x^{6}a^{2}b+2079A\,x^{3}a^{2}b+693a^{3}B\,x^{3}+1155a^{3}A)}{10395}$	80
trager	$\frac{2x^{\frac{9}{2}}(315B\,b^{3}x^{12}+385A\,x^{9}b^{3}+1155B\,x^{9}a\,b^{2}+1485A\,x^{6}a\,b^{2}+1485B\,x^{6}a^{2}b+2079A\,x^{3}a^{2}b+693a^{3}B\,x^{3}+1155a^{3}A)}{10395}$	80
risch	$\frac{2x^{\frac{9}{2}}(315B\,b^{3}x^{12}+385A\,x^{9}b^{3}+1155B\,x^{9}a\,b^{2}+1485A\,x^{6}a\,b^{2}+1485B\,x^{6}a^{2}b+2079A\,x^{3}a^{2}b+693a^{3}B\,x^{3}+1155a^{3}A)}{10395}$	80

input
```
int(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/33*b^3*B*x^(33/2)+2/27*(A*b^3+3*B*a*b^2)*x^(27/2)+2/21*(3*A*a*b^2+3*B*a^
2*b)*x^(21/2)+2/15*(3*A*a^2*b+B*a^3)*x^(15/2)+2/9*a^3*A*x^(9/2)
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{7/2}(a+bx^{3})^{3}\,(A+Bx^{3})\;dx=\frac{2}{10395}\left(315\,Bb^{3}x^{16}+385\left(3\,Bab^{2}+Ab^{3}\right)x^{13}+1485\left(Ba^{2}b+Aab^{2}\right)x^{10}+1155\,Aa^{3}x^{4}+693\,(B\right.$$

input
```
integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/10395*(315*B*b^3*x^16 + 385*(3*B*a*b^2 + A*b^3)*x^13 + 1485*(B*a^2*b + A
*a*b^2)*x^10 + 1155*A*a^3*x^4 + 693*(B*a^3 + 3*A*a^2*b)*x^7)*sqrt(x)
```

3.147.6 Sympy [A] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{7/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2Aa^3x^{\frac{9}{2}}}{9}+\frac{2Aa^2bx^{\frac{15}{2}}}{5}+\frac{2Aab^2x^{\frac{21}{2}}}{7}+\frac{2Ab^3x^{\frac{27}{2}}}{27}+\frac{2Ba^3x^{\frac{15}{2}}}{15}+\frac{2Ba^2bx^{\frac{21}{2}}}{7}+\frac{2Bab^2x^{\frac{27}{2}}}{9}+\frac{2Bb^3x^{\frac{33}{2}}}{33}$$

input
```
integrate(x**(7/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

output
```
2*A*a**3*x**(9/2)/9 + 2*A*a**2*b*x**(15/2)/5 + 2*A*a*b**2*x**(21/2)/7 + 2*
A*b**3*x**(27/2)/27 + 2*B*a**3*x**(15/2)/15 + 2*B*a**2*b*x**(21/2)/7 + 2*B
*a*b**2*x**(27/2)/9 + 2*B*b**3*x**(33/2)/33
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{7/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{33}\,Bb^3x^{\frac{33}{2}}+\frac{2}{27}\left(3\,Bab^2+Ab^3\right)x^{\frac{27}{2}}+\frac{2}{7}\left(Ba^2b+Aab^2\right)x^{\frac{21}{2}}+\frac{2}{9}\,Aa^3x^{\frac{9}{2}}+\frac{2}{15}\left(Ba^3+3\,Aa^2b\right)x^{\frac{15}{2}}$$

input
```
integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/33*B*b^3*x^(33/2) + 2/27*(3*B*a*b^2 + A*b^3)*x^(27/2) + 2/7*(B*a^2*b + A
*a*b^2)*x^(21/2) + 2/9*A*a^3*x^(9/2) + 2/15*(B*a^3 + 3*A*a^2*b)*x^(15/2)
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{7/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{33}\,Bb^3x^{\frac{33}{2}}+\frac{2}{9}\,Bab^2x^{\frac{27}{2}}+\frac{2}{27}\,Ab^3x^{\frac{27}{2}}+\frac{2}{7}\,Ba^2bx^{\frac{21}{2}}+\frac{2}{7}\,Aab^2x^{\frac{21}{2}}+\frac{2}{15}\,Ba^3x^{\frac{15}{2}}+\frac{2}{5}\,Aa^2bx^{\frac{15}{2}}+\frac{2}{9}\,Aa^3x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

output
```
2/33*B*b^3*x^(33/2) + 2/9*B*a*b^2*x^(27/2) + 2/27*A*b^3*x^(27/2) + 2/7*B*a
^2*b*x^(21/2) + 2/7*A*a*b^2*x^(21/2) + 2/15*B*a^3*x^(15/2) + 2/5*A*a^2*b*x
^(15/2) + 2/9*A*a^3*x^(9/2)
```

3.147.9 Mupad [B] (verification not implemented)

Time = 6.94 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{7/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=x^{15/2}\left(\frac{2\,B\,a^3}{15}+\frac{2\,A\,b\,a^2}{5}\right)+x^{27/2}\left(\frac{2\,A\,b^3}{27}+\frac{2\,B\,a\,b^2}{9}\right)+\frac{2\,A\,a^3\,x^{9/2}}{9}+\frac{2\,B\,b^3\,x^{33/2}}{33}+\frac{2\,a\,b\,x^{21/2}\,(A\,b+B\,a)}{7}$$

input
```
int(x^(7/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

output
```
x^(15/2)*((2*B*a^3)/15 + (2*A*a^2*b)/5) + x^(27/2)*((2*A*b^3)/27 + (2*B*a*
b^2)/9) + (2*A*a^3*x^(9/2))/9 + (2*B*b^3*x^(33/2))/33 + (2*a*b*x^(21/2)*(A
*b + B*a))/7
```

3.148 $\int x^{5/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx$

3.148.1 Optimal result

Integrand size = 22, antiderivative size = 85

$$\int x^{5/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{7}a^3Ax^{7/2}+\frac{2}{13}a^2(3Ab+aB)x^{13/2}+\frac{6}{19}ab(Ab+aB)x^{19/2}+\frac{2}{25}b^2(Ab+3aB)x^{25/2}+\frac{2}{31}b^3Bx^{31/2}$$

output

```
2/7*a^3*A*x^(7/2)+2/13*a^2*(3*A*b+B*a)*x^(13/2)+6/19*a*b*(A*b+B*a)*x^(19/2
)+2/25*b^2*(A*b+3*B*a)*x^(25/2)+2/31*b^3*B*x^(31/2)
```

3.148.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07

$$\int x^{5/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{91}a^3x^{7/2}\left(13A+7Bx^3\right)+\frac{6}{247}a^2bx^{13/2}\left(19A+13Bx^3\right)+\frac{6}{475}ab^2x^{19/2}\left(25A+19Bx^3\right)+\frac{2}{775}b^3x^{25/2}\left(31A+25Bx^3\right)$$

input

```
Integrate[x^(5/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

output

```
(2*a^3*x^(7/2)*(13*A + 7*B*x^3))/91 + (6*a^2*b*x^(13/2)*(19*A + 13*B*x^3))
/247 + (6*a*b^2*x^(19/2)*(25*A + 19*B*x^3))/475 + (2*b^3*x^(25/2)*(31*A +
25*B*x^3))/775
```

3.148.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^3Ax^{5/2}+a^2x^{11/2}(aB+3Ab)+b^2x^{23/2}(3aB+Ab)+3abx^{17/2}(aB+Ab)+b^3Bx^{29/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{25}b^2x^{25/2}(3aB+Ab)+\frac{6}{19}abx^{19/2}(aB+Ab)+\frac{2}{31}b^3Bx^{31/2}$$

input
```
Int[x^(5/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

output
```
(2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*
B)*x^(19/2))/19 + (2*b^2*(A*b + 3*a*B)*x^(25/2))/25 + (2*b^3*B*x^(31/2))/3
1
```

3.148.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.148.4 Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2b^{3} B \,x^{\frac{31}{2}}}{31}+\frac{2\left(b^{3} A +3 a \,b^{2} B \right) x^{\frac{25}{2}}}{25}+\frac{2\left(3 a \,b^{2} A +3 a^{2} b B \right) x^{\frac{19}{2}}}{19}+\frac{2\left(3 a^{2} b A +a^{3} B \right) x^{\frac{13}{2}}}{13}+\frac{2 a^{3} A \,x^{\frac{7}{2}}}{7}$
default	$\frac{2b^{3} B \,x^{\frac{31}{2}}}{31}+\frac{2\left(b^{3} A +3 a \,b^{2} B \right) x^{\frac{25}{2}}}{25}+\frac{2\left(3 a \,b^{2} A +3 a^{2} b B \right) x^{\frac{19}{2}}}{19}+\frac{2\left(3 a^{2} b A +a^{3} B \right) x^{\frac{13}{2}}}{13}+\frac{2 a^{3} A \,x^{\frac{7}{2}}}{7}$
gosper	$\frac{2 x^{\frac{7}{2}} (43225 B \,b^{3} x^{12}+53599 A \,x^{9} b^{3}+160797 B \,x^{9} a \,b^{2}+211575 A \,x^{6} a \,b^{2}+211575 B \,x^{6} a^{2} b +309225 A \,x^{3} a^{2} b +103075 a^{3} B \,x^{3}}{1339975}$
trager	$\frac{2 x^{\frac{7}{2}} (43225 B \,b^{3} x^{12}+53599 A \,x^{9} b^{3}+160797 B \,x^{9} a \,b^{2}+211575 A \,x^{6} a \,b^{2}+211575 B \,x^{6} a^{2} b +309225 A \,x^{3} a^{2} b +103075 a^{3} B \,x^{3}}{1339975}$
risch	$\frac{2 x^{\frac{7}{2}} (43225 B \,b^{3} x^{12}+53599 A \,x^{9} b^{3}+160797 B \,x^{9} a \,b^{2}+211575 A \,x^{6} a \,b^{2}+211575 B \,x^{6} a^{2} b +309225 A \,x^{3} a^{2} b +103075 a^{3} B \,x^{3}}{1339975}$

input
```
int(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/31*b^3*B*x^(31/2)+2/25*(A*b^3+3*B*a*b^2)*x^(25/2)+2/19*(3*A*a*b^2+3*B*a^
2*b)*x^(19/2)+2/13*(3*A*a^2*b+B*a^3)*x^(13/2)+2/7*a^3*A*x^(7/2)
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{5/2}\left(a+bx^3\right)^3 (A+Bx^3)\ dx = \frac{2}{1339975}\,(43225\,Bb^3x^{15}+53599\left(3\,Bab^2+Ab^3\right)x^{12}+211575\left(Ba^2b+Aab^2\right)x^9+191425\,Aa^3x$$

input
```
integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/1339975*(43225*B*b^3*x^15 + 53599*(3*B*a*b^2 + A*b^3)*x^12 + 211575*(B*a
^2*b + A*a*b^2)*x^9 + 191425*A*a^3*x^3 + 103075*(B*a^3 + 3*A*a^2*b)*x^6)*s
qrt(x)
```

3.148.6 Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{5/2}(a+bx^3)^3\,(A+Bx^3)\,dx = \frac{2Aa^3x^{\frac{7}{2}}}{7} + \frac{6Aa^2bx^{\frac{13}{2}}}{13} + \frac{6Aab^2x^{\frac{19}{2}}}{19} + \frac{2Ab^3x^{\frac{25}{2}}}{25} + \frac{2Ba^3x^{\frac{13}{2}}}{13} + \frac{6Ba^2bx^{\frac{19}{2}}}{19} + \frac{6Bab^2x^{\frac{25}{2}}}{25} + \frac{2Bb^3x^{\frac{31}{2}}}{31}$$

input
```
integrate(x**(5/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

output
```
2*A*a**3*x**(7/2)/7 + 6*A*a**2*b*x**(13/2)/13 + 6*A*a*b**2*x**(19/2)/19 +
2*A*b**3*x**(25/2)/25 + 2*B*a**3*x**(13/2)/13 + 6*B*a**2*b*x**(19/2)/19 +
6*B*a*b**2*x**(25/2)/25 + 2*B*b**3*x**(31/2)/31
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{5/2}(a+bx^3)^3\,(A+Bx^3)\,dx = \frac{2}{31}\,Bb^3x^{\frac{31}{2}} + \frac{2}{25}\,(3\,Bab^2 + Ab^3)x^{\frac{25}{2}} + \frac{6}{19}\,(Ba^2b + Aab^2)x^{\frac{19}{2}} + \frac{2}{7}\,Aa^3x^{\frac{7}{2}} + \frac{2}{13}\,(Ba^3 + 3\,Aa^2b)x^{\frac{13}{2}}$$

input
```
integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/31*B*b^3*x^(31/2) + 2/25*(3*B*a*b^2 + A*b^3)*x^(25/2) + 6/19*(B*a^2*b +
A*a*b^2)*x^(19/2) + 2/7*A*a^3*x^(7/2) + 2/13*(B*a^3 + 3*A*a^2*b)*x^(13/2)
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{5/2}(a+bx^3)^3\,(A+Bx^3)\,dx = \frac{2}{31}\,Bb^3x^{\frac{31}{2}} + \frac{6}{25}\,Bab^2x^{\frac{25}{2}} + \frac{2}{25}\,Ab^3x^{\frac{25}{2}} + \frac{6}{19}\,Ba^2bx^{\frac{19}{2}} + \frac{6}{19}\,Aab^2x^{\frac{19}{2}} + \frac{2}{13}\,Ba^3x^{\frac{13}{2}} + \frac{6}{13}\,Aa^2bx^{\frac{13}{2}} + \frac{2}{7}\,Aa^3x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

output
```
2/31*B*b^3*x^(31/2) + 6/25*B*a*b^2*x^(25/2) + 2/25*A*b^3*x^(25/2) + 6/19*B
*a^2*b*x^(19/2) + 6/19*A*a*b^2*x^(19/2) + 2/13*B*a^3*x^(13/2) + 6/13*A*a^2
*b*x^(13/2) + 2/7*A*a^3*x^(7/2)
```

3.148.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{5/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx=x^{13/2}\left(\frac{2\,B\,a^3}{13}+\frac{6\,A\,b\,a^2}{13}\right)+x^{25/2}\left(\frac{2\,A\,b^3}{25}+\frac{6\,B\,a\,b^2}{25}\right)+\frac{2\,A\,a^3\,x^{7/2}}{7}+\frac{2\,B\,b^3\,x^{31/2}}{31}+\frac{6\,a\,b\,x^{19/2}\,\left(A\,b+B\,a\right)}{19}$$

input
```
int(x^(5/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

output
```
x^(13/2)*((2*B*a^3)/13 + (6*A*a^2*b)/13) + x^(25/2)*((2*A*b^3)/25 + (6*B*a
*b^2)/25) + (2*A*a^3*x^(7/2))/7 + (2*B*b^3*x^(31/2))/31 + (6*a*b*x^(19/2)*
(A*b + B*a))/19
```

3.149 $\int x^{3/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx$

3.149.1 Optimal result

Integrand size = 22, antiderivative size = 85

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{5}a^3Ax^{5/2}+\frac{2}{11}a^2(3Ab+aB)x^{11/2}+\frac{6}{17}ab(Ab+aB)x^{17/2}+\frac{2}{23}b^2(Ab+3aB)x^{23/2}+\frac{2}{29}b^3Bx^{29/2}$$

output
```
2/5*a^3*A*x^(5/2)+2/11*a^2*(3*A*b+B*a)*x^(11/2)+6/17*a*b*(A*b+B*a)*x^(17/2
)+2/23*b^2*(A*b+3*B*a)*x^(23/2)+2/29*b^3*B*x^(29/2)
```

3.149.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2x^{5/2}(11339a^3(11A+5Bx^3)+10005a^2bx^3(17A+11Bx^3)+4785ab^2x^6(23A+17Bx^3)+935b}{623645}$$

input
```
Integrate[x^(3/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

output
```
(2*x^(5/2)*(11339*a^3*(11*A + 5*B*x^3) + 10005*a^2*b*x^3*(17*A + 11*B*x^3)
 + 4785*a*b^2*x^6*(23*A + 17*B*x^3) + 935*b^3*x^9*(29*A + 23*B*x^3)))/6236
45
```

3.149.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2} \left(a+bx^3\right)^3 \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow 950$$

$$\int \left(a^3Ax^{3/2}+a^2x^{9/2}(aB+3Ab)+b^2x^{21/2}(3aB+Ab)+3abx^{15/2}(aB+Ab)+b^3Bx^{27/2}\right)dx$$

$$\Big\downarrow 2009$$

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{23}b^2x^{23/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{29}b^3Bx^{29/2}$$

input
```
Int[x^(3/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

output
```
(2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (6*a*b*(A*b + a*
B)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(23/2))/23 + (2*b^3*B*x^(29/2))/2
9
```

3.149.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.149.4 Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2b^{3}B\,x^{\frac{29}{2}}}{29}+\frac{2\left(b^{3}A+3a\,b^{2}B\right)x^{\frac{23}{2}}}{23}+\frac{2\left(3a\,b^{2}A+3a^{2}bB\right)x^{\frac{17}{2}}}{17}+\frac{2\left(3a^{2}bA+a^{3}B\right)x^{\frac{11}{2}}}{11}+\frac{2a^{3}A\,x^{\frac{5}{2}}}{5}$
default	$\frac{2b^{3}B\,x^{\frac{29}{2}}}{29}+\frac{2\left(b^{3}A+3a\,b^{2}B\right)x^{\frac{23}{2}}}{23}+\frac{2\left(3a\,b^{2}A+3a^{2}bB\right)x^{\frac{17}{2}}}{17}+\frac{2\left(3a^{2}bA+a^{3}B\right)x^{\frac{11}{2}}}{11}+\frac{2a^{3}A\,x^{\frac{5}{2}}}{5}$
gosper	$\frac{2x^{\frac{5}{2}}(21505B\,b^{3}x^{12}+27115A\,x^{9}b^{3}+81345B\,x^{9}a\,b^{2}+110055A\,x^{6}a\,b^{2}+110055B\,x^{6}a^{2}b+170085A\,x^{3}a^{2}b+56695a^{3}B\,x^{3}+1}{623645}$
trager	$\frac{2x^{\frac{5}{2}}(21505B\,b^{3}x^{12}+27115A\,x^{9}b^{3}+81345B\,x^{9}a\,b^{2}+110055A\,x^{6}a\,b^{2}+110055B\,x^{6}a^{2}b+170085A\,x^{3}a^{2}b+56695a^{3}B\,x^{3}+1}{623645}$
risch	$\frac{2x^{\frac{5}{2}}(21505B\,b^{3}x^{12}+27115A\,x^{9}b^{3}+81345B\,x^{9}a\,b^{2}+110055A\,x^{6}a\,b^{2}+110055B\,x^{6}a^{2}b+170085A\,x^{3}a^{2}b+56695a^{3}B\,x^{3}+1}{623645}$

input
```
int(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/29*b^3*B*x^(29/2)+2/23*(A*b^3+3*B*a*b^2)*x^(23/2)+2/17*(3*A*a*b^2+3*B*a^
2*b)*x^(17/2)+2/11*(3*A*a^2*b+B*a^3)*x^(11/2)+2/5*a^3*A*x^(5/2)
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{3/2}\left(a+bx^{3}\right)^{3}\left(A\right.$$
$$\left.+Bx^{3}\right)\,dx=\frac{2}{623645}\,(21505\,Bb^{3}x^{14}+27115\left(3\,Bab^{2}+Ab^{3}\right)x^{11}+110055\left(Ba^{2}b+Aab^{2}\right)x^{8}+124729\,Aa^{3}x^{2}$$

input
```
integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/623645*(21505*B*b^3*x^14 + 27115*(3*B*a*b^2 + A*b^3)*x^11 + 110055*(B*a^
2*b + A*a*b^2)*x^8 + 124729*A*a^3*x^2 + 56695*(B*a^3 + 3*A*a^2*b)*x^5)*sqr
t(x)
```

3.149.6 Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2Aa^3x^{\frac{5}{2}}}{5}+\frac{6Aa^2bx^{\frac{11}{2}}}{11}+\frac{6Aab^2x^{\frac{17}{2}}}{17}+\frac{2Ab^3x^{\frac{23}{2}}}{23}+\frac{2Ba^3x^{\frac{11}{2}}}{11}+\frac{6Ba^2bx^{\frac{17}{2}}}{17}+\frac{6Bab^2x^{\frac{23}{2}}}{23}+\frac{2Bb^3x^{\frac{29}{2}}}{29}$$

input
```
integrate(x**(3/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

output
```
2*A*a**3*x**(5/2)/5 + 6*A*a**2*b*x**(11/2)/11 + 6*A*a*b**2*x**(17/2)/17 +
2*A*b**3*x**(23/2)/23 + 2*B*a**3*x**(11/2)/11 + 6*B*a**2*b*x**(17/2)/17 +
6*B*a*b**2*x**(23/2)/23 + 2*B*b**3*x**(29/2)/29
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{29}\,Bb^3x^{\frac{29}{2}}+\frac{2}{23}\left(3\,Bab^2+Ab^3\right)x^{\frac{23}{2}}+\frac{6}{17}\left(Ba^2b+Aab^2\right)x^{\frac{17}{2}}+\frac{2}{5}\,Aa^3x^{\frac{5}{2}}+\frac{2}{11}\left(Ba^3+3\,Aa^2b\right)x^{\frac{11}{2}}$$

input
```
integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/29*B*b^3*x^(29/2) + 2/23*(3*B*a*b^2 + A*b^3)*x^(23/2) + 6/17*(B*a^2*b +
A*a*b^2)*x^(17/2) + 2/5*A*a^3*x^(5/2) + 2/11*(B*a^3 + 3*A*a^2*b)*x^(11/2)
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{29}\,Bb^3x^{\frac{29}{2}}+\frac{6}{23}\,Bab^2x^{\frac{23}{2}}+\frac{2}{23}\,Ab^3x^{\frac{23}{2}}+\frac{6}{17}\,Ba^2bx^{\frac{17}{2}}+\frac{6}{17}\,Aab^2x^{\frac{17}{2}}+\frac{2}{11}\,Ba^3x^{\frac{11}{2}}+\frac{6}{11}\,Aa^2bx^{\frac{11}{2}}+\frac{2}{5}\,Aa^3x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

output
```
2/29*B*b^3*x^(29/2) + 6/23*B*a*b^2*x^(23/2) + 2/23*A*b^3*x^(23/2) + 6/17*B
*a^2*b*x^(17/2) + 6/17*A*a*b^2*x^(17/2) + 2/11*B*a^3*x^(11/2) + 6/11*A*a^2
*b*x^(11/2) + 2/5*A*a^3*x^(5/2)
```

3.149.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{3/2}(a+bx^3)^3\,(A+Bx^3)\,dx = x^{11/2}\left(\frac{2\,B\,a^3}{11}+\frac{6\,A\,b\,a^2}{11}\right) + x^{23/2}\left(\frac{2\,A\,b^3}{23}+\frac{6\,B\,a\,b^2}{23}\right)+\frac{2\,A\,a^3\,x^{5/2}}{5}+\frac{2\,B\,b^3\,x^{29/2}}{29}+\frac{6\,a\,b\,x^{17/2}\,(A\,b+B\,a)}{17}$$

input
```
int(x^(3/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

output
```
x^(11/2)*((2*B*a^3)/11 + (6*A*a^2*b)/11) + x^(23/2)*((2*A*b^3)/23 + (6*B*a
*b^2)/23) + (2*A*a^3*x^(5/2))/5 + (2*B*b^3*x^(29/2))/29 + (6*a*b*x^(17/2)*
(A*b + B*a))/17
```

3.150 $\int \sqrt{x}(a+bx^3)^3\,(A+Bx^3)\,dx$

3.150.1 Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \sqrt{x}(a+bx^3)^3\,(A+Bx^3)\,dx = \frac{2}{3}a^3Ax^{3/2}+\frac{2}{9}a^2(3Ab+aB)x^{9/2} +\frac{2}{5}ab(Ab+aB)x^{15/2}+\frac{2}{21}b^2(Ab+3aB)x^{21/2}+\frac{2}{27}b^3Bx^{27/2}$$

output
```
2/3*a^3*A*x^(3/2)+2/9*a^2*(3*A*b+B*a)*x^(9/2)+2/5*a*b*(A*b+B*a)*x^(15/2)+2
/21*b^2*(A*b+3*B*a)*x^(21/2)+2/27*b^3*B*x^(27/2)
```

3.150.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \sqrt{x}(a+bx^3)^3\,(A+Bx^3)\,dx = \frac{2}{945}x^{3/2}\left(105a^3(3A+Bx^3)+63a^2bx^3(5A+3Bx^3) +27ab^2x^6(7A+5Bx^3)+5b^3x^9(9A+7Bx^3)\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3),x]
```

output
```
(2*x^(3/2)*(105*a^3*(3*A + B*x^3) + 63*a^2*b*x^3*(5*A + 3*B*x^3) + 27*a*b^
2*x^6*(7*A + 5*B*x^3) + 5*b^3*x^9*(9*A + 7*B*x^3)))/945
```

3.150.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \,950$$

$$\int \left(a^3A\sqrt{x}+a^2x^{7/2}(aB+3Ab)+b^2x^{19/2}(3aB+Ab)+3abx^{13/2}(aB+Ab)+b^3Bx^{25/2}\right)dx$$

$$\Big\downarrow \,2009$$

$$\frac{2}{3}a^3Ax^{3/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{27}b^3Bx^{27/2}$$

input
```
Int[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3),x]
```

output
```
(2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (2*a*b*(A*b + a*B)
*x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(27/2))/27
```

3.150.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.150.4 Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2b^3Bx^{\frac{27}{2}}}{27}+\frac{2(b^3A+3ab^2B)x^{\frac{21}{2}}}{21}+\frac{2(3ab^2A+3a^2bB)x^{\frac{15}{2}}}{15}+\frac{2(3a^2bA+a^3B)x^{\frac{9}{2}}}{9}+\frac{2a^3Ax^{\frac{3}{2}}}{3}$	76
default	$\frac{2b^3Bx^{\frac{27}{2}}}{27}+\frac{2(b^3A+3ab^2B)x^{\frac{21}{2}}}{21}+\frac{2(3ab^2A+3a^2bB)x^{\frac{15}{2}}}{15}+\frac{2(3a^2bA+a^3B)x^{\frac{9}{2}}}{9}+\frac{2a^3Ax^{\frac{3}{2}}}{3}$	76
gosper	$\frac{2x^{\frac{3}{2}}(35Bb^3x^{12}+45Ax^9b^3+135Bx^9ab^2+189Ax^6ab^2+189Bx^6a^2b+315Ax^3a^2b+105a^3Bx^3+315a^3A)}{945}$	80
trager	$\frac{2x^{\frac{3}{2}}(35Bb^3x^{12}+45Ax^9b^3+135Bx^9ab^2+189Ax^6ab^2+189Bx^6a^2b+315Ax^3a^2b+105a^3Bx^3+315a^3A)}{945}$	80
risch	$\frac{2x^{\frac{3}{2}}(35Bb^3x^{12}+45Ax^9b^3+135Bx^9ab^2+189Ax^6ab^2+189Bx^6a^2b+315Ax^3a^2b+105a^3Bx^3+315a^3A)}{945}$	80

input
```
int((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/27*b^3*B*x^(27/2)+2/21*(A*b^3+3*B*a*b^2)*x^(21/2)+2/15*(3*A*a*b^2+3*B*a^
2*b)*x^(15/2)+2/9*(3*A*a^2*b+B*a^3)*x^(9/2)+2/3*a^3*A*x^(3/2)
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \sqrt{x}(a+bx^3)^3\left(A+Bx^3\right)\,dx$$
$$=\frac{2}{945}\left(35\,Bb^3x^{13}+45\left(3\,Bab^2+Ab^3\right)x^{10}+189\left(Ba^2b+Aab^2\right)x^7+315\,Aa^3x+105\left(Ba^3+3\,Aa^2b\right)x^4\right)\sqrt{x}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
```

output
```
2/945*(35*B*b^3*x^13 + 45*(3*B*a*b^2 + A*b^3)*x^10 + 189*(B*a^2*b + A*a*b^
2)*x^7 + 315*A*a^3*x + 105*(B*a^3 + 3*A*a^2*b)*x^4)*sqrt(x)
```

3.150.6 Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \sqrt{x}(a+bx^3)^3\left(A+Bx^3\right)\,dx=\frac{2Aa^3x^{\frac{3}{2}}}{3}+\frac{2Aa^2bx^{\frac{9}{2}}}{3}+\frac{2Aab^2x^{\frac{15}{2}}}{5}+\frac{2Ab^3x^{\frac{21}{2}}}{21}+\frac{2Ba^3x^{\frac{9}{2}}}{9}+\frac{2Ba^2bx^{\frac{15}{2}}}{5}+\frac{2Bab^2x^{\frac{21}{2}}}{7}+\frac{2Bb^3x^{\frac{27}{2}}}{27}$$

input
```
integrate((b*x**3+a)**3*(B*x**3+A)*x**(1/2),x)
```

output
```
2*A*a**3*x**(3/2)/3 + 2*A*a**2*b*x**(9/2)/3 + 2*A*a*b**2*x**(15/2)/5 + 2*A
*b**3*x**(21/2)/21 + 2*B*a**3*x**(9/2)/9 + 2*B*a**2*b*x**(15/2)/5 + 2*B*a*
b**2*x**(21/2)/7 + 2*B*b**3*x**(27/2)/27
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(a+bx^3)^3\left(A+Bx^3\right)\,dx=\frac{2}{27}\,Bb^3x^{\frac{27}{2}}+\frac{2}{21}\,(3\,Bab^2+Ab^3)x^{\frac{21}{2}}+\frac{2}{5}\,(Ba^2b+Aab^2)x^{\frac{15}{2}}+\frac{2}{3}\,Aa^3x^{\frac{3}{2}}+\frac{2}{9}\,(Ba^3+3\,Aa^2b)x^{\frac{9}{2}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
```

output
```
2/27*B*b^3*x^(27/2) + 2/21*(3*B*a*b^2 + A*b^3)*x^(21/2) + 2/5*(B*a^2*b + A
*a*b^2)*x^(15/2) + 2/3*A*a^3*x^(3/2) + 2/9*(B*a^3 + 3*A*a^2*b)*x^(9/2)
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \sqrt{x}(a+bx^3)^3\left(A+Bx^3\right)\,dx=\frac{2}{27}\,Bb^3x^{\frac{27}{2}}+\frac{2}{7}\,Bab^2x^{\frac{21}{2}}+\frac{2}{21}\,Ab^3x^{\frac{21}{2}}+\frac{2}{5}\,Ba^2bx^{\frac{15}{2}}+\frac{2}{5}\,Aab^2x^{\frac{15}{2}}+\frac{2}{9}\,Ba^3x^{\frac{9}{2}}+\frac{2}{3}\,Aa^2bx^{\frac{9}{2}}+\frac{2}{3}\,Aa^3x^{\frac{3}{2}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

output
```
2/27*B*b^3*x^(27/2) + 2/7*B*a*b^2*x^(21/2) + 2/21*A*b^3*x^(21/2) + 2/5*B*a
^2*b*x^(15/2) + 2/5*A*a*b^2*x^(15/2) + 2/9*B*a^3*x^(9/2) + 2/3*A*a^2*b*x^(
9/2) + 2/3*A*a^3*x^(3/2)
```

3.150.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(a+bx^3)^3\left(A+Bx^3\right)\,dx = x^{9/2}\left(\frac{2\,B\,a^3}{9}+\frac{2\,A\,b\,a^2}{3}\right)+x^{21/2}\left(\frac{2\,A\,b^3}{21}+\frac{2\,B\,a\,b^2}{7}\right) + \frac{2\,A\,a^3\,x^{3/2}}{3}+\frac{2\,B\,b^3\,x^{27/2}}{27}+\frac{2\,a\,b\,x^{15/2}\,(A\,b+B\,a)}{5}$$

input
```
int(x^(1/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

output
```
x^(9/2)*((2*B*a^3)/9 + (2*A*a^2*b)/3) + x^(21/2)*((2*A*b^3)/21 + (2*B*a*b^
2)/7) + (2*A*a^3*x^(3/2))/3 + (2*B*b^3*x^(27/2))/27 + (2*a*b*x^(15/2)*(A*b
 + B*a))/5
```

3.151 $\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx$

3.151.1 Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx = 2a^3A\sqrt{x}+\frac{2}{7}a^2(3Ab+aB)x^{7/2} + \frac{6}{13}ab(Ab+aB)x^{13/2}+\frac{2}{19}b^2(Ab+3aB)x^{19/2}+\frac{2}{25}b^3Bx^{25/2}$$

output

```
2/7*a^2*(3*A*b+B*a)*x^(7/2)+6/13*a*b*(A*b+B*a)*x^(13/2)+2/19*b^2*(A*b+3*B*
a)*x^(19/2)+2/25*b^3*B*x^(25/2)+2*a^3*A*x^(1/2)
```

3.151.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}(6175a^3(7A+Bx^3)+1425a^2bx^3(13A+7Bx^3)+525ab^2x^6(19A+13Bx^3)+91b^3x^9(25A+19Bx^3))}{43225}$$

input

```
Integrate[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x],x]
```

output

```
(2*Sqrt[x]*(6175*a^3*(7*A + B*x^3) + 1425*a^2*b*x^3*(13*A + 7*B*x^3) + 525
*a*b^2*x^6*(19*A + 13*B*x^3) + 91*b^3*x^9*(25*A + 19*B*x^3)))/43225
```

3.151.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^3A}{\sqrt{x}}+a^2x^{5/2}(aB+3Ab)+b^2x^{17/2}(3aB+Ab)+3abx^{11/2}(aB+Ab)+b^3Bx^{23/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a^3A\sqrt{x}+\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

input
```
Int[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x],x]
```

output
```
2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*x^(
13/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(25/2))/25
```

3.151.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.151.4 Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{2b^3Bx^{\frac{25}{2}}}{25}+\frac{2(b^3A+3a\,b^2B)x^{\frac{19}{2}}}{19}+\frac{2(3a\,b^2A+3a^2bB)x^{\frac{13}{2}}}{13}+\frac{2(3a^2bA+a^3B)x^{\frac{7}{2}}}{7}+2a^3A\sqrt{x}$
default	$\frac{2b^3Bx^{\frac{25}{2}}}{25}+\frac{2(b^3A+3a\,b^2B)x^{\frac{19}{2}}}{19}+\frac{2(3a\,b^2A+3a^2bB)x^{\frac{13}{2}}}{13}+\frac{2(3a^2bA+a^3B)x^{\frac{7}{2}}}{7}+2a^3A\sqrt{x}$
trager	$\left(\frac{2}{25}B\,b^3x^{12}+\frac{2}{19}A\,x^9b^3+\frac{6}{19}B\,x^9a\,b^2+\frac{6}{13}A\,x^6a\,b^2+\frac{6}{13}B\,x^6a^2b+\frac{6}{7}A\,x^3a^2b+\frac{2}{7}a^3B\,x^3+$
gosper	$\frac{2\sqrt{x}\,(1729B\,b^3x^{12}+2275A\,x^9b^3+6825B\,x^9a\,b^2+9975A\,x^6a\,b^2+9975B\,x^6a^2b+18525A\,x^3a^2b+6175a^3B\,x^3+43225a^3A)}{43225}$
risch	$\frac{2\sqrt{x}\,(1729B\,b^3x^{12}+2275A\,x^9b^3+6825B\,x^9a\,b^2+9975A\,x^6a\,b^2+9975B\,x^6a^2b+18525A\,x^3a^2b+6175a^3B\,x^3+43225a^3A)}{43225}$

input
```
int((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/25*b^3*B*x^(25/2)+2/19*(A*b^3+3*B*a*b^2)*x^(19/2)+2/13*(3*A*a*b^2+3*B*a^
2*b)*x^(13/2)+2/7*(3*A*a^2*b+B*a^3)*x^(7/2)+2*a^3*A*x^(1/2)
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int\frac{(a+bx^3)^3\,(A+Bx^3)}{\sqrt{x}}\,dx$$
$$=\frac{2}{43225}\left(1729\,Bb^3x^{12}+2275\left(3\,Bab^2+Ab^3\right)x^9+9975\left(Ba^2b+Aab^2\right)x^6+43225\,Aa^3+6175\left(Ba^3+3\,Aa\right.\right.$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

output
```
2/43225*(1729*B*b^3*x^12 + 2275*(3*B*a*b^2 + A*b^3)*x^9 + 9975*(B*a^2*b +
A*a*b^2)*x^6 + 43225*A*a^3 + 6175*(B*a^3 + 3*A*a^2*b)*x^3)*sqrt(x)
```

3.151.6 Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx = 2Aa^3\sqrt{x} + \frac{6Aa^2bx^{\frac{7}{2}}}{7} + \frac{6Aab^2x^{\frac{13}{2}}}{13} + \frac{2Ab^3x^{\frac{19}{2}}}{19} + \frac{2Ba^3x^{\frac{7}{2}}}{7} + \frac{6Ba^2bx^{\frac{13}{2}}}{13} + \frac{6Bab^2x^{\frac{19}{2}}}{19} + \frac{2Bb^3x^{\frac{25}{2}}}{25}$$

input
```
integrate((b*x**3+a)**3*(B*x**3+A)/x**(1/2),x)
```

output
```
2*A*a**3*sqrt(x) + 6*A*a**2*b*x**(7/2)/7 + 6*A*a*b**2*x**(13/2)/13 + 2*A*b
**3*x**(19/2)/19 + 2*B*a**3*x**(7/2)/7 + 6*B*a**2*b*x**(13/2)/13 + 6*B*a*b
**2*x**(19/2)/19 + 2*B*b**3*x**(25/2)/25
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{25}\,Bb^3x^{\frac{25}{2}} + \frac{2}{19}\left(3\,Bab^2 + Ab^3\right)x^{\frac{19}{2}} + \frac{6}{13}\left(Ba^2b + Aab^2\right)x^{\frac{13}{2}} + 2\,Aa^3\sqrt{x} + \frac{2}{7}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{7}{2}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

output
```
2/25*B*b^3*x^(25/2) + 2/19*(3*B*a*b^2 + A*b^3)*x^(19/2) + 6/13*(B*a^2*b +
A*a*b^2)*x^(13/2) + 2*A*a^3*sqrt(x) + 2/7*(B*a^3 + 3*A*a^2*b)*x^(7/2)
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{25}\,Bb^3x^{\frac{25}{2}} + \frac{6}{19}\,Bab^2x^{\frac{19}{2}} + \frac{2}{19}\,Ab^3x^{\frac{19}{2}} + \frac{6}{13}\,Ba^2bx^{\frac{13}{2}} + \frac{6}{13}\,Aab^2x^{\frac{13}{2}} + \frac{2}{7}\,Ba^3x^{\frac{7}{2}} + \frac{6}{7}\,Aa^2bx^{\frac{7}{2}} + 2\,Aa^3\sqrt{x}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

output
```
2/25*B*b^3*x^(25/2) + 6/19*B*a*b^2*x^(19/2) + 2/19*A*b^3*x^(19/2) + 6/13*B
*a^2*b*x^(13/2) + 6/13*A*a*b^2*x^(13/2) + 2/7*B*a^3*x^(7/2) + 6/7*A*a^2*b*
x^(7/2) + 2*A*a^3*sqrt(x)
```

3.151.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{\sqrt{x}}\,dx = x^{7/2}\left(\frac{2\,B\,a^3}{7}+\frac{6\,A\,b\,a^2}{7}\right)+x^{19/2}\left(\frac{2\,A\,b^3}{19}+\frac{6\,B\,a\,b^2}{19}\right)+2\,A\,a^3\,\sqrt{x}+\frac{2\,B\,b^3\,x^{25/2}}{25}+\frac{6\,a\,b\,x^{13/2}\,(A\,b+B\,a)}{13}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^3)/x^(1/2),x)
```

output
```
x^(7/2)*((2*B*a^3)/7 + (6*A*a^2*b)/7) + x^(19/2)*((2*A*b^3)/19 + (6*B*a*b^
2)/19) + 2*A*a^3*x^(1/2) + (2*B*b^3*x^(25/2))/25 + (6*a*b*x^(13/2)*(A*b +
B*a))/13
```

3.152 $\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx$

3.152.1 Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2a^3A}{\sqrt{x}} + \frac{2}{5}a^2(3Ab+aB)x^{5/2} + \frac{6}{11}ab(Ab+aB)x^{11/2} + \frac{2}{17}b^2(Ab+3aB)x^{17/2} + \frac{2}{23}b^3Bx^{23/2}$$

output

```
2/5*a^2*(3*A*b+B*a)*x^(5/2)+6/11*a*b*(A*b+B*a)*x^(11/2)+2/17*b^2*(A*b+3*B*
a)*x^(17/2)+2/23*b^3*B*x^(23/2)-2*a^3*A/x^(1/2)
```

3.152.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2(21505a^3A - 12903a^2Abx^3 - 4301a^3Bx^3 - 5865aAb^2x^6 - 5865a^2bBx^6 - 1265Ab^3x^9 - 3795ab^2Bx^9 - 9}{21505\sqrt{x}}$$

input

```
Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(3/2),x]
```

output

```
(-2*(21505*a^3*A - 12903*a^2*A*b*x^3 - 4301*a^3*B*x^3 - 5865*a*A*b^2*x^6 -
 5865*a^2*b*B*x^6 - 1265*A*b^3*x^9 - 3795*a*b^2*B*x^9 - 935*b^3*B*x^12))/(
21505*Sqrt[x])
```

3.152.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^3\left(A+Bx^3\right)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^3A}{x^{3/2}}+a^2x^{3/2}(aB+3Ab)+b^2x^{15/2}(3aB+Ab)+3abx^{9/2}(aB+Ab)+b^3Bx^{21/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3A}{\sqrt{x}}+\frac{2}{5}a^2x^{5/2}(aB+3Ab)+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

input
```
Int[((a + b*x^3)^3*(A + B*x^3))/x^(3/2),x]
```

output
```
(-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (6*a*b*(A*b + a*B)*
x^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(23/2))/23
```

3.152.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.152.4 Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{2b^3 B\,x^{\frac{23}{2}}}{23}+\frac{2A\,b^3 x^{\frac{17}{2}}}{17}+\frac{6Ba\,b^2 x^{\frac{17}{2}}}{17}+\frac{6Aa\,b^2 x^{\frac{11}{2}}}{11}+\frac{6B\,a^2 b\,x^{\frac{11}{2}}}{11}+\frac{6A\,a^2 b\,x^{\frac{5}{2}}}{5}+\frac{2B\,a^3 x^{\frac{5}{2}}}{5}-\frac{2a^3 A}{\sqrt{x}}$
default	$\frac{2b^3 B\,x^{\frac{23}{2}}}{23}+\frac{2A\,b^3 x^{\frac{17}{2}}}{17}+\frac{6Ba\,b^2 x^{\frac{17}{2}}}{17}+\frac{6Aa\,b^2 x^{\frac{11}{2}}}{11}+\frac{6B\,a^2 b\,x^{\frac{11}{2}}}{11}+\frac{6A\,a^2 b\,x^{\frac{5}{2}}}{5}+\frac{2B\,a^3 x^{\frac{5}{2}}}{5}-\frac{2a^3 A}{\sqrt{x}}$
gosper	$-\frac{2\left(-935B\,b^3 x^{12}-1265A\,x^9 b^3-3795B\,x^9 a\,b^2-5865A\,x^6 a\,b^2-5865B\,x^6 a^2 b-12903A\,x^3 a^2 b-4301a^3 B\,x^3+21505a^3 A\right)}{21505\sqrt{x}}$
trager	$-\frac{2\left(-935B\,b^3 x^{12}-1265A\,x^9 b^3-3795B\,x^9 a\,b^2-5865A\,x^6 a\,b^2-5865B\,x^6 a^2 b-12903A\,x^3 a^2 b-4301a^3 B\,x^3+21505a^3 A\right)}{21505\sqrt{x}}$
risch	$-\frac{2\left(-935B\,b^3 x^{12}-1265A\,x^9 b^3-3795B\,x^9 a\,b^2-5865A\,x^6 a\,b^2-5865B\,x^6 a^2 b-12903A\,x^3 a^2 b-4301a^3 B\,x^3+21505a^3 A\right)}{21505\sqrt{x}}$

input
```
int((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/23*b^3*B*x^(23/2)+2/17*A*b^3*x^(17/2)+6/17*B*a*b^2*x^(17/2)+6/11*A*a*b^2
*x^(11/2)+6/11*B*a^2*b*x^(11/2)+6/5*A*a^2*b*x^(5/2)+2/5*B*a^3*x^(5/2)-2*a^
3*A/x^(1/2)
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+bx^3\right)^3\left(A+Bx^3\right)}{x^{3/2}}\,dx=\frac{2\left(935\,Bb^3x^{12}+1265\,(3\,Bab^2+Ab^3)x^9+5865\,(Ba^2b+Aab^2)x^6-21505\,Aa^3\right.}{21505\,\sqrt{x}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

output
```
2/21505*(935*B*b^3*x^12 + 1265*(3*B*a*b^2 + A*b^3)*x^9 + 5865*(B*a^2*b + A
*a*b^2)*x^6 - 21505*A*a^3 + 4301*(B*a^3 + 3*A*a^2*b)*x^3)/sqrt(x)
```

3.152.6 Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{3/2}}\,dx = -\frac{2Aa^3}{\sqrt{x}} + \frac{6Aa^2bx^{\frac{5}{2}}}{5} + \frac{6Aab^2x^{\frac{11}{2}}}{11} + \frac{2Ab^3x^{\frac{17}{2}}}{17} + \frac{2Ba^3x^{\frac{5}{2}}}{5} + \frac{6Ba^2bx^{\frac{11}{2}}}{11} + \frac{6Bab^2x^{\frac{17}{2}}}{17} + \frac{2Bb^3x^{\frac{23}{2}}}{23}$$

input
```
integrate((b*x**3+a)**3*(B*x**3+A)/x**(3/2),x)
```

output
```
-2*A*a**3/sqrt(x) + 6*A*a**2*b*x**(5/2)/5 + 6*A*a*b**2*x**(11/2)/11 + 2*A*
b**3*x**(17/2)/17 + 2*B*a**3*x**(5/2)/5 + 6*B*a**2*b*x**(11/2)/11 + 6*B*a*
b**2*x**(17/2)/17 + 2*B*b**3*x**(23/2)/23
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{3/2}}\,dx = \frac{2}{23}\,Bb^3x^{\frac{23}{2}} + \frac{2}{17}\,(3\,Bab^2 + Ab^3)x^{\frac{17}{2}} + \frac{6}{11}\,(Ba^2b + Aab^2)x^{\frac{11}{2}} - \frac{2\,Aa^3}{\sqrt{x}} + \frac{2}{5}\,(Ba^3 + 3\,Aa^2b)x^{\frac{5}{2}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
```

output
```
2/23*B*b^3*x^(23/2) + 2/17*(3*B*a*b^2 + A*b^3)*x^(17/2) + 6/11*(B*a^2*b +
A*a*b^2)*x^(11/2) - 2*A*a^3/sqrt(x) + 2/5*(B*a^3 + 3*A*a^2*b)*x^(5/2)
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{3/2}}\,dx = \frac{2}{23}\,Bb^3x^{\frac{23}{2}} + \frac{6}{17}\,Bab^2x^{\frac{17}{2}} + \frac{2}{17}\,Ab^3x^{\frac{17}{2}} + \frac{6}{11}\,Ba^2bx^{\frac{11}{2}} + \frac{6}{11}\,Aab^2x^{\frac{11}{2}} + \frac{2}{5}\,Ba^3x^{\frac{5}{2}} + \frac{6}{5}\,Aa^2bx^{\frac{5}{2}} - \frac{2\,Aa^3}{\sqrt{x}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

output
```
2/23*B*b^3*x^(23/2) + 6/17*B*a*b^2*x^(17/2) + 2/17*A*b^3*x^(17/2) + 6/11*B
*a^2*b*x^(11/2) + 6/11*A*a*b^2*x^(11/2) + 2/5*B*a^3*x^(5/2) + 6/5*A*a^2*b*
x^(5/2) - 2*A*a^3/sqrt(x)
```

3.152.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{3/2}}\,dx = x^{5/2}\left(\frac{2\,B\,a^3}{5}+\frac{6\,A\,b\,a^2}{5}\right) + x^{17/2}\left(\frac{2\,A\,b^3}{17}+\frac{6\,B\,a\,b^2}{17}\right) - \frac{2\,A\,a^3}{\sqrt{x}} + \frac{2\,B\,b^3\,x^{23/2}}{23} + \frac{6\,a\,b\,x^{11/2}\,(A\,b+B\,a)}{11}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^3)/x^(3/2),x)
```

output
```
x^(5/2)*((2*B*a^3)/5 + (6*A*a^2*b)/5) + x^(17/2)*((2*A*b^3)/17 + (6*B*a*b^
2)/17) - (2*A*a^3)/x^(1/2) + (2*B*b^3*x^(23/2))/23 + (6*a*b*x^(11/2)*(A*b
+ B*a))/11
```

3.153 $\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{5/2}}\,dx$

3.153.1 Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{5/2}}\,dx = -\frac{2a^3A}{3x^{3/2}} + \frac{2}{3}a^2(3Ab+aB)x^{3/2} + \frac{2}{3}ab(Ab+aB)x^{9/2} + \frac{2}{15}b^2(Ab+3aB)x^{15/2} + \frac{2}{21}b^3Bx^{21/2}$$

output
```
-2/3*a^3*A/x^(3/2)+2/3*a^2*(3*A*b+B*a)*x^(3/2)+2/3*a*b*(A*b+B*a)*x^(9/2)+2
/15*b^2*(A*b+3*B*a)*x^(15/2)+2/21*b^3*B*x^(21/2)
```

3.153.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{5/2}}\,dx = \frac{2(-35a^3(A-Bx^3)+35a^2bx^3(3A+Bx^3)+7ab^2x^6(5A+3Bx^3)+b^3x^9(7A-}{105x^{3/2}}$$

input
```
Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(5/2),x]
```

output
```
(2*(-35*a^3*(A - B*x^3) + 35*a^2*b*x^3*(3*A + B*x^3) + 7*a*b^2*x^6*(5*A +
3*B*x^3) + b^3*x^9*(7*A + 5*B*x^3)))/(105*x^(3/2))
```

3.153.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^3A}{x^{5/2}}+a^2\sqrt{x}(aB+3Ab)+b^2x^{13/2}(3aB+Ab)+3abx^{7/2}(aB+Ab)+b^3Bx^{19/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3A}{3x^{3/2}}+\frac{2}{3}a^2x^{3/2}(aB+3Ab)+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

input
```
Int[((a + b*x^3)^3*(A + B*x^3))/x^(5/2),x]
```

output
```
(-2*a^3*A)/(3*x^(3/2)) + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (2*a*b*(A*b + a
*B)*x^(9/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*B*x^(21/2))/21
```

3.153.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.153.4 Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{2b^{3}B\,x^{\frac{21}{2}}}{21}+\frac{2A\,b^{3}x^{\frac{15}{2}}}{15}+\frac{2Ba\,b^{2}x^{\frac{15}{2}}}{5}+\frac{2Aa\,b^{2}x^{\frac{9}{2}}}{3}+\frac{2B\,a^{2}b\,x^{\frac{9}{2}}}{3}+2A\,a^{2}b\,x^{\frac{3}{2}}+\frac{2B\,a^{3}x^{\frac{3}{2}}}{3}-\frac{2a^{3}A}{3x^{\frac{3}{2}}}$	78
default	$\frac{2b^{3}B\,x^{\frac{21}{2}}}{21}+\frac{2A\,b^{3}x^{\frac{15}{2}}}{15}+\frac{2Ba\,b^{2}x^{\frac{15}{2}}}{5}+\frac{2Aa\,b^{2}x^{\frac{9}{2}}}{3}+\frac{2B\,a^{2}b\,x^{\frac{9}{2}}}{3}+2A\,a^{2}b\,x^{\frac{3}{2}}+\frac{2B\,a^{3}x^{\frac{3}{2}}}{3}-\frac{2a^{3}A}{3x^{\frac{3}{2}}}$	78
gosper	$-\frac{2\left(-5B\,b^{3}x^{12}-7A\,x^{9}b^{3}-21B\,x^{9}a\,b^{2}-35A\,x^{6}a\,b^{2}-35B\,x^{6}a^{2}b-105A\,x^{3}a^{2}b-35a^{3}B\,x^{3}+35a^{3}A\right)}{105x^{\frac{3}{2}}}$	80
trager	$-\frac{2\left(-5B\,b^{3}x^{12}-7A\,x^{9}b^{3}-21B\,x^{9}a\,b^{2}-35A\,x^{6}a\,b^{2}-35B\,x^{6}a^{2}b-105A\,x^{3}a^{2}b-35a^{3}B\,x^{3}+35a^{3}A\right)}{105x^{\frac{3}{2}}}$	80
risch	$-\frac{2\left(-5B\,b^{3}x^{12}-7A\,x^{9}b^{3}-21B\,x^{9}a\,b^{2}-35A\,x^{6}a\,b^{2}-35B\,x^{6}a^{2}b-105A\,x^{3}a^{2}b-35a^{3}B\,x^{3}+35a^{3}A\right)}{105x^{\frac{3}{2}}}$	80

input
```
int((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*b^3*B*x^(21/2)+2/15*A*b^3*x^(15/2)+2/5*B*a*b^2*x^(15/2)+2/3*A*a*b^2*x
^(9/2)+2/3*B*a^2*b*x^(9/2)+2*A*a^2*b*x^(3/2)+2/3*B*a^3*x^(3/2)-2/3*a^3*A/x
^(3/2)
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+bx^{3}\right)^{3}\left(A+Bx^{3}\right)}{x^{5/2}}\,dx=\frac{2\,(5\,Bb^{3}x^{12}+7\,(3\,Bab^{2}+Ab^{3})x^{9}+35\,(Ba^{2}b+Aab^{2})x^{6}-35\,Aa^{3}+35\,(Ba^{3}}{105\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

output
```
2/105*(5*B*b^3*x^12 + 7*(3*B*a*b^2 + A*b^3)*x^9 + 35*(B*a^2*b + A*a*b^2)*x
^6 - 35*A*a^3 + 35*(B*a^3 + 3*A*a^2*b)*x^3)/x^(3/2)
```

3.153.6 Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}}\,dx = -\frac{2Aa^3}{3x^{\frac{3}{2}}} + 2Aa^2bx^{\frac{3}{2}} + \frac{2Aab^2x^{\frac{9}{2}}}{3} + \frac{2Ab^3x^{\frac{15}{2}}}{15} + \frac{2Ba^3x^{\frac{3}{2}}}{3} + \frac{2Ba^2bx^{\frac{9}{2}}}{3} + \frac{2Bab^2x^{\frac{15}{2}}}{5} + \frac{2Bb^3x^{\frac{21}{2}}}{21}$$

input
```
integrate((b*x**3+a)**3*(B*x**3+A)/x**(5/2),x)
```

output
```
-2*A*a**3/(3*x**(3/2)) + 2*A*a**2*b*x**(3/2) + 2*A*a*b**2*x**(9/2)/3 + 2*A
*b**3*x**(15/2)/15 + 2*B*a**3*x**(3/2)/3 + 2*B*a**2*b*x**(9/2)/3 + 2*B*a*b
**2*x**(15/2)/5 + 2*B*b**3*x**(21/2)/21
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}}\,dx = \frac{2}{21}\,Bb^3x^{\frac{21}{2}} + \frac{2}{15}\left(3\,Bab^2 + Ab^3\right)x^{\frac{15}{2}} + \frac{2}{3}\left(Ba^2b + Aab^2\right)x^{\frac{9}{2}} - \frac{2\,Aa^3}{3\,x^{\frac{3}{2}}} + \frac{2}{3}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{3}{2}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

output
```
2/21*B*b^3*x^(21/2) + 2/15*(3*B*a*b^2 + A*b^3)*x^(15/2) + 2/3*(B*a^2*b + A
*a*b^2)*x^(9/2) - 2/3*A*a^3/x^(3/2) + 2/3*(B*a^3 + 3*A*a^2*b)*x^(3/2)
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}}\,dx = \frac{2}{21}\,Bb^3x^{\frac{21}{2}} + \frac{2}{5}\,Bab^2x^{\frac{15}{2}} + \frac{2}{15}\,Ab^3x^{\frac{15}{2}} + \frac{2}{3}\,Ba^2bx^{\frac{9}{2}} + \frac{2}{3}\,Aab^2x^{\frac{9}{2}} + \frac{2}{3}\,Ba^3x^{\frac{3}{2}} + 2\,Aa^2bx^{\frac{3}{2}} - \frac{2\,Aa^3}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

output
```
2/21*B*b^3*x^(21/2) + 2/5*B*a*b^2*x^(15/2) + 2/15*A*b^3*x^(15/2) + 2/3*B*a
^2*b*x^(9/2) + 2/3*A*a*b^2*x^(9/2) + 2/3*B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2)
 - 2/3*A*a^3/x^(3/2)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+bx^3\right)^3\left(A+Bx^3\right)}{x^{5/2}}\,dx = x^{3/2}\left(\frac{2\,B\,a^3}{3}+2\,A\,b\,a^2\right) + x^{15/2}\left(\frac{2\,A\,b^3}{15}+\frac{2\,B\,a\,b^2}{5}\right)-\frac{2\,A\,a^3}{3\,x^{3/2}}+\frac{2\,B\,b^3\,x^{21/2}}{21}+\frac{2\,a\,b\,x^{9/2}\,(A\,b+B\,a)}{3}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^3)/x^(5/2),x)
```

output
```
x^(3/2)*((2*B*a^3)/3 + 2*A*a^2*b) + x^(15/2)*((2*A*b^3)/15 + (2*B*a*b^2)/5
) - (2*A*a^3)/(3*x^(3/2)) + (2*B*b^3*x^(21/2))/21 + (2*a*b*x^(9/2)*(A*b +
B*a))/3
```

3.154 $\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx$

3.154.1 Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx = -\frac{2a^3A}{5x^{5/2}} + 2a^2(3Ab+aB)\sqrt{x} + \frac{6}{7}ab(Ab+aB)x^{7/2} + \frac{2}{13}b^2(Ab+3aB)x^{13/2} + \frac{2}{19}b^3Bx^{19/2}$$

output
```
-2/5*a^3*A/x^(5/2)+6/7*a*b*(A*b+B*a)*x^(7/2)+2/13*b^2*(A*b+3*B*a)*x^(13/2)
+2/19*b^3*B*x^(19/2)+2*a^2*(3*A*b+B*a)*x^(1/2)
```

3.154.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx = \frac{-3458a^3(A-5Bx^3) + 7410a^2bx^3(7A+Bx^3) + 570ab^2x^6(13A+7Bx^3) + 70}{8645x^{5/2}}$$

input
```
Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(7/2),x]
```

output
```
(-3458*a^3*(A - 5*B*x^3) + 7410*a^2*b*x^3*(7*A + B*x^3) + 570*a*b^2*x^6*(1
3*A + 7*B*x^3) + 70*b^3*x^9*(19*A + 13*B*x^3))/(8645*x^(5/2))
```

3.154.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{7/2}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^3 A}{x^{7/2}}+\frac{a^2(aB+3Ab)}{\sqrt{x}}+b^2 x^{11/2}(3aB+Ab)+3abx^{5/2}(aB+Ab)+b^3 B x^{17/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3 A}{5x^{5/2}}+2a^2\sqrt{x}(aB+3Ab)+\frac{2}{13}b^2 x^{13/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{19}b^3 B x^{19/2}$$

input
```
Int[((a + b*x^3)^3*(A + B*x^3))/x^(7/2),x]
```

output
```
(-2*a^3*A)/(5*x^(5/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*b + a*B)*
x^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x^(19/2))/19
```

3.154.3.1 Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.154.4 Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{2b^3 B\,x^{\frac{19}{2}}}{19}+\frac{2A\,b^3 x^{\frac{13}{2}}}{13}+\frac{6Ba\,b^2 x^{\frac{13}{2}}}{13}+\frac{6Aa\,b^2 x^{\frac{7}{2}}}{7}+\frac{6B\,a^2 b\,x^{\frac{7}{2}}}{7}+6A\,a^2 b\sqrt{x}+2B\,a^3\sqrt{x}-\frac{2a^3 A}{5x^{\frac{5}{2}}}$
default	$\frac{2b^3 B\,x^{\frac{19}{2}}}{19}+\frac{2A\,b^3 x^{\frac{13}{2}}}{13}+\frac{6Ba\,b^2 x^{\frac{13}{2}}}{13}+\frac{6Aa\,b^2 x^{\frac{7}{2}}}{7}+\frac{6B\,a^2 b\,x^{\frac{7}{2}}}{7}+6A\,a^2 b\sqrt{x}+2B\,a^3\sqrt{x}-\frac{2a^3 A}{5x^{\frac{5}{2}}}$
gosper	$-\frac{2\left(-455B\,b^3 x^{12}-665A\,x^9 b^3-1995B\,x^9 a\,b^2-3705A\,x^6 a\,b^2-3705B\,x^6 a^2 b-25935A\,x^3 a^2 b-8645a^3 B\,x^3+1729a^3 A\right)}{8645x^{\frac{5}{2}}}$
trager	$-\frac{2\left(-455B\,b^3 x^{12}-665A\,x^9 b^3-1995B\,x^9 a\,b^2-3705A\,x^6 a\,b^2-3705B\,x^6 a^2 b-25935A\,x^3 a^2 b-8645a^3 B\,x^3+1729a^3 A\right)}{8645x^{\frac{5}{2}}}$
risch	$-\frac{2\left(-455B\,b^3 x^{12}-665A\,x^9 b^3-1995B\,x^9 a\,b^2-3705A\,x^6 a\,b^2-3705B\,x^6 a^2 b-25935A\,x^3 a^2 b-8645a^3 B\,x^3+1729a^3 A\right)}{8645x^{\frac{5}{2}}}$

input

```
int((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

output

```
2/19*b^3*B*x^(19/2)+2/13*A*b^3*x^(13/2)+6/13*B*a*b^2*x^(13/2)+6/7*A*a*b^2*
x^(7/2)+6/7*B*a^2*b*x^(7/2)+6*A*a^2*b*x^(1/2)+2*B*a^3*x^(1/2)-2/5*a^3*A/x^
(5/2)
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^3\,(A+Bx^3)}{x^{7/2}}\,dx=\frac{2\,(455\,Bb^3x^{12}+665\,(3\,Bab^2+Ab^3)x^9+3705\,(Ba^2b+Aab^2)x^6-1729\,Aa^3+}{8645\,x^{\frac{5}{2}}}$$

input

```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

output

```
2/8645*(455*B*b^3*x^12 + 665*(3*B*a*b^2 + A*b^3)*x^9 + 3705*(B*a^2*b + A*a
*b^2)*x^6 - 1729*A*a^3 + 8645*(B*a^3 + 3*A*a^2*b)*x^3)/x^(5/2)
```

3.154.6 Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx = -\frac{2Aa^3}{5x^{\frac{5}{2}}} + 6Aa^2b\sqrt{x} + \frac{6Aab^2x^{\frac{7}{2}}}{7} + \frac{2Ab^3x^{\frac{13}{2}}}{13} + 2Ba^3\sqrt{x} + \frac{6Ba^2bx^{\frac{7}{2}}}{7} + \frac{6Bab^2x^{\frac{13}{2}}}{13} + \frac{2Bb^3x^{\frac{19}{2}}}{19}$$

input
```
integrate((b*x**3+a)**3*(B*x**3+A)/x**(7/2),x)
```

output
```
-2*A*a**3/(5*x**(5/2)) + 6*A*a**2*b*sqrt(x) + 6*A*a*b**2*x**(7/2)/7 + 2*A*
b**3*x**(13/2)/13 + 2*B*a**3*sqrt(x) + 6*B*a**2*b*x**(7/2)/7 + 6*B*a*b**2*
x**(13/2)/13 + 2*B*b**3*x**(19/2)/19
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{19}\,Bb^3x^{\frac{19}{2}} + \frac{2}{13}\left(3\,Bab^2 + Ab^3\right)x^{\frac{13}{2}} + \frac{6}{7}\left(Ba^2b + Aab^2\right)x^{\frac{7}{2}} - \frac{2\,Aa^3}{5\,x^{\frac{5}{2}}} + 2\left(Ba^3 + 3\,Aa^2b\right)\sqrt{x}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

output
```
2/19*B*b^3*x^(19/2) + 2/13*(3*B*a*b^2 + A*b^3)*x^(13/2) + 6/7*(B*a^2*b + A
*a*b^2)*x^(7/2) - 2/5*A*a^3/x^(5/2) + 2*(B*a^3 + 3*A*a^2*b)*sqrt(x)
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{19}\,Bb^3x^{\frac{19}{2}} + \frac{6}{13}\,Bab^2x^{\frac{13}{2}} + \frac{2}{13}\,Ab^3x^{\frac{13}{2}} + \frac{6}{7}\,Ba^2bx^{\frac{7}{2}} + \frac{6}{7}\,Aab^2x^{\frac{7}{2}} + 2\,Ba^3\sqrt{x} + 6\,Aa^2b\sqrt{x} - \frac{2\,Aa^3}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

output
```
2/19*B*b^3*x^(19/2) + 6/13*B*a*b^2*x^(13/2) + 2/13*A*b^3*x^(13/2) + 6/7*B*
a^2*b*x^(7/2) + 6/7*A*a*b^2*x^(7/2) + 2*B*a^3*sqrt(x) + 6*A*a^2*b*sqrt(x)
- 2/5*A*a^3/x^(5/2)
```

3.154.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{7/2}}\,dx = \sqrt{x}\left(2\,B\,a^3+6\,A\,b\,a^2\right) + x^{13/2}\left(\frac{2\,A\,b^3}{13}+\frac{6\,B\,a\,b^2}{13}\right) - \frac{2\,A\,a^3}{5\,x^{5/2}} + \frac{2\,B\,b^3\,x^{19/2}}{19} + \frac{6\,a\,b\,x^{7/2}\,(A\,b+B\,a)}{7}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^3)/x^(7/2),x)
```

output
```
x^(1/2)*(2*B*a^3 + 6*A*a^2*b) + x^(13/2)*((2*A*b^3)/13 + (6*B*a*b^2)/13) -
 (2*A*a^3)/(5*x^(5/2)) + (2*B*b^3*x^(19/2))/19 + (6*a*b*x^(7/2)*(A*b + B*a
))/7
```

3.155 $\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx$

3.155.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx=\frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{9/2}}{9b}-\frac{2\sqrt{a}(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}$$

output
```
2/3*(A*b-B*a)*x^(3/2)/b^2+2/9*B*x^(9/2)/b-2/3*(A*b-B*a)*arctan(x^(3/2)*b^(
1/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

3.155.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx=\frac{2x^{3/2}(3Ab-3aB+bBx^3)}{9b^2}+\frac{2\sqrt{a}(-Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(2*x^(3/2)*(3*A*b - 3*a*B + b*B*x^3))/(9*b^2) + (2*Sqrt[a]*(-(A*b) + a*B)*
ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(5/2))
```

3.155.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {959, 843, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{x^{7/2}}{bx^3+a}dx}{b}+\frac{2Bx^{9/2}}{9b}$$

$$\Big\downarrow \text{843}$$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{bx^3+a}dx}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$$\Big\downarrow \text{851}$$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\int \frac{x}{bx^3+a}d\sqrt{x}}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$$\Big\downarrow \text{807}$$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\int \frac{1}{a+bx}dx^{3/2}}{3b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$$\Big\downarrow \text{218}$$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{3/2}}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

input
```
Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(2*B*x^(9/2))/(9*b) + ((A*b - a*B)*((2*x^(3/2))/(3*b) - (2*Sqrt[a]*ArcTan[
(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(3/2))))/b
```

3.155. $\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}dx$

3.155.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.155. $\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx$

3.155.4 Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{2x^{\frac{3}{2}}(bB\,x^3+3Ab-3Ba)}{9b^2}-\frac{2a(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b^2\sqrt{ab}}$	55
derivativedivides	$\frac{\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}-\frac{2Ba\,x^{\frac{3}{2}}}{3}}{b^2}-\frac{2a(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b^2\sqrt{ab}}$	58
default	$\frac{\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}-\frac{2Ba\,x^{\frac{3}{2}}}{3}}{b^2}-\frac{2a(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b^2\sqrt{ab}}$	58

input
```
int(x^(7/2)*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^(3/2)*(B*b*x^3+3*A*b-3*B*a)/b^2-2/3*a*(A*b-B*a)/b^2/(a*b)^(1/2)*arct
an(b*x^(3/2)/(a*b)^(1/2))
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.96

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = \left[-\frac{3\,(Ba-Ab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^3-2\,bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}-a}{bx^3+a}\right)-2\,(Bbx^4-3\,(Ba-Ab)x)\sqrt{x}}{9\,b^2},\ 2\Big(3\,(\right.$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/9*(3*(B*a - A*b)*sqrt(-a/b)*log((b*x^3 - 2*b*x^(3/2)*sqrt(-a/b) - a)/(
b*x^3 + a)) - 2*(B*b*x^4 - 3*(B*a - A*b)*x)*sqrt(x))/b^2, 2/9*(3*(B*a - A*
b)*sqrt(a/b)*arctan(b*x^(3/2)*sqrt(a/b)/a) + (B*b*x^4 - 3*(B*a - A*b)*x)*s
qrt(x))/b^2]
```

3.155.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(73) = 146$.

Time = 62.16 (sec) , antiderivative size = 428, normalized size of antiderivative = 5.86

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = \begin{cases} \tilde{\infty}\left(\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{9}{2}}}{9}\right) \\ \frac{\frac{2Ax^{\frac{9}{2}}}{9}+\frac{2Bx^{\frac{15}{2}}}{15}}{a} \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{9}{2}}}{9}}{b} \\ -\frac{Aa\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}}+\frac{Aa\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}}+\frac{Aa\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}}- Aa\log\Big(\end{cases}$$

input
```
integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a),x)
```

output
```
Piecewise((zoo*(2*A*x**(3/2)/3 + 2*B*x**(9/2)/9), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(9/2)/9 + 2*B*x**(15/2)/15)/a, Eq(b, 0)), ((2*A*x**(3/2)/3 + 2*B*x*
*(9/2)/9)/b, Eq(a, 0)), (-A*a*log(sqrt(x) - (-a/b)**(1/6))/(3*b**2*sqrt(-a
/b)) + A*a*log(sqrt(x) + (-a/b)**(1/6))/(3*b**2*sqrt(-a/b)) + A*a*log(-4*s
qrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**2*sqrt(-a/b)) - A*a*lo
g(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**2*sqrt(-a/b)) + 2
*A*x**(3/2)/(3*b) + B*a**2*log(sqrt(x) - (-a/b)**(1/6))/(3*b**3*sqrt(-a/b)
) - B*a**2*log(sqrt(x) + (-a/b)**(1/6))/(3*b**3*sqrt(-a/b)) - B*a**2*log(-
4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**3*sqrt(-a/b)) + B*a
**2*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**3*sqrt(-a/b
)) - 2*B*a*x**(3/2)/(3*b**2) + 2*B*x**(9/2)/(9*b), True))
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,(Ba^2-Aab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b^2}+\frac{2\left(Bbx^{\frac{9}{2}}-3\,(Ba-Ab)x^{\frac{3}{2}}\right)}{9\,b^2}$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
2/3*(B*a^2 - A*a*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/9*(B*b
*x^(9/2) - 3*(B*a - A*b)*x^(3/2))/b^2
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,(Ba^2-Aab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}b^2}+\frac{2\left(Bb^2x^{\frac{9}{2}}-3\,Babx^{\frac{3}{2}}+3\,Ab^2x^{\frac{3}{2}}\right)}{9\,b^3}$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*(B*a^2 - A*a*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/9*(B*b
^2*x^(9/2) - 3*B*a*b*x^(3/2) + 3*A*b^2*x^(3/2))/b^3
```

3.155.9 Mupad [B] (verification not implemented)

Time = 7.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.52

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = x^{3/2}\left(\frac{2\,A}{3\,b}-\frac{2\,B\,a}{3\,b^2}\right)+\frac{2\,B\,x^{9/2}}{9\,b} - \frac{2\,\sqrt{a}\,\mathrm{atan}\left(\frac{72\,b^{3/2}\,x^{3/2}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{\sqrt{a}\,\left(72\,A\,a^2\,b^2-72\,B\,a^3\,b\right)\,\left(A\,b-B\,a\right)}\right)\,\left(A\,b-B\,a\right)}{3\,b^{5/2}}$$

input
```
int((x^(7/2)*(A + B*x^3))/(a + b*x^3),x)
```

output
```
x^(3/2)*((2*A)/(3*b) - (2*B*a)/(3*b^2)) + (2*B*x^(9/2))/(9*b) - (2*a^(1/2)
*atan((72*b^(3/2)*x^(3/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/(a^(1/2)*
(72*A*a^2*b^2 - 72*B*a^3*b)*(A*b - B*a)))*(A*b - B*a))/(3*b^(5/2))
```

3.156 $\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx$

3.156.1 Optimal result

Integrand size = 22, antiderivative size = 288

$$\begin{aligned}\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx &= \frac{2(Ab-aB)\sqrt{x}}{b^2}+\frac{2Bx^{7/2}}{7b}\\
&+\frac{\sqrt[6]{a}(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}}-\frac{\sqrt[6]{a}(Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}}\\
&-\frac{2\sqrt[6]{a}(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}}+\frac{\sqrt[6]{a}(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}b^{13/6}}\\
&-\frac{\sqrt[6]{a}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}b^{13/6}}\end{aligned}$$

output

```
2/7*B*x^(7/2)/b-2/3*a^(1/6)*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/b^(1
3/6)-1/3*a^(1/6)*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/b^(1
3/6)-1/3*a^(1/6)*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/b^(13
/6)+1/6*a^(1/6)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(
1/2))/b^(13/6)*3^(1/2)-1/6*a^(1/6)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*
b^(1/6)*3^(1/2)*x^(1/2))/b^(13/6)*3^(1/2)+2*(A*b-B*a)*x^(1/2)/b^2
```

3.156.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.60

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{6\sqrt[6]{b}\sqrt{x}(7Ab-7aB+bBx^3)+14\sqrt[6]{a}(-Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-7\sqrt[6]{a}(-Ab+a}{21b^{13/6}}$$

input
```
Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(6*b^(1/6)*Sqrt[x]*(7*A*b - 7*a*B + b*B*x^3) + 14*a^(1/6)*(-(A*b) + a*B)*A
rcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - 7*a^(1/6)*(-(A*b) + a*B)*ArcTan[(a^(1/3
) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + 7*Sqrt[3]*a^(1/6)*(-(A*b) + a*
B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(21*b
^(13/6))
```

3.156.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {959, 843, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(Ab-aB)\int \frac{x^{5/2}}{bx^3+a}dx}{b}+\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \; 843$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(bx^3+a)}dx}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \; 851$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{bx^3+a}d\sqrt{x}}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \text{ 753}$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \text{ 1142}$$

3.156. $\int \frac{x^{5/2}\left(A+Bx^3\right)}{a+bx^3}\,dx$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+}}{}\right.}{b}\right.}{b}$$

$$\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow 25$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{}}}{}\right.}{b}\right.}{b}$$

$$\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow 27$$

3.156. $\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+}}{}\right.}{b}\right.}{b}$$

$$\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int\frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int\frac{1}{-x-\frac{1}{3}}}{}}{6a^{5/6}}\right.}{b}\right.}{b}$$

$$\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \text{217}$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan}{}}{6a^{5/6}}\right.}{b}\right.}{b}$$

$$\frac{2Bx^{7/2}}{7b}$$

3.156. $\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)\right.}{\sqrt[6]{b}}}{}\right.}{b}\right.}{b}$$

$$\frac{2Bx^{7/2}}{7b}$$

input
```
Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(2*B*x^(7/2))/(7*b) + ((A*b - a*B)*((2*Sqrt[x])/b - (2*a*(ArcTan[(b^(1/6)*
Sqrt[x])/a^(1/6)]/(3*a^(5/6)*b^(1/6)) + (-(ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*
Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(
1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6)) + (ArcTan[Sqrt
[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) + (Sqrt[3]*Log[a^
(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5
/6))))/b))/b
```

3.156.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.156. $\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.156.4 Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.72

method	result
risch	$\frac{2\left(bB\,x^{3}+7Ab-7Ba\right)\sqrt{x}}{7b^{2}}-\frac{a(Ab-Ba)\left(\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3a}+\frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}}\mathrm{arc}}{}\right.}{b^{2}}$
derivativedivides	$\frac{\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}-2Ba\sqrt{x}}{b^{2}}-\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\right.}{6a}\right.}{b^{2}}$
default	$\frac{\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}-2Ba\sqrt{x}}{b^{2}}-\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\right.}{6a}\right.}{b^{2}}$

input
```
int(x^(5/2)*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/7*(B*b*x^3+7*A*b-7*B*a)*x^(1/2)/b^2-a*(A*b-B*a)/b^2*(1/6/a*3^(1/2)*(a/b)
^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/3/a*(a/b)^(1/6)*arc
tan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))+2/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^
(1/6))-1/6/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1
/3))+1/3/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6)))
```

3.156.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1289 vs. $2(204) = 408$.

Time = 0.30 (sec) , antiderivative size = 1289, normalized size of antiderivative = 4.48

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/42*(14*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3
*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*l
og(b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^
3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6) - (B*a -
 A*b)*sqrt(x)) - 14*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 -
20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^1
3)^(1/6)*log(-b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3
*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/
6) - (B*a - A*b)*sqrt(x)) + 7*(sqrt(-3)*b^2 + b^2)*(-(B^6*a^7 - 6*A*B^5*a^
6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5
*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(-2*(B*a - A*b)*sqrt(x) + (sqrt(-3)
*b^2 + b^2)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a
^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)) -
7*(sqrt(-3)*b^2 + b^2)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 2
0*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13
)^(1/6)*log(-2*(B*a - A*b)*sqrt(x) - (sqrt(-3)*b^2 + b^2)*(-(B^6*a^7 - 6*A
*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4
- 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)) + 7*(sqrt(-3)*b^2 - b^2)*(-(B^
6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B
^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(-2*(B*a - A*b...
```

3.156.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(267) = 534$.

Time = 29.28 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.10

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \begin{cases} \tilde{\infty}\left(2A\sqrt{x}+\frac{2Bx^{\frac{7}{2}}}{7}\right) \\ \frac{\frac{2Ax^{\frac{7}{2}}}{7}+\frac{2Bx^{\frac{13}{2}}}{13}}{a} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{7}{2}}}{7}}{b} \\ \frac{2A\sqrt{x}}{b}+\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b}-\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b}+\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+\right.}{6b} \end{cases}$$

input
```
integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a),x)
```

output
```
Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(7/2)/7), Eq(a, 0) & Eq(b, 0)), ((2*A
*x**(7/2)/7 + 2*B*x**(13/2)/13)/a, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(7/2)
/7)/b, Eq(a, 0)), (2*A*sqrt(x)/b + A*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(
1/6))/(3*b) - A*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3*b) + A*(-a/b
)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) - A*(
-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) -
sqrt(3)*A*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)
/3)/(3*b) - sqrt(3)*A*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6
)) + sqrt(3)/3)/(3*b) - 2*B*a*sqrt(x)/b**2 - B*a*(-a/b)**(1/6)*log(sqrt(x)
 - (-a/b)**(1/6))/(3*b**2) + B*a*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6)
)/(3*b**2) - B*a*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/
b)**(1/3))/(6*b**2) + B*a*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x
+ 4*(-a/b)**(1/3))/(6*b**2) + sqrt(3)*B*a*(-a/b)**(1/6)*atan(2*sqrt(3)*sqr
t(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b**2) + sqrt(3)*B*a*(-a/b)**(1/6)*a
tan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b**2) + 2*B*x**(7/
2)/(7*b), True))
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.02

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{\left(\frac{\sqrt{3}(Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}}\right.}{}$$

$$+ \frac{2\left(Bbx^{\frac{7}{2}} - 7\,(Ba-Ab)\sqrt{x}\right)}{7\,b^2}$$

input
```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
 a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(
1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) - A
*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*s
qrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((
sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(
1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*ar
ctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))
/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))*a/b^2 + 2/7*(B*b*x^(7/2) - 7*(B*a - A*
b)*sqrt(x))/b^2
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx &= \frac{\sqrt{3}\left((ab^5)^{\frac{1}{6}}Ba - (ab^5)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,b^3} \\ &- \frac{\sqrt{3}\left((ab^5)^{\frac{1}{6}}Ba - (ab^5)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,b^3} \\ &+ \frac{\left((ab^5)^{\frac{1}{6}}Ba - (ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,b^3} \\ &+ \frac{\left((ab^5)^{\frac{1}{6}}Ba - (ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,b^3} \\ &+ \frac{2\left((ab^5)^{\frac{1}{6}}Ba - (ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,b^3} + \frac{2\left(Bb^6x^{\frac{7}{2}} - 7\,Bab^5\sqrt{x} + 7\,Ab^6\sqrt{x}\right)}{7\,b^7}\end{aligned}$$

input

```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output

```
1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a
/b)^(1/6) + x + (a/b)^(1/3))/b^3 - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5
)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/b^3 + 1/3
*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*s
qrt(x))/(a/b)^(1/6))/b^3 + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arc
tan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/b^3 + 2/3*((a*b^5)^(1/
6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/b^3 + 2/7*(B*b^6*x
^(7/2) - 7*B*a*b^5*sqrt(x) + 7*A*b^6*sqrt(x))/b^7
```

3.156.9 Mupad [B] (verification not implemented)

Time = 7.33 (sec) , antiderivative size = 1933, normalized size of antiderivative = 6.71

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \text{Too large to display}$$

input

```
int((x^(5/2)*(A + B*x^3))/(a + b*x^3),x)
```

output

```
x^(1/2)*((2*A)/b - (2*B*a)/b^2) + (2*B*x^(7/2))/(7*b) + ((-a)^(1/6)*atan((
((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^
6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*(A*b - B*a)
*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/
(3*b^(13/6)) + ((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4
 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1
/6)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))
/b^(19/6))*1i)/(3*b^(13/6)))/(((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^
8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^
3 - (96*(-a)^(1/6)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*
A^2*B*a^5*b^2))/b^(19/6)))/(3*b^(13/6)) - ((-a)^(1/6)*(A*b - B*a)*((96*x^(
1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*
a^5*b^3))/b^3 + (96*(-a)^(1/6)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^
2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6)))/(3*b^(13/6))))*(A*b - B*a)*2i)/(3*b
^(13/6)) + ((-a)^(1/6)*atan((((-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a
)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b
 - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a
)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)
/(3*b^(13/6)) + ((-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*((96*x^(1/2
)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*...
```

3.157 $\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx$

3.157.1 Optimal result

Integrand size = 22, antiderivative size = 270

$$\begin{aligned}\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx &= \frac{2Bx^{5/2}}{5b} - \frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} \\ &+ \frac{(Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} \\ &+ \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}} \\ &- \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}}\end{aligned}$$

output
```
2/5*B*x^(5/2)/b+2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(1/6)/b^(1
1/6)+1/3*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(1/6)/b^(1
1/6)+1/3*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(1/6)/b^(11
/6)+1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^
(1/6)/b^(11/6)*3^(1/2)-1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*
3^(1/2)*x^(1/2))/a^(1/6)/b^(11/6)*3^(1/2)
```

3.157.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.57

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{6\sqrt[6]{a}b^{5/6}Bx^{5/2}+10(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-5(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right) -}{15\sqrt[6]{a}b^{11/6}}$$

input
```
Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(6*a^(1/6)*b^(5/6)*B*x^(5/2) + 10*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(
1/6)] - 5*(A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x
])] - 5*Sqrt[3]*(A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(
1/3) + b^(1/3)*x)])/(15*a^(1/6)*b^(11/6))
```

3.157.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {959, 851, 824, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{x^{3/2}}{bx^3+a}dx}{b}+\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \text{851}$$

$$\frac{2(Ab-aB)\int \frac{x^2}{bx^3+a}d\sqrt{x}}{b}+\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \text{824}$$

$$\frac{2(Ab-aB)\left(\frac{\int\frac{1}{\sqrt[3]{b}_{x+}\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}+\frac{\int-\frac{\sqrt[6]{a}_{-\sqrt{3}}\sqrt[6]{b}_{\sqrt{x}}}{2\left(\sqrt[3]{b}_{x-\sqrt{3}}\sqrt[6]{a}\sqrt[6]{b}_{\sqrt{x}+}\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}+\frac{\int-\frac{\sqrt{3}\sqrt[6]{b}_{\sqrt{x}+}\sqrt[6]{a}}{2\left(\sqrt[3]{b}_{x+\sqrt{3}}\sqrt[6]{a}\sqrt[6]{b}_{\sqrt{x}+}\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}\right)}{b}+\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2(Ab-aB)\left(\frac{\int\frac{1}{\sqrt[3]{b}_{x+}\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}-\frac{\int\frac{\sqrt[6]{a}_{-\sqrt{3}}\sqrt[6]{b}_{\sqrt{x}}}{\sqrt[3]{b}_{x-\sqrt{3}}\sqrt[6]{a}\sqrt[6]{b}_{\sqrt{x}+}\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int\frac{\sqrt{3}\sqrt[6]{b}_{\sqrt{x}+}\sqrt[6]{a}}{\sqrt[3]{b}_{x+\sqrt{3}}\sqrt[6]{a}\sqrt[6]{b}_{\sqrt{x}+}\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}\right)}{b}+\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \text{218}$$

$$\frac{2(Ab-aB)\left(-\frac{\int\frac{\sqrt[6]{a}_{-\sqrt{3}}\sqrt[6]{b}_{\sqrt{x}}}{\sqrt[3]{b}_{x-\sqrt{3}}\sqrt[6]{a}\sqrt[6]{b}_{\sqrt{x}+}\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int\frac{\sqrt{3}\sqrt[6]{b}_{\sqrt{x}+}\sqrt[6]{a}}{\sqrt[3]{b}_{x+\sqrt{3}}\sqrt[6]{a}\sqrt[6]{b}_{\sqrt{x}+}\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}+\frac{\arctan\left(\frac{\sqrt[6]{b}_{\sqrt{x}}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}\right)}{b}+\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{2(Ab-aB)\left(-\frac{-\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}_{x-\sqrt{3}}\sqrt[6]{a}\sqrt[6]{b}_{\sqrt{x}+}\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int-\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}_{\sqrt{x}}\right)}{\sqrt[3]{b}_{x-\sqrt{3}}\sqrt[6]{a}\sqrt[6]{b}_{\sqrt{x}+}\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(2\sqrt[6]{b}_{\sqrt{x}+\sqrt{3}}\sqrt[6]{a}\right)}{\sqrt[3]{b}_{x+\sqrt{3}}\sqrt[6]{a}\sqrt[6]{b}_{\sqrt{x}+}\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6}\right.}{b}$$

$$\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \text{25}$$

3.157. $\int\frac{x^{3/2}\left(A+Bx^3\right)}{a+bx^3}\,dx$

$$\frac{2(Ab-aB)\left(-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}}{6\sqrt[6]{a}}\right.}{b}$$

$$\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2(Ab-aB)\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d}{6}\right.}{b}$$

$$\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2(Ab-aB)\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}\right.}{b}$$

$$\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \text{217}$$

$$\frac{2(Ab-aB)\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\right.}{}}{6\sqrt[6]{a}b^{2/3}}\right.}{b}$$

$$\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \text{1103}$$

3.157. $\int \frac{x^{3/2}\left(A+Bx^3\right)}{a+bx^3}\,dx$

$$\frac{2(Ab-aB)\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}\right.}{2\sqrt[6]{b}}}{6\sqrt[6]{a}}\right.}{b}$$

$$\frac{2Bx^{5/2}}{5b}$$

input
```
Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(2*B*x^(5/2))/(5*b) + (2*(A*b - a*B)*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3
*a^(1/6)*b^(5/6)) - (ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1
/6)))]/b^(1/6) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] +
b^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3)) - (-(ArcTan[Sqrt[3]*(1 + (2*b
^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) + (Sqrt[3]*Log[a^(1/3) + Sqrt
[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3))
))/b
```

3.157.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.157. $\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx$

3.157.4 Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.70

method	result
risch	$\frac{2B\,x^{\frac{5}{2}}}{5b}+\frac{(Ab-Ba)\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{2\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\right.}{b}$
derivativedivides	$\frac{2B\,x^{\frac{5}{2}}}{5b}+\frac{2\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\right.}{b}$
default	$\frac{2B\,x^{\frac{5}{2}}}{5b}+\frac{2\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\right.}{b}$

input
```
int(x^(3/2)*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/5*B*x^(5/2)/b+(A*b-B*a)/b*(-1/6/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)
^(1/6)*x^(1/2)+(a/b)^(1/3))+1/3/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)
+3^(1/2))+2/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))+1/6/a*3^(1/2)*(a/b
)^(5/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/3/b/(a/b)^(1/6)*ar
ctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6)))
```

3.157.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(188) = 376$.

Time = 0.31 (sec) , antiderivative size = 1603, normalized size of antiderivative = 5.94

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/30*(12*B*x^(5/2) + 10*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2
- 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^
11))^(1/6)*log(a*b^9*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*
A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^
(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3
 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - 10*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1
5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^
5 + A^6*b^6)/(a*b^11))^(1/6)*log(-a*b^9*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^
2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
A^6*b^6)/(a*b^11))^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 -
 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - 5*(sqrt(-3)*b -
b)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*log((sqrt(-3
)*a*b^9 + a*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*
B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6
) - 2*(B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 +
 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) + 5*(sqrt(-3)*b - b)*(-(B^6*a^6 - 6*A*B
^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*log(-(sqrt(-3)*a*b^9 + a*b^9)*(-(
B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*...
```

3.157.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(258) = 516$.

Time = 11.67 (sec) , antiderivative size = 581, normalized size of antiderivative = 2.15

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{5}{2}}}{5}\right) \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{11}{2}}}{11}}{a} \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{5}{2}}}{5}}{b} \\ \frac{A\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt[6]{-\frac{a}{b}}} - \frac{A\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt[6]{-\frac{a}{b}}} + \frac{A\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6b\sqrt[6]{-\frac{a}{b}}} - \end{cases}$$

input
```
integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a),x)
```

output
```
Piecewise((zoo*(-2*A/sqrt(x) + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((2*
A*x**(5/2)/5 + 2*B*x**(11/2)/11)/a, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*x**(5/
2)/5)/b, Eq(a, 0)), (A*log(sqrt(x) - (-a/b)**(1/6))/(3*b*(-a/b)**(1/6)) -
A*log(sqrt(x) + (-a/b)**(1/6))/(3*b*(-a/b)**(1/6)) + A*log(-4*sqrt(x)*(-a/
b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) - A*log(4*sqrt(x)*(
-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) + sqrt(3)*A*atan
(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b*(-a/b)**(1/6)) + sq
rt(3)*A*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b*(-a/b)*
*(1/6)) - B*a*log(sqrt(x) - (-a/b)**(1/6))/(3*b**2*(-a/b)**(1/6)) + B*a*lo
g(sqrt(x) + (-a/b)**(1/6))/(3*b**2*(-a/b)**(1/6)) - B*a*log(-4*sqrt(x)*(-a
/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b**2*(-a/b)**(1/6)) + B*a*log(4*sqr
t(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b**2*(-a/b)**(1/6)) - sqrt(
3)*B*a*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b**2*(-a/b
)**(1/6)) - sqrt(3)*B*a*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)
/3)/(3*b**2*(-a/b)**(1/6)) + 2*B*x**(5/2)/(5*b), True))
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.79

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,Bx^{\frac{5}{2}}}{5\,b}$$
$$+\frac{(Ba-Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\sqrt{3}a\right.}{b^{\frac{2}{3}}\sqrt{a}}\right.}{6\,b}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
2/5*B*x^(5/2)/b + 1/6*(B*a - A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqr
t(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/
6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sq
rt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)
*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*s
qrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(
b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/b
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx &= \frac{2\,Bx^{\frac{5}{2}}}{5\,b} - \frac{(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,(ab^5)^{\frac{1}{6}}\,b} \\ &\quad - \frac{(Ba-Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,(ab^5)^{\frac{1}{6}}\,b} - \frac{2\left(Ba\left(\frac{a}{b}\right)^{\frac{5}{6}} - Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab} \\ &\quad + \frac{\sqrt{3}\left((ab^5)^{\frac{5}{6}}\,Ba - (ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^6} \\ &\quad - \frac{\sqrt{3}\left((ab^5)^{\frac{5}{6}}\,Ba - (ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^6}\end{aligned}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/5*B*x^(5/2)/b - 1/3*(B*a - A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))
/(a/b)^(1/6))/((a*b^5)^(1/6)*b) - 1/3*(B*a - A*b)*arctan(-(sqrt(3)*(a/b)^(
1/6) - 2*sqrt(x))/(a/b)^(1/6))/((a*b^5)^(1/6)*b) - 2/3*(B*a*(a/b)^(5/6) -
A*b*(a/b)^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/(a*b) + 1/6*sqrt(3)*((a*b^5)^
(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)
^(1/3))/(a*b^6) - 1/6*sqrt(3)*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(
-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^6)
```

3.157.9 Mupad [B] (verification not implemented)

Time = 7.26 (sec) , antiderivative size = 1640, normalized size of antiderivative = 6.07

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
int((x^(3/2)*(A + B*x^3))/(a + b*x^3),x)
```

output

```
(2*B*x^(5/2))/(5*b) + (atan((((A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6 +
 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4
 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a
)^(1/3)*b^(11/3)) + ((A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2
*a^5*b + 96*A^2*B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^
2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b
^(11/3)))/(((A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6 + 96*A*B^2*a^5*b -
96*A^2*B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2
 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3)) - (
(A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B*a^4
*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*
a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3))))*(A*b - B*a)*2
i)/(3*(-a)^(1/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^
2*(32*A^3*a^3*b^3 - 32*B^3*a^6 + 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1
/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2
- 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3)) +
 (((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96
*A*B^2*a^5*b + 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B
*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)
*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3)))/((((3^(1/2)*1i)/2 - 1/2)^2*(A*b ...
```

3.157. $\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx$

3.158 $\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx$

3.158.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx=\frac{2Bx^{3/2}}{3b}+\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}$$

output
```
2/3*B*x^(3/2)/b+2/3*(A*b-B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/b^(3/2)/a^(1
/2)
```

3.158.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx=\frac{2Bx^{3/2}}{3b}-\frac{2(-Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(2*B*x^(3/2))/(3*b) - (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])
/(3*Sqrt[a]*b^(3/2))
```

3.158.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {959, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}\left(A+Bx^3\right)}{a+bx^3}dx$$

$$\Big\downarrow \; 959$$

$$\frac{(Ab-aB)\int \frac{\sqrt{x}}{bx^3+a}dx}{b}+\frac{2Bx^{3/2}}{3b}$$

$$\Big\downarrow \; 851$$

$$\frac{2(Ab-aB)\int \frac{x}{bx^3+a}d\sqrt{x}}{b}+\frac{2Bx^{3/2}}{3b}$$

$$\Big\downarrow \; 807$$

$$\frac{2(Ab-aB)\int \frac{1}{a+bx}dx^{3/2}}{3b}+\frac{2Bx^{3/2}}{3b}$$

$$\Big\downarrow \; 218$$

$$\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}+\frac{2Bx^{3/2}}{3b}$$

input
```
Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]
```

output
```
(2*B*x^(3/2))/(3*b) + (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3
*Sqrt[a]*b^(3/2))
```

3.158.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.158.4 Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2 B x^{\frac{3}{2}}}{3 b}+\frac{2 (A b -B a) \arctan\left(\frac{b x^{\frac{3}{2}}}{\sqrt{a b}}\right)}{3 b \sqrt{a b}}$	40
default	$\frac{2 B x^{\frac{3}{2}}}{3 b}+\frac{2 (A b -B a) \arctan\left(\frac{b x^{\frac{3}{2}}}{\sqrt{a b}}\right)}{3 b \sqrt{a b}}$	40
risch	$\frac{2 B x^{\frac{3}{2}}}{3 b}+\frac{2 (A b -B a) \arctan\left(\frac{b x^{\frac{3}{2}}}{\sqrt{a b}}\right)}{3 b \sqrt{a b}}$	40

input
```
int((B*x^3+A)*x^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*B*x^(3/2)/b+2/3*(A*b-B*a)/b/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \left[\frac{2\,Babx^{\frac{3}{2}} + (Ba-Ab)\sqrt{-ab}\log\left(\frac{bx^3-2\,\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)}{3\,ab^2}, \frac{2\left(Babx^{\frac{3}{2}} - (Ba-Ab)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x^{\frac{3}{2}}}{a}\right)\right)}{3\,ab^2}\right]$$

input
```
integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/3*(2*B*a*b*x^(3/2) + (B*a - A*b)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x
^(3/2) - a)/(b*x^3 + a)))/(a*b^2), 2/3*(B*a*b*x^(3/2) - (B*a - A*b)*sqrt(a
*b)*arctan(sqrt(a*b)*x^(3/2)/a))/(a*b^2)]
```

3.158.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(51) = 102.

Time = 4.68 (sec) , antiderivative size = 381, normalized size of antiderivative = 7.19

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}} + \frac{2Bx^{\frac{3}{2}}}{3}\right) \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{9}{2}}}{9}}{a} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}}{b} \\ \frac{A\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}} - \frac{A\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}} - \frac{A\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}} + \frac{A\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}} - B \end{cases}$$

input
```
integrate((B*x**3+A)*x**(1/2)/(b*x**3+a),x)
```

output
```
Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*x**(3/2)/3), Eq(a, 0) & Eq(b, 0)),
 ((2*A*x**(3/2)/3 + 2*B*x**(9/2)/9)/a, Eq(b, 0)), ((-2*A/(3*x**(3/2)) + 2*
B*x**(3/2)/3)/b, Eq(a, 0)), (A*log(sqrt(x) - (-a/b)**(1/6))/(3*b*sqrt(-a/b
)) - A*log(sqrt(x) + (-a/b)**(1/6))/(3*b*sqrt(-a/b)) - A*log(-4*sqrt(x)*(-
a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b*sqrt(-a/b)) + A*log(4*sqrt(x)*(-
a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b*sqrt(-a/b)) - B*a*log(sqrt(x) -
(-a/b)**(1/6))/(3*b**2*sqrt(-a/b)) + B*a*log(sqrt(x) + (-a/b)**(1/6))/(3*b
**2*sqrt(-a/b)) + B*a*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3)
)/(3*b**2*sqrt(-a/b)) - B*a*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**
(1/3))/(3*b**2*sqrt(-a/b)) + 2*B*x**(3/2)/(3*b), True))
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,b} - \frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b}$$

input
```
integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
2/3*B*x^(3/2)/b - 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b
)
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,b} - \frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b}$$

input
```
integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*B*x^(3/2)/b - 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b
)
```

3.158.9 Mupad [B] (verification not implemented)

Time = 7.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,B\,x^{3/2}}{3\,b} - \frac{2\,\mathrm{atan}\left(\frac{3\,\sqrt{a}\,b^{3/2}\,x^{3/2}\,\left(24\,A^2\,b^3-48\,A\,B\,a\,b^2+24\,B^2\,a^2\,b\right)}{\left(72\,B\,a^2\,b^2-72\,A\,a\,b^3\right)\,\left(A\,b-B\,a\right)}\right)\,\left(A\,b-B\,a\right)}{3\,\sqrt{a}\,b^{3/2}}$$

input
```
int((x^(1/2)*(A + B*x^3))/(a + b*x^3),x)
```

output
```
(2*B*x^(3/2))/(3*b) - (2*atan((3*a^(1/2)*b^(3/2)*x^(3/2)*(24*A^2*b^3 + 24*
B^2*a^2*b - 48*A*B*a*b^2))/((72*B*a^2*b^2 - 72*A*a*b^3)*(A*b - B*a)))*(A*b
 - B*a))/(3*a^(1/2)*b^(3/2))
```

3.159 $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)}\,dx$

3.159.1 Optimal result

Integrand size = 22, antiderivative size = 268

$$\begin{aligned}\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)}\,dx &= \frac{2B\sqrt{x}}{b}-\frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}\\ &\quad+\frac{(Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}+\frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}\\ &\quad-\frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}\\ &\quad+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}\end{aligned}$$

output

```
2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(7/6)+1/3*(A*b-B*a
)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(7/6)+1/3*(A*b-B*a)
*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(7/6)-1/6*(A*b-B*a)*l
n(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(5/6)/b^(7/6)*3^(1/
2)+1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(
5/6)/b^(7/6)*3^(1/2)+2*B*x^(1/2)/b
```

3.159.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx$$
$$= \frac{6a^{5/6}\sqrt[6]{b}B\sqrt{x}+2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)+\sqrt{3}(Ab-aB)\text{arctanh}(}{3a^{5/6}b^{7/6}}$$

input
```
Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)),x]
```

output
```
(6*a^(5/6)*b^(1/6)*B*Sqrt[x] + 2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1
/6)] - (A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])]
 + Sqrt[3]*(A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3)
+ b^(1/3)*x)])/(3*a^(5/6)*b^(7/6))
```

3.159.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {959, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx$$
$$\Big\downarrow \text{959}$$
$$\frac{(Ab-aB)\int \frac{1}{\sqrt{x}(bx^3+a)}dx}{b}+\frac{2B\sqrt{x}}{b}$$
$$\Big\downarrow \text{851}$$
$$\frac{2(Ab-aB)\int \frac{1}{bx^3+a}d\sqrt{x}}{b}+\frac{2B\sqrt{x}}{b}$$
$$\Big\downarrow \text{753}$$

$$\frac{2(Ab-aB)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{b}+\frac{2B\sqrt{x}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{2(Ab-aB)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{b}+\frac{2B\sqrt{x}}{b}$$

$$\Big\downarrow \; 218$$

$$\frac{2(Ab-aB)\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{b}+\frac{2B\sqrt{x}}{b}$$

$$\Big\downarrow \; 1142$$

$$\frac{2(Ab-aB)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5}}\right.}{b}$$

$$\frac{2B\sqrt{x}}{b}$$

$$\Big\downarrow \; 25$$

3.159. $\int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)}\,dx$

$$\frac{2(Ab-aB)\Bigg(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+}{6a^{5/6}}}{b}$$

$$\frac{2B\sqrt{x}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{2(Ab-aB)\Bigg(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}}{}}{b}$$

$$\frac{2B\sqrt{x}}{b}$$

$$\Big\downarrow \; 1082$$

$$\frac{2(Ab-aB)\Bigg(\frac{\frac{\int\frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-}{6a^{5/6}}}{b}$$

$$\frac{2B\sqrt{x}}{b}$$

$$\Big\downarrow \; 217$$

$$\frac{2(Ab-aB)\Bigg(\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+}{6a^{5/6}}}{b}$$

$$\frac{2B\sqrt{x}}{b}$$

$$\Big\downarrow \; 1103$$

3.159. $\int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)}\,dx$

$$\frac{2(Ab-aB)\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{b}}+\frac{\sqrt{3}}{}}{6}\right.}{b}$$

$$\frac{2B\sqrt{x}}{b}$$

input
```
Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)),x]
```

output
```
(2*B*Sqrt[x])/b + (2*(A*b - a*B)*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(
5/6)*b^(1/6)) + (-(ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6
)))]/b^(1/6)) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b
^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6)) + (ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt
[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*
b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6))))/b
```

3.159.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.159. $\int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)}\,dx$

3.159.4 Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.71

method	result
risch	$\frac{2B\sqrt{x}}{b} + \frac{(Ab-Ba)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3a} + \frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a} - \frac{\sqrt{3}\,(}{}\right.}{b}$
derivativedivides	$\frac{2B\sqrt{x}}{b} + \frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a} - \frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6a} + \frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln}{}\right.}{b}$
default	$\frac{2B\sqrt{x}}{b} + \frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a} - \frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6a} + \frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln}{}\right.}{b}$

input
```
int((B*x^3+A)/(b*x^3+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*B*x^(1/2)/b+(A*b-B*a)/b*(1/6/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1
/6)*x^(1/2)+(a/b)^(1/3))+1/3/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^
(1/2))+2/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1/6/a*3^(1/2)*(a/b)^(
1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/3/a*(a/b)^(1/6)*arcta
n(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6)))
```

3.159.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1245 vs. 2(188) = 376.

Time = 0.30 (sec) , antiderivative size = 1245, normalized size of antiderivative = 4.65

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(a
*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6) - (B*a - A*
b)*sqrt(x)) - 2*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3
*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1
/6)*log(-a*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*
a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6) -
 (B*a - A*b)*sqrt(x)) + (sqrt(-3)*b + b)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A
^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
 A^6*b^6)/(a^5*b^7))^(1/6)*log(-2*(B*a - A*b)*sqrt(x) + (sqrt(-3)*a*b + a*
b)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)) - (sqrt(-3
)*b + b)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(-
2*(B*a - A*b)*sqrt(x) - (sqrt(-3)*a*b + a*b)*(-(B^6*a^6 - 6*A*B^5*a^5*b +
15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b
^5 + A^6*b^6)/(a^5*b^7))^(1/6)) + (sqrt(-3)*b - b)*(-(B^6*a^6 - 6*A*B^5*a^
5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5
*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(-2*(B*a - A*b)*sqrt(x) + (sqrt...
```

3.159.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(257) = 514$.

Time = 5.31 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.08

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{2A}{5x^{\frac{5}{2}}}+2B\sqrt{x}\right) \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}}+2B\sqrt{x}}{b} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{7}{2}}}{7}}{a} \\ -\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3a}+\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3a}-\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6a}+A\sqrt[6]{-\frac{a}{b}}\log\left(4\right. \end{cases}$$

input
```
integrate((B*x**3+A)/(b*x**3+a)/x**(1/2),x)
```

output
```
Piecewise((zoo*(-2*A/(5*x**(5/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((
-2*A/(5*x**(5/2)) + 2*B*sqrt(x))/b, Eq(a, 0)), ((2*A*sqrt(x) + 2*B*x**(7/2
)/7)/a, Eq(b, 0)), (-A*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(3*a) +
A*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3*a) - A*(-a/b)**(1/6)*log(-
4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a) + A*(-a/b)**(1/6)*l
og(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a) + sqrt(3)*A*(-a/
b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a) + sq
rt(3)*A*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3
)/(3*a) + 2*B*sqrt(x)/b + B*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(3*
b) - B*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3*b) + B*(-a/b)**(1/6)*
log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) - B*(-a/b)**(1
/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) - sqrt(3)*B
*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b)
 - sqrt(3)*B*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt
(3)/3)/(3*b), True))
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = \frac{2\,B\sqrt{x}}{b}$$

$$-\frac{\frac{\sqrt{3}(Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}(Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+}{6\,b}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="maxima")
```

output
```
2*B*sqrt(x)/b - 1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(
x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqr
t(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*
(B*a*b^(1/3) - A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a
^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b
^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*
b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(
1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a
^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/b
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = \frac{2\,B\sqrt{x}}{b} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}\,Ba - \left(ab^5\right)^{\frac{1}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^2} + \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}\,Ba - \left(ab^5\right)^{\frac{1}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^2} - \frac{\left(\left(ab^5\right)^{\frac{1}{6}}\,Ba - \left(ab^5\right)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^2} - \frac{\left(\left(ab^5\right)^{\frac{1}{6}}\,Ba - \left(ab^5\right)^{\frac{1}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^2} - \frac{2\left(\left(ab^5\right)^{\frac{1}{6}}\,Ba - \left(ab^5\right)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^2}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="giac")
```

output
```
2*B*sqrt(x)/b - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sq
rt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^2) + 1/6*sqrt(3)*((a*b^5
)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a
/b)^(1/3))/(a*b^2) - 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((s
qrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^2) - 1/3*((a*b^5)^(1/6)*
B*a - (a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(
1/6))/(a*b^2) - 2/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)
/(a/b)^(1/6))/(a*b^2)
```

3.159.9 Mupad [B] (verification not implemented)

Time = 7.34 (sec) , antiderivative size = 1915, normalized size of antiderivative = 7.15

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^3)/(x^(1/2)*(a + b*x^3)),x)
```

output

```
(2*B*x^(1/2))/b + (atan((((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^
2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((A*b - B*a)*(288*A^3*a
*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(
5/6)*b^(7/6)))*(A*b - B*a)*1i)/(3*(-a)^(5/6)*b^(7/6)) + ((x^(1/2)*(96*A^4*
b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3
*b^2) + ((A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3
- 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*(A*b - B*a)*1i)/(3*(-a)^(5/6
)*b^(7/6)))/(((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 -
384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((A*b - B*a)*(288*A^3*a*b^5 - 288*B
^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)
))*(A*b - B*a))/(3*(-a)^(5/6)*b^(7/6)) - ((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^
4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + ((A*b -
 B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2
*b^4))/(3*(-a)^(5/6)*b^(7/6)))*(A*b - B*a))/(3*(-a)^(5/6)*b^(7/6))))*(A*b
- B*a)*2i)/(3*(-a)^(5/6)*b^(7/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)*(A*b -
B*a)*(x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B
*a*b^4 - 384*A*B^3*a^3*b^2) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3
*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)
^(5/6)*b^(7/6)))*1i)/(3*(-a)^(5/6)*b^(7/6)) + (((3^(1/2)*1i)/2 - 1/2)*(A*b
 - B*a)*(x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384...
```

3.159. $\int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)}\,dx$

3.160 $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx$

3.160.1 Optimal result

Integrand size = 22, antiderivative size = 268

$$\begin{aligned}\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx=&-\frac{2A}{a\sqrt{x}}+\frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}\\&-\frac{(Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}-\frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}\\&-\frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}\\&+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}\end{aligned}$$

output
```
-2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(7/6)/b^(5/6)-1/3*(A*b-B*
a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(7/6)/b^(5/6)-1/3*(A*b-B*a
)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(7/6)/b^(5/6)-1/6*(A*b-B*a)*
ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(7/6)/b^(5/6)*3^(1
/2)+1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^
(7/6)/b^(5/6)*3^(1/2)-2*A/a/x^(1/2)
```

3.160.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx=\frac{-\frac{6\sqrt[6]{a}A}{\sqrt{x}}+\frac{2(-Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{b^{5/6}}+\frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{b^{5/6}}+\frac{\sqrt{3}(Ab-aB)\operatorname{arctanh}(}{b^{5/6}}}{3a^{7/6}}$$

input
```
Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)),x]
```

output
```
((-6*a^(1/6)*A)/Sqrt[x] + (2*(-(A*b) + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/
6)])/b^(5/6) + ((A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*
Sqrt[x])])/b^(5/6) + (Sqrt[3]*(A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)
*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/b^(5/6))/(3*a^(7/6))
```

3.160.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {955, 851, 824, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(Ab-aB)\int \frac{x^{3/2}}{bx^3+a}dx}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{851}$$

$$-\frac{2(Ab-aB)\int \frac{x^2}{bx^3+a}d\sqrt{x}}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{824}$$

$$-\frac{2(Ab-aB)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}+\frac{\int -\frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}+\frac{\int -\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}\right)}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2(Ab-aB)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}\right)}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \; 218$$

$$-\frac{2(Ab-aB)\left(-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}\right)}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \; 1142$$

$$-\frac{2(Ab-aB)\left(-\frac{-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d}{2\sqrt[6]{b}}\right.}{a}$$

$$\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \; 25$$

3.160. $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx$

$$-\frac{2(Ab-aB)\left(-\frac{\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-}{6\sqrt[6]{}}\right.}{a}$$

$$\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow 27$$

$$-\frac{2(Ab-aB)\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}}{}\right.}{a}$$

$$\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow 1082$$

$$-\frac{2(Ab-aB)\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int\frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\int\frac{1}{-x-\frac{1}{3}}d\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}\right.}{a}$$

$$\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow 217$$

$$-\frac{2(Ab-aB)\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\right.}{}}{6\sqrt[6]{a}b^{2/3}}\right.}{a}$$

$$\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow 1103$$

3.160. $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx$

$$-\frac{2(Ab-aB)\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\right.}{2\sqrt[6]{b}}}{6}\right.}{a}$$

$$\frac{2A}{a\sqrt{x}}$$

input
```
Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)),x]
```

output
```
(-2*A)/(a*Sqrt[x]) - (2*(A*b - a*B)*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*
a^(1/6)*b^(5/6)) - (ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/
6)))]/b^(1/6) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b
^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3)) - (-(ArcTan[Sqrt[3]*(1 + (2*b^
(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) + (Sqrt[3]*Log[a^(1/3) + Sqrt[
3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3)))
)/a
```

3.160.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.160. $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx$

3.160.4 Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{}}{\left(\frac{a}{b}\right)}\right)}{6b\left(\frac{a}{b}\right)}\right)}{a}$
default	$-\frac{2\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{}}{\left(\frac{a}{b}\right)}\right)}{6b\left(\frac{a}{b}\right)}\right)}{a}$
risch	$-\frac{2A}{a\sqrt{x}}-\frac{(Ab-Ba)\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{2\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\right.}{}\right)}{a}$

input
```
int((B*x^3+A)/x^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-2*(1/12/a*3^(1/2)*(a/b)^(5/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3
))+1/6/b/(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)
*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/
6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))+1/3/b/(a/b)^(1/6)*arctan(x^(1/2)/
(a/b)^(1/6)))*(A*b-B*a)/a-2*A/a/x^(1/2)
```

3.160.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1636 vs. 2(188) = 376.

Time = 0.28 (sec) , antiderivative size = 1636, normalized size of antiderivative = 6.10

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx=\text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*(2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a
^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*lo
g(a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3
*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6) - (B
^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 + 5*A^4*B
*a*b^4 - A^5*b^5)*sqrt(x)) - 2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4
*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b
^6)/(a^7*b^5))^(1/6)*log(-a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*
a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^
6)/(a^7*b^5))^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A
^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - (sqrt(-3)*a*x - a*x)*
(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*
A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*log((sqrt(-3)*
a^6*b^4 + a^6*b^4)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^
3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(
5/6) - 2*(B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^
3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) + (sqrt(-3)*a*x - a*x)*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b
^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*log(-(sqrt(-3)*a^6*b^4 + a^
6*b^4)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3...
```

3.160.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(257) = 514$.

Time $= 9.21$ (sec) , antiderivative size $= 561$, normalized size of antiderivative $= 2.09$

$$\int \frac{A+Bx^3}{x^{3/2}\,(a+bx^3)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{\sqrt{x}}\right) \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{\sqrt{x}}}{b} \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{5}{2}}}{5}}{a} \\ -\frac{A\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt[6]{-\frac{a}{b}}}+\frac{A\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt[6]{-\frac{a}{b}}}-\frac{A\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6a\sqrt[6]{-\frac{a}{b}}}+ \end{cases}$$

input
```
integrate((B*x**3+A)/x**(3/2)/(b*x**3+a),x)
```

output
```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((
-2*A/(7*x**(7/2)) - 2*B/sqrt(x))/b, Eq(a, 0)), ((-2*A/sqrt(x) + 2*B*x**(5/
2)/5)/a, Eq(b, 0)), (-A*log(sqrt(x) - (-a/b)**(1/6))/(3*a*(-a/b)**(1/6)) +
 A*log(sqrt(x) + (-a/b)**(1/6))/(3*a*(-a/b)**(1/6)) - A*log(-4*sqrt(x)*(-a
/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a*(-a/b)**(1/6)) + A*log(4*sqrt(x)*
(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a*(-a/b)**(1/6)) - sqrt(3)*A*ata
n(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a*(-a/b)**(1/6)) - s
qrt(3)*A*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*a*(-a/b)
**(1/6)) - 2*A/(a*sqrt(x)) + B*log(sqrt(x) - (-a/b)**(1/6))/(3*b*(-a/b)**(
1/6)) - B*log(sqrt(x) + (-a/b)**(1/6))/(3*b*(-a/b)**(1/6)) + B*log(-4*sqrt
(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) - B*log(4*s
qrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) + sqrt(3
)*B*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b*(-a/b)**(1/
6)) + sqrt(3)*B*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b
*(-a/b)**(1/6)), True))
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx = -\frac{(Ba-Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\sqrt{3}a\right.}{b^{\frac{2}{3}}\sqrt{a}}\right.}{6a}-\frac{2A}{a\sqrt{x}}$$

input
```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/6*(B*a - A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x
+ a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x
) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(
1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(
1/3))) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1
/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/s
qrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a - 2*A/(a*sqrt(x))
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx = \frac{(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,\left(ab^5\right)^{\frac{1}{6}}\,a} + \frac{(Ba-Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,\left(ab^5\right)^{\frac{1}{6}}\,a} + \frac{2\left(Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}-Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^2} - \frac{2\,A}{a\sqrt{x}} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}\,Ba-\left(ab^5\right)^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^2b^5} + \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}\,Ba-\left(ab^5\right)^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^2b^5}$$

input

```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="giac")
```

output

```
1/3*(B*a - A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/((a*
b^5)^(1/6)*a) + 1/3*(B*a - A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/
(a/b)^(1/6))/((a*b^5)^(1/6)*a) + 2/3*(B*a*(a/b)^(5/6) - A*b*(a/b)^(5/6))*a
rctan(sqrt(x)/(a/b)^(1/6))/a^2 - 2*A/(a*sqrt(x)) - 1/6*sqrt(3)*((a*b^5)^(5
/6)*B*a - (a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(
1/3))/(a^2*b^5) + 1/6*sqrt(3)*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(
-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^5)
```

3.160.9 Mupad [B] (verification not implemented)

Time = 7.29 (sec) , antiderivative size = 1700, normalized size of antiderivative = 6.34

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx = \text{Too large to display}$$

input

```
int((A + B*x^3)/(x^(3/2)*(a + b*x^3)),x)
```

output

```
(atan((((A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^
4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a
^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(
5/3)) + ((A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b
^4 - 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*
a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^
(5/3)))/(((A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*
b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2
*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5
/3)) - ((A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^
4 - 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a
^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3
))))*(A*b - B*a)*2i)/(3*(-a)^(7/6)*b^(5/6)) - (2*A)/(a*x^(1/2)) + (atan(((
((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 -
 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(
A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*
(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*
(A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A
^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^10*
b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1...
```

3.160. $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx$

3.161 $\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)}\,dx$

3.161.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx=-\frac{2A}{3ax^{3/2}}-\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}$$

output
```
-2/3*A/a/x^(3/2)-2/3*(A*b-B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(3/2)/b^(
1/2)
```

3.161.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx=-\frac{2A}{3ax^{3/2}}+\frac{2(-Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)),x]
```

output
```
(-2*A)/(3*a*x^(3/2)) + (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]]
)/(3*a^(3/2)*Sqrt[b])
```

3.161.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(Ab-aB)\int \frac{\sqrt{x}}{bx^3+a}dx}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \ 851$$

$$-\frac{2(Ab-aB)\int \frac{x}{bx^3+a}d\sqrt{x}}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \ 807$$

$$-\frac{2(Ab-aB)\int \frac{1}{a+bx}dx^{3/2}}{3a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \ 218$$

$$-\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}-\frac{2A}{3ax^{3/2}}$$

input
```
Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)),x]
```

output
```
(-2*A)/(3*a*x^(3/2)) - (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(
3*a^(3/2)*Sqrt[b])
```

3.161.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.161.4 Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3a\sqrt{ab}}$	40
default	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3a\sqrt{ab}}$	40
risch	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3a\sqrt{ab}}$	40

input
```
int((B*x^3+A)/x^(5/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*A/a/x^(3/2)-2/3*(A*b-B*a)/a/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.26

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx = \left[\frac{(Ba-Ab)\sqrt{-ab}x^2\log\left(\frac{bx^3+2\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\,Aab\sqrt{x}}{3\,a^2bx^2}, \frac{2\left((Ba-Ab)\sqrt{ab}x^2\arctan\left(\right.\right.}{3\,a^2bx^2}\right.$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/3*((B*a - A*b)*sqrt(-a*b)*x^2*log((b*x^3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b
*x^3 + a)) - 2*A*a*b*sqrt(x))/(a^2*b*x^2), 2/3*((B*a - A*b)*sqrt(a*b)*x^2*
arctan(sqrt(a*b)*x^(3/2)/a) - A*a*b*sqrt(x))/(a^2*b*x^2)]
```

3.161.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(53) = 106.

Time = 19.50 (sec) , antiderivative size = 371, normalized size of antiderivative = 7.00

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) \\ \frac{-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}}{a} \\ -\frac{A\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt{-\frac{a}{b}}}+\frac{A\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt{-\frac{a}{b}}}+\frac{A\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3a\sqrt{-\frac{a}{b}}}-\frac{A\log\left(4\sqrt{x}\right.}{} \end{cases}$$

input
```
integrate((B*x**3+A)/x**(5/2)/(b*x**3+a),x)
```

output
```
Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0)
), ((-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2)))/b, Eq(a, 0)), ((-2*A/(3*x**(3/2
)) + 2*B*x**(3/2)/3)/a, Eq(b, 0)), (-A*log(sqrt(x) - (-a/b)**(1/6))/(3*a*s
qrt(-a/b)) + A*log(sqrt(x) + (-a/b)**(1/6))/(3*a*sqrt(-a/b)) + A*log(-4*sq
rt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*a*sqrt(-a/b)) - A*log(4*sq
rt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*a*sqrt(-a/b)) - 2*A/(3*a*x
**(3/2)) + B*log(sqrt(x) - (-a/b)**(1/6))/(3*b*sqrt(-a/b)) - B*log(sqrt(x)
 + (-a/b)**(1/6))/(3*b*sqrt(-a/b)) - B*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x
+ 4*(-a/b)**(1/3))/(3*b*sqrt(-a/b)) + B*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x
+ 4*(-a/b)**(1/3))/(3*b*sqrt(-a/b)), True))
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx=\frac{2\left(Ba-Ab\right)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}a}-\frac{2A}{3ax^{\frac{3}{2}}}$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a) - 2/3*A/(a*x^(3/
2))
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx=\frac{2\left(Ba-Ab\right)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}a}-\frac{2A}{3ax^{\frac{3}{2}}}$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a) - 2/3*A/(a*x^(3/
2))
```

3.161.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx^3}{x^{5/2}\,(a+bx^3)}\,dx = -\frac{2\,A}{3\,a\,x^{3/2}} - \frac{2\,\mathrm{atan}\left(\frac{3\,a^{3/2}\,\sqrt{b}\,x^{3/2}\,\left(24\,A^2\,a^3\,b^5-48\,A\,B\,a^4\,b^4+24\,B^2\,a^5\,b^3\right)}{\left(A\,b-B\,a\right)\,\left(72\,A\,a^5\,b^4-72\,B\,a^6\,b^3\right)}\right)\,\left(A\,b-B\,a\right)}{3\,a^{3/2}\,\sqrt{b}}$$

input
```
int((A + B*x^3)/(x^(5/2)*(a + b*x^3)),x)
```

output
```
- (2*A)/(3*a*x^(3/2)) - (2*atan((3*a^(3/2)*b^(1/2)*x^(3/2)*(24*A^2*a^3*b^5
 + 24*B^2*a^5*b^3 - 48*A*B*a^4*b^4))/((A*b - B*a)*(72*A*a^5*b^4 - 72*B*a^6
*b^3)))*(A*b - B*a))/(3*a^(3/2)*b^(1/2))
```

3.162 $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)}\,dx$

3.162.1 Optimal result

Integrand size = 22, antiderivative size = 270

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx = -\frac{2A}{5ax^{5/2}} + \frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{(Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}$$

output

```
-2/5*A/a/x^(5/2)-2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^
(1/6)-1/3*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^
(1/6)-1/3*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(
1/6)+1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a
^(11/6)/b^(1/6)*3^(1/2)-1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)
*3^(1/2)*x^(1/2))/a^(11/6)/b^(1/6)*3^(1/2)
```

3.162.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx=\frac{-\frac{6a^{5/6}A}{x^{5/2}}+\frac{10(-Ab+aB)\arctan\left(\frac{\sqrt[6]{b}_{\sqrt{x}}}{\sqrt[6]{a}}\right)}{\sqrt[6]{b}}+\frac{5(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}_{-}\sqrt[3]{b}_{x}}{\sqrt[6]{a}\sqrt[6]{b}_{\sqrt{x}}}\right)}{\sqrt[6]{b}}+\frac{5\sqrt{3}(-Ab+aB)\text{arctan}}{\sqrt[6]{}}}{15a^{11/6}}$$

input
```
Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]
```

output
```
((-6*a^(5/6)*A)/x^(5/2) + (10*(-(A*b) + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1
/6)])/b^(1/6) + (5*(A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/
6)*Sqrt[x])])/b^(1/6) + (5*Sqrt[3]*(-(A*b) + a*B)*ArcTanh[(Sqrt[3]*a^(1/6)
*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/b^(1/6))/(15*a^(11/6))
```

3.162.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {955, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(Ab-aB)\int \frac{1}{\sqrt{x}\left(bx^3+a\right)}dx}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \text{ 851}$$

$$-\frac{2(Ab-aB)\int \frac{1}{bx^3+a}d\sqrt{x}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \text{ 753}$$

$$-\frac{2(Ab-aB)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2(Ab-aB)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 218$$

$$-\frac{2(Ab-aB)\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 1142$$

$$-\frac{2(Ab-aB)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d}{6}\right.}{a}$$

$$\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 25$$

3.162. $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)}\,dx$

$$-\frac{2(Ab-aB)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{}}{6a^{5}}\right.}{a}$$

$$\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2(Ab-aB)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d}{}\right.}{a}$$

$$\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2(Ab-aB)\left(\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(2\right.}{\sqrt{3}}}{6a^{5/6}}\right.}{a}$$

$$\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2(Ab-aB)\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\right.}{}}{6a^{5/6}}\right.}{a}$$

$$\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \text{1103}$$

3.162. $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)}\,dx$

$$-\frac{2(Ab-aB)\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{b}}+}{}\right.}{a}$$

$$\frac{2A}{5ax^{5/2}}$$

input
```
Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]
```

output
```
(-2*A)/(5*a*x^(5/2)) - (2*(A*b - a*B)*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(
3*a^(5/6)*b^(1/6)) + (-(ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a
^(1/6)))]/b^(1/6)) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x
] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6)) + (ArcTan[Sqrt[3]*(1 + (2*b^(1/6)
*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(
1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6))))/a
```

3.162.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.162. $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)}\,dx$

3.162.4 Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2A}{5a\,x^{\frac{5}{2}}}+\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6a}+\right.}{a}$
default	$-\frac{2A}{5a\,x^{\frac{5}{2}}}+\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6a}+\right.}{a}$
risch	$-\frac{2A}{5a\,x^{\frac{5}{2}}}-\frac{(Ab-Ba)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3a}+\frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\right.}{a}$

input
```
int((B*x^3+A)/x^(7/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-2/5*A/a/x^(5/2)+2*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1/12/a*3
^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/
b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)
*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*
x^(1/2)/(a/b)^(1/6)+3^(1/2)))*(-A*b+B*a)/a
```

3.162.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1290 vs. 2(188) = 376.

Time = 0.34 (sec) , antiderivative size = 1290, normalized size of antiderivative = 4.78

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx=\text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/30*(10*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B
^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)
*log(a^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6) - (B*a
 - A*b)*sqrt(x)) - 10*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^
2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^
11*b))^(1/6)*log(-a^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20
*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))
^(1/6) - (B*a - A*b)*sqrt(x)) + 5*(sqrt(-3)*a*x^3 + a*x^3)*(-(B^6*a^6 - 6*
A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4
 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(-2*(B*a - A*b)*sqrt(x) + (
sqrt(-3)*a^2 + a^2)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(
1/6)) - 5*(sqrt(-3)*a*x^3 + a*x^3)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4
*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b
^6)/(a^11*b))^(1/6)*log(-2*(B*a - A*b)*sqrt(x) - (sqrt(-3)*a^2 + a^2)*(-(B
^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*
B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)) + 5*(sqrt(-3)*a*x^
3 - a*x^3)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^
3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*l...
```

3.162.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(258) = 516$.

Time = 47.88 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.17

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{5x^{\frac{5}{2}}}\right) \\ \frac{-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{5x^{\frac{5}{2}}}}{b} \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}}+2B\sqrt{x}}{a} \\ -\frac{2A}{5ax^{\frac{5}{2}}}+\frac{Ab\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3a^2}-\frac{Ab\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3a^2}+\frac{Ab\sqrt[6]{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-}\right.}{6a^2} \end{cases}$$

input
```
integrate((B*x**3+A)/x**(7/2)/(b*x**3+a),x)
```

output
```
Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(5*x**(5/2))), Eq(a, 0) & Eq(b,
0)), ((-2*A/(11*x**(11/2)) - 2*B/(5*x**(5/2)))/b, Eq(a, 0)), ((-2*A/(5*x**
(5/2)) + 2*B*sqrt(x))/a, Eq(b, 0)), (-2*A/(5*a*x**(5/2)) + A*b*(-a/b)**(1/
6)*log(sqrt(x) - (-a/b)**(1/6))/(3*a**2) - A*b*(-a/b)**(1/6)*log(sqrt(x) +
 (-a/b)**(1/6))/(3*a**2) + A*b*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6)
+ 4*x + 4*(-a/b)**(1/3))/(6*a**2) - A*b*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)
**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**2) - sqrt(3)*A*b*(-a/b)**(1/6)*atan
(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a**2) - sqrt(3)*A*b*(
-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*a**2
) - B*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(3*a) + B*(-a/b)**(1/6)*l
og(sqrt(x) + (-a/b)**(1/6))/(3*a) - B*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)*
*(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a) + B*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/
b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a) + sqrt(3)*B*(-a/b)**(1/6)*atan(2*
sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a) + sqrt(3)*B*(-a/b)**(
1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*a), True))
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx = \frac{\frac{\sqrt{3}(Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\mathrm{arct}}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a}}}{} - \frac{2A}{5ax^{\frac{5}{2}}}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
 a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(
1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) - A
*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*s
qrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((
sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(
1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*ar
ctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))
/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/a - 2/5*A/(a*x^(5/2))
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx = \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^2b}$$
$$-\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^2b}$$
$$+\frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^2b}$$
$$+\frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^2b}$$
$$+\frac{2\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^2b}-\frac{2\,A}{5\,ax^{\frac{5}{2}}}$$

input

```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="giac")
```

output

```
1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a
/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b) - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a
*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*
b) + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/
6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1
/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) +
2/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a
^2*b) - 2/5*A/(a*x^(5/2))
```

3.162.9 Mupad [B] (verification not implemented)

Time = 7.38 (sec) , antiderivative size = 2023, normalized size of antiderivative = 7.49

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx = \text{Too large to display}$$

input

```
int((A + B*x^3)/(x^(7/2)*(a + b*x^3)),x)
```

output

```
- (2*A)/(5*a*x^(5/2)) - (atan((((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5
+ 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - ((A*b - B
*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^
8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)) +
 ((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*
B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3
*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)
))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)))/(((x^(1/2)*(96*A^4*a^5*b^9 + 9
6*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^
8) - ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6
- 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)
*b^(1/6)) - ((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b
^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + ((A*b - B*a)*(288*A^3*a^7*b^
8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11
/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6))))*(A*b - B*a)*2i)/(3*(-
a)^(11/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(96*A^4*a^5*b
^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*
a^6*b^8) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*
a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6))
)*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)) + (((3^(1/2)*1i)/2 - 1/2)*(x^...
```

3.162. $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)}\,dx$

3.163 $\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$

3.163.1 Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx=-\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\,(a+bx^3)}+\frac{(Ab-3aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{5/2}}$$

output
```
-1/3*(A*b-3*B*a)*x^(3/2)/a/b^2+1/3*(A*b-B*a)*x^(9/2)/a/b/(b*x^3+a)+1/3*(A*
b-3*B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/b^(5/2)/a^(1/2)
```

3.163.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx=\frac{x^{3/2}(-Ab+3aB+2bBx^3)}{3b^2\,(a+bx^3)}+\frac{(Ab-3aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{5/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
(x^(3/2)*(-(A*b) + 3*a*B + 2*b*B*x^3))/(3*b^2*(a + b*x^3)) + ((A*b - 3*a*B
)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*Sqrt[a]*b^(5/2))
```

3.163.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {957, 843, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \; 957$$

$$\frac{x^{9/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-3aB)\int \frac{x^{7/2}}{bx^3+a}dx}{2ab}$$

$$\Big\downarrow \; 843$$

$$\frac{x^{9/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-3aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{bx^3+a}dx}{b}\right)}{2ab}$$

$$\Big\downarrow \; 851$$

$$\frac{x^{9/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-3aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\int \frac{x}{bx^3+a}d\sqrt{x}}{b}\right)}{2ab}$$

$$\Big\downarrow \; 807$$

$$\frac{x^{9/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-3aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\int \frac{1}{a+bx}dx^{3/2}}{3b}\right)}{2ab}$$

$$\Big\downarrow \; 218$$

$$\frac{x^{9/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-3aB)\left(\frac{2x^{3/2}}{3b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{3/2}}\right)}{2ab}$$

input

```
Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

output

```
((A*b - a*B)*x^(9/2))/(3*a*b*(a + b*x^3)) - ((A*b - 3*a*B)*((2*x^(3/2))/(3
*b) - (2*Sqrt[a]*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(3/2))))/(2*a*b)
```

3.163.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.163. $\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$

3.163.4 Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{2B\,x^{\frac{3}{2}}}{3b^{2}}+\frac{\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x^{\frac{3}{2}}}{3\left(b\,x^{3}+a\right)}+\frac{\left(Ab-3Ba\right)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}}}{b^{2}}$	65
default	$\frac{2B\,x^{\frac{3}{2}}}{3b^{2}}+\frac{\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x^{\frac{3}{2}}}{3\left(b\,x^{3}+a\right)}+\frac{\left(Ab-3Ba\right)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}}}{b^{2}}$	65
risch	$\frac{2B\,x^{\frac{3}{2}}}{3b^{2}}+\frac{\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x^{\frac{3}{2}}}{3\left(b\,x^{3}+a\right)}+\frac{\left(Ab-3Ba\right)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}}}{b^{2}}$	65

input
```
int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3/b^2*B*x^(3/2)+2/3/b^2*((-1/2*A*b+1/2*B*a)*x^(3/2)/(b*x^3+a)+1/2*(A*b-3
*B*a)/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2)))
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.34

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \left[\frac{((3\,Bab-Ab^2)x^3+3\,Ba^2-Aab)\sqrt{-ab}\log\left(\frac{bx^3-2\,\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)+2\,(2\,Bab^2x^4+(3}{6\,(ab^4x^3+a^2b^3)}\right.$$

$$\left.-\frac{((3\,Bab-Ab^2)x^3+3\,Ba^2-Aab)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x^{\frac{3}{2}}}{a}\right)-(2\,Bab^2x^4+(3\,Ba^2b-Aab^2)x)\sqrt{x}}{3\,(ab^4x^3+a^2b^3)}\right]$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/6*(((3*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)*sqrt(-a*b)*log((b*x^3 - 2*
sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) + 2*(2*B*a*b^2*x^4 + (3*B*a^2*b - A*a
*b^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3), -1/3*(((3*B*a*b - A*b^2)*x^3 + 3*
B*a^2 - A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*B*a*b^2*x^4 + (3
*B*a^2*b - A*a*b^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3)]
```

3.163.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba-Ab)x^{\frac{3}{2}}}{3\,(b^3x^3+ab^2)} + \frac{2\,Bx^{\frac{3}{2}}}{3\,b^2} - \frac{(3\,Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b^2}$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(B*a - A*b)*x^(3/2)/(b^3*x^3 + a*b^2) + 2/3*B*x^(3/2)/b^2 - 1/3*(3*B*a
 - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,b^2} - \frac{(3\,Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b^2} + \frac{Bax^{\frac{3}{2}} - Abx^{\frac{3}{2}}}{3\,(bx^3+a)b^2}$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
2/3*B*x^(3/2)/b^2 - 1/3*(3*B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*
b)*b^2) + 1/3*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^3 + a)*b^2)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 7.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.22

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{2\,B\,x^{3/2}}{3\,b^2}-\frac{x^{3/2}\,\left(\frac{A\,b}{3}-\frac{B\,a}{3}\right)}{b^3\,x^3+a\,b^2}+\frac{\mathrm{atan}\left(\frac{36\,\sqrt{a}\,b^{3/2}\,x^{3/2}\,\left(A^2\,b^2-6\,A\,B\,a\,b+9\,B^2\,a^2\right)}{\left(A\,b-3\,B\,a\right)\,\left(36\,A\,a\,b^2-108\,B\,a^2\,b\right)}\right)\,\left(A\,b-3\,B\,a\right)}{3\,\sqrt{a}\,b^{5/2}}$$

input
```
int((x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(2*B*x^(3/2))/(3*b^2) - (x^(3/2)*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) +
(atan((36*a^(1/2)*b^(3/2)*x^(3/2)*(A^2*b^2 + 9*B^2*a^2 - 6*A*B*a*b))/((A*b
 - 3*B*a)*(36*A*a*b^2 - 108*B*a^2*b)))*(A*b - 3*B*a))/(3*a^(1/2)*b^(5/2))
```

3.164 $\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

3.164.1 Optimal result

Integrand size = 22, antiderivative size = 312

$$\begin{aligned}\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx &= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2}+\frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)}\\ &\quad-\frac{(Ab-7aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}}+\frac{(Ab-7aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}}\\ &\quad+\frac{(Ab-7aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}}-\frac{(Ab-7aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}\\ &\quad+\frac{(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}\end{aligned}$$

output

```
1/3*(A*b-B*a)*x^(7/2)/a/b/(b*x^3+a)+1/9*(A*b-7*B*a)*arctan(b^(1/6)*x^(1/2)
/a^(1/6))/a^(5/6)/b^(13/6)+1/18*(A*b-7*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1
/2)/a^(1/6))/a^(5/6)/b^(13/6)+1/18*(A*b-7*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^
(1/2)/a^(1/6))/a^(5/6)/b^(13/6)-1/36*(A*b-7*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1
/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(5/6)/b^(13/6)*3^(1/2)+1/36*(A*b-7*B*a)*ln(
a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(5/6)/b^(13/6)*3^(1/2
)-1/3*(A*b-7*B*a)*x^(1/2)/a/b^2
```

3.164.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.58

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\frac{6\sqrt[6]{b}\sqrt{x}(-Ab+7aB+6bBx^3)}{a+bx^3} + \frac{2(Ab-7aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{a^{5/6}} + \frac{(-Ab+7aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{a^{5/6}} + \sqrt{3}}{18b^{13/6}}$$

input
```
Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((6*b^(1/6)*Sqrt[x]*(-(A*b) + 7*a*B + 6*b*B*x^3))/(a + b*x^3) + (2*(A*b -
7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/a^(5/6) + ((-(A*b) + 7*a*B)*ArcT
an[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/a^(5/6) + (Sqrt[3]*(A
*b - 7*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x
)])/a^(5/6))/(18*b^(13/6))
```

3.164.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {957, 843, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{7/2}(Ab-aB)}{3ab\,(a+bx^3)} - \frac{(Ab-7aB)\int \frac{x^{5/2}}{bx^3+a}dx}{6ab}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^{7/2}(Ab-aB)}{3ab\,(a+bx^3)} - \frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b} - \frac{a\int \frac{1}{\sqrt{x}\left(bx^3+a\right)}dx}{b}\right)}{6ab}$$

$$\Big\downarrow \text{851}$$

$$\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{bx^3+a}d\sqrt{x}}{b}\right)}{6ab}$$

$$\Big\downarrow\ 753$$

$$\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int\frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{b}\right)}{6ab}$$

$$\Big\downarrow\ 27$$

$$\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int\frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{b}\right)}{6ab}$$

$$\Big\downarrow\ 218$$

$$\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{b}\right)}{6ab}$$

$$\Big\downarrow\ 1142$$

3.164. $\int\frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

$$\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int-\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}-}{}\right.}{b}\right.}{6ab}$$

$$\Big\downarrow \; 25$$

$$\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+}{}\right.}{b}\right.}{6ab}$$

$$\Big\downarrow \; 27$$

3.164. $\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

$$\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+}{}}{}\right.}{b}\right.}{6ab}$$

$$\Big\downarrow \text{1082}$$

$$\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int\frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int\frac{1}{-x-}}{}}{6a^{5/6}}\right.}{b}\right.}{6ab}$$

$$\Big\downarrow \text{217}$$

$$\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\text{arcta}}{}}{6a^{5/6}}\right.}{b}\right.}{6ab}$$

$$\Big\downarrow \text{1103}$$

3.164. $\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

$$\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}-\frac{(Ab-7aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{b}}\right)}{b}\right)}{6ab}$$

input
```
Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^(7/2))/(3*a*b*(a + b*x^3)) - ((A*b - 7*a*B)*((2*Sqrt[x])/b
- (2*a*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(5/6)*b^(1/6)) + (-(ArcTan[
Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) - (Sqrt[3]*L
og[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6
*a^(5/6)) + (ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b
^(1/6) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*
x])/(2*b^(1/6)))/(6*a^(5/6))))/b))/(6*a*b)
```

3.164.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.164. $\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.164.4 Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{6}+\frac{Ba}{6}\right)\sqrt{x}}{b\,x^3+a}+\frac{(Ab-7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\right.}{6a}\right.}{3}}{b^2}$
default	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{6}+\frac{Ba}{6}\right)\sqrt{x}}{b\,x^3+a}+\frac{(Ab-7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\right.}{6a}\right.}{3}}{b^2}$
risch	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{6}+\frac{Ba}{6}\right)\sqrt{x}}{b\,x^3+a}+\frac{(Ab-7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\right.}{6a}\right.}{3}}{b^2}$

input

```
int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output

```
2*B/b^2*x^(1/2)+2/b^2*((-1/6*A*b+1/6*B*a)*x^(1/2)/(b*x^3+a)+1/6*(A*b-7*B*a
)*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(1/6
)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(-
3^(1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/
b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/
6)+3^(1/2))))
```

3.164.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1426 vs. $2(232) = 464$.

Time = 0.28 (sec) , antiderivative size = 1426, normalized size of antiderivative = 4.57

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input

```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output

```
1/36*(2*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A
^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b
^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(a*b^2*(-(117649*B^6*a^6 - 100842*A*B^5
*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^
4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6) - (7*B*a - A*b)*sqrt(x)) -
 2*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^
4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 +
A^6*b^6)/(a^5*b^13))^(1/6)*log(-a*b^2*(-(117649*B^6*a^6 - 100842*A*B^5*a^5
*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 -
42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6) - (7*B*a - A*b)*sqrt(x)) + (b^
3*x^3 + a*b^2 + sqrt(-3)*(b^3*x^3 + a*b^2))*(-(117649*B^6*a^6 - 100842*A*B
^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*
b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(-(7*B*a - A*b)*sqrt(
x) + 1/2*(sqrt(-3)*a*b^2 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b +
 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A
^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)) - (b^3*x^3 + a*b^2 + sqrt(-3)*(b^
3*x^3 + a*b^2))*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4
*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b
^6)/(a^5*b^13))^(1/6)*log(-(7*B*a - A*b)*sqrt(x) - 1/2*(sqrt(-3)*a*b^2 + a
*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - ...
```

3.164.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1658 vs. $2(299) = 598$.

Time = 167.74 (sec) , antiderivative size = 1658, normalized size of antiderivative = 5.31

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(5*x**(5/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(7/2)/7 + 2*B*x**(13/2)/13)/a**2, Eq(b, 0)), ((-2*A/(5*x**(5/2)) +
2*B*sqrt(x))/b**2, Eq(a, 0)), (-12*A*a*b*sqrt(x)/(36*a**2*b**2 + 36*a*b**3
*x**3) - 2*A*a*b*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2
+ 36*a*b**3*x**3) + 2*A*a*b*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(36
*a**2*b**2 + 36*a*b**3*x**3) - A*a*b*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**
(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2 + 36*a*b**3*x**3) + A*a*b*(-a
/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b
**2 + 36*a*b**3*x**3) + 2*sqrt(3)*A*a*b*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(
x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**2*b**2 + 36*a*b**3*x**3) + 2*sqrt
(3)*A*a*b*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)
/3)/(36*a**2*b**2 + 36*a*b**3*x**3) - 2*A*b**2*x**3*(-a/b)**(1/6)*log(sqrt
(x) - (-a/b)**(1/6))/(36*a**2*b**2 + 36*a*b**3*x**3) + 2*A*b**2*x**3*(-a/b
)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**2*b**2 + 36*a*b**3*x**3) - A*
b**2*x**3*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/
3))/(36*a**2*b**2 + 36*a*b**3*x**3) + A*b**2*x**3*(-a/b)**(1/6)*log(4*sqrt
(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2 + 36*a*b**3*x**3)
 + 2*sqrt(3)*A*b**2*x**3*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(
1/6)) - sqrt(3)/3)/(36*a**2*b**2 + 36*a*b**3*x**3) + 2*sqrt(3)*A*b**2*x**3
*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3...
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba-Ab)\sqrt{x}}{3\,(b^3x^3+ab^2)} + \frac{2\,B\sqrt{x}}{b^2}$$

$$-\frac{\frac{\sqrt{3}(7\,Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(7\,Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(7\,Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + 2(7}{36\,b^2}$$

input
```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output

```
1/3*(B*a - A*b)*sqrt(x)/(b^3*x^3 + a*b^2) + 2*B*sqrt(x)/b^2 - 1/36*(sqrt(3
)*(7*B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))
/(a^(5/6)*b^(1/6)) - sqrt(3)*(7*B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sq
rt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(7*B*a*b^(1/3) - A*b^(4
/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a
^(1/3)*b^(1/3))) + 2*(7*B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((sqr
t(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3
)*sqrt(a^(1/3)*b^(1/3))) + 2*(7*B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arc
tan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/
(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/b^2
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = {} & \frac{2\,B\sqrt{x}}{b^2} \\ & - \frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}\,Ba - (ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,ab^3} \\ & + \frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}\,Ba - (ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,ab^3} \\ & + \frac{Ba\sqrt{x} - Ab\sqrt{x}}{3\,(bx^3+a)b^2} - \frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba - (ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} + 2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,ab^3} \\ & - \frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba - (ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} - 2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,ab^3} \\ & - \frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba - (ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,ab^3}\end{aligned}$$

input

```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output

```
2*B*sqrt(x)/b^2 - 1/36*sqrt(3)*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*l
og(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^3) + 1/36*sqrt(3)*(
7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6)
+ x + (a/b)^(1/3))/(a*b^3) + 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*
b^2) - 1/18*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b
)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^3) - 1/18*(7*(a*b^5)^(1/6)*B*a - (a
*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a
*b^3) - 1/9*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)
^(1/6))/(a*b^3)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 7.35 (sec) , antiderivative size = 1884, normalized size of antiderivative = 6.04

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input

```
int((x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

output

```
(2*B*x^(1/2))/b^2 - (x^(1/2)*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) - (ata
n(((((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3
*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a
*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b
 - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((2*x^(1/2)*(A^4*b^4 + 2401*B^4*
a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) +
 (2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^
2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6
)))/((((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B
^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3
*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A
*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6)) - (((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a
^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) +
(2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2
*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6)))
*(A*b - 7*B*a)*1i)/(9*(-a)^(5/6)*b^(13/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2
)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a
^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)
*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^
(5/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((3^(1/2...
```

3.165 $\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$

3.165.1 Optimal result

Integrand size = 22, antiderivative size = 289

$$\begin{aligned}\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{(Ab-aB)x^{5/2}}{3ab\,(a+bx^3)} - \frac{(Ab+5aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}} \\ &+ \frac{(Ab+5aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}} \\ &+ \frac{(Ab+5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}} \\ &- \frac{(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}}\end{aligned}$$

output
```
1/3*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)+1/9*(A*b+5*B*a)*arctan(b^(1/6)*x^(1/2)
/a^(1/6))/a^(7/6)/b^(11/6)+1/18*(A*b+5*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1
/2)/a^(1/6))/a^(7/6)/b^(11/6)+1/18*(A*b+5*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^
(1/2)/a^(1/6))/a^(7/6)/b^(11/6)+1/36*(A*b+5*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1
/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(7/6)/b^(11/6)*3^(1/2)-1/36*(A*b+5*B*a)*ln(
a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(7/6)/b^(11/6)*3^(1/2
)
```

3.165.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.58

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{-\frac{6\sqrt[6]{a}b^{5/6}(-Ab+aB)x^{5/2}}{a+bx^3} + 2(Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right) - (Ab+5aB)\arctan\left(\frac{\sqrt[3]{a}-}{\sqrt[6]{a}\sqrt[6]{}}\right.}{18a^{7/6}b^{11/6}}$$

input
```
Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((-6*a^(1/6)*b^(5/6)*(-(A*b) + a*B)*x^(5/2))/(a + b*x^3) + 2*(A*b + 5*a*B)
*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (A*b + 5*a*B)*ArcTan[(a^(1/3) - b^(1/
3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] - Sqrt[3]*(A*b + 5*a*B)*ArcTanh[(Sqrt[3]*
a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(18*a^(7/6)*b^(11/6))
```

3.165.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {957, 851, 824, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(5aB+Ab)\int \frac{x^{3/2}}{bx^3+a}dx}{6ab} + \frac{x^{5/2}(Ab-aB)}{3ab\,(a+bx^3)}$$

$$\Big\downarrow \text{851}$$

$$\frac{(5aB+Ab)\int \frac{x^2}{bx^3+a}d\sqrt{x}}{3ab} + \frac{x^{5/2}(Ab-aB)}{3ab\,(a+bx^3)}$$

$$\Big\downarrow \text{824}$$

$$\frac{(5aB+Ab)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}+\frac{\int -\frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}+\frac{\int -\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}\right)}{3ab}+\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{(5aB+Ab)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}\right)}{3ab}+\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \ 218$$

$$\frac{(5aB+Ab)\left(-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}\right)}{3ab}+\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \ 1142$$

$$\frac{(5aB+Ab)\left(-\frac{-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6}\right.}{3ab}$$

$$\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \ 25$$

3.165. $\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

$$\frac{(5aB+Ab)\left(-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}}{6\sqrt[6]{a}}\right.}{3ab}$$

$$\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$\Big\downarrow$ 27

$$\frac{(5aB+Ab)\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{}}{6}\right.}{3ab}$$

$$\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$\Big\downarrow$ 1082

$$\frac{(5aB+Ab)\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}\right.}{3ab}$$

$$\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$\Big\downarrow$ 217

$$\frac{(5aB+Ab)\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\right.}{}}{6\sqrt[6]{a}b^{2/3}}\right.}{3ab}$$

$$\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$\Big\downarrow$ 1103

3.165. $\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}dx$

$$\frac{(5aB+Ab)\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}\right.}{2\sqrt[6]{b}}}{6\sqrt[6]{a}}\right.}{3ab}$$

$$\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

input
```
Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^(5/2))/(3*a*b*(a + b*x^3)) + ((A*b + 5*a*B)*(ArcTan[(b^(1/6
)*Sqrt[x])/a^(1/6)]/(3*a^(1/6)*b^(5/6)) - (ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*
Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1
/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3)) - (-(Ar
cTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) + (Sqrt
[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)
))/(6*a^(1/6)*b^(2/3))))/(3*a*b)
```

3.165.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

---

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.165. $\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$

3.165.4 Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{(Ab-Ba)x^{\frac{5}{2}}}{3ab(b\,x^{3}+a)}+\frac{(Ab+5Ba)\left(\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{}\right.}{12a}\right.}{3ab}$
default	$\frac{(Ab-Ba)x^{\frac{5}{2}}}{3ab(b\,x^{3}+a)}+\frac{(Ab+5Ba)\left(\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{}\right.}{12a}\right.}{3ab}$

input
```
int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)+1/3*(A*b+5*B*a)/a/b*(1/12/a*3^(1/2)*(a
/b)^(5/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*
arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(
1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(
a/b)^(1/6)+3^(1/2))+1/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6)))
```

3.165.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1773 vs. 2(207) = 414.

Time = 0.29 (sec) , antiderivative size = 1773, normalized size of antiderivative = 6.13

$$\int \frac{x^{3/2}(A+Bx^3)}{\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

output

```
-1/36*(12*(B*a - A*b)*x^(5/2) - 2*(a*b^2*x^3 + a^2*b)*(-(15625*B^6*a^6 + 1
8750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B
^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(a^6*b^9*(-(15
625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*
b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) +
(3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^3*B^2*a^2*
b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) + 2*(a*b^2*x^3 + a^2*b)*(-(15625*
B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3
+ 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(-a
^6*b^9*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*
A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^1
1))^(5/6) + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*
A^3*B^2*a^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) - (a*b^2*x^3 + a^2*b
- sqrt(-3)*(a*b^2*x^3 + a^2*b))*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 937
5*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*
a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(1/2*(sqrt(-3)*a^6*b^9 + a^6*b^9)*(-
(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a
^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6)
 + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^3*B^2*a
^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) + (a*b^2*x^3 + a^2*b - sqrt...
```

3.165.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1885 vs. $2(277) = 554$.

Time = 112.41 (sec) , antiderivative size = 1885, normalized size of antiderivative = 6.52

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input

```
integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(5/2)/5 + 2*B*x**(11/2)/11)/a**2, Eq(b, 0)), ((-2*A/(7*x**(7/2)) -
2*B/sqrt(x))/b**2, Eq(a, 0)), (2*A*a*b*log(sqrt(x) - (-a/b)**(1/6))/(36*a*
*2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - 2*A*a*b*log(sqrt(x
) + (-a/b)**(1/6))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1
/6)) + A*a*b*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**
2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - A*a*b*log(4*sqrt(x)
*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a
*b**3*x**3*(-a/b)**(1/6)) + 2*sqrt(3)*A*a*b*atan(2*sqrt(3)*sqrt(x)/(3*(-a/
b)**(1/6)) - sqrt(3)/3)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b
)**(1/6)) + 2*sqrt(3)*A*a*b*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqr
t(3)/3)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 12*A
*b**2*x**(5/2)*(-a/b)**(1/6)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*
(-a/b)**(1/6)) + 2*A*b**2*x**3*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2*
(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - 2*A*b**2*x**3*log(sqrt(x)
+ (-a/b)**(1/6))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6
)) + A*b**2*x**3*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36
*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - A*b**2*x**3*log
(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2*(-a/b)**(1
/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 2*sqrt(3)*A*b**2*x**3*atan(2*sqrt...
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.81

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ba-Ab)x^{\frac{5}{2}}}{3\,(ab^2x^3+a^2b)} - \frac{(5\,Ba+Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} - \right.}{36\,ab}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(B*a - A*b)*x^(5/2)/(a*b^2*x^3 + a^2*b) - 1/36*(5*B*a + A*b)*(sqrt(3)
*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/
6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/
(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))
/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(
3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sq
rt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^
(2/3)*sqrt(a^(1/3)*b^(1/3))))/(a*b)
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{(5\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,(ab^5)^{\frac{1}{6}}\,ab} \\
&+ \frac{(5\,Ba+Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,(ab^5)^{\frac{1}{6}}\,ab} + \frac{\left(5\,Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}+Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^2b} \\
&- \frac{Bax^{\frac{5}{2}}-Abx^{\frac{5}{2}}}{3\,(bx^3+a)ab} - \frac{\sqrt{3}\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^6} \\
&+ \frac{\sqrt{3}\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^6}
\end{aligned}
$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/18*(5*B*a + A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(
(a*b^5)^(1/6)*a*b) + 1/18*(5*B*a + A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*s
qrt(x))/(a/b)^(1/6))/((a*b^5)^(1/6)*a*b) + 1/9*(5*B*a*(a/b)^(5/6) + A*b*(a
/b)^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b) - 1/3*(B*a*x^(5/2) - A*b*x^
(5/2))/((b*x^3 + a)*a*b) - 1/36*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + (a*b^5)^(5/
6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^6) + 1/3
6*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(
a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^6)
```

3.165.9 Mupad [B] (verification not implemented)

Time = 7.29 (sec) , antiderivative size = 1578, normalized size of antiderivative = 5.46

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(x^(5/2)*(A*b - B*a))/(3*a*b*(a + b*x^3)) - (atan(((((3^(1/2)*1i)/2 - 1/2)
^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20
*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4
 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*
(-a)^(7/3)*b^(11/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b
^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(1/2)*((3^
(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B
*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)))/((((3^
(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 10
0*A*B^2*a^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*
B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^
(11/6))))/(324*(-a)^(7/3)*b^(11/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B
*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2
+ (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^
3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11
/3))))*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*1i)/(9*(-a)^(7/6)*b^(11/6)) -
(atan(((((3^(1/2)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3
*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 + 1/
2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(
-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)) - (((3^(1/2)*1i)/2 +...
```

3.166 $\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2} dx$

3.166.1 Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2} dx = \frac{(Ab-aB)x^{3/2}}{3ab(a+bx^3)} + \frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}$$

output
```
1/3*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)+1/3*(A*b+B*a)*arctan(x^(3/2)*b^(1/2)/a
^(1/2))/a^(3/2)/b^(3/2)
```

3.166.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2} dx = -\frac{(-Ab+aB)x^{3/2}}{3ab(a+bx^3)} + \frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
-1/3*((-(A*b) + a*B)*x^(3/2))/(a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqr
t[b]*x^(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

3.166.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {957, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}dx$$

$$\Big\downarrow \; 957$$

$$\frac{(aB+Ab)\int \frac{\sqrt{x}}{bx^3+a}dx}{2ab}+\frac{x^{3/2}(Ab-aB)}{3ab(a+bx^3)}$$

$$\Big\downarrow \; 851$$

$$\frac{(aB+Ab)\int \frac{x}{bx^3+a}d\sqrt{x}}{ab}+\frac{x^{3/2}(Ab-aB)}{3ab(a+bx^3)}$$

$$\Big\downarrow \; 807$$

$$\frac{(aB+Ab)\int \frac{1}{a+bx}dx^{3/2}}{3ab}+\frac{x^{3/2}(Ab-aB)}{3ab(a+bx^3)}$$

$$\Big\downarrow \; 218$$

$$\frac{(aB+Ab)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}+\frac{x^{3/2}(Ab-aB)}{3ab(a+bx^3)}$$

input
```
Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]
```

output
```
((A*b - a*B)*x^(3/2))/(3*a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x
^(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

3.166.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.166.4 Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{(Ab-Ba)x^{\frac{3}{2}}}{3ab(bx^3+a)}+\frac{(Ab+Ba)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3ab\sqrt{ab}}$	61
default	$\frac{(Ab-Ba)x^{\frac{3}{2}}}{3ab(bx^3+a)}+\frac{(Ab+Ba)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3ab\sqrt{ab}}$	61

input
```
int((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)+1/3*(A*b+B*a)/a/b/(a*b)^(1/2)*arctan(b
*x^(3/2)/(a*b)^(1/2))
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx = \left[-\frac{2\,(Ba^2b-Aab^2)x^{\frac{3}{2}}+((Bab+Ab^2)x^3+Ba^2+Aab)\sqrt{-ab}\log\left(\frac{bx^3-2\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)}{6\,(a^2b^3x^3+a^3b^2)},\; -\frac{(Ba^2b-Aab^2)x^{\frac{3}{2}}-((Bab+Ab^2)x^3+Ba^2+Aab)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x^{\frac{3}{2}}}{a}\right)}{3\,(a^2b^3x^3+a^3b^2)}\right]$$

input
```
integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/6*(2*(B*a^2*b - A*a*b^2)*x^(3/2) + ((B*a*b + A*b^2)*x^3 + B*a^2 + A*a*
b)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)))/(a^2*b^
3*x^3 + a^3*b^2), -1/3*((B*a^2*b - A*a*b^2)*x^(3/2) - ((B*a*b + A*b^2)*x^3
 + B*a^2 + A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a))/(a^2*b^3*x^3 + a^
3*b^2)]
```

3.166.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(61) = 122$.

Time = 67.82 (sec) , antiderivative size = 1042, normalized size of antiderivative = 14.68

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{9}{2}}}{9}}{a^2} \\ \frac{-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b^2} \\ \frac{2Aabx^{\frac{3}{2}}}{6a^3b+6a^2b^2x^3}-\frac{Aab\sqrt{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3}+\frac{Aab\sqrt{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3}+\frac{Aab\sqrt{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3}- \end{cases}$$

input
```
integrate((B*x**3+A)*x**(1/2)/(b*x**3+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0)
), ((2*A*x**(3/2)/3 + 2*B*x**(9/2)/9)/a**2, Eq(b, 0)), ((-2*A/(9*x**(9/2))
 - 2*B/(3*x**(3/2)))/b**2, Eq(a, 0)), (2*A*a*b*x**(3/2)/(6*a**3*b + 6*a**2
*b**2*x**3) - A*a*b*sqrt(-a/b)*log(sqrt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*
a**2*b**2*x**3) + A*a*b*sqrt(-a/b)*log(sqrt(x) + (-a/b)**(1/6))/(6*a**3*b
+ 6*a**2*b**2*x**3) + A*a*b*sqrt(-a/b)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x
+ 4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3) - A*a*b*sqrt(-a/b)*log(4*
sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**
3) - A*b**2*x**3*sqrt(-a/b)*log(sqrt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*a**
2*b**2*x**3) + A*b**2*x**3*sqrt(-a/b)*log(sqrt(x) + (-a/b)**(1/6))/(6*a**3
*b + 6*a**2*b**2*x**3) + A*b**2*x**3*sqrt(-a/b)*log(-4*sqrt(x)*(-a/b)**(1/
6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3) - A*b**2*x**3*sq
rt(-a/b)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b +
6*a**2*b**2*x**3) - 2*B*a**2*x**(3/2)/(6*a**3*b + 6*a**2*b**2*x**3) - B*a*
*2*sqrt(-a/b)*log(sqrt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x**3) +
 B*a**2*sqrt(-a/b)*log(sqrt(x) + (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x*
*3) + B*a**2*sqrt(-a/b)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/
3))/(6*a**3*b + 6*a**2*b**2*x**3) - B*a**2*sqrt(-a/b)*log(4*sqrt(x)*(-a/b)
**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3) - B*a*b*x**
3*sqrt(-a/b)*log(sqrt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x**3)...
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ba-Ab)x^{\frac{3}{2}}}{3\,(ab^2x^3+a^2b)} + \frac{(Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}ab}$$

input
```
integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(B*a - A*b)*x^(3/2)/(a*b^2*x^3 + a^2*b) + 1/3*(B*a + A*b)*arctan(b*x^
(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b)
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}ab} - \frac{Bax^{\frac{3}{2}} - Abx^{\frac{3}{2}}}{3\,(bx^3+a)ab}$$

input
```
integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*(B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/3*(B*a*x^(
3/2) - A*b*x^(3/2))/((b*x^3 + a)*a*b)
```

3.166.9 Mupad [B] (verification not implemented)

Time = 7.02 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx$$
$$= \frac{B\,a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right) + A\,b^2\,x^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right) + A\,a\,b\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right) + A\,\sqrt{a}\,b^{3/2}\,x^{3/2} - B\,a^{3/2}\,\sqrt{b}\,x^{3/2} + B\,a}{3\,a^{5/2}\,b^{3/2} + 3\,a^{3/2}\,b^{5/2}\,x^3}$$

input
```
int((x^(1/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

output
```
(B*a^2*atan((b^(1/2)*x^(3/2))/a^(1/2)) + A*b^2*x^3*atan((b^(1/2)*x^(3/2))/
a^(1/2)) + A*a*b*atan((b^(1/2)*x^(3/2))/a^(1/2)) + A*a^(1/2)*b^(3/2)*x^(3/
2) - B*a^(3/2)*b^(1/2)*x^(3/2) + B*a*b*x^3*atan((b^(1/2)*x^(3/2))/a^(1/2))
)/(3*a^(5/2)*b^(3/2) + 3*a^(3/2)*b^(5/2)*x^3)
```

3.167 $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx$

3.167.1 Optimal result

Integrand size = 22, antiderivative size = 289

$$\begin{aligned}\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx &= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)} - \frac{(5Ab+aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}} \\ &\quad + \frac{(5Ab+aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}} \\ &\quad + \frac{(5Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}} \\ &\quad - \frac{(5Ab+aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}} \\ &\quad + \frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}\end{aligned}$$

output
```
1/9*(5*A*b+B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(7/6)+1/18*(5*A
*b+B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(7/6)+1/18*(
5*A*b+B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(7/6)-1/36
*(5*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(11/6
)/b^(7/6)*3^(1/2)+1/36*(5*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^
(1/2)*x^(1/2))/a^(11/6)/b^(7/6)*3^(1/2)+1/3*(A*b-B*a)*x^(1/2)/a/b/(b*x^3+a
)
```

3.167.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx$$
$$= \frac{-\frac{6a^{5/6}\sqrt[6]{b}(-Ab+aB)\sqrt{x}}{a+bx^3}+2(5Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-(5Ab+aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)+\sqrt{3}(5Ab+aB}{18a^{11/6}b^{7/6}}$$

input
```
Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```

output
```
((-6*a^(5/6)*b^(1/6)*(-(A*b) + a*B)*Sqrt[x])/(a + b*x^3) + 2*(5*A*b + a*B)
*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (5*A*b + a*B)*ArcTan[(a^(1/3) - b^(1/
3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + Sqrt[3]*(5*A*b + a*B)*ArcTanh[(Sqrt[3]*
a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(18*a^(11/6)*b^(7/6))
```

3.167.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {957, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB+5Ab)\int \frac{1}{\sqrt{x}\left(bx^3+a\right)}dx}{6ab}+\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{851}$$

$$\frac{(aB+5Ab)\int \frac{1}{bx^3+a}d\sqrt{x}}{3ab}+\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{753}$$

$$\frac{(aB+5Ab)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{3ab}+\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow 27$$

$$\frac{(aB+5Ab)\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{3ab}+\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow 218$$

$$\frac{(aB+5Ab)\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{3ab}+\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow 1142$$

$$\frac{(aB+5Ab)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/}}\right.}{3ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow 25$$

3.167. $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx$

$$\frac{(aB+5Ab)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+}{6a^{5/6}}\right.}{3ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(aB+5Ab)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a}\right.}{3ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{(aB+5Ab)\left(\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-}{6a^{5/6}}\right.}{3ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{(aB+5Ab)\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+}{6a^{5/6}}\right.}{3ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1103$$

3.167. $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx$

$$\frac{(aB+5Ab)\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{b}}+\sqrt{3}}{6a}\right.}{3ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```

output
```
((A*b - a*B)*Sqrt[x])/(3*a*b*(a + b*x^3)) + ((5*A*b + a*B)*(ArcTan[(b^(1/6
)*Sqrt[x])/a^(1/6)]/(3*a^(5/6)*b^(1/6)) + (-(ArcTan[Sqrt[3]*(1 - (2*b^(1/6
)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a
^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6)) + (ArcTan[Sq
rt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) + (Sqrt[3]*Log[
a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^
(5/6))))/(3*a*b)
```

3.167.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.167. $\int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)^2}\,dx$

3.167.4 Maple [A] (verified)

Time = 4.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{(Ab-Ba)\sqrt{x}}{3ab(b\,x^3+a)}+\frac{(5Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{6a}\right.}{3ab}$
default	$\frac{(Ab-Ba)\sqrt{x}}{3ab(b\,x^3+a)}+\frac{(5Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{6a}\right.}{3ab}$

input
```
int((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(A*b-B*a)*x^(1/2)/a/b/(b*x^3+a)+1/3*(5*A*b+B*a)/a/b*(1/3/a*(a/b)^(1/6)
*arctan(x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(
1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a
/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/
b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2)))
```

3.167.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1417 vs. 2(207) = 414.

Time = 0.32 (sec) , antiderivative size = 1417, normalized size of antiderivative = 4.90

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="fricas")
```

output

```
1/36*(2*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*
b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15
625*A^6*b^6)/(a^11*b^7))^(1/6)*log(a^2*b*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375
*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5
*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6) + (B*a + 5*A*b)*sqrt(x)) - 2*(
a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 250
0*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b
^6)/(a^11*b^7))^(1/6)*log(-a^2*b*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4
*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5
 + 15625*A^6*b^6)/(a^11*b^7))^(1/6) + (B*a + 5*A*b)*sqrt(x)) + (a*b^2*x^3
+ a^2*b + sqrt(-3)*(a*b^2*x^3 + a^2*b))*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*
A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*
B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*log((B*a + 5*A*b)*sqrt(x) + 1/2
*(sqrt(-3)*a^2*b + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^
2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 1562
5*A^6*b^6)/(a^11*b^7))^(1/6)) - (a*b^2*x^3 + a^2*b + sqrt(-3)*(a*b^2*x^3 +
 a^2*b))*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*
a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*
b^7))^(1/6)*log((B*a + 5*A*b)*sqrt(x) - 1/2*(sqrt(-3)*a^2*b + a^2*b)*(-(B^
6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9...
```

3.167.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1632 vs. 2(277) = 554.

Time = 90.34 (sec) , antiderivative size = 1632, normalized size of antiderivative = 5.65

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input

```
integrate((B*x**3+A)/(b*x**3+a)**2/x**(1/2),x)
```

output
```
Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(5*x**(5/2))), Eq(a, 0) & Eq(b,
0)), ((2*A*sqrt(x) + 2*B*x**(7/2)/7)/a**2, Eq(b, 0)), ((-2*A/(11*x**(11/2)
) - 2*B/(5*x**(5/2)))/b**2, Eq(a, 0)), (12*A*a*b*sqrt(x)/(36*a**3*b + 36*a
**2*b**2*x**3) - 10*A*a*b*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a
**3*b + 36*a**2*b**2*x**3) + 10*A*a*b*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**
(1/6))/(36*a**3*b + 36*a**2*b**2*x**3) - 5*A*a*b*(-a/b)**(1/6)*log(-4*sqrt
(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b + 36*a**2*b**2*x**3)
 + 5*A*a*b*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/
3))/(36*a**3*b + 36*a**2*b**2*x**3) + 10*sqrt(3)*A*a*b*(-a/b)**(1/6)*atan(
2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b + 36*a**2*b**2
*x**3) + 10*sqrt(3)*A*a*b*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**
(1/6)) + sqrt(3)/3)/(36*a**3*b + 36*a**2*b**2*x**3) - 10*A*b**2*x**3*(-a/b
)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**3*b + 36*a**2*b**2*x**3) + 10
*A*b**2*x**3*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b + 36*a*
*2*b**2*x**3) - 5*A*b**2*x**3*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) +
 4*x + 4*(-a/b)**(1/3))/(36*a**3*b + 36*a**2*b**2*x**3) + 5*A*b**2*x**3*(-
a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*
b + 36*a**2*b**2*x**3) + 10*sqrt(3)*A*b**2*x**3*(-a/b)**(1/6)*atan(2*sqrt(
3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b + 36*a**2*b**2*x**3)
+ 10*sqrt(3)*A*b**2*x**3*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)...
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx = -\frac{(Ba-Ab)\sqrt{x}}{3\,(ab^2x^3+a^2b)} + \frac{\frac{\sqrt{3}(Ba+5\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(Ba+5\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}+5\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} +}{36\,ab}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(B*a - A*b)*sqrt(x)/(a*b^2*x^3 + a^2*b) + 1/36*(sqrt(3)*(B*a + 5*A*b)
*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/
6)) - sqrt(3)*(B*a + 5*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)
*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) + 5*A*b^(4/3))*arctan(b^(
1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))
) + 2*(B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^
(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*
b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a
^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt
(a^(1/3)*b^(1/3))))/(a*b)
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.04

$$
\begin{aligned}
\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx = {} & \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^2} \\
& - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^2} \\
& - \frac{Ba\sqrt{x}-Ab\sqrt{x}}{3\,(bx^3+a)ab} \\
& + \frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^2b^2} \\
& + \frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^2b^2} \\
& + \frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^2b^2}
\end{aligned}
$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="giac")
```

output
```
1/36*sqrt(3)*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)
*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^2) - 1/36*sqrt(3)*((a*b^5)^(1/6)*B*
a + 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3
))/(a^2*b^2) - 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a*b) + 1/18*((
a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sq
rt(x))/(a/b)^(1/6))/(a^2*b^2) + 1/18*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*
A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^2) + 1/
9*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a
^2*b^2)
```

3.167.9 Mupad [B] (verification not implemented)

Time = 7.32 (sec) , antiderivative size = 1922, normalized size of antiderivative = 6.65

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^3)/(x^(1/2)*(a + b*x^3)^2),x)
```

output
```
(atan(((((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A
^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*(5*A*b + B*a)*(125*A^3*b^5 +
 B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)
))*(5*A*b + B*a)*1i)/(18*(-a)^(11/6)*b^(7/6)) + (((2*x^(1/2)*(625*A^4*b^5
+ B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(
27*a^4) + (2*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 1
5*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*(5*A*b + B*a)*1i)/(18*(-a)^(11
/6)*b^(7/6)))/((((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3
 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*(5*A*b + B*a)*(125*A
^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)
*b^(7/6)))*(5*A*b + B*a))/(18*(-a)^(11/6)*b^(7/6)) - (((2*x^(1/2)*(625*A^4
*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^
2))/(27*a^4) + (2*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^
4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*(5*A*b + B*a))/(18*(-a)^(
11/6)*b^(7/6))))*(5*A*b + B*a)*1i)/(9*(-a)^(11/6)*b^(7/6)) + (atan(((((3^(
1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150
*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*((3^
(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b
^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(18*(-a)^(11/6)*b^(7
/6)) + (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 +...
```

3.168 $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx$

3.168.1 Optimal result

Integrand size = 22, antiderivative size = 318

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx &= -\frac{7Ab-aB}{3a^2b\sqrt{x}}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} \\
&+\frac{(7Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}} \\
&-\frac{(7Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}} \\
&+\frac{(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}
\end{aligned}
$$

output

```
-1/9*(7*A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(5/6)-1/18*(7*
A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(5/6)-1/18*
(7*A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(5/6)-1/3
6*(7*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(13/
6)/b^(5/6)*3^(1/2)+1/36*(7*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3
^(1/2)*x^(1/2))/a^(13/6)/b^(5/6)*3^(1/2)+1/3*(-7*A*b+B*a)/a^2/b/x^(1/2)+1/
3*(A*b-B*a)/a/b/(b*x^3+a)/x^(1/2)
```

3.168.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \frac{\frac{6\sqrt[6]{a}\left(-6aA-7Abx^3+aBx^3\right)}{\sqrt{x}\left(a+bx^3\right)} + \frac{2(-7Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{b^{5/6}} + \frac{(7Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{b^{5/6}} + \sqrt{3}}{18a^{13/6}}$$

input
```
Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x]
```

output
```
((6*a^(1/6)*(-6*a*A - 7*A*b*x^3 + a*B*x^3))/(Sqrt[x]*(a + b*x^3)) + (2*(-7
*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/b^(5/6) + ((7*A*b - a*B)*Ar
cTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/b^(5/6) + (Sqrt[3]*
(7*A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)
*x)])/b^(5/6))/(18*a^(13/6))
```

3.168.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {957, 847, 851, 824, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{(7Ab-aB)\int \frac{1}{x^{3/2}\left(bx^3+a\right)}dx}{6ab} + \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{(7Ab-aB)\left(-\frac{b\int \frac{x^{3/2}}{bx^3+a}dx}{a} - \frac{2}{a\sqrt{x}}\right)}{6ab} + \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(7Ab-aB)\left(-\frac{2b\int \frac{x^2}{bx^3+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 824}$$

$$\frac{(7Ab-aB)\left(-\frac{2b\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}+\frac{\int -\frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}+\frac{\int -\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(7Ab-aB)\left(-\frac{2b\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(7Ab-aB)\left(-\frac{2b\left(-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

3.168. $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}dx$

$$\Big\downarrow \text{1142}$$

$$\frac{(7Ab-aB)\left(-\frac{2b\left(-\frac{-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d}{2\sqrt[6]{b}}}{}\right)}{a}\right)}{6ab}$$

$$\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(7Ab-aB)\left(-\frac{2b\left(-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6}\right)}{a}\right)}{6ab}$$

$$\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

3.168. $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx$

$$\frac{(7Ab-aB)\left(-\frac{2b\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d}{6}\right.}{a}\right.}{6ab}$$

$$\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

$$\Big\downarrow \textbf{1082}$$

$$\frac{(7Ab-aB)\left(-\frac{2b\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}\right.}{a}\right.}{6ab}$$

$$\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{(7Ab-aB)\left(-\frac{2b\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\arctan\left(\sqrt{}\right.}{6\sqrt[6]{a}b^{2/3}}\right.}{a}\right.}{6ab}$$

$$\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

3.168. $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{(7Ab-aB)\left(-\frac{2b\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{}\right.}{2\sqrt[6]{b}}}{6\sqrt[6]{}}\right.}{a}\right.}{6ab}$$

$$\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x]
```

output
```
(A*b - a*B)/(3*a*b*Sqrt[x]*(a + b*x^3)) + ((7*A*b - a*B)*(-2/(a*Sqrt[x]) -
 (2*b*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(1/6)*b^(5/6)) - (ArcTan[Sqr
t[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) - (Sqrt[3]*Log[a
^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(
1/6)*b^(2/3)) - (-(ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6
)))]/b^(1/6)) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b
^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3))))/a))/(6*a*b)
```

3.168.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.168. $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.168.4 Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)x^{\frac{5}{2}}}{b\,x^{3}+a}+\left(\frac{7Ab}{6}-\frac{Ba}{6}\right)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\right.\right.}{12a}\right.\right.}{a^{2}}$
default	$-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)x^{\frac{5}{2}}}{b\,x^{3}+a}+\left(\frac{7Ab}{6}-\frac{Ba}{6}\right)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\right.\right.}{12a}\right.\right.}{a^{2}}$
risch	$-\frac{2A}{a^{2}\sqrt{x}}-\frac{\frac{2\left(\frac{Ab}{6}-\frac{Ba}{6}\right)x^{\frac{5}{2}}}{b\,x^{3}+a}+2\left(\frac{7Ab}{6}-\frac{Ba}{6}\right)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\right.}{a^{2}}$

input
```
int((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/a^2*((1/6*A*b-1/6*B*a)*x^(5/2)/(b*x^3+a)+(7/6*A*b-1/6*B*a)*(1/12/a*3^(1
/2)*(a/b)^(5/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/b/(a/b)^
(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(5/6)*ln
(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(2*x^(
1/2)/(a/b)^(1/6)+3^(1/2))+1/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))))-
2*A/a^2/x^(1/2)
```

3.168.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1788 vs. $2(226) = 452$.

Time = 0.37 (sec) , antiderivative size = 1788, normalized size of antiderivative = 5.62

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input

```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output

```
-1/36*(2*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4
*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 +
 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(a^11*b^4*(-(B^6*a^6 - 42*A*B^5*a^5*
b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 1
00842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a^5 - 35*A*B^
4*a^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^4*B*a*b^4 -
 16807*A^5*b^5)*sqrt(x)) - 2*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5
*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 -
100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(-a^11*b^4*(-(B^
6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 3601
5*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6)
 - (B^5*a^5 - 35*A*B^4*a^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3
+ 12005*A^4*B*a*b^4 - 16807*A^5*b^5)*sqrt(x)) + (a^2*b*x^4 + a^3*x - sqrt(
-3)*(a^2*b*x^4 + a^3*x))*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2
 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117
649*A^6*b^6)/(a^13*b^5))^(1/6)*log(1/2*(sqrt(-3)*a^11*b^4 + a^11*b^4)*(-(B
^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 360
15*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6
) - (B^5*a^5 - 35*A*B^4*a^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3
 + 12005*A^4*B*a*b^4 - 16807*A^5*b^5)*sqrt(x)) - (a^2*b*x^4 + a^3*x - s...
```

3.168.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2200 vs. $2(299) = 598$.

Time = 151.56 (sec) , antiderivative size = 2200, normalized size of antiderivative = 6.92

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(13*x**(13/2)) - 2*B/(7*x**(7/2))), Eq(a, 0) & Eq(b,
0)), ((-2*A/sqrt(x) + 2*B*x**(5/2)/5)/a**2, Eq(b, 0)), ((-2*A/(13*x**(13/2
)) - 2*B/(7*x**(7/2)))/b**2, Eq(a, 0)), (-14*A*a*b*sqrt(x)*log(sqrt(x) - (
-a/b)**(1/6))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a
/b)**(1/6)) + 14*A*a*b*sqrt(x)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b*sqr
t(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 7*A*a*b*sqrt(x
)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)
*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 7*A*a*b*sqrt(x)*lo
g(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/
b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 14*sqrt(3)*A*a*b*sqrt(x
)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b*sqrt(x)
*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 14*sqrt(3)*A*a*b*s
qrt(x)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**3*b*sq
rt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 72*A*a*b*(-a/
b)**(1/6)/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)*
*(1/6)) - 14*A*b**2*x**(7/2)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**3*b*sqrt(
x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 14*A*b**2*x**(7/
2)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2
*b**2*x**(7/2)*(-a/b)**(1/6)) - 7*A*b**2*x**(7/2)*log(-4*sqrt(x)*(-a/b)**(
1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**...
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \frac{(Ba-7\,Ab)x^3-6\,Aa}{3\left(a^2bx^{\frac{7}{2}}+a^3\sqrt{x}\right)}$$

$$-\frac{(Ba-7\,Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\right)}{36\,a^2}$$

input
```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*((B*a - 7*A*b)*x^3 - 6*A*a)/(a^2*b*x^(7/2) + a^3*sqrt(x)) - 1/36*(B*a
- 7*A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3
))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1
/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2
*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) -
 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1
/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1
/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a^2
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.89

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = {} & \frac{(Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,\left(ab^5\right)^{\frac{1}{6}}a^2} \\
& + \frac{(Ba-7\,Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,\left(ab^5\right)^{\frac{1}{6}}a^2} \\
& + \frac{\left(Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}-7\,Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^3} + \frac{Bax^3-7\,Abx^3-6\,Aa}{3\left(bx^{\frac{7}{2}}+a\sqrt{x}\right)a^2} \\
& - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba-7\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b^5} \\
& + \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba-7\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b^5}
\end{aligned}
$$

input
```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/18*(B*a - 7*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(
(a*b^5)^(1/6)*a^2) + 1/18*(B*a - 7*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*s
qrt(x))/(a/b)^(1/6))/((a*b^5)^(1/6)*a^2) + 1/9*(B*a*(a/b)^(5/6) - 7*A*b*(a
/b)^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/a^3 + 1/3*(B*a*x^3 - 7*A*b*x^3 - 6*
A*a)/((b*x^(7/2) + a*sqrt(x))*a^2) - 1/36*sqrt(3)*((a*b^5)^(5/6)*B*a - 7*(
a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*
b^5) + 1/36*sqrt(3)*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*log(-sqrt(3)
*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^5)
```

3.168.9 Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 1757, normalized size of antiderivative = 5.53

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx=\text{Too large to display}$$

input

```
int((A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x)
```

output

```
(atan((((7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2
*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^
17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^
(5/6)))*1i)/((-a)^(13/3)*b^(5/3)) + ((7*A*b - B*a)^2*(27783*A^3*a^15*b^6 -
 81*B^3*a^18*b^3 + 1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*(
7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^
18*b^5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)))/(((7*A*b -
 B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2*a^17*b^4 + 1190
7*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*
B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(
13/3)*b^(5/3)) - ((7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 +
1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147
208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)
^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3))))*(7*A*b - B*a)*1i)/(9*(-a)^(13/6
)*b^(5/6)) - ((2*A)/a + (x^3*(7*A*b - B*a))/(3*a^2))/(a*x^(1/2) + b*x^(7/2
)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 2
7783*A^3*a^15*b^6 - 1701*A*B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*
((3^(1/2)*1i)/2 - 1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a
^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13
/3)*b^(5/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^1...
```

3.169 $\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2} \, dx$

3.169.1 Optimal result

Integrand size = 22, antiderivative size = 96

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2} \, dx = \frac{-3Ab+aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}-\frac{(3Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}$$

output
```
1/3*(-3*A*b+B*a)/a^2/b/x^(3/2)+1/3*(A*b-B*a)/a/b/x^(3/2)/(b*x^3+a)-1/3*(3*
A*b-B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

3.169.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2} \, dx = \frac{-2aA-3Abx^3+aBx^3}{3a^2x^{3/2}\left(a+bx^3\right)}+\frac{(-3Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x]
```

output
```
(-2*a*A - 3*A*b*x^3 + a*B*x^3)/(3*a^2*x^(3/2)*(a + b*x^3)) + ((-3*A*b + a*
B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(5/2)*Sqrt[b])
```

3.169.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {957, 847, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(3Ab-aB)\int \frac{1}{x^{5/2}(bx^3+a)}dx}{2ab}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{847}$$

$$\frac{(3Ab-aB)\left(-\frac{b\int \frac{\sqrt{x}}{bx^3+a}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{851}$$

$$\frac{(3Ab-aB)\left(-\frac{2b\int \frac{x}{bx^3+a}d\sqrt{x}}{a}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{807}$$

$$\frac{(3Ab-aB)\left(-\frac{2b\int \frac{1}{a+bx}dx^{3/2}}{3a}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(3Ab-aB)\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x]
```

output
```
(A*b - a*B)/(3*a*b*x^(3/2)*(a + b*x^3)) + ((3*A*b - a*B)*(-2/(3*a*x^(3/2))
 - (2*Sqrt[b]*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(3/2))))/(2*a*b)
```

3.169.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.169.4 Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{2A}{3a^{2}x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x^{\frac{3}{2}}}{b\,x^{3}+a}+\frac{(3Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{3a^{2}}$	66
default	$-\frac{2A}{3a^{2}x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x^{\frac{3}{2}}}{b\,x^{3}+a}+\frac{(3Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{3a^{2}}$	66
risch	$-\frac{2A}{3a^{2}x^{\frac{3}{2}}}-\frac{\frac{2\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x^{\frac{3}{2}}}{3\left(b\,x^{3}+a\right)}+\frac{(3Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}}}{a^{2}}$	67

input
```
int((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*A/a^2/x^(3/2)-2/3/a^2*((1/2*A*b-1/2*B*a)*x^(3/2)/(b*x^3+a)+1/2*(3*A*b
-B*a)/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2)))
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.42

$$\int \frac{A+Bx^{3}}{x^{5/2}\left(a+bx^{3}\right)^{2}}\,dx = \Bigg[\frac{((Bab-3\,Ab^{2})x^{5}+(Ba^{2}-3\,Aab)x^{2})\sqrt{-ab}\log\left(\frac{bx^{3}+2\,\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^{3}+a}\right)-2\,(2\,Aa^{2}b-(}{6\,(a^{3}b^{2}x^{5}+a^{4}bx^{2})}$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/6*(((B*a*b - 3*A*b^2)*x^5 + (B*a^2 - 3*A*a*b)*x^2)*sqrt(-a*b)*log((b*x^
3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) - 2*(2*A*a^2*b - (B*a^2*b - 3*A
*a*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2), 1/3*(((B*a*b - 3*A*b^2)*x
^5 + (B*a^2 - 3*A*a*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*A*a
^2*b - (B*a^2*b - 3*A*a*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2)]
```

3.169.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx = \frac{(Ba-3\,Ab)x^3-2\,Aa}{3\left(a^2bx^{\frac{9}{2}}+a^3x^{\frac{3}{2}}\right)} + \frac{(Ba-3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a^2}$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*((B*a - 3*A*b)*x^3 - 2*A*a)/(a^2*b*x^(9/2) + a^3*x^(3/2)) + 1/3*(B*a -
 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx = \frac{(Ba-3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a^2} + \frac{Bax^3-3\,Abx^3-2\,Aa}{3\left(bx^{\frac{9}{2}}+ax^{\frac{3}{2}}\right)a^2}$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*(B*a - 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(B*a*x
^3 - 3*A*b*x^3 - 2*A*a)/((b*x^(9/2) + a*x^(3/2))*a^2)
```

3.169.9 Mupad [B] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx =$$

$$-\frac{2\,A\,a^{3/2}\,\sqrt{b}-B\,a^2\,x^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+3\,A\,b^2\,x^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+3\,A\,\sqrt{a}\,b^{3/2}\,x^3-B\,a^{3/2}\,\sqrt{b}\,x^3+3\,A\,a\,b}{3\,a^{7/2}\,\sqrt{b}\,x^{3/2}+3\,a^{5/2}\,b^{3/2}\,x^{9/2}}$$

input

```
int((A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x)
```

output

```
-(2*A*a^(3/2)*b^(1/2) - B*a^2*x^(3/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) + 3*
A*b^2*x^(9/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) + 3*A*a^(1/2)*b^(3/2)*x^3 -
B*a^(3/2)*b^(1/2)*x^3 + 3*A*a*b*x^(3/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) -
B*a*b*x^(9/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)))/(3*a^(7/2)*b^(1/2)*x^(3/2)
+ 3*a^(5/2)*b^(3/2)*x^(9/2))
```

3.170 $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^2}\,dx$

3.170.1 Optimal result

Integrand size = 22, antiderivative size = 318

$$\begin{aligned}\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^2}\,dx &= -\frac{11Ab-5aB}{15a^2bx^{5/2}}+\frac{Ab-aB}{3abx^{5/2}(a+bx^3)}\\ &+\frac{(11Ab-5aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}}-\frac{(11Ab-5aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}}\\ &-\frac{(11Ab-5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}}+\frac{(11Ab-5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}\\ &-\frac{(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}\end{aligned}$$

output
```
1/15*(-11*A*b+5*B*a)/a^2/b/x^(5/2)+1/3*(A*b-B*a)/a/b/x^(5/2)/(b*x^3+a)-1/9
*(11*A*b-5*B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(1/6)-1/18*(11*
A*b-5*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(1/6)-1/1
8*(11*A*b-5*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(1/6
)+1/36*(11*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2)
)/a^(17/6)/b^(1/6)*3^(1/2)-1/36*(11*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6
)*b^(1/6)*3^(1/2)*x^(1/2))/a^(17/6)/b^(1/6)*3^(1/2)
```

3.170.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx=\frac{\frac{6a^{5/6}\left(-6aA-11Abx^3+5aBx^3\right)}{x^{5/2}\left(a+bx^3\right)}+\frac{10(-11Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{\sqrt[6]{b}}+\frac{5(11Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{\sqrt[6]{b}}}{90a^{17/6}}$$

input
```
Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x]
```

output
```
((6*a^(5/6)*(-6*a*A - 11*A*b*x^3 + 5*a*B*x^3))/(x^(5/2)*(a + b*x^3)) + (10
*(-11*A*b + 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/b^(1/6) + (5*(11*A*b
 - 5*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/b^(1/6)
 + (5*Sqrt[3]*(-11*A*b + 5*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/
(a^(1/3) + b^(1/3)*x)])/b^(1/6))/(90*a^(17/6))
```

3.170.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {957, 847, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(11Ab-5aB)\int \frac{1}{x^{7/2}\left(bx^3+a\right)}dx}{6ab}+\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{847}$$

$$\frac{(11Ab-5aB)\left(-\frac{b\int \frac{1}{\sqrt{x}\left(bx^3+a\right)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{6ab}+\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{851}$$

$$\frac{(11Ab-5aB)\left(-\frac{2b\int \frac{1}{bx^3+a}d\sqrt{x}}{a}-\frac{2}{5ax^{5/2}}\right)}{6ab}+\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{753}$$

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{6ab}+\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{6ab}+\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{6ab}+\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

3.170. $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx$

$$\Big\downarrow \text{1142}$$

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int-\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{}}}{}\right)}{a}\right)}{6ab}$$

$$\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}}{}\right)}{a}\right)}{6ab}$$

$$\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

3.170. $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx$

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+}}{}\right.}{a}\right.}{6ab}$$

$$\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$$\Big\downarrow 1082$$

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\frac{\int\frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int\frac{1}{-x-\frac{1}{3}}d}{}}{6a^{5/6}}\right.}{a}\right.}{6ab}$$

$$\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

$$\Big\downarrow 217$$

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\right.}{}}{6a^{5/6}}\right.}{a}\right.}{6ab}$$

$$\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

3.170. $\int\frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{(11Ab-5aB)\left(-\frac{2b\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{b}}}{}\right)}{a}\right.}{6ab}$$

$$\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

input
```
Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x]
```

output
```
(A*b - a*B)/(3*a*b*x^(5/2)*(a + b*x^3)) + ((11*A*b - 5*a*B)*(-2/(5*a*x^(5/
2)) - (2*b*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(5/6)*b^(1/6)) + (-(Arc
Tan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) - (Sqrt[
3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6))
)/(6*a^(5/6)) + (ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6))
)]/b^(1/6) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1
/3)*x])/(2*b^(1/6)))/(6*a^(5/6))))/a))/(6*a*b)
```

3.170.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.170. $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^2}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.170.4 Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{2A}{5a^{2}x^{\frac{5}{2}}}-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)\sqrt{x}}{b\,x^{3}+a}+\frac{(11Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\mathrm{arct}}{}\right.}{}\right.}{a^{2}}$
default	$-\frac{2A}{5a^{2}x^{\frac{5}{2}}}-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)\sqrt{x}}{b\,x^{3}+a}+\frac{(11Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\mathrm{arct}}{}\right.}{}\right.}{a^{2}}$
risch	$-\frac{2A}{5a^{2}x^{\frac{5}{2}}}-\frac{\frac{2\left(\frac{Ab}{6}-\frac{Ba}{6}\right)\sqrt{x}}{b\,x^{3}+a}+\frac{(11Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan}{}\right.}{3}}{a^{2}}$

input
```
int((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5*A/a^2/x^(5/2)-2/a^2*((1/6*A*b-1/6*B*a)*x^(1/2)/(b*x^3+a)+1/6*(11*A*b-
5*B*a)*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)
^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arc
tan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2
)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b
)^(1/6)+3^(1/2))))
```

3.170.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1463 vs. $2(232) = 464$.

Time = 0.43 (sec) , antiderivative size = 1463, normalized size of antiderivative = 4.60

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/180*(10*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1
134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4
 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*log(a^3*(-(15625
*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*
a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)
/(a^17*b))^(1/6) - (5*B*a - 11*A*b)*sqrt(x)) - 10*(a^2*b*x^6 + a^3*x^3)*(-
(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^
3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^
6*b^6)/(a^17*b))^(1/6)*log(-a^3*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 11
34375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4
- 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6) - (5*B*a - 11*A*b
)*sqrt(x)) + 5*(a^2*b*x^6 + a^3*x^3 + sqrt(-3)*(a^2*b*x^6 + a^3*x^3))*(-(1
5625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*
B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*
b^6)/(a^17*b))^(1/6)*log(-(5*B*a - 11*A*b)*sqrt(x) + 1/2*(sqrt(-3)*a^3 + a
^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327
500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771
561*A^6*b^6)/(a^17*b))^(1/6)) - 5*(a^2*b*x^6 + a^3*x^3 + sqrt(-3)*(a^2*b*x
^6 + a^3*x^3))*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4
*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*...
```

3.170.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx = \frac{(5\,Ba-11\,Ab)x^3-6\,Aa}{15\left(a^2bx^{\frac{11}{2}}+a^3x^{\frac{5}{2}}\right)} + \frac{\frac{\sqrt{3}(5\,Ba-11\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(5\,Ba-11\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(5\,Bab^{\frac{1}{3}}-11\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{36\,a^2}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/15*((5*B*a - 11*A*b)*x^3 - 6*A*a)/(a^2*b*x^(11/2) + a^3*x^(5/2)) + 1/36*
(sqrt(3)*(5*B*a - 11*A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x
+ a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(5*B*a - 11*A*b)*log(-sqrt(3)*a^(1/
6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(5*B*a*b^(
1/3) - 11*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3
)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 11*A*a^(1/3)*b
^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*
b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 11*
A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/s
qrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/a^2
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx &= \frac{\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b}\\
&-\frac{\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b}\\
&+\frac{Ba\sqrt{x}-Ab\sqrt{x}}{3\left(bx^3+a\right)a^2}+\frac{\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^3b}\\
&+\frac{\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^3b}\\
&+\frac{\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^3b}-\frac{2\,A}{5\,a^2x^{\frac{5}{2}}}\end{aligned}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/36*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt
(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b) - 1/36*sqrt(3)*(5*(a*b^5)^(1/6)
*B*a - 11*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^
(1/3))/(a^3*b) + 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a^2) + 1/18*
(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) +
 2*sqrt(x))/(a/b)^(1/6))/(a^3*b) + 1/18*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^
(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b)
+ 1/9*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1
/6))/(a^3*b) - 2/5*A/(a^2*x^(5/2))
```

3.170.9 Mupad [B] (verification not implemented)

Time = 7.18 (sec) , antiderivative size = 2080, normalized size of antiderivative = 6.54

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x)
```

output
```
- ((2*A)/(5*a) + (x^3*(11*A*b - 5*B*a))/(15*a^2))/(a*x^(5/2) + b*x^(11/2))
 - (atan((((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 2646270
0*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) - (
(11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*
A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A
*b - 5*B*a)*1i)/(18*(-a)^(17/6)*b^(1/6)) + ((x^(1/2)*(21346578*A^4*a^10*b^
9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b
^6 - 38811960*A^3*B*a^11*b^8) + ((11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 -
 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7)
)/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)^(17/6)*b^(1/6)))
/(((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^
2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) - ((11*A*b
- 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^
15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B
*a))/(18*(-a)^(17/6)*b^(1/6)) - ((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*
B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 388119
60*A^3*B*a^11*b^8) + ((11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^
3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^
(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(17/6)*b^(1/6))))*(11*A*b - 5*
B*a)*1i)/(9*(-a)^(17/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(x^(1...
```

3.170. $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx$

3.171 $\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.171.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx=\frac{(Ab-aB)x^{9/2}}{6ab\,(a+bx^3)^2}-\frac{(Ab+3aB)x^{3/2}}{12ab^2\,(a+bx^3)}+\frac{(Ab+3aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}$$

output
```
1/6*(A*b-B*a)*x^(9/2)/a/b/(b*x^3+a)^2-1/12*(A*b+3*B*a)*x^(3/2)/a/b^2/(b*x^
3+a)+1/12*(A*b+3*B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

3.171.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx=-\frac{x^{3/2}(aAb+3a^2B-Ab^2x^3+5abBx^3)}{12ab^2\,(a+bx^3)^2}+\frac{(Ab+3aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
-1/12*(x^(3/2)*(a*A*b + 3*a^2*B - A*b^2*x^3 + 5*a*b*B*x^3))/(a*b^2*(a + b*
x^3)^2) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(3/2)*b^
(5/2))
```

3.171.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {957, 817, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(3aB+Ab)\int \frac{x^{7/2}}{\left(bx^3+a\right)^2}dx}{4ab}+\frac{x^{9/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{817}$$

$$\frac{(3aB+Ab)\left(\frac{\int \frac{\sqrt{x}}{bx^3+a}dx}{2b}-\frac{x^{3/2}}{3b(a+bx^3)}\right)}{4ab}+\frac{x^{9/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{(3aB+Ab)\left(\frac{\int \frac{x}{bx^3+a}d\sqrt{x}}{b}-\frac{x^{3/2}}{3b(a+bx^3)}\right)}{4ab}+\frac{x^{9/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{807}$$

$$\frac{(3aB+Ab)\left(\frac{\int \frac{1}{a+bx}dx^{3/2}}{3b}-\frac{x^{3/2}}{3b(a+bx^3)}\right)}{4ab}+\frac{x^{9/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{(3aB+Ab)\left(\frac{\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}-\frac{x^{3/2}}{3b(a+bx^3)}\right)}{4ab}+\frac{x^{9/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input

```
Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^(9/2))/(6*a*b*(a + b*x^3)^2) + ((A*b + 3*a*B)*(-1/3*x^(3/2)
/(b*(a + b*x^3)) + ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]]/(3*Sqrt[a]*b^(3/2))))
/(4*a*b)
```

3.171.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.171. $\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.171.4 Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{(Ab-5Ba)x^{\frac{9}{2}}}{12ab}-\frac{(Ab+3Ba)x^{\frac{3}{2}}}{12b^{2}}}{(b\,x^{3}+a)^{2}}+\frac{(Ab+3Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12b^{2}a\sqrt{ab}}$	81
default	$\frac{\frac{(Ab-5Ba)x^{\frac{9}{2}}}{12ab}-\frac{(Ab+3Ba)x^{\frac{3}{2}}}{12b^{2}}}{(b\,x^{3}+a)^{2}}+\frac{(Ab+3Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12b^{2}a\sqrt{ab}}$	81

input
```
int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*(1/8*(A*b-5*B*a)/a/b*x^(9/2)-1/8*(A*b+3*B*a)/b^2*x^(3/2))/(b*x^3+a)^2+
1/12*(A*b+3*B*a)/b^2/a/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.02

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \left[-\frac{((3\,Bab^2+Ab^3)x^6+3\,Ba^3+Aa^2b+2\,(3\,Ba^2b+Aab^2)x^3)\sqrt{-ab}\log\left(\frac{bx^3-2\sqrt{-ab}}{bx^3+a}\right.}{24\,(a^2b^5x^6+2\,a^3b^4x^3+a^4}\right.$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[-1/24*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*
b^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) +
 2*((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(x))/(a^2
*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), 1/12*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*
a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^
(3/2)/a) - ((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(
x))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

3.171.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(5\,Bab-Ab^2)x^{\frac{9}{2}}+(3\,Ba^2+Aab)x^{\frac{3}{2}}}{12\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)}+\frac{(3\,Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}ab^2}$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/12*((5*B*a*b - A*b^2)*x^(9/2) + (3*B*a^2 + A*a*b)*x^(3/2))/(a*b^4*x^6 +
 2*a^2*b^3*x^3 + a^3*b^2) + 1/12*(3*B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))
/(sqrt(a*b)*a*b^2)
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(3\,Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}ab^2}-\frac{5\,Babx^{\frac{9}{2}}-Ab^2x^{\frac{9}{2}}+3\,Ba^2x^{\frac{3}{2}}+Aabx^{\frac{3}{2}}}{12\,(bx^3+a)^2ab^2}$$

input
```
integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/12*(3*B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/12*(5
*B*a*b*x^(9/2) - A*b^2*x^(9/2) + 3*B*a^2*x^(3/2) + A*a*b*x^(3/2))/((b*x^3
+ a)^2*a*b^2)
```

3.171.9 Mupad [B] (verification not implemented)

Time = 7.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.28

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\operatorname{atan}\left(\frac{9\,b^{3/2}\,x^{3/2}\,\left(A^2\,b^2+6\,A\,B\,a\,b+9\,B^2\,a^2\right)}{\sqrt{a}\,\left(9\,A\,b^2+27\,B\,a\,b\right)\,\left(A\,b+3\,B\,a\right)}\right)\,\left(A\,b+3\,B\,a\right)}{12\,a^{3/2}\,b^{5/2}} - \frac{\frac{x^{3/2}\,\left(A\,b+3\,B\,a\right)}{12\,b^2}-\frac{x^{9/2}\,\left(A\,b-5\,B\,a\right)}{12\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

input

```
int((x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

output

```
(atan((9*b^(3/2)*x^(3/2)*(A^2*b^2 + 9*B^2*a^2 + 6*A*B*a*b))/(a^(1/2)*(9*A*
b^2 + 27*B*a*b)*(A*b + 3*B*a)))*(A*b + 3*B*a))/(12*a^(3/2)*b^(5/2)) - ((x^
(3/2)*(A*b + 3*B*a))/(12*b^2) - (x^(9/2)*(A*b - 5*B*a))/(12*a*b))/(a^2 + b
^2*x^6 + 2*a*b*x^3)
```

3.172 $\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

3.172.1 Optimal result

Integrand size = 22, antiderivative size = 327

$$\begin{aligned}\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2}-\frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)}\\ &-\frac{(5Ab+7aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}}+\frac{(5Ab+7aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}}\\ &+\frac{(5Ab+7aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}}-\frac{(5Ab+7aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}\\ &+\frac{(5Ab+7aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}\end{aligned}$$

output

```
1/6*(A*b-B*a)*x^(7/2)/a/b/(b*x^3+a)^2+1/108*(5*A*b+7*B*a)*arctan(b^(1/6)*x
^(1/2)/a^(1/6))/a^(11/6)/b^(13/6)+1/216*(5*A*b+7*B*a)*arctan(-3^(1/2)+2*b^
(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(13/6)+1/216*(5*A*b+7*B*a)*arctan(3^(1/2
)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(13/6)-1/432*(5*A*b+7*B*a)*ln(a^(1
/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(11/6)/b^(13/6)*3^(1/2)+1
/432*(5*A*b+7*B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a
^(11/6)/b^(13/6)*3^(1/2)-1/36*(5*A*b+7*B*a)*x^(1/2)/a/b^2/(b*x^3+a)
```

3.172.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.59

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{-\frac{6a^{5/6}\sqrt[6]{b}\sqrt{x}(7a^2B-Ab^2x^3+ab(5A+13Bx^3))}{(a+bx^3)^2}+2(5Ab+7aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-(5Ab+7a}{216a^{11/6}b^{13/6}}$$

input
```
Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((-6*a^(5/6)*b^(1/6)*Sqrt[x]*(7*a^2*B - A*b^2*x^3 + a*b*(5*A + 13*B*x^3)))
/(a + b*x^3)^2 + 2*(5*A*b + 7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (5*
A*b + 7*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + Sqr
t[3]*(5*A*b + 7*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) +
b^(1/3)*x)]])/(216*a^(11/6)*b^(13/6))
```

3.172.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {957, 817, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(7aB+5Ab)\int \frac{x^{5/2}}{(bx^3+a)^2}dx}{12ab}+\frac{x^{7/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{817}$$

$$\frac{(7aB+5Ab)\left(\frac{\int \frac{1}{\sqrt{x}\left(bx^3+a\right)}dx}{6b}-\frac{\sqrt{x}}{3b(a+bx^3)}\right)}{12ab}+\frac{x^{7/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{(7aB+5Ab)\left(\frac{\int \frac{1}{bx^3+a}d\sqrt{x}}{3b}-\frac{\sqrt{x}}{3b(a+bx^3)}\right)}{12ab}+\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{753}$$

$$\frac{(7aB+5Ab)\left(\frac{\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}}{3b}-\frac{\sqrt{x}}{3b(a+bx^3)}\right)}{12ab}+\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(7aB+5Ab)\left(\frac{\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}}{3b}-\frac{\sqrt{x}}{3b(a+bx^3)}\right)}{12ab}+\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{(7aB+5Ab)\left(\frac{\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}}{3b}-\frac{\sqrt{x}}{3b(a+bx^3)}\right)}{12ab}+\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1142}$$

3.172. $\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{(7aB+5Ab)\left(\frac{\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+}{6a^{5/6}}}{3b}\right.}{12ab}$$

$$\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{(7aB+5Ab)\left(\frac{\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+}{6a^{5/6}}}{3b}\right.}{12ab}$$

$$\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(7aB+5Ab)\left(\frac{\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}}{6a^{5/6}}}{3b}\right.}{12ab}$$

$$\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 1082$$

3.172. $\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{(7aB+5Ab)\left(\frac{\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(\frac{2\sqrt[6]{b}\sqrt{}}{\sqrt{3}\sqrt[6]{}}\right.}{\sqrt{3}\sqrt[6]{b}}}{6a^{5/6}}}{3b}\right.}{12ab}$$

$$\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{(7aB+5Ab)\left(\frac{\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{}}{\sqrt{3}}\right.\right.}{\sqrt[6]{b}}}{}}{3b}\right.}{12ab}$$

$$\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{(7aB+5Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{b}}+\frac{\sqrt{3}\log\left(\right.}{}}{6a^{5/6}}}{3b}\right.}{12ab}$$

$$\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

3.172. $\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

output
```
((A*b - a*B)*x^(7/2))/(6*a*b*(a + b*x^3)^2) + ((5*A*b + 7*a*B)*(-1/3*Sqrt[
x]/(b*(a + b*x^3)) + (ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(5/6)*b^(1/6)
) + (-(ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)
) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(
2*b^(1/6)))/(6*a^(5/6)) + (ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3
]*a^(1/6)))]/b^(1/6) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6)))/(3*b)))/(12*a*b)
```

3.172.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.172. $\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.172.4 Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\frac{(Ab-13Ba)x^{\frac{7}{2}}}{36ab}-\frac{(5Ab+7Ba)\sqrt{x}}{36b^{2}}}{(b\,x^{3}+a)^{2}}+\frac{(5Ab+7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}a}{}\right.}{3}$
default	$\frac{\frac{(Ab-13Ba)x^{\frac{7}{2}}}{36ab}-\frac{(5Ab+7Ba)\sqrt{x}}{36b^{2}}}{(b\,x^{3}+a)^{2}}+\frac{(5Ab+7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}a}{}\right.}{3}$

input
```
int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/72*(A*b-13*B*a)/a/b*x^(7/2)-1/72*(5*A*b+7*B*a)/b^2*x^(1/2))/(b*x^3+a)
^2+1/36*(5*A*b+7*B*a)/b^2/a*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))
-1/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+
1/6/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a
/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*
arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2)))
```

3.172.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. 2(241) = 482.

Time = 0.40 (sec) , antiderivative size = 1618, normalized size of antiderivative = 4.95

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output

```
1/432*(2*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*
A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4
*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*log(
a^2*b^2*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 +
857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 156
25*A^6*b^6)/(a^11*b^13))^(1/6) + (7*B*a + 5*A*b)*sqrt(x)) - 2*(a*b^4*x^6 +
 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*
A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250
*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*log(-a^2*b^2*(-(117649*B^
6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b
^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^
13))^(1/6) + (7*B*a + 5*A*b)*sqrt(x)) + (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b
^2 + sqrt(-3)*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2))*(-(117649*B^6*a^6 + 5
04210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 4593
75*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6
)*log((7*B*a + 5*A*b)*sqrt(x) + 1/2*(sqrt(-3)*a^2*b^2 + a^2*b^2)*(-(117649
*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^
3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11
*b^13))^(1/6)) - (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2 + sqrt(-3)*(a*b^4*x^
6 + 2*a^2*b^3*x^3 + a^3*b^2))*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + ...
```

3.172.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input

```
integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

output

```
Timed out
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.04

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(13\,Bab-Ab^2)x^{\frac{7}{2}}+(7\,Ba^2+5\,Aab)\sqrt{x}}{36\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)} + \frac{\frac{\sqrt{3}(7\,Ba+5\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(7\,Ba+5\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(7\,Bab^{\frac{1}{3}}+5\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} +}{432\,ab^2}$$

input
```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/36*((13*B*a*b - A*b^2)*x^(7/2) + (7*B*a^2 + 5*A*a*b)*sqrt(x))/(a*b^4*x^
6 + 2*a^2*b^3*x^3 + a^3*b^2) + 1/432*(sqrt(3)*(7*B*a + 5*A*b)*log(sqrt(3)*
a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)
*(7*B*a + 5*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3
))/(a^(5/6)*b^(1/6)) + 4*(7*B*a*b^(1/3) + 5*A*b^(4/3))*arctan(b^(1/3)*sqrt
(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(7*
B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) +
 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3)
)) + 2*(7*B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6
)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1
/3)*b^(1/3))))/(a*b^2)
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}\,Ba+5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^2b^3}$$
$$-\frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}\,Ba+5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^2b^3}$$
$$+\frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba+5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^2b^3}$$
$$+\frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba+5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^2b^3}$$
$$+\frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba+5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^2b^3}-\frac{13\,Babx^{\frac{7}{2}}-Ab^2x^{\frac{7}{2}}+7\,Ba^2\sqrt{x}+5\,Aab\sqrt{x}}{36\,(bx^3+a)^2ab^2}$$

input
```
integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/432*sqrt(3)*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt
(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^3) - 1/432*sqrt(3)*(7*(a*b^5)^(1
/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b
)^(1/3))/(a^2*b^3) + 1/216*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arc
tan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^3) + 1/216*(7*(a
*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sq
rt(x))/(a/b)^(1/6))/(a^2*b^3) + 1/108*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/
6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b^3) - 1/36*(13*B*a*b*x^(7/2) - A
*b^2*x^(7/2) + 7*B*a^2*sqrt(x) + 5*A*a*b*sqrt(x))/((b*x^3 + a)^2*a*b^2)
```

3.172.9 Mupad [B] (verification not implemented)

Time = 7.39 (sec) , antiderivative size = 1944, normalized size of antiderivative = 5.94

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
int((x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
(atan((((((5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 5
25*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)) - (x^(1/2)*(625*A^4*b^4 + 2
401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))
/(279936*a^4*b^3))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)) - ((((5*
A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^
2))/(279936*(-a)^(23/6)*b^(19/6)) + (x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 +
 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*
b^3))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)))/(((((5*A*b + 7*B*a)*
(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(
-a)^(23/6)*b^(19/6)) - (x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2
*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(5*A*b
+ 7*B*a))/(216*(-a)^(11/6)*b^(13/6)) + ((((5*A*b + 7*B*a)*(125*A^3*b^3 + 3
43*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19
/6)) + (x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*
A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(5*A*b + 7*B*a))/(216*(
-a)^(11/6)*b^(13/6))))*(5*A*b + 7*B*a)*1i)/(108*(-a)^(11/6)*b^(13/6)) - ((
x^(1/2)*(5*A*b + 7*B*a))/(36*b^2) - (x^(7/2)*(A*b - 13*B*a))/(36*a*b))/(a^
2 + b^2*x^6 + 2*a*b*x^3) + (atan(((((3^(1/2)*1i)/2 - 1/2)*((x^(1/2)*(625*A
^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3
*B*a*b^3))/(279936*a^4*b^3) - (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + 7*B*a)*(...
```

3.172. $\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.173 $\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

3.173.1 Optimal result

Integrand size = 22, antiderivative size = 327

$$
\begin{aligned}
\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2}+\frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)} \\
&\quad-\frac{(7Ab+5aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}}+\frac{(7Ab+5aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}} \\
&\quad+\frac{(7Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}}+\frac{(7Ab+5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}} \\
&\quad-\frac{(7Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}}
\end{aligned}
$$

output
```
1/6*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)^2+1/36*(7*A*b+5*B*a)*x^(5/2)/a^2/b/(b*
x^3+a)+1/108*(7*A*b+5*B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(11/
6)+1/216*(7*A*b+5*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)
/b^(11/6)+1/216*(7*A*b+5*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^
(13/6)/b^(11/6)+1/432*(7*A*b+5*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3
^(1/2)*x^(1/2))/a^(13/6)/b^(11/6)*3^(1/2)-1/432*(7*A*b+5*B*a)*ln(a^(1/3)+b
^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(13/6)/b^(11/6)*3^(1/2)
```

3.173.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.59

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\frac{6\sqrt[6]{a}b^{5/6}x^{5/2}(-a^2B+7Ab^2x^3+ab(13A+5Bx^3))}{(a+bx^3)^2}+2(7Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-(7Ab+5a}{216a^{13/6}b^{11/6}}$$

input
```
Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((6*a^(1/6)*b^(5/6)*x^(5/2)*(-(a^2*B) + 7*A*b^2*x^3 + a*b*(13*A + 5*B*x^3)
))/(a + b*x^3)^2 + 2*(7*A*b + 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (
7*A*b + 5*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] - S
qrt[3]*(7*A*b + 5*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3)
+ b^(1/3)*x)])/(216*a^(13/6)*b^(11/6))
```

3.173.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {957, 819, 851, 824, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(5aB+7Ab)\int \frac{x^{3/2}}{(bx^3+a)^2}dx}{12ab}+\frac{x^{5/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{(5aB+7Ab)\left(\frac{\int \frac{x^{3/2}}{bx^3+a}dx}{6a}+\frac{x^{5/2}}{3a(a+bx^3)}\right)}{12ab}+\frac{x^{5/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{(5aB+7Ab)\left(\frac{\int \frac{x^2}{bx^3+a}d\sqrt{x}}{3a}+\frac{x^{5/2}}{3a(a+bx^3)}\right)}{12ab}+\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow 824$$

$$\frac{(5aB+7Ab)\left(\frac{\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}+\frac{\int -\frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}+\frac{\int -\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}}{3a}+\frac{x^{5/2}}{3a(a+bx^3)}\right)}{12ab}+\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow 27$$

$$\frac{(5aB+7Ab)\left(\frac{\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}}{3a}+\frac{x^{5/2}}{3a(a+bx^3)}\right)}{12ab}+\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow 218$$

$$\frac{(5aB+7Ab)\left(\frac{-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}}{3a}+\frac{x^{5/2}}{3a(a+bx^3)}\right)}{12ab}+\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow 1142$$

3.173. $\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}dx$

$$\frac{(5aB+7Ab)\left(\frac{-\frac{-\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int-\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-}{6\sqrt[6]{}}}{3a}\right.}{12ab}$$

$$\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{(5aB+7Ab)\left(\frac{-\frac{\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}}{6\sqrt[6]{a}b^{2}}}{3a}\right.}{12ab}$$

$$\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(5aB+7Ab)\left(\frac{-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{1}{2}}{6\sqrt[6]{a}}}{3a}\right.}{12ab}$$

$$\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1082$$

3.173. $\int\frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{(5aB+7Ab)\left(\frac{-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int\frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\int\frac{1}{-x-\frac{1}{3}}d\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+}}{6\sqrt[6]{a}b^{2/3}}}{3a}\right.}{12ab}$$

$$\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{(5aB+7Ab)\left(\frac{-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\arctan\left(\sqrt{3}\left(\right.\right.}{6\sqrt[6]{a}b^{2/3}}}{3a}\right.}{12ab}$$

$$\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{(5aB+7Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}}{3a}\right.}{12ab}$$

$$\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

3.173. $\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

output
```
((A*b - a*B)*x^(5/2))/(6*a*b*(a + b*x^3)^2) + ((7*A*b + 5*a*B)*(x^(5/2)/(3
*a*(a + b*x^3)) + (ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(1/6)*b^(5/6)) -
 (ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) - (S
qrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1
/6)))/(6*a^(1/6)*b^(2/3)) - (-(ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sq
rt[3]*a^(1/6)))]/b^(1/6)) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)
*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3)))/(3*a)))/(12*a*b)
```

3.173.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.173. $\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.173.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\frac{(7Ab+5Ba)x^{\frac{11}{2}}}{36a^{2}}+\frac{(13Ab-Ba)x^{\frac{5}{2}}}{36ab}}{(b\,x^{3}+a)^{2}}+\frac{(7Ab+5Ba)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\left(\frac{a}{b}\right.\right.}{36a^{2}b}$
default	$\frac{\frac{(7Ab+5Ba)x^{\frac{11}{2}}}{36a^{2}}+\frac{(13Ab-Ba)x^{\frac{5}{2}}}{36ab}}{(b\,x^{3}+a)^{2}}+\frac{(7Ab+5Ba)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\left(\frac{a}{b}\right.\right.}{36a^{2}b}$

input
```
int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/72*(7*A*b+5*B*a)/a^2*x^(11/2)+1/72*(13*A*b-B*a)/a/b*x^(5/2))/(b*x^3+a
)^2+1/36*(7*A*b+5*B*a)/a^2/b*(1/12/a*3^(1/2)*(a/b)^(5/6)*ln(3^(1/2)*(a/b)^
(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(
a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a
/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))+1/3/b/(
a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6)))
```

3.173.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1959 vs. 2(241) = 482.

Time = 0.43 (sec) , antiderivative size = 1959, normalized size of antiderivative = 5.99

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

output

```
1/432*(2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 + 131250*A
*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*
B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)*log(
a^11*b^9*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 +
857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117
649*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^4*b + 6125
0*A^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*A^5*
b^5)*sqrt(x)) - 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 +
 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 90
0375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(
1/6)*log(-a^11*b^9*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*
a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a
*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^
4*b + 61250*A^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 +
16807*A^5*b^5)*sqrt(x)) + (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b - sqrt(-3)*
(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b))*(-(15625*B^6*a^6 + 131250*A*B^5*a^5
*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*
b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)*log(1/2*(sqr
t(-3)*a^11*b^9 + a^11*b^9)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*
A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504...
```

3.173.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input

```
integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

output

```
Timed out
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.83

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(5\,Bab+7\,Ab^2)x^{\frac{11}{2}}-(Ba^2-13\,Aab)x^{\frac{5}{2}}}{36\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)}$$

$$-\frac{(5\,Ba+7\,Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\right.}{b}\right.}{432\,a^2b}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/36*((5*B*a*b + 7*A*b^2)*x^(11/2) - (B*a^2 - 13*A*a*b)*x^(5/2))/(a^2*b^3*
x^6 + 2*a^3*b^2*x^3 + a^4*b) - 1/432*(5*B*a + 7*A*b)*(sqrt(3)*log(sqrt(3)*
a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)
*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5
/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)
*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^(1/6)*b^
(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^
(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^
(1/3)*b^(1/3))))/(a^2*b)
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.96

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(5\,Ba+7\,Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,\left(ab^5\right)^{\frac{1}{6}}a^2b}$$

$$+\frac{(5\,Ba+7\,Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,\left(ab^5\right)^{\frac{1}{6}}a^2b}+\frac{\left(5\,Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}+7\,Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^3b}$$

$$+\frac{5\,Babx^{\frac{11}{2}}+7\,Ab^2x^{\frac{11}{2}}-Ba^2x^{\frac{5}{2}}+13\,Aabx^{\frac{5}{2}}}{36\,(bx^3+a)^2a^2b}$$

$$-\frac{\sqrt{3}\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+7\,(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^6}$$

$$+\frac{\sqrt{3}\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+7\,(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^6}$$

input
```
integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/216*(5*B*a + 7*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6)
)/((a*b^5)^(1/6)*a^2*b) + 1/216*(5*B*a + 7*A*b)*arctan(-(sqrt(3)*(a/b)^(1/
6) - 2*sqrt(x))/(a/b)^(1/6))/((a*b^5)^(1/6)*a^2*b) + 1/108*(5*B*a*(a/b)^(5
/6) + 7*A*b*(a/b)^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/(a^3*b) + 1/36*(5*B*a
*b*x^(11/2) + 7*A*b^2*x^(11/2) - B*a^2*x^(5/2) + 13*A*a*b*x^(5/2))/((b*x^3
 + a)^2*a^2*b) - 1/432*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(5/6)*A*b)
*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^6) + 1/432*sqrt
(3)*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)
^(1/6) + x + (a/b)^(1/3))/(a^3*b^6)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 7.35 (sec) , antiderivative size = 1672, normalized size of antiderivative = 5.11

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
int((x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
((x^(11/2)*(7*A*b + 5*B*a))/(36*a^2) + (x^(5/2)*(13*A*b - B*a))/(36*a*b))/
(a^2 + b^2*x^6 + 2*a*b*x^3) + (atan(((((343*A^3*b^3 + 125*B^3*a^3 + 525*A*
B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*(7*A*b + 5*B*a)*(49*A^2
*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*(7*A*b
 + 5*B*a)^2*1i)/(46656*(-a)^(13/3)*b^(11/3)) - (((343*A^3*b^3 + 125*B^3*a^
3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*(7*A*b + 5*B*
a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6
)))*(7*A*b + 5*B*a)^2*1i)/(46656*(-a)^(13/3)*b^(11/3)))/((((343*A^3*b^3 +
125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*(7*
A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/
6)*b^(11/6)))*(7*A*b + 5*B*a)^2)/(46656*(-a)^(13/3)*b^(11/3)) + (((343*A^3
*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1
/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-
a)^(19/6)*b^(11/6)))*(7*A*b + 5*B*a)^2)/(46656*(-a)^(13/3)*b^(11/3))))*(7*
A*b + 5*B*a)*1i)/(108*(-a)^(13/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 - 1/
2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735
*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)
*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6))
)*1i)/(46656*(-a)^(13/3)*b^(11/3)) - ((((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b + 5*
B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)...
```

3.173. $\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.174 $\int \frac{\sqrt{x}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

3.174.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{\sqrt{x}(A+Bx^3)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)}+\frac{(3Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}$$

output
```
1/6*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)^2+1/12*(3*A*b+B*a)*x^(3/2)/a^2/b/(b*x^
3+a)+1/12*(3*A*b+B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

3.174.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{x}(A+Bx^3)}{\left(a+bx^3\right)^3}\,dx$$
$$= -\frac{x^{3/2}(-5aAb+a^2B-3Ab^2x^3-abBx^3)}{12a^2b\left(a+bx^3\right)^2}+\frac{(3Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
-1/12*(x^(3/2)*(-5*a*A*b + a^2*B - 3*A*b^2*x^3 - a*b*B*x^3))/(a^2*b*(a + b
*x^3)^2) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(5/2)*b
^(3/2))
```

3.174.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {957, 819, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB+3Ab)\int \frac{\sqrt{x}}{\left(bx^3+a\right)^2}dx}{4ab}+\frac{x^{3/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{\sqrt{x}}{bx^3+a}dx}{2a}+\frac{x^{3/2}}{3a(a+bx^3)}\right)}{4ab}+\frac{x^{3/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{x}{bx^3+a}d\sqrt{x}}{a}+\frac{x^{3/2}}{3a(a+bx^3)}\right)}{4ab}+\frac{x^{3/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{807}$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{1}{a+bx}dx^{3/2}}{3a}+\frac{x^{3/2}}{3a(a+bx^3)}\right)}{4ab}+\frac{x^{3/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{(aB+3Ab)\left(\frac{\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}+\frac{x^{3/2}}{3a(a+bx^3)}\right)}{4ab}+\frac{x^{3/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input

```
Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]
```

output
```
((A*b - a*B)*x^(3/2))/(6*a*b*(a + b*x^3)^2) + ((3*A*b + a*B)*(x^(3/2)/(3*a
*(a + b*x^3)) + ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]]/(3*a^(3/2)*Sqrt[b])))/(4
*a*b)
```

3.174.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.174. $\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx$

3.174.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{(3Ab+Ba)x^{\frac{9}{2}}}{12a^2}+\frac{(5Ab-Ba)x^{\frac{3}{2}}}{12ab}}{(b\,x^3+a)^2}+\frac{(3Ab+Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12a^2b\sqrt{ab}}$	82
default	$\frac{\frac{(3Ab+Ba)x^{\frac{9}{2}}}{12a^2}+\frac{(5Ab-Ba)x^{\frac{3}{2}}}{12ab}}{(b\,x^3+a)^2}+\frac{(3Ab+Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12a^2b\sqrt{ab}}$	82

input
```
int((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*(1/8*(3*A*b+B*a)/a^2*x^(9/2)+1/8*(5*A*b-B*a)/a/b*x^(3/2))/(b*x^3+a)^2+
1/12*(3*A*b+B*a)/a^2/b/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.01

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx = \left[-\frac{((Bab^2+3\,Ab^3)x^6+Ba^3+3\,Aa^2b+2\,(Ba^2b+3\,Aab^2)x^3)\sqrt{-ab}\log\left(\frac{bx^3-2\,\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\,((Ba^2b^2+}{24\,(a^3b^4x^6+2\,a^4b^3x^3+a^5b^2)}\right.$$

input
```
integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[-1/24*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*
b^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) -
 2*((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a^3
*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2), 1/12*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^
3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^
(3/2)/a) + ((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(
x))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

3.174.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)*x**(1/2)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Bab+3\,Ab^2)x^{\frac{9}{2}}-(Ba^2-5\,Aab)x^{\frac{3}{2}}}{12\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)}+\frac{(Ba+3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}a^2b}$$

input
```
integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/12*((B*a*b + 3*A*b^2)*x^(9/2) - (B*a^2 - 5*A*a*b)*x^(3/2))/(a^2*b^3*x^6
+ 2*a^3*b^2*x^3 + a^4*b) + 1/12*(B*a + 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/
(sqrt(a*b)*a^2*b)
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ba+3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}a^2b}+\frac{Babx^{\frac{9}{2}}+3\,Ab^2x^{\frac{9}{2}}-Ba^2x^{\frac{3}{2}}+5\,Aabx^{\frac{3}{2}}}{12\,(bx^3+a)^2a^2b}$$

input
```
integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/12*(B*a + 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/12*(B
*a*b*x^(9/2) + 3*A*b^2*x^(9/2) - B*a^2*x^(3/2) + 5*A*a*b*x^(3/2))/((b*x^3
+ a)^2*a^2*b)
```

3.174.9 Mupad [B] (verification not implemented)

Time = 7.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\frac{x^{9/2}\,(3\,A\,b+B\,a)}{12\,a^2}+\frac{x^{3/2}\,(5\,A\,b-B\,a)}{12\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \frac{\operatorname{atan}\left(\frac{b^{3/2}\,x^{3/2}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{\sqrt{a}\,(3\,A\,b+B\,a)\,\left(3\,A\,b^3+B\,a\,b^2\right)}\right)\,(3\,A\,b+B\,a)}{12\,a^{5/2}\,b^{3/2}}$$

input
```
int((x^(1/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

output
```
((x^(9/2)*(3*A*b + B*a))/(12*a^2) + (x^(3/2)*(5*A*b - B*a))/(12*a*b))/(a^2
 + b^2*x^6 + 2*a*b*x^3) + (atan((b^(3/2)*x^(3/2)*(9*A^2*b^3 + B^2*a^2*b +
6*A*B*a*b^2))/(a^(1/2)*(3*A*b + B*a)*(3*A*b^3 + B*a*b^2)))*(3*A*b + B*a))/
(12*a^(5/2)*b^(3/2))
```

3.175 $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx$

3.175.1 Optimal result

Integrand size = 22, antiderivative size = 321

$$\begin{aligned}\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx &= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)} \\ &\quad - \frac{5(11Ab+aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}} \\ &\quad + \frac{5(11Ab+aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}} \\ &\quad + \frac{5(11Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}} \\ &\quad - \frac{5(11Ab+aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}} \\ &\quad + \frac{5(11Ab+aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}}\end{aligned}$$

output
```
5/108*(11*A*b+B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(7/6)+5/216*
(11*A*b+B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(7/6)+5
/216*(11*A*b+B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(7/
6)-5/432*(11*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2)
)/a^(17/6)/b^(7/6)*3^(1/2)+5/432*(11*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)
*b^(1/6)*3^(1/2)*x^(1/2))/a^(17/6)/b^(7/6)*3^(1/2)+1/6*(A*b-B*a)*x^(1/2)/a
/b/(b*x^3+a)^2+1/36*(11*A*b+B*a)*x^(1/2)/a^2/b/(b*x^3+a)
```

3.175.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx = \frac{\frac{6a^{5/6}\sqrt[6]{b}\sqrt{x}(-5a^2B+11Ab^2x^3+ab(17A+Bx^3))}{(a+bx^3)^2}+10(11Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-5(11Ab+aB)\arctan\left(\frac{\sqrt[3]{a}-}{\sqrt[6]{a}}\right.}{216a^{17/6}b^{7/6}}$$

input
```
Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3),x]
```

output
```
((6*a^(5/6)*b^(1/6)*Sqrt[x]*(-5*a^2*B + 11*A*b^2*x^3 + a*b*(17*A + B*x^3))
)/(a + b*x^3)^2 + 10*(11*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - 5*
(11*A*b + a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + 5
*Sqrt[3]*(11*A*b + a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3)
 + b^(1/3)*x)])/(216*a^(17/6)*b^(7/6))
```

3.175.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {957, 819, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB+11Ab)\int \frac{1}{\sqrt{x}\left(bx^3+a\right)^2}dx}{12ab}+\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \textcolor{blue}{819}$$

$$\frac{(aB+11Ab)\left(\frac{5\int \frac{1}{\sqrt{x}\left(bx^3+a\right)}dx}{6a}+\frac{\sqrt{x}}{3a(a+bx^3)}\right)}{12ab}+\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \textcolor{blue}{851}$$

$$\frac{(aB+11Ab)\left(\frac{5\int \frac{1}{bx^3+a}d\sqrt{x}}{3a}+\frac{\sqrt{x}}{3a(a+bx^3)}\right)}{12ab}+\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \textcolor{blue}{753}$$

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{3a}+\frac{\sqrt{x}}{3a(a+bx^3)}\right)}{12ab}+\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{3a}+\frac{\sqrt{x}}{3a(a+bx^3)}\right)}{12ab}+\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \textcolor{blue}{218}$$

3.175. $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx$

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{3a}+\frac{\sqrt{x}}{3a(a+bx^3)}\right)}{12ab}+\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d}{6a}\right.}{3a}\right.}{12ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

3.175. $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx$

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/}}\right.\right.}{3a}\right.}{12ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a}\right.\right.}{3a}\right.}{12ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1082$$

3.175. $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx$

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(\frac{2\sqrt[6]{}}{\sqrt{3}}\right.}{\sqrt{3}\sqrt[6]{b}}}{6a^{5/6}}\right.}{3a}\right.}{12ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(\right.\right.}{\sqrt[6]{}}}{6a^{5/6}}\right.}{3a}\right.}{12ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1103$$

3.175. $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx$

$$\frac{(aB+11Ab)\left(\frac{5\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{b}}+\sqrt{3}}{6a}\right)}{3a}\right)}{12ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3),x]
```

output
```
((A*b - a*B)*Sqrt[x])/(6*a*b*(a + b*x^3)^2) + ((11*A*b + a*B)*(Sqrt[x]/(3*
a*(a + b*x^3)) + (5*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)]/(3*a^(5/6)*b^(1/6))
 + (-(ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6))
 - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2
*b^(1/6)))/(6*a^(5/6)) + (ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*Sqrt[x])/(Sqrt[3]
*a^(1/6)))]/b^(1/6) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[
x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6))))/(3*a)))/(12*a*b)
```

3.175.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.175. $\int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)^3}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.175.4 Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{\frac{(11Ab+Ba)x^{\frac{7}{2}}}{36a^{2}}+\frac{(17Ab-5Ba)\sqrt{x}}{36ab}}{(b\,x^{3}+a)^{2}}+\frac{5(11Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}}{}\right.}{}$
default	$\frac{\frac{(11Ab+Ba)x^{\frac{7}{2}}}{36a^{2}}+\frac{(17Ab-5Ba)\sqrt{x}}{36ab}}{(b\,x^{3}+a)^{2}}+\frac{5(11Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}}{}\right.}{}$

input
```
int((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1/72*(11*A*b+B*a)/a^2*x^(7/2)+1/72*(17*A*b-5*B*a)/a/b*x^(1/2))/(b*x^3+a
)^2+5/36*(11*A*b+B*a)/a^2/b*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))
-1/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+
1/6/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a
/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*
arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2)))
```

3.175.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1588 vs. $2(235) = 470$.

Time = 0.42 (sec) , antiderivative size = 1588, normalized size of antiderivative = 4.95

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx = \text{Too large to display}$$

input

```
integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="fricas")
```

output

```
1/432*(10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*
b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4
+ 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*log(5*a^3*b*(-(B
^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 2
19615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^
(1/6) + 5*(B*a + 11*A*b)*sqrt(x)) - 10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*
b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*
b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17
*b^7))^(1/6)*log(-5*a^3*b*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b
^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 +
 1771561*A^6*b^6)/(a^17*b^7))^(1/6) + 5*(B*a + 11*A*b)*sqrt(x)) + 5*(a^2*b
^3*x^6 + 2*a^3*b^2*x^3 + a^4*b + sqrt(-3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a
^4*b))*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*
a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(
a^17*b^7))^(1/6)*log(5*(B*a + 11*A*b)*sqrt(x) + 5/2*(sqrt(-3)*a^3*b + a^3*
b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*
b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17
*b^7))^(1/6)) - 5*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b + sqrt(-3)*(a^2*b^3
*x^6 + 2*a^3*b^2*x^3 + a^4*b))*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*
a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B...
```

3.175.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx = \text{Timed out}$$

input

```
integrate((B*x**3+A)/(b*x**3+a)**3/x**(1/2),x)
```

output

```
Timed out
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx = \frac{(Bab+11\,Ab^2)x^{\frac{7}{2}}-(5\,Ba^2-17\,Aab)\sqrt{x}}{36\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)} + \frac{5\left(\frac{\sqrt{3}(Ba+11\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}(Ba+11\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(Bab^{\frac{1}{3}}+11\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right.}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right.}{432\,a^2b}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="maxima")
```

output
```
1/36*((B*a*b + 11*A*b^2)*x^(7/2) - (5*B*a^2 - 17*A*a*b)*sqrt(x))/(a^2*b^3*
x^6 + 2*a^3*b^2*x^3 + a^4*b) + 5/432*(sqrt(3)*(B*a + 11*A*b)*log(sqrt(3)*a
^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*
(B*a + 11*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))
/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) + 11*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)
/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(
4/3)*b^(1/3) + 11*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b
^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) +
 2*(B*a^(4/3)*b^(1/3) + 11*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(
1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b
^(1/3))))/(a^2*b)
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx = \frac{5\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^2}$$
$$-\frac{5\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^2}$$
$$+\frac{5\left(\left(ab^5\right)^{\frac{1}{6}}Ba+11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^3b^2}$$
$$+\frac{5\left(\left(ab^5\right)^{\frac{1}{6}}Ba+11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^3b^2}$$
$$+\frac{5\left(\left(ab^5\right)^{\frac{1}{6}}Ba+11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^3b^2}$$
$$+\frac{Babx^{\frac{7}{2}}+11\,Ab^2x^{\frac{7}{2}}-5\,Ba^2\sqrt{x}+17\,Aab\sqrt{x}}{36\left(bx^3+a\right)^2a^2b}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="giac")
```

output
```
5/432*sqrt(3)*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(
x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^2) - 5/432*sqrt(3)*((a*b^5)^(1/6)
*B*a + 11*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^
(1/3))/(a^3*b^2) + 5/216*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*arctan
((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^2) + 5/216*((a*b^5)
^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x
))/(a/b)^(1/6))/(a^3*b^2) + 5/108*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*
b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^3*b^2) + 1/36*(B*a*b*x^(7/2) + 11*A*b^2*
x^(7/2) - 5*B*a^2*sqrt(x) + 17*A*a*b*sqrt(x))/((b*x^3 + a)^2*a^2*b)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 1952, normalized size of antiderivative = 6.08

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^3)/(x^(1/2)*(a + b*x^3)^3),x)
```

output
```
((x^(7/2)*(11*A*b + B*a))/(36*a^2) + (x^(1/2)*(17*A*b - 5*B*a))/(36*a*b))/
(a^2 + b^2*x^6 + 2*a*b*x^3) - (atan(((((625*x^(1/2)*(14641*A^4*b^5 + B^4*a
^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936
*a^8) - (625*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4
+ 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a)*5i)/(216
*(-a)^(17/6)*b^(7/6)) + (((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^
2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*
(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^
2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a)*5i)/(216*(-a)^(17/6)*
b^(7/6)))/((5*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b
^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*(11*A*b + B
*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(27
9936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a))/(216*(-a)^(17/6)*b^(7/6)) - (5*
((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*
B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*(11*A*b + B*a)*(1331*A^3*
b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/
6)*b^(7/6)))*(11*A*b + B*a))/(216*(-a)^(17/6)*b^(7/6))))*(11*A*b + B*a)*5i
)/(108*(-a)^(17/6)*b^(7/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a
)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^
3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*((3^(1/2)*1i)/2 - 1/...
```

3.176 $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx$

3.176.1 Optimal result

Integrand size = 22, antiderivative size = 351

$$\begin{aligned}\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx &= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}\\ &+\frac{7(13Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{19/6}b^{5/6}}-\frac{7(13Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{19/6}b^{5/6}}\\ &-\frac{7(13Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{19/6}b^{5/6}}-\frac{7(13Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}}\\ &+\frac{7(13Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}}\end{aligned}$$

output
```
-7/108*(13*A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(19/6)/b^(5/6)-7/216
*(13*A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(19/6)/b^(5/6)-
7/216*(13*A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(19/6)/b^(5
/6)-7/432*(13*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2
))/a^(19/6)/b^(5/6)*3^(1/2)+7/432*(13*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6
)*b^(1/6)*3^(1/2)*x^(1/2))/a^(19/6)/b^(5/6)*3^(1/2)-7/36*(13*A*b-B*a)/a^3/
b/x^(1/2)+1/6*(A*b-B*a)/a/b/(b*x^3+a)^2/x^(1/2)+1/36*(13*A*b-B*a)/a^2/b/(b
*x^3+a)/x^(1/2)
```

3.176.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx=\frac{-\frac{6\sqrt[6]{a}\left(91Ab^2x^6+a^2\left(72A-13Bx^3\right)+abx^3\left(169A-7Bx^3\right)\right)}{\sqrt{x}\left(a+bx^3\right)^2}+\frac{14(-13Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{b^{5/6}}+7(13Ab-a}{216a^{19/6}}$$

input
```
Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x]
```

output
```
((-6*a^(1/6)*(91*A*b^2*x^6 + a^2*(72*A - 13*B*x^3) + a*b*x^3*(169*A - 7*B*
x^3)))/(Sqrt[x]*(a + b*x^3)^2) + (14*(-13*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[
x])/a^(1/6)])/b^(5/6) + (7*(13*A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^
(1/6)*b^(1/6)*Sqrt[x])])/b^(5/6) + (7*Sqrt[3]*(13*A*b - a*B)*ArcTanh[(Sqrt
[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/b^(5/6))/(216*a^(19/6
))
```

3.176.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.95, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {957, 819, 847, 851, 824, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(13Ab-aB)\int \frac{1}{x^{3/2}\left(bx^3+a\right)^2}dx}{12ab}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{(13Ab-aB)\left(\frac{7\int \frac{1}{x^{3/2}\left(bx^3+a\right)}dx}{6a}+\frac{1}{3a\sqrt{x}\left(a+bx^3\right)}\right)}{12ab}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{847}$$

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{b\int\frac{x^{3/2}}{bx^3+a}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+bx^3)}\right)}{12ab}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\int\frac{x^{2}}{bx^3+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+bx^3)}\right)}{12ab}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{824}$$

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\left(\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}+\frac{\int-\frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}+\frac{\int-\frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3\sqrt[6]{a}b^{2/3}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{6a}+\right)}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{27}$$

3.176. $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^3}\,dx$

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3b^{2/3}}-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+bx}\right.}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\left(-\frac{\int \frac{\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+b}\right.}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1142$$

3.176. $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^3}\,dx$

$$(13Ab-aB)\frac{\left(7\frac{\left(-\frac{2b\left(-\frac{-\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int-\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{}}}{2\sqrt[6]{b}}\right.}{a}\right.}{6a}\right.}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow 25$$

3.176. $\int\frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx$

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\left(-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}-\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d}{2\sqrt[6]{b}}\right.}{a}\right.}{6a}\right.}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

3.176. $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx$

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+}}{}\right.}{a}\right.}{6a}\right.}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow 1082$$

3.176. $\int\frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx$

$$(13Ab-aB)\frac{\Bigg(\frac{7\Bigg(-\frac{2b\Bigg(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int\frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\int\frac{1}{-x-\frac{1}{3}}d\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}}}{6\sqrt[6]{a}b^{2/3}}}{a}}{6a}}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow 217$$

3.176. $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx$

$$\frac{(13Ab-aB)\left(\frac{7\left(-\frac{2b\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan}{}}{6\sqrt[6]{a}b^{2/3}}}{a}}{6a}}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1103}$$

3.176. $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx$

$$\frac{(13Ab-aB)\Bigg(\frac{7\Bigg(-\frac{2b\Bigg(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}}-\frac{\frac{\sqrt{3}\log\big(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}-}{2\sqrt[6]{b}}}{6}}{a}}{6a}}{12ab}$$

$$\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

input
```
Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x]
```

output
```
(A*b - a*B)/(6*a*b*Sqrt[x]*(a + b*x^3)^2) + ((13*A*b - a*B)*(1/(3*a*Sqrt[x
]*(a + b*x^3)) + (7*(-2/(a*Sqrt[x]) - (2*b*(ArcTan[(b^(1/6)*Sqrt[x])/a^(1/
6)]/(3*a^(1/6)*b^(5/6)) - (ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/(Sqrt[3
]*a^(1/6)))]/b^(1/6) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3)) - (-(ArcTan[Sqrt[3]*(1
+ (2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6)) + (Sqrt[3]*Log[a^(1/3)
+ Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(1/6)*b^
(2/3))))/a))/(6*a)))/(12*a*b)
```

3.176. $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^3}\,dx$

3.176.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.176. $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^3}\,dx$

3.176.4 Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.67

method	result
derivativedivides	$-\frac{2A}{a^{3}\sqrt{x}}-\frac{2\left(\frac{\left(\frac{19}{72}b^{2}A-\frac{7}{72}abB\right)x^{\frac{11}{2}}+\frac{a(25Ab-13Ba)x^{\frac{5}{2}}}{72}}{\left(b\,x^{3}+a\right)^{2}}+\left(\frac{91Ab}{72}-\frac{7Ba}{72}\right)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan}{}\right.\right.}{a^{3}}$
default	$-\frac{2A}{a^{3}\sqrt{x}}-\frac{2\left(\frac{\left(\frac{19}{72}b^{2}A-\frac{7}{72}abB\right)x^{\frac{11}{2}}+\frac{a(25Ab-13Ba)x^{\frac{5}{2}}}{72}}{\left(b\,x^{3}+a\right)^{2}}+\left(\frac{91Ab}{72}-\frac{7Ba}{72}\right)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan}{}\right.\right.}{a^{3}}$
risch	$-\frac{2A}{a^{3}\sqrt{x}}-\frac{\frac{2\left(\frac{19}{72}b^{2}A-\frac{7}{72}abB\right)x^{\frac{11}{2}}+\frac{a(25Ab-13Ba)x^{\frac{5}{2}}}{36}}{\left(b\,x^{3}+a\right)^{2}}+2\left(\frac{91Ab}{72}-\frac{7Ba}{72}\right)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\right.}{}\right.}{a^{3}}$

input
```
int((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*A/a^3/x^(1/2)-2/a^3*(((19/72*b^2*A-7/72*a*b*B)*x^(11/2)+1/72*a*(25*A*b-
13*B*a)*x^(5/2))/(b*x^3+a)^2+(91/72*A*b-7/72*B*a)*(1/12/a*3^(1/2)*(a/b)^(5
/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan
(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(
a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(
1/6)+3^(1/2))+1/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))))
```

3.176.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1934 vs. 2(254) = 508.

Time = 0.37 (sec) , antiderivative size = 1934, normalized size of antiderivative = 5.51

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

output

```
-1/432*(14*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5*a^5*b
 + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 -
 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*log(16807*a^16*b
^4*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*
b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^1
9*b^5))^(5/6) - 16807*(B^5*a^5 - 65*A*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 2
1970*A^3*B^2*a^2*b^3 + 142805*A^4*B*a*b^4 - 371293*A^5*b^5)*sqrt(x)) - 14*
(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2
*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^
5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*log(-16807*a^16*b^4*(-(B^6*
a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 4284
15*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5
/6) - 16807*(B^5*a^5 - 65*A*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B
^2*a^2*b^3 + 142805*A^4*B*a*b^4 - 371293*A^5*b^5)*sqrt(x)) + 7*(a^3*b^2*x^
7 + 2*a^4*b*x^4 + a^5*x - sqrt(-3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x))*(-
(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 +
 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5
))^(1/6)*log(16807/2*(sqrt(-3)*a^16*b^4 + a^16*b^4)*(-(B^6*a^6 - 78*A*B^5*
a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*
b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5/6) - 16807*...
```

3.176.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx=\text{Timed out}$$

input

```
integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**3,x)
```

output

```
Timed out
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx = \frac{7\,(Bab-13\,Ab^2)x^6+13\,(Ba^2-13\,Aab)x^3-72\,Aa^2}{36\,\left(a^3b^2x^{\frac{13}{2}}+2\,a^4bx^{\frac{7}{2}}+a^5\sqrt{x}\right)}$$

$$-\frac{7\,(Ba-13\,Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-2\arctan\left(\right.\right.}{432\,a^3}$$

input
```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/36*(7*(B*a*b - 13*A*b^2)*x^6 + 13*(B*a^2 - 13*A*a*b)*x^3 - 72*A*a^2)/(a^
3*b^2*x^(13/2) + 2*a^4*b*x^(7/2) + a^5*sqrt(x)) - 7/432*(B*a - 13*A*b)*(sq
rt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*
b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1
/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqr
t(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(
sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/
3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3))
)/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a^3
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx = \frac{7\,(Ba-13\,Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,\left(ab^5\right)^{\frac{1}{6}}a^3}$$
$$+\frac{7\,(Ba-13\,Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,\left(ab^5\right)^{\frac{1}{6}}a^3}+\frac{7\,\left(Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}-13\,Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^4}$$
$$-\frac{2\,A}{a^3\sqrt{x}}+\frac{7\,Babx^{\frac{11}{2}}-19\,Ab^2x^{\frac{11}{2}}+13\,Ba^2x^{\frac{5}{2}}-25\,Aabx^{\frac{5}{2}}}{36\,\left(bx^3+a\right)^2a^3}$$
$$-\frac{7\,\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba-13\,\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b^5}$$
$$+\frac{7\,\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba-13\,\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b^5}$$

input
```
integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
7/216*(B*a - 13*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))
/((a*b^5)^(1/6)*a^3) + 7/216*(B*a - 13*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) -
 2*sqrt(x))/(a/b)^(1/6))/((a*b^5)^(1/6)*a^3) + 7/108*(B*a*(a/b)^(5/6) - 13
*A*b*(a/b)^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/a^4 - 2*A/(a^3*sqrt(x)) + 1/
36*(7*B*a*b*x^(11/2) - 19*A*b^2*x^(11/2) + 13*B*a^2*x^(5/2) - 25*A*a*b*x^(
5/2))/((b*x^3 + a)^2*a^3) - 7/432*sqrt(3)*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^
(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b^5) +
7/432*sqrt(3)*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt
(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b^5)
```

3.176.9 Mupad [B] (verification not implemented)

Time = 7.38 (sec) , antiderivative size = 1786, normalized size of antiderivative = 5.09

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

input
```
int((A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x)
```

output

```
(atan((((13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^
21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (3
43*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 8294063127920
64*B^2*a^26*b^4 - 2156456413259364*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b
^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)) + ((13*A*b - B*a)^2*(62019785528064*A^3
*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 1431
2258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(14016966686185881
6*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 2156456413259364*A*B*a^25
*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)))/(((13*A*
b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 11009
42938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13
*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^
4 - 2156456413259364*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))/((-a
)^(19/3)*b^(5/3)) - ((13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229
306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*
a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 +
829406312792064*B^2*a^26*b^4 - 2156456413259364*A*B*a^25*b^5))/(10077696*
(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3))))*(13*A*b - B*a)*7i)/(108*(-a
)^(19/6)*b^(5/6)) - ((2*A)/a + (13*x^3*(13*A*b - B*a))/(36*a^2) + (7*b*x^6
*(13*A*b - B*a))/(36*a^3))/(a^2*x^(1/2) + b^2*x^(13/2) + 2*a*b*x^(7/2))...
```

3.176. $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^3}\,dx$

3.177 $\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx$

3.177.1 Optimal result

Integrand size = 22, antiderivative size = 129

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx=\frac{-5Ab+aB}{4a^3bx^{3/2}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{(5Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}$$

output

```
1/4*(-5*A*b+B*a)/a^3/b/x^(3/2)+1/6*(A*b-B*a)/a/b/x^(3/2)/(b*x^3+a)^2+1/12*
(5*A*b-B*a)/a^2/b/x^(3/2)/(b*x^3+a)-1/4*(5*A*b-B*a)*arctan(x^(3/2)*b^(1/2)
/a^(1/2))/a^(7/2)/b^(1/2)
```

3.177.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx=\frac{-8a^2A-25aAbx^3+5a^2Bx^3-15Ab^2x^6+3abBx^6}{12a^3x^{3/2}\left(a+bx^3\right)^2}+\frac{(-5Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}$$

input

```
Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x]
```

output
```
(-8*a^2*A - 25*a*A*b*x^3 + 5*a^2*B*x^3 - 15*A*b^2*x^6 + 3*a*b*B*x^6)/(12*a
^3*x^(3/2)*(a + b*x^3)^2) + ((-5*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[
a]])/(4*a^(7/2)*Sqrt[b])
```

3.177.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {957, 819, 847, 851, 807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(5Ab-aB)\int \frac{1}{x^{5/2}\left(bx^3+a\right)^2}dx}{4ab}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{(5Ab-aB)\left(\frac{3\int \frac{1}{x^{5/2}\left(bx^3+a\right)}dx}{2a}+\frac{1}{3ax^{3/2}\left(a+bx^3\right)}\right)}{4ab}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{847}$$

$$\frac{(5Ab-aB)\left(\frac{3\left(-\frac{b\int \frac{\sqrt{x}}{bx^3+a}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{3ax^{3/2}\left(a+bx^3\right)}\right)}{4ab}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{(5Ab-aB)\left(\frac{3\left(-\frac{2b\int \frac{x}{bx^3+a}d\sqrt{x}}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{3ax^{3/2}\left(a+bx^3\right)}\right)}{4ab}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{807}$$

$$\frac{(5Ab-aB)\left(\frac{3\left(-\frac{2b\int\frac{1}{a+bx}dx^{3/2}}{3a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx^3)}\right)}{4ab}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow 218$$

$$\frac{(5Ab-aB)\left(\frac{3\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx^3)}\right)}{4ab}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

input
```
Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x]
```

output
```
(A*b - a*B)/(6*a*b*x^(3/2)*(a + b*x^3)^2) + ((5*A*b - a*B)*(1/(3*a*x^(3/2)
*(a + b*x^3)) + (3*(-2/(3*a*x^(3/2)) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*x^(3/2))
/Sqrt[a]])/(3*a^(3/2))))/(2*a)))/(4*a*b)
```

3.177.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.177.4 Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$-\frac{2\left(\frac{\left(\frac{7}{8}b^{2}A-\frac{3}{8}abB\right)x^{\frac{9}{2}}+\frac{a(9Ab-5Ba)x^{\frac{3}{2}}}{8}}{\left(b\,x^{3}+a\right)^{2}}+\frac{3(5Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{3a^{3}}-\frac{2A}{3a^{3}x^{\frac{3}{2}}}$	86
default	$-\frac{2\left(\frac{\left(\frac{7}{8}b^{2}A-\frac{3}{8}abB\right)x^{\frac{9}{2}}+\frac{a(9Ab-5Ba)x^{\frac{3}{2}}}{8}}{\left(b\,x^{3}+a\right)^{2}}+\frac{3(5Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{3a^{3}}-\frac{2A}{3a^{3}x^{\frac{3}{2}}}$	86
risch	$-\frac{2A}{3a^{3}x^{\frac{3}{2}}}-\frac{\frac{\frac{2\left(\frac{7}{8}b^{2}A-\frac{3}{8}abB\right)x^{\frac{9}{2}}}{3}+\frac{a(9Ab-5Ba)x^{\frac{3}{2}}}{12}}{\left(b\,x^{3}+a\right)^{2}}+\frac{(5Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab}}}{a^{3}}$	87

input
```
int((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3/a^3*(((7/8*b^2*A-3/8*a*b*B)*x^(9/2)+1/8*a*(9*A*b-5*B*a)*x^(3/2))/(b*x
^3+a)^2+3/8*(5*A*b-B*a)/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2)))-2/3*A/a
^3/x^(3/2)
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.69

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = \Bigg[\frac{3\,((Bab^2-5\,Ab^3)x^8+2\,(Ba^2b-5\,Aab^2)x^5+(Ba^3-5\,Aa^2b)x^2)\sqrt{-ab}\log\Big(\frac{bx^3+2}{b}}{24\,(a^4b^3x^8+2\,a^5b^2}$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/24*(3*((B*a*b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 -
 5*A*a^2*b)*x^2)*sqrt(-a*b)*log((b*x^3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3
+ a)) + 2*(3*(B*a^2*b^2 - 5*A*a*b^3)*x^6 - 8*A*a^3*b + 5*(B*a^3*b - 5*A*a^
2*b^2)*x^3)*sqrt(x))/(a^4*b^3*x^8 + 2*a^5*b^2*x^5 + a^6*b*x^2), 1/12*(3*((
B*a*b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 - 5*A*a^2*b)
*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) + (3*(B*a^2*b^2 - 5*A*a*b^3)*x
^6 - 8*A*a^3*b + 5*(B*a^3*b - 5*A*a^2*b^2)*x^3)*sqrt(x))/(a^4*b^3*x^8 + 2*
a^5*b^2*x^5 + a^6*b*x^2)]
```

3.177.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.177.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = \frac{3\,(Bab-5\,Ab^2)x^6+5\,(Ba^2-5\,Aab)x^3-8\,Aa^2}{12\left(a^3b^2x^{\frac{15}{2}}+2\,a^4bx^{\frac{9}{2}}+a^5x^{\frac{3}{2}}\right)} + \frac{(Ba-5\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3}$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/12*(3*(B*a*b - 5*A*b^2)*x^6 + 5*(B*a^2 - 5*A*a*b)*x^3 - 8*A*a^2)/(a^3*b^
2*x^(15/2) + 2*a^4*b*x^(9/2) + a^5*x^(3/2)) + 1/4*(B*a - 5*A*b)*arctan(b*x
^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

3.177.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = \frac{(Ba-5\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3} - \frac{2\,A}{3\,a^3x^{\frac{3}{2}}} + \frac{3\,Babx^{\frac{9}{2}}-7\,Ab^2x^{\frac{9}{2}}+5\,Ba^2x^{\frac{3}{2}}-9\,Aabx^{\frac{3}{2}}}{12\,(bx^3+a)^2a^3}$$

input
```
integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/4*(B*a - 5*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/3*A/(a^3
*x^(3/2)) + 1/12*(3*B*a*b*x^(9/2) - 7*A*b^2*x^(9/2) + 5*B*a^2*x^(3/2) - 9*
A*a*b*x^(3/2))/((b*x^3 + a)^2*a^3)
```

3.177.9 Mupad [B] (verification not implemented)

Time = 7.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = -\frac{\frac{2\,A}{3\,a}+\frac{5\,x^3\,(5\,A\,b-B\,a)}{12\,a^2}+\frac{b\,x^6\,(5\,A\,b-B\,a)}{4\,a^3}}{a^2\,x^{3/2}+b^2\,x^{15/2}+2\,a\,b\,x^{9/2}} - \frac{\mathrm{atan}\left(\frac{8\,a^{7/2}\,\sqrt{b}\,x^{3/2}\,\left(86400\,A^2\,a^9\,b^5-34560\,A\,B\,a^{10}\,b^4+3456\,B^2\,a^{11}\,b^3\right)}{(5\,A\,b-B\,a)\,\left(138240\,A\,a^{13}\,b^4-27648\,B\,a^{14}\,b^3\right)}\right)\,(5\,A\,b-B\,a)}{4\,a^{7/2}\,\sqrt{b}}$$

input
```
int((A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x)
```

output
```
- ((2*A)/(3*a) + (5*x^3*(5*A*b - B*a))/(12*a^2) + (b*x^6*(5*A*b - B*a))/(4
*a^3))/(a^2*x^(3/2) + b^2*x^(15/2) + 2*a*b*x^(9/2)) - (atan((8*a^(7/2)*b^(
1/2)*x^(3/2)*(86400*A^2*a^9*b^5 + 3456*B^2*a^11*b^3 - 34560*A*B*a^10*b^4))
/((5*A*b - B*a)*(138240*A*a^13*b^4 - 27648*B*a^14*b^3)))*(5*A*b - B*a))/(4
*a^(7/2)*b^(1/2))
```

3.178 $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^3}\,dx$

3.178.1 Optimal result

Integrand size = 22, antiderivative size = 351

$$\begin{aligned}\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx &= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}\\ &+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}+\frac{11(17Ab-5aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{23/6}\sqrt[6]{b}}\\ &-\frac{11(17Ab-5aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{23/6}\sqrt[6]{b}}-\frac{11(17Ab-5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{23/6}\sqrt[6]{b}}\\ &+\frac{11(17Ab-5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}}\\ &-\frac{11(17Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}}\end{aligned}$$

output

```
-11/180*(17*A*b-5*B*a)/a^3/b/x^(5/2)+1/6*(A*b-B*a)/a/b/x^(5/2)/(b*x^3+a)^2
+1/36*(17*A*b-5*B*a)/a^2/b/x^(5/2)/(b*x^3+a)-11/108*(17*A*b-5*B*a)*arctan(
b^(1/6)*x^(1/2)/a^(1/6))/a^(23/6)/b^(1/6)-11/216*(17*A*b-5*B*a)*arctan(-3^
(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(23/6)/b^(1/6)-11/216*(17*A*b-5*B*a)*ar
ctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(23/6)/b^(1/6)+11/432*(17*A*b-5*
B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(23/6)/b^(1/6
)*3^(1/2)-11/432*(17*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/
2)*x^(1/2))/a^(23/6)/b^(1/6)*3^(1/2)
```

3.178.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx=\frac{-\frac{6a^{5/6}\left(187Ab^2x^6+a^2\left(72A-85Bx^3\right)+abx^3\left(289A-55Bx^3\right)\right)}{x^{5/2}\left(a+bx^3\right)^2}+\frac{110(-17Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{\sqrt[6]{b}}+55(17}{1080a^{23/6}}$$

input
```
Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x]
```

output
```
((-6*a^(5/6)*(187*A*b^2*x^6 + a^2*(72*A - 85*B*x^3) + a*b*x^3*(289*A - 55*
B*x^3)))/(x^(5/2)*(a + b*x^3)^2) + (110*(-17*A*b + 5*a*B)*ArcTan[(b^(1/6)*
Sqrt[x])/a^(1/6)])/b^(1/6) + (55*(17*A*b - 5*a*B)*ArcTan[(a^(1/3) - b^(1/3
)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/b^(1/6) + (55*Sqrt[3]*(-17*A*b + 5*a*B)*A
rcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/b^(1/6))/
(1080*a^(23/6))
```

3.178.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {957, 819, 847, 851, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{(17Ab-5aB)\int \frac{1}{x^{7/2}\left(bx^3+a\right)^2}dx}{12ab}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 819$$

$$\frac{(17Ab-5aB)\left(\frac{11\int \frac{1}{x^{7/2}\left(bx^3+a\right)}dx}{6a}+\frac{1}{3ax^{5/2}\left(a+bx^3\right)}\right)}{12ab}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 847$$

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{b\int \frac{1}{\sqrt{x}\left(bx^3+a\right)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{6a}+\frac{1}{3ax^{5/2}(a+bx^3)}\right)}{12ab}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\int \frac{1}{bx^3+a}d\sqrt{x}}{a}-\frac{2}{5ax^{5/2}}\right)}{6a}+\frac{1}{3ax^{5/2}(a+bx^3)}\right)}{12ab}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{753}$$

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{2\left(\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{2\left(\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}\right)}d\sqrt{x}}{3a^{5/6}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{6a}+\right)}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{27}$$

3.178. $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^3}\,dx$

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}d\sqrt{x}}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{6a}+\frac{}{3ax^{5/2}}\right.}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{b}\sqrt{x}+2\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{6a}+\frac{1}{3ax^{5/2}(}\right.}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1142$$

3.178. $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx$

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+}}{}\right.}{a}\right.}{6a}\right.}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow 25$$

3.178. $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^3}\,dx$

$$\frac{(17Ab-5aB)\Bigg(\frac{11\bigg(-\frac{2b\Big(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}\right)}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+}}{}}{a}}{6a}}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow 27$$

3.178. $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx$

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+}}{}}{a}}{6a}}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1082}$$

3.178. $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx$

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\int\frac{1}{-x-}}{}}{6a^{5/6}}\right.}{a}\right.}{6a}\right.}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 217$$

3.178. $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx$

$$(17Ab-5aB)\frac{11\left(-\frac{2b\left(\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{b}x-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{b}\sqrt{x}+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{b}x+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}}d\sqrt{x}+\frac{\mathrm{arcta}}{}}{6a^{5/6}}}{a}}{6a}}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1103}$$

3.178. $\int\frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx$

$$\frac{(17Ab-5aB)\left(\frac{11\left(-\frac{2b\left(\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}+\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt[6]{b}}}{6a^{5/6}}+\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{b}}\right)}{a}\right)}{6a}\right)}{12ab}$$

$$\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

input
```
Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x]
```

output
```
(A*b - a*B)/(6*a*b*x^(5/2)*(a + b*x^3)^2) + ((17*A*b - 5*a*B)*(1/(3*a*x^(5
/2)*(a + b*x^3)) + (11*(-2/(5*a*x^(5/2)) - (2*b*(ArcTan[(b^(1/6)*Sqrt[x])/
a^(1/6)]/(3*a^(5/6)*b^(1/6)) + (-(ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*Sqrt[x])/
(Sqrt[3]*a^(1/6)))]/b^(1/6)) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1
/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6)) + (ArcTan[Sqrt[3]*(1 +
(2*b^(1/6)*Sqrt[x])/(Sqrt[3]*a^(1/6)))]/b^(1/6) + (Sqrt[3]*Log[a^(1/3) + S
qrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*b^(1/6)))/(6*a^(5/6))))/a)
)/(6*a)))/(12*a*b)
```

3.178. $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^3}\,dx$

3.178.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.178. $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^3}\,dx$

3.178.4 Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{2A}{5a^{3}x^{\frac{5}{2}}}-\frac{2\left(\frac{\left(\frac{23}{72}b^{2}A-\frac{11}{72}abB\right)x^{\frac{7}{2}}+\frac{a(29Ab-17Ba)\sqrt{x}}{72}}{\left(b\,x^{3}+a\right)^{2}}+\frac{11(17Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}}{12a}\right.}{}\right.}{}$
default	$-\frac{2A}{5a^{3}x^{\frac{5}{2}}}-\frac{2\left(\frac{\left(\frac{23}{72}b^{2}A-\frac{11}{72}abB\right)x^{\frac{7}{2}}+\frac{a(29Ab-17Ba)\sqrt{x}}{72}}{\left(b\,x^{3}+a\right)^{2}}+\frac{11(17Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}}{12a}\right.}{}\right.}{}$
risch	$-\frac{2A}{5a^{3}x^{\frac{5}{2}}}-\frac{\frac{2\left(\frac{23}{72}b^{2}A-\frac{11}{72}abB\right)x^{\frac{7}{2}}+\frac{a(29Ab-17Ba)\sqrt{x}}{36}}{\left(b\,x^{3}+a\right)^{2}}+\frac{11(17Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{}\right.}{12a}\right.}{}}{}$

input
```
int((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/5*A/a^3/x^(5/2)-2/a^3*(((23/72*b^2*A-11/72*a*b*B)*x^(7/2)+1/72*a*(29*A*
b-17*B*a)*x^(1/2))/(b*x^3+a)^2+11/72*(17*A*b-5*B*a)*(1/3/a*(a/b)^(1/6)*arc
tan(x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)
*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^
(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(
1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))))
```

3.178.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. $2(261) = 522$.

Time = 0.28 (sec) , antiderivative size = 1608, normalized size of antiderivative = 4.58

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/2160*(110*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6*a^6 - 3187
50*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 3132
0375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^
(1/6)*log(11*a^4*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a
^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^
5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6) - 11*(5*B*a - 17*A*b)*sqrt(x
)) - 110*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6*a^6 - 318750*A
*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375
*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6
)*log(-11*a^4*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*
b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B
*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6) - 11*(5*B*a - 17*A*b)*sqrt(x))
 + 55*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3 + sqrt(-3)*(a^3*b^2*x^9 + 2*a^4*
b*x^6 + a^5*x^3))*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*
a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A
^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6)*log(-11*(5*B*a - 17*A*b)*sq
rt(x) + 11/2*(sqrt(-3)*a^4 + a^4)*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b +
2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*
b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6)) - 55*(a^3*
b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3 + sqrt(-3)*(a^3*b^2*x^9 + 2*a^4*b*x^6 +...
```

3.178.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.178.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx = \frac{11\,(5\,Bab-17\,Ab^2)x^6+17\,(5\,Ba^2-17\,Aab)x^3-72\,Aa^2}{180\,\left(a^3b^2x^{\frac{17}{2}}+2\,a^4bx^{\frac{11}{2}}+a^5x^{\frac{5}{2}}\right)}$$

$$+\frac{11\left(\frac{\sqrt{3}(5\,Ba-17\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}(5\,Ba-17\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(5\,Bab^{\frac{1}{3}}-17\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}}{\sqrt{a^{\frac{1}{3}}}}\right.}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right.}{432\,a^3}$$

input
```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/180*(11*(5*B*a*b - 17*A*b^2)*x^6 + 17*(5*B*a^2 - 17*A*a*b)*x^3 - 72*A*a^
2)/(a^3*b^2*x^(17/2) + 2*a^4*b*x^(11/2) + a^5*x^(5/2)) + 11/432*(sqrt(3)*(
5*B*a - 17*A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))
/(a^(5/6)*b^(1/6)) - sqrt(3)*(5*B*a - 17*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)
*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(5*B*a*b^(1/3) - 17*
A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*
sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 17*A*a^(1/3)*b^(4/3))*ar
ctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/
(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 17*A*a^(1/3)*
b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3
)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/a^3
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx = \frac{11\,\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-17\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b}$$
$$-\frac{11\,\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-17\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b}$$
$$+\frac{11\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-17\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^4b}$$
$$+\frac{11\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-17\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^4b}$$
$$+\frac{11\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-17\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^4b}$$
$$+\frac{11\,Babx^{\frac{7}{2}}-23\,Ab^2x^{\frac{7}{2}}+17\,Ba^2\sqrt{x}-29\,Aab\sqrt{x}}{36\left(bx^3+a\right)^2a^3}-\frac{2\,A}{5\,a^3x^{\frac{5}{2}}}$$

input

```
integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="giac")
```

output

```
11/432*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sq
rt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b) - 11/432*sqrt(3)*(5*(a*b^5)^(
1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a
/b)^(1/3))/(a^4*b) + 11/216*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*a
rctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^4*b) + 11/216*(5*(
a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*
sqrt(x))/(a/b)^(1/6))/(a^4*b) + 11/108*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(
1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^4*b) + 1/36*(11*B*a*b*x^(7/2) - 2
3*A*b^2*x^(7/2) + 17*B*a^2*sqrt(x) - 29*A*a*b*sqrt(x))/((b*x^3 + a)^2*a^3)
 - 2/5*A/(a^3*x^(5/2))
```

3.178.9 Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 2109, normalized size of antiderivative = 6.01

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

input

```
int((A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x)
```

output

```
- ((2*A)/(5*a) + (17*x^3*(17*A*b - 5*B*a))/(180*a^2) + (11*b*x^6*(17*A*b -
 5*B*a))/(180*a^3))/(a^2*x^(5/2) + b^2*x^(17/2) + 2*a*b*x^(11/2)) - (atan(
(((x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^
5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6
 - 521928791337000960*A^3*B*a^16*b^8) - (11*(17*A*b - 5*B*a)*(512439176949
055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*
A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1
/6)))*(17*A*b - 5*B*a)*11i)/(216*(-a)^(23/6)*b^(1/6)) + ((x^(1/2)*(4436394
72636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441
600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 5219287913370009
60*A^3*B*a^16*b^8) + (11*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8
 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 45
2152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*(17*A*b - 5*B
*a)*11i)/(216*(-a)^(23/6)*b^(1/6)))/((11*(x^(1/2)*(443639472636450816*A^4*
a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17
*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^
8) - (11*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 1303783780147
2000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960
*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*(17*A*b - 5*B*a))/(216*(-a)^(
23/6)*b^(1/6)) - (11*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 331981...
```

3.179 $\int x^8\sqrt{a+bx^3}(A+Bx^3)\,dx$

3.179.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int x^8\sqrt{a+bx^3}(A+Bx^3)\,dx=\frac{2a^2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4}+\frac{2(Ab-3aB)\left(a+bx^3\right)^{7/2}}{21b^4}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^4}$$

output
```
2/9*a^2*(A*b-B*a)*(b*x^3+a)^(3/2)/b^4-2/15*a*(2*A*b-3*B*a)*(b*x^3+a)^(5/2)
/b^4+2/21*(A*b-3*B*a)*(b*x^3+a)^(7/2)/b^4+2/27*B*(b*x^3+a)^(9/2)/b^4
```

3.179.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.73

$$\int x^8\sqrt{a+bx^3}(A+Bx^3)\,dx=\frac{2\left(a+bx^3\right)^{3/2}\left(-16a^3B+24a^2b(A+Bx^3)-6ab^2x^3(6A+5Bx^3)+5b^3x^6(9A+7Bx^3)\right)}{945b^4}$$

input
```
Integrate[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*(a + b*x^3)^(3/2)*(-16*a^3*B + 24*a^2*b*(A + B*x^3) - 6*a*b^2*x^3*(6*A
+ 5*B*x^3) + 5*b^3*x^6*(9*A + 7*B*x^3)))/(945*b^4)
```

3.179.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \sqrt{a+bx^3} \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int x^6 \sqrt{bx^3+a} \left(Bx^3+A\right) dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3} \int \left(\frac{B\left(bx^3+a\right)^{7/2}}{b^3}+\frac{(Ab-3aB)\left(bx^3+a\right)^{5/2}}{b^3}+\frac{a(3aB-2Ab)\left(bx^3+a\right)^{3/2}}{b^3}-\frac{a^2(aB-Ab)\sqrt{bx^3+a}}{b^3}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{2a^2\left(a+bx^3\right)^{3/2}(Ab-aB)}{3b^4}+\frac{2\left(a+bx^3\right)^{7/2}(Ab-3aB)}{7b^4}-\frac{2a\left(a+bx^3\right)^{5/2}(2Ab-3aB)}{5b^4}+\frac{2B\left(a+bx^3\right)^{9/2}}{9b^4}\right)$$

input
```
Int[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
((2*a^2*(A*b - a*B)*(a + b*x^3)^(3/2))/(3*b^4) - (2*a*(2*A*b - 3*a*B)*(a +
 b*x^3)^(5/2))/(5*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(7/2))/(7*b^4) + (2*
B*(a + b*x^3)^(9/2))/(9*b^4))/3
```

3.179.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.179.4 Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{16\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(\frac{15x^{6}\left(\frac{7x^{3}B}{9}+A\right)b^{3}}{8}-\frac{3x^{3}\left(\frac{5x^{3}B}{6}+A\right)a\,b^{2}}{2}+a^{2}\left(x^{3}B+A\right)b-\frac{2a^{3}B}{3}\right)}{315b^{4}}$
gosper	$\frac{2\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(35b^{3}B\,x^{9}+45x^{6}b^{3}A-30B\,x^{6}a\,b^{2}-36aA\,b^{2}x^{3}+24B\,a^{2}b\,x^{3}+24a^{2}bA-16a^{3}B\right)}{945b^{4}}$
trager	$\frac{2\left(35B\,b^{4}x^{12}+45A\,b^{4}x^{9}+5Ba\,b^{3}x^{9}+9Aa\,b^{3}x^{6}-6B\,a^{2}b^{2}x^{6}-12A\,a^{2}b^{2}x^{3}+8B\,a^{3}b\,x^{3}+24A\,a^{3}b-16B\,a^{4}\right)\sqrt{b\,x^{3}+a}}{945b^{4}}$
risch	$\frac{2\left(35B\,b^{4}x^{12}+45A\,b^{4}x^{9}+5Ba\,b^{3}x^{9}+9Aa\,b^{3}x^{6}-6B\,a^{2}b^{2}x^{6}-12A\,a^{2}b^{2}x^{3}+8B\,a^{3}b\,x^{3}+24A\,a^{3}b-16B\,a^{4}\right)\sqrt{b\,x^{3}+a}}{945b^{4}}$
elliptic	$\frac{2B\,x^{12}\sqrt{b\,x^{3}+a}}{27}+\frac{2\left(Ab+\frac{Ba}{9}\right)x^{9}\sqrt{b\,x^{3}+a}}{21b}+\frac{2\left(Aa-\frac{6a\left(Ab+\frac{Ba}{9}\right)}{7b}\right)x^{6}\sqrt{b\,x^{3}+a}}{15b}-\frac{8a\left(Aa-\frac{6a\left(Ab+\frac{Ba}{9}\right)}{7b}\right)x^{3}\sqrt{b\,x^{3}+a}}{45b^{2}}$
default	$A\left(\frac{2x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{2a\,x^{6}\sqrt{b\,x^{3}+a}}{105b}-\frac{8a^{2}x^{3}\sqrt{b\,x^{3}+a}}{315b^{2}}+\frac{16a^{3}\sqrt{b\,x^{3}+a}}{315b^{3}}\right)+B\left(\frac{2x^{12}\sqrt{b\,x^{3}+a}}{27}+\frac{2a\,x^{9}\sqrt{b\,x^{3}+a}}{189b}-\right.$

input
```
int(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
16/315*(b*x^3+a)^(3/2)*(15/8*x^6*(7/9*x^3*B+A)*b^3-3/2*x^3*(5/6*x^3*B+A)*a
*b^2+a^2*(B*x^3+A)*b-2/3*a^3*B)/b^4
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{2\,(35\,Bb^4x^{12}+5\,(Bab^3+9\,Ab^4)x^9-3\,(2\,Ba^2b^2-3\,Aab^3)x^6-16\,Ba^4+24\,Aa^3b+4\,(2\,Ba^3b-3\,Aa^2b^2)x}{945\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/945*(35*B*b^4*x^12 + 5*(B*a*b^3 + 9*A*b^4)*x^9 - 3*(2*B*a^2*b^2 - 3*A*a*
b^3)*x^6 - 16*B*a^4 + 24*A*a^3*b + 4*(2*B*a^3*b - 3*A*a^2*b^2)*x^3)*sqrt(b
*x^3 + a)/b^4
```

3.179.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(100) = 200.

Time = 0.40 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.13

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\begin{cases}\frac{16Aa^3\sqrt{a+bx^3}}{315b^3}-\frac{8Aa^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2Aax^6\sqrt{a+bx^3}}{105b}+\frac{2Ax^9\sqrt{a+bx^3}}{21}-\frac{32Ba^4\sqrt{a+bx^3}}{945b^4}+\frac{16Ba^3x^3\sqrt{a+bx^3}}{945b^3}-\frac{4Ba^2x^6\sqrt{a+bx^3}}{315b^2}+ \\ \sqrt{a}\left(\frac{Ax^9}{9}+\frac{Bx^{12}}{12}\right)\end{cases}$$

input
```
integrate(x**8*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((16*A*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*A*a**2*x**3*sqrt(a +
b*x**3)/(315*b**2) + 2*A*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*A*x**9*sqrt(a
 + b*x**3)/21 - 32*B*a**4*sqrt(a + b*x**3)/(945*b**4) + 16*B*a**3*x**3*sqr
t(a + b*x**3)/(945*b**3) - 4*B*a**2*x**6*sqrt(a + b*x**3)/(315*b**2) + 2*B
*a*x**9*sqrt(a + b*x**3)/(189*b) + 2*B*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)
), (sqrt(a)*(A*x**9/9 + B*x**12/12), True))
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{2}{945}\,B\left(\frac{35\,(bx^3+a)^{\frac{9}{2}}}{b^4}-\frac{135\,(bx^3+a)^{\frac{7}{2}}a}{b^4}+\frac{189\,(bx^3+a)^{\frac{5}{2}}a^2}{b^4}-\frac{105\,(bx^3+a)^{\frac{3}{2}}a^3}{b^4}\right)$$
$$+\frac{2}{315}\,A\left(\frac{15\,(bx^3+a)^{\frac{7}{2}}}{b^3}-\frac{42\,(bx^3+a)^{\frac{5}{2}}a}{b^3}+\frac{35\,(bx^3+a)^{\frac{3}{2}}a^2}{b^3}\right)$$

input
```
integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/945*B*(35*(b*x^3 + a)^(9/2)/b^4 - 135*(b*x^3 + a)^(7/2)*a/b^4 + 189*(b*x
^3 + a)^(5/2)*a^2/b^4 - 105*(b*x^3 + a)^(3/2)*a^3/b^4) + 2/315*A*(15*(b*x^
3 + a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3 + a)^(3/2)*a^2/b
^3)
```

3.179.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{2\left(35\,(bx^3+a)^{\frac{9}{2}}B-135\,(bx^3+a)^{\frac{7}{2}}Ba+189\,(bx^3+a)^{\frac{5}{2}}Ba^2-105\,(bx^3+a)^{\frac{3}{2}}Ba^3+45\,(bx^3+a)^{\frac{7}{2}}Ab-1\right.}{945\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/945*(35*(b*x^3 + a)^(9/2)*B - 135*(b*x^3 + a)^(7/2)*B*a + 189*(b*x^3 + a
)^(5/2)*B*a^2 - 105*(b*x^3 + a)^(3/2)*B*a^3 + 45*(b*x^3 + a)^(7/2)*A*b - 1
26*(b*x^3 + a)^(5/2)*A*a*b + 105*(b*x^3 + a)^(3/2)*A*a^2*b)/b^4
```

3.179.9 Mupad [B] (verification not implemented)

Time = 7.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\,B\,x^{12}\,\sqrt{b\,x^3+a}}{27}+\frac{x^9\,\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{21\,b}+\frac{8\,a^2\,\left(2\,A\,a-\frac{6\,a\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{45\,b^3}+\frac{x^6\,\left(2\,A\,a-\frac{6\,a\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{15\,b}-\frac{4\,a\,x^3\,\left(2\,A\,a-\frac{6\,a\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{45\,b^2}$$

input

```
int(x^8*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

output

```
(2*B*x^12*(a + b*x^3)^(1/2))/27 + (x^9*(a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/
9))/(21*b) + (8*a^2*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x^3)^
(1/2))/(45*b^3) + (x^6*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x^
3)^(1/2))/(15*b) - (4*a*x^3*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a +
 b*x^3)^(1/2))/(45*b^2)
```

3.180 $\int x^5\sqrt{a+bx^3}(A+Bx^3)\,dx$

3.180.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=-\frac{2a(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}$$

output
```
-2/9*a*(A*b-B*a)*(b*x^3+a)^(3/2)/b^3+2/15*(A*b-2*B*a)*(b*x^3+a)^(5/2)/b^3+
2/21*B*(b*x^3+a)^(7/2)/b^3
```

3.180.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\left(a+bx^3\right)^{3/2}\left(-14aAb+8a^2B+21Ab^2x^3-12abBx^3+15b^2Bx^6\right)}{315b^3}$$

input
```
Integrate[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*(a + b*x^3)^(3/2)*(-14*a*A*b + 8*a^2*B + 21*A*b^2*x^3 - 12*a*b*B*x^3 +
15*b^2*B*x^6))/(315*b^3)
```

3.180.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int x^3\sqrt{bx^3+a}\left(Bx^3+A\right)dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{B\left(bx^3+a\right)^{5/2}}{b^2}+\frac{(Ab-2aB)\left(bx^3+a\right)^{3/2}}{b^2}+\frac{a(aB-Ab)\sqrt{bx^3+a}}{b^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2\left(a+bx^3\right)^{5/2}(Ab-2aB)}{5b^3}-\frac{2a\left(a+bx^3\right)^{3/2}(Ab-aB)}{3b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{7b^3}\right)$$

input
```
Int[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
((-2*a*(A*b - a*B)*(a + b*x^3)^(3/2))/(3*b^3) + (2*(A*b - 2*a*B)*(a + b*x^
3)^(5/2))/(5*b^3) + (2*B*(a + b*x^3)^(7/2))/(7*b^3))/3
```

3.180.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.180.4 Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-\frac{4\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(-\frac{3x^{3}\left(\frac{5x^{3}B}{7}+A\right)b^{2}}{2}+a\left(\frac{6x^{3}B}{7}+A\right)b-\frac{4a^{2}B}{7}\right)}{45b^{3}}$
gosper	$-\frac{2\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(-15b^{2}B\,x^{6}-21A\,b^{2}x^{3}+12Bab\,x^{3}+14abA-8a^{2}B\right)}{315b^{3}}$
trager	$-\frac{2\left(-15b^{3}B\,x^{9}-21x^{6}b^{3}A-3B\,x^{6}a\,b^{2}-7aA\,b^{2}x^{3}+4B\,a^{2}b\,x^{3}+14a^{2}bA-8a^{3}B\right)\sqrt{b\,x^{3}+a}}{315b^{3}}$
risch	$-\frac{2\left(-15b^{3}B\,x^{9}-21x^{6}b^{3}A-3B\,x^{6}a\,b^{2}-7aA\,b^{2}x^{3}+4B\,a^{2}b\,x^{3}+14a^{2}bA-8a^{3}B\right)\sqrt{b\,x^{3}+a}}{315b^{3}}$
elliptic	$\frac{2B\,x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{2\left(Ab+\frac{Ba}{7}\right)x^{6}\sqrt{b\,x^{3}+a}}{15b}+\frac{2\left(Aa-\frac{4a\left(Ab+\frac{Ba}{7}\right)}{5b}\right)x^{3}\sqrt{b\,x^{3}+a}}{9b}-\frac{4a\left(Aa-\frac{4a\left(Ab+\frac{Ba}{7}\right)}{5b}\right)\sqrt{b\,x^{3}+a}}{9b^{2}}$
default	$B\left(\frac{2x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{2a\,x^{6}\sqrt{b\,x^{3}+a}}{105b}-\frac{8a^{2}x^{3}\sqrt{b\,x^{3}+a}}{315b^{2}}+\frac{16a^{3}\sqrt{b\,x^{3}+a}}{315b^{3}}\right)+A\left(\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15}+\frac{2a\,x^{3}\sqrt{b\,x^{3}+a}}{45b}-\right.$

input
```
int(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/45*(b*x^3+a)^(3/2)*(-3/2*x^3*(5/7*x^3*B+A)*b^2+a*(6/7*x^3*B+A)*b-4/7*a^
2*B)/b^3
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{2\,(15\,Bb^3x^9+3\,(Bab^2+7\,Ab^3)x^6+8\,Ba^3-14\,Aa^2b-(4\,Ba^2b-7\,Aab^2)x^3)\sqrt{bx^3+a}}{315\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(15*B*b^3*x^9 + 3*(B*a*b^2 + 7*A*b^3)*x^6 + 8*B*a^3 - 14*A*a^2*b - (
4*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^3
```

3.180.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(70) = 140.

Time = 0.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.30

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\begin{cases}-\frac{4Aa^2\sqrt{a+bx^3}}{45b^2}+\frac{2Aax^3\sqrt{a+bx^3}}{45b}+\frac{2Ax^6\sqrt{a+bx^3}}{15}+\frac{16Ba^3\sqrt{a+bx^3}}{315b^3}-\frac{8Ba^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2Bax^6\sqrt{a+bx^3}}{105b}+\frac{2Bx^9\sqrt{a+bx^3}}{21} & \text{for} \\ \sqrt{a}\left(\frac{Ax^6}{6}+\frac{Bx^9}{9}\right) & \text{otl}\end{cases}$$

input
```
integrate(x**5*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((-4*A*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*A*a*x**3*sqrt(a + b*x*
*3)/(45*b) + 2*A*x**6*sqrt(a + b*x**3)/15 + 16*B*a**3*sqrt(a + b*x**3)/(31
5*b**3) - 8*B*a**2*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*B*a*x**6*sqrt(a +
b*x**3)/(105*b) + 2*B*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt(a)*(A*x**
6/6 + B*x**9/9), True))
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2}{315}\,B\left(\frac{15\,(bx^3+a)^{\frac{7}{2}}}{b^3}-\frac{42\,(bx^3+a)^{\frac{5}{2}}a}{b^3}+\frac{35\,(bx^3+a)^{\frac{3}{2}}a^2}{b^3}\right)+\frac{2}{45}\,A\left(\frac{3\,(bx^3+a)^{\frac{5}{2}}}{b^2}-\frac{5\,(bx^3+a)^{\frac{3}{2}}a}{b^2}\right)$$

input
```
integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/315*B*(15*(b*x^3 + a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3
 + a)^(3/2)*a^2/b^3) + 2/45*A*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/
2)*a/b^2)
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\left(15\,(bx^3+a)^{\frac{7}{2}}B-42\,(bx^3+a)^{\frac{5}{2}}Ba+35\,(bx^3+a)^{\frac{3}{2}}Ba^2+21\,(bx^3+a)^{\frac{5}{2}}Ab-35\,(bx^3+a)^{\frac{3}{2}}Aab\right)}{315\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/315*(15*(b*x^3 + a)^(7/2)*B - 42*(b*x^3 + a)^(5/2)*B*a + 35*(b*x^3 + a)^
(3/2)*B*a^2 + 21*(b*x^3 + a)^(5/2)*A*b - 35*(b*x^3 + a)^(3/2)*A*a*b)/b^3
```

3.180.9 Mupad [B] (verification not implemented)

Time = 7.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.56

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\,B\,x^9\,\sqrt{b\,x^3+a}}{21}+\frac{x^6\,\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{15\,b}-\frac{2\,a\,\left(2\,A\,a-\frac{4\,a\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{5\,b}\right)\,\sqrt{b\,x^3+a}}{9\,b^2}+\frac{x^3\,\left(2\,A\,a-\frac{4\,a\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{5\,b}\right)\,\sqrt{b\,x^3+a}}{9\,b}$$

input
```
int(x^5*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

output
```
(2*B*x^9*(a + b*x^3)^(1/2))/21 + (x^6*(a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/7
))/(15*b) - (2*a*(2*A*a - (4*a*(2*A*b + (2*B*a)/7))/(5*b))*(a + b*x^3)^(1/
2))/(9*b^2) + (x^3*(2*A*a - (4*a*(2*A*b + (2*B*a)/7))/(5*b))*(a + b*x^3)^(
1/2))/(9*b)
```

3.181 $\int x^2\sqrt{a+bx^3}(A+Bx^3)\,dx$

3.181.1 Optimal result

Integrand size = 22, antiderivative size = 46

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^2}$$

output
```
2/9*(A*b-B*a)*(b*x^3+a)^(3/2)/b^2+2/15*B*(b*x^3+a)^(5/2)/b^2
```

3.181.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\left(a+bx^3\right)^{3/2}\left(5Ab-2aB+3bBx^3\right)}{45b^2}$$

input
```
Integrate[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*(a + b*x^3)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^3))/(45*b^2)
```

3.181.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a+bx^3}\left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3} \int \sqrt{bx^3+a}\left(Bx^3+A\right)dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3} \int \left(\frac{B\left(bx^3+a\right)^{3/2}}{b}+\frac{(Ab-aB)\sqrt{bx^3+a}}{b}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2\left(a+bx^3\right)^{3/2}(Ab-aB)}{3b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{5b^2}\right)$$

input
```
Int[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
((2*(A*b - a*B)*(a + b*x^3)^(3/2))/(3*b^2) + (2*B*(a + b*x^3)^(5/2))/(5*b^
2))/3
```

3.181.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.181.4 Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{2\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(3bB\,x^{3}+5Ab-2Ba\right)}{45b^{2}}$	31
pseudoelliptic	$\frac{2\left(\left(3x^{3}B+5A\right)b-2Ba\right)\left(b\,x^{3}+a\right)^{\frac{3}{2}}}{45b^{2}}$	32
trager	$\frac{2\left(3b^{2}B\,x^{6}+5A\,b^{2}x^{3}+Bab\,x^{3}+5abA-2a^{2}B\right)\sqrt{b\,x^{3}+a}}{45b^{2}}$	52
risch	$\frac{2\left(3b^{2}B\,x^{6}+5A\,b^{2}x^{3}+Bab\,x^{3}+5abA-2a^{2}B\right)\sqrt{b\,x^{3}+a}}{45b^{2}}$	52
default	$B\left(\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15}+\frac{2a\,x^{3}\sqrt{b\,x^{3}+a}}{45b}-\frac{4a^{2}\sqrt{b\,x^{3}+a}}{45b^{2}}\right)+\frac{2A\left(b\,x^{3}+a\right)^{\frac{3}{2}}}{9b}$	69
elliptic	$\frac{2B\,x^{6}\sqrt{b\,x^{3}+a}}{15}+\frac{2\left(Ab+\frac{Ba}{5}\right)x^{3}\sqrt{b\,x^{3}+a}}{9b}+\frac{2\left(Aa-\frac{2a\left(Ab+\frac{Ba}{5}\right)}{3b}\right)\sqrt{b\,x^{3}+a}}{3b}$	74

input

```
int(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/45*(b*x^3+a)^(3/2)*(3*B*b*x^3+5*A*b-2*B*a)/b^2
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\,(3\,Bb^2x^6+(Bab+5\,Ab^2)x^3-2\,Ba^2+5\,Aab)\sqrt{bx^3+a}}{45\,b^2}$$

input

```
integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*B*b^2*x^6 + (B*a*b + 5*A*b^2)*x^3 - 2*B*a^2 + 5*A*a*b)*sqrt(b*x^3
+ a)/b^2
```

3.181.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(44) = 88$.

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.54

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \begin{cases} \frac{2Aa\sqrt{a+bx^3}}{9b} + \frac{2Ax^3\sqrt{a+bx^3}}{9} - \frac{4Ba^2\sqrt{a+bx^3}}{45b^2} + \frac{2Bax^3\sqrt{a+bx^3}}{45b} + \frac{2Bx^6\sqrt{a+bx^3}}{15} & \text{for } b \neq 0 \\ \sqrt{a}\left(\frac{Ax^3}{3} + \frac{Bx^6}{6}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((2*A*a*sqrt(a + b*x**3)/(9*b) + 2*A*x**3*sqrt(a + b*x**3)/9 - 4*
B*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*B*a*x**3*sqrt(a + b*x**3)/(45*b) + 2
*B*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*(A*x**3/3 + B*x**6/6), Tr
ue))
```

3.181.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2}{45}\,B\left(\frac{3\,(bx^3+a)^{\frac{5}{2}}}{b^2} - \frac{5\,(bx^3+a)^{\frac{3}{2}}a}{b^2}\right) + \frac{2\,(bx^3+a)^{\frac{3}{2}}A}{9\,b}$$

input
```
integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/45*B*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/2)*a/b^2) + 2/9*(b*x^3
+ a)^(3/2)*A/b
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\left(3\,(bx^3+a)^{\frac{5}{2}}B-5\,(bx^3+a)^{\frac{3}{2}}Ba+5\,(bx^3+a)^{\frac{3}{2}}Ab\right)}{45\,b^2}$$

input
```
integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/45*(3*(b*x^3 + a)^(5/2)*B - 5*(b*x^3 + a)^(3/2)*B*a + 5*(b*x^3 + a)^(3/2
)*A*b)/b^2
```

3.181.9 Mupad [B] (verification not implemented)

Time = 6.98 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{6\,B\,(b\,x^3+a)^{5/2}+10\,A\,b\,(b\,x^3+a)^{3/2}-10\,B\,a\,(b\,x^3+a)^{3/2}}{45\,b^2}$$

input
```
int(x^2*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

output
```
(6*B*(a + b*x^3)^(5/2) + 10*A*b*(a + b*x^3)^(3/2) - 10*B*a*(a + b*x^3)^(3/
2))/(45*b^2)
```

3.182 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx$

3.182.1 Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{2}{3}A\sqrt{a+bx^3}+\frac{2B(a+bx^3)^{3/2}}{9b}-\frac{2}{3}\sqrt{a}A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

output
```
2/9*B*(b*x^3+a)^(3/2)/b-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/3
*A*(b*x^3+a)^(1/2)
```

3.182.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{2\sqrt{a+bx^3}(3Ab+aB+bBx^3)}{9b}-\frac{2}{3}\sqrt{a}A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]
```

output
```
(2*Sqrt[a + b*x^3]*(3*A*b + a*B + b*B*x^3))/(9*b) - (2*Sqrt[a]*A*ArcTanh[S
qrt[a + b*x^3]/Sqrt[a]])/3
```

3.182.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {948, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{bx^3+a}\left(Bx^3+A\right)}{x^3}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(A\int \frac{\sqrt{bx^3+a}}{x^3}dx^3+\frac{2B\left(a+bx^3\right)^{3/2}}{3b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(A\left(a\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+2\sqrt{a+bx^3}\right)+\frac{2B\left(a+bx^3\right)^{3/2}}{3b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(A\left(\frac{2a\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}+2\sqrt{a+bx^3}\right)+\frac{2B\left(a+bx^3\right)^{3/2}}{3b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(A\left(2\sqrt{a+bx^3}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)+\frac{2B\left(a+bx^3\right)^{3/2}}{3b}\right)$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]
```

output
```
((2*B*(a + b*x^3)^(3/2))/(3*b) + A*(2*Sqrt[a + b*x^3] - 2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x^3]/Sqrt[a]]))/3
```

3.182.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.182.4 Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{2B(b\,x^3+a)^{\frac{3}{2}}}{9b}+A\left(-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)\sqrt{a}}{3}+\frac{2\sqrt{b\,x^3+a}}{3}\right)$	50
elliptic	$\frac{2B\,x^3\sqrt{b\,x^3+a}}{9}+\frac{2\left(Ab+\frac{Ba}{3}\right)\sqrt{b\,x^3+a}}{3b}-\frac{2A\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)\sqrt{a}}{3}$	59
pseudoelliptic	$\frac{2Bb\,x^3\sqrt{b\,x^3+a}-6\sqrt{a}\,bA\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)+6Ab\sqrt{b\,x^3+a}+2Ba\sqrt{b\,x^3+a}}{9b}$	70

input
```
int((B*x^3+A)*(b*x^3+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/9*B*(b*x^3+a)^(3/2)/b+A*(-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2
/3*(b*x^3+a)^(1/2))
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \left[\frac{3\,A\sqrt{a}b\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(Bbx^3+Ba+3\,Ab)\sqrt{bx^3+a}}{9\,b}, \frac{2\left(3\,A\sqrt{-a}b\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+\right.}{9\,b}\right.$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/9*(3*A*sqrt(a)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2
*(B*b*x^3 + B*a + 3*A*b)*sqrt(b*x^3 + a))/b, 2/9*(3*A*sqrt(-a)*b*arctan(sq
rt(b*x^3 + a)*sqrt(-a)/a) + (B*b*x^3 + B*a + 3*A*b)*sqrt(b*x^3 + a))/b]
```

3.182.6 Sympy [A] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{A\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+bx^3} & \text{for } b\neq 0\\ -\sqrt{a}\log\left(\frac{1}{x^3}\right) & \text{otherwise}\end{cases}\right)}{3} - \frac{B\left(\begin{cases}-\sqrt{a}x^3 & \text{for } b=0\\ -\frac{2(a+bx^3)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)}{3}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x,x)
```

output
```
A*Piecewise((2*a*atan(sqrt(a + b*x**3)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b*x
**3), Ne(b, 0)), (-sqrt(a)*log(x**(-3)), True))/3 - B*Piecewise((-sqrt(a)*
x**3, Eq(b, 0)), (-2*(a + b*x**3)**(3/2)/(3*b), True))/3
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{1}{3}\left(\sqrt{a}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+2\sqrt{bx^3+a}\right)A+\frac{2\,(bx^3+a)^{\frac{3}{2}}B}{9\,b}$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/3*(sqrt(a)*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))
+ 2*sqrt(b*x^3 + a))*A + 2/9*(b*x^3 + a)^(3/2)*B/b
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{2\,Aa\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}} + \frac{2\left((bx^3+a)^{\frac{3}{2}}Bb^2+3\,\sqrt{bx^3+a}Ab^3\right)}{9\,b^3}$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

output
```
2/3*A*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/9*((b*x^3 + a)^(3/2)
*B*b^2 + 3*sqrt(b*x^3 + a)*A*b^3)/b^3
```

3.182.9 Mupad [B] (verification not implemented)

Time = 7.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{2\,B\,x^3\,\sqrt{b\,x^3+a}}{9} + \frac{\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{3}\right)}{3\,b} + \frac{A\,\sqrt{a}\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x,x)
```

output
```
(2*B*x^3*(a + b*x^3)^(1/2))/9 + ((a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/3))/(3
*b) + (A*a^(1/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) +
 a^(1/2)))/x^6))/3
```

3.183 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^4}\,dx$

3.183.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^4}\,dx = \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a} - \frac{A\left(a+bx^3\right)^{3/2}}{3ax^3} - \frac{(Ab+2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

output
```
-1/3*A*(b*x^3+a)^(3/2)/a/x^3-1/3*(A*b+2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/
2))/a^(1/2)+1/3*(A*b+2*B*a)*(b*x^3+a)^(1/2)/a
```

3.183.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^4}\,dx = \frac{\sqrt{a+bx^3}\left(-A+2Bx^3\right)}{3x^3} + \frac{(-Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]
```

output
```
(Sqrt[a + b*x^3]*(-A + 2*B*x^3))/(3*x^3) + ((-(A*b) - 2*a*B)*ArcTanh[Sqrt[
a + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

3.183.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {948, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^4} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{\sqrt{bx^3+a}\left(Bx^3+A\right)}{x^6}dx^3$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{3}\left(\frac{(2aB+Ab)\int \frac{\sqrt{bx^3+a}}{x^3}dx^3}{2a}-\frac{A\left(a+bx^3\right)^{3/2}}{ax^3}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}\left(\frac{(2aB+Ab)\left(a\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+2\sqrt{a+bx^3}\right)}{2a}-\frac{A\left(a+bx^3\right)^{3/2}}{ax^3}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{(2aB+Ab)\left(\frac{2a\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}+2\sqrt{a+bx^3}\right)}{2a}-\frac{A\left(a+bx^3\right)^{3/2}}{ax^3}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{3}\left(\frac{(2aB+Ab)\left(2\sqrt{a+bx^3}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)}{2a}-\frac{A\left(a+bx^3\right)^{3/2}}{ax^3}\right)$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]
```

output
```
(-((A*(a + b*x^3)^(3/2))/(a*x^3)) + ((A*b + 2*a*B)*(2*Sqrt[a + b*x^3] - 2*
Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]))/(2*a))/3
```

3.183. $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^4} \, dx$

3.183.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.183. $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^4}\,dx$

3.183.4 Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{A\sqrt{b\,x^3+a}}{3x^3}+\frac{2B\sqrt{b\,x^3+a}}{3}-\frac{(Ab+2Ba)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	56
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{3x^3}+\frac{2B\sqrt{b\,x^3+a}}{3}-\frac{2\left(\frac{Ab}{2}+Ba\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	56
pseudoelliptic	$-\frac{(Ab+2Ba)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)x^3+\sqrt{b\,x^3+a}\,(-2x^3B+A)\sqrt{a}}{3\sqrt{a}\,x^3}$	57
default	$B\left(-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)\sqrt{a}}{3}+\frac{2\sqrt{b\,x^3+a}}{3}\right)+A\left(-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{\sqrt{b\,x^3+a}}{3x^3}\right)$	72

input

```
int((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output

```
-1/3*A*(b*x^3+a)^(1/2)/x^3+2/3*B*(b*x^3+a)^(1/2)-1/3*(A*b+2*B*a)*arctanh((
b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx$$

$$=\left[\frac{(2\,Ba+Ab)\sqrt{a}x^3\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(2\,Bax^3-Aa)\sqrt{bx^3+a}}{6\,ax^3},\frac{(2\,Ba+Ab)\sqrt{-a}x^3\arctan\left(\sqrt{b}\right.}{}\right.$$

input

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")
```

output

```
[1/6*((2*B*a + A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2
*a)/x^3) + 2*(2*B*a*x^3 - A*a)*sqrt(b*x^3 + a))/(a*x^3), 1/3*((2*B*a + A*b
)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (2*B*a*x^3 - A*a)*sqrt
(b*x^3 + a))/(a*x^3)]
```

3.183.6 Sympy [A] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}} - \frac{2B\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{2Ba}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2B\sqrt{b}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**4,x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - A*b*asinh(sqrt(a)/(sqrt(b)*
x**(3/2)))/(3*sqrt(a)) - 2*B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 +
 2*B*a/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*B*sqrt(b)*x**(3/2)/(3
*sqrt(a/(b*x**3) + 1))
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx = \frac{1}{6}\left(\frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\sqrt{bx^3+a}}{x^3}\right)A + \frac{1}{3}\left(\sqrt{a}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 2\sqrt{bx^3+a}\right)B$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/6*(b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a
) - 2*sqrt(b*x^3 + a)/x^3)*A + 1/3*(sqrt(a)*log((sqrt(b*x^3 + a) - sqrt(a)
)/(sqrt(b*x^3 + a) + sqrt(a))) + 2*sqrt(b*x^3 + a))*B
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx = \frac{2\sqrt{bx^3+a}Bb + \frac{(2\,Bab+Ab^2)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx^3+a}Ab}{x^3}}{3\,b}$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/3*(2*sqrt(b*x^3 + a)*B*b + (2*B*a*b + A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt
(-a))/sqrt(-a) - sqrt(b*x^3 + a)*A*b/x^3)/b
```

3.183.9 Mupad [B] (verification not implemented)

Time = 7.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx = \frac{2\,B\,\sqrt{b\,x^3+a}}{3} - \frac{A\,\sqrt{b\,x^3+a}}{3\,x^3} + \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\left(\frac{A\,b}{2}+B\,a\right)}{3\,\sqrt{a}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^4,x)
```

output
```
(2*B*(a + b*x^3)^(1/2))/3 - (A*(a + b*x^3)^(1/2))/(3*x^3) + (log((((a + b*
x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*((A*b)/2 + B*a
))/(3*a^(1/2))
```

3.184 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx$

3.184.1 Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A(a+bx^3)^{3/2}}{6ax^6} + \frac{b(Ab-4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$

output
```
-1/6*A*(b*x^3+a)^(3/2)/a/x^6+1/12*b*(A*b-4*B*a)*arctanh((b*x^3+a)^(1/2)/a^
(1/2))/a^(3/2)+1/12*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a/x^3
```

3.184.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = \frac{\sqrt{a+bx^3}(-2aA-Abx^3-4aBx^3)}{12ax^6} - \frac{b(-Ab+4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]
```

output
```
(Sqrt[a + b*x^3]*(-2*a*A - A*b*x^3 - 4*a*B*x^3))/(12*a*x^6) - (b*(-(A*b) +
 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2))
```

3.184.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {948, 87, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^7} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{bx^3+a}\left(Bx^3+A\right)}{x^9}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(-\frac{(Ab-4aB)\int \frac{\sqrt{bx^3+a}}{x^6}dx^3}{4a}-\frac{A\left(a+bx^3\right)^{3/2}}{2ax^6}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}\left(-\frac{(Ab-4aB)\left(\frac{1}{2}b\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3-\frac{\sqrt{a+bx^3}}{x^3}\right)}{4a}-\frac{A\left(a+bx^3\right)^{3/2}}{2ax^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{(Ab-4aB)\left(\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}-\frac{\sqrt{a+bx^3}}{x^3}\right)}{4a}-\frac{A\left(a+bx^3\right)^{3/2}}{2ax^6}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{(Ab-4aB)\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^3}}{x^3}\right)}{4a}-\frac{A\left(a+bx^3\right)^{3/2}}{2ax^6}\right)$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]
```

output
```
(-1/2*(A*(a + b*x^3)^(3/2))/(a*x^6) - ((A*b - 4*a*B)*(-(Sqrt[a + b*x^3]/x^
3) - (b*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/Sqrt[a]))/(4*a))/3
```

3.184.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.184. $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx$

3.184.4 Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(A b\,x^{3}+4 B a\,x^{3}+2 A a\right)}{12 x^{6} a}+\frac{b \left(A b -4 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{3}{2}}}$	65
elliptic	$-\frac{A \sqrt{b\,x^{3}+a}}{6 x^{6}}-\frac{\left(A b +4 B a \right) \sqrt{b\,x^{3}+a}}{12 a\,x^{3}}+\frac{b \left(A b -4 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{3}{2}}}$	70
pseudoelliptic	$-\frac{-b\,x^{6} \left(A b -4 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)+\left(\left(4 x^{3} B +2 A \right) a^{\frac{3}{2}}+A \sqrt{a}\,b\,x^{3}\right) \sqrt{b\,x^{3}+a}}{12 a^{\frac{3}{2}} x^{6}}$	72
default	$A \left(\frac{b^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^{3}+a}}{6 x^{6}}-\frac{b \sqrt{b\,x^{3}+a}}{12 a\,x^{3}}\right)+B \left(-\frac{b \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3 \sqrt{a}}-\frac{\sqrt{b\,x^{3}+a}}{3 x^{3}}\right)$	96

input

```
int((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output

```
-1/12*(b*x^3+a)^(1/2)*(A*b*x^3+4*B*a*x^3+2*A*a)/x^6/a+1/12*b*(A*b-4*B*a)*a
rctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx$$

$$= \left[-\frac{(4\,Bab - Ab^2)\sqrt{a}x^6 \log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right) + 2\,((4\,Ba^2 + Aab)x^3 + 2\,Aa^2)\sqrt{bx^3+a}}{24\,a^2x^6}, \; (4\,Bab - Ab^2)\right.$$

input

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

output

```
[-1/24*((4*B*a*b - A*b^2)*sqrt(a)*x^6*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(
a) + 2*a)/x^3) + 2*((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^2
*x^6), 1/12*((4*B*a*b - A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^3 + a)*sqrt(-a
)/a) - ((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^2*x^6)]
```

3.184.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(76) = 152$.

Time = 33.57 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = -\frac{Aa}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{A\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{Ab^{\frac{3}{2}}}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**7,x)
```

output
```
-A*a/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - A*sqrt(b)/(4*x**(9/2)*sq
rt(a/(b*x**3) + 1)) - A*b**(3/2)/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + A*
b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2)) - B*sqrt(b)*sqrt(a/(b
*x**3) + 1)/(3*x**(3/2)) - B*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a
))
```

3.184.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(74) = 148$.

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = -\frac{1}{24}\left(\frac{b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2\left((bx^3+a)^{\frac{3}{2}}b^2 + \sqrt{bx^3+a}\,ab^2\right)}{(bx^3+a)^2a - 2\,(bx^3+a)a^2 + a^3}\right)A + \frac{1}{6}\left(\frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\sqrt{bx^3+a}}{x^3}\right)B$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/24*(b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^
(3/2) + 2*((b*x^3 + a)^(3/2)*b^2 + sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a
 - 2*(b*x^3 + a)*a^2 + a^3))*A + 1/6*(b*log((sqrt(b*x^3 + a) - sqrt(a))/(s
qrt(b*x^3 + a) + sqrt(a)))/sqrt(a) - 2*sqrt(b*x^3 + a)/x^3)*B
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = \frac{\frac{(4\,Bab^2-Ab^3)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{4\,(bx^3+a)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx^3+a}Ba^2b^2+(bx^3+a)^{\frac{3}{2}}Ab^3+\sqrt{bx^3+a}Aab^3}{ab^2x^6}}{12\,b}$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="giac")
```

output
```
1/12*((4*B*a*b^2 - A*b^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) -
(4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a^2*b^2 + (b*x^3 + a)^(
3/2)*A*b^3 + sqrt(b*x^3 + a)*A*a*b^3)/(a*b^2*x^6))/b
```

3.184.9 Mupad [B] (verification not implemented)

Time = 7.60 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = \frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)(A\,b-4\,B\,a)}{24\,a^{3/2}} - \frac{(4\,B\,a^2+A\,b\,a)\,\sqrt{b\,x^3+a}}{12\,a^2\,x^3} - \frac{A\,\sqrt{b\,x^3+a}}{6\,x^6}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^7,x)
```

output
```
(b*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6
)*(A*b - 4*B*a))/(24*a^(3/2)) - ((4*B*a^2 + A*a*b)*(a + b*x^3)^(1/2))/(12*
a^2*x^3) - (A*(a + b*x^3)^(1/2))/(6*x^6)
```

3.185 $\int x^3\sqrt{a+bx^3}\,(A+Bx^3)\ dx$

3.185.1 Optimal result

Integrand size = 22, antiderivative size = 303

$$\int x^3\sqrt{a+bx^3}\,(A+Bx^3)\ dx$$

$$= \frac{6a(17Ab-8aB)x\sqrt{a+bx^3}}{935b^2}+\frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b}+\frac{2Bx^4(a+bx^3)^{3/2}}{17b}$$

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2(17Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\ \text{EllipticF}\Big(\arcsin\Big(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}}{935b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/17*B*x^4*(b*x^3+a)^(3/2)/b+6/935*a*(17*A*b-8*B*a)*x*(b*x^3+a)^(1/2)/b^2+
2/187*(17*A*b-8*B*a)*x^4*(b*x^3+a)^(1/2)/b-4/935*3^(3/4)*a^2*(17*A*b-8*B*a
)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(
7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)
```

3.185.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.49 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.29

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{2x\sqrt{a+bx^3}\left(-\left(\left(a+bx^3\right)\left(-17Ab+8aB-11bBx^3\right)\right)+\frac{a(-17Ab+8aB)\text{ Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{187b^2}$$

input
```
Integrate[x^3*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(-((a + b*x^3)*(-17*A*b + 8*a*B - 11*b*B*x^3)) + (a*(
-17*A*b + 8*a*B)*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 +
 (b*x^3)/a]))/(187*b^2)
```

3.185.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {959, 811, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$\Big\downarrow \text{959}$$
$$\frac{(17Ab-8aB)\int x^3\sqrt{bx^3+a}dx}{17b}+\frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}$$
$$\Big\downarrow \text{811}$$
$$\frac{(17Ab-8aB)\left(\frac{3}{11}a\int \frac{x^3}{\sqrt{bx^3+a}}dx+\frac{2}{11}x^4\sqrt{a+bx^3}\right)}{17b}+\frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}$$
$$\Big\downarrow \text{843}$$
$$\frac{(17Ab-8aB)\left(\frac{3}{11}a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{2a\int \frac{1}{\sqrt{bx^3+a}}dx}{5b}\right)+\frac{2}{11}x^4\sqrt{a+bx^3}\right)}{17b}+\frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}$$

$$\downarrow 759$$

$$\frac{(17Ab-8aB)\left(\frac{3}{11}a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.\right.}{17b}$$

$$\frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}$$

input
```
Int[x^3*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*B*x^4*(a + b*x^3)^(3/2))/(17*b) + ((17*A*b - 8*a*B)*((2*x^4*Sqrt[a + b*
x^3])/11 + (3*a*((2*x*Sqrt[a + b*x^3])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/
4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3])))/11))/(17*b)
```

3.185.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.185.4 Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.15

method	result
risch	$\frac{2x(55b^2Bx^6+85Ab^2x^3+15Babx^3+51abA-24a^2B)\sqrt{bx^3+a}}{935b^2}+\frac{4ia^2(17Ab-8Ba)\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)}{(-ab^2)^{\frac{1}{3}}}}}{}$
elliptic	$\frac{2Bx^7\sqrt{bx^3+a}}{17}+\frac{2\left(Ab+\frac{3Ba}{17}\right)x^4\sqrt{bx^3+a}}{11b}+\frac{2\left(Aa-\frac{8a\left(Ab+\frac{3Ba}{17}\right)}{11b}\right)x\sqrt{bx^3+a}}{5b}+\frac{4ia\left(Aa-\frac{8a\left(Ab+\frac{3Ba}{17}\right)}{11b}\right)\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x\right.}{}}}{}$
default	$B\left(\frac{2x^7\sqrt{bx^3+a}}{17}+\frac{6ax^4\sqrt{bx^3+a}}{187b}-\frac{48a^2x\sqrt{bx^3+a}}{935b^2}-\frac{32ia^3\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3(-ab^2}{2b}}}}{}\right.$

input
```
int(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/935*x*(55*B*b^2*x^6+85*A*b^2*x^3+15*B*a*b*x^3+51*A*a*b-24*B*a^2)*(b*x^3+
a)^(1/2)/b^2+4/935*I*a^2*(17*A*b-8*B*a)/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.185. $\int x^3\sqrt{a+bx^3}(A+Bx^3)\,dx$

3.185.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.30

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$=\frac{2\,\Big(6\,(8\,Ba^3-17\,Aa^2b)\sqrt{b}\mathrm{weierstrassPInverse}\big(0,-\tfrac{4\,a}{b},x\big)+(55\,Bb^3x^7+5\,(3\,Bab^2+17\,Ab^3)x^4-3\,(8\,Ba}{935\,b^3}$$

input
```
integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/935*(6*(8*B*a^3 - 17*A*a^2*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x)
+ (55*B*b^3*x^7 + 5*(3*B*a*b^2 + 17*A*b^3)*x^4 - 3*(8*B*a^2*b - 17*A*a*b^2
)*x)*sqrt(b*x^3 + a))/b^3
```

3.185.6 Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.27

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{A\sqrt{a}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{B\sqrt{a}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**3*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

output
```
A*sqrt(a)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(7/3)) + B*sqrt(a)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

3.185.7 Maxima [F]

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int\left(Bx^3+A\right)\sqrt{bx^3+a}x^3\,dx$$

input
```
integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^3, x)
```

3.185.8 Giac [F]

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int\left(Bx^3+A\right)\sqrt{bx^3+a}x^3\,dx$$

input
```
integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^3, x)
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int x^3\,\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\,dx$$

input
```
int(x^3*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

output
```
int(x^3*(A + B*x^3)*(a + b*x^3)^(1/2), x)
```

3.186 $\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

3.186.1 Optimal result

Integrand size = 19, antiderivative size = 268

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2(11Ab-2aB)x\sqrt{a+bx^3}}{55b}+\frac{2Bx\left(a+bx^3\right)^{3/2}}{11b}$$
$$+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a(11Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}\right.\right.}{55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/11*B*x*(b*x^3+a)^(3/2)/b+2/55*(11*A*b-2*B*a)*x*(b*x^3+a)^(1/2)/b+2/55*3^
(3/4)*a*(11*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2
*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.186.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.57 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.28

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2x\sqrt{a+bx^3}\left(B(a+bx^3)+\frac{(11Ab-2aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{11b}$$

input

```
Integrate[Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output

```
(2*x*Sqrt[a + b*x^3]*(B*(a + b*x^3) + ((11*A*b - 2*a*B)*Hypergeometric2F1[
-1/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(11*b)
```

3.186.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {913, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(11Ab-2aB)\int \sqrt{bx^3+a}dx}{11b}+\frac{2Bx\left(a+bx^3\right)^{3/2}}{11b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(11Ab-2aB)\left(\frac{3}{5}a\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{2}{5}x\sqrt{a+bx^3}\right)}{11b}+\frac{2Bx\left(a+bx^3\right)^{3/2}}{11b}$$

$$\Big\downarrow \text{759}$$

$$\frac{(11Ab-2aB)\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2}{5}x\sqrt{a}\right.}{11b}$$

$$\frac{2Bx\left(a+bx^3\right)^{3/2}}{11b}$$

input
```
Int[Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*B*x*(a + b*x^3)^(3/2))/(11*b) + ((11*A*b - 2*a*B)*((2*x*Sqrt[a + b*x^3]
)/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(11*b)
```

3.186.3.1 Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.186.4 Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.21

method	result
risch	$\frac{2x(5bBx^3+11Ab+3Ba)\sqrt{bx^3+a}}{55b}-\frac{2ia(11Ab-2Ba)\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}b}{\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\right.}{2b}}}}{}$
elliptic	$\frac{2Bx^4\sqrt{bx^3+a}}{11}+\frac{2\left(Ab+\frac{3Ba}{11}\right)x\sqrt{bx^3+a}}{5b}-\frac{2i\left(Aa-\frac{2a\left(Ab+\frac{3Ba}{11}\right)}{5b}\right)\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}b}{\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{}{-\frac{3\left(-\right.}{}}}}{}$
default	$A\left(\frac{2x\sqrt{bx^3+a}}{5}-\frac{2ia\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}b}{\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}+i\right.}{\left(-ab\right.}}}{5b\sqrt{bx^3+}}\right.$

input
```
int((B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/55*x*(5*B*b*x^3+11*A*b+3*B*a)/b*(b*x^3+a)^(1/2)-2/55*I*a*(11*A*b-2*B*a)/
b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.186.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = -\frac{2\left(3\left(2\,Ba^2-11\,Aab\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\left(5\,Bb^2x^4+\left(3\,Bab+11\,Ab^2\right)x\right)\sqrt{bx^3+a}\right)}{55\,b^2}$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/55*(3*(2*B*a^2 - 11*A*a*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) -
(5*B*b^2*x^4 + (3*B*a*b + 11*A*b^2)*x)*sqrt(b*x^3 + a))/b^2
```

3.186.6 Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.31

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{A\sqrt{a}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{B\sqrt{a}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2),x)
```

output
```
A*sqrt(a)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + B*sqrt(a)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

3.186.7 Maxima [F]

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a), x)
```

3.186.8 Giac [F]

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a), x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((A + B*x^3)*(a + b*x^3)^(1/2),x)
```

output
```
int((A + B*x^3)*(a + b*x^3)^(1/2), x)
```

3.187 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^3}\,dx$

3.187.1 Optimal result

Integrand size = 22, antiderivative size = 269

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^3}\,dx=\frac{(5Ab+4aB)x\sqrt{a+bx^3}}{10a}-\frac{A\left(a+bx^3\right)^{3/2}}{2ax^2}$$
$$+\frac{3^{3/4}\sqrt{2+\sqrt{3}}(5Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),\right.}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/2*A*(b*x^3+a)^(3/2)/a/x^2+1/10*(5*A*b+4*B*a)*x*(b*x^3+a)^(1/2)/a+1/10*3
^(3/4)*(5*A*b+4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3
^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2
^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.187.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx$$
$$=\frac{\sqrt{a+bx^3}\left(-A(a+bx^3)+\frac{(5Ab+4aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{2ax^2}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^3,x]
```

output
```
(Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) + ((5*A*b + 4*a*B)*x^3*Hypergeometric2F
1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(2*a*x^2)
```

3.187.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {955, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx$$
$$\Big\downarrow \text{955}$$
$$\frac{(4aB+5Ab)\int \sqrt{bx^3+a}dx}{4a}-\frac{A\left(a+bx^3\right)^{3/2}}{2ax^2}$$
$$\Big\downarrow \text{748}$$
$$\frac{(4aB+5Ab)\left(\frac{3}{5}a\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{2}{5}x\sqrt{a+bx^3}\right)}{4a}-\frac{A\left(a+bx^3\right)^{3/2}}{2ax^2}$$
$$\Big\downarrow \text{759}$$

$$\frac{(4aB+5Ab)\left(\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2}{5}x\sqrt{a+}\right.}{4a}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{2ax^2}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(A*(a + b*x^3)^(3/2))/(a*x^2) + ((5*A*b + 4*a*B)*((2*x*Sqrt[a + b*x^3
])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(4*a)
```

3.187.3.1 Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.187.4 Maple [A] (verified)

Time = 4.41 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-4x^{3}B+5A\right)}{10x^{2}}-\frac{2i\left(\frac{3Ab}{4}+\frac{3Ba}{5}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{}}{}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{2x^{2}}+\frac{2Bx\sqrt{b\,x^{3}+a}}{5}-\frac{2i\left(\frac{3Ab}{4}+\frac{3Ba}{5}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)}{2b}}}}{}$
default	$B\left(\frac{2x\sqrt{b\,x^{3}+a}}{5}-\frac{2ia\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+i\right.}{\left(-a\,b\right.}}}{5b\sqrt{b\,x^{3}+}}\right.$

input
```
int((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/10*(b*x^3+a)^(1/2)*(-4*B*x^3+5*A)/x^2-2/3*I*(3/4*A*b+3/5*B*a)*3^(1/2)/b
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.187.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \frac{3\,(4\,Ba+5\,Ab)\sqrt{b}x^2\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(4\,Bbx^3-5\,Ab)\sqrt{bx^3+a}}{10\,bx^2}$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/10*(3*(4*B*a + 5*A*b)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) + (4
*B*b*x^3 - 5*A*b)*sqrt(b*x^3 + a))/(b*x^2)
```

3.187.6 Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{B\sqrt{a}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**3,x)
```

output
```
A*sqrt(a)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**2*gamma(1/3)) + B*sqrt(a)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3))
```

3.187.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^3}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^3, x)
```

3.187.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^3}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^3, x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^3}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^3,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^3, x)
```

3.188 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx$

3.188.1 Optimal result

Integrand size = 22, antiderivative size = 272

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx=\frac{(Ab-10aB)\sqrt{a+bx^3}}{20ax^2}-\frac{A(a+bx^3)^{3/2}}{5ax^5}$$
$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}(Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Bigg(\arcsin\Bigg(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}}{20a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/5*A*(b*x^3+a)^(3/2)/a/x^5+1/20*(A*b-10*B*a)*(b*x^3+a)^(1/2)/a/x^2-1/20*
3^(3/4)*b^(2/3)*(A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1
/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/
2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.188.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \frac{\sqrt{a+bx^3}\left(-2A(a+bx^3)+\frac{\left(\frac{Ab}{2}-5aB\right)x^3\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{10ax^5}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^6,x]
```

output
```
(Sqrt[a + b*x^3]*(-2*A*(a + b*x^3) + (((A*b)/2 - 5*a*B)*x^3*Hypergeometric
2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(10*a*x^5)
```

3.188.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {955, 809, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(Ab-10aB)\int \frac{\sqrt{bx^3+a}}{x^3}dx}{10a}-\frac{A\left(a+bx^3\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{809}$$

$$-\frac{(Ab-10aB)\left(\frac{3}{4}b\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{2x^2}\right)}{10a}-\frac{A\left(a+bx^3\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{759}$$

$$-\frac{(Ab-10aB)\left(\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+b}}{2x^2}}{10a}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{5ax^5}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^6,x]
```

output
```
-1/5*(A*(a + b*x^3)^(3/2))/(a*x^5) - ((A*b - 10*a*B)*(-1/2*Sqrt[a + b*x^3]
/x^2 + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(10*a)
```

3.188.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.188.4 Maple [A] (verified)

Time = 4.47 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,(3Ab\,x^{3}+10Ba\,x^{3}+4Aa)}{20x^{5}a}+\frac{i(Ab-10Ba)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a}{2b}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{5x^{5}}-\frac{(3Ab+10Ba)\sqrt{b\,x^{3}+a}}{20a\,x^{2}}-\frac{2i\left(Bb-\frac{b(3Ab+10Ba)}{40a}\right)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x}{-\frac{3(-a\,b^{2}}{2b}}}}{}$
default	$B\left(-\frac{\sqrt{b\,x^{3}+a}}{2x^{2}}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}\right.}{(-a\,b^{2}}}}{2\sqrt{b\,x^{3}+a}}\right.$

input
```
int((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/20*(b*x^3+a)^(1/2)*(3*A*b*x^3+10*B*a*x^3+4*A*a)/x^5/a+1/20*I*(A*b-10*B*
a)/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.188.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \frac{3\,(10\,Ba-Ab)\sqrt{b}x^5\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\left((10\,Ba+3\,Ab)x^3+4\,Aa\right)\sqrt{bx^3+a}}{20\,ax^5}$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
1/20*(3*(10*B*a - A*b)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) - ((1
0*B*a + 3*A*b)*x^3 + 4*A*a)*sqrt(b*x^3 + a))/(a*x^5)
```

3.188.6 Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**6,x)
```

output
```
A*sqrt(a)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**5*gamma(-2/3)) + B*sqrt(a)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3))
```

3.188.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx=\int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^6}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^6, x)
```

3.188.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx=\int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^6}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^6, x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx=\int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^6}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^6,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^6, x)
```

3.189 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^9}\,dx$

3.189.1 Optimal result

Integrand size = 22, antiderivative size = 305

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx$$

$$= \frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5} + \frac{3b(7Ab-16aB)\sqrt{a+bx^3}}{320a^2x^2} - \frac{A(a+bx^3)^{3/2}}{8ax^8}$$

$$+ \frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}(7Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}\right.\right.}{320a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/8*A*(b*x^3+a)^(3/2)/a/x^8+1/80*(7*A*b-16*B*a)*(b*x^3+a)^(1/2)/a/x^5+3/3
20*b*(7*A*b-16*B*a)*(b*x^3+a)^(1/2)/a^2/x^2+1/320*3^(3/4)*b^(5/3)*(7*A*b-1
6*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2
)/a^2/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

3.189.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \frac{\sqrt{a+bx^3}\left(-5A(a+bx^3)+\frac{\left(\frac{7Ab}{2}-8aB\right)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{2},-\frac{2}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{40ax^8}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^9,x]
```

output
```
(Sqrt[a + b*x^3]*(-5*A*(a + b*x^3) + (((7*A*b)/2 - 8*a*B)*x^3*Hypergeometr
ic2F1[-5/3, -1/2, -2/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(40*a*x^8)
```

3.189.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 809, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(7Ab-16aB)\int \frac{\sqrt{bx^3+a}}{x^6}dx}{16a}-\frac{A\left(a+bx^3\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \text{ 809}$$

$$-\frac{(7Ab-16aB)\left(\frac{3}{10}b\int \frac{1}{x^3\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{5x^5}\right)}{16a}-\frac{A\left(a+bx^3\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{(7Ab-16aB)\left(\frac{3}{10}b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)-\frac{\sqrt{a+bx^3}}{5x^5}\right)}{16a}-\frac{A\left(a+bx^3\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \text{759}$$

$$-\frac{(7Ab-16aB)\left(\frac{3}{10}b\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\right.\right.}{16a}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{8ax^8}$$

input

```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^9,x]
```

output

```
-1/8*(A*(a + b*x^3)^(3/2))/(a*x^8) - ((7*A*b - 16*a*B)*(-1/5*Sqrt[a + b*x^
3]/x^5 + (3*b*(-1/2*Sqrt[a + b*x^3]/(a*x^2) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
 + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^
(1/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*Sqrt[a + b*x^3])))/10))/(16*a)
```

3.189.3.1 Defintions of rubi rules used

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.189.4 Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-21A\,b^2x^6+48B\,x^6ab+12aAb\,x^3+64a^2B\,x^3+40a^2A)}{320x^8a^2}-\frac{ib(7Ab-16Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right.}{(-a\,b^2)^{\frac{1}{3}}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{8x^8}-\frac{(3Ab+16Ba)\sqrt{b\,x^3+a}}{80a\,x^5}+\frac{3b(7Ab-16Ba)\sqrt{b\,x^3+a}}{320a^2x^2}-\frac{ib(7Ab-16Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right.}{(-a\,b^2)^{\frac{1}{3}}}}}{}$
default	$A\left(-\frac{\sqrt{b\,x^3+a}}{8x^8}-\frac{3b\sqrt{b\,x^3+a}}{80a\,x^5}+\frac{21b^2\sqrt{b\,x^3+a}}{320a^2x^2}-\frac{7ib^2\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{}}{}}}}{}\right.$

input
```
int((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/320*(b*x^3+a)^(1/2)*(-21*A*b^2*x^6+48*B*a*b*x^6+12*A*a*b*x^3+64*B*a^2*x
^3+40*A*a^2)/x^8/a^2-1/320*I*b*(7*A*b-16*B*a)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.189. $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^9}\,dx$

3.189.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = -\frac{3\,(16\,Bab-7\,Ab^2)\sqrt{b}x^8\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(3\,(16\,Bab-7\,Ab^2)x^6+4\,(16\,Ba^2+3\,Aab)x}{320\,a^2x^8}$$

input

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="fricas")
```

output

```
-1/320*(3*(16*B*a*b - 7*A*b^2)*sqrt(b)*x^8*weierstrassPInverse(0, -4*a/b,
x) + (3*(16*B*a*b - 7*A*b^2)*x^6 + 4*(16*B*a^2 + 3*A*a*b)*x^3 + 40*A*a^2)*
sqrt(b*x^3 + a))/(a^2*x^8)
```

3.189.6 Sympy [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

input

```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**9,x)
```

output

```
A*sqrt(a)*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**8*gamma(-5/3)) + B*sqrt(a)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3))
```

3.189.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^9}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^9, x)
```

3.189.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^9}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^9, x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^9}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^9,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^9, x)
```

3.190 $\int x^4\sqrt{a+bx^3}(A+Bx^3)\,dx$

3.190.1 Optimal result

Integrand size = 22, antiderivative size = 581

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2}$$

$$+\frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b}-\frac{24a^2(19Ab-10aB)\sqrt{a+bx^3}}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

$$+\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{8\sqrt{2}3^{3/4}a^{7/3}(19Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/19*B*x^5*(b*x^3+a)^(3/2)/b+6/1729*a*(19*A*b-10*B*a)*x^2*(b*x^3+a)^(1/2)/
b^2+2/247*(19*A*b-10*B*a)*x^5*(b*x^3+a)^(1/2)/b-24/1729*a^2*(19*A*b-10*B*a
)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-8/1729*3^(3/4)*a
^(7/3)*(19*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/
b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)+12/1729*3^(1/4)*a^(7/3)*(19*A*b-10*B*a)*(a^(1/3)+b^(1/
3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(
1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.190.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.82 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.16

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{2x^2\sqrt{a+bx^3}\left(-\left(\left(a+bx^3\right)\left(-19Ab+10aB-13bBx^3\right)\right)+\frac{a(-19Ab+10aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{247b^2}$$

input
```
Integrate[x^4*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*x^2*Sqrt[a + b*x^3]*(-((a + b*x^3)*(-19*A*b + 10*a*B - 13*b*B*x^3)) + (
a*(-19*A*b + 10*a*B)*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt
[1 + (b*x^3)/a]))/(247*b^2)
```

3.190.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {959, 811, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt{a+bx^3}\left(A+Bx^3\right)\, dx$$

$$\Big\downarrow \ 959$$

$$\frac{(19Ab-10aB)\int x^4\sqrt{bx^3+a}dx}{19b}+\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

$$\Big\downarrow \ 811$$

$$\frac{(19Ab-10aB)\left(\frac{3}{13}a\int \frac{x^4}{\sqrt{bx^3+a}}dx+\frac{2}{13}x^5\sqrt{a+bx^3}\right)}{19b}+\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

$$\Big\downarrow \ 843$$

$$\frac{(19Ab-10aB)\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\int \frac{x}{\sqrt{bx^3+a}}dx}{7b}\right)+\frac{2}{13}x^5\sqrt{a+bx^3}\right)}{19b}+\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

$$\Big\downarrow \ 832$$

$$\frac{(19Ab-10aB)\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b}\right)+\frac{2}{13}x^5\sqrt{a+bx^3}\right)}{19b}+\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

$$\Big\downarrow \ 759$$

3.190. $\int x^4\sqrt{a+bx^3}(A+Bx^3)\ dx$

$$\frac{(19Ab-10aB)\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{Elliptic}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}\right.}{7b}\right.\right.}{19b}$$

$$\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

$$\Big\downarrow \text{2416}$$

$$\frac{(19Ab-10aB)\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+}}}{\sqrt[3]{b}}\right.}{}\right.\right.}{}$$

$$\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

3.190. $\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

input
```
Int[x^4*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*B*x^5*(a + b*x^3)^(3/2))/(19*b) + ((19*A*b - 10*a*B)*((2*x^5*Sqrt[a + b
*x^3])/13 + (3*a*((2*x^2*Sqrt[a + b*x^3])/(7*b) - (4*a*(((2*Sqrt[a + b*x^3
])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[
3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + S
qrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(7*b)))/13))/(19*b)
```

3.190.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.190.4 Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.87

method	result
risch	$\frac{2x^2\left(91b^2Bx^6+133Ab^2x^3+21Babx^3+57abA-30a^2B\right)\sqrt{bx^3+a}}{1729b^2}+\frac{8ia^2(19Ab-10Ba)\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\right)}{2b}\right.}{\left(-ab^2\right)^{\frac{1}{3}}}}}{}$
elliptic	$\frac{2Bx^8\sqrt{bx^3+a}}{19}+\frac{2\left(Ab+\frac{3Ba}{19}\right)x^5\sqrt{bx^3+a}}{13b}+\frac{2\left(Aa-\frac{10a\left(Ab+\frac{3Ba}{19}\right)}{13b}\right)x^2\sqrt{bx^3+a}}{7b}+\frac{8ia\left(Aa-\frac{10a\left(Ab+\frac{3Ba}{19}\right)}{13b}\right)\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\sqrt{}}{}$
default	Expression too large to display

input
```
int(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

3.190. $\int x^4\sqrt{a+bx^3}(A+Bx^3)\,dx$

output
```
2/1729*x^2*(91*B*b^2*x^6+133*A*b^2*x^3+21*B*a*b*x^3+57*A*a*b-30*B*a^2)/b^2
*(b*x^3+a)^(1/2)+8/1729*I*a^2*(19*A*b-10*B*a)/b^3*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.190.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.18

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx =$$

$$-\frac{2\left(12\left(10\,Ba^3-19\,Aa^2b\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-\left(91\,Bb^3x^8+7\right.}{1729\,b^3}$$

input
```
integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/1729*(12*(10*B*a^3 - 19*A*a^2*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, wei
erstrassPInverse(0, -4*a/b, x)) - (91*B*b^3*x^8 + 7*(3*B*a*b^2 + 19*A*b^3)
*x^5 - 3*(10*B*a^2*b - 19*A*a*b^2)*x^2)*sqrt(b*x^3 + a))/b^3
```

3.190.6 Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.14

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{A\sqrt{a}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{B\sqrt{a}x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**4*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

output
```
A*sqrt(a)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(8/3)) + B*sqrt(a)*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3))
```

3.190.7 Maxima [F]

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\sqrt{bx^3+a}\,x^4\,dx$$

input
```
integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^4, x)
```

3.190.8 Giac [F]

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\sqrt{bx^3+a}\,x^4\,dx$$

input
```
integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^4, x)
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int x^4\,\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\,dx$$

input
```
int(x^4*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

output
```
int(x^4*(A + B*x^3)*(a + b*x^3)^(1/2), x)
```

3.191 $\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

3.191.1 Optimal result

Integrand size = 20, antiderivative size = 548

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$= \frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b} + \frac{6a(13Ab-4aB)\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\Bigg(\arcsin\Bigg(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\Bigg)|-7}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{2\sqrt{2}3^{3/4}a^{4/3}(13Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\Bigg(\arcsin\Bigg(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\Bigg),-}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/13*B*x^2*(b*x^3+a)^(3/2)/b+2/91*(13*A*b-4*B*a)*x^2*(b*x^3+a)^(1/2)/b+6/9
1*a*(13*A*b-4*B*a)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
+2/91*3^(3/4)*a^(4/3)*(13*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^
(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/91*3^(1/4)*a^(4/3)*(13*A*b-4*B*a)*(a^(
1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(
b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

3.191.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.14

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{x^2\sqrt{a+bx^3}\left(4B(a+bx^3)+\frac{(13Ab-4aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{26b}$$

input
```
Integrate[x*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(x^2*Sqrt[a + b*x^3]*(4*B*(a + b*x^3) + ((13*A*b - 4*a*B)*Hypergeometric2F
1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(26*b)
```

3.191.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {959, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(13Ab-4aB)\int x\sqrt{bx^3+a}dx}{13b}+\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}$$

$$\Big\downarrow \; 811$$

$$\frac{(13Ab-4aB)\left(\frac{3}{7}a\int\frac{x}{\sqrt{bx^3+a}}dx+\frac{2}{7}x^2\sqrt{a+bx^3}\right)}{13b}+\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}$$

$$\Big\downarrow \; 832$$

$$\frac{(13Ab-4aB)\left(\frac{3}{7}a\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{7}x^2\sqrt{a+bx^3}\right)}{13b}+\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}$$

$$\Big\downarrow \; 759$$

$$(13Ab-4aB)\Bigg(\frac{3}{7}a\Bigg(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\Bigg(\arcsin\Bigg(\frac{\sqrt[3]{b}}{\sqrt[3]{b}}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$13b$$

$$\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}$$

$$\Big\downarrow \; 2416$$

3.191. $\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

$$(13Ab-4aB)\left(\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.$$

$$\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}$$

input
```
Int[x*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*B*x^2*(a + b*x^3)^(3/2))/(13*b) + ((13*A*b - 4*a*B)*((2*x^2*Sqrt[a + b*
x^3])/7 + (3*a*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(
1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3])))/7))/(13*b)
```

3.191.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.191.4 Maple [A] (verified)

Time = 4.61 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.87

method	result
risch	$\frac{2x^{2}\left(7bB\,x^{3}+13Ab+3Ba\right)\sqrt{b\,x^{3}+a}}{91b}-\frac{2ia(13Ab-4Ba)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+i\sqrt{3}}}}{}$
elliptic	$\frac{2B\,x^{5}\sqrt{b\,x^{3}+a}}{13}+\frac{2\left(Ab+\frac{3Ba}{13}\right)x^{2}\sqrt{b\,x^{3}+a}}{7b}-\frac{2i\left(Aa-\frac{4a\left(Ab+\frac{3Ba}{13}\right)}{7b}\right)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{}{-3}}}{}$
default	Expression too large to display

input
```
int(x*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

3.191. $\int x\sqrt{a+bx^3}(A+Bx^3)\,dx$

output
```
2/91*x^2*(7*B*b*x^3+13*A*b+3*B*a)/b*(b*x^3+a)^(1/2)-2/91*I*a*(13*A*b-4*B*a
)/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.191.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.14

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{2\Big(3\left(4\,Ba^2-13\,Aab\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(7\,Bb^2x^5+\left(3\,Bab+\right.\right.}{91\,b^2}$$

input
```
integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/91*(3*(4*B*a^2 - 13*A*a*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstras
sPInverse(0, -4*a/b, x)) + (7*B*b^2*x^5 + (3*B*a*b + 13*A*b^2)*x^2)*sqrt(b
*x^3 + a))/b^2
```

3.191.6 Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.15

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{A\sqrt{a}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{B\sqrt{a}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

output
```
A*sqrt(a)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(5/3)) + B*sqrt(a)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3))
```

3.191.7 Maxima [F]

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int\left(Bx^3+A\right)\sqrt{bx^3+a}x\,dx$$

input
```
integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x, x)
```

3.191.8 Giac [F]

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int\left(Bx^3+A\right)\sqrt{bx^3+a}x\,dx$$

input
```
integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x, x)
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int x\,\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\,dx$$

input
```
int(x*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

output
```
int(x*(A + B*x^3)*(a + b*x^3)^(1/2), x)
```

3.192 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^2}\,dx$

3.192.1 Optimal result

Integrand size = 22, antiderivative size = 545

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^2}\,dx$$

$$= \frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a} + \frac{3(7Ab+2aB)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A\left(a+bx^3\right)^{3/2}}{ax}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\Bigg(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{\sqrt{2}3^{3/4}\sqrt[3]{a}(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Bigg(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-A*(b*x^3+a)^(3/2)/a/x+1/7*(7*A*b+2*B*a)*x^2*(b*x^3+a)^(1/2)/a+3/7*(7*A*b+
2*B*a)*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/7*3^(3/4)
*a^(1/3)*(7*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/
b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)-3/14*3^(1/4)*a^(1/3)*(7*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)
*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/
(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.192.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \frac{\sqrt{a+bx^3}\left(-2A(a+bx^3)+\frac{(7Ab+2aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{2ax}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^2,x]
```

output
```
(Sqrt[a + b*x^3]*(-2*A*(a + b*x^3) + ((7*A*b + 2*a*B)*x^3*Hypergeometric2F
1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(2*a*x)
```

3.192.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {955, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^2}dx$$

$$\Big\downarrow 955$$

$$\frac{(2aB+7Ab)\int x\sqrt{bx^3+a}dx}{2a}-\frac{A\left(a+bx^3\right)^{3/2}}{ax}$$

$$\Big\downarrow 811$$

$$\frac{(2aB+7Ab)\left(\frac{3}{7}a\int \frac{x}{\sqrt{bx^3+a}}dx+\frac{2}{7}x^2\sqrt{a+bx^3}\right)}{2a}-\frac{A\left(a+bx^3\right)^{3/2}}{ax}$$

$$\Big\downarrow 832$$

$$\frac{(2aB+7Ab)\left(\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{7}x^2\sqrt{a+bx^3}\right)}{2a}-\frac{A\left(a+bx^3\right)^{3/2}}{ax}$$

$$\Big\downarrow 759$$

$$\frac{(2aB+7Ab)\left(\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{b}x}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.}{2a}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{ax}$$

$$\Big\downarrow 2416$$

3.192. $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^2}dx$

$$(2aB+7Ab)\left(\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{ax}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^2,x]
```

output
```
-((A*(a + b*x^3)^(3/2))/(a*x)) + ((7*A*b + 2*a*B)*((2*x^2*Sqrt[a + b*x^3])
/7 + (3*a*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3)
- (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])))/7))/(2*a)
```

3.192.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.192.4 Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 469, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-2x^3B+7A)}{7x}-\frac{2i\left(\frac{3Ab}{2}+\frac{3Ba}{7}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{x}+\frac{2B\sqrt{b\,x^3+a}\,x^2}{7}-\frac{2i\left(\frac{3Ab}{2}+\frac{3Ba}{7}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+}}}{}$
default	Expression too large to display

input
```
int((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

3.192. $\int\frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^2}\,dx$

output

```
-1/7*(b*x^3+a)^(1/2)*(-2*B*x^3+7*A)/x-2/3*I*(3/2*A*b+3/7*B*a)*3^(1/2)/b*(-
a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2
)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.192.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = -\frac{3\,(2\,Ba+7\,Ab)\sqrt{b}x\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-(2\,Bbx^3-7\,Ab)\sqrt{bx^3+}}{7\,bx}$$

input

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")
```

output

```
-1/7*(3*(2*B*a + 7*A*b)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weierstrassPI
nverse(0, -4*a/b, x)) - (2*B*b*x^3 - 7*A*b)*sqrt(b*x^3 + a))/(b*x)
```

3.192.6 Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{B\sqrt{a}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**2,x)
```

output
```
A*sqrt(a)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x*gamma(2/3)) + B*sqrt(a)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3))
```

3.192.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^2}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^2, x)
```

3.192.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^2}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^2, x)
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^2}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^2,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^2, x)
```

3.193 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5} \, dx$

3.193.1 Optimal result

Integrand size = 22, antiderivative size = 546

$$
\begin{aligned}
&\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5} \, dx \\
&= -\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax} + \frac{3\sqrt[3]{b}(Ab+8aB)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A(a+bx^3)^{3/2}}{4ax^4} \\
&\quad - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\Bigg(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} \\
&\quad + \frac{3^{3/4}\sqrt[3]{b}(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\Bigg(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4}{4\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}
$$

output
```
-1/4*A*(b*x^3+a)^(3/2)/a/x^4-1/8*(A*b+8*B*a)*(b*x^3+a)^(1/2)/a/x+3/8*b^(1/
3)*(A*b+8*B*a)*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/8*3^(3/
4)*b^(1/3)*(A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)*
2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)-3/16*3^(1/4)*b^(1/3)*(A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*E
llipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I
*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.193.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx = \frac{\sqrt{a+bx^3}\left(-A(a+bx^3)-\frac{(Ab+8aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{4ax^4}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^5,x]
```

output
```
(Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) - ((A*b + 8*a*B)*x^3*Hypergeometric2F1[
-1/2, -1/3, 2/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(4*a*x^4)
```

3.193.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 809, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^5}\,dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(8aB+Ab)\int \frac{\sqrt{bx^3+a}}{x^2}dx}{8a}-\frac{A\left(a+bx^3\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{809}$$

$$\frac{(8aB+Ab)\left(\frac{3}{2}b\int \frac{x}{\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{x}\right)}{8a}-\frac{A\left(a+bx^3\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{832}$$

$$\frac{(8aB+Ab)\left(\frac{3}{2}b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)-\frac{\sqrt{a+bx^3}}{x}\right)}{8a}-\frac{A\left(a+bx^3\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{759}$$

$$\frac{(8aB+Ab)\left(\frac{3}{2}b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(}{\sqrt[3]{b}x+(}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.}{8a}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{2416}$$

3.193. $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^5}\,dx$

$$\frac{(8aB+Ab)\left(\frac{3}{2}b\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.}{8}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{4ax^4}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^5,x]
```

output
```
-1/4*(A*(a + b*x^3)^(3/2))/(a*x^4) + ((A*b + 8*a*B)*(-(Sqrt[a + b*x^3]/x)
+ (3*b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (
2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))
)/2))/(8*a)
```

3.193.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.193.4 Maple [A] (verified)

Time = 4.25 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(3Ab\,x^3+8Ba\,x^3+2Aa)}{8x^4 a}-$ $i(Ab+8Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2}{2b}}}$ $2i\left(Bb+\frac{b(3Ab+8Ba)}{16a}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}}}$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{4x^4}-\frac{(3Ab+8Ba)\sqrt{b\,x^3+a}}{8ax}-$
default	Expression too large to display

input

```
int((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

3.193. $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^5}\,dx$

output

```
-1/8*(b*x^3+a)^(1/2)*(3*A*b*x^3+8*B*a*x^3+2*A*a)/x^4/a-1/8*I*(A*b+8*B*a)/a
*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.193.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx = -\frac{3\,(8\,Ba+Ab)\sqrt{b}x^4\text{weierstrassZeta}\big(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\big(0,-\frac{4\,a}{b},x\big)\big)+((8\,Ba+3\,Ab)x^3+2\,A}{8\,ax^4}$$

input

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="fricas")
```

output

```
-1/8*(3*(8*B*a + A*b)*sqrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstrassPI
nverse(0, -4*a/b, x)) + ((8*B*a + 3*A*b)*x^3 + 2*A*a)*sqrt(b*x^3 + a))/(a*
x^4)
```

3.193.6 Sympy [A] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**5,x)
```

output
```
A*sqrt(a)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**4*gamma(-1/3)) + B*sqrt(a)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3))
```

3.193.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^5}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^5, x)
```

3.193.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^5}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^5, x)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^5}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^5,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^5, x)
```

3.194 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx$

3.194.1 Optimal result

Integrand size = 22, antiderivative size = 581

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx = \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4}$$

$$+\frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x}-\frac{3b^{4/3}(5Ab-14aB)\sqrt{a+bx^3}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{A(a+bx^3)^{3/2}}{7ax^7}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(5Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\Bigg(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3^{3/4}b^{4/3}(5Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\Bigg(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-}{56\sqrt{2}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/7*A*(b*x^3+a)^(3/2)/a/x^7+1/56*(5*A*b-14*B*a)*(b*x^3+a)^(1/2)/a/x^4+3/1
12*b*(5*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^2/x-3/112*b^(4/3)*(5*A*b-14*B*a)*(b*
x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-1/112*3^(3/4)*b^(4/3)*(5*
A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)*2^(1/2)/(b*
x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)+3/224*3^(1/4)*b^(4/3)*(5*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*Elliptic
E((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*
(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.194.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx = \frac{\sqrt{a+bx^3}\left(-4A(a+bx^3)+\frac{\left(\frac{5Ab}{2}-7aB\right)x^3\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{1}{2},-\frac{1}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{28ax^7}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^8,x]
```

output
```
(Sqrt[a + b*x^3]*(-4*A*(a + b*x^3) + (((5*A*b)/2 - 7*a*B)*x^3*Hypergeometr
ic2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(28*a*x^7)
```

3.194.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 575, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 809, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^8}\,dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(5Ab-14aB)\int \frac{\sqrt{bx^3+a}}{x^5}dx}{14a}-\frac{A\left(a+bx^3\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \ 809$$

$$-\frac{(5Ab-14aB)\left(\frac{3}{8}b\int \frac{1}{x^2\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{4x^4}\right)}{14a}-\frac{A\left(a+bx^3\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \ 847$$

$$-\frac{(5Ab-14aB)\left(\frac{3}{8}b\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)-\frac{\sqrt{a+bx^3}}{4x^4}\right)}{14a}-\frac{A\left(a+bx^3\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \ 832$$

$$-\frac{(5Ab-14aB)\left(\frac{3}{8}b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)-\frac{\sqrt{a+bx^3}}{4x^4}\right)}{14a}-\frac{A\left(a+bx^3\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \ 759$$

$$-\frac{(5Ab-14aB)\left(\frac{3}{8}b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(}{\sqrt[3]{b}x+(}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{2a}\right.\right.}{14a}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{7ax^7}$$

3.194. $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^8}\,dx$

$$\Big\downarrow 2416$$

$$-\frac{(5Ab-14aB)\left(\frac{3}{8}b\left(b\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.\right.}{}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{7ax^7}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^8,x]
```

3.194. $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^8}\,dx$

output
```
-1/7*(A*(a + b*x^3)^(3/2))/(a*x^7) - ((5*A*b - 14*a*B)*(-1/4*Sqrt[a + b*x^
3]/x^4 + (3*b*(-(Sqrt[a + b*x^3]/(a*x)) + (b*(((2*Sqrt[a + b*x^3])/(b^(1/3
)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b
^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^
(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(2*a)))/8))/(14*a)
```

3.194.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

3.194. $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^8}\,dx$

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.194.4 Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-15A\,b^{2}x^{6}+42B\,x^{6}ab+6aAb\,x^{3}+28a^{2}B\,x^{3}+16a^{2}A\right)}{112x^{7}a^{2}}+\frac{ib(5Ab-14Ba)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{ib(5Ab-14Ba)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right.}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{7x^{7}}-\frac{(3Ab+14Ba)\sqrt{b\,x^{3}+a}}{56a\,x^{4}}+\frac{3b(5Ab-14Ba)\sqrt{b\,x^{3}+a}}{112a^{2}x}+$
default	Expression too large to display

input
```
int((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

3.194. $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^8}\,dx$

output
```
-1/112*(b*x^3+a)^(1/2)*(-15*A*b^2*x^6+42*B*a*b*x^6+6*A*a*b*x^3+28*B*a^2*x^
3+16*A*a^2)/x^7/a^2+1/112*I*b*(5*A*b-14*B*a)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2)))
```

3.194.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx =$$
$$-\frac{3\,(14\,Bab-5\,Ab^2)\sqrt{b}x^7\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+(3\,(14\,Bab-5\,Ab^2}{112\,a^2x^7}$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
-1/112*(3*(14*B*a*b - 5*A*b^2)*sqrt(b)*x^7*weierstrassZeta(0, -4*a/b, weie
rstrassPInverse(0, -4*a/b, x)) + (3*(14*B*a*b - 5*A*b^2)*x^6 + 2*(14*B*a^2
 + 3*A*a*b)*x^3 + 16*A*a^2)*sqrt(b*x^3 + a))/(a^2*x^7)
```

3.194.6 Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**8,x)
```

output
```
A*sqrt(a)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**7*gamma(-4/3)) + B*sqrt(a)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3))
```

3.194.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^8}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^8, x)
```

3.194.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^8}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^8, x)
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx=\int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^8}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^8,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^8, x)
```

3.195 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx$

3.195.1 Optimal result

Integrand size = 22, antiderivative size = 614

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7}+\frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4}$$

$$-\frac{3b^2(11Ab-20aB)\sqrt{a+bx^3}}{448a^3x}+\frac{3b^{7/3}(11Ab-20aB)\sqrt{a+bx^3}}{448a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{A(a+bx^3)^{3/2}}{10ax^{10}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}(11Ab-20aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-\right.}{896a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3^{3/4}b^{7/3}(11Ab-20aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7\right.}{224\sqrt{2}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/10*A*(b*x^3+a)^(3/2)/a/x^10+1/140*(11*A*b-20*B*a)*(b*x^3+a)^(1/2)/a/x^7
+3/1120*b*(11*A*b-20*B*a)*(b*x^3+a)^(1/2)/a^2/x^4-3/448*b^2*(11*A*b-20*B*a
)*(b*x^3+a)^(1/2)/a^3/x+3/448*b^(7/3)*(11*A*b-20*B*a)*(b*x^3+a)^(1/2)/a^3/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/448*3^(3/4)*b^(7/3)*(11*A*b-20*B*a)*(a^(
1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(8/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^
(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/896*3
^(1/4)*b^(7/3)*(11*A*b-20*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^
(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(
1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)/a^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.195.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \frac{\sqrt{a+bx^3}\left(-7A(a+bx^3)+\frac{\left(\frac{11Ab}{2}-10aB\right)x^3\,\mathrm{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{1}{2},-\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{70ax^{10}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^11,x]
```

output
```
(Sqrt[a + b*x^3]*(-7*A*(a + b*x^3) + (((11*A*b)/2 - 10*a*B)*x^3*Hypergeome
tric2F1[-7/3, -1/2, -4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(70*a*x^10)
```

3.195.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 605, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {955, 809, 847, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11}}\,dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(11Ab-20aB)\int \frac{\sqrt{bx^3+a}}{x^8}dx}{20a}-\frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}}$$

$$\Big\downarrow \ 809$$

$$-\frac{(11Ab-20aB)\left(\frac{3}{14}b\int \frac{1}{x^5\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{7x^7}\right)}{20a}-\frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}}$$

$$\Big\downarrow \ 847$$

$$-\frac{(11Ab-20aB)\left(\frac{3}{14}b\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+a}}dx}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)-\frac{\sqrt{a+bx^3}}{7x^7}\right)}{20a}-\frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}}$$

$$\Big\downarrow \ 847$$

$$-\frac{(11Ab-20aB)\left(\frac{3}{14}b\left(-\frac{5b\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)-\frac{\sqrt{a+bx^3}}{7x^7}\right)}{20a}-\frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}}$$

$$\Big\downarrow \ 832$$

$$-\frac{(11Ab-20aB)\left(\frac{3}{14}b\left(-\frac{5b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)-\frac{\sqrt{a+bx^3}}{7x^7}\right)}{20a}-\frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}}$$

$$\Big\downarrow \ 759$$

3.195. $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11}}\,dx$

$$-\frac{(11Ab-20aB)\left(\frac{3}{14}b\left(-\frac{5b\left(\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.}{2a}\right.\right.}{8a}\right.\right.}{20a}$$

$$\frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}}$$

$$\Big\downarrow 2416$$

3.195. $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11}}\,dx$

$$-\frac{(11Ab-20aB)\left(\frac{3}{14}b\left(-\frac{5b\left(\frac{b\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{b}x+}{\sqrt[3]{b}x+}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}\right.}{}\right.\right.}{}$$

3.195. $\int\frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11}}\,dx$ $\qquad\frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}}$

input

```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^11,x]
```

output

```
-1/10*(A*(a + b*x^3)^(3/2))/(a*x^10) - ((11*A*b - 20*a*B)*(-1/7*Sqrt[a + b
*x^3]/x^7 + (3*b*(-1/4*Sqrt[a + b*x^3]/(a*x^4) - (5*b*(-(Sqrt[a + b*x^3]/(
a*x)) + (b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3)
 - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])))/(2*a)))/(8*a)))/14))/(20*a)
```

3.195.3.1 Defintions of rubi rules used

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.195.4 Maple [A] (verified)

Time = 4.49 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.86

method	result
risch	$ib^2(11Ab-20Ba)\sqrt{3}\,\left(-a\,b^2\right)$ $-\frac{\sqrt{b\,x^3+a}\,\left(165A\,x^9b^3-300B\,x^9a\,b^2-66A\,x^6a\,b^2+120B\,x^6a^2b+48A\,x^3a^2b+320a^3B\,x^3+224a^3A\right)}{2240x^{10}a^3}-$ $ib^2(11Ab-20Ba)\sqrt{3}\,\left(-a\,b\right.$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{10x^{10}}-\frac{(3Ab+20Ba)\sqrt{b\,x^3+a}}{140a\,x^7}+\frac{3b(11Ab-20Ba)\sqrt{b\,x^3+a}}{1120a^2x^4}-\frac{3b^2(11Ab-20Ba)\sqrt{b\,x^3+a}}{448a^3x}-$
default	Expression too large to display

input
```
int((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

3.195. $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^{11}}\,dx$

output
```
-1/2240*(b*x^3+a)^(1/2)*(165*A*b^3*x^9-300*B*a*b^2*x^9-66*A*a*b^2*x^6+120*
B*a^2*b*x^6+48*A*a^2*b*x^3+320*B*a^3*x^3+224*A*a^3)/x^10/a^3-1/448*I*b^2*(
11*A*b-20*B*a)/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2
)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(
-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.195.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx$$
$$= \frac{15\,(20\,Bab^2-11\,Ab^3)\sqrt{b}x^{10}\text{weierstrassZeta}\big(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\big(0,-\frac{4\,a}{b},x\big)\big)+(15\,(20\,Bab^2-11}{2240\,a^3x^{10}}$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="fricas")
```

output
```
1/2240*(15*(20*B*a*b^2 - 11*A*b^3)*sqrt(b)*x^10*weierstrassZeta(0, -4*a/b,
 weierstrassPInverse(0, -4*a/b, x)) + (15*(20*B*a*b^2 - 11*A*b^3)*x^9 - 6*
(20*B*a^2*b - 11*A*a*b^2)*x^6 - 224*A*a^3 - 16*(20*B*a^3 + 3*A*a^2*b)*x^3)
*sqrt(b*x^3 + a))/(a^3*x^10)
```

3.195.6 Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{10}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{10}{3}, -\frac{1}{2} \\ -\frac{7}{3} \end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{7}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{7}{3}, -\frac{1}{2} \\ -\frac{4}{3} \end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**11,x)
```

output
```
A*sqrt(a)*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*x**10*gamma(-7/3)) + B*sqrt(a)*gamma(-7/3)*hyper((-7/3, -1/2), (-4
/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3))
```

3.195.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{11}}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^11, x)
```

3.195.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{11}}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^11, x)
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx=\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{x^{11}} \,d x$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^11,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^11, x)
```

3.196 $\int x^8(a+bx^3)^{3/2}(A+Bx^3)\,dx$

3.196.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int x^8(a+bx^3)^{3/2}(A+Bx^3)\,dx=\frac{2a^2(Ab-aB)(a+bx^3)^{5/2}}{15b^4}-\frac{2a(2Ab-3aB)(a+bx^3)^{7/2}}{21b^4}+\frac{2(Ab-3aB)(a+bx^3)^{9/2}}{27b^4}+\frac{2B(a+bx^3)^{11/2}}{33b^4}$$

output
```
2/15*a^2*(A*b-B*a)*(b*x^3+a)^(5/2)/b^4-2/21*a*(2*A*b-3*B*a)*(b*x^3+a)^(7/2
)/b^4+2/27*(A*b-3*B*a)*(b*x^3+a)^(9/2)/b^4+2/33*B*(b*x^3+a)^(11/2)/b^4
```

3.196.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int x^8(a+bx^3)^{3/2}(A+Bx^3)\,dx=\frac{2(a+bx^3)^{5/2}(88a^2Ab-48a^3B-220aAb^2x^3+120a^2bBx^3+385Ab^3x^6-210ab^2Bx^6+315b^3}{10395b^4}$$

input
```
Integrate[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*(a + b*x^3)^(5/2)*(88*a^2*A*b - 48*a^3*B - 220*a*A*b^2*x^3 + 120*a^2*b*
B*x^3 + 385*A*b^3*x^6 - 210*a*b^2*B*x^6 + 315*b^3*B*x^9))/(10395*b^4)
```

3.196.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int x^6 \left(bx^3+a\right)^{3/2} \left(Bx^3+A\right)dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{B\left(bx^3+a\right)^{9/2}}{b^3}+\frac{(Ab-3aB)\left(bx^3+a\right)^{7/2}}{b^3}+\frac{a(3aB-2Ab)\left(bx^3+a\right)^{5/2}}{b^3}-\frac{a^2(aB-Ab)\left(bx^3+a\right)^{3/2}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2a^2\left(a+bx^3\right)^{5/2}(Ab-aB)}{5b^4}+\frac{2\left(a+bx^3\right)^{9/2}(Ab-3aB)}{9b^4}-\frac{2a\left(a+bx^3\right)^{7/2}(2Ab-3aB)}{7b^4}+\frac{2B\left(a+bx^3\right)^{11/2}}{11b^4}\right)$$

input
```
Int[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
((2*a^2*(A*b - a*B)*(a + b*x^3)^(5/2))/(5*b^4) - (2*a*(2*A*b - 3*a*B)*(a +
 b*x^3)^(7/2))/(7*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(9/2))/(9*b^4) + (2*
B*(a + b*x^3)^(11/2))/(11*b^4))/3
```

3.196.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.196.4 Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{16\left(b\,x^{3}+a\right)^{\frac{5}{2}}\left(\frac{35x^{6}\left(\frac{9x^{3}B}{11}+A\right)b^{3}}{8}-\frac{5x^{3}\left(\frac{21x^{3}B}{22}+A\right)a\,b^{2}}{2}+a^{2}\left(\frac{15x^{3}B}{11}+A\right)b-\frac{6a^{3}B}{11}\right)}{945b^{4}}$
gosper	$\frac{2\left(b\,x^{3}+a\right)^{\frac{5}{2}}\left(315b^{3}B\,x^{9}+385x^{6}b^{3}A-210B\,x^{6}a\,b^{2}-220aA\,b^{2}x^{3}+120B\,a^{2}b\,x^{3}+88a^{2}bA-48a^{3}B\right)}{10395b^{4}}$
trager	$\frac{2(315b^{5}B\,x^{15}+385b^{5}A\,x^{12}+420a\,b^{4}B\,x^{12}+550a\,b^{4}A\,x^{9}+15a^{2}b^{3}B\,x^{9}+33a^{2}A\,b^{3}x^{6}-18a^{3}b^{2}B\,x^{6}-44a^{3}A\,b^{2}x^{3}+24B\,a^{4}b\,x^{3}+}{10395b^{4}}$
risch	$\frac{2(315b^{5}B\,x^{15}+385b^{5}A\,x^{12}+420a\,b^{4}B\,x^{12}+550a\,b^{4}A\,x^{9}+15a^{2}b^{3}B\,x^{9}+33a^{2}A\,b^{3}x^{6}-18a^{3}b^{2}B\,x^{6}-44a^{3}A\,b^{2}x^{3}+24B\,a^{4}b\,x^{3}+}{10395b^{4}}$
default	$A\left(\frac{2b\,x^{12}\sqrt{b\,x^{3}+a}}{27}+\frac{20a\,x^{9}\sqrt{b\,x^{3}+a}}{189}+\frac{2a^{2}x^{6}\sqrt{b\,x^{3}+a}}{315b}-\frac{8a^{3}x^{3}\sqrt{b\,x^{3}+a}}{945b^{2}}+\frac{16a^{4}\sqrt{b\,x^{3}+a}}{945b^{3}}\right)+B\left(\frac{2b\,x^{15}\sqrt{b\,x^{3}+a}}{33}\right.$
elliptic	$\frac{2Bb\,x^{15}\sqrt{b\,x^{3}+a}}{33}+\frac{2\left(b^{2}A+\frac{12}{11}abB\right)x^{12}\sqrt{b\,x^{3}+a}}{27b}+\frac{2\left(2abA+a^{2}B-\frac{8a\left(b^{2}A+\frac{12}{11}abB\right)}{9b}\right)x^{9}\sqrt{b\,x^{3}+a}}{21b}+\frac{2\left(a^{2}A-\frac{6a\left(2abA\right.}{}\right.}{}$

input
```
int(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
16/945*(b*x^3+a)^(5/2)*(35/8*x^6*(9/11*x^3*B+A)*b^3-5/2*x^3*(21/22*x^3*B+A
)*a*b^2+a^2*(15/11*x^3*B+A)*b-6/11*a^3*B)/b^4
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.20

$$\int x^8\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\,(315\,Bb^5x^{15}+35\,(12\,Bab^4+11\,Ab^5)x^{12}+5\,(3\,Ba^2b^3+110\,Aab^4)x^9-3\,(6\,Ba^3b^2-11\,Aa^2b^3}{10395\,b^4}$$

input
```
integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/10395*(315*B*b^5*x^15 + 35*(12*B*a*b^4 + 11*A*b^5)*x^12 + 5*(3*B*a^2*b^3
 + 110*A*a*b^4)*x^9 - 3*(6*B*a^3*b^2 - 11*A*a^2*b^3)*x^6 - 48*B*a^5 + 88*A
*a^4*b + 4*(6*B*a^4*b - 11*A*a^3*b^2)*x^3)*sqrt(b*x^3 + a)/b^4
```

3.196.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(100) = 200.

Time = 0.58 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.59

$$\int x^8\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\begin{cases}\frac{16Aa^4\sqrt{a+bx^3}}{945b^3}-\frac{8Aa^3x^3\sqrt{a+bx^3}}{945b^2}+\frac{2Aa^2x^6\sqrt{a+bx^3}}{315b}+\frac{20Aax^9\sqrt{a+bx^3}}{189}+\frac{2Abx^{12}\sqrt{a+bx^3}}{27}-\frac{32Ba^5\sqrt{a+bx^3}}{3465b^4}+ \\ a^{\frac{3}{2}}\left(\frac{Ax^9}{9}+\frac{Bx^{12}}{12}\right)\end{cases}$$

input
```
integrate(x**8*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
Piecewise((16*A*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*A*a**3*x**3*sqrt(a +
b*x**3)/(945*b**2) + 2*A*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*A*a*x**9*
sqrt(a + b*x**3)/189 + 2*A*b*x**12*sqrt(a + b*x**3)/27 - 32*B*a**5*sqrt(a
+ b*x**3)/(3465*b**4) + 16*B*a**4*x**3*sqrt(a + b*x**3)/(3465*b**3) - 4*B*
a**3*x**6*sqrt(a + b*x**3)/(1155*b**2) + 2*B*a**2*x**9*sqrt(a + b*x**3)/(6
93*b) + 8*B*a*x**12*sqrt(a + b*x**3)/99 + 2*B*b*x**15*sqrt(a + b*x**3)/33,
 Ne(b, 0)), (a**(3/2)*(A*x**9/9 + B*x**12/12), True))
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int x^8(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2}{945}\left(\frac{35\left(bx^3+a\right)^{\frac{9}{2}}}{b^3}-\frac{90\left(bx^3+a\right)^{\frac{7}{2}}a}{b^3}+\frac{63\left(bx^3+a\right)^{\frac{5}{2}}a^2}{b^3}\right)A+\frac{2}{3465}\left(\frac{105\left(bx^3+a\right)^{\frac{11}{2}}}{b^4}-\frac{385\left(bx^3+a\right)^{\frac{9}{2}}a}{b^4}+\frac{495\left(bx^3+a\right)^{\frac{7}{2}}a^2}{b^4}-\frac{231\left(bx^3+a\right)^{\frac{5}{2}}a^3}{b^4}\right)B$$

input
```
integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/945*(35*(b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 +
 a)^(5/2)*a^2/b^3)*A + 2/3465*(105*(b*x^3 + a)^(11/2)/b^4 - 385*(b*x^3 + a
)^(9/2)*a/b^4 + 495*(b*x^3 + a)^(7/2)*a^2/b^4 - 231*(b*x^3 + a)^(5/2)*a^3/
b^4)*B
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int x^8\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\left(315\left(bx^3+a\right)^{\frac{11}{2}}B-1155\left(bx^3+a\right)^{\frac{9}{2}}Ba+1485\left(bx^3+a\right)^{\frac{7}{2}}Ba^2-693\left(bx^3+a\right)^{\frac{5}{2}}Ba^3+385\left(\right.\right.}{10395\,b^4}$$

input
```
integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
2/10395*(315*(b*x^3 + a)^(11/2)*B - 1155*(b*x^3 + a)^(9/2)*B*a + 1485*(b*x
^3 + a)^(7/2)*B*a^2 - 693*(b*x^3 + a)^(5/2)*B*a^3 + 385*(b*x^3 + a)^(9/2)*
A*b - 990*(b*x^3 + a)^(7/2)*A*a*b + 693*(b*x^3 + a)^(5/2)*A*a^2*b)/b^4
```

3.196.9 Mupad [B] (verification not implemented)

Time = 6.99 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.00

$$\int x^8 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx = \frac{20\,A\,a\,x^9\,\sqrt{b\,x^3+a}}{189}+\frac{2\,A\,b\,x^{12}\,\sqrt{b\,x^3+a}}{27} + \frac{8\,B\,a\,x^{12}\,\sqrt{b\,x^3+a}}{99}+\frac{2\,B\,b\,x^{15}\,\sqrt{b\,x^3+a}}{33}+\frac{16\,A\,a^4\,\sqrt{b\,x^3+a}}{945\,b^3} - \frac{32\,B\,a^5\,\sqrt{b\,x^3+a}}{3465\,b^4}-\frac{8\,A\,a^3\,x^3\,\sqrt{b\,x^3+a}}{945\,b^2}+\frac{2\,A\,a^2\,x^6\,\sqrt{b\,x^3+a}}{315\,b} + \frac{16\,B\,a^4\,x^3\,\sqrt{b\,x^3+a}}{3465\,b^3}-\frac{4\,B\,a^3\,x^6\,\sqrt{b\,x^3+a}}{1155\,b^2}+\frac{2\,B\,a^2\,x^9\,\sqrt{b\,x^3+a}}{693\,b}$$

input
```
int(x^8*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

output
```
(20*A*a*x^9*(a + b*x^3)^(1/2))/189 + (2*A*b*x^12*(a + b*x^3)^(1/2))/27 + (
8*B*a*x^12*(a + b*x^3)^(1/2))/99 + (2*B*b*x^15*(a + b*x^3)^(1/2))/33 + (16
*A*a^4*(a + b*x^3)^(1/2))/(945*b^3) - (32*B*a^5*(a + b*x^3)^(1/2))/(3465*b
^4) - (8*A*a^3*x^3*(a + b*x^3)^(1/2))/(945*b^2) + (2*A*a^2*x^6*(a + b*x^3)
^(1/2))/(315*b) + (16*B*a^4*x^3*(a + b*x^3)^(1/2))/(3465*b^3) - (4*B*a^3*x
^6*(a + b*x^3)^(1/2))/(1155*b^2) + (2*B*a^2*x^9*(a + b*x^3)^(1/2))/(693*b)
```

3.197 $\int x^5(a+bx^3)^{3/2}(A+Bx^3)\,dx$

3.197.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int x^5\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=-\frac{2a(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{7/2}}{21b^3}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^3}$$

output
```
-2/15*a*(A*b-B*a)*(b*x^3+a)^(5/2)/b^3+2/21*(A*b-2*B*a)*(b*x^3+a)^(7/2)/b^3
+2/27*B*(b*x^3+a)^(9/2)/b^3
```

3.197.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int x^5\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\left(a+bx^3\right)^{5/2}\left(-18aAb+8a^2B+45Ab^2x^3-20abBx^3+35b^2Bx^6\right)}{945b^3}$$

input
```
Integrate[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*(a + b*x^3)^(5/2)*(-18*a*A*b + 8*a^2*B + 45*A*b^2*x^3 - 20*a*b*B*x^3 +
35*b^2*B*x^6))/(945*b^3)
```

3.197.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (a+bx^3)^{3/2} (A+Bx^3) \, dx$$

$$\Big\downarrow \, 948$$

$$\frac{1}{3}\int x^3 (bx^3+a)^{3/2} (Bx^3+A) \, dx^3$$

$$\Big\downarrow \, 86$$

$$\frac{1}{3}\int \left(\frac{B(bx^3+a)^{7/2}}{b^2}+\frac{(Ab-2aB)(bx^3+a)^{5/2}}{b^2}+\frac{a(aB-Ab)(bx^3+a)^{3/2}}{b^2}\right)dx^3$$

$$\Big\downarrow \, 2009$$

$$\frac{1}{3}\left(\frac{2(a+bx^3)^{7/2}(Ab-2aB)}{7b^3}-\frac{2a(a+bx^3)^{5/2}(Ab-aB)}{5b^3}+\frac{2B(a+bx^3)^{9/2}}{9b^3}\right)$$

input
```
Int[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
((-2*a*(A*b - a*B)*(a + b*x^3)^(5/2))/(5*b^3) + (2*(A*b - 2*a*B)*(a + b*x^
3)^(7/2))/(7*b^3) + (2*B*(a + b*x^3)^(9/2))/(9*b^3))/3
```

3.197.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.197.4 Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-\frac{4\left(b\,x^{3}+a\right)^{\frac{5}{2}}\left(-\frac{5x^{3}\left(\frac{7x^{3}B}{9}+A\right)b^{2}}{2}+a\left(\frac{10x^{3}B}{9}+A\right)b-\frac{4a^{2}B}{9}\right)}{105b^{3}}$
gosper	$-\frac{2\left(b\,x^{3}+a\right)^{\frac{5}{2}}\left(-35b^{2}B\,x^{6}-45A\,b^{2}x^{3}+20Bab\,x^{3}+18abA-8a^{2}B\right)}{945b^{3}}$
trager	$-\frac{2\left(-35B\,b^{4}x^{12}-45A\,b^{4}x^{9}-50Ba\,b^{3}x^{9}-72Aa\,b^{3}x^{6}-3B\,a^{2}b^{2}x^{6}-9A\,a^{2}b^{2}x^{3}+4B\,a^{3}b\,x^{3}+18A\,a^{3}b-8B\,a^{4}\right)\sqrt{b\,x^{3}+a}}{945b^{3}}$
risch	$-\frac{2\left(-35B\,b^{4}x^{12}-45A\,b^{4}x^{9}-50Ba\,b^{3}x^{9}-72Aa\,b^{3}x^{6}-3B\,a^{2}b^{2}x^{6}-9A\,a^{2}b^{2}x^{3}+4B\,a^{3}b\,x^{3}+18A\,a^{3}b-8B\,a^{4}\right)\sqrt{b\,x^{3}+a}}{945b^{3}}$
default	$B\left(\frac{2b\,x^{12}\sqrt{b\,x^{3}+a}}{27}+\frac{20a\,x^{9}\sqrt{b\,x^{3}+a}}{189}+\frac{2a^{2}x^{6}\sqrt{b\,x^{3}+a}}{315b}-\frac{8a^{3}x^{3}\sqrt{b\,x^{3}+a}}{945b^{2}}+\frac{16a^{4}\sqrt{b\,x^{3}+a}}{945b^{3}}\right)+A\left(\frac{2b\,x^{9}\sqrt{b\,x^{3}+a}}{21}\right.$
elliptic	$\frac{2Bb\,x^{12}\sqrt{b\,x^{3}+a}}{27}+\frac{2\left(b^{2}A+\frac{10}{9}abB\right)x^{9}\sqrt{b\,x^{3}+a}}{21b}+\frac{2\left(2abA+a^{2}B-\frac{6a\left(b^{2}A+\frac{10}{9}abB\right)}{7b}\right)x^{6}\sqrt{b\,x^{3}+a}}{15b}+2\left(a^{2}A-\frac{4a\left(2abA+\right.}{}\right.$

input
```
int(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
-4/105*(b*x^3+a)^(5/2)*(-5/2*x^3*(7/9*x^3*B+A)*b^2+a*(10/9*x^3*B+A)*b-4/9*
a^2*B)/b^3
```

3.197.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int x^5\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\,(35\,Bb^4x^{12}+5\,(10\,Bab^3+9\,Ab^4)x^9+3\,(Ba^2b^2+24\,Aab^3)x^6+8\,Ba^4-18\,Aa^3b-(4\,Ba^3b-}{945\,b^3}$$

input
```
integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/945*(35*B*b^4*x^12 + 5*(10*B*a*b^3 + 9*A*b^4)*x^9 + 3*(B*a^2*b^2 + 24*A*
a*b^3)*x^6 + 8*B*a^4 - 18*A*a^3*b - (4*B*a^3*b - 9*A*a^2*b^2)*x^3)*sqrt(b*
x^3 + a)/b^3
```

3.197.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(70) = 140.

Time = 0.42 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.96

$$\int x^5\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\begin{cases}-\frac{4Aa^3\sqrt{a+bx^3}}{105b^2}+\frac{2Aa^2x^3\sqrt{a+bx^3}}{105b}+\frac{16Aax^6\sqrt{a+bx^3}}{105}+\frac{2Abx^9\sqrt{a+bx^3}}{21}+\frac{16Ba^4\sqrt{a+bx^3}}{945b^3}-\frac{8Ba^3x^3\sqrt{a+bx^3}}{945b^2}+ \\ a^{\frac{3}{2}}\left(\frac{Ax^6}{6}+\frac{Bx^9}{9}\right)\end{cases}$$

input
```
integrate(x**5*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
Piecewise((-4*A*a**3*sqrt(a + b*x**3)/(105*b**2) + 2*A*a**2*x**3*sqrt(a +
b*x**3)/(105*b) + 16*A*a*x**6*sqrt(a + b*x**3)/105 + 2*A*b*x**9*sqrt(a + b
*x**3)/21 + 16*B*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*B*a**3*x**3*sqrt(a +
 b*x**3)/(945*b**2) + 2*B*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*B*a*x**9
*sqrt(a + b*x**3)/189 + 2*B*b*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (a**(3
/2)*(A*x**6/6 + B*x**9/9), True))
```

3.197.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int x^5\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2}{105}\left(\frac{5\left(bx^3+a\right)^{\frac{7}{2}}}{b^2}-\frac{7\left(bx^3+a\right)^{\frac{5}{2}}a}{b^2}\right)A+\frac{2}{945}\left(\frac{35\left(bx^3+a\right)^{\frac{9}{2}}}{b^3}-\frac{90\left(bx^3+a\right)^{\frac{7}{2}}a}{b^3}+\frac{63\left(bx^3+a\right)^{\frac{5}{2}}a^2}{b^3}\right)B$$

input

```
integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output

```
2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a)^(5/2)*a/b^2)*A + 2/945*(35*
(b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 + a)^(5/2)*
a^2/b^3)*B
```

3.197.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^5\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\left(35\left(bx^3+a\right)^{\frac{9}{2}}B-90\left(bx^3+a\right)^{\frac{7}{2}}Ba+63\left(bx^3+a\right)^{\frac{5}{2}}Ba^2+45\left(bx^3+a\right)^{\frac{7}{2}}Ab-63\left(bx^3+a\right)^{\frac{5}{2}}A}{945\,b^3}$$

input

```
integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output

```
2/945*(35*(b*x^3 + a)^(9/2)*B - 90*(b*x^3 + a)^(7/2)*B*a + 63*(b*x^3 + a)^
(5/2)*B*a^2 + 45*(b*x^3 + a)^(7/2)*A*b - 63*(b*x^3 + a)^(5/2)*A*a*b)/b^3
```

3.197.9 Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.89

$$\int x^5 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx = \frac{x^6\,\sqrt{b\,x^3+a}\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\,\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{15\,b} - \frac{2\,a\,\left(2\,A\,a^2-\frac{4\,a\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\,\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{5\,b}\right)\,\sqrt{b\,x^3+a}}{9\,b^2} + \frac{2\,B\,b\,x^{12}\,\sqrt{b\,x^3+a}}{27} + \frac{x^3\,\left(2\,A\,a^2-\frac{4\,a\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\,\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{5\,b}\right)\,\sqrt{b\,x^3+a}}{9\,b} + \frac{x^9\,\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)\,\sqrt{b\,x^3+a}}{21\,b}$$

input
```
int(x^5*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

output
```
(x^6*(a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2 + (20*B*a*b)/9))
/(7*b)))/(15*b) - (2*a*(2*A*a^2 - (4*a*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2
+ (20*B*a*b)/9))/(7*b)))/(5*b))*(a + b*x^3)^(1/2))/(9*b^2) + (2*B*b*x^12*(
a + b*x^3)^(1/2))/27 + (x^3*(2*A*a^2 - (4*a*(2*B*a^2 + 4*A*a*b - (6*a*(2*A
*b^2 + (20*B*a*b)/9))/(7*b)))/(5*b))*(a + b*x^3)^(1/2))/(9*b) + (x^9*(2*A*
b^2 + (20*B*a*b)/9)*(a + b*x^3)^(1/2))/(21*b)
```

3.198 $\int x^2(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx$

3.198.1 Optimal result

Integrand size = 22, antiderivative size = 46

$$\int x^2\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^2}$$

output
```
2/15*(A*b-B*a)*(b*x^3+a)^(5/2)/b^2+2/21*B*(b*x^3+a)^(7/2)/b^2
```

3.198.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int x^2\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\left(a+bx^3\right)^{5/2}\left(7Ab-2aB+5bBx^3\right)}{105b^2}$$

input
```
Integrate[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*(a + b*x^3)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^3))/(105*b^2)
```

3.198.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \left(bx^3+a\right)^{3/2}\left(Bx^3+A\right)dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{B\left(bx^3+a\right)^{5/2}}{b}+\frac{(Ab-aB)\left(bx^3+a\right)^{3/2}}{b}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2\left(a+bx^3\right)^{5/2}(Ab-aB)}{5b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{7b^2}\right)$$

input
```
Int[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
((2*(A*b - a*B)*(a + b*x^3)^(5/2))/(5*b^2) + (2*B*(a + b*x^3)^(7/2))/(7*b^
2))/3
```

3.198.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.198.4 Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result
gosper	$\frac{2\left(b\,x^{3}+a\right)^{\frac{5}{2}}\left(5bB\,x^{3}+7Ab-2Ba\right)}{105b^{2}}$
pseudoelliptic	$\frac{2\left(\left(5x^{3}B+7A\right)b-2Ba\right)\left(b\,x^{3}+a\right)^{\frac{5}{2}}}{105b^{2}}$
trager	$\frac{2\left(5b^{3}B\,x^{9}+7x^{6}b^{3}A+8B\,x^{6}a\,b^{2}+14aA\,b^{2}x^{3}+B\,a^{2}b\,x^{3}+7a^{2}bA-2a^{3}B\right)\sqrt{b\,x^{3}+a}}{105b^{2}}$
risch	$\frac{2\left(5b^{3}B\,x^{9}+7x^{6}b^{3}A+8B\,x^{6}a\,b^{2}+14aA\,b^{2}x^{3}+B\,a^{2}b\,x^{3}+7a^{2}bA-2a^{3}B\right)\sqrt{b\,x^{3}+a}}{105b^{2}}$
default	$B\left(\frac{2b\,x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{16a\,x^{6}\sqrt{b\,x^{3}+a}}{105}+\frac{2a^{2}x^{3}\sqrt{b\,x^{3}+a}}{105b}-\frac{4a^{3}\sqrt{b\,x^{3}+a}}{105b^{2}}\right)+\frac{2A\left(b\,x^{3}+a\right)^{\frac{5}{2}}}{15b}$
elliptic	$\frac{2Bb\,x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{2\left(b^{2}A+\frac{8}{7}abB\right)x^{6}\sqrt{b\,x^{3}+a}}{15b}+\frac{2\left(2abA+a^{2}B-\frac{4a\left(b^{2}A+\frac{8}{7}abB\right)}{5b}\right)x^{3}\sqrt{b\,x^{3}+a}}{9b}+\frac{2\left(a^{2}A-\frac{2a\left(2abA+a^{2}\right.}{}\right.}{}$

input
```
int(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
2/105*(b*x^3+a)^(5/2)*(5*B*b*x^3+7*A*b-2*B*a)/b^2
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int x^2\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\,(5\,Bb^3x^9+(8\,Bab^2+7\,Ab^3)x^6-2\,Ba^3+7\,Aa^2b+(Ba^2b+14\,Aab^2)x^3)\sqrt{bx^3+a}}{105\,b^2}$$

input
```
integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/105*(5*B*b^3*x^9 + (8*B*a*b^2 + 7*A*b^3)*x^6 - 2*B*a^3 + 7*A*a^2*b + (B*
a^2*b + 14*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^2
```

3.198.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(44) = 88.

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.59

$$\int x^2\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\begin{cases}\frac{2Aa^2\sqrt{a+bx^3}}{15b}+\frac{4Aax^3\sqrt{a+bx^3}}{15}+\frac{2Abx^6\sqrt{a+bx^3}}{15}-\frac{4Ba^3\sqrt{a+bx^3}}{105b^2}+\frac{2Ba^2x^3\sqrt{a+bx^3}}{105b}+\frac{16Bax^6\sqrt{a+bx^3}}{105}+\frac{2Bbx^9\sqrt{}}{21} \\ a^{\frac{3}{2}}\left(\frac{Ax^3}{3}+\frac{Bx^6}{6}\right)\end{cases}$$

input
```
integrate(x**2*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
Piecewise((2*A*a**2*sqrt(a + b*x**3)/(15*b) + 4*A*a*x**3*sqrt(a + b*x**3)/
15 + 2*A*b*x**6*sqrt(a + b*x**3)/15 - 4*B*a**3*sqrt(a + b*x**3)/(105*b**2)
 + 2*B*a**2*x**3*sqrt(a + b*x**3)/(105*b) + 16*B*a*x**6*sqrt(a + b*x**3)/1
05 + 2*B*b*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (a**(3/2)*(A*x**3/3 + B*x*
*6/6), True))
```

3.198.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int x^2 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx = \frac{2\,(bx^3+a)^{\frac{5}{2}}A}{15\,b} + \frac{2}{105}\left(\frac{5\,(bx^3+a)^{\frac{7}{2}}}{b^2} - \frac{7\,(bx^3+a)^{\frac{5}{2}}a}{b^2}\right)B$$

input
```
integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
2/15*(b*x^3 + a)^(5/2)*A/b + 2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a
)^(5/2)*a/b^2)*B
```

3.198.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x^2 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx = \frac{2\left(5\,(bx^3+a)^{\frac{7}{2}}B - 7\,(bx^3+a)^{\frac{5}{2}}Ba + 7\,(bx^3+a)^{\frac{5}{2}}Ab\right)}{105\,b^2}$$

input
```
integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
2/105*(5*(b*x^3 + a)^(7/2)*B - 7*(b*x^3 + a)^(5/2)*B*a + 7*(b*x^3 + a)^(5/
2)*A*b)/b^2
```

3.198.9 Mupad [B] (verification not implemented)

Time = 6.91 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.26

$$\int x^2 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx = \frac{\left(2\,A\,a^2 - \frac{2\,a\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{4\,a\,\left(2\,A\,b^2+\frac{16\,B\,a\,b}{7}\right)}{5\,b}\right)}{3\,b}\right)\sqrt{b\,x^3+a}}{3\,b} + \frac{x^3\,\sqrt{b\,x^3+a}\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{4\,a\,\left(2\,A\,b^2+\frac{16\,B\,a\,b}{7}\right)}{5\,b}\right)}{9\,b} + \frac{2\,B\,b\,x^9\,\sqrt{b\,x^3+a}}{21} + \frac{x^6\,\left(2\,A\,b^2+\frac{16\,B\,a\,b}{7}\right)\,\sqrt{b\,x^3+a}}{15\,b}$$

input
```
int(x^2*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

output
```
((2*A*a^2 - (2*a*(2*B*a^2 + 4*A*a*b - (4*a*(2*A*b^2 + (16*B*a*b)/7))/(5*b)
))/(3*b))*(a + b*x^3)^(1/2))/(3*b) + (x^3*(a + b*x^3)^(1/2)*(2*B*a^2 + 4*A
*a*b - (4*a*(2*A*b^2 + (16*B*a*b)/7))/(5*b)))/(9*b) + (2*B*b*x^9*(a + b*x^
3)^(1/2))/21 + (x^6*(2*A*b^2 + (16*B*a*b)/7)*(a + b*x^3)^(1/2))/(15*b)
```

3.198. $\int x^2(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx$

3.199 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx$

3.199.1 Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \frac{2}{3}aA\sqrt{a+bx^3} + \frac{2}{9}A(a+bx^3)^{3/2} + \frac{2B(a+bx^3)^{5/2}}{15b} - \frac{2}{3}a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

output
```
2/9*A*(b*x^3+a)^(3/2)+2/15*B*(b*x^3+a)^(5/2)/b-2/3*a^(3/2)*A*arctanh((b*x^
3+a)^(1/2)/a^(1/2))+2/3*a*A*(b*x^3+a)^(1/2)
```

3.199.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \frac{2\sqrt{a+bx^3}(20aAb+3a^2B+5Ab^2x^3+6abBx^3+3b^2Bx^6)}{45b} - \frac{2}{3}a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]
```

output
```
(2*Sqrt[a + b*x^3]*(20*a*A*b + 3*a^2*B + 5*A*b^2*x^3 + 6*a*b*B*x^3 + 3*b^2
*B*x^6))/(45*b) - (2*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

3.199.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {948, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{3/2}(Bx^3+A)}{x^3}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(A\int \frac{(bx^3+a)^{3/2}}{x^3}dx^3+\frac{2B(a+bx^3)^{5/2}}{5b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(A\left(a\int \frac{\sqrt{bx^3+a}}{x^3}dx^3+\frac{2}{3}(a+bx^3)^{3/2}\right)+\frac{2B(a+bx^3)^{5/2}}{5b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(A\left(a\left(a\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+2\sqrt{a+bx^3}\right)+\frac{2}{3}(a+bx^3)^{3/2}\right)+\frac{2B(a+bx^3)^{5/2}}{5b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(A\left(a\left(\frac{2a\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}+2\sqrt{a+bx^3}\right)+\frac{2}{3}(a+bx^3)^{3/2}\right)+\frac{2B(a+bx^3)^{5/2}}{5b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(A\left(a\left(2\sqrt{a+bx^3}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx^3)^{3/2}\right)+\frac{2B(a+bx^3)^{5/2}}{5b}\right)$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]
```

3.199. $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}dx$

output
```
((2*B*(a + b*x^3)^(5/2))/(5*b) + A*((2*(a + b*x^3)^(3/2))/3 + a*(2*Sqrt[a
+ b*x^3] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])))/3
```

3.199.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.199. $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx$

3.199.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

method	result
default	$\frac{2B(b\,x^{3}+a)^{\frac{5}{2}}}{15b}+A\left(\frac{2b\,x^{3}\sqrt{b\,x^{3}+a}}{9}+\frac{8a\sqrt{b\,x^{3}+a}}{9}-\frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}\right)$
pseudoelliptic	$\frac{-\frac{2a^{\frac{3}{2}}bA\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}+\frac{8\left(\frac{x^{3}\left(\frac{3x^{3}B}{5}+A\right)b^{2}}{4}+a\left(\frac{3x^{3}B}{10}+A\right)b+\frac{3a^{2}B}{20}\right)\sqrt{b\,x^{3}+a}}{9}}{b}$
elliptic	$\frac{2Bb\,x^{6}\sqrt{b\,x^{3}+a}}{15}+\frac{2(b^{2}A+\frac{6}{5}abB)x^{3}\sqrt{b\,x^{3}+a}}{9b}+\frac{2\left(2abA+a^{2}B-\frac{2\left(b^{2}A+\frac{6}{5}abB\right)a}{3b}\right)\sqrt{b\,x^{3}+a}}{3b}-\frac{2a^{\frac{3}{2}}A\operatorname{arctanh}\Big(\frac{\sqrt{b\,x^{3}+}}{\sqrt{a}}}{3}$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

output
```
2/15*B*(b*x^3+a)^(5/2)/b+A*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2
)-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.12

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \Bigg[\frac{15\,Aa^{\frac{3}{2}}b\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(3\,Bb^2x^6+(6\,Bab+5\,Ab^2)x^3+3\,Ba}{45\,b}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="fricas")
```

output
```
[1/45*(15*A*a^(3/2)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) +
 2*(3*B*b^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3
 + a))/b, 2/45*(15*A*sqrt(-a)*a*b*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*
B*b^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3 + a))
/b]
```

3.199.6 Sympy [A] (verification not implemented)

Time = 10.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \frac{\begin{cases} \frac{2Aa^2\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2Aa\sqrt{a+bx^3} + \frac{2A(a+bx^3)^{\frac{3}{2}}}{3} + \frac{2B(a+bx^3)^{\frac{5}{2}}}{5b} & \text{for } b \neq 0 \\ Aa^{\frac{3}{2}}\log\left(Ba^{\frac{3}{2}}x^3\right) + Ba^{\frac{3}{2}}x^3 & \text{otherwise} \end{cases}}{3}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x,x)
```

output
```
Piecewise((2*A*a**2*atan(sqrt(a + b*x**3)/sqrt(-a))/sqrt(-a) + 2*A*a*sqrt(
a + b*x**3) + 2*A*(a + b*x**3)**(3/2)/3 + 2*B*(a + b*x**3)**(5/2)/(5*b), N
e(b, 0)), (A*a**(3/2)*log(B*a**(3/2)*x**3) + B*a**(3/2)*x**3, True))/3
```

3.199.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \frac{2\,(bx^3+a)^{\frac{5}{2}}B}{15\,b} + \frac{1}{9}\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 2\,(bx^3+a)^{\frac{3}{2}} + 6\,\sqrt{bx^3+a}a\right)A$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="maxima")
```

output
```
2/15*(b*x^3 + a)^(5/2)*B/b + 1/9*(3*a^(3/2)*log((sqrt(b*x^3 + a) - sqrt(a)
)/(sqrt(b*x^3 + a) + sqrt(a))) + 2*(b*x^3 + a)^(3/2) + 6*sqrt(b*x^3 + a)*a
)*A
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \frac{2\,Aa^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}} + \frac{2\left(3\,(bx^3+a)^{\frac{5}{2}}Bb^4+5\,(bx^3+a)^{\frac{3}{2}}Ab^5+15\,\sqrt{bx^3+a}Aab^5\right)}{45\,b^5}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="giac")
```

output
```
2/3*A*a^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/45*(3*(b*x^3 + a)^
(5/2)*B*b^4 + 5*(b*x^3 + a)^(3/2)*A*b^5 + 15*sqrt(b*x^3 + a)*A*a*b^5)/b^5
```

3.199.9 Mupad [B] (verification not implemented)

Time = 6.91 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \frac{A\,a^{3/2}\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3} + \frac{\sqrt{b\,x^3+a}\left(2\,B\,a^2+4\,A\,a\,b-\frac{2\,a\left(2\,A\,b^2+\frac{12\,B\,a\,b}{5}\right)}{3\,b}\right)}{3\,b} + \frac{2\,B\,b\,x^6\,\sqrt{b\,x^3+a}}{15} + \frac{x^3\left(2\,A\,b^2+\frac{12\,B\,a\,b}{5}\right)\,\sqrt{b\,x^3+a}}{9\,b}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x,x)
```

output
```
(A*a^(3/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/
2)))/x^6))/3 + ((a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a*b - (2*a*(2*A*b^2 + (12
*B*a*b)/5))/(3*b)))/(3*b) + (2*B*b*x^6*(a + b*x^3)^(1/2))/15 + (x^3*(2*A*b
^2 + (12*B*a*b)/5)*(a + b*x^3)^(1/2))/(9*b)
```

3.200 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx$

3.200.1 Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = \frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3}+\frac{(3Ab+2aB)(a+bx^3)^{3/2}}{9a} - \frac{A(a+bx^3)^{5/2}}{3ax^3}-\frac{1}{3}\sqrt{a}(3Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

output
```
1/9*(3*A*b+2*B*a)*(b*x^3+a)^(3/2)/a-1/3*A*(b*x^3+a)^(5/2)/a/x^3-1/3*(3*A*b
+2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+1/3*(3*A*b+2*B*a)*(b*x^3+
a)^(1/2)
```

3.200.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = \frac{\sqrt{a+bx^3}(-3aA+6Abx^3+8aBx^3+2bBx^6)}{9x^3} - \frac{1}{3}\sqrt{a}(3Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]
```

output

```
(Sqrt[a + b*x^3]*(-3*a*A + 6*A*b*x^3 + 8*a*B*x^3 + 2*b*B*x^6))/(9*x^3) - (
Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

3.200.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^4}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)^{3/2}\left(Bx^3+A\right)}{x^6}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{(2aB+3Ab)\int \frac{\left(bx^3+a\right)^{3/2}}{x^3}dx^3}{2a}-\frac{A\left(a+bx^3\right)^{5/2}}{ax^3}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{(2aB+3Ab)\left(a\int \frac{\sqrt{bx^3+a}}{x^3}dx^3+\frac{2}{3}\left(a+bx^3\right)^{3/2}\right)}{2a}-\frac{A\left(a+bx^3\right)^{5/2}}{ax^3}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{(2aB+3Ab)\left(a\left(a\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+2\sqrt{a+bx^3}\right)+\frac{2}{3}\left(a+bx^3\right)^{3/2}\right)}{2a}-\frac{A\left(a+bx^3\right)^{5/2}}{ax^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{(2aB+3Ab)\left(a\left(\frac{2a\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}+2\sqrt{a+bx^3}\right)+\frac{2}{3}\left(a+bx^3\right)^{3/2}\right)}{2a}-\frac{A\left(a+bx^3\right)^{5/2}}{ax^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{(2aB+3Ab)\left(a\left(2\sqrt{a+bx^3}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+bx^3\right)^{3/2}\right)}{2a}-\frac{A\left(a+bx^3\right)^{5/2}}{ax^3}\right)$$

input

```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]
```

output

```
(-((A*(a + b*x^3)^(5/2))/(a*x^3)) + ((3*A*b + 2*a*B)*((2*(a + b*x^3)^(3/2)
)/3 + a*(2*Sqrt[a + b*x^3] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])))
/(2*a))/3
```

3.200.3.1 Defintions of rubi rules used

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.200.4 Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{a\,x^{3} \left(A b+\frac{2 B a}{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)-\frac{2 \sqrt{b\,x^{3}+a}\, \left(\left(\frac{4 x^{3} B}{3}-\frac{A}{2}\right) a^{\frac{3}{2}}+b\,x^{3} \sqrt{a}\, \left(\frac{x^{3} B}{3}+A\right)\right)}{3}}{\sqrt{a}\,x^{3}}$
elliptic	$-\frac{a A \sqrt{b\,x^{3}+a}}{3 x^{3}}+\frac{2 B b\,x^{3} \sqrt{b\,x^{3}+a}}{9}+\frac{2 \left(b^{2} A+\frac{4}{3} a b B\right) \sqrt{b\,x^{3}+a}}{3 b}-\frac{2 \left(\frac{3}{2} a b A+a^{2} B\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3 \sqrt{a}}$
default	$B \left(\frac{2 b\,x^{3} \sqrt{b\,x^{3}+a}}{9}+\frac{8 a \sqrt{b\,x^{3}+a}}{9}-\frac{2 a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}\right)+A \left(-\frac{a \sqrt{b\,x^{3}+a}}{3 x^{3}}+\frac{2 b \sqrt{b\,x^{3}+a}}{3}-b \operatorname{arctanh}\right.$
risch	$-\frac{a A \sqrt{b\,x^{3}+a}}{3 x^{3}}+B\,b^{2} \left(\frac{2 x^{3} \sqrt{b\,x^{3}+a}}{9 b}-\frac{4 a \sqrt{b\,x^{3}+a}}{9 b^{2}}\right)+\frac{2 A b \sqrt{b\,x^{3}+a}}{3}+\frac{4 B a \sqrt{b\,x^{3}+a}}{3}-\frac{\left(3 A b+2 B a\right) \operatorname{arctanh}\left(\sqrt{b}\right.}{3}$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/a^(1/2)*(a*x^3*(A*b+2/3*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))-2/3*(b*x^
3+a)^(1/2)*((4/3*x^3*B-1/2*A)*a^(3/2)+b*x^3*a^(1/2)*(1/3*x^3*B+A)))/x^3
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.54

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^4}\,dx=\left[\frac{3\,(2\,Ba+3\,Ab)\sqrt{a}x^3\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(2\,Bbx^6+2\,(4\,Ba+3\,A}{18\,x^3}\right.$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="fricas")
```

output
```
[1/18*(3*(2*B*a + 3*A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a
) + 2*a)/x^3) + 2*(2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 +
 a))/x^3, 1/9*(3*(2*B*a + 3*A*b)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(
-a)/a) + (2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 + a))/x^3]
```

3.200.6 Sympy [A] (verification not implemented)

Time = 14.42 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.03

$$\begin{aligned}\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx &= -A\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)\\ &\quad-\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}+\frac{2Aa\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2Ab^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}-\frac{2Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3}\\ &\quad+\frac{2Ba^2}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2Ba\sqrt{b}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}+Bb\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)\end{aligned}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**4,x)
```

output
```
-A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - A*a*sqrt(b)*sqrt(a/(b*x**
3) + 1)/(3*x**(3/2)) + 2*A*a*sqrt(b)/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2
*A*b**(3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) - 2*B*a**(3/2)*asinh(sqrt(a)
/(sqrt(b)*x**(3/2)))/3 + 2*B*a**2/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)
) + 2*B*a*sqrt(b)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + B*b*Piecewise((sqrt(
a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

3.200.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx &= \frac{1}{6}\left(3\sqrt{a}b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+4\sqrt{bx^3+a}b-\frac{2\sqrt{bx^3+a}a}{x^3}\right)A\\ &\quad+\frac{1}{9}\left(3a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+2\left(bx^3+a\right)^{\frac{3}{2}}+6\sqrt{bx^3+a}a\right)B\end{aligned}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="maxima")
```

output
```
1/6*(3*sqrt(a)*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a
))) + 4*sqrt(b*x^3 + a)*b - 2*sqrt(b*x^3 + a)*a/x^3)*A + 1/9*(3*a^(3/2)*lo
g((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) + 2*(b*x^3 + a)
^(3/2) + 6*sqrt(b*x^3 + a)*a)*B
```

3.200.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = \frac{2\left(bx^3+a\right)^{\frac{3}{2}}Bb + 6\sqrt{bx^3+a}Bab + 6\sqrt{bx^3+a}Ab^2 + \frac{3\left(2Ba^2b+3Aab^2\right)\arctan\left(\right.}{\sqrt{-a}}}{9b}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="giac")
```

output
```
1/9*(2*(b*x^3 + a)^(3/2)*B*b + 6*sqrt(b*x^3 + a)*B*a*b + 6*sqrt(b*x^3 + a)
*A*b^2 + 3*(2*B*a^2*b + 3*A*a*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-
a) - 3*sqrt(b*x^3 + a)*A*a*b/x^3)/b
```

3.200.9 Mupad [B] (verification not implemented)

Time = 7.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\left(3\,A\,b+2\,B\,a\right)\,\sqrt{\frac{a}{4}}}{3} + \frac{\left(2\,A\,b^2+\frac{8\,B\,a\,b}{3}\right)\,\sqrt{b\,x^3+a}}{3\,b} - \frac{A\,a\,\sqrt{b\,x^3+a}}{3\,x^3} + \frac{2\,B\,b\,x^3\,\sqrt{b\,x^3+a}}{9}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^4,x)
```

output
```
(log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*
(3*A*b + 2*B*a)*(a/4)^(1/2))/3 + ((2*A*b^2 + (8*B*a*b)/3)*(a + b*x^3)^(1/2
))/(3*b) - (A*a*(a + b*x^3)^(1/2))/(3*x^3) + (2*B*b*x^3*(a + b*x^3)^(1/2))
/9
```

3.201 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx$

3.201.1 Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)(a+bx^3)^{3/2}}{12ax^3} - \frac{A(a+bx^3)^{5/2}}{6ax^6} - \frac{b(Ab+4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

output
```
-1/12*(A*b+4*B*a)*(b*x^3+a)^(3/2)/a/x^3-1/6*A*(b*x^3+a)^(5/2)/a/x^6-1/4*b*
(A*b+4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+1/4*b*(A*b+4*B*a)*(b*
x^3+a)^(1/2)/a
```

3.201.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = \frac{\sqrt{a+bx^3}(-2aA-5Abx^3-4aBx^3+8bBx^6)}{12x^6} - \frac{b(Ab+4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]
```

output
```
(Sqrt[a + b*x^3]*(-2*a*A - 5*A*b*x^3 - 4*a*B*x^3 + 8*b*B*x^6))/(12*x^6) -
(b*(A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a])
```

3.201.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {948, 87, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)}{x^7} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{\left(bx^3+a\right)^{3/2} \left(Bx^3+A\right)}{x^9} dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3} \left(\frac{(4aB+Ab) \int \frac{\left(bx^3+a\right)^{3/2}}{x^6} dx^3}{4a} - \frac{A\left(a+bx^3\right)^{5/2}}{2ax^6}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3} \left(\frac{(4aB+Ab) \left(\frac{3}{2} b \int \frac{\sqrt{bx^3+a}}{x^3} dx^3 - \frac{\left(a+bx^3\right)^{3/2}}{x^3}\right)}{4a} - \frac{A\left(a+bx^3\right)^{5/2}}{2ax^6}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3} \left(\frac{(4aB+Ab) \left(\frac{3}{2} b \left(a \int \frac{1}{x^3 \sqrt{bx^3+a}} dx^3 + 2\sqrt{a+bx^3}\right) - \frac{\left(a+bx^3\right)^{3/2}}{x^3}\right)}{4a} - \frac{A\left(a+bx^3\right)^{5/2}}{2ax^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3} \left(\frac{(4aB+Ab) \left(\frac{3}{2} b \left(\frac{2a \int \frac{1}{\frac{x^6}{b}-\frac{a}{b}} d\sqrt{bx^3+a}}{b} + 2\sqrt{a+bx^3}\right) - \frac{\left(a+bx^3\right)^{3/2}}{x^3}\right)}{4a} - \frac{A\left(a+bx^3\right)^{5/2}}{2ax^6}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{3}\left(\frac{(4aB+Ab)\left(\frac{3}{2}b\left(2\sqrt{a+bx^3}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)-\frac{(a+bx^3)^{3/2}}{x^3}\right)}{4a}-\frac{A(a+bx^3)^{5/2}}{2ax^6}\right)$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]
```

output
```
(-1/2*(A*(a + b*x^3)^(5/2))/(a*x^6) + ((A*b + 4*a*B)*(-((a + b*x^3)^(3/2)/
x^3) + (3*b*(2*Sqrt[a + b*x^3] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]
]))/2))/(4*a))/3
```

3.201.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.201.4 Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{b\,x^{6}\left(Ab+4Ba\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)+\frac{5\sqrt{b\,x^{3}+a}\,\left(\frac{2\left(2x^{3}B+A\right)a^{\frac{3}{2}}}{5}+b\,x^{3}\sqrt{a}\,\left(-\frac{8x^{3}B}{5}+A\right)\right)}{3}}{4\sqrt{a}\,x^{6}}$
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(5Ab\,x^{3}+4Ba\,x^{3}+2Aa\right)}{12x^{6}}+\frac{b\left(\frac{16B\sqrt{b\,x^{3}+a}}{3}-\frac{2\left(3Ab+12Ba\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}\right)}{8}$
elliptic	$-\frac{Aa\sqrt{b\,x^{3}+a}}{6x^{6}}-\frac{\left(\frac{5Ab}{4}+Ba\right)\sqrt{b\,x^{3}+a}}{3x^{3}}+\frac{2Bb\sqrt{b\,x^{3}+a}}{3}-\frac{2\left(\frac{3}{8}b^{2}A+\frac{3}{2}abB\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$
default	$A\left(-\frac{a\sqrt{b\,x^{3}+a}}{6x^{6}}-\frac{5b\sqrt{b\,x^{3}+a}}{12x^{3}}-\frac{b^{2}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4\sqrt{a}}\right)+B\Big(-\frac{a\sqrt{b\,x^{3}+a}}{3x^{3}}+\frac{2b\sqrt{b\,x^{3}+a}}{3}-b\operatorname{arctanh}\Big($

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^(1/2)*(b*x^6*(A*b+4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))+5/3*(b*x^
3+a)^(1/2)*(2/5*(2*B*x^3+A)*a^(3/2)+b*x^3*a^(1/2)*(-8/5*x^3*B+A)))/x^6
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = \left[\frac{3\,(4\,Bab+Ab^2)\sqrt{a}x^6\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(8\,Babx^6-(4\,Ba^2+5}{24\,ax^6}\right.$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="fricas")
```

output
```
[1/24*(3*(4*B*a*b + A*b^2)*sqrt(a)*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt
(a) + 2*a)/x^3) + 2*(8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*sqrt
(b*x^3 + a))/(a*x^6), 1/12*(3*(4*B*a*b + A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b
*x^3 + a)*sqrt(-a)/a) + (8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*
sqrt(b*x^3 + a))/(a*x^6)]
```

3.201.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(102) = 204$.

Time = 38.80 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.11

$$\begin{aligned}\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx &= -\frac{Aa^2}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Aa\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}\\ &\quad-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}-\frac{Ab^{\frac{3}{2}}}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}}\\ &\quad-B\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)-\frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}+\frac{2Ba\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2Bb^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}\end{aligned}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**7,x)
```

output
```
-A*a**2/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - A*a*sqrt(b)/(4*x**(9/
2)*sqrt(a/(b*x**3) + 1)) - A*b**(3/2)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) -
A*b**(3/2)/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - A*b**2*asinh(sqrt(a)/(sqrt
(b)*x**(3/2)))/(4*sqrt(a)) - B*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))
 - B*a*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*B*a*sqrt(b)/(3*x**(3/
2)*sqrt(a/(b*x**3) + 1)) + 2*B*b**(3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1))
```

3.201.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = \frac{1}{24}\left(\frac{3\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\left(5\,(bx^3+a)^{\frac{3}{2}}b^2 - 3\,\sqrt{bx^3+a}ab^2\right)}{(bx^3+a)^2 - 2\,(bx^3+a)a + a^2}\right)A + \frac{1}{6}\left(3\,\sqrt{a}b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 4\,\sqrt{bx^3+a}b - \frac{2\,\sqrt{bx^3+a}a}{x^3}\right)B$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="maxima")
```

output
```
1/24*(3*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/s
qrt(a) - 2*(5*(b*x^3 + a)^(3/2)*b^2 - 3*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a
)^2 - 2*(b*x^3 + a)*a + a^2))*A + 1/6*(3*sqrt(a)*b*log((sqrt(b*x^3 + a) -
sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) + 4*sqrt(b*x^3 + a)*b - 2*sqrt(b*x^3
 + a)*a/x^3)*B
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = \frac{8\,\sqrt{bx^3+a}Bb^2 + \frac{3\,(4\,Bab^2+Ab^3)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{4\,(bx^3+a)^{\frac{3}{2}}Bab^2 - 4\,\sqrt{bx^3+a}Ba^2b^2+5}{b^2x^6}}{12\,b}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="giac")
```

output
```
1/12*(8*sqrt(b*x^3 + a)*B*b^2 + 3*(4*B*a*b^2 + A*b^3)*arctan(sqrt(b*x^3 +
a)/sqrt(-a))/sqrt(-a) - (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B
*a^2*b^2 + 5*(b*x^3 + a)^(3/2)*A*b^3 - 3*sqrt(b*x^3 + a)*A*a*b^3)/(b^2*x^6
))/b
```

3.201.9 Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = \frac{2\,B\,b\,\sqrt{b\,x^3+a}}{3} - \frac{\sqrt{b\,x^3+a}\,(4\,B\,a^3+5\,A\,b\,a^2)}{12\,a^2\,x^3} - \frac{A\,a\,\sqrt{b\,x^3+a}}{6\,x^6} + \frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)(A\,b+4\,B\,a)}{8\,\sqrt{a}}$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^7,x)
```

output
```
(2*B*b*(a + b*x^3)^(1/2))/3 - ((a + b*x^3)^(1/2)*(4*B*a^3 + 5*A*a^2*b))/(1
2*a^2*x^3) - (A*a*(a + b*x^3)^(1/2))/(6*x^6) + (b*log((((a + b*x^3)^(1/2)
- a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*(A*b + 4*B*a))/(8*a^(1/2)
)
```

3.202 $\int x^3(a+bx^3)^{3/2}(A+Bx^3)\,dx$

3.202.1 Optimal result

Integrand size = 22, antiderivative size = 336

$$\int x^3\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{54a^2(23Ab-8aB)x\sqrt{a+bx^3}}{21505b^2}$$
$$+\frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b}+\frac{2(23Ab-8aB)x^4\left(a+bx^3\right)^{3/2}}{391b}+\frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b}$$
$$-\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3(23Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{21505b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/391*(23*A*b-8*B*a)*x^4*(b*x^3+a)^(3/2)/b+2/23*B*x^4*(b*x^3+a)^(5/2)/b+54
/21505*a^2*(23*A*b-8*B*a)*x*(b*x^3+a)^(1/2)/b^2+18/4301*a*(23*A*b-8*B*a)*x
^4*(b*x^3+a)^(1/2)/b-36/21505*3^(3/4)*a^3*(23*A*b-8*B*a)*(a^(1/3)+b^(1/3)*
x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(7/3)/(b*x^3+a)^(1/2
)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.202.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.28

$$\int x^3\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2x\sqrt{a+bx^3}\left(-\left(a+bx^3\right)^2\left(-23Ab+8aB-17bBx^3\right)+\frac{a^2(-23Ab+8aB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right.}{\sqrt{1+\frac{bx^3}{a}}}\right.}{391b^2}$$

input
```
Integrate[x^3*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*(-23*A*b + 8*a*B - 17*b*B*x^3)) + (a
^2*(-23*A*b + 8*a*B)*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt
[1 + (b*x^3)/a]))/(391*b^2)
```

3.202.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {959, 811, 811, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \textbf{959}$$

$$\frac{(23Ab-8aB)\int x^3\left(bx^3+a\right)^{3/2}dx}{23b}+\frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b}$$

$$\Big\downarrow \textbf{811}$$

$$\frac{(23Ab-8aB)\left(\frac{9}{17}a\int x^3\sqrt{bx^3+a}dx+\frac{2}{17}x^4\left(a+bx^3\right)^{3/2}\right)}{23b}+\frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b}$$

$$\Big\downarrow \textbf{811}$$

$$\frac{(23Ab-8aB)\left(\frac{9}{17}a\left(\frac{3}{11}a\int\frac{x^3}{\sqrt{bx^3+a}}dx+\frac{2}{11}x^4\sqrt{a+bx^3}\right)+\frac{2}{17}x^4\left(a+bx^3\right)^{3/2}\right)}{23b}+\frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b}$$

$$\Big\downarrow \text{843}$$

$$\frac{(23Ab-8aB)\left(\frac{9}{17}a\left(\frac{3}{11}a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{2a\int\frac{1}{\sqrt{bx^3+a}}dx}{5b}\right)+\frac{2}{11}x^4\sqrt{a+bx^3}\right)+\frac{2}{17}x^4\left(a+bx^3\right)^{3/2}\right)}{23b}+\frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b}$$

$$\Big\downarrow \text{759}$$

$$\frac{(23Ab-8aB)\left(\frac{9}{17}a\left(\frac{3}{11}a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.\right.}{23b}$$

$$\frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b}$$

input
```
Int[x^3*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*B*x^4*(a + b*x^3)^(5/2))/(23*b) + ((23*A*b - 8*a*B)*((2*x^4*(a + b*x^3)
^(3/2))/17 + (9*a*((2*x^4*Sqrt[a + b*x^3]))/11 + (3*a*((2*x*Sqrt[a + b*x^3]
)/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/11))/
17))/(23*b)
```

3.202.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

---

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.202.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.11

method	result
risch	$\frac{2x(935b^3B\,x^9+1265x^6b^3A+1430B\,x^6a\,b^2+2300aA\,b^2x^3+135B\,a^2b\,x^3+621a^2bA-216a^3B)\sqrt{b\,x^3+a}}{21505b^2}+\frac{36ia^3(23Ab-8Ba)\sqrt{3}\,(-a}{}$
elliptic	$\frac{2Bb\,x^{10}\sqrt{b\,x^3+a}}{23}+\frac{2(b^2A+\frac{26}{23}abB)x^7\sqrt{b\,x^3+a}}{17b}+\frac{2\left(2abA+a^2B-\frac{14a\left(b^2A+\frac{26}{23}abB\right)}{17b}\right)x^4\sqrt{b\,x^3+a}}{11b}+\frac{2\,(a^2A-\frac{8a\,(2abA+a^2B-\frac{1}{}}{1}}{}$
default	$B\,(\frac{2b\,x^{10}\sqrt{b\,x^3+a}}{23}+\frac{52a\,x^7\sqrt{b\,x^3+a}}{391}+\frac{54a^2x^4\sqrt{b\,x^3+a}}{4301b}-\frac{432a^3x\sqrt{b\,x^3+a}}{21505b^2}-\frac{288ia^4\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\,(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(}{}}{(-a\,b^2)^{\frac{1}{3}}}}}{}$

input
```
int(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

3.202. $\int x^3(a+bx^3)^{3/2}\,(A+Bx^3)\;dx$

output
```
2/21505/b^2*x*(935*B*b^3*x^9+1265*A*b^3*x^6+1430*B*a*b^2*x^6+2300*A*a*b^2*
x^3+135*B*a^2*b*x^3+621*A*a^2*b-216*B*a^3)*(b*x^3+a)^(1/2)+36/21505*I*a^3*
(23*A*b-8*B*a)/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2
)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))
```

3.202.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.34

$$\int x^3\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\left(54\left(8\,Ba^4-23\,Aa^3b\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(935\,Bb^4x^{10}+55\left(26\,Bab^3+23\,A\right.\right.\right.}{21505\,b^3}$$

input
```
integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/21505*(54*(8*B*a^4 - 23*A*a^3*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b,
x) + (935*B*b^4*x^10 + 55*(26*B*a*b^3 + 23*A*b^4)*x^7 + 5*(27*B*a^2*b^2 +
460*A*a*b^3)*x^4 - 27*(8*B*a^3*b - 23*A*a^2*b^2)*x)*sqrt(b*x^3 + a))/b^3
```

3.202.6 Sympy [A] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.51

$$\int x^3\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \frac{Aa^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{A\sqrt{a}bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{B\sqrt{a}bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate(x**3*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
A*a**(3/2)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(7/3)) + A*sqrt(a)*b*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + B*a**(3/2)*x**7*gamma(7/3
)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) +
B*sqrt(a)*b*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*gamma(13/3))
```

3.202.7 Maxima [F]

$$\int x^3\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^3, x)
```

3.202.8 Giac [F]

$$\int x^3\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^3, x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int x^3\,\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{3/2}\,dx$$

input
```
int(x^3*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

output
```
int(x^3*(A + B*x^3)*(a + b*x^3)^(3/2), x)
```

3.203 $\int (a+bx^3)^{3/2} (A+Bx^3)\, dx$

3.203.1 Optimal result

Integrand size = 19, antiderivative size = 299

$$\int (a+bx^3)^{3/2} (A+Bx^3)\, dx = \frac{18a(17Ab-2aB)x\sqrt{a+bx^3}}{935b}$$

$$+ \frac{2(17Ab-2aB)x(a+bx^3)^{3/2}}{187b} + \frac{2Bx(a+bx^3)^{5/2}}{17b}$$

$$+ \frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2(17Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{935b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
2/187*(17*A*b-2*B*a)*x*(b*x^3+a)^(3/2)/b+2/17*B*x*(b*x^3+a)^(5/2)/b+18/935
*a*(17*A*b-2*B*a)*x*(b*x^3+a)^(1/2)/b+18/935*3^(3/4)*a^2*(17*A*b-2*B*a)*(a
^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(4/3)
/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)
```

3.203.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.26

$$\int \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx = \frac{2x\sqrt{a+bx^3}\left(B\left(a+bx^3\right)^2-\frac{a\left(-\frac{17Ab}{2}+aB\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{17b}$$

input
```
Integrate[(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2 - (a*((-17*A*b)/2 + a*B)*Hypergeomet
ric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(17*b)
```

3.203.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {913, 748, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(17Ab-2aB)\int \left(bx^3+a\right)^{3/2}dx}{17b}+\frac{2Bx\left(a+bx^3\right)^{5/2}}{17b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(17Ab-2aB)\left(\frac{9}{11}a\int \sqrt{bx^3+a}dx+\frac{2}{11}x\left(a+bx^3\right)^{3/2}\right)}{17b}+\frac{2Bx\left(a+bx^3\right)^{5/2}}{17b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(17Ab-2aB)\left(\frac{9}{11}a\left(\frac{3}{5}a\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{2}{5}x\sqrt{a+bx^3}\right)+\frac{2}{11}x\left(a+bx^3\right)^{3/2}\right)}{17b}+\frac{2Bx\left(a+bx^3\right)^{5/2}}{17b}$$

$$\Big\downarrow \text{759}$$

$$\frac{(17Ab-2aB)\Bigg(\frac{9}{11}a\Bigg(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+}{17b}$$

$$\frac{2Bx\left(a+bx^3\right)^{5/2}}{17b}$$

input
```
Int[(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*B*x*(a + b*x^3)^(5/2))/(17*b) + ((17*A*b - 2*a*B)*((2*x*(a + b*x^3)^(3/
2))/11 + (9*a*((2*x*Sqrt[a + b*x^3])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])))/11))/(17*b)
```

3.203.3.1 Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
 + 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.203.4 Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.17

method	result
risch	$\frac{2x\left(55b^2Bx^6+85Ab^2x^3+100Babx^3+238abA+27a^2B\right)\sqrt{bx^3+a}}{935b}-\frac{18ia^2(17Ab-2Ba)\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-ab^2\right)}{2b}\right.}{\left(-ab^2\right)^{\frac{1}{3}}}}}{}$
elliptic	$\frac{2Bbx^7\sqrt{bx^3+a}}{17}+\frac{2\left(b^2A+\frac{20}{17}abB\right)x^4\sqrt{bx^3+a}}{11b}+\frac{2\left(2abA+a^2B-\frac{8a\left(b^2A+\frac{20}{17}abB\right)}{11b}\right)x\sqrt{bx^3+a}}{5b}-\frac{2i\left(a^2A-\frac{2a\left(2abA+a^2B-\frac{8a(}{}\right.}{5b}\right.}{}$
default	$A\left(\frac{2bx^4\sqrt{bx^3+a}}{11}+\frac{28ax\sqrt{bx^3+a}}{55}-\frac{18ia^2\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}}{2b}}}}{}\right.$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output

```
2/935/b*x*(55*B*b^2*x^6+85*A*b^2*x^3+100*B*a*b*x^3+238*A*a*b+27*B*a^2)*(b*
x^3+a)^(1/2)-18/935*I*a^2*(17*A*b-2*B*a)/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.203.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.30

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx =$$

$$-\frac{2\left(27\left(2\,Ba^3-17\,Aa^2b\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\left(55\,Bb^3x^7+5\left(20\,Bab^2+17\,Ab^3\right)x^4+(27\,B\right.\right.}{935\,b^2}$$

input

```
integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output

```
-2/935*(27*(2*B*a^3 - 17*A*a^2*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x
) - (55*B*b^3*x^7 + 5*(20*B*a*b^2 + 17*A*b^3)*x^4 + (27*B*a^2*b + 238*A*a*
b^2)*x)*sqrt(b*x^3 + a))/b^2
```

3.203.6 Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.57

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{Aa^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{A\sqrt{a}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{Ba^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{B\sqrt{a}bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
A*a**(3/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + A*sqrt(a)*b*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + B*a**(3/2)*x**4*gamma(4/3)*hyp
er((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + B*sqrt(
a)*b*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(10/3))
```

3.203.7 Maxima [F]

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2), x)
```

3.203.8 Giac [F]

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2), x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{3/2}\,dx$$

input
```
int((A + B*x^3)*(a + b*x^3)^(3/2),x)
```

output
```
int((A + B*x^3)*(a + b*x^3)^(3/2), x)
```

3.204 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx$

3.204.1 Optimal result

Integrand size = 22, antiderivative size = 295

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = \frac{9}{110}(11Ab+4aB)x\sqrt{a+bx^3}$$

$$+\frac{(11Ab+4aB)x(a+bx^3)^{3/2}}{22a}-\frac{A(a+bx^3)^{5/2}}{2ax^2}$$

$$+\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}a(11Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{110\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
1/22*(11*A*b+4*B*a)*x*(b*x^3+a)^(3/2)/a-1/2*A*(b*x^3+a)^(5/2)/a/x^2+9/110*
(11*A*b+4*B*a)*x*(b*x^3+a)^(1/2)+9/110*3^(3/4)*a*(11*A*b+4*B*a)*(a^(1/3)+b
^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(b*x^3+
a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

3.204.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.58 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.28

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = -\frac{A(a+bx^3)^{5/2}}{2ax^2} - \frac{\left(-\frac{11Ab}{2}-2aB\right)x\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

input

```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^3,x]
```

output

```
-1/2*(A*(a + b*x^3)^(5/2))/(a*x^2) - (((-11*A*b)/2 - 2*a*B)*x*Sqrt[a + b*x
^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a
])
```

3.204.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 748, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(4aB+11Ab)\int (bx^3+a)^{3/2}dx}{4a} - \frac{A(a+bx^3)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{748}$$

$$\frac{(4aB+11Ab)\left(\frac{9}{11}a\int \sqrt{bx^3+a}dx+\frac{2}{11}x(a+bx^3)^{3/2}\right)}{4a} - \frac{A(a+bx^3)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{748}$$

$$\frac{(4aB+11Ab)\left(\frac{9}{11}a\left(\frac{3}{5}a\int\frac{1}{\sqrt{bx^3+a}}dx+\frac{2}{5}x\sqrt{a+bx^3}\right)+\frac{2}{11}x\left(a+bx^3\right)^{3/2}\right)}{4a}-\frac{A\left(a+bx^3\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{759}$$

$$\frac{(4aB+11Ab)\left(\frac{9}{11}a\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\right.\right.}{4a}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{2ax^2}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^3,x]
```

output
```
-1/2*(A*(a + b*x^3)^(5/2))/(a*x^2) + ((11*A*b + 4*a*B)*((2*x*(a + b*x^3)^(
3/2))/11 + (9*a*((2*x*Sqrt[a + b*x^3])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*b
^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3])))/11))/(4*a)
```

3.204.3.1 Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
 + 1)), x] + Simp[a*n*(p/(n*p + 1))    Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.204.4 Maple [A] (verified)

Time = 4.25 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\sqrt{b x^3+a}\,(-20bB x^6-44Ab x^3-56Ba x^3+55Aa)}{110x^2}-\frac{9ia(11Ab+4Ba)\sqrt{3}\,(-a b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a b^2)^{\frac{1}{3}}}}\sqrt{\frac{}{-\frac{3(}{}}}}{}$
elliptic	$-\frac{Aa\sqrt{b x^3+a}}{2x^2}+\frac{2Bb x^4\sqrt{b x^3+a}}{11}+\frac{2(b^2A+\frac{14}{11}abB)x\sqrt{b x^3+a}}{5b}-\frac{2i\left(\frac{7abA}{4}+a^2B-\frac{2a\left(b^2A+\frac{14}{11}abB\right)}{5b}\right)\sqrt{3}\,(-a b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a}{2}\right.}{}}}{}$
default	$B\left(\frac{2b x^4\sqrt{b x^3+a}}{11}+\frac{28ax\sqrt{b x^3+a}}{55}-\frac{18ia^2\sqrt{3}\,(-a b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}}}}{}\right.$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/110*(b*x^3+a)^(1/2)*(-20*B*b*x^6-44*A*b*x^3-56*B*a*x^3+55*A*a)/x^2-9/11
0*I*a*(11*A*b+4*B*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),
(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))
```

3.204.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^3}\,dx = \frac{27\left(4\,Ba^2+11\,Aab\right)\sqrt{b}x^2\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(20\,Bb^2x^6+4\,(}{110\,bx^2}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="fricas")
```

output
```
1/110*(27*(4*B*a^2 + 11*A*a*b)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b,
x) + (20*B*b^2*x^6 + 4*(14*B*a*b + 11*A*b^2)*x^3 - 55*A*a*b)*sqrt(b*x^3 +
a))/(b*x^2)
```

3.204.6 Sympy [A] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{A\sqrt{a}bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{Ba^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{B\sqrt{a}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**3,x)
```

output
```
A*a**(3/2)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**2*gamma(1/3)) + A*sqrt(a)*b*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + B*a**(3/2)*x*gamma(1/3)*hype
r((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + B*sqrt(a
)*b*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(7/3))
```

3.204.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^3, x)
```

3.204.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^3, x)
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^3}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^3,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^3, x)
```

3.205 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx$

3.205.1 Optimal result

Integrand size = 22, antiderivative size = 297

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = \frac{9b(Ab+2aB)x\sqrt{a+bx^3}}{20a} - \frac{(Ab+2aB)(a+bx^3)^{3/2}}{4ax^2} - \frac{A(a+bx^3)^{5/2}}{5ax^5}$$

$$+ \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{20\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/4*(A*b+2*B*a)*(b*x^3+a)^(3/2)/a/x^2-1/5*A*(b*x^3+a)^(5/2)/a/x^5+9/20*b*
(A*b+2*B*a)*x*(b*x^3+a)^(1/2)/a+9/20*3^(3/4)*b^(2/3)*(A*b+2*B*a)*(a^(1/3)+
b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2
)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.205.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.28

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx=\frac{\sqrt{a+bx^3}\left(-\frac{2A(a+bx^3)^2}{a}-\frac{5(Ab+2aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{10x^5}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^6,x]
```

output
```
(Sqrt[a + b*x^3]*((-2*A*(a + b*x^3)^2)/a - (5*(A*b + 2*a*B)*x^3*Hypergeome
tric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(10*x^5)
```

3.205.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 809, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx$$

$$\Big\downarrow 955$$

$$\frac{(2aB+Ab)\int \frac{(bx^3+a)^{3/2}}{x^3}dx}{2a}-\frac{A(a+bx^3)^{5/2}}{5ax^5}$$

$$\Big\downarrow 809$$

$$\frac{(2aB+Ab)\left(\frac{9}{4}b\int \sqrt{bx^3+a}dx-\frac{(a+bx^3)^{3/2}}{2x^2}\right)}{2a}-\frac{A(a+bx^3)^{5/2}}{5ax^5}$$

$$\Big\downarrow 748$$

$$\frac{(2aB+Ab)\left(\frac{9}{4}b\left(\frac{3}{5}a\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{2}{5}x\sqrt{a+bx^3}\right)-\frac{(a+bx^3)^{3/2}}{2x^2}\right)}{2a}-\frac{A(a+bx^3)^{5/2}}{5ax^5}$$

$$\Big\downarrow 759$$

$$\frac{(2aB+Ab)\left(\frac{9}{4}b\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2}{5}x\sqrt{\right.\right.\right.}{2a}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{5ax^5}$$

input

```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^6,x]
```

output

```
-1/5*(A*(a + b*x^3)^(5/2))/(a*x^5) + ((A*b + 2*a*B)*(-1/2*(a + b*x^3)^(3/2
)/x^2 + (9*b*((2*x*Sqrt[a + b*x^3])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*b^(1
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])))/4))/(2*a)
```

3.205.3.1 Defintions of rubi rules used

rule 748

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.205.4 Maple [A] (verified)

Time = 4.49 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-8bB\,x^6+13Ab\,x^3+10Ba\,x^3+4Aa)}{20x^5}-\frac{9i(Ab+2Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3(-a\,b^2)}{2b}}}}{}$
elliptic	$-\frac{Aa\sqrt{b\,x^3+a}}{5x^5}-\frac{\left(\frac{13Ab}{10}+Ba\right)\sqrt{b\,x^3+a}}{2x^2}+\frac{2Bbx\sqrt{b\,x^3+a}}{5}-\frac{2i\left(b^2A+\frac{8abB}{5}-\frac{b(13Ab+10Ba)}{40}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\right)}{(-a\,b^2}}}{}$
default	$B\left(-\frac{a\sqrt{b\,x^3+a}}{2x^2}+\frac{2bx\sqrt{b\,x^3+a}}{5}-\frac{9ia\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-}}{}\right.$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/20*(b*x^3+a)^(1/2)*(-8*B*b*x^6+13*A*b*x^3+10*B*a*x^3+4*A*a)/x^5-9/20*I*
(A*b+2*B*a)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))
```

3.205. $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx$

3.205.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.23

$$\int \frac{(a+bx^3)^{3/2}\,(A+Bx^3)}{x^6}\,dx = \frac{27\,(2\,Ba+Ab)\sqrt{b}x^5\text{weierstrassPInverse}\big(0,-\frac{4\,a}{b},x\big)+\big(8\,Bbx^6-(10\,Ba+}{20\,x^5}$$

input

```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="fricas")
```

output

```
1/20*(27*(2*B*a + A*b)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) + (8*
B*b*x^6 - (10*B*a + 13*A*b)*x^3 - 4*A*a)*sqrt(b*x^3 + a))/x^5
```

3.205.6 Sympy [A] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.62

$$\begin{aligned}\int \frac{(a+bx^3)^{3/2}\,(A+Bx^3)}{x^6}\,dx &= \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}\\&+\frac{A\sqrt{a}b\Gamma\left(-\frac{2}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}\\&+\frac{B\sqrt{a}bx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}\end{aligned}$$

input

```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**6,x)
```

output

```
A*a**(3/2)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**5*gamma(-2/3)) + A*sqrt(a)*b*gamma(-2/3)*hyper((-2/3, -1/2), (1/
3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + B*a**(3/2)*gamma(-2/3
)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)
) + B*sqrt(a)*b*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I
*pi)/a)/(3*gamma(4/3))
```

3.205.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^6, x)
```

3.205.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^6, x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^6}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^6,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^6, x)
```

3.206 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx$

3.206.1 Optimal result

Integrand size = 22, antiderivative size = 302

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \frac{9b(Ab-16aB)\sqrt{a+bx^3}}{320ax^2} + \frac{(Ab-16aB)(a+bx^3)^{3/2}}{80ax^5} - \frac{A(a+bx^3)^{5/2}}{8ax^8} - \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}(Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{320a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/80*(A*b-16*B*a)*(b*x^3+a)^(3/2)/a/x^5-1/8*A*(b*x^3+a)^(5/2)/a/x^8+9/320*
b*(A*b-16*B*a)*(b*x^3+a)^(1/2)/a/x^2-9/320*3^(3/4)*b^(5/3)*(A*b-16*B*a)*(a
^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a/(b*x^
3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

3.206.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.27

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \frac{\sqrt{a+bx^3}\left(-\frac{5A(a+bx^3)^2}{a}+\frac{\left(\frac{Ab}{2}-8aB\right)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{3}{2},-\frac{2}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{40x^8}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^9,x]
```

output
```
(Sqrt[a + b*x^3]*((-5*A*(a + b*x^3)^2)/a + (((A*b)/2 - 8*a*B)*x^3*Hypergeo
metric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(40*x^8)
```

3.206.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 809, 809, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(Ab-16aB)\int \frac{(bx^3+a)^{3/2}}{x^6}dx}{16a}-\frac{A(a+bx^3)^{5/2}}{8ax^8}$$

$$\Big\downarrow \text{809}$$

$$-\frac{(Ab-16aB)\left(\frac{9}{10}b\int \frac{\sqrt{bx^3+a}}{x^3}dx-\frac{(a+bx^3)^{3/2}}{5x^5}\right)}{16a}-\frac{A(a+bx^3)^{5/2}}{8ax^8}$$

$$\Big\downarrow \text{809}$$

$$-\frac{(Ab-16aB)\left(\frac{9}{10}b\left(\frac{3}{4}b\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{2x^2}\right)-\frac{(a+bx^3)^{3/2}}{5x^5}\right)}{16a}-\frac{A(a+bx^3)^{5/2}}{8ax^8}$$

$$\Big\downarrow \text{759}$$

$$-\frac{(Ab-16aB)\left(\frac{9}{10}b\left(\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\right.\right.}{16a}$$

$$\frac{A(a+bx^3)^{5/2}}{8ax^8}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^9,x]
```

output
```
-1/8*(A*(a + b*x^3)^(5/2))/(a*x^8) - ((A*b - 16*a*B)*(-1/5*(a + b*x^3)^(3/
2)/x^5 + (9*b*(-1/2*Sqrt[a + b*x^3]/x^2 + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/
3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3])))/10))/(16*a)
```

3.206.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.206.4 Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(27A\,b^{2}x^{6}+208B\,x^{6}ab+76aAb\,x^{3}+64a^{2}B\,x^{3}+40a^{2}A\right)}{320x^{8}a}+\frac{9ib(Ab-16Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
elliptic	$-\frac{Aa\sqrt{b\,x^{3}+a}}{8x^{8}}-\frac{\left(\frac{19Ab}{16}+Ba\right)\sqrt{b\,x^{3}+a}}{5x^{5}}-\frac{b(27Ab+208Ba)\sqrt{b\,x^{3}+a}}{320a\,x^{2}}-\frac{2i\left(B\,b^{2}-\frac{b^{2}(27Ab+208Ba)}{640a}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right.}{2b}\right.}{}}}{}$
default	$A\left(-\frac{a\sqrt{b\,x^{3}+a}}{8x^{8}}-\frac{19b\sqrt{b\,x^{3}+a}}{80x^{5}}-\frac{27b^{2}\sqrt{b\,x^{3}+a}}{320a\,x^{2}}+\frac{9ib^{2}\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\right.}{}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+}}}{}\right.$

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/320*(b*x^3+a)^(1/2)*(27*A*b^2*x^6+208*B*a*b*x^6+76*A*a*b*x^3+64*B*a^2*x
^3+40*A*a^2)/x^8/a+9/320*I*b*(A*b-16*B*a)/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.206.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.29

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^9}\,dx = \frac{27\,(16\,Bab-Ab^2)\sqrt{b}x^8\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-((208\,Bab+27\,A}{320\,ax^8}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="fricas")
```

output
```
1/320*(27*(16*B*a*b - A*b^2)*sqrt(b)*x^8*weierstrassPInverse(0, -4*a/b, x)
 - ((208*B*a*b + 27*A*b^2)*x^6 + 4*(16*B*a^2 + 19*A*a*b)*x^3 + 40*A*a^2)*s
qrt(b*x^3 + a))/(a*x^8)
```

3.206.6 Sympy [A] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.65

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{B\sqrt{a}b\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**9,x)
```

output
```
A*a**(3/2)*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**8*gamma(-5/3)) + A*sqrt(a)*b*gamma(-5/3)*hyper((-5/3, -1/2), (-2
/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + B*a**(3/2)*gamma(-5
/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-
2/3)) + B*sqrt(a)*b*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x**2*gamma(1/3))
```

3.206.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^9, x)
```

3.206.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^9, x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^9}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^9,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^9, x)
```

3.207 $\int x^4(a+bx^3)^{3/2}(A+Bx^3)\,dx$

3.207.1 Optimal result

Integrand size = 22, antiderivative size = 614

$$\int x^4(a+bx^3)^{3/2}(A+Bx^3)\,dx=\frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2}$$

$$+\frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b}-\frac{216a^3(5Ab-2aB)\sqrt{a+bx^3}}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{2(5Ab-2aB)x^5(a+bx^3)^{3/2}}{95b}+\frac{2Bx^5(a+bx^3)^{5/2}}{25b}$$

$$+\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7\right.}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{72\sqrt{2}3^{3/4}a^{10/3}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\right.}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/95*(5*A*b-2*B*a)*x^5*(b*x^3+a)^(3/2)/b+2/25*B*x^5*(b*x^3+a)^(5/2)/b+54/8
645*a^2*(5*A*b-2*B*a)*x^2*(b*x^3+a)^(1/2)/b^2+18/1235*a*(5*A*b-2*B*a)*x^5*
(b*x^3+a)^(1/2)/b-216/8645*a^3*(5*A*b-2*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))-72/8645*3^(3/4)*a^(10/3)*(5*A*b-2*B*a)*(a^(1/3)
+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3
)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+108/8645*3^
(1/4)*a^(10/3)*(5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1
/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/
2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*
x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.207.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.81 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.16

$$\int x^4(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2x^2\sqrt{a+bx^3}\left(-(a+bx^3)^2(-25Ab+10aB-19bBx^3)+\frac{5a^2(-5Ab+2aB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},-\right.}{\sqrt{1+\frac{bx^3}{a}}}\right.}{475b^2}$$

input
```
Integrate[x^4*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*x^2*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*(-25*A*b + 10*a*B - 19*b*B*x^3)) +
 (5*a^2*(-5*A*b + 2*a*B)*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/
Sqrt[1 + (b*x^3)/a]))/(475*b^2)
```

3.207.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 601, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {959, 811, 811, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(5Ab-2aB)\int x^4 \left(bx^3+a\right)^{3/2}dx}{5b}+\frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b}$$

$$\Big\downarrow \text{811}$$

$$\frac{(5Ab-2aB)\left(\frac{9}{19}a\int x^4\sqrt{bx^3+a}dx+\frac{2}{19}x^5\left(a+bx^3\right)^{3/2}\right)}{5b}+\frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b}$$

$$\Big\downarrow \text{811}$$

$$\frac{(5Ab-2aB)\left(\frac{9}{19}a\left(\frac{3}{13}a\int \frac{x^4}{\sqrt{bx^3+a}}dx+\frac{2}{13}x^5\sqrt{a+bx^3}\right)+\frac{2}{19}x^5\left(a+bx^3\right)^{3/2}\right)}{5b}+\frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b}$$

$$\Big\downarrow \text{843}$$

$$\frac{(5Ab-2aB)\left(\frac{9}{19}a\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\int \frac{x}{\sqrt{bx^3+a}}dx}{7b}\right)+\frac{2}{13}x^5\sqrt{a+bx^3}\right)+\frac{2}{19}x^5\left(a+bx^3\right)^{3/2}\right)}{5b}+$$
$$\frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b}$$

$$\Big\downarrow \text{832}$$

$$\frac{(5Ab-2aB)\left(\frac{9}{19}a\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b}\right)+\frac{2}{13}x^5\sqrt{a+bx^3}\right)+\frac{2}{19}x^5\right.}{5b}$$
$$\frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b}$$

$$\Big\downarrow \text{759}$$

$$\frac{(5Ab-2aB)\left(\frac{9}{19}a\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{El}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}}}\right.}{7b}\right.\right.\right.}{5b}$$

$$\frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b}$$

$$\Big\downarrow \color{blue}{2416}$$

$$(5Ab-2aB)\left(\frac{9}{19}a\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(a\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}}{\sqrt[3]{b}}\right.}{}\right.\right.\right.$$

$$\frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b}$$

3.207. $\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$

input
```
Int[x^4*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*B*x^5*(a + b*x^3)^(5/2))/(25*b) + ((5*A*b - 2*a*B)*((2*x^5*(a + b*x^3)^
(3/2))/19 + (9*a*((2*x^5*Sqrt[a + b*x^3]))/13 + (3*a*((2*x^2*Sqrt[a + b*x^3
])/(7*b) - (4*a*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^
(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])))/(7*b)))/13))/19))/(5*b)
```

3.207.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.207.4 Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 527, normalized size of antiderivative = 0.86

method	result
risch	$\frac{2x^2\left(1729b^3Bx^9+2275x^6b^3A+2548Bx^6ab^2+3850aAb^2x^3+189Ba^2bx^3+675a^2bA-270a^3B\right)\sqrt{bx^3+a}}{43225b^2}+\frac{72ia^3(5Ab-2Ba)\sqrt{3}\,(-}{}$
elliptic	$\frac{2Bbx^{11}\sqrt{bx^3+a}}{25}+\frac{2\left(b^2A+\frac{28}{25}abB\right)x^8\sqrt{bx^3+a}}{19b}+\frac{2\left(2abA+a^2B-\frac{16a\left(b^2A+\frac{28}{25}abB\right)}{19b}\right)x^5\sqrt{bx^3+a}}{13b}+\frac{2\left(a^2A-\frac{10a\left(2abA+a^2B-\right.}{1}\right.}{}$
default	Expression too large to display

input
```
int(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

3.207. $\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$

output

```
2/43225/b^2*x^2*(1729*B*b^3*x^9+2275*A*b^3*x^6+2548*B*a*b^2*x^6+3850*A*a*b
^2*x^3+189*B*a^2*b*x^3+675*A*a^2*b-270*B*a^3)*(b*x^3+a)^(1/2)+72/8645*I*a^
3*(5*A*b-2*B*a)/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^
2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-
I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*
(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.207.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.21

$$\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = -\frac{2\Big(540\left(2\,Ba^4-5\,Aa^3b\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-\left(1729\,Bb^4x^{11}+91\right.}{43225\,b^3}$$

input

```
integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output

```
-2/43225*(540*(2*B*a^4 - 5*A*a^3*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, wei
erstrassPInverse(0, -4*a/b, x)) - (1729*B*b^4*x^11 + 91*(28*B*a*b^3 + 25*A
*b^4)*x^8 + 7*(27*B*a^2*b^2 + 550*A*a*b^3)*x^5 - 135*(2*B*a^3*b - 5*A*a^2*
b^2)*x^2)*sqrt(b*x^3 + a))/b^3
```

3.207.6 Sympy [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.28

$$\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{Aa^{\frac{3}{2}}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{A\sqrt{a}bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}+\frac{Ba^{\frac{3}{2}}x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}+\frac{B\sqrt{a}bx^{11}\Gamma\left(\frac{11}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{11}{3}\\\frac{14}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{14}{3}\right)}$$

input
```
integrate(x**4*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
A*a**(3/2)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(8/3)) + A*sqrt(a)*b*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + B*a**(3/2)*x**8*gamma(8/3
)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) +
B*sqrt(a)*b*x**11*gamma(11/3)*hyper((-1/2, 11/3), (14/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*gamma(14/3))
```

3.207.7 Maxima [F]

$$\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^4, x)
```

3.207.8 Giac [F]

$$\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^4, x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int x^4\,\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{3/2}\,dx$$

input
```
int(x^4*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

output
```
int(x^4*(A + B*x^3)*(a + b*x^3)^(3/2), x)
```

3.208 $\int x(a+bx^3)^{3/2}(A+Bx^3)\,dx$

3.208.1 Optimal result

Integrand size = 20, antiderivative size = 581

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b}$$

$$+\frac{54a^2(19Ab-4aB)\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2(19Ab-4aB)x^2\left(a+bx^3\right)^{3/2}}{247b}+\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{18\sqrt{2}3^{3/4}a^{7/3}(19Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/247*(19*A*b-4*B*a)*x^2*(b*x^3+a)^(3/2)/b+2/19*B*x^2*(b*x^3+a)^(5/2)/b+18
/1729*a*(19*A*b-4*B*a)*x^2*(b*x^3+a)^(1/2)/b+54/1729*a^2*(19*A*b-4*B*a)*(b
*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+18/1729*3^(3/4)*a^(7
/3)*(19*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5
/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)-27/1729*3^(1/4)*a^(7/3)*(19*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)
*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/
(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.208.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.13

$$\int x(a+bx^3)^{3/2}(A+Bx^3)\,dx=\frac{x^2\sqrt{a+bx^3}\left(4B(a+bx^3)^2+\frac{a(19Ab-4aB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{38b}$$

input
```
Integrate[x*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(x^2*Sqrt[a + b*x^3]*(4*B*(a + b*x^3)^2 + (a*(19*A*b - 4*a*B)*Hypergeometr
ic2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(38*b)
```

3.208.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 571, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {959, 811, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \ 959$$

$$\frac{(19Ab-4aB)\int x\left(bx^3+a\right)^{3/2}dx}{19b}+\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}$$

$$\Big\downarrow \ 811$$

$$\frac{(19Ab-4aB)\left(\frac{9}{13}a\int x\sqrt{bx^3+a}dx+\frac{2}{13}x^2\left(a+bx^3\right)^{3/2}\right)}{19b}+\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}$$

$$\Big\downarrow \ 811$$

$$\frac{(19Ab-4aB)\left(\frac{9}{13}a\left(\frac{3}{7}a\int\frac{x}{\sqrt{bx^3+a}}dx+\frac{2}{7}x^2\sqrt{a+bx^3}\right)+\frac{2}{13}x^2\left(a+bx^3\right)^{3/2}\right)}{19b}+\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}$$

$$\Big\downarrow \ 832$$

$$\frac{(19Ab-4aB)\left(\frac{9}{13}a\left(\frac{3}{7}a\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{7}x^2\sqrt{a+bx^3}\right)+\frac{2}{13}x^2\left(a+bx^3\right)^{3/2}\right)}{19b}$$

$$\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}$$

$$\Big\downarrow \ 759$$

$$\frac{(19Ab-4aB)\left(\frac{9}{13}a\left(\frac{3}{7}a\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{arcsin}\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.\right.}{19b}$$

$$\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}$$

$$\Big\downarrow \ 2416$$

3.208. $\int x(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx$

$$(19Ab-4aB)\left(\frac{9}{13}a\left(\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.\right.$$

$$\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}$$

input
```
Int[x*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*B*x^2*(a + b*x^3)^(5/2))/(19*b) + ((19*A*b - 4*a*B)*((2*x^2*(a + b*x^3)
^(3/2))/13 + (9*a*((2*x^2*Sqrt[a + b*x^3])/7 + (3*a*(((2*Sqrt[a + b*x^3])/
(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]
*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[
3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt
[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/7))/13))/(19*b)
```

3.208.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.208.4 Maple [A] (verified)

Time = 4.29 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.87

method	result
risch	$\frac{2x^2(91b^2Bx^6+133Ab^2x^3+154Babx^3+304abA+27a^2B)\sqrt{bx^3+a}}{1729b} - \frac{18ia^2(19Ab-4Ba)\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab\right.}{2b}\right.}{\left(-ab^2\right)^{\frac{1}{3}}}}}{}$
elliptic	$\frac{2Bbx^8\sqrt{bx^3+a}}{19}+\frac{2\left(b^2A+\frac{22}{19}abB\right)x^5\sqrt{bx^3+a}}{13b}+\frac{2\left(2abA+a^2B-\frac{10a\left(b^2A+\frac{22}{19}abB\right)}{13b}\right)x^2\sqrt{bx^3+a}}{7b}-\frac{2i\left(a^2A-\frac{4a\left(2abA+a^2B-\frac{10}{}\right.}{7}\right.}{}$
default	Expression too large to display

input
```
int(x*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

3.208. $\int x(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx$

output
```
2/1729/b*x^2*(91*B*b^2*x^6+133*A*b^2*x^3+154*B*a*b*x^3+304*A*a*b+27*B*a^2)
*(b*x^3+a)^(1/2)-18/1729*I*a^2*(19*A*b-4*B*a)/b^2*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.208.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.17

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\left(27\left(4\,Ba^3-19\,Aa^2b\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(91\,Bb^3\right.\right.}{1729\,b^2}$$

input
```
integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
2/1729*(27*(4*B*a^3 - 19*A*a^2*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weier
strassPInverse(0, -4*a/b, x)) + (91*B*b^3*x^8 + 7*(22*B*a*b^2 + 19*A*b^3)*
x^5 + (27*B*a^2*b + 304*A*a*b^2)*x^2)*sqrt(b*x^3 + a))/b^2
```

3.208.6 Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.30

$$\int x(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{Aa^{\frac{3}{2}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{A\sqrt{a}bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{B\sqrt{a}bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
A*a**(3/2)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(5/3)) + A*sqrt(a)*b*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + B*a**(3/2)*x**5*gamma(5/3)*
hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + B*sq
rt(a)*b*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(11/3))
```

3.208.7 Maxima [F]

$$\int x(a+bx^3)^{3/2}(A+Bx^3)\,dx = \int (Bx^3+A)(bx^3+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x, x)
```

3.208.8 Giac [F]

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x, x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int x\,\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{3/2}\,dx$$

input
```
int(x*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

output
```
int(x*(A + B*x^3)*(a + b*x^3)^(3/2), x)
```

3.209 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx$

3.209.1 Optimal result

Integrand size = 22, antiderivative size = 573

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3}$$

$$+\frac{27a(13Ab+2aB)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{(13Ab+2aB)x^2(a+bx^3)^{3/2}}{13a}-\frac{A(a+bx^3)^{5/2}}{ax}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{9\sqrt{2}3^{3/4}a^{4/3}(13Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/13*(13*A*b+2*B*a)*x^2*(b*x^3+a)^(3/2)/a-A*(b*x^3+a)^(5/2)/a/x+9/91*(13*A
*b+2*B*a)*x^2*(b*x^3+a)^(1/2)+27/91*a*(13*A*b+2*B*a)*(b*x^3+a)^(1/2)/b^(2/
3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+9/91*3^(3/4)*a^(4/3)*(13*A*b+2*B*a)*(a^
(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/182
*3^(1/4)*a^(4/3)*(13*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^
(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.209.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.63 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.14

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = -\frac{A(a+bx^3)^{5/2}}{ax} - \frac{\left(-\frac{13Ab}{2}-aB\right)x^2\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^2,x]
```

output
```
-((A*(a + b*x^3)^(5/2))/(a*x)) - (((-13*A*b)/2 - a*B)*x^2*Sqrt[a + b*x^3]*
Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])
```

3.209.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 569, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 811, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^2}\,dx$$

$$\Big\downarrow \; 955$$

$$\frac{(2aB+13Ab)\int x\left(bx^3+a\right)^{3/2}dx}{2a}-\frac{A\left(a+bx^3\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 811$$

$$\frac{(2aB+13Ab)\left(\frac{9}{13}a\int x\sqrt{bx^3+a}dx+\frac{2}{13}x^2\left(a+bx^3\right)^{3/2}\right)}{2a}-\frac{A\left(a+bx^3\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 811$$

$$\frac{(2aB+13Ab)\left(\frac{9}{13}a\left(\frac{3}{7}a\int\frac{x}{\sqrt{bx^3+a}}dx+\frac{2}{7}x^2\sqrt{a+bx^3}\right)+\frac{2}{13}x^2\left(a+bx^3\right)^{3/2}\right)}{2a}-\frac{A\left(a+bx^3\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 832$$

$$\frac{(2aB+13Ab)\left(\frac{9}{13}a\left(\frac{3}{7}a\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{7}x^2\sqrt{a+bx^3}\right)+\frac{2}{13}x^2\left(a+bx^3\right)^{3/2}\right)}{2a}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 759$$

$$\frac{(2aB+13Ab)\Bigg(\frac{9}{13}a\Bigg(\frac{3}{7}a\Bigg(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Bigg(\text{arcsi}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2a}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 2416$$

3.209. $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^2}\,dx$

$$\frac{(2aB+13Ab)\left(\frac{9}{13}a\left(\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.\right.}{}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{ax}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^2,x]
```

output
```
-((A*(a + b*x^3)^(5/2))/(a*x)) + ((13*A*b + 2*a*B)*((2*x^2*(a + b*x^3)^(3/
2))/13 + (9*a*((2*x^2*Sqrt[a + b*x^3])/7 + (3*a*(((2*Sqrt[a + b*x^3])/(b^(
1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(
1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]
*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[
3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/7))/13))/(2*a)
```

3.209.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.209.4 Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 486, normalized size of antiderivative = 0.85

method	result
risch	$9ia(13Ab+2Ba)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$ $-\frac{\sqrt{b\,x^3+a}\left(-14bB\,x^6-26Ab\,x^3-32Ba\,x^3+91Aa\right)}{91x}-$ $2i\left(\frac{5abA}{2}+a^2B-\frac{4a\left(b^2A+\frac{16}{13}abB\right)}{7b}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}$
elliptic	$-\frac{Aa\sqrt{b\,x^3+a}}{x}+\frac{2Bb\,x^5\sqrt{b\,x^3+a}}{13}+\frac{2\left(b^2A+\frac{16}{13}abB\right)x^2\sqrt{b\,x^3+a}}{7b}-$
default	Expression too large to display

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
```

3.209. $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^2}\,dx$

output

```
-1/91*(b*x^3+a)^(1/2)*(-14*B*b*x^6-26*A*b*x^3-32*B*a*x^3+91*A*a)/x-9/91*I*
a*(13*A*b+2*B*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2
)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(
-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.209.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.15

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx =$$

$$-\frac{27\,(2\,Ba^2+13\,Aab)\sqrt{b}x\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-(14\,Bb^2x^6+2\,(16\,Ba}{91\,bx}$$

input

```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="fricas")
```

output

```
-1/91*(27*(2*B*a^2 + 13*A*a*b)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weiers
trassPInverse(0, -4*a/b, x)) - (14*B*b^2*x^6 + 2*(16*B*a*b + 13*A*b^2)*x^3
 - 91*A*a*b)*sqrt(b*x^3 + a))/(b*x)
```

3.209.6 Sympy [A] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{A\sqrt{a}bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{B\sqrt{a}bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**2,x)
```

output
```
A*a**(3/2)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x*gamma(2/3)) + A*sqrt(a)*b*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + B*a**(3/2)*x**2*gamma(2/3)*h
yper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + B*sqr
t(a)*b*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(8/3))
```

3.209.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^2, x)
```

3.209.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^2, x)
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^2}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^2,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^2, x)
```

3.210 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx$

3.210.1 Optimal result

Integrand size = 22, antiderivative size = 578

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = \frac{9b(7Ab+8aB)x^2\sqrt{a+bx^3}}{56a}$$

$$+\frac{27\sqrt[3]{b}(7Ab+8aB)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{(7Ab+8aB)\left(a+bx^3\right)^{3/2}}{8ax}-\frac{A\left(a+bx^3\right)^{5/2}}{4ax^4}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}(7Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\Bigg(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{9\ 3^{3/4}\sqrt[3]{a}\sqrt[3]{b}(7Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\Bigg(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-}{28\sqrt{2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/8*(7*A*b+8*B*a)*(b*x^3+a)^(3/2)/a/x-1/4*A*(b*x^3+a)^(5/2)/a/x^4+9/56*b*
(7*A*b+8*B*a)*x^2*(b*x^3+a)^(1/2)/a+27/56*b^(1/3)*(7*A*b+8*B*a)*(b*x^3+a)^
(1/2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+9/56*3^(3/4)*a^(1/3)*b^(1/3)*(7*A*b+
8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/2)/(b*x^3+a)^(1/2)/
(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/1
12*3^(1/4)*a^(1/3)*b^(1/3)*(7*A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^
(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I
)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.210.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.15

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^5}\,dx=-\frac{A\left(a+bx^3\right)^{5/2}}{4ax^4}+\frac{\left(-\frac{7Ab}{2}-4aB\right)\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{4x\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^5,x]
```

output
```
-1/4*(A*(a + b*x^3)^(5/2))/(a*x^4) + (((-7*A*b)/2 - 4*a*B)*Sqrt[a + b*x^3]
*Hypergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)])/(4*x*Sqrt[1 + (b*x^3)/a
])
```

3.210.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 569, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 809, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(8aB+7Ab)\int \frac{(bx^3+a)^{3/2}}{x^2}dx}{8a}-\frac{A(a+bx^3)^{5/2}}{4ax^4}$$

$$\Big\downarrow \text{809}$$

$$\frac{(8aB+7Ab)\left(\frac{9}{2}b\int x\sqrt{bx^3+a}dx-\frac{(a+bx^3)^{3/2}}{x}\right)}{8a}-\frac{A(a+bx^3)^{5/2}}{4ax^4}$$

$$\Big\downarrow \text{811}$$

$$\frac{(8aB+7Ab)\left(\frac{9}{2}b\left(\frac{3}{7}a\int \frac{x}{\sqrt{bx^3+a}}dx+\frac{2}{7}x^2\sqrt{a+bx^3}\right)-\frac{(a+bx^3)^{3/2}}{x}\right)}{8a}-\frac{A(a+bx^3)^{5/2}}{4ax^4}$$

$$\Big\downarrow \text{832}$$

$$\frac{(8aB+7Ab)\left(\frac{9}{2}b\left(\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{(1-\sqrt{3})\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{7}x^2\sqrt{a+bx^3}\right)-\frac{(a+bx^3)^{3/2}}{x}\right)}{8a}-\frac{A(a+bx^3)^{5/2}}{4ax^4}$$

$$\Big\downarrow \text{759}$$

$$\frac{(8aB+7Ab)\left(\frac{9}{2}b\left(\frac{3}{7}a\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.\right.}{8a}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{4ax^4}$$

$$\Big\downarrow 2416$$

$$\frac{(8aB+7Ab)\left(\frac{9}{2}b\left(\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.\right.}{}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{4ax^4}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^5,x]
```

3.210. $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^5}\,dx$

output
```
-1/4*(A*(a + b*x^3)^(5/2))/(a*x^4) + ((7*A*b + 8*a*B)*(-((a + b*x^3)^(3/2)
/x) + (9*b*((2*x^2*Sqrt[a + b*x^3])/7 + (3*a*(((2*Sqrt[a + b*x^3])/(b^(1/3
)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b
^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^
(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/7))/2))/(8*a)
```

3.210.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.210.4 Maple [A] (verified)

Time = 4.49 (sec) , antiderivative size = 482, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-16bB\,x^6+77Ab\,x^3+56Ba\,x^3+14Aa)}{56x^4}-\frac{9i(7Ab+8Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{-\frac{3(-a}{2}}}{2i(\frac{27}{16}b^2A+\frac{27}{14}abB)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right.}{(-a\,b^2)^{\frac{1}{3}}}}}$
elliptic	$-\frac{Aa\sqrt{b\,x^3+a}}{4x^4}-\frac{\left(\frac{11Ab}{8}+Ba\right)\sqrt{b\,x^3+a}}{x}+\frac{2Bb\,x^2\sqrt{b\,x^3+a}}{7}-$
default	Expression too large to display

input

```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
```

3.210. $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx$

output
```
-1/56*(b*x^3+a)^(1/2)*(-16*B*b*x^6+77*A*b*x^3+56*B*a*x^3+14*A*a)/x^4-9/56*
I*(7*A*b+8*B*a)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*
b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.210.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.13

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = -\frac{27\,(8\,Ba+7\,Ab)\sqrt{b}x^4\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-(16\,Bbx^6-7\,(8\,Ba+1}{56\,x^4}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="fricas")
```

output
```
-1/56*(27*(8*B*a + 7*A*b)*sqrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstra
ssPInverse(0, -4*a/b, x)) - (16*B*b*x^6 - 7*(8*B*a + 11*A*b)*x^3 - 14*A*a)
*sqrt(b*x^3 + a))/x^4
```

3.210.6 Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.31

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^5}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{B\sqrt{a}bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**5,x)
```

output
```
A*a**(3/2)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**4*gamma(-1/3)) + A*sqrt(a)*b*gamma(-1/3)*hyper((-1/2, -1/3), (2/
3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) + B*a**(3/2)*gamma(-1/3)*h
yper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) + B*
sqrt(a)*b*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(5/3))
```

3.210.7 Maxima [F]

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^5}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^5, x)
```

3.210.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^5, x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^5}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^5,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^5, x)
```

3.211 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx$

3.211.1 Optimal result

Integrand size = 22, antiderivative size = 576

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = -\frac{9b(Ab+14aB)\sqrt{a+bx^3}}{112ax}$$

$$+\frac{27b^{4/3}(Ab+14aB)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{(Ab+14aB)\left(a+bx^3\right)^{3/2}}{56ax^4}-\frac{A(a+bx^3)^{5/2}}{7ax^7}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(Ab+14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\Bigg(\arcsin\Bigg(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\Bigg)|-7-}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{9\ 3^{3/4}b^{4/3}(Ab+14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\Bigg(\arcsin\Bigg(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\Bigg),-7-4}{56\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/56*(A*b+14*B*a)*(b*x^3+a)^(3/2)/a/x^4-1/7*A*(b*x^3+a)^(5/2)/a/x^7-9/112
*b*(A*b+14*B*a)*(b*x^3+a)^(1/2)/a/x+27/112*b^(4/3)*(A*b+14*B*a)*(b*x^3+a)^
(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+9/112*3^(3/4)*b^(4/3)*(A*b+14*B*a)
*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)*2^(1/2)/(b*x^3+a)^(1/2
)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-27
/224*3^(1/4)*b^(4/3)*(A*b+14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.211.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.14

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx=\frac{\sqrt{a+bx^3}\left(-\frac{4A(a+bx^3)^2}{a}-\frac{(Ab+14aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{4}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{28x^7}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^8,x]
```

output
```
(Sqrt[a + b*x^3]*((-4*A*(a + b*x^3)^2)/a - ((A*b + 14*a*B)*x^3*Hypergeomet
ric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(28*x^7)
```

3.211.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 568, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {955, 809, 809, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(14aB+Ab)\int \frac{\left(bx^3+a\right)^{3/2}}{x^5}dx}{14a}-\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{809}$$

$$\frac{(14aB+Ab)\left(\frac{9}{8}b\int \frac{\sqrt{bx^3+a}}{x^2}dx-\frac{\left(a+bx^3\right)^{3/2}}{4x^4}\right)}{14a}-\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{809}$$

$$\frac{(14aB+Ab)\left(\frac{9}{8}b\left(\frac{3}{2}b\int \frac{x}{\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{x}\right)-\frac{\left(a+bx^3\right)^{3/2}}{4x^4}\right)}{14a}-\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{832}$$

$$\frac{(14aB+Ab)\left(\frac{9}{8}b\left(\frac{3}{2}b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)-\frac{\sqrt{a+bx^3}}{x}\right)-\frac{\left(a+bx^3\right)^{3/2}}{4x^4}\right)}{14a}-\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{759}$$

$$\frac{(14aB+Ab)\left(\frac{9}{8}b\left(\frac{3}{2}b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.\right.}{14a}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{2416}$$

3.211. $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^8}\,dx$

$$(14aB+Ab)\left(\frac{9}{8}b\left(\frac{3}{2}b\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.\right.$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^8,x]
```

output
```
-1/7*(A*(a + b*x^3)^(5/2))/(a*x^7) + ((A*b + 14*a*B)*(-1/4*(a + b*x^3)^(3/
2)/x^4 + (9*b*(-(Sqrt[a + b*x^3]/x) + (3*b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1
/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/2))/8))/(14*a)
```

3.211.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.211.4 Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.87

method	result
risch	$9ib(Ab+14Ba)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}$ $-\frac{\sqrt{b\,x^{3}+a}\,\left(27A\,b^{2}x^{6}+154B\,x^{6}ab+34aAb\,x^{3}+28a^{2}B\,x^{3}+16a^{2}A\right)}{112x^{7}a}-$ $2i\left(B\,b^{2}+\frac{b^{2}(27Ab+154Ba)}{224a}\right)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right.}{2b}\right.}{}}$
elliptic	$-\frac{Aa\sqrt{b\,x^{3}+a}}{7x^{7}}-\frac{\left(\frac{17Ab}{14}+Ba\right)\sqrt{b\,x^{3}+a}}{4x^{4}}-\frac{b(27Ab+154Ba)\sqrt{b\,x^{3}+a}}{112ax}-$
default	Expression too large to display

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x,method=_RETURNVERBOSE)
```

3.211. $\int \frac{\left(a+bx^{3}\right)^{3/2}\left(A+Bx^{3}\right)}{x^{8}}\,dx$

output
```
-1/112*(b*x^3+a)^(1/2)*(27*A*b^2*x^6+154*B*a*b*x^6+34*A*a*b*x^3+28*B*a^2*x
^3+16*A*a^2)/x^7/a-9/112*I*b*(A*b+14*B*a)/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)))
```

3.211.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.16

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^8}\,dx =$$

$$-\frac{27\,(14\,Bab+Ab^2)\sqrt{b}x^7\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+((154\,Bab+27\,Ab^2)x^6}{112\,ax^7}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="fricas")
```

output
```
-1/112*(27*(14*B*a*b + A*b^2)*sqrt(b)*x^7*weierstrassZeta(0, -4*a/b, weier
strassPInverse(0, -4*a/b, x)) + ((154*B*a*b + 27*A*b^2)*x^6 + 2*(14*B*a^2
+ 17*A*a*b)*x^3 + 16*A*a^2)*sqrt(b*x^3 + a))/(a*x^7)
```

3.211.6 Sympy [A] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.34

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{3}, -\frac{1}{2} \\ -\frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{B\sqrt{a}b\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**8,x)
```

output
```
A*a**(3/2)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**7*gamma(-4/3)) + A*sqrt(a)*b*gamma(-4/3)*hyper((-4/3, -1/2), (-1
/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + B*a**(3/2)*gamma(-4
/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-
1/3)) + B*sqrt(a)*b*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x*gamma(2/3))
```

3.211.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^8, x)
```

3.211.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^8, x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^8}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^8,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^8, x)
```

3.212 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx$

3.212.1 Optimal result

Integrand size = 22, antiderivative size = 608

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx = \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4}+\frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x}$$

$$-\frac{27b^{7/3}(Ab-4aB)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{(Ab-4aB)\left(a+bx^3\right)^{3/2}}{28ax^7}-\frac{A(a+bx^3)^{5/2}}{10ax^{10}}$$

$$+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\right.}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{9\ 3^{3/4}b^{7/3}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\right.}{224\sqrt{2}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
1/28*(A*b-4*B*a)*(b*x^3+a)^(3/2)/a/x^7-1/10*A*(b*x^3+a)^(5/2)/a/x^10+9/224
*b*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a/x^4+27/448*b^2*(A*b-4*B*a)*(b*x^3+a)^(1/2
)/a^2/x-27/448*b^(7/3)*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))-9/448*3^(3/4)*b^(7/3)*(A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*Ellipti
cF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/
2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)/a^(5/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*
x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+27/896*3^(1/4)*b^(7/3)*(A*b-4*
B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/
a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)
```

3.212.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.13

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^{11}}\,dx=\frac{\sqrt{a+bx^3}\left(-\frac{7A\left(a+bx^3\right)^2}{a}+\frac{5(Ab-4aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{3}{2},-\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{70x^{10}}$$

input

```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^11,x]
```

output

```
(Sqrt[a + b*x^3]*((-7*A*(a + b*x^3)^2)/a + (5*(A*b - 4*a*B)*x^3*Hypergeome
tric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(70*x^1
0)
```

3.212.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 598, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {955, 809, 809, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^{11}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(Ab-4aB)\int \frac{\left(bx^3+a\right)^{3/2}}{x^8}dx}{4a}-\frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \text{809}$$

$$-\frac{(Ab-4aB)\left(\frac{9}{14}b\int \frac{\sqrt{bx^3+a}}{x^5}dx-\frac{\left(a+bx^3\right)^{3/2}}{7x^7}\right)}{4a}-\frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \text{809}$$

$$-\frac{(Ab-4aB)\left(\frac{9}{14}b\left(\frac{3}{8}b\int \frac{1}{x^2\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{4x^4}\right)-\frac{\left(a+bx^3\right)^{3/2}}{7x^7}\right)}{4a}-\frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(Ab-4aB)\left(\frac{9}{14}b\left(\frac{3}{8}b\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)-\frac{\sqrt{a+bx^3}}{4x^4}\right)-\frac{\left(a+bx^3\right)^{3/2}}{7x^7}\right)}{4a}-\frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \text{832}$$

$$-\frac{(Ab-4aB)\left(\frac{9}{14}b\left(\frac{3}{8}b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)-\frac{\sqrt{a+bx^3}}{4x^4}\right)-\frac{\left(a+bx^3\right)^{3/2}}{7x^7}\right)}{4a}-\frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \text{759}$$

3.212. $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^{11}}\,dx$

$$-\frac{(Ab-4aB)\Bigg(\frac{9}{14}b\Bigg(\frac{3}{8}b\Bigg(\frac{b\Bigg(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Bigg(\arcsin\Bigg(\frac{\sqrt[3]{b}}{\sqrt[3]{b}}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2a}}{4a}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \text{2416}$$

$$-\frac{(Ab-4aB)\Bigg(\frac{9}{14}b\Bigg(\frac{3}{8}b\Bigg(\frac{b\Bigg(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\Bigg(\arcsin\Bigg(\frac{\sqrt[3]{b}x+\left(1-\sqrt{}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{}\right.}}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}}{}}{}$$

$$\frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}}$$

3.212. $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^{11}}\,dx$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^11,x]
```

output
```
-1/10*(A*(a + b*x^3)^(5/2))/(a*x^10) - ((A*b - 4*a*B)*(-1/7*(a + b*x^3)^(3
/2)/x^7 + (9*b*(-1/4*Sqrt[a + b*x^3]/x^4 + (3*b*(-(Sqrt[a + b*x^3]/(a*x))
+ (b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
 (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*
(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/
(2*a)))/8))/14))/(4*a)
```

3.212.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.212.4 Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 529, normalized size of antiderivative = 0.87

method	result
risch	$9ib^2(Ab-4Ba)\sqrt{3}\,(-a\,b^2$ $-\frac{\sqrt{b\,x^3+a}\,(-135A\,x^9b^3+540B\,x^9a\,b^2+54A\,x^6a\,b^2+680B\,x^6a^2b+368A\,x^3a^2b+320a^3B\,x^3+224a^3A)}{2240x^{10}a^2}+$ $9ib^2(Ab-4Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}$
elliptic	$-\frac{Aa\sqrt{b\,x^3+a}}{10x^{10}}-\frac{\left(\frac{23Ab}{20}+Ba\right)\sqrt{b\,x^3+a}}{7x^7}-\frac{b(27Ab+340Ba)\sqrt{b\,x^3+a}}{1120a\,x^4}+\frac{27b^2(Ab-4Ba)\sqrt{b\,x^3+a}}{448a^2x}+$
default	Expression too large to display

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x,method=_RETURNVERBOSE)
```

3.212. $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^{11}}\,dx$

output
```
-1/2240*(b*x^3+a)^(1/2)*(-135*A*b^3*x^9+540*B*a*b^2*x^9+54*A*a*b^2*x^6+680
*B*a^2*b*x^6+368*A*a^2*b*x^3+320*B*a^3*x^3+224*A*a^3)/x^10/a^2+9/448*I*b^2
*(A*b-4*B*a)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a
*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.212.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.20

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^{11}}\,dx =$$

$$-\frac{135\left(4\,Bab^2-Ab^3\right)\sqrt{b}x^{10}\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(135\left(4\,Bab^2-Ab^3\right)\right.}{2240\,a^2x^{10}}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="fricas")
```

output
```
-1/2240*(135*(4*B*a*b^2 - A*b^3)*sqrt(b)*x^10*weierstrassZeta(0, -4*a/b, w
eierstrassPInverse(0, -4*a/b, x)) + (135*(4*B*a*b^2 - A*b^3)*x^9 + 2*(340*
B*a^2*b + 27*A*a*b^2)*x^6 + 224*A*a^3 + 16*(20*B*a^3 + 23*A*a^2*b)*x^3)*sq
rt(b*x^3 + a))/(a^2*x^10)
```

3.212.6 Sympy [A] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.33

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{10}{3},-\frac{1}{2}\\-\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)} + \frac{B\sqrt{a}b\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{4}\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**11,x)
```

output
```
A*a**(3/2)*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*p
i)/a)/(3*x**10*gamma(-7/3)) + A*sqrt(a)*b*gamma(-7/3)*hyper((-7/3, -1/2),
(-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + B*a**(3/2)*gamma
(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamm
a(-4/3)) + B*sqrt(a)*b*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp
_polar(I*pi)/a)/(3*x**4*gamma(-1/3))
```

3.212.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^11, x)
```

3.212.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^11, x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx=\int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^{11}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^11,x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^11, x)
```

3.213 $\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

3.213.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2a^2(Ab-aB)\sqrt{a+bx^3}}{3b^4} - \frac{2a(2Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4} + \frac{2(Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4} + \frac{2B\left(a+bx^3\right)^{7/2}}{21b^4}$$

output
```
-2/9*a*(2*A*b-3*B*a)*(b*x^3+a)^(3/2)/b^4+2/15*(A*b-3*B*a)*(b*x^3+a)^(5/2)/
b^4+2/21*B*(b*x^3+a)^(7/2)/b^4+2/3*a^2*(A*b-B*a)*(b*x^3+a)^(1/2)/b^4
```

3.213.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}(56a^2Ab-48a^3B-28aAb^2x^3+24a^2bBx^3+21Ab^3x^6-18ab^2Bx^6+15b^3Bx^9)}{315b^4}$$

input
```
Integrate[(x^8*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3]*(56*a^2*A*b - 48*a^3*B - 28*a*A*b^2*x^3 + 24*a^2*b*B*x^
3 + 21*A*b^3*x^6 - 18*a*b^2*B*x^6 + 15*b^3*B*x^9))/(315*b^4)
```

3.213.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (A+Bx^3)}{\sqrt{a+bx^3}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^6 (Bx^3+A)}{\sqrt{bx^3+a}} dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3} \int \left(\frac{B (bx^3+a)^{5/2}}{b^3} + \frac{(Ab-3aB) (bx^3+a)^{3/2}}{b^3} + \frac{a(3aB-2Ab)\sqrt{bx^3+a}}{b^3} - \frac{a^2(aB-Ab)}{b^3\sqrt{bx^3+a}} \right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{2a^2\sqrt{a+bx^3}(Ab-aB)}{b^4} + \frac{2(a+bx^3)^{5/2} (Ab-3aB)}{5b^4} - \frac{2a(a+bx^3)^{3/2} (2Ab-3aB)}{3b^4} + \frac{2B(a+bx^3)^{7/2}}{7b^4} \right)$$

input
```
Int[(x^8*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
((2*a^2*(A*b - a*B)*Sqrt[a + b*x^3])/b^4 - (2*a*(2*A*b - 3*a*B)*(a + b*x^3
)^(3/2))/(3*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(5/2))/(5*b^4) + (2*B*(a +
 b*x^3)^(7/2))/(7*b^4))/3
```

3.213.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.213.4 Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{16\sqrt{b\,x^{3}+a}\,\left(\frac{3x^{6}\left(\frac{5x^{3}B}{7}+A\right)b^{3}}{8}-\frac{x^{3}\left(\frac{9x^{3}B}{14}+A\right)a\,b^{2}}{2}+a^{2}\left(\frac{3x^{3}B}{7}+A\right)b-\frac{6a^{3}B}{7}\right)}{45b^{4}}$
gosper	$\frac{2\sqrt{b\,x^{3}+a}\,\left(15b^{3}B\,x^{9}+21x^{6}b^{3}A-18B\,x^{6}a\,b^{2}-28aA\,b^{2}x^{3}+24B\,a^{2}b\,x^{3}+56a^{2}bA-48a^{3}B\right)}{315b^{4}}$
trager	$\frac{2\sqrt{b\,x^{3}+a}\,\left(15b^{3}B\,x^{9}+21x^{6}b^{3}A-18B\,x^{6}a\,b^{2}-28aA\,b^{2}x^{3}+24B\,a^{2}b\,x^{3}+56a^{2}bA-48a^{3}B\right)}{315b^{4}}$
risch	$\frac{2\sqrt{b\,x^{3}+a}\,\left(15b^{3}B\,x^{9}+21x^{6}b^{3}A-18B\,x^{6}a\,b^{2}-28aA\,b^{2}x^{3}+24B\,a^{2}b\,x^{3}+56a^{2}bA-48a^{3}B\right)}{315b^{4}}$
elliptic	$\frac{2B\,x^{9}\sqrt{b\,x^{3}+a}}{21b}+\frac{2\left(A-\frac{6aB}{7b}\right)x^{6}\sqrt{b\,x^{3}+a}}{15b}-\frac{8a\left(A-\frac{6aB}{7b}\right)x^{3}\sqrt{b\,x^{3}+a}}{45b^{2}}+\frac{16a^{2}\left(A-\frac{6aB}{7b}\right)\sqrt{b\,x^{3}+a}}{45b^{3}}$
default	$A\left(\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15b}-\frac{8a\,x^{3}\sqrt{b\,x^{3}+a}}{45b^{2}}+\frac{16a^{2}\sqrt{b\,x^{3}+a}}{45b^{3}}\right)+B\left(\frac{2x^{9}\sqrt{b\,x^{3}+a}}{21b}-\frac{4a\,x^{6}\sqrt{b\,x^{3}+a}}{35b^{2}}+\frac{16a^{2}x^{3}\sqrt{b\,x^{3}+a}}{105b^{3}}-\right.$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
16/45*(b*x^3+a)^(1/2)*(3/8*x^6*(5/7*x^3*B+A)*b^3-1/2*x^3*(9/14*x^3*B+A)*a*
b^2+a^2*(3/7*x^3*B+A)*b-6/7*a^3*B)/b^4
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$$
$$=\frac{2\,(15\,Bb^3x^9-3\,(6\,Bab^2-7\,Ab^3)x^6-48\,Ba^3+56\,Aa^2b+4\,(6\,Ba^2b-7\,Aab^2)x^3)\sqrt{bx^3+a}}{315\,b^4}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(15*B*b^3*x^9 - 3*(6*B*a*b^2 - 7*A*b^3)*x^6 - 48*B*a^3 + 56*A*a^2*b
+ 4*(6*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^4
```

3.213.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.70

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{16Aa^2\sqrt{a+bx^3}}{45b^3} - \frac{8Aax^3\sqrt{a+bx^3}}{45b^2} + \frac{2Ax^6\sqrt{a+bx^3}}{15b} - \frac{32Ba^3\sqrt{a+bx^3}}{105b^4} + \frac{16Ba^2x^3\sqrt{a+bx^3}}{105b^3} - \frac{4Bax^6\sqrt{a+bx^3}}{35b^2} + \frac{2Bx^9\sqrt{a+bx^3}}{21b} & \text{fo} \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{\sqrt{a}} & \text{ot} \end{cases}$$

input
```
integrate(x**8*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((16*A*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*A*a*x**3*sqrt(a + b*x*
*3)/(45*b**2) + 2*A*x**6*sqrt(a + b*x**3)/(15*b) - 32*B*a**3*sqrt(a + b*x*
*3)/(105*b**4) + 16*B*a**2*x**3*sqrt(a + b*x**3)/(105*b**3) - 4*B*a*x**6*s
qrt(a + b*x**3)/(35*b**2) + 2*B*x**9*sqrt(a + b*x**3)/(21*b), Ne(b, 0)), (
(A*x**9/9 + B*x**12/12)/sqrt(a), True))
```

3.213.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\begin{aligned} \int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx &= \frac{2}{105}\,B\left(\frac{5\,(bx^3+a)^{\frac{7}{2}}}{b^4} - \frac{21\,(bx^3+a)^{\frac{5}{2}}a}{b^4} + \frac{35\,(bx^3+a)^{\frac{3}{2}}a^2}{b^4} - \frac{35\,\sqrt{bx^3+a}a^3}{b^4}\right) \\ &\quad + \frac{2}{45}\,A\left(\frac{3\,(bx^3+a)^{\frac{5}{2}}}{b^3} - \frac{10\,(bx^3+a)^{\frac{3}{2}}a}{b^3} + \frac{15\,\sqrt{bx^3+a}a^2}{b^3}\right) \end{aligned}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output

```
2/105*B*(5*(b*x^3 + a)^(7/2)/b^4 - 21*(b*x^3 + a)^(5/2)*a/b^4 + 35*(b*x^3
+ a)^(3/2)*a^2/b^4 - 35*sqrt(b*x^3 + a)*a^3/b^4) + 2/45*A*(3*(b*x^3 + a)^(
5/2)/b^3 - 10*(b*x^3 + a)^(3/2)*a/b^3 + 15*sqrt(b*x^3 + a)*a^2/b^3)
```

3.213.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = -\frac{2\,(Ba^3-Aa^2b)\sqrt{bx^3+a}}{3\,b^4} + \frac{2\left(15\,(bx^3+a)^{\frac{7}{2}}B - 63\,(bx^3+a)^{\frac{5}{2}}Ba + 105\,(bx^3+a)^{\frac{3}{2}}Ba^2 + 21\,(bx^3+a)^{\frac{5}{2}}Ab - 70\,(bx^3+a)^{\frac{3}{2}}Aab\right)}{315\,b^4}$$

input

```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output

```
-2/3*(B*a^3 - A*a^2*b)*sqrt(b*x^3 + a)/b^4 + 2/315*(15*(b*x^3 + a)^(7/2)*B
 - 63*(b*x^3 + a)^(5/2)*B*a + 105*(b*x^3 + a)^(3/2)*B*a^2 + 21*(b*x^3 + a)
^(5/2)*A*b - 70*(b*x^3 + a)^(3/2)*A*a*b)/b^4
```

3.213.9 Mupad [B] (verification not implemented)

Time = 7.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{8\,a^2\,\sqrt{b\,x^3+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{45\,b^3} + \frac{x^6\,\sqrt{b\,x^3+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{15\,b} + \frac{2\,B\,x^9\,\sqrt{b\,x^3+a}}{21\,b} - \frac{4\,a\,x^3\,\sqrt{b\,x^3+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{45\,b^2}$$

input

```
int((x^8*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

output

```
(8*a^2*(a + b*x^3)^(1/2)*(2*A - (12*B*a)/(7*b)))/(45*b^3) + (x^6*(a + b*x^
3)^(1/2)*(2*A - (12*B*a)/(7*b)))/(15*b) + (2*B*x^9*(a + b*x^3)^(1/2))/(21*
b) - (4*a*x^3*(a + b*x^3)^(1/2)*(2*A - (12*B*a)/(7*b)))/(45*b^2)
```

3.214 $\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

3.214.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = -\frac{2a(Ab-aB)\sqrt{a+bx^3}}{3b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2B(a+bx^3)^{5/2}}{15b^3}$$

output
```
2/9*(A*b-2*B*a)*(b*x^3+a)^(3/2)/b^3+2/15*B*(b*x^3+a)^(5/2)/b^3-2/3*a*(A*b-
B*a)*(b*x^3+a)^(1/2)/b^3
```

3.214.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}(-10aAb+8a^2B+5Ab^2x^3-4abBx^3+3b^2Bx^6)}{45b^3}$$

input
```
Integrate[(x^5*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3]*(-10*a*A*b + 8*a^2*B + 5*A*b^2*x^3 - 4*a*b*B*x^3 + 3*b^
2*B*x^6))/(45*b^3)
```

3.214.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \textbf{948}$$

$$\frac{1}{3}\int \frac{x^3(Bx^3+A)}{\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \textbf{86}$$

$$\frac{1}{3}\int \left(\frac{B(bx^3+a)^{3/2}}{b^2}+\frac{(Ab-2aB)\sqrt{bx^3+a}}{b^2}+\frac{a(aB-Ab)}{b^2\sqrt{bx^3+a}}\right)dx^3$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{1}{3}\left(\frac{2(a+bx^3)^{3/2}(Ab-2aB)}{3b^3}-\frac{2a\sqrt{a+bx^3}(Ab-aB)}{b^3}+\frac{2B(a+bx^3)^{5/2}}{5b^3}\right)$$

input
```
Int[(x^5*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
((-2*a*(A*b - a*B)*Sqrt[a + b*x^3])/b^3 + (2*(A*b - 2*a*B)*(a + b*x^3)^(3/
2))/(3*b^3) + (2*B*(a + b*x^3)^(5/2))/(5*b^3))/3
```

3.214.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.214.4 Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$-\frac{4\left(-\frac{x^{3}\left(\frac{3x^{3}B}{5}+A\right)b^{2}}{2}+a\left(\frac{2x^{3}B}{5}+A\right)b-\frac{4a^{2}B}{5}\right)\sqrt{b\,x^{3}+a}}{9b^{3}}$	49
gosper	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-3b^{2}B\,x^{6}-5A\,b^{2}x^{3}+4Bab\,x^{3}+10abA-8a^{2}B\right)}{45b^{3}}$	53
trager	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-3b^{2}B\,x^{6}-5A\,b^{2}x^{3}+4Bab\,x^{3}+10abA-8a^{2}B\right)}{45b^{3}}$	53
risch	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-3b^{2}B\,x^{6}-5A\,b^{2}x^{3}+4Bab\,x^{3}+10abA-8a^{2}B\right)}{45b^{3}}$	53
elliptic	$\frac{2B\,x^{6}\sqrt{b\,x^{3}+a}}{15b}+\frac{2\left(A-\frac{4aB}{5b}\right)x^{3}\sqrt{b\,x^{3}+a}}{9b}-\frac{4a\left(A-\frac{4aB}{5b}\right)\sqrt{b\,x^{3}+a}}{9b^{2}}$	70
default	$B\left(\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15b}-\frac{8a\,x^{3}\sqrt{b\,x^{3}+a}}{45b^{2}}+\frac{16a^{2}\sqrt{b\,x^{3}+a}}{45b^{3}}\right)+A\left(\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b}-\frac{4a\sqrt{b\,x^{3}+a}}{9b^{2}}\right)$	92

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/9*(-1/2*x^3*(3/5*x^3*B+A)*b^2+a*(2/5*x^3*B+A)*b-4/5*a^2*B)*(b*x^3+a)^(1
/2)/b^3
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{2\,(3\,Bb^2x^6-(4\,Bab-5\,Ab^2)x^3+8\,Ba^2-10\,Aab)\sqrt{bx^3+a}}{45\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*B*b^2*x^6 - (4*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 10*A*a*b)*sqrt(b*x
^3 + a)/b^3
```

3.214.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.70

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \begin{cases} -\frac{4Aa\sqrt{a+bx^3}}{9b^2} + \frac{2Ax^3\sqrt{a+bx^3}}{9b} + \frac{16Ba^2\sqrt{a+bx^3}}{45b^3} - \frac{8Bax^3\sqrt{a+bx^3}}{45b^2} + \frac{2Bx^6\sqrt{a+bx^3}}{15b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((-4*A*a*sqrt(a + b*x**3)/(9*b**2) + 2*A*x**3*sqrt(a + b*x**3)/(9
*b) + 16*B*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*B*a*x**3*sqrt(a + b*x**3)/(
45*b**2) + 2*B*x**6*sqrt(a + b*x**3)/(15*b), Ne(b, 0)), ((A*x**6/6 + B*x**
9/9)/sqrt(a), True))
```

3.214.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2}{45}\,B\left(\frac{3\,(bx^3+a)^{\frac{5}{2}}}{b^3} - \frac{10\,(bx^3+a)^{\frac{3}{2}}a}{b^3} + \frac{15\,\sqrt{bx^3+a}a^2}{b^3}\right) + \frac{2}{9}\,A\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^2} - \frac{3\,\sqrt{bx^3+a}a}{b^2}\right)$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/45*B*(3*(b*x^3 + a)^(5/2)/b^3 - 10*(b*x^3 + a)^(3/2)*a/b^3 + 15*sqrt(b*x
^3 + a)*a^2/b^3) + 2/9*A*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2)
```

3.214.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,\sqrt{bx^3+a}(Ba^2-Aab)}{3\,b^3} + \frac{2\left(3\,(bx^3+a)^{\frac{5}{2}}B-10\,(bx^3+a)^{\frac{3}{2}}Ba+5\,(bx^3+a)^{\frac{3}{2}}Ab\right)}{45\,b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(b*x^3 + a)*(B*a^2 - A*a*b)/b^3 + 2/45*(3*(b*x^3 + a)^(5/2)*B - 10
*(b*x^3 + a)^(3/2)*B*a + 5*(b*x^3 + a)^(3/2)*A*b)/b^3
```

3.214.9 Mupad [B] (verification not implemented)

Time = 7.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,\sqrt{b\,x^3+a}\,(8\,B\,a^2-4\,B\,a\,b\,x^3-10\,A\,a\,b+3\,B\,b^2\,x^6+5\,A\,b^2\,x^3)}{45\,b^3}$$

input
```
int((x^5*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

output
```
(2*(a + b*x^3)^(1/2)*(8*B*a^2 + 5*A*b^2*x^3 + 3*B*b^2*x^6 - 10*A*a*b - 4*B
*a*b*x^3))/(45*b^3)
```

3.215 $\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

3.215.1 Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{2(Ab-aB)\sqrt{a+bx^3}}{3b^2}+\frac{2B(a+bx^3)^{3/2}}{9b^2}$$

output
```
2/9*B*(b*x^3+a)^(3/2)/b^2+2/3*(A*b-B*a)*(b*x^3+a)^(1/2)/b^2
```

3.215.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{2\sqrt{a+bx^3}(3Ab-2aB+bBx^3)}{9b^2}$$

input
```
Integrate[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3]*(3*A*b - 2*a*B + b*B*x^3))/(9*b^2)
```

3.215.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^3)}{\sqrt{a+bx^3}} \, dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3} \int \frac{Bx^3+A}{\sqrt{bx^3+a}} dx^3$$

$$\Big\downarrow \; 53$$

$$\frac{1}{3} \int \left(\frac{\sqrt{bx^3+a}B}{b} + \frac{Ab-aB}{b\sqrt{bx^3+a}}\right) dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3} \left(\frac{2\sqrt{a+bx^3}(Ab-aB)}{b^2} + \frac{2B(a+bx^3)^{3/2}}{3b^2}\right)$$

input
```
Int[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
((2*(A*b - a*B)*Sqrt[a + b*x^3])/b^2 + (2*B*(a + b*x^3)^(3/2))/(3*b^2))/3
```

3.215.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.215.4 Maple [A] (verified)

Time = 4.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{2\sqrt{b\,x^{3}+a}\,\left(bB\,x^{3}+3Ab-2Ba\right)}{9b^{2}}$	30
trager	$\frac{2\sqrt{b\,x^{3}+a}\,\left(bB\,x^{3}+3Ab-2Ba\right)}{9b^{2}}$	30
risch	$\frac{2\sqrt{b\,x^{3}+a}\,\left(bB\,x^{3}+3Ab-2Ba\right)}{9b^{2}}$	30
pseudoelliptic	$\frac{2\sqrt{b\,x^{3}+a}\,\left(\left(\frac{x^{3}B}{3}+A\right)b-\frac{2Ba}{3}\right)}{3b^{2}}$	30
elliptic	$\frac{2B\,x^{3}\sqrt{b\,x^{3}+a}}{9b}+\frac{2\left(A-\frac{2aB}{3b}\right)\sqrt{b\,x^{3}+a}}{3b}$	43
default	$B\left(\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b}-\frac{4a\sqrt{b\,x^{3}+a}}{9b^{2}}\right)+\frac{2A\sqrt{b\,x^{3}+a}}{3b}$	52

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(b*x^3+a)^(1/2)*(B*b*x^3+3*A*b-2*B*a)/b^2
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{2\left(Bbx^3-2\,Ba+3\,Ab\right)\sqrt{bx^3+a}}{9\,b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(B*b*x^3 - 2*B*a + 3*A*b)*sqrt(b*x^3 + a)/b^2
```

3.215.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{2A\sqrt{a+bx^3}}{3b} - \frac{4Ba\sqrt{a+bx^3}}{9b^2} + \frac{2Bx^3\sqrt{a+bx^3}}{9b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input

```
integrate(x**2*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output

```
Piecewise((2*A*sqrt(a + b*x**3)/(3*b) - 4*B*a*sqrt(a + b*x**3)/(9*b**2) +
2*B*x**3*sqrt(a + b*x**3)/(9*b), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/sqrt(a)
, True))
```

3.215.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2}{9}\,B\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^2} - \frac{3\,\sqrt{bx^3+a}a}{b^2}\right) + \frac{2\,\sqrt{bx^3+a}A}{3\,b}$$

input

```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output

```
2/9*B*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2) + 2/3*sqrt(b*x^3 +
 a)*A/b
```

3.215.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,(bx^3+a)^{\frac{3}{2}}B}{9\,b^2} - \frac{2\,\sqrt{bx^3+a}(Ba-Ab)}{3\,b^2}$$

input

```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output

```
2/9*(b*x^3 + a)^(3/2)*B/b^2 - 2/3*sqrt(b*x^3 + a)*(B*a - A*b)/b^2
```

3.215.9 Mupad [B] (verification not implemented)

Time = 7.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{2\,\sqrt{b\,x^3+a}\,\left(B\,b\,x^3+3\,A\,b-2\,B\,a\right)}{9\,b^2}$$

input
```
int((x^2*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

output
```
(2*(a + b*x^3)^(1/2)*(3*A*b - 2*B*a + B*b*x^3))/(9*b^2)
```

3.216 $\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx$

3.216.1 Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx=\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

output
```
-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+2/3*B*(b*x^3+a)^(1/2)/b
```

3.216.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx=\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Integrate[(A + B*x^3)/(x*Sqrt[a + b*x^3]),x]
```

output
```
(2*B*Sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sq
rt[a])
```

3.216.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^3\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(A\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+\frac{2B\sqrt{a+bx^3}}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2A\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}+\frac{2B\sqrt{a+bx^3}}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{2B\sqrt{a+bx^3}}{b}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

input
```
Int[(A + B*x^3)/(x*Sqrt[a + b*x^3]),x]
```

output
```
((2*B*Sqrt[a + b*x^3])/b - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/Sqrt[a])
/3
```

3.216.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.216.4 Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{2A\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{2B\sqrt{b\,x^{3}+a}}{3b}$	37
elliptic	$-\frac{2A\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{2B\sqrt{b\,x^{3}+a}}{3b}$	37
pseudoelliptic	$-\frac{2\left(Ab\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)-B\sqrt{b\,x^{3}+a}\,\sqrt{a}\right)}{3\sqrt{a}\,b}$	42

input
```
int((B*x^3+A)/x/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+2/3*B*(b*x^3+a)^(1/2)/b
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.19

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx = \left[\frac{A\sqrt{a}b\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)+2\sqrt{bx^3+a}Ba}{3ab}, \frac{2\left(A\sqrt{-a}b\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+\sqrt{bx^3+a}Ba\right)}{3ab}\right]$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(A*sqrt(a)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*s
qrt(b*x^3 + a)*B*a)/(a*b), 2/3*(A*sqrt(-a)*b*arctan(sqrt(b*x^3 + a)*sqrt(-
a)/a) + sqrt(b*x^3 + a)*B*a)/(a*b)]
```

3.216.6 Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx = \frac{A\left(\begin{cases}\frac{2\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for}\ b\neq 0\\ -\frac{\log\left(\frac{1}{x^3}\right)}{\sqrt{a}} & \text{otherwise}\end{cases}\right)}{3} - \frac{B\left(\begin{cases}-\frac{x^3}{\sqrt{a}} & \text{for}\ b=0\\ -\frac{2\sqrt{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)}{3}$$

input
```
integrate((B*x**3+A)/x/(b*x**3+a)**(1/2),x)
```

output
```
A*Piecewise((2*atan(sqrt(a + b*x**3)/sqrt(-a))/sqrt(-a), Ne(b, 0)), (-log(
x**(-3))/sqrt(a), True))/3 - B*Piecewise((-x**3/sqrt(a), Eq(b, 0)), (-2*sq
rt(a + b*x**3)/b, True))/3
```

3.216.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx=\frac{A\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3\sqrt{a}}+\frac{2\sqrt{bx^3+a}B}{3b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*A*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
 + 2/3*sqrt(b*x^3 + a)*B/b
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx=\frac{2A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}}+\frac{2\sqrt{bx^3+a}B}{3b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/3*sqrt(b*x^3 + a)*B/b
```

3.216.9 Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx=\frac{2B\sqrt{b\,x^3+a}}{3b}+\frac{A\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\sqrt{a}}$$

input
```
int((A + B*x^3)/(x*(a + b*x^3)^(1/2)),x)
```

output
```
(2*B*(a + b*x^3)^(1/2))/(3*b) + (A*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((
a + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(1/2))
```

3.217 $\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx$

3.217.1 Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx=-\frac{A\sqrt{a+bx^3}}{3ax^3}+\frac{(Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

output
```
1/3*(A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/3*A*(b*x^3+a)^(
1/2)/a/x^3
```

3.217.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx=-\frac{A\sqrt{a+bx^3}}{3ax^3}+\frac{(Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

input
```
Integrate[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]
```

output
```
-1/3*(A*Sqrt[a + b*x^3])/(a*x^3) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/
Sqrt[a]])/(3*a^(3/2))
```

3.217.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^6\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(-\frac{(Ab-2aB)\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{2a}-\frac{A\sqrt{a+bx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{(Ab-2aB)\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{ab}-\frac{A\sqrt{a+bx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{A\sqrt{a+bx^3}}{ax^3}\right)$$

input
```
Int[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]
```

output
```
(-((A*Sqrt[a + b*x^3])/(a*x^3)) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]])/a^(3/2))/3
```

3.217.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.217.4 Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}-\frac{A\sqrt{b\,x^{3}+a}}{3a\,x^{3}}$	47
elliptic	$\frac{(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}-\frac{A\sqrt{b\,x^{3}+a}}{3a\,x^{3}}$	47
pseudoelliptic	$\frac{(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}-\frac{A\sqrt{b\,x^{3}+a}}{3a\,x^{3}}$	47
default	$-\frac{2B\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+A\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^{3}+a}}{3a\,x^{3}}\right)$	62

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/3*A*(b*x^3+a)^(
1/2)/a/x^3
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.17

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx$$
$$=\left[-\frac{(2\,Ba-Ab)\sqrt{a}x^3\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,\sqrt{bx^3+a}Aa}{6\,a^2x^3},\frac{(2\,Ba-Ab)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)}{3\,a^2x^3}\right.$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/6*((2*B*a - A*b)*sqrt(a)*x^3*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) +
2*a)/x^3) + 2*sqrt(b*x^3 + a)*A*a)/(a^2*x^3), 1/3*((2*B*a - A*b)*sqrt(-a)*
x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) - sqrt(b*x^3 + a)*A*a)/(a^2*x^3)]
```

3.217.6 Sympy [A] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx=-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ax^{\frac{3}{2}}}+\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}}-\frac{2B\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

input
```
integrate((B*x**3+A)/x**4/(b*x**3+a)**(1/2),x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*x**(3/2)) + A*b*asinh(sqrt(a)/(sqrt(b
)*x**(3/2)))/(3*a**(3/2)) - 2*B*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(
a))
```

3.217.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(47) = 94$.

Time $= 0.29$ (sec) , antiderivative size $= 109$, normalized size of antiderivative $= 1.88$

$$\int \frac{A + Bx^3}{x^4\sqrt{a + bx^3}}\,dx = -\frac{1}{6}\,A\left(\frac{2\,\sqrt{bx^3 + a}b}{(bx^3 + a)a - a^2} + \frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right) + \frac{B\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3\,\sqrt{a}}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*A*(2*sqrt(b*x^3 + a)*b/((b*x^3 + a)*a - a^2) + b*log((sqrt(b*x^3 + a)
 - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)) + 1/3*B*log((sqrt(b*x^3
+ a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
```

3.217.8 Giac [A] (verification not implemented)

Time $= 0.28$ (sec) , antiderivative size $= 62$, normalized size of antiderivative $= 1.07$

$$\int \frac{A + Bx^3}{x^4\sqrt{a + bx^3}}\,dx = \frac{\frac{(2\,Bab - Ab^2)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{\sqrt{bx^3+a}Ab}{ax^3}}{3\,b}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/3*((2*B*a*b - A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - sqr
t(b*x^3 + a)*A*b/(a*x^3))/b
```

3.217.9 Mupad [B] (verification not implemented)

Time $= 7.30$ (sec) , antiderivative size $= 67$, normalized size of antiderivative $= 1.16$

$$\int \frac{A + Bx^3}{x^4\sqrt{a + bx^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)(A\,b - 2\,B\,a)}{6\,a^{3/2}} - \frac{A\,\sqrt{b\,x^3 + a}}{3\,a\,x^3}$$

input
```
int((A + B*x^3)/(x^4*(a + b*x^3)^(1/2)),x)
```

output
```
(log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*
(A*b - 2*B*a))/(6*a^(3/2)) - (A*(a + b*x^3)^(1/2))/(3*a*x^3)
```

3.217. $\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx$

3.218 $\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx$

3.218.1 Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx=-\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3}-\frac{b(3Ab-4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}$$

output
```
-1/12*b*(3*A*b-4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)-1/6*A*(b*x^
3+a)^(1/2)/a/x^6+1/12*(3*A*b-4*B*a)*(b*x^3+a)^(1/2)/a^2/x^3
```

3.218.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx=\frac{\sqrt{a+bx^3}(-2aA+3Abx^3-4aBx^3)}{12a^2x^6}+\frac{b(-3Ab+4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}$$

input
```
Integrate[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]),x]
```

output
```
(Sqrt[a + b*x^3]*(-2*a*A + 3*A*b*x^3 - 4*a*B*x^3))/(12*a^2*x^6) + (b*(-3*A
*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(5/2))
```

3.218.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {948, 87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^9\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(-\frac{(3Ab-4aB)\int \frac{1}{x^6\sqrt{bx^3+a}}dx^3}{4a}-\frac{A\sqrt{a+bx^3}}{2ax^6}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{(3Ab-4aB)\left(-\frac{b\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{2a}-\frac{\sqrt{a+bx^3}}{ax^3}\right)}{4a}-\frac{A\sqrt{a+bx^3}}{2ax^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{(3Ab-4aB)\left(-\frac{\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{a}-\frac{\sqrt{a+bx^3}}{ax^3}\right)}{4a}-\frac{A\sqrt{a+bx^3}}{2ax^6}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{(3Ab-4aB)\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^3}}{ax^3}\right)}{4a}-\frac{A\sqrt{a+bx^3}}{2ax^6}\right)$$

input
```
Int[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]),x]
```

output
```
(-1/2*(A*Sqrt[a + b*x^3])/(a*x^6) - ((3*A*b - 4*a*B)*(-(Sqrt[a + b*x^3]/(a
*x^3)) + (b*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/a^(3/2)))/(4*a))/3
```

3.218.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.218.4 Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-3 A b\,x^{3}+4 B a\,x^{3}+2 A a \right)}{12 a^{2} x^{6}}-\frac{b \left(3 A b -4 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{5}{2}}}$	67
pseudoelliptic	$\frac{-x^{6} \left(A b -\frac{4 B a}{3}\right) b \,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)+\left(\frac{2 \left(-2 x^{3} B -A \right) a^{\frac{3}{2}}}{3}+A \sqrt{a}\, b\,x^{3}\right) \sqrt{b\,x^{3}+a}}{4 a^{\frac{5}{2}} x^{6}}$	73
elliptic	$-\frac{b \left(3 A b -4 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{5}{2}}}-\frac{A \sqrt{b\,x^{3}+a}}{6 a\,x^{6}}+\frac{\left(3 A b -4 B a \right) \sqrt{b\,x^{3}+a}}{12 a^{2} x^{3}}$	75
default	$A \left(-\frac{b^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4 a^{\frac{5}{2}}}-\frac{\sqrt{b\,x^{3}+a}}{6 a\,x^{6}}+\frac{b \sqrt{b\,x^{3}+a}}{4 a^{2} x^{3}}\right)+B \left(\frac{b \,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3 a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^{3}+a}}{3 a\,x^{3}}\right)$	102

input

```
int((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/12*(b*x^3+a)^(1/2)*(-3*A*b*x^3+4*B*a*x^3+2*A*a)/a^2/x^6-1/12*b*(3*A*b-4
*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx$$
$$= \left[-\frac{(4\,Bab - 3\,Ab^{2})\sqrt{a}x^{6}\log\left(\frac{bx^{3} - 2\,\sqrt{bx^{3} + a}\sqrt{a} + 2\,a}{x^{3}}\right) + 2\,((4\,Ba^{2} - 3\,Aab)x^{3} + 2\,Aa^{2})\sqrt{bx^{3} + a}}{24\,a^{3}x^{6}},\right.$$
$$\left.-\frac{(4\,Bab - 3\,Ab^{2})\sqrt{-a}x^{6}\arctan\left(\frac{\sqrt{bx^{3} + a}\sqrt{-a}}{a}\right) + ((4\,Ba^{2} - 3\,Aab)x^{3} + 2\,Aa^{2})\sqrt{bx^{3} + a}}{12\,a^{3}x^{6}}\right]$$

input

```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/24*((4*B*a*b - 3*A*b^2)*sqrt(a)*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqr
t(a) + 2*a)/x^3) + 2*((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/
(a^3*x^6), -1/12*((4*B*a*b - 3*A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^3 + a)*
sqrt(-a)/a) + ((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^3*x^
6)]
```

3.218.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(80) = 160$.

Time = 14.46 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.81

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx = -\frac{A}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{A\sqrt{b}}{12ax^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{Ab^{\frac{3}{2}}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4a^{\frac{5}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ax^{\frac{3}{2}}} + \frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}}$$

input
```
integrate((B*x**3+A)/x**7/(b*x**3+a)**(1/2),x)
```

output
```
-A/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + A*sqrt(b)/(12*a*x**(9/2)*s
qrt(a/(b*x**3) + 1)) + A*b**(3/2)/(4*a**2*x**(3/2)*sqrt(a/(b*x**3) + 1)) -
 A*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(5/2)) - B*sqrt(b)*sqrt(a/
(b*x**3) + 1)/(3*a*x**(3/2)) + B*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*a*
*(3/2))
```

3.218.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(74) = 148$.

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.98

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx = -\frac{1}{6}B\left(\frac{2\sqrt{bx^3+a}b}{(bx^3+a)a-a^2} + \frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right) + \frac{1}{24}A\left(\frac{3b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2\left(3(bx^3+a)^{\frac{3}{2}}b^2-5\sqrt{bx^3+a}ab^2\right)}{(bx^3+a)^2a^2-2(bx^3+a)a^3+a^4}\right)$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*B*(2*sqrt(b*x^3 + a)*b/((b*x^3 + a)*a - a^2) + b*log((sqrt(b*x^3 + a)
 - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)) + 1/24*A*(3*b^2*log((sqr
t(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2) + 2*(3*(b*x^3
 + a)^(3/2)*b^2 - 5*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a^2 - 2*(b*x^3 +
 a)*a^3 + a^4))
```

3.218.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx = -\frac{\frac{(4\,Bab^2-3\,Ab^3)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{4\,(bx^3+a)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx^3+a}Ba^2b^2-3\,(bx^3+a)^{\frac{3}{2}}Ab^3+5\,\sqrt{bx^3+a}Aab^3}{a^2b^2x^6}}{12\,b}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
-1/12*((4*B*a*b^2 - 3*A*b^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^
2) + (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a^2*b^2 - 3*(b*x^3
 + a)^(3/2)*A*b^3 + 5*sqrt(b*x^3 + a)*A*a*b^3)/(a^2*b^2*x^6))/b
```

3.218.9 Mupad [B] (verification not implemented)

Time = 7.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx = \frac{\sqrt{b\,x^3+a}\,(3\,A\,b-4\,B\,a)}{12\,a^2\,x^3}-\frac{A\,\sqrt{b\,x^3+a}}{6\,a\,x^6}+\frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,(3\,A\,b-4\,B\,a)}{24\,a^{5/2}}$$

input
```
int((A + B*x^3)/(x^7*(a + b*x^3)^(1/2)),x)
```

output

```
((a + b*x^3)^(1/2)*(3*A*b - 4*B*a))/(12*a^2*x^3) - (A*(a + b*x^3)^(1/2))/(
6*a*x^6) + (b*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^
(1/2)))/x^6)*(3*A*b - 4*B*a))/(24*a^(5/2))
```

3.218. $\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx$

3.219 $\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

3.219.1 Optimal result

Integrand size = 22, antiderivative size = 270

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{2(11Ab-8aB)x\sqrt{a+bx^3}}{55b^2}+\frac{2Bx^4\sqrt{a+bx^3}}{11b}$$
$$-\frac{4\sqrt{2+\sqrt{3}}a(11Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-}{55\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/55*(11*A*b-8*B*a)*x*(b*x^3+a)^(1/2)/b^2+2/11*B*x^4*(b*x^3+a)^(1/2)/b-4/1
65*a*(11*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)*3^(3/4)/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.219.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.33

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2x\left(-((a+bx^3)\,(-11Ab+8aB-5bBx^3))+a(-11Ab+8aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\right.\right.}{55b^2\sqrt{a+bx^3}}$$

input
```
Integrate[(x^3*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*x*(-((a + b*x^3)*(-11*A*b + 8*a*B - 5*b*B*x^3)) + a*(-11*A*b + 8*a*B)*S
qrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(55*b^
2*Sqrt[a + b*x^3])
```

3.219.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx^3\right)}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(11Ab-8aB)\int \frac{x^3}{\sqrt{bx^3+a}}dx}{11b}+\frac{2Bx^4\sqrt{a+bx^3}}{11b}$$

$$\Big\downarrow \text{843}$$

$$\frac{(11Ab-8aB)\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{2a\int \frac{1}{\sqrt{bx^3+a}}dx}{5b}\right)}{11b}+\frac{2Bx^4\sqrt{a+bx^3}}{11b}$$

$$\Big\downarrow \text{759}$$

$$\frac{(11Ab-8aB)\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{11b}$$

$$\frac{2Bx^4\sqrt{a+bx^3}}{11b}$$

input
```
Int[(x^3*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*B*x^4*Sqrt[a + b*x^3])/(11*b) + ((11*A*b - 8*a*B)*((2*x*Sqrt[a + b*x^3]
)/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(11*b
)
```

3.219.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

3.219. $\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.219.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.20

method	result
risch	$\frac{2x(5bBx^3+11Ab-8Ba)\sqrt{bx^3+a}}{55b^2}+\frac{4i(11Ab-8Ba)a\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-ab^2)^{\frac{1}{3}}}{b}}{-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a}{2b}}}}{}$
elliptic	$\frac{2Bx^4\sqrt{bx^3+a}}{11b}+\frac{2\left(A-\frac{8aB}{11b}\right)x\sqrt{bx^3+a}}{5b}+\frac{4ia\left(A-\frac{8aB}{11b}\right)\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-ab}{b}}{-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+i}}}{}$
default	$B\left(\frac{2x^4\sqrt{bx^3+a}}{11b}-\frac{16ax\sqrt{bx^3+a}}{55b^2}-\frac{32ia^2\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-ab^2)^{\frac{1}{3}}}{b}}{-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}}}}{}\right.$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/55*x*(5*B*b*x^3+11*A*b-8*B*a)*(b*x^3+a)^(1/2)/b^2+4/165*I*(11*A*b-8*B*a)
*a/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.219.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$$
$$=\frac{2\left(2\,(8\,Ba^2-11\,Aab)\sqrt{b}\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(5\,Bb^2x^4-(8\,Bab-11\,Ab^2)x)\sqrt{bx^3+a}\right)}{55\,b^3}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/55*(2*(8*B*a^2 - 11*A*a*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (
5*B*b^2*x^4 - (8*B*a*b - 11*A*b^2)*x)*sqrt(b*x^3 + a))/b^3
```

3.219.6 Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.30

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)}+\frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**3*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
A*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*s
qrt(a)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(10/3))
```

3.219.7 Maxima [F]

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x^3}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^3/sqrt(b*x^3 + a), x)
```

3.219.8 Giac [F]

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x^3}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^3/sqrt(b*x^3 + a), x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{x^3\,(B\,x^3+A)}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

output
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(1/2), x)
```

3.220 $\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx$

3.220.1 Optimal result

Integrand size = 19, antiderivative size = 239

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \frac{2Bx\sqrt{a+bx^3}}{5b} + \frac{2\sqrt{2+\sqrt{3}}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/5*B*x*(b*x^3+a)^(1/2)/b+2/15*(5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF
((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(4/3)/(b*x^3+a)^(1/2)/(a
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.220.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \frac{2Bx(a+bx^3)+(5Ab-2aB)x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/Sqrt[a + b*x^3],x]
```

output
```
(2*B*x*(a + b*x^3) + (5*A*b - 2*a*B)*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(5*b*Sqrt[a + b*x^3])
```

3.220.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {913, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(5Ab-2aB)\int \frac{1}{\sqrt{bx^3+a}}dx}{5b}+\frac{2Bx\sqrt{a+bx^3}}{5b}$$

$$\Big\downarrow \text{759}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{2Bx\sqrt{a+bx^3}}{5b}$$

input
```
Int[(A + B*x^3)/Sqrt[a + b*x^3],x]
```

output
```
(2*B*x*Sqrt[a + b*x^3])/(5*b) + (2*Sqrt[2 + Sqrt[3]]*(5*A*b - 2*a*B)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4
)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])
```

3.220.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.220.4 Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.29

method	result
risch	$\frac{2Bx\sqrt{b\,x^3+a}}{5b}-\frac{2i(5Ab-2Ba)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right.}{2b}\right.}{}}}{15b^2\sqrt{b\,x}}$
elliptic	$\frac{2Bx\sqrt{b\,x^3+a}}{5b}-\frac{2i\left(A-\frac{2aB}{5b}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)}{2b}\right.}{}}}{3b\sqrt{b\,x^3}}$
default	$-\frac{2iA\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{3b\sqrt{b\,x^3+a}}$

input
```
int((B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/5*B*x*(b*x^3+a)^(1/2)/b-2/15*I*(5*A*b-2*B*a)/b^2*3^(1/2)*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.220.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.18

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx=\frac{2\left(\sqrt{bx^3+a}Bbx-(2\,Ba-5\,Ab)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{5\,b^2}$$

input

```
integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output

```
2/5*(sqrt(b*x^3 + a)*B*b*x - (2*B*a - 5*A*b)*sqrt(b)*weierstrassPInverse(0
, -4*a/b, x))/b^2
```

3.220.6 Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.33

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx=\frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)}$$

input

```
integrate((B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output

```
A*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*gamma(7/3))
```

3.220.7 Maxima [F]

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/sqrt(b*x^3 + a), x)
```

3.220.8 Giac [F]

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/sqrt(b*x^3 + a), x)
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/(a + b*x^3)^(1/2),x)
```

output
```
int((A + B*x^3)/(a + b*x^3)^(1/2), x)
```

3.221 $\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx$

3.221.1 Optimal result

Integrand size = 22, antiderivative size = 243

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{2ax^2}$$
$$-\frac{\sqrt{2+\sqrt{3}}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\right.}{2\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/2*A*(b*x^3+a)^(1/2)/a/x^2-1/6*(A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF
((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(1/3)/(b*x^3+a)^(1/2)/
(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.221.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.32

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = \frac{-2A(a+bx^3)+(-Ab+4aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{4ax^2\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^3*Sqrt[a + b*x^3]),x]
```

output
```
(-2*A*(a + b*x^3) + (-(A*b) + 4*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometri
c2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(4*a*x^2*Sqrt[a + b*x^3])
```

3.221.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(Ab-4aB)\int \frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{A\sqrt{a+bx^3}}{2ax^2}$$

$$\Big\downarrow \; 759$$

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{A\sqrt{a+bx^3}}{2ax^2}$$

input
```
Int[(A + B*x^3)/(x^3*Sqrt[a + b*x^3]),x]
```

output
```
-1/2*(A*Sqrt[a + b*x^3])/(a*x^2) - (Sqrt[2 + Sqrt[3]]*(A*b - 4*a*B)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)
*a*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

3.221.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.221.4 Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.28

method	result
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{2a\,x^2}-\frac{2i\left(B-\frac{Ab}{4a}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{}}}{3b\sqrt{b\,x^3+}}$
risch	$-\frac{A\sqrt{b\,x^3+a}}{2a\,x^2}+\frac{i(Ab-4Ba)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{(}}}{6ab\sqrt{b\,x^3}}$
default	$-\frac{2iB\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{3b\sqrt{b\,x^3+a}}$

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*A*(b*x^3+a)^(1/2)/a/x^2-2/3*I*(B-1/4*A/a*b)*3^(1/2)/b*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.221.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.21

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = \frac{(4\,Ba-Ab)\sqrt{b}x^2\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\sqrt{bx^3+a}Ab}{2\,abx^2}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((4*B*a - A*b)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) - sqrt(b*
x^3 + a)*A*b)/(a*b*x^2)
```

3.221.6 Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.34

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**3/(b*x**3+a)**(1/2),x)
```

output
```
A*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

3.221.7 Maxima [F]

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}x^3}\,dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^3), x)
```

3.221.8 Giac [F]

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}x^3}\,dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^3), x)
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{x^3\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(1/2)), x)
```

3.222 $\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx$

3.222.1 Optimal result

Integrand size = 22, antiderivative size = 274

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{5ax^5}+\frac{(7Ab-10aB)\sqrt{a+bx^3}}{20a^2x^2}$$
$$+\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}(7Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),}{20\sqrt[4]{3}a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/5*A*(b*x^3+a)^(1/2)/a/x^5+1/20*(7*A*b-10*B*a)*(b*x^3+a)^(1/2)/a^2/x^2+1
/60*b^(2/3)*(7*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3
)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^2/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.222.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.28

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = \frac{-4A(a+bx^3)+(7Ab-10aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{20ax^5\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^6*Sqrt[a + b*x^3]),x]
```

output
```
(-4*A*(a + b*x^3) + (7*A*b - 10*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometri
c2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])/(20*a*x^5*Sqrt[a + b*x^3])
```

3.222.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(7Ab-10aB)\int \frac{1}{x^3\sqrt{bx^3+a}}dx}{10a}-\frac{A\sqrt{a+bx^3}}{5ax^5}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(7Ab-10aB)\left(-\frac{b\int \frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{10a}-\frac{A\sqrt{a+bx^3}}{5ax^5}$$

$$\Big\downarrow \text{759}$$

$$-\frac{(7Ab-10aB)\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+bx}}{2ax^2}\right.}{10a}$$

$$\frac{A\sqrt{a+bx^3}}{5ax^5}$$

input
```
Int[(A + B*x^3)/(x^6*Sqrt[a + b*x^3]),x]
```

output
```
-1/5*(A*Sqrt[a + b*x^3])/(a*x^5) - ((7*A*b - 10*a*B)*(-1/2*Sqrt[a + b*x^3]
/(a*x^2) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(10*a)
```

3.222.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.222.4 Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-7Ab\,x^{3}+10Ba\,x^{3}+4Aa\right)}{20a^{2}x^{5}}-\frac{i(7Ab-10Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{5a\,x^{5}}+\frac{(7Ab-10Ba)\sqrt{b\,x^{3}+a}}{20a^{2}x^{2}}-\frac{i(7Ab-10Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+}}}{}$
default	$B\left(-\frac{\sqrt{b\,x^{3}+a}}{2a\,x^{2}}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}\right)}{\left(-a\,b^{2}\right)}}}{6a\sqrt{b\,x^{3}+}}\right.$

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(b*x^3+a)^(1/2)*(-7*A*b*x^3+10*B*a*x^3+4*A*a)/a^2/x^5-1/60*I*(7*A*b-
10*B*a)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))
```

3.222.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.23

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = -\frac{(10\,Ba-7\,Ab)\sqrt{b}x^5\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left((10\,Ba-7\,Ab)x^3+4\,Aa\right)\sqrt{bx^3+a}}{20\,a^2x^5}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/20*((10*B*a - 7*A*b)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) + ((
10*B*a - 7*A*b)*x^3 + 4*A*a)*sqrt(b*x^3 + a))/(a^2*x^5)
```

3.222.6 Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.33

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{1}{2}\\ -\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{B\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**6/(b*x**3+a)**(1/2),x)
```

output
```
A*gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqr
t(a)*x**5*gamma(-2/3)) + B*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*x**2*gamma(1/3))
```

3.222.7 Maxima [F]

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+ax^6}}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^6), x)
```

3.222.8 Giac [F]

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+ax^6}}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^6), x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{x^6\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(1/2)), x)
```

3.223 $\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

3.223.1 Optimal result

Integrand size = 22, antiderivative size = 548

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2} + \frac{2Bx^5\sqrt{a+bx^3}}{13b} - \frac{8a(13Ab-10aB)\sqrt{a+bx^3}}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+ \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\Bigg(\arcsin\Bigg(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\Bigg)|-}{91b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{8\sqrt{2}a^{4/3}(13Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\Bigg(\arcsin\Bigg(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\Bigg),-7}{91\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/91*(13*A*b-10*B*a)*x^2*(b*x^3+a)^(1/2)/b^2+2/13*B*x^5*(b*x^3+a)^(1/2)/b-
8/91*a*(13*A*b-10*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))-8/273*a^(4/3)*(13*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1
/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))^2)^(1/2)*3^(3/4)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/91*3^(1/4)*a^(4/3)*(13*A*b-10*B*
a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^
(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

3.223.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.17

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2x^2\left(-((a+bx^3)(-13Ab+10aB-7bBx^3))+a(-13Ab+10aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3}\right.\right.}{91b^2\sqrt{a+bx^3}}$$

input
```
Integrate[(x^4*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*x^2*(-((a + b*x^3)*(-13*A*b + 10*a*B - 7*b*B*x^3)) + a*(-13*A*b + 10*a*
B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(9
1*b^2*Sqrt[a + b*x^3])
```

3.223.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(13Ab-10aB)\int \frac{x^4}{\sqrt{bx^3+a}}dx}{13b}+\frac{2Bx^5\sqrt{a+bx^3}}{13b}$$

$$\Big\downarrow \; 843$$

$$\frac{(13Ab-10aB)\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\int \frac{x}{\sqrt{bx^3+a}}dx}{7b}\right)}{13b}+\frac{2Bx^5\sqrt{a+bx^3}}{13b}$$

$$\Big\downarrow \; 832$$

$$\frac{(13Ab-10aB)\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b}\right)}{13b}+\frac{2Bx^5\sqrt{a+bx^3}}{13b}$$

$$\Big\downarrow \; 759$$

$$\frac{(13Ab-10aB)\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\text{arcsi}\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{7b}\right.}{13b}$$

$$\frac{2Bx^5\sqrt{a+bx^3}}{13b}$$

$$\Big\downarrow \; 2416$$

3.223. $\int \frac{x^4\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$

$$\frac{(13Ab-10aB)\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}}{\sqrt[3]{b}}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{\sqrt[3]{b}}\right.}{13}$$

$$\frac{2Bx^5\sqrt{a+bx^3}}{13b}$$

input
```
Int[(x^4*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*B*x^5*Sqrt[a + b*x^3])/(13*b) + ((13*A*b - 10*a*B)*((2*x^2*Sqrt[a + b*x
^3])/(7*b) - (4*a*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/
b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]))/(7*b)))/(13*b)
```

3.223. $\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

3.223.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.223. $\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

3.223.4 Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.87

method	result
risch	$\frac{2x^2\left(7bB\,x^3+13Ab-10Ba\right)\sqrt{b\,x^3+a}}{91b^2}+\frac{8i(13Ab-10Ba)a\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}}}}{}$
elliptic	$\frac{2B\,x^5\sqrt{b\,x^3+a}}{13b}+\frac{2\left(A-\frac{10aB}{13b}\right)x^2\sqrt{b\,x^3+a}}{7b}+\frac{8ia\left(A-\frac{10aB}{13b}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}}{}$
default	Expression too large to display

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/91*x^2*(7*B*b*x^3+13*A*b-10*B*a)/b^2*(b*x^3+a)^(1/2)+8/273*I*(13*A*b-10*
B*a)*a/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^
(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.223.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.14

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = -\frac{2\left(4\left(10\,Ba^2-13\,Aab\right)\sqrt{b}\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-\left(7\,Bb^2x^5-\left(10\,B\right.\right.}{91\,b^3}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/91*(4*(10*B*a^2 - 13*A*a*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstr
assPInverse(0, -4*a/b, x)) - (7*B*b^2*x^5 - (10*B*a*b - 13*A*b^2)*x^2)*sqr
t(b*x^3 + a))/b^3
```

3.223.6 Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{Ax^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}+\frac{Bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**4*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
A*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*s
qrt(a)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(11/3))
```

3.223.7 Maxima [F]

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)x^4}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^4/sqrt(b*x^3 + a), x)
```

3.223.8 Giac [F]

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)x^4}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^4/sqrt(b*x^3 + a), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{x^4\,(B\,x^3+A)}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

output
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(1/2), x)
```

3.224 $\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

3.224.1 Optimal result

Integrand size = 20, antiderivative size = 517

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2Bx^2\sqrt{a+bx^3}}{7b} + \frac{2(7Ab-4aB)\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2}\sqrt[3]{a}(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/7*B*x^2*(b*x^3+a)^(1/2)/b+2/7*(7*A*b-4*B*a)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))+2/21*a^(1/3)*(7*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)
*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/7*3^(1/4)*a^
(1/3)*(7*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.224.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.15

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{x^2\left(4B(a+bx^3)+(7Ab-4aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{14b\sqrt{a+bx^3}}$$

input
```
Integrate[(x*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(x^2*(4*B*(a + b*x^3) + (7*A*b - 4*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(14*b*Sqrt[a + b*x^3])
```

3.224.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {959, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \ 959$$

$$\frac{(7Ab-4aB)\int \frac{x}{\sqrt{bx^3+a}}dx}{7b}+\frac{2Bx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \ 832$$

$$\frac{(7Ab-4aB)\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b}+\frac{2Bx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \ 759$$

$$\frac{(7Ab-4aB)\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{\right.}}{\sqrt[3]{b}x+\left(1+\sqrt{\right.}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{7b}$$

$$\frac{2Bx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \ 2416$$

$$\frac{(7Ab-4aB)\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{\right.}\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{7b}$$

$$\frac{2Bx^2\sqrt{a+bx^3}}{7b}$$

input
```
Int[(x*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

3.224. $\int \frac{x\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$

output
```
(2*B*x^2*Sqrt[a + b*x^3])/(7*b) + ((7*A*b - 4*a*B)*(((2*Sqrt[a + b*x^3])/(
b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*
a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3
]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[
3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(7*b)
```

3.224.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.224.4 Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.90

method	result
risch	$\frac{2B\,x^2\sqrt{b\,x^3+a}}{7b} - \frac{2i(7Ab-4Ba)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\right.}{2}\right.}{}}}{}$
elliptic	$\frac{2B\,x^2\sqrt{b\,x^3+a}}{7b} - \frac{2i\left(A-\frac{4aB}{7b}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b\right.}{2b}\right.}{}}}{}$
default	Expression too large to display

input

```
int(x*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

3.224. $\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

output
```
2/7*B*x^2*(b*x^3+a)^(1/2)/b-2/21*I*(7*A*b-4*B*a)/b^2*3^(1/2)*(-a*b^2)^(1/3
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.224.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.10

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt{bx^3+a}Bbx^2+(4\,Ba-7\,Ab)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)\right)}{7\,b^2}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/7*(sqrt(b*x^3 + a)*B*b*x^2 + (4*B*a - 7*A*b)*sqrt(b)*weierstrassZeta(0,
-4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b^2
```

3.224.6 Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{Ax^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{Bx^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
A*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*s
qrt(a)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))
```

3.224.7 Maxima [F]

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)x}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x/sqrt(b*x^3 + a), x)
```

3.224.8 Giac [F]

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)x}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x/sqrt(b*x^3 + a), x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{x\,(B\,x^3+A)}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((x*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

output
```
int((x*(A + B*x^3))/(a + b*x^3)^(1/2), x)
```

3.225 $\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx$

3.225.1 Optimal result

Integrand size = 22, antiderivative size = 509

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{ax}+\frac{(Ab+2aB)\sqrt{a+bx^3}}{ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\Bigg(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}}{2a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\sqrt{2}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-A*(b*x^3+a)^(1/2)/a/x+(A*b+2*B*a)*(b*x^3+a)^(1/2)/a/b^(2/3)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))+1/3*(A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1
/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))^2)^(1/2)*3^(3/4)/a^(2/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/2*3^(1/4)*(A*b+2*B*a)*(a
^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)
/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)
```

3.225.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = \frac{-4A(a+bx^3)+(Ab+2aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{4ax\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^2*Sqrt[a + b*x^3]),x]
```

output
```
(-4*A*(a + b*x^3) + (A*b + 2*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F
1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(4*a*x*Sqrt[a + b*x^3])
```

3.225.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(2aB+Ab)\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{A\sqrt{a+bx^3}}{ax}$$

$$\Big\downarrow \text{832}$$

$$\frac{(2aB+Ab)\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{A\sqrt{a+bx^3}}{ax}$$

$$\Big\downarrow \text{759}$$

$$\frac{(2aB+Ab)\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right.}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{2a}$$

$$\frac{A\sqrt{a+bx^3}}{ax}$$

$$\Big\downarrow \text{2416}$$

$$\frac{(2aB+Ab)\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{2a}$$

$$\frac{A\sqrt{a+bx^3}}{ax}$$

input

```
Int[(A + B*x^3)/(x^2*Sqrt[a + b*x^3]),x]
```

3.225. $\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx$

output
```
-((A*Sqrt[a + b*x^3])/(a*x)) + ((A*b + 2*a*B)*(((2*Sqrt[a + b*x^3])/(b^(1/
3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/
3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a
^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]
])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(2*a)
```

3.225.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.225.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.91

method	result
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{a x}-\frac{2 i \left(B+\frac{A b}{2 a}\right) \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3 \left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}}}\,\sqrt{-\frac{i \left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right.}{}}}{}$
risch	$-\frac{A\sqrt{b\,x^{3}+a}}{a x}-\frac{i \left(A b+2 B a\right) \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3 \left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}}}\,\sqrt{-\frac{i \left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right.}{}}}{}$
default	Expression too large to display

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

3.225. $\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx$

output

```
-A*(b*x^3+a)^(1/2)/a/x-2/3*I*(B+1/2*A/a*b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2))
```

3.225.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.11

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = -\frac{(2\,Ba+Ab)\sqrt{b}x\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\sqrt{bx^3+a}Ab}{abx}$$

input

```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output

```
-((2*B*a + A*b)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0
, -4*a/b, x)) + sqrt(b*x^3 + a)*A*b)/(a*b*x)
```

3.225.6 Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.16

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\Gamma\left(\frac{2}{3}\right)}+\frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**2/(b*x**3+a)**(1/2),x)
```

output
```
A*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

3.225.7 Maxima [F]

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^2), x)
```

3.225.8 Giac [F]

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^2), x)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = \int \frac{B\,x^3+A}{x^2\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(1/2)), x)
```

3.226 $\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx$

3.226.1 Optimal result

Integrand size = 22, antiderivative size = 550

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{4ax^4} + \frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x} - \frac{\sqrt[3]{b}(5Ab-8aB)\sqrt{a+bx^3}}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-}{16a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt[3]{b}(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}}{4\sqrt{2}\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/4*A*(b*x^3+a)^(1/2)/a/x^4+1/8*(5*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^2/x-1/8*b
^(1/3)*(5*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-1
/24*b^(1/3)*(5*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/
4)/a^(5/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/16*3^(1/4)*b^(1/3)*(5*A*b-8*B*a)*(a^(1/3)
+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^
3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

3.226.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = \frac{-2A(a+bx^3)+(5Ab-8aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{8ax^4\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^5*Sqrt[a + b*x^3]),x]
```

output
```
(-2*A*(a + b*x^3) + (5*A*b - 8*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/(8*a*x^4*Sqrt[a + b*x^3])
```

3.226.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(5Ab-8aB)\int \frac{1}{x^2\sqrt{bx^3+a}}dx}{8a}-\frac{A\sqrt{a+bx^3}}{4ax^4}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(5Ab-8aB)\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{A\sqrt{a+bx^3}}{4ax^4}$$

$$\Big\downarrow \text{832}$$

$$-\frac{(5Ab-8aB)\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{A\sqrt{a+bx^3}}{4ax^4}$$

$$\Big\downarrow \text{759}$$

$$-\frac{(5Ab-8aB)\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{2a}\right.}{8a}$$

$$\frac{A\sqrt{a+bx^3}}{4ax^4}$$

$$\Big\downarrow \text{2416}$$

3.226. $\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx$

$$-\frac{(5Ab-8aB)\left(\frac{b\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{2}\right.}{8a}$$

$$\frac{A\sqrt{a+bx^3}}{4ax^4}$$

input
```
Int[(A + B*x^3)/(x^5*Sqrt[a + b*x^3]),x]
```

output
```
-1/4*(A*Sqrt[a + b*x^3])/(a*x^4) - ((5*A*b - 8*a*B)*(-(Sqrt[a + b*x^3]/(a*
x)) + (b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) -
 (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
)))/(2*a)))/(8*a)
```

3.226. $\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx$

3.226.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.226.4 Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 481, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,\left(-5Ab\,x^3+8Ba\,x^3+2Aa\right)}{8a^2x^4}+\frac{i(5Ab-8Ba)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-\right.}{2}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{4a\,x^4}+\frac{(5Ab-8Ba)\sqrt{b\,x^3+a}}{8a^2x}+\frac{i(5Ab-8Ba)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-\right.}{}}}}{}$
default	Expression too large to display

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^3+a)^(1/2)*(-5*A*b*x^3+8*B*a*x^3+2*A*a)/a^2/x^4+1/24*I*(5*A*b-8*
B*a)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.226.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = -\frac{(8\,Ba-5\,Ab)\sqrt{b}x^4\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+((8\,Ba-5\,Ab)x^3+2\,A}{8\,a^2x^4}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/8*((8*B*a - 5*A*b)*sqrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstrassPI
nverse(0, -4*a/b, x)) + ((8*B*a - 5*A*b)*x^3 + 2*A*a)*sqrt(b*x^3 + a))/(a^
2*x^4)
```

3.226.6 Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.16

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{B\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**5/(b*x**3+a)**(1/2),x)
```

output
```
A*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqr
t(a)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*x*gamma(2/3))
```

3.226.7 Maxima [F]

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+ax^5}}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^5), x)
```

3.226.8 Giac [F]

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+ax^5}}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^5), x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{x^5\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(1/2)), x)
```

3.227 $\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx$

3.227.1 Optimal result

Integrand size = 22, antiderivative size = 581

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{7ax^7} + \frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4}$$

$$-\frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x} + \frac{5b^{4/3}(11Ab-14aB)\sqrt{a+bx^3}}{112a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}(11Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-}{224a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{5b^{4/3}(11Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-}{56\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/7*A*(b*x^3+a)^(1/2)/a/x^7+1/56*(11*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^2/x^4-
5/112*b*(11*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^3/x+5/112*b^(4/3)*(11*A*b-14*B*a
)*(b*x^3+a)^(1/2)/a^3/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+5/336*b^(4/3)*(11*A*
b-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)*2^(1/
2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)-5/224*3^(1/4)*b^(4/3)*(11*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*E
llipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I
*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(8/3)/(b*x^3+a)^(1/2)/(a
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.227.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx = \frac{-8A(a+bx^3)+(11Ab-14aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},-\frac{bx^3}{a}\right)}{56ax^7\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^8*Sqrt[a + b*x^3]),x]
```

output
```
(-8*A*(a + b*x^3) + (11*A*b - 14*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometr
ic2F1[-4/3, 1/2, -1/3, -((b*x^3)/a)])/(56*a*x^7*Sqrt[a + b*x^3])
```

3.227.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 847, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(11Ab-14aB)\int \frac{1}{x^5\sqrt{bx^3+a}}dx}{14a}-\frac{A\sqrt{a+bx^3}}{7ax^7}$$

$$\Big\downarrow \; 847$$

$$-\frac{(11Ab-14aB)\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+a}}dx}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)}{14a}-\frac{A\sqrt{a+bx^3}}{7ax^7}$$

$$\Big\downarrow \; 847$$

$$-\frac{(11Ab-14aB)\left(-\frac{5b\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)}{14a}-\frac{A\sqrt{a+bx^3}}{7ax^7}$$

$$\Big\downarrow \; 832$$

$$-\frac{(11Ab-14aB)\left(-\frac{5b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)}{14a}-\frac{A\sqrt{a+bx^3}}{7ax^7}$$

$$\Big\downarrow \; 759$$

3.227. $\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}dx$

$$-\frac{(11Ab-14aB)\left(-\frac{5b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+}{\sqrt[3]{b}x+}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{2a}\right.}{8a}\right.}{14a}$$

$$\frac{A\sqrt{a+bx^3}}{7ax^7}$$

$$\Big\downarrow 2416$$

3.227. $\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx$

$$-\frac{(11Ab-14aB)\left(-\frac{5b\left(\frac{b\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{\sqrt[3]{b}}\right.}{}\right.}{}$$

3.227. $\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx$ $\frac{A\sqrt{a+bx^3}}{7ax^7}$

input

```
Int[(A + B*x^3)/(x^8*Sqrt[a + b*x^3]),x]
```

output

```
-1/7*(A*Sqrt[a + b*x^3])/(a*x^7) - ((11*A*b - 14*a*B)*(-1/4*Sqrt[a + b*x^3
]/(a*x^4) - (5*b*(-(Sqrt[a + b*x^3]/(a*x)) + (b*(((2*Sqrt[a + b*x^3])/(b^(
1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(
1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]
*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[
3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(2*a)))/(8*a)))/(14*a)
```

3.227.3.1 Defintions of rubi rules used

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.227.4 Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,\left(55A\,b^2x^6-70B\,x^6ab-22aAb\,x^3+28a^2B\,x^3+16a^2A\right)}{112a^3x^7}-\frac{5ib(11Ab-14Ba)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{5ib(11Ab-14Ba)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-i\sqrt{3}\left(-\right.\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{7a\,x^7}+\frac{(11Ab-14Ba)\sqrt{b\,x^3+a}}{56a^2x^4}-\frac{5b(11Ab-14Ba)\sqrt{b\,x^3+a}}{112a^3x}-$
default	Expression too large to display

input
```
int((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

3.227. $\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx$

output
```
-1/112*(b*x^3+a)^(1/2)*(55*A*b^2*x^6-70*B*a*b*x^6-22*A*a*b*x^3+28*B*a^2*x^
3+16*A*a^2)/a^3/x^7-5/336*I*b*(11*A*b-14*B*a)/a^3*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.227.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx$$

$$= \frac{5\,(14\,Bab-11\,Ab^2)\sqrt{b}x^7\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(5\,(14\,Bab-11\,Ab^2\right.}{112\,a^3x^7}$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
1/112*(5*(14*B*a*b - 11*A*b^2)*sqrt(b)*x^7*weierstrassZeta(0, -4*a/b, weie
rstrassPInverse(0, -4*a/b, x)) + (5*(14*B*a*b - 11*A*b^2)*x^6 - 2*(14*B*a^
2 - 11*A*a*b)*x^3 - 16*A*a^2)*sqrt(b*x^3 + a))/(a^3*x^7)
```

3.227.6 Sympy [A] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.16

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{B\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**8/(b*x**3+a)**(1/2),x)
```

output
```
A*gamma(-7/3)*hyper((-7/3, 1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqr
t(a)*x**7*gamma(-4/3)) + B*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*x**3*
exp_polar(I*pi)/a)/(3*sqrt(a)*x**4*gamma(-1/3))
```

3.227.7 Maxima [F]

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}x^8}\,dx$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^8), x)
```

3.227.8 Giac [F]

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}x^8}\,dx$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^8), x)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{x^8\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/(x^8*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/(x^8*(a + b*x^3)^(1/2)), x)
```

3.228 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.228.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}} - \frac{2a(2Ab-3aB)\sqrt{a+bx^3}}{3b^4} + \frac{2(Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4} + \frac{2B(a+bx^3)^{5/2}}{15b^4}$$

output
```
2/9*(A*b-3*B*a)*(b*x^3+a)^(3/2)/b^4+2/15*B*(b*x^3+a)^(5/2)/b^4-2/3*a^2*(A*
b-B*a)/b^4/(b*x^3+a)^(1/2)-2/3*a*(2*A*b-3*B*a)*(b*x^3+a)^(1/2)/b^4
```

3.228.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(48a^3B-8a^2b(5A-3Bx^3)+b^3x^6(5A+3Bx^3)-2ab^2x^3(10A+3Bx^3))}{45b^4\sqrt{a+bx^3}}$$

input
```
Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*(48*a^3*B - 8*a^2*b*(5*A - 3*B*x^3) + b^3*x^6*(5*A + 3*B*x^3) - 2*a*b^2
*x^3*(10*A + 3*B*x^3)))/(45*b^4*Sqrt[a + b*x^3])
```

3.228.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (A+Bx^3)}{(a+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \textbf{948}$$

$$\frac{1}{3}\int \frac{x^6 (Bx^3+A)}{(bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \textbf{86}$$

$$\frac{1}{3}\int \left(-\frac{(aB-Ab)a^2}{b^3 (bx^3+a)^{3/2}}+\frac{(3aB-2Ab)a}{b^3\sqrt{bx^3+a}}+\frac{B (bx^3+a)^{3/2}}{b^3}+\frac{(Ab-3aB)\sqrt{bx^3+a}}{b^3}\right)dx^3$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{1}{3}\left(-\frac{2a^2(Ab-aB)}{b^4\sqrt{a+bx^3}}+\frac{2(a+bx^3)^{3/2} (Ab-3aB)}{3b^4}-\frac{2a\sqrt{a+bx^3}(2Ab-3aB)}{b^4}+\frac{2B (a+bx^3)^{5/2}}{5b^4}\right)$$

input
```
Int[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
((-2*a^2*(A*b - a*B))/(b^4*Sqrt[a + b*x^3]) - (2*a*(2*A*b - 3*a*B)*Sqrt[a
+ b*x^3])/b^4 + (2*(A*b - 3*a*B)*(a + b*x^3)^(3/2))/(3*b^4) + (2*B*(a + b*
x^3)^(5/2))/(5*b^4))/3
```

3.228.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.228.4 Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{16\left(-\frac{x^{6}\left(\frac{3x^{3}B}{5}+A\right)b^{3}}{8}+\frac{x^{3}\left(\frac{3x^{3}B}{10}+A\right)a\,b^{2}}{2}+a^{2}\left(-\frac{3x^{3}B}{5}+A\right)b-\frac{6a^{3}B}{5}\right)}{9\sqrt{b\,x^{3}+a}\,b^{4}}$
gosper	$-\frac{2\left(-3b^{3}B\,x^{9}-5x^{6}b^{3}A+6B\,x^{6}a\,b^{2}+20aA\,b^{2}x^{3}-24B\,a^{2}b\,x^{3}+40a^{2}bA-48a^{3}B\right)}{45\sqrt{b\,x^{3}+a}\,b^{4}}$
trager	$-\frac{2\left(-3b^{3}B\,x^{9}-5x^{6}b^{3}A+6B\,x^{6}a\,b^{2}+20aA\,b^{2}x^{3}-24B\,a^{2}b\,x^{3}+40a^{2}bA-48a^{3}B\right)}{45\sqrt{b\,x^{3}+a}\,b^{4}}$
risch	$-\frac{2\left(-3b^{2}B\,x^{6}-5A\,b^{2}x^{3}+9Bab\,x^{3}+25abA-33a^{2}B\right)\sqrt{b\,x^{3}+a}}{45b^{4}}-\frac{2a^{2}\left(Ab-Ba\right)}{3b^{4}\sqrt{b\,x^{3}+a}}$
default	$A\left(-\frac{2a^{2}}{3b^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b^{2}}-\frac{10a\sqrt{b\,x^{3}+a}}{9b^{3}}\right)+B\Bigg(\frac{2a^{3}}{3b^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15b^{2}}-\frac{2a\,x^{3}\sqrt{b\,x^{3}+a}}{5b^{3}}+$
elliptic	$-\frac{2a^{2}\left(Ab-Ba\right)}{3b^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2B\,x^{6}\sqrt{b\,x^{3}+a}}{15b^{2}}+\frac{2\left(\frac{Ab-Ba}{b^{2}}-\frac{4Ba}{5b^{2}}\right)x^{3}\sqrt{b\,x^{3}+a}}{9b}+\frac{2\left(-\frac{a\left(Ab-Ba\right)}{b^{3}}-\frac{2\left(\frac{Ab-Ba}{b^{2}}-\frac{4Ba}{5b^{2}}\right)a}{3b}\right)\sqrt{b\,x^{3}+a}}{3b}$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-16/9*(-1/8*x^6*(3/5*x^3*B+A)*b^3+1/2*x^3*(3/10*x^3*B+A)*a*b^2+a^2*(-3/5*x
^3*B+A)*b-6/5*a^3*B)/(b*x^3+a)^(1/2)/b^4
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(3\,Bb^3x^9-(6\,Bab^2-5\,Ab^3)x^6+48\,Ba^3-40\,Aa^2b+4\,(6\,Ba^2b-5\,Aab^2)x^3)\sqrt{bx^3+}}{45\,(b^5x^3+ab^4)}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/45*(3*B*b^3*x^9 - (6*B*a*b^2 - 5*A*b^3)*x^6 + 48*B*a^3 - 40*A*a^2*b + 4*
(6*B*a^2*b - 5*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/(b^5*x^3 + a*b^4)
```

3.228.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.70

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \begin{cases} -\frac{16Aa^2}{9b^3\sqrt{a+bx^3}} - \frac{8Aax^3}{9b^2\sqrt{a+bx^3}} + \frac{2Ax^6}{9b\sqrt{a+bx^3}} + \frac{32Ba^3}{15b^4\sqrt{a+bx^3}} + \frac{16Ba^2x^3}{15b^3\sqrt{a+bx^3}} - \frac{4Bax^6}{15b^2\sqrt{a+bx^3}} + \frac{2Bx^9}{15b\sqrt{a+bx}} \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{3}{2}}} \end{cases}$$

input
```
integrate(x**8*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((-16*A*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*A*a*x**3/(9*b**2*sqrt(
a + b*x**3)) + 2*A*x**6/(9*b*sqrt(a + b*x**3)) + 32*B*a**3/(15*b**4*sqrt(a
 + b*x**3)) + 16*B*a**2*x**3/(15*b**3*sqrt(a + b*x**3)) - 4*B*a*x**6/(15*b
**2*sqrt(a + b*x**3)) + 2*B*x**9/(15*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x
**9/9 + B*x**12/12)/a**(3/2), True))
```

3.228.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2}{15}\,B\left(\frac{(bx^3+a)^{\frac{5}{2}}}{b^4} - \frac{5\,(bx^3+a)^{\frac{3}{2}}a}{b^4} + \frac{15\,\sqrt{bx^3+a}a^2}{b^4} + \frac{5\,a^3}{\sqrt{bx^3+a}b^4}\right) + \frac{2}{9}\,A\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^3} - \frac{6\,\sqrt{bx^3+a}a}{b^3} - \frac{3\,a^2}{\sqrt{bx^3+a}b^3}\right)$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
2/15*B*((b*x^3 + a)^(5/2)/b^4 - 5*(b*x^3 + a)^(3/2)*a/b^4 + 15*sqrt(b*x^3
+ a)*a^2/b^4 + 5*a^3/(sqrt(b*x^3 + a)*b^4)) + 2/9*A*((b*x^3 + a)^(3/2)/b^3
 - 6*sqrt(b*x^3 + a)*a/b^3 - 3*a^2/(sqrt(b*x^3 + a)*b^3))
```

3.228.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(Ba^3-Aa^2b)}{3\,\sqrt{bx^3+a}b^4} + \frac{2\left(3\,(bx^3+a)^{\frac{5}{2}}Bb^{16}-15\,(bx^3+a)^{\frac{3}{2}}Bab^{16}+45\,\sqrt{bx^3+a}Ba^2b^{16}+5\,(bx^3+a)^{\frac{3}{2}}Ab^{17}-30\,\sqrt{bx^3+a}Aab^{17}\right)}{45\,b^{20}}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/3*(B*a^3 - A*a^2*b)/(sqrt(b*x^3 + a)*b^4) + 2/45*(3*(b*x^3 + a)^(5/2)*B*
b^16 - 15*(b*x^3 + a)^(3/2)*B*a*b^16 + 45*sqrt(b*x^3 + a)*B*a^2*b^16 + 5*(
b*x^3 + a)^(3/2)*A*b^17 - 30*sqrt(b*x^3 + a)*A*a*b^17)/b^20
```

3.228.9 Mupad [B] (verification not implemented)

Time = 7.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.48

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,(B\,a^2-A\,a\,b)}{b^3}-\frac{2\,a\,\left(\frac{2\,\left(A\,b^2-B\,a\,b\right)}{b^3}-\frac{8\,B\,a}{5\,b^2}\right)}{3\,b}\right)}{3\,b} + \frac{x^3\,\sqrt{b\,x^3+a}\,\left(\frac{2\,(A\,b^2-B\,a\,b)}{b^3}-\frac{8\,B\,a}{5\,b^2}\right)}{9\,b} - \frac{a^2\,\left(\frac{2\,A}{3\,b}-\frac{2\,B\,a}{3\,b^2}\right)}{b^2\,\sqrt{b\,x^3+a}} + \frac{2\,B\,x^6\,\sqrt{b\,x^3+a}}{15\,b^2}$$

input
```
int((x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

output
```
((a + b*x^3)^(1/2)*((2*(B*a^2 - A*a*b))/b^3 - (2*a*((2*(A*b^2 - B*a*b))/b^
3 - (8*B*a)/(5*b^2)))/(3*b)))/(3*b) + (x^3*(a + b*x^3)^(1/2)*((2*(A*b^2 -
B*a*b))/b^3 - (8*B*a)/(5*b^2)))/(9*b) - (a^2*((2*A)/(3*b) - (2*B*a)/(3*b^2
)))/(b^2*(a + b*x^3)^(1/2)) + (2*B*x^6*(a + b*x^3)^(1/2))/(15*b^2)
```

3.228. $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.229 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.229.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}} + \frac{2(Ab-2aB)\sqrt{a+bx^3}}{3b^3} + \frac{2B(a+bx^3)^{3/2}}{9b^3}$$

output
```
2/9*B*(b*x^3+a)^(3/2)/b^3+2/3*a*(A*b-B*a)/b^3/(b*x^3+a)^(1/2)+2/3*(A*b-2*B
*a)*(b*x^3+a)^(1/2)/b^3
```

3.229.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(6aAb-8a^2B+3Ab^2x^3-4abBx^3+b^2Bx^6)}{9b^3\sqrt{a+bx^3}}$$

input
```
Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*(6*a*A*b - 8*a^2*B + 3*A*b^2*x^3 - 4*a*b*B*x^3 + b^2*B*x^6))/(9*b^3*Sqr
t[a + b*x^3])
```

3.229.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^3)}{(a+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3 (Bx^3+A)}{(bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{\sqrt{bx^3+a}B}{b^2}+\frac{Ab-2aB}{b^2\sqrt{bx^3+a}}+\frac{a(aB-Ab)}{b^2\left(bx^3+a\right)^{3/2}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2\sqrt{a+bx^3}(Ab-2aB)}{b^3}+\frac{2a(Ab-aB)}{b^3\sqrt{a+bx^3}}+\frac{2B\left(a+bx^3\right)^{3/2}}{3b^3}\right)$$

input
```
Int[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
((2*a*(A*b - a*B))/(b^3*Sqrt[a + b*x^3]) + (2*(A*b - 2*a*B)*Sqrt[a + b*x^3
])/b^3 + (2*B*(a + b*x^3)^(3/2))/(3*b^3))/3
```

3.229.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.229.4 Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{\frac{2x^{3}\left(\frac{x^{3}B}{3}+A\right)b^{2}}{3}+\frac{4a\left(-\frac{2x^{3}B}{3}+A\right)b}{3}-\frac{16a^{2}B}{9}}{\sqrt{b\,x^{3}+a}\,b^{3}}$	49
gosper	$\frac{\frac{2}{9}b^{2}B\,x^{6}+\frac{2}{3}A\,b^{2}x^{3}-\frac{8}{9}Bab\,x^{3}+\frac{4}{3}abA-\frac{16}{9}a^{2}B}{\sqrt{b\,x^{3}+a}\,b^{3}}$	52
trager	$\frac{\frac{2}{9}b^{2}B\,x^{6}+\frac{2}{3}A\,b^{2}x^{3}-\frac{8}{9}Bab\,x^{3}+\frac{4}{3}abA-\frac{16}{9}a^{2}B}{\sqrt{b\,x^{3}+a}\,b^{3}}$	52
risch	$\frac{2\left(bB\,x^{3}+3Ab-5Ba\right)\sqrt{b\,x^{3}+a}}{9b^{3}}+\frac{2a\left(Ab-Ba\right)}{3b^{3}\sqrt{b\,x^{3}+a}}$	54
elliptic	$\frac{2a\left(Ab-Ba\right)}{3b^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2B\,x^{3}\sqrt{b\,x^{3}+a}}{9b^{2}}+\frac{2\left(\frac{Ab-Ba}{b^{2}}-\frac{2Ba}{3b^{2}}\right)\sqrt{b\,x^{3}+a}}{3b}$	81
default	$B\left(-\frac{2a^{2}}{3b^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b^{2}}-\frac{10a\sqrt{b\,x^{3}+a}}{9b^{3}}\right)+A\left(\frac{2a}{3b^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2\sqrt{b\,x^{3}+a}}{3b^{2}}\right)$	94

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/3*(1/2*x^3*(1/3*x^3*B+A)*b^2+a*(-2/3*x^3*B+A)*b-4/3*a^2*B)/(b*x^3+a)^(1/
2)/b^3
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(Bb^2x^6-(4\,Bab-3\,Ab^2)x^3-8\,Ba^2+6\,Aab)\sqrt{bx^3+a}}{9\,(b^4x^3+ab^3)}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/9*(B*b^2*x^6 - (4*B*a*b - 3*A*b^2)*x^3 - 8*B*a^2 + 6*A*a*b)*sqrt(b*x^3 +
 a)/(b^4*x^3 + a*b^3)
```

3.229.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.70

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \begin{cases} \frac{4Aa}{3b^2\sqrt{a+bx^3}}+\frac{2Ax^3}{3b\sqrt{a+bx^3}}-\frac{16Ba^2}{9b^3\sqrt{a+bx^3}}-\frac{8Bax^3}{9b^2\sqrt{a+bx^3}}+\frac{2Bx^6}{9b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((4*A*a/(3*b**2*sqrt(a + b*x**3)) + 2*A*x**3/(3*b*sqrt(a + b*x**3
)) - 16*B*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*B*a*x**3/(9*b**2*sqrt(a + b*x
**3)) + 2*B*x**6/(9*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**6/6 + B*x**9/9)
/a**(3/2), True))
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2}{9}\,B\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^3}-\frac{6\,\sqrt{bx^3+a}a}{b^3}-\frac{3\,a^2}{\sqrt{bx^3+a}b^3}\right) + \frac{2}{3}\,A\left(\frac{\sqrt{bx^3+a}}{b^2}+\frac{a}{\sqrt{bx^3+a}b^2}\right)$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
2/9*B*((b*x^3 + a)^(3/2)/b^3 - 6*sqrt(b*x^3 + a)*a/b^3 - 3*a^2/(sqrt(b*x^3
 + a)*b^3)) + 2/3*A*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2))
```

3.229.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2\,(Ba^2-Aab)}{3\,\sqrt{bx^3+a}b^3} + \frac{2\left((bx^3+a)^{\frac{3}{2}}Bb^6 - 6\,\sqrt{bx^3+a}Bab^6 + 3\,\sqrt{bx^3+a}Ab^7\right)}{9\,b^9}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
-2/3*(B*a^2 - A*a*b)/(sqrt(b*x^3 + a)*b^3) + 2/9*((b*x^3 + a)^(3/2)*B*b^6
- 6*sqrt(b*x^3 + a)*B*a*b^6 + 3*sqrt(b*x^3 + a)*A*b^7)/b^9
```

3.229.9 Mupad [B] (verification not implemented)

Time = 7.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,B\,(b\,x^3+a)^2 - 6\,B\,a^2 + 6\,A\,b\,(b\,x^3+a) - 12\,B\,a\,(b\,x^3+a) + 6\,A\,a\,b}{9\,b^3\,\sqrt{b\,x^3+a}}$$

input
```
int((x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

output
```
(2*B*(a + b*x^3)^2 - 6*B*a^2 + 6*A*b*(a + b*x^3) - 12*B*a*(a + b*x^3) + 6*
A*a*b)/(9*b^3*(a + b*x^3)^(1/2))
```

3.230 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.230.1 Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}+\frac{2B\sqrt{a+bx^3}}{3b^2}$$

output
```
-2/3*(A*b-B*a)/b^2/(b*x^3+a)^(1/2)+2/3*B*(b*x^3+a)^(1/2)/b^2
```

3.230.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(-Ab+2aB+bBx^3)}{3b^2\sqrt{a+bx^3}}$$

input
```
Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*(-(A*b) + 2*a*B + b*B*x^3))/(3*b^2*Sqrt[a + b*x^3])
```

3.230.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^3)}{(a+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3} \int \frac{Bx^3+A}{(bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3} \int \left(\frac{B}{b\sqrt{bx^3+a}}+\frac{Ab-aB}{b\,(bx^3+a)^{3/2}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2B\sqrt{a+bx^3}}{b^2}-\frac{2(Ab-aB)}{b^2\sqrt{a+bx^3}}\right)$$

input
```
Int[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
((-2*(A*b - a*B))/(b^2*Sqrt[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/b^2)/3
```

3.230.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.230.4 Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{2\left(-bB\,x^{3}+Ab-2Ba\right)}{3\sqrt{b\,x^{3}+a}\,b^{2}}$	30
trager	$-\frac{2\left(-bB\,x^{3}+Ab-2Ba\right)}{3\sqrt{b\,x^{3}+a}\,b^{2}}$	30
pseudoelliptic	$-\frac{2\left(\left(-x^{3}B+A\right)b-2Ba\right)}{3\sqrt{b\,x^{3}+a}\,b^{2}}$	30
risch	$-\frac{2\left(Ab-Ba\right)}{3b^{2}\sqrt{b\,x^{3}+a}}+\frac{2B\sqrt{b\,x^{3}+a}}{3b^{2}}$	39
elliptic	$-\frac{2\left(Ab-Ba\right)}{3b^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2B\sqrt{b\,x^{3}+a}}{3b^{2}}$	43
default	$B\left(\frac{2a}{3b^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2\sqrt{b\,x^{3}+a}}{3b^{2}}\right)-\frac{2A}{3b\sqrt{b\,x^{3}+a}}$	53

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(b*x^3+a)^(1/2)*(-B*b*x^3+A*b-2*B*a)/b^2
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(Bbx^3+2\,Ba-Ab)\sqrt{bx^3+a}}{3\,(b^3x^3+ab^2)}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(B*b*x^3 + 2*B*a - A*b)*sqrt(b*x^3 + a)/(b^3*x^3 + a*b^2)
```

3.230.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \begin{cases} -\frac{2A}{3b\sqrt{a+bx^3}} + \frac{4Ba}{3b^2\sqrt{a+bx^3}} + \frac{2Bx^3}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((-2*A/(3*b*sqrt(a + b*x**3)) + 4*B*a/(3*b**2*sqrt(a + b*x**3)) +
 2*B*x**3/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/a**(3/
2), True))
```

3.230.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2}{3}\,B\left(\frac{\sqrt{bx^3+a}}{b^2} + \frac{a}{\sqrt{bx^3+a}b^2}\right) - \frac{2\,A}{3\,\sqrt{bx^3+a}b}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
2/3*B*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2)) - 2/3*A/(sqrt(b*x^3
+ a)*b)
```

3.230.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,\sqrt{bx^3+a}B}{3\,b^2} + \frac{2\,(Ba-Ab)}{3\,\sqrt{bx^3+a}b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/3*sqrt(b*x^3 + a)*B/b^2 + 2/3*(B*a - A*b)/(sqrt(b*x^3 + a)*b^2)
```

3.230.9 Mupad [B] (verification not implemented)

Time = 7.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,B\,a-2\,A\,b+2\,B\,(b\,x^3+a)}{3\,b^2\,\sqrt{b\,x^3+a}}$$

input
```
int((x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

output
```
(2*B*a - 2*A*b + 2*B*(a + b*x^3))/(3*b^2*(a + b*x^3)^(1/2))
```

3.231 $\int \frac{A+Bx^3}{x(a+bx^3)^{3/2}}\,dx$

3.231.1 Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx=\frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

output
```
-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)+2/3*(A*b-B*a)/a/b/(b*x^3+a
)^(1/2)
```

3.231.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx=\frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

input
```
Integrate[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]
```

output
```
(2*(A*b - a*B))/(3*a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqr
t[a]])/(3*a^(3/2))
```

3.231.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^3\left(bx^3+a\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{A\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{a}+\frac{2(Ab-aB)}{ab\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2A\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{ab}+\frac{2(Ab-aB)}{ab\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{2(Ab-aB)}{ab\sqrt{a+bx^3}}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

input
```
Int[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]
```

output
```
((2*(A*b - a*B))/(a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/a^(3/2))/3
```

3.231.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.231.4 Maple [A] (verified)

Time = 4.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

method	result	size
elliptic	$\frac{\frac{2Ab}{3}-\frac{2Ba}{3}}{ba\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2A\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}$	51
default	$-\frac{2B}{3b\sqrt{b\,x^{3}+a}}+A\left(\frac{2}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)$	57
pseudoelliptic	$-\frac{2\left(A\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)b\sqrt{b\,x^{3}+a}-Ab\sqrt{a}+B\,a^{\frac{3}{2}}\right)}{3a^{\frac{3}{2}}\sqrt{b\,x^{3}+a}\,b}$	57

input
```
int((B*x^3+A)/x/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b*(A*b-B*a)/a/((x^3+a/b)*b)^(1/2)-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2
))/a^(3/2)
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.93

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx = \left[\frac{\left(Ab^2x^3+Aab\right)\sqrt{a}\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)-2\sqrt{bx^3+a}\left(Ba^2-Aab\right)}{3\left(a^2b^2x^3+a^3b\right)},\ 2\left(\left(Ab^2x^3+\right.\right.\right.$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/3*((A*b^2*x^3 + A*a*b)*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) +
 2*a)/x^3) - 2*sqrt(b*x^3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b), 2/3
*((A*b^2*x^3 + A*a*b)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) - sqrt(b
*x^3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b)]
```

3.231.6 Sympy [A] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{Ab\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3a\sqrt{-a}}-\frac{-Ab+Ba}{3a\sqrt{a+bx^3}}\right)}{b} & \text{for}\ b\neq 0 \\ \frac{A\log\left(Bx^3\right)+Bx^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x**3+A)/x/(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((2*(A*b*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*a*sqrt(-a)) - (-A*b +
 B*a)/(3*a*sqrt(a + b*x**3)))/b, Ne(b, 0)), ((A*log(B*x**3) + B*x**3)/(3*a
**(3/2)), True))
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx = \frac{1}{3}\,A\left(\frac{\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{bx^3+a}a}\right) - \frac{2\,B}{3\,\sqrt{bx^3+a}b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
1/3*A*(log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2
) + 2/(sqrt(b*x^3 + a)*a)) - 2/3*B/(sqrt(b*x^3 + a)*b)
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx = \frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a} - \frac{2\,(Ba-Ab)}{3\,\sqrt{bx^3+a}ab}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - 2/3*(B*a - A*b)/(sqr
t(b*x^3 + a)*a*b)
```

3.231.9 Mupad [B] (verification not implemented)

Time = 7.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx = \frac{\frac{2\,A}{3\,a}-\frac{2\,B}{3\,b}}{\sqrt{b\,x^3+a}} + \frac{A\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{3/2}}$$

input
```
int((A + B*x^3)/(x*(a + b*x^3)^(3/2)),x)
```

output
```
((2*A)/(3*a) - (2*B)/(3*b))/(a + b*x^3)^(1/2) + (A*log((((a + b*x^3)^(1/2)
 - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(3/2))
```

3.232 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx$

3.232.1 Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx=\frac{-3Ab+2aB}{3a^2\sqrt{a+bx^3}}-\frac{A}{3ax^3\sqrt{a+bx^3}}+\frac{(3Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

output
```
1/3*(3*A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)+1/3*(-3*A*b+2*B
*a)/a^2/(b*x^3+a)^(1/2)-1/3*A/a/x^3/(b*x^3+a)^(1/2)
```

3.232.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx=\frac{-aA-3Abx^3+2aBx^3}{3a^2x^3\sqrt{a+bx^3}}+\frac{(3Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

input
```
Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]
```

output
```
(-(a*A) - 3*A*b*x^3 + 2*a*B*x^3)/(3*a^2*x^3*Sqrt[a + b*x^3]) + ((3*A*b - 2
*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

3.232.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {948, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^6\left(bx^3+a\right)^{3/2}}dx^3$$

$$\Big\downarrow \; 87$$

$$\frac{1}{3}\left(-\frac{(3Ab-2aB)\int \frac{1}{x^3\left(bx^3+a\right)^{3/2}}dx^3}{2a}-\frac{A}{ax^3\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{3}\left(-\frac{(3Ab-2aB)\left(\frac{\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{a}+\frac{2}{a\sqrt{a+bx^3}}\right)}{2a}-\frac{A}{ax^3\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(-\frac{(3Ab-2aB)\left(\frac{2\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{ab}+\frac{2}{a\sqrt{a+bx^3}}\right)}{2a}-\frac{A}{ax^3\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(-\frac{(3Ab-2aB)\left(\frac{2}{a\sqrt{a+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{A}{ax^3\sqrt{a+bx^3}}\right)$$

input

```
Int[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]
```

output
```
(-(A/(a*x^3*Sqrt[a + b*x^3])) - ((3*A*b - 2*a*B)*(2/(a*Sqrt[a + b*x^3]) -
(2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/a^(3/2)))/(2*a))/3
```

3.232.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.232.4 Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

method	result
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{3a^{2}x^{3}}-\frac{2(Ab-Ba)}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{(3Ab-2Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}$
pseudoelliptic	$\frac{\sqrt{b\,x^{3}+a}\,x^{3}\left(Ab-\frac{2Ba}{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)+\frac{\left(2x^{3}B-A\right)a^{\frac{3}{2}}}{3}-A\sqrt{a}\,b\,x^{3}}{\sqrt{b\,x^{3}+a}\,a^{\frac{5}{2}}x^{3}}$
risch	$-\frac{A\sqrt{b\,x^{3}+a}}{3a^{2}x^{3}}-\frac{-\frac{2bA}{3\sqrt{b\,x^{3}+a}}+a(3Ab-2Ba)\left(\frac{2}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)}{2a^{2}}$
default	$B\left(\frac{2}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)+A\left(-\frac{\sqrt{b\,x^{3}+a}}{3a^{2}x^{3}}-\frac{2b}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2*A*(b*x^3+a)^(1/2)/x^3-2/3*(A*b-B*a)/a^2/((x^3+a/b)*b)^(1/2)+1/3*(
3*A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.71

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx=\left[-\frac{\left((2\,Bab-3\,Ab^2)x^6+(2\,Ba^2-3\,Aab)x^3\right)\sqrt{a}\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)-2\,((2\,Ba^2}{6\,(a^3bx^6+a^4x^3)}\right.$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/6*(((2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3)*sqrt(a)*log((b*
x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) - 2*((2*B*a^2 - 3*A*a*b)*x^3 -
 A*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3), 1/3*(((2*B*a*b - 3*A*b^2)*
x^6 + (2*B*a^2 - 3*A*a*b)*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a)
 + ((2*B*a^2 - 3*A*a*b)*x^3 - A*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3
)]
```

3.232.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(73) = 146$.

Time = 23.53 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.07

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx = A\left(-\frac{1}{3a\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}}{a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{a^{\frac{5}{2}}}\right) + B\left(\frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3} + \frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3} + \frac{a^2bx^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3} - \frac{2a^2bx^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}\right)$$

input
```
integrate((B*x**3+A)/x**4/(b*x**3+a)**(3/2),x)
```

output
```
A*(-1/(3*a*sqrt(b)*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(a**2*x**(3/2)
*sqrt(a/(b*x**3) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/a**(5/2)) + B
*(2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*
x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) +
 1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/
2) + 3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**
(9/2) + 3*a**(7/2)*b*x**3))
```

3.232.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.67

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx = -\frac{1}{6}A\left(\frac{2\left(3\left(bx^3+a\right)b-2\,ab\right)}{\left(bx^3+a\right)^{\frac{3}{2}}a^2-\sqrt{bx^3+a}\,a^3} + \frac{3\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right) + \frac{1}{3}B\left(\frac{\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{bx^3+a}\,a}\right)$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/6*A*(2*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^(3/2)*a^2 - sqrt(b*x^3 +
a)*a^3) + 3*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))
/a^(5/2)) + 1/3*B*(log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt
(a)))/a^(3/2) + 2/(sqrt(b*x^3 + a)*a))
```

3.232.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx = \frac{\left(2\,Ba-3\,Ab\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a^2} + \frac{2\,(bx^3+a)Ba-2\,Ba^2-3\,(bx^3+a)Ab+2\,Aab}{3\,\left((bx^3+a)^{\frac{3}{2}}-\sqrt{bx^3+a}a\right)a^2}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
1/3*(2*B*a - 3*A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/3*
(2*(b*x^3 + a)*B*a - 2*B*a^2 - 3*(b*x^3 + a)*A*b + 2*A*a*b)/(((b*x^3 + a)^
(3/2) - sqrt(b*x^3 + a)*a)*a^2)
```

3.232.9 Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.52

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\,{\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}^3}{x^6}\right)\,\left(3\,A\,b-2\,B\,a\right)}{6\,a^{5/2}} - \frac{\frac{2\,B\,a^2-3\,A\,a\,b}{2\,a^3}-\frac{a\,\left(\frac{A\,b^2}{3\,a^3}+\frac{5\,b\,\left(2\,B\,a^2-3\,A\,a\,b\right)}{6\,a^4}\right)}{b}}{\sqrt{b\,x^3+a}} - \frac{A\,\sqrt{b\,x^3+a}}{3\,a^2\,x^3}$$

input
```
int((A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x)
```

output
```
(log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*
(3*A*b - 2*B*a))/(6*a^(5/2)) - ((2*B*a^2 - 3*A*a*b)/(2*a^3) - (a*((A*b^2)/
(3*a^3) + (5*b*(2*B*a^2 - 3*A*a*b))/(6*a^4)))/b)/(a + b*x^3)^(1/2) - (A*(a
 + b*x^3)^(1/2))/(3*a^2*x^3)
```

3.232. $\int \frac{A+Bx^3}{x^4(a+bx^3)^{3/2}}\,dx$

3.233 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx$

3.233.1 Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \frac{b(5Ab-4aB)}{4a^3\sqrt{a+bx^3}} - \frac{A}{6ax^6\sqrt{a+bx^3}} + \frac{5Ab-4aB}{12a^2x^3\sqrt{a+bx^3}} - \frac{b(5Ab-4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}$$

output

```
-1/4*b*(5*A*b-4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(7/2)+1/4*b*(5*A*b
-4*B*a)/a^3/(b*x^3+a)^(1/2)-1/6*A/a/x^6/(b*x^3+a)^(1/2)+1/12*(5*A*b-4*B*a)
/a^2/x^3/(b*x^3+a)^(1/2)
```

3.233.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \frac{-2a^2A+5aAbx^3-4a^2Bx^3+15Ab^2x^6-12abBx^6}{12a^3x^6\sqrt{a+bx^3}} + \frac{b(-5Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}$$

input

```
Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*a^2*A + 5*a*A*b*x^3 - 4*a^2*B*x^3 + 15*A*b^2*x^6 - 12*a*b*B*x^6)/(12*a
^3*x^6*Sqrt[a + b*x^3]) + (b*(-5*A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/(4*a^(7/2))
```

3.233.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^7 \left(a+bx^3\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{Bx^3+A}{x^9 \left(bx^3+a\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3} \left(-\frac{(5Ab-4aB) \int \frac{1}{x^6 \left(bx^3+a\right)^{3/2}}dx^3}{4a}-\frac{A}{2ax^6 \sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3} \left(-\frac{(5Ab-4aB) \left(-\frac{3b \int \frac{1}{x^3 \left(bx^3+a\right)^{3/2}}dx^3}{2a}-\frac{1}{ax^3 \sqrt{a+bx^3}}\right)}{4a}-\frac{A}{2ax^6 \sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3} \left(-\frac{(5Ab-4aB) \left(-\frac{3b \left(\frac{\int \frac{1}{x^3 \sqrt{bx^3+a}}dx^3}{a}+\frac{2}{a \sqrt{a+bx^3}}\right)}{2a}-\frac{1}{ax^3 \sqrt{a+bx^3}}\right)}{4a}-\frac{A}{2ax^6 \sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \textcolor{blue}{73}$$

$$\frac{1}{3}\left(-\frac{(5Ab-4aB)\left(-\frac{3b\left(\frac{2\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{ab}+\frac{2}{a\sqrt{a+bx^3}}\right)}{2a}-\frac{1}{ax^3\sqrt{a+bx^3}}\right)}{4a}-\frac{A}{2ax^6\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$\frac{1}{3}\left(-\frac{(5Ab-4aB)\left(-\frac{3b\left(\frac{2}{a\sqrt{a+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{1}{ax^3\sqrt{a+bx^3}}\right)}{4a}-\frac{A}{2ax^6\sqrt{a+bx^3}}\right)$$

input

```
Int[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x]
```

output

```
(-1/2*A/(a*x^6*Sqrt[a + b*x^3]) - ((5*A*b - 4*a*B)*(-(1/(a*x^3*Sqrt[a + b*
x^3])) - (3*b*(2/(a*Sqrt[a + b*x^3]) - (2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]
)/a^(3/2)))/(2*a)))/(4*a))/3
```

3.233.3.1 Defintions of rubi rules used

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.233. $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx$

3.233.4 Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{5\left(-\frac{x^{3} b \left(-\frac{12 x^{3} B}{5}+A \right) a^{\frac{3}{2}}}{3}+\frac{2 \left(2 x^{3} B +A \right) a^{\frac{5}{2}}}{15}+x^{6} \left(-A b \sqrt{a}+\left(A b -\frac{4 B a}{5}\right) \sqrt{b \,x^{3}+a}\, \operatorname{arctanh}\left(\frac{\sqrt{b \,x^{3}+a}}{\sqrt{a}}\right)\right) b \right)}{4 \sqrt{b \,x^{3}+a}\, a^{\frac{7}{2}} x^{6}}$
elliptic	$-\frac{A \sqrt{b \,x^{3}+a}}{6 a^{2} x^{6}}+\frac{\left(7 A b -4 B a \right) \sqrt{b \,x^{3}+a}}{12 a^{3} x^{3}}+\frac{2 b \left(A b -B a \right)}{3 a^{3} \sqrt{\left(x^{3}+\frac{a}{b}\right) b}}-\frac{b \left(5 A b -4 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{b \,x^{3}+a}}{\sqrt{a}}\right)}{4 a^{\frac{7}{2}}}$
risch	$-\frac{\sqrt{b \,x^{3}+a}\, \left(-7 A b \,x^{3}+4 B a \,x^{3}+2 A a \right)}{12 a^{3} x^{6}}+\frac{b \left(-\frac{2 \left(7 A b -4 B a \right)}{3 \sqrt{b \,x^{3}+a}}+3 a \left(5 A b -4 B a \right) \left(\frac{2}{3 a \sqrt{\left(x^{3}+\frac{a}{b}\right) b}}-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b \,x^{3}+a}}{\sqrt{a}}\right)}{3 a^{\frac{3}{2}}}\right)\right)}{8 a^{3}}$
default	$A \left(-\frac{\sqrt{b \,x^{3}+a}}{6 a^{2} x^{6}}+\frac{7 b \sqrt{b \,x^{3}+a}}{12 a^{3} x^{3}}+\frac{2 b^{2}}{3 a^{3} \sqrt{\left(x^{3}+\frac{a}{b}\right) b}}-\frac{5 b^{2} \operatorname{arctanh}\left(\frac{\sqrt{b \,x^{3}+a}}{\sqrt{a}}\right)}{4 a^{\frac{7}{2}}}\right)+B \left(-\frac{\sqrt{b \,x^{3}+a}}{3 a^{2} x^{3}}-\frac{2 b}{3 a^{2} \sqrt{\left(x^{3}+\frac{a}{b}\right.}}\right.$

input
```
int((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-5/4*(-1/3*x^3*b*(-12/5*x^3*B+A)*a^(3/2)+2/15*(2*B*x^3+A)*a^(5/2)+x^6*(-A*
b*a^(1/2)+(A*b-4/5*B*a)*(b*x^3+a)^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))*
b)/(b*x^3+a)^(1/2)/a^(7/2)/x^6
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.45

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \left[-\frac{3\,((4\,Bab^2-5\,Ab^3)x^9+(4\,Ba^2b-5\,Aab^2)x^6)\sqrt{a}\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(3}{24\,(a^4bx^9+a^5x^6)}\right.$$

$$-\frac{3\,((4\,Bab^2-5\,Ab^3)x^9+(4\,Ba^2b-5\,Aab^2)x^6)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+(3\,(4\,Ba^2b-5\,Aab^2)x^6+2\,Aa^3}{12\,(a^4bx^9+a^5x^6)}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output

```
[-1/24*(3*((4*B*a*b^2 - 5*A*b^3)*x^9 + (4*B*a^2*b - 5*A*a*b^2)*x^6)*sqrt(a
)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*(4*B*a^2*b - 5
*A*a*b^2)*x^6 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4
*b*x^9 + a^5*x^6), -1/12*(3*((4*B*a*b^2 - 5*A*b^3)*x^9 + (4*B*a^2*b - 5*A*
a*b^2)*x^6)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*(4*B*a^2*b -
5*A*a*b^2)*x^6 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^
4*b*x^9 + a^5*x^6)]
```

3.233.6 Sympy [A] (verification not implemented)

Time = 59.40 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.63

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = A\left(-\frac{1}{6a\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{5\sqrt{b}}{12a^2x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{5b^{\frac{3}{2}}}{4a^3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{5b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4a^{\frac{7}{2}}}\right)+B\left(-\frac{1}{3a\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{\sqrt{b}}{a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{a^{\frac{5}{2}}}\right)$$

input

```
integrate((B*x**3+A)/x**7/(b*x**3+a)**(3/2),x)
```

output

```
A*(-1/(6*a*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + 5*sqrt(b)/(12*a**2*x*
*(9/2)*sqrt(a/(b*x**3) + 1)) + 5*b**(3/2)/(4*a**3*x**(3/2)*sqrt(a/(b*x**3)
 + 1)) - 5*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(7/2))) + B*(-1/(3
*a*sqrt(b)*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(a**2*x**(3/2)*sqrt(a/
(b*x**3) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/a**(5/2))
```

3.233.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(98) = 196.

Time = 0.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.82

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \frac{1}{24}\,A\left(\frac{2\left(15\left(bx^3+a\right)^2b^2-25\left(bx^3+a\right)ab^2+8\,a^2b^2\right)}{\left(bx^3+a\right)^{\frac{5}{2}}a^3-2\left(bx^3+a\right)^{\frac{3}{2}}a^4+\sqrt{bx^3+a}\,aa^5}+\frac{15\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-\frac{1}{6}\,B\left(\frac{2\left(3\left(bx^3+a\right)b-2\,ab\right)}{\left(bx^3+a\right)^{\frac{3}{2}}a^2-\sqrt{bx^3+a}\,aa^3}+\frac{3\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
1/24*A*(2*(15*(b*x^3 + a)^2*b^2 - 25*(b*x^3 + a)*a*b^2 + 8*a^2*b^2)/((b*x^
3 + a)^(5/2)*a^3 - 2*(b*x^3 + a)^(3/2)*a^4 + sqrt(b*x^3 + a)*a^5) + 15*b^2
*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(7/2)) - 1
/6*B*(2*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^(3/2)*a^2 - sqrt(b*x^3 + a)
*a^3) + 3*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a
^(5/2))
```

3.233.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = -\frac{\left(4\,Bab-5\,Ab^2\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3} - \frac{2\,\left(Bab-Ab^2\right)}{3\,\sqrt{bx^3+a}a^3} - \frac{4\,\left(bx^3+a\right)^{\frac{3}{2}}Bab-4\,\sqrt{bx^3+a}Ba^2b-7\,\left(bx^3+a\right)^{\frac{3}{2}}Ab^2+9\,\sqrt{bx^3+a}Aab^2}{12\,a^3b^2x^6}$$

input
```
integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
-1/4*(4*B*a*b - 5*A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) -
 2/3*(B*a*b - A*b^2)/(sqrt(b*x^3 + a)*a^3) - 1/12*(4*(b*x^3 + a)^(3/2)*B*a
*b - 4*sqrt(b*x^3 + a)*B*a^2*b - 7*(b*x^3 + a)^(3/2)*A*b^2 + 9*sqrt(b*x^3
+ a)*A*a*b^2)/(a^3*b^2*x^6)
```

3.233.9 Mupad [B] (verification not implemented)

Time = 7.69 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \frac{b\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,\left(5\,A\,b-4\,B\,a\right)}{8\,a^{7/2}} - \frac{\left(4\,B\,a^2-7\,A\,a\,b\right)\,\sqrt{b\,x^3+a}}{12\,a^4\,x^3} - \frac{A\,\sqrt{b\,x^3+a}}{6\,a^2\,x^6} - \frac{\frac{a\,\left(\frac{7\,A\,b^3-4\,B\,a\,b^2}{12\,a^4}-\frac{5\,b^2\,\left(5\,A\,b-4\,B\,a\right)}{8\,a^4}\right)}{b}+\frac{3\,b\,\left(5\,A\,b-4\,B\,a\right)}{8\,a^3}}{\sqrt{b\,x^3+a}}$$

input
```
int((A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x)
```

output
```
(b*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6
)*(5*A*b - 4*B*a))/(8*a^(7/2)) - ((4*B*a^2 - 7*A*a*b)*(a + b*x^3)^(1/2))/(
12*a^4*x^3) - (A*(a + b*x^3)^(1/2))/(6*a^2*x^6) - ((a*((7*A*b^3 - 4*B*a*b^
2)/(12*a^4) - (5*b^2*(5*A*b - 4*B*a))/(8*a^4)))/b + (3*b*(5*A*b - 4*B*a))/
(8*a^3))/(a + b*x^3)^(1/2)
```

3.233. $\int \frac{A+Bx^3}{x^7(a+bx^3)^{3/2}}\,dx$

3.234 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.234.1 Optimal result

Integrand size = 22, antiderivative size = 300

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2(11Ab-14aB)x^4}{33b^2\sqrt{a+bx^3}}+\frac{2Bx^7}{11b\sqrt{a+bx^3}}+\frac{16(11Ab-14aB)x\sqrt{a+bx^3}}{165b^3}$$

$$-\frac{32\sqrt{2+\sqrt{3}}a(11Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Bigg(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-}{165\sqrt[4]{3}b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-2/33*(11*A*b-14*B*a)*x^4/b^2/(b*x^3+a)^(1/2)+2/11*B*x^7/b/(b*x^3+a)^(1/2)
+16/165*(11*A*b-14*B*a)*x*(b*x^3+a)^(1/2)/b^3-32/495*a*(11*A*b-14*B*a)*(a^
(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
b^(10/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)
```

3.234.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.34

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx=\frac{2x\left(-112a^2B+3b^2x^3(11A+5Bx^3)+a(88Ab-42bBx^3)+8a(-11Ab+14aB)\sqrt{1+}\right.}{165b^3\sqrt{a+bx^3}}$$

input
```
Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*x*(-112*a^2*B + 3*b^2*x^3*(11*A + 5*B*x^3) + a*(88*A*b - 42*b*B*x^3) +
8*a*(-11*A*b + 14*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3
, -((b*x^3)/a)]))/(165*b^3*Sqrt[a + b*x^3])
```

3.234.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {959, 817, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 959}$$

$$\frac{(11Ab-14aB)\int \frac{x^6}{\left(bx^3+a\right)^{3/2}}dx}{11b}+\frac{2Bx^7}{11b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 817}$$

$$\frac{(11Ab-14aB)\left(\frac{8\int \frac{x^3}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x^4}{3b\sqrt{a+bx^3}}\right)}{11b}+\frac{2Bx^7}{11b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{(11Ab-14aB)\left(\frac{8\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{2a\int\frac{1}{\sqrt{bx^3+a}}dx}{5b}\right)}{3b}-\frac{2x^4}{3b\sqrt{a+bx^3}}\right)}{11b}+\frac{2Bx^7}{11b\sqrt{a+bx^3}}$$

$$\Big\downarrow 759$$

$$\frac{(11Ab-14aB)\left(\frac{8\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{3b}-}{11b}$$

$$\frac{2Bx^7}{11b\sqrt{a+bx^3}}$$

input
```
Int[(x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*B*x^7)/(11*b*Sqrt[a + b*x^3]) + ((11*A*b - 14*a*B)*((-2*x^4)/(3*b*Sqrt[
a + b*x^3]) + (8*((2*x*Sqrt[a + b*x^3])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1
/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(3*b)))/(11*b)
```

3.234. $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.234.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.234.4 Maple [A] (verified)

Time = 4.81 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.35

method	result
elliptic	$\frac{2xa(Ab-Ba)}{3b^3\sqrt{(x^3+\frac{a}{b})b}}+\frac{2B\,x^4\sqrt{b\,x^3+a}}{11b^2}+\frac{2\left(\frac{Ab-Ba}{b^2}-\frac{8Ba}{11b^2}\right)x\sqrt{b\,x^3+a}}{5b}-\frac{2i\left(-\frac{2a(Ab-Ba)}{3b^3}-\frac{2\left(\frac{Ab-Ba}{b^2}-\frac{8Ba}{11b^2}\right)a}{5b}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x\right.}{}}}{}$
default	$B\left(-\frac{2a^2x}{3b^3\sqrt{(x^3+\frac{a}{b})b}}+\frac{2x^4\sqrt{b\,x^3+a}}{11b^2}-\frac{38ax\sqrt{b\,x^3+a}}{55b^3}-\frac{448ia^2\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{}{-\frac{3(-}{}}}}{}\right.$
risch	Expression too large to display

input

```
int(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/3/b^3*x*a*(A*b-B*a)/((x^3+a/b)*b)^(1/2)+2/11*B/b^2*x^4*(b*x^3+a)^(1/2)+2
/5*((A*b-B*a)/b^2-8/11*B/b^2*a)/b*x*(b*x^3+a)^(1/2)-2/3*I*(-2/3*a*(A*b-B*a
)/b^3-2/5*((A*b-B*a)/b^2-8/11*B/b^2*a)/b*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.234. $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.234.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.41

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(16\left(14\,Ba^3-11\,Aa^2b+(14\,Ba^2b-11\,Aab^2)x^3\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)}{165\,(b^5x^3+}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/165*(16*(14*B*a^3 - 11*A*a^2*b + (14*B*a^2*b - 11*A*a*b^2)*x^3)*sqrt(b)*
weierstrassPInverse(0, -4*a/b, x) + (15*B*b^3*x^7 - 3*(14*B*a*b^2 - 11*A*b
^3)*x^4 - 8*(14*B*a^2*b - 11*A*a*b^2)*x)*sqrt(b*x^3 + a))/(b^5*x^3 + a*b^4
)
```

3.234.6 Sympy [A] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{Ax^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)} + \frac{Bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate(x**6*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
A*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(3/2)*gamma(10/3)) + B*x**10*gamma(10/3)*hyper((3/2, 10/3), (13/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(13/3))
```

3.234.7 Maxima [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(3/2), x)
```

3.234.8 Giac [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(3/2), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^6\,(B\,x^3+A)}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

output
```
int((x^6*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

3.235 $\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

3.235.1 Optimal result

Integrand size = 22, antiderivative size = 269

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2(5Ab-8aB)x}{15b^2\sqrt{a+bx^3}}+\frac{2Bx^4}{5b\sqrt{a+bx^3}}$$

$$+\frac{4\sqrt{2+\sqrt{3}}(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\right.}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/15*(5*A*b-8*B*a)*x/b^2/(b*x^3+a)^(1/2)+2/5*B*x^4/b/(b*x^3+a)^(1/2)+4/45
*(5*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2)
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)*3^(3/4)/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.235.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.29

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2x\left(-5Ab+8aB+3bBx^3+(5Ab-8aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\right.\right.}{15b^2\sqrt{a+bx^3}}$$

input
```
Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*x*(-5*A*b + 8*a*B + 3*b*B*x^3 + (5*A*b - 8*a*B)*Sqrt[1 + (b*x^3)/a]*Hyp
ergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(15*b^2*Sqrt[a + b*x^3])
```

3.235.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 817, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(5Ab-8aB)\int \frac{x^3}{(bx^3+a)^{3/2}}dx}{5b}+\frac{2Bx^4}{5b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(5Ab-8aB)\left(\frac{2\int \frac{1}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x}{3b\sqrt{a+bx^3}}\right)}{5b}+\frac{2Bx^4}{5b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{759}$$

$$\frac{(5Ab-8aB)\left(\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x}{3b\sqrt{a+bx^3}}\right)}{5b}+\frac{2Bx^4}{5b\sqrt{a+bx^3}}$$

input
```
Int[(x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*B*x^4)/(5*b*Sqrt[a + b*x^3]) + ((5*A*b - 8*a*B)*((-2*x)/(3*b*Sqrt[a + b
*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(5*b)
```

3.235.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.235.4 Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.29

method	result
elliptic	$-\frac{2x(Ab-Ba)}{3b^2\sqrt{(x^3+\frac{a}{b})b}}+\frac{2Bx\sqrt{b\,x^3+a}}{5b^2}-\frac{2i\left(\frac{\frac{2Ab}{3}-\frac{2Ba}{3}}{b^2}-\frac{2Ba}{5b^2}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+}}}{}$
default	$B\left(\frac{2ax}{3b^2\sqrt{(x^3+\frac{a}{b})b}}+\frac{2x\sqrt{b\,x^3+a}}{5b^2}+\frac{32ia\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}}{}\right.$
risch	$\frac{2Bx\sqrt{b\,x^3+a}}{5b^2}+\frac{-2a^2B\left(\frac{2x}{3a\sqrt{(x^3+\frac{a}{b})b}}-\frac{2i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{\frac{i}{-}}}{}\right.}{}$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

3.235. $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

output

```
-2/3/b^2*x*(A*b-B*a)/((x^3+a/b)*b)^(1/2)+2/5*B*x/b^2*(b*x^3+a)^(1/2)-2/3*I
*(2/3*(A*b-B*a)/b^2-2/5*B/b^2*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.235.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.35

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx =$$

$$-\frac{2\left(2\left((8\,Bab-5\,Ab^2)x^3+8\,Ba^2-5\,Aab\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\left(3\,Bb^2x^4+(8\,Bab-5\,Ab^2\right.\right.}{15\left(b^4x^3+ab^3\right)}$$

input

```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output

```
-2/15*(2*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*sqrt(b)*weierstrass
PInverse(0, -4*a/b, x) - (3*B*b^2*x^4 + (8*B*a*b - 5*A*b^2)*x)*sqrt(b*x^3
+ a))/(b^4*x^3 + a*b^3)
```

3.235.6 Sympy [A] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.30

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{3}{2}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

input

```
integrate(x**3*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
A*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(3/2)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(10/3))
```

3.235.7 Maxima [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(3/2), x)
```

3.235.8 Giac [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(3/2), x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^3\,(B\,x^3+A)}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

output
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

3.236 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^{3/2}}\,dx$

3.236.1 Optimal result

Integrand size = 19, antiderivative size = 251

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{2(Ab-aB)x}{3ab\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*(A*b-B*a)*x/a/b/(b*x^3+a)^(1/2)+2/9*(A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*El
lipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*
3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(4/3)/(b*x^3+a)
^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/
2)
```

3.236.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.29

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{x\left(2Ab-2aB+(Ab+2aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{3ab\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(a + b*x^3)^(3/2),x]
```

output
```
(x*(2*A*b - 2*a*B + (A*b + 2*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/
3, 1/2, 4/3, -((b*x^3)/a)]))/(3*a*b*Sqrt[a + b*x^3])
```

3.236.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {910, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(2aB+Ab)\int \frac{1}{\sqrt{bx^3+a}}dx}{3ab}+\frac{2x(Ab-aB)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{759}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x(Ab-aB)}{3ab\sqrt{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(a + b*x^3)^(3/2),x]
```

output
```
(2*(A*b - a*B)*x)/(3*a*b*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(A*b + 2*
a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(3*3^(1/4)*a*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

3.236.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.236.4 Maple [A] (verified)

Time = 4.30 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.34

method	result
elliptic	$\frac{2x(Ab-Ba)}{3ba\sqrt{(x^3+\frac{a}{b})b}}-\frac{2i\left(\frac{B}{b}+\frac{Ab-Ba}{3ab}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-}{}\right.}{}}}{3b\sqrt{b}}$
default	$A\left(\frac{2x}{3a\sqrt{(x^3+\frac{a}{b})b}}-\frac{2i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\right.}{(-a}}}{9ab\sqrt{b\,x}}\right.$

input
```
int((B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b*x/a*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-2/3*I*(B/b+1/3*(A*b-B*a)/a/b)*3^(1
/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.236. $\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx$

3.236.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{bx^3+a}(Bab-Ab^2)x-((2\,Bab+Ab^2)x^3+2\,Ba^2+Aab)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{3\,(ab^3x^3+a^2b^2)}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(b*x^3 + a)*(B*a*b - A*b^2)*x - ((2*B*a*b + A*b^2)*x^3 + 2*B*a^2
 + A*a*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x))/(a*b^3*x^3 + a^2*b^2)
```

3.236.6 Sympy [A] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.31

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
A*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3))
```

3.236.7 Maxima [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(b*x^3 + a)^(3/2), x)
```

3.236.8 Giac [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(b*x^3 + a)^(3/2), x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \int \frac{B\,x^3+A}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((A + B*x^3)/(a + b*x^3)^(3/2),x)
```

output
```
int((A + B*x^3)/(a + b*x^3)^(3/2), x)
```

3.237 $\int \frac{A+Bx^3}{x^3(a+bx^3)^{3/2}}\,dx$

3.237.1 Optimal result

Integrand size = 22, antiderivative size = 272

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{2ax^2\sqrt{a+bx^3}}-\frac{(7Ab-4aB)x}{6a^2\sqrt{a+bx^3}}$$

$$-\frac{\sqrt{2+\sqrt{3}}(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-}{6\sqrt[4]{3}a^2\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/2*A/a/x^2/(b*x^3+a)^(1/2)-1/6*(7*A*b-4*B*a)*x/a^2/(b*x^3+a)^(1/2)-1/18*
(7*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2))
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)*3^(3/4)/a^2/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.237.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.32

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx = \frac{-6aA-14Abx^3+8aBx^3+(-7Ab+4aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3}}{12a^2x^2\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x]
```

output
```
(-6*a*A - 14*A*b*x^3 + 8*a*B*x^3 + (-7*A*b + 4*a*B)*x^3*Sqrt[1 + (b*x^3)/a
]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(12*a^2*x^2*Sqrt[a + b*x
^3])
```

3.237.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {955, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(7Ab-4aB)\int \frac{1}{\left(bx^3+a\right)^{3/2}}dx}{4a}-\frac{A}{2ax^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(7Ab-4aB)\left(\frac{\int \frac{1}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x}{3a\sqrt{a+bx^3}}\right)}{4a}-\frac{A}{2ax^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{(7Ab-4aB)\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x}{3a\sqrt{a+bx^3}}\right)}{4a}$$

$$\frac{A}{2ax^2\sqrt{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x]
```

output
```
-1/2*A/(a*x^2*Sqrt[a + b*x^3]) - ((7*A*b - 4*a*B)*((2*x)/(3*a*Sqrt[a + b*x
^3]) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(4*a)
```

3.237.3.1 Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.237.4 Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.29

method	result
elliptic	$-\frac{2x(Ab-Ba)}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^{3}+a}}{2a^{2}x^{2}}-\frac{2i\left(-\frac{Ab-Ba}{3a^{2}}-\frac{Ab}{4a^{2}}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(}{}}}}{}$
default	$B\left(\frac{2x}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\right.}{(-a}}}{9ab\sqrt{b\,x}}\right.$
risch	Expression too large to display

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x/a^2*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-1/2/a^2*A*(b*x^3+a)^(1/2)/x^2-2/3
*I*(-1/3*(A*b-B*a)/a^2-1/4/a^2*A*b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.237.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.38

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx = \frac{((4\,Bab-7\,Ab^2)x^5+(4\,Ba^2-7\,Aab)x^2)\sqrt{b}\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+((4\,B}{6\,(a^2b^2x^5+a^3bx^2)}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(((4*B*a*b - 7*A*b^2)*x^5 + (4*B*a^2 - 7*A*a*b)*x^2)*sqrt(b)*weierstra
ssPInverse(0, -4*a/b, x) + ((4*B*a*b - 7*A*b^2)*x^3 - 3*A*a*b)*sqrt(b*x^3
+ a))/(a^2*b^2*x^5 + a^3*b*x^2)
```

3.237.6 Sympy [A] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Bx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**3/(b*x**3+a)**(3/2),x)
```

output
```
A*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3))
```

3.237.7 Maxima [F]

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^3), x)
```

3.237.8 Giac [F]

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^3), x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{B\,x^3+A}{x^3\,\left(b\,x^3+a\right)^{3/2}}\,dx$$

input
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(3/2)), x)
```

3.238 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx$

3.238.1 Optimal result

Integrand size = 22, antiderivative size = 304

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx=-\frac{A}{5ax^5\sqrt{a+bx^3}}-\frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}}+\frac{7(13Ab-10aB)\sqrt{a+bx^3}}{60a^3x^2}$$

$$+\frac{7\sqrt{2+\sqrt{3}}b^{2/3}(13Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),}{60\sqrt[4]{3}a^3\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/5*A/a/x^5/(b*x^3+a)^(1/2)+1/15*(-13*A*b+10*B*a)/a^2/x^2/(b*x^3+a)^(1/2)
+7/60*(13*A*b-10*B*a)*(b*x^3+a)^(1/2)/a^3/x^2+7/180*b^(2/3)*(13*A*b-10*B*a
)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(
3/4)/a^3/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)
```

3.238.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.24

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx = \frac{-4aA+(13Ab-10aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{20a^2x^5\sqrt{a+bx^3}}$$

input

```
Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x]
```

output

```
(-4*a*A + (13*A*b - 10*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3
, 3/2, 1/3, -((b*x^3)/a)])/(20*a^2*x^5*Sqrt[a + b*x^3])
```

3.238.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 819, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(13Ab-10aB)\int \frac{1}{x^3\left(bx^3+a\right)^{3/2}}dx}{10a}-\frac{A}{5ax^5\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(13Ab-10aB)\left(\frac{7\int \frac{1}{x^3\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ax^2\sqrt{a+bx^3}}\right)}{10a}-\frac{A}{5ax^5\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(13Ab-10aB)\left(\frac{7\left(-\frac{b\int \frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{3a}+\frac{2}{3ax^2\sqrt{a+bx^3}}\right)}{10a}-\frac{A}{5ax^5\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{(13Ab-10aB)\left(\frac{7\left(-\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{3a}-\right.}{10a}$$
$$\frac{A}{5ax^5\sqrt{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x]
```

output
```
-1/5*A/(a*x^5*Sqrt[a + b*x^3]) - ((13*A*b - 10*a*B)*(2/(3*a*x^2*Sqrt[a + b
*x^3]) + (7*(-1/2*Sqrt[a + b*x^3]/(a*x^2) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1
/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a + b*x^3])))/(3*a)))/(10*a)
```

3.238.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.238.4 Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.27

method	result
elliptic	$\frac{2bx(Ab-Ba)}{3a^3\sqrt{(x^3+\frac{a}{b})b}}-\frac{A\sqrt{b\,x^3+a}}{5a^2x^5}+\frac{(17Ab-10Ba)\sqrt{b\,x^3+a}}{20a^3x^2}-\frac{2i\left(\frac{(Ab-Ba)b}{3a^3}+\frac{b(17Ab-10Ba)}{40a^3}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(}{}\right.}{(-a\,b^2)^{\frac{1}{3}}}}}{}$
default	$B\left(-\frac{2bx}{3a^2\sqrt{(x^3+\frac{a}{b})b}}-\frac{\sqrt{b\,x^3+a}}{2a^2x^2}+\frac{7i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-}}{}\right.$
risch	Expression too large to display

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*b*x/a^3*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-1/5*A/a^2*(b*x^3+a)^(1/2)/x^5+1/
20/a^3*(17*A*b-10*B*a)*(b*x^3+a)^(1/2)/x^2-2/3*I*(1/3*(A*b-B*a)*b/a^3+1/40
*b*(17*A*b-10*B*a)/a^3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*
(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2))
```

3.238. $\int \frac{A+Bx^3}{x^6(a+bx^3)^{3/2}}\,dx$

3.238.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx =$$
$$-\frac{7\,((10\,Bab-13\,Ab^2)x^8+(10\,Ba^2-13\,Aab)x^5)\sqrt{b}\text{weierstrassPInverse}\big(0,-\frac{4\,a}{b},x\big)+(7\,(10\,Bab-13\,Ab^2)}{60\,(a^3bx^8+a^4x^5)}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/60*(7*((10*B*a*b - 13*A*b^2)*x^8 + (10*B*a^2 - 13*A*a*b)*x^5)*sqrt(b)*w
eierstrassPInverse(0, -4*a/b, x) + (7*(10*B*a*b - 13*A*b^2)*x^6 + 3*(10*B*
a^2 - 13*A*a*b)*x^3 + 12*A*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^8 + a^4*x^5)
```

3.238.6 Sympy [A] (verification not implemented)

Time = 24.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{3}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{B\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**6/(b*x**3+a)**(3/2),x)
```

output
```
A*gamma(-5/3)*hyper((-5/3, 3/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(3/2)*x**5*gamma(-2/3)) + B*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(3/2)*x**2*gamma(1/3))
```

3.238.7 Maxima [F]

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^6), x)
```

3.238.8 Giac [F]

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^6), x)
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{B\,x^3+A}{x^6\,\left(b\,x^3+a\right)^{3/2}}\,dx$$

input
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(3/2)), x)
```

3.239 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.239.1 Optimal result

Integrand size = 22, antiderivative size = 547

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2(7Ab-10aB)x^2}{21b^2\sqrt{a+bx^3}} + \frac{2Bx^5}{7b\sqrt{a+bx^3}} + \frac{8(7Ab-10aB)\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\surd}{7\ 3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{8\sqrt{2}\sqrt[3]{a}(7Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\surd}{21\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-2/21*(7*A*b-10*B*a)*x^2/b^2/(b*x^3+a)^(1/2)+2/7*B*x^5/b/(b*x^3+a)^(1/2)+8
/21*(7*A*b-10*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
+8/63*a^(1/3)*(7*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1
/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)*3^(3/4)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-4/21*a^(1/3)*(7*A*b-10*B*a)*(a^(1/3)+b^(
1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(8/3)/(
b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

3.239.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.14

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2x^2\left(7Ab-10aB+bBx^3+(-7Ab+10aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},-\right.\right.}{7b^2\sqrt{a+bx^3}}$$

input

```
Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output

```
(2*x^2*(7*A*b - 10*a*B + b*B*x^3 + (-7*A*b + 10*a*B)*Sqrt[1 + (b*x^3)/a]*H
ypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(7*b^2*Sqrt[a + b*x^3])
```

3.239.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 817, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(7Ab-10aB)\int \frac{x^4}{(bx^3+a)^{3/2}}dx}{7b}+\frac{2Bx^5}{7b\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 817$$

$$\frac{(7Ab-10aB)\left(\frac{4\int \frac{x}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x^2}{3b\sqrt{a+bx^3}}\right)}{7b}+\frac{2Bx^5}{7b\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 832$$

$$\frac{(7Ab-10aB)\left(\frac{4\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{3b}-\frac{2x^2}{3b\sqrt{a+bx^3}}\right)}{7b}+\frac{2Bx^5}{7b\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 759$$

$$(7Ab-10aB)\Bigg(\frac{4\Bigg(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{3b}$$

$$7b$$

$$\frac{2Bx^5}{7b\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 2416$$

3.239. $\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

$$\frac{(7Ab-10aB)\left(\frac{4\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{\sqrt[3]{b}}}{3b}\right.}{7b}$$

$$\frac{2Bx^5}{7b\sqrt{a+bx^3}}$$

input
```
Int[(x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*B*x^5)/(7*b*Sqrt[a + b*x^3]) + ((7*A*b - 10*a*B)*((-2*x^2)/(3*b*Sqrt[a
+ b*x^3]) + (4*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(
1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3])))/(3*b)))/(7*b)
```

3.239. $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.239.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.239. $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.239.4 Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.92

method	result
elliptic	$-\frac{2x^2(Ab-Ba)}{3b^2\sqrt{(x^3+\frac{a}{b})b}}+\frac{2B\,x^2\sqrt{b\,x^3+a}}{7b^2}-\frac{2i\left(\frac{\frac{4Ab}{3}-\frac{4Ba}{3}}{b^2}-\frac{4Ba}{7b^2}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}}}}{}$
default	Expression too large to display
risch	Expression too large to display

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b^2*x^2*(A*b-B*a)/((x^3+a/b)*b)^(1/2)+2/7*B*x^2/b^2*(b*x^3+a)^(1/2)-2
/3*I*(4/3*(A*b-B*a)/b^2-4/7*B/b^2*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)))
```

3.239.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.19

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(4\,((10\,Bab-7\,Ab^2)x^3+10\,Ba^2-7\,Aab)\sqrt{b}\text{weierstrassZeta}\,(0,-\frac{4\,a}{b},\text{weierstrassPI}}{21\,(b^4x^3+ab^3)}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/21*(4*((10*B*a*b - 7*A*b^2)*x^3 + 10*B*a^2 - 7*A*a*b)*sqrt(b)*weierstras
sZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (3*B*b^2*x^5 + (10*B
*a*b - 7*A*b^2)*x^2)*sqrt(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

3.239.6 Sympy [A] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{Ax^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)} + \frac{Bx^8\Gamma\left(\frac{8}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**4*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
A*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(3/2)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((3/2, 8/3), (11/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(11/3))
```

3.239.7 Maxima [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(3/2), x)
```

3.239.8 Giac [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(3/2), x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^4\,(B\,x^3+A)}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

output
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

3.240 $\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

3.240.1 Optimal result

Integrand size = 20, antiderivative size = 524

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}} - \frac{2(Ab-4aB)\sqrt{a+bx^3}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{\sqrt{2-\sqrt{3}}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{2}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)^(1/2)-2/3*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a/b^
(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-2/9*(A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*
EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^
(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/3*(A*
b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)*3^(1/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.240.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.14

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx=\frac{x^2\left(4aB+(Ab-4aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2ab\sqrt{a+bx^3}}$$

input
```
Integrate[(x*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(x^2*(4*a*B + (A*b - 4*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2
, 5/3, -((b*x^3)/a)]))/(2*a*b*Sqrt[a + b*x^3])
```

3.240.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {957, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{2x^2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{(Ab-4aB)\int \frac{x}{\sqrt{bx^3+a}}dx}{3ab}$$

$$\Big\downarrow \text{832}$$

$$\frac{2x^2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{(Ab-4aB)\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{3ab}$$

$$\Big\downarrow \text{759}$$

$$\frac{2x^2(Ab-aB)}{3ab\sqrt{a+bx^3}}-$$

$$\frac{(Ab-4aB)\Bigg(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\Bigg(\arcsin\Bigg(\frac{\sqrt[3]{b}x+(1-\sqrt{3}}{\sqrt[3]{b}x+(1+\sqrt{3}}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{3ab}$$

$$\Big\downarrow \text{2416}$$

$$\frac{2x^2(Ab-aB)}{3ab\sqrt{a+bx^3}}-$$

$$\frac{(Ab-4aB)\Bigg(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}}{3ab}$$

input
```
Int[(x*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

3.240. $\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}dx$

output

```
(2*(A*b - a*B)*x^2)/(3*a*b*Sqrt[a + b*x^3]) - ((A*b - 4*a*B)*(((2*Sqrt[a +
 b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 -
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt
[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(3*a*b)
```

3.240.3.1 Defintions of rubi rules used

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 957

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.240.4 Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 490, normalized size of antiderivative = 0.94

method	result
elliptic	$\frac{2x^2(Ab-Ba)}{3ba\sqrt{\left(x^3+\frac{a}{b}\right)b}} - \frac{2i\left(\frac{B}{b}-\frac{Ab-Ba}{3ab}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\right.}{}}}{}$
default	Expression too large to display

input
```
int(x*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b*x^2/a*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-2/3*I*(B/b-1/3*(A*b-B*a)/a/b)*3^
(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.240.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.18

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{bx^3+a}(Bab-Ab^2)x^2+((4\,Bab-Ab^2)x^3+4\,Ba^2-Aab)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPI}\right.\right.}{3\,(ab^3x^3+a^2b^2)}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(b*x^3 + a)*(B*a*b - A*b^2)*x^2 + ((4*B*a*b - A*b^2)*x^3 + 4*B*a
^2 - A*a*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a
/b, x)))/(a*b^3*x^3 + a^2*b^2)
```

3.240.6 Sympy [A] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}+\frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
A*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(3/2)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3))
```

3.240.7 Maxima [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x/(b*x^3 + a)^(3/2), x)
```

3.240.8 Giac [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x/(b*x^3 + a)^(3/2), x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x\,(B\,x^3+A)}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((x*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

output
```
int((x*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

3.241 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx$

3.241.1 Optimal result

Integrand size = 22, antiderivative size = 548

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{ax\sqrt{a+bx^3}} - \frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}} + \frac{(5Ab-2aB)\sqrt{a+bx^3}}{3a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt{2-\sqrt{3}}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\sqrt{2}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-A/a/x/(b*x^3+a)^(1/2)-1/3*(5*A*b-2*B*a)*x^2/a^2/(b*x^3+a)^(1/2)+1/3*(5*A*
b-2*B*a)*(b*x^3+a)^(1/2)/a^2/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/9*(
5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^
(5/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)-1/6*(5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE(
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5/3)/b^(2/3)/(b*x^3+a)^(
1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.241.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx=\frac{-4aA+(-5Ab+2aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a}\right)}{4a^2x\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x]
```

output
```
(-4*a*A + (-5*A*b + 2*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3,
3/2, 5/3, -((b*x^3)/a)])/(4*a^2*x*Sqrt[a + b*x^3])
```

3.241.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {955, 819, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(5Ab-2aB)\int \frac{x}{(bx^3+a)^{3/2}}dx}{2a}-\frac{A}{ax\sqrt{a+bx^3}}$$

$$\Big\downarrow 819$$

$$-\frac{(5Ab-2aB)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\int \frac{x}{\sqrt{bx^3+a}}dx}{3a}\right)}{2a}-\frac{A}{ax\sqrt{a+bx^3}}$$

$$\Big\downarrow 832$$

$$-\frac{(5Ab-2aB)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}\right)}{2a}-\frac{A}{ax\sqrt{a+bx^3}}$$

$$\Big\downarrow 759$$

$$-\frac{(5Ab-2aB)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{3a}\right.}{2a}$$

$$\frac{A}{ax\sqrt{a+bx^3}}$$

$$\Big\downarrow 2416$$

3.241. $\int \frac{A+Bx^3}{x^2(a+bx^3)^{3/2}}\,dx$

$$-\frac{(5Ab-2aB)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+(1-}{\sqrt[3]{b}x+(1+}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{2a}$$

$$\frac{A}{ax\sqrt{a+bx^3}}$$

input
```
Int[(A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x]
```

output
```
-(A/(a*x*Sqrt[a + b*x^3])) - ((5*A*b - 2*a*B)*((2*x^2)/(3*a*Sqrt[a + b*x^3
]) - (((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
 (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*
(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(
3*a)))/(2*a)
```

3.241. $\int \frac{A+Bx^3}{x^2(a+bx^3)^{3/2}}\,dx$

3.241.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.241. $\int \frac{A+Bx^3}{x^2(a+bx^3)^{3/2}}\,dx$

3.241.4 Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.92

method	result
elliptic	$-\frac{2x^2(Ab-Ba)}{3a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{a^2x}-\frac{2i\left(\frac{Ab-Ba}{3a^2}+\frac{Ab}{2a^2}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-}{2}}}}{}$
default	Expression too large to display
risch	Expression too large to display

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x^2/a^2*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-1/a^2*A*(b*x^3+a)^(1/2)/x-2/3*I
*(1/3*(A*b-B*a)/a^2+1/2/a^2*A*b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)))
```

3.241.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.19

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \frac{((2\,Bab-5\,Ab^2)x^4+(2\,Ba^2-5\,Aab)x)\sqrt{b}\text{weierstrassZeta}(0,-\frac{4\,a}{b},\text{weierstrassPInv}}{3\,(a^2b^2x^4+a^3bx)}$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(((2*B*a*b - 5*A*b^2)*x^4 + (2*B*a^2 - 5*A*a*b)*x)*sqrt(b)*weierstrass
Zeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + ((2*B*a*b - 5*A*b^2)*
x^3 - 3*A*a*b)*sqrt(b*x^3 + a))/(a^2*b^2*x^4 + a^3*b*x)
```

3.241.6 Sympy [A] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{2}{3}\right)} + \frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**2/(b*x**3+a)**(3/2),x)
```

output
```
A*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))
```

3.241.7 Maxima [F]

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^2), x)
```

3.241.8 Giac [F]

$$\int \frac{A+Bx^3}{x^2 \left(a+bx^3\right)^{3/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^2), x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^2 \left(a+bx^3\right)^{3/2}}\,dx=\int \frac{B\,x^3+A}{x^2\,{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

input
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(3/2)), x)
```

3.242 $\int \frac{A+Bx^3}{x^5(a+bx^3)^{3/2}}\,dx$

3.242.1 Optimal result

Integrand size = 22, antiderivative size = 580

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx=-\frac{A}{4ax^4\sqrt{a+bx^3}}-\frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}}$$

$$+\frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x}-\frac{5\sqrt[3]{b}(11Ab-8aB)\sqrt{a+bx^3}}{24a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}(11Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right.}{16\ 3^{3/4}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{5\sqrt[3]{b}(11Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-1/4*A/a/x^4/(b*x^3+a)^(1/2)+1/12*(-11*A*b+8*B*a)/a^2/x/(b*x^3+a)^(1/2)+5/
24*(11*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^3/x-5/24*b^(1/3)*(11*A*b-8*B*a)*(b*x^3
+a)^(1/2)/a^3/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-5/72*b^(1/3)*(11*A*b-8*B*a)*
(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)*2^(1/2)/(b*x^3+
a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)+5/48*b^(1/3)*(11*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6
^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1
/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.242.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.12

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx=\frac{-2aA+(11Ab-8aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{8a^2x^4\sqrt{a+bx^3}}$$

input

```
Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x]
```

output

```
(-2*a*A + (11*A*b - 8*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3,
 3/2, 2/3, -((b*x^3)/a)])/(8*a^2*x^4*Sqrt[a + b*x^3])
```

3.242.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {955, 819, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(11Ab-8aB)\int \frac{1}{x^2\left(bx^3+a\right)^{3/2}}dx}{8a}-\frac{A}{4ax^4\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(11Ab-8aB)\left(\frac{5\int \frac{1}{x^2\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ax\sqrt{a+bx^3}}\right)}{8a}-\frac{A}{4ax^4\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(11Ab-8aB)\left(\frac{5\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{3a}+\frac{2}{3ax\sqrt{a+bx^3}}\right)}{8a}-\frac{A}{4ax^4\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{832}$$

$$-\frac{(11Ab-8aB)\left(\frac{5\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{3a}+\frac{2}{3ax\sqrt{a+bx^3}}\right)}{8a}-\frac{A}{4ax^4\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{759}$$

3.242. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx$

$$-\frac{(11Ab-8aB)\left(\frac{5\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(1-}{\sqrt[3]{b}x+(1+}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{2a}\right.}{3a}\right.}{8a}$$

$$\frac{A}{4ax^4\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2416}$$

3.242. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx$

$$-\frac{(11Ab-8aB)\left(\frac{5\left(\frac{b\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{}\right.}{}\right.}{}$$

3.242. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx$ $\frac{A}{4ax^4\sqrt{a+bx^3}}$

input
```
Int[(A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x]
```

output
```
-1/4*A/(a*x^4*Sqrt[a + b*x^3]) - ((11*A*b - 8*a*B)*(2/(3*a*x*Sqrt[a + b*x^
3]) + (5*(-(Sqrt[a + b*x^3]/(a*x)) + (b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)
*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(2*a)))/(3*a)))/(8*a)
```

3.242.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.242.4 Maple [A] (verified)

Time = 5.36 (sec) , antiderivative size = 540, normalized size of antiderivative = 0.93

method	result
	$2i\left(-\frac{(Ab-Ba)b}{3a^3}-\frac{b(13Ab-8Ba)}{16a^3}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(}{}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$
elliptic	$\frac{2b\,x^2(Ab-Ba)}{3a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{4a^2x^4}+\frac{(13Ab-8Ba)\sqrt{b\,x^3+a}}{8a^3x}-$
default	Expression too large to display
risch	Expression too large to display

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*b*x^2/a^3*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-1/4/a^2*A*(b*x^3+a)^(1/2)/x^4+
1/8/a^3*(13*A*b-8*B*a)*(b*x^3+a)^(1/2)/x-2/3*I*(-1/3*(A*b-B*a)*b/a^3-1/16*
b*(13*A*b-8*B*a)/a^3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-
a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+
1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
)
```

3.242. $\int \frac{A+Bx^3}{x^5(a+bx^3)^{3/2}}\,dx$

3.242.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.22

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx = -\frac{5\left((8\,Bab-11\,Ab^2)x^7+(8\,Ba^2-11\,Aab)x^4\right)\sqrt{b}\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{24\left(a^3bx^7+a^4x^4\right)}$$

input

```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output

```
-1/24*(5*((8*B*a*b - 11*A*b^2)*x^7 + (8*B*a^2 - 11*A*a*b)*x^4)*sqrt(b)*wei
erstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (5*(8*B*a*b -
 11*A*b^2)*x^6 + 3*(8*B*a^2 - 11*A*a*b)*x^3 + 6*A*a^2)*sqrt(b*x^3 + a))/(a
^3*b*x^7 + a^4*x^4)
```

3.242.6 Sympy [A] (verification not implemented)

Time = 16.46 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{4}{3},\frac{3}{2}\\-\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{B\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{3}{2}\\\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{2}{3}\right)}$$

input

```
integrate((B*x**3+A)/x**5/(b*x**3+a)**(3/2),x)
```

output

```
A*gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(3/2)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(3/2)*x*gamma(2/3))
```

3.242.7 Maxima [F]

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^5), x)
```

3.242.8 Giac [F]

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^5), x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{B\,x^3+A}{x^5\,\left(b\,x^3+a\right)^{3/2}}\,dx$$

input
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(3/2)), x)
```

3.243 $\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx$

3.243.1 Optimal result

Integrand size = 22, antiderivative size = 611

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{7ax^7\sqrt{a+bx^3}}-\frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}}+\frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4}$$

$$-\frac{55b(17Ab-14aB)\sqrt{a+bx^3}}{336a^4x}+\frac{55b^{4/3}(17Ab-14aB)\sqrt{a+bx^3}}{336a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{55\sqrt{2-\sqrt{3}}b^{4/3}(17Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{224\ 3^{3/4}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{55b^{4/3}(17Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\right.}{168\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/7*A/a/x^7/(b*x^3+a)^(1/2)+1/21*(-17*A*b+14*B*a)/a^2/x^4/(b*x^3+a)^(1/2)
+11/168*(17*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^3/x^4-55/336*b*(17*A*b-14*B*a)*(
b*x^3+a)^(1/2)/a^4/x+55/336*b^(4/3)*(17*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^4/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))+55/1008*b^(4/3)*(17*A*b-14*B*a)*(a^(1/3)+b^(
1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(11/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-55/672
*b^(4/3)*(17*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(11/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.243.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.12

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx = \frac{-8aA+(17Ab-14aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{3}{2},-\frac{1}{3},-\frac{bx^3}{a}\right)}{56a^2x^7\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x]
```

output
```
(-8*a*A + (17*A*b - 14*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-4/3
, 3/2, -1/3, -((b*x^3)/a)])/(56*a^2*x^7*Sqrt[a + b*x^3])
```

3.243.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {955, 819, 847, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^8 \left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(17Ab-14aB)\int \frac{1}{x^5 \left(bx^3+a\right)^{3/2}}dx}{14a}-\frac{A}{7ax^7\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{(17Ab-14aB)\left(\frac{11\int \frac{1}{x^5\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ax^4\sqrt{a+bx^3}}\right)}{14a}-\frac{A}{7ax^7\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{(17Ab-14aB)\left(\frac{11\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+a}}dx}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)}{3a}+\frac{2}{3ax^4\sqrt{a+bx^3}}\right)}{14a}-\frac{A}{7ax^7\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{(17Ab-14aB)\left(\frac{11\left(-\frac{5b\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)}{3a}+\frac{2}{3ax^4\sqrt{a+bx^3}}\right)}{14a}-\frac{A}{7ax^7\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 832}$$

3.243. $\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx$

$$-\frac{(17Ab-14aB)\left(\frac{11\left(-\frac{5b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)}{3a}+\frac{2}{3ax^4\sqrt{a+bx^3}}\right)}{14a}-\frac{A}{7ax^7\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 759$$

3.243. $\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx$

$$(17Ab-14aB)\left(\frac{11\left(-\frac{\left(5b\,\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{2a}\right.}{8a}\right.}{3a}\right.$$

3.243. $\int\frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx$

$\downarrow$ 2416

3.243. $\int \frac{A+Bx^3}{x^8(a+bx^3)^{3/2}}\,dx$

$$(17Ab-14aB)\Bigg(\frac{11\Bigg(-\frac{5b\Bigg(\frac{b\Bigg(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\Bigg(\arcsin\Bigg(\frac{\sqrt[3]{b}x+(1-}{\sqrt[3]{b}x+(1+}}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}}{}}{}}{}$$

3.243. $\int \frac{A+Bx^3}{x^8(a+bx^3)^{3/2}}\,dx$

input

```
Int[(A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x]
```

output

```
-1/7*A/(a*x^7*Sqrt[a + b*x^3]) - ((17*A*b - 14*a*B)*(2/(3*a*x^4*Sqrt[a + b
*x^3]) + (11*(-1/4*Sqrt[a + b*x^3]/(a*x^4) - (5*b*(-(Sqrt[a + b*x^3]/(a*x)
) + (b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (
2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))
)/(2*a)))/(8*a)))/(3*a)))/(14*a)
```

3.243.3.1 Defintions of rubi rules used

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.243.4 Maple [A] (verified)

Time = 5.22 (sec) , antiderivative size = 573, normalized size of antiderivative = 0.94

method	result
	$2i\left(\frac{b^2(Ab-Ba)}{3a^4}+\frac{b^2(237Ab-182Ba)}{224a^4}\right.$
elliptic	$-\frac{2b^2x^2(Ab-Ba)}{3a^4\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{7a^2x^7}+\frac{(25Ab-14Ba)\sqrt{b\,x^3+a}}{56a^3x^4}-\frac{(237Ab-182Ba)b\sqrt{b\,x^3+a}}{112a^4x}-$
risch	Expression too large to display
default	Expression too large to display

input
```
int((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*b^2*x^2/a^4*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-1/7*A/a^2*(b*x^3+a)^(1/2)/x
^7+1/56/a^3*(25*A*b-14*B*a)*(b*x^3+a)^(1/2)/x^4-1/112/a^4*(237*A*b-182*B*a
)*b*(b*x^3+a)^(1/2)/x-2/3*I*(1/3*b^2*(A*b-B*a)/a^4+1/224*b^2*(237*A*b-182*
B*a)/a^4)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.243. $\int \frac{A+Bx^3}{x^8(a+bx^3)^{3/2}}\,dx$

3.243.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.26

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx = \frac{55\,\left((14\,Bab^2-17\,Ab^3)x^{10}+(14\,Ba^2b-17\,Aab^2)x^7\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{wei}\right.}{}$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
1/336*(55*((14*B*a*b^2 - 17*A*b^3)*x^10 + (14*B*a^2*b - 17*A*a*b^2)*x^7)*s
qrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (55
*(14*B*a*b^2 - 17*A*b^3)*x^9 + 33*(14*B*a^2*b - 17*A*a*b^2)*x^6 - 48*A*a^3
 - 6*(14*B*a^3 - 17*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b*x^10 + a^5*x^7)
```

3.243.6 Sympy [A] (verification not implemented)

Time = 43.68 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{7}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{7}{3},\frac{3}{2}\\-\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{B\Gamma\left(-\frac{4}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{4}{3},\frac{3}{2}\\-\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**8/(b*x**3+a)**(3/2),x)
```

output
```
A*gamma(-7/3)*hyper((-7/3, 3/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(3/2)*x**7*gamma(-4/3)) + B*gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(3/2)*x**4*gamma(-1/3))
```

3.243.7 Maxima [F]

$$\int \frac{A+Bx^3}{x^8 (a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^8), x)
```

3.243.8 Giac [F]

$$\int \frac{A+Bx^3}{x^8 (a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^8), x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^8 (a+bx^3)^{3/2}}\,dx = \int \frac{B\,x^3+A}{x^8\,(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/(x^8*(a + b*x^3)^(3/2)), x)
```

3.244 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

3.244.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2a^2(Ab-aB)}{9b^4\,(a+bx^3)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}} + \frac{2(Ab-3aB)\sqrt{a+bx^3}}{3b^4}+\frac{2B(a+bx^3)^{3/2}}{9b^4}$$

output
```
-2/9*a^2*(A*b-B*a)/b^4/(b*x^3+a)^(3/2)+2/9*B*(b*x^3+a)^(3/2)/b^4+2/3*a*(2*
A*b-3*B*a)/b^4/(b*x^3+a)^(1/2)+2/3*(A*b-3*B*a)*(b*x^3+a)^(1/2)/b^4
```

3.244.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.71

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(-16a^3B+8a^2b(A-3Bx^3)-6ab^2x^3(-2A+Bx^3)+b^3x^6(3A+Bx^3))}{9b^4\,(a+bx^3)^{3/2}}$$

input
```
Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(-16*a^3*B + 8*a^2*b*(A - 3*B*x^3) - 6*a*b^2*x^3*(-2*A + B*x^3) + b^3*x
^6*(3*A + B*x^3)))/(9*b^4*(a + b*x^3)^(3/2))
```

3.244.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (A+Bx^3)}{(a+bx^3)^{5/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6 (Bx^3+A)}{(bx^3+a)^{5/2}}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(-\frac{(aB-Ab)a^2}{b^3 (bx^3+a)^{5/2}}+\frac{(3aB-2Ab)a}{b^3 (bx^3+a)^{3/2}}+\frac{B\sqrt{bx^3+a}}{b^3}+\frac{Ab-3aB}{b^3\sqrt{bx^3+a}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{2a^2(Ab-aB)}{3b^4 (a+bx^3)^{3/2}}+\frac{2a(2Ab-3aB)}{b^4\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}(Ab-3aB)}{b^4}+\frac{2B (a+bx^3)^{3/2}}{3b^4}\right)$$

input
```
Int[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
((-2*a^2*(A*b - a*B))/(3*b^4*(a + b*x^3)^(3/2)) + (2*a*(2*A*b - 3*a*B))/(b
^4*Sqrt[a + b*x^3]) + (2*(A*b - 3*a*B)*Sqrt[a + b*x^3])/b^4 + (2*B*(a + b*
x^3)^(3/2))/(3*b^4))/3
```

3.244.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.244.4 Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{\left(2B\,x^{9}+6A\,x^{6}\right)b^{3}+24x^{3}\left(-\frac{x^{3}B}{2}+A\right)a\,b^{2}+16a^{2}\left(-3x^{3}B+A\right)b-32a^{3}B}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{4}}$
risch	$\frac{2\left(bB\,x^{3}+3Ab-8Ba\right)\sqrt{b\,x^{3}+a}}{9b^{4}}+\frac{2a\left(6A\,b^{2}x^{3}-9Bab\,x^{3}+5abA-8a^{2}B\right)}{9b^{4}\left(b\,x^{3}+a\right)^{\frac{3}{2}}}$
gosper	$\frac{\frac{2}{9}b^{3}B\,x^{9}+\frac{2}{3}x^{6}b^{3}A-\frac{4}{3}B\,x^{6}a\,b^{2}+\frac{8}{3}aA\,b^{2}x^{3}-\frac{16}{3}B\,a^{2}b\,x^{3}+\frac{16}{9}a^{2}bA-\frac{32}{9}a^{3}B}{\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{4}}$
trager	$\frac{\frac{2}{9}b^{3}B\,x^{9}+\frac{2}{3}x^{6}b^{3}A-\frac{4}{3}B\,x^{6}a\,b^{2}+\frac{8}{3}aA\,b^{2}x^{3}-\frac{16}{3}B\,a^{2}b\,x^{3}+\frac{16}{9}a^{2}bA-\frac{32}{9}a^{3}B}{\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{4}}$
elliptic	$-\frac{2a^{2}(Ab-Ba)\sqrt{b\,x^{3}+a}}{9b^{6}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2(2Ab-3Ba)a}{3b^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2B\,x^{3}\sqrt{b\,x^{3}+a}}{9b^{3}}+\frac{2\left(\frac{Ab-2Ba}{b^{3}}-\frac{2Ba}{3b^{3}}\right)\sqrt{b\,x^{3}+a}}{3b}$
default	$A\left(-\frac{2a^{2}\sqrt{b\,x^{3}+a}}{9b^{5}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{4a}{3b^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2\sqrt{b\,x^{3}+a}}{3b^{3}}\right)+B\left(\frac{2a^{3}\sqrt{b\,x^{3}+a}}{9b^{6}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2a^{2}}{b^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b^{3}}-16a$

input
```
int(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*((2*B*x^9+6*A*x^6)*b^3+24*x^3*(-1/2*x^3*B+A)*a*b^2+16*a^2*(-3*B*x^3+A)
*b-32*a^3*B)/(b*x^3+a)^(3/2)/b^4
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\,(Bb^3x^9 - 3\,(2\,Bab^2 - Ab^3)x^6 - 16\,Ba^3 + 8\,Aa^2b - 12\,(2\,Ba^2b - Aab^2)x^3)\sqrt{bx^3+a}}{9\,(b^6x^6 + 2\,ab^5x^3 + a^2b^4)}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/9*(B*b^3*x^9 - 3*(2*B*a*b^2 - A*b^3)*x^6 - 16*B*a^3 + 8*A*a^2*b - 12*(2*
B*a^2*b - A*a*b^2)*x^3)*sqrt(b*x^3 + a)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

3.244.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(99) = 198$.

Time = 0.59 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.28

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \begin{cases} \frac{16Aa^2b}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} + \frac{24Aab^2x^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} + \frac{6Ab^3x^6}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} - \frac{3}{9ab^4\sqrt{a+bx^3}} \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{5}{2}}} \end{cases}$$

input
```
integrate(x**8*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
Piecewise((16*A*a**2*b/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b
*x**3)) + 24*A*a*b**2*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a
 + b*x**3)) + 6*A*b**3*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(
a + b*x**3)) - 32*B*a**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
 b*x**3)) - 48*B*a**2*b*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt
(a + b*x**3)) - 12*B*a*b**2*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*
sqrt(a + b*x**3)) + 2*B*b**3*x**9/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3
*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**9/9 + B*x**12/12)/a**(5/2), True))
```

3.244.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2}{9}\,B\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^4} - \frac{9\,\sqrt{bx^3+a}a}{b^4} - \frac{9\,a^2}{\sqrt{bx^3+a}b^4} + \frac{a^3}{(bx^3+a)^{\frac{3}{2}}b^4}\right) + \frac{2}{9}\,A\left(\frac{3\,\sqrt{bx^3+a}}{b^3} + \frac{6\,a}{\sqrt{bx^3+a}b^3} - \frac{a^2}{(bx^3+a)^{\frac{3}{2}}b^3}\right)$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
2/9*B*((b*x^3 + a)^(3/2)/b^4 - 9*sqrt(b*x^3 + a)*a/b^4 - 9*a^2/(sqrt(b*x^3
 + a)*b^4) + a^3/((b*x^3 + a)^(3/2)*b^4)) + 2/9*A*(3*sqrt(b*x^3 + a)/b^3 +
 6*a/(sqrt(b*x^3 + a)*b^3) - a^2/((b*x^3 + a)^(3/2)*b^3))
```

3.244.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\,(9\,(bx^3+a)Ba^2 - Ba^3 - 6\,(bx^3+a)Aab + Aa^2b)}{9\,(bx^3+a)^{\frac{3}{2}}b^4} + \frac{2\left((bx^3+a)^{\frac{3}{2}}Bb^8 - 9\,\sqrt{bx^3+a}Bab^8 + 3\,\sqrt{bx^3+a}Ab^9\right)}{9\,b^{12}}$$

input
```
integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
-2/9*(9*(b*x^3 + a)*B*a^2 - B*a^3 - 6*(b*x^3 + a)*A*a*b + A*a^2*b)/((b*x^3
 + a)^(3/2)*b^4) + 2/9*((b*x^3 + a)^(3/2)*B*b^8 - 9*sqrt(b*x^3 + a)*B*a*b^
8 + 3*sqrt(b*x^3 + a)*A*b^9)/b^12
```

3.244.9 Mupad [B] (verification not implemented)

Time = 7.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.41

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,(A\,b-2\,B\,a)}{b^3}-\frac{4\,B\,a}{3\,b^3}\right)}{3\,b} - \frac{\frac{2\,B\,a^2-2\,A\,a\,b}{3\,b^4}-\frac{a\,\left(\frac{2\,A\,b^2-2\,B\,a\,b}{3\,b^4}-\frac{2\,B\,a}{3\,b^3}\right)}{b}}{\sqrt{b\,x^3+a}} - \frac{a^2\,\left(\frac{2\,A}{9\,b}-\frac{2\,B\,a}{9\,b^2}\right)}{b^2\,{\left(b\,x^3+a\right)}^{3/2}} + \frac{2\,B\,x^3\,\sqrt{b\,x^3+a}}{9\,b^3}$$

input

```
int((x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output

```
((a + b*x^3)^(1/2)*((2*(A*b - 2*B*a))/b^3 - (4*B*a)/(3*b^3)))/(3*b) - ((2*
B*a^2 - 2*A*a*b)/(3*b^4) - (a*((2*A*b^2 - 2*B*a*b)/(3*b^4) - (2*B*a)/(3*b^
3)))/b)/(a + b*x^3)^(1/2) - (a^2*((2*A)/(9*b) - (2*B*a)/(9*b^2)))/(b^2*(a
+ b*x^3)^(3/2)) + (2*B*x^3*(a + b*x^3)^(1/2))/(9*b^3)
```

3.245 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

3.245.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2a(Ab-aB)}{9b^3(a+bx^3)^{3/2}} - \frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}} + \frac{2B\sqrt{a+bx^3}}{3b^3}$$

output
```
2/9*a*(A*b-B*a)/b^3/(b*x^3+a)^(3/2)-2/3*(A*b-2*B*a)/b^3/(b*x^3+a)^(1/2)+2/
3*B*(b*x^3+a)^(1/2)/b^3
```

3.245.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(-2aAb+8a^2B-3Ab^2x^3+12abBx^3+3b^2Bx^6)}{9b^3(a+bx^3)^{3/2}}$$

input
```
Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(-2*a*A*b + 8*a^2*B - 3*A*b^2*x^3 + 12*a*b*B*x^3 + 3*b^2*B*x^6))/(9*b^3
*(a + b*x^3)^(3/2))
```

3.245.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^3)}{(a+bx^3)^{5/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^3 (Bx^3+A)}{(bx^3+a)^{5/2}} dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3} \int \left(\frac{B}{b^2\sqrt{bx^3+a}}+\frac{Ab-2aB}{b^2 (bx^3+a)^{3/2}}+\frac{a(aB-Ab)}{b^2 (bx^3+a)^{5/2}}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{2(Ab-2aB)}{b^3\sqrt{a+bx^3}}+\frac{2a(Ab-aB)}{3b^3 (a+bx^3)^{3/2}}+\frac{2B\sqrt{a+bx^3}}{b^3}\right)$$

input
```
Int[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
((2*a*(A*b - a*B))/(3*b^3*(a + b*x^3)^(3/2)) - (2*(A*b - 2*a*B))/(b^3*Sqrt
[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/b^3)/3
```

3.245.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.245.4 Maple [A] (verified)

Time = 4.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$-\frac{4\left(\frac{3x^{3}\left(-x^{3}B+A\right)b^{2}}{2}+a\left(-6x^{3}B+A\right)b-4a^{2}B\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{3}}$	49
gosper	$-\frac{2\left(-3b^{2}B\,x^{6}+3A\,b^{2}x^{3}-12Bab\,x^{3}+2abA-8a^{2}B\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{3}}$	53
trager	$-\frac{2\left(-3b^{2}B\,x^{6}+3A\,b^{2}x^{3}-12Bab\,x^{3}+2abA-8a^{2}B\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{3}}$	53
risch	$\frac{2B\sqrt{b\,x^{3}+a}}{3b^{3}}-\frac{2\left(3A\,b^{2}x^{3}-6Bab\,x^{3}+2abA-5a^{2}B\right)}{9b^{3}\left(b\,x^{3}+a\right)^{\frac{3}{2}}}$	60
elliptic	$\frac{2\left(Ab-Ba\right)a\sqrt{b\,x^{3}+a}}{9b^{5}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2\left(Ab-2Ba\right)}{3b^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2B\sqrt{b\,x^{3}+a}}{3b^{3}}$	77
default	$B\left(-\frac{2a^{2}\sqrt{b\,x^{3}+a}}{9b^{5}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{4a}{3b^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2\sqrt{b\,x^{3}+a}}{3b^{3}}\right)+A\left(\frac{2a\sqrt{b\,x^{3}+a}}{9b^{4}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2}{3b^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}\right)$	113

input
```
int(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/9/(b*x^3+a)^(3/2)*(3/2*x^3*(-B*x^3+A)*b^2+a*(-6*B*x^3+A)*b-4*a^2*B)/b^3
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\,(3\,Bb^2x^6+3\,(4\,Bab-Ab^2)x^3+8\,Ba^2-2\,Aab)\sqrt{bx^3+a}}{9\,(b^5x^6+2\,ab^4x^3+a^2b^3)}$$

input

```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output

```
2/9*(3*B*b^2*x^6 + 3*(4*B*a*b - A*b^2)*x^3 + 8*B*a^2 - 2*A*a*b)*sqrt(b*x^3
 + a)/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

3.245.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(70) = 140$.

Time = 0.44 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.29

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \begin{cases} -\frac{4Aab}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} - \frac{6Ab^2x^3}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} + \frac{16Ba^2}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} + \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{5}{2}}} \end{cases}$$

input

```
integrate(x**5*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output

```
Piecewise((-4*A*a*b/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x*
*3)) - 6*A*b**2*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x
**3)) + 16*B*a**2/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3
)) + 24*B*a*b*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**
3)) + 6*B*b**2*x**6/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x*
*3)), Ne(b, 0)), ((A*x**6/6 + B*x**9/9)/a**(5/2), True))
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2}{9}\,B\left(\frac{3\,\sqrt{bx^3+a}}{b^3} + \frac{6\,a}{\sqrt{bx^3+a}b^3} - \frac{a^2}{(bx^3+a)^{\frac{3}{2}}b^3}\right) - \frac{2}{9}\,A\left(\frac{3}{\sqrt{bx^3+a}b^2} - \frac{a}{(bx^3+a)^{\frac{3}{2}}b^2}\right)$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
2/9*B*(3*sqrt(b*x^3 + a)/b^3 + 6*a/(sqrt(b*x^3 + a)*b^3) - a^2/((b*x^3 + a
)^(3/2)*b^3)) - 2/9*A*(3/(sqrt(b*x^3 + a)*b^2) - a/((b*x^3 + a)^(3/2)*b^2)
)
```

3.245.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\,\sqrt{bx^3+a}B}{3\,b^3} + \frac{2\,(6\,(bx^3+a)Ba - Ba^2 - 3\,(bx^3+a)Ab + Aab)}{9\,(bx^3+a)^{\frac{3}{2}}b^3}$$

input
```
integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
2/3*sqrt(b*x^3 + a)*B/b^3 + 2/9*(6*(b*x^3 + a)*B*a - B*a^2 - 3*(b*x^3 + a)
*A*b + A*a*b)/((b*x^3 + a)^(3/2)*b^3)
```

3.245.9 Mupad [B] (verification not implemented)

Time = 7.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{6\,B\,(b\,x^3+a)^2 - 2\,B\,a^2 - 6\,A\,b\,(b\,x^3+a) + 12\,B\,a\,(b\,x^3+a) + 2\,A\,a\,b}{9\,b^3\,(b\,x^3+a)^{3/2}}$$

input
```
int((x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output
```
(6*B*(a + b*x^3)^2 - 2*B*a^2 - 6*A*b*(a + b*x^3) + 12*B*a*(a + b*x^3) + 2*
A*a*b)/(9*b^3*(a + b*x^3)^(3/2))
```

3.246 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

3.246.1 Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2(Ab-aB)}{9b^2(a+bx^3)^{3/2}} - \frac{2B}{3b^2\sqrt{a+bx^3}}$$

output
```
-2/9*(A*b-B*a)/b^2/(b*x^3+a)^(3/2)-2/3*B/b^2/(b*x^3+a)^(1/2)
```

3.246.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2(Ab+2aB+3bBx^3)}{9b^2(a+bx^3)^{3/2}}$$

input
```
Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(-2*(A*b + 2*a*B + 3*b*B*x^3))/(9*b^2*(a + b*x^3)^(3/2))
```

3.246.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow 946$$

$$\frac{1}{3}\int \frac{Bx^3+A}{(bx^3+a)^{5/2}}dx^3$$

$$\Big\downarrow 53$$

$$\frac{1}{3}\int \left(\frac{B}{b\,(bx^3+a)^{3/2}}+\frac{Ab-aB}{b\,(bx^3+a)^{5/2}}\right)dx^3$$

$$\Big\downarrow 2009$$

$$\frac{1}{3}\left(-\frac{2(Ab-aB)}{3b^2\,(a+bx^3)^{3/2}}-\frac{2B}{b^2\sqrt{a+bx^3}}\right)$$

input
```
Int[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
((-2*(A*b - a*B))/(3*b^2*(a + b*x^3)^(3/2)) - (2*B)/(b^2*Sqrt[a + b*x^3]))
/3
```

3.246.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.246.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{2(3bB\,x^3+Ab+2Ba)}{9(b\,x^3+a)^{\frac{3}{2}}\,b^2}$	30
trager	$-\frac{2(3bB\,x^3+Ab+2Ba)}{9(b\,x^3+a)^{\frac{3}{2}}\,b^2}$	30
pseudoelliptic	$-\frac{2((3x^3B+A)b+2Ba)}{9(b\,x^3+a)^{\frac{3}{2}}\,b^2}$	30
elliptic	$-\frac{2(Ab-Ba)\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}-\frac{2B}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}$	54
default	$B\left(\frac{2a\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}-\frac{2}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}\right)-\frac{2A}{9b(b\,x^3+a)^{\frac{3}{2}}}$	64

input
```
int(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9/(b*x^3+a)^(3/2)*(3*B*b*x^3+A*b+2*B*a)/b^2
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\,(3\,Bbx^3+2\,Ba+Ab)\sqrt{bx^3+a}}{9\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/9*(3*B*b*x^3 + 2*B*a + A*b)*sqrt(b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^
2*b^2)
```

3.246.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(44) = 88$.

Time = 0.34 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.13

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \begin{cases} -\frac{2Ab}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} - \frac{4Ba}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} - \frac{6Bbx^3}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} & \text{for } b \neq \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{a^{\frac{5}{2}}} & \text{otherwis} \end{cases}$$

input
```
integrate(x**2*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
Piecewise((-2*A*b/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3
)) - 4*B*a/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)) - 6*
B*b*x**3/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)), Ne(b,
 0)), ((A*x**3/3 + B*x**6/6)/a**(5/2), True))
```

3.246.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2}{9}B\left(\frac{3}{\sqrt{bx^3+a}b^2} - \frac{a}{(bx^3+a)^{\frac{3}{2}}b^2}\right) - \frac{2A}{9(bx^3+a)^{\frac{3}{2}}b}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
-2/9*B*(3/(sqrt(b*x^3 + a)*b^2) - a/((b*x^3 + a)^(3/2)*b^2)) - 2/9*A/((b*x
^3 + a)^(3/2)*b)
```

3.246.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\,(3\,(bx^3+a)B-Ba+Ab)}{9\,(bx^3+a)^{\frac{3}{2}}b^2}$$

input
```
integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
-2/9*(3*(b*x^3 + a)*B - B*a + A*b)/((b*x^3 + a)^(3/2)*b^2)
```

3.246.9 Mupad [B] (verification not implemented)

Time = 6.96 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\,A\,b-2\,B\,a+6\,B\,(b\,x^3+a)}{9\,b^2\,(b\,x^3+a)^{3/2}}$$

input
```
int((x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output
```
-(2*A*b - 2*B*a + 6*B*(a + b*x^3))/(9*b^2*(a + b*x^3)^(3/2))
```

3.247 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx$

3.247.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx=\frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

output
```
2/9*(A*b-B*a)/a/b/(b*x^3+a)^(3/2)-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a
^(5/2)+2/3*A/a^2/(b*x^3+a)^(1/2)
```

3.247.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx=-\frac{2(-4aAb+a^2B-3Ab^2x^3)}{9a^2b\left(a+bx^3\right)^{3/2}}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

input
```
Integrate[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]
```

output
```
(-2*(-4*a*A*b + a^2*B - 3*A*b^2*x^3))/(9*a^2*b*(a + b*x^3)^(3/2)) - (2*A*A
rcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

3.247.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {948, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^3\left(bx^3+a\right)^{5/2}}dx^3$$

$$\Big\downarrow \; 87$$

$$\frac{1}{3}\left(\frac{A\int \frac{1}{x^3\left(bx^3+a\right)^{3/2}}dx^3}{a}+\frac{2(Ab-aB)}{3ab\left(a+bx^3\right)^{3/2}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{3}\left(\frac{A\left(\frac{\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{a}+\frac{2}{a\sqrt{a+bx^3}}\right)}{a}+\frac{2(Ab-aB)}{3ab\left(a+bx^3\right)^{3/2}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{A\left(\frac{2\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{ab}+\frac{2}{a\sqrt{a+bx^3}}\right)}{a}+\frac{2(Ab-aB)}{3ab\left(a+bx^3\right)^{3/2}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(\frac{A\left(\frac{2}{a\sqrt{a+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{a}+\frac{2(Ab-aB)}{3ab\left(a+bx^3\right)^{3/2}}\right)$$

input
```
Int[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]
```

output
```
((2*(A*b - a*B))/(3*a*b*(a + b*x^3)^(3/2)) + (A*(2/(a*Sqrt[a + b*x^3]) - (
2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/a^(3/2)))/a)/3
```

3.247.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.247. $\int \frac{A+Bx^3}{x(a+bx^3)^{5/2}}\,dx$

3.247.4 Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

method	result	size
pseudoelliptic	$-\frac{2\left(-3A\sqrt{a}\,b^{2}x^{3}+3A\left(b\,x^{3}+a\right)^{\frac{3}{2}}b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)-4A\,a^{\frac{3}{2}}b+B\,a^{\frac{5}{2}}\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{\frac{5}{2}}b}$	70
elliptic	$\frac{2(Ab-Ba)\sqrt{b\,x^{3}+a}}{9b^{3}a\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2A}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2A\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}$	77
default	$-\frac{2B}{9b\left(b\,x^{3}+a\right)^{\frac{3}{2}}}+A\left(\frac{2\sqrt{b\,x^{3}+a}}{9a\,b^{2}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}\right)$	85

input
```
int((B*x^3+A)/x/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(-3*A*a^(1/2)*b^2*x^3+3*A*(b*x^3+a)^(3/2)*b*arctanh((b*x^3+a)^(1/2)/a
^(1/2))-4*A*a^(3/2)*b+B*a^(5/2))/(b*x^3+a)^(3/2)/a^(5/2)/b
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.16

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx=\left[\frac{3\left(Ab^3x^6+2\,Aab^2x^3+Aa^2b\right)\sqrt{a}\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\left(3\,Aab^2x^3-Ba^3+4\right.}{9\left(a^3b^3x^6+2\,a^4b^2x^3+a^5b\right)}\right.$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/9*(3*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*sqrt(a)*log((b*x^3 - 2*sqrt(
b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*A*a*b^2*x^3 - B*a^3 + 4*A*a^2*b)*sqr
t(b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b), 2/9*(3*(A*b^3*x^6 + 2
*A*a*b^2*x^3 + A*a^2*b)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*A
*a*b^2*x^3 - B*a^3 + 4*A*a^2*b)*sqrt(b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*
x^3 + a^5*b)]
```

3.247.6 Sympy [A] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{Ab}{3a^2\sqrt{a+bx^3}}+\frac{Ab\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3a^2\sqrt{-a}}-\frac{-Ab+Ba}{9a\left(a+bx^3\right)^{\frac{3}{2}}}\right)}{b} & \text{for } b\neq 0 \\ \frac{A\log\left(Bx^3\right)+Bx^3}{3a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input

```
integrate((B*x**3+A)/x/(b*x**3+a)**(5/2),x)
```

output

```
Piecewise((2*(A*b/(3*a**2*sqrt(a + b*x**3)) + A*b*atan(sqrt(a + b*x**3)/sq
rt(-a))/(3*a**2*sqrt(-a)) - (-A*b + B*a)/(9*a*(a + b*x**3)**(3/2)))/b, Ne(
b, 0)), ((A*log(B*x**3) + B*x**3)/(3*a**(5/2)), True))
```

3.247.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx = \frac{1}{9}A\left(\frac{3\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(3\,bx^3+4\,a\right)}{\left(bx^3+a\right)^{\frac{3}{2}}a^2}\right)-\frac{2\,B}{9\left(bx^3+a\right)^{\frac{3}{2}}b}$$

input

```
integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output

```
1/9*A*(3*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5
/2) + 2*(3*b*x^3 + 4*a)/((b*x^3 + a)^(3/2)*a^2)) - 2/9*B/((b*x^3 + a)^(3/2
)*b)
```

3.247.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx = \frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}a^2}-\frac{2\left(Ba^2-3\left(bx^3+a\right)Ab-Aab\right)}{9\left(bx^3+a\right)^{\frac{3}{2}}a^2b}$$

input
```
integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 2/9*(B*a^2 - 3*(b*
x^3 + a)*A*b - A*a*b)/((b*x^3 + a)^(3/2)*a^2*b)
```

3.247.9 Mupad [B] (verification not implemented)

Time = 7.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx=\frac{\frac{2\,A}{9\,a}-\frac{2\,B}{9\,b}}{{\left(b\,x^3+a\right)}^{3/2}}+\frac{2\,A}{3\,a^2\,\sqrt{b\,x^3+a}}+\frac{A\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{5/2}}$$

input
```
int((A + B*x^3)/(x*(a + b*x^3)^(5/2)),x)
```

output
```
((2*A)/(9*a) - (2*B)/(9*b))/(a + b*x^3)^(3/2) + (2*A)/(3*a^2*(a + b*x^3)^(
1/2)) + (A*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/
2)))/x^6))/(3*a^(5/2))
```

3.248 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx$

3.248.1 Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx=\frac{-5Ab+2aB}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}+\frac{(5Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}$$

output
```
1/9*(-5*A*b+2*B*a)/a^2/(b*x^3+a)^(3/2)-1/3*A/a/x^3/(b*x^3+a)^(3/2)+1/3*(5*
A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(7/2)+1/3*(-5*A*b+2*B*a)/a^3
/(b*x^3+a)^(1/2)
```

3.248.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx=\frac{-3a^2A-20aAbx^3+8a^2Bx^3-15Ab^2x^6+6abBx^6}{9a^3x^3\left(a+bx^3\right)^{3/2}}+\frac{(5Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}$$

input
```
Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]
```

output
```
(-3*a^2*A - 20*a*A*b*x^3 + 8*a^2*B*x^3 - 15*A*b^2*x^6 + 6*a*b*B*x^6)/(9*a^
3*x^3*(a + b*x^3)^(3/2)) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a
]])/(3*a^(7/2))
```

3.248.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{Bx^3+A}{x^6\left(bx^3+a\right)^{5/2}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(-\frac{(5Ab-2aB)\int \frac{1}{x^3\left(bx^3+a\right)^{5/2}}dx^3}{2a}-\frac{A}{ax^3\left(a+bx^3\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(-\frac{(5Ab-2aB)\left(\frac{\int \frac{1}{x^3\left(bx^3+a\right)^{3/2}}dx^3}{a}+\frac{2}{3a\left(a+bx^3\right)^{3/2}}\right)}{2a}-\frac{A}{ax^3\left(a+bx^3\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(-\frac{(5Ab-2aB)\left(\frac{\frac{\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{a}+\frac{2}{a\sqrt{a+bx^3}}}{a}+\frac{2}{3a\left(a+bx^3\right)^{3/2}}\right)}{2a}-\frac{A}{ax^3\left(a+bx^3\right)^{3/2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{(5Ab-2aB)\left(\frac{\frac{2\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{ab}+\frac{2}{a\sqrt{a+bx^3}}}{a}+\frac{2}{3a(a+bx^3)^{3/2}}\right)}{2a}-\frac{A}{ax^3\left(a+bx^3\right)^{3/2}}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{3}\left(-\frac{(5Ab-2aB)\left(\frac{\frac{2}{a\sqrt{a+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a+bx^3)^{3/2}}\right)}{2a}-\frac{A}{ax^3\left(a+bx^3\right)^{3/2}}\right)$$

input
```
Int[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]
```

output
```
(-(A/(a*x^3*(a + b*x^3)^(3/2))) - ((5*A*b - 2*a*B)*(2/(3*a*(a + b*x^3)^(3/
2)) + (2/(a*Sqrt[a + b*x^3]) - (2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/a^(3/2
))/a))/(2*a))/3
```

3.248.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.248.4 Maple [A] (verified)

Time = 4.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{A\sqrt{b\,x^{3}+a}}{3a^{3}x^{3}}-\frac{-\frac{2\left(5Ab-2Ba\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{\frac{8Ab}{3}-\frac{4Ba}{3}}{\sqrt{b\,x^{3}+a}}+\frac{4a\left(Ab-Ba\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}}}{2a^{3}}$
pseudoelliptic	$-\frac{-5\left(b\,x^{3}+a\right)^{\frac{3}{2}}x^{3}\left(Ab-\frac{2Ba}{5}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)+\frac{20x^{3}b\left(-\frac{3x^{3}B}{10}+A\right)a^{\frac{3}{2}}}{3}+\left(-\frac{8x^{3}B}{3}+A\right)a^{\frac{5}{2}}+5A\sqrt{a}\,b^{2}x^{6}}{3\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{\frac{7}{2}}x^{3}}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{3a^{3}x^{3}}-\frac{2\left(Ab-Ba\right)\sqrt{b\,x^{3}+a}}{9a^{2}b^{2}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2\left(2Ab-Ba\right)}{3a^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{\left(5Ab-2Ba\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{7}{2}}}$
default	$B\left(\frac{2\sqrt{b\,x^{3}+a}}{9a\,b^{2}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}\right)+A\left(-\frac{\sqrt{b\,x^{3}+a}}{3a^{3}x^{3}}-\frac{2\sqrt{b\,x^{3}+a}}{9a^{2}b\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{4b}{3a^{3}\sqrt{(x^{3}}}\right.$

input
```
int((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^3*A*(b*x^3+a)^(1/2)/x^3-1/2/a^3*(-2/3*(5*A*b-2*B*a)*arctanh((b*x^3+
a)^(1/2)/a^(1/2))/a^(1/2)+4/3*(2*A*b-B*a)/(b*x^3+a)^(1/2)+4/9*a*(A*b-B*a)/
(b*x^3+a)^(3/2))
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.11

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx = \left[-\frac{3\left(\left(2\,Bab^2-5\,Ab^3\right)x^9+2\left(2\,Ba^2b-5\,Aab^2\right)x^6+\left(2\,Ba^3-5\,Aa^2b\right)x^3\right)\sqrt{a}\log\left(\frac{bx^3}{}\right.}{18\left(a^4b^2x^9\right.}\right.$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/18*(3*((2*B*a*b^2 - 5*A*b^3)*x^9 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^6 + (2*
B*a^3 - 5*A*a^2*b)*x^3)*sqrt(a)*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2
*a)/x^3) - 2*(3*(2*B*a^2*b - 5*A*a*b^2)*x^6 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a
^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3), 1/9*(3*
((2*B*a*b^2 - 5*A*b^3)*x^9 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^6 + (2*B*a^3 - 5*
A*a^2*b)*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*(2*B*a^2*b
- 5*A*a*b^2)*x^6 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))
/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)]
```

3.248.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. 2(107) = 214.

Time = 126.77 (sec) , antiderivative size = 1608, normalized size of antiderivative = 14.23

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x**3+A)/x**4/(b*x**3+a)**(5/2),x)
```

output
```
A*(-6*a**17*sqrt(1 + b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 +
54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) - 46*a**16*b*x**3*sqrt(1
 + b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*
x**9 + 18*a**(33/2)*b**3*x**12) - 15*a**16*b*x**3*log(b*x**3/a)/(18*a**(39
/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**
3*x**12) + 30*a**16*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(18*a**(39/2)*x**3
+ 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12)
- 70*a**15*b**2*x**6*sqrt(1 + b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*
b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) - 45*a**15*b**2
*x**6*log(b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2
)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) + 90*a**15*b**2*x**6*log(sqrt(1 + b
*x**3/a) + 1)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2
*x**9 + 18*a**(33/2)*b**3*x**12) - 30*a**14*b**3*x**9*sqrt(1 + b*x**3/a)/(
18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(
33/2)*b**3*x**12) - 45*a**14*b**3*x**9*log(b*x**3/a)/(18*a**(39/2)*x**3 +
54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) +
90*a**14*b**3*x**9*log(sqrt(1 + b*x**3/a) + 1)/(18*a**(39/2)*x**3 + 54*a**
(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) - 15*a**
13*b**4*x**12*log(b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*
a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) + 30*a**13*b**4*x**12*lo...
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx =$$
$$-\frac{1}{18}\,A\left(\frac{2\left(15\,(bx^3+a)^2b-10\,(bx^3+a)ab-2\,a^2b\right)}{(bx^3+a)^{\frac{5}{2}}a^3-(bx^3+a)^{\frac{3}{2}}a^4}+\frac{15\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)$$
$$+\frac{1}{9}\,B\left(\frac{3\,\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\,(3\,bx^3+4\,a)}{(bx^3+a)^{\frac{3}{2}}a^2}\right)$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/18*A*(2*(15*(b*x^3 + a)^2*b - 10*(b*x^3 + a)*a*b - 2*a^2*b)/((b*x^3 + a
)^(5/2)*a^3 - (b*x^3 + a)^(3/2)*a^4) + 15*b*log((sqrt(b*x^3 + a) - sqrt(a)
)/(sqrt(b*x^3 + a) + sqrt(a)))/a^(7/2)) + 1/9*B*(3*log((sqrt(b*x^3 + a) -
sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2) + 2*(3*b*x^3 + 4*a)/((b*x^3
+ a)^(3/2)*a^2))
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx = \frac{(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a^3} + \frac{2\,(3\,(bx^3+a)Ba+Ba^2-6\,(bx^3+a)Ab-Aab)}{9\,(bx^3+a)^{\frac{3}{2}}a^3} - \frac{\sqrt{bx^3+a}A}{3\,a^3x^3}$$

input
```
integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*(2*B*a - 5*A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2/9*
(3*(b*x^3 + a)*B*a + B*a^2 - 6*(b*x^3 + a)*A*b - A*a*b)/((b*x^3 + a)^(3/2)
*a^3) - 1/3*sqrt(b*x^3 + a)*A/(a^3*x^3)
```

3.248.9 Mupad [B] (verification not implemented)

Time = 7.23 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)\,\left(5\,A\,b-2\,B\,a\right)}{6\,a^{7/2}} - \frac{\frac{2\,B\,a^2-5\,A\,a\,b}{2\,a^4}-\frac{a\,\left(\frac{A\,b^2}{3\,a^4}+\frac{5\,b\,\left(2\,B\,a^2-5\,A\,a\,b\right)}{6\,a^5}\right)}{b}}{\sqrt{b\,x^3+a}} - \frac{\frac{2\,B\,a^3-5\,A\,a^2\,b}{4\,a^4}-\frac{a\,\left(\frac{13\,b\,\left(2\,B\,a^3-5\,A\,a^2\,b\right)}{36\,a^5}+\frac{A\,b^2}{3\,a^3}\right)}{b}}{{\left(b\,x^3+a\right)}^{3/2}} - \frac{A\,\sqrt{b\,x^3+a}}{3\,a^3\,x^3}$$

input
```
int((A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x)
```

output
```
(log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*
(5*A*b - 2*B*a))/(6*a^(7/2)) - ((2*B*a^2 - 5*A*a*b)/(2*a^4) - (a*((A*b^2)/
(3*a^4) + (5*b*(2*B*a^2 - 5*A*a*b))/(6*a^5)))/b)/(a + b*x^3)^(1/2) - ((2*B
*a^3 - 5*A*a^2*b)/(4*a^4) - (a*((13*b*(2*B*a^3 - 5*A*a^2*b))/(36*a^5) + (A
*b^2)/(3*a^3)))/b)/(a + b*x^3)^(3/2) - (A*(a + b*x^3)^(1/2))/(3*a^3*x^3)
```

3.248. $\int \frac{A+Bx^3}{x^4(a+bx^3)^{5/2}}\,dx$

3.249 $\int \frac{x^6 \left(A+B x^3\right)}{\left(a+b x^3\right)^{5/2}} \, dx$

3.249.1 Optimal result

Integrand size = 22, antiderivative size = 299

$$\int \frac{x^6(A+Bx^3)}{\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2(5Ab-14aB)x^4}{45b^2\left(a+bx^3\right)^{3/2}}+\frac{2Bx^7}{5b\left(a+bx^3\right)^{3/2}}-\frac{16(5Ab-14aB)x}{135b^3\sqrt{a+bx^3}}$$
$$+\frac{32\sqrt{2+\sqrt{3}}(5Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7\right.}{135\sqrt[4]{3}b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/45*(5*A*b-14*B*a)*x^4/b^2/(b*x^3+a)^(3/2)+2/5*B*x^7/b/(b*x^3+a)^(3/2)-1
6/135*(5*A*b-14*B*a)*x/b^3/(b*x^3+a)^(1/2)+32/405*(5*A*b-14*B*a)*(a^(1/3)+
b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(10/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

3.249.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.36

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2x\left(112a^2B+b^2x^3(-55A+27Bx^3)+a(-40Ab+154bBx^3)+8(5Ab-14aB)\,(a+b\right.}{135b^3\,(a+bx^3)^{3/2}}$$

input
```
Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*x*(112*a^2*B + b^2*x^3*(-55*A + 27*B*x^3) + a*(-40*A*b + 154*b*B*x^3) +
 8*(5*A*b - 14*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3,
 1/2, 4/3, -((b*x^3)/a)]))/(135*b^3*(a + b*x^3)^(3/2))
```

3.249.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {959, 817, 817, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(A+Bx^3\right)}{(a+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(5Ab-14aB)\int \frac{x^6}{(bx^3+a)^{5/2}}dx}{5b}+\frac{2Bx^7}{5b\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \; 817$$

$$\frac{(5Ab-14aB)\left(\frac{8\int \frac{x^3}{\left(bx^3+a\right)^{3/2}}dx}{9b}-\frac{2x^4}{9b(a+bx^3)^{3/2}}\right)}{5b}+\frac{2Bx^7}{5b\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \; 817$$

$$\frac{(5Ab-14aB)\left(\frac{8\left(\frac{2\int \frac{1}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x}{3b\sqrt{a+bx^3}}\right)}{9b}-\frac{2x^4}{9b(a+bx^3)^{3/2}}\right)}{5b}+\frac{2Bx^7}{5b\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 759$$

$$\frac{(5Ab-14aB)\left(\frac{8\left(\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x}{3b\sqrt{a+bx^3}}\right)}{9b}-\frac{}{9b(a}\right.}{5b}$$

$$\frac{2Bx^7}{5b\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*B*x^7)/(5*b*(a + b*x^3)^(3/2)) + ((5*A*b - 14*a*B)*((-2*x^4)/(9*b*(a +
b*x^3)^(3/2)) + (8*((-2*x)/(3*b*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(
1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(9*b)))/(5*b)
```

3.249. $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

3.249.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.249.4 Maple [A] (verified)

Time = 5.36 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.33

method	result
elliptic	$\frac{2xa(Ab-Ba)\sqrt{b\,x^3+a}}{9b^5\left(x^3+\frac{a}{b}\right)^2}-\frac{2x(11Ab-20Ba)}{27b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2Bx\sqrt{b\,x^3+a}}{5b^3}-\frac{2i\left(\frac{Ab-2Ba}{b^3}-\frac{11Ab-20Ba}{27b^3}-\frac{2Ba}{5b^3}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\right.}{(-}}}{}$
default	$B\left(-\frac{2x\,a^2\sqrt{b\,x^3+a}}{9b^5\left(x^3+\frac{a}{b}\right)^2}+\frac{40ax}{27b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2x\sqrt{b\,x^3+a}}{5b^3}+\frac{448ia\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{}{-\frac{3(-a}{2}}}}{}\right.$
risch	Expression too large to display

input
```
int(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x*a/b^5*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2/27/b^3*x*(11*A*b-20*B*
a)/((x^3+a/b)*b)^(1/2)+2/5*B/b^3*x*(b*x^3+a)^(1/2)-2/3*I*((A*b-2*B*a)/b^3-
1/27/b^3*(11*A*b-20*B*a)-2/5*B/b^3*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.249. $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

3.249.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.51

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx =$$

$$-\frac{2\Big(16\left(\left(14\,Bab^2-5\,Ab^3\right)x^6+14\,Ba^3-5\,Aa^2b+2\left(14\,Ba^2b-5\,Aab^2\right)x^3\right)\sqrt{b}\,\mathrm{weierstrassPInverse}(0,-\frac{4\,a}{b},}{135\left(b^6x^6+2\,ab^5x^3+a^2b^4\right)}$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/135*(16*((14*B*a*b^2 - 5*A*b^3)*x^6 + 14*B*a^3 - 5*A*a^2*b + 2*(14*B*a^
2*b - 5*A*a*b^2)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - (27*B*b^
3*x^7 + 11*(14*B*a*b^2 - 5*A*b^3)*x^4 + 8*(14*B*a^2*b - 5*A*a*b^2)*x)*sqrt
(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

3.249.6 Sympy [A] (verification not implemented)

Time = 61.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{Ax^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{7}{3},\frac{5}{2}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{10}{3}\right)} + \frac{Bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate(x**6*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
A*x**7*gamma(7/3)*hyper((7/3, 5/2), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(5/2)*gamma(10/3)) + B*x**10*gamma(10/3)*hyper((5/2, 10/3), (13/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(13/3))
```

3.249.7 Maxima [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(5/2), x)
```

3.249.8 Giac [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(5/2), x)
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{x^6\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output
```
int((x^6*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

3.250 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

3.250.1 Optimal result

Integrand size = 22, antiderivative size = 283

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)x^4}{9ab\,(a+bx^3)^{3/2}} - \frac{2(Ab+8aB)x}{27ab^2\sqrt{a+bx^3}} + \frac{4\sqrt{2+\sqrt{3}}(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4}{27\sqrt[4]{3}ab^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*(A*b-B*a)*x^4/a/b/(b*x^3+a)^(3/2)-2/27*(A*b+8*B*a)*x/a/b^2/(b*x^3+a)^(
1/2)+4/81*(A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*
2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)*3^(3/4)/a/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.250.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.35

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2x\left(-8a^2B+2Ab^2x^3-ab(A+11Bx^3)+(Ab+8aB)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeon}\right.}{27ab^2\left(a+bx^3\right)^{3/2}}$$

input
```
Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*x*(-8*a^2*B + 2*A*b^2*x^3 - a*b*(A + 11*B*x^3) + (A*b + 8*a*B)*(a + b*x
^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(
27*a*b^2*(a + b*x^3)^(3/2))
```

3.250.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {957, 817, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{(8aB+Ab)\int \frac{x^3}{\left(bx^3+a\right)^{3/2}}dx}{9ab}+\frac{2x^4(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 817$$

$$\frac{(8aB+Ab)\left(\frac{2\int \frac{1}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x}{3b\sqrt{a+bx^3}}\right)}{9ab}+\frac{2x^4(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 759$$

$$\frac{(8aB+Ab)\left(\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x}{3b\sqrt{a+bx^3}}\right)}{9ab}+\frac{2x^4(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

input

```
Int[(x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output

```
(2*(A*b - a*B)*x^4)/(9*a*b*(a + b*x^3)^(3/2)) + ((A*b + 8*a*B)*((-2*x)/(3*
b*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
)))/(9*a*b)
```

3.250.3.1 Defintions of rubi rules used

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.250.4 Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.31

method	result
elliptic	$-\frac{2x(Ab-Ba)\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}+\frac{2x(2Ab-11Ba)}{27b^2a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\left(\frac{B}{b^2}+\frac{2Ab-11Ba}{27b^2a}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{}{-\frac{3(-}{}}}}{}$
default	$B\left(\frac{2ax\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}-\frac{22x}{27b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{32i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}}{}\right.$

input
```
int(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*x/b^4*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27/b^2*x/a*(2*A*b-11*B*
a)/((x^3+a/b)*b)^(1/2)-2/3*I*(B/b^2+1/27/b^2/a*(2*A*b-11*B*a))*3^(1/2)/b*(
-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.250.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.50

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\left(2\left((8\,Bab^2+Ab^3)x^6+8\,Ba^3+Aa^2b+2\,(8\,Ba^2b+Aab^2)x^3\right)\sqrt{b}\mathrm{weierstrassPInvers}\right.}{27\left(ab^5x^6+2\,a^2b^4x^3+\right.}$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/27*(2*((8*B*a*b^2 + A*b^3)*x^6 + 8*B*a^3 + A*a^2*b + 2*(8*B*a^2*b + A*a*
b^2)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - ((11*B*a*b^2 - 2*A*b
^3)*x^4 + (8*B*a^2*b + A*a*b^2)*x)*sqrt(b*x^3 + a))/(a*b^5*x^6 + 2*a^2*b^4
*x^3 + a^3*b^3)
```

3.250.6 Sympy [A] (verification not implemented)

Time = 38.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.28

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{7}{3},\frac{5}{2}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**3*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
A*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(5/2)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((7/3, 5/2), (10/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(10/3))
```

3.250.7 Maxima [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(5/2), x)
```

3.250.8 Giac [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(5/2), x)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{x^3\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output
```
int((x^3*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

3.251 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^{5/2}} \, dx$

3.251.1 Optimal result

Integrand size = 19, antiderivative size = 283

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{5/2}} \, dx = \frac{2(Ab-aB)x}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2(7Ab+2aB)x}{27a^2b\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\right.}{27\sqrt[4]{3}a^2b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*(A*b-B*a)*x/a/b/(b*x^3+a)^(3/2)+2/27*(7*A*b+2*B*a)*x/a^2/b/(b*x^3+a)^(
1/2)+2/81*(7*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(
1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/
2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)*3^(3/4)/a^2/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b
^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.251.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{5/2}}\,dx=\frac{-2a^2Bx+14Ab^2x^4+4abx(5A+Bx^3)+(7Ab+2aB)x(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeo}}{27a^2b\left(a+bx^3\right)^{3/2}}$$

input
```
Integrate[(A + B*x^3)/(a + b*x^3)^(5/2),x]
```

output
```
(-2*a^2*B*x + 14*A*b^2*x^4 + 4*a*b*x*(5*A + B*x^3) + (7*A*b + 2*a*B)*x*(a
+ b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)
])/(27*a^2*b*(a + b*x^3)^(3/2))
```

3.251.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {910, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 910$$

$$\frac{(2aB+7Ab)\int \frac{1}{\left(bx^3+a\right)^{3/2}}dx}{9ab}+\frac{2x(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 749$$

$$\frac{(2aB+7Ab)\left(\frac{\int \frac{1}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x}{3a\sqrt{a+bx^3}}\right)}{9ab}+\frac{2x(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 759$$

$$\frac{(2aB+7Ab)\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x}{3a\sqrt{a+bx^3}}\right)}{9ab}+\frac{2x(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

input

```
Int[(A + B*x^3)/(a + b*x^3)^(5/2),x]
```

output

```
(2*(A*b - a*B)*x)/(9*a*b*(a + b*x^3)^(3/2)) + ((7*A*b + 2*a*B)*((2*x)/(3*a
*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(1/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])))/(9*a*b)
```

3.251.3.1 Defintions of rubi rules used

rule 749

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.251.4 Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.29

method	result
elliptic	$\frac{2x(Ab-Ba)\sqrt{b\,x^3+a}}{9a\,b^3\left(x^3+\frac{a}{b}\right)^2}+\frac{2x(7Ab+2Ba)}{27b\,a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i(7Ab+2Ba)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\left(-a\right.}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+}}}{}$
default	$A\left(\frac{2x\sqrt{b\,x^3+a}}{9a\,b^2\left(x^3+\frac{a}{b}\right)^2}+\frac{14x}{27a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{14i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}}{}\right.$

input
```
int((B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x/a/b^3*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27/b*x/a^2*(7*A*b+2*B*
a)/((x^3+a/b)*b)^(1/2)-2/81*I*(7*A*b+2*B*a)/a^2/b^2*3^(1/2)*(-a*b^2)^(1/3)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.251.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2\left(\left(\left(2\,Bab^2+7\,Ab^3\right)x^6+2\,Ba^3+7\,Aa^2b+2\left(2\,Ba^2b+7\,Aab^2\right)x^3\right)\sqrt{b}\text{weierstrassPInve}}{27\left(a^2b^4x^6+2\,a^3b^3x^3\right.}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/27*(((2*B*a*b^2 + 7*A*b^3)*x^6 + 2*B*a^3 + 7*A*a^2*b + 2*(2*B*a^2*b + 7*
A*a*b^2)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + ((2*B*a*b^2 + 7*
A*b^3)*x^4 - (B*a^2*b - 10*A*a*b^2)*x)*sqrt(b*x^3 + a))/(a^2*b^4*x^6 + 2*a
^3*b^3*x^3 + a^4*b^2)
```

3.251.6 Sympy [A] (verification not implemented)

Time = 24.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.28

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{4}{3}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
A*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
5/2)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(5/2)*gamma(7/3))
```

3.251.7 Maxima [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(b*x^3 + a)^(5/2), x)
```

3.251.8 Giac [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx=\int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(b*x^3 + a)^(5/2), x)
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx=\int \frac{B\,x^3+A}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((A + B*x^3)/(a + b*x^3)^(5/2),x)
```

output
```
int((A + B*x^3)/(a + b*x^3)^(5/2), x)
```

3.252 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx$

3.252.1 Optimal result

Integrand size = 22, antiderivative size = 300

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx=-\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}-\frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}}-\frac{7(13Ab-4aB)x}{54a^3\sqrt{a+bx^3}}$$
$$-\frac{7\sqrt{2+\sqrt{3}}(13Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-}{54\sqrt[4]{3}a^3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/2*A/a/x^2/(b*x^3+a)^(3/2)-1/18*(13*A*b-4*B*a)*x/a^2/(b*x^3+a)^(3/2)-7/5
4*(13*A*b-4*B*a)*x/a^3/(b*x^3+a)^(1/2)-7/162*(13*A*b-4*B*a)*(a^(1/3)+b^(1/
3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^3/b^(1/3)
/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)
```

3.252.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx = \frac{-182Ab^2x^6+a^2(-54A+80Bx^3)+a(-260Abx^3+56bBx^6)+7(-13Ab+4aB)x^3(}{108a^3x^2\left(a+bx^3\right)^{3/2}}$$

input
```
Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x]
```

output
```
(-182*A*b^2*x^6 + a^2*(-54*A + 80*B*x^3) + a*(-260*A*b*x^3 + 56*b*B*x^6) +
 7*(-13*A*b + 4*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1
[1/3, 1/2, 4/3, -((b*x^3)/a)])/(108*a^3*x^2*(a + b*x^3)^(3/2))
```

3.252.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 749, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(13Ab-4aB)\int \frac{1}{\left(bx^3+a\right)^{5/2}}dx}{4a}-\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(13Ab-4aB)\left(\frac{7\int \frac{1}{\left(bx^3+a\right)^{3/2}}dx}{9a}+\frac{2x}{9a\left(a+bx^3\right)^{3/2}}\right)}{4a}-\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(13Ab-4aB)\left(\frac{7\left(\frac{\int \frac{1}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x}{3a\sqrt{a+bx^3}}\right)}{9a}+\frac{2x}{9a(a+bx^3)^{3/2}}\right)}{4a}-\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{(13Ab-4aB)\left(\frac{7\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x}{3a\sqrt{a+bx^3}}\right)}{9a}+\right.}{4a}$$

$$\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x]
```

output
```
-1/2*A/(a*x^2*(a + b*x^3)^(3/2)) - ((13*A*b - 4*a*B)*((2*x)/(9*a*(a + b*x^
3)^(3/2)) + (7*((2*x)/(3*a*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*
a*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])))/(9*a)))/(4*a)
```

3.252. $\int \frac{A+Bx^3}{x^3(a+bx^3)^{5/2}}\,dx$

3.252.3.1 Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.252.4 Maple [A] (verified)

Time = 5.54 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.30

method	result
elliptic	$-\frac{2x(Ab-Ba)\sqrt{b\,x^3+a}}{9a^2b^2\left(x^3+\frac{a}{b}\right)^2}-\frac{2x(16Ab-7Ba)}{27a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{2a^3x^2}-\frac{2i\left(-\frac{16Ab-7Ba}{27a^3}-\frac{Ab}{4a^3}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)}{2b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{}$
default	$B\left(\frac{2x\sqrt{b\,x^3+a}}{9a\,b^2\left(x^3+\frac{a}{b}\right)^2}+\frac{14x}{27a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{14i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}}{}\right.$
risch	Expression too large to display

input
```
int((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*x/a^2/b^2*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2/27*x/a^3*(16*A*b-7*
B*a)/((x^3+a/b)*b)^(1/2)-1/2/a^3*A*(b*x^3+a)^(1/2)/x^2-2/3*I*(-1/27/a^3*(1
6*A*b-7*B*a)-1/4/a^3*A*b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2))
```

3.252. $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx$

3.252.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.54

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx = \frac{7\left(\left(4\,Bab^2-13\,Ab^3\right)x^8+2\left(4\,Ba^2b-13\,Aab^2\right)x^5+\left(4\,Ba^3-13\,Aa^2b\right)x^2\right)\sqrt{b}\mathrm{weierstr}}{54\left(a^3b^3\right.}$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
1/54*(7*((4*B*a*b^2 - 13*A*b^3)*x^8 + 2*(4*B*a^2*b - 13*A*a*b^2)*x^5 + (4*
B*a^3 - 13*A*a^2*b)*x^2)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (7*(4
*B*a*b^2 - 13*A*b^3)*x^6 - 27*A*a^2*b + 10*(4*B*a^2*b - 13*A*a*b^2)*x^3)*s
qrt(b*x^3 + a))/(a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)
```

3.252.6 Sympy [A] (verification not implemented)

Time = 81.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.27

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx = \frac{A\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Bx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**3/(b*x**3+a)**(5/2),x)
```

output
```
A*gamma(-2/3)*hyper((-2/3, 5/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
5/2)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(5/2)*gamma(4/3))
```

3.252.7 Maxima [F]

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^3), x)
```

3.252.8 Giac [F]

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^3), x)
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{B\,x^3+A}{x^3\,\left(b\,x^3+a\right)^{5/2}}\,dx$$

input
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x)
```

output
```
int((A + B*x^3)/(x^3*(a + b*x^3)^(5/2)), x)
```

3.253 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx$

3.253.1 Optimal result

Integrand size = 22, antiderivative size = 334

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx=-\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}-\frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}}$$
$$-\frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}}+\frac{91(19Ab-10aB)\sqrt{a+bx^3}}{540a^4x^2}$$
$$+\frac{91\sqrt{2+\sqrt{3}}b^{2/3}(19Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{540\sqrt[4]{3}a^4\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/5*A/a/x^5/(b*x^3+a)^(3/2)+1/45*(-19*A*b+10*B*a)/a^2/x^2/(b*x^3+a)^(3/2)
-13/135*(19*A*b-10*B*a)/a^3/x^2/(b*x^3+a)^(1/2)+91/540*(19*A*b-10*B*a)*(b*
x^3+a)^(1/2)/a^4/x^2+91/1620*b^(2/3)*(19*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*E
llipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I
*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^4/(b*x^3+a)^(1/2
)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.253.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.25

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx = \frac{-2a^2A+\left(\frac{19Ab}{2}-5aB\right)x^3(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{5}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{10a^3x^5\left(a+bx^3\right)^{3/2}}$$

input

```
Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x]
```

output

```
(-2*a^2*A + ((19*A*b)/2 - 5*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyper
geometric2F1[-2/3, 5/2, 1/3, -((b*x^3)/a)])/(10*a^3*x^5*(a + b*x^3)^(3/2))
```

3.253.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 819, 819, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(19Ab-10aB)\int \frac{1}{x^3\left(bx^3+a\right)^{5/2}}dx}{10a}-\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(19Ab-10aB)\left(\frac{13\int \frac{1}{x^3\left(bx^3+a\right)^{3/2}}dx}{9a}+\frac{2}{9ax^2\left(a+bx^3\right)^{3/2}}\right)}{10a}-\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(19Ab-10aB)\left(\frac{13\left(\frac{7\int \frac{1}{x^3\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ax^2\sqrt{a+bx^3}}\right)}{9a}+\frac{2}{9ax^2(a+bx^3)^{3/2}}\right)}{10a}-\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 847$$

$$-\frac{(19Ab-10aB)\left(\frac{13\left(\frac{7\left(-\frac{b\int \frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{3a}+\frac{2}{3ax^2\sqrt{a+bx^3}}\right)}{9a}+\frac{2}{9ax^2(a+bx^3)^{3/2}}\right)}{10a}-\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 759$$

3.253. $\int \frac{A+Bx^3}{x^6(a+bx^3)^{5/2}}\,dx$

$$-\frac{(19Ab-10aB)\left(\frac{13\left(\frac{7\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{2ax^2}\right.}{3a}\right.}{9a}\right.}{10a}$$

$$\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x]
```

output
```
-1/5*A/(a*x^5*(a + b*x^3)^(3/2)) - ((19*A*b - 10*a*B)*(2/(9*a*x^2*(a + b*x
^3)^(3/2)) + (13*(2/(3*a*x^2*Sqrt[a + b*x^3]) + (7*(-1/2*Sqrt[a + b*x^3]/(
a*x^2) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(3*a)))/
(9*a)))/(10*a)
```

3.253. $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx$

3.253.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.253.4 Maple [A] (verified)

Time = 5.58 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.27

method	result
elliptic	$\frac{2x(Ab-Ba)\sqrt{b\,x^3+a}}{9a^3b\left(x^3+\frac{a}{b}\right)^2}+\frac{2bx(25Ab-16Ba)}{27a^4\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{5a^3x^5}+\frac{(27Ab-10Ba)\sqrt{b\,x^3+a}}{20a^4x^2}-\frac{2i\left(\frac{b(25Ab-16Ba)}{27a^4}+\frac{b(27Ab-10Ba)}{40a^4}\right)\sqrt{3}\,(-}{}$
default	$B\left(-\frac{2x\sqrt{b\,x^3+a}}{9a^2b\left(x^3+\frac{a}{b}\right)^2}-\frac{32bx}{27a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{\sqrt{b\,x^3+a}}{2a^3x^2}+\frac{91i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}}{}\right.$
risch	Expression too large to display

input
```
int((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x/a^3/b*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27*b*x/a^4*(25*A*b-16*
B*a)/((x^3+a/b)*b)^(1/2)-1/5*A/a^3*(b*x^3+a)^(1/2)/x^5+1/20/a^4*(27*A*b-10
*B*a)*(b*x^3+a)^(1/2)/x^2-2/3*I*(1/27*b/a^4*(25*A*b-16*B*a)+1/40*b*(27*A*b
-10*B*a)/a^4)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1
/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))
```

3.253.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.53

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx =$$

$$-\frac{91\,((10\,Bab^2-19\,Ab^3)x^{11}+2\,(10\,Ba^2b-19\,Aab^2)x^8+(10\,Ba^3-19\,Aa^2b)x^5)\sqrt{b}\text{weierstrassPInverse}(0,}{540\,(a^4b^2x^1}$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/540*(91*((10*B*a*b^2 - 19*A*b^3)*x^11 + 2*(10*B*a^2*b - 19*A*a*b^2)*x^8
 + (10*B*a^3 - 19*A*a^2*b)*x^5)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x)
+ (91*(10*B*a*b^2 - 19*A*b^3)*x^9 + 130*(10*B*a^2*b - 19*A*a*b^2)*x^6 + 10
8*A*a^3 + 27*(10*B*a^3 - 19*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b^2*x^11 +
 2*a^5*b*x^8 + a^6*x^5)
```

3.253.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/x**6/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

3.253.7 Maxima [F]

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^6}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^6), x)
```

3.253.8 Giac [F]

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^6}\,dx$$

input
```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^6), x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{B\,x^3+A}{x^6\,\left(b\,x^3+a\right)^{5/2}}\,dx$$

input
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x)
```

output
```
int((A + B*x^3)/(x^6*(a + b*x^3)^(5/2)), x)
```

3.254 $\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

3.254.1 Optimal result

Integrand size = 22, antiderivative size = 577

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2(7Ab-16aB)x^5}{63b^2\left(a+bx^3\right)^{3/2}}+\frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}}$$

$$-\frac{20(7Ab-16aB)x^2}{189b^3\sqrt{a+bx^3}}+\frac{80(7Ab-16aB)\sqrt{a+bx^3}}{189b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{40\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\right.}{63\ 3^{3/4}b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{80\sqrt{2}\sqrt[3]{a}(7Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\right.}{189\sqrt[4]{3}b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-2/63*(7*A*b-16*B*a)*x^5/b^2/(b*x^3+a)^(3/2)+2/7*B*x^8/b/(b*x^3+a)^(3/2)-2
0/189*(7*A*b-16*B*a)*x^2/b^3/(b*x^3+a)^(1/2)+80/189*(7*A*b-16*B*a)*(b*x^3+
a)^(1/2)/b^(11/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+80/567*a^(1/3)*(7*A*b-16
*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(11/3)/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))^2)^(1/2)-40/189*a^(1/3)*(7*A*b-16*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b
^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*
I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(11/3)/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.254.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.19

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2x^2\Big(-32a^2B+2ab(7A-8Bx^3)+b^2x^3(7A+Bx^3)+2(-7Ab+16aB)\,(a+bx^3)\,\sqrt{}}{7b^3\,(a+bx^3)^{3/2}}$$

input

```
Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output

```
(2*x^2*(-32*a^2*B + 2*a*b*(7*A - 8*B*x^3) + b^2*x^3*(7*A + B*x^3) + 2*(-7*
A*b + 16*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 5/2,
5/3, -((b*x^3)/a)]))/(7*b^3*(a + b*x^3)^(3/2))
```

3.254.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {959, 817, 817, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(7Ab-16aB)\int \frac{x^7}{\left(bx^3+a\right)^{5/2}}dx}{7b}+\frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 817$$

$$\frac{(7Ab-16aB)\left(\frac{10\int \frac{x^4}{\left(bx^3+a\right)^{3/2}}dx}{9b}-\frac{2x^5}{9b\left(a+bx^3\right)^{3/2}}\right)}{7b}+\frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 817$$

$$\frac{(7Ab-16aB)\left(\frac{10\left(\frac{4\int \frac{x}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x^2}{3b\sqrt{a+bx^3}}\right)}{9b}-\frac{2x^5}{9b\left(a+bx^3\right)^{3/2}}\right)}{7b}+\frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 832$$

$$\frac{(7Ab-16aB)\left(\frac{10\left(\frac{4\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{3b}-\frac{2x^2}{3b\sqrt{a+bx^3}}\right)}{9b}-\frac{2x^5}{9b\left(a+bx^3\right)^{3/2}}\right)}{7b}+\frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 759$$

3.254. $\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

$$\frac{(7Ab-16aB)\left(\frac{10\left(\frac{4\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{\right.}}{\sqrt[3]{b}x+\left(1+\sqrt{\right.}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{3b}\right.}{9b}\right.}{7b}$$

$$\frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 2416}$$

3.254. $\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

$$(7Ab-16aB)\left(\frac{10\left(\frac{4\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{\sqrt[3]{b}}\right.}{}\right.}{}\right.$$

3.254. $\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$ $\frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}}$

input
```
Int[(x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*B*x^8)/(7*b*(a + b*x^3)^(3/2)) + ((7*A*b - 16*a*B)*((-2*x^5)/(9*b*(a +
b*x^3)^(3/2)) + (10*((-2*x^2)/(3*b*Sqrt[a + b*x^3]) + (4*(((2*Sqrt[a + b*x
^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 +
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(3*b)))/(9*b)))/(7*b)
```

3.254.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.254.4 Maple [A] (verified)

Time = 5.56 (sec) , antiderivative size = 555, normalized size of antiderivative = 0.96

method	result
elliptic	$\frac{2x^2a(Ab-Ba)\sqrt{b\,x^3+a}}{9b^5\left(x^3+\frac{a}{b}\right)^2}-\frac{2x^2(13Ab-22Ba)}{27b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2B\,x^2\sqrt{b\,x^3+a}}{7b^3}-\frac{2i\left(\frac{Ab-2Ba}{b^3}+\frac{13Ab-22Ba}{27b^3}-\frac{4Ba}{7b^3}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)}{2b}\right.}{}}}{}$
default	Expression too large to display
risch	Expression too large to display

input
```
int(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
2/9*x^2*a/b^5*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2/27/b^3*x^2*(13*A*b-2
2*B*a)/((x^3+a/b)*b)^(1/2)+2/7*B/b^3*x^2*(b*x^3+a)^(1/2)-2/3*I*((A*b-2*B*a
)/b^3+1/27/b^3*(13*A*b-22*B*a)-4/7*B/b^3*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)))
```

3.254.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.28

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\Big(40\left(\left(16\,Bab^2-7\,Ab^3\right)x^6+16\,Ba^3-7\,Aa^2b+2\left(16\,Ba^2b-7\,Aab^2\right)x^3\right)\sqrt{b}\text{weierstra}}{}$$

input

```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output

```
2/189*(40*((16*B*a*b^2 - 7*A*b^3)*x^6 + 16*B*a^3 - 7*A*a^2*b + 2*(16*B*a^2
*b - 7*A*a*b^2)*x^3)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInvers
e(0, -4*a/b, x)) + (27*B*b^3*x^8 + 13*(16*B*a*b^2 - 7*A*b^3)*x^5 + 10*(16*
B*a^2*b - 7*A*a*b^2)*x^2)*sqrt(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^
4)
```

3.254.6 Sympy [A] (verification not implemented)

Time = 79.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{Ax^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{11}{3}\right)} + \frac{Bx^{11}\Gamma\left(\frac{11}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{11}{3}\\ \frac{14}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{14}{3}\right)}$$

input
```
integrate(x**7*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
A*x**8*gamma(8/3)*hyper((5/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(5/2)*gamma(11/3)) + B*x**11*gamma(11/3)*hyper((5/2, 11/3), (14/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(14/3))
```

3.254.7 Maxima [F]

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^7}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^7/(b*x^3 + a)^(5/2), x)
```

3.254.8 Giac [F]

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^7}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^7/(b*x^3 + a)^(5/2), x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{x^7\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output
```
int((x^7*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

3.255 $\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

3.255.1 Optimal result

Integrand size = 22, antiderivative size = 559

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\frac{2(Ab-aB)x^5}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2(Ab-10aB)x^2}{27ab^2\sqrt{a+bx^3}}-\frac{8(Ab-10aB)\sqrt{a+bx^3}}{27ab^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{4\sqrt{2-\sqrt{3}}(Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{8\sqrt{2}(Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*(A*b-B*a)*x^5/a/b/(b*x^3+a)^(3/2)+2/27*(A*b-10*B*a)*x^2/a/b^2/(b*x^3+a
)^(1/2)-8/27*(A*b-10*B*a)*(b*x^3+a)^(1/2)/a/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))-8/81*(A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/
3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)*3^(3/4)/a^(2/3)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/27*(A*b-10*B*a)*(a^(1/3)+b^(1/3
)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(8
/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)
```

3.255.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.16

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2x^2\left(-aAb+5aB(2a+bx^3)+(Ab-10aB)\,(a+bx^3)\,\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\right.}{5ab^2\,(a+bx^3)^{3/2}}$$

input
```
Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*x^2*(-(a*A*b) + 5*a*B*(2*a + b*x^3) + (A*b - 10*a*B)*(a + b*x^3)*Sqrt[1
 + (b*x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)]))/(5*a*b^2*(a
 + b*x^3)^(3/2))
```

3.255.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {957, 817, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{2x^5(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}-\frac{(Ab-10aB)\int \frac{x^4}{\left(bx^3+a\right)^{3/2}}dx}{9ab}$$

$$\Big\downarrow \text{817}$$

$$\frac{2x^5(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}-\frac{(Ab-10aB)\left(\frac{4\int \frac{x}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x^2}{3b\sqrt{a+bx^3}}\right)}{9ab}$$

$$\Big\downarrow \text{832}$$

$$\frac{2x^5(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}-\frac{(Ab-10aB)\left(\frac{4\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{3b}-\frac{2x^2}{3b\sqrt{a+bx^3}}\right)}{9ab}$$

$$\Big\downarrow \text{759}$$

$$\frac{2x^5(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}-$$

$$\frac{(Ab-10aB)\left(\frac{4\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{3b}\right.}{9ab}$$

$$\Big\downarrow \text{2416}$$

3.255. $\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

$$\frac{2x^5(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}-$$

$$\frac{(Ab-10aB)\left(\frac{4\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}\right.}{3b}}{9ab}$$

input
```
Int[(x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(A*b - a*B)*x^5)/(9*a*b*(a + b*x^3)^(3/2)) - ((A*b - 10*a*B)*((-2*x^2)/
(3*b*Sqrt[a + b*x^3]) + (4*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)
*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])))/(3*b)))/(9*a*b)
```

3.255. $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

3.255.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.255. $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

3.255.4 Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 528, normalized size of antiderivative = 0.94

method	result
elliptic	$2i\left(\frac{B}{b^{2}}-\frac{4Ab-13Ba}{27a\,b^{2}}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{3(-}{}}$ $-\frac{2x^{2}(Ab-Ba)\sqrt{b\,x^{3}+a}}{9b^{4}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2x^{2}(4Ab-13Ba)}{27b^{2}a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-$
default	Expression too large to display

input
```
int(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*x^2/b^4*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27/b^2*x^2/a*(4*A*b-1
3*B*a)/((x^3+a/b)*b)^(1/2)-2/3*I*(B/b^2-1/27*(4*A*b-13*B*a)/a/b^2)*3^(1/2)
/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.255.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.28

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx =$$

$$-\frac{2\left(4\left((10\,Bab^2-Ab^3)x^6+10\,Ba^3-Aa^2b+2\,(10\,Ba^2b-Aab^2)x^3\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrass}\right.\right.}{27\,(ab^5x^6+2\,a^2b^4x^3+a^3b}$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/27*(4*((10*B*a*b^2 - A*b^3)*x^6 + 10*B*a^3 - A*a^2*b + 2*(10*B*a^2*b -
A*a*b^2)*x^3)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4
*a/b, x)) + ((13*B*a*b^2 - 4*A*b^3)*x^5 + (10*B*a^2*b - A*a*b^2)*x^2)*sqrt
(b*x^3 + a))/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)
```

3.255.6 Sympy [A] (verification not implemented)

Time = 39.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{Ax^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{5}{3},\frac{5}{2}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)} + \frac{Bx^8\Gamma\left(\frac{8}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{5}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**4*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
A*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(5/2)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((5/2, 8/3), (11/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(11/3))
```

3.255.7 Maxima [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(5/2), x)
```

3.255.8 Giac [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(5/2), x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\int \frac{x^4\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output
```
int((x^4*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

3.256 $\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

3.256.1 Optimal result

Integrand size = 20, antiderivative size = 563

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)x^2}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}} - \frac{2(5Ab+4aB)\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{\sqrt{2-\sqrt{3}}(5Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{2}(5Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*(A*b-B*a)*x^2/a/b/(b*x^3+a)^(3/2)+2/27*(5*A*b+4*B*a)*x^2/a^2/b/(b*x^3+
a)^(1/2)-2/27*(5*A*b+4*B*a)*(b*x^3+a)^(1/2)/a^2/b^(5/3)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))-2/81*(5*A*b+4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)*3^(3/4)/a^(5/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/27*(5*A*b+4*B*a)*(a^(1/3)+
b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5/3
)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)
```

3.256.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.14

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{x^2\left(-4a^2B+(5Ab+4aB)\,(a+bx^3)\,\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{2},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{10a^2b\,(a+bx^3)^{3/2}}$$

input
```
Integrate[(x*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(x^2*(-4*a^2*B + (5*A*b + 4*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeom
etric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)]))/(10*a^2*b*(a + b*x^3)^(3/2))
```

3.256.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {957, 819, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow 957$$

$$\frac{(4aB+5Ab)\int \frac{x}{\left(bx^3+a\right)^{3/2}}dx}{9ab}+\frac{2x^2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow 819$$

$$\frac{(4aB+5Ab)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\int \frac{x}{\sqrt{bx^3+a}}dx}{3a}\right)}{9ab}+\frac{2x^2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow 832$$

$$\frac{(4aB+5Ab)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}\right)}{9ab}+\frac{2x^2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow 759$$

$$\frac{(4aB+5Ab)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{b}x}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{3a}\right)}{9ab}$$

$$\frac{2x^2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow 2416$$

3.256. $\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}dx$

$$\frac{(4aB+5Ab)\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right.}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{9ab}$$

$$\frac{2x^2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(x*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(A*b - a*B)*x^2)/(9*a*b*(a + b*x^3)^(3/2)) + ((5*A*b + 4*a*B)*((2*x^2)/
(3*a*Sqrt[a + b*x^3]) - (((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3]))/(3*a)))/(9*a*b)
```

3.256. $\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

3.256.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.256. $\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

3.256.4 Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 520, normalized size of antiderivative = 0.92

method	result
elliptic	$\frac{2x^2(Ab-Ba)\sqrt{b\,x^3+a}}{9a\,b^3\left(x^3+\frac{a}{b}\right)^2}+\frac{2x^2(5Ab+4Ba)}{27b\,a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+$ $2i(5Ab+4Ba)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$
default	Expression too large to display

input
```
int(x*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^2/a/b^3*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27/b*x^2/a^2*(5*A*b+
4*B*a)/((x^3+a/b)*b)^(1/2)+2/81*I/b^2/a^2*(5*A*b+4*B*a)*3^(1/2)*(-a*b^2)^(
1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.256.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.27

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\Big(((4\,Bab^2+5\,Ab^3)x^6+4\,Ba^3+5\,Aa^2b+2\,(4\,Ba^2b+5\,Aab^2)x^3)\sqrt{b}\mathrm{weierstrassZeta}(}{27\,(a^2b^4x}$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/27*(((4*B*a*b^2 + 5*A*b^3)*x^6 + 4*B*a^3 + 5*A*a^2*b + 2*(4*B*a^2*b + 5*
A*a*b^2)*x^3)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4
*a/b, x)) + ((4*B*a*b^2 + 5*A*b^3)*x^5 + (B*a^2*b + 8*A*a*b^2)*x^2)*sqrt(b
*x^3 + a))/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)
```

3.256.6 Sympy [A] (verification not implemented)

Time = 24.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{3},\frac{5}{2}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
A*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(5/2)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(5/2)*gamma(8/3))
```

3.256.7 Maxima [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*x/(b*x^3 + a)^(5/2), x)
```

3.256.8 Giac [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*x/(b*x^3 + a)^(5/2), x)
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\int \frac{x\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((x*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

output
```
int((x*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

3.257 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx$

3.257.1 Optimal result

Integrand size $= 22$, antiderivative size $= 578$

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx = -\frac{A}{ax\left(a+bx^3\right)^{3/2}} - \frac{(11Ab-2aB)x^2}{9a^2\left(a+bx^3\right)^{3/2}}$$

$$-\frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}} + \frac{5(11Ab-2aB)\sqrt{a+bx^3}}{27a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{5\sqrt{2-\sqrt{3}}(11Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{18\ 3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{5\sqrt{2}(11Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-A/a/x/(b*x^3+a)^(3/2)-1/9*(11*A*b-2*B*a)*x^2/a^2/(b*x^3+a)^(3/2)-5/27*(11
*A*b-2*B*a)*x^2/a^3/(b*x^3+a)^(1/2)+5/27*(11*A*b-2*B*a)*(b*x^3+a)^(1/2)/a^
3/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+5/81*(11*A*b-2*B*a)*(a^(1/3)+b^(
1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)/b^(2/3)/(b*x^3+a)^(
1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
-5/54*(11*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3
^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.257.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{5/2}}\,dx = -\frac{A}{ax(a+bx^3)^{3/2}} - \frac{\left(\frac{11Ab}{2}-aB\right)x^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{2},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a^3\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x]
```

output
```
-(A/(a*x*(a + b*x^3)^(3/2))) - (((11*A*b)/2 - a*B)*x^2*Sqrt[1 + (b*x^3)/a]
*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(2*a^3*Sqrt[a + b*x^3])
```

3.257.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 819, 819, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(11Ab-2aB)\int \frac{x}{\left(bx^3+a\right)^{5/2}}dx}{2a}-\frac{A}{ax\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 819$$

$$-\frac{(11Ab-2aB)\left(\frac{5\int \frac{x}{\left(bx^3+a\right)^{3/2}}dx}{9a}+\frac{2x^2}{9a\left(a+bx^3\right)^{3/2}}\right)}{2a}-\frac{A}{ax\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 819$$

$$-\frac{(11Ab-2aB)\left(\frac{5\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\int \frac{x}{\sqrt{bx^3+a}}dx}{3a}\right)}{9a}+\frac{2x^2}{9a\left(a+bx^3\right)^{3/2}}\right)}{2a}-\frac{A}{ax\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 832$$

$$-\frac{(11Ab-2aB)\left(\frac{5\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}\right)}{9a}+\frac{2x^2}{9a\left(a+bx^3\right)^{3/2}}\right)}{2a}-\frac{A}{ax\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 759$$

3.257. $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx$

$$-\frac{(11Ab-2aB)\Bigg(\frac{5\Bigg(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\Bigg(\arcsin\Bigg(}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{3a}}{9a}}{2a}$$

$$\frac{A}{ax\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{2416}$$

3.257. $\int\frac{A+Bx^3}{x^2(a+bx^3)^{5/2}}\,dx$

$$-\frac{(11Ab-2aB)\left(\frac{5\left(\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+(1}{\sqrt[3]{b}x+(1}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{}\right.}{\frac{A}{ax\left(a+bx^3\right)^{3/2}}}$$

input
```
Int[(A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x]
```

3.257. $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx$

output

```
-(A/(a*x*(a + b*x^3)^(3/2))) - ((11*A*b - 2*a*B)*((2*x^2)/(9*a*(a + b*x^3)
^(3/2)) + (5*((2*x^2)/(3*a*Sqrt[a + b*x^3]) - (((2*Sqrt[a + b*x^3])/(b^(1/
3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/
3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a
^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]
])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(3*a)))/(9*a)))/(2*a)
```

3.257.3.1 Defintions of rubi rules used

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.257.4 Maple [A] (verified)

Time = 5.22 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.94

method	result
	$2i\left(\frac{14Ab-5Ba}{27a^3}+\frac{Ab}{2a^3}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)}{2b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$
elliptic	$-\frac{2x^2(Ab-Ba)\sqrt{b\,x^3+a}}{9a^2b^2\left(x^3+\frac{a}{b}\right)^2}-\frac{2x^2(14Ab-5Ba)}{27a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{a^3x}-$
default	Expression too large to display
risch	Expression too large to display

input
```
int((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-2/9*x^2/a^2/b^2*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2/27*x^2/a^3*(14*A*
b-5*B*a)/((x^3+a/b)*b)^(1/2)-1/a^3*A*(b*x^3+a)^(1/2)/x-2/3*I*(1/27/a^3*(14
*A*b-5*B*a)+1/2/a^3*A*b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)))
```

3.257.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.29

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx = \frac{5\left((2\,Bab^2-11\,Ab^3)x^7+2\,(2\,Ba^2b-11\,Aab^2)x^4+(2\,Ba^3-11\,Aa^2b)x\right)\sqrt{b}\text{weierstra}}{}$$

input

```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output

```
1/27*(5*((2*B*a*b^2 - 11*A*b^3)*x^7 + 2*(2*B*a^2*b - 11*A*a*b^2)*x^4 + (2*
B*a^3 - 11*A*a^2*b)*x)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInve
rse(0, -4*a/b, x)) + (5*(2*B*a*b^2 - 11*A*b^3)*x^6 - 27*A*a^2*b + 8*(2*B*a
^2*b - 11*A*a*b^2)*x^3)*sqrt(b*x^3 + a))/(a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^
5*b*x)
```

3.257.6 Sympy [A] (verification not implemented)

Time = 52.47 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx=\frac{A\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x\Gamma\left(\frac{2}{3}\right)}+\frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**2/(b*x**3+a)**(5/2),x)
```

output
```
A*gamma(-1/3)*hyper((-1/3, 5/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
5/2)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(5/2)*gamma(5/3))
```

3.257.7 Maxima [F]

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^2), x)
```

3.257.8 Giac [F]

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^2), x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{B\,x^3+A}{x^2\,{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

input
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x)
```

output
```
int((A + B*x^3)/(x^2*(a + b*x^3)^(5/2)), x)
```

3.258 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx$

3.258.1 Optimal result

Integrand size = 22, antiderivative size = 610

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx=-\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}-\frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}}$$

$$-\frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}}+\frac{55(17Ab-8aB)\sqrt{a+bx^3}}{216a^4x}-\frac{55\sqrt[3]{b}(17Ab-8aB)\sqrt{a+bx^3}}{216a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{55\sqrt{2-\sqrt{3}}\sqrt[3]{b}(17Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\surd\right.}{144\ 3^{3/4}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{55\sqrt[3]{b}(17Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\surd\right.}{108\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/4*A/a/x^4/(b*x^3+a)^(3/2)+1/36*(-17*A*b+8*B*a)/a^2/x/(b*x^3+a)^(3/2)-11
/108*(17*A*b-8*B*a)/a^3/x/(b*x^3+a)^(1/2)+55/216*(17*A*b-8*B*a)*(b*x^3+a)^
(1/2)/a^4/x-55/216*b^(1/3)*(17*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^4/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))-55/648*b^(1/3)*(17*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*Elli
pticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^
(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)*3^(3/4)/a^(11/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1
/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+55/432*b^(1/3)*(17
*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(1/4)/a^(11/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.258.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^3}{x^5 \left(a+bx^3\right)^{5/2}}\,dx = \frac{-a^2A+\left(\frac{17Ab}{2}-4aB\right)x^3(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{4a^3x^4\left(a+bx^3\right)^{3/2}}$$

input
```
Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x]
```

output
```
(-(a^2*A) + ((17*A*b)/2 - 4*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyper
geometric2F1[-1/3, 5/2, 2/3, -((b*x^3)/a)])/(4*a^3*x^4*(a + b*x^3)^(3/2))
```

3.258.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {955, 819, 819, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(17Ab-8aB)\int \frac{1}{x^2\left(bx^3+a\right)^{5/2}}dx}{8a}-\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 819$$

$$-\frac{(17Ab-8aB)\left(\frac{11\int \frac{1}{x^2\left(bx^3+a\right)^{3/2}}dx}{9a}+\frac{2}{9ax\left(a+bx^3\right)^{3/2}}\right)}{8a}-\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 819$$

$$-\frac{(17Ab-8aB)\left(\frac{11\left(\frac{5\int \frac{1}{x^2\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ax\sqrt{a+bx^3}}\right)}{9a}+\frac{2}{9ax\left(a+bx^3\right)^{3/2}}\right)}{8a}-\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 847$$

$$-\frac{(17Ab-8aB)\left(\frac{11\left(\frac{5\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{3a}+\frac{2}{3ax\sqrt{a+bx^3}}\right)}{9a}+\frac{2}{9ax\left(a+bx^3\right)^{3/2}}\right)}{8a}-\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 832$$

3.258. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx$

$$-\frac{(17Ab-8aB)\left(\frac{11\left(\frac{5\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{3a}+\frac{2}{3ax\sqrt{a+bx^3}}\right)}{9a}+\frac{2}{9ax(a+bx^3)^{3/2}}\right)}{8a}-\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow 759$$

3.258. $\int \frac{A+Bx^3}{x^5(a+bx^3)^{5/2}}\,dx$

$$(17Ab-8aB)\,\frac{11\,\frac{5\,\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+}{\sqrt[3]{b}x+}\right.\right.}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{2a}}{3a}}{9a}$$

3.258. $\int\frac{A+Bx^3}{x^5(a+bx^3)^{5/2}}\,dx$

$\downarrow$ 2416

3.258. $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx$

$$(17Ab-8aB)\left(\frac{11\left(\frac{5\left(\frac{b\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{\sqrt[3]{b}}\right.}{}\right.}{}\right.}{}\right.$$

3.258. $\int \frac{A+Bx^3}{x^5(a+bx^3)^{5/2}}\,dx$

input
```
Int[(A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x]
```

output
```
-1/4*A/(a*x^4*(a + b*x^3)^(3/2)) - ((17*A*b - 8*a*B)*(2/(9*a*x*(a + b*x^3)
^(3/2)) + (11*(2/(3*a*x*Sqrt[a + b*x^3]) + (5*(-(Sqrt[a + b*x^3]/(a*x)) +
(b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (
3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1
 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(2
*a)))/(3*a)))/(9*a)))/(8*a)
```

3.258.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.258.4 Maple [A] (verified)

Time = 5.88 (sec) , antiderivative size = 581, normalized size of antiderivative = 0.95

method	result
elliptic	$\frac{2x^{2}(Ab-Ba)\sqrt{b\,x^{3}+a}}{9a^{3}b\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2b\,x^{2}(23Ab-14Ba)}{27a^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^{3}+a}}{4a^{3}x^{4}}+\frac{(21Ab-8Ba)\sqrt{b\,x^{3}+a}}{8a^{4}x}-\frac{2i\left(-\frac{b(23Ab-14Ba)}{27a^{4}}-\frac{b(21Ab-8Ba)}{16a^{4}}\right)\sqrt{3}}{}$
default	Expression too large to display
risch	Expression too large to display

input
```
int((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^2/a^3/b*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27*b*x^2/a^4*(23*A*b
-14*B*a)/((x^3+a/b)*b)^(1/2)-1/4/a^3*A*(b*x^3+a)^(1/2)/x^4+1/8/a^4*(21*A*b
-8*B*a)*(b*x^3+a)^(1/2)/x-2/3*I*(-1/27*b/a^4*(23*A*b-14*B*a)-1/16*b*(21*A*
b-8*B*a)/a^4)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*
b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.258. $\int \frac{A+Bx^{3}}{x^{5}\left(a+bx^{3}\right)^{5/2}}\,dx$

3.258.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx =$$

$$-\frac{55\left((8\,Bab^2-17\,Ab^3)x^{10}+2\,(8\,Ba^2b-17\,Aab^2)x^7+(8\,Ba^3-17\,Aa^2b)x^4\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{we}\right.}{216}$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/216*(55*((8*B*a*b^2 - 17*A*b^3)*x^10 + 2*(8*B*a^2*b - 17*A*a*b^2)*x^7 +
 (8*B*a^3 - 17*A*a^2*b)*x^4)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstras
sPInverse(0, -4*a/b, x)) + (55*(8*B*a*b^2 - 17*A*b^3)*x^9 + 88*(8*B*a^2*b
- 17*A*a*b^2)*x^6 + 54*A*a^3 + 27*(8*B*a^3 - 17*A*a^2*b)*x^3)*sqrt(b*x^3 +
 a))/(a^4*b^2*x^10 + 2*a^5*b*x^7 + a^6*x^4)
```

3.258.6 Sympy [A] (verification not implemented)

Time = 156.89 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx = \frac{A\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{5}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{B\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{2}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((B*x**3+A)/x**5/(b*x**3+a)**(5/2),x)
```

output
```
A*gamma(-4/3)*hyper((-4/3, 5/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(5/2)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 5/2), (2/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(5/2)*x*gamma(2/3))
```

3.258.7 Maxima [F]

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^5), x)
```

3.258.8 Giac [F]

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^5}\,dx$$

input
```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^5), x)
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{B\,x^3+A}{x^5\,\left(b\,x^3+a\right)^{5/2}}\,dx$$

input
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x)
```

output
```
int((A + B*x^3)/(x^5*(a + b*x^3)^(5/2)), x)
```

3.259 $\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx$

3.259.1 Optimal result

Integrand size = 26, antiderivative size = 97

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{32c^2\sqrt{c+dx^3}}{3d^3} - \frac{10c(c+dx^3)^{3/2}}{9d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} - \frac{32c^{5/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3}$$

output
```
-10/9*c*(d*x^3+c)^(3/2)/d^3+2/15*(d*x^3+c)^(5/2)/d^3-32/3*c^(5/2)*arctan(1
/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))/d^3*3^(1/2)+32/3*c^2*(d*x^3+c)^(1/2)/d
^3
```

3.259.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2\sqrt{c+dx^3}(218c^2-19cdx^3+3d^2x^6)}{45d^3} - \frac{32c^{5/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3}$$

input
```
Integrate[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
(2*Sqrt[c + d*x^3]*(218*c^2 - 19*c*d*x^3 + 3*d^2*x^6))/(45*d^3) - (32*c^(5
/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^3)
```

3.259.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}dx$$

$$\Big\downarrow \textcolor{blue}{948}$$

$$\frac{1}{3}\int \frac{x^6\sqrt{dx^3+c}}{dx^3+4c}dx^3$$

$$\Big\downarrow \textcolor{blue}{99}$$

$$\frac{1}{3}\int \left(\frac{16\sqrt{dx^3+c}c^2}{d^2\left(dx^3+4c\right)}-\frac{5\sqrt{dx^3+c}c}{d^2}+\frac{\left(dx^3+c\right)^{3/2}}{d^2}\right)dx^3$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{1}{3}\left(-\frac{32\sqrt{3}c^{5/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{d^3}+\frac{32c^2\sqrt{c+dx^3}}{d^3}-\frac{10c\left(c+dx^3\right)^{3/2}}{3d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{5d^3}\right)$$

input
```
Int[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
((32*c^2*Sqrt[c + d*x^3])/d^3 - (10*c*(c + d*x^3)^(3/2))/(3*d^3) + (2*(c +
 d*x^3)^(5/2))/(5*d^3) - (32*Sqrt[3]*c^(5/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[
3]*Sqrt[c])])/d^3)/3
```

3.259.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.259.4 Maple [A] (verified)

Time = 8.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{-480c^{\frac{5}{2}}\sqrt{3}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)+\left(6d^2x^6-38cdx^3+436c^2\right)\sqrt{dx^3+c}}{45d^3}$
risch	$\frac{2\left(3d^2x^6-19cdx^3+218c^2\right)\sqrt{dx^3+c}}{45d^3}-\frac{32c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{3d^3}$
default	$\frac{\frac{2x^6\sqrt{dx^3+c}}{15}+\frac{2cx^3\sqrt{dx^3+c}}{45d}-\frac{4c^2\sqrt{dx^3+c}}{45d^2}}{d}-\frac{8c\left(dx^3+c\right)^{\frac{3}{2}}}{9d^3}+\frac{16c^2\left(2\sqrt{dx^3+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\right)}{3d^3}$
elliptic	$\frac{2x^6\sqrt{dx^3+c}}{15d}-\frac{38cx^3\sqrt{dx^3+c}}{45d^2}+\frac{436c^2\sqrt{dx^3+c}}{45d^3}+$ $16ic^2\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\left(-cd^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\right.}{}\right.}{\left(-c\right.}}\right.$

input
```
int(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
1/45*(-480*c^(5/2)*3^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))+(6*
d^2*x^6-38*c*d*x^3+436*c^2)*(d*x^3+c)^(1/2))/d^3
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.61

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \left[\frac{2\left(120\sqrt{3}\sqrt{-c}c^2\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+(3d^2x^6-19cdx^3+218c^2)\sqrt{dx^3+c}\right)}{45d^3},\ -\frac{2\left(240\sqrt{3}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-(3d^2x^6-19cdx^3+218c^2)\sqrt{dx^3+c}\right)}{45d^3}\right]$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
[2/45*(120*sqrt(3)*sqrt(-c)*c^2*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqr
t(-c) - 2*c)/(d*x^3 + 4*c)) + (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(d*x^
3 + c))/d^3, -2/45*(240*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)
/sqrt(c)) - (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(d*x^3 + c))/d^3]
```

3.259.6 Sympy [A] (verification not implemented)

Time = 8.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{16\sqrt{3}c^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}+\frac{16c^2\sqrt{c+dx^3}}{3}-\frac{5c\left(c+dx^3\right)^{\frac{3}{2}}}{9}+\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{x^9}{36\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Piecewise((2*(-16*sqrt(3)*c**(5/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c
)))/3 + 16*c**2*sqrt(c + d*x**3)/3 - 5*c*(c + d*x**3)**(3/2)/9 + (c + d*x*
*3)**(5/2)/15)/d**3, Ne(d, 0)), (x**9/(36*sqrt(c)), True))
```

3.259.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.71

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\frac{2\left(240\,\sqrt{3}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)-3\,(dx^3+c)^{\frac{5}{2}}+25\,(dx^3+c)^{\frac{3}{2}}c-240\,\sqrt{dx^3+c}c^2\right)}{45\,d^3}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
-2/45*(240*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3
*(d*x^3 + c)^(5/2) + 25*(d*x^3 + c)^(3/2)*c - 240*sqrt(d*x^3 + c)*c^2)/d^3
```

3.259.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\frac{32\,\sqrt{3}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{3\,d^3} + \frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}d^{12}-25\,(dx^3+c)^{\frac{3}{2}}cd^{12}+240\,\sqrt{dx^3+c}c^2d^{12}\right)}{45\,d^{15}}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
-32/3*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^3 + 2/
45*(3*(d*x^3 + c)^(5/2)*d^12 - 25*(d*x^3 + c)^(3/2)*c*d^12 + 240*sqrt(d*x^
3 + c)*c^2*d^12)/d^15
```

3.259.9 Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.12

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{436\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3}+\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d}-\frac{38\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2} + \frac{\sqrt{3}\,c^{5/2}\,\ln\left(\frac{2\,\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,16\mathrm{i}}{3\,d^3}$$

input
```
int((x^8*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

output
```
(436*c^2*(c + d*x^3)^(1/2))/(45*d^3) + (2*x^6*(c + d*x^3)^(1/2))/(15*d) -
(38*c*x^3*(c + d*x^3)^(1/2))/(45*d^2) + (3^(1/2)*c^(5/2)*log((2*3^(1/2)*c
+ c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*16i)/(3*d^3
)
```

3.260 $\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx$

3.260.1 Optimal result

Integrand size = 26, antiderivative size = 76

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx=-\frac{8c\sqrt{c+dx^3}}{3d^2}+\frac{2(c+dx^3)^{3/2}}{9d^2}+\frac{8c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2}$$

output
```
2/9*(d*x^3+c)^(3/2)/d^2+8/3*c^(3/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(
1/2))/d^2*3^(1/2)-8/3*c*(d*x^3+c)^(1/2)/d^2
```

3.260.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{2(-11c+dx^3)\sqrt{c+dx^3}}{9d^2}+\frac{8c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2}$$

input
```
Integrate[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
(2*(-11*c + d*x^3)*Sqrt[c + d*x^3])/(9*d^2) + (8*c^(3/2)*ArcTan[Sqrt[c + d
*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^2)
```

3.260.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {948, 90, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^3\sqrt{dx^3+c}}{dx^3+4c}dx^3$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3d^2}-\frac{4c\int \frac{\sqrt{dx^3+c}}{dx^3+4c}dx^3}{d}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3d^2}-\frac{4c\left(\frac{2\sqrt{c+dx^3}}{d}-3c\int \frac{1}{\sqrt{dx^3+c}\left(dx^3+4c\right)}dx^3\right)}{d}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3d^2}-\frac{4c\left(\frac{2\sqrt{c+dx^3}}{d}-\frac{6c\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3d^2}-\frac{4c\left(\frac{2\sqrt{c+dx^3}}{d}-\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{d}\right)}{d}\right)$$

input
```
Int[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
((2*(c + d*x^3)^(3/2))/(3*d^2) - (4*c*((2*Sqrt[c + d*x^3])/d - (2*Sqrt[3]*
Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/d))/d)/3
```

3.260.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.260. $\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx$

3.260.4 Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{24c^{\frac{3}{2}}\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)-2\sqrt{d\,x^{3}+c}\,\left(-d\,x^{3}+11c\right)}{9d^{2}}$
risch	$-\frac{2\left(-d\,x^{3}+11c\right)\sqrt{d\,x^{3}+c}}{9d^{2}}+\frac{8c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{3d^{2}}$
default	$\frac{2\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9d^{2}}-\frac{4c\left(2\sqrt{d\,x^{3}+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\right)}{3d^{2}}$
elliptic	$\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d}-\frac{22c\sqrt{d\,x^{3}+c}}{9d^{2}}-\frac{4ic\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{}{-3(}}}{}\right.}{}$

input
```
int(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
1/9*(24*c^(3/2)*3^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))-2*(d*x
^3+c)^(1/2)*(-d*x^3+11*c))/d^2
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.70

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx$$
$$=\left[\frac{2\left(6\sqrt{3}\sqrt{-c}c\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+\sqrt{dx^3+c}(dx^3-11c)\right)}{9d^2},\frac{2\left(12\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\right.}{9d^2}\right.$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
[2/9*(6*sqrt(3)*sqrt(-c)*c*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c)
 - 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c)*(d*x^3 - 11*c))/d^2, 2/9*(12*sqrt
(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt(d*x^3 + c)*
(d*x^3 - 11*c))/d^2]
```

3.260.6 Sympy [A] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = \begin{cases} \frac{2\cdot\left(\frac{4\sqrt{3}c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3} - \frac{4c\sqrt{c+dx^3}}{3} + \frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^2} & \text{for } d \neq 0 \\ \frac{x^6}{24\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Piecewise((2*(4*sqrt(3)*c**(3/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c))
)/3 - 4*c*sqrt(c + d*x**3)/3 + (c + d*x**3)**(3/2)/9)/d**2, Ne(d, 0)), (x*
*6/(24*sqrt(c)), True))
```

3.260.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2\left(12\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right) + (dx^3+c)^{\frac{3}{2}} - 12\sqrt{dx^3+c}\,c\right)}{9\,d^2}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
2/9*(12*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + (d*x
^3 + c)^(3/2) - 12*sqrt(d*x^3 + c)*c)/d^2
```

3.260.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{8\,\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{3\,d^2} + \frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^4 - 12\,\sqrt{dx^3+c}cd^4\right)}{9\,d^6}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
8/3*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^2 + 2/9*
((d*x^3 + c)^(3/2)*d^4 - 12*sqrt(d*x^3 + c)*c*d^4)/d^6
```

3.260.9 Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

$$= \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d} - \frac{22\,c\,\sqrt{d\,x^3+c}}{9\,d^2} + \frac{\sqrt{3}\,c^{3/2}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,4\mathrm{i}}{3\,d^2}$$

input
```
int((x^5*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

output
```
(2*x^3*(c + d*x^3)^(1/2))/(9*d) - (22*c*(c + d*x^3)^(1/2))/(9*d^2) + (3^(1
/2)*c^(3/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^
3)/(4*c + d*x^3))*4i)/(3*d^2)
```

3.261 $\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx$

3.261.1 Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{2\sqrt{c+dx^3}}{3d}-\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d}$$

output
```
-2/3*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*c^(1/2)/d*3^(1/2)+2/3*(d*
x^3+c)^(1/2)/d
```

3.261.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{2\left(\sqrt{c+dx^3}-\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)\right)}{3d}$$

input
```
Integrate[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
(2*(Sqrt[c + d*x^3] - Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c]])]))/(3*d)
```

3.261.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {946, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{dx^3+4c}dx^3$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{d}-3c\int \frac{1}{\sqrt{dx^3+c}\,(dx^3+4c)}dx^3\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{d}-\frac{6c\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}}{d}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{d}-\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{d}\right)$$

input
```
Int[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
((2*Sqrt[c + d*x^3])/d - (2*Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Sqrt[c])])/d)/3
```

3.261.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.261.4 Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result
default	$\frac{2\sqrt{d\,x^3+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)}{3d}$
pseudoelliptic	$\frac{2\sqrt{d\,x^3+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)}{3d}$
risch	$-\frac{2\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{c}\,\sqrt{3}}{3d}+\frac{2\sqrt{d\,x^3+c}}{3d}$
elliptic	$\frac{2\sqrt{d\,x^3+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{i d\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$

input
```
int(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(2*(d*x^3+c)^(1/2)-2*c^(1/2)*3^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1
/2)/c^(1/2)))
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.93

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx=\left[\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\,\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right)+2\,\sqrt{dx^3+c}}{3\,d},\right.$$
$$\left.-\frac{2\left(\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)-\sqrt{dx^3+c}\right)}{3\,d}\right]$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
[1/3*(sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2
*c)/(d*x^3 + 4*c)) + 2*sqrt(d*x^3 + c))/d, -2/3*(sqrt(3)*sqrt(c)*arctan(1/
3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - sqrt(d*x^3 + c))/d]
```

3.261.6 Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{\sqrt{3}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}+\frac{\sqrt{c+dx^3}}{3}\right)}{d} & \text{for } d\neq 0 \\ \frac{x^3}{12\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Piecewise((2*(-sqrt(3)*sqrt(c)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/
3 + sqrt(c + d*x**3)/3)/d, Ne(d, 0)), (x**3/(12*sqrt(c)), True))
```

3.261.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\frac{2\left(\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-\sqrt{dx^3+c}\right)}{3\,d}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
-2/3*(sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - sqrt(d
*x^3 + c))/d
```

3.261.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3\,d}+\frac{2\sqrt{dx^3+c}}{3\,d}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d + 2/3*s
qrt(d*x^3 + c)/d
```

3.261.9 Mupad [B] (verification not implemented)

Time = 8.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,d}+\frac{\sqrt{3}\,\sqrt{c}\,\ln\left(\frac{2\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{3\,d}$$

input
```
int((x^2*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*d) + (3^(1/2)*c^(1/2)*log((2*3^(1/2)*c + c^(1/2)*
(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(3*d)
```

3.262 $\int \frac{\sqrt{c+dx^3}}{x\left(4c+dx^3\right)}\,dx$

3.262.1 Optimal result

Integrand size = 26, antiderivative size = 65

$$\int \frac{\sqrt{c+dx^3}}{x\left(4c+dx^3\right)}\,dx=\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

output
```
-1/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+1/6*arctan(1/3*(d*x^3+c)^(1/
2)*3^(1/2)/c^(1/2))*3^(1/2)/c^(1/2)
```

3.262.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+dx^3}}{x\left(4c+dx^3\right)}\,dx=\frac{\sqrt{3}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]
```

output
```
(Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^
3]/Sqrt[c]])/(6*Sqrt[c])
```

3.262.3 Rubi [A] (verified)

Time $=0.20$ (sec) , antiderivative size $=69$, normalized size of antiderivative $=1.06$, number of steps used $=6$, number of rules used $=5$, $\frac{\text{number of rules}}{\text{integrand size}}=0.192$, Rules used $=\{948, 94, 73, 216, 221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x\left(4c+dx^3\right)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^3\left(dx^3+4c\right)}dx^3$$

$$\Big\downarrow \text{94}$$

$$\frac{1}{3}\left(\frac{1}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{3}{4}d\int \frac{1}{\sqrt{dx^3+c}\left(dx^3+4c\right)}dx^3\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}+\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{c}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]
```

output
```
((Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(2*Sqrt[c]) - ArcTanh
[Sqrt[c + d*x^3]/Sqrt[c]]/(2*Sqrt[c]))/3
```

3.262.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.262.4 Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}$	45
default	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\sqrt{c}}{3}}{4c}-\frac{2\sqrt{d\,x^{3}+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)}{12c}$	81
elliptic	Expression too large to display	1502

input
```
int((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*3^(1/2)-arctanh((d*x^3+c)
^(1/2)/c^(1/2)))/c^(1/2)
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.26

$$\int \frac{\sqrt{c+dx^3}}{x\,(4c+dx^3)}\,dx = \left[\frac{2\,\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)+\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{12\,c},\right.$$
$$\left.-\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\,\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right)-2\,\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)}{12\,c}\right]$$

input
```
integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
[1/12*(2*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqr
t(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c, -1/12*(sqrt(3)
*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 +
4*c)) - 2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/c]
```

3.262.6 Sympy [A] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{c+dx^3}}{x\,(4c+dx^3)}\,dx = \begin{cases} \frac{2\left(\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12\sqrt{-c}}+\frac{\sqrt{3}d\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{12\sqrt{c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{\log\left(x^3\right)}{12\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**3+c)**(1/2)/x/(d*x**3+4*c),x)
```

output
```
Piecewise((2*(d*atan(sqrt(c + d*x**3)/sqrt(-c))/(12*sqrt(-c)) + sqrt(3)*d*
atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(12*sqrt(c)))/d, Ne(d, 0)), (lo
g(x**3)/(12*sqrt(c)), True))
```

3.262.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x}\,dx$$

input

```
integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="maxima")
```

output

```
integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x), x)
```

3.262.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{c+dx^3}}{x\left(4c+dx^3\right)}\,dx=\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{6\sqrt{c}}+\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\sqrt{-c}}$$

input

```
integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="giac")
```

output

```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/sqrt(c) + 1/6*arct
an(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c)
```

3.262.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{c+dx^3}}{x\left(4c+dx^3\right)}\,dx=\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{12\sqrt{c}}+\frac{\sqrt{3}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)\,1{}\mathrm{i}}{12\sqrt{c}}$$

input

```
int((c + d*x^3)^(1/2)/(x*(4*c + d*x^3)),x)
```

output

```
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(
12*c^(1/2)) + (3^(1/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3
^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(12*c^(1/2))
```

3.263 $\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}\,dx$

3.263.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}\,dx=-\frac{\sqrt{c+dx^3}}{12cx^3}-\frac{d\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}}-\frac{d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}$$

output
```
-1/24*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/24*d*arctan(1/3*(d*x^3+
c)^(1/2)*3^(1/2)/c^(1/2))/c^(3/2)*3^(1/2)-1/12*(d*x^3+c)^(1/2)/c/x^3
```

3.263.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}\,dx=-\frac{\sqrt{c+dx^3}}{12cx^3}-\frac{d\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}}-\frac{d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]
```

output
```
-1/12*Sqrt[c + d*x^3]/(c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c]
)])/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2)
)
```

3.263.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {948, 110, 27, 174, 73, 216, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^6\left(dx^3+4c\right)}dx^3$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\left(\frac{\int \frac{d\left(2c-dx^3\right)}{2x^3\sqrt{dx^3+c}\left(dx^3+4c\right)}dx^3}{4c}-\frac{\sqrt{c+dx^3}}{4cx^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{d\int \frac{2c-dx^3}{x^3\sqrt{dx^3+c}\left(dx^3+4c\right)}dx^3}{8c}-\frac{\sqrt{c+dx^3}}{4cx^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{d\left(\frac{1}{2}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3-\frac{3}{2}d\int \frac{1}{\sqrt{dx^3+c}\left(dx^3+4c\right)}dx^3\right)}{8c}-\frac{\sqrt{c+dx^3}}{4cx^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{d\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{d}-3\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}\right)}{8c}-\frac{\sqrt{c+dx^3}}{4cx^3}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\left(\frac{d\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{d}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{c}}\right)}{8c}-\frac{\sqrt{c+dx^3}}{4cx^3}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{3}\left(\frac{d\left(-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{8c}-\frac{\sqrt{c+dx^3}}{4cx^3}\right)$$

input

```
Int[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]
```

output

```
(-1/4*Sqrt[c + d*x^3]/(c*x^3) + (d*(-((Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqr
t[3]*Sqrt[c])])/Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/Sqrt[c]))/(8*c
))/3
```

3.263.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

3.263. $\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)}\,dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.263.4 Maple [A] (verified)

Time = 4.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{24c^{\frac{3}{2}}}-\frac{d\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{24c^{\frac{3}{2}}}-\frac{\sqrt{d\,x^{3}+c}}{12c\,x^{3}}$
pseudoelliptic	$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)d\,x^{3}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d\,x^{3}+2\sqrt{d\,x^{3}+c}\,\sqrt{c}}{24c^{\frac{3}{2}}\,x^{3}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3x^{3}}-\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}}{4c}-\frac{d\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\sqrt{c}}{3}\right)}{16c^{2}}+\frac{d\bigg(2\sqrt{d\,x^{3}+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\bigg(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}}{48c^{2}}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-1/24*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/24*d*arctan(1/3*(d*x^3+
c)^(1/2)*3^(1/2)/c^(1/2))/c^(3/2)*3^(1/2)-1/12*(d*x^3+c)^(1/2)/c/x^3
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.20

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}\,dx = \left[-\frac{2\,\sqrt{3}\sqrt{c}dx^3\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)-\sqrt{c}dx^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+4\,\sqrt{dx^3+c}c}{48\,c^2x^3}, -\frac{\sqrt{3}\sqrt{-c}dx^3\log\left(\frac{dx^3+2\,\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right)-2\,\sqrt{-c}dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+4\,\sqrt{dx^3+c}c}{48\,c^2x^3}\right]$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
[-1/48*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)
) - sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 4*s
qrt(d*x^3 + c)*c)/(c^2*x^3), -1/48*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3 + 2*
sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) - 2*sqrt(-c)*d*x^3*
arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]
```

3.263.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}\,dx = \int \frac{\sqrt{c+dx^3}}{x^4\cdot\left(4c+dx^3\right)}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**4/(d*x**3+4*c),x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**4*(4*c + d*x**3)), x)
```

3.263.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(4c+dx^3)}\,dx=\int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x^4}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^4), x)
```

3.263.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(4c+dx^3)}\,dx=-\frac{\sqrt{3}d\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{24\,c^{\frac{3}{2}}}+\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{24\,\sqrt{-c}c}-\frac{\sqrt{dx^3+c}}{12\,cx^3}$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="giac")
```

output
```
-1/24*sqrt(3)*d*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) + 1/24
*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/12*sqrt(d*x^3 + c)/(c
*x^3)
```

3.263.9 Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(4c+dx^3)}\,dx=\frac{d\,\ln\left(\frac{{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{48\,c^{3/2}}-\frac{\sqrt{d\,x^3+c}}{12\,c\,x^3}+\frac{\sqrt{3}\,d\,\ln\left(\frac{2\,\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)\,1{}\mathrm{i}}{48\,c^{3/2}}$$

input
```
int((c + d*x^3)^(1/2)/(x^4*(4*c + d*x^3)),x)
```

output

```
(d*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6
))/(48*c^(3/2)) - (c + d*x^3)^(1/2)/(12*c*x^3) + (3^(1/2)*d*log((2*3^(1/2)
*c + c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(48*
c^(3/2))
```

3.263. $\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)}\,dx$

3.264 $\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx$

3.264.1 Optimal result

Integrand size = 26, antiderivative size = 689

$$
\begin{aligned}
\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = {} & \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{50c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
& - \frac{2\sqrt[3]{2}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}d^{5/3}} + \frac{2\sqrt[3]{2}c^{7/6}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^{5/3}} \\
& - \frac{2\sqrt[3]{2}c^{7/6}\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{2\sqrt[3]{2}c^{7/6}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3d^{5/3}} \\
& + \frac{25\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
& - \frac{50\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

output
```
-2*2^(1/3)*c^(7/6)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(
1/2))/d^(5/3)+2/3*2^(1/3)*c^(7/6)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)
-2/3*2^(1/3)*c^(7/6)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d
*x^3+c)^(1/2))/d^(5/3)*3^(1/2)+2/3*2^(1/3)*c^(7/6)*arctan(1/3*(d*x^3+c)^(1
/2)*3^(1/2)/c^(1/2))/d^(5/3)*3^(1/2)+2/7*x^2*(d*x^3+c)^(1/2)/d-50/7*c*(d*x
^3+c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-50/21*c^(4/3)*(c^(1/3)
+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)
/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+25/
7*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1
/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1
/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2
)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.264.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.19

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx$$
$$= \frac{8x^2(c+dx^3) - 8cx^2\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right) - 5dx^5\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\right.}{28d\sqrt{c+dx^3}}$$

input
```
Integrate[(x^4*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
(8*x^2*(c + d*x^3) - 8*c*x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3
, -((d*x^3)/c), -1/4*(d*x^3)/c] - 5*d*x^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(28*d*Sqrt[c + d*x^3])
```

3.264.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {978, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}dx$$

$$\Big\downarrow \text{978}$$

$$\frac{2x^2\sqrt{c+dx^3}}{7d}-\frac{2\int \frac{cx\left(25dx^3+16c\right)}{2\sqrt{dx^3+c}\left(dx^3+4c\right)}dx}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x^2\sqrt{c+dx^3}}{7d}-\frac{c\int \frac{x\left(25dx^3+16c\right)}{\sqrt{dx^3+c}\left(dx^3+4c\right)}dx}{7d}$$

$$\Big\downarrow \text{1054}$$

$$\frac{2x^2\sqrt{c+dx^3}}{7d}-\frac{c\int \left(\frac{25x}{\sqrt{dx^3+c}}-\frac{84cx}{\sqrt{dx^3+c}\left(dx^3+4c\right)}\right)dx}{7d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x^2\sqrt{c+dx^3}}{7d}-$$

$$c\left(\frac{50\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{25\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}(\sqrt[3]{c}+\sqrt[3]{d}}{}\right.$$

input
```
Int[(x^4*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output

```
(2*x^2*Sqrt[c + d*x^3])/(7*d) - (c*((50*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)) + (14*2^(1/3)*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)
*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) - (14*
2^(1/3)*c^(1/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^(2/3
)) + (14*2^(1/3)*c^(1/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/S
qrt[c + d*x^3]])/d^(2/3) - (14*2^(1/3)*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/Sqr
t[c]])/(3*d^(2/3)) - (25*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1
/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1
/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
 d*x^3]) + (50*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/
3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(7*d)
```

3.264.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.264.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.85 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.26

method	result	size
elliptic	Expression too large to display	867
risch	Expression too large to display	872
default	Expression too large to display	1309

input
```
int(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^2*(d*x^3+c)^(1/2)/d+50/21*I*c/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)))-4/3*I*c/d^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^
(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^
2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/
2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3...
```

3.264.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.12 (sec) , antiderivative size = 2442, normalized size of antiderivative = 3.54

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
1/42*(12*sqrt(d*x^3 + c)*d*x^2 - 14*(4/27)^(1/6)*d^2*(-c^7/d^10)^(1/6)*log
(32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*d^
8)*(-c^7/d^10)^(5/6) - 96*sqrt(1/3)*(c^3*d^7*x^7 - c^4*d^6*x^4 - 2*c^5*d^5
*x)*sqrt(-c^7/d^10) + 4*(9*4^(2/3)*c^2*d^8*x^5*(-c^7/d^10)^(2/3) + 2*c^6*d
^2*x^7 - 32*c^7*d*x^4 - 16*c^8*x + 4^(1/3)*(5*c^4*d^5*x^6 - 20*c^5*d^4*x^3
 - 16*c^6*d^3)*(-c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 24*(4/27)^(1/6)*(c^5*d
^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(-c^7/d^10)^(1/6))/(d^3*x^9 + 12*c
*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 14*(4/27)^(1/6)*d^2*(-c^7/d^10)^(1/6)
*log(-32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c
^3*d^8)*(-c^7/d^10)^(5/6) - 96*sqrt(1/3)*(c^3*d^7*x^7 - c^4*d^6*x^4 - 2*c^
5*d^5*x)*sqrt(-c^7/d^10) - 4*(9*4^(2/3)*c^2*d^8*x^5*(-c^7/d^10)^(2/3) + 2*
c^6*d^2*x^7 - 32*c^7*d*x^4 - 16*c^8*x + 4^(1/3)*(5*c^4*d^5*x^6 - 20*c^5*d^
4*x^3 - 16*c^6*d^3)*(-c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 24*(4/27)^(1/6)*(
c^5*d^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(-c^7/d^10)^(1/6))/(d^3*x^9 +
 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 300*c*sqrt(d)*weierstrassZeta(0,
 -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 7*(4/27)^(1/6)*(sqrt(-3)*d^2
 - d^2)*(-c^7/d^10)^(1/6)*log(32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6
 - 72*c^2*d^9*x^3 - 32*c^3*d^8 + sqrt(-3)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c
^2*d^9*x^3 - 32*c^3*d^8))*(-c^7/d^10)^(5/6) + 192*sqrt(1/3)*(c^3*d^7*x^7 -
 c^4*d^6*x^4 - 2*c^5*d^5*x)*sqrt(-c^7/d^10) + 4*(4*c^6*d^2*x^7 - 64*c^7...
```

3.264.6 Sympy [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

input
```
integrate(x**4*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Integral(x**4*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

3.264.7 Maxima [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+c}x^4}{dx^3+4\,c}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 + 4*c), x)
```

3.264.8 Giac [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+c}x^4}{dx^3+4\,c}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 + 4*c), x)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{x^4\,\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

input
```
int((x^4*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

output
```
int((x^4*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)
```

3.265 $\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$

3.265.1 Optimal result

Integrand size = 24, antiderivative size = 659

$$\begin{aligned}
\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx &= \frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}\sqrt{3}d^{2/3}} \\
&\quad-\frac{\sqrt[6]{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2^{2/3}\sqrt{3}d^{2/3}}+\frac{\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}d^{2/3}}-\frac{\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3\ 2^{2/3}d^{2/3}} \\
&\quad-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad+\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
1/2*c^(1/6)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2
^(1/3)/d^(2/3)-1/6*c^(1/6)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/d^(2/3
)+1/6*c^(1/6)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^3+c)
^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)-1/6*c^(1/6)*arctan(1/3*(d*x^3+c)^(1/2)*3^(
1/2)/c^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)+2*(d*x^3+c)^(1/2)/d^(2/3)/(d^(1/3)*x
+c^(1/3)*(1+3^(1/2)))+2/3*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x
+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/
2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)^2)^(1/2)*3^(3/4)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3^(1/4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)
*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2
/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(2/3)/(d*x^3+c)^(1/2)/
(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.265.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.10

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{8\sqrt{c+dx^3}}$$

input
```
Integrate[(x*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
(x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), -1/4*(d
*x^3)/c])/(8*Sqrt[c + d*x^3])
```

3.265.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {984, 832, 759, 986, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}dx$$

$$\Big\downarrow \ 984$$

$$\int \frac{x}{\sqrt{dx^3+c}}dx-3c\int \frac{x}{\sqrt{dx^3+c}\,(dx^3+4c)}dx$$

$$\Big\downarrow \ 832$$

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}\int \frac{1}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}+\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-3c\int \frac{x}{\sqrt{dx^3+c}\,(dx^3+4c)}dx$$

$$\Big\downarrow \ 759$$

$$\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-3c\int \frac{x}{\sqrt{dx^3+c}\,(dx^3+4c)}dx-$$
$$\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\Big\downarrow \ 986$$

$$\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-$$
$$\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-$$
$$3c\left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right.}{9\ 2^{2/3}c^{5/6}d^{2/3}}\right.$$

$$\Big\downarrow \ 2416$$

3.265. $\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$

$$-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-$$

$$3c\left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}\right.$$

input
```
Int[(x*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
-3*c*(-1/3*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c +
 d*x^3]]/(2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTan[Sqrt[c + d*x^3]/(Sqrt[
3]*Sqrt[c])]/(3*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/6)*(c^(1/
3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3*2^(2/3)*c^(5/6)*d^(2/3)) + Ar
cTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(9*2^(2/3)*c^(5/6)*d^(2/3))) + ((2*Sqrt[c +
 d*x^3])/(d^(1/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (3^(1/4)*Sqrt[2 -
 Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
+ d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1
 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7
- 4*Sqrt[3]])/(d^(1/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/d^(1/3) - (2*(1 - Sqrt[3])*Sqrt
[2 + Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

3.265. $\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$

3.265.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 986
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.265. $\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$

3.265.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.21 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.29

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

input
```
int(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3*2^(
1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(
-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-
(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2
)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-...
```

3.265.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.84 (sec) , antiderivative size = 2202, normalized size of antiderivative = 3.34

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
-1/12*((1/432)^(1/6)*(sqrt(-3)*d - d)*(-c/d^4)^(1/6)*log(1/2*(36*(1/432)^(
5/6)*(d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 + sqrt(-3)*(d^6
*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3))*(-c/d^4)^(5/6) + 24*sq
rt(1/3)*(c*d^4*x^7 - c^2*d^3*x^4 - 2*c^3*d^2*x)*sqrt(-c/d^4) + (2*c*d^2*x^
7 - 32*c^2*d*x^4 - 16*c^3*x + 18*(1/2)^(2/3)*(sqrt(-3)*c*d^4*x^5 - c*d^4*x
^5)*(-c/d^4)^(2/3) - (1/2)^(1/3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d
+ sqrt(-3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d))*(-c/d^4)^(1/3))*sqrt
(d*x^3 + c) - 6*(1/432)^(1/6)*(c*d^3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2 - s
qrt(-3)*(c*d^3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2))*(-c/d^4)^(1/6))/(d^3*x^
9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - (1/432)^(1/6)*(sqrt(-3)*d - d
)*(-c/d^4)^(1/6)*log(-1/2*(36*(1/432)^(5/6)*(d^6*x^9 - 66*c*d^5*x^6 - 72*c
^2*d^4*x^3 - 32*c^3*d^3 + sqrt(-3)*(d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^
3 - 32*c^3*d^3))*(-c/d^4)^(5/6) + 24*sqrt(1/3)*(c*d^4*x^7 - c^2*d^3*x^4 -
2*c^3*d^2*x)*sqrt(-c/d^4) - (2*c*d^2*x^7 - 32*c^2*d*x^4 - 16*c^3*x + 18*(1
/2)^(2/3)*(sqrt(-3)*c*d^4*x^5 - c*d^4*x^5)*(-c/d^4)^(2/3) - (1/2)^(1/3)*(5
*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d + sqrt(-3)*(5*c*d^3*x^6 - 20*c^2*d^
2*x^3 - 16*c^3*d))*(-c/d^4)^(1/3))*sqrt(d*x^3 + c) - 6*(1/432)^(1/6)*(c*d^
3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2 - sqrt(-3)*(c*d^3*x^8 - 7*c^2*d^2*x^5
- 8*c^3*d*x^2))*(-c/d^4)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 6
4*c^3)) - (1/432)^(1/6)*(sqrt(-3)*d + d)*(-c/d^4)^(1/6)*log(1/2*(36*(1/...
```

3.265.6 Sympy [F]

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

input
```
integrate(x*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Integral(x*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

3.265.7 Maxima [F]

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{\sqrt{dx^3+cx}}{dx^3+4\,c}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(d*x^3 + 4*c), x)
```

3.265.8 Giac [F]

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{\sqrt{dx^3+cx}}{dx^3+4\,c}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(d*x^3 + 4*c), x)
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{x\,\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

input
```
int((x*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

output
```
int((x*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)
```

3.266 $\int \frac{\sqrt{c+dx^3}}{x^2(4c+dx^3)}\,dx$

3.266.1 Optimal result

Integrand size = 26, antiderivative size = 697

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx$$

$$= -\frac{\sqrt{c+dx^3}}{4cx}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{4c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\ 2^{2/3}\sqrt{3}c^{5/6}}$$

$$+\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{4\ 2^{2/3}\sqrt{3}c^{5/6}}-\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\ 2^{2/3}c^{5/6}}+\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\ 2^{2/3}c^{5/6}}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-1/8*d^(1/3)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*
2^(1/3)/c^(5/6)+1/24*d^(1/3)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/c^(5
/6)-1/24*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^3
+c)^(1/2))*2^(1/3)/c^(5/6)*3^(1/2)+1/24*d^(1/3)*arctan(1/3*(d*x^3+c)^(1/2)
*3^(1/2)/c^(1/2))*2^(1/3)/c^(5/6)*3^(1/2)-1/4*(d*x^3+c)^(1/2)/c/x+1/4*d^(1
/3)*(d*x^3+c)^(1/2)/c/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/12*d^(1/3)*(c^(1/3
)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2
)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/
8*3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1
/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1
/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2
)))^2)^(1/2)/c^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.266.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx$$

$$= \frac{-40c(c+dx^3)+25cdx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+d^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3}\right.}{160c^2x\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^2*(4*c + d*x^3)),x]
```

output
```
(-40*c*(c + d*x^3) + 25*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[
5/3, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(160*c^2*x*Sqrt[c + d*x^3
])
```

3.266.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 690, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {975, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}dx$$

$$\Big\downarrow \; 975$$

$$\frac{\int \frac{dx\left(dx^3+10c\right)}{2\sqrt{dx^3+c}\left(dx^3+4c\right)}dx}{4c}-\frac{\sqrt{c+dx^3}}{4cx}$$

$$\Big\downarrow \; 27$$

$$\frac{d\int \frac{x\left(dx^3+10c\right)}{\sqrt{dx^3+c}\left(dx^3+4c\right)}dx}{8c}-\frac{\sqrt{c+dx^3}}{4cx}$$

$$\Big\downarrow \; 1054$$

$$\frac{d\int \left(\frac{x}{\sqrt{dx^3+c}}+\frac{6cx}{\sqrt{dx^3+c}\left(dx^3+4c\right)}\right)dx}{8c}-\frac{\sqrt{c+dx^3}}{4cx}$$

$$\Big\downarrow \; 2009$$

$$d\left(\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{}\right.$$

$$\frac{\sqrt{c+dx^3}}{4cx}$$

input
```
Int[Sqrt[c + d*x^3]/(x^2*(4*c + d*x^3)),x]
```

output
```
-1/4*Sqrt[c + d*x^3]/(c*x) + (d*((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)) - (2^(1/3)*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/
3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) + (2^(1/3)*c^
(1/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^(2/3)) - (2^(1
/3)*c^(1/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3
]])/d^(2/3) + (2^(1/3)*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*d^(2/3
)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1
/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[
2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqr
t[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(8*c)
```

3.266.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.266. $\int \frac{\sqrt{c+dx^3}}{x^2(4c+dx^3)}\,dx$

3.266.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.01 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.25

method	result	size
elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	1306

input

```
int((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output

```
-1/4*(d*x^3+c)^(1/2)/c/x-1/12*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1
/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)))-1/12*I/d^2/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*
(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_al...
```

3.266.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.53 (sec) , antiderivative size = 2253, normalized size of antiderivative = 3.23

$$\int \frac{\sqrt{c+dx^3}}{x^2(4c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
1/48*(2*(1/432)^(1/6)*c*x*(-d^2/c^5)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 - 7
2*c^2*d^2*x^3 - 32*c^3*d + 48*(1/2)^(2/3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6
*x)*(-d^2/c^5)^(2/3) + 12*(1/2)^(1/3)*(c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4
*d*x^2)*(-d^2/c^5)^(1/3) + 6*(1296*(1/432)^(5/6)*c^5*d*x^5*(-d^2/c^5)^(5/6
) + sqrt(1/3)*(5*c^3*d^2*x^6 - 20*c^4*d*x^3 - 16*c^5)*sqrt(-d^2/c^5) + 2*(
1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-d^2/c^5)^(1/6))*sq
rt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 2*(1/43
2)^(1/6)*c*x*(-d^2/c^5)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 - 72*c^2*d^2*x^3
 - 32*c^3*d + 48*(1/2)^(2/3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6*x)*(-d^2/c^5
)^(2/3) + 12*(1/2)^(1/3)*(c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4*d*x^2)*(-d^2
/c^5)^(1/3) - 6*(1296*(1/432)^(5/6)*c^5*d*x^5*(-d^2/c^5)^(5/6) + sqrt(1/3)
*(5*c^3*d^2*x^6 - 20*c^4*d*x^3 - 16*c^5)*sqrt(-d^2/c^5) + 2*(1/432)^(1/6)*
(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-d^2/c^5)^(1/6))*sqrt(d*x^3 + c)
)/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 12*sqrt(d)*x*weierst
rassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - (1/432)^(1/6)*(sq
rt(-3)*c*x + c*x)*(-d^2/c^5)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 - 72*c^2*d^
2*x^3 - 32*c^3*d - 24*(1/2)^(2/3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6*x + sqr
t(-3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6*x))*(-d^2/c^5)^(2/3) - 6*(1/2)^(1/3
)*(c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4*d*x^2 - sqrt(-3)*(c^2*d^3*x^8 - 7*c
^3*d^2*x^5 - 8*c^4*d*x^2))*(-d^2/c^5)^(1/3) + 6*sqrt(d*x^3 + c)*(648*(1...
```

3.266.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2(4c+dx^3)}\,dx = \int \frac{\sqrt{c+dx^3}}{x^2\cdot(4c+dx^3)}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**2/(d*x**3+4*c),x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**2*(4*c + d*x**3)), x)
```

3.266.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^2), x)
```

3.266.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^2), x)
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^2\,\left(d\,x^3+4\,c\right)}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^2*(4*c + d*x^3)),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^2*(4*c + d*x^3)), x)
```

3.267 $\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$

3.267.1 Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{x^4\sqrt{c+dx^3}\operatorname{AppellF1}\left(\frac{4}{3},1,-\frac{1}{2},\frac{7}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/16*x^4*AppellF1(4/3,-1/2,1,7/3,-d*x^3/c,-1/4*d*x^3/c)*(d*x^3+c)^(1/2)/c/
(1+d*x^3/c)^(1/2)
```

3.267.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 236 vs. $2(66) = 132$.

Time = 6.34 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.58

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{x\left(-17x^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+32\left(\frac{c}{d}+x^3+\frac{6}{d(4c+dx^3)\left(-16c\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx}{4c}\right.\right.}\right.\right.}{80\sqrt{c+dx^3}}$$

input
```
Integrate[(x^3*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
(x*(-17*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -
1/4*(d*x^3)/c] + 32*(c/d + x^3 + (64*c^3*AppellF1[1/3, 1/2, 1, 4/3, -((d*x
^3)/c), -1/4*(d*x^3)/c])/(d*(4*c + d*x^3)*(-16*c*AppellF1[1/3, 1/2, 1, 4/3
, -((d*x^3)/c), -1/4*(d*x^3)/c] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d
*x^3)/c), -1/4*(d*x^3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -1/
4*(d*x^3)/c]))))))/(80*Sqrt[c + d*x^3])
```

3.267.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}dx$$

$$\Big\downarrow \; 1013$$

$$\frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{\frac{dx^3}{c}+1}}{dx^3+4c}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \; 1012$$

$$\frac{x^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},1,-\frac{1}{2},\frac{7}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x^3*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

output
```
(x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -1/2, 7/3, -1/4*(d*x^3)/c, -((d*x^3)
/c)])/(16*c*Sqrt[1 + (d*x^3)/c])
```

3.267.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.267.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.74 (sec) , antiderivative size = 713, normalized size of antiderivative = 10.80

method	result
elliptic	$\frac{2x\sqrt{d\,x^{3}+c}}{5d}+\frac{34ic\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-\right.}{2}\right.}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{15d^{2}\sqrt{d\,x^{3}+c}}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
2/5*x*(d*x^3+c)^(1/2)/d+34/15*I*c/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-4/3*I*c/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2
)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3
^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*
(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

3.267.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2387 vs. $2(52) = 104$.

Time = 1.87 (sec) , antiderivative size = 2387, normalized size of antiderivative = 36.17

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output

```
-1/60*(10*(16/27)^(1/6)*d^2*(-c^5/d^8)^(1/6)*log((27*(16/27)^(5/6)*(d^9*x^
8 - 7*c*d^8*x^5 - 8*c^2*d^7*x^2)*(-c^5/d^8)^(5/6) + 96*sqrt(1/3)*(c^2*d^6*
x^7 - c^3*d^5*x^4 - 2*c^4*d^4*x)*sqrt(-c^5/d^8) + 4*(2*c^4*d^2*x^7 - 18*2^
(1/3)*c^3*d^4*x^5*(-c^5/d^8)^(1/3) - 32*c^5*d*x^4 - 16*c^6*x - 2^(2/3)*(5*
c*d^7*x^6 - 20*c^2*d^6*x^3 - 16*c^3*d^5)*(-c^5/d^8)^(2/3))*sqrt(d*x^3 + c)
 - 2*(16/27)^(1/6)*(c^3*d^4*x^9 - 66*c^4*d^3*x^6 - 72*c^5*d^2*x^3 - 32*c^6
*d)*(-c^5/d^8)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) -
10*(16/27)^(1/6)*d^2*(-c^5/d^8)^(1/6)*log(-(27*(16/27)^(5/6)*(d^9*x^8 - 7*
c*d^8*x^5 - 8*c^2*d^7*x^2)*(-c^5/d^8)^(5/6) + 96*sqrt(1/3)*(c^2*d^6*x^7 -
c^3*d^5*x^4 - 2*c^4*d^4*x)*sqrt(-c^5/d^8) - 4*(2*c^4*d^2*x^7 - 18*2^(1/3)*
c^3*d^4*x^5*(-c^5/d^8)^(1/3) - 32*c^5*d*x^4 - 16*c^6*x - 2^(2/3)*(5*c*d^7*
x^6 - 20*c^2*d^6*x^3 - 16*c^3*d^5)*(-c^5/d^8)^(2/3))*sqrt(d*x^3 + c) - 2*(
16/27)^(1/6)*(c^3*d^4*x^9 - 66*c^4*d^3*x^6 - 72*c^5*d^2*x^3 - 32*c^6*d)*(-
c^5/d^8)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 24*sqr
t(d*x^3 + c)*d*x + 168*c*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) - 5*(16
/27)^(1/6)*(sqrt(-3)*d^2 - d^2)*(-c^5/d^8)^(1/6)*log((27*(16/27)^(5/6)*(d^
9*x^8 - 7*c*d^8*x^5 - 8*c^2*d^7*x^2 + sqrt(-3)*(d^9*x^8 - 7*c*d^8*x^5 - 8*
c^2*d^7*x^2))*(-c^5/d^8)^(5/6) - 192*sqrt(1/3)*(c^2*d^6*x^7 - c^3*d^5*x^4
- 2*c^4*d^4*x)*sqrt(-c^5/d^8) + 4*(4*c^4*d^2*x^7 - 64*c^5*d*x^4 - 32*c^6*x
 + 2^(2/3)*(5*c*d^7*x^6 - 20*c^2*d^6*x^3 - 16*c^3*d^5 - sqrt(-3)*(5*c*d...
```

3.267.6 Sympy [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

input

```
integrate(x**3*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output

```
Integral(x**3*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

3.267.7 Maxima [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{\sqrt{dx^3+c}x^3}{dx^3+4\,c}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x^3/(d*x^3 + 4*c), x)
```

3.267.8 Giac [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{\sqrt{dx^3+c}x^3}{dx^3+4\,c}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x^3/(d*x^3 + 4*c), x)
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{x^3\,\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

input
```
int((x^3*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

output
```
int((x^3*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)
```

3.268 $\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx$

3.268.1 Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{x\sqrt{c+dx^3}\,\mathrm{AppellF1}\left(\frac{1}{3},1,-\frac{1}{2},\frac{4}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/4*x*AppellF1(1/3,-1/2,1,4/3,-d*x^3/c,-1/4*d*x^3/c)*(d*x^3+c)^(1/2)/c/(1+
d*x^3/c)^(1/2)
```

3.268.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(64) = 128.

Time = 10.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.58

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx$$
$$=\frac{16cx\sqrt{c+dx^3}\,\mathrm{AppellF1}\left(\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{(4c+dx^3)\,\big(16c\,\mathrm{AppellF1}\left(\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-3dx^3\,\big(\mathrm{AppellF1}\left(\frac{4}{3},-\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-2\,\mathrm{Appell}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(4*c + d*x^3),x]
```

output
```
(16*c*x*Sqrt[c + d*x^3]*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -1/4*(d*
x^3)/c])/((4*c + d*x^3)*(16*c*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -1
/4*(d*x^3)/c] - 3*d*x^3*(AppellF1[4/3, -1/2, 2, 7/3, -((d*x^3)/c), -1/4*(d
*x^3)/c] - 2*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c])))
```

3.268.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{c+dx^3}\int \frac{\sqrt{\frac{dx^3}{c}+1}}{dx^3+4c}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{c+dx^3}\operatorname{AppellF1}\left(\frac{1}{3},1,-\frac{1}{2},\frac{4}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(4*c + d*x^3),x]
```

output
```
(x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -1/2, 4/3, -1/4*(d*x^3)/c, -((d*x^3)/c
)])/(4*c*Sqrt[1 + (d*x^3)/c])
```

3.268.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.268.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.65 (sec) , antiderivative size = 696, normalized size of antiderivative = 10.88

method	result
default	$-\frac{2 i \sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}-\frac{i \sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}\right) \sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3 \left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}+\frac{i \sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}}}\, \sqrt{-\frac{i \left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}+\frac{i \sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}\right) \sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\, F}{3 d \sqrt{d\,x^{3}+c}}$
elliptic	$-\frac{2 i \sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}-\frac{i \sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}\right) \sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3 \left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}+\frac{i \sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}}}\, \sqrt{-\frac{i \left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}+\frac{i \sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}\right) \sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\, F}{3 d \sqrt{d\,x^{3}+c}}$

input
```
int((d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

3.268. $\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx$

output
```
-2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
))+1/3*I/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c
*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d
*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/
3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

3.268.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2240 vs. $2(50) = 100$.

Time = 0.51 (sec) , antiderivative size = 2240, normalized size of antiderivative = 35.00

$$\int \frac{\sqrt{c + dx^3}}{4c + dx^3} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
1/24*((1/108)^(1/6)*(sqrt(-3)*d + d)*(-1/(c*d^2))^(1/6)*log((d^3*x^9 - 66*
c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 + 12*(1/4)^(2/3)*(c*d^4*x^8 - 7*c^2*d^3*
x^5 - 8*c^3*d^2*x^2 + sqrt(-3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2)
)*(-1/(c*d^2))^(2/3) + 24*(1/4)^(1/3)*(c*d^3*x^7 - c^2*d^2*x^4 - 2*c^3*d*x
 - sqrt(-3)*(c*d^3*x^7 - c^2*d^2*x^4 - 2*c^3*d*x))*(-1/(c*d^2))^(1/3) + 6*
sqrt(d*x^3 + c)*(18*(1/108)^(5/6)*(c*d^4*x^7 - 16*c^2*d^3*x^4 - 8*c^3*d^2*
x - sqrt(-3)*(c*d^4*x^7 - 16*c^2*d^3*x^4 - 8*c^3*d^2*x))*(-1/(c*d^2))^(5/6
) - sqrt(1/3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d)*sqrt(-1/(c*d^2)) +
 9*(1/108)^(1/6)*(sqrt(-3)*c*d^2*x^5 + c*d^2*x^5)*(-1/(c*d^2))^(1/6)))/(d^
3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - (1/108)^(1/6)*(sqrt(-3)*d
 + d)*(-1/(c*d^2))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c
^3 + 12*(1/4)^(2/3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2 + sqrt(-3)*
(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2))*(-1/(c*d^2))^(2/3) + 24*(1/4)
^(1/3)*(c*d^3*x^7 - c^2*d^2*x^4 - 2*c^3*d*x - sqrt(-3)*(c*d^3*x^7 - c^2*d^
2*x^4 - 2*c^3*d*x))*(-1/(c*d^2))^(1/3) - 6*sqrt(d*x^3 + c)*(18*(1/108)^(5/
6)*(c*d^4*x^7 - 16*c^2*d^3*x^4 - 8*c^3*d^2*x - sqrt(-3)*(c*d^4*x^7 - 16*c^
2*d^3*x^4 - 8*c^3*d^2*x))*(-1/(c*d^2))^(5/6) - sqrt(1/3)*(5*c*d^3*x^6 - 20
*c^2*d^2*x^3 - 16*c^3*d)*sqrt(-1/(c*d^2)) + 9*(1/108)^(1/6)*(sqrt(-3)*c*d^
2*x^5 + c*d^2*x^5)*(-1/(c*d^2))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d
*x^3 + 64*c^3)) - (1/108)^(1/6)*(sqrt(-3)*d - d)*(-1/(c*d^2))^(1/6)*log...
```

3.268.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

output
```
Integral(sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

3.268.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{\sqrt{dx^3+c}}{dx^3+4\,c}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/(d*x^3 + 4*c), x)
```

3.268.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{\sqrt{dx^3+c}}{dx^3+4\,c}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/(d*x^3 + 4*c), x)
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(4*c + d*x^3),x)
```

output
```
int((c + d*x^3)^(1/2)/(4*c + d*x^3), x)
```

3.269 $\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)} \, dx$

3.269.1 Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)} \, dx = -\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-1/8*AppellF1(-2/3,-1/2,1,1/3,-d*x^3/c,-1/4*d*x^3/c)*(d*x^3+c)^(1/2)/c/x^2
/(1+d*x^3/c)^(1/2)
```

3.269.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 244 vs. $2(66) = 132$.

Time = 11.15 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.70

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)} \, dx$$
$$= \frac{-32c(c+dx^3) - d^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right) + \frac{2048c^3}{(4c+dx^3)\left(16c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-}}{256c^2x^2\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^3*(4*c + d*x^3)),x]
```

output
```
(-32*c*(c + d*x^3) - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3
, -((d*x^3)/c), -1/4*(d*x^3)/c] + (2048*c^3*d*x^3*AppellF1[1/3, 1/2, 1, 4/
3, -((d*x^3)/c), -1/4*(d*x^3)/c])/((4*c + d*x^3)*(16*c*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c] - 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3
, -((d*x^3)/c), -1/4*(d*x^3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c
), -1/4*(d*x^3)/c]))))/(256*c^2*x^2*Sqrt[c + d*x^3])
```

3.269.3 Rubi [A] (verified)

Time $= 0.21$ (sec) , antiderivative size $= 66$, normalized size of antiderivative $= 1.00$, number of steps used $= 2$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used $= \{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{c+dx^3}\int \frac{\sqrt{\frac{dx^3}{c}+1}}{x^3(dx^3+4c)}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(x^3*(4*c + d*x^3)),x]
```

output
```
-1/8*(Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -1/2, 1/3, -1/4*(d*x^3)/c, -((d*x^
3)/c)])/(c*x^2*Sqrt[1 + (d*x^3)/c])
```

3.269.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.269.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.64 (sec) , antiderivative size = 716, normalized size of antiderivative = 10.85

method	result
elliptic	$-\frac{\sqrt{d\,x^3+c}}{8c\,x^2}+\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-c\,d^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-c\,d^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-c\,d^2)^{\frac{1}{3}}}{d}}{-\frac{3(-c\,d^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{(-c\,d^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,(-c\,d^2}{2d}\right.}{(-c\,d^2)^{\frac{1}{3}}}}}{24c\sqrt{d\,x^3+c}}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(d*x^3+c)^(1/2)/c/x^2+1/24*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/12*I/d^2/c*2^(1/2)*sum(1/_alpha^2*(-c*d^2)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^
(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(
-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

3.269.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2361 vs. $2(52) = 104$.

Time = 0.99 (sec) , antiderivative size = 2361, normalized size of antiderivative = 35.77

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="fricas")
```

output
```
-1/96*(2*(1/108)^(1/6)*c*x^2*(-d^4/c^7)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6
- 72*c^2*d^4*x^3 - 32*c^3*d^3 - 24*(1/4)^(2/3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^
5 - 8*c^7*d*x^2)*(-d^4/c^7)^(2/3) - 48*(1/4)^(1/3)*(c^3*d^4*x^7 - c^4*d^3*
x^4 - 2*c^5*d^2*x)*(-d^4/c^7)^(1/3) + 6*(18*(1/108)^(1/6)*c^2*d^4*x^5*(-d^
4/c^7)^(1/6) + 36*(1/108)^(5/6)*(c^6*d^2*x^7 - 16*c^7*d*x^4 - 8*c^8*x)*(-d
^4/c^7)^(5/6) + sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt
(-d^4/c^7))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c
^3)) - 2*(1/108)^(1/6)*c*x^2*(-d^4/c^7)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6
- 72*c^2*d^4*x^3 - 32*c^3*d^3 - 24*(1/4)^(2/3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^
5 - 8*c^7*d*x^2)*(-d^4/c^7)^(2/3) - 48*(1/4)^(1/3)*(c^3*d^4*x^7 - c^4*d^3*
x^4 - 2*c^5*d^2*x)*(-d^4/c^7)^(1/3) - 6*(18*(1/108)^(1/6)*c^2*d^4*x^5*(-d^
4/c^7)^(1/6) + 36*(1/108)^(5/6)*(c^6*d^2*x^7 - 16*c^7*d*x^4 - 8*c^8*x)*(-d
^4/c^7)^(5/6) + sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt
(-d^4/c^7))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c
^3)) - 12*sqrt(d)*x^2*weierstrassPInverse(0, -4*c/d, x) + (1/108)^(1/6)*(s
qrt(-3)*c*x^2 + c*x^2)*(-d^4/c^7)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6 - 72*c
^2*d^4*x^3 - 32*c^3*d^3 + 12*(1/4)^(2/3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^5 - 8*
c^7*d*x^2 + sqrt(-3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^5 - 8*c^7*d*x^2))*(-d^4/c^
7)^(2/3) + 24*(1/4)^(1/3)*(c^3*d^4*x^7 - c^4*d^3*x^4 - 2*c^5*d^2*x - sqrt(
-3)*(c^3*d^4*x^7 - c^4*d^3*x^4 - 2*c^5*d^2*x))*(-d^4/c^7)^(1/3) + 6*sqr...
```

3.269.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{c+dx^3}}{x^3\cdot\left(4c+dx^3\right)}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**3/(d*x**3+4*c),x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**3*(4*c + d*x**3)), x)
```

3.269.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x^3}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^3), x)
```

3.269.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x^3}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^3), x)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^3\,\left(d\,x^3+4\,c\right)}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^3*(4*c + d*x^3)),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^3*(4*c + d*x^3)), x)
```

3.270 $\int \frac{x^8}{\sqrt{c+dx^3}(4c+dx^3)} dx$

3.270.1 Optimal result

Integrand size = 26, antiderivative size = 78

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{32c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^3}$$

output
```
2/9*(d*x^3+c)^(3/2)/d^3+32/9*c^(3/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^
(1/2))/d^3*3^(1/2)-10/3*c*(d*x^3+c)^(1/2)/d^3
```

3.270.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2(-14c+dx^3)\sqrt{c+dx^3}+32\sqrt{3}c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{9d^3}$$

input
```
Integrate[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(2*(-14*c + d*x^3)*Sqrt[c + d*x^3] + 32*Sqrt[3]*c^(3/2)*ArcTan[Sqrt[c + d*
x^3]/(Sqrt[3]*Sqrt[c])])/(9*d^3)
```

3.270.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}dx$$
$$\Big\downarrow \; 948$$
$$\frac{1}{3}\int \frac{x^6}{\sqrt{dx^3+c}\,(dx^3+4c)}dx^3$$
$$\Big\downarrow \; 99$$
$$\frac{1}{3}\int \left(\frac{16c^2}{d^2\sqrt{dx^3+c}\,(dx^3+4c)}-\frac{5c}{d^2\sqrt{dx^3+c}}+\frac{\sqrt{dx^3+c}}{d^2}\right)dx^3$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{3}\left(\frac{32c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3}-\frac{10c\sqrt{c+dx^3}}{d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{3d^3}\right)$$

input
```
Int[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
((-10*c*Sqrt[c + d*x^3])/d^3 + (2*(c + d*x^3)^(3/2))/(3*d^3) + (32*c^(3/2)
*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^3))/3
```

3.270.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.270.4 Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{32c^{\frac{3}{2}}\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)-2\sqrt{d\,x^{3}+c}\,\left(-d\,x^{3}+14c\right)}{9d^{3}}$
risch	$-\frac{2\left(-d\,x^{3}+14c\right)\sqrt{d\,x^{3}+c}}{9d^{3}}+\frac{32c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{9d^{3}}$
default	$\frac{\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d}-\frac{4c\sqrt{d\,x^{3}+c}}{9d^{2}}}{d}-\frac{8c\sqrt{d\,x^{3}+c}}{3d^{3}}+\frac{32c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{9d^{3}}$
elliptic	$\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d^{2}}-\frac{28c\sqrt{d\,x^{3}+c}}{9d^{3}}-\frac{16ic\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{}{-3}}}{}\right.}{}$

input
```
int(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(32*c^(3/2)*3^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))-2*(d*x
^3+c)^(1/2)*(-d*x^3+14*c))/d^3
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.65

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$
$$= \left[\frac{2\left(8\sqrt{3}\sqrt{-c}c\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+\sqrt{dx^3+c}(dx^3-14c)\right)}{9d^3}, \frac{2\left(16\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\right.}{9d^3}\right.$$

input
```
integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[2/9*(8*sqrt(3)*sqrt(-c)*c*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c)
 - 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c)*(d*x^3 - 14*c))/d^3, 2/9*(16*sqrt
(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt(d*x^3 + c)*
(d*x^3 - 14*c))/d^3]
```

3.270.6 Sympy [A] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \begin{cases} \frac{2\cdot\left(\frac{16\sqrt{3}c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9}-\frac{5c\sqrt{c+dx^3}}{3}+\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^3} & \text{for}\ d\neq 0 \\ \frac{x^9}{36c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((2*(16*sqrt(3)*c**(3/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)
))/9 - 5*c*sqrt(c + d*x**3)/3 + (c + d*x**3)**(3/2)/9)/d**3, Ne(d, 0)), (x
**9/(36*c**(3/2)), True))
```

3.270.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.68

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\left(16\,\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)+(dx^3+c)^{\frac{3}{2}}-15\,\sqrt{dx^3+c}c\right)}{9\,d^3}$$

input
```
integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(16*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + (d*x
^3 + c)^(3/2) - 15*sqrt(d*x^3 + c)*c)/d^3
```

3.270.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{32\,\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{9\,d^3}+\frac{2\left((dx^3+c)^{\frac{3}{2}}d^6-15\,\sqrt{dx^3+c}cd^6\right)}{9\,d^9}$$

input
```
integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
32/9*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^3 + 2/9
*((d*x^3 + c)^(3/2)*d^6 - 15*sqrt(d*x^3 + c)*c*d^6)/d^9
```

3.270.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2}-\frac{28\,c\,\sqrt{d\,x^3+c}}{9\,d^3}+\frac{\sqrt{3}\,c^{3/2}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,16\mathrm{i}}{9\,d^3}$$

input
```
int(x^8/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
(2*x^3*(c + d*x^3)^(1/2))/(9*d^2) - (28*c*(c + d*x^3)^(1/2))/(9*d^3) + (3^
(1/2)*c^(3/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*
x^3)/(4*c + d*x^3))*16i)/(9*d^3)
```

3.270. $\int \frac{x^8}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

3.271 $\int \frac{x^5}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

3.271.1 Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \frac{x^5}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx=\frac{2\sqrt{c+dx^3}}{3d^2}-\frac{8\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^2}$$

output
```
-8/9*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*c^(1/2)/d^2*3^(1/2)+2/3*(
d*x^3+c)^(1/2)/d^2
```

3.271.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx=\frac{6\sqrt{c+dx^3}-8\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{9d^2}$$

input
```
Integrate[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(6*Sqrt[c + d*x^3] - 8*Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqr
t[c])])/(9*d^2)
```

3.271.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {948, 90, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt{dx^3+c}\,(dx^3+4c)}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{d^2}-\frac{4c\int \frac{1}{\sqrt{dx^3+c}(dx^3+4c)}dx^3}{d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{d^2}-\frac{8c\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}}{d^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{d^2}-\frac{8\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2}\right)$$

input
```
Int[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
((2*Sqrt[c + d*x^3])/d^2 - (8*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c])])/(Sqrt[3]*d^2))/3
```

3.271.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.271.4 Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{-8\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)+6\sqrt{d\,x^3+c}}{9d^2}$
default	$-\frac{8\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{c}\,\sqrt{3}}{9d^2}+\frac{2\sqrt{d\,x^3+c}}{3d^2}$
risch	$-\frac{8\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{c}\,\sqrt{3}}{9d^2}+\frac{2\sqrt{d\,x^3+c}}{3d^2}$
elliptic	$\frac{2\sqrt{d\,x^3+c}}{3d^2}+\frac{4i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)}}}{}\right)}{}$

input
```
int(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(-8*c^(1/2)*3^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))+6*(d*x
^3+c)^(1/2))/d^2
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.90

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \left[\frac{2\left(2\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+3\sqrt{dx^3+c}\right)}{9\,d^2},\; -\frac{2\left(4\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\sqrt{dx^3+c}\right)}{9\,d^2}\right]$$

input
```
integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[2/9*(2*sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) -
 2*c)/(d*x^3 + 4*c)) + 3*sqrt(d*x^3 + c))/d^2, -2/9*(4*sqrt(3)*sqrt(c)*arc
tan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*sqrt(d*x^3 + c))/d^2]
```

3.271.6 Sympy [A] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \begin{cases} \frac{2\left(-\frac{4\sqrt{3}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9}+\frac{\sqrt{c+dx^3}}{3}\right)}{d^2} & \text{for } d\neq 0 \\ \frac{x^6}{24c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((2*(-4*sqrt(3)*sqrt(c)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c))
)/9 + sqrt(c + d*x**3)/3)/d**2, Ne(d, 0)), (x**6/(24*c**(3/2)), True))
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{2\left(4\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\sqrt{dx^3+c}\right)}{9\,d^2}$$

input
```
integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-2/9*(4*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*sq
rt(d*x^3 + c))/d^2
```

3.271.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{2\left(\frac{4\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{d}-\frac{3\sqrt{dx^3+c}}{d}\right)}{9\,d}$$

input
```
integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-2/9*(4*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d - 3*
sqrt(d*x^3 + c)/d)/d
```

3.271.9 Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,d^2}+\frac{\sqrt{3}\,\sqrt{c}\,\ln\left(\frac{2\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,4\mathrm{i}}{9\,d^2}$$

input
```
int(x^5/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*d^2) + (3^(1/2)*c^(1/2)*log((2*3^(1/2)*c + c^(1/2
)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*4i)/(9*d^2)
```

3.272 $\int \frac{x^2}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

3.272.1 Optimal result

Integrand size = 26, antiderivative size = 40

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}d}$$

output
```
2/9*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))/d*3^(1/2)/c^(1/2)
```

3.272.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}d}$$

input
```
Integrate[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)
```

3.272.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {946, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3}\int \frac{1}{\sqrt{dx^3+c}\,(dx^3+4c)}dx^3$$

$$\Big\downarrow \; 73$$

$$\frac{2\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}}{3d}$$

$$\Big\downarrow \; 216$$

$$\frac{2\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}d}$$

input
```
Int[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)
```

3.272.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.272.4 Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

method	result
default	$\frac{2\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{9d\sqrt{c}}$
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{9d\sqrt{c}}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{id\left(2x+\right.}{}}}{}\right)}{}$

input
```
int(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))/d*3^(1/2)/c^(1/2)
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.18

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \left[-\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)}{9\,cd}, \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}d}\right]$$

input
```
integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/9*sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2
*c)/(d*x^3 + 4*c))/(c*d), 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/s
qrt(c))/(sqrt(c)*d)]
```

3.272.6 Sympy [A] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \begin{cases} \frac{2\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d} & \text{for}\ d \neq 0 \\ \frac{x^3}{12c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((2*sqrt(3)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(9*sqrt(c)
*d), Ne(d, 0)), (x**3/(12*c**(3/2)), True))
```

3.272.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

input
```
integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)
```

3.272.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

input
```
integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)
```

3.272.9 Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{\sqrt{3}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{2\,d\,x^3+8\,c}\right)\,1\mathrm{i}}{9\,\sqrt{c}\,d}$$

input
```
int(x^2/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
(3^(1/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)/
(8*c + 2*d*x^3))*1i)/(9*c^(1/2)*d)
```

3.273 $\int \frac{1}{x\sqrt{c+dx^3}(4c+dx^3)}\,dx$

3.273.1 Optimal result

Integrand size = 26, antiderivative size = 65

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}c^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

output
```
-1/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/18*arctan(1/3*(d*x^3+c)^(1
/2)*3^(1/2)/c^(1/2))/c^(3/2)*3^(1/2)
```

3.273.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right) + 3\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{18c^{3/2}}$$

input
```
Integrate[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
-1/18*(Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])] + 3*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/c^(3/2)
```

3.273.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {948, 97, 73, 216, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3\sqrt{dx^3+c}\,(dx^3+4c)}dx^3$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{4c}-\frac{d\int \frac{1}{\sqrt{dx^3+c}(dx^3+4c)}dx^3}{4c}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2cd}-\frac{\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}}{2c}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2cd}-\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}c^{3/2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}c^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2c^{3/2}}\right)$$

input
```
Int[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(-1/2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(Sqrt[3]*c^(3/2)) - ArcTan
h[Sqrt[c + d*x^3]/Sqrt[c]]/(2*c^(3/2)))/3
```

3.273.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.273.4 Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}+3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{18c^{\frac{3}{2}}}$	45
default	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6c^{\frac{3}{2}}}-\frac{\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{18c^{\frac{3}{2}}}$	47
elliptic	Expression too large to display	1508

input
```
int(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/18*(arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*3^(1/2)+3*arctanh((d*x^
3+c)^(1/2)/c^(1/2)))/c^(3/2)
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.28

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$
$$= \left[-\frac{2\,\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)-3\,\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{36\,c^2},\right.$$
$$\left.-\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3+2\,\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right)-6\,\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)}{36\,c^2}\right]$$

input
```
integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/36*(2*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*
sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, -1/36*(sq
rt(3)*sqrt(-c)*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x
^3 + 4*c)) - 6*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/c^2]
```

3.273.6 Sympy [A] (verification not implemented)

Time = 3.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \begin{cases} \frac{2\left(\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12c\sqrt{-c}}-\frac{\sqrt{3}d\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{\frac{3}{2}}}\right)}{d} & \text{for}\ d\neq 0\\ \frac{\log\left(x^3\right)}{12c^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

input
```
integrate(1/x/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((2*(d*atan(sqrt(c + d*x**3)/sqrt(-c))/(12*c*sqrt(-c)) - sqrt(3)*
d*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(36*c**(3/2)))/d, Ne(d, 0)),
(log(x**3)/(12*c**(3/2)), True))
```

3.273.7 Maxima [F]

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x}\,dx$$

input
```
integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x), x)
```

3.273.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{18\,c^{\frac{3}{2}}} + \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\sqrt{-c}c}$$

input
```
integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-1/18*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) + 1/6*ar
ctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)
```

3.273.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{12\,c^{3/2}} + \frac{\sqrt{3}\,\ln\left(\frac{2\,\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{36\,c^{3/2}}$$

input
```
int(1/(x*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(
12*c^(3/2)) + (3^(1/2)*log((2*3^(1/2)*c + c^(1/2)*(c + d*x^3)^(1/2)*6i - 3
^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(36*c^(3/2))
```

3.273. $\int \frac{1}{x\sqrt{c+dx^3}(4c+dx^3)}\,dx$

3.274 $\int \frac{1}{x^4\sqrt{c+dx^3}(4c+dx^3)}\,dx$

3.274.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=-\frac{\sqrt{c+dx^3}}{12c^2x^3}+\frac{d\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}}+\frac{d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}$$

output
```
1/8*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/72*d*arctan(1/3*(d*x^3+c)
^(1/2)*3^(1/2)/c^(1/2))/c^(5/2)*3^(1/2)-1/12*(d*x^3+c)^(1/2)/c^2/x^3
```

3.274.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=-\frac{\sqrt{c+dx^3}}{12c^2x^3}+\frac{d\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}}+\frac{d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}$$

input
```
Integrate[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
-1/12*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[
c])])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/
2))
```

3.274.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {948, 114, 27, 174, 73, 216, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^6\sqrt{dx^3+c}\,(dx^3+4c)}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{d(dx^3+6c)}{2x^3\sqrt{dx^3+c}(dx^3+4c)}dx^3}{4c^2}-\frac{\sqrt{c+dx^3}}{4c^2x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\int \frac{dx^3+6c}{x^3\sqrt{dx^3+c}(dx^3+4c)}dx^3}{8c^2}-\frac{\sqrt{c+dx^3}}{4c^2x^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{3}{2}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3-\frac{1}{2}d\int \frac{1}{\sqrt{dx^3+c}(dx^3+4c)}dx^3\right)}{8c^2}-\frac{\sqrt{c+dx^3}}{4c^2x^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{3\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{d}-\int \frac{1}{x^6+3c}d\sqrt{dx^3+c}\right)}{8c^2}-\frac{\sqrt{c+dx^3}}{4c^2x^3}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{3\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{d}-\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\sqrt{c}}\right)}{8c^2}-\frac{\sqrt{c+dx^3}}{4c^2x^3}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\sqrt{c}}-\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{8c^2}-\frac{\sqrt{c+dx^3}}{4c^2x^3}\right)$$

input
```
Int[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(-1/4*Sqrt[c + d*x^3]/(c^2*x^3) - (d*(-(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sq
rt[c])]/(Sqrt[3]*Sqrt[c])) - (3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c])
)/(8*c^2))/3
```

3.274.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.274.4 Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{8 c^{\frac{5}{2}}}+\frac{d \arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3 \sqrt{c}}\right) \sqrt{3}}{72 c^{\frac{5}{2}}}-\frac{\sqrt{d\,x^{3}+c}}{12 c^{2} x^{3}}$	66
pseudoelliptic	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3 \sqrt{c}}\right) d\,x^{3}+9\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right) d\,x^{3}-6 \sqrt{d\,x^{3}+c}\,\sqrt{c}}{72 c^{\frac{5}{2}} x^{3}}$	70
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3 c\,x^{3}}+\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3 c^{\frac{3}{2}}}}{4 c}+\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{24 c^{\frac{5}{2}}}+\frac{d \arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3 \sqrt{c}}\right) \sqrt{3}}{72 c^{\frac{5}{2}}}$	92
elliptic	Expression too large to display	1523

input
```
int(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/72*d*arctan(1/3*(d*x^3+c)
^(1/2)*3^(1/2)/c^(1/2))/c^(5/2)*3^(1/2)-1/12*(d*x^3+c)^(1/2)/c^2/x^3
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \left[\frac{2\sqrt{3}\sqrt{c}dx^3\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+9\sqrt{c}dx^3\log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-12\sqrt{dx^3+c}c}{144\,c^3x^3}, -\frac{\sqrt{3}\sqrt{-c}dx^3\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+18\sqrt{-c}dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+12\sqrt{dx^3+c}c}{144\,c^3x^3}\right]$$

input
```
integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/144*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)
) + 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 1
2*sqrt(d*x^3 + c)*c)/(c^3*x^3), -1/144*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3
- 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) + 18*sqrt(-c)*d
*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]
```

3.274.6 Sympy [F]

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{x^4\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

input
```
integrate(1/x**4/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

3.274.7 Maxima [F]

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x^4}\,dx$$

input
```
integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^4), x)
```

3.274.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{\sqrt{3}d\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{72\,c^{\frac{5}{2}}} - \frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}c^2} - \frac{\sqrt{dx^3+c}}{12\,c^2x^3}$$

input
```
integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
1/72*sqrt(3)*d*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(5/2) - 1/8*d
*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/12*sqrt(d*x^3 + c)/(c
^2*x^3)
```

3.274.9 Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{d\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)^3}{x^6}\right)}{16\,c^{5/2}} - \frac{\sqrt{d\,x^3+c}}{12\,c^2\,x^3} + \frac{\sqrt{3}\,d\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)\,1{}\mathrm{i}}{144\,c^{5/2}}$$

input
```
int(1/(x^4*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output

```
(d*log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6
))/(16*c^(5/2)) - (c + d*x^3)^(1/2)/(12*c^2*x^3) + (3^(1/2)*d*log((c^(1/2)
*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(1
44*c^(5/2))
```

3.274. $\int \frac{1}{x^4\sqrt{c+dx^3}(4c+dx^3)}\,dx$

3.275 $\int \frac{x^4}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

3.275.1 Optimal result

Integrand size = 26, antiderivative size = 667

$$\begin{aligned}
\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx &= \frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&+ \frac{2\sqrt[3]{2}\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} - \frac{2\sqrt[3]{2}\sqrt[6]{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^{5/3}} \\
&+ \frac{2\sqrt[3]{2}\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3d^{5/3}} - \frac{2\sqrt[3]{2}\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9d^{5/3}} \\
&- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
2/3*2^(1/3)*c^(1/6)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^
(1/2))/d^(5/3)-2/9*2^(1/3)*c^(1/6)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/d^(5/3
)+2/9*2^(1/3)*c^(1/6)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(
d*x^3+c)^(1/2))/d^(5/3)*3^(1/2)-2/9*2^(1/3)*c^(1/6)*arctan(1/3*(d*x^3+c)^(
1/2)*3^(1/2)/c^(1/2))/d^(5/3)*3^(1/2)+2*(d*x^3+c)^(1/2)/d^(5/3)/(d^(1/3)*x
+c^(1/3)*(1+3^(1/2)))+2/3*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x
+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/
2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3^(1/4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)
*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2
/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/
(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.275.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.10

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{x^5\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{20c\sqrt{c+dx^3}}$$

input
```
Integrate[x^4/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*
x^3)/c])/(20*c*Sqrt[c + d*x^3])
```

3.275.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {983, 832, 759, 986, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$\Big\downarrow \; 983$$

$$\frac{\int \frac{x}{\sqrt{dx^3+c}}dx}{d}-\frac{4c\int \frac{x}{\sqrt{dx^3+c}(dx^3+4c)}dx}{d}$$

$$\Big\downarrow \; 832$$

$$\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}\int \frac{1}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}}{d}-\frac{4c\int \frac{x}{\sqrt{dx^3+c}(dx^3+4c)}dx}{d}$$

$$\Big\downarrow \; 759$$

$$\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right.}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{d}$$

$$\frac{4c\int \frac{x}{\sqrt{dx^3+c}(dx^3+4c)}dx}{d}$$

$$\Big\downarrow \; 986$$

$$\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right.}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{d}$$

$$\frac{4c\left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}\right)}{d}$$

$$\Big\downarrow \; 2416$$

3.275. $\int \frac{x^4}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

$$\frac{\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{}}{}}{d}$$

$$\frac{4c\left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}\right)}{d}$$

input
```
Int[x^4/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(-4*c*(-1/3*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c
+ d*x^3]]/(2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTan[Sqrt[c + d*x^3]/(Sqrt
[3]*Sqrt[c])]/(3*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/6)*(c^(1
/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3*2^(2/3)*c^(5/6)*d^(2/3)) + A
rcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(9*2^(2/3)*c^(5/6)*d^(2/3))))/d + (((2*Sqr
t[c + d*x^3])/(d^(1/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (3^(1/4)*Sqr
t[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/
3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(d^(1/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/d^(1/3) - (2*(1 - Sqrt[3])
*Sqrt[2 + Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[A
rcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x
))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/d
```

3.275. $\int \frac{x^4}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

3.275.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 986
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.275. $\int \frac{x^4}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

3.275.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.38 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.27

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

input

```
int(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^
(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+4/9*I/d^4*2
^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3
*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^
2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1
/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*...
```

3.275.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.64 (sec) , antiderivative size = 2268, normalized size of antiderivative = 3.40

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/18*(2*(4/27)^(1/6)*d^2*(-c/d^10)^(1/6)*log(32*(9*(4/27)^(5/6)*(d^11*x^9
- 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*d^8)*(-c/d^10)^(5/6) - 96*sqrt(1
/3)*(c*d^7*x^7 - c^2*d^6*x^4 - 2*c^3*d^5*x)*sqrt(-c/d^10) + 4*(9*4^(2/3)*c
*d^8*x^5*(-c/d^10)^(2/3) + 2*c*d^2*x^7 - 32*c^2*d*x^4 - 16*c^3*x + 4^(1/3)
*(5*c*d^5*x^6 - 20*c^2*d^4*x^3 - 16*c^3*d^3)*(-c/d^10)^(1/3))*sqrt(d*x^3 +
 c) - 24*(4/27)^(1/6)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2)*(-c/d^10
)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 2*(4/27)^(1/6
)*d^2*(-c/d^10)^(1/6)*log(-32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 -
72*c^2*d^9*x^3 - 32*c^3*d^8)*(-c/d^10)^(5/6) - 96*sqrt(1/3)*(c*d^7*x^7 - c
^2*d^6*x^4 - 2*c^3*d^5*x)*sqrt(-c/d^10) - 4*(9*4^(2/3)*c*d^8*x^5*(-c/d^10)
^(2/3) + 2*c*d^2*x^7 - 32*c^2*d*x^4 - 16*c^3*x + 4^(1/3)*(5*c*d^5*x^6 - 20
*c^2*d^4*x^3 - 16*c^3*d^3)*(-c/d^10)^(1/3))*sqrt(d*x^3 + c) - 24*(4/27)^(1
/6)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2)*(-c/d^10)^(1/6))/(d^3*x^9
+ 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - (4/27)^(1/6)*(sqrt(-3)*d^2 - d^
2)*(-c/d^10)^(1/6)*log(32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c
^2*d^9*x^3 - 32*c^3*d^8 + sqrt(-3)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*
x^3 - 32*c^3*d^8))*(-c/d^10)^(5/6) + 192*sqrt(1/3)*(c*d^7*x^7 - c^2*d^6*x^
4 - 2*c^3*d^5*x)*sqrt(-c/d^10) + 4*(4*c*d^2*x^7 - 64*c^2*d*x^4 - 32*c^3*x
+ 9*4^(2/3)*(sqrt(-3)*c*d^8*x^5 - c*d^8*x^5)*(-c/d^10)^(2/3) - 4^(1/3)*(5*
c*d^5*x^6 - 20*c^2*d^4*x^3 - 16*c^3*d^3 + sqrt(-3)*(5*c*d^5*x^6 - 20*c^...
```

3.275.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^4}{\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

input
```
integrate(x**4/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**4/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

3.275.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{x^4}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

3.275.8 Giac [F]

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{x^4}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{x^4}{\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

input
```
int(x^4/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
int(x^4/((c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

3.276 $\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

3.276.1 Optimal result

Integrand size = 24, antiderivative size = 206

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}$$

output
```
-1/6*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2^(1/3)/
c^(5/6)/d^(2/3)+1/18*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/c^(5/6)/d^(2
/3)-1/18*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2
))*2^(1/3)/c^(5/6)/d^(2/3)*3^(1/2)+1/18*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)
/c^(1/2))*2^(1/3)/c^(5/6)/d^(2/3)*3^(1/2)
```

3.276.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.33

$$\int \frac{x}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx=\frac{x^2\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{8c\sqrt{c+dx^3}}$$

input

```
Integrate[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output

```
(x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -1/4*(d*
x^3)/c])/(8*c*Sqrt[c + d*x^3])
```

3.276.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {986}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$$

$$\Big\downarrow \text{986}$$

$$-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}$$

input

```
Int[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output

```
-1/3*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3
]]/(2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqr
t[c])]/(3*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/6)*(c^(1/3) - 2
^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3*2^(2/3)*c^(5/6)*d^(2/3)) + ArcTanh[
Sqrt[c + d*x^3]/Sqrt[c]]/(9*2^(2/3)*c^(5/6)*d^(2/3))
```

3.276.3.1 Defintions of rubi rules used

rule 986
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

3.276.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.35 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.02

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\right.}{}\right.}{2\left(-c\right.}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\right.}{}\right.}{2\left(-c\right.}}}{}\right.}{}$

input
```
int(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

3.276. $\int \frac{x}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

output
```
-1/9*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d
^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)
+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d
^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

3.276.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2289 vs. $2(141) = 282$.

Time = 0.67 (sec) , antiderivative size = 2289, normalized size of antiderivative = 11.11

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/36*(1/432)^(1/6)*(sqrt(-3) + 1)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*
c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 - 24*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*
x^4 - 2*c^6*d^3*x + sqrt(-3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x))*(-
1/(c^5*d^4))^(2/3) - 6*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^
2*x^2 - sqrt(-3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2))*(-1/(c^5*d
^4))^(1/3) + 6*sqrt(d*x^3 + c)*(648*(1/432)^(5/6)*(sqrt(-3)*c^5*d^5*x^5 -
c^5*d^5*x^5)*(-1/(c^5*d^4))^(5/6) + sqrt(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*
x^3 - 16*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d
^2*x^4 - 8*c^3*d*x + sqrt(-3)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x))*(-
1/(c^5*d^4))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 1
/36*(1/432)^(1/6)*(sqrt(-3) + 1)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*c*
d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 - 24*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*x^
4 - 2*c^6*d^3*x + sqrt(-3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x))*(-1/
(c^5*d^4))^(2/3) - 6*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*
x^2 - sqrt(-3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2))*(-1/(c^5*d^4
))^(1/3) - 6*sqrt(d*x^3 + c)*(648*(1/432)^(5/6)*(sqrt(-3)*c^5*d^5*x^5 - c^
5*d^5*x^5)*(-1/(c^5*d^4))^(5/6) + sqrt(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^
3 - 16*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2
*x^4 - 8*c^3*d*x + sqrt(-3)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x))*(-1/
(c^5*d^4))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - ...
```

3.276.6 Sympy [F]

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{x}{\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

input
```
integrate(x/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

3.276.7 Maxima [F]

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{x}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

3.276.8 Giac [F]

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{x}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

3.276.9 Mupad [B] (verification not implemented)

Time = 29.70 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.20

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$=\frac{\sqrt{3}\,314928^{1/3}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{3}\,\sqrt{-c}-2^{1/3}\,\sqrt{3}\,(-c)^{1/6}\,d^{1/3}\,x\right)^3\,\left(54\,\sqrt{d\,x^3+c}-54\,\sqrt{3}\,\sqrt{-c}+54\,2^{1/3}\,\sqrt{3}\,(-c)^{1/6}\,d^{1/3}\,x\right)}{\left(d^{1/3}\,x-2^{2/3}\,(-c)^{1/3}\right)^6}\right)}{2916\,(-c)^{5/6}\,d^{2/3}}$$

$$+\frac{\sqrt{3}\,314928^{1/3}\,\ln\Bigg(\frac{\left(2\,\sqrt{3}\,\sqrt{-c}-2\,\sqrt{d\,x^3+c}+2^{1/3}\,\sqrt{3}\,(-c)^{1/6}\,d^{1/3}\,x+2^{1/3}\,(-c)^{1/6}\,d^{1/3}\,x\,3\mathrm{i}\right)^3\,\Big(108\,\sqrt{d\,x^3+c}+108\,\sqrt{3}\,\sqrt{-c}+54\,2^{1/3}\,\sqrt{3}}{\left(2\,d^{1/3}\,x+2^{2/3}\,(-c)^{1/3}-2^{2/3}\,\sqrt{3}\,(-c)^{1/3}\,1\mathrm{i}\right)^6}}{2916\,(-c)^{5/6}\,d^{2/3}}$$

$$+\frac{\sqrt{3}\,314928^{1/3}\,\ln\Bigg(\frac{\left(2\,\sqrt{d\,x^3+c}+2\,\sqrt{3}\,\sqrt{-c}+2^{1/3}\,\sqrt{3}\,(-c)^{1/6}\,d^{1/3}\,x-2^{1/3}\,(-c)^{1/6}\,d^{1/3}\,x\,3\mathrm{i}\right)^3\,\Big(108\,\sqrt{d\,x^3+c}-108\,\sqrt{3}\,\sqrt{-c}-54\,2^{1/3}\,\sqrt{3}}{\left(2\,d^{1/3}\,x+2^{2/3}\,(-c)^{1/3}+2^{2/3}\,\sqrt{3}\,(-c)^{1/3}\,1\mathrm{i}\right)^6}}{2916\,(-c)^{5/6}\,d^{2/3}}$$

input
```
int(x/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
(3^(1/2)*314928^(1/3)*log((((c + d*x^3)^(1/2) + 3^(1/2)*(-c)^(1/2) - 2^(1/
3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(54*(c + d*x^3)^(1/2) - 54*3^(1/2)*(-c)
^(1/2) + 54*2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x))/(d^(1/3)*x - 2^(2/3)*(-
c)^(1/3))^6))/(2916*(-c)^(5/6)*d^(2/3)) + (3^(1/2)*314928^(1/3)*log(((2*3^
(1/2)*(-c)^(1/2) - 2*(c + d*x^3)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*3i +
 2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(108*(c + d*x^3)^(1/2) + 108*3^(1
/2)*(-c)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*162i + 54*2^(1/3)*3^(1/2)*(-
c)^(1/6)*d^(1/3)*x))/(2*d^(1/3)*x + 2^(2/3)*(-c)^(1/3) - 2^(2/3)*3^(1/2)*(
-c)^(1/3)*1i)^6)*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(2916*(-c)^(5/6)*d^(2/3)) +
 (3^(1/2)*314928^(1/3)*log(((2*(c + d*x^3)^(1/2) + 2*3^(1/2)*(-c)^(1/2) -
2^(1/3)*(-c)^(1/6)*d^(1/3)*x*3i + 2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*
(108*(c + d*x^3)^(1/2) - 108*3^(1/2)*(-c)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/
3)*x*162i - 54*2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x))/(2*d^(1/3)*x + 2^(2/
3)*(-c)^(1/3) + 2^(2/3)*3^(1/2)*(-c)^(1/3)*1i)^6)*((3^(1/2)*1i)/2 + 1/2)^(
1/2)*1i)/(2916*(-c)^(5/6)*d^(2/3))
```

3.276. $\int \frac{x}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

3.277 $\int \frac{1}{x^2\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

3.277.1 Optimal result

Integrand size = 26, antiderivative size = 697

$$\begin{aligned}
&\int \frac{1}{x^2\sqrt{c+dx^3}\,\left(4c+dx^3\right)}\,dx \\
&= -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{4c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\ 2^{2/3}\sqrt{3}c^{11/6}} \\
&\quad -\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{12\ 2^{2/3}\sqrt{3}c^{11/6}}+\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\ 2^{2/3}c^{11/6}}-\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{36\ 2^{2/3}c^{11/6}} \\
&\quad -\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad +\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
1/24*d^(1/3)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*
2^(1/3)/c^(11/6)-1/72*d^(1/3)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/c^(
11/6)+1/72*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x
^3+c)^(1/2))*2^(1/3)/c^(11/6)*3^(1/2)-1/72*d^(1/3)*arctan(1/3*(d*x^3+c)^(1
/2)*3^(1/2)/c^(1/2))*2^(1/3)/c^(11/6)*3^(1/2)-1/4*(d*x^3+c)^(1/2)/c^2/x+1/
4*d^(1/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/12*d^(1/3)
*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^
2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3
+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)-1/8*3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)
*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-
1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/
(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.277.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$= \frac{-40c(c+dx^3)+5cdx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+d^2x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\big(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},}{160c^3x\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^2*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(-40*c*(c + d*x^3) + 5*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5
/3, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(160*c^3*x*Sqrt[c + d*x^3]
)
```

3.277.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {980, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}dx$$

$$\Big\downarrow \text{ 980}$$

$$\frac{\int \frac{dx(dx^3+2c)}{2\sqrt{dx^3+c}(dx^3+4c)}dx}{4c^2}-\frac{\sqrt{c+dx^3}}{4c^2x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\int \frac{x(dx^3+2c)}{\sqrt{dx^3+c}(dx^3+4c)}dx}{8c^2}-\frac{\sqrt{c+dx^3}}{4c^2x}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{d\int \left(\frac{x}{\sqrt{dx^3+c}}-\frac{2cx}{\sqrt{dx^3+c}(dx^3+4c)}\right)dx}{8c^2}-\frac{\sqrt{c+dx^3}}{4c^2x}$$

$$\Big\downarrow \text{ 2009}$$

$$d\left(\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{}\right.$$

$$\frac{\sqrt{c+dx^3}}{4c^2x}$$

input
```
Int[1/(x^2*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output

```
-1/4*Sqrt[c + d*x^3]/(c^2*x) + (d*((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)) + (2^(1/3)*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(2/3)) - (2^(1/3
)*c^(1/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^(2/3)) +
 (2^(1/3)*c^(1/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c +
 d*x^3]])/(3*d^(2/3)) - (2^(1/3)*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(9*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1
/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
+ (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
-7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(8*c^2)
```

3.277.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.277.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.94 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.25

method	result	size
elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	874

input
```
int(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(d*x^3+c)^(1/2)/c^2/x-1/12*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)))+1/36*I/d^2/c^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))
^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1
/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/...
```

3.277.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.70 (sec) , antiderivative size = 2293, normalized size of antiderivative = 3.29

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/144*(2*(1/432)^(1/6)*c^2*x*(-d^2/c^11)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^
6 - 72*c^2*d^2*x^3 - 32*c^3*d + 48*(1/2)^(2/3)*(c^8*d^2*x^7 - c^9*d*x^4 -
2*c^10*x)*(-d^2/c^11)^(2/3) + 12*(1/2)^(1/3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5
- 8*c^6*d*x^2)*(-d^2/c^11)^(1/3) + 6*(1296*(1/432)^(5/6)*c^10*d*x^5*(-d^2/
c^11)^(5/6) + sqrt(1/3)*(5*c^6*d^2*x^6 - 20*c^7*d*x^3 - 16*c^8)*sqrt(-d^2/
c^11) + 2*(1/432)^(1/6)*(c^2*d^3*x^7 - 16*c^3*d^2*x^4 - 8*c^4*d*x)*(-d^2/c
^11)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c
^3)) - 2*(1/432)^(1/6)*c^2*x*(-d^2/c^11)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6
 - 72*c^2*d^2*x^3 - 32*c^3*d + 48*(1/2)^(2/3)*(c^8*d^2*x^7 - c^9*d*x^4 - 2
*c^10*x)*(-d^2/c^11)^(2/3) + 12*(1/2)^(1/3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5 -
 8*c^6*d*x^2)*(-d^2/c^11)^(1/3) - 6*(1296*(1/432)^(5/6)*c^10*d*x^5*(-d^2/c
^11)^(5/6) + sqrt(1/3)*(5*c^6*d^2*x^6 - 20*c^7*d*x^3 - 16*c^8)*sqrt(-d^2/c
^11) + 2*(1/432)^(1/6)*(c^2*d^3*x^7 - 16*c^3*d^2*x^4 - 8*c^4*d*x)*(-d^2/c^
11)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^
3)) + 36*sqrt(d)*x*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/
d, x)) - (1/432)^(1/6)*(sqrt(-3)*c^2*x + c^2*x)*(-d^2/c^11)^(1/6)*log((d^4
*x^9 - 66*c*d^3*x^6 - 72*c^2*d^2*x^3 - 32*c^3*d - 24*(1/2)^(2/3)*(c^8*d^2*
x^7 - c^9*d*x^4 - 2*c^10*x + sqrt(-3)*(c^8*d^2*x^7 - c^9*d*x^4 - 2*c^10*x)
)*(-d^2/c^11)^(2/3) - 6*(1/2)^(1/3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5 - 8*c^6*d
*x^2 - sqrt(-3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5 - 8*c^6*d*x^2))*(-d^2/c^11...
```

3.277.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{x^2\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

input
```
integrate(1/x**2/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

3.277.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x^2}\,dx$$

input
```
integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^2), x)
```

3.277.8 Giac [F]

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x^2}\,dx$$

input
```
integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^2), x)
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{x^2\,\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

input
```
int(1/(x^2*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
int(1/(x^2*(c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

3.278 $\int \frac{x^3}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

3.278.1 Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}$$

output
```
1/16*x^4*AppellF1(4/3,1/2,1,7/3,-d*x^3/c,-1/4*d*x^3/c)*(1+d*x^3/c)^(1/2)/c
/(d*x^3+c)^(1/2)
```

3.278.2 Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{x^4\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{16c\sqrt{c+dx^3}}$$

input
```
Integrate[x^3/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*
x^3)/c])/(16*c*Sqrt[c + d*x^3])
```

3.278.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$\Big\downarrow \; 1013$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{(dx^3+4c)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 1012$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}$$

input
```
Int[x^3/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, -1/4*(d*x^3)/c, -((d*x
^3)/c)])/(16*c*Sqrt[c + d*x^3])
```

3.278.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.278.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.36 (sec) , antiderivative size = 696, normalized size of antiderivative = 10.55

method	result
default	$-\frac{2 i \sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,F}{3d^{2}\sqrt{d\,x^{3}+c}}$
elliptic	$-\frac{2 i \sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,F}{3d^{2}\sqrt{d\,x^{3}+c}}$

input

```
int(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2))+4/9*I/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(
-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/
c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

3.278.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2300 vs. $2(52) = 104$.

Time = 0.71 (sec) , antiderivative size = 2300, normalized size of antiderivative = 34.85

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/36*(2*(16/27)^(1/6)*d^2*(-1/(c*d^8))^(1/6)*log((4*d^3*x^9 - 264*c*d^2*x^
6 - 288*c^2*d*x^3 - 128*c^3 - 24*2^(2/3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*c^
3*d^6*x^2)*(-1/(c*d^8))^(2/3) - 96*2^(1/3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*c^
3*d^3*x)*(-1/(c*d^8))^(1/3) + 3*(72*(16/27)^(1/6)*c*d^3*x^5*(-1/(c*d^8))^(
1/6) + 9*(16/27)^(5/6)*(c*d^9*x^7 - 16*c^2*d^8*x^4 - 8*c^3*d^7*x)*(-1/(c*d
^8))^(5/6) + 8*sqrt(1/3)*(5*c*d^6*x^6 - 20*c^2*d^5*x^3 - 16*c^3*d^4)*sqrt(
-1/(c*d^8)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*
c^3)) - 2*(16/27)^(1/6)*d^2*(-1/(c*d^8))^(1/6)*log((4*d^3*x^9 - 264*c*d^2*
x^6 - 288*c^2*d*x^3 - 128*c^3 - 24*2^(2/3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*
c^3*d^6*x^2)*(-1/(c*d^8))^(2/3) - 96*2^(1/3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*
c^3*d^3*x)*(-1/(c*d^8))^(1/3) - 3*(72*(16/27)^(1/6)*c*d^3*x^5*(-1/(c*d^8))
^(1/6) + 9*(16/27)^(5/6)*(c*d^9*x^7 - 16*c^2*d^8*x^4 - 8*c^3*d^7*x)*(-1/(c
*d^8))^(5/6) + 8*sqrt(1/3)*(5*c*d^6*x^6 - 20*c^2*d^5*x^3 - 16*c^3*d^4)*sqr
t(-1/(c*d^8)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 6
4*c^3)) + (16/27)^(1/6)*(sqrt(-3)*d^2 + d^2)*(-1/(c*d^8))^(1/6)*log((8*d^3
*x^9 - 528*c*d^2*x^6 - 576*c^2*d*x^3 - 256*c^3 + 24*2^(2/3)*(c*d^8*x^8 - 7
*c^2*d^7*x^5 - 8*c^3*d^6*x^2 + sqrt(-3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*c^3
*d^6*x^2))*(-1/(c*d^8))^(2/3) + 96*2^(1/3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*c^
3*d^3*x - sqrt(-3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*c^3*d^3*x))*(-1/(c*d^8))^(
1/3) + 3*sqrt(d*x^3 + c)*(9*(16/27)^(5/6)*(c*d^9*x^7 - 16*c^2*d^8*x^4 -...
```

3.278.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^3}{\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

input
```
integrate(x**3/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

3.278.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^3}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

3.278.8 Giac [F]

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^3}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^3}{\sqrt{d\,x^3+c}\,(d\,x^3+4\,c)}\,dx$$

input
```
int(x^3/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
int(x^3/((c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

3.279 $\int \frac{1}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

3.279.1 Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \frac{1}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}$$

output
```
1/4*x*AppellF1(1/3,1/2,1,4/3,-d*x^3/c,-1/4*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/(d
*x^3+c)^(1/2)
```

3.279.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(64) = 128.

Time = 10.05 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.58

$$\int \frac{1}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$$

$$=\frac{16cx\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{\sqrt{c+dx^3}\left(4c+dx^3\right)\Big(16c\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-3dx^3\Big(\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+2}$$

input
```
Integrate[1/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(16*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(Sqrt[c
+ d*x^3]*(4*c + d*x^3)*(16*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -1/4
*(d*x^3)/c] - 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -1/4*(d*x^
3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c])))
```

3.279.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$
$$\Big\downarrow \text{937}$$
$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(dx^3+4c)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$
$$\Big\downarrow \text{936}$$
$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}$$

input
```
Int[1/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, -1/4*(d*x^3)/c, -((d*x^3
)/c)])/(4*c*Sqrt[c + d*x^3])
```

3.279.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.279.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.33 (sec) , antiderivative size = 416, normalized size of antiderivative = 6.50

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\right.}{}\right.}{2\left(-\right.}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\right.}{}\right.}{2\left(-\right.}}}{}\right.}{}$

input
```
int(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

3.279. $\int \frac{1}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

output
```
-1/9*I/d^3/c*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c
*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d
*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/
3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

3.279.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2347 vs. $2(50) = 100$.

Time = 0.72 (sec) , antiderivative size = 2347, normalized size of antiderivative = 36.67

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output

```
-1/72*(2*(1/108)^(1/6)*c*d*(-1/(c^7*d^2))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^
6 - 72*c^2*d*x^3 - 32*c^3 - 24*(1/4)^(2/3)*(c^5*d^4*x^8 - 7*c^6*d^3*x^5 -
8*c^7*d^2*x^2)*(-1/(c^7*d^2))^(2/3) - 48*(1/4)^(1/3)*(c^3*d^3*x^7 - c^4*d^
2*x^4 - 2*c^5*d*x)*(-1/(c^7*d^2))^(1/3) + 6*(18*(1/108)^(1/6)*c^2*d^2*x^5*
(-1/(c^7*d^2))^(1/6) + 36*(1/108)^(5/6)*(c^6*d^4*x^7 - 16*c^7*d^3*x^4 - 8*
c^8*d^2*x)*(-1/(c^7*d^2))^(5/6) + sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2*x^
3 - 16*c^6*d)*sqrt(-1/(c^7*d^2)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6
 + 48*c^2*d*x^3 + 64*c^3)) - 2*(1/108)^(1/6)*c*d*(-1/(c^7*d^2))^(1/6)*log(
(d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 - 24*(1/4)^(2/3)*(c^5*d^4*
x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(-1/(c^7*d^2))^(2/3) - 48*(1/4)^(1/3)
*(c^3*d^3*x^7 - c^4*d^2*x^4 - 2*c^5*d*x)*(-1/(c^7*d^2))^(1/3) - 6*(18*(1/1
08)^(1/6)*c^2*d^2*x^5*(-1/(c^7*d^2))^(1/6) + 36*(1/108)^(5/6)*(c^6*d^4*x^7
 - 16*c^7*d^3*x^4 - 8*c^8*d^2*x)*(-1/(c^7*d^2))^(5/6) + sqrt(1/3)*(5*c^4*d
^3*x^6 - 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt(-1/(c^7*d^2)))*sqrt(d*x^3 + c))/(
d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (1/108)^(1/6)*(sqrt(-3)
*c*d + c*d)*(-1/(c^7*d^2))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^
3 - 32*c^3 + 12*(1/4)^(2/3)*(c^5*d^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2 +
 sqrt(-3)*(c^5*d^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2))*(-1/(c^7*d^2))^(2
/3) + 24*(1/4)^(1/3)*(c^3*d^3*x^7 - c^4*d^2*x^4 - 2*c^5*d*x - sqrt(-3)*(c^
3*d^3*x^7 - c^4*d^2*x^4 - 2*c^5*d*x))*(-1/(c^7*d^2))^(1/3) + 6*sqrt(d*x...
```

3.279.6 Sympy [F]

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{\sqrt{c+dx^3}\cdot (4c+dx^3)}\,dx$$

input

```
integrate(1/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

output

```
Integral(1/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

3.279.7 Maxima [F]

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

3.279.8 Giac [F]

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{1}{\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

input
```
int(1/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
int(1/((c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

3.280 $\int \frac{1}{x^3\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

3.280.1 Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{1}{x^3\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}$$

output
```
-1/8*AppellF1(-2/3,1/2,1,1/3,-d*x^3/c,-1/4*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/x^
2/(d*x^3+c)^(1/2)
```

3.280.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 243 vs. $2(66) = 132$.

Time = 11.18 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.68

$$\int \frac{1}{x^3\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$$
$$=\frac{-\frac{32(c+dx^3)}{c^2}-\frac{d^2x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{c^3}+\frac{2048dx^3\operatorname{AppellF1}\bigl(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},}{(4c+dx^3)\bigl(-16c\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+3dx^3\bigl(\operatorname{AppellF1}\bigl(\frac{4}{3},\frac{1}{2}}}{256x^2\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^3*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
((-32*(c + d*x^3))/c^2 - (d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1
, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/c^3 + (2048*d*x^3*AppellF1[1/3, 1/2,
 1, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/((4*c + d*x^3)*(-16*c*AppellF1[1/3
, 1/2, 1, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + 3*d*x^3*(AppellF1[4/3, 1/2,
 2, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d
*x^3)/c), -1/4*(d*x^3)/c]))))/(256*x^2*Sqrt[c + d*x^3])
```

3.280.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{c+dx^3}\left(4c+dx^3\right)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^3(dx^3+4c)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

output
```
-1/8*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -1/4*(d*x^3)/c, -((d
*x^3)/c)])/(c*x^2*Sqrt[c + d*x^3])
```

3.280.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.280.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.04 (sec) , antiderivative size = 716, normalized size of antiderivative = 10.85

method	result
elliptic	$-\frac{\sqrt{d\,x^3+c}}{8c^2x^2}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right.}{2d}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{24c^2\sqrt{d\,x^3+c}}$
risch	Expression too large to display
default	$\frac{-\frac{\sqrt{d\,x^3+c}}{2c\,x^2}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{6c\sqrt{d\,x^3+c}}}{4c}$

input
```
int(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

3.280. $\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

output
```
-1/8*(d*x^3+c)^(1/2)/c^2/x^2+1/24*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/36*I/d^2/c^2*2^(1/2)*sum(1/_alpha^2*(-
c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))
/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1
/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_al
pha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alph
a*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c
*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+
4*c))
```

3.280.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2381 vs. $2(52) = 104$.

Time = 1.42 (sec) , antiderivative size = 2381, normalized size of antiderivative = 36.08

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output

```
1/288*(2*(1/108)^(1/6)*c^2*x^2*(-d^4/c^13)^(1/6)*log((d^6*x^9 - 66*c*d^5*x
^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 - 24*(1/4)^(2/3)*(c^9*d^3*x^8 - 7*c^10*d^
2*x^5 - 8*c^11*d*x^2)*(-d^4/c^13)^(2/3) - 48*(1/4)^(1/3)*(c^5*d^4*x^7 - c^
6*d^3*x^4 - 2*c^7*d^2*x)*(-d^4/c^13)^(1/3) + 6*(18*(1/108)^(1/6)*c^3*d^4*x
^5*(-d^4/c^13)^(1/6) + 36*(1/108)^(5/6)*(c^11*d^2*x^7 - 16*c^12*d*x^4 - 8*
c^13*x)*(-d^4/c^13)^(5/6) + sqrt(1/3)*(5*c^7*d^3*x^6 - 20*c^8*d^2*x^3 - 16
*c^9*d)*sqrt(-d^4/c^13))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2
*d*x^3 + 64*c^3)) - 2*(1/108)^(1/6)*c^2*x^2*(-d^4/c^13)^(1/6)*log((d^6*x^9
 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 - 24*(1/4)^(2/3)*(c^9*d^3*x^
8 - 7*c^10*d^2*x^5 - 8*c^11*d*x^2)*(-d^4/c^13)^(2/3) - 48*(1/4)^(1/3)*(c^5
*d^4*x^7 - c^6*d^3*x^4 - 2*c^7*d^2*x)*(-d^4/c^13)^(1/3) - 6*(18*(1/108)^(1
/6)*c^3*d^4*x^5*(-d^4/c^13)^(1/6) + 36*(1/108)^(5/6)*(c^11*d^2*x^7 - 16*c^
12*d*x^4 - 8*c^13*x)*(-d^4/c^13)^(5/6) + sqrt(1/3)*(5*c^7*d^3*x^6 - 20*c^8
*d^2*x^3 - 16*c^9*d)*sqrt(-d^4/c^13))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2
*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 60*sqrt(d)*x^2*weierstrassPInverse(0, -4*
c/d, x) + (1/108)^(1/6)*(sqrt(-3)*c^2*x^2 + c^2*x^2)*(-d^4/c^13)^(1/6)*log
((d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 + 12*(1/4)^(2/3)*(c
^9*d^3*x^8 - 7*c^10*d^2*x^5 - 8*c^11*d*x^2 + sqrt(-3)*(c^9*d^3*x^8 - 7*c^1
0*d^2*x^5 - 8*c^11*d*x^2))*(-d^4/c^13)^(2/3) + 24*(1/4)^(1/3)*(c^5*d^4*x^7
 - c^6*d^3*x^4 - 2*c^7*d^2*x - sqrt(-3)*(c^5*d^4*x^7 - c^6*d^3*x^4 - 2*...
```

3.280.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{x^3\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

input

```
integrate(1/x**3/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

output

```
Integral(1/(x**3*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

3.280.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x^3}\,dx$$

input
```
integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^3), x)
```

3.280.8 Giac [F]

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x^3}\,dx$$

input
```
integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^3), x)
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{x^3\,\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

input
```
int(1/(x^3*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

output
```
int(1/(x^3*(c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

3.281 $\int \frac{x}{\sqrt{1-x^3}(4-x^3)}\,dx$

3.281.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\operatorname{arctanh}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

output

```
-1/6*arctanh((1+2^(1/3)*x)/(-x^3+1)^(1/2))*2^(1/3)+1/18*arctanh((-x^3+1)^(
1/2))*2^(1/3)-1/18*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*2^(1/3)*3^
(1/2)+1/18*arctan(1/3*(-x^3+1)^(1/2)*3^(1/2))*2^(1/3)*3^(1/2)
```

3.281.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.22

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \frac{1}{8}x^2\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{4}\right)$$

input

```
Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

output
```
(x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8
```

3.281.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {986}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx$$

$$\Big\downarrow \textcolor{blue}{986}$$

$$-\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\text{arctanh}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\text{arctanh}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

input
```
Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

output
```
-1/3*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(2^(2/3)*Sqrt[3]) + A
rcTan[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)
/Sqrt[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

3.281.3.1 Defintions of rubi rules used

rule 986
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

3.281.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 13.97 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.29

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{3}-4\right)}\frac{\_\alpha^{2}\sqrt{2}\,\sqrt{i\left(-i\sqrt{3}+2x+1\right)}\,\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\,\sqrt{-\frac{i\left(i\sqrt{3}+2x+1\right)}{2}}\,\left(-2\_\alpha^{2}+\_\alpha+1+i\sqrt{3}\,(1-\_\alpha)\right)\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right.}{2\sqrt{-x^{3}+1}}\right.}{36}$
elliptic	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{3}-4\right)}\frac{\_\alpha^{2}\sqrt{2}\,\sqrt{i\left(-i\sqrt{3}+2x+1\right)}\,\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\,\sqrt{-\frac{i\left(i\sqrt{3}+2x+1\right)}{2}}\,\left(-2\_\alpha^{2}+\_\alpha+1+i\sqrt{3}\,(1-\_\alpha)\right)\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right.}{2\sqrt{-x^{3}+1}}\right.}{36}$
trager	Expression too large to display

input
```
int(x/(-x^3+4)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(-I*3^(1/2)+2*x+1))^(1/2)*((-1+x)/(I*3^
(1/2)-3))^(1/2)*(-1/2*I*(I*3^(1/2)+2*x+1))^(1/2)/(-x^3+1)^(1/2)*(-2*_alpha
^2+_alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),1/2*_alpha-1/3*I*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*
3^(1/2)+1/6*I*3^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootO
f(_Z^3-4))
```

3.281.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 1116, normalized size of antiderivative = 8.79

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output

```
-1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log(-(72*x^9 + 4752*x^6 - 518
4*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2 + sqrt(-3)*(x^8 + 7*x^
5 - 8*x^2)) - 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 - sqrt(-3)*(x^7 + x^4 - 2*
x) - 2*x) + sqrt(-x^3 + 1)*(432^(5/6)*(-1)^(1/6)*(x^7 + 16*x^4 - sqrt(-3)*
(x^7 + 16*x^4 - 8*x) - 8*x) + 648*432^(1/6)*(-1)^(5/6)*(sqrt(-3)*x^5 + x^5
) + 144*sqrt(3)*(5*I*x^6 + 20*I*x^3 - 16*I)) + 2304)/(x^9 - 12*x^6 + 48*x^
3 - 64)) + 1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log(-(72*x^9 + 4752
*x^6 - 5184*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2 + sqrt(-3)*(
x^8 + 7*x^5 - 8*x^2)) - 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 - sqrt(-3)*(x^7
+ x^4 - 2*x) - 2*x) - sqrt(-x^3 + 1)*(432^(5/6)*(-1)^(1/6)*(x^7 + 16*x^4 -
 sqrt(-3)*(x^7 + 16*x^4 - 8*x) - 8*x) + 648*432^(1/6)*(-1)^(5/6)*(sqrt(-3)
*x^5 + x^5) - 144*sqrt(3)*(-5*I*x^6 - 20*I*x^3 + 16*I)) + 2304)/(x^9 - 12*
x^6 + 48*x^3 - 64)) + 1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*log(-(72
*x^9 + 4752*x^6 - 5184*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2 -
 sqrt(-3)*(x^8 + 7*x^5 - 8*x^2)) - 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 + sqr
t(-3)*(x^7 + x^4 - 2*x) - 2*x) + sqrt(-x^3 + 1)*(432^(5/6)*(-1)^(1/6)*(x^7
 + 16*x^4 + sqrt(-3)*(x^7 + 16*x^4 - 8*x) - 8*x) - 648*432^(1/6)*(-1)^(5/6
)*(sqrt(-3)*x^5 - x^5) + 144*sqrt(3)*(5*I*x^6 + 20*I*x^3 - 16*I)) + 2304)/
(x^9 - 12*x^6 + 48*x^3 - 64)) - 1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1
)*log(-(72*x^9 + 4752*x^6 - 5184*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*...
```

3.281.6 Sympy [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = -\int \frac{x}{x^3\sqrt{1-x^3}-4\sqrt{1-x^3}}\,dx$$

input

```
integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)
```

output

```
-Integral(x/(x**3*sqrt(1 - x**3) - 4*sqrt(1 - x**3)), x)
```

3.281.7 Maxima [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \int -\frac{x}{(x^3-4)\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

3.281.8 Giac [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \int -\frac{x}{(x^3-4)\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

3.281.9 Mupad [B] (verification not implemented)

Time = 7.16 (sec) , antiderivative size = 653, normalized size of antiderivative = 5.14

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \text{Too large to display}$$

input
```
int(-x/((1 - x^3)^(1/2)*(x^3 - 4)),x)
```

output

```
- (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 +
1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3
^(1/2)*1i)/2 + 3/2)/(2^(2/3) - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2
)*(2^(2/3) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^
(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3
/2)/(2^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/
2)*1i)/2 + 1/2)*(1 - x^3)^(1/2)*(2^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 1)*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x
^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*(
(x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(2/3)*((3^(1/
2)*1i)/2 - 1/2) - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 - 1/2)*(...
```

3.281. $\int \frac{x}{\sqrt{1-x^3}(4-x^3)} \, dx$

3.282 $\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx$

3.282.1 Optimal result

Integrand size = 27, antiderivative size = 111

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{1024c^3\sqrt{c+dx^3}}{3d^4} - \frac{38c^2(c+dx^3)^{3/2}}{3d^4} - \frac{4c(c+dx^3)^{5/2}}{5d^4} - \frac{2(c+dx^3)^{7/2}}{21d^4} + \frac{1024c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

output
```
-38/3*c^2*(d*x^3+c)^(3/2)/d^4-4/5*c*(d*x^3+c)^(5/2)/d^4-2/21*(d*x^3+c)^(7/
2)/d^4+1024*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4-1024/3*c^3*(d
*x^3+c)^(1/2)/d^4
```

3.282.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}(18632c^3+764c^2dx^3+57cd^2x^6+5d^3x^9)}{105d^4} + \frac{1024c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

input
```
Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(18632*c^3 + 764*c^2*d*x^3 + 57*c*d^2*x^6 + 5*d^3*x^9)
)/(105*d^4) + (1024*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

3.282.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx$$
$$\Big\downarrow \text{948}$$
$$\frac{1}{3}\int \frac{x^9\sqrt{dx^3+c}}{8c-dx^3}dx^3$$
$$\Big\downarrow \text{99}$$
$$\frac{1}{3}\int \left(\frac{512\sqrt{dx^3+c}c^3}{d^3\left(8c-dx^3\right)}-\frac{57\sqrt{dx^3+c}c^2}{d^3}-\frac{6\left(dx^3+c\right)^{3/2}c}{d^3}-\frac{\left(dx^3+c\right)^{5/2}}{d^3}\right)dx^3$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}\left(\frac{3072c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{1024c^3\sqrt{c+dx^3}}{d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{d^4}-\frac{12c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{7d^4}\right)$$

input
```
Int[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
((-1024*c^3*Sqrt[c + d*x^3])/d^4 - (38*c^2*(c + d*x^3)^(3/2))/d^4 - (12*c*
(c + d*x^3)^(5/2))/(5*d^4) - (2*(c + d*x^3)^(7/2))/(7*d^4) + (3072*c^(7/2)
*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4)/3
```

3.282.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.282.4 Maple [A] (verified)

Time = 3.65 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{1024c^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)-\frac{2\left(5d^3x^9+57c\,d^2x^6+764c^2d\,x^3+18632c^3\right)\sqrt{d\,x^3+c}}{105}}{d^4}$
risch	$-\frac{2(5d^3x^9+57c\,d^2x^6+764c^2d\,x^3+18632c^3)\sqrt{d\,x^3+c}}{105d^4}+\frac{1024c^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^4}$
default	$-\frac{\frac{2x^9\sqrt{d\,x^3+c}}{21}+\frac{2c\,x^6\sqrt{d\,x^3+c}}{105d}-\frac{8c^2x^3\sqrt{d\,x^3+c}}{315d^2}+\frac{16c^3\sqrt{d\,x^3+c}}{315d^3}}{d}-\frac{8c\left(\frac{2x^6\sqrt{d\,x^3+c}}{15}+\frac{2c\,x^3\sqrt{d\,x^3+c}}{45d}-\frac{4c^2\sqrt{d\,x^3+c}}{45d^2}\right)}{d^2}-$ $128c^2($
elliptic	$-\frac{2x^9\sqrt{d\,x^3+c}}{21d}-\frac{38c\,x^6\sqrt{d\,x^3+c}}{35d^2}-\frac{1528c^2x^3\sqrt{d\,x^3+c}}{105d^3}-\frac{37264c^3\sqrt{d\,x^3+c}}{105d^4}-$ $512ic^3\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\right.$ $(-$

input
```
int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
2/105*(53760*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-(5*d^3*x^9+57*c*
d^2*x^6+764*c^2*d*x^3+18632*c^3)*(d*x^3+c)^(1/2))/d^4
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.52

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx$$
$$= \left[\frac{2\left(26880\,c^{\frac{7}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(5\,d^3x^9+57\,cd^2x^6+764\,c^2dx^3+18632\,c^3)\sqrt{dx^3+c}\right)}{105\,d^4},\right.$$
$$\left.-\frac{2\left(53760\sqrt{-c}c^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(5\,d^3x^9+57\,cd^2x^6+764\,c^2dx^3+18632\,c^3)\sqrt{dx^3+c}\right)}{105\,d^4}\right]$$

input
```
integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[2/105*(26880*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^
3 - 8*c)) - (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*
x^3 + c))/d^4, -2/105*(53760*sqrt(-c)*c^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(
-c)/c) + (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*x^3
 + c))/d^4]
```

3.282.6 Sympy [A] (verification not implemented)

Time = 18.48 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx$$
$$= \begin{cases} \frac{2\left(-\frac{512c^4\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{512c^3\sqrt{c+dx^3}}{3}-\frac{19c^2\left(c+dx^3\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+dx^3\right)^{\frac{5}{2}}}{5}-\frac{\left(c+dx^3\right)^{\frac{7}{2}}}{21}\right)}{d^4} & \text{for } d\neq 0 \\ \frac{x^{12}}{96\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-512*c**4*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 512
*c**3*sqrt(c + d*x**3)/3 - 19*c**2*(c + d*x**3)**(3/2)/3 - 2*c*(c + d*x**3
)**(5/2)/5 - (c + d*x**3)**(7/2)/21)/d**4, Ne(d, 0)), (x**12/(96*sqrt(c)),
 True))
```

3.282.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\left(26880\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+5\,(dx^3+c)^{\frac{7}{2}}+42\,(dx^3+c)^{\frac{5}{2}}c+665\,(dx^3+c)^{\frac{3}{2}}c^2+17920\,\sqrt{dx^3+c}c^3\right)}{105\,d^4}$$

input
```
integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-2/105*(26880*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) +
 3*sqrt(c))) + 5*(d*x^3 + c)^(7/2) + 42*(d*x^3 + c)^(5/2)*c + 665*(d*x^3 +
 c)^(3/2)*c^2 + 17920*sqrt(d*x^3 + c)*c^3)/d^4
```

3.282.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{1024\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^4} - \frac{2\left(5\,(dx^3+c)^{\frac{7}{2}}d^{24}+42\,(dx^3+c)^{\frac{5}{2}}cd^{24}+665\,(dx^3+c)^{\frac{3}{2}}c^2d^{24}+17920\,\sqrt{dx^3+c}c^3d^{24}\right)}{105\,d^{28}}$$

input
```
integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-1024*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/105*(5*(
d*x^3 + c)^(7/2)*d^24 + 42*(d*x^3 + c)^(5/2)*c*d^24 + 665*(d*x^3 + c)^(3/2
)*c^2*d^24 + 17920*sqrt(d*x^3 + c)*c^3*d^24)/d^28
```

3.282.9 Mupad [B] (verification not implemented)

Time = 7.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{512\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4} - \frac{37264\,c^3\,\sqrt{d\,x^3+c}}{105\,d^4} - \frac{2\,x^9\,\sqrt{d\,x^3+c}}{21\,d} - \frac{38\,c\,x^6\,\sqrt{d\,x^3+c}}{35\,d^2} - \frac{1528\,c^2\,x^3\,\sqrt{d\,x^3+c}}{105\,d^3}$$

input

```
int((x^11*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

output

```
(512*c^(7/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/d^4 - (37264*c^3*(c + d*x^3)^(1/2))/(105*d^4) - (2*x^9*(c + d*x^3)^(1/
2))/(21*d) - (38*c*x^6*(c + d*x^3)^(1/2))/(35*d^2) - (1528*c^2*x^3*(c + d*
x^3)^(1/2))/(105*d^3)
```

3.283 $\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx$

3.283.1 Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c(c+dx^3)^{3/2}}{9d^3} -\frac{2(c+dx^3)^{5/2}}{15d^3}+\frac{128c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

output
```
-14/9*c*(d*x^3+c)^(3/2)/d^3-2/15*(d*x^3+c)^(5/2)/d^3+128*c^(5/2)*arctanh(1
/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3-128/3*c^2*(d*x^3+c)^(1/2)/d^3
```

3.283.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.79

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}(998c^2+41cdx^3+3d^2x^6)}{45d^3}+\frac{128c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

input
```
Integrate[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(998*c^2 + 41*c*d*x^3 + 3*d^2*x^6))/(45*d^3) + (128*c^
(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

3.283.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6\sqrt{dx^3+c}}{8c-dx^3}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{64\sqrt{dx^3+c}c^2}{d^2\left(8c-dx^3\right)}-\frac{7\sqrt{dx^3+c}c}{d^2}-\frac{\left(dx^3+c\right)^{3/2}}{d^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{384c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{128c^2\sqrt{c+dx^3}}{d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{3d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{5d^3}\right)$$

input
```
Int[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
((-128*c^2*Sqrt[c + d*x^3])/d^3 - (14*c*(c + d*x^3)^(3/2))/(3*d^3) - (2*(c
 + d*x^3)^(5/2))/(5*d^3) + (384*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)])/d^3)/3
```

3.283.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.283.4 Maple [A] (verified)

Time = 4.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{\frac{2\left(-3d^{2}x^{6}-41cd\,x^{3}-998c^{2}\right)\sqrt{d\,x^{3}+c}}{45}+128c^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{d^{3}}$
risch	$-\frac{2\left(3d^{2}x^{6}+41cd\,x^{3}+998c^{2}\right)\sqrt{d\,x^{3}+c}}{45d^{3}}+\frac{128c^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{d^{3}}$
default	$-\frac{\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15}+\frac{2c\,x^{3}\sqrt{d\,x^{3}+c}}{45d}-\frac{4c^{2}\sqrt{d\,x^{3}+c}}{45d^{2}}}{d}-\frac{16c\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9d^{3}}+\frac{64c^{2}\left(-2\sqrt{d\,x^{3}+c}+6\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}\right)}{3d^{3}}$
elliptic	$-\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15d}-\frac{82c\,x^{3}\sqrt{d\,x^{3}+c}}{45d^{2}}-\frac{1996c^{2}\sqrt{d\,x^{3}+c}}{45d^{3}}-$ $64ic^{2}\sqrt{2}$ $\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}$ $\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}$ $id\left(2x+\frac{-i\sqrt{3}\,(}{}\right.$

input
```
int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
2/45*((-3*d^2*x^6-41*c*d*x^3-998*c^2)*(d*x^3+c)^(1/2)+2880*c^(5/2)*arctanh
(1/3*(d*x^3+c)^(1/2)/c^(1/2)))/d^3
```

3.283.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.63

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = \left[\frac{2\left(1440\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(3\,d^2x^6+41\,cdx^3+998\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^3},\right.$$
$$\left.-\frac{2\left(2880\,\sqrt{-c}c^2\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+\left(3\,d^2x^6+41\,cdx^3+998\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^3}\right]$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[2/45*(1440*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
- 8*c)) - (3*d^2*x^6 + 41*c*d*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3, -2/45*(
2880*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^2*x^6 + 41
*c*d*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3]
```

3.283.6 Sympy [A] (verification not implemented)

Time = 8.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{64c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{64c^2\sqrt{c+dx^3}}{3}-\frac{7c\left(c+dx^3\right)^{\frac{3}{2}}}{9}-\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{x^9}{72\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-64*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 64*c
**2*sqrt(c + d*x**3)/3 - 7*c*(c + d*x**3)**(3/2)/9 - (c + d*x**3)**(5/2)/1
5)/d**3, Ne(d, 0)), (x**9/(72*sqrt(c)), True))
```

3.283.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\left(1440\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+3\,(dx^3+c)^{\frac{5}{2}}+35\,(dx^3+c)^{\frac{3}{2}}c+960\,\sqrt{dx^3+c}c^2\right)}{45\,d^3}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-2/45*(1440*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3
*sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 35*(d*x^3 + c)^(3/2)*c + 960*sqrt(d*x^3
 + c)*c^2)/d^3
```

3.283.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{128\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d^3} - \frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}d^{12}+35\,(dx^3+c)^{\frac{3}{2}}cd^{12}+960\,\sqrt{dx^3+c}c^2d^{12}\right)}{45\,d^{15}}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-128*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/45*(3*(d*
x^3 + c)^(5/2)*d^12 + 35*(d*x^3 + c)^(3/2)*c*d^12 + 960*sqrt(d*x^3 + c)*c^
2*d^12)/d^15
```

3.283.9 Mupad [B] (verification not implemented)

Time = 7.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{64\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3} - \frac{1996\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3} - \frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d} - \frac{82\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2}$$

input

```
int((x^8*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

output

```
(64*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^3 - (1996*c^2*(c + d*x^3)^(1/2))/(45*d^3) - (2*x^6*(c + d*x^3)^(1/2))
/(15*d) - (82*c*x^3*(c + d*x^3)^(1/2))/(45*d^2)
```

3.284 $\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx$

3.284.1 Optimal result

Integrand size = 27, antiderivative size = 69

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx=-\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2(c+dx^3)^{3/2}}{9d^2}+\frac{16c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

output
```
-2/9*(d*x^3+c)^(3/2)/d^2+16*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d
^2-16/3*c*(d*x^3+c)^(1/2)/d^2
```

3.284.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx=-\frac{2\sqrt{c+dx^3}(25c+dx^3)}{9d^2}+\frac{16c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

input
```
Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(25*c + d*x^3))/(9*d^2) + (16*c^(3/2)*ArcTanh[Sqrt[c +
 d*x^3]/(3*Sqrt[c])])/d^2
```

3.284.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {948, 90, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3\sqrt{dx^3+c}}{8c-dx^3}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{8c\int \frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3}{d}-\frac{2\left(c+dx^3\right)^{3/2}}{3d^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{8c\left(9c\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3-\frac{2\sqrt{c+dx^3}}{d}\right)}{d}-\frac{2\left(c+dx^3\right)^{3/2}}{3d^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{8c\left(\frac{18c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)}{d}-\frac{2\left(c+dx^3\right)^{3/2}}{3d^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{8c\left(\frac{6\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)}{d}-\frac{2\left(c+dx^3\right)^{3/2}}{3d^2}\right)$$

input
```
Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output

```
((-2*(c + d*x^3)^(3/2))/(3*d^2) + (8*c*((-2*Sqrt[c + d*x^3])/d + (6*Sqrt[c
]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d))/d)/3
```

3.284.3.1 Defintions of rubi rules used

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.284.4 Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{16c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-\frac{2\left(d\,x^{3}+25c\right)\sqrt{d\,x^{3}+c}}{9}}{d^{2}}$
risch	$-\frac{2\left(d\,x^{3}+25c\right)\sqrt{d\,x^{3}+c}}{9d^{2}}+\frac{16c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{d^{2}}$
default	$-\frac{2\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9d^{2}}+\frac{8c\left(-2\sqrt{d\,x^{3}+c}+6\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}\right)}{3d^{2}}$
elliptic	$-\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d}-\frac{50c\sqrt{d\,x^{3}+c}}{9d^{2}}-\frac{8ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$

input
```
int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
2/9*(72*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-(d*x^3+25*c)*(d*x^3+c
)^(1/2))/d^2
```

3.284.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.75

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx = \left[\frac{2\left(36\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(dx^3+25\,c)\sqrt{dx^3+c}\right)}{9\,d^2},\right.$$
$$\left.-\frac{2\left(72\,\sqrt{-c}c\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(dx^3+25\,c)\sqrt{dx^3+c}\right)}{9\,d^2}\right]$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[2/9*(36*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8
*c)) - (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2, -2/9*(72*sqrt(-c)*c*arctan(1/3
*sqrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2]
```

3.284.6 Sympy [A] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx=\begin{cases}\frac{2\left(-\frac{8c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{8c\sqrt{c+dx^3}}{3}-\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^2} & \text{for } d\neq 0\\ \frac{x^6}{48\sqrt{c}} & \text{otherwise}\end{cases}$$

input
```
integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-8*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 8*c*s
qrt(c + d*x**3)/3 - (c + d*x**3)**(3/2)/9)/d**2, Ne(d, 0)), (x**6/(48*sqrt
(c)), True))
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx=-\frac{2\left(36\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{3}{2}}+24\sqrt{dx^3+c}c\right)}{9\,d^2}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-2/9*(36*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + (d*x^3 + c)^(3/2) + 24*sqrt(d*x^3 + c)*c)/d^2
```

3.284.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{16\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d^2} - \frac{2\left((dx^3+c)^{\frac{3}{2}}d^4+24\,\sqrt{dx^3+c}cd^4\right)}{9\,d^6}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-16*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 2/9*((d*x^3
+ c)^(3/2)*d^4 + 24*sqrt(d*x^3 + c)*c*d^4)/d^6
```

3.284.9 Mupad [B] (verification not implemented)

Time = 7.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{8\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2} - \frac{50\,c\,\sqrt{d\,x^3+c}}{9\,d^2} - \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d}$$

input
```
int((x^5*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

output
```
(8*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/d^2 - (50*c*(c + d*x^3)^(1/2))/(9*d^2) - (2*x^3*(c + d*x^3)^(1/2))/(9*d)
```

3.285 $\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx$

3.285.1 Optimal result

Integrand size = 27, antiderivative size = 50

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}}{3d}+\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

output
```
2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d-2/3*(d*x^3+c)^(1/2)/d
```

3.285.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\left(\sqrt{c+dx^3}-3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{3d}$$

input
```
Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(-2*(Sqrt[c + d*x^3] - 3*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(3
*d)
```

3.285.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {946, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(9c\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3-\frac{2\sqrt{c+dx^3}}{d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{18c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{6\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)$$

input
```
Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
((-2*Sqrt[c + d*x^3])/d + (6*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/d)/3
```

3.285.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.285.4 Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

method	result
default	$\frac{-2\sqrt{d\,x^3+c}+6\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{c}}{3d}$
pseudoelliptic	$\frac{-2\sqrt{d\,x^3+c}+6\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{c}}{3d}$
risch	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{c}}{d}-\frac{2\sqrt{d\,x^3+c}}{3d}$
elliptic	$-\frac{2\sqrt{d\,x^3+c}}{3d}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right.}}}{}\right.}{}$

input
```
int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(-2*(d*x^3+c)^(1/2)+6*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.02

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx=\left[\frac{3\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-2\sqrt{dx^3+c}}{3\,d},\ -\frac{2\left(3\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+\sqrt{dx^3+c}\right)}{3\,d}\right]$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[1/3*(3*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - 2*sqrt(d*x^3 + c))/d, -2/3*(3*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sq
rt(-c)/c) + sqrt(d*x^3 + c))/d]
```

3.285.6 Sympy [A] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3}\right)}{d} & \text{for } d\neq 0 \\ \frac{x^3}{24\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - sqrt(c + d
*x**3)/3)/d, Ne(d, 0)), (x**3/(24*sqrt(c)), True))
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{3\,\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+2\,\sqrt{dx^3+c}}{3\,d}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 2*sqrt(d*x^3 + c))/d
```

3.285.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d} - \frac{2\sqrt{dx^3+c}}{3\,d}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-2*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 2/3*sqrt(d*x^3 +
c)/d
```

3.285.9 Mupad [B] (verification not implemented)

Time = 7.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d} - \frac{2\sqrt{d\,x^3+c}}{3\,d}$$

input
```
int((x^2*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

output
```
(c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/
d - (2*(c + d*x^3)^(1/2))/(3*d)
```

3.286 $\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)}\,dx$

3.286.1 Optimal result

Integrand size = 27, antiderivative size = 58

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

output
```
1/4*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/12*arctanh((d*x^3+c)^(1
/2)/c^(1/2))/c^(1/2)
```

3.286.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)}\,dx = \frac{3\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
```

output
```
(3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]
)/(12*Sqrt[c])
```

3.286.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {948, 94, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^3\left(8c-dx^3\right)}dx^3$$

$$\Big\downarrow \text{94}$$

$$\frac{1}{3}\left(\frac{1}{8}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{9}{8}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{9}{4}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4d}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4d}+\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{4\sqrt{c}}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
```

output
```
((3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(4*Sqrt[c]) - ArcTanh[Sqrt[c + d
*x^3]/Sqrt[c]]/(4*Sqrt[c]))/3
```

3.286.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.286.4 Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$-\frac{-3 \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{12 \sqrt{c}}$	38
default	$\frac{\frac{2 \sqrt{d\,x^{3}+c}}{3}-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right) \sqrt{c}}{3}}{8 c}+\frac{-2 \sqrt{d\,x^{3}+c}+6 \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right) \sqrt{c}}{24 c}$	75
elliptic	Expression too large to display	1502

input
```
int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+arctanh((d*x^3+c)^(1/2)/c^(
1/2)))/c^(1/2)
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{c+dx^3}}{x\,(8c-dx^3)}\,dx = \left[\frac{3\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{24\,c}, \frac{\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-3\sqrt{-c}\arctan\left(\right.}{12\,c}\right.$$

input
```
integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8
*c)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c, 1/12
*(sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(-c)*arctan(1/3*sqrt
(d*x^3 + c)*sqrt(-c)/c))/c]
```

3.286.6 Sympy [A] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{c+dx^3}}{x\,(8c-dx^3)}\,dx = \begin{cases} \frac{2\left(-\frac{d\,\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{8\sqrt{-c}}+\frac{d\,\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24\sqrt{-c}}\right)}{d} & \text{for}\ d\neq 0 \\ \frac{\log\left(x^3\right)}{24\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-d*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(8*sqrt(-c)) + d*atan
(sqrt(c + d*x**3)/sqrt(-c))/(24*sqrt(-c)))/d, Ne(d, 0)), (log(x**3)/(24*sq
rt(c)), True))
```

3.286.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)}\,dx=\int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x), x)
```

3.286.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)}\,dx=\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}}-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{4\sqrt{-c}}$$

input
```
integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 1/4*arctan(1/3*sqrt(d*x^3
 + c)/sqrt(-c))/sqrt(-c)
```

3.286.9 Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)}\,dx =\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\,\left(6\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}\right)^3\,\left(24\,c^2-24\,c^{3/2}\,\sqrt{d\,x^3+c}+d^2\,x^6-20\,c\,d\,x^3\right)^3}{x^{15}\,\left(8\,c-d\,x^3\right)^3\,\left(24\,c-d\,x^3\right)^3}\right)}{24\,\sqrt{c}}$$

input
```
int((c + d*x^3)^(1/2)/(x*(8*c - d*x^3)),x)
```

output
```
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))*(6*c +
d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))^3*(24*c^2 - 24*c^(3/2)*(c + d*x^3)^(1
/2) + d^2*x^6 - 20*c*d*x^3)^3)/(x^15*(8*c - d*x^3)^3*(24*c - d*x^3)^3))/(2
4*c^(1/2))
```

3.287 $\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)}\,dx$

3.287.1 Optimal result

Integrand size = 27, antiderivative size = 81

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)}\,dx=-\frac{\sqrt{c+dx^3}}{24cx^3}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}}-\frac{5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

output
```
1/32*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-5/96*d*arctanh((d*x^3+
c)^(1/2)/c^(1/2))/c^(3/2)-1/24*(d*x^3+c)^(1/2)/c/x^3
```

3.287.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)}\,dx=-\frac{\sqrt{c+dx^3}}{24cx^3}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}}-\frac{5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]
```

output
```
-1/24*Sqrt[c + d*x^3]/(c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(
32*c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))
```

3.287.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {948, 110, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^4 (8c-dx^3)} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{\sqrt{dx^3+c}}{x^6 (8c-dx^3)} dx^3$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3} \left(\frac{\int \frac{d(dx^3+10c)}{2x^3(8c-dx^3)\sqrt{dx^3+c}} dx^3}{8c} - \frac{\sqrt{c+dx^3}}{8cx^3} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(\frac{d\int \frac{dx^3+10c}{x^3(8c-dx^3)\sqrt{dx^3+c}} dx^3}{16c} - \frac{\sqrt{c+dx^3}}{8cx^3} \right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3} \left(\frac{d\left(\frac{5}{4}\int \frac{1}{x^3\sqrt{dx^3+c}} dx^3 + \frac{9}{4} d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}} dx^3\right)}{16c} - \frac{\sqrt{c+dx^3}}{8cx^3} \right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3} \left(\frac{d\left(\frac{9}{2}\int \frac{1}{9c-x^6} d\sqrt{dx^3+c} + \frac{5\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}} d\sqrt{dx^3+c}}{2d}\right)}{16c} - \frac{\sqrt{c+dx^3}}{8cx^3} \right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{d\left(\frac{5\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{d\left(\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}-\frac{5\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]
```

output
```
(-1/8*Sqrt[c + d*x^3]/(c*x^3) + (d*((3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/(2*Sqrt[c]) - (5*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c])))/(16*c)
)/3
```

3.287.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.287.4 Maple [A] (verified)

Time = 4.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{\sqrt{d x^3+c}}{24 c x^3}+\frac{d\left(-\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)}{16c}$
pseudoelliptic	$\frac{-5\,\mathrm{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right) d x^3+3\,\mathrm{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right) d x^3-4\sqrt{d x^3+c}\,\sqrt{c}}{96 c^{\frac{3}{2}} x^3}$
default	$\frac{-\frac{\sqrt{d x^3+c}}{3x^3}-\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3\sqrt{c}}}{8c}+\frac{d\left(\frac{2\sqrt{d x^3+c}}{3}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)\sqrt{c}}{3}\right)}{64c^2}+\frac{d\left(-2\sqrt{d x^3+c}+6\,\mathrm{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)\right.}{192c^2}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(d*x^3+c)^(1/2)/c/x^3+1/16*d/c*(-5/6*arctanh((d*x^3+c)^(1/2)/c^(1/2)
)/c^(1/2)+1/2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.30

$$\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)}\,dx = \left[\frac{3\sqrt{c}dx^3\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+5\sqrt{c}dx^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-8\sqrt{dx^3+c}c}{192\,c^2x^3}, \frac{5\sqrt{-c}dx^3\arctan\left(\right.}{}\right.$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[1/192*(3*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*
x^3 - 8*c)) + 5*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c
)/x^3) - 8*sqrt(d*x^3 + c)*c)/(c^2*x^3), 1/96*(5*sqrt(-c)*d*x^3*arctan(sqr
t(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqr
t(-c)/c) - 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]
```

3.287.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)}\,dx = -\int \frac{\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c),x)
```

output
```
-Integral(sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x)
```

3.287.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^4}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^4), x)
```

3.287.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)}\,dx = \frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}c} - \frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{32\,\sqrt{-c}c} - \frac{\sqrt{dx^3+c}}{24\,cx^3}$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
5/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/32*d*arctan(1/3*s
qrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/24*sqrt(d*x^3 + c)/(c*x^3)
```

3.287.9 Mupad [B] (verification not implemented)

Time = 7.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)}\,dx = \frac{d\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{32\,\sqrt{c^3}} - \frac{5\,d\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{96\,\sqrt{c^3}} - \frac{\sqrt{d\,x^3+c}}{24\,c\,x^3}$$

input
```
int((c + d*x^3)^(1/2)/(x^4*(8*c - d*x^3)),x)
```

output
```
(d*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2))))/(32*(c^3)^(1/2)) - (5*d*a
tanh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2)))/(96*(c^3)^(1/2)) - (c + d*x^3)^(1
/2)/(24*c*x^3)
```

3.288 $\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx$

3.288.1 Optimal result

Integrand size = 27, antiderivative size = 107

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx=-\frac{\sqrt{c+dx^3}}{48cx^6}-\frac{d\sqrt{c+dx^3}}{64c^2x^3}+\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}}+\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}$$

output
```
1/256*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/256*d^2*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/48*(d*x^3+c)^(1/2)/c/x^6-1/64*d*(d*x^3+c
)^(1/2)/c^2/x^3
```

3.288.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx=\frac{\left(-4c-3dx^3\right)\sqrt{c+dx^3}}{192c^2x^6}+\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}}+\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]
```

output
```
((-4*c - 3*d*x^3)*Sqrt[c + d*x^3])/(192*c^2*x^6) + (d^2*ArcTanh[Sqrt[c + d
*x^3]/(3*Sqrt[c])])/(256*c^(5/2)) + (d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(256*c^(5/2))
```

3.288.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 110, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^9\left(8c-dx^3\right)}dx^3$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\left(\frac{\int \frac{3d\left(dx^3+4c\right)}{2x^6\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{16c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{3d\int \frac{dx^3+4c}{x^6\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(\frac{3d\left(-\frac{\int \frac{2cd\left(2c-dx^3\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{8c^2}-\frac{\sqrt{c+dx^3}}{2cx^3}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{3d\left(-\frac{d\int \frac{2c-dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{4c}-\frac{\sqrt{c+dx^3}}{2cx^3}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{3d\left(-\frac{d\left(\frac{1}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3-\frac{3}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3\right)}{4c}-\frac{\sqrt{c+dx^3}}{2cx^3}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{3}\left(\frac{3d\left(-\frac{d\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{3}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}\right)}{4c}-\frac{\sqrt{c+dx^3}}{2cx^3}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

$$\Big\downarrow \textbf{219}$$

$$\frac{1}{3}\left(\frac{3d\left(-\frac{d\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)}{4c}-\frac{\sqrt{c+dx^3}}{2cx^3}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

$$\Big\downarrow \textbf{221}$$

$$\frac{1}{3}\left(\frac{3d\left(-\frac{d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{4c}-\frac{\sqrt{c+dx^3}}{2cx^3}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6}\right)$$

3.288. $\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)}\,dx$

input
```
Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]
```

output
```
(-1/16*Sqrt[c + d*x^3]/(c*x^6) + (3*d*(-1/2*Sqrt[c + d*x^3]/(c*x^3) - (d*(
-1/2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/Sqrt[c] - ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]]/(2*Sqrt[c])))/(4*c)))/(32*c))/3
```

3.288.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.288.4 Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\sqrt{dx^3+c}\,(3dx^3+4c)}{192c^2x^6}-\frac{3d^2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{6\sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{6\sqrt{c}}\right)}{128c^2}$
pseudoelliptic	$\frac{3\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)d^2x^6+3\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)d^2x^6-12dx^3\sqrt{dx^3+c}\,\sqrt{c}-16\sqrt{dx^3+c}\,c^{\frac{3}{2}}}{768c^{\frac{5}{2}}x^6}$
default	$\frac{-\frac{\sqrt{dx^3+c}}{6x^6}-\frac{d\sqrt{dx^3+c}}{12c\,x^3}+\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{12c^{\frac{3}{2}}}}{8c}+\frac{d\left(-\frac{\sqrt{dx^3+c}}{3x^3}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3\sqrt{c}}\right)}{64c^2}+\frac{d^2\left(\frac{2\sqrt{dx^3+c}}{3}-\frac{2\operatorname{arctanh}\left(\sqrt{d}\right.}{3}\right.}{512c^3}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(d*x^3+c)^(1/2)*(3*d*x^3+4*c)/c^2/x^6-3/128*d^2/c^2*(-1/6*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/6*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c
^(1/2))
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)}\,dx = \left[\frac{3\sqrt{c}d^2x^6\log\left(\frac{d^2x^6+24\,cdx^3+8\,(dx^3+4\,c)\sqrt{dx^3+c}\sqrt{c}+32\,c^2}{dx^6-8\,cx^3}\right)-8\,(3\,cdx^3+4\,c^2)\sqrt{dx^3+c}}{1536\,c^3x^6}, -\frac{3\sqrt{-c}d^2x^6\arctan\left(\frac{(dx^3+4\,c)\sqrt{dx^3+c}\sqrt{-c}}{4\,(cdx^3+c^2)}\right)+4\,(3\,cdx^3+4\,c^2)\sqrt{dx^3+c}}{768\,c^3x^6}\right]$$

input
```
integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[1/1536*(3*sqrt(c)*d^2*x^6*log((d^2*x^6 + 24*c*d*x^3 + 8*(d*x^3 + 4*c)*sqr
t(d*x^3 + c)*sqrt(c) + 32*c^2)/(d*x^6 - 8*c*x^3)) - 8*(3*c*d*x^3 + 4*c^2)*
sqrt(d*x^3 + c))/(c^3*x^6), -1/768*(3*sqrt(-c)*d^2*x^6*arctan(1/4*(d*x^3 +
 4*c)*sqrt(d*x^3 + c)*sqrt(-c)/(c*d*x^3 + c^2)) + 4*(3*c*d*x^3 + 4*c^2)*sq
rt(d*x^3 + c))/(c^3*x^6)]
```

3.288.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)}\,dx = -\int \frac{\sqrt{c+dx^3}}{-8cx^7+dx^{10}}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c),x)
```

output
```
-Integral(sqrt(c + d*x**3)/(-8*c*x**7 + d*x**10), x)
```

3.288.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx=\int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^7}\,dx$$

input

```
integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
```

output

```
-integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^7), x)
```

3.288.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx=-\frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{256\,\sqrt{-c}c^2}-\frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{256\,\sqrt{-c}c^2}-\frac{3\,(dx^3+c)^{\frac{3}{2}}d^2+\sqrt{dx^3+c}cd^2}{192\,c^2d^2x^6}$$

input

```
integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")
```

output

```
-1/256*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/256*d^2*arc
tan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/192*(3*(d*x^3 + c)^(3
/2)*d^2 + sqrt(d*x^3 + c)*c*d^2)/(c^2*d^2*x^6)
```

3.288.9 Mupad [B] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx=\frac{d^2\,\mathrm{atanh}\left(\frac{d^4\,\sqrt{d\,x^3+c}}{2048\,c^{7/2}\,\left(\frac{d^4}{2048\,c^3}+\frac{d^5\,x^3}{8192\,c^4}\right)}\right)}{256\,c^{5/2}}-\frac{\sqrt{d\,x^3+c}}{192\,c\,x^6}-\frac{{\left(d\,x^3+c\right)}^{3/2}}{64\,c^2\,x^6}$$

input

```
int((c + d*x^3)^(1/2)/(x^7*(8*c - d*x^3)),x)
```

output

```
(d^2*atanh((d^4*(c + d*x^3)^(1/2))/(2048*c^(7/2)*(d^4/(2048*c^3) + (d^5*x^
3)/(8192*c^4)))))/(256*c^(5/2)) - (c + d*x^3)^(1/2)/(192*c*x^6) - (c + d*x
^3)^(3/2)/(64*c^2*x^6)
```

3.289 $\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx$

3.289.1 Optimal result

Integrand size = 27, antiderivative size = 648

$$\begin{aligned}
\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx &= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d} \\
&\quad -\frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{32\sqrt{3}c^{13/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}} \\
&\quad +\frac{32c^{13/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{32c^{13/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}} \\
&\quad +\frac{6124\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{91d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad -\frac{12248\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{91\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
32*c^(13/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(
8/3)-32*c^(13/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-32*c^(13/6)*
arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(8/3
)-214/91*c*x^2*(d*x^3+c)^(1/2)/d^2-2/13*x^5*(d*x^3+c)^(1/2)/d-12248/91*c^2
*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-12248/273*c^(7/3)
*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^
(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3
+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)+6124/91*3^(1/4)*c^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(
1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1
/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/
3)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)
*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.289.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.23

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx$$
$$=\frac{-20(107c^2x^2+114cdx^5+7d^2x^8)+2140c^2x^2\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+1531cdx^5\sqrt{1+}}{910d^2\sqrt{c+dx^3}}$$

input
```
Integrate[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(-20*(107*c^2*x^2 + 114*c*d*x^5 + 7*d^2*x^8) + 2140*c^2*x^2*Sqrt[1 + (d*x^
3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 1531*c*d*x
^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8
*c)])/(910*d^2*Sqrt[c + d*x^3])
```

3.289.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {978, 27, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}dx$$

$$\Big\downarrow \text{ 978}$$

$$\frac{2\int \frac{cx^4\left(107dx^3+80c\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{13d}-\frac{2x^5\sqrt{c+dx^3}}{13d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\int \frac{x^4\left(107dx^3+80c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{13d}-\frac{2x^5\sqrt{c+dx^3}}{13d}$$

$$\Big\downarrow \text{ 1052}$$

$$\frac{c\left(\frac{2\int \frac{2cdx\left(1531dx^3+856c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d^2}-\frac{214x^2\sqrt{c+dx^3}}{7d}\right)}{13d}-\frac{2x^5\sqrt{c+dx^3}}{13d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\left(\frac{4c\int \frac{x\left(1531dx^3+856c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d}-\frac{214x^2\sqrt{c+dx^3}}{7d}\right)}{13d}-\frac{2x^5\sqrt{c+dx^3}}{13d}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{c\left(\frac{4c\int \left(\frac{13104cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}-\frac{1531x}{\sqrt{dx^3+c}}\right)dx}{7d}-\frac{214x^2\sqrt{c+dx^3}}{7d}\right)}{13d}-\frac{2x^5\sqrt{c+dx^3}}{13d}$$

$$\Big\downarrow \text{ 2009}$$

$$c\left(\frac{4c\left(-\frac{3062\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{1531\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{}\right.}{}\right.$$

$$\frac{2x^5\sqrt{c+dx^3}}{13d}$$

input

```
Int[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output

```
(-2*x^5*Sqrt[c + d*x^3])/(13*d) + (c*((-214*x^2*Sqrt[c + d*x^3])/(7*d) + (
4*c*((-3062*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x))
 - (728*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqr
t[c + d*x^3]])/d^(2/3) + (728*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c
^(1/6)*Sqrt[c + d*x^3])])/d^(2/3) - (728*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])])/d^(2/3) + (1531*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) +
d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(
c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3]) - (3062*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) -
 c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) +
 d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(7*
d)))/(13*d)
```

3.289. $\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx$

3.289.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.289.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.91 (sec) , antiderivative size = 884, normalized size of antiderivative = 1.36

method	result	size
risch	Expression too large to display	884
elliptic	Expression too large to display	889
default	Expression too large to display	1788

input
```
int(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-2/91*x^2*(7*d*x^3+107*c)*(d*x^3+c)^(1/2)/d^2-4/91/d^2*c^2*(-3062/3*I*3^(1
/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3
+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Elliptic
E(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1456/3*I/d^3*2^(1/2)*sum
(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^
2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)
^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1
/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_...
```

3.289.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.58 (sec) , antiderivative size = 2442, normalized size of antiderivative = 3.77

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output

```
2/273*(728*d^3*(c^13/d^16)^(1/6)*log(33554432*((d^16*x^9 + 318*c*d^15*x^6
+ 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c^13/d^16)^(5/6) + 6*(c^11*d^2*x^7 +
80*c^12*d*x^4 + 160*c^13*x + 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2)*(c^13/d^
16)^(2/3) + (7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5)*(c^13/d^16)^(1/
3))*sqrt(d*x^3 + c) + 18*(5*c^5*d^10*x^7 + 64*c^6*d^9*x^4 + 32*c^7*d^8*x)*
sqrt(c^13/d^16) + 18*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*x^2)*(c^
13/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 728*
d^3*(c^13/d^16)^(1/6)*log(-33554432*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2
*d^14*x^3 + 640*c^3*d^13)*(c^13/d^16)^(5/6) - 6*(c^11*d^2*x^7 + 80*c^12*d*
x^4 + 160*c^13*x + 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2)*(c^13/d^16)^(2/3)
+ (7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5)*(c^13/d^16)^(1/3))*sqrt(d
*x^3 + c) + 18*(5*c^5*d^10*x^7 + 64*c^6*d^9*x^4 + 32*c^7*d^8*x)*sqrt(c^13/
d^16) + 18*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*x^2)*(c^13/d^16)^(
1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 18372*c^2*sqrt
(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 364*(s
qrt(-3)*d^3 - d^3)*(c^13/d^16)^(1/6)*log(33554432*((d^16*x^9 + 318*c*d^15*
x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x
^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^13/d^16)^(5/6) + 6*(2*c^11*d^2*
x^7 + 160*c^12*d*x^4 + 320*c^13*x - 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2 -
sqrt(-3)*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2))*(c^13/d^16)^(2/3) - (7*c^7...
```

3.289.6 Sympy [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx=-\int \frac{x^7\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

input

```
integrate(x**7*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

output

```
-Integral(x**7*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

3.289.7 Maxima [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx=\int -\frac{\sqrt{dx^3+c}x^7}{dx^3-8\,c}\,dx$$

input
```
integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c), x)
```

3.289.8 Giac [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx=\int -\frac{\sqrt{dx^3+c}x^7}{dx^3-8\,c}\,dx$$

input
```
integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c), x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx=\int \frac{x^7\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\,dx$$

input
```
int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

output
```
int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3), x)
```

3.290 $\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx$

3.290.1 Optimal result

Integrand size = 27, antiderivative size = 624

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{4\sqrt{3}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}$$

$$+ \frac{4c^{7/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}} - \frac{4c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}$$

$$+ \frac{59\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{118\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
4*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5/
3)-4*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-4*c^(7/6)*arctan
(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(5/3)-2/7*
x^2*(d*x^3+c)^(1/2)/d-118/7*c*(d*x^3+c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2)))-118/21*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3
)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1
/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+59/7*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*E
llipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I
*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3
)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c
^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.290.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.21

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{x^2\left(-80(c+dx^3)+80c\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+59dx^3\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},\right.\right.}{280d\sqrt{c+dx^3}}$$

input
```
Integrate[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(x^2*(-80*(c + d*x^3) + 80*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3
, -((d*x^3)/c), (d*x^3)/(8*c)] + 59*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(280*d*Sqrt[c + d*x^3])
```

3.290.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {978, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx$$

$$\Big\downarrow \; 978$$

$$\frac{2\int \frac{cx\left(59dx^3+32c\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d}-\frac{2x^2\sqrt{c+dx^3}}{7d}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{x\left(59dx^3+32c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d}-\frac{2x^2\sqrt{c+dx^3}}{7d}$$

$$\Big\downarrow \; 1054$$

$$\frac{c\int \left(\frac{504cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}-\frac{59x}{\sqrt{dx^3+c}}\right)dx}{7d}-\frac{2x^2\sqrt{c+dx^3}}{7d}$$

$$\Big\downarrow \; 2009$$

$$c\left(-\frac{118\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{59\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\right.}{}\right.$$

$$\frac{2x^2\sqrt{c+dx^3}}{7d}$$

input `Int[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3),x]`

output

```
(-2*x^2*Sqrt[c + d*x^3])/(7*d) + (c*((-118*Sqrt[c + d*x^3])/(d^(2/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (28*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1
/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(2/3) + (28*c^(1/6)*ArcTanh
[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(2/3) - (28*c^(1/
6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(2/3) + (59*3^(1/4)*Sqrt[2 - Sq
rt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4
*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (118*Sqrt[2]*c^(1/3)*(c^(1/3) + d
^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*
x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)
*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^
2]*Sqrt[c + d*x^3])))/(7*d)
```

3.290.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.290 $\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx$

3.290.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.84 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.39

method	result	size
elliptic	Expression too large to display	867
risch	Expression too large to display	872
default	Expression too large to display	1310

input
```
int(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-2/7*x^2*(d*x^3+c)^(1/2)/d+118/21*I*c/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)))-8/3*I*c/d^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I
*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))
)^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(
1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^...
```

3.290.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.95 (sec) , antiderivative size = 2428, normalized size of antiderivative = 3.89

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
-1/21*(6*sqrt(d*x^3 + c)*d*x^2 - 14*d^2*(c^7/d^10)^(1/6)*log(1024*((d^11*x
^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8)*(c^7/d^10)^(5/6) + 6
*(c^6*d^2*x^7 + 80*c^7*d*x^4 + 160*c^8*x + 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*x
^2)*(c^7/d^10)^(2/3) + (7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3)*(c^7
/d^10)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^3*d^7*x^7 + 64*c^4*d^6*x^4 + 32*c^
5*d^5*x)*sqrt(c^7/d^10) + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^
2)*(c^7/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) +
 14*d^2*(c^7/d^10)^(1/6)*log(-1024*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*
d^9*x^3 + 640*c^3*d^8)*(c^7/d^10)^(5/6) - 6*(c^6*d^2*x^7 + 80*c^7*d*x^4 +
160*c^8*x + 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2)*(c^7/d^10)^(2/3) + (7*c^4*d
^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3)*(c^7/d^10)^(1/3))*sqrt(d*x^3 + c) +
 18*(5*c^3*d^7*x^7 + 64*c^4*d^6*x^4 + 32*c^5*d^5*x)*sqrt(c^7/d^10) + 18*(c
^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2)*(c^7/d^10)^(1/6))/(d^3*x^9 -
 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 354*c*sqrt(d)*weierstrassZeta(
0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 7*(sqrt(-3)*d^2 - d^2)*(c^
7/d^10)^(1/6)*log(1024*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 64
0*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c
^3*d^8))*(c^7/d^10)^(5/6) + 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x -
 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2 - sqrt(-3)*(5*c^2*d^8*x^5 + 32*c^3*d^7*
x^2))*(c^7/d^10)^(2/3) - (7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3 ...
```

3.290.6 Sympy [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\int \frac{x^4\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

input
```
integrate(x**4*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

output
```
-Integral(x**4*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

3.290.7 Maxima [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx=\int -\frac{\sqrt{dx^3+cx^4}}{dx^3-8\,c}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c), x)
```

3.290.8 Giac [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx=\int -\frac{\sqrt{dx^3+cx^4}}{dx^3-8\,c}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c), x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx=\int \frac{x^4\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\,dx$$

input
```
int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

output
```
int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3), x)
```

3.291 $\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx$

3.291.1 Optimal result

Integrand size = 25, antiderivative size = 601

$$\begin{aligned}\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = &-\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}}\\
&+\frac{\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}}-\frac{\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2d^{2/3}}\\
&+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)\middle|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\\
&-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\end{aligned}$$

output
```
1/2*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(
2/3)-1/2*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(2/3)-1/2*c^(1/6)*
arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(2/3
)-2*(d*x^3+c)^(1/2)/d^(2/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-2/3*c^(1/3)*(c
^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(2/3)/(d*x^3+c)
^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)+3^(1/4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(
1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(
1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/
2)))^2)^(1/2)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.291.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.10

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{16\sqrt{c+dx^3}}$$

input
```
Integrate[(x*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
(x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)
/(8*c)])/(16*Sqrt[c + d*x^3])
```

3.291.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {984, 832, 759, 988, 946, 73, 219, 2416, 2563, 219, 2570, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx$$

$$\Big\downarrow \color{blue}{984}$$

$$9c\int \frac{x}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx-\int \frac{x}{\sqrt{dx^3+c}}dx$$

$$\Big\downarrow \color{blue}{832}$$

$$\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}\int \frac{1}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}+9c\int \frac{x}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx$$

$$\Big\downarrow \color{blue}{759}$$

$$-\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}+9c\int \frac{x}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx+\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\Big\downarrow \color{blue}{988}$$

$$9c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{d}\int \frac{x^2}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{4\sqrt[3]{c}}\right)-\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}+\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\Big\downarrow \color{blue}{946}$$

3.291. $\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx$

$$9c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{d}\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{12\sqrt[3]{c}}\right)-$$

$$\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}+$$

$$\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 73}$$

$$9c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{6\sqrt[3]{c}d^{2/3}}\right)-$$

$$\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}+$$

$$\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 219}$$

3.291. $\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx$

$$9c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)-\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}+\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 2416$$

$$9c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)+\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}$$

$$\Big\downarrow \; 2563$$

3.291. $\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx$

$$9c\left(\frac{\int \frac{1}{9-\frac{\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^4}{\sqrt[3]{c}\left(dx^3+c\right)}}d\frac{\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^2}{c^{2/3}\sqrt{dx^3+c}}}{6\sqrt[3]{c}d^{2/3}}-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)+\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}$$

$$\Big\downarrow \text{219}$$

$$9c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)+\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}$$

$$\Big\downarrow \text{2570}$$

3.291. $\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx$

$$9c\left(\frac{d^{4/3}\int \frac{1}{-\frac{2d^2}{c}-\frac{6\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^2d^2}{c^{2/3}\left(dx^3+c\right)}}d\frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\sqrt[3]{c}\sqrt{dx^3+c}}}{3c^{4/3}}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)+$$

$$\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-$$

$$\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}$$

$$\Big\downarrow 218$$

$$\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-$$

$$\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}+$$

$$9c\left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)$$

3.291. $\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx$

input
```
Int[(x*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

output
```
9*c*(-1/6*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/
(Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqr
t[c + d*x^3])]/(18*c^(5/6)*d^(2/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]
/(18*c^(5/6)*d^(2/3))) - ((2*Sqrt[c + d*x^3])/(d^(1/3)*((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*
x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(1/3)*Sqrt[(c^(1/3)*
(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x
^3]))/d^(1/3) + (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1
/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sq
rt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
Sqrt[c + d*x^3])
```

3.291.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 988
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[d*(q/(4*b))   Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x
^3]), x], x] + (-Simp[q^2/(12*b)   Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]
), x], x] + Simp[1/(12*b*c)   Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x +
 q^2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[8*b*c + a*d, 0]
```

3.291. $\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

3.291.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.20 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.41

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

input
```
int(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/3*I/d^3*2^(1
/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-
c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(
-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(...
```

3.291.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 2194, normalized size of antiderivative = 3.65

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output

```
-1/24*((sqrt(-3)*d - d)*(c/d^4)^(1/6)*log(1/4*((d^6*x^9 + 318*c*d^5*x^6 +
1200*c^2*d^4*x^3 + 640*c^3*d^3 + sqrt(-3)*(d^6*x^9 + 318*c*d^5*x^6 + 1200*
c^2*d^4*x^3 + 640*c^3*d^3))*(c/d^4)^(5/6) + 6*(2*c*d^2*x^7 + 160*c^2*d*x^4
 + 320*c^3*x - 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 - sqrt(-3)*(5*c*d^4*x^5 + 3
2*c^2*d^3*x^2))*(c/d^4)^(2/3) - (7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d
+ sqrt(-3)*(7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d))*(c/d^4)^(1/3))*sqrt
(d*x^3 + c) - 36*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*x)*sqrt(c/d^4)
 + 18*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2 - sqrt(-3)*(c*d^3*x^8 + 3
8*c^2*d^2*x^5 + 64*c^3*d*x^2))*(c/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 19
2*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*d - d)*(c/d^4)^(1/6)*log(-1/4*((d^6*x^
9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + sqrt(-3)*(d^6*x^9 + 3
18*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3))*(c/d^4)^(5/6) - 6*(2*c*d^2
*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 - sqrt(
-3)*(5*c*d^4*x^5 + 32*c^2*d^3*x^2))*(c/d^4)^(2/3) - (7*c*d^3*x^6 + 152*c^2
*d^2*x^3 + 64*c^3*d + sqrt(-3)*(7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d))
*(c/d^4)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^
3*d^2*x)*sqrt(c/d^4) + 18*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2 - sqr
t(-3)*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2))*(c/d^4)^(1/6))/(d^3*x^9
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*d + d)*(c/d^4)^(1/
6)*log(1/4*((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 -...
```

3.291.6 Sympy [F]

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx=-\int \frac{x\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

input

```
integrate(x*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

output

```
-Integral(x*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

3.291.7 Maxima [F]

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx=\int -\frac{\sqrt{dx^3+c}x}{dx^3-8\,c}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c), x)
```

3.291.8 Giac [F]

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx=\int -\frac{\sqrt{dx^3+c}x}{dx^3-8\,c}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-sqrt(d*x^3 + c)*x/(d*x^3 - 8*c), x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx=\int \frac{x\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\,dx$$

input
```
int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

output
```
int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3), x)
```

3.292 $\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx$

3.292.1 Optimal result

Integrand size = 27, antiderivative size = 632

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)}\,dx$$

$$= -\frac{\sqrt{c+dx^3}}{8cx}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16c^{5/6}}$$
$$+\frac{\sqrt[3]{d}\,\mathrm{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16c^{5/6}}-\frac{\sqrt[3]{d}\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{16c^{5/6}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
1/16*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^
(5/6)-1/16*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)-1/16*d^(1/
3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/c^(
5/6)-1/8*(d*x^3+c)^(1/2)/c/x+1/8*d^(1/3)*(d*x^3+c)^(1/2)/c/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))+1/24*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(
1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)
-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3
^(3/4)/c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/16*3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*x
)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(
2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(2/3)/(d*x^3+c)^(1/2)
/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.292.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(8c-dx^3)}\,dx = \frac{-80c(c+dx^3)+65cdx^3\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\Big(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\Big)-d^2x^6\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\Big(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-}{640c^2x\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)),x]
```

output
```
(-80*c*(c + d*x^3) + 65*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5
/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(640*c^2*x*Sqrt[c + d*x^3])
```

3.292.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 625, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {975, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^2 (8c-dx^3)} \, dx$$

$$\Big\downarrow \ 975$$

$$\frac{\int \frac{dx(26c-dx^3)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{8c} - \frac{\sqrt{c+dx^3}}{8cx}$$

$$\Big\downarrow \ 27$$

$$\frac{d\int \frac{x(26c-dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{16c} - \frac{\sqrt{c+dx^3}}{8cx}$$

$$\Big\downarrow \ 1054$$

$$\frac{d\int \left(\frac{18cx}{(8c-dx^3)\sqrt{dx^3+c}} + \frac{x}{\sqrt{dx^3+c}}\right)dx}{16c} - \frac{\sqrt{c+dx^3}}{8cx}$$

$$\Big\downarrow \ 2009$$

$$d\left(\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\,\cdots}{\cdots}\right.$$

$$\frac{\sqrt{c+dx^3}}{8cx}$$

input
```
Int[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)),x]
```

output

```
-1/8*Sqrt[c + d*x^3]/(c*x) + (d*((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/
3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(2/3) + (c^(1/6)*ArcTanh[(c^(1/3) + d
^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(2/3) - (c^(1/6)*ArcTanh[Sqrt[
c + d*x^3]/(3*Sqrt[c])])/d^(2/3) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(
1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)
*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^
2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/
3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/
3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))
/(16*c)
```

3.292.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.292. $\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx$

3.292.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.11 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	1306

input

```
int((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output

```
-1/8*(d*x^3+c)^(1/2)/c/x-1/24*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1
/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)))-1/24*I/d^2/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*
(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alp
ha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_...
```

3.292.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 2219, normalized size of antiderivative = 3.51

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)}\,dx = \text{Too large to display}$$

input

```
integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="fricas")
```

output

```
1/192*(2*c*x*(d^2/c^5)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x
^3 + 640*c^3*d + 18*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x)*(d^2/c^5)^(2
/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2)*(d^2/c^5)^(5/6) + (7
*c^3*d^2*x^6 + 152*c^4*d*x^3 + 64*c^5)*sqrt(d^2/c^5) + (c*d^3*x^7 + 80*c^2
*d^2*x^4 + 160*c^3*d*x)*(d^2/c^5)^(1/6)) + 18*(c^2*d^3*x^8 + 38*c^3*d^2*x^
5 + 64*c^4*d*x^2)*(d^2/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3
 - 512*c^3)) - 2*c*x*(d^2/c^5)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c
^2*d^2*x^3 + 640*c^3*d + 18*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x)*(d^2
/c^5)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2)*(d^2/c^5)^(5
/6) + (7*c^3*d^2*x^6 + 152*c^4*d*x^3 + 64*c^5)*sqrt(d^2/c^5) + (c*d^3*x^7
 + 80*c^2*d^2*x^4 + 160*c^3*d*x)*(d^2/c^5)^(1/6)) + 18*(c^2*d^3*x^8 + 38*c^
3*d^2*x^5 + 64*c^4*d*x^2)*(d^2/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c
^2*d*x^3 - 512*c^3)) - 24*sqrt(d)*x*weierstrassZeta(0, -4*c/d, weierstrass
PInverse(0, -4*c/d, x)) + (sqrt(-3)*c*x + c*x)*(d^2/c^5)^(1/6)*log((d^4*x^
9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^4*d^2*x^7 + 64*c
^5*d*x^4 + 32*c^6*x + sqrt(-3)*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x))*
(d^2/c^5)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2 - sqrt(-3
)*(5*c^5*d*x^5 + 32*c^6*x^2))*(d^2/c^5)^(5/6) - 2*(7*c^3*d^2*x^6 + 152*c^4
*d*x^3 + 64*c^5)*sqrt(d^2/c^5) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x
 + sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x))*(d^2/c^5)^(1/6)...
```

3.292.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)}\,dx = -\int \frac{\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx$$

input

```
integrate((d*x**3+c)**(1/2)/x**2/(-d*x**3+8*c),x)
```

output

```
-Integral(sqrt(c + d*x**3)/(-8*c*x**2 + d*x**5), x)
```

3.292.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^2), x)
```

3.292.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^2), x)
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{x^2\,(8\,c-d\,x^3)}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)), x)
```

3.293 $\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)}\,dx$

3.293.1 Optimal result

Integrand size = 27, antiderivative size = 654

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)}\,dx &= -\frac{\sqrt{c+dx^3}}{32cx^4}-\frac{d\sqrt{c+dx^3}}{16c^2x} \\
&\quad +\frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{11/6}} \\
&\quad +\frac{d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{11/6}}-\frac{d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{11/6}} \\
&\quad -\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad +\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output

```
1/128*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(11/6)-1/128*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-1/128*
d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2
)/c^(11/6)-1/32*(d*x^3+c)^(1/2)/c/x^4-1/16*d*(d*x^3+c)^(1/2)/c^2/x+1/16*d^
(4/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/48*d^(4/3)*(c^
(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(
d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c)^
(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2
)-1/32*3^(1/4)*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1
-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2
*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.293.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)}\,dx$$

$$= \frac{125cd^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\Big(40c(c^2+3cdx^3+2d^2x^6)+d^3x^9\sqrt{1+\frac{dx^3}{c}}\,\text{Appell}}{5120c^3x^4\sqrt{c+dx^3}}$$

input

```
Integrate[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)),x]
```

output

```
(125*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, (d*x^3)/(8*c)] - 4*(40*c*(c^2 + 3*c*d*x^3 + 2*d^2*x^6) + d^3*x^9*Sqrt[1
+ (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(51
20*c^3*x^4*Sqrt[c + d*x^3])
```

3.293.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = $\{975, 27, 1053, 27, 1054, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)}dx$$

$$\Big\downarrow \text{ 975}$$

$$\frac{\int \frac{d\left(5dx^3+32c\right)}{2x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{\sqrt{c+dx^3}}{32cx^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\int \frac{5dx^3+32c}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{64c}-\frac{\sqrt{c+dx^3}}{32cx^4}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{d\left(-\frac{\int -\frac{8cdx\left(25c-2dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{4\sqrt{c+dx^3}}{cx}\right)}{64c}-\frac{\sqrt{c+dx^3}}{32cx^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\left(\frac{d\int \frac{x\left(25c-2dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{4\sqrt{c+dx^3}}{cx}\right)}{64c}-\frac{\sqrt{c+dx^3}}{32cx^4}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{d\left(\frac{d\int \left(\frac{9cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{2x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{4\sqrt{c+dx^3}}{cx}\right)}{64c}-\frac{\sqrt{c+dx^3}}{32cx^4}$$

$$\Big\downarrow \text{ 2009}$$

$$d\left(\frac{d\left(\frac{4\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{2\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{c^{2/3}}}{d^{2/3}}\right.}{}\right.$$

$$\frac{\sqrt{c+dx^3}}{32cx^4}$$

input
```
Int[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)),x]
```

output
```
-1/32*Sqrt[c + d*x^3]/(c*x^4) + (d*((-4*Sqrt[c + d*x^3])/(c*x) + (d*((4*Sq
rt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*c^
(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*
d^(2/3)) + (c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*
x^3])])/(2*d^(2/3)) - (c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*d^
(2/3)) - (2*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3)
+ d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (4
*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 -
 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/c))/(64*c)
```

3.293. $\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)}\,dx$

3.293.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.293.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.79 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.35

method	result	size
risch	Expression too large to display	882
elliptic	Expression too large to display	887
default	Expression too large to display	1782

input
```
int((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(d*x^3+c)^(1/2)*(2*d*x^3+c)/c^2/x^4+1/64*d^2/c^2*(-4/3*I*3^(1/2)/d*(
-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/3*I/d^3*2^(1/2)*sum(1/_alpha*
(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^
(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_
alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_al
pha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^...
```

3.293.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.74 (sec) , antiderivative size = 2401, normalized size of antiderivative = 3.67

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)}\,dx=\text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
1/1536*(2*c^2*x^4*(d^8/c^11)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2
*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x)
*(d^8/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^8
/c^11)^(5/6) + (7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^1
1) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x)*(d^8/c^11)^(1/6)) + 18
*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2)*(d^8/c^11)^(1/3))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*c^2*x^4*(d^8/c^11)^(1/6)*
log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^8*
d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x)*(d^8/c^11)^(2/3) - 6*sqrt(d*x^3 +
c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^8/c^11)^(5/6) + (7*c^6*d^4*x^6 + 152
*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4
+ 160*c^4*d^5*x)*(d^8/c^11)^(1/6)) + 18*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64
*c^6*d^4*x^2)*(d^8/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
512*c^3)) - 96*d^(3/2)*x^4*weierstrassZeta(0, -4*c/d, weierstrassPInverse(
0, -4*c/d, x)) + (sqrt(-3)*c^2*x^4 + c^2*x^4)*(d^8/c^11)^(1/6)*log((d^9*x^
9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^8*d^3*x^7 + 64
*c^9*d^2*x^4 + 32*c^10*d*x + sqrt(-3)*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32
*c^10*d*x))*(d^8/c^11)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^1
1*x^2 - sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^8/c^11)^(5/6) - 2*(7*c^6
*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 ...
```

3.293.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)}\,dx=-\int \frac{\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**5/(-d*x**3+8*c),x)
```

output
```
-Integral(sqrt(c + d*x**3)/(-8*c*x**5 + d*x**8), x)
```

3.293.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^5}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^5), x)
```

3.293.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^5}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^5), x)
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{x^5\,\left(8\,c-d\,x^3\right)}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)), x)
```

3.294 $\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx$

3.294.1 Optimal result

Integrand size = 27, antiderivative size = 678

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx = &-\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}\\
&-\frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{17/6}}\\
&+\frac{d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{17/6}}-\frac{d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{17/6}}\\
&+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)\middle|-7-4\sqrt{3}\right)}{224c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\\
&-\frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{56\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\end{aligned}$$

output

```
1/1024*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
c^(17/6)-1/1024*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/10
24*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(
1/2)/c^(17/6)-1/56*(d*x^3+c)^(1/2)/c/x^7-19/1792*d*(d*x^3+c)^(1/2)/c^2/x^4
+1/112*d^2*(d*x^3+c)^(1/2)/c^3/x-1/112*d^(7/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))-1/336*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1
/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2
)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*
x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/224*3^(1/4)*d^(7/3)*(c^(1/3)
+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(
1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(8/3)/(d*x^
3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

3.294.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx$$
$$= \frac{-160c(32c^3+51c^2dx^3+3cd^2x^6-16d^3x^9)-325cd^3x^9\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+32d^4x}{286720c^4x^7\sqrt{c+dx^3}}$$

input

```
Integrate[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)),x]
```

output

```
(-160*c*(32*c^3 + 51*c^2*d*x^3 + 3*c*d^2*x^6 - 16*d^3*x^9) - 325*c*d^3*x^9
*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c
)] + 32*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/
c), (d*x^3)/(8*c)])/(286720*c^4*x^7*Sqrt[c + d*x^3])
```

3.294.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {975, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{d\left(11dx^3+38c\right)}{2x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{56c}-\frac{\sqrt{c+dx^3}}{56cx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{11dx^3+38c}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{112c}-\frac{\sqrt{c+dx^3}}{56cx^7}$$

$$\Big\downarrow \text{1053}$$

$$\frac{d\left(-\frac{\int \frac{cd\left(256c-95dx^3\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{19\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{\sqrt{c+dx^3}}{56cx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(-\frac{d\int \frac{256c-95dx^3}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{19\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{\sqrt{c+dx^3}}{56cx^7}$$

$$\Big\downarrow \text{1053}$$

$$\frac{d\left(-\frac{d\left(-\frac{\int -\frac{8cdx\left(65c-16dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{32\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{19\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{\sqrt{c+dx^3}}{56cx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(-\frac{d\left(\frac{d\int \frac{x\left(65c-16dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{32\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{19\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{\sqrt{c+dx^3}}{56cx^7}$$

$$\Big\downarrow \ 1054$$

$$\frac{d\left(-\frac{d\left(\frac{d\int \left(\frac{16x}{\sqrt{dx^3+c}}-\frac{63cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}\right)dx}{c}-\frac{32\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{19\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{\sqrt{c+dx^3}}{56cx^7}$$

$$\Big\downarrow \ 2009$$

3.294. $\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)}\,dx$

$$d\Biggl(-\frac{d\Bigl(d\Bigl(\frac{32\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{16\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{}}{}}{}$$

$$\frac{\sqrt{c+dx^3}}{56cx^7}$$

input
```
Int[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)),x]
```

3.294. $\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)}\,dx$

output
```
-1/56*Sqrt[c + d*x^3]/(c*x^7) + (d*((-19*Sqrt[c + d*x^3])/(16*c*x^4) - (d*
((-32*Sqrt[c + d*x^3])/(c*x) + (d*((32*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) + (7*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(
c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*d^(2/3)) - (7*c^(1/6)*ArcTanh[(
c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(2*d^(2/3)) + (7*c^(1
/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*d^(2/3)) - (16*3^(1/4)*Sqrt[2
 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[(
(1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (32*Sqrt[2]*c^(1/3)*(c^(1/3)
 + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1
/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(
2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*Sqrt[c + d*x^3])))/c))/(32*c)))/(112*c)
```

3.294.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.294. $\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)}\,dx$

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.294.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.03 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.32

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	906
default	Expression too large to display	2280

input
```
int((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/1792*(d*x^3+c)^(1/2)*(-16*d^2*x^6+19*c*d*x^3+32*c^2)/c^3/x^7-1/3584*d^3
/c^3*(-32/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(
1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x
+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/
3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*
d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+7/3*I/
d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3)
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha
^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^...
```

3.294.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.92 (sec) , antiderivative size = 2436, normalized size of antiderivative = 3.59

$$\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="fricas")
```

output

```
1/86016*(14*c^3*x^7*(d^14/c^17)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 120
0*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*
c^14*d^2*x)*(d^14/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^
16*x^2)*(d^14/c^17)^(5/6) + (7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^
4)*sqrt(d^14/c^17) + (c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x)*(d^1
4/c^17)^(1/6)) + 18*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2)*(d^14/
c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 14*c^3*
x^7*(d^14/c^17)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 +
 640*c^3*d^11 + 18*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x)*(d^1
4/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^14/c^
17)^(5/6) + (7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*sqrt(d^14/c^1
7) + (c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x)*(d^14/c^17)^(1/6)) +
 18*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2)*(d^14/c^17)^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 768*d^(5/2)*x^7*weierst
rassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 7*(sqrt(-3)*c^3*x
^7 + c^3*x^7)*(d^14/c^17)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*
d^12*x^3 + 640*c^3*d^11 - 9*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^
2*x + sqrt(-3)*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x))*(d^14/c
^17)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(
5*c^15*d*x^5 + 32*c^16*x^2))*(d^14/c^17)^(5/6) - 2*(7*c^9*d^6*x^6 + 152...
```

3.294.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8\,(8c-dx^3)}\,dx = -\int \frac{\sqrt{c+dx^3}}{-8cx^8+dx^{11}}\,dx$$

input

```
integrate((d*x**3+c)**(1/2)/x**8/(-d*x**3+8*c),x)
```

output

```
-Integral(sqrt(c + d*x**3)/(-8*c*x**8 + d*x**11), x)
```

3.294.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx=\int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^8}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^8), x)
```

3.294.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx=\int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^8}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^8), x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^8\,\left(8\,c-d\,x^3\right)}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)), x)
```

3.295 $\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$

3.295.1 Optimal result

Integrand size = 27, antiderivative size = 130

$$\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx=-\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4}-\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+dx^3\right)^{9/2}}{27d^4}+\frac{9216c^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

output
```
-1024/9*c^3*(d*x^3+c)^(3/2)/d^4-38/5*c^2*(d*x^3+c)^(5/2)/d^4-4/7*c*(d*x^3+
c)^(7/2)/d^4-2/27*(d*x^3+c)^(9/2)/d^4+9216*c^(9/2)*arctanh(1/3*(d*x^3+c)^(
1/2)/c^(1/2))/d^4-3072*c^4*(d*x^3+c)^(1/2)/d^4
```

3.295.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.72

$$\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx=-\frac{2\sqrt{c+dx^3}\left(1509176c^4+61892c^3dx^3+4611c^2d^2x^6+410cd^3x^9+35d^4x^{12}\right)}{945d^4}+\frac{9216c^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

input
```
Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(1509176*c^4 + 61892*c^3*d*x^3 + 4611*c^2*d^2*x^6 + 41
0*c*d^3*x^9 + 35*d^4*x^12))/(945*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x
^3]/(3*Sqrt[c])])/d^4
```

3.295.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{8c-dx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9\left(dx^3+c\right)^{3/2}}{8c-dx^3}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(-\frac{\left(dx^3+c\right)^{7/2}}{d^3}-\frac{6c\left(dx^3+c\right)^{5/2}}{d^3}+\frac{512c^3\left(dx^3+c\right)^{3/2}}{d^3\left(8c-dx^3\right)}-\frac{57c^2\left(dx^3+c\right)^{3/2}}{d^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{27648c^{9/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{9216c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{114c^2\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{12c\left(c+dx^3\right)^{7/}}{7d^4}\right.$$

input
```
Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
((-9216*c^4*Sqrt[c + d*x^3])/d^4 - (1024*c^3*(c + d*x^3)^(3/2))/(3*d^4) -
(114*c^2*(c + d*x^3)^(5/2))/(5*d^4) - (12*c*(c + d*x^3)^(7/2))/(7*d^4) - (
2*(c + d*x^3)^(9/2))/(9*d^4) + (27648*c^(9/2)*ArcTanh[Sqrt[c + d*x^3]/(3*S
qrt[c])])/d^4)/3
```

3.295.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.295.4 Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$\frac{\frac{2\left(-35d^4x^{12}-410d^3x^9c-4611c^2x^6d^2-61892c^3d\,x^3-1509176c^4\right)\sqrt{d\,x^3+c}}{945}+9216c^{\frac{9}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^4}$
risch	$-\frac{2\left(35d^4x^{12}+410d^3x^9c+4611c^2x^6d^2+61892c^3d\,x^3+1509176c^4\right)\sqrt{d\,x^3+c}}{945d^4}+\frac{9216c^{\frac{9}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^4}$
default	$-\frac{\frac{2d\,x^{12}\sqrt{d\,x^3+c}}{27}+\frac{20c\,x^9\sqrt{d\,x^3+c}}{189}+\frac{2c^2x^6\sqrt{d\,x^3+c}}{315d}-\frac{8c^3x^3\sqrt{d\,x^3+c}}{945d^2}+\frac{16c^4\sqrt{d\,x^3+c}}{945d^3}}{d}-\frac{8c\left(\frac{2d\,x^9\sqrt{d\,x^3+c}}{21}+\frac{16c\,x^6\sqrt{d\,x^3+c}}{105}+2c^2\right.}{d^2}$ $1536ic^4\surd$
elliptic	$-\frac{2x^{12}\sqrt{d\,x^3+c}}{27}-\frac{164c\,x^9\sqrt{d\,x^3+c}}{189d}-\frac{3074c^2x^6\sqrt{d\,x^3+c}}{315d^2}-\frac{123784c^3x^3\sqrt{d\,x^3+c}}{945d^3}-\frac{3018352c^4\sqrt{d\,x^3+c}}{945d^4}-$

3.295. $\int\frac{x^{11}\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$

input
```
int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
2/945*((-35*d^4*x^12-410*c*d^3*x^9-4611*c^2*d^2*x^6-61892*c^3*d*x^3-150917
6*c^4)*(d*x^3+c)^(1/2)+4354560*c^(9/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2)
))/d^4
```

3.295.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.47

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \Bigg[\frac{2\left(2177280\,c^{\frac{9}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right) - (35\,d^4x^{12}+410\,cd^3x^9+4611\,c^2d^2x^6+6}{945\,d^4}$$

$$-\frac{2\left(4354560\,\sqrt{-c}c^4\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(35\,d^4x^{12}+410\,cd^3x^9+4611\,c^2d^2x^6+61892\,c^3dx^3+1509176\,c\right.}{945\,d^4}$$

input
```
integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[2/945*(2177280*c^(9/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*
x^3 - 8*c)) - (35*d^4*x^12 + 410*c*d^3*x^9 + 4611*c^2*d^2*x^6 + 61892*c^3*
d*x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4, -2/945*(4354560*sqrt(-c)*c^4*ar
ctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (35*d^4*x^12 + 410*c*d^3*x^9 + 4611
*c^2*d^2*x^6 + 61892*c^3*d*x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4]
```

3.295.6 Sympy [A] (verification not implemented)

Time = 49.71 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{4608c^5\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^3\sqrt{-c}}-\frac{1536c^4\sqrt{c+dx^3}}{d^3}-\frac{512c^3\left(c+dx^3\right)^{\frac{3}{2}}}{9d^3}-\frac{19c^2\left(c+dx^3\right)^{\frac{5}{2}}}{5d^3}-\frac{2c\left(c+dx^3\right)^{\frac{7}{2}}}{7d^3}-\frac{\left(c+dx^3\right)^{\frac{9}{2}}}{27d^3}\right)}{d} & \\ \frac{\sqrt{c}x^{12}}{96} & \end{cases}$$

input
```
integrate(x**11*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-4608*c**5*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d**3*sqrt(-c
)) - 1536*c**4*sqrt(c + d*x**3)/d**3 - 512*c**3*(c + d*x**3)**(3/2)/(9*d**
3) - 19*c**2*(c + d*x**3)**(5/2)/(5*d**3) - 2*c*(c + d*x**3)**(7/2)/(7*d**
3) - (c + d*x**3)**(9/2)/(27*d**3))/d, Ne(d, 0)), (sqrt(c)*x**12/96, True)
)
```

3.295.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx =$$

$$-\frac{2\left(2177280\,c^{\frac{9}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+35\,(dx^3+c)^{\frac{9}{2}}+270\,(dx^3+c)^{\frac{7}{2}}c+3591\,(dx^3+c)^{\frac{5}{2}}c^2+53760\,(dx^3+c}{945\,d^4}$$

input
```
integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-2/945*(2177280*c^(9/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c)
 + 3*sqrt(c))) + 35*(d*x^3 + c)^(9/2) + 270*(d*x^3 + c)^(7/2)*c + 3591*(d*
x^3 + c)^(5/2)*c^2 + 53760*(d*x^3 + c)^(3/2)*c^3 + 1451520*sqrt(d*x^3 + c)
*c^4)/d^4
```

3.295.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{9216\,c^5\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d^4}$$

$$-\frac{2\left(35\,(dx^3+c)^{\frac{9}{2}}d^{32}+270\,(dx^3+c)^{\frac{7}{2}}cd^{32}+3591\,(dx^3+c)^{\frac{5}{2}}c^2d^{32}+53760\,(dx^3+c)^{\frac{3}{2}}c^3d^{32}+1451520\,\sqrt{dx}}{945\,d^{36}}$$

input
```
integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-9216*c^5*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/945*(35*
(d*x^3 + c)^(9/2)*d^32 + 270*(d*x^3 + c)^(7/2)*c*d^32 + 3591*(d*x^3 + c)^(
5/2)*c^2*d^32 + 53760*(d*x^3 + c)^(3/2)*c^3*d^32 + 1451520*sqrt(d*x^3 + c)
*c^4*d^32)/d^36
```

3.295.9 Mupad [B] (verification not implemented)

Time = 7.46 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{4608\,c^{9/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4} - \frac{2\,x^{12}\,\sqrt{d\,x^3+c}}{27} - \frac{3018352\,c^4\,\sqrt{d\,x^3+c}}{945\,d^4} - \frac{164\,c\,x^9\,\sqrt{d\,x^3+c}}{189\,d} - \frac{123784\,c^3\,x^3\,\sqrt{d\,x^3+c}}{945\,d^3} - \frac{3074\,c^2\,x^6\,\sqrt{d\,x^3+c}}{315\,d^2}$$

input

```
int((x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

output

```
(4608*c^(9/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^
3)))/d^4 - (2*x^12*(c + d*x^3)^(1/2))/27 - (3018352*c^4*(c + d*x^3)^(1/2))
/(945*d^4) - (164*c*x^9*(c + d*x^3)^(1/2))/(189*d) - (123784*c^3*x^3*(c +
d*x^3)^(1/2))/(945*d^3) - (3074*c^2*x^6*(c + d*x^3)^(1/2))/(315*d^2)
```

3.296 $\int \frac{x^8\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$

3.296.1 Optimal result

Integrand size = 27, antiderivative size = 109

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2(c+dx^3)^{3/2}}{9d^3} -\frac{14c(c+dx^3)^{5/2}}{15d^3}-\frac{2(c+dx^3)^{7/2}}{21d^3}+\frac{1152c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

output
```
-128/9*c^2*(d*x^3+c)^(3/2)/d^3-14/15*c*(d*x^3+c)^(5/2)/d^3-2/21*(d*x^3+c)^
(7/2)/d^3+1152*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3-384*c^3*(d
*x^3+c)^(1/2)/d^3
```

3.296.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}(62882c^3+2579c^2dx^3+192cd^2x^6+15d^3x^9)}{315d^3} +\frac{1152c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

input
```
Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(62882*c^3 + 2579*c^2*d*x^3 + 192*c*d^2*x^6 + 15*d^3*x
^9))/(315*d^3) + (1152*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

3.296.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 \left(c+dx^3\right)^{3/2}}{8c-dx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6 \left(dx^3+c\right)^{3/2}}{8c-dx^3}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(-\frac{\left(dx^3+c\right)^{5/2}}{d^2}+\frac{64c^2\left(dx^3+c\right)^{3/2}}{d^2\left(8c-dx^3\right)}-\frac{7c\left(dx^3+c\right)^{3/2}}{d^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3456c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{1152c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2\left(c+dx^3\right)^{3/2}}{3d^3}-\frac{14c\left(c+dx^3\right)^{5/2}}{5d^3}-\frac{2\left(c+dx^3\right)^{7/2}}{7d^3}\right)$$

input
```
Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
((-1152*c^3*Sqrt[c + d*x^3])/d^3 - (128*c^2*(c + d*x^3)^(3/2))/(3*d^3) - (
14*c*(c + d*x^3)^(5/2))/(5*d^3) - (2*(c + d*x^3)^(7/2))/(7*d^3) + (3456*c^
(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3)/3
```

3.296.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.296.4 Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{\frac{2\left(-15d^3x^9-192c\,d^2x^6-2579c^2d\,x^3-62882c^3\right)\sqrt{d\,x^3+c}}{315}+1152c^{\frac{7}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^3}$
risch	$-\frac{2\left(15d^3x^9+192c\,d^2x^6+2579c^2d\,x^3+62882c^3\right)\sqrt{d\,x^3+c}}{315d^3}+\frac{1152c^{\frac{7}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^3}$
default	$-\frac{\frac{2d\,x^9\sqrt{d\,x^3+c}}{21}+\frac{16c\,x^6\sqrt{d\,x^3+c}}{105}+\frac{2c^2x^3\sqrt{d\,x^3+c}}{105d}-\frac{4c^3\sqrt{d\,x^3+c}}{105d^2}}{d}-\frac{16c\left(d\,x^3+c\right)^{\frac{5}{2}}}{15d^3}+\frac{128c^2\left(81c^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)-(d\right.}{9d^3}$
elliptic	$-\frac{2x^9\sqrt{d\,x^3+c}}{21}-\frac{128c\,x^6\sqrt{d\,x^3+c}}{105d}-\frac{5158c^2x^3\sqrt{d\,x^3+c}}{315d^2}-\frac{125764c^3\sqrt{d\,x^3+c}}{315d^3}-\frac{192ic^3\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\right.}{}$

input
```
int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
2/315*((-15*d^3*x^9-192*c*d^2*x^6-2579*c^2*d*x^3-62882*c^3)*(d*x^3+c)^(1/2
)+181440*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2)))/d^3
```

3.296.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.55

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \left[\frac{2\left(90720\,c^{\frac{7}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(15\,d^3x^9+192\,cd^2x^6+2579\,c^2dx^3+62882}{315\,d^3}\right.$$

$$\left.-\frac{2\left(181440\sqrt{-c}c^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(15\,d^3x^9+192\,cd^2x^6+2579\,c^2dx^3+62882\,c^3)\sqrt{dx^3+c}\right)}{315\,d^3}\right]$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[2/315*(90720*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^
3 - 8*c)) - (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqrt
(d*x^3 + c))/d^3, -2/315*(181440*sqrt(-c)*c^3*arctan(1/3*sqrt(d*x^3 + c)*s
qrt(-c)/c) + (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqr
t(d*x^3 + c))/d^3]
```

3.296.6 Sympy [A] (verification not implemented)

Time = 27.49 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{576c^4\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^2\sqrt{-c}}-\frac{192c^3\sqrt{c+dx^3}}{d^2}-\frac{64c^2\left(c+dx^3\right)^{\frac{3}{2}}}{9d^2}-\frac{7c\left(c+dx^3\right)^{\frac{5}{2}}}{15d^2}-\frac{\left(c+dx^3\right)^{\frac{7}{2}}}{21d^2}\right)}{d} & \text{for } d\neq 0 \\ \frac{\sqrt{c}x^9}{72} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-576*c**4*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d**2*sqrt(-c)
) - 192*c**3*sqrt(c + d*x**3)/d**2 - 64*c**2*(c + d*x**3)**(3/2)/(9*d**2)
- 7*c*(c + d*x**3)**(5/2)/(15*d**2) - (c + d*x**3)**(7/2)/(21*d**2))/d, Ne
(d, 0)), (sqrt(c)*x**9/72, True))
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2\Big(90720\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+15\,(dx^3+c)^{\frac{7}{2}}+147\,(dx^3+c)^{\frac{5}{2}}c+2240\,(dx^3+c)^{\frac{3}{2}}c^2+60480\,\sqrt{dx^3+c}c^3}{315\,d^3}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-2/315*(90720*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) +
 3*sqrt(c))) + 15*(d*x^3 + c)^(7/2) + 147*(d*x^3 + c)^(5/2)*c + 2240*(d*x^
3 + c)^(3/2)*c^2 + 60480*sqrt(d*x^3 + c)*c^3)/d^3
```

3.296.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{1152\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^3} - \frac{2\left(15\,(dx^3+c)^{\frac{7}{2}}d^{18}+147\,(dx^3+c)^{\frac{5}{2}}cd^{18}+2240\,(dx^3+c)^{\frac{3}{2}}c^2d^{18}+60480\,\sqrt{dx^3+c}c^3d^{18}\right)}{315\,d^{21}}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-1152*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/315*(15*
(d*x^3 + c)^(7/2)*d^18 + 147*(d*x^3 + c)^(5/2)*c*d^18 + 2240*(d*x^3 + c)^(
3/2)*c^2*d^18 + 60480*sqrt(d*x^3 + c)*c^3*d^18)/d^21
```

3.296.9 Mupad [B] (verification not implemented)

Time = 7.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx=\frac{576\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3}-\frac{2\,x^9\,\sqrt{d\,x^3+c}}{21}-\frac{125764\,c^3\,\sqrt{d\,x^3+c}}{315\,d^3}-\frac{128\,c\,x^6\,\sqrt{d\,x^3+c}}{105\,d}-\frac{5158\,c^2\,x^3\,\sqrt{d\,x^3+c}}{315\,d^2}$$

input
```
int((x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

output
```
(576*c^(7/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/d^3 - (2*x^9*(c + d*x^3)^(1/2))/21 - (125764*c^3*(c + d*x^3)^(1/2))/(3
15*d^3) - (128*c*x^6*(c + d*x^3)^(1/2))/(105*d) - (5158*c^2*x^3*(c + d*x^3
)^(1/2))/(315*d^2)
```

3.297 $\int \frac{x^5 \left(c+d x^3\right)^{3/2}}{8 c-d x^3} \, dx$

3.297.1 Optimal result

Integrand size = 27, antiderivative size = 88

$$\int \frac{x^5 \left(c+d x^3\right)^{3/2}}{8 c-d x^3} \, dx=-\frac{48 c^2 \sqrt{c+d x^3}}{d^2}-\frac{16 c \left(c+d x^3\right)^{3/2}}{9 d^2}-\frac{2 \left(c+d x^3\right)^{5/2}}{15 d^2}+\frac{144 c^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right)}{d^2}$$

output
```
-16/9*c*(d*x^3+c)^(3/2)/d^2-2/15*(d*x^3+c)^(5/2)/d^2+144*c^(5/2)*arctanh(1
/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2-48*c^2*(d*x^3+c)^(1/2)/d^2
```

3.297.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \frac{x^5 \left(c+d x^3\right)^{3/2}}{8 c-d x^3} \, dx=-\frac{2 \sqrt{c+d x^3} \left(1123 c^2+46 c d x^3+3 d^2 x^6\right)}{45 d^2}+\frac{144 c^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right)}{d^2}$$

input
```
Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(1123*c^2 + 46*c*d*x^3 + 3*d^2*x^6))/(45*d^2) + (144*c
^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2
```

3.297.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {948, 90, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(c+d x^3\right)^{3/2}}{8 c-d x^3} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^3 \left(d x^3+c\right)^{3/2}}{8 c-d x^3}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3} \left(\frac{8 c \int \frac{\left(d x^3+c\right)^{3/2}}{8 c-d x^3}dx^3}{d}-\frac{2 \left(c+d x^3\right)^{5/2}}{5 d^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3} \left(\frac{8 c \left(9 c \int \frac{\sqrt{d x^3+c}}{8 c-d x^3}dx^3-\frac{2 \left(c+d x^3\right)^{3/2}}{3 d}\right)}{d}-\frac{2 \left(c+d x^3\right)^{5/2}}{5 d^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3} \left(\frac{8 c \left(9 c \left(9 c \int \frac{1}{\left(8 c-d x^3\right) \sqrt{d x^3+c}}dx^3-\frac{2 \sqrt{c+d x^3}}{d}\right)-\frac{2 \left(c+d x^3\right)^{3/2}}{3 d}\right)}{d}-\frac{2 \left(c+d x^3\right)^{5/2}}{5 d^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3} \left(\frac{8 c \left(9 c \left(\frac{18 c \int \frac{1}{9 c-x^6}d\sqrt{d x^3+c}}{d}-\frac{2 \sqrt{c+d x^3}}{d}\right)-\frac{2 \left(c+d x^3\right)^{3/2}}{3 d}\right)}{d}-\frac{2 \left(c+d x^3\right)^{5/2}}{5 d^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{8c\left(9c\left(\frac{6\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2(c+dx^3)^{3/2}}{3d}\right)}{d}-\frac{2(c+dx^3)^{5/2}}{5d^2}\right)$$

input
```
Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
((-2*(c + d*x^3)^(5/2))/(5*d^2) + (8*c*((-2*(c + d*x^3)^(3/2))/(3*d) + 9*c
*((-2*Sqrt[c + d*x^3])/d + (6*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]
)/d)))/d)/3
```

3.297.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.297.4 Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{6480c^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\left(-6d^2x^6-92cdx^3-2246c^2\right)\sqrt{dx^3+c}}{45d^2}$
risch	$-\frac{2\left(3d^2x^6+46cdx^3+1123c^2\right)\sqrt{dx^3+c}}{45d^2}+\frac{144c^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{d^2}$
default	$-\frac{2\left(dx^3+c\right)^{\frac{5}{2}}}{15d^2}+\frac{16c\left(81c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)-\left(dx^3+28c\right)\sqrt{dx^3+c}\right)}{9d^2}$
elliptic	$-\frac{2x^6\sqrt{dx^3+c}}{15}-\frac{92cx^3\sqrt{dx^3+c}}{45d}-\frac{2246c^2\sqrt{dx^3+c}}{45d^2}-\frac{24ic^2\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(\right.}{}\right.}{\left(\right.}}}{}\right.}{}$

input
```
int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
1/45*(6480*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+(-6*d^2*x^6-92*c*d
*x^3-2246*c^2)*(d*x^3+c)^(1/2))/d^2
```

3.297.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.67

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \left[\frac{2\left(1620\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(3\,d^2x^6+46\,cdx^3+1123\,c^2)\sqrt{dx^3+c}\right)}{45\,d^2},\right.$$
$$\left.-\frac{2\left(3240\,\sqrt{-c}c^2\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(3\,d^2x^6+46\,cdx^3+1123\,c^2)\sqrt{dx^3+c}\right)}{45\,d^2}\right]$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[2/45*(1620*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
- 8*c)) - (3*d^2*x^6 + 46*c*d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2, -2/45*
(3240*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^2*x^6 + 4
6*c*d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2]
```

3.297.6 Sympy [A] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{72c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d\sqrt{-c}}-\frac{24c^2\sqrt{c+dx^3}}{d}-\frac{8c\left(c+dx^3\right)^{\frac{3}{2}}}{9d}-\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15d}\right)}{d} & \text{for } d\neq 0 \\ \frac{\sqrt{c}x^6}{48} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-72*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d*sqrt(-c)) -
24*c**2*sqrt(c + d*x**3)/d - 8*c*(c + d*x**3)**(3/2)/(9*d) - (c + d*x**3)*
*(5/2)/(15*d))/d, Ne(d, 0)), (sqrt(c)*x**6/48, True))
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{x^5 \left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx = -\frac{2\left(1620\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\left(dx^3+c\right)^{\frac{5}{2}}+40\left(dx^3+c\right)^{\frac{3}{2}}c+1080\sqrt{dx^3+c}c^2\right)}{45\,d^2}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-2/45*(1620*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3
*sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 40*(d*x^3 + c)^(3/2)*c + 1080*sqrt(d*x^
3 + c)*c^2)/d^2
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{x^5 \left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx = -\frac{144\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^2} - \frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^8+40\left(dx^3+c\right)^{\frac{3}{2}}cd^8+1080\sqrt{dx^3+c}c^2d^8\right)}{45\,d^{10}}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-144*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 2/45*(3*(d*
x^3 + c)^(5/2)*d^8 + 40*(d*x^3 + c)^(3/2)*c*d^8 + 1080*sqrt(d*x^3 + c)*c^2
*d^8)/d^10
```

3.297.9 Mupad [B] (verification not implemented)

Time = 7.44 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{72\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2} - \frac{2\,x^6\,\sqrt{d\,x^3+c}}{15} - \frac{2246\,c^2\,\sqrt{d\,x^3+c}}{45\,d^2} - \frac{92\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d}$$

input
```
int((x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

output
```
(72*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^2 - (2*x^6*(c + d*x^3)^(1/2))/15 - (2246*c^2*(c + d*x^3)^(1/2))/(45*d
^2) - (92*c*x^3*(c + d*x^3)^(1/2))/(45*d)
```

3.298 $\int \frac{x^2 \left(c+d x^3\right)^{3/2}}{8 c-d x^3} \, dx$

3.298.1 Optimal result

Integrand size = 27, antiderivative size = 67

$$\int \frac{x^2 \left(c+d x^3\right)^{3/2}}{8 c-d x^3} \, dx=-\frac{6 c \sqrt{c+d x^3}}{d}-\frac{2 \left(c+d x^3\right)^{3/2}}{9 d}+\frac{18 c^{3/2} \text{arctanh}\left(\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right)}{d}$$

output
```
-2/9*(d*x^3+c)^(3/2)/d+18*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d-6
*c*(d*x^3+c)^(1/2)/d
```

3.298.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x^2 \left(c+d x^3\right)^{3/2}}{8 c-d x^3} \, dx=-\frac{2 \sqrt{c+d x^3} \left(28 c+d x^3\right)}{9 d}+\frac{18 c^{3/2} \text{arctanh}\left(\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right)}{d}$$

input
```
Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(28*c + d*x^3))/(9*d) + (18*c^(3/2)*ArcTanh[Sqrt[c + d
*x^3]/(3*Sqrt[c])])/d
```

3.298.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {946, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (c+dx^3)^{3/2}}{8c-dx^3} dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3} \int \frac{(dx^3+c)^{3/2}}{8c-dx^3} dx^3$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3} \left(9c \int \frac{\sqrt{dx^3+c}}{8c-dx^3} dx^3 - \frac{2(c+dx^3)^{3/2}}{3d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3} \left(9c \left(9c \int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}} dx^3 - \frac{2\sqrt{c+dx^3}}{d}\right) - \frac{2(c+dx^3)^{3/2}}{3d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3} \left(9c \left(\frac{18c \int \frac{1}{9c-x^6} d\sqrt{dx^3+c}}{d} - \frac{2\sqrt{c+dx^3}}{d}\right) - \frac{2(c+dx^3)^{3/2}}{3d}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3} \left(9c \left(\frac{6\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d} - \frac{2\sqrt{c+dx^3}}{d}\right) - \frac{2(c+dx^3)^{3/2}}{3d}\right)$$

input
```
Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
((-2*(c + d*x^3)^(3/2))/(3*d) + 9*c*((-2*Sqrt[c + d*x^3])/d + (6*Sqrt[c]*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d))/3
```

3.298.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.298.4 Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

method	result
default	$\frac{18c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-\frac{2\left(d\,x^{3}+28c\right)\sqrt{d\,x^{3}+c}}{9}}{d}$
pseudoelliptic	$\frac{18c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-\frac{2\left(d\,x^{3}+28c\right)\sqrt{d\,x^{3}+c}}{9}}{d}$
risch	$-\frac{2(d\,x^{3}+28c)\sqrt{d\,x^{3}+c}}{9d}+\frac{18c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{d}$
elliptic	$-\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9}-\frac{56c\sqrt{d\,x^{3}+c}}{9d}-\frac{3ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$

input

```
int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output

```
2/9*(81*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-(d*x^3+28*c)*(d*x^3+c
)^(1/2))/d
```

3.298.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.81

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \left[\frac{81\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-2\,(dx^3+28\,c)\sqrt{dx^3+c}}{9\,d},\right.$$
$$\left.-\frac{2\left(81\,\sqrt{-c}c\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(dx^3+28\,c)\sqrt{dx^3+c}\right)}{9\,d}\right]$$

input

```
integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output

```
[1/9*(81*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8
*c)) - 2*(d*x^3 + 28*c)*sqrt(d*x^3 + c))/d, -2/9*(81*sqrt(-c)*c*arctan(1/3
*sqrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 28*c)*sqrt(d*x^3 + c))/d]
```

3.298.6 Sympy [A] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{9c^2 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}} - 3c\sqrt{c+dx^3} - \frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d} & \text{for } d \neq 0 \\ \frac{\sqrt{c}x^3}{24} & \text{otherwise} \end{cases}$$

input

```
integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

output

```
Piecewise((2*(-9*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 3*c*s
qrt(c + d*x**3) - (c + d*x**3)**(3/2)/9)/d, Ne(d, 0)), (sqrt(c)*x**3/24, T
rue))
```

3.298.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{81\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right) + 2\,(dx^3+c)^{\frac{3}{2}} + 54\,\sqrt{dx^3+c}c}{9\,d}$$

input

```
integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output

```
-1/9*(81*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + 2*(d*x^3 + c)^(3/2) + 54*sqrt(d*x^3 + c)*c)/d
```

3.298.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{18\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d} - \frac{2\left((dx^3+c)^{\frac{3}{2}}d^2+27\sqrt{dx^3+c}cd^2\right)}{9\,d^3}$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
-18*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 2/9*((d*x^3 +
c)^(3/2)*d^2 + 27*sqrt(d*x^3 + c)*c*d^2)/d^3
```

3.298.9 Mupad [B] (verification not implemented)

Time = 7.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{9\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d} - \frac{56\,c\,\sqrt{d\,x^3+c}}{9\,d} - \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9}$$

input
```
int((x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

output
```
(9*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/d - (56*c*(c + d*x^3)^(1/2))/(9*d) - (2*x^3*(c + d*x^3)^(1/2))/9
```

3.299 $\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx$

3.299.1 Optimal result

Integrand size = 27, antiderivative size = 73

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx=-\frac{2}{3}\sqrt{c+dx^3}+\frac{9}{4}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

output
```
9/4*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)-1/12*arctanh((d*x^3+c)^(1
/2)/c^(1/2))*c^(1/2)-2/3*(d*x^3+c)^(1/2)
```

3.299.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx=-\frac{2}{3}\sqrt{c+dx^3}+\frac{9}{4}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]
```

output
```
(-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
4 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12
```

3.299.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {948, 95, 25, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(dx^3+c)^{3/2}}{x^3(8c-dx^3)}dx^3$$

$$\Big\downarrow \text{95}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{cd(10dx^3+c)}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{d}-2\sqrt{c+dx^3}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{\int \frac{cd(10dx^3+c)}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{d}-2\sqrt{c+dx^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(c\int \frac{10dx^3+c}{x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3-2\sqrt{c+dx^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(c\left(\frac{1}{8}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{81}{8}d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3\right)-2\sqrt{c+dx^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(c\left(\frac{81}{4}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4d}\right)-2\sqrt{c+dx^3}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(c\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4d}+\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}\right)-2\sqrt{c+dx^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(c\left(\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{4\sqrt{c}}\right)-2\sqrt{c+dx^3}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]
```

output
```
(-2*Sqrt[c + d*x^3] + c*((27*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(4*Sqrt
[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(4*Sqrt[c])))/3
```

3.299.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

---

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.299.4 Maple [A] (verified)

Time = 4.41 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$\frac{9\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}}{4}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\sqrt{c}}{12}-\frac{2\sqrt{d\,x^{3}+c}}{3}$	52
default	$\frac{\frac{2d\,x^{3}\sqrt{d\,x^{3}+c}}{9}+\frac{8c\sqrt{d\,x^{3}+c}}{9}-\frac{2c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}}{8c}+\frac{81c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-\left(d\,x^{3}+28c\right)\sqrt{d\,x^{3}+c}}{36c}$	100
elliptic	Expression too large to display	1506

input
```
int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
9/4*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)-1/12*arctanh((d*x^3+c)^(1
/2)/c^(1/2))*c^(1/2)-2/3*(d*x^3+c)^(1/2)
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.08

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx = \left[\frac{9}{8}\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right) + \frac{1}{24}\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right) - \frac{2}{3}\sqrt{dx^3+c}, \frac{1}{12}\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - \frac{9}{4}\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right) - \frac{2}{3}\sqrt{dx^3+c}\right]$$

input
```
integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[9/8*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c))
 + 1/24*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2/3*s
qrt(d*x^3 + c), 1/12*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 9/4*sqr
t(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 2/3*sqrt(d*x^3 + c)]
```

3.299.6 Sympy [A] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx = \begin{cases} \frac{2\left(-\frac{9cd\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{8\sqrt{-c}} + \frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24\sqrt{-c}} - \frac{d\sqrt{c+dx^3}}{3}\right)}{d} & \text{for } d \neq 0 \\ \frac{\sqrt{c}\log(x^3)}{24} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c),x)
```

output
```
Piecewise((2*(-9*c*d*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(8*sqrt(-c)) + c*
d*atan(sqrt(c + d*x**3)/sqrt(-c))/(24*sqrt(-c)) - d*sqrt(c + d*x**3)/3)/d,
 Ne(d, 0)), (sqrt(c)*log(x**3)/24, True))
```

3.299.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x\,(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x), x)
```

3.299.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx^3)^{3/2}}{x\,(8c-dx^3)}\,dx = \frac{c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\,\sqrt{-c}} - \frac{9\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{4\,\sqrt{-c}} - \frac{2}{3}\,\sqrt{dx^3+c}$$

input
```
integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
1/12*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 9/4*c*arctan(1/3*sqrt(d
*x^3 + c)/sqrt(-c))/sqrt(-c) - 2/3*sqrt(d*x^3 + c)
```

3.299.9 Mupad [B] (verification not implemented)

Time = 9.97 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx^3)^{3/2}}{x\,(8c-dx^3)}\,dx = \frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\,\left(10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}\right)^{27}}{x^6\,\left(8\,c-d\,x^3\right)^{27}}\right)}{24} - \frac{2\,\sqrt{d\,x^3+c}}{3}$$

input
```
int((c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x)
```

output
```
(c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)
)*(10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))^27)/(x^6*(8*c - d*x^3)^27))
)/24 - (2*(c + d*x^3)^(1/2))/3
```

3.300 $\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx$

3.300.1 Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx=-\frac{\sqrt{c+dx^3}}{24x^3}+\frac{9d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{13d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

output
```
9/32*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-13/96*d*arctanh((d*x^3
+c)^(1/2)/c^(1/2))/c^(1/2)-1/24*(d*x^3+c)^(1/2)/x^3
```

3.300.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx=-\frac{\sqrt{c+dx^3}}{24x^3}+\frac{9d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{13d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x]
```

output
```
-1/24*Sqrt[c + d*x^3]/x^3 + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32
*Sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c])
```

3.300.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {948, 109, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(8c-dx^3\right)}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{\left(dx^3+c\right)^{3/2}}{x^6\left(8c-dx^3\right)}dx^3$$

$$\Big\downarrow \; 109$$

$$\frac{1}{3}\left(-\frac{\int -\frac{cd\left(17dx^3+26c\right)}{2x^3(8c-dx^3)\sqrt{dx^3+c}}dx^3}{8c}-\frac{\sqrt{c+dx^3}}{8x^3}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{1}{16}d\int \frac{17dx^3+26c}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3-\frac{\sqrt{c+dx^3}}{8x^3}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{13}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{81}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3\right)-\frac{\sqrt{c+dx^3}}{8x^3}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{81}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{13\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}\right)-\frac{\sqrt{c+dx^3}}{8x^3}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{13\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)-\frac{\sqrt{c+dx^3}}{8x^3}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}-\frac{13\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)-\frac{\sqrt{c+dx^3}}{8x^3}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x]
```

output
```
(-1/8*Sqrt[c + d*x^3]/x^3 + (d*((27*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(2*Sqrt[c]) - (13*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c])))/16)/3
```

3.300.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.300. $\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.300.4 Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}}{24 x^{3}}+\frac{d\left(-\frac{13\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{9\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)}{16}$
pseudoelliptic	$\frac{-13\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d\,x^{3}+27\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)d\,x^{3}-4\sqrt{d\,x^{3}+c}\,\sqrt{c}}{96 x^{3}\sqrt{c}}$
default	$\frac{-\frac{c\sqrt{d\,x^{3}+c}}{3 x^{3}}+\frac{2 d\sqrt{d\,x^{3}+c}}{3}-\sqrt{c}\,d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{8 c}+\frac{d\left(\frac{2 d\,x^{3}\sqrt{d\,x^{3}+c}}{9}+\frac{8 c\sqrt{d\,x^{3}+c}}{9}-\frac{2 c^{\frac{3}{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}\right)}{64 c^{2}}+\frac{d\left(81c\right.}{}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(d*x^3+c)^(1/2)/x^3+1/16*d*(-13/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c
^(1/2)+9/2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))
```

3.300.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.38

$$\int \frac{\left(c+dx^{3}\right)^{3/2}}{x^{4}\left(8c-dx^{3}\right)}\,dx=\left[\frac{27\sqrt{c}dx^{3}\log\left(\frac{dx^{3}+6\sqrt{dx^{3}+c}\sqrt{c}+10\,c}{dx^{3}-8\,c}\right)+13\sqrt{c}dx^{3}\log\left(\frac{dx^{3}-2\sqrt{dx^{3}+c}\sqrt{c}+2\,c}{x^{3}}\right)-8\sqrt{dx^{3}+}}{192\,cx^{3}}\right.$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[1/192*(27*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d
*x^3 - 8*c)) + 13*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2
*c)/x^3) - 8*sqrt(d*x^3 + c)*c)/(c*x^3), 1/96*(13*sqrt(-c)*d*x^3*arctan(sq
rt(d*x^3 + c)*sqrt(-c)/c) - 27*sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*s
qrt(-c)/c) - 4*sqrt(d*x^3 + c)*c)/(c*x^3)]
```

3.300.6 Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx = -\int \frac{c\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c),x)
```

output
```
-Integral(c*sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x) - Integral(d*x**3*sq
rt(c + d*x**3)/(-8*c*x**4 + d*x**7), x)
```

3.300.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8c)x^4}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^4), x)
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx = \frac{13\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}} - \frac{9\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{32\sqrt{-c}} - \frac{\sqrt{dx^3+c}}{24\,x^3}$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
13/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 9/32*d*arctan(1/3*sqrt
(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 1/24*sqrt(d*x^3 + c)/x^3
```

3.300.9 Mupad [B] (verification not implemented)

Time = 7.73 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.72

$$\int \frac{(c+dx^3)^{3/2}}{x^4\,(8c-dx^3)}\,dx=\frac{9\,d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^3+c}}{3\,\sqrt{c}}\right)}{32\,\sqrt{c}}-\frac{13\,d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{96\,\sqrt{c}}-\frac{\sqrt{d\,x^3+c}}{24\,x^3}$$

input
```
int((c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x)
```

output
```
(9*d*atanh((c + d*x^3)^(1/2)/(3*c^(1/2))))/(32*c^(1/2)) - (13*d*atanh((c +
 d*x^3)^(1/2)/c^(1/2)))/(96*c^(1/2)) - (c + d*x^3)^(1/2)/(24*x^3)
```

3.301 $\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)} dx$

3.301.1 Optimal result

Integrand size = 27, antiderivative size = 104

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)} dx = -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{11d\sqrt{c+dx^3}}{192cx^3} + \frac{9d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}} - \frac{37d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}$$

output
```
9/256*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-37/768*d^2*arctanh(
(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/48*(d*x^3+c)^(1/2)/x^6-11/192*d*(d*x^3+
c)^(1/2)/c/x^3
```

3.301.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)} dx = \frac{(-4c-11dx^3)\sqrt{c+dx^3}}{192cx^6} + \frac{9d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}} - \frac{37d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]
```

output
```
((-4*c - 11*d*x^3)*Sqrt[c + d*x^3])/(192*c*x^6) + (9*d^2*ArcTanh[Sqrt[c +
d*x^3]/(3*Sqrt[c])])/(256*c^(3/2)) - (37*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[
c]])/(768*c^(3/2))
```

3.301.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 109, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^7\left(8c-dx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\left(dx^3+c\right)^{3/2}}{x^9\left(8c-dx^3\right)}dx^3$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{cd\left(35dx^3+44c\right)}{2x^6\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{16c}-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{32}d\int \frac{35dx^3+44c}{x^6\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(-\frac{\int -\frac{2cd\left(11dx^3+74c\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{8c^2}-\frac{11\sqrt{c+dx^3}}{2cx^3}\right)-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{d\int \frac{11dx^3+74c}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{4c}-\frac{11\sqrt{c+dx^3}}{2cx^3}\right)-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{d\left(\frac{37}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{81}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3\right)}{4c}-\frac{11\sqrt{c+dx^3}}{2cx^3}\right)-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{d\left(\frac{81}{2}\int\frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{37\int\frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}\right)}{4c}-\frac{11\sqrt{c+dx^3}}{2cx^3}\right)-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{d\left(\frac{37\int\frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)}{4c}-\frac{11\sqrt{c+dx^3}}{2cx^3}\right)-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{d\left(\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}-\frac{37\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{4c}-\frac{11\sqrt{c+dx^3}}{2cx^3}\right)-\frac{\sqrt{c+dx^3}}{16x^6}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]
```

output
```
(-1/16*Sqrt[c + d*x^3]/x^6 + (d*((-11*Sqrt[c + d*x^3])/(2*c*x^3) + (d*((27
*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*Sqrt[c]) - (37*ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]])/(2*Sqrt[c])))/(4*c)))/32)/3
```

3.301.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.301. $\int \frac{\left(c+dx^3\right)^{3/2}}{x^7\left(8c-dx^3\right)}\,dx$

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.301. $\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx$

3.301.4 Maple [A] (verified)

Time = 4.74 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(11d\,x^{3}+4c\right)}{192x^{6}c}+\frac{d^{2}\left(-\frac{37\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{9\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)}{128c}$
pseudoelliptic	$\frac{-37\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d^{2}x^{6}+27\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)d^{2}x^{6}-44d\,x^{3}\sqrt{d\,x^{3}+c}\,\sqrt{c}-16\sqrt{d\,x^{3}+c}\,c^{\frac{3}{2}}}{768c^{\frac{3}{2}}x^{6}}$
default	$\frac{-\frac{c\sqrt{d\,x^{3}+c}}{6x^{6}}-\frac{5d\sqrt{d\,x^{3}+c}}{12x^{3}}-\frac{d^{2}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4\sqrt{c}}}{8c}+\frac{d\left(-\frac{c\sqrt{d\,x^{3}+c}}{3x^{3}}+\frac{2d\sqrt{d\,x^{3}+c}}{3}-\sqrt{c}\,d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\right)}{64c^{2}}+ d^{2}\left(2d\,x^{3}\right.$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(d*x^3+c)^(1/2)*(11*d*x^3+4*c)/x^6/c+1/128*d^2/c*(-37/6*arctanh((d*
x^3+c)^(1/2)/c^(1/2))/c^(1/2)+9/2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(
1/2))
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.10

$$\int \frac{\left(c+dx^{3}\right)^{3/2}}{x^{7}\left(8c-dx^{3}\right)}\,dx=\left[\frac{27\,\sqrt{c}d^{2}x^{6}\log\left(\frac{dx^{3}+6\,\sqrt{dx^{3}+c}\sqrt{c}+10\,c}{dx^{3}-8\,c}\right)+37\,\sqrt{c}d^{2}x^{6}\log\left(\frac{dx^{3}-2\,\sqrt{dx^{3}+c}\sqrt{c}+2\,c}{x^{3}}\right)-8\,(11\,cdx}{1536\,c^{2}x^{6}}\right.$$

input
```
integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
[1/1536*(27*sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)
/(d*x^3 - 8*c)) + 37*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c
) + 2*c)/x^3) - 8*(11*c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^2*x^6), 1/768*(
37*sqrt(-c)*d^2*x^6*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 27*sqrt(-c)*d^2*x
^6*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(11*c*d*x^3 + 4*c^2)*sqrt(d*
x^3 + c))/(c^2*x^6)]
```

3.301.6 Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = -\int \frac{c\sqrt{c+dx^3}}{-8cx^7+dx^{10}}\,dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^7+dx^{10}}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c),x)
```

output
```
-Integral(c*sqrt(c + d*x**3)/(-8*c*x**7 + d*x**10), x) - Integral(d*x**3*s
qrt(c + d*x**3)/(-8*c*x**7 + d*x**10), x)
```

3.301.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^7}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^7), x)
```

3.301.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = \frac{37\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{768\,\sqrt{-c}c} - \frac{9\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{256\,\sqrt{-c}c} - \frac{11\,(dx^3+c)^{\frac{3}{2}}d^2 - 7\,\sqrt{dx^3+c}cd^2}{192\,cd^2x^6}$$

input
```
integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
37/768*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 9/256*d^2*arcta
n(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/192*(11*(d*x^3 + c)^(3/2)
*d^2 - 7*sqrt(d*x^3 + c)*c*d^2)/(c*d^2*x^6)
```

3.301.9 Mupad [B] (verification not implemented)

Time = 7.91 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = \frac{7\sqrt{d\,x^3+c}}{192\,x^6} - \frac{37\,d^2\,\mathrm{atanh}\left(\frac{c\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{768\,\sqrt{c^3}} + \frac{9\,d^2\,\mathrm{atanh}\left(\frac{c\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{256\,\sqrt{c^3}} - \frac{11\,(d\,x^3+c)^{3/2}}{192\,c\,x^6}$$

input
```
int((c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x)
```

output
```
(7*(c + d*x^3)^(1/2))/(192*x^6) - (37*d^2*atanh((c*(c + d*x^3)^(1/2))/(c^3
)^(1/2)))/(768*(c^3)^(1/2)) + (9*d^2*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^
(1/2))))/(256*(c^3)^(1/2)) - (11*(c + d*x^3)^(3/2))/(192*c*x^6)
```

3.302 $\int \frac{x^7 \left(c+dx^3\right)^{3/2}}{8c-dx^3} \, dx$

3.302.1 Optimal result

Integrand size = 27, antiderivative size = 669

$$\begin{aligned}
\int \frac{x^7 (c+dx^3)^{3/2}}{8c-dx^3} \, dx &= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} \\
&- \frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{288\sqrt{3}c^{19/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}} \\
&+ \frac{288c^{19/6}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}} - \frac{288c^{19/6}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}} \\
&+ \frac{1047324\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)\middle|-7-4\sqrt{3}\right)}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&- \frac{698216\sqrt{2}3^{3/4}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
288*c^(19/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^
(8/3)-288*c^(19/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-288*c^(19/
6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(
8/3)-36534/1729*c^2*x^2*(d*x^3+c)^(1/2)/d^2-348/247*c*x^5*(d*x^3+c)^(1/2)/
d-2/19*x^8*(d*x^3+c)^(1/2)-2094648/1729*c^3*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))-698216/1729*3^(3/4)*c^(10/3)*(c^(1/3)+d^(1/3)*x)
*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+
d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+1047324/1729*3^(1/4)*c
^(10/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(
2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2)))^2)^(1/2)
```

3.302.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.83 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.24

$$\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{-20x^2(18267c^3+19485c^2dx^3+1309cd^2x^6+91d^3x^9)+365340c^3x^2\sqrt{1+\frac{dx^3}{c}}\ \text{Appel}}{17290d^2\sqrt{}}$$

input
```
Integrate[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-20*x^2*(18267*c^3 + 19485*c^2*d*x^3 + 1309*c*d^2*x^6 + 91*d^3*x^9) + 365
340*c^3*x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (
d*x^3)/(8*c)] + 261831*c^2*d*x^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1,
 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(17290*d^2*Sqrt[c + d*x^3])
```

3.302.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {977, 27, 1052, 27, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \left(c+dx^3\right)^{3/2}}{8c-dx^3} dx$$

$$\Big\downarrow \ 977$$

$$-\frac{2\int -\frac{3cdx^7\left(58dx^3+49c\right)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{19d}-\frac{2}{19}x^8\sqrt{c+dx^3}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{19}c\int \frac{x^7\left(58dx^3+49c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx-\frac{2}{19}x^8\sqrt{c+dx^3}$$

$$\Big\downarrow \ 1052$$

$$\frac{3}{19}c\left(\frac{2\int \frac{cdx^4\left(6089dx^3+4640c\right)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{13d^2}-\frac{116x^5\sqrt{c+dx^3}}{13d}\right)-\frac{2}{19}x^8\sqrt{c+dx^3}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{19}c\left(\frac{c\int \frac{x^4\left(6089dx^3+4640c\right)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{13d}-\frac{116x^5\sqrt{c+dx^3}}{13d}\right)-\frac{2}{19}x^8\sqrt{c+dx^3}$$

$$\Big\downarrow \ 1052$$

$$\frac{3}{19}c\left(\frac{c\left(\frac{2\int \frac{2cdx\left(87277dx^3+48712c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d^2}-\frac{12178x^2\sqrt{c+dx^3}}{7d}\right)}{13d}-\frac{116x^5\sqrt{c+dx^3}}{13d}\right)-\frac{2}{19}x^8\sqrt{c+dx^3}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{19}c\left(\frac{c\left(\frac{4c\int \frac{x\left(87277dx^3+48712c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d}-\frac{12178x^2\sqrt{c+dx^3}}{7d}\right)}{13d}-\frac{116x^5\sqrt{c+dx^3}}{13d}\right)-\frac{2}{19}x^8\sqrt{c+dx^3}$$

$$\Big\downarrow \text{1054}$$

$$\frac{3}{19}c\left(\frac{c\left(\frac{4c\int \left(\frac{746928cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}-\frac{87277x}{\sqrt{dx^3+c}}\right)dx}{7d}-\frac{12178x^2\sqrt{c+dx^3}}{7d}\right)}{13d}-\frac{116x^5\sqrt{c+dx^3}}{13d}\right)-\frac{2}{19}x^8\sqrt{c+dx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{19}c\left(\frac{c\left(4c\left(-\frac{174554\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{87277\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\right.}{}\right.\right.}{}\right.$$

$$\frac{2}{19}x^8\sqrt{c+dx^3}$$

3.302. $\int \frac{x^7\left(c+dx^3\right)^{3/2}}{8c-dx^3}dx$

input
```
Int[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-2*x^8*Sqrt[c + d*x^3])/19 + (3*c*((-116*x^5*Sqrt[c + d*x^3])/(13*d) + (c
*((-12178*x^2*Sqrt[c + d*x^3])/(7*d) + (4*c*((-174554*Sqrt[c + d*x^3])/(d^
(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (41496*Sqrt[3]*c^(1/6)*ArcTan
[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(2/3) + (4149
6*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^
(2/3) - (41496*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(2/3) + (87
277*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3
)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (174554*Sq
rt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^
(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*
Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(7*d)))/(13*d)))/19
```

3.302.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 977
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c -
a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p,
q, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.302.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.75 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.34

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	906
default	Expression too large to display	1840

input
```
int(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output

```
-2/1729*x^2*(91*d^2*x^6+1218*c*d*x^3+18267*c^2)*(d*x^3+c)^(1/2)/d^2-12/172
9*c^3/d^2*(-174554/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(
-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
))+27664*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c
*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d
*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/
3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-...
```

3.302.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 24.90 (sec) , antiderivative size = 2453, normalized size of antiderivative = 3.67

$$\int \frac{x^7 (c+dx^3)^{3/2}}{8c-dx^3}\,dx = \text{Too large to display}$$

input

```
integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output

```
2/1729*(1047324*c^3*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse
(0, -4*c/d, x)) + 41496*(c^19/d^16)^(1/6)*d^3*log(1981355655168*((d^16*x^9
 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c^19/d^16)^(5/6) +
6*(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x + 6*(5*c^4*d^12*x^5 + 32*c^5*
d^11*x^2)*(c^19/d^16)^(2/3) + (7*c^10*d^7*x^6 + 152*c^11*d^6*x^3 + 64*c^12
*d^5)*(c^19/d^16)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^7*d^10*x^7 + 64*c^8*d^9
*x^4 + 32*c^9*d^8*x)*sqrt(c^19/d^16) + 18*(c^13*d^5*x^8 + 38*c^14*d^4*x^5
+ 64*c^15*d^3*x^2)*(c^19/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*
x^3 - 512*c^3)) - 41496*(c^19/d^16)^(1/6)*d^3*log(-1981355655168*((d^16*x^
9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c^19/d^16)^(5/6) -
 6*(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x + 6*(5*c^4*d^12*x^5 + 32*c^5
*d^11*x^2)*(c^19/d^16)^(2/3) + (7*c^10*d^7*x^6 + 152*c^11*d^6*x^3 + 64*c^1
2*d^5)*(c^19/d^16)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^7*d^10*x^7 + 64*c^8*d^
9*x^4 + 32*c^9*d^8*x)*sqrt(c^19/d^16) + 18*(c^13*d^5*x^8 + 38*c^14*d^4*x^5
 + 64*c^15*d^3*x^2)*(c^19/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d
*x^3 - 512*c^3)) - 20748*(sqrt(-3)*d^3 - d^3)*(c^19/d^16)^(1/6)*log(198135
5655168*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + s
qrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c
^19/d^16)^(5/6) + 6*(2*c^16*d^2*x^7 + 160*c^17*d*x^4 + 320*c^18*x - 6*(5*c
^4*d^12*x^5 + 32*c^5*d^11*x^2 - sqrt(-3)*(5*c^4*d^12*x^5 + 32*c^5*d^11*...
```

3.302.6 Sympy [F]

$$\int \frac{x^7 \left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx = -\int \frac{cx^7\sqrt{c+dx^3}}{-8c+dx^3}\,dx - \int \frac{dx^{10}\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

input

```
integrate(x**7*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

output

```
-Integral(c*x**7*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**10*s
qrt(c + d*x**3)/(-8*c + d*x**3), x)
```

3.302.7 Maxima [F]

$$\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}x^7}{dx^3-8\,c}\,dx$$

input
```
integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c), x)
```

3.302.8 Giac [F]

$$\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}x^7}{dx^3-8\,c}\,dx$$

input
```
integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-(d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c), x)
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int \frac{x^7\,(d\,x^3+c)^{3/2}}{8\,c-d\,x^3}\,dx$$

input
```
int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

output
```
int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)
```

3.303 $\int \frac{x^4\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$

3.303.1 Optimal result

Integrand size = 27, antiderivative size = 645

$$\begin{aligned}
\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx &= -\frac{240cx^2\sqrt{c+dx^3}}{91d}-\frac{2}{13}x^5\sqrt{c+dx^3} \\
&\quad -\frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{36\sqrt{3}c^{13/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} \\
&\quad +\frac{36c^{13/6}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{36c^{13/6}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}} \\
&\quad +\frac{6891\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad -\frac{4594\sqrt{2}3^{3/4}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
36*c^(13/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(
5/3)-36*c^(13/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-36*c^(13/6)*
arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(5/3
)-240/91*c*x^2*(d*x^3+c)^(1/2)/d-2/13*x^5*(d*x^3+c)^(1/2)-13782/91*c^2*(d*
x^3+c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-4594/91*3^(3/4)*c^(7/
3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+
d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1
/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+
6891/91*3^(1/4)*c^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(
1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/
2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.303.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.59 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.23

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{-80(120c^2x^2+127cdx^5+7d^2x^8)+9600c^2x^2\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\right.}{3640d\sqrt{c+dx^3}}$$

input
```
Integrate[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-80*(120*c^2*x^2 + 127*c*d*x^5 + 7*d^2*x^8) + 9600*c^2*x^2*Sqrt[1 + (d*x^
3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 6891*c*d*x
^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8
*c)])/(3640*d*Sqrt[c + d*x^3])
```

3.303.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {977, 27, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(c+dx^3\right)^{3/2}}{8c-dx^3} \, dx$$

$$\Big\downarrow \text{977}$$

$$-\frac{2\int -\frac{3cdx^4\left(40dx^3+31c\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{13d}-\frac{2}{13}x^5\sqrt{c+dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{13}c\int \frac{x^4\left(40dx^3+31c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx-\frac{2}{13}x^5\sqrt{c+dx^3}$$

$$\Big\downarrow \text{1052}$$

$$\frac{3}{13}c\left(\frac{2\int \frac{cdx\left(2297dx^3+1280c\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d^2}-\frac{80x^2\sqrt{c+dx^3}}{7d}\right)-\frac{2}{13}x^5\sqrt{c+dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{13}c\left(\frac{c\int \frac{x\left(2297dx^3+1280c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d}-\frac{80x^2\sqrt{c+dx^3}}{7d}\right)-\frac{2}{13}x^5\sqrt{c+dx^3}$$

$$\Big\downarrow \text{1054}$$

$$\frac{3}{13}c\left(\frac{c\int \left(\frac{19656cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}-\frac{2297x}{\sqrt{dx^3+c}}\right)dx}{7d}-\frac{80x^2\sqrt{c+dx^3}}{7d}\right)-\frac{2}{13}x^5\sqrt{c+dx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{13}c\left(\frac{c\left(-\frac{4594\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{2297\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{}\right.}{}\right.$$

$$\frac{2}{13}x^5\sqrt{c+dx^3}$$

input
```
Int[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(-2*x^5*Sqrt[c + d*x^3])/13 + (3*c*((-80*x^2*Sqrt[c + d*x^3])/(7*d) + (c*(
(-4594*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (1
092*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c
+ d*x^3]])/d^(2/3) + (1092*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1
/6)*Sqrt[c + d*x^3])])/d^(2/3) - (1092*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*
Sqrt[c])])/d^(2/3) + (2297*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^
(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^
(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c
 + d*x^3]) - (4594*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c
^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d
^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(7*d)
))/13
```

3.303. $\int \frac{x^4\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$

3.303.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 977
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c -
a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p,
q, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.303.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.89 (sec) , antiderivative size = 884, normalized size of antiderivative = 1.37

3.303. $\int \frac{x^4 (c+dx^3)^{3/2}}{8c-dx^3} \, dx$

method	result	size
risch	Expression too large to display	884
elliptic	Expression too large to display	886
default	Expression too large to display	1344

input

```
int(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output

```
-2/91*x^2*(7*d*x^3+120*c)/d*(d*x^3+c)^(1/2)-3/91*c^2/d*(-4594/3*I*3^(1/2)/
d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+728*I/d^3*2^(1/2)*sum(1/_alp
ha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I
*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*
_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(
2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I...
```

3.303.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.32 (sec) , antiderivative size = 2442, normalized size of antiderivative = 3.79

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \text{Too large to display}$$

input

```
integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
1/91*(13782*c^2*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0,
-4*c/d, x)) + 546*(c^13/d^10)^(1/6)*d^2*log(60466176*((d^11*x^9 + 318*c*d^
10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8)*(c^13/d^10)^(5/6) + 6*(c^11*d^2*x
^7 + 80*c^12*d*x^4 + 160*c^13*x + 6*(5*c^3*d^8*x^5 + 32*c^4*d^7*x^2)*(c^13
/d^10)^(2/3) + (7*c^7*d^5*x^6 + 152*c^8*d^4*x^3 + 64*c^9*d^3)*(c^13/d^10)^
(1/3))*sqrt(d*x^3 + c) + 18*(5*c^5*d^7*x^7 + 64*c^6*d^6*x^4 + 32*c^7*d^5*x
)*sqrt(c^13/d^10) + 18*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2)*(
c^13/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 54
6*(c^13/d^10)^(1/6)*d^2*log(-60466176*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c
^2*d^9*x^3 + 640*c^3*d^8)*(c^13/d^10)^(5/6) - 6*(c^11*d^2*x^7 + 80*c^12*d*
x^4 + 160*c^13*x + 6*(5*c^3*d^8*x^5 + 32*c^4*d^7*x^2)*(c^13/d^10)^(2/3) +
(7*c^7*d^5*x^6 + 152*c^8*d^4*x^3 + 64*c^9*d^3)*(c^13/d^10)^(1/3))*sqrt(d*x
^3 + c) + 18*(5*c^5*d^7*x^7 + 64*c^6*d^6*x^4 + 32*c^7*d^5*x)*sqrt(c^13/d^1
0) + 18*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2)*(c^13/d^10)^(1/6
))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 273*(c^13/d^10)^(
1/6)*(sqrt(-3)*d^2 - d^2)*log(60466176*((d^11*x^9 + 318*c*d^10*x^6 + 1200*
c^2*d^9*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2
*d^9*x^3 + 640*c^3*d^8))*(c^13/d^10)^(5/6) + 6*(2*c^11*d^2*x^7 + 160*c^12*
d*x^4 + 320*c^13*x - 6*(5*c^3*d^8*x^5 + 32*c^4*d^7*x^2 - sqrt(-3)*(5*c^3*d
^8*x^5 + 32*c^4*d^7*x^2))*(c^13/d^10)^(2/3) - (7*c^7*d^5*x^6 + 152*c^8*...
```

3.303.6 Sympy [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\int \frac{cx^4\sqrt{c+dx^3}}{-8c+dx^3}\,dx - \int \frac{dx^7\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

input
```
integrate(x**4*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

output
```
-Integral(c*x**4*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**7*sq
rt(c + d*x**3)/(-8*c + d*x**3), x)
```

3.303.7 Maxima [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}x^4}{dx^3-8\,c}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c), x)
```

3.303.8 Giac [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}x^4}{dx^3-8\,c}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-(d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c), x)
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int \frac{x^4\,(d\,x^3+c)^{3/2}}{8\,c-d\,x^3}\,dx$$

input
```
int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

output
```
int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)
```

3.304 $\int \frac{x\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$

3.304.1 Optimal result

Integrand size = 25, antiderivative size = 627

$$\begin{aligned}
\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx &= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&- \frac{9\sqrt{3}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} \\
&+ \frac{9c^{7/6}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}} - \frac{9c^{7/6}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2d^{2/3}} \\
&+ \frac{66\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&- \frac{44\sqrt{2}3^{3/4}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
9/2*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(
2/3)-9/2*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(2/3)-9/2*c^(7/6)*
arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(2/3
)-2/7*x^2*(d*x^3+c)^(1/2)-132/7*c*(d*x^3+c)^(1/2)/d^(2/3)/(d^(1/3)*x+c^(1/
3)*(1+3^(1/2)))-44/7*3^(3/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3
)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^
(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/
2)))^2)^(1/2)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+66/7*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)
*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2
/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(2/3)/(d*x^3+c)^(1/2)/
(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.304.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 9.98 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.20

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{x^2\Bigg(-160(c+dx^3)+195c\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+132dx^3\sqrt{1+}}{560\sqrt{c+dx^3}}$$

input
```
Integrate[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

output
```
(x^2*(-160*(c + d*x^3) + 195*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 132*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[
5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(560*Sqrt[c + d*x^3])
```

3.304.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {977, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx^3\right)^{3/2}}{8c-dx^3}dx$$

$$\Big\downarrow \; 977$$

$$-\frac{2\int -\frac{3cdx\left(22dx^3+13c\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d}-\frac{2}{7}x^2\sqrt{c+dx^3}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{7}c\int \frac{x\left(22dx^3+13c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx-\frac{2}{7}x^2\sqrt{c+dx^3}$$

$$\Big\downarrow \; 1054$$

$$\frac{3}{7}c\int \left(\frac{189cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}-\frac{22x}{\sqrt{dx^3+c}}\right)dx-\frac{2}{7}x^2\sqrt{c+dx^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{3}{7}c\left(-\frac{44\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{22\sqrt[4]{3}}{}\right.$$

$$\frac{2}{7}x^2\sqrt{c+dx^3}$$

input
```
Int[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

3.304. $\int \frac{x\left(c+dx^3\right)^{3/2}}{8c-dx^3}dx$

output
```
(-2*x^2*Sqrt[c + d*x^3])/7 + (3*c*((-44*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)) - (21*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)
*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*d^(2/3)) + (21*c^(1/6)*ArcTan
h[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(2*d^(2/3)) - (21*
c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*d^(2/3)) + (22*3^(1/4)*Sq
rt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (44*Sqrt[2]*c^(1/3)*(c^(
1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*Sqrt[c + d*x^3]))/7
```

3.304.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 977
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c -
a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p,
q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.304. $\int \frac{x\left(c+dx^3\right)^{3/2}}{8c-dx^3} dx$

3.304.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.85 (sec) , antiderivative size = 864, normalized size of antiderivative = 1.38

method	result	size
default	Expression too large to display	864
elliptic	Expression too large to display	864
risch	Expression too large to display	866

input

```
int(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output

```
-2/7*x^2*(d*x^3+c)^(1/2)+44/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)))-3*I*c/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(
x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*
(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c
*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_a...
```

3.304.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.81 (sec) , antiderivative size = 2368, normalized size of antiderivative = 3.78

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
-1/56*(16*sqrt(d*x^3 + c)*d*x^2 - 1056*c*sqrt(d)*weierstrassZeta(0, -4*c/d
, weierstrassPInverse(0, -4*c/d, x)) + 21*(c^7/d^4)^(1/6)*(sqrt(-3)*d - d)
*log(59049/4*((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 +
sqrt(-3)*(d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3))*(c^7/
d^4)^(5/6) + 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^4*x
^5 + 32*c^3*d^3*x^2 - sqrt(-3)*(5*c^2*d^4*x^5 + 32*c^3*d^3*x^2))*(c^7/d^4)
^(2/3) - (7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d + sqrt(-3)*(7*c^4*d^3
*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d))*(c^7/d^4)^(1/3))*sqrt(d*x^3 + c) - 36*
(5*c^3*d^4*x^7 + 64*c^4*d^3*x^4 + 32*c^5*d^2*x)*sqrt(c^7/d^4) + 18*(c^5*d^
3*x^8 + 38*c^6*d^2*x^5 + 64*c^7*d*x^2 - sqrt(-3)*(c^5*d^3*x^8 + 38*c^6*d^2
*x^5 + 64*c^7*d*x^2))*(c^7/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d
*x^3 - 512*c^3)) - 21*(c^7/d^4)^(1/6)*(sqrt(-3)*d - d)*log(-59049/4*((d^6*
x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + sqrt(-3)*(d^6*x^9 +
 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3))*(c^7/d^4)^(5/6) - 6*(2*c
^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^4*x^5 + 32*c^3*d^3*x^2
 - sqrt(-3)*(5*c^2*d^4*x^5 + 32*c^3*d^3*x^2))*(c^7/d^4)^(2/3) - (7*c^4*d^3
*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d + sqrt(-3)*(7*c^4*d^3*x^6 + 152*c^5*d^2*
x^3 + 64*c^6*d))*(c^7/d^4)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^4*x^7 + 64
*c^4*d^3*x^4 + 32*c^5*d^2*x)*sqrt(c^7/d^4) + 18*(c^5*d^3*x^8 + 38*c^6*d^2*
x^5 + 64*c^7*d*x^2 - sqrt(-3)*(c^5*d^3*x^8 + 38*c^6*d^2*x^5 + 64*c^7*d*...
```

3.304.6 Sympy [F]

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\int \frac{cx\sqrt{c+dx^3}}{-8c+dx^3}\,dx - \int \frac{dx^4\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

input
```
integrate(x*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

output
```
-Integral(c*x*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**4*sqrt(
c + d*x**3)/(-8*c + d*x**3), x)
```

3.304.7 Maxima [F]

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}x}{dx^3-8\,c}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c), x)
```

3.304.8 Giac [F]

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}x}{dx^3-8\,c}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-(d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c), x)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int \frac{x\,(d\,x^3+c)^{3/2}}{8\,c-d\,x^3}\,dx$$

input
```
int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

output
```
int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)
```

3.305 $\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx$

3.305.1 Optimal result

Integrand size = 27, antiderivative size = 626

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{8x} - \frac{15\sqrt[3]{d}\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$-\frac{9}{16}\sqrt{3}\sqrt[6]{c}\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)$$

$$+\frac{9}{16}\sqrt[6]{c}\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right) - \frac{9}{16}\sqrt[6]{c}\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)$$

$$+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{5\ 3^{3/4}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
9/16*c^(1/6)*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(
1/2))-9/16*c^(1/6)*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-9/16*c^(1/
6)*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(
1/2)-1/8*(d*x^3+c)^(1/2)/x-15/8*d^(1/3)*(d*x^3+c)^(1/2)/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2)))-5/8*3^(3/4)*c^(1/3)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d
^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*
I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)^2)^(1/2)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x
+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+15/16*3^(1/4)*c^(1/3)*d^(1/3)*(c^(1/3)+d^(1
/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*
x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/(d*x^3+c)^(1/2)/(c
^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.305.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.22

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = \frac{-16c(c+dx^3)+21cdx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3d^2x^6\sqrt{1+\frac{dx^3}{c}}}{128cx\sqrt{c+dx^3}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x]
```

output
```
(-16*c*(c + d*x^3) + 21*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1
[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(128*c*x*Sqrt[c + d*x^3])
```

3.305.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 620, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {974, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)}dx$$

$$\Big\downarrow \; 974$$

$$\frac{\int \frac{3cdx\left(5dx^3+14c\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c}-\frac{\sqrt{c+dx^3}}{8x}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{16}d\int \frac{x\left(5dx^3+14c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx-\frac{\sqrt{c+dx^3}}{8x}$$

$$\Big\downarrow \; 1054$$

$$\frac{3}{16}d\int \left(\frac{54cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}-\frac{5x}{\sqrt{dx^3+c}}\right)dx-\frac{\sqrt{c+dx^3}}{8x}$$

$$\Big\downarrow \; 2009$$

$$\frac{3}{16}d\left(-\frac{10\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\right. \quad 5\sqrt[4]{3}$$

$$\frac{\sqrt{c+dx^3}}{8x}$$

input
```
Int[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x]
```

3.305. $\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)}dx$

output

```
-1/8*Sqrt[c + d*x^3]/x + (3*d*((-10*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)) - (3*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(2/3) + (3*c^(1/6)*ArcTanh[(c^(1/3)
 + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(2/3) - (3*c^(1/6)*ArcTanh
[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(2/3) + (5*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1
/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)
/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^
(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/
(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*Sqrt[c + d*x^3]) - (10*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
d*x^3])))/16
```

3.305.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 974

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.305. $\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)}\,dx$

3.305.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.89 (sec) , antiderivative size = 859, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	859
risch	Expression too large to display	866
default	Expression too large to display	1339

input

```
int((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output

```
-1/8*(d*x^3+c)^(1/2)/x+5/8*I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1
/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/
d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(
1/2)))-3/8*I/d^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*...
```

3.305.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
Timed out
```

3.305.6 Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = -\int \frac{c\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**2/(-d*x**3+8*c),x)
```

output
```
-Integral(c*sqrt(c + d*x**3)/(-8*c*x**2 + d*x**5), x) - Integral(d*x**3*sq
rt(c + d*x**3)/(-8*c*x**2 + d*x**5), x)
```

3.305.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^2), x)
```

3.305.8 Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^2), x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^2\,(8\,c-d\,x^3)}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)), x)
```

3.306 $\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx$

3.306.1 Optimal result

Integrand size = 27, antiderivative size = 651

$$\begin{aligned}\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx &= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}\\
&+\frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{9\sqrt{3}d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{5/6}}\\
&+\frac{9d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{5/6}}-\frac{9d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{5/6}}\\
&-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\\
&+\frac{3^{3/4}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\end{aligned}$$

output

```
9/128*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(5/6)-9/128*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)-9/128*d^
(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/
c^(5/6)-1/32*(d*x^3+c)^(1/2)/x^4-3/16*d*(d*x^3+c)^(1/2)/c/x+3/16*d^(4/3)*(
d*x^3+c)^(1/2)/c/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/16*3^(3/4)*d^(4/3)*(c^(
1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^
(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/32*3^
(1/4)*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2))
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))
*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)/c^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.306.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.24

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = \frac{645cd^2x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\Big(40c(c^2+7cdx^3+6d^2x^6)+3}{5120c^2x^4\sqrt{c+dx^3}}$$

input

```
Integrate[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x]
```

output

```
(645*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, (d*x^3)/(8*c)] - 4*(40*c*(c^2 + 7*c*d*x^3 + 6*d^2*x^6) + 3*d^3*x^9*Sqrt[
1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(
5120*c^2*x^4*Sqrt[c + d*x^3])
```

3.306.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {974, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^5\left(8c-dx^3\right)}dx$$

$$\Big\downarrow \text{ 974}$$

$$\frac{\int \frac{3cd\left(23dx^3+32c\right)}{2x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{\sqrt{c+dx^3}}{32x^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{64}d\int \frac{23dx^3+32c}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx-\frac{\sqrt{c+dx^3}}{32x^4}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{3}{64}d\left(-\frac{\int -\frac{8cdx\left(43c-2dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{4\sqrt{c+dx^3}}{cx}\right)-\frac{\sqrt{c+dx^3}}{32x^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{64}d\left(\frac{d\int \frac{x\left(43c-2dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{4\sqrt{c+dx^3}}{cx}\right)-\frac{\sqrt{c+dx^3}}{32x^4}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{3}{64}d\left(\frac{d\int \left(\frac{27cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{2x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{4\sqrt{c+dx^3}}{cx}\right)-\frac{\sqrt{c+dx^3}}{32x^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{64}d\left(d\left(\frac{4\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{2\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\right.}{}\right.\right.$$

$$\frac{\sqrt{c+dx^3}}{32x^4}$$

input
```
Int[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x]
```

output
```
-1/32*Sqrt[c + d*x^3]/x^4 + (3*d*((-4*Sqrt[c + d*x^3])/(c*x) + (d*((4*Sqrt
[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (3*Sqrt[3]*c^
(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*
d^(2/3)) + (3*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c +
d*x^3])])/(2*d^(2/3)) - (3*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(
2*d^(2/3)) - (2*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1
/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
+ (4*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
-7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/c))/64
```

3.306. $\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx$

3.306.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.306.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.96 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.35

method	result	size
risch	Expression too large to display	882
elliptic	Expression too large to display	884
default	Expression too large to display	1810

input
```
int((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(d*x^3+c)^(1/2)*(6*d*x^3+c)/x^4/c+3/64*d^2/c*(-4/3*I*3^(1/2)/d*(-c*d
^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/
2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^
(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*...
```

3.306.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 2369, normalized size of antiderivative = 3.64

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
-1/512*(96*d^(3/2)*x^4*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -
4*c/d, x)) - 6*(d^8/c^5)^(1/6)*c*x^4*log(6561*(d^9*x^9 + 318*c*d^8*x^6 + 1
200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 + 32*c^
6*d*x)*(d^8/c^5)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2)*(
d^8/c^5)^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(d^8/c
^5) + (c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^3*d^5*x)*(d^8/c^5)^(1/6)) + 18*(
c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2)*(d^8/c^5)^(1/3))/(d^3*x^9 -
 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 6*(d^8/c^5)^(1/6)*c*x^4*log(65
61*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^4*d
^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x)*(d^8/c^5)^(2/3) - 6*sqrt(d*x^3 + c)*
(6*(5*c^5*d*x^5 + 32*c^6*x^2)*(d^8/c^5)^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d
^3*x^3 + 64*c^5*d^2)*sqrt(d^8/c^5) + (c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^3
*d^5*x)*(d^8/c^5)^(1/6)) + 18*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x
^2)*(d^8/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) -
 3*(sqrt(-3)*c*x^4 + c*x^4)*(d^8/c^5)^(1/6)*log(6561*(d^9*x^9 + 318*c*d^8*
x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 +
 32*c^6*d*x + sqrt(-3)*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x))*(d^8
/c^5)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2 - sqrt(-3)*(5
*c^5*d*x^5 + 32*c^6*x^2))*(d^8/c^5)^(5/6) - 2*(7*c^3*d^4*x^6 + 152*c^4*d^3
*x^3 + 64*c^5*d^2)*sqrt(d^8/c^5) + (c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^...
```

3.306.6 Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = -\int \frac{c\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**5/(-d*x**3+8*c),x)
```

output
```
-Integral(c*sqrt(c + d*x**3)/(-8*c*x**5 + d*x**8), x) - Integral(d*x**3*sq
rt(c + d*x**3)/(-8*c*x**5 + d*x**8), x)
```

3.306.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^5}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^5), x)
```

3.306.8 Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^5}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^5), x)
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^5\,(8\,c-d\,x^3)}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)), x)
```

3.307 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)}\,dx$

3.307.1 Optimal result

Integrand size = 27, antiderivative size = 675

$$\begin{aligned}
\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)}\,dx &= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}\\
&+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{9\sqrt{3}d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}\\
&+\frac{9d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}-\frac{9d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{11/6}}\\
&-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{112c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\\
&+\frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{28\sqrt{2}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
9/1024*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
c^(11/6)-9/1024*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-9/10
24*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(
1/2)/c^(11/6)-1/56*(d*x^3+c)^(1/2)/x^7-75/1792*d*(d*x^3+c)^(1/2)/c/x^4-3/5
6*d^2*(d*x^3+c)^(1/2)/c^2/x+3/56*d^(7/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))+1/56*3^(3/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(
1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)
*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/112*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3
)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+
d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1
/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.307.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.25

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)}\,dx=\frac{6675cd^3x^9\sqrt{1+\frac{dx^3}{c}}\,\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-32\Big(5c(32c^3+107c^2dx^3+171c}{286720c^3x^7\sqrt{c+d}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x]
```

output
```
(6675*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c
), (d*x^3)/(8*c)] - 32*(5*c*(32*c^3 + 107*c^2*d*x^3 + 171*c*d^2*x^6 + 96*d
^3*x^9) + 6*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x
^3)/c), (d*x^3)/(8*c)]))/(286720*c^3*x^7*Sqrt[c + d*x^3])
```

3.307.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {974, 27, 1053, 25, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)}dx$$

$$\Big\downarrow \text{ 974}$$

$$\frac{\int \frac{3cd\left(41dx^3+50c\right)}{2x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{56c}-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{112}d\int \frac{41dx^3+50c}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{3}{112}d\left(-\frac{\int -\frac{cd\left(125dx^3+512c\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{25\sqrt{c+dx^3}}{16cx^4}\right)-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3}{112}d\left(\frac{\int \frac{cd\left(125dx^3+512c\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{25\sqrt{c+dx^3}}{16cx^4}\right)-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{112}d\left(\frac{d\int \frac{125dx^3+512c}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{25\sqrt{c+dx^3}}{16cx^4}\right)-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{3}{112}d\left(\frac{d\left(-\frac{\int -\frac{8cdx\left(445c-32dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{64\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{25\sqrt{c+dx^3}}{16cx^4}\right)-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{112}d\left(\frac{d\left(\frac{d\int \frac{x\left(445c-32dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{64\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{25\sqrt{c+dx^3}}{16cx^4}\right)-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{3}{112}d\left(\frac{d\left(\frac{d\int \left(\frac{189cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{32x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{64\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{25\sqrt{c+dx^3}}{16cx^4}\right)-\frac{\sqrt{c+dx^3}}{56x^7}$$

$$\Big\downarrow \text{ 2009}$$

3.307. $\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)}\,dx$

$$\frac{3}{112}d\left(\frac{d\left(\frac{d\left(\frac{64\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{32\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{\right.}{}\right.}{}\right.}{}\right.}{}\right.$$

$$\frac{\sqrt{c+dx^3}}{56x^7}$$

input
```
Int[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x]
```

3.307. $\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx$

output
```
-1/56*Sqrt[c + d*x^3]/x^7 + (3*d*((-25*Sqrt[c + d*x^3])/(16*c*x^4) + (d*((
-64*Sqrt[c + d*x^3])/(c*x) + (d*((64*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) - (21*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c
^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*d^(2/3)) + (21*c^(1/6)*ArcTanh[(
c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(2*d^(2/3)) - (21*c^(
1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*d^(2/3)) - (32*3^(1/4)*Sqrt[
2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
-7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (64*Sqrt[2]*c^(1/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(
1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^
(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3])))/c))/(32*c)))/112
```

3.307.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.307.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.93 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.33

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	903
default	Expression too large to display	2306

input
```
int((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

output
```
-1/1792*(d*x^3+c)^(1/2)*(96*d^2*x^6+75*c*d*x^3+32*c^2)/x^7/c^2+3/3584*d^3/
c^2*(-64/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1
/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/
3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-7*I/d^3
*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(
-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*
d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/...
```

3.307.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.51 (sec) , antiderivative size = 2442, normalized size of antiderivative = 3.62

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)}\,dx=\text{Too large to display}$$

input
```
integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="fricas")
```

output
```
-1/28672*(1536*d^(5/2)*x^7*weierstrassZeta(0, -4*c/d, weierstrassPInverse(
0, -4*c/d, x)) - 42*(d^14/c^11)^(1/6)*c^2*x^7*log(6561*(d^14*x^9 + 318*c*d
^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^8*d^4*x^7 + 64*c^9*d^
3*x^4 + 32*c^10*d^2*x)*(d^14/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*
x^5 + 32*c^11*x^2)*(d^14/c^11)^(5/6) + (7*c^6*d^6*x^6 + 152*c^7*d^5*x^3 +
64*c^8*d^4)*sqrt(d^14/c^11) + (c^2*d^11*x^7 + 80*c^3*d^10*x^4 + 160*c^4*d^
9*x)*(d^14/c^11)^(1/6)) + 18*(c^4*d^9*x^8 + 38*c^5*d^8*x^5 + 64*c^6*d^7*x^
2)*(d^14/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
+ 42*(d^14/c^11)^(1/6)*c^2*x^7*log(6561*(d^14*x^9 + 318*c*d^13*x^6 + 1200*
c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^8*d^4*x^7 + 64*c^9*d^3*x^4 + 32*c^10
*d^2*x)*(d^14/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x
^2)*(d^14/c^11)^(5/6) + (7*c^6*d^6*x^6 + 152*c^7*d^5*x^3 + 64*c^8*d^4)*sqr
t(d^14/c^11) + (c^2*d^11*x^7 + 80*c^3*d^10*x^4 + 160*c^4*d^9*x)*(d^14/c^11
)^(1/6)) + 18*(c^4*d^9*x^8 + 38*c^5*d^8*x^5 + 64*c^6*d^7*x^2)*(d^14/c^11)^
(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 21*(sqrt(-3)*
c^2*x^7 + c^2*x^7)*(d^14/c^11)^(1/6)*log(6561*(d^14*x^9 + 318*c*d^13*x^6 +
 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^8*d^4*x^7 + 64*c^9*d^3*x^4 + 32
*c^10*d^2*x + sqrt(-3)*(5*c^8*d^4*x^7 + 64*c^9*d^3*x^4 + 32*c^10*d^2*x))*(
d^14/c^11)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt
(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^14/c^11)^(5/6) - 2*(7*c^6*d^6*x^6...
```

3.307.6 Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = -\int \frac{c\sqrt{c+dx^3}}{-8cx^8+dx^{11}}\,dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^8+dx^{11}}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**8/(-d*x**3+8*c),x)
```

output
```
-Integral(c*sqrt(c + d*x**3)/(-8*c*x**8 + d*x**11), x) - Integral(d*x**3*s
qrt(c + d*x**3)/(-8*c*x**8 + d*x**11), x)
```

3.307.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^8}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="maxima")
```

output
```
-integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^8), x)
```

3.307.8 Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^8}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="giac")
```

output
```
integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^8), x)
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^8\,(8\,c-d\,x^3)}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)), x)
```

3.308 $\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.308.1 Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{38c^2\sqrt{c+dx^3}}{d^4} - \frac{4c(c+dx^3)^{3/2}}{3d^4} - \frac{2(c+dx^3)^{5/2}}{15d^4} + \frac{1024c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}$$

output
```
-4/3*c*(d*x^3+c)^(3/2)/d^4-2/15*(d*x^3+c)^(5/2)/d^4+1024/9*c^(5/2)*arctanh
(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4-38*c^2*(d*x^3+c)^(1/2)/d^4
```

3.308.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{-6\sqrt{c+dx^3}(296c^2+12cdx^3+d^2x^6)+5120c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{45d^4}$$

input
```
Integrate[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-6*Sqrt[c + d*x^3]*(296*c^2 + 12*c*d*x^3 + d^2*x^6) + 5120*c^(5/2)*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^4)
```

3.308.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9}{(8c-dx^3)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{512c^3}{d^3(8c-dx^3)\sqrt{dx^3+c}}-\frac{57c^2}{d^3\sqrt{dx^3+c}}-\frac{6\sqrt{dx^3+c}c}{d^3}-\frac{(dx^3+c)^{3/2}}{d^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1024c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3d^4}-\frac{114c^2\sqrt{c+dx^3}}{d^4}-\frac{4c(c+dx^3)^{3/2}}{d^4}-\frac{2(c+dx^3)^{5/2}}{5d^4}\right)$$

input
```
Int[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
((-114*c^2*Sqrt[c + d*x^3])/d^4 - (4*c*(c + d*x^3)^(3/2))/d^4 - (2*(c + d*
x^3)^(5/2))/(5*d^4) + (1024*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(3*d^4))/3
```

3.308.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.308.4 Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{5120c^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)-6\sqrt{dx^3+c}\,(d^2x^6+12cdx^3+296c^2)}{45d^4}$
risch	$-\frac{2(d^2x^6+12cdx^3+296c^2)\sqrt{dx^3+c}}{15d^4}+\frac{1024c^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9d^4}$
default	$-\frac{\frac{2x^6\sqrt{dx^3+c}}{15d}-\frac{8cx^3\sqrt{dx^3+c}}{45d^2}+\frac{16c^2\sqrt{dx^3+c}}{45d^3}}{d}-\frac{8c\left(\frac{2x^3\sqrt{dx^3+c}}{9d}-\frac{4c\sqrt{dx^3+c}}{9d^2}\right)}{d^2}-\frac{128c^2\sqrt{dx^3+c}}{3d^4}+\frac{1024c^{\frac{5}{2}}\operatorname{arctanh}\left(\sqrt{\right.}{9d^4}$
elliptic	$-\frac{2x^6\sqrt{dx^3+c}}{15d^2}-\frac{8cx^3\sqrt{dx^3+c}}{5d^3}-\frac{592c^2\sqrt{dx^3+c}}{15d^4}-\frac{512ic^2\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-cd^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-\right.}{}\right.}{\left(-\right.}}}{}\right.}{}$

input
```
int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/45*(5120*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-6*(d*x^3+c)^(1/2)*
(d^2*x^6+12*c*d*x^3+296*c^2))/d^4
```

3.308.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{2\left(1280\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\,(d^2x^6+12\,cdx^3+296\,c^2)\sqrt{dx^3+c}\right)}{45\,d^4},\right.$$
$$\left.-\frac{2\left(2560\,\sqrt{-c}c^2\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\,(d^2x^6+12\,cdx^3+296\,c^2)\sqrt{dx^3+c}\right)}{45\,d^4}\right]$$

input
```
integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[2/45*(1280*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
- 8*c)) - 3*(d^2*x^6 + 12*c*d*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4, -2/45*(
2560*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(d^2*x^6 + 12
*c*d*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4]
```

3.308.6 Sympy [A] (verification not implemented)

Time = 16.60 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\left(-\frac{512c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9\sqrt{-c}}-19c^2\sqrt{c+dx^3}-\frac{2c\left(c+dx^3\right)^{\frac{3}{2}}}{3}-\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^4} & \text{for } d\neq 0 \\ \frac{x^{12}}{96c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((2*(-512*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*sqrt(-c)) -
 19*c**2*sqrt(c + d*x**3) - 2*c*(c + d*x**3)**(3/2)/3 - (c + d*x**3)**(5/2
)/15)/d**4, Ne(d, 0)), (x**12/(96*c**(3/2)), True))
```

3.308.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(1280\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+3\,(dx^3+c)^{\frac{5}{2}}+30\,(dx^3+c)^{\frac{3}{2}}c+855\,\sqrt{dx^3+c}c^2\right)}{45\,d^4}$$

input
```
integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-2/45*(1280*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3
*sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 30*(d*x^3 + c)^(3/2)*c + 855*sqrt(d*x^3
 + c)*c^2)/d^4
```

3.308.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{1024\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{9\,\sqrt{-c}d^4} - \frac{2\left((dx^3+c)^{\frac{5}{2}}d^{16}+10\,(dx^3+c)^{\frac{3}{2}}cd^{16}+285\,\sqrt{dx^3+c}c^2d^{16}\right)}{15\,d^{20}}$$

input
```
integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-1024/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/15*((d
*x^3 + c)^(5/2)*d^16 + 10*(d*x^3 + c)^(3/2)*c*d^16 + 285*sqrt(d*x^3 + c)*c
^2*d^16)/d^20
```

3.308.9 Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{512\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^4}-\frac{592\,c^2\,\sqrt{d\,x^3+c}}{15\,d^4} -\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d^2}-\frac{8\,c\,x^3\,\sqrt{d\,x^3+c}}{5\,d^3}$$

input
```
int(x^11/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
(512*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/(9*d^4) - (592*c^2*(c + d*x^3)^(1/2))/(15*d^4) - (2*x^6*(c + d*x^3)^(1
/2))/(15*d^2) - (8*c*x^3*(c + d*x^3)^(1/2))/(5*d^3)
```

3.309 $\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.309.1 Optimal result

Integrand size = 27, antiderivative size = 71

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{14c\sqrt{c+dx^3}}{3d^3} - \frac{2(c+dx^3)^{3/2}}{9d^3} + \frac{128c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

output
```
-2/9*(d*x^3+c)^(3/2)/d^3+128/9*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2)
)/d^3-14/3*c*(d*x^3+c)^(1/2)/d^3
```

3.309.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{-2\sqrt{c+dx^3}(22c+dx^3)+128c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

input
```
Integrate[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-2*Sqrt[c + d*x^3]*(22*c + d*x^3) + 128*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])])/(9*d^3)
```

3.309.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{(8c-dx^3)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{64c^2}{d^2(8c-dx^3)\sqrt{dx^3+c}}-\frac{7c}{d^2\sqrt{dx^3+c}}-\frac{\sqrt{dx^3+c}}{d^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{128c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3d^3}-\frac{14c\sqrt{c+dx^3}}{d^3}-\frac{2(c+dx^3)^{3/2}}{3d^3}\right)$$

input
```
Int[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
((-14*c*Sqrt[c + d*x^3])/d^3 - (2*(c + d*x^3)^(3/2))/(3*d^3) + (128*c^(3/2
)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^3))/3
```

3.309.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.309.4 Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{\frac{128c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9}-\frac{2\left(dx^3+22c\right)\sqrt{dx^3+c}}{9}}{d^3}$
risch	$-\frac{2(dx^3+22c)\sqrt{dx^3+c}}{9d^3}+\frac{128c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9d^3}$
default	$-\frac{\frac{2x^3\sqrt{dx^3+c}}{9d}-\frac{4c\sqrt{dx^3+c}}{9d^2}}{d}-\frac{16c\sqrt{dx^3+c}}{3d^3}+\frac{128c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9d^3}$
elliptic	$-\frac{2x^3\sqrt{dx^3+c}}{9d^2}-\frac{44c\sqrt{dx^3+c}}{9d^3}-\frac{64ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-cd^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{}}{}\right.}{}$

input
```
int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(64*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-(d*x^3+22*c)*(d*x^3+c
)^(1/2))/d^3
```

3.309.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{2\left(32\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(dx^3+22\,c)\sqrt{dx^3+c}\right)}{9\,d^3},\right.$$
$$\left.-\frac{2\left(64\,\sqrt{-c}c\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(dx^3+22\,c)\sqrt{dx^3+c}\right)}{9\,d^3}\right]$$

input

```
integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output

```
[2/9*(32*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8
*c)) - (d*x^3 + 22*c)*sqrt(d*x^3 + c))/d^3, -2/9*(64*sqrt(-c)*c*arctan(1/3
*sqrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 22*c)*sqrt(d*x^3 + c))/d^3]
```

3.309.6 Sympy [A] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\left(-\frac{64c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9\sqrt{-c}}-\frac{7c\sqrt{c+dx^3}}{3}-\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{x^9}{72c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input

```
integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output

```
Piecewise((2*(-64*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*sqrt(-c)) -
7*c*sqrt(c + d*x**3)/3 - (c + d*x**3)**(3/2)/9)/d**3, Ne(d, 0)), (x**9/(72
*c**(3/2)), True))
```

3.309.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(32\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+(dx^3+c)^{\frac{3}{2}}+21\sqrt{dx^3+c}c\right)}{9\,d^3}$$

input
```
integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-2/9*(32*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + (d*x^3 + c)^(3/2) + 21*sqrt(d*x^3 + c)*c)/d^3
```

3.309.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{128\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d^3}-\frac{2\left((dx^3+c)^{\frac{3}{2}}d^6+21\sqrt{dx^3+c}cd^6\right)}{9\,d^9}$$

input
```
integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-128/9*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/9*((d*x
^3 + c)^(3/2)*d^6 + 21*sqrt(d*x^3 + c)*c*d^6)/d^9
```

3.309.9 Mupad [B] (verification not implemented)

Time = 7.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{64\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^3}-\frac{44\,c\,\sqrt{d\,x^3+c}}{9\,d^3}-\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2}$$

input
```
int(x^8/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
(64*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/(9*d^3) - (44*c*(c + d*x^3)^(1/2))/(9*d^3) - (2*x^3*(c + d*x^3)^(1/2))/
(9*d^2)
```

3.310 $\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.310.1 Optimal result

Integrand size = 27, antiderivative size = 52

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\sqrt{c+dx^3}}{3d^2}+\frac{16\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

output
```
16/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^2-2/3*(d*x^3+c)^(1/2)/
d^2
```

3.310.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(3\sqrt{c+dx^3}-8\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{9d^2}$$

input
```
Integrate[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-2*(3*Sqrt[c + d*x^3] - 8*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/
(9*d^2)
```

3.310.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {948, 90, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(8c-dx^3)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{8c\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{d}-\frac{2\sqrt{c+dx^3}}{d^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{16c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d^2}-\frac{2\sqrt{c+dx^3}}{d^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{16\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3d^2}-\frac{2\sqrt{c+dx^3}}{d^2}\right)$$

input
```
Int[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
((-2*Sqrt[c + d*x^3])/d^2 + (16*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)])/(3*d^2))/3
```

3.310.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.310.4 Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{-\frac{2\sqrt{d\,x^{3}+c}}{3}+\frac{16\ \mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}}{9}}{d^{2}}$
default	$\frac{16\ \mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}}{9d^{2}}-\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}$
risch	$\frac{16\ \mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}}{9d^{2}}-\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}$
elliptic	$-\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}-\frac{8i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d\right.}}}{}\right.}{}$

input
```
int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(8*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)-3*(d*x^3+c)^(1/2))/d^2
```

3.310.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.98

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{2\left(4\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-3\sqrt{dx^3+c}\right)}{9d^2},\ -\frac{2\left(8\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+3\sqrt{dx^3+c}\right)}{9d^2}\right]$$

input
```
integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[2/9*(4*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - 3*sqrt(d*x^3 + c))/d^2, -2/9*(8*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*
sqrt(-c)/c) + 3*sqrt(d*x^3 + c))/d^2]
```

3.310.6 Sympy [A] (verification not implemented)

Time = 4.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\left(-\frac{8c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9\sqrt{-c}} - \frac{\sqrt{c+dx^3}}{3}\right)}{d^2} & \text{for } d \neq 0 \\ \frac{x^6}{48c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((2*(-8*c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*sqrt(-c)) - sqrt
(c + d*x**3)/3)/d**2, Ne(d, 0)), (x**6/(48*c**(3/2)), True))
```

3.310.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(4\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right) + 3\sqrt{dx^3+c}\right)}{9\,d^2}$$

input
```
integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-2/9*(4*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 3*sqrt(d*x^3 + c))/d^2
```

3.310.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(\frac{8\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d}+\frac{3\,\sqrt{dx^3+c}}{d}\right)}{9\,d}$$

input
```
integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-2/9*(8*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 3*sqrt(d*x^3
 + c)/d)/d
```

3.310.9 Mupad [B] (verification not implemented)

Time = 7.49 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{8\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^2}-\frac{2\,\sqrt{d\,x^3+c}}{3\,d^2}$$

input
```
int(x^5/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
(8*c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/(9*d^2) - (2*(c + d*x^3)^(1/2))/(3*d^2)
```

3.311 $\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.311.1 Optimal result

Integrand size = 27, antiderivative size = 33

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

output
```
2/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d/c^(1/2)
```

3.311.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

input
```
Integrate[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)
```

3.311.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {946, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{3d}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

input

```
Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output

```
(2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)
```

3.311.3.1 Defintions of rubi rules used

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.311.4 Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

method	result
default	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}$
pseudoelliptic	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\right.}{}}}{}\right.}{}$

input
```
int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d/c^(1/2)
```

3.311.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)}{9\sqrt{c}d}, -\frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)}{9\,cd}\right]$$

input
```
integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/9*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c))/(sqrt(c
)*d), -2/9*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c)/(c*d)]
```

3.311.6 Sympy [A] (verification not implemented)

Time = 3.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \begin{cases} -\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9d\sqrt{-c}} & \text{for } d \neq 0 \\ \frac{x^3}{24c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((-2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*d*sqrt(-c)), Ne(d, 0)
), (x**3/(24*c**(3/2)), True))
```

3.311.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{9\sqrt{c}d}$$

input
```
integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/9*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/(sqr
t(c)*d)
```

3.311.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d}$$

input
```
integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-2/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d)
```

3.311.9 Mupad [B] (verification not implemented)

Time = 7.50 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,\sqrt{c}\,d}$$

input
```
int(x^2/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(9*c^(1/2)
*d)
```

3.312 $\int \frac{1}{x(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.312.1 Optimal result

Integrand size = 27, antiderivative size = 58

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

output
```
1/36*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/12*arctanh((d*x^3+c)^(
1/2)/c^(1/2))/c^(3/2)
```

3.312.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-3\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{36c^{3/2}}$$

input
```
Integrate[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]
)/(36*c^(3/2))
```

3.312.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {948, 97, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{ 97}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{8c}+\frac{d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{8c}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{4c}+\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4cd}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4cd}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{12c^{3/2}}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{3}\left(\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{12c^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{4c^{3/2}}\right)$$

input

```
Int[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output

```
(ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(12*c^(3/2)) - ArcTanh[Sqrt[c + d*x^
3]/Sqrt[c]]/(4*c^(3/2)))/3
```

3.312.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.312.4 Maple [A] (verified)

Time = 4.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-3\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{36c^{\frac{3}{2}}}$	38
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{36c^{\frac{3}{2}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{12c^{\frac{3}{2}}}$	41
elliptic	Expression too large to display	1508

input
```
int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/36*(arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-3*arctanh((d*x^3+c)^(1/2)/c^(1/
2)))/c^(3/2)
```

3.312.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.40

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$
$$=\left[\frac{\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{72\,c^2},\frac{3\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\sqrt{-c}\arctan\left(\right.}{36\,c^2}\right.$$

input
```
integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/72*(sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c
)) + 3*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, 1/
36*(3*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(-c)*arctan(1/3*sq
rt(d*x^3 + c)*sqrt(-c)/c))/c^2]
```

3.312.6 Sympy [A] (verification not implemented)

Time = 3.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\begin{cases}\frac{2\left(-\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{72c\sqrt{-c}}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24c\sqrt{-c}}\right)}{d} & \text{for } d\neq 0\\ \frac{\log\left(x^3\right)}{24c^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

input
```
integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((2*(-d*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(72*c*sqrt(-c)) + d*a
tan(sqrt(c + d*x**3)/sqrt(-c))/(24*c*sqrt(-c)))/d, Ne(d, 0)), (log(x**3)/(
24*c**(3/2)), True))
```

3.312.7 Maxima [F]

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x}\,dx$$

input
```
integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x), x)
```

3.312.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}c}-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{36\sqrt{-c}c}$$

input
```
integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/36*arctan(1/3*sqrt(
d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)
```

3.312.9 Mupad [B] (verification not implemented)

Time = 7.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{3\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)-\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{36\,\sqrt{c^3}}$$

input
```
int(1/(x*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
-(3*atanh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2)) - atanh((c*(c + d*x^3)^(1/2))
/(3*(c^3)^(1/2))))/(36*(c^3)^(1/2))
```

3.313 $\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.313.1 Optimal result

Integrand size = 27, antiderivative size = 81

$$\int \frac{1}{x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{c+dx^3}}{24c^2x^3}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

output
```
1/288*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/32*d*arctanh((d*x^3
+c)^(1/2)/c^(1/2))/c^(5/2)-1/24*(d*x^3+c)^(1/2)/c^2/x^3
```

3.313.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{c+dx^3}}{24c^2x^3}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

input
```
Integrate[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/24*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/(288*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))
```

3.313.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {948, 114, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (8c-dx^3) \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^6 (8c-dx^3) \sqrt{dx^3+c}} dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3} \left(-\frac{\int \frac{d(6c-dx^3)}{2x^3(8c-dx^3)\sqrt{dx^3+c}} dx^3}{8c^2} - \frac{\sqrt{c+dx^3}}{8c^2x^3} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{d \int \frac{6c-dx^3}{x^3(8c-dx^3)\sqrt{dx^3+c}} dx^3}{16c^2} - \frac{\sqrt{c+dx^3}}{8c^2x^3} \right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3} \left(-\frac{d \left(\frac{3}{4} \int \frac{1}{x^3\sqrt{dx^3+c}} dx^3 - \frac{1}{4} d \int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}} dx^3 \right)}{16c^2} - \frac{\sqrt{c+dx^3}}{8c^2x^3} \right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3} \left(-\frac{d \left(\frac{3 \int \frac{1}{\frac{x^6}{d}-\frac{c}{d}} d\sqrt{dx^3+c}}{2d} - \frac{1}{2} \int \frac{1}{9c-x^6} d\sqrt{dx^3+c} \right)}{16c^2} - \frac{\sqrt{c+dx^3}}{8c^2x^3} \right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{3\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3}\right)$$

input

```
Int[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output

```
(-1/8*Sqrt[c + d*x^3]/(c^2*x^3) - (d*(-1/6*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt
[c])]/Sqrt[c] - (3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c])))/(16*c^2
))/3
```

3.313.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.313.4 Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{9 \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right) d\,x^{3}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right) d\,x^{3}-12 \sqrt{d\,x^{3}+c}\,\sqrt{c}}{288 c^{\frac{5}{2}} x^{3}}$	64
risch	$-\frac{\sqrt{d\,x^{3}+c}}{24 c^{2} x^{3}}-\frac{d \left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{2 \sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{18 \sqrt{c}}\right)}{16 c^{2}}$	65
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3 c\,x^{3}}+\frac{d \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3 c^{\frac{3}{2}}}}{8 c}-\frac{d \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{96 c^{\frac{5}{2}}}+\frac{d \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{288 c^{\frac{5}{2}}}$	86
elliptic	Expression too large to display	1523

input
```
int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/288/c^(5/2)*(9*arctanh((d*x^3+c)^(1/2)/c^(1/2))*d*x^3+arctanh(1/3*(d*x^3
+c)^(1/2)/c^(1/2))*d*x^3-12*(d*x^3+c)^(1/2)*c^(1/2))/x^3
```

3.313.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \left[\frac{\sqrt{c}dx^3\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+9\,\sqrt{c}dx^3\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\,\sqrt{dx^3+c}c}{576\,c^3x^3}, -\frac{9\,\sqrt{-c}dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\sqrt{-c}dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+12\,\sqrt{dx^3+c}c}{288\,c^3x^3}\right]$$

input
```
integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/576*(sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^
3 - 8*c)) + 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/
x^3) - 24*sqrt(d*x^3 + c)*c)/(c^3*x^3), -1/288*(9*sqrt(-c)*d*x^3*arctan(sq
rt(d*x^3 + c)*sqrt(-c)/c) + sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt
(-c)/c) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]
```

3.313.6 Sympy [F]

$$\int \frac{1}{x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = -\int \frac{1}{-8cx^4\sqrt{c+dx^3}+dx^7\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(1/(-8*c*x**4*sqrt(c + d*x**3) + d*x**7*sqrt(c + d*x**3)), x)
```

3.313.7 Maxima [F]

$$\int \frac{1}{x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^4}\,dx$$

input
```
integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^4), x)
```

3.313.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{32\sqrt{-c}c^2}-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{288\sqrt{-c}c^2}-\frac{\sqrt{dx^3+c}}{24\,c^2x^3}$$

input
```
integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-1/32*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/288*d*arctan(1
/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/24*sqrt(d*x^3 + c)/(c^2*x^
3)
```

3.313.9 Mupad [B] (verification not implemented)

Time = 7.59 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\frac{d\,\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{32\,\sqrt{c^5}}+\frac{d\,\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)}{288\,\sqrt{c^5}}-\frac{\sqrt{d\,x^3+c}}{24\,c^2\,x^3}$$

input
```
int(1/(x^4*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
(d*atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2)))/(32*(c^5)^(1/2)) + (d*atanh
((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2))))/(288*(c^5)^(1/2)) - (c + d*x^3)
^(1/2)/(24*c^2*x^3)
```

3.314 $\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.314.1 Optimal result

Integrand size = 27, antiderivative size = 107

$$\int \frac{1}{x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{c+dx^3}}{48c^2x^6}+\frac{5d\sqrt{c+dx^3}}{192c^3x^3}+\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}}-\frac{7d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}$$

output
```
1/2304*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-7/256*d^2*arctanh(
(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-1/48*(d*x^3+c)^(1/2)/c^2/x^6+5/192*d*(d*x
^3+c)^(1/2)/c^3/x^3
```

3.314.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\frac{\sqrt{c+dx^3}\left(-4c+5dx^3\right)}{192c^3x^6}+\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}}-\frac{7d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}$$

input
```
Integrate[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(Sqrt[c + d*x^3]*(-4*c + 5*d*x^3))/(192*c^3*x^6) + (d^2*ArcTanh[Sqrt[c + d
*x^3]/(3*Sqrt[c])])/(2304*c^(7/2)) - (7*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c
]])/(256*c^(7/2))
```

3.314.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 114, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (8c-dx^3) \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^9 (8c-dx^3) \sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{d(20c-3dx^3)}{2x^6(8c-dx^3)\sqrt{dx^3+c}}dx^3}{16c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\int \frac{20c-3dx^3}{x^6(8c-dx^3)\sqrt{dx^3+c}}dx^3}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{\int \frac{2cd\left(42c-5dx^3\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{8c^2}-\frac{5\sqrt{c+dx^3}}{2cx^3}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\int \frac{42c-5dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{4c}-\frac{5\sqrt{c+dx^3}}{2cx^3}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6}\right)$$

$$\Big\downarrow \text{ 174}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{21}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{1}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3\right)}{4c}-\frac{5\sqrt{c+dx^3}}{2cx^3}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{1}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{21\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}\right)}{4c}-\frac{5\sqrt{c+dx^3}}{2cx^3}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{21\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}\right)}{4c}-\frac{5\sqrt{c+dx^3}}{2cx^3}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6}\right)$$

$$\Big\downarrow \text{ 221}$$

3.314. $\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{21\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{4c}-\frac{5\sqrt{c+dx^3}}{2cx^3}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6}\right)$$

input
```
Int[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-1/16*Sqrt[c + d*x^3]/(c^2*x^6) - (d*((-5*Sqrt[c + d*x^3])/(2*c*x^3) - (d
*(ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(6*Sqrt[c]) - (21*ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]])/(2*Sqrt[c])))/(4*c)))/(32*c^2))/3
```

3.314.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.314.4 Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(-5d\,x^{3}+4c\right)}{192c^{3}x^{6}}+\frac{d^{2}\left(-\frac{7\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{18\sqrt{c}}\right)}{128c^{3}}$
pseudoelliptic	$\frac{-63\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d^{2}x^{6}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)d^{2}x^{6}+60d\,x^{3}\sqrt{d\,x^{3}+c}\,\sqrt{c}-48\sqrt{d\,x^{3}+c}\,c^{\frac{3}{2}}}{2304c^{\frac{7}{2}}\,x^{6}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{6c\,x^{6}}+\frac{d\sqrt{d\,x^{3}+c}}{4c^{2}x^{3}}-\frac{d^{2}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4c^{\frac{5}{2}}}}{8c}+\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{3c\,x^{3}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{64c^{2}}-\frac{d^{2}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{768c^{\frac{7}{2}}}+$
elliptic	Expression too large to display

input
```
int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(d*x^3+c)^(1/2)*(-5*d*x^3+4*c)/c^3/x^6+1/128*d^2/c^3*(-7/2*arctanh(
(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+1/18*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))
/c^(1/2))
```

3.314.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.03

$$\int \frac{1}{x^{7}\left(8c-dx^{3}\right)\sqrt{c+dx^{3}}}\,dx$$
$$=\left[\frac{\sqrt{c}d^{2}x^{6}\log\left(\frac{dx^{3}+6\sqrt{dx^{3}+c}\sqrt{c}+10\,c}{dx^{3}-8\,c}\right)+63\,\sqrt{c}d^{2}x^{6}\log\left(\frac{dx^{3}-2\sqrt{dx^{3}+c}\sqrt{c}+2\,c}{x^{3}}\right)+24\left(5\,cdx^{3}-4\,c^{2}\right)\sqrt{dx^{3}+c}}{4608\,c^{4}x^{6}},\ 63\right.$$

input
```
integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4608*(sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d
*x^3 - 8*c)) + 63*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) +
 2*c)/x^3) + 24*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^4*x^6), 1/2304*(63
*sqrt(-c)*d^2*x^6*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(-c)*d^2*x^6*ar
ctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 +
 c))/(c^4*x^6)]
```

3.314.6 Sympy [F]

$$\int \frac{1}{x^7 (8c - dx^3) \sqrt{c + dx^3}}\, dx = -\int \frac{1}{-8cx^7\sqrt{c + dx^3} + dx^{10}\sqrt{c + dx^3}}\, dx$$

input
```
integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(1/(-8*c*x**7*sqrt(c + d*x**3) + d*x**10*sqrt(c + d*x**3)), x)
```

3.314.7 Maxima [F]

$$\int \frac{1}{x^7 (8c - dx^3) \sqrt{c + dx^3}}\, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^7}\, dx$$

input
```
integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^7), x)
```

3.314.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^7 (8c - dx^3) \sqrt{c + dx^3}}\, dx = \frac{7\,d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{256\,\sqrt{-c}c^3} - \frac{d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{2304\,\sqrt{-c}c^3} + \frac{5\,(dx^3 + c)^{\frac{3}{2}}d^2 - 9\,\sqrt{dx^3 + c}cd^2}{192\,c^3d^2x^6}$$

input
```
integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
7/256*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/2304*d^2*arc
tan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 1/192*(5*(d*x^3 + c)^(3
/2)*d^2 - 9*sqrt(d*x^3 + c)*c*d^2)/(c^3*d^2*x^6)
```

3.314.9 Mupad [B] (verification not implemented)

Time = 7.67 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^7 (8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{d^2\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)}{2304\,\sqrt{c^7}} - \frac{7\,d^2\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)}{256\,\sqrt{c^7}} - \frac{3\,\sqrt{d\,x^3+c}}{64\,c^2\,x^6} + \frac{5\,{\left(d\,x^3+c\right)}^{3/2}}{192\,c^3\,x^6}$$

input
```
int(1/(x^7*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
(d^2*atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2))))/(2304*(c^7)^(1/2)) -
(7*d^2*atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2)))/(256*(c^7)^(1/2)) - (3*
(c + d*x^3)^(1/2))/(64*c^2*x^6) + (5*(c + d*x^3)^(3/2))/(192*c^3*x^6)
```

3.315 $\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.315.1 Optimal result

Integrand size = 27, antiderivative size = 630

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$\begin{aligned}
&= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{32c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}} \\
&\quad + \frac{32c^{7/6}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}} - \frac{32c^{7/6}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}} \\
&\quad + \frac{52\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad - \frac{104\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output

```
32/9*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^
(8/3)-32/9*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-32/9*c^(7/
6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)*3^(
1/2)-2/7*x^2*(d*x^3+c)^(1/2)/d^2-104/7*c*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))-104/21*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/
3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2
^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+52/7*3^(1/4)*c^(4/3)*(c^(1/3)+d
^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/
3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+
c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

3.315.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.21

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{x^2\left(-20(c+dx^3)+20c\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+13dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},\right.\right.}{70d^2\sqrt{c+dx^3}}$$

input

```
Integrate[x^7/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output

```
(x^2*(-20*(c + d*x^3) + 20*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3
, -((d*x^3)/c), (d*x^3)/(8*c)] + 13*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(70*d^2*Sqrt[c + d*x^3])
```

3.315.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {979, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \ 979$$

$$\frac{2\int \frac{2cx(13dx^3+8c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{7d^2}-\frac{2x^2\sqrt{c+dx^3}}{7d^2}$$

$$\Big\downarrow \ 27$$

$$\frac{4c\int \frac{x(13dx^3+8c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{7d^2}-\frac{2x^2\sqrt{c+dx^3}}{7d^2}$$

$$\Big\downarrow \ 1054$$

$$\frac{4c\int \left(\frac{112cx}{(8c-dx^3)\sqrt{dx^3+c}}-\frac{13x}{\sqrt{dx^3+c}}\right)dx}{7d^2}-\frac{2x^2\sqrt{c+dx^3}}{7d^2}$$

$$\Big\downarrow \ 2009$$

$$4c\left(-\frac{26\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{13\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\right.}{}\right.$$

$$\frac{2x^2\sqrt{c+dx^3}}{7d^2}$$

input
```
Int[x^7/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

3.315. $\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}dx$

output

```
(-2*x^2*Sqrt[c + d*x^3])/(7*d^2) + (4*c*((-26*Sqrt[c + d*x^3])/(d^(2/3)*((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (56*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(
c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(2/3)) + (56*c^(1/6)*
ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(9*d^(2/3))
- (56*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^(2/3)) + (13*3^(1
/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (26*Sqrt[2]*c^(1/3
)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1
/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3
^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(7*d^2)
```

3.315.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 979

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.315. $\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.315.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.87 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.38

method	result	size
elliptic	Expression too large to display	867
risch	Expression too large to display	872
default	Expression too large to display	1308

input

```
int(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2/7*x^2*(d*x^3+c)^(1/2)/d^2+104/21*I/d^3*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)))-64/27*I*c/d^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1
/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3
^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d...
```

3.315.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.43 (sec) , antiderivative size = 2428, normalized size of antiderivative = 3.85

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
2/189*(56*d^3*(c^7/d^16)^(1/6)*log(33554432/3*((d^16*x^9 + 318*c*d^15*x^6
+ 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c^7/d^16)^(5/6) + 6*(c^6*d^2*x^7 + 80
*c^7*d*x^4 + 160*c^8*x + 6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2)*(c^7/d^16)^(
2/3) + (7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64*c^6*d^5)*(c^7/d^16)^(1/3))*sq
rt(d*x^3 + c) + 18*(5*c^3*d^10*x^7 + 64*c^4*d^9*x^4 + 32*c^5*d^8*x)*sqrt(c
^7/d^16) + 18*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3*x^2)*(c^7/d^16)^(
1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 56*d^3*(c^7/d^
16)^(1/6)*log(-33554432/3*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3
+ 640*c^3*d^13)*(c^7/d^16)^(5/6) - 6*(c^6*d^2*x^7 + 80*c^7*d*x^4 + 160*c^8
*x + 6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2)*(c^7/d^16)^(2/3) + (7*c^4*d^7*x^
6 + 152*c^5*d^6*x^3 + 64*c^6*d^5)*(c^7/d^16)^(1/3))*sqrt(d*x^3 + c) + 18*(
5*c^3*d^10*x^7 + 64*c^4*d^9*x^4 + 32*c^5*d^8*x)*sqrt(c^7/d^16) + 18*(c^5*d
^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3*x^2)*(c^7/d^16)^(1/6))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 27*sqrt(d*x^3 + c)*d*x^2 + 1404*c*
sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 28
*(sqrt(-3)*d^3 - d^3)*(c^7/d^16)^(1/6)*log(33554432/3*((d^16*x^9 + 318*c*d
^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^
15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^7/d^16)^(5/6) + 6*(2*c^6*d^
2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2 -
sqrt(-3)*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2))*(c^7/d^16)^(2/3) - (7*c^4*...
```

3.315.6 Sympy [F]

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{x^7}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**7/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(x**7/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

3.315.7 Maxima [F]

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\int -\frac{x^7}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

3.315.8 Giac [F]

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\int -\frac{x^7}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\int \frac{x^7}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)}\,dx$$

input
```
int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.316 $\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.316.1 Optimal result

Integrand size = 27, antiderivative size = 601

$$\begin{aligned}
&\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx \\
&= -\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{4\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} \\
&\quad + \frac{4\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{5/3}} - \frac{4\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{5/3}} \\
&\quad + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)\middle|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad - \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
4/9*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(
5/3)-4/9*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-4/9*c^(1/6)*
arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(5/3)*3^(1/2
)-2*(d*x^3+c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-2/3*c^(1/3)*(c
^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)
^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)+3^(1/4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(
1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(
1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/
2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.316.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.11

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\frac{x^5\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{40c\sqrt{c+dx^3}}$$

input
```
Integrate[x^4/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/
(8*c)])/(40*c*Sqrt[c + d*x^3])
```

3.316.3 Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {983, 832, 759, 988, 946, 73, 219, 2416, 2563, 219, 2570, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{983}$$

$$\frac{8c\int \frac{x}{(8c-dx^3)\sqrt{dx^3+c}}dx}{d}-\frac{\int \frac{x}{\sqrt{dx^3+c}}dx}{d}$$

$$\Big\downarrow \text{832}$$

$$\frac{8c\int \frac{x}{(8c-dx^3)\sqrt{dx^3+c}}dx}{d}-\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}\int \frac{1}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}}{d}$$

$$\Big\downarrow \text{759}$$

$$\frac{8c\int \frac{x}{(8c-dx^3)\sqrt{dx^3+c}}dx}{d}-\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{d}$$

$$\Big\downarrow \text{988}$$

$$\frac{8c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{d}\int \frac{x^2}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{4\sqrt[3]{c}}\right)}{d}-\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{d}$$

$$\Big\downarrow \text{946}$$

3.316. $\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}dx$

$$\frac{8c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{d}\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{12\sqrt[3]{c}}\right)}{d}-$$

$$\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right.}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{d}$$

$$\Big\downarrow \; 73$$

$$\frac{8c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{6\sqrt[3]{c}d^{2/3}}\right)}{d}-$$

$$\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right.}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{d}$$

$$\Big\downarrow \; 219$$

3.316. $\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

$$\frac{8c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)}{d}-$$

$$\frac{\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{d}$$

$$\Big\downarrow \; 2416$$

$$\frac{8c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)}{d}-$$

$$\frac{\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2}}{}}{d}$$

$$\Big\downarrow \; 2563$$

3.316. $\int \frac{x^4}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$

$$\frac{8c\left(\frac{\int \frac{1}{9-\frac{\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^4}{\sqrt[3]{c}\left(dx^3+c\right)}}d\frac{\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^2}{c^{2/3}\sqrt{dx^3+c}}}{6\sqrt[3]{c}d^{2/3}}-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)}{d}-$$

$$\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-2\left(1-\sqrt{3}\right)$$

$$d$$

$$\Big\downarrow \; 219$$

$$\frac{8c\left(-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)}{d}-$$

$$\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-2\left(1-\sqrt{3}\right)$$

$$d$$

$$\Big\downarrow \; 2570$$

3.316. $\int \frac{x^4}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$

$$\frac{\frac{8c\left(\frac{d^{4/3}\int \frac{1}{-\frac{2d^2}{c}-\frac{6\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^2 d^2}{c^{2/3}\left(dx^3+c\right)}}d\frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\sqrt[3]{c}\sqrt{dx^3+c}}}{3c^{4/3}}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)}{d}-\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{}}{}}{d}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{8c\left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}\right)}{d}-\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{}}{}}{d}$$

input

```
Int[x^4/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

3.316. $\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

output
```
(8*c*(-1/6*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]
/(Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sq
rt[c + d*x^3])]/(18*c^(5/6)*d^(2/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
]/(18*c^(5/6)*d^(2/3))))/d - (((2*Sqrt[c + d*x^3])/(d^(1/3)*((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(
1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)
/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(1/3)*Sqrt[(c^(
1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c
+ d*x^3]))/d^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*c^(1/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3
)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/
3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x
)^2]*Sqrt[c + d*x^3]))/d
```

3.316.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 988
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[d*(q/(4*b))   Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x
^3]), x], x] + (-Simp[q^2/(12*b)   Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]
), x], x] + Simp[1/(12*b*c)   Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x +
 q^2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[8*b*c + a*d, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.316. $\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

3.316.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.45 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.41

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

input
```
int(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-8/27*I/d^4*2
^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3
*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^
2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^
(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/...
```

3.316.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.63 (sec) , antiderivative size = 2254, normalized size of antiderivative = 3.75

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input

```
integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output

```
1/27*(2*d^2*(c/d^10)^(1/6)*log(1024/3*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c
^2*d^9*x^3 + 640*c^3*d^8)*(c/d^10)^(5/6) + 6*(c*d^2*x^7 + 80*c^2*d*x^4 + 1
60*c^3*x + 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2)*(c/d^10)^(2/3) + (7*c*d^5*x^6
+ 152*c^2*d^4*x^3 + 64*c^3*d^3)*(c/d^10)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c*
d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/d^10) + 18*(c*d^4*x^8 + 38
*c^2*d^3*x^5 + 64*c^3*d^2*x^2)*(c/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 1
92*c^2*d*x^3 - 512*c^3)) - 2*d^2*(c/d^10)^(1/6)*log(-1024/3*((d^11*x^9 + 3
18*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8)*(c/d^10)^(5/6) - 6*(c*d^2*
x^7 + 80*c^2*d*x^4 + 160*c^3*x + 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2)*(c/d^10)
^(2/3) + (7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3)*(c/d^10)^(1/3))*sqrt
(d*x^3 + c) + 18*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/d^10
) + 18*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2)*(c/d^10)^(1/6))/(d^3*
x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*d^2 - d^2)*(c/d
^10)^(1/6)*log(1024/3*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640
*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^
3*d^8))*(c/d^10)^(5/6) + 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5
*c*d^8*x^5 + 32*c^2*d^7*x^2 - sqrt(-3)*(5*c*d^8*x^5 + 32*c^2*d^7*x^2))*(c/
d^10)^(2/3) - (7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3 + sqrt(-3)*(7*c*
d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3))*(c/d^10)^(1/3))*sqrt(d*x^3 + c) -
 36*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/d^10) + 18*(c*...
```

3.316.6 Sympy [F]

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{x^4}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

input

```
integrate(x**4/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output

```
-Integral(x**4/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

3.316.7 Maxima [F]

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x^4}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

3.316.8 Giac [F]

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x^4}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

input
```
int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.317 $\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.317.1 Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

output
```
1/18*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d^
(2/3)-1/18*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(2/3)-1/18*arcta
n(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(2/3)*3^(
1/2)
```

3.317.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{16c\sqrt{c+dx^3}}$$

input
```
Integrate[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/
(8*c)])/(16*c*Sqrt[c + d*x^3])
```

3.317.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {988, 946, 73, 219, 2563, 219, 2570, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{988}$$

$$-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{d}\int \frac{x^2}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{4\sqrt[3]{c}}$$

$$\Big\downarrow \text{946}$$

$$-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{d}\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{12\sqrt[3]{c}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{6\sqrt[3]{c}d^{2/3}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\int \frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(2\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{dx^3+c}}dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

$$\Big\downarrow \text{2563}$$

$$\frac{\int \frac{1}{9-\frac{\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^4}{\sqrt[3]{c}\left(dx^3+c\right)}}d\frac{\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^2}{c^{2/3}\sqrt{dx^3+c}}}{6\sqrt[3]{c}d^{2/3}}-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\int \frac{-\frac{d^{4/3}x^2}{\sqrt[3]{c}}-2dx+2\sqrt[3]{c}d^{2/3}}{\left(\frac{d^{2/3}x^2}{c^{2/3}}+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+4\right)\sqrt{dx^3+c}}dx}{12cd}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

$$\Big\downarrow \text{2570}$$

$$\frac{d^{4/3}\int \frac{1}{-\frac{2d^2}{c}-\frac{6\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)^2d^2}{c^{2/3}\left(dx^3+c\right)}}d\frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\sqrt[3]{c}\sqrt{dx^3+c}}}{3c^{4/3}}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

input
```
Int[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/6*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/(Sqrt
[3]*c^(5/6)*d^(2/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c +
 d*x^3])]/(18*c^(5/6)*d^(2/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(18*
c^(5/6)*d^(2/3))
```

3.317. $\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.317.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 988
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[d*(q/(4*b))   Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x
^3]), x], x] + (-Simp[q^2/(12*b)   Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]
), x], x] + Simp[1/(12*b*c)   Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x +
 q^2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[8*b*c + a*d, 0]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

3.317.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.23 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.95

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\right.}{}\right.}{2\left(-c\right.}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\right.}{}\right.}{2\left(-c\right.}}}{}\right.}{}$

input

```
int(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*
d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*
(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3
)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

3.317.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2285 vs. $2(95) = 190$.

Time = 0.54 (sec) , antiderivative size = 2285, normalized size of antiderivative = 16.21

$$\int \frac{x}{(8c - dx^3)\sqrt{c + dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/216*(sqrt(-3) + 1)*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 12
00*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x
+ sqrt(-3)*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x))*(1/(c^5*d^4))^
(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2 - sqrt(-3)*(5
*c^5*d^5*x^5 + 32*c^6*d^4*x^2))*(1/(c^5*d^4))^(5/6) - 2*(7*c^3*d^4*x^6 + 1
52*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x
^4 + 160*c^3*d*x + sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x))*(1
/(c^5*d^4))^(1/6)) - 9*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2 - sq
rt(-3)*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2))*(1/(c^5*d^4))^(1/3
))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 1/216*(sqrt(-3) +
 1)*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 64
0*c^3 - 9*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x + sqrt(-3)*(5*c^4
*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x))*(1/(c^5*d^4))^(2/3) - 3*sqrt(d*
x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2 - sqrt(-3)*(5*c^5*d^5*x^5 + 32
*c^6*d^4*x^2))*(1/(c^5*d^4))^(5/6) - 2*(7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 +
64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x
+ sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x))*(1/(c^5*d^4))^(1/6)
) - 9*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2 - sqrt(-3)*(c^2*d^4*x
^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2))*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 1/216*(sqrt(-3) - 1)*(1/(c^5*d^...
```

3.317.6 Sympy [F]

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{x}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(x/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

3.317.7 Maxima [F]

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

3.317.8 Giac [F]

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

3.317.9 Mupad [B] (verification not implemented)

Time = 45.12 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.93

$$\begin{aligned}
&\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx \\
&= \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(\sqrt{d\,x^3+c}-\sqrt{c}+2\,c^{1/6}\,d^{1/3}\,x\right)^3}{x^3\,\left(d^{1/3}\,x-2\,c^{1/3}\right)^3}\right)}{54\,c^{5/6}\,d^{2/3}} \\
&\quad + \frac{\sqrt{2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(-\sqrt{3}\,c^{1/6}\,d^{1/3}\,x+\sqrt{d\,x^3+c}\,1\mathrm{i}+\sqrt{c}\,1\mathrm{i}+c^{1/6}\,d^{1/3}\,x\,1\mathrm{i}\right)^3}{x^3\,\left(d^{1/3}\,x+c^{1/3}-\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)^3}\right)\,\sqrt{-1+\sqrt{3}\,1\mathrm{i}}}{108\,c^{5/6}\,d^{2/3}} \\
&\quad + \frac{\sqrt{2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(\sqrt{3}\,c^{1/6}\,d^{1/3}\,x-\sqrt{d\,x^3+c}\,1\mathrm{i}+\sqrt{c}\,1\mathrm{i}+c^{1/6}\,d^{1/3}\,x\,1\mathrm{i}\right)^3}{x^3\,\left(d^{1/3}\,x+c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)^3}\right)\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{108\,c^{5/6}\,d^{2/3}}
\end{aligned}$$

input
```
int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
log((((c + d*x^3)^(1/2) + c^(1/2))*((c + d*x^3)^(1/2) - c^(1/2) + 2*c^(1/6
)*d^(1/3)*x)^3)/(x^3*(d^(1/3)*x - 2*c^(1/3))^3))/(54*c^(5/6)*d^(2/3)) + (2
^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2)*1i + c^(1/2)*
1i + c^(1/6)*d^(1/3)*x*1i - 3^(1/2)*c^(1/6)*d^(1/3)*x)^3)/(x^3*(d^(1/3)*x
- 3^(1/2)*c^(1/3)*1i + c^(1/3))^3))*(3^(1/2)*1i - 1)^(1/2))/(108*c^(5/6)*d
^(2/3)) + (2^(1/2)*log((((c + d*x^3)^(1/2) + c^(1/2))*(c^(1/2)*1i - (c + d
*x^3)^(1/2)*1i + c^(1/6)*d^(1/3)*x*1i + 3^(1/2)*c^(1/6)*d^(1/3)*x)^3)/(x^3
*(3^(1/2)*c^(1/3)*1i + d^(1/3)*x + c^(1/3))^3))*(3^(1/2)*1i + 1)^(1/2)*1i)
/(108*c^(5/6)*d^(2/3))
```

3.317. $\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.318 $\int \frac{1}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.318.1 Optimal result

Integrand size = 27, antiderivative size = 632

$$\begin{aligned}
\int \frac{1}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= -\frac{\sqrt{c+dx^3}}{8c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&\quad -\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}}+\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{11/6}}-\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{11/6}} \\
&\quad -\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad +\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
1/144*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(11/6)-1/144*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-1/144*
d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/
6)*3^(1/2)-1/8*(d*x^3+c)^(1/2)/c^2/x+1/8*d^(1/3)*(d*x^3+c)^(1/2)/c^2/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2)))+1/24*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(
1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)
*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)
*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/16*3^(1/4)*d^(1/3)*(c^(1/3)
+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(
1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(5/3)/(d*x^
3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

3.318.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$
$$=\frac{-80c(c+dx^3)+25cdx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-d^2x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\right.}{640c^3x\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^2*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-80*c*(c + d*x^3) + 25*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5
/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(640*c^3*x*Sqrt[c + d*x^3])
```

3.318.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {980, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{ 980}$$

$$\frac{\int \frac{dx\left(10c-dx^3\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{\sqrt{c+dx^3}}{8c^2x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\int \frac{x\left(10c-dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{d\int \left(\frac{2cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{x}{\sqrt{dx^3+c}}\right)dx}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x}$$

$$\Big\downarrow \text{ 2009}$$

$$d\left(\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{}\right.$$

$$\frac{\sqrt{c+dx^3}}{8c^2x}$$

input
```
Int[1/(x^2*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/8*Sqrt[c + d*x^3]/(c^2*x) + (d*((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)) - (c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d
^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(2/3)) + (c^(1/6)*ArcTanh[(c^(1/
3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9*d^(2/3)) - (c^(1/6)*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]
]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3
)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[
3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt
[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*
x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(
c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])))/(16*c^2)
```

3.318.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.318. $\int \frac{1}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.318.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.89 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	874

input

```
int(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/8*(d*x^3+c)^(1/2)/c^2/x-1/24*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)))-1/216*I/d^2/c^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I
*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))
)^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(
1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^...
```

3.318.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 2259, normalized size of antiderivative = 3.57

$$\int \frac{1}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/1728*(2*c^2*x*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d
^2*x^3 + 640*c^3*d + 18*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x)*(d^2/c^
11)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^2/c^11)^(
5/6) + (7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) + (c^2*d^3*
x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x)*(d^2/c^11)^(1/6)) + 18*(c^4*d^3*x^8 +
38*c^5*d^2*x^5 + 64*c^6*d*x^2)*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
 192*c^2*d*x^3 - 512*c^3)) - 2*c^2*x*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c
*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d + 18*(5*c^8*d^2*x^7 + 64*c^9*d*x^4
 + 32*c^10*x)*(d^2/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c
^11*x^2)*(d^2/c^11)^(5/6) + (7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(
d^2/c^11) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x)*(d^2/c^11)^(1/6))
 + 18*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2)*(d^2/c^11)^(1/3))/(d^3
*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 216*sqrt(d)*x*weierstras
sZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + (sqrt(-3)*c^2*x + c^
2*x)*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 64
0*c^3*d - 9*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x + sqrt(-3)*(5*c^8*d^
2*x^7 + 64*c^9*d*x^4 + 32*c^10*x))*(d^2/c^11)^(2/3) + 3*sqrt(d*x^3 + c)*(6
*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^2
/c^11)^(5/6) - 2*(7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) +
 (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + ...
```

3.318.6 Sympy [F]

$$\int \frac{1}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = -\int \frac{1}{-8cx^2\sqrt{c+dx^3}+dx^5\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**2/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(1/(-8*c*x**2*sqrt(c + d*x**3) + d*x**5*sqrt(c + d*x**3)), x)
```

3.318.7 Maxima [F]

$$\int \frac{1}{x^2 (8c-dx^3) \sqrt{c+dx^3}} \, dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^2} \, dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^2), x)
```

3.318.8 Giac [F]

$$\int \frac{1}{x^2 (8c-dx^3) \sqrt{c+dx^3}} \, dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^2} \, dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^2), x)
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (8c-dx^3) \sqrt{c+dx^3}} \, dx = \int \frac{1}{x^2\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)} \, dx$$

input
```
int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.319 $\int \frac{1}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.319.1 Optimal result

Integrand size = 27, antiderivative size = 654

$$\int \frac{1}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{d\sqrt{c+dx^3}}{16c^3x}-\frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$-\frac{d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{384\sqrt{3}c^{17/6}}+\frac{d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1152c^{17/6}}-\frac{d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{17/6}}$$
$$+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
1/1152*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
c^(17/6)-1/1152*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/11
52*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(
17/6)*3^(1/2)-1/32*(d*x^3+c)^(1/2)/c^2/x^4+1/16*d*(d*x^3+c)^(1/2)/c^3/x-1/
16*d^(4/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-1/48*d^(4/3
)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x
+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x
^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*x^
3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)+1/32*3^(1/4)*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/
3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2
)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2)))^2)^(1/2)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.319.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$

$$= \frac{160c(-c^2+cdx^3+2d^2x^6)-75cd^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+4d^3x^9\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF}}{5120c^4x^4\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^5*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(160*c*(-c^2 + c*d*x^3 + 2*d^2*x^6) - 75*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*App
ellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 4*d^3*x^9*Sqrt[1 +
(d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(5120*
c^4*x^4*Sqrt[c + d*x^3])
```

3.319.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {980, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (8c-dx^3) \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{980}$$

$$\frac{\int -\frac{d(32c-5dx^3)}{2x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c^2}-\frac{\sqrt{c+dx^3}}{32c^2x^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{32c-5dx^3}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{64c^2}-\frac{\sqrt{c+dx^3}}{32c^2x^4}$$

$$\Big\downarrow \text{1053}$$

$$-\frac{d\left(-\frac{\int -\frac{8cdx\left(15c-2dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{4\sqrt{c+dx^3}}{cx}\right)}{64c^2}-\frac{\sqrt{c+dx^3}}{32c^2x^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\left(\frac{d\int \frac{x\left(15c-2dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{4\sqrt{c+dx^3}}{cx}\right)}{64c^2}-\frac{\sqrt{c+dx^3}}{32c^2x^4}$$

$$\Big\downarrow \text{1054}$$

$$-\frac{d\left(\frac{d\int \left(\frac{2x}{\sqrt{dx^3+c}}-\frac{cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}\right)dx}{c}-\frac{4\sqrt{c+dx^3}}{cx}\right)}{64c^2}-\frac{\sqrt{c+dx^3}}{32c^2x^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\left(\frac{d\left(\frac{4\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{2\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2}}{\left(\right.}}}{d^{2/3}}\right.}{}\right.}{}$$

$$\frac{\sqrt{c+dx^3}}{32c^2x^4}$$

input
```
Int[1/(x^5*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/32*Sqrt[c + d*x^3]/(c^2*x^4) - (d*((-4*Sqrt[c + d*x^3])/(c*x) + (d*((4*
Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (c^(1/6)*
ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(6*Sqrt[3
]*d^(2/3)) - (c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c +
d*x^3])])/(18*d^(2/3)) + (c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1
8*d^(2/3)) - (2*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1
/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 + (4*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
-7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/c))/(64*c^2)
```

3.319. $\int \frac{1}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.319.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.319.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.93 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.35

method	result	size
risch	Expression too large to display	882
elliptic	Expression too large to display	887
default	Expression too large to display	1351

3.319. $\int \frac{1}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx$

input
```
int(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(d*x^3+c)^(1/2)*(-2*d*x^3+c)/c^3/x^4-1/64/c^3*d^2*(-4/3*I*3^(1/2)/d*
(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/27*I/d^3*2^(1/2)*sum(1/_alph
a*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*
3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_
alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2
*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*...
```

3.319.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.11 (sec) , antiderivative size = 2403, normalized size of antiderivative = 3.67

$$\int \frac{1}{x^5 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/13824*(2*c^3*x^4*(d^8/c^17)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^
2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d
*x)*(d^8/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(
d^8/c^17)^(5/6) + (7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^
8/c^17) + (c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d^5*x)*(d^8/c^17)^(1/6))
 + 18*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2)*(d^8/c^17)^(1/3))/(d
^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*c^3*x^4*(d^8/c^17)^(
1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5
*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x)*(d^8/c^17)^(2/3) - 6*sqrt(d
*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^8/c^17)^(5/6) + (7*c^9*d^4*x^
6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) + (c^3*d^7*x^7 + 80*c^4
*d^6*x^4 + 160*c^5*d^5*x)*(d^8/c^17)^(1/6)) + 18*(c^6*d^6*x^8 + 38*c^7*d^5
*x^5 + 64*c^8*d^4*x^2)*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2
*d*x^3 - 512*c^3)) + 864*d^(3/2)*x^4*weierstrassZeta(0, -4*c/d, weierstras
sPInverse(0, -4*c/d, x)) + (sqrt(-3)*c^3*x^4 + c^3*x^4)*(d^8/c^17)^(1/6)*l
og((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^12*d
^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x + sqrt(-3)*(5*c^12*d^3*x^7 + 64*c^1
3*d^2*x^4 + 32*c^14*d*x))*(d^8/c^17)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^15*
d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^8/c^17)^(5
/6) - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17)...
```

3.319.6 Sympy [F]

$$\int \frac{1}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=-\int \frac{1}{-8cx^5\sqrt{c+dx^3}+dx^8\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**5/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(1/(-8*c*x**5*sqrt(c + d*x**3) + d*x**8*sqrt(c + d*x**3)), x)
```

3.319.7 Maxima [F]

$$\int \frac{1}{x^5 (8c - dx^3) \sqrt{c + dx^3}}\, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^5}\, dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^5), x)
```

3.319.8 Giac [F]

$$\int \frac{1}{x^5 (8c - dx^3) \sqrt{c + dx^3}}\, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^5}\, dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^5), x)
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (8c - dx^3) \sqrt{c + dx^3}}\, dx = \int \frac{1}{x^5\,\sqrt{d\,x^3 + c}\,\left(8\,c - d\,x^3\right)}\, dx$$

input
```
int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.320 $\int \frac{1}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.320.1 Optimal result

Integrand size = 27, antiderivative size = 678

$$\begin{aligned}\int \frac{1}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x} \\ &\quad +\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{23/6}} \\ &\quad +\frac{d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{23/6}}-\frac{d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9216c^{23/6}} \\ &\quad -\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{112c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\ &\quad +\frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{28\sqrt{2}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\end{aligned}$$

output
```
1/9216*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
c^(23/6)-1/9216*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-1/92
16*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(
23/6)*3^(1/2)-1/56*(d*x^3+c)^(1/2)/c^2/x^7+37/1792*d*(d*x^3+c)^(1/2)/c^3/x
^4-3/56*d^2*(d*x^3+c)^(1/2)/c^4/x+3/56*d^(7/3)*(d*x^3+c)^(1/2)/c^4/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))+1/56*3^(3/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*Elliptic
F((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*
x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/112*3^(1/4)*d^(7/3)*(c^(1/3)
+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(
1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(11/3)/(d*x
^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

3.320.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^8 (8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$= \frac{3875cd^3x^9\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-32\Big(5c(32c^3-5c^2dx^3+59cd^2x^6+96d^3x^9)+6d^4x}{286720c^5x^7\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^8*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(3875*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c
), (d*x^3)/(8*c)] - 32*(5*c*(32*c^3 - 5*c^2*d*x^3 + 59*c*d^2*x^6 + 96*d^3*
x^9) + 6*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)
/c), (d*x^3)/(8*c)]))/(286720*c^5*x^7*Sqrt[c + d*x^3])
```

3.320.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {980, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 \left(8c-dx^3\right) \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \ 980$$

$$\frac{\int -\frac{d\left(74c-11dx^3\right)}{2x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{56c^2}-\frac{\sqrt{c+dx^3}}{56c^2x^7}$$

$$\Big\downarrow \ 27$$

$$-\frac{d\int \frac{74c-11dx^3}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{112c^2}-\frac{\sqrt{c+dx^3}}{56c^2x^7}$$

$$\Big\downarrow \ 1053$$

$$-\frac{d\left(-\frac{\int \frac{cd\left(1536c-185dx^3\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{37\sqrt{c+dx^3}}{16cx^4}\right)}{112c^2}-\frac{\sqrt{c+dx^3}}{56c^2x^7}$$

$$\Big\downarrow \ 27$$

$$-\frac{d\left(-\frac{d\int \frac{1536c-185dx^3}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{37\sqrt{c+dx^3}}{16cx^4}\right)}{112c^2}-\frac{\sqrt{c+dx^3}}{56c^2x^7}$$

$$\Big\downarrow \ 1053$$

$$-\frac{d\left(-\frac{d\left(-\frac{\int -\frac{8cdx\left(775c-96dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{192\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{37\sqrt{c+dx^3}}{16cx^4}\right)}{112c^2}-\frac{\sqrt{c+dx^3}}{56c^2x^7}$$

$$\Big\downarrow \ 27$$

$$-\frac{d\left(-\frac{d\left(\frac{d\int \frac{x\left(775c-96dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{192\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{37\sqrt{c+dx^3}}{16cx^4}\right)}{112c^2}-\frac{\sqrt{c+dx^3}}{56c^2x^7}$$

$$\Big\downarrow \text{1054}$$

$$-\frac{d\left(-\frac{d\left(\frac{d\int \left(\frac{7cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{96x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{192\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{37\sqrt{c+dx^3}}{16cx^4}\right)}{112c^2}-\frac{\sqrt{c+dx^3}}{56c^2x^7}$$

$$\Big\downarrow \text{2009}$$

3.320. $\int \frac{1}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx$

$$-\frac{d\left(-\frac{d\left(\frac{d\left(\frac{64\sqrt{2}3^{3/4}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{96\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\right.}{}\right.}{}\right.}{}\right.}{}$$

$$\frac{\sqrt{c+dx^3}}{56c^2x^7}$$

input
```
Int[1/(x^8*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

3.320. $\int \frac{1}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx$

output
```
-1/56*Sqrt[c + d*x^3]/(c^2*x^7) - (d*((-37*Sqrt[c + d*x^3])/(16*c*x^4) - (
d*((-192*Sqrt[c + d*x^3])/(c*x) + (d*((192*Sqrt[c + d*x^3])/(d^(2/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (7*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1
/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(6*Sqrt[3]*d^(2/3)) + (7*c^(1/6)*ArcTa
nh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(18*d^(2/3)) - (7
*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(18*d^(2/3)) - (96*3^(1/4)*
Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^
(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (64*Sqrt[2]*3^(3/4)*c^
(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^
2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)^2]*Sqrt[c + d*x^3]))/c))/(32*c)))/(112*c^2)
```

3.320.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.320. $\int \frac{1}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx$

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.320.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.02 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.32

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	906
default	Expression too large to display	1849

input
```
int(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/1792*(d*x^3+c)^(1/2)*(96*d^2*x^6-37*c*d*x^3+32*c^2)/c^4/x^7+1/3584*d^3/
c^4*(-64*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3
))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2
)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-7/27*I/d^
3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2
*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2...
```

3.320.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.62 (sec) , antiderivative size = 2436, normalized size of antiderivative = 3.59

$$\int \frac{1}{x^8 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \text{Too large to display}$$

input

```
integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/774144*(14*c^4*x^7*(d^14/c^23)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 12
00*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32
*c^18*d^2*x)*(d^14/c^23)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c
^21*x^2)*(d^14/c^23)^(5/6) + (7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*
d^4)*sqrt(d^14/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x)*(d
^14/c^23)^(1/6)) + 18*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2)*(d^
14/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 14*c
^4*x^7*(d^14/c^23)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^
3 + 640*c^3*d^11 + 18*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x)*(
d^14/c^23)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2)*(d^14
/c^23)^(5/6) + (7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*d^4)*sqrt(d^14
/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x)*(d^14/c^23)^(1/6
)) + 18*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2)*(d^14/c^23)^(1/3)
)/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 41472*d^(5/2)*x^7*
weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 7*(sqrt(-3
)*c^4*x^7 + c^4*x^7)*(d^14/c^23)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 12
00*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*
c^18*d^2*x + sqrt(-3)*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x))*
(d^14/c^23)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqr
t(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^14/c^23)^(5/6) - 2*(7*c^12*d^6*x...
```

3.320.6 Sympy [F]

$$\int \frac{1}{x^8 (8c - dx^3) \sqrt{c + dx^3}} \, dx = -\int \frac{1}{-8cx^8\sqrt{c + dx^3} + dx^{11}\sqrt{c + dx^3}} \, dx$$

input
```
integrate(1/x**8/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(1/(-8*c*x**8*sqrt(c + d*x**3) + d*x**11*sqrt(c + d*x**3)), x)
```

3.320.7 Maxima [F]

$$\int \frac{1}{x^8 (8c-dx^3) \sqrt{c+dx^3}}\,dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^8}\,dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^8), x)
```

3.320.8 Giac [F]

$$\int \frac{1}{x^8 (8c-dx^3) \sqrt{c+dx^3}}\,dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^8}\,dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^8), x)
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (8c-dx^3) \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^8\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

input
```
int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.321 $\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.321.1 Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}$$

output
```
1/32*x^4*AppellF1(4/3,1/2,1,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/
(d*x^3+c)^(1/2)
```

3.321.2 Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32c\sqrt{c+dx^3}}$$

input
```
Integrate[x^3/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/
(8*c)])/(32*c*Sqrt[c + d*x^3])
```

3.321.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{(8c-dx^3)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}$$

input
```
Int[x^3/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(32*c*Sqrt[c + d*x^3])
```

3.321.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.321.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.36 (sec) , antiderivative size = 696, normalized size of antiderivative = 10.55

method	result
default	$\frac{2i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,F\Bigg(}{3d^{2}\sqrt{d\,x^{3}+c}}$
elliptic	$\frac{2i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,F\Bigg(}{3d^{2}\sqrt{d\,x^{3}+c}}$

input
```
int(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*
(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2))-8/27*I/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d
)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

3.321.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2284 vs. $2(52) = 104$.

Time = 0.56 (sec) , antiderivative size = 2284, normalized size of antiderivative = 34.61

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/54*(2*d^2*(1/(c*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^
3 + 640*c^3 + 18*(c*d^8*x^8 + 38*c^2*d^7*x^5 + 64*c^3*d^6*x^2)*(1/(c*d^8))
^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^9*x^7 + 80*c^2*d^8*x^4 + 160*c^3*d^7*x)*(
1/(c*d^8))^(5/6) + (7*c*d^6*x^6 + 152*c^2*d^5*x^3 + 64*c^3*d^4)*sqrt(1/(c*
d^8)) + 6*(5*c*d^3*x^5 + 32*c^2*d^2*x^2)*(1/(c*d^8))^(1/6)) + 18*(5*c*d^5*
x^7 + 64*c^2*d^4*x^4 + 32*c^3*d^3*x)*(1/(c*d^8))^(1/3))/(d^3*x^9 - 24*c*d^
2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*d^2*(1/(c*d^8))^(1/6)*log((d^3*x^9 +
 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c*d^8*x^8 + 38*c^2*d^7*x^5
 + 64*c^3*d^6*x^2)*(1/(c*d^8))^(2/3) - 6*sqrt(d*x^3 + c)*((c*d^9*x^7 + 80*
c^2*d^8*x^4 + 160*c^3*d^7*x)*(1/(c*d^8))^(5/6) + (7*c*d^6*x^6 + 152*c^2*d^
5*x^3 + 64*c^3*d^4)*sqrt(1/(c*d^8)) + 6*(5*c*d^3*x^5 + 32*c^2*d^2*x^2)*(1/
(c*d^8))^(1/6)) + 18*(5*c*d^5*x^7 + 64*c^2*d^4*x^4 + 32*c^3*d^3*x)*(1/(c*d
^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (sqrt(-3
)*d^2 + d^2)*(1/(c*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x
^3 + 640*c^3 - 9*(c*d^8*x^8 + 38*c^2*d^7*x^5 + 64*c^3*d^6*x^2 + sqrt(-3)*(
c*d^8*x^8 + 38*c^2*d^7*x^5 + 64*c^3*d^6*x^2))*(1/(c*d^8))^(2/3) + 3*sqrt(d
*x^3 + c)*((c*d^9*x^7 + 80*c^2*d^8*x^4 + 160*c^3*d^7*x - sqrt(-3)*(c*d^9*x
^7 + 80*c^2*d^8*x^4 + 160*c^3*d^7*x))*(1/(c*d^8))^(5/6) - 2*(7*c*d^6*x^6 +
 152*c^2*d^5*x^3 + 64*c^3*d^4)*sqrt(1/(c*d^8)) + 6*(5*c*d^3*x^5 + 32*c^2*d
^2*x^2 + sqrt(-3)*(5*c*d^3*x^5 + 32*c^2*d^2*x^2))*(1/(c*d^8))^(1/6)) - ...
```

3.321.6 Sympy [F]

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{x^3}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**3/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(x**3/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

3.321.7 Maxima [F]

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\int -\frac{x^3}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

3.321.8 Giac [F]

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\int -\frac{x^3}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)}\,dx$$

input
```
int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.322 $\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.322.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}$$

output

```
1/8*x*AppellF1(1/3,1/2,1,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/(d*
x^3+c)^(1/2)
```

3.322.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 166 vs. 2(64) = 128.

Time = 10.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.59

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$=\frac{32cx\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\sqrt{c+dx^3}\,\bigl(32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\,\bigl(\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\,\text{App}}$$

input

```
Integrate[1/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(32*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d
*x^3)*Sqrt[c + d*x^3]*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^
3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c
)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))
```

3.322.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(8c-dx^3)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}$$

input
```
Int[1/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(8*c*Sqrt[c + d*x^3])
```

3.322.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.322.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.22 (sec) , antiderivative size = 416, normalized size of antiderivative = 6.50

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c}{}\right)}{2\left(-c\right)}}}{}\right)}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c}{}\right)}{2\left(-c\right)}}}{}\right)}{}$

input
```
int(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

3.322. $\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

output

```
-1/27*I/d^3/c*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-
c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*
d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2
/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*
(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)
/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

3.322.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2319 vs. $2(50) = 100$.

Time = 0.59 (sec) , antiderivative size = 2319, normalized size of antiderivative = 36.23

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input

```
integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/432*(2*c*d*(1/(c^7*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d
*x^3 + 640*c^3 + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2)*(1/(c^
7*d^2))^(2/3) + 6*sqrt(d*x^3 + c)*((c^6*d^4*x^7 + 80*c^7*d^3*x^4 + 160*c^8
*d^2*x)*(1/(c^7*d^2))^(5/6) + (7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d)
*sqrt(1/(c^7*d^2)) + 6*(5*c^2*d^2*x^5 + 32*c^3*d*x^2)*(1/(c^7*d^2))^(1/6))
 + 18*(5*c^3*d^3*x^7 + 64*c^4*d^2*x^4 + 32*c^5*d*x)*(1/(c^7*d^2))^(1/3))/(
d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*c*d*(1/(c^7*d^2))^(
1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c^5*d^4
*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2)*(1/(c^7*d^2))^(2/3) - 6*sqrt(d*x^3
 + c)*((c^6*d^4*x^7 + 80*c^7*d^3*x^4 + 160*c^8*d^2*x)*(1/(c^7*d^2))^(5/6)
+ (7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d)*sqrt(1/(c^7*d^2)) + 6*(5*c^
2*d^2*x^5 + 32*c^3*d*x^2)*(1/(c^7*d^2))^(1/6)) + 18*(5*c^3*d^3*x^7 + 64*c^
4*d^2*x^4 + 32*c^5*d*x)*(1/(c^7*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192
*c^2*d*x^3 - 512*c^3)) + (sqrt(-3)*c*d + c*d)*(1/(c^7*d^2))^(1/6)*log((d^3
*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^5*d^4*x^8 + 38*c^6*
d^3*x^5 + 64*c^7*d^2*x^2 + sqrt(-3)*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7
*d^2*x^2))*(1/(c^7*d^2))^(2/3) + 3*sqrt(d*x^3 + c)*((c^6*d^4*x^7 + 80*c^7*
d^3*x^4 + 160*c^8*d^2*x - sqrt(-3)*(c^6*d^4*x^7 + 80*c^7*d^3*x^4 + 160*c^8
*d^2*x))*(1/(c^7*d^2))^(5/6) - 2*(7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6
*d)*sqrt(1/(c^7*d^2)) + 6*(5*c^2*d^2*x^5 + 32*c^3*d*x^2 + sqrt(-3)*(5*c...
```

3.322.6 Sympy [F]

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{1}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(1/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

3.322.7 Maxima [F]

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

3.322.8 Giac [F]

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)}\,dx$$

input
```
int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.323 $\int \frac{1}{x^3(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.323.1 Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16cx^2\sqrt{c+dx^3}}$$

output
```
-1/16*AppellF1(-2/3,1/2,1,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/x^
2/(d*x^3+c)^(1/2)
```

3.323.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 242 vs. $2(66) = 132$.

Time = 11.24 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.67

$$\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$
$$=\frac{-\frac{64(c+dx^3)}{c^2}+\frac{d^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{4096dx^3\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\right.}{(-8c+dx^3)\left(32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\right.\right.\right.}}{1024x^2\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
((-64*(c + d*x^3))/c^2 + (d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1
, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c^3 + (4096*d*x^3*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((-8*c + d*x^3)*(32*c*AppellF1[1/3,
1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2,
 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3
)/c), (d*x^3)/(8*c)]))))/(1024*x^2*Sqrt[c + d*x^3])
```

3.323.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (8c-dx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^3(8c-dx^3)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16cx^2\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/16*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, (d*x^3)/(8*c), -((d
*x^3)/c)])/(c*x^2*Sqrt[c + d*x^3])
```

3.323.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.323.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.90 (sec) , antiderivative size = 716, normalized size of antiderivative = 10.85

method	result
elliptic	$-\frac{\sqrt{d\,x^{3}+c}}{16 c^{2} x^{2}}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{48c^{2}\sqrt{d\,x^{3}+c}}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(d*x^3+c)^(1/2)/c^2/x^2+1/48*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/216*I/d^2/c^2*2^(1/2)*sum(1/_alpha^2*
(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^
(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_
alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_al
pha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/
2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^
3*d-8*c))
```

3.323.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2373 vs. $2(52) = 104$.

Time = 0.99 (sec) , antiderivative size = 2373, normalized size of antiderivative = 35.95

$$\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3456*(2*c^2*x^2*(d^4/c^13)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2
*d^4*x^3 + 640*c^3*d^3 + 18*(c^9*d^3*x^8 + 38*c^10*d^2*x^5 + 64*c^11*d*x^2
)*(d^4/c^13)^(2/3) + 6*sqrt(d*x^3 + c)*((c^11*d^2*x^7 + 80*c^12*d*x^4 + 16
0*c^13*x)*(d^4/c^13)^(5/6) + (7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*
sqrt(d^4/c^13) + 6*(5*c^3*d^4*x^5 + 32*c^4*d^3*x^2)*(d^4/c^13)^(1/6)) + 18
*(5*c^5*d^4*x^7 + 64*c^6*d^3*x^4 + 32*c^7*d^2*x)*(d^4/c^13)^(1/3))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*c^2*x^2*(d^4/c^13)^(1/6)*
log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + 18*(c^9*d^
3*x^8 + 38*c^10*d^2*x^5 + 64*c^11*d*x^2)*(d^4/c^13)^(2/3) - 6*sqrt(d*x^3 +
 c)*((c^11*d^2*x^7 + 80*c^12*d*x^4 + 160*c^13*x)*(d^4/c^13)^(5/6) + (7*c^7
*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(d^4/c^13) + 6*(5*c^3*d^4*x^5 +
 32*c^4*d^3*x^2)*(d^4/c^13)^(1/6)) + 18*(5*c^5*d^4*x^7 + 64*c^6*d^3*x^4 +
32*c^7*d^2*x)*(d^4/c^13)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
512*c^3)) - 144*sqrt(d)*x^2*weierstrassPInverse(0, -4*c/d, x) + (sqrt(-3)*
c^2*x^2 + c^2*x^2)*(d^4/c^13)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^
2*d^4*x^3 + 640*c^3*d^3 - 9*(c^9*d^3*x^8 + 38*c^10*d^2*x^5 + 64*c^11*d*x^2
 + sqrt(-3)*(c^9*d^3*x^8 + 38*c^10*d^2*x^5 + 64*c^11*d*x^2))*(d^4/c^13)^(2
/3) + 3*sqrt(d*x^3 + c)*((c^11*d^2*x^7 + 80*c^12*d*x^4 + 160*c^13*x - sqrt
(-3)*(c^11*d^2*x^7 + 80*c^12*d*x^4 + 160*c^13*x))*(d^4/c^13)^(5/6) - 2*(7*
c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(d^4/c^13) + 6*(5*c^3*d^4...
```

3.323.6 Sympy [F]

$$\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=-\int \frac{1}{-8cx^3\sqrt{c+dx^3}+dx^6\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**3/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(1/(-8*c*x**3*sqrt(c + d*x**3) + d*x**6*sqrt(c + d*x**3)), x)
```

3.323.7 Maxima [F]

$$\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^3}\,dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^3), x)
```

3.323.8 Giac [F]

$$\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^3}\,dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^3), x)
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\int \frac{1}{x^3\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

input
```
int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.324 $\int \frac{1}{x^6(8c-dx^3)\sqrt{c+dx^3}}\,dx$

3.324.1 Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{5}{3},1,\frac{1}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40cx^5\sqrt{c+dx^3}}$$

output
```
-1/40*AppellF1(-5/3,1/2,1,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/x
^5/(d*x^3+c)^(1/2)
```

3.324.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 261 vs. 2(66) = 132.

Time = 11.22 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.95

$$\int \frac{1}{x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$

$$=\frac{-23d^3x^9\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+64c\Big(-16c^2+7cdx^3+23d^2x^6+\frac{}{(8c-dx^3)\Big(32c\,\text{AppellF1}(}}{40960c^4x^5\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-23*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
(d*x^3)/(8*c)] + 64*c*(-16*c^2 + 7*c*d*x^3 + 23*d^2*x^6 + (3264*c^2*d^2*x^
6*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*
(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(A
ppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3
/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))))/(40960*c^4*x^5*Sqrt[c + d*x^
3])
```

3.324.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (8c-dx^3) \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{x^6(8c-dx^3)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1} \operatorname{AppellF1}\left(-\frac{5}{3},1,\frac{1}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40cx^5\sqrt{c+dx^3}}$$

input
```
Int[1/(x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/40*(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 1, 1/2, -2/3, (d*x^3)/(8*c), -((
d*x^3)/c)])/(c*x^5*Sqrt[c + d*x^3])
```

3.324.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.324.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.92 (sec) , antiderivative size = 732, normalized size of antiderivative = 11.09

method	result	size
risch	Expression too large to display	732
elliptic	Expression too large to display	735
default	Expression too large to display	1047

input
```
int(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/640*(d*x^3+c)^(1/2)*(-23*d*x^3+16*c)/c^3/x^5+1/1280/c^3*d^2*(-46/3*I*3^
(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-20/27*
I/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(
1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1
/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3
+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_a
lpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

3.324.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2417 vs. $2(52) = 104$.

Time = 2.51 (sec) , antiderivative size = 2417, normalized size of antiderivative = 36.62

$$\int \frac{1}{x^6 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/138240*(10*c^3*x^5*(d^10/c^19)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 12
00*c^2*d^9*x^3 + 640*c^3*d^8 + 18*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^1
5*d^2*x^2)*(d^10/c^19)^(2/3) + 6*sqrt(d*x^3 + c)*((c^16*d^2*x^7 + 80*c^17*
d*x^4 + 160*c^18*x)*(d^10/c^19)^(5/6) + (7*c^10*d^5*x^6 + 152*c^11*d^4*x^3
 + 64*c^12*d^3)*sqrt(d^10/c^19) + 6*(5*c^4*d^8*x^5 + 32*c^5*d^7*x^2)*(d^10
/c^19)^(1/6)) + 18*(5*c^7*d^7*x^7 + 64*c^8*d^6*x^4 + 32*c^9*d^5*x)*(d^10/c
^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 10*c^3*x
^5*(d^10/c^19)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 6
40*c^3*d^8 + 18*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2)*(d^10/c
^19)^(2/3) - 6*sqrt(d*x^3 + c)*((c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x
)*(d^10/c^19)^(5/6) + (7*c^10*d^5*x^6 + 152*c^11*d^4*x^3 + 64*c^12*d^3)*sq
rt(d^10/c^19) + 6*(5*c^4*d^8*x^5 + 32*c^5*d^7*x^2)*(d^10/c^19)^(1/6)) + 18
*(5*c^7*d^7*x^7 + 64*c^8*d^6*x^4 + 32*c^9*d^5*x)*(d^10/c^19)^(1/3))/(d^3*x
^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 5328*d^(3/2)*x^5*weierstra
ssPInverse(0, -4*c/d, x) + 5*(sqrt(-3)*c^3*x^5 + c^3*x^5)*(d^10/c^19)^(1/6
)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^1
3*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2 + sqrt(-3)*(c^13*d^4*x^8 + 3
8*c^14*d^3*x^5 + 64*c^15*d^2*x^2))*(d^10/c^19)^(2/3) + 3*sqrt(d*x^3 + c)*(
(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x - sqrt(-3)*(c^16*d^2*x^7 + 80*c
^17*d*x^4 + 160*c^18*x))*(d^10/c^19)^(5/6) - 2*(7*c^10*d^5*x^6 + 152*c^...
```

3.324.6 Sympy [F]

$$\int \frac{1}{x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = -\int \frac{1}{-8cx^6\sqrt{c+dx^3}+dx^9\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**6/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

output
```
-Integral(1/(-8*c*x**6*sqrt(c + d*x**3) + d*x**9*sqrt(c + d*x**3)), x)
```

3.324.7 Maxima [F]

$$\int \frac{1}{x^6 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^6} \, dx$$

input
```
integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^6), x)
```

3.324.8 Giac [F]

$$\int \frac{1}{x^6 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^6} \, dx$$

input
```
integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^6), x)
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^6 \sqrt{d\,x^3 + c}\,(8\,c - d\,x^3)} \, dx$$

input
```
int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.325 $\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.325.1 Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{4c\sqrt{c+dx^3}}{d^4} - \frac{2(c+dx^3)^{3/2}}{9d^4} + \frac{1024c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}$$

output
```
-2/9*(d*x^3+c)^(3/2)/d^4+1024/81*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/
2))/d^4+2/27*c^2/d^4/(d*x^3+c)^(1/2)-4*c*(d*x^3+c)^(1/2)/d^4
```

3.325.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\left(-\frac{3(56c^2+60cdx^3+3d^2x^6)}{\sqrt{c+dx^3}} + 512c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^4}$$

input
```
Integrate[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2*((-3*(56*c^2 + 60*c*d*x^3 + 3*d^2*x^6))/Sqrt[c + d*x^3] + 512*c^(3/2)*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(81*d^4)
```

3.325.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \textcolor{blue}{948}$$

$$\frac{1}{3}\int \frac{x^9}{(8c-dx^3)(dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \textcolor{blue}{98}$$

$$\frac{1}{3}\int \left(-\frac{x^3}{d^2\sqrt{dx^3+c}}+\frac{512c^2}{9d^3(8c-dx^3)\sqrt{dx^3+c}}-\frac{7c}{d^3\sqrt{dx^3+c}}-\frac{c^2}{9d^3(dx^3+c)^{3/2}}\right)dx^3$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{1}{3}\left(\frac{1024c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27d^4}+\frac{2c^2}{9d^4\sqrt{c+dx^3}}-\frac{12c\sqrt{c+dx^3}}{d^4}-\frac{2(c+dx^3)^{3/2}}{3d^4}\right)$$

input
```
Int[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((2*c^2)/(9*d^4*Sqrt[c + d*x^3]) - (12*c*Sqrt[c + d*x^3])/d^4 - (2*(c + d*
x^3)^(3/2))/(3*d^4) + (1024*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(27*d^4))/3
```

3.325.3.1 Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.325.4 Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{-\frac{2 d^{2} x^{6}}{9}+\frac{1024 c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right) \sqrt{d\,x^{3}+c}}{81}-\frac{40 c d\,x^{3}}{9}-\frac{112 c^{2}}{27}}{\sqrt{d\,x^{3}+c}\, d^{4}}$
risch	$-\frac{2\left(d\,x^{3}+19 c\right) \sqrt{d\,x^{3}+c}}{9 d^{4}}-\frac{c^{2}\left(-\frac{2}{27 d \sqrt{d\,x^{3}+c}}-\frac{1024 \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{81 d \sqrt{c}}\right)}{d^{3}}$
default	$-\frac{-\frac{2 c^{2}}{3 d^{3} \sqrt{\left(x^{3}+\frac{c}{d}\right) d}}+\frac{2 x^{3} \sqrt{d\,x^{3}+c}}{9 d^{2}}-\frac{10 c \sqrt{d\,x^{3}+c}}{9 d^{3}}}{d}-\frac{8 c\left(\frac{2 c}{3 d^{2} \sqrt{\left(x^{3}+\frac{c}{d}\right) d}}+\frac{2 \sqrt{d\,x^{3}+c}}{3 d^{2}}\right)}{d^{2}}+\frac{128 c^{2}}{3 d^{4} \sqrt{d\,x^{3}+c}}-$
elliptic	$\frac{2 c^{2}}{27 d^{4} \sqrt{\left(x^{3}+\frac{c}{d}\right) d}}-\frac{2 x^{3} \sqrt{d\,x^{3}+c}}{9 d^{3}}-\frac{38 c \sqrt{d\,x^{3}+c}}{9 d^{4}}-$

input
```
int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/81*(-9*d^2*x^6+512*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*(d*x^3+c
)^(1/2)-180*c*d*x^3-168*c^2)/(d*x^3+c)^(1/2)/d^4
```

3.325.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.10

$$\int \frac{x^{11}}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = \left[\frac{2\left(256\left(cdx^3+c^2\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\left(3\,d^2x^6+60\,cdx^3+56\right.\right.}{81\left(d^5x^3+cd^4\right)}\right.$$

$$\left.-\frac{2\left(512\left(cdx^3+c^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\left(3\,d^2x^6+60\,cdx^3+56\,c^2\right)\sqrt{dx^3+c}\right)}{81\left(d^5x^3+cd^4\right)}\right]$$

input
```
integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[2/81*(256*(c*d*x^3 + c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c)
+ 10*c)/(d*x^3 - 8*c)) - 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*sqrt(d*x^3 +
c))/(d^5*x^3 + c*d^4), -2/81*(512*(c*d*x^3 + c^2)*sqrt(-c)*arctan(1/3*sqrt
(d*x^3 + c)*sqrt(-c)/c) + 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*sqrt(d*x^3 +
 c))/(d^5*x^3 + c*d^4)]
```

3.325.6 Sympy [A] (verification not implemented)

Time = 21.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^2}{27\sqrt{c+dx^3}}-\frac{512c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81\sqrt{-c}}-2c\sqrt{c+dx^3}-\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^4} & \text{for } d\neq 0 \\ \frac{x^{12}}{96c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
Piecewise((2*(c**2/(27*sqrt(c + d*x**3)) - 512*c**2*atan(sqrt(c + d*x**3)/
(3*sqrt(-c)))/(81*sqrt(-c)) - 2*c*sqrt(c + d*x**3) - (c + d*x**3)**(3/2)/9
)/d**4, Ne(d, 0)), (x**12/(96*c**(5/2)), True))
```

3.325.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = -\frac{2\left(256\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+9\,(dx^3+c)^{\frac{3}{2}}+162\,\sqrt{dx^3+c}c-\frac{3\,c^2}{\sqrt{dx^3+c}}\right)}{81\,d^4}$$

input
```
integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-2/81*(256*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 9*(d*x^3 + c)^(3/2) + 162*sqrt(d*x^3 + c)*c - 3*c^2/sqrt(d*x^3
 + c))/d^4
```

3.325.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = -\frac{1024\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{81\,\sqrt{-c}d^4} + \frac{2\,c^2}{27\,\sqrt{dx^3+c}d^4} - \frac{2\left((dx^3+c)^{\frac{3}{2}}d^8+18\,\sqrt{dx^3+c}cd^8\right)}{9\,d^{12}}$$

input
```
integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-1024/81*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) + 2/27*c^
2/(sqrt(d*x^3 + c)*d^4) - 2/9*((d*x^3 + c)^(3/2)*d^8 + 18*sqrt(d*x^3 + c)*
c*d^8)/d^12
```

3.325.9 Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{512\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^4} - \frac{38\,c\,\sqrt{d\,x^3+c}}{9\,d^4} + \frac{2\,c^2}{27\,d^4\,\sqrt{d\,x^3+c}} - \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^3}$$

input
```
int(x^11/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
(512*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/(81*d^4) - (38*c*(c + d*x^3)^(1/2))/(9*d^4) + (2*c^2)/(27*d^4*(c + d*x
^3)^(1/2)) - (2*x^3*(c + d*x^3)^(1/2))/(9*d^3)
```

3.326 $\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.326.1 Optimal result

Integrand size = 27, antiderivative size = 71

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2c}{27d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{128\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

output
```
128/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^3-2/27*c/d^3/(d*x^3+
c)^(1/2)-2/3*(d*x^3+c)^(1/2)/d^3
```

3.326.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\left(-\frac{3(10c+9dx^3)}{\sqrt{c+dx^3}}+64\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^3}$$

input
```
Integrate[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2*((-3*(10*c + 9*d*x^3))/Sqrt[c + d*x^3] + 64*Sqrt[c]*ArcTanh[Sqrt[c + d*
x^3]/(3*Sqrt[c])]))/(81*d^3)
```

3.326.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{(8c-dx^3)(dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{98}$$

$$\frac{1}{3}\int \left(\frac{64c}{9d^2(8c-dx^3)\sqrt{dx^3+c}}+\frac{c}{9d^2(dx^3+c)^{3/2}}-\frac{1}{d^2\sqrt{dx^3+c}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{128\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27d^3}-\frac{2c}{9d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{d^3}\right)$$

input
```
Int[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((-2*c)/(9*d^3*Sqrt[c + d*x^3]) - (2*Sqrt[c + d*x^3])/d^3 + (128*Sqrt[c]*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(27*d^3))/3
```

3.326.3.1 Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.326.4 Maple [A] (verified)

Time = 4.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$-\frac{2\left(-64\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}\,\sqrt{d\,x^{3}+c}+27d\,x^{3}+30c\right)}{81\sqrt{d\,x^{3}+c}\,d^{3}}$
risch	$-\frac{2\sqrt{d\,x^{3}+c}}{3d^{3}}-\frac{c\left(\frac{2}{27d\sqrt{d\,x^{3}+c}}-\frac{128\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{81d\sqrt{c}}\right)}{d^{2}}$
default	$-\frac{\frac{2c}{3d^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}}{d}+\frac{16c}{3d^{3}\sqrt{d\,x^{3}+c}}-\frac{128\sqrt{c}\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{d\,x^{3}+c}}{3}+\sqrt{c}\right)}{27d^{3}\sqrt{d\,x^{3}+c}}$
elliptic	$-\frac{2c}{27d^{3}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{2\sqrt{d\,x^{3}+c}}{3d^{3}}-\frac{64i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$

input
```
int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/81*(-64*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)*(d*x^3+c)^(1/2)+27
*d*x^3+30*c)/(d*x^3+c)^(1/2)/d^3
```

3.326.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.27

$$\int \frac{x^8}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \left[\frac{2\left(32\,(dx^3+c)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\,(9\,dx^3+10\,c)\sqrt{dx^3+c}\right)}{81\,(d^4x^3+cd^3)},\right.$$
$$\left.-\frac{2\left(64\,(dx^3+c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\,(9\,dx^3+10\,c)\sqrt{dx^3+c}\right)}{81\,(d^4x^3+cd^3)}\right]$$

input
```
integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[2/81*(32*(d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) - 3*(9*d*x^3 + 10*c)*sqrt(d*x^3 + c))/(d^4*x^3 + c*d^3),
 -2/81*(64*(d*x^3 + c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3
*(9*d*x^3 + 10*c)*sqrt(d*x^3 + c))/(d^4*x^3 + c*d^3)]
```

3.326.6 Sympy [A] (verification not implemented)

Time = 11.70 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{x^8}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{c}{27\sqrt{c+dx^3}}-\frac{64c\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{x^9}{72c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
Piecewise((2*(-c/(27*sqrt(c + d*x**3)) - 64*c*atan(sqrt(c + d*x**3)/(3*sqr
t(-c)))/(81*sqrt(-c)) - sqrt(c + d*x**3)/3)/d**3, Ne(d, 0)), (x**9/(72*c**
(5/2)), True))
```

3.326.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{x^8}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = -\frac{2\left(32\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+27\sqrt{dx^3+c}+\frac{3c}{\sqrt{dx^3+c}}\right)}{81\,d^3}$$

input
```
integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-2/81*(32*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 27*sqrt(d*x^3 + c) + 3*c/sqrt(d*x^3 + c))/d^3
```

3.326.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = -\frac{128\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}d^3} - \frac{2\sqrt{dx^3+c}}{3\,d^3} - \frac{2\,c}{27\sqrt{dx^3+c}d^3}$$

input
```
integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-128/81*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/3*sqrt(d
*x^3 + c)/d^3 - 2/27*c/(sqrt(d*x^3 + c)*d^3)
```

3.326.9 Mupad [B] (verification not implemented)

Time = 7.92 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{x^8}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = \frac{64\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\sqrt{c}\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^3} - \frac{2\,c}{27\,d^3\sqrt{d\,x^3+c}} - \frac{2\sqrt{d\,x^3+c}}{3\,d^3}$$

input
```
int(x^8/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
(64*c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/(81*d^3) - (2*c)/(27*d^3*(c + d*x^3)^(1/2)) - (2*(c + d*x^3)^(1/2))/(3*
d^3)
```

3.327 $\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.327.1 Optimal result

Integrand size = 27, antiderivative size = 52

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{16\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

output
```
16/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2/c^(1/2)+2/27/d^2/(d*x^3+c)^
(1/2)
```

3.327.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\left(\frac{3}{\sqrt{c+dx^3}} + \frac{8\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{81d^2}$$

input
```
Integrate[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2*(3/Sqrt[c + d*x^3] + (8*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))
/(81*d^2)
```

3.327.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {948, 87, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(8c-dx^3)(dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{8\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9d}+\frac{2}{9d^2\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{16\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9d^2}+\frac{2}{9d^2\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{16\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27\sqrt{c}d^2}+\frac{2}{9d^2\sqrt{c+dx^3}}\right)$$

input
```
Int[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2/(9*d^2*Sqrt[c + d*x^3]) + (16*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(27
*Sqrt[c]*d^2))/3
```

3.327.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.327.4 Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$\frac{\frac{16\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{d\,x^3+c}}{81}+\frac{2\sqrt{c}}{27}}{\sqrt{d\,x^3+c}\,\sqrt{c}\,d^2}$
default	$\frac{2}{3d^2\sqrt{d\,x^3+c}}-\frac{16\left(-\frac{\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{d\,x^3+c}}{3}+\sqrt{c}\right)}{27\sqrt{c}\,d^2\sqrt{d\,x^3+c}}$
elliptic	$\frac{2}{27d^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{8i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)}{d}\right.}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,(}}}{}\right.}{}$

input
```
int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/27*(8/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*(d*x^3+c)^(1/2)+c^(1/2))/(d
*x^3+c)^(1/2)/c^(1/2)/d^2
```

3.327.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.87

$$\int \frac{x^5}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx=\left[\frac{2\left(4\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\sqrt{dx^3+c}c\right)}{81\left(cd^3x^3+c^2d^2\right)},\right.$$
$$\left.-\frac{2\left(8\left(dx^3+c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)-3\sqrt{dx^3+c}c\right)}{81\left(cd^3x^3+c^2d^2\right)}\right]$$

input
```
integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[2/81*(4*(d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) + 3*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 + c^2*d^2), -2/81*(8*(d
*x^3 + c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(d*x^3 +
 c)*c)/(c*d^3*x^3 + c^2*d^2)]
```

3.327.6 Sympy [A] (verification not implemented)

Time = 7.90 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \begin{cases} \frac{2\cdot\left(\frac{1}{27\sqrt{c+dx^3}}-\frac{8\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81\sqrt{-c}}\right)}{d^2} & \text{for } d \neq 0 \\ \frac{x^6}{48c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
Piecewise((2*(1/(27*sqrt(c + d*x**3)) - 8*atan(sqrt(c + d*x**3)/(3*sqrt(-c
)))/(81*sqrt(-c)))/d**2, Ne(d, 0)), (x**6/(48*c**(5/2)), True))
```

3.327.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = -\frac{2\left(\frac{4\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}}-\frac{3}{\sqrt{dx^3+c}}\right)}{81\,d^2}$$

input
```
integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-2/81*(4*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/
sqrt(c) - 3/sqrt(d*x^3 + c))/d^2
```

3.327.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = -\frac{2\left(\frac{8\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}-\frac{3}{\sqrt{dx^3+c}d}\right)}{81\,d}$$

input
```
integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-2/81*(8*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 3/(sqrt(d*x^3
 + c)*d))/d
```

3.327.9 Mupad [B] (verification not implemented)

Time = 7.89 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{2}{27\,d^2\,\sqrt{d\,x^3+c}}+\frac{8\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,\sqrt{c}\,d^2}$$

input
```
int(x^5/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
2/(27*d^2*(c + d*x^3)^(1/2)) + (8*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3
)^(1/2))/(8*c - d*x^3)))/(81*c^(1/2)*d^2)
```

3.328 $\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.328.1 Optimal result

Integrand size = 27, antiderivative size = 55

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2}{27cd\sqrt{c+dx^3}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

output
```
2/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)/d-2/27/c/d/(d*x^3+c)^(1/
2)
```

3.328.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\left(-\frac{3\sqrt{c}}{\sqrt{c+dx^3}}+\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81c^{3/2}d}$$

input
```
Integrate[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2*((-3*Sqrt[c])/Sqrt[c + d*x^3] + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/
(81*c^(3/2)*d)
```

3.328.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {946, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3}\int \frac{1}{(8c-dx^3)(dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \; 61$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9c}-\frac{2}{9cd\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{2\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9cd}-\frac{2}{9cd\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27c^{3/2}d}-\frac{2}{9cd\sqrt{c+dx^3}}\right)$$

input
```
Int[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(-2/(9*c*d*Sqrt[c + d*x^3]) + (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(27
*c^(3/2)*d))/3
```

3.328.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.328.4 Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

method	result
default	$-\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{d\,x^{3}+c}}{3}+\sqrt{c}\right)}{27\sqrt{d\,x^{3}+c}\,c^{\frac{3}{2}}\,d}$
pseudoelliptic	$-\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{d\,x^{3}+c}}{3}+\sqrt{c}\right)}{27\sqrt{d\,x^{3}+c}\,c^{\frac{3}{2}}\,d}$
elliptic	$-\frac{2}{27dc\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right.}{d}\right.}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}}}}{}\right.}{}$

input
```
int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/27/(d*x^3+c)^(1/2)/c^(3/2)*(-1/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*(
d*x^3+c)^(1/2)+c^(1/2))/d
```

3.328.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.67

$$\int \frac{x^2}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \left[\frac{(dx^3+c)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\sqrt{dx^3+c}c}{81\,(c^2d^2x^3+c^3d)},\right.$$
$$\left.-\frac{2\left((dx^3+c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\sqrt{dx^3+c}c\right)}{81\,(c^2d^2x^3+c^3d)}\right]$$

input
```
integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/81*((d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/
(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 + c^3*d), -2/81*((d*x^3
 + c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*sqrt(d*x^3 + c)*
c)/(c^2*d^2*x^3 + c^3*d)]
```

3.328.6 Sympy [A] (verification not implemented)

Time = 6.51 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{1}{27c\sqrt{c+dx^3}}-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81c\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{x^3}{24c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
Piecewise((2*(-1/(27*c*sqrt(c + d*x**3)) - atan(sqrt(c + d*x**3)/(3*sqrt(-
c)))/(81*c*sqrt(-c)))/d, Ne(d, 0)), (x**3/(24*c**(5/2)), True))
```

3.328.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = -\frac{\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{6}{\sqrt{dx^3+c}c}}{81\,d}$$

input
```
integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/81*(log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^
(3/2) + 6/(sqrt(d*x^3 + c)*c))/d
```

3.328.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}cd} - \frac{2}{27\sqrt{dx^3+c}cd}$$

input
```
integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-2/81*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) - 2/27/(sqrt(d*x
^3 + c)*c*d)
```

3.328.9 Mupad [B] (verification not implemented)

Time = 7.89 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,c^{3/2}\,d} - \frac{2}{27\,c\,d\,\sqrt{d\,x^3+c}}$$

input
```
int(x^2/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(81*c^(3/2
)*d) - 2/(27*c*d*(c + d*x^3)^(1/2))
```

3.329 $\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.329.1 Optimal result

Integrand size = 27, antiderivative size = 76

$$\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{2}{27c^2\sqrt{c+dx^3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}$$

output
```
1/324*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/12*arctanh((d*x^3+c)^
(1/2)/c^(1/2))/c^(5/2)+2/27/c^2/(d*x^3+c)^(1/2)
```

3.329.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{\frac{24\sqrt{c}}{\sqrt{c+dx^3}}+\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-27\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{324c^{5/2}}$$

input
```
Integrate[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((24*Sqrt[c])/Sqrt[c + d*x^3] + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 27*
ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(324*c^(5/2))
```

3.329.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {948, 96, 25, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{96}$$

$$\frac{1}{3}\left(\frac{2}{9c^2\sqrt{c+dx^3}}-\frac{\int -\frac{d\left(9c-dx^3\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c^2d}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{\int \frac{d\left(9c-dx^3\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c^2d}+\frac{2}{9c^2\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{9c-dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c^2}+\frac{2}{9c^2\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{\frac{9}{8}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{1}{8}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c^2}+\frac{2}{9c^2\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\frac{1}{4}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{9\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4d}}{9c^2}+\frac{2}{9c^2\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\frac{9\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{4d}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{12\sqrt{c}}}{9c^2}+\frac{2}{9c^2\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{3}\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{12\sqrt{c}}-\frac{9\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{4\sqrt{c}}}{9c^2}+\frac{2}{9c^2\sqrt{c+dx^3}}\right)$$

input
```
Int[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2/(9*c^2*Sqrt[c + d*x^3]) + (ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(12*Sqr
t[c]) - (9*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(4*Sqrt[c]))/(9*c^2))/3
```

3.329.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.329.4 Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{d\,x^{3}+c}-27\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\sqrt{d\,x^{3}+c}+24\sqrt{c}}{324c^{\frac{5}{2}}\sqrt{d\,x^{3}+c}}$	71
default	$\frac{\frac{2}{3c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{8c}-\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{d\,x^{3}+c}}{3}+\sqrt{c}}{108c^{\frac{5}{2}}\sqrt{d\,x^{3}+c}}$	89
elliptic	Expression too large to display	1526

input
```
int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/324/c^(5/2)*(arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*(d*x^3+c)^(1/2)-27*arc
tanh((d*x^3+c)^(1/2)/c^(1/2))*(d*x^3+c)^(1/2)+24*c^(1/2))/(d*x^3+c)^(1/2)
```

3.329.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.80

$$\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \left[\frac{\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+27\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}}{x^3}\right.}{648\left(c^3dx^3+c^4\right)}\right.$$

input
```
integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/648*((d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)
/(d*x^3 - 8*c)) + 27*(d*x^3 + c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sq
rt(c) + 2*c)/x^3) + 48*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 + c^4), 1/324*(27*(d*
x^3 + c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (d*x^3 + c)*sqrt(-c
)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 24*sqrt(d*x^3 + c)*c)/(c^3*d*x^
3 + c^4)]
```

3.329.6 Sympy [A] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

$$\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{d}{27c^2\sqrt{c+dx^3}}-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{648c^2\sqrt{-c}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24c^2\sqrt{-c}}\right)}{d} & \text{for } d \neq 0 \\ \frac{\log\left(x^3\right)}{24c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
Piecewise((2*(d/(27*c**2*sqrt(c + d*x**3)) - d*atan(sqrt(c + d*x**3)/(3*sq
rt(-c)))/(648*c**2*sqrt(-c)) + d*atan(sqrt(c + d*x**3)/sqrt(-c))/(24*c**2*
sqrt(-c)))/d, Ne(d, 0)), (log(x**3)/(24*c**(5/2)), True))
```

3.329.7 Maxima [F]

$$\int \frac{1}{x\left(8c-dx^{3}\right)\left(c+dx^{3}\right)^{3/2}}\,dx=\int -\frac{1}{\left(dx^{3}+c\right)^{\frac{3}{2}}\left(dx^{3}-8\,c\right)x}\,dx$$

input
```
integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x), x)
```

3.329.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(8c-dx^{3}\right)\left(c+dx^{3}\right)^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}c^{2}}-\frac{\arctan\left(\frac{\sqrt{dx^{3}+c}}{3\sqrt{-c}}\right)}{324\sqrt{-c}c^{2}}+\frac{2}{27\sqrt{dx^{3}+c}c^{2}}$$

input
```
integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/324*arctan(1/3*sq
rt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) + 2/27/(sqrt(d*x^3 + c)*c^2)
```

3.329.9 Mupad [B] (verification not implemented)

Time = 7.94 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(8c-dx^{3}\right)\left(c+dx^{3}\right)^{3/2}}\,dx=\frac{2}{27\,c^{2}\sqrt{d\,x^{3}+c}}-\frac{\mathrm{atanh}\left(\frac{c^{2}\sqrt{d\,x^{3}+c}}{\sqrt{c^{5}}}\right)}{12\sqrt{c^{5}}}+\frac{\mathrm{atanh}\left(\frac{c^{2}\sqrt{d\,x^{3}+c}}{3\sqrt{c^{5}}}\right)}{324\sqrt{c^{5}}}$$

input
```
int(1/(x*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
2/(27*c^2*(c + d*x^3)^(1/2)) - atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2))/
(12*(c^5)^(1/2)) + atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))/(324*(c^
5)^(1/2))
```

3.330 $\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.330.1 Optimal result

Integrand size = 27, antiderivative size = 100

$$\int \frac{1}{x^4\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=-\frac{25d}{216c^3\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\sqrt{c+dx^3}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}}+\frac{11d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

output
```
1/2592*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+11/96*d*arctanh((d*x
^3+c)^(1/2)/c^(1/2))/c^(7/2)-25/216*d/c^3/(d*x^3+c)^(1/2)-1/24/c^2/x^3/(d*
x^3+c)^(1/2)
```

3.330.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{-\frac{12\sqrt{c}(9c+25dx^3)}{x^3\sqrt{c+dx^3}}+d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+297d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2592c^{7/2}}$$

input
```
Integrate[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((-12*Sqrt[c]*(9*c + 25*d*x^3))/(x^3*Sqrt[c + d*x^3]) + d*ArcTanh[Sqrt[c +
 d*x^3]/(3*Sqrt[c])] + 297*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2592*c^(7/
2))
```

3.330.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 114, 27, 169, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (8c - dx^3) (c + dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3} \int \frac{1}{x^6 (8c - dx^3) (dx^3 + c)^{3/2}} dx^3$$

$$\Big\downarrow \; 114$$

$$\frac{1}{3} \left(-\frac{\int \frac{d(22c-3dx^3)}{2x^3(8c-dx^3)(dx^3+c)^{3/2}} dx^3}{8c^2} - \frac{1}{8c^2 x^3 \sqrt{c+dx^3}} \right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3} \left(-\frac{d \int \frac{22c-3dx^3}{x^3(8c-dx^3)(dx^3+c)^{3/2}} dx^3}{16c^2} - \frac{1}{8c^2 x^3 \sqrt{c+dx^3}} \right)$$

$$\Big\downarrow \; 169$$

$$\frac{1}{3} \left(-\frac{d \left(\frac{2 \int \frac{cd\left(198c-25dx^3\right)}{2x^3\left(8c-dx^3\right)\sqrt{dx^3+c}} dx^3}{9c^2 d} + \frac{50}{9c\sqrt{c+dx^3}} \right)}{16c^2} - \frac{1}{8c^2 x^3 \sqrt{c+dx^3}} \right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3} \left(-\frac{d \left(\frac{\int \frac{198c-25dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}} dx^3}{9c} + \frac{50}{9c\sqrt{c+dx^3}} \right)}{16c^2} - \frac{1}{8c^2 x^3 \sqrt{c+dx^3}} \right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{\frac{99}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3-\frac{1}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{50}{9c\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{\frac{99\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{1}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9c}+\frac{50}{9c\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{\frac{99\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{9c}+\frac{50}{9c\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{99\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{9c}+\frac{50}{9c\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\sqrt{c+dx^3}}\right)$$

input
```
Int[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(-1/8*1/(c^2*x^3*Sqrt[c + d*x^3]) - (d*(50/(9*c*Sqrt[c + d*x^3]) + (-1/6*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/Sqrt[c] - (99*ArcTanh[Sqrt[c + d*x^3]/
Sqrt[c]])/(2*Sqrt[c]))/(9*c)))/(16*c^2))/3
```

3.330. $\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.330.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.330. $\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.330.4 Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}}{24c^{3}x^{3}}-\frac{d\left(-\frac{11\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{32}{27\sqrt{d\,x^{3}+c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{162\sqrt{c}}\right)}{16c^{3}}$
pseudoelliptic	$\frac{297\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d\,x^{3}\sqrt{d\,x^{3}+c}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)d\,x^{3}\sqrt{d\,x^{3}+c}-300d\,x^{3}\sqrt{c}-108c^{\frac{3}{2}}}{2592c^{\frac{7}{2}}x^{3}\sqrt{d\,x^{3}+c}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3c^{2}x^{3}}-\frac{2d}{3c^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{8c}+\frac{d\left(\frac{2}{3c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{64c^{2}}-\frac{d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right.}{3}\right.}{864c^{\frac{7}{2}}\sqrt{d\,x}}$
elliptic	Expression too large to display

input
```
int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24/c^3*(d*x^3+c)^(1/2)/x^3-1/16/c^3*d*(-11/6*arctanh((d*x^3+c)^(1/2)/c^
(1/2))/c^(1/2)+32/27/(d*x^3+c)^(1/2)-1/162*arctanh(1/3*(d*x^3+c)^(1/2)/c^(
1/2))/c^(1/2))
```

3.330.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.72

$$\int \frac{1}{x^4 (8c-dx^3) (c+dx^3)^{3/2}}\,dx = \left[\frac{(d^2x^6+cdx^3)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+297\,(d^2x^6+cdx^3)\sqrt{c}\log\left(\right.}{5184\,(c^4dx^6+c^5x^3)}\right.$$

$$-\frac{297\,(d^2x^6+cdx^3)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+(d^2x^6+cdx^3)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+12\,(25\,cdx^3+9\,c^2}{2592\,(c^4dx^6+c^5x^3)}$$

input
```
integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/5184*((d^2*x^6 + c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c
) + 10*c)/(d*x^3 - 8*c)) + 297*(d^2*x^6 + c*d*x^3)*sqrt(c)*log((d*x^3 + 2*
sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(25*c*d*x^3 + 9*c^2)*sqrt(d*x^3 +
 c))/(c^4*d*x^6 + c^5*x^3), -1/2592*(297*(d^2*x^6 + c*d*x^3)*sqrt(-c)*arct
an(sqrt(d*x^3 + c)*sqrt(-c)/c) + (d^2*x^6 + c*d*x^3)*sqrt(-c)*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + 12*(25*c*d*x^3 + 9*c^2)*sqrt(d*x^3 + c))/(c^4
*d*x^6 + c^5*x^3)]
```

3.330.6 Sympy [F]

$$\int \frac{1}{x^4 (8c-dx^3) (c+dx^3)^{3/2}}\,dx = -\int \frac{1}{-8c^2x^4\sqrt{c+dx^3}-7cdx^7\sqrt{c+dx^3}+d^2x^{10}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
-Integral(1/(-8*c**2*x**4*sqrt(c + d*x**3) - 7*c*d*x**7*sqrt(c + d*x**3) +
 d**2*x**10*sqrt(c + d*x**3)), x)
```

3.330.7 Maxima [F]

$$\int \frac{1}{x^4\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\int -\frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^4}\,dx$$

input
```
integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^4), x)
```

3.330.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=-\frac{11\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}c^3}-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{2592\,\sqrt{-c}c^3}-\frac{25\,(dx^3+c)d-16\,cd}{216\,\left(\left(dx^3+c\right)^{\frac{3}{2}}-\sqrt{dx^3+c}c\right)c^3}$$

input
```
integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-11/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/2592*d*arctan
(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/216*(25*(d*x^3 + c)*d -
16*c*d)/(((d*x^3 + c)^(3/2) - sqrt(d*x^3 + c)*c)*c^3)
```

3.330.9 Mupad [B] (verification not implemented)

Time = 8.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{11\,d\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)}{96\,\sqrt{c^7}}-\frac{25\,d}{216\,c^3\,\sqrt{d\,x^3+c}}+\frac{d\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)}{2592\,\sqrt{c^7}}-\frac{1}{24\,c^2\,x^3\,\sqrt{d\,x^3+c}}$$

input
```
int(1/(x^4*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
(11*d*atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2)))/(96*(c^7)^(1/2)) - (25*d
)/(216*c^3*(c + d*x^3)^(1/2)) + (d*atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^
(1/2))))/(2592*(c^7)^(1/2)) - 1/(24*c^2*x^3*(c + d*x^3)^(1/2))
```

3.330. $\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.331 $\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.331.1 Optimal result

Integrand size = 27, antiderivative size = 128

$$\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}} - \frac{109d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}}$$

output
```
1/20736*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)-109/768*d^2*arcta
nh((d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)+245/1728*d^2/c^4/(d*x^3+c)^(1/2)-1/48/
c^2/x^6/(d*x^3+c)^(1/2)+3/64*d/c^3/x^3/(d*x^3+c)^(1/2)
```

3.331.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{\frac{12\sqrt{c}(-36c^2+81cdx^3+245d^2x^6)}{x^6\sqrt{c+dx^3}} + d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 2943d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right.}{20736c^{9/2}}$$

input
```
Integrate[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((12*Sqrt[c]*(-36*c^2 + 81*c*d*x^3 + 245*d^2*x^6))/(x^6*Sqrt[c + d*x^3]) +
 d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 2943*d^2*ArcTanh[Sqrt[c + d*x^
3]/Sqrt[c]])/(20736*c^(9/2))
```

3.331.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {948, 114, 27, 168, 27, 169, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (8c-dx^3) (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^9 (8c-dx^3) (dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3} \left(-\frac{\int \frac{d(36c-5dx^3)}{2x^6(8c-dx^3)(dx^3+c)^{3/2}}dx^3}{16c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{d\int \frac{36c-5dx^3}{x^6(8c-dx^3)(dx^3+c)^{3/2}}dx^3}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3} \left(-\frac{d\left(-\frac{\int \frac{2cd\left(218c-27dx^3\right)}{x^3\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx^3}{8c^2}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{d\left(-\frac{d\int \frac{218c-27dx^3}{x^3\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx^3}{4c}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{2\int \frac{cd\left(1962c-245dx^3\right)}{2x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c^2d}+\frac{490}{9c\sqrt{c+dx^3}}\right)}{4c}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow 27$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\int \frac{1962c-245dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{490}{9c\sqrt{c+dx^3}}\right)}{4c}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow 174$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{981}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{1}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{490}{9c\sqrt{c+dx^3}}\right)}{4c}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow 73$$

3.331. $\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{1}{2}\int\frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{981\int\frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{9c}+\frac{490}{9c\sqrt{c+dx^3}}\right)}{4c}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{981\int\frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{9c}+\frac{490}{9c\sqrt{c+dx^3}}\right)}{4c}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{221}$$

3.331. $\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{981\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{9c}+\frac{490}{9c\sqrt{c+dx^3}}\right)}{4c}-\frac{9}{2cx^3\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\sqrt{c+dx^3}}\right)$$

input
```
Int[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(-1/16*1/(c^2*x^6*Sqrt[c + d*x^3]) - (d*(-9/(2*c*x^3*Sqrt[c + d*x^3]) - (d
*(490/(9*c*Sqrt[c + d*x^3]) + (ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(6*Sqr
t[c]) - (981*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(9*c)))/(4*c))
)/(32*c^2))/3
```

3.331.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.331. $\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.331.4 Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(-13d\,x^{3}+4c\right)}{192c^{4}x^{6}}+\frac{d^{2}\left(-\frac{109\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{162\sqrt{c}}+\frac{256}{27\sqrt{d\,x^{3}+c}}\right)}{128c^{4}}$
pseudoelliptic	$\frac{-2943\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d^{2}x^{6}\sqrt{d\,x^{3}+c}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)d^{2}x^{6}\sqrt{d\,x^{3}+c}+2940d^{2}x^{6}\sqrt{c}+972c^{\frac{3}{2}}d\,x^{3}-432c^{\frac{5}{2}}}{20736c^{\frac{9}{2}}x^{6}\sqrt{d\,x^{3}+c}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{6c^{2}x^{6}}+\frac{7d\sqrt{d\,x^{3}+c}}{12c^{3}x^{3}}+\frac{2d^{2}}{3c^{3}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{5d^{2}\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4c^{\frac{7}{2}}}}{8c}+\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{3c^{2}x^{3}}-\frac{2d}{3c^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{d\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)}{64c^{2}}$
elliptic	Expression too large to display

input
```
int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(d*x^3+c)^(1/2)*(-13*d*x^3+4*c)/c^4/x^6+1/128*d^2/c^4*(-109/6*arcta
nh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+1/162*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1
/2))/c^(1/2)+256/27/(d*x^3+c)^(1/2))
```

3.331.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.37

$$\int \frac{1}{x^7\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\left[\frac{(d^3x^9+cd^2x^6)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+2943\,(d^3x^9+cd^2x^6)\sqrt{c}\log}{41472\,(c^5dx^9}\right.$$

input
```
integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/41472*((d^3*x^9 + c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + 2943*(d^3*x^9 + c*d^2*x^6)*sqrt(c)*log((d*x^
3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(245*c*d^2*x^6 + 81*c^2*d*x
^3 - 36*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6), 1/20736*(2943*(d^3*x^
9 + c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (d^3*x^9 + c*
d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(245*c*d^2*x
^6 + 81*c^2*d*x^3 - 36*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6)]
```

3.331.6 Sympy [F]

$$\int \frac{1}{x^7 (8c-dx^3) (c+dx^3)^{3/2}}\,dx = -\int \frac{1}{-8c^2x^7\sqrt{c+dx^3}-7cdx^{10}\sqrt{c+dx^3}+d^2x^{13}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
-Integral(1/(-8*c**2*x**7*sqrt(c + d*x**3) - 7*c*d*x**10*sqrt(c + d*x**3)
+ d**2*x**13*sqrt(c + d*x**3)), x)
```

3.331.7 Maxima [F]

$$\int \frac{1}{x^7 (8c-dx^3) (c+dx^3)^{3/2}}\,dx = \int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^7}\,dx$$

input
```
integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^7), x)
```

3.331.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^7 (8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{109\,d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{768\sqrt{-c}c^4} - \frac{d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{20736\sqrt{-c}c^4} + \frac{2\,d^2}{27\sqrt{dx^3+c}c^4} + \frac{13\,(dx^3+c)^{\frac{3}{2}}d^2 - 17\sqrt{dx^3+c}cd^2}{192\,c^4d^2x^6}$$

input
```
integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
109/768*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/20736*d^2*
arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) + 2/27*d^2/(sqrt(d*x^3
 + c)*c^4) + 1/192*(13*(d*x^3 + c)^(3/2)*d^2 - 17*sqrt(d*x^3 + c)*c*d^2)/(
c^4*d^2*x^6)
```

3.331.9 Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^7 (8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{245\,d^2}{1728\,c^4\sqrt{d\,x^3+c}} - \frac{109\,d^2\,\mathrm{atanh}\left(\frac{c^4\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)}{768\sqrt{c^9}} + \frac{d^2\,\mathrm{atanh}\left(\frac{c^4\sqrt{d\,x^3+c}}{3\sqrt{c^9}}\right)}{20736\sqrt{c^9}} - \frac{1}{48\,c^2\,x^6\sqrt{d\,x^3+c}} + \frac{3\,d}{64\,c^3\,x^3\sqrt{d\,x^3+c}}$$

input
```
int(1/(x^7*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
(245*d^2)/(1728*c^4*(c + d*x^3)^(1/2)) - (109*d^2*atanh((c^4*(c + d*x^3)^(
1/2))/(c^9)^(1/2)))/(768*(c^9)^(1/2)) + (d^2*atanh((c^4*(c + d*x^3)^(1/2))
/(3*(c^9)^(1/2))))/(20736*(c^9)^(1/2)) - 1/(48*c^2*x^6*(c + d*x^3)^(1/2))
+ (3*d)/(64*c^3*x^3*(c + d*x^3)^(1/2))
```

3.332 $\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.332.1 Optimal result

Integrand size = 27, antiderivative size = 629

$$\begin{aligned}
\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx &= \frac{2x^2}{27d^2\sqrt{c+dx^3}} \\
&- \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{32\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}} \\
&+ \frac{32\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81d^{8/3}} - \frac{32\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^{8/3}} \\
&+ \frac{28\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&- \frac{56\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
32/81*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d
^(8/3)-32/81*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-32/81*c^
(1/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)*
3^(1/2)+2/27*x^2/d^2/(d*x^3+c)^(1/2)-56/27*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))-56/81*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1
/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2)))^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*
x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+28/27*c^(1/3)*(c^(1/3)+d^(1/3)
*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2
))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d
^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/d^(8/3)/(d*x^
3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

3.332.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.54 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.20

$$\int \frac{x^7}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{x^2\Bigg(20c-20c\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+7dx^3\sqrt{1+\frac{dx^3}{c}}\,\mathrm{Ap}}{270cd^2\sqrt{c+dx^3}}$$

input
```
Integrate[x^7/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x^2*(20*c - 20*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)
/c), (d*x^3)/(8*c)] + 7*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/
3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(270*c*d^2*Sqrt[c + d*x^3])
```

3.332.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {970, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \; 970$$

$$\frac{2x^2}{27d^2\sqrt{c+dx^3}}-\frac{2\int \frac{2cx\left(8c-7dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27cd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{2x^2}{27d^2\sqrt{c+dx^3}}-\frac{4\int \frac{x\left(8c-7dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27d^2}$$

$$\Big\downarrow \; 1054$$

$$\frac{2x^2}{27d^2\sqrt{c+dx^3}}-\frac{4\int \left(\frac{7x}{\sqrt{dx^3+c}}-\frac{48cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}\right)dx}{27d^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{2x^2}{27d^2\sqrt{c+dx^3}}-$$

$$4\left(\frac{14\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{7\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{}\right.$$

input
```
Int[x^7/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

3.332. $\int \frac{x^7}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}dx$

output

```
(2*x^2)/(27*d^2*Sqrt[c + d*x^3]) - (4*((14*Sqrt[c + d*x^3])/(d^(2/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (8*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1
/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) - (8*c^(1/6)*ArcTanh
[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(3*d^(2/3)) + (8*c^
(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) - (7*3^(1/4)*Sqrt[
2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
-7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (14*Sqrt[2]*c^(1/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(
1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^
(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3]))/(27*d^2)
```

3.332.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 970

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.332. $\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.332.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.32 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.38

method	result	size
elliptic	Expression too large to display	869
default	Expression too large to display	1810

input
```
int(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/27/d^2*x^2/((x^3+c/d)*d)^(1/2)+56/81*I/d^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)))-64/243*I/d^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1
/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3
^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d...
```

3.332.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.57 (sec) , antiderivative size = 2384, normalized size of antiderivative = 3.79

$$\int \frac{x^7}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
2/243*(9*sqrt(d*x^3 + c)*d*x^2 + 252*(d*x^3 + c)*sqrt(d)*weierstrassZeta(0
, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 4*(d^4*x^3 + c*d^3 - sqrt(-
3)*(d^4*x^3 + c*d^3))*(c/d^16)^(1/6)*log(33554432/3*((d^16*x^9 + 318*c*d^1
5*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15
*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d^16)^(5/6) + 6*(2*c*d^2*x^7
+ 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5 + 32*c^2*d^11*x^2 - sqrt(-3)
*(5*c*d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2/3) - (7*c*d^7*x^6 + 152*c^2
*d^6*x^3 + 64*c^3*d^5 + sqrt(-3)*(7*c*d^7*x^6 + 152*c^2*d^6*x^3 + 64*c^3*d
^5))*(c/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^10*x^7 + 64*c^2*d^9*x^4 +
 32*c^3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*
x^2 - sqrt(-3)*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2))*(c/d^16)^(1/
6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 4*(d^4*x^3 + c*d
^3 - sqrt(-3)*(d^4*x^3 + c*d^3))*(c/d^16)^(1/6)*log(-33554432/3*((d^16*x^9
 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9
+ 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d^16)^(5/6) - 6*(
2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5 + 32*c^2*d^11*x^
2 - sqrt(-3)*(5*c*d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2/3) - (7*c*d^7*x
^6 + 152*c^2*d^6*x^3 + 64*c^3*d^5 + sqrt(-3)*(7*c*d^7*x^6 + 152*c^2*d^6*x^
3 + 64*c^3*d^5))*(c/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^10*x^7 + 64*c
^2*d^9*x^4 + 32*c^3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^...
```

3.332.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**7/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
Timed out
```

3.332.7 Maxima [F]

$$\int \frac{x^7}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int -\frac{x^7}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

3.332.8 Giac [F]

$$\int \frac{x^7}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int -\frac{x^7}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^7}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)}\,dx$$

input
```
int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.333 $\int \frac{x^4}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx$

3.333.1 Optimal result

Integrand size = 27, antiderivative size = 635

$$\begin{aligned}
\int \frac{x^4}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx &= -\frac{2x^2}{27cd\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&\quad -\frac{4\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}}+\frac{4\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}}-\frac{4\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}} \\
&\quad -\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad +\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
4/81*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d^
(5/3)-4/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(5/3)-4/81*arcta
n(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(5/3)*3^(
1/2)-2/27*x^2/c/d/(d*x^3+c)^(1/2)+2/27*(d*x^3+c)^(1/2)/c/d^(5/3)/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))+2/81*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/
3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c
^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(
1/2)*3^(3/4)/c^(2/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/
(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/27*(c^(1/3)+d^(1/3)*x)*Elliptic
E((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/
(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(2/3)/d^(5/3)/(d*x^3+c)
^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)
```

3.333.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.20

$$\int \frac{x^4}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = -\frac{x^2\left(80c-80c\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{1080c^2d\sqrt{c+dx^3}}$$

input
```
Integrate[x^4/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
-1/1080*(x^2*(80*c - 80*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -
((d*x^3)/c), (d*x^3)/(8*c)] + d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2,
 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(c^2*d*Sqrt[c + d*x^3])
```

3.333.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {971, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 971$$

$$\frac{2\int \frac{x(32c-dx^3)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{27cd}-\frac{2x^2}{27cd\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{x(32c-dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{27cd}-\frac{2x^2}{27cd\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 1054$$

$$\frac{\int \left(\frac{24cx}{(8c-dx^3)\sqrt{dx^3+c}}+\frac{x}{\sqrt{dx^3+c}}\right)dx}{27cd}-\frac{2x^2}{27cd\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^2}{(}}}{d^{2/3}}}{}$$

$$\frac{2x^2}{27cd\sqrt{c+dx^3}}$$

input `Int[x^4/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]`

3.333. $\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

output
```
(-2*x^2)/(27*c*d*Sqrt[c + d*x^3]) + ((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)) - (4*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3)
 + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) + (4*c^(1/6)*ArcTanh[(c
^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)) - (4*c^(1/
6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) - (3^(1/4)*Sqrt[2 - S
qrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
 Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 -
4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^
(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*
Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*Sqrt[c + d*x^3]))/(27*c*d)
```

3.333.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.333. $\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.333.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.36 (sec) , antiderivative size = 878, normalized size of antiderivative = 1.38

method	result	size
elliptic	Expression too large to display	878
default	Expression too large to display	1346

input

```
int(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-2/27/d*x^2/c/((x^3+c/d)*d)^(1/2)-2/81*I/d^2/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)))-8/243*I/d^4/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)
*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)
^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*
d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3
)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-...
```

3.333.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 2525, normalized size of antiderivative = 3.98

$$\int \frac{x^4}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/243*(18*sqrt(d*x^3 + c)*d*x^2 + 18*(d*x^3 + c)*sqrt(d)*weierstrassZeta(
0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - (c*d^3*x^3 + c^2*d^2 + sqr
t(-3)*(c*d^3*x^3 + c^2*d^2))*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2
*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c
^6*d^7*x + sqrt(-3)*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x))*(1/(c
^5*d^10))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2 -
sqrt(-3)*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2))*(1/(c^5*d^10))^(5/6) - 2*(7*c^
3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c^5*d^10)) + (c*d^4*x^7
+ 80*c^2*d^3*x^4 + 160*c^3*d^2*x + sqrt(-3)*(c*d^4*x^7 + 80*c^2*d^3*x^4 +
160*c^3*d^2*x))*(1/(c^5*d^10))^(1/6)) - 9*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 +
64*c^4*d^4*x^2 - sqrt(-3)*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2))
*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
 + (c*d^3*x^3 + c^2*d^2 + sqrt(-3)*(c*d^3*x^3 + c^2*d^2))*(1/(c^5*d^10))^(
1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^
9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x + sqrt(-3)*(5*c^4*d^9*x^7 + 64*c^5*d
^8*x^4 + 32*c^6*d^7*x))*(1/(c^5*d^10))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^5
*d^10*x^5 + 32*c^6*d^9*x^2 - sqrt(-3)*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2))*(
1/(c^5*d^10))^(5/6) - 2*(7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqr
t(1/(c^5*d^10)) + (c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x + sqrt(-3)*(
c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x))*(1/(c^5*d^10))^(1/6)) - 9*...
```

3.333.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
Timed out
```

3.333.7 Maxima [F]

$$\int \frac{x^4}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx=\int -\frac{x^4}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

3.333.8 Giac [F]

$$\int \frac{x^4}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx=\int -\frac{x^4}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{x^4}{\left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

input
```
int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.334 $\int \frac{x}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

3.334.1 Optimal result

Integrand size = 25, antiderivative size = 632

$$\int \frac{x}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{2x^2}{27c^2\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{54\sqrt{3}c^{11/6}d^{2/3}}+\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{162c^{11/6}d^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{162c^{11/6}d^{2/3}}$$
$$+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
1/162*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)/
d^(2/3)-1/162*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)/d^(2/3)-1/162*
arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)/d^(2/
3)*3^(1/2)+2/27*x^2/c^2/(d*x^3+c)^(1/2)-2/27*(d*x^3+c)^(1/2)/c^2/d^(2/3)/(
d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-2/81*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)
*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(
1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2
)))^2)^(1/2)*3^(3/4)/c^(5/3)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(
1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/27*(c^(1/3)+d^(1/3)*x)*
EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(5/3)/d^(2/3)/(
d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

3.334.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.20

$$\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{x^2\left(160c-25c\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+2dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{A}\right.}{2160c^3\sqrt{c+dx^3}}$$

input
```
Integrate[x/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x^2*(160*c - 25*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3
)/c), (d*x^3)/(8*c)] + 2*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8
/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(2160*c^3*Sqrt[c + d*x^3])
```

3.334.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {972, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \; 972$$

$$\frac{2x^2}{27c^2\sqrt{c+dx^3}}-\frac{2\int \frac{dx\left(5c-dx^3\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27c^2d}$$

$$\Big\downarrow \; 27$$

$$\frac{2x^2}{27c^2\sqrt{c+dx^3}}-\frac{\int \frac{x\left(5c-dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27c^2}$$

$$\Big\downarrow \; 1054$$

$$\frac{2x^2}{27c^2\sqrt{c+dx^3}}-\frac{\int \left(\frac{x}{\sqrt{dx^3+c}}-\frac{3cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}\right)dx}{27c^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{2x^2}{27c^2\sqrt{c+dx^3}}-$$

$$\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-$$

$$\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)$$

$$d^{2/3}$$

input
```
Int[x/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

3.334. $\int \frac{x}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}dx$

output
```
(2*x^2)/(27*c^2*Sqrt[c + d*x^3]) - ((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) + (c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) +
d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*Sqrt[3]*d^(2/3)) - (c^(1/6)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/3)) + (c^(1/6)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3
]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqr
t[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)
*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqr
t[c + d*x^3]))/(27*c^2)
```

3.334.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.334. $\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.334.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.55 (sec) , antiderivative size = 875, normalized size of antiderivative = 1.38

method	result	size
default	Expression too large to display	875
elliptic	Expression too large to display	875

input

```
int(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/27*x^2/c^2/((x^3+c/d)*d)^(1/2)+2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)))-1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3
)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(
1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2
)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)
*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(...
```

3.334.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 2525, normalized size of antiderivative = 4.00

$$\int \frac{x}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
1/1944*(144*sqrt(d*x^3 + c)*d*x^2 + 144*(d*x^3 + c)*sqrt(d)*weierstrassZet
a(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + (c^2*d^2*x^3 + c^3*d + s
qrt(-3)*(c^2*d^2*x^3 + c^3*d))*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 + 318*c*d
^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32
*c^10*d^3*x + sqrt(-3)*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x))*(
1/(c^11*d^4))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x
^2 - sqrt(-3)*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2))*(1/(c^11*d^4))^(5/6) - 2
*(7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^11*d^4)) + (c^2*
d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^
2*x^4 + 160*c^4*d*x))*(1/(c^11*d^4))^(1/6)) - 9*(c^4*d^4*x^8 + 38*c^5*d^3*
x^5 + 64*c^6*d^2*x^2 - sqrt(-3)*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2
*x^2))*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512
*c^3)) - (c^2*d^2*x^3 + c^3*d + sqrt(-3)*(c^2*d^2*x^3 + c^3*d))*(1/(c^11*d
^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*
c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x + sqrt(-3)*(5*c^8*d^5*x^7 + 6
4*c^9*d^4*x^4 + 32*c^10*d^3*x))*(1/(c^11*d^4))^(2/3) - 3*sqrt(d*x^3 + c)*(
6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2 - sqrt(-3)*(5*c^10*d^5*x^5 + 32*c^11*d
^4*x^2))*(1/(c^11*d^4))^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^
8*d^2)*sqrt(1/(c^11*d^4)) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x +
sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x))*(1/(c^11*d^4))^(...
```

3.334.6 Sympy [F]

$$\int \frac{x}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = -\int \frac{x}{-8c^2\sqrt{c+dx^3}-7cdx^3\sqrt{c+dx^3}+d^2x^6\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
-Integral(x/(-8*c**2*sqrt(c + d*x**3) - 7*c*d*x**3*sqrt(c + d*x**3) + d**2
*x**6*sqrt(c + d*x**3)), x)
```

3.334.7 Maxima [F]

$$\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int -\frac{x}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

input
```
integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

3.334.8 Giac [F]

$$\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int -\frac{x}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

input
```
integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)}\,dx$$

input
```
int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.335 $\int \frac{1}{x^2\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

3.335.1 Optimal result

Integrand size = 27, antiderivative size = 653

$$
\begin{aligned}
&\int \frac{1}{x^2\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{2}{27c^2x\sqrt{c+dx^3}}-\frac{43\sqrt{c+dx^3}}{216c^3x} \\
&+\frac{43\sqrt[3]{d}\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{17/6}} \\
&+\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1296c^{17/6}}-\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1296c^{17/6}} \\
&-\frac{43\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{144\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+\frac{43\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

output
```
1/1296*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
c^(17/6)-1/1296*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/12
96*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(
17/6)*3^(1/2)+2/27/c^2/x/(d*x^3+c)^(1/2)-43/216*(d*x^3+c)^(1/2)/c^3/x+43/2
16*d^(1/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+43/648*d^(1
/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)
*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*
x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)-43/432*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(
1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/
2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2)))^2)^(1/2)*3^(1/4)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/
3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.335.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^2\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{-80c(27c+43dx^3)+875cdx^3\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{17280c^4x\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^2*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(-80*c*(27*c + 43*d*x^3) + 875*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1
/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 43*d^2*x^6*Sqrt[1 + (d*x^3)/c]*
AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(17280*c^4*x*Sqrt
[c + d*x^3])
```

3.335.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {972, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (8c-dx^3) (c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{2}{27c^2x\sqrt{c+dx^3}}-\frac{2\int -\frac{d(43c-5dx^3)}{2x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{27c^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{43c-5dx^3}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{27c^2}+\frac{2}{27c^2x\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int -\frac{cdx\left(350c-43dx^3\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{43\sqrt{c+dx^3}}{8cx}}{27c^2}+\frac{2}{27c^2x\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{d\int \frac{x\left(350c-43dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{16c}-\frac{43\sqrt{c+dx^3}}{8cx}}{27c^2}+\frac{2}{27c^2x\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1054}$$

$$\frac{\frac{d\int \left(\frac{6cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{43x}{\sqrt{dx^3+c}}\right)dx}{16c}-\frac{43\sqrt{c+dx^3}}{8cx}}{27c^2}+\frac{2}{27c^2x\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{2009}$$

$$d\left(\frac{86\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{43\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{2/3}}\right.$$

$$\frac{2}{27c^2x\sqrt{c+dx^3}}$$

input
```
Int[1/(x^2*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
2/(27*c^2*x*Sqrt[c + d*x^3]) + ((-43*Sqrt[c + d*x^3])/(8*c*x) + (d*((86*Sq
rt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (c^(1/6)*Ar
cTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^
(2/3)) + (c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^
3])])/(3*d^(2/3)) - (c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2
/3)) - (43*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) +
 d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (86
*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 -
 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(16*c))/(27*c^2)
```

3.335.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.335.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.42 (sec) , antiderivative size = 890, normalized size of antiderivative = 1.36

method	result	size
elliptic	Expression too large to display	890
risch	Expression too large to display	1334
default	Expression too large to display	1361

input
```
int(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/27*d/c^3*x^2/((x^3+c/d)*d)^(1/2)-1/8*(d*x^3+c)^(1/2)/c^3/x-43/648*I/c^3
*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/1944*I/c^3/d^2*2^(1
/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-
c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(
-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^...
```

3.335.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 2379, normalized size of antiderivative = 3.64

$$\int \frac{1}{x^2\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output

```
-1/15552*(3096*(d*x^4 + c*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstras
sPInverse(0, -4*c/d, x)) - (c^3*d*x^4 + c^4*x + sqrt(-3)*(c^3*d*x^4 + c^4*
x))*(d^2/c^17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640
*c^3*d - 9*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x + sqrt(-3)*(5*c^12*
d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x))*(d^2/c^17)^(2/3) + 3*sqrt(d*x^3 + c)
*(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(
d^2/c^17)^(5/6) - 2*(7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^
17) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x + sqrt(-3)*(c^3*d^3*x^7
+ 80*c^4*d^2*x^4 + 160*c^5*d*x))*(d^2/c^17)^(1/6)) - 9*(c^6*d^3*x^8 + 38*c
^7*d^2*x^5 + 64*c^8*d*x^2 - sqrt(-3)*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^
8*d*x^2))*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*
c^3)) + (c^3*d*x^4 + c^4*x + sqrt(-3)*(c^3*d*x^4 + c^4*x))*(d^2/c^17)^(1/6
)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^12*
d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x + sqrt(-3)*(5*c^12*d^2*x^7 + 64*c^13*d
*x^4 + 32*c^14*x))*(d^2/c^17)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 +
 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^2/c^17)^(5/6) - 2
*(7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^17) + (c^3*d^3*x^7
+ 80*c^4*d^2*x^4 + 160*c^5*d*x + sqrt(-3)*(c^3*d^3*x^7 + 80*c^4*d^2*x^4 +
160*c^5*d*x))*(d^2/c^17)^(1/6)) - 9*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8
*d*x^2 - sqrt(-3)*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2))*(d^2/c...
```

3.335.6 Sympy [F]

$$\int \frac{1}{x^2\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = -\int \frac{1}{-8c^2x^2\sqrt{c+dx^3}-7cdx^5\sqrt{c+dx^3}+d^2x^8\sqrt{c+dx^3}}\,dx$$

input

```
integrate(1/x**2/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output

```
-Integral(1/(-8*c**2*x**2*sqrt(c + d*x**3) - 7*c*d*x**5*sqrt(c + d*x**3) +
 d**2*x**8*sqrt(c + d*x**3)), x)
```

3.335.7 Maxima [F]

$$\int \frac{1}{x^2\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^2}\,dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^2), x)
```

3.335.8 Giac [F]

$$\int \frac{1}{x^2\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^2}\,dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^2), x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^2\,{\left(d\,x^3+c\right)}^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

input
```
int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.336 $\int \frac{1}{x^5(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.336.1 Optimal result

Integrand size = 27, antiderivative size = 675

$$
\begin{aligned}
\int \frac{1}{x^5\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{2}{27c^2x^4\sqrt{c+dx^3}}-\frac{91\sqrt{c+dx^3}}{864c^3x^4}+\frac{113d\sqrt{c+dx^3}}{432c^4x} \\
&- \frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3456\sqrt{3}c^{23/6}} \\
&+ \frac{d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{10368c^{23/6}}-\frac{d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{23/6}} \\
&+ \frac{113\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{288\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&- \frac{113d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

output
```
1/10368*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))
/c^(23/6)-1/10368*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-1/
10368*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/
c^(23/6)*3^(1/2)+2/27/c^2/x^4/(d*x^3+c)^(1/2)-91/864*(d*x^3+c)^(1/2)/c^3/x
^4+113/432*d*(d*x^3+c)^(1/2)/c^4/x-113/432*d^(4/3)*(d*x^3+c)^(1/2)/c^4/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))-113/1296*d^(4/3)*(c^(1/3)+d^(1/3)*x)*Elliptic
F((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)*3^(3/4)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+
d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+113/864*d^(4/3)*(c^(1/
3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d
^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(
11/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

3.336.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^5\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{160c\left(-27c^2+135cdx^3+226d^2x^6\right)-9025cd^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\big(\frac{2}{3},\frac{1}{2}}{138240c^5x^4\sqrt{}}$$

input
```
Integrate[1/(x^5*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(160*c*(-27*c^2 + 135*c*d*x^3 + 226*d^2*x^6) - 9025*c*d^2*x^6*Sqrt[1 + (d*
x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 452*d^3*
x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(138240*c^5*x^4*Sqrt[c + d*x^3])
```

3.336.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {972, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (8c-dx^3) (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{972}$$

$$\frac{2}{27c^2x^4\sqrt{c+dx^3}}-\frac{2\int -\frac{d(91c-11dx^3)}{2x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{27c^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{91c-11dx^3}{x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{27c^2}+\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int \frac{cd\left(3616c-455dx^3\right)}{2x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{91\sqrt{c+dx^3}}{32cx^4}}{27c^2}+\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{d\int \frac{3616c-455dx^3}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{64c}-\frac{91\sqrt{c+dx^3}}{32cx^4}}{27c^2}+\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{d\left(-\frac{\int -\frac{8cdx\left(1805c-226dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{452\sqrt{c+dx^3}}{cx}\right)}{64c}-\frac{91\sqrt{c+dx^3}}{32cx^4}}{27c^2}+\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{d\left(\frac{d\int \frac{x\left(1805c-226dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{452\sqrt{c+dx^3}}{cx}\right)}{64c}-\frac{91\sqrt{c+dx^3}}{32cx^4}}{27c^2}+\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1054}$$

$$\frac{-\frac{d\left(\frac{d\int \left(\frac{226x}{\sqrt{dx^3+c}}-\frac{3cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}\right)dx}{c}-\frac{452\sqrt{c+dx^3}}{cx}\right)}{64c}-\frac{91\sqrt{c+dx^3}}{32cx^4}}{27c^2}+\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\left(\frac{d\left(\frac{452\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{226\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{2/}}}{}}{}$$

$$\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

input
```
Int[1/(x^5*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

3.336. $\int \frac{1}{x^5(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

output
```
2/(27*c^2*x^4*Sqrt[c + d*x^3]) + ((-91*Sqrt[c + d*x^3])/(32*c*x^4) - (d*((
-452*Sqrt[c + d*x^3])/(c*x) + (d*((452*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) + (c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) +
d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*Sqrt[3]*d^(2/3)) - (c^(1/6)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/3)) + (c^(1/6)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*d^(2/3)) - (226*3^(1/4)*Sqrt[2 - Sq
rt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4
*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (452*Sqrt[2]*c^(1/3)*(c^(1/3) + d
^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*
x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)
*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^
2]*Sqrt[c + d*x^3])))/c))/(64*c))/(27*c^2)
```

3.336.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.336.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.48 (sec) , antiderivative size = 911, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	911
risch	Expression too large to display	1344
default	Expression too large to display	1864

input
```
int(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/32*(d*x^3+c)^(1/2)/c^3/x^4+3/16*d*(d*x^3+c)^(1/2)/c^4/x+2/27*d^2/c^4*x^
2/((x^3+c/d)*d)^(1/2)+113/1296*I*d/c^4*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2))-1/15552*I/d/c^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*
I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(
1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c
*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^
(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3...
```

3.336.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.74 (sec) , antiderivative size = 2529, normalized size of antiderivative = 3.75

$$\int \frac{1}{x^5 (8c-dx^3) (c+dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input

```
integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output

```
1/124416*(32544*(d^2*x^7 + c*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, wei
erstrassPInverse(0, -4*c/d, x)) + (c^4*d*x^7 + c^5*x^4 + sqrt(-3)*(c^4*d*x
^7 + c^5*x^4))*(d^8/c^23)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^
7*x^3 + 640*c^3*d^6 - 9*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x +
sqrt(-3)*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x))*(d^8/c^23)^(2/3
) + 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqrt(-3)*(5*c^20*d*
x^5 + 32*c^21*x^2))*(d^8/c^23)^(5/6) - 2*(7*c^12*d^4*x^6 + 152*c^13*d^3*x^
3 + 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*
d^5*x + sqrt(-3)*(c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x))*(d^8/c^23
)^(1/6)) - 9*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2 - sqrt(-3)*(c
^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2))*(d^8/c^23)^(1/3))/(d^3*x^9
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^4*d*x^7 + c^5*x^4 + sqrt(
-3)*(c^4*d*x^7 + c^5*x^4))*(d^8/c^23)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 +
 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32
*c^18*d*x + sqrt(-3)*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x))*(d^
8/c^23)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqrt(-3
)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^8/c^23)^(5/6) - 2*(7*c^12*d^4*x^6 + 152
*c^13*d^3*x^3 + 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^6*x^
4 + 160*c^6*d^5*x + sqrt(-3)*(c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x
))*(d^8/c^23)^(1/6)) - 9*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^...
```

3.336.6 Sympy [F]

$$\int \frac{1}{x^5\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=-\int \frac{1}{-8c^2x^5\sqrt{c+dx^3}-7cdx^8\sqrt{c+dx^3}+d^2x^{11}\sqrt{c+dx^3}}\,dx$$

input

```
integrate(1/x**5/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output

```
-Integral(1/(-8*c**2*x**5*sqrt(c + d*x**3) - 7*c*d*x**8*sqrt(c + d*x**3) +
 d**2*x**11*sqrt(c + d*x**3)), x)
```

3.336.7 Maxima [F]

$$\int \frac{1}{x^5 (8c-dx^3) (c+dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^5} \, dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^5), x)
```

3.336.8 Giac [F]

$$\int \frac{1}{x^5 (8c-dx^3) (c+dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^5} \, dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^5), x)
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (8c-dx^3) (c+dx^3)^{3/2}} \, dx = \int \frac{1}{x^5\,(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)} \, dx$$

input
```
int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.337 $\int \frac{1}{x^8\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

3.337.1 Optimal result

Integrand size = 27, antiderivative size = 699

$$\begin{aligned}\int \frac{1}{x^8\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{2}{27c^2x^7\sqrt{c+dx^3}}-\frac{139\sqrt{c+dx^3}}{1512c^3x^7}\\
&+\frac{6095d\sqrt{c+dx^3}}{48384c^4x^4}-\frac{953d^2\sqrt{c+dx^3}}{3024c^5x}+\frac{953d^{7/3}\sqrt{c+dx^3}}{3024c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}\\
&-\frac{d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{29/6}}+\frac{d^{7/3}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{29/6}}-\frac{d^{7/3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{82944c^{29/6}}\\
&-\frac{953\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2016\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\\
&+\frac{953d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{1512\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\end{aligned}$$

output

```
1/82944*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))
/c^(29/6)-1/82944*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(29/6)-1/
82944*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/
c^(29/6)*3^(1/2)+2/27/c^2/x^7/(d*x^3+c)^(1/2)-139/1512*(d*x^3+c)^(1/2)/c^3
/x^7+6095/48384*d*(d*x^3+c)^(1/2)/c^4/x^4-953/3024*d^2*(d*x^3+c)^(1/2)/c^5
/x+953/3024*d^(7/3)*(d*x^3+c)^(1/2)/c^5/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+95
3/9072*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)
))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3
)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(14/3)
*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2)))^2)^(1/2)-953/6048*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/
3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(
1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(14/3)/(d*x^3+c)^(1/2)/(c^(1/3)
*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.337.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^8\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{610025cd^3x^9\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-32\Big(5c(864c^3-}{7}$$

input

```
Integrate[1/(x^8*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output

```
(610025*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)
/c), (d*x^3)/(8*c)] - 32*(5*c*(864*c^3 - 1647*c^2*d*x^3 + 9153*c*d^2*x^6 +
 15248*d^3*x^9) + 953*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8
/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(7741440*c^6*x^7*Sqrt[c + d*x^3])
```

3.337.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {972, 27, 1053, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 \left(8c-dx^3\right) \left(c+dx^3\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{972}$$

$$\frac{2}{27c^2x^7\sqrt{c+dx^3}}-\frac{2\int -\frac{d\left(139c-17dx^3\right)}{2x^8\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27c^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{139c-17dx^3}{x^8\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27c^2}+\frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int \frac{cd\left(12190c-1529dx^3\right)}{2x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{56c^2}-\frac{139\sqrt{c+dx^3}}{56cx^7}}{27c^2}+\frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{d\int \frac{12190c-1529dx^3}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{112c}-\frac{139\sqrt{c+dx^3}}{56cx^7}}{27c^2}+\frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{d\left(-\frac{\int \frac{cd\left(243968c-30475dx^3\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{6095\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{139\sqrt{c+dx^3}}{56cx^7}}{27c^2}+\frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{d\left(-\frac{d\int \frac{243968c-30475dx^3}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{6095\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{139\sqrt{c+dx^3}}{56cx^7}}{27c^2}+\frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{d\left(-\frac{d\left(-\frac{\int -\frac{8cdx\left(122005c-15248dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{30496\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{6095\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{139\sqrt{c+dx^3}}{56cx^7}}{27c^2}+\frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{d\left(-\frac{d\left(\frac{d\int \frac{x\left(122005c-15248dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{30496\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{6095\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{139\sqrt{c+dx^3}}{56cx^7}}{27c^2}+\frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1054}$$

$$\frac{-\frac{d\left(-\frac{d\left(\frac{d\int \left(\frac{21cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{15248x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{30496\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{6095\sqrt{c+dx^3}}{16cx^4}\right)}{112c}-\frac{139\sqrt{c+dx^3}}{56cx^7}}{27c^2}+\frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{2009}$$

3.337. $\int \frac{1}{x^8(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

$$-\frac{d\left(-\frac{d\left(\frac{d\left(\frac{30496\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{15248\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\right.}{}\right.}{}\right.}{}\right.}{}$$

$$\frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

input
```
Int[1/(x^8*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

3.337. $\int \frac{1}{x^8(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

output

```
2/(27*c^2*x^7*Sqrt[c + d*x^3]) + ((-139*Sqrt[c + d*x^3])/(56*c*x^7) - (d*(
(-6095*Sqrt[c + d*x^3])/(16*c*x^4) - (d*((-30496*Sqrt[c + d*x^3])/(c*x) +
(d*((30496*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x))
- (7*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3
]])/(2*Sqrt[3]*d^(2/3)) + (7*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/3)) - (7*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(6*d^(2/3)) - (15248*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1
/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d
^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*
Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*Sqrt[c + d*x^3]) + (30496*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^
(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]
)))/c))/(32*c)))/(112*c))/(27*c^2)
```

3.337.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.337.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.48 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	930
risch	Expression too large to display	1357
default	Expression too large to display	2389

input
```
int(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/27*d^3*x^2/c^5/((x^3+c/d)*d)^(1/2)-1/56*(d*x^3+c)^(1/2)/c^3/x^7+93/1792
*d*(d*x^3+c)^(1/2)/c^4/x^4-27/112*d^2*(d*x^3+c)^(1/2)/c^5/x-953/9072*I*d^2
/c^5*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/124416*I/c^5*2^
(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2
-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/...
```

3.337.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.26 (sec) , antiderivative size = 2564, normalized size of antiderivative = 3.67

$$\int \frac{1}{x^8\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/6967296*(2195712*(d^3*x^10 + c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4*c
/d, weierstrassPInverse(0, -4*c/d, x)) - 7*(c^5*d*x^10 + c^6*x^7 + sqrt(-3
)*(c^5*d*x^10 + c^6*x^7))*(d^14/c^29)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6
 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4
+ 32*c^22*d^2*x + sqrt(-3)*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2
*x))*(d^14/c^29)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2
- sqrt(-3)*(5*c^25*d*x^5 + 32*c^26*x^2))*(d^14/c^29)^(5/6) - 2*(7*c^15*d^6
*x^6 + 152*c^16*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 8
0*c^6*d^10*x^4 + 160*c^7*d^9*x + sqrt(-3)*(c^5*d^11*x^7 + 80*c^6*d^10*x^4
+ 160*c^7*d^9*x))*(d^14/c^29)^(1/6)) - 9*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 +
 64*c^12*d^7*x^2 - sqrt(-3)*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12*d^7*
x^2))*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)) + 7*(c^5*d*x^10 + c^6*x^7 + sqrt(-3)*(c^5*d*x^10 + c^6*x^7))*(d^14/c^29
)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11
- 9*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x + sqrt(-3)*(5*c^20*d
^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x))*(d^14/c^29)^(2/3) - 3*sqrt(d*x^
3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2 - sqrt(-3)*(5*c^25*d*x^5 + 32*c^26*x
^2))*(d^14/c^29)^(5/6) - 2*(7*c^15*d^6*x^6 + 152*c^16*d^5*x^3 + 64*c^17*d^
4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x + sqr
t(-3)*(c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x))*(d^14/c^29)^(1/...
```

3.337.6 Sympy [F]

$$\int \frac{1}{x^8\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=-\int \frac{1}{-8c^2x^8\sqrt{c+dx^3}-7cdx^{11}\sqrt{c+dx^3}+d^2x^{14}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**8/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
-Integral(1/(-8*c**2*x**8*sqrt(c + d*x**3) - 7*c*d*x**11*sqrt(c + d*x**3)
+ d**2*x**14*sqrt(c + d*x**3)), x)
```

3.337.7 Maxima [F]

$$\int \frac{1}{x^8 (8c-dx^3) (c+dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^8} \, dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^8), x)
```

3.337.8 Giac [F]

$$\int \frac{1}{x^8 (8c-dx^3) (c+dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^8} \, dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^8), x)
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (8c-dx^3) (c+dx^3)^{3/2}} \, dx = \int \frac{1}{x^8\,(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)} \, dx$$

input
```
int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.338 $\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.338.1 Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{x^4\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},1,\frac{3}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}$$

output
```
1/32*x^4*AppellF1(4/3,3/2,1,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^
2/(d*x^3+c)^(1/2)
```

3.338.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(66) = 132.

Time = 8.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.53

$$\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{x\left(x^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+\frac{64c\left(-1+\frac{}{(8c-dx^3)\left(32c\operatorname{AppellF1}\left(\right.\right.}\right.}{}\right.}{864c^2\sqrt{c}}$$

input
```
Integrate[x^3/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x*(x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^
3)/(8*c)] + (64*c*(-1 + (256*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
(d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/
c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))
))/d))/(864*c^2*Sqrt[c + d*x^3])
```

3.338.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{(8c-dx^3)\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{3}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}$$

input
```
Int[x^3/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 3/2, 7/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(32*c^2*Sqrt[c + d*x^3])
```

3.338.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.338.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.24 (sec) , antiderivative size = 724, normalized size of antiderivative = 10.97

method	result	size
elliptic	Expression too large to display	724
default	Expression too large to display	1038

input
```
int(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/27/d*x/c/((x^3+c/d)*d)^(1/2)+2/81*I/d^2/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-8/243*I/d^4/c*2^(1/2)*sum(1/_alpha^
2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*
3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_
alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2
*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(
1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_
Z^3*d-8*c))
```

3.338.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2535 vs. $2(52) = 104$.

Time = 0.78 (sec) , antiderivative size = 2535, normalized size of antiderivative = 38.41

$$\int \frac{x^3}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output

```
-1/486*(36*sqrt(d*x^3 + c)*d*x - 36*(d*x^3 + c)*sqrt(d)*weierstrassPInvers
e(0, -4*c/d, x) - (c*d^3*x^3 + c^2*d^2 + sqrt(-3)*(c*d^3*x^3 + c^2*d^2))*(
1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3
 - 9*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2 + sqrt(-3)*(c^5*d^8*x^
8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2))*(1/(c^7*d^8))^(2/3) + 3*sqrt(d*x^3 +
 c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x - sqrt(-3)*(c^6*d^9*x^7
 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x))*(1/(c^7*d^8))^(5/6) - 2*(7*c^4*d^6*x^6
 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32
*c^3*d^2*x^2 + sqrt(-3)*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2))*(1/(c^7*d^8))^(1
/6)) - 9*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x - sqrt(-3)*(5*c^3*
d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x))*(1/(c^7*d^8))^(1/3))/(d^3*x^9 -
24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c*d^3*x^3 + c^2*d^2 + sqrt(-3)
*(c*d^3*x^3 + c^2*d^2))*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 +
 1200*c^2*d*x^3 + 640*c^3 - 9*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x
^2 + sqrt(-3)*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2))*(1/(c^7*d^8
))^(2/3) - 3*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*
x - sqrt(-3)*(c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x))*(1/(c^7*d^8))
^(5/6) - 2*(7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)
) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2 + sqrt(-3)*(5*c^2*d^3*x^5 + 32*c^3*d
^2*x^2))*(1/(c^7*d^8))^(1/6)) - 9*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*...
```

3.338.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = \text{Timed out}$$

input

```
integrate(x**3/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output

```
Timed out
```

3.338.7 Maxima [F]

$$\int \frac{x^3}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int -\frac{x^3}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

3.338.8 Giac [F]

$$\int \frac{x^3}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int -\frac{x^3}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

input
```
integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)}\,dx$$

input
```
int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.339 $\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

3.339.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{3}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}$$

output
```
1/8*x*AppellF1(1/3,3/2,1,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/(
d*x^3+c)^(1/2)
```

3.339.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 230 vs. 2(64) = 128.

Time = 10.16 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.59

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{x\left(-\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+64\left(\frac{1}{c^2}+\frac{}{(8c-dx^3)\left(32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3}\right.\right.}\right.\right.}{864\sqrt{c+}}$$

input
```
Integrate[1/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x*(-((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
(d*x^3)/(8*c)])/c^3) + 64*(c^(-2) + (176*AppellF1[1/3, 1/2, 1, 4/3, -((d*x
^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -(
(d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)
/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(
8*c)]))))))/(864*Sqrt[c + d*x^3])
```

3.339.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(8c-dx^3)\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\operatorname{AppellF1}\left(\frac{1}{3},1,\frac{3}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}$$

input
```
Int[1/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 3/2, 4/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(8*c^2*Sqrt[c + d*x^3])
```

3.339.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.339.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.41 (sec) , antiderivative size = 721, normalized size of antiderivative = 11.27

method	result	size
default	Expression too large to display	721
elliptic	Expression too large to display	721

input
```
int(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/27*x/c^2/((x^3+c/d)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha
^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I
*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*
_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(
2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^
(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*d-8*c))
```

3.339.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2483 vs. $2(50) = 100$.

Time = 0.92 (sec) , antiderivative size = 2483, normalized size of antiderivative = 38.80

$$\int \frac{1}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output

```
1/3888*(288*sqrt(d*x^3 + c)*d*x + 360*(d*x^3 + c)*sqrt(d)*weierstrassPInve
rse(0, -4*c/d, x) + (c^2*d^2*x^3 + c^3*d + sqrt(-3)*(c^2*d^2*x^3 + c^3*d))
*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*
c^3 - 9*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2 + sqrt(-3)*(c^9*d
^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2))*(1/(c^13*d^2))^(2/3) + 3*sqrt
(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x - sqrt(-3)*(
c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x))*(1/(c^13*d^2))^(5/6) - 2
*(7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(1/(c^13*d^2)) + 6*(5*c^
3*d^2*x^5 + 32*c^4*d*x^2 + sqrt(-3)*(5*c^3*d^2*x^5 + 32*c^4*d*x^2))*(1/(c^
13*d^2))^(1/6)) - 9*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x - sqrt(-3
)*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x))*(1/(c^13*d^2))^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^2*d^2*x^3 + c^3*d +
sqrt(-3)*(c^2*d^2*x^3 + c^3*d))*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*
d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64
*c^11*d^2*x^2 + sqrt(-3)*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2)
)*(1/(c^13*d^2))^(2/3) - 3*sqrt(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x^
4 + 160*c^13*d^2*x - sqrt(-3)*(c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d
^2*x))*(1/(c^13*d^2))^(5/6) - 2*(7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*
d)*sqrt(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2 + sqrt(-3)*(5*c^3*
d^2*x^5 + 32*c^4*d*x^2))*(1/(c^13*d^2))^(1/6)) - 9*(5*c^5*d^3*x^7 + 64*...
```

3.339.6 Sympy [F]

$$\int \frac{1}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = -\int \frac{1}{-8c^2\sqrt{c+dx^3}-7cdx^3\sqrt{c+dx^3}+d^2x^6\sqrt{c+dx^3}}\,dx$$

input

```
integrate(1/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output

```
-Integral(1/(-8*c**2*sqrt(c + d*x**3) - 7*c*d*x**3*sqrt(c + d*x**3) + d**2
*x**6*sqrt(c + d*x**3)), x)
```

3.339.7 Maxima [F]

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

3.339.8 Giac [F]

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\int \frac{1}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)}\,dx$$

input
```
int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.340 $\int \frac{1}{x^3\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

3.340.1 Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^3\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{3}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c+dx^3}}$$

output
```
-1/16*AppellF1(-2/3,3/2,1,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/
x^2/(d*x^3+c)^(1/2)
```

3.340.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 248 vs. 2(66) = 132.

Time = 11.19 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.76

$$\int \frac{1}{x^3\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{59d^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+64c\Big(-27c-59dx^3-}{}$$

input
```
Integrate[1/(x^3*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(59*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (
d*x^3)/(8*c)] + 64*c*(-27*c - 59*d*x^3 - (7360*c^2*d*x^3*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3,
 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2
, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^
3)/c), (d*x^3)/(8*c)])))))/(27648*c^4*x^2*Sqrt[c + d*x^3])
```

3.340.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (8c-dx^3) (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^3(8c-dx^3)\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{3}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
-1/16*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 3/2, 1/3, (d*x^3)/(8*c), -((d
*x^3)/c)])/(c^2*x^2*Sqrt[c + d*x^3])
```

3.340.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.340.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.20 (sec) , antiderivative size = 736, normalized size of antiderivative = 11.15

method	result	size
elliptic	Expression too large to display	736
risch	Expression too large to display	1028
default	Expression too large to display	1053

input
```
int(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/27*d/c^3*x/((x^3+c/d)*d)^(1/2)-1/16*(d*x^3+c)^(1/2)/c^3/x^2+59/1296*I/c
^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/1
944*I/c^3/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-
c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*
d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2
/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*
(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)
/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

3.340.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2495 vs. $2(52) = 104$.

Time = 1.59 (sec) , antiderivative size = 2495, normalized size of antiderivative = 37.80

$$\int \frac{1}{x^3 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output

```
-1/31104*(4176*(d*x^5 + c*x^2)*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) -
 (c^3*d*x^5 + c^4*x^2 + sqrt(-3)*(c^3*d*x^5 + c^4*x^2))*(d^4/c^19)^(1/6)*l
og((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - 9*(c^13*d^3
*x^8 + 38*c^14*d^2*x^5 + 64*c^15*d*x^2 + sqrt(-3)*(c^13*d^3*x^8 + 38*c^14*
d^2*x^5 + 64*c^15*d*x^2))*(d^4/c^19)^(2/3) + 3*sqrt(d*x^3 + c)*((c^16*d^2*
x^7 + 80*c^17*d*x^4 + 160*c^18*x - sqrt(-3)*(c^16*d^2*x^7 + 80*c^17*d*x^4
+ 160*c^18*x))*(d^4/c^19)^(5/6) - 2*(7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 6
4*c^12*d)*sqrt(d^4/c^19) + 6*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2 + sqrt(-3)*(5
*c^4*d^4*x^5 + 32*c^5*d^3*x^2))*(d^4/c^19)^(1/6)) - 9*(5*c^7*d^4*x^7 + 64*
c^8*d^3*x^4 + 32*c^9*d^2*x - sqrt(-3)*(5*c^7*d^4*x^7 + 64*c^8*d^3*x^4 + 32
*c^9*d^2*x))*(d^4/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 5
12*c^3)) + (c^3*d*x^5 + c^4*x^2 + sqrt(-3)*(c^3*d*x^5 + c^4*x^2))*(d^4/c^1
9)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - 9
*(c^13*d^3*x^8 + 38*c^14*d^2*x^5 + 64*c^15*d*x^2 + sqrt(-3)*(c^13*d^3*x^8
+ 38*c^14*d^2*x^5 + 64*c^15*d*x^2))*(d^4/c^19)^(2/3) - 3*sqrt(d*x^3 + c)*(
(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x - sqrt(-3)*(c^16*d^2*x^7 + 80*c
^17*d*x^4 + 160*c^18*x))*(d^4/c^19)^(5/6) - 2*(7*c^10*d^3*x^6 + 152*c^11*d
^2*x^3 + 64*c^12*d)*sqrt(d^4/c^19) + 6*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2 + s
qrt(-3)*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2))*(d^4/c^19)^(1/6)) - 9*(5*c^7*d^4
*x^7 + 64*c^8*d^3*x^4 + 32*c^9*d^2*x - sqrt(-3)*(5*c^7*d^4*x^7 + 64*c^8...
```

3.340.6 Sympy [F]

$$\int \frac{1}{x^3\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=-\int \frac{1}{-8c^2x^3\sqrt{c+dx^3}-7cdx^6\sqrt{c+dx^3}+d^2x^9\sqrt{c+dx^3}}\,dx$$

input

```
integrate(1/x**3/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output

```
-Integral(1/(-8*c**2*x**3*sqrt(c + d*x**3) - 7*c*d*x**6*sqrt(c + d*x**3) +
 d**2*x**9*sqrt(c + d*x**3)), x)
```

3.340.7 Maxima [F]

$$\int \frac{1}{x^3\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^3}\,dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^3), x)
```

3.340.8 Giac [F]

$$\int \frac{1}{x^3\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^3}\,dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^3), x)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^3\,\left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

input
```
int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.341 $\int \frac{1}{x^6\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

3.341.1 Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^6\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{5}{3},1,\frac{3}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40c^2x^5\sqrt{c+dx^3}}$$

output
```
-1/40*AppellF1(-5/3,3/2,1,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2
/x^5/(d*x^3+c)^(1/2)
```

3.341.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 261 vs. 2(66) = 132.

Time = 11.26 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.95

$$\int \frac{1}{x^6\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{-2981d^3x^9\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+64c\Bigg(-432c^2+1}{}$$

input
```
Integrate[1/(x^6*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(-2981*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c)
, (d*x^3)/(8*c)] + 64*c*(-432*c^2 + 1269*c*d*x^3 + 2981*d^2*x^6 + (382528*
c^2*d^2*x^6*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c
 - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] +
3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*Appel
lF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))))/(1105920*c^5*x^5*S
qrt[c + d*x^3])
```

3.341.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(8 c-d x^3\right) \left(c+d x^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{d x^3}{c}+1} \int \frac{1}{x^6 \left(8 c-d x^3\right) \left(\frac{d x^3}{c}+1\right)^{3/2}}dx}{c \sqrt{c+d x^3}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt{\frac{d x^3}{c}+1} \operatorname{AppellF1}\left(-\frac{5}{3},1,\frac{3}{2},-\frac{2}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right)}{40 c^2 x^5 \sqrt{c+d x^3}}$$

input
```
Int[1/(x^6*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
-1/40*(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 1, 3/2, -2/3, (d*x^3)/(8*c), -((
d*x^3)/c)])/(c^2*x^5*Sqrt[c + d*x^3])
```

3.341.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.341.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.29 (sec) , antiderivative size = 757, normalized size of antiderivative = 11.47

method	result	size
elliptic	Expression too large to display	757
risch	Expression too large to display	1040
default	Expression too large to display	1402

input
```
int(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/27*d^2/c^4*x/((x^3+c/d)*d)^(1/2)-1/40*(d*x^3+c)^(1/2)/c^3/x^5+63/640*d*(
d*x^3+c)^(1/2)/c^4/x^2-2981/51840*I*d/c^4*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/15552*I/d/c^4*2^(1/2)*sum(1/_alpha^2
*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3
^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*
_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_a
lpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1
/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z
^3*d-8*c))
```

3.341.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2543 vs. $2(52) = 104$.

Time = 4.22 (sec) , antiderivative size = 2543, normalized size of antiderivative = 38.53

$$\int \frac{1}{x^6 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
1/1244160*(214992*(d^2*x^8 + c*d*x^5)*sqrt(d)*weierstrassPInverse(0, -4*c/
d, x) + 5*(c^4*d*x^8 + c^5*x^5 + sqrt(-3)*(c^4*d*x^8 + c^5*x^5))*(d^10/c^2
5)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 -
 9*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2 + sqrt(-3)*(c^17*d^4*
x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2))*(d^10/c^25)^(2/3) + 3*sqrt(d*x^3
 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x - sqrt(-3)*(c^21*d^2*x^7
 + 80*c^22*d*x^4 + 160*c^23*x))*(d^10/c^25)^(5/6) - 2*(7*c^13*d^5*x^6 + 15
2*c^14*d^4*x^3 + 64*c^15*d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x^5 + 32*c^6*
d^7*x^2 + sqrt(-3)*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2))*(d^10/c^25)^(1/6)) -
9*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x - sqrt(-3)*(5*c^9*d^7*x
^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x))*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c*
d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 5*(c^4*d*x^8 + c^5*x^5 + sqrt(-3)*(c
^4*d*x^8 + c^5*x^5))*(d^10/c^25)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 12
00*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19
*d^2*x^2 + sqrt(-3)*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2))*(d
^10/c^25)^(2/3) - 3*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c
^23*x - sqrt(-3)*(c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x))*(d^10/c^25)^
(5/6) - 2*(7*c^13*d^5*x^6 + 152*c^14*d^4*x^3 + 64*c^15*d^3)*sqrt(d^10/c^25
) + 6*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2 + sqrt(-3)*(5*c^5*d^8*x^5 + 32*c^6*d
^7*x^2))*(d^10/c^25)^(1/6)) - 9*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c...
```

3.341.6 Sympy [F]

$$\int \frac{1}{x^6\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = -\int \frac{1}{-8c^2x^6\sqrt{c+dx^3}-7cdx^9\sqrt{c+dx^3}+d^2x^{12}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**6/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

output
```
-Integral(1/(-8*c**2*x**6*sqrt(c + d*x**3) - 7*c*d*x**9*sqrt(c + d*x**3) +
 d**2*x**12*sqrt(c + d*x**3)), x)
```

3.341.7 Maxima [F]

$$\int \frac{1}{x^6 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c) x^6} \, dx$$

input
```
integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^6), x)
```

3.341.8 Giac [F]

$$\int \frac{1}{x^6 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c) x^6} \, dx$$

input
```
integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^6), x)
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int \frac{1}{x^6 \left(d\,x^3 + c\right)^{3/2} \left(8\,c - d\,x^3\right)} \, dx$$

input
```
int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

output
```
int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.342 $\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$

3.342.1 Optimal result

Integrand size = 33, antiderivative size = 737

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$$

$$= \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$

$$+ \frac{\sqrt[6]{a}\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$

$$+ \frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1
/2))*2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1-3^
(1/2))*(b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+1/
4*3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/
2))*2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctanh(1
/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(b*x^3+a)^(1
/2))/b^(2/3)*2^(1/2)+2*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))+2/3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/
2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4
)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/
3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(
1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)
+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.342.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20+12\sqrt{3}\right)\sqrt{a+bx^3}}$$

input
```
Integrate[(x*Sqrt[a + b*x^3])/(2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x^
3)/(10*a + 6*Sqrt[3]*a))])/((20 + 12*Sqrt[3])*Sqrt[a + b*x^3])
```

3.342.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {984, 832, 759, 989, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}dx$$

$$\Big\downarrow \text{ 984}$$

$$\int \frac{x}{\sqrt{bx^3+a}}dx-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{bx^3+a}\left(bx^3+2\left(5+3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \text{ 832}$$

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{bx^3+a}\left(bx^3+2\left(5+3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{bx^3+a}\left(bx^3+2\left(5+3\sqrt{3}\right)a\right)}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 989}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-$$

$$\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-$$

$$3\left(3+2\sqrt{3}\right)a\left(-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{}}{}\right.$$

$$\Big\downarrow \text{2416}$$

$$-3\left(3+2\sqrt{3}\right)a\left(-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt{2}\sqrt{bx^3+a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{bx^3+a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2-}{}\right.$$

$$\frac{\frac{2\sqrt{bx^3+a}}{\sqrt[3]{b}\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}}}{\sqrt[3]{b}}-$$

$$\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}}$$

input
```
Int[(x*Sqrt[a + b*x^3])/(2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

3.342. $\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$

output

```
-3*(3 + 2*Sqrt[3])*a*(-1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^
(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*3^(3/4)*
a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a + b*x^3])/(
Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqr
t[3])*ArcTanh[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqr
t[2]*Sqrt[a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3
])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))) + ((2*Sqrt[a + b*x
^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 +
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

3.342.3.1 Defintions of rubi rules used

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

3.342. $\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$

rule 984

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 989

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

rule 2416

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.342.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.02 (sec) , antiderivative size = 977, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	977
default	Expression too large to display	995

input

```
int(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^(
1/2)*sum(1/_alpha*(3+2*3^(1/2))*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)
^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)
^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x
+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3
*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(
1/3)*_alpha*b-6*b^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)+3
*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(...
```

3.342.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.43 (sec) , antiderivative size = 4847, normalized size of antiderivative = 6.58

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="fricas
")
```

output
```
-1/8*((1/72)^(1/6)*(sqrt(-3)*b - b)*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^
(5/6)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x + sqrt(
-3)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x) + 3*sqrt
(3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x + sqrt(-3)*(b
^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)
^(5/6) - 4*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^
3*b^2*x^2 + sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b
^2*x^2))*sqrt(-sqrt(3)*a/b^4) + 6*(12*a*b^2*x^8 - 48*a^2*b*x^5 - 384*a^3*x
^2 + 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 + sqrt(3)*(b^
5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2 - sqrt(-3)*(b^5*x^9 -
30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*x^9 + 96*a
^2*b^3*x^3 + 16*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) + (1/9)^(1/3)*(b^4*x^10 +
 240*a^2*b^2*x^4 + 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 16
0*a^3*b*x) - 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x + sqrt(-3)*
(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) + 8*sqrt(
3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 + a) + (1/72)^(1/6)*
(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 -
 sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 25
6*a^4 - sqrt(-3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*
x^3 + 256*a^4)) - 3*sqrt(-3)*(b^4*x^12 - 4*a*b^3*x^9 + 360*a^2*b^2*x^6 ...
```

3.342.6 Sympy [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\sqrt{a+bx^3}}{10a+6\sqrt{3}a+bx^3}\,dx$$

input
```
integrate(x*(b*x**3+a)**(1/2)/(b*x**3+2*a*(5+3*3**(1/2))),x)
```

output
```
Integral(x*sqrt(a + b*x**3)/(10*a + 6*sqrt(3)*a + b*x**3), x)
```

3.342.7 Maxima [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{bx^3+a}x}{bx^3+2\,a\left(3\sqrt{3}+5\right)}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

3.342.8 Giac [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{bx^3+a}x}{bx^3+2\,a\left(3\sqrt{3}+5\right)}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\,\sqrt{b\,x^3+a}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}+5\right)}\,dx$$

input
```
int((x*(a + b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)),x)
```

output
```
int((x*(a + b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)), x)
```

3.343 $\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$

3.343.1 Optimal result

Integrand size = 35, antiderivative size = 757

$$
\begin{aligned}
&\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx \\
&= \frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}+\frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} \\
&+\frac{\sqrt[6]{a}\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} \\
&+\frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{\sqrt{2}b^{2/3}} \\
&-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} \\
&+\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}
\end{aligned}
$$

output
```
1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1
/2))*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1-3
^(1/2))*(-b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+
1/4*3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(
1/2))*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctan
h(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(-b*x^3+a)
^(1/2))/b^(2/3)*2^(1/2)+2*(-b*x^3+a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))+2/3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/
3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2
)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*Ellipt
icE((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^
(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^
(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.343.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx=\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20+12\sqrt{3}\right)\sqrt{a-bx^3}}$$

input
```
Integrate[(x*Sqrt[a - b*x^3])/(2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

output
```
(x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(1
0*a + 6*Sqrt[3]*a)])/((20 + 12*Sqrt[3])*Sqrt[a - b*x^3])
```

3.343.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {984, 832, 759, 989, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}dx$$

$$\Big\downarrow \text{ 984}$$

$$\int \frac{x}{\sqrt{a-bx^3}}dx-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}dx$$

$$\Big\downarrow \text{ 832}$$

$$\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}dx$$

$$\Big\downarrow \text{ 759}$$

$$-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{ 989}$$

$$-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-$$

$$\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$3\left(3+2\sqrt{3}\right)a\left(-\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{\right.}}{}\right.$$

$$\Big\downarrow \color{blue}{2416}$$

$$-3\left(3+2\sqrt{3}\right)a\left(-\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\right.}{}\right.$$

$$\frac{\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}}{\sqrt[3]{b}}-$$

$$\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

input
```
Int[(x*Sqrt[a - b*x^3])/(2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

3.343. $\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$

output
```
-3*(3 + 2*Sqrt[3])*a*(-1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^
(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(Sqrt[2]*3^(3/4)*
a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a - b*x^3])/(
Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqr
t[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[
2]*Sqrt[a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])
*ArcTanh[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*
Sqrt[a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))) - ((-2*Sqrt[a - b*
x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1/4)*Sqrt[2 - Sq
rt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2
+ Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

3.343.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

3.343. $\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.343.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.28 (sec) , antiderivative size = 924, normalized size of antiderivative = 1.22

method	result	size
elliptic	Expression too large to display	924
default	Expression too large to display	942

input
```
int(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^
3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(
1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(3+
2*3^(1/2))*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^
2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3)
-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1/
3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*(3*I*(a*b^2)^(1/3
)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2/3)*3^(1/2)-2*3^(1
/2)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-6*b^2*_alpha^2-2*3^(
1/2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3)*_alpha*b+3*(a*b^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*
b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/2)*(a*b^2)^...
```

3.343.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.37 (sec) , antiderivative size = 4855, normalized size of antiderivative = 6.41

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="fric
as")
```

output
```
-1/8*((1/72)^(1/6)*(sqrt(-3)*b - b)*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^
(5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x + sqrt(
-3)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x) + 3*sqrt
(3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x + sqrt(-3)*(b
^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)
^(5/6) + 4*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^
3*b^2*x^2 + sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b
^2*x^2))*sqrt(-sqrt(3)*a/b^4) + 6*(12*a*b^2*x^8 + 48*a^2*b*x^5 - 384*a^3*x
^2 - 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 + sqrt(3)*(b^
5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2 - sqrt(-3)*(b^5*x^9 +
30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*x^9 + 96*a
^2*b^3*x^3 - 16*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) + (1/9)^(1/3)*(b^4*x^10 +
 240*a^2*b^2*x^4 - 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 16
0*a^3*b*x) + 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x + sqrt(-3)*
(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) + 8*sqrt(
3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 + a) + (1/72)^(1/6)
*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4
- 3*sqrt(-3)*(b^4*x^12 + 4*a*b^3*x^9 + 360*a^2*b^2*x^6 - 736*a^3*b*x^3 + 1
28*a^4) - sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b
*x^3 + 256*a^4 - sqrt(-3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 -...
```

3.343.6 Sympy [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = -\int \frac{x\sqrt{a-bx^3}}{-6\sqrt{3}a-10a+bx^3}\,dx$$

input
```
integrate(x*(-b*x**3+a)**(1/2)/(-b*x**3+2*a*(5+3*3**(1/2))),x)
```

output
```
-Integral(x*sqrt(a - b*x**3)/(-6*sqrt(3)*a - 10*a + b*x**3), x)
```

3.343.7 Maxima [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{\sqrt{-bx^3+ax}}{bx^3-2\,a\left(3\,\sqrt{3}+5\right)}\,dx$$

input
```
integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="maxi
ma")
```

output
```
-integrate(sqrt(-b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

3.343.8 Giac [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{\sqrt{-bx^3+ax}}{bx^3-2\,a\left(3\,\sqrt{3}+5\right)}\,dx$$

input
```
integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="giac
")
```

output
```
integrate(-sqrt(-b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx=-\int \frac{x\,\sqrt{a-b\,x^3}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)}\,dx$$

input
```
int(-(x*(a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)),x)
```

output
```
-int((x*(a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)), x)
```

3.344 $\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$

3.344.1 Optimal result

Integrand size $= 35$, antiderivative size $= 774$

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$$

$$= -\frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$

$$+ \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$

$$+ \frac{3^{3/4}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}} - \frac{\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$

$$+ \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$- \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
1/4*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1
/2))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctan(1
/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(b*x^3-a)^(1/
2))/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/
3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)-1/6*a^(
1/6)*arctanh(1/6*(1-3^(1/2))*(b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3
/4)/b^(2/3)*2^(1/2)-2*(b*x^3-a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/
2)))-2/3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(
3/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(
1/3)*(1-3^(1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((
-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(
1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1
/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)
*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.344.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=-\frac{x^2\sqrt{-a+bx^3}\,\mathrm{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)}{4\left(5+3\sqrt{3}\right)a\sqrt{1-\frac{bx^3}{a}}}$$

input
```
Integrate[(x*Sqrt[-a + b*x^3])/(-2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

output
```
-1/4*(x^2*Sqrt[-a + b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/
(10*a + 6*Sqrt[3]*a)])/((5 + 3*Sqrt[3])*a*Sqrt[1 - (b*x^3)/a])
```

3.344.3 Rubi [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 852, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {984, 25, 833, 760, 990, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{bx^3-a}}{bx^3-2\left(5+3\sqrt{3}\right)a}dx$$

$$\Big\downarrow \ 984$$

$$\int \frac{x}{\sqrt{bx^3-a}}dx+3\left(3+2\sqrt{3}\right)a\int -\frac{x}{\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \ 25$$

$$\int \frac{x}{\sqrt{bx^3-a}}dx-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \ 833$$

$$\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}-\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \ 760$$

$$-\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}dx-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \ 990$$

$$-\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-$$

$$3\left(3+2\sqrt{3}\right)a\left(-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right.}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$$\Big\downarrow \text{2418}$$

$$-3\left(3+2\sqrt{3}\right)a\left(-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(2\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}{\sqrt{2}\sqrt{bx^3-a}}\right.}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$$\frac{\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}}{\sqrt[3]{b}}-$$

$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

input

```
Int[(x*Sqrt[-a + b*x^3])/(-2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

3.344. $\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$

output
```
-3*(3 + 2*Sqrt[3])*a*(-1/6*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^
(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(Sqrt[2]*3^(1/4)
*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*
a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^
(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^
(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6
)*b^(2/3)) + ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt
[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))) - ((2*Sqrt[-a
+ b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4)*Sqrt[2
+ Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((
1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7
 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[
3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]))/b^(1/3) - (2*Sqrt[2 - Sqrt
[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[A
rcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)
*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)
*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

3.344.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.344.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.97 (sec) , antiderivative size = 926, normalized size of antiderivative = 1.20

method	result	size
elliptic	Expression too large to display	926
default	Expression too large to display	944

input
```
int(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3
-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(
1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1
/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*
b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(3+2
*3^(1/2))*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2
)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3)-
I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1/3
)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*(3*I*(a*b^2)^(1/3)*
_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2/3)*3^(1/2)-2*3^(1/2
)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-6*b^2*_alpha^2-2*3^(1/
2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3)*_alpha*b+3*(a*b^2)^(2/3
))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^
2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/2)*(a*b^2)^(1...
```

3.344.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.74 (sec) , antiderivative size = 4963, normalized size of antiderivative = 6.41

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="fricas
")
```

output
```
1/8*((1/72)^(1/6)*(sqrt(-3)*b + b)*(sqrt(3)*a/b^4)^(1/6)*log(-(12*(1/9)^(2
/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 + sqrt(3)*(b^5*x^9 + 30*a*b^
4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2 + sqrt(-3)*(b^5*x^9 + 30*a*b^4*x^6 -
144*a^2*b^3*x^3 + 32*a^3*b^2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 - 16
*a^3*b^2))*sqrt(b*x^3 - a)*(sqrt(3)*a/b^4)^(2/3) + 72*(1/72)^(5/6)*(7*b^6*
x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^
10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x) + 3*sqrt(3)*(b^6*x^10
 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 + 12*
a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) + 6*(1/
9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x - sqrt(-3)*(b^4*x^10 +
240*a^2*b^2*x^4 - 160*a^3*b*x) + 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4
*a^3*b*x - sqrt(-3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt(b*x^3 -
 a)*(sqrt(3)*a/b^4)^(1/3) - 4*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a
^2*b^3*x^5 - 624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*
b^3*x^5 + 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) - 24*(3*a*b^2*x^8 + 12*a^2
*b*x^5 - 96*a^3*x^2 + 2*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sq
rt(b*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6
 - 2208*a^3*b*x^3 + 384*a^4 + 3*sqrt(-3)*(b^4*x^12 + 4*a*b^3*x^9 + 360*a^2
*b^2*x^6 - 736*a^3*b*x^3 + 128*a^4) - sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 +
744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12 - 124*a...
```

3.344.6 Sympy [F]

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\sqrt{-a+bx^3}}{-6\sqrt{3}a-10a+bx^3}\,dx$$

input
```
integrate(x*(b*x**3-a)**(1/2)/(b*x**3-2*a*(5+3*3**(1/2))),x)
```

output
```
Integral(x*sqrt(-a + b*x**3)/(-6*sqrt(3)*a - 10*a + b*x**3), x)
```

3.344.7 Maxima [F]

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{bx^3-a}x}{bx^3-2\,a\left(3\,\sqrt{3}+5\right)}\,dx$$

input
```
integrate(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

3.344.8 Giac [F]

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{bx^3-a}x}{bx^3-2\,a\left(3\,\sqrt{3}+5\right)}\,dx$$

input
```
integrate(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\,\sqrt{b\,x^3-a}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)}\,dx$$

input
```
int((x*(b*x^3 - a)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)),x)
```

output
```
int((x*(b*x^3 - a)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)), x)
```

3.345 $\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$

3.345.1 Optimal result

Integrand size $= 37$, antiderivative size $= 768$

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$$

$$= -\frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}b^{2/3}}$$

$$+ \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$

$$+ \frac{3^{3/4}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} - \frac{\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$

$$+ \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$- \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
1/4*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1
/2))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctan(
1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(-b*x^3-a)^
(1/2))/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^
(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)-1/6
*a^(1/6)*arctanh(1/6*(1-3^(1/2))*(-b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))
*3^(3/4)/b^(2/3)*2^(1/2)-2*(-b*x^3-a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))-2/3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3
^(3/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE
((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(
1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/
2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.345.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.12

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = -\frac{x^2\sqrt{-a-bx^3}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)}{4\left(5+3\sqrt{3}\right)a\sqrt{\frac{a+bx^3}{a}}}$$

input
```
Integrate[(x*Sqrt[-a - b*x^3])/(-2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

output
```
-1/4*(x^2*Sqrt[-a - b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*
x^3)/(10*a + 6*Sqrt[3]*a))])/((5 + 3*Sqrt[3])*a*Sqrt[(a + b*x^3)/a])
```

3.345.3 Rubi [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {984, 25, 833, 760, 990, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}dx$$

$$\Big\downarrow \textcolor{blue}{984}$$

$$\int \frac{x}{\sqrt{-bx^3-a}}dx+3\left(3+2\sqrt{3}\right)a\int -\frac{x}{\sqrt{-bx^3-a}\left(bx^3+2\left(5+3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$\int \frac{x}{\sqrt{-bx^3-a}}dx-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{-bx^3-a}\left(bx^3+2\left(5+3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \textcolor{blue}{833}$$

$$-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}+\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{-bx^3-a}\left(bx^3+2\left(5+3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \textcolor{blue}{760}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}-3\left(3+2\sqrt{3}\right)a\int \frac{x}{\sqrt{-bx^3-a}\left(bx^3+2\left(5+3\sqrt{3}\right)a\right)}dx-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \textcolor{blue}{990}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}-$$

$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-$$

$$3\left(3+2\sqrt{3}\right)a\left(-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right.}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$$\Big\downarrow \text{2418}$$

$$-3\left(3+2\sqrt{3}\right)a\left(-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{b}x+\right.}{\sqrt{2}\sqrt{-bx^3-a}}\right.}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$$\frac{\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{-bx^3-a}}-\frac{2\sqrt{-bx^3-a}}{\sqrt[3]{b}\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)}}{\sqrt[3]{b}}-$$

$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{-bx^3-a}}$$

input
```
Int[(x*Sqrt[-a - b*x^3])/(-2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

3.345. $\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$

output

```
-3*(3 + 2*Sqrt[3])*a*(-1/3*((2 - Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sq
rt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(Sqrt[2]*3^(1/
4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)
*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^
(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^
(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6
)*b^(2/3)) + ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt
[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))) + ((-2*Sqrt[-a
 - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1/4)*Sqrt[2
 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]))/b^(1/3) - (2*Sqrt[2 - Sqr
t[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[
ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

3.345.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 760

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

3.345. $\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.345.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.09 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.28

method	result	size
elliptic	Expression too large to display	983
default	Expression too large to display	1001

input
```
int(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^
(1/2)*sum(1/_alpha*(3+2*3^(1/2))*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2
)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2
)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*
x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-b
*x^3-a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)
+3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)
^(1/3)*_alpha*b-6*b^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)
+3*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)...
```

3.345.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.28 (sec) , antiderivative size = 4981, normalized size of antiderivative = 6.49

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="fric
as")
```

output
```
-1/8*((1/72)^(1/6)*(sqrt(-3)*b + b)*(sqrt(3)*a/b^4)^(1/6)*log((12*(1/9)^(2
/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 + sqrt(3)*(b^5*x^9 - 30*a*b^
4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2 + sqrt(-3)*(b^5*x^9 - 30*a*b^4*x^6 -
144*a^2*b^3*x^3 - 32*a^3*b^2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 + 16
*a^3*b^2))*sqrt(-b*x^3 - a)*(sqrt(3)*a/b^4)^(2/3) + 72*(1/72)^(5/6)*(7*b^6
*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x
^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x) + 3*sqrt(3)*(b^6*x^1
0 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 - 12
*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) - 6*(1
/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x - sqrt(-3)*(b^4*x^10 +
 240*a^2*b^2*x^4 + 160*a^3*b*x) - 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 +
4*a^3*b*x - sqrt(-3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt(-b*x^3
 - a)*(sqrt(3)*a/b^4)^(1/3) + 4*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360
*a^2*b^3*x^5 + 624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^
2*b^3*x^5 - 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) + 24*(3*a*b^2*x^8 - 12*a
^2*b*x^5 - 96*a^3*x^2 + 2*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*
sqrt(-b*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*
x^6 + 2208*a^3*b*x^3 + 384*a^4 - sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a
^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12 + 124*a*b^3*x^9
 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4)) + 3*sqrt(-3)*(b^4*x^12 ...
```

3.345.6 Sympy [F]

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx=-\int \frac{x\sqrt{-a-bx^3}}{10a+6\sqrt{3}a+bx^3}\,dx$$

input
```
integrate(x*(-b*x**3-a)**(1/2)/(-b*x**3-2*a*(5+3*3**(1/2))),x)
```

output
```
-Integral(x*sqrt(-a - b*x**3)/(10*a + 6*sqrt(3)*a + b*x**3), x)
```

3.345.7 Maxima [F]

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{\sqrt{-bx^3-ax}}{bx^3+2\,a\left(3\,\sqrt{3}+5\right)}\,dx$$

input
```
integrate(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="maxi
ma")
```

output
```
-integrate(sqrt(-b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

3.345.8 Giac [F]

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{\sqrt{-bx^3-ax}}{bx^3+2\,a\left(3\,\sqrt{3}+5\right)}\,dx$$

input
```
integrate(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="giac
")
```

output
```
integrate(-sqrt(-b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{x\,\sqrt{-b\,x^3-a}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}+5\right)}\,dx$$

input
```
int(-(x*(- a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)),x)
```

output
```
int(-(x*(- a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)), x)
```

3.346 $\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$

3.346.1 Optimal result

Integrand size = 33, antiderivative size = 738

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$$

$$= \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$

$$- \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$

$$+ \frac{3^{3/4}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/2*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1-3
^(1/2)))*2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)-1/4*3^(1/4)*a^(1/6)*arct
an(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3+a)^(
1/2))/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(
1/3)+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a
^(1/6)*arctanh(1/6*(1+3^(1/2))*(b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^
(3/4)/b^(2/3)*2^(1/2)+2*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))+2/3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1
/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/
4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1
/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3
)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.346.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20-12\sqrt{3}\right)\sqrt{a+bx^3}}$$

input
```
Integrate[(x*Sqrt[a + b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x^
3)/(10*a - 6*Sqrt[3]*a))])/((20 - 12*Sqrt[3])*Sqrt[a + b*x^3])
```

3.346.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {984, 832, 759, 989, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}dx$$

$$\Big\downarrow \text{984}$$

$$\int \frac{x}{\sqrt{bx^3+a}}dx-3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{bx^3+a}\left(bx^3+2\left(5-3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \text{832}$$

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{bx^3+a}\left(bx^3+2\left(5-3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \text{759}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{bx^3+a}\left(bx^3+2\left(5-3\sqrt{3}\right)a\right)}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{989}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-$$

$$\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-$$

$$3\left(3-2\sqrt{3}\right)a\left(-\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right.}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$$\Big\downarrow \text{2416}$$

$$-3\left(3-2\sqrt{3}\right)a\left(-\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3+a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{b}x+\right.}{\sqrt{2}\sqrt{bx^3+a}}\right.}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$$\frac{\frac{2\sqrt{bx^3+a}}{\sqrt[3]{b}\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}}}{\sqrt[3]{b}}-$$

$$\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}}$$

input
```
Int[(x*Sqrt[a + b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

3.346. $\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$

output
```
-3*(3 - 2*Sqrt[3])*a*(-1/3*((2 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sq
rt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*3^(1/4
)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*
(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5
/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1
/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b
^(2/3)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*
3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))) + ((2*Sqrt[a + b*x
^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 +
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

3.346.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

3.346. $\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.346.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.29 (sec) , antiderivative size = 977, normalized size of antiderivative = 1.32

method	result	size
elliptic	Expression too large to display	977
default	Expression too large to display	995

input
```
int(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x,method=_RETURNVERBOSE)
```

output

```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^(
1/2)*sum(1/_alpha*(2*3^(1/2)-3)*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)
^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)
^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x
+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*
I*(-a*b^2)^(2/3)*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(-a*b^2)^(1
/3)*_alpha*b+6*b^2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*3^(1/2)*(-a*b^2)^(2/3)-3*
(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1...
```

3.346.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.39 (sec) , antiderivative size = 4931, normalized size of antiderivative = 6.68

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \text{Too large to display}$$

input

```
integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="fricas
")
```

output
```
1/8*((1/72)^(1/6)*(sqrt(-3)*b + b)*(sqrt(3)*a/b^4)^(1/6)*log((12*(1/9)^(2/
3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 - sqrt(3)*(b^5*x^9 - 30*a*b^4
*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2 + sqrt(-3)*(b^5*x^9 - 30*a*b^4*x^6 - 1
44*a^2*b^3*x^3 - 32*a^3*b^2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 + 16*
a^3*b^2))*sqrt(b*x^3 + a)*(sqrt(3)*a/b^4)^(2/3) + 72*(1/72)^(5/6)*(7*b^6*x
^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^1
0 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x) - 3*sqrt(3)*(b^6*x^10
- 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 - 12*a
*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) + 6*(1/9
)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x - sqrt(-3)*(b^4*x^10 + 2
40*a^2*b^2*x^4 + 160*a^3*b*x) + 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*
a^3*b*x - sqrt(-3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt(b*x^3 +
a)*(sqrt(3)*a/b^4)^(1/3) - 4*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^
2*b^3*x^5 + 624*a^3*b^2*x^2 - sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b
^3*x^5 - 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) + 24*(3*a*b^2*x^8 - 12*a^2*
b*x^5 - 96*a^3*x^2 - 2*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqr
t(b*x^3 + a) + (1/72)^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6
+ 2208*a^3*b*x^3 + 384*a^4 + sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b
^2*x^6 + 1120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12 + 124*a*b^3*x^9 + 7
44*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4)) + 3*sqrt(-3)*(b^4*x^12 - 4*...
```

3.346.6 Sympy [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\sqrt{a+bx^3}}{-6\sqrt{3}a+10a+bx^3}\,dx$$

input
```
integrate(x*(b*x**3+a)**(1/2)/(b*x**3+2*a*(5-3*3**(1/2))),x)
```

output
```
Integral(x*sqrt(a + b*x**3)/(-6*sqrt(3)*a + 10*a + b*x**3), x)
```

3.346.7 Maxima [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \int \frac{\sqrt{bx^3+a}x}{bx^3-2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

3.346.8 Giac [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \int \frac{\sqrt{bx^3+a}x}{bx^3-2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

3.346.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \int \frac{x\,\sqrt{b\,x^3+a}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
int((x*(a + b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)),x)
```

output
```
int((x*(a + b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)), x)
```

3.347 $\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$

3.347.1 Optimal result

Integrand size = 35, antiderivative size = 758

$$
\begin{aligned}
&\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx \\
&= \frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} \\
&\quad-\frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{\sqrt{2}b^{2/3}} \\
&\quad+\frac{3^{3/4}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} \\
&\quad-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} \\
&\quad+\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}
\end{aligned}
$$

output
```
-1/2*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1-3^
(1/2))*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)-1/4*3^(1/4)*a^(1/6)*arct
an(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3+a)^
(1/2))/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^
(1/3)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6
*a^(1/6)*arctanh(1/6*(1+3^(1/2))*(-b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))
*3^(3/4)/b^(2/3)*2^(1/2)+2*(-b*x^3+a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))+2/3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(
1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2
/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/
2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*Ellip
ticE((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(-b*x^3+a)^(1/2)/(a
^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.347.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20-12\sqrt{3}\right)\sqrt{a-bx^3}}$$

input
```
Integrate[(x*Sqrt[a - b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

output
```
(x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(1
0*a - 6*Sqrt[3]*a)])/((20 - 12*Sqrt[3])*Sqrt[a - b*x^3])
```

3.347.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {984, 832, 759, 989, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}dx$$

$$\Big\downarrow \text{ 984}$$

$$\int \frac{x}{\sqrt{a-bx^3}}dx-3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}dx$$

$$\Big\downarrow \text{ 832}$$

$$\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}dx$$

$$\Big\downarrow \text{ 759}$$

$$-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{ 989}$$

$$-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-$$

$$\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} -$$

$$3\left(3-2\sqrt{3}\right)a\left(-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right.}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$$\Big\downarrow \text{2416}$$

$$-3\left(3-2\sqrt{3}\right)a\left(-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(2\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{}\right.}{\sqrt{2}\sqrt{a-bx^3}}\right.}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\right.$$

$$\frac{\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}}{\sqrt[3]{b}}-$$

$$\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

input
```
Int[(x*Sqrt[a - b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

3.347. $\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$

output

```
-3*(3 - 2*Sqrt[3])*a*(-1/6*((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^
(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(Sqrt[2]*3^(1/4)*
a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a
^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5
/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1
/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b
^(2/3)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*
3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))) - ((-2*Sqrt[a - b*
x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1/4)*Sqrt[2 - Sq
rt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2
+ Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

3.347.3.1 Defintions of rubi rules used

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

3.347. $\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.347.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.00 (sec) , antiderivative size = 924, normalized size of antiderivative = 1.22

method	result	size
elliptic	Expression too large to display	924
default	Expression too large to display	942

input
```
int(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^
3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(
1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(2*
3^(1/2)-3)*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^
2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3)
-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1/
3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*(-3*I*(a*b^2)^(1/
3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^(2/3)*3^(1/2)-6*I*(
a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b+6*b^2*_alpha^2+6*I*
(a*b^2)^(2/3)-2*3^(1/2)*(a*b^2)^(2/3)-3*(a*b^2)^(1/3)*_alpha*b-3*(a*b^2)^(
2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/2)*(a*b^2)...
```

3.347.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.38 (sec) , antiderivative size = 4953, normalized size of antiderivative = 6.53

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="fric
as")
```

output
```
-1/8*((1/72)^(1/6)*(sqrt(-3)*b + b)*(sqrt(3)*a/b^4)^(1/6)*log(-(12*(1/9)^(
2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 - sqrt(3)*(b^5*x^9 + 30*a*b
^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2 + sqrt(-3)*(b^5*x^9 + 30*a*b^4*x^6 -
 144*a^2*b^3*x^3 + 32*a^3*b^2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 - 1
6*a^3*b^2))*sqrt(-b*x^3 + a)*(sqrt(3)*a/b^4)^(2/3) + 72*(1/72)^(5/6)*(7*b^
6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x - sqrt(-3)*(7*b^6*
x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x) - 3*sqrt(3)*(b^6*x^
10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 + 1
2*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) - 6*(
1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x - sqrt(-3)*(b^4*x^10
+ 240*a^2*b^2*x^4 - 160*a^3*b*x) - 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 +
 4*a^3*b*x - sqrt(-3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt(-b*x^
3 + a)*(sqrt(3)*a/b^4)^(1/3) + 4*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 36
0*a^2*b^3*x^5 - 624*a^3*b^2*x^2 - sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a
^2*b^3*x^5 + 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) - 24*(3*a*b^2*x^8 + 12*
a^2*b*x^5 - 96*a^3*x^2 - 2*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))
*sqrt(-b*x^3 + a) + (1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2
*x^6 - 2208*a^3*b*x^3 + 384*a^4 + 3*sqrt(-3)*(b^4*x^12 + 4*a*b^3*x^9 + 360
*a^2*b^2*x^6 - 736*a^3*b*x^3 + 128*a^4) + sqrt(3)*(b^4*x^12 - 124*a*b^3*x^
9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12 - 1...
```

3.347.6 Sympy [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=-\int \frac{x\sqrt{a-bx^3}}{-10a+6\sqrt{3}a+bx^3}\,dx$$

input
```
integrate(x*(-b*x**3+a)**(1/2)/(-b*x**3+2*a*(5-3*3**(1/2))),x)
```

output
```
-Integral(x*sqrt(a - b*x**3)/(-10*a + 6*sqrt(3)*a + b*x**3), x)
```

3.347.7 Maxima [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{\sqrt{-bx^3+ax}}{bx^3+2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxi
ma")
```

output
```
-integrate(sqrt(-b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

3.347.8 Giac [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{\sqrt{-bx^3+ax}}{bx^3+2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac
")
```

output
```
integrate(-sqrt(-b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{x\,\sqrt{a-b\,x^3}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
int(-(x*(a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)),x)
```

output
```
int(-(x*(a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)), x)
```

3.348 $\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$

3.348.1 Optimal result

Integrand size = 36, antiderivative size = 774

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$$

$$= \frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$

$$+ \frac{\sqrt[6]{a}\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$

$$+ \frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$

$$- \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output

```
-1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(
1/2))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1+3
^(1/2))*(b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+1
/2*3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/4*3^(1/4)*a^(1/6)*arctan
h(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3-a)^(1
/2))/b^(2/3)*2^(1/2)+2*(b*x^3-a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1
/2)))+2/3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^
(3/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^
(1/3)*(1-3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE(
(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^
(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3
)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.348.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=-\frac{x^2\sqrt{-a+bx^3}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},-\frac{bx^3}{-10a+6\sqrt{3}a}\right)}{4\left(-5+3\sqrt{3}\right)a\sqrt{\frac{a-bx^3}{a}}}$$

input

```
Integrate[(x*Sqrt[-a + b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

output

```
-1/4*(x^2*Sqrt[-a + b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, -((b*x^3
)/(-10*a + 6*Sqrt[3]*a))])/((-5 + 3*Sqrt[3])*a*Sqrt[(a - b*x^3)/a])
```

3.348.3 Rubi [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 843, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {984, 833, 760, 990, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{bx^3-a}}{2\left(5-3\sqrt{3}\right)a-bx^3}dx$$

$$\Big\downarrow \; 984$$

$$3\left(3-2\sqrt{3}\right)a\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}dx-\int \frac{x}{\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \; 833$$

$$-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}+\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}+3\left(3-2\sqrt{3}\right)a\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \; 760$$

$$\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}+3\left(3-2\sqrt{3}\right)a\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}dx+\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \; 990$$

$$\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}+$$

$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}} -$$

$$3\left(3-2\sqrt{3}\right)a\left(\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3})\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-(2+\sqrt{3})\right.$$

$$\Big\downarrow \text{2418}$$

$$3\left(3-2\sqrt{3}\right)a\left(\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3})\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-(2+\sqrt{3})\right.$$

$$\frac{\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}}{\sqrt[3]{b}}+$$

$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

input
```
Int[(x*Sqrt[-a + b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

3.348. $\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$

output
```
3*(3 - 2*Sqrt[3])*a*(((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*
(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(
5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a + b*x^3])/(Sq
rt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[
3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]
*Sqrt[-a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*
ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*S
qrt[-a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))) + ((2*Sqrt[-a + b*
x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4)*Sqrt[2 + Sq
rt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 +
Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4
*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*
a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]))/b^(1/3) + (2*Sqrt[2 - Sqrt[3]]
*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)]
, -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))
/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

3.348.3.1 Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

3.348. $\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.348.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.74 (sec) , antiderivative size = 926, normalized size of antiderivative = 1.20

method	result	size
elliptic	Expression too large to display	926
default	Expression too large to display	944

input
```
int(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2
/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^
3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(
1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^(1/2)*sum(1/_alpha*(2*
3^(1/2)-3)*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^
2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3)
-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1/
3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*(-3*I*(a*b^2)^(1/3
)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^(2/3)*3^(1/2)-6*I*(a
*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b+6*b^2*_alpha^2+6*I*(
a*b^2)^(2/3)-2*3^(1/2)*(a*b^2)^(2/3)-3*(a*b^2)^(1/3)*_alpha*b-3*(a*b^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*
b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/2)*(a*b^2)^...
```

3.348.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.39 (sec) , antiderivative size = 4867, normalized size of antiderivative = 6.29

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="frica
s")
```

output
```
-1/8*((1/72)^(1/6)*(sqrt(-3)*b - b)*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^
(5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x + sqrt(
-3)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x) - 3*sqrt
(3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x + sqrt(-3)*(b
^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)
^(5/6) - 4*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^
3*b^2*x^2 - sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b
^2*x^2))*sqrt(-sqrt(3)*a/b^4) + 6*(12*a*b^2*x^8 + 48*a^2*b*x^5 - 384*a^3*x
^2 - 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 - sqrt(3)*(b^
5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2 - sqrt(-3)*(b^5*x^9 +
30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*x^9 + 96*a
^2*b^3*x^3 - 16*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) - (1/9)^(1/3)*(b^4*x^10 +
 240*a^2*b^2*x^4 - 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 16
0*a^3*b*x) - 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x + sqrt(-3)*
(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) - 8*sqrt(
3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 - a) + (1/72)^(1/6)*
(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 -
 3*sqrt(-3)*(b^4*x^12 + 4*a*b^3*x^9 + 360*a^2*b^2*x^6 - 736*a^3*b*x^3 + 12
8*a^4) + sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*
x^3 + 256*a^4 - sqrt(-3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - ...
```

3.348.6 Sympy [F]

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=-\int \frac{x\sqrt{-a+bx^3}}{-10a+6\sqrt{3}a+bx^3}\,dx$$

input
```
integrate(x*(b*x**3-a)**(1/2)/(-b*x**3+2*a*(5-3*3**(1/2))),x)
```

output
```
-Integral(x*sqrt(-a + b*x**3)/(-10*a + 6*sqrt(3)*a + b*x**3), x)
```

3.348.7 Maxima [F]

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{\sqrt{bx^3-a}x}{bx^3+2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxim
a")
```

output
```
-integrate(sqrt(b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

3.348.8 Giac [F]

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{\sqrt{bx^3-a}x}{bx^3+2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac"
)
```

output
```
integrate(-sqrt(b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{x\,\sqrt{b\,x^3-a}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
int(-(x*(b*x^3 - a)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)),x)
```

output
```
int(-(x*(b*x^3 - a)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)), x)
```

3.349 $\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$

3.349.1 Optimal result

Integrand size $= 36$, antiderivative size $= 768$

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$$

$$= \frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$

$$+ \frac{\sqrt[6]{a}\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}b^{2/3}}$$

$$+ \frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$

$$- \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
-1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(
1/2))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1+
3^(1/2))*(-b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)
+1/2*3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/4*3^(1/4)*a^(1/6)*ar
ctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3-
a)^(1/2))/b^(2/3)*2^(1/2)+2*(-b*x^3-a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))+2/3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*
3^(3/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*Elliptic
E((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1
/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3
)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.349.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.12

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=-\frac{x^2\sqrt{-a-bx^3}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{4\left(-5+3\sqrt{3}\right)a\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(x*Sqrt[-a - b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

output
```
-1/4*(x^2*Sqrt[-a - b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*
x^3)/(10*a - 6*Sqrt[3]*a))])/((-5 + 3*Sqrt[3])*a*Sqrt[1 + (b*x^3)/a])
```

3.349.3 Rubi [A] (warning: unable to verify)

Time = 0.82 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {984, 833, 760, 990, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}dx$$

$$\Big\downarrow \text{984}$$

$$3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{-bx^3-a}\left(bx^3+2\left(5-3\sqrt{3}\right)a\right)}dx-\int \frac{x}{\sqrt{-bx^3-a}}dx$$

$$\Big\downarrow \text{833}$$

$$\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}+3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{-bx^3-a}\left(bx^3+2\left(5-3\sqrt{3}\right)a\right)}dx$$

$$\Big\downarrow \text{760}$$

$$-\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}+3\left(3-2\sqrt{3}\right)a\int \frac{x}{\sqrt{-bx^3-a}\left(bx^3+2\left(5-3\sqrt{3}\right)a\right)}dx+\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{990}$$

$$-\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}+$$

$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-$$

$$3\left(3-2\sqrt{3}\right)a\left(\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3})\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3}}{}\right.$$

$$\Big\downarrow \text{2418}$$

$$3\left(3-2\sqrt{3}\right)a\left(\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt{2}\sqrt{-bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3})\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3}}{}\right.$$

$$\frac{\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{-bx^3-a}}-\frac{2\sqrt{-bx^3-a}}{\sqrt[3]{b}\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)}}{\sqrt[3]{b}}+$$

$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{-bx^3-a}}$$

input
```
Int[(x*Sqrt[-a - b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

3.349. $\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$

output
```
3*(3 - 2*Sqrt[3])*a*(((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*
(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(
5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a - b*x^3])/(Sq
rt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[
3])*ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[
2]*Sqrt[-a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3]
)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*S
qrt[-a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))) - ((-2*Sqrt[-a - b
*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1/4)*Sqrt[2 + S
qrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 +
4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]))/b^(1/3) + (2*Sqrt[2 - Sqrt[3]
]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

3.349.3.1 Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

3.349. $\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.349.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.77 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.28

method	result	size
elliptic	Expression too large to display	983
default	Expression too large to display	1001

input
```
int(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(
1/2)*sum(1/_alpha*(2*3^(1/2)-3)*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)
^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)
^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x
+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-b*
x^3-a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3
*I*(-a*b^2)^(2/3)*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(-a*b^2)^(
1/3)*_alpha*b+6*b^2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*3^(1/2)*(-a*b^2)^(2/3)-3
*(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(...
```

3.349.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.41 (sec) , antiderivative size = 4875, normalized size of antiderivative = 6.35

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="frica
s")
```

output
```
-1/8*((1/72)^(1/6)*(sqrt(-3)*b - b)*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^
(5/6)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x + sqrt(
-3)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x) - 3*sqrt
(3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x + sqrt(-3)*(b
^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)
^(5/6) + 4*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^
3*b^2*x^2 - sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b
^2*x^2))*sqrt(-sqrt(3)*a/b^4) + 6*(12*a*b^2*x^8 - 48*a^2*b*x^5 - 384*a^3*x
^2 + 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 - sqrt(3)*(b^
5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2 - sqrt(-3)*(b^5*x^9 -
30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*x^9 + 96*a
^2*b^3*x^3 + 16*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) - (1/9)^(1/3)*(b^4*x^10 +
 240*a^2*b^2*x^4 + 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 16
0*a^3*b*x) + 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x + sqrt(-3)*
(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) - 8*sqrt(
3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 - a) + (1/72)^(1/6)
*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4
+ sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 2
56*a^4 - sqrt(-3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b
*x^3 + 256*a^4)) - 3*sqrt(-3)*(b^4*x^12 - 4*a*b^3*x^9 + 360*a^2*b^2*x^6...
```

3.349.6 Sympy [F]

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \int \frac{x\sqrt{-a-bx^3}}{-6\sqrt{3}a+10a+bx^3}\,dx$$

input
```
integrate(x*(-b*x**3-a)**(1/2)/(b*x**3+2*a*(5-3*3**(1/2))),x)
```

output
```
Integral(x*sqrt(-a - b*x**3)/(-6*sqrt(3)*a + 10*a + b*x**3), x)
```

3.349.7 Maxima [F]

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{-bx^3-ax}}{bx^3-2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

3.349.8 Giac [F]

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{-bx^3-ax}}{bx^3-2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
integrate(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac"
)
```

output
```
integrate(sqrt(-b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\,\sqrt{-b\,x^3-a}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

input
```
int((x*(- a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)),x)
```

output
```
int((x*(- a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)), x)
```

3.350 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

3.350.1 Optimal result

Integrand size = 33, antiderivative size = 318

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx &= -\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} \\ &\quad -\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} \\ &\quad -\frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} \\ &\quad -\frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}\end{aligned}$$

output
```
-1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(
b*x^3+a)^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1/
6*(1-3^(1/2))*(b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/4)
/a^(5/6)/b^(2/3)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)
*x)*(1-3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/
3)*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))*2^(1/2)/(b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

3.350.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=\frac{x^2\sqrt{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20a+12\sqrt{3}a\right)\sqrt{a+bx^3}}$$

input
```
Integrate[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3
)/(10*a + 6*Sqrt[3]*a))])/((20*a + 12*Sqrt[3]*a)*Sqrt[a + b*x^3])
```

3.350.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$$

$$\Big\downarrow \text{989}$$

$$-\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

input
```
Int[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

output
```
-1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/
3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2
 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a
])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)
*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]
)/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1
 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*
Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))
```

3.350.3.1 Defintions of rubi rules used

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

3.350.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.83 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.69

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{ib\left(2x+\frac{\left(-\right.}{}\right.}{}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{ib\left(2x+\frac{\left(-\right.}{}\right.}{}}}{}\right.}{}$

input
```
int(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

3.350. $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

output
```
-1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b
^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b
^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(
2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(
b*x^3+a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2
)+3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2
)^(1/3)*_alpha*b-6*b^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3
)+3*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),-1/6/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^
2)^(2/3)*_alpha-4*I*(-a*b^2)^(1/3)*_alpha^2*b+2*3^(1/2)*(-a*b^2)^(2/3)*_al
pha+I*3^(1/2)*a*b+2*I*(-a*b^2)^(2/3)*_alpha+2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)
*_alpha-2*I*a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a
+10*a))
```

3.350.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5563 vs. $2(211) = 422$.

Time = 3.56 (sec) , antiderivative size = 5563, normalized size of antiderivative = 17.49

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

3.350.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{a+bx^3}\cdot\left(10a+6\sqrt{3}a+bx^3\right)}\,dx$$

input
```
integrate(x/(b*x**3+2*a*(5+3*3**(1/2)))/(b*x**3+a)**(1/2),x)
```

output
```
Integral(x/(sqrt(a + b*x**3)*(10*a + 6*sqrt(3)*a + b*x**3)), x)
```

3.350.7 Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=\int \frac{x}{\left(bx^3+2\,a\left(3\,\sqrt{3}+5\right)\right)\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 + a)), x)
```

3.350.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

3.350.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=\int \frac{x}{\sqrt{b\,x^3+a}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

input
```
int(x/((a + b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))),x)
```

output
```
int(x/((a + b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))), x)
```

3.351 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

3.351.1 Optimal result

Integrand size = 35, antiderivative size = 324

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = -\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

output
```
-1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(
-b*x^3+a)^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1
/6*(1-3^(1/2))*(-b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/
4)/a^(5/6)/b^(2/3)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/
3)*x)*(1-3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^
(2/3)*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))*2^(1/2)/(-b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/
2)
```

3.351.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20a+12\sqrt{3}a\right)\sqrt{a-bx^3}}$$

input
```
Integrate[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

output
```
(x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10
*a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[a - b*x^3])
```

3.351.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}dx$$

$$\Big\downarrow \text{989}$$

$$-\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\,\mathrm{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{(2-\sqrt{3})\,\mathrm{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

input
```
Int[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

output
```
-1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/
3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])]])/(Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2
 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a
]])])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)
*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])]])/
(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(1
/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])]])/(3*
Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))
```

3.351.3.1 Defintions of rubi rules used

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

3.351. $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

3.351.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.69 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.57

method	result
default	$i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a-10a\right)}\frac{(a\,b^{2})^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\,(a\,b^{2})^{\frac{1}{3}}+(a\,b^{2})^{\frac{1}{3}}}{b}\right)}{2(a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{(a\,b^{2})^{\frac{1}{3}}}{b}\right)}{-3(a\,b^{2})^{\frac{1}{3}}-i\sqrt{3}\,(a\,b^{2})^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\,(a\,b^{2})^{\frac{1}{3}}}{b}\right.}{(a\,b^{2})^{\frac{1}{3}}}}}{}\right.$
elliptic	$i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a-10a\right)}\frac{(a\,b^{2})^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\,(a\,b^{2})^{\frac{1}{3}}+(a\,b^{2})^{\frac{1}{3}}}{b}\right)}{2(a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{(a\,b^{2})^{\frac{1}{3}}}{b}\right)}{-3(a\,b^{2})^{\frac{1}{3}}-i\sqrt{3}\,(a\,b^{2})^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\,(a\,b^{2})^{\frac{1}{3}}}{b}\right.}{(a\,b^{2})^{\frac{1}{3}}}}}{}\right.$

input
```
int(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1
/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(
1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*
(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^
(1/2)*(3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^
2)^(2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha
*b-6*b^2*_alpha^2-2*3^(1/2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3
)*_alpha*b+3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-
2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha+4*I*(a
*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b+2*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*I*(a
*b^2)^(2/3)*_alpha-2*3^(1/2)*a*b-2*I*a*b-3*(a*b^2)^(2/3)*_alpha+3*a*b)/a,(
-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(
1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a-10*a))
```

3.351.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5587 vs. $2(218) = 436$.

Time = 3.50 (sec) , antiderivative size = 5587, normalized size of antiderivative = 17.24

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="fric
as")
```

output
```
Too large to include
```

3.351.6 Sympy [F]

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$$
$$= -\int \frac{x}{-6\sqrt{3}a\sqrt{a-bx^3}-10a\sqrt{a-bx^3}+bx^3\sqrt{a-bx^3}}\,dx$$

input
```
integrate(x/(-b*x**3+2*a*(5+3*3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

output
```
-Integral(x/(-6*sqrt(3)*a*sqrt(a - b*x**3) - 10*a*sqrt(a - b*x**3) + b*x**
3*sqrt(a - b*x**3)), x)
```

3.351.7 Maxima [F]

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx=\int -\frac{x}{\left(bx^3-2\,a\left(3\,\sqrt{3}+5\right)\right)\sqrt{-bx^3+a}}\,dx$$

input
```
integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
-integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 + a)), x)
```

3.351.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

3.351.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx=-\int \frac{x}{\sqrt{a-b\,x^3}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

input
```
int(-x/((a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))),x)
```

output
```
-int(x/((a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)
```

3.352 $\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

3.352.1 Optimal result

Integrand size = 35, antiderivative size = 328

$$\begin{aligned}\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx &= \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} \\ &\quad+\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} \\ &\quad+\frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}\end{aligned}$$

output

```
1/36*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b
*x^3-a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctan(1/2
*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(b*x^3-a)^(1/2)
)*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*a^(
1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3-a)^(1/2))*(2-3^(1/2))*
3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/6*(1-3^(1/2))*(b*x^3-a)^(1/
2)*3^(1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

3.352.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = -\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20a+12\sqrt{3}a\right)\sqrt{-a+bx^3}}$$

input
```
Integrate[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

output
```
-((x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(
10*a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a + b*x^3]))
```

3.352.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {990}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{bx^3-a}\left(bx^3-2\left(5+3\sqrt{3}\right)a\right)}\,dx$$

$$\Big\downarrow \, 990$$

$$\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

input
```
Int[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

output
```
((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x)
)/(Sqrt[2]*Sqrt[-a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 -
 Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(
Sqrt[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sq
rt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt
[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3
])*ArcTanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3
*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))
```

3.352.3.1 Defintions of rubi rules used

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

3.352.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.86 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.55

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a\,b^{2}\right.}{}\right.}{\left(a\,b^{2}\right)}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a\,b^{2}\right.}{}\right.}{\left(a\,b^{2}\right)}}}{}\right.}{}$

input
```
int(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

3.352. $\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

output
```
-1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(
1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^
(1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b
*(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^
(1/2)*(3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^
2)^(2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha
*b-6*b^2*_alpha^2-2*3^(1/2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3
)*_alpha*b+3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-
2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha+4*I*(a
*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b+2*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*I*(a
*b^2)^(2/3)*_alpha-2*3^(1/2)*a*b-2*I*a*b-3*(a*b^2)^(2/3)*_alpha+3*a*b)/a,(
-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(
1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a-10*a))
```

3.352.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5667 vs. $2(222) = 444$.

Time = 3.69 (sec) , antiderivative size = 5667, normalized size of antiderivative = 17.28

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

3.352.6 Sympy [F]

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{-a+bx^3}\left(-6\sqrt{3}a-10a+bx^3\right)}\,dx$$

input
```
integrate(x/(b*x**3-2*a*(5+3*3**(1/2)))/(b*x**3-a)**(1/2),x)
```

output
```
Integral(x/(sqrt(-a + b*x**3)*(-6*sqrt(3)*a - 10*a + b*x**3)), x)
```

3.352.7 Maxima [F]

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=\int \frac{x}{\left(bx^3-2\,a\left(3\,\sqrt{3}+5\right)\right)\sqrt{bx^3-a}}\,dx$$

input
```
integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 - a)), x)
```

3.352.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

3.352.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=\int \frac{x}{\sqrt{b\,x^3-a}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

input
```
int(x/((b*x^3 - a)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))),x)
```

output
```
int(x/((b*x^3 - a)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)
```

3.353 $\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

3.353.1 Optimal result

Integrand size = 37, antiderivative size = 330

$$\begin{aligned}
&\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx \\
&= \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} \\
&\quad+\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} \\
&\quad+\frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}
\end{aligned}$$

output
```
1/36*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-
b*x^3-a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctan(1/
2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(-b*x^3-a)^(1
/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*
a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3-a)^(1/2))*(2-3^(1/
2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/6*(1-3^(1/2))*(-b*x^3-a
)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/
2)
```

3.353.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = -\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20a+12\sqrt{3}a\right)\sqrt{-a-bx^3}}$$

input
```
Integrate[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

output
```
-((x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x
^3)/(10*a + 6*Sqrt[3]*a))])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a - b*x^3]))
```

3.353.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {990}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$$

$$\Big\downarrow \text{990}$$

$$\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

input
```
Int[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

---

output

```
((2 - Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*
x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2
 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(
Sqrt[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sq
rt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt
[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3
])*ArcTanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3
*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))
```

3.353.3.1 Defintions of rubi rules used

rule 990

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

3.353.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.75 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.64

method	result
default	$i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{ib\left(2x+\left(-a\,b\right.\right.}{2}}}{}\right.$
elliptic	$i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{ib\left(2x+\left(-a\,b\right.\right.}{2}}}{}\right.$

input
```
int(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

3.353. $\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

output
```
1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^
2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^
2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2
*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-
b*x^3-a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2
)+3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2
)^(1/3)*_alpha*b-6*b^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3
)+3*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),-1/6/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^
2)^(2/3)*_alpha-4*I*(-a*b^2)^(1/3)*_alpha^2*b+2*3^(1/2)*(-a*b^2)^(2/3)*_al
pha+I*3^(1/2)*a*b+2*I*(-a*b^2)^(2/3)*_alpha+2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)
*_alpha-2*I*a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a
+10*a))
```

3.353.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5679 vs. $2(223) = 446$.

Time = 3.55 (sec) , antiderivative size = 5679, normalized size of antiderivative = 17.21

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fric
as")
```

output
```
Too large to include
```

3.353.6 Sympy [F]

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$$
$$= -\int \frac{x}{10a\sqrt{-a-bx^3}+6\sqrt{3}a\sqrt{-a-bx^3}+bx^3\sqrt{-a-bx^3}}\,dx$$

input
```
integrate(x/(-b*x**3-2*a*(5+3*3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

output
```
-Integral(x/(10*a*sqrt(-a - b*x**3) + 6*sqrt(3)*a*sqrt(-a - b*x**3) + b*x*
*3*sqrt(-a - b*x**3)), x)
```

3.353.7 Maxima [F]

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx=\int -\frac{x}{\left(bx^3+2\,a\left(3\,\sqrt{3}+5\right)\right)\sqrt{-bx^3-a}}\,dx$$

input
```
integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxi
ma")
```

output
```
-integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 - a)), x)
```

3.353.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

3.353.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx=\int -\frac{x}{\sqrt{-b\,x^3-a}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

input
```
int(-x/((- a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))),x)
```

output
```
int(-x/((- a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))), x)
```

3.354 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

3.354.1 Optimal result

Integrand size = 33, antiderivative size = 310

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

output
```
-1/18*arctan(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1-3^(1/2)))*2^(1/2
)/(b*x^3+a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/36*arctan
(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3+a)^(1/
2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*a
^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))*(2+3^(1/2)
)*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctanh(1/6*(1+3^(1/2))*(b*x^3+a)^(
1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

3.354.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.27

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx=\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{a+bx^3}}$$

input
```
Integrate[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3
)/(10*a - 6*Sqrt[3]*a))])/((20*a - 12*Sqrt[3]*a)*Sqrt[a + b*x^3])
```

3.354.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}dx$$

$$\Big\downarrow \text{989}$$

$$-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

input
```
Int[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

output
```
-1/3*((2 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(
1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - (
(2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))
/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + S
qrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3
])*ArcTanh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*
Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))
```

3.354.3.1 Defintions of rubi rules used

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

3.354.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.72 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.74

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{ib\left(2x+\frac{\left(-a\,b\right.}{}\right.}{2}}}{}\right.}{}$
elliptic	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{ib\left(2x+\frac{\left(-a\,b\right.}{}\right.}{2}}}{}\right.}{}$

input
```
int(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

3.354. $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

output

```
1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^
2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^
2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2
*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-
3*I*(-a*b^2)^(2/3)*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(-a*b^2)^
(1/3)*_alpha*b+6*b^2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*3^(1/2)*(-a*b^2)^(2/3)-
3*(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),-1/6/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)
^(2/3)*_alpha+4*I*(-a*b^2)^(1/3)*_alpha^2*b-2*I*(-a*b^2)^(2/3)*_alpha-2*3^
(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha+2*I*a*b-
2*3^(1/2)*a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a+1
0*a))
```

3.354.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5631 vs. $2(210) = 420$.

Time = 3.43 (sec) , antiderivative size = 5631, normalized size of antiderivative = 18.16

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Too large to display}$$

input

```
integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas
")
```

output

```
Too large to include
```

3.354.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{a+bx^3}\left(-6\sqrt{3}a+10a+bx^3\right)}\,dx$$

input

```
integrate(x/(b*x**3+2*a*(5-3*3**(1/2)))/(b*x**3+a)**(1/2),x)
```

output

```
Integral(x/(sqrt(a + b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)
```

3.354.7 Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx=\int \frac{x}{\left(bx^3-2\,a\left(3\,\sqrt{3}-5\right)\right)\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(x/((b*x^3 - 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 + a)), x)
```

3.354.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

3.354.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx=\int \frac{x}{\sqrt{b\,x^3+a}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

input
```
int(x/((a + b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))),x)
```

output
```
int(x/((a + b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))), x)
```

3.355 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx$

3.355.1 Optimal result

Integrand size = 35, antiderivative size = 316

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

output
```
-1/18*arctan(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1-3^(1/2)))*2^(1/2)
/(-b*x^3+a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/36*arctan
(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3+a)^(1
/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*
a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2))*(2+3^(1/
2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctanh(1/6*(1+3^(1/2))*(-b*x^3+a
)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/
2)
```

3.355.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx=\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{a-bx^3}}$$

input
```
Integrate[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]
```

output
```
(x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10
*a - 6*Sqrt[3]*a)])/((20*a - 12*Sqrt[3]*a)*Sqrt[a - b*x^3])
```

3.355.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}dx$$

$$\Big\downarrow \text{989}$$

$$-\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{(2+\sqrt{3})\,\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}+\frac{(2+\sqrt{3})\,\text{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

input

```
Int[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]
```

output

```
-1/6*((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/
3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])]])/(Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2
 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))
/(Sqrt[2]*Sqrt[a - b*x^3])]])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + S
qrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqr
t[2]*Sqrt[a - b*x^3])]])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3
])*ArcTanh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])]])/(3*
Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3))
```

3.355.3.1 Defintions of rubi rules used

rule 989

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

3.355.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.99 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.61

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a\,b^{2}\right.}{}\right.}{\left(a\,b^{2}\right)}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a\,b^{2}\right.}{}\right.}{\left(a\,b^{2}\right)}}}{}\right.}{}$

input
```
int(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

3.355. $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx$

output
```
-1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(
1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^
(1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b
*(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)
^(1/2)*(-3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*
b^2)^(2/3)*3^(1/2)-6*I*(a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alp
ha*b+6*b^2*_alpha^2+6*I*(a*b^2)^(2/3)-2*3^(1/2)*(a*b^2)^(2/3)-3*(a*b^2)^(1
/3)*_alpha*b-3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*
(-2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-4*I*
(a*b^2)^(1/3)*_alpha^2*b+2*I*(a*b^2)^(2/3)*_alpha-2*3^(1/2)*(a*b^2)^(2/3)*
_alpha+I*3^(1/2)*a*b-3*(a*b^2)^(2/3)*_alpha+2*I*a*b+2*3^(1/2)*a*b+3*a*b)/a
,(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)
^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a-10*a))
```

3.355.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5667 vs. $2(219) = 438$.

Time = 3.44 (sec) , antiderivative size = 5667, normalized size of antiderivative = 17.93

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="fric
as")
```

output
```
Too large to include
```

3.355.6 Sympy [F]

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = -\int \frac{x}{-10a\sqrt{a-bx^3}+6\sqrt{3}a\sqrt{a-bx^3}+bx^3\sqrt{a-bx^3}}\,dx$$

input
```
integrate(x/(-b*x**3+2*a*(5-3*3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

output
```
-Integral(x/(-10*a*sqrt(a - b*x**3) + 6*sqrt(3)*a*sqrt(a - b*x**3) + b*x**
3*sqrt(a - b*x**3)), x)
```

3.355.7 Maxima [F]

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = \int -\frac{x}{\left(bx^3+2\,a\left(3\,\sqrt{3}-5\right)\right)\sqrt{-bx^3+a}}\,dx$$

input
```
integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
-integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(-b*x^3 + a)), x)
```

3.355.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

3.355.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = \int -\frac{x}{\sqrt{a-b\,x^3}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

input
```
int(-x/((a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))),x)
```

output
```
int(-x/((a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)
```

3.356 $\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx$

3.356.1 Optimal result

Integrand size = 36, antiderivative size = 320

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx = \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

output
```
1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b
*x^3-a)^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1/6
*(1+3^(1/2))*(b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4)/
a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1
/3)*(1-3^(1/2)))*2^(1/2)/(b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2
/3)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2
))*2^(1/2)/(b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

3.356.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.26

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx=\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{-a+bx^3}}$$

input
```
Integrate[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]
```

output
```
(x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10
*a - 6*Sqrt[3]*a)])/((20*a - 12*Sqrt[3]*a)*Sqrt[-a + b*x^3])
```

3.356.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {990}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{bx^3-a}}\,dx$$

$$\Big\downarrow \text{990}$$

$$\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

input
```
Int[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]
```

output
```
((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x)
)/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 +
 Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a]
)])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*
(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/
(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1
/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(3
*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))
```

3.356.3.1 Defintions of rubi rules used

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

3.356.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.85 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.59

method	result
default	$-\frac{i\sqrt{2}\Big(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}+\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}}}\,\sqrt{2}\,\sqrt{\frac{ib\Big(2x+\frac{-i\sqrt{3}\,\big(a\,b^2}{}}{\big(a\,b^2\big)}}}{}}{}$
elliptic	$-\frac{i\sqrt{2}\Big(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}+\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}}}\,\sqrt{2}\,\sqrt{\frac{ib\Big(2x+\frac{-i\sqrt{3}\,\big(a\,b^2}{}}{\big(a\,b^2\big)}}}{}}{}$

input
```
int(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

3.356. $\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx$

output
```
-1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(
1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^
(1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b
*(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^
(1/2)*(-3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b
^2)^(2/3)*3^(1/2)-6*I*(a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alph
a*b+6*b^2*_alpha^2+6*I*(a*b^2)^(2/3)-2*3^(1/2)*(a*b^2)^(2/3)-3*(a*b^2)^(1/
3)*_alpha*b-3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(
-2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-4*I*(
a*b^2)^(1/3)*_alpha^2*b+2*I*(a*b^2)^(2/3)*_alpha-2*3^(1/2)*(a*b^2)^(2/3)*_
alpha+I*3^(1/2)*a*b-3*(a*b^2)^(2/3)*_alpha+2*I*a*b+2*3^(1/2)*a*b+3*a*b)/a,
(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^
(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a-10*a))
```

3.356.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5599 vs. $2(223) = 446$.

Time = 3.57 (sec) , antiderivative size = 5599, normalized size of antiderivative = 17.50

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="frica
s")
```

output
```
Too large to include
```

3.356.6 Sympy [F]

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx = -\int \frac{x}{-10a\sqrt{-a+bx^3}+6\sqrt{3}a\sqrt{-a+bx^3}+bx^3\sqrt{-a+bx^3}}\,dx$$

input
```
integrate(x/(-b*x**3+2*a*(5-3*3**(1/2)))/(b*x**3-a)**(1/2),x)
```

output
```
-Integral(x/(-10*a*sqrt(-a + b*x**3) + 6*sqrt(3)*a*sqrt(-a + b*x**3) + b*x
**3*sqrt(-a + b*x**3)), x)
```

3.356.7 Maxima [F]

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{x}{\left(bx^3+2\,a\left(3\,\sqrt{3}-5\right)\right)\sqrt{bx^3-a}}\,dx$$

input
```
integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="maxim
a")
```

output
```
-integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 - a)), x)
```

3.356.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

3.356.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{x}{\sqrt{b\,x^3-a}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

input
```
int(-x/((b*x^3 - a)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))),x)
```

output
```
int(-x/((b*x^3 - a)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)
```

3.357 $\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

3.357.1 Optimal result

Integrand size = 36, antiderivative size = 322

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

output
```
1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-
b*x^3-a)^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1/
6*(1+3^(1/2))*(-b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4
)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))*2^(1/2)/(-b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/
b^(2/3)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^
(1/2))*2^(1/2)/(-b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/
2)
```

3.357.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.27

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{-a-bx^3}}$$

input
```
Integrate[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3
)/(10*a - 6*Sqrt[3]*a))])/((20*a - 12*Sqrt[3]*a)*Sqrt[-a - b*x^3])
```

3.357.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {990}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$$

$$\Big\downarrow \text{990}$$

$$\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

input

```
Int[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

output

```
((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x)
)/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 +
 Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a]
)])/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*
a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])]
)/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1
 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(6
*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3))
```

3.357.3.1 Defintions of rubi rules used

rule 990

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

3.357.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.97 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.68

method	result
default	$i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{ib\left(2x+\frac{(-a}{}\right.}{2}}}{}\right.$
elliptic	$i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{ib\left(2x+\frac{(-a}{}\right.}{2}}}{}\right.$

input

```
int(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

3.357. $\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

output

```
1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^
2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^
2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2
*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-
b*x^3-a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)
-3*I*(-a*b^2)^(2/3)*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(-a*b^2)
^(1/3)*_alpha*b+6*b^2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*3^(1/2)*(-a*b^2)^(2/3)
-3*(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),-1/6/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2
)^(2/3)*_alpha+4*I*(-a*b^2)^(1/3)*_alpha^2*b-2*I*(-a*b^2)^(2/3)*_alpha-2*3
^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha+2*I*a*b
-2*3^(1/2)*a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a+
10*a))
```

3.357.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5599 vs. $2(222) = 444$.

Time = 3.60 (sec) , antiderivative size = 5599, normalized size of antiderivative = 17.39

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Too large to display}$$

input

```
integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="frica
s")
```

output

```
Too large to include
```

3.357.6 Sympy [F]

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{-a-bx^3}\left(-6\sqrt{3}a+10a+bx^3\right)}\,dx$$

input

```
integrate(x/(b*x**3+2*a*(5-3*3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

output

```
Integral(x/(sqrt(-a - b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)
```

3.357.7 Maxima [F]

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\left(bx^3-2\,a\left(3\,\sqrt{3}-5\right)\right)\sqrt{-bx^3-a}}\,dx$$

input
```
integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(x/((b*x^3 - 2*a*(3*sqrt(3) - 5))*sqrt(-b*x^3 - a)), x)
```

3.357.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

3.357.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{-b\,x^3-a}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

input
```
int(x/((- a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))),x)
```

output
```
int(x/((- a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))), x)
```

3.358 $\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx$

3.358.1 Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}-\frac{2a^2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

output
```
-2/9*(a*d+b*c)*(d*x^3+c)^(3/2)/b^2/d^2+2/15*(d*x^3+c)^(5/2)/b/d^2-2/3*a^2*
arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(7/2)
+2/3*a^2*(d*x^3+c)^(1/2)/b^3
```

3.358.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{2\sqrt{c+dx^3}\left(15a^2d^2-5abd(c+dx^3)+b^2(-2c^2+cdx^3+3d^2x^6)\right)}{45b^3d^2}-\frac{2a^2\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}}$$

input
```
Integrate[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
(2*Sqrt[c + d*x^3]*(15*a^2*d^2 - 5*a*b*d*(c + d*x^3) + b^2*(-2*c^2 + c*d*x
^3 + 3*d^2*x^6)))/(45*b^3*d^2) - (2*a^2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]
*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(7/2))
```

3.358.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6\sqrt{dx^3+c}}{bx^3+a}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{\sqrt{dx^3+c}a^2}{b^2\left(bx^3+a\right)}+\frac{\left(dx^3+c\right)^{3/2}}{bd}+\frac{(-bc-ad)\sqrt{dx^3+c}}{b^2d}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{2a^2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{2a^2\sqrt{c+dx^3}}{b^3}-\frac{2\left(c+dx^3\right)^{3/2}(ad+bc)}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{5bd^2}\right)$$

input
```
Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
((2*a^2*Sqrt[c + d*x^3])/b^3 - (2*(b*c + a*d)*(c + d*x^3)^(3/2))/(3*b^2*d^
2) + (2*(c + d*x^3)^(5/2))/(5*b*d^2) - (2*a^2*Sqrt[b*c - a*d]*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(7/2))/3
```

3.358.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.358.4 Maple [A] (verified)

Time = 5.61 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.99

method	result
risch	$\frac{2\left(3b^{2}d^{2}x^{6}-5x^{3}ab\,d^{2}+x^{3}b^{2}cd+15a^{2}d^{2}-5abcd-2b^{2}c^{2}\right)\sqrt{d\,x^{3}+c}}{45d^{2}b^{3}}-\frac{2a^{2}(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3b^{3}\sqrt{(ad-bc)b}}$
pseudoelliptic	$-\frac{2\left(-\sqrt{(ad-bc)b}\left(-\frac{2\left(d\,x^{3}+c\right)\left(-\frac{3d\,x^{3}}{2}+c\right)b^{2}}{15}-\frac{\left(d\,x^{3}+c\right)abd}{3}+a^{2}d^{2}\right)\sqrt{d\,x^{3}+c}+a^{2}d^{2}(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)\right)}{3\sqrt{(ad-bc)b}\,d^{2}b^{3}}$
default	$\frac{\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15}+\frac{2c\,x^{3}\sqrt{d\,x^{3}+c}}{45d}-\frac{4c^{2}\sqrt{d\,x^{3}+c}}{45d^{2}}}{b}-\frac{2a\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9b^{2}d}+\frac{2a^{2}\left(\sqrt{d\,x^{3}+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3b^{3}}$
elliptic	$\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15b}+\frac{2\left(-\frac{ad-bc}{b^{2}}-\frac{4c}{5b}\right)x^{3}\sqrt{d\,x^{3}+c}}{9d}+\frac{2\left(\frac{(ad-bc)a}{b^{3}}-\frac{2\left(-\frac{ad-bc}{b^{2}}-\frac{4c}{5b}\right)c}{3d}\right)\sqrt{d\,x^{3}+c}}{3d}+$ $i a^{2}\sqrt{2}\,\Bigg(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z\right.}$

input

```
int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output

```
2/45*(3*b^2*d^2*x^6-5*a*b*d^2*x^3+b^2*c*d*x^3+15*a^2*d^2-5*a*b*c*d-2*b^2*c
^2)*(d*x^3+c)^(1/2)/d^2/b^3-2/3*a^2*(a*d-b*c)/b^3/((a*d-b*c)*b)^(1/2)*arct
an(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

3.358. $\int \frac{x^{8}\sqrt{c+dx^{3}}}{a+bx^{3}}\,dx$

3.358.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.24

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx$$

$$= \left[\frac{15\,a^2d^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,(3\,b^2d^2x^6-2\,b^2c^2-5\,abcd+15\,a^2d^2+(b^2cd-5\,a}{45\,b^3d^2} \right.$$

$$-\frac{2\left(15\,a^2d^2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(3\,b^2d^2x^6-2\,b^2c^2-5\,abcd+15\,a^2d^2+(b^2cd-5\,ab\right.}{45\,b^3d^2}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/45*(15*a^2*d^2*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(
d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(3*b^2*d^2*x^6 - 2*b^2*
c^2 - 5*a*b*c*d + 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c))
/(b^3*d^2), -2/45*(15*a^2*d^2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (3*b^2*d^2*x^6 - 2*b^2*c^2 - 5*a*b*
c*d + 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^3*d^2)]
```

3.358.6 Sympy [A] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.38

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx$$

$$= \begin{cases} \frac{2\left(\frac{a^2d^3\sqrt{c+dx^3}}{3b^3}-\frac{a^2d^3(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^3\right)^{\frac{5}{2}}}{15b}+\frac{\left(c+dx^3\right)^{\frac{3}{2}}\left(-ad^2-bcd\right)}{9b^2}\right)}{d^3} & \text{for } d\neq 0 \\ \sqrt{c}\left(\frac{a^2\left(\begin{cases}\frac{x^3}{a} & \text{for } b=0\\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right)}{3b^2}-\frac{ax^3}{3b^2}+\frac{x^6}{6b}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

output
```
Piecewise((2*(a**2*d**3*sqrt(c + d*x**3)/(3*b**3) - a**2*d**3*(a*d - b*c)*
atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**4*sqrt((a*d - b*c)/b)) +
d*(c + d*x**3)**(5/2)/(15*b) + (c + d*x**3)**(3/2)*(-a*d**2 - b*c*d)/(9*b*
*2))/d**3, Ne(d, 0)), (sqrt(c)*(a**2*Piecewise((x**3/a, Eq(b, 0)), (log(a
+ b*x**3)/b, True))/(3*b**2) - a*x**3/(3*b**2) + x**6/(6*b)), True))
```

3.358.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.358.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.11

$$\begin{aligned}&\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx\\&=\frac{2\,(a^2bc-a^3d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^3}\\&\quad+\frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}b^4d^8-5\,(dx^3+c)^{\frac{3}{2}}b^4cd^8-5\,(dx^3+c)^{\frac{3}{2}}ab^3d^9+15\,\sqrt{dx^3+c}a^2b^2d^{10}\right)}{45\,b^5d^{10}}\end{aligned}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*(a^2*b*c - a^3*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt
(-b^2*c + a*b*d)*b^3) + 2/45*(3*(d*x^3 + c)^(5/2)*b^4*d^8 - 5*(d*x^3 + c)^
(3/2)*b^4*c*d^8 - 5*(d*x^3 + c)^(3/2)*a*b^3*d^9 + 15*sqrt(d*x^3 + c)*a^2*b
^2*d^10)/(b^5*d^10)
```

3.358.9 Mupad [B] (verification not implemented)

Time = 10.53 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.41

$$\begin{aligned}\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx &= \frac{2\,a^2\,\sqrt{d\,x^3+c}}{3\,b^3}+\frac{2\,{\left(d\,x^3+c\right)}^{5/2}}{15\,b\,d^2}-\frac{2\,a\,{\left(d\,x^3+c\right)}^{3/2}}{9\,b^2\,d}-\frac{2\,c\,{\left(d\,x^3+c\right)}^{3/2}}{9\,b\,d^2}\\ &\quad+\frac{a^2\,\ln\left(\frac{a^2\,d^2\,1\mathrm{i}+b^2\,c^2\,2\mathrm{i}-2\,\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}-a\,b\,d^2\,x^3\,1\mathrm{i}+b^2\,c\,d\,x^3\,1\mathrm{i}-a\,b\,c\,d\,3\mathrm{i}}{2\,b\,x^3+2\,a}\right)\,\sqrt{a\,d-b\,c}\,1\mathrm{i}}{3\,b^{7/2}}\end{aligned}$$

input
```
int((x^8*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

output
```
(2*a^2*(c + d*x^3)^(1/2))/(3*b^3) + (2*(c + d*x^3)^(5/2))/(15*b*d^2) - (2*
a*(c + d*x^3)^(3/2))/(9*b^2*d) - (2*c*(c + d*x^3)^(3/2))/(9*b*d^2) + (a^2*
log((a^2*d^2*1i + b^2*c^2*2i - 2*b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/
2) - a*b*d^2*x^3*1i + b^2*c*d*x^3*1i - a*b*c*d*3i)/(2*a + 2*b*x^3))*(a*d -
 b*c)^(1/2)*1i)/(3*b^(7/2))
```

3.359 $\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx$

3.359.1 Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx=-\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2(c+dx^3)^{3/2}}{9bd}+\frac{2a\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

output
```
2/9*(d*x^3+c)^(3/2)/b/d+2/3*a*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(
1/2))*(-a*d+b*c)^(1/2)/b^(5/2)-2/3*a*(d*x^3+c)^(1/2)/b^2
```

3.359.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{2\sqrt{c+dx^3}(-3ad+b(c+dx^3))}{9b^2d}+\frac{2a\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}}$$

input
```
Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
(2*Sqrt[c + d*x^3]*(-3*a*d + b*(c + d*x^3)))/(9*b^2*d) + (2*a*Sqrt[-(b*c)
+ a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2))
```

3.359.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {948, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}dx$$

$$\Big\downarrow \textcolor{blue}{948}$$

$$\frac{1}{3}\int \frac{x^3\sqrt{dx^3+c}}{bx^3+a}dx^3$$

$$\Big\downarrow \textcolor{blue}{90}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3bd}-\frac{a\int \frac{\sqrt{dx^3+c}}{bx^3+a}dx^3}{b}\right)$$

$$\Big\downarrow \textcolor{blue}{60}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3bd}-\frac{a\left(\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{b}+\frac{2\sqrt{c+dx^3}}{b}\right)}{b}\right)$$

$$\Big\downarrow \textcolor{blue}{73}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3bd}-\frac{a\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)}{b}\right)$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{3/2}}{3bd}-\frac{a\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}\right)$$

input

```
Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
((2*(c + d*x^3)^(3/2))/(3*b*d) - (a*((2*Sqrt[c + d*x^3])/b - (2*Sqrt[b*c -
 a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2)))/b)/3
```

3.359.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.359. $\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx$

3.359.4 Maple [A] (verified)

Time = 4.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

method	result
default	$\frac{2(dx^3+c)^{\frac{3}{2}}}{9bd}-\frac{2a\left(\sqrt{dx^3+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3b^2}$
pseudoelliptic	$\frac{-\frac{2\sqrt{dx^3+c}\left(-bdx^3+3ad-bc\right)}{9}+\frac{2ad(ad-bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}}{b^2d}$
risch	$-\frac{2\left(-bdx^3+3ad-bc\right)\sqrt{dx^3+c}}{9d\,b^2}+\frac{2(ad-bc)a\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b^2\sqrt{(ad-bc)b}}$
elliptic	$\frac{2x^3\sqrt{dx^3+c}}{9b}+\frac{2\left(-\frac{ad-bc}{b^2}-\frac{2c}{3b}\right)\sqrt{dx^3+c}}{3d}-\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)}{d}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$

input
```
int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/9*(d*x^3+c)^(3/2)/b/d-2/3*a/b^2*((d*x^3+c)^(1/2)-(a*d-b*c)*arctan(b*(d*x
^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2))
```

3.359.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.10

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx$$

$$= \left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,(bdx^3+bc-3\,ad)\sqrt{dx^3+c}}{9\,b^2d}, \frac{2\left(3\,ad\sqrt{-\frac{bc-ad}{b}}\arctan\right.}{}\right.$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/9*(3*a*d*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3
+ c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(b*d*x^3 + b*c - 3*a*d)*sqrt(
d*x^3 + c))/(b^2*d), 2/9*(3*a*d*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 +
c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b*d*x^3 + b*c - 3*a*d)*sqrt(d*x^
3 + c))/(b^2*d)]
```

3.359.6 Sympy [A] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.38

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = \begin{cases} \frac{2\left(-\frac{ad^2\sqrt{c+dx^3}}{3b^2}+\frac{ad^2(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^3\right)^{\frac{3}{2}}}{9b}\right)}{d^2} & \text{for } d\neq 0 \\ \sqrt{c}\left(-\frac{a\left(\begin{cases}\frac{x^3}{a} & \text{for } b=0\\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right)}{3b}+\frac{x^3}{3b}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

output
```
Piecewise((2*(-a*d**2*sqrt(c + d*x**3)/(3*b**2) + a*d**2*(a*d - b*c)*atan(
sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**3*sqrt((a*d - b*c)/b)) + d*(c
+ d*x**3)**(3/2)/(9*b))/d**2, Ne(d, 0)), (sqrt(c)*(-a*Piecewise((x**3/a, E
q(b, 0)), (log(a + b*x**3)/b, True))/(3*b) + x**3/(3*b)), True))
```

3.359.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.359.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = -\frac{2\,(abc-a^2d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^2} + \frac{2\,\left((dx^3+c)^{\frac{3}{2}}b^2d^2-3\,\sqrt{dx^3+c}abd^3\right)}{9\,b^3d^3}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
-2/3*(a*b*c - a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(
-b^2*c + a*b*d)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2*d^2 - 3*sqrt(d*x^3 + c)*
a*b*d^3)/(b^3*d^3)
```

3.359.9 Mupad [B] (verification not implemented)

Time = 10.42 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.46

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\,{\left(d\,x^3+c\right)}^{3/2}}{9\,b\,d} - \frac{2\,a\,\sqrt{d\,x^3+c}}{3\,b^2} + \frac{a\,\ln\left(\frac{a^2\,d^2\,1\mathrm{i}+b^2\,c^2\,2\mathrm{i}+2\,\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}-a\,b\,d^2\,x^3\,1\mathrm{i}+b^2\,c\,d\,x^3\,1\mathrm{i}-a\,b\,c\,d\,3\mathrm{i}}{2\,b\,x^3+2\,a}\right)\,\sqrt{a\,d-b\,c}\,1\mathrm{i}}{3\,b^{5/2}}$$

input
```
int((x^5*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

output
```
(2*(c + d*x^3)^(3/2))/(9*b*d) - (2*a*(c + d*x^3)^(1/2))/(3*b^2) + (a*log((
a^2*d^2*1i + b^2*c^2*2i + 2*b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2) -
a*b*d^2*x^3*1i + b^2*c*d*x^3*1i - a*b*c*d*3i)/(2*a + 2*b*x^3))*(a*d - b*c)
^(1/2)*1i)/(3*b^(5/2))
```

3.359. $\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx$

3.360 $\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx$

3.360.1 Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\sqrt{c+dx^3}}{3b} - \frac{2\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

output
```
-2/3*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^
(3/2)+2/3*(d*x^3+c)^(1/2)/b
```

3.360.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{b} - \frac{2\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}\right)$$

input
```
Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
((2*Sqrt[c + d*x^3])/b - (2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[-(b*c) + a*d]])/b^(3/2))/3
```

3.360.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {946, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{bx^3+a}dx^3$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{(bc-ad)\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{b}+\frac{2\sqrt{c+dx^3}}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)$$

input
```
Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
((2*Sqrt[c + d*x^3])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^
3])/Sqrt[b*c - a*d]])/b^(3/2))/3
```

3.360.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.360.4 Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

method	result
default	$\frac{\frac{2\sqrt{dx^3+c}}{3}-\frac{2(ad-bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}}{b}$
pseudoelliptic	$\frac{\frac{2\sqrt{dx^3+c}}{3}-\frac{2(ad-bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}}{b}$
risch	$\frac{2\sqrt{dx^3+c}}{3b}-\frac{2(ad-bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b\sqrt{(ad-bc)b}}$
elliptic	$\frac{2\sqrt{dx^3+c}}{3b}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-cd^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}}{}\right.}{}$

input
```
int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/3/b*((d*x^3+c)^(1/2)-(a*d-b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1
/2))/((a*d-b*c)*b)^(1/2))
```

3.360.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.23

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}}{3\,b},\right.$$
$$\left.-\frac{2\left(\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx^3+c}\right)}{3\,b}\right]$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/3*(sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b
*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c))/b, -2/3*(sqrt(-(b*
c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) -
sqrt(d*x^3 + c))/b]
```

3.360.6 Sympy [A] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \begin{cases} \frac{2\left(\frac{d\sqrt{c+dx^3}}{3b} - \frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for } d \neq 0 \\ \sqrt{c}\left(\begin{cases} \frac{x^3}{3a} & \text{for } b = 0 \\ \frac{\log\left(3a+3bx^3\right)}{3b} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

output
```
Piecewise((2*(d*sqrt(c + d*x**3)/(3*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**
3)/sqrt((a*d - b*c)/b))/(3*b**2*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (sqrt(
c)*Piecewise((x**3/(3*a), Eq(b, 0)), (log(3*a + 3*b*x**3)/(3*b), True)), T
rue))
```

3.360.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.360.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\,(bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b}+\frac{2\sqrt{dx^3+c}}{3\,b}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*(b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*b) + 2/3*sqrt(d*x^3 + c)/b
```

3.360.9 Mupad [B] (verification not implemented)

Time = 10.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,b}+\frac{\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\sqrt{a\,d-b\,c}\,1\mathrm{i}}{3\,b^{3/2}}$$

input
```
int((x^2*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*b) + (log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2
)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(3*b^
(3/2))
```

3.361 $\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx$

3.361.1 Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx=-\frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}$$

output
```
-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a+2/3*arctanh(b^(1/2)*(d*x^3
+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a/b^(1/2)
```

3.361.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx=\frac{2\left(\frac{\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}}-\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)\right)}{3a}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]
```

output
```
(2*((Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d
]])/Sqrt[b] - Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]))/(3*a)
```

3.361.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x\,(a+bx^3)}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^3\,(bx^3+a)}dx^3$$

$$\Big\downarrow \; 94$$

$$\frac{1}{3}\left(\frac{c\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{(bc-ad)\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{a}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{2c\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]
```

output
```
((-2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/a + (2*Sqrt[b*c - a*d]*ArcT
anh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/3
```

3.361.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.361.4 Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\sqrt{c}}{3}}{a}-\frac{2\left(\sqrt{d\,x^{3}+c}-\frac{(ad-bc)\operatorname{arctan}\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3a}$	98
pseudoelliptic	$\frac{2\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)ad-2\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)bc-2\sqrt{c}\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\sqrt{(ad-bc)b}}{3a\sqrt{(ad-bc)b}}$	106
elliptic	Expression too large to display	1543

input
```
int((d*x^3+c)^(1/2)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))-2/3
/a*((d*x^3+c)^(1/2)-(a*d-b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)
)/((a*d-b*c)*b)^(1/2))
```

3.361.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.51

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)}\,dx = \left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{3\,a}, \frac{2\,\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{}}{bc-a}\right.}{}\right.$$

input
```
integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/3*(sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b
*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c
)*sqrt(c) + 2*c)/x^3))/a, 1/3*(2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 +
 c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^
3 + c)*sqrt(c) + 2*c)/x^3))/a, 1/3*(2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt
(-c)/c) + sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 +
c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)))/a, 2/3*(sqrt(-(b*c - a*d)/b)*arcta
n(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(-c)*arctan(s
qrt(d*x^3 + c)*sqrt(-c)/c))/a]
```

3.361.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(73) = 146$.

Time = 4.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)}\,dx = \begin{cases} \frac{2\left(\frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}+\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3ab\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for } d\neq 0 \\ \sqrt{c}\left(-\frac{2b\left(\begin{cases}\frac{\frac{a}{2b}+x^3}{a} & \text{for } b=0\\ -\frac{\log\left(a-2b\left(\frac{a}{2b}+x^3\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{3a}-\frac{2b\left(\begin{cases}\frac{\frac{a}{2b}+x^3}{a} & \text{for } b=0\\ \frac{\log\left(a+2b\left(\frac{a}{2b}+x^3\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{3a}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**3+c)**(1/2)/x/(b*x**3+a),x)
```

output
```
Piecewise((2*(c*d*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c)) + d*(a*d
- b*c)*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*b*sqrt((a*d - b*c)/
b)))/d, Ne(d, 0)), (sqrt(c)*(-2*b*Piecewise(((a/(2*b) + x**3)/a, Eq(b, 0))
, (-log(a - 2*b*(a/(2*b) + x**3))/(2*b), True))/(3*a) - 2*b*Piecewise(((a/
(2*b) + x**3)/a, Eq(b, 0)), (log(a + 2*b*(a/(2*b) + x**3))/(2*b), True))/(
3*a)), True))
```

3.361.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)}\,dx = \int \frac{\sqrt{dx^3+c}}{(bx^3+a)x}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x), x)
```

3.361.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^3}}{x\,(a+bx^3)}\,dx = -\frac{2\,(bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}a} + \frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}$$

input

```
integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="giac")
```

output

```
-2/3*(b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2
*c + a*b*d)*a) + 2/3*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c))
```

3.361.9 Mupad [B] (verification not implemented)

Time = 12.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{c+dx^3}}{x\,(a+bx^3)}\,dx = \frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,1{}\mathrm{i}}{3\,a\,\sqrt{b}}$$

input

```
int((c + d*x^3)^(1/2)/(x*(a + b*x^3)),x)
```

output

```
(c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)
))/x^6))/(3*a) + (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)
^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(3*a*b^(1/2))
```

3.362 $\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx$

3.362.1 Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{3ax^3}+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} - \frac{2\sqrt{b}\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2}$$

output
```
1/3*(-a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(1/2)-2/3*arctanh(
b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)*(-a*d+b*c)^(1/2)/a^2-1/3
*(d*x^3+c)^(1/2)/a/x^3
```

3.362.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx = \frac{-\frac{a\sqrt{c+dx^3}}{x^3}-2\sqrt{b}\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^2}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]
```

output
```
(-((a*Sqrt[c + d*x^3])/x^3) - 2*Sqrt[b]*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]
*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]] + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*
x^3]/Sqrt[c]])/Sqrt[c])/(3*a^2)
```

3.362.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^6\left(bx^3+a\right)}dx^3$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\left(\frac{\int -\frac{bdx^3+2bc-ad}{2x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}-\frac{\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{bdx^3+2bc-ad}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{2a}-\frac{\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(-\frac{\frac{(2bc-ad)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{2b(bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{\frac{2(2bc-ad)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{4b(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{\frac{4\sqrt{b}\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a}-\frac{2(2bc-ad)\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3}\right)$$

input

```
Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]
```

output

```
(-(Sqrt[c + d*x^3]/(a*x^3)) - ((-2*(2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/S
qrt[c]])/(a*Sqrt[c]) + (4*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^3])/Sqrt[b*c - a*d]])/a)/(2*a))/3
```

3.362.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.362.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{-\frac{2(ad-bc)b\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{\sqrt{dx^3+c}\,a}{x^3}-\frac{(ad-2bc)\,\text{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^2}$
risch	$-\frac{\sqrt{dx^3+c}}{3a\,x^3}-\frac{-\frac{2(-ad+2bc)\,\text{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{4(ad-bc)b\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3a\sqrt{(ad-bc)b}}}{2a}$
default	$\frac{-\frac{\sqrt{dx^3+c}}{3x^3}-\frac{d\ \text{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3\sqrt{c}}}{a}-\frac{b\left(\frac{2\sqrt{dx^3+c}}{3}-\frac{2\ \text{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)\sqrt{c}}{3}\right)}{a^2}+\frac{2b\left(\sqrt{dx^3+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right.}{\sqrt{(ad-bc)b}}\right.}{3a^2}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*(-2*(a*d-b*c)*b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d
-b*c)*b)^(1/2))-(d*x^3+c)^(1/2)*a/x^3-(a*d-2*b*c)/c^(1/2)*arctanh((d*x^3+c
)^(1/2)/c^(1/2)))
```

3.362.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 513, normalized size of antiderivative = 4.46

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx$$

$$= \left[\frac{2\sqrt{b^2c-abd}cx^3\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-(2\,bc-ad)\sqrt{c}x^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-2\,\sqrt{dx^3}}{6\,a^2cx^3}\right.$$

$$-\frac{(2\,bc-ad)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\sqrt{b^2c-abd}cx^3\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+\sqrt{dx^3+c}}{3\,a^2cx^3}$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(2*sqrt(b^2*c - a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^
3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - (2*b*c - a*d)*sqrt(c)*x^3*log((
d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^
2*c*x^3), 1/6*(4*sqrt(-b^2*c + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2*b*c - a*d)*sqrt(c)*x^3*log((d*x^3 - 2*
sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3),
-1/3*((2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt
(b^2*c - a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(
b^2*c - a*b*d))/(b*x^3 + a)) + sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3), 1/3*(2*sq
rt(-b^2*c + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*
x^3 + b*c)) - (2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c
) - sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3)]
```

3.362.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx=\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a),x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**4*(a + b*x**3)), x)
```

3.362.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx = \int \frac{\sqrt{dx^3+c}}{(bx^3+a)x^4}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^4), x)
```

3.362.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx &= \frac{2\left(b^2c-abd\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^2} \\ &\quad - \frac{\left(2\,bc-ad\right)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}} - \frac{\sqrt{dx^3+c}}{3\,ax^3}\end{aligned}$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*(b^2*c - a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*a^2) - 1/3*(2*b*c - a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(
a^2*sqrt(-c)) - 1/3*sqrt(d*x^3 + c)/(a*x^3)
```

3.362.9 Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx &= \frac{\ln\left(\frac{a\,d-2\,b\,c+2\,\sqrt{d\,x^3+c}\,\sqrt{b^2\,c-a\,b\,d}-b\,d\,x^3}{b\,x^3+a}\right)\,\sqrt{b^2\,c-a\,b\,d}}{3\,a^2} \\ &\quad - \frac{\sqrt{d\,x^3+c}}{3\,a\,x^3} + \frac{\ln\left(\frac{{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)\,\left(a\,d-2\,b\,c\right)}{6\,a^2\,\sqrt{c}}\end{aligned}$$

input
```
int((c + d*x^3)^(1/2)/(x^4*(a + b*x^3)),x)
```

output
```
(log((a*d - 2*b*c + 2*(c + d*x^3)^(1/2)*(b^2*c - a*b*d)^(1/2) - b*d*x^3)/(
a + b*x^3))*(b^2*c - a*b*d)^(1/2))/(3*a^2) - (c + d*x^3)^(1/2)/(3*a*x^3) +
 (log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)
*(a*d - 2*b*c))/(6*a^2*c^(1/2))
```

3.362. $\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)}\,dx$

3.363 $\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx$

3.363.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{x^4\sqrt{c+dx^3}\operatorname{AppellF1}\left(\frac{4}{3},1,-\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/4*x^4*AppellF1(4/3,1,-1/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*
x^3/c)^(1/2)
```

3.363.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 241 vs. $2(64) = 128$.

Time = 6.50 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.77

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx$$
$$=\frac{x\left(\frac{(3bc-5ad)x^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+8\left(c+dx^3+\frac{8a^2c^2\operatorname{AppellF}}{(a+bx^3)\left(-8ac\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bc\,\mathrm{A}\right.\right.}\right.\right.}{20b\sqrt{c+dx^3}}$$

input
```
Integrate[(x^3*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
(x*(((3*b*c - 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -(
(d*x^3)/c), -((b*x^3)/a)])/a + 8*(c + d*x^3 + (8*a^2*c^2*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3,
1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2,
 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d
*x^3)/c), -((b*x^3)/a)]))))))/(20*b*Sqrt[c + d*x^3])
```

3.363.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{\frac{dx^3}{c}+1}}{bx^3+a}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},1,-\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x^3*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
(x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)])/(4*a*Sqrt[1 + (d*x^3)/c])
```

3.363.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.363.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.09 (sec) , antiderivative size = 741, normalized size of antiderivative = 11.58

method	result	size
elliptic	Expression too large to display	741
risch	Expression too large to display	757
default	Expression too large to display	1012

input
```
int(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/5*x/b*(d*x^3+c)^(1/2)-2/3*I*(-(a*d-b*c)/b^2-2/5*c/b)*3^(1/2)/d*(-c*d^2)^
(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/3*I*a/b^2/d^2*2^(1/2)
*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c
*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*
(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-
c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2
)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.363.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.363.6 Sympy [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx$$

input
```
integrate(x**3*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

output
```
Integral(x**3*sqrt(c + d*x**3)/(a + b*x**3), x)
```

3.363.7 Maxima [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{bx^3+a}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a), x)
```

3.363.8 Giac [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{bx^3+a}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a), x)
```

3.363.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{x^3\,\sqrt{d\,x^3+c}}{b\,x^3+a}\,dx$$

input
```
int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

output
```
int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3), x)
```

3.364 $\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx$

3.364.1 Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{x^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},1,-\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/2*x^2*AppellF1(2/3,1,-1/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*
x^3/c)^(1/2)
```

3.364.2 Mathematica [A] (verified)

Time = 9.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{x^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{2a\sqrt{\frac{c+dx^3}{c}}}$$

input
```
Integrate[(x*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output
```
(x^2*Sqrt[c + d*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a
)])/(2*a*Sqrt[(c + d*x^3)/c])
```

3.364.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{c+dx^3}\int \frac{x\sqrt{\frac{dx^3}{c}+1}}{bx^3+a}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},1,-\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

input

```
Int[(x*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

output

```
(x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 1, -1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a*Sqrt[1 + (d*x^3)/c])
```

3.364.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.364.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.44 (sec) , antiderivative size = 857, normalized size of antiderivative = 13.39

method	result	size
default	Expression too large to display	857
elliptic	Expression too large to display	857

input
```
int(x*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/b/d^2*2
^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3
*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^
2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^
(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*...
```

3.364.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.364.6 Sympy [F]

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx$$

input
```
integrate(x*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

output
```
Integral(x*sqrt(c + d*x**3)/(a + b*x**3), x)
```

3.364.7 Maxima [F]

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{dx^3+c}x}{bx^3+a}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a), x)
```

3.364.8 Giac [F]

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{dx^3+c}x}{bx^3+a}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a), x)
```

3.364.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx=\int \frac{x\,\sqrt{d\,x^3+c}}{b\,x^3+a}\,dx$$

input
```
int((x*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

output
```
int((x*(c + d*x^3)^(1/2))/(a + b*x^3), x)
```

3.365 $\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx$

3.365.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{x\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},1,-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}$$

output
```
x*AppellF1(1/3,1,-1/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*x^3/c)
^(1/2)
```

3.365.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. $2(59) = 118$.

Time = 10.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx$$
$$=\frac{8acx\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\,\big(8ac\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\,\big(-2bc\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+ad\,\text{Ap}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(a + b*x^3),x]
```

output
```
(8*a*c*x*Sqrt[c + d*x^3]*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -((b*x^
3)/a)])/((a + b*x^3)*(8*a*c*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -((b
*x^3)/a)] + 3*x^3*(-2*b*c*AppellF1[4/3, -1/2, 2, 7/3, -((d*x^3)/c), -((b*x
^3)/a)] + a*d*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

3.365.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{c+dx^3}\int \frac{\sqrt{\frac{dx^3}{c}+1}}{bx^3+a}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{c+dx^3}\operatorname{AppellF1}\left(\frac{1}{3},1,-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(a + b*x^3),x]
```

output
```
(x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(a*Sqrt[1 + (d*x^3)/c])
```

3.365.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.365.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.28 (sec) , antiderivative size = 705, normalized size of antiderivative = 11.95

method	result
default	$-\frac{2 i \sqrt{3}\, \left(-c\, d^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x +\frac{\left(-c\, d^{2}\right)^{\frac{1}{3}}}{2 d}-\frac{i \sqrt{3}\, \left(-c\, d^{2}\right)^{\frac{1}{3}}}{2 d}\right) \sqrt{3}\, d}{\left(-c\, d^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x -\frac{\left(-c\, d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3 \left(-c\, d^{2}\right)^{\frac{1}{3}}}{2 d}+\frac{i \sqrt{3}\, \left(-c\, d^{2}\right)^{\frac{1}{3}}}{2 d}}}\, \sqrt{-\frac{i \left(x +\frac{\left(-c\, d^{2}\right)^{\frac{1}{3}}}{2 d}+\frac{i \sqrt{3}\, \left(-c\, d^{2}\right)^{\frac{1}{3}}}{2 d}\right) \sqrt{3}\, d}{\left(-c\, d^{2}\right)^{\frac{1}{3}}}}\, F}{3 b \sqrt{d\, x^{3}+c}}$
elliptic	$-\frac{2 i \sqrt{3}\, \left(-c\, d^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x +\frac{\left(-c\, d^{2}\right)^{\frac{1}{3}}}{2 d}-\frac{i \sqrt{3}\, \left(-c\, d^{2}\right)^{\frac{1}{3}}}{2 d}\right) \sqrt{3}\, d}{\left(-c\, d^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x -\frac{\left(-c\, d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3 \left(-c\, d^{2}\right)^{\frac{1}{3}}}{2 d}+\frac{i \sqrt{3}\, \left(-c\, d^{2}\right)^{\frac{1}{3}}}{2 d}}}\, \sqrt{-\frac{i \left(x +\frac{\left(-c\, d^{2}\right)^{\frac{1}{3}}}{2 d}+\frac{i \sqrt{3}\, \left(-c\, d^{2}\right)^{\frac{1}{3}}}{2 d}\right) \sqrt{3}\, d}{\left(-c\, d^{2}\right)^{\frac{1}{3}}}}\, F}{3 b \sqrt{d\, x^{3}+c}}$

input
```
int((d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

3.365. $\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx$

output
```
-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
))+1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d
)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.365.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.365.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/(b*x**3+a),x)
```

output
```
Integral(sqrt(c + d*x**3)/(a + b*x**3), x)
```

3.365.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx=\int \frac{\sqrt{dx^3+c}}{bx^3+a}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/(b*x^3 + a), x)
```

3.365.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx=\int \frac{\sqrt{dx^3+c}}{bx^3+a}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/(b*x^3 + a), x)
```

3.365.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx=\int \frac{\sqrt{d\,x^3+c}}{b\,x^3+a}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(a + b*x^3),x)
```

output
```
int((c + d*x^3)^(1/2)/(a + b*x^3), x)
```

3.366 $\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx$

3.366.1 Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}\,dx=-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},1,-\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-AppellF1(-1/3,1,-1/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x/(1+d*x^3/
c)^(1/2)
```

3.366.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 139 vs. $2(62) = 124$.

Time = 10.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.24

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}\,dx$$
$$=\frac{-20a(c+dx^3)+5(-2bc+3ad)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bdx^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF}}{20a^2x\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)),x]
```

output
```
(-20*a*(c + d*x^3) + 5*(-2*b*c + 3*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2
/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^6*Sqrt[1 + (d*x^3)/
c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(20*a^2*x*Sqrt[
c + d*x^3])
```

3.366.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^2 \left(a+bx^3\right)} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{c+dx^3} \int \frac{\sqrt{\frac{dx^3}{c}+1}}{x^2 (bx^3+a)}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{c+dx^3} \operatorname{AppellF1}\left(-\frac{1}{3},1,-\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)),x]
```

output
```
-((Sqrt[c + d*x^3]*AppellF1[-1/3, 1, -1/2, 2/3, -((b*x^3)/a), -((d*x^3)/c)
])/(a*x*Sqrt[1 + (d*x^3)/c]))
```

3.366.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.366.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.24 (sec) , antiderivative size = 892, normalized size of antiderivative = 14.39

method	result	size
elliptic	Expression too large to display	892
risch	Expression too large to display	893
default	Expression too large to display	1314

input
```
int((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(d*x^3+c)^(1/2)/x-1/3*I/a*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x
-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)))+1/3*I/a/d^2*2^(1/2)*sum((-a*d+b*c)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3
)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(
1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2
)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)
*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(...
```

3.366.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Not
integrable (provided residues have no relations)
```

3.366.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}\,dx=\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**2/(b*x**3+a),x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**2*(a + b*x**3)), x)
```

3.366.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}\,dx=\int \frac{\sqrt{dx^3+c}}{(bx^3+a)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^2), x)
```

3.366.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}\,dx=\int \frac{\sqrt{dx^3+c}}{(bx^3+a)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^2), x)
```

3.366.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{x^2\,\left(b\,x^3+a\right)}\,dx$$

input

```
int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)),x)
```

output

```
int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)), x)
```

3.367 $\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)}\,dx$

3.367.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx=-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-1/2*AppellF1(-2/3,1,-1/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x^2/(1+
d*x^3/c)^(1/2)
```

3.367.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 335 vs. 2(64) = 128.

Time = 10.27 (sec) , antiderivative size = 335, normalized size of antiderivative = 5.23

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx$$
$$=\frac{-bdx^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a(32ac(2ac+6bcx^3-adx^3+2bdx^6)\,\text{AppellF1}(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a})-2}{(a+bx^3)(-8ac\,\text{AppellF1}(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a})+3}}{16a^2x^2\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)),x]
```

output
```
(-(b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)]) + (a*(32*a*c*(2*a*c + 6*b*c*x^3 - a*d*x^3 + 2*b*d*x^6)*Appell
F1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(a + b*x^3)*(c +
 d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*
d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(
-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b
*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4
/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(16*a^2*x^2*Sqrt[c + d*x^
3])
```

3.367.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{c+dx^3}\int \frac{\sqrt{\frac{dx^3}{c}+1}}{x^3\left(bx^3+a\right)}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)),x]
```

output
```
-1/2*(Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -1/2, 1/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*x^2*Sqrt[1 + (d*x^3)/c])
```

3.367.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.367.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.35 (sec) , antiderivative size = 740, normalized size of antiderivative = 11.56

method	result	size
elliptic	Expression too large to display	740
risch	Expression too large to display	741
default	Expression too large to display	1010

input
```
int((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output

```
-1/2/a*(d*x^3+c)^(1/2)/x^2+1/6*I/a*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I/a/d^2*2^(1/2)*sum((-a*d+b*c)/_alpha^2/(
a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(
1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^
(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2
*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
a+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*b+a))
```

3.367.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   Not
integrable (provided residues have no relations)
```

3.367.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx=\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**3/(b*x**3+a),x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**3*(a + b*x**3)), x)
```

3.367.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx=\int \frac{\sqrt{dx^3+c}}{(bx^3+a)x^3}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^3), x)
```

3.367.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx=\int \frac{\sqrt{dx^3+c}}{(bx^3+a)x^3}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^3), x)
```

3.367.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^3\,\left(b\,x^3+a\right)}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)), x)
```

3.368 $\int \frac{x^8\left(c+dx^3\right)^{3/2}}{a+bx^3} dx$

3.368.1 Optimal result

Integrand size = 24, antiderivative size = 154

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3} dx = \frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4} + \frac{2a^2(c+dx^3)^{3/2}}{9b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2} + \frac{2(c+dx^3)^{7/2}}{21bd^2} - \frac{2a^2(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}$$

output
```
2/9*a^2*(d*x^3+c)^(3/2)/b^3-2/15*(a*d+b*c)*(d*x^3+c)^(5/2)/b^2/d^2+2/21*(d
*x^3+c)^(7/2)/b/d^2-2/3*a^2*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/
2)/(-a*d+b*c)^(1/2))/b^(9/2)+2/3*a^2*(-a*d+b*c)*(d*x^3+c)^(1/2)/b^4
```

3.368.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3} dx = \frac{2\sqrt{c+dx^3}\Big(-105a^3d^3-21ab^2d(c+dx^3)^2-3b^3(2c-5dx^3)\left(c+dx^3\right)^2+35a^2bd^2(4c}{315b^4d^2} + \frac{2a^2(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{9/2}}$$

input
```
Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
(2*Sqrt[c + d*x^3]*(-105*a^3*d^3 - 21*a*b^2*d*(c + d*x^3)^2 - 3*b^3*(2*c -
 5*d*x^3)*(c + d*x^3)^2 + 35*a^2*b*d^2*(4*c + d*x^3)))/(315*b^4*d^2) + (2*
a^2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*
d]])/(3*b^(9/2))
```

3.368.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 \left(c+d x^3\right)^{3/2}}{a+b x^3} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^6 \left(d x^3+c\right)^{3/2}}{b x^3+a}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3} \int \left(\frac{\left(d x^3+c\right)^{5/2}}{b d}+\frac{(-b c-a d) \left(d x^3+c\right)^{3/2}}{b^2 d}+\frac{a^2 \left(d x^3+c\right)^{3/2}}{b^2 \left(b x^3+a\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{2 a^2 (b c-a d)^{3/2} \text{arctanh}\left(\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right)}{b^{9/2}}+\frac{2 a^2 \sqrt{c+d x^3} (b c-a d)}{b^4}+\frac{2 a^2 \left(c+d x^3\right)^{3/2}}{3 b^3}-\frac{2 \left(c+d x^3\right)^{5/2} (a d+b c)}{5 b^2 d^2}\right.$$

input
```
Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
((2*a^2*(b*c - a*d)*Sqrt[c + d*x^3])/b^4 + (2*a^2*(c + d*x^3)^(3/2))/(3*b^
3) - (2*(b*c + a*d)*(c + d*x^3)^(5/2))/(5*b^2*d^2) + (2*(c + d*x^3)^(7/2))
/(7*b*d^2) - (2*a^2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sq
rt[b*c - a*d]])/b^(9/2))/3
```

3.368.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.368.4 Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(\frac{2\left(-\frac{5d\,x^3}{2}+c\right)\left(d\,x^3+c\right)^2b^3}{35}+\frac{ad\left(d\,x^3+c\right)^2b^2}{5}-\frac{4\left(\frac{d\,x^3}{4}+c\right)d^2a^2b}{3}+a^3d^3\right)\sqrt{(ad-bc)b}\,\sqrt{d\,x^3+c}-a^2d^2(ad-bc)^2\arctan\left(\right.\right.\right.}{3\sqrt{(ad-bc)b}\,d^2b^4}$
default	$\frac{\frac{2d\,x^9\sqrt{d\,x^3+c}}{21}+\frac{16c\,x^6\sqrt{d\,x^3+c}}{105}+\frac{2c^2x^3\sqrt{d\,x^3+c}}{105d}-\frac{4c^3\sqrt{d\,x^3+c}}{105d^2}}{b}-\frac{2a\left(d\,x^3+c\right)^{\frac{5}{2}}}{15b^2d}-\frac{2a^2\left(-(ad-bc)^2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\right.\right.}{3b^4\sqrt{(}}$
risch	$-\frac{2\left(-15b^3d^3x^9+21a\,b^2d^3x^6-24b^3c\,d^2x^6-35a^2b\,d^3x^3+42a\,b^2c\,d^2x^3-3b^3c^2d\,x^3+105a^3d^3-140a^2bc\,d^2+21a\,b^2c^2d+6b^3c^3\right)}{315d^2b^4}$
elliptic	$\frac{2d\,x^9\sqrt{d\,x^3+c}}{21b}+\frac{2\left(-\frac{d(ad-2bc)}{b^2}-\frac{6cd}{7b}\right)x^6\sqrt{d\,x^3+c}}{15d}+\frac{2\left(\frac{a^2d^2-2abcd+b^2c^2}{b^3}-\frac{4\left(-\frac{d(ad-2bc)}{b^2}-\frac{6cd}{7b}\right)c}{5d}\right)x^3\sqrt{d\,x^3+c}}{9d}+$

input
```
int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2/35*(-5/2*d*x^3+c)*(d*x^3+c)^2*b^3+1/5*a*d*(d*x^3+c)^2*b^2-4/3*(1/
4*d*x^3+c)*d^2*a^2*b+a^3*d^3)*((a*d-b*c)*b)^(1/2)*(d*x^3+c)^(1/2)-a^2*d^2*
(a*d-b*c)^2*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)))/((a*d-b*c)*b)^(
1/2)/d^2/b^4
```

3.368. $\int \frac{x^8\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx$

3.368.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.66

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \Bigg[-\frac{105\,(a^2bcd^2-a^3d^3)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\,(15\,b^3d^3x^9+3\,(}{}$$

$$-\frac{2\left(105\,(a^2bcd^2-a^3d^3)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(15\,b^3d^3x^9+3\,(8\,b^3cd^2-7\,ab^2d^3)x^6-6\,b^3c\right.}{315\,b^4d^2}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/315*(105*(a^2*b*c*d^2 - a^3*d^3)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*
b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(15*
b^3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c^3 - 21*a*b^2*c^2
*d + 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*b^2*c*d^2 + 35*a^
2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*d^2), -2/315*(105*(a^2*b*c*d^2 - a^3*d
^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b
*c - a*d)) - (15*b^3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c
^3 - 21*a*b^2*c^2*d + 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*
b^2*c*d^2 + 35*a^2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*d^2)]
```

3.368.6 Sympy [A] (verification not implemented)

Time = 33.41 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.31

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \begin{cases} \frac{2\left(\frac{a^2d\left(c+dx^3\right)^{\frac{3}{2}}}{9b^3}+\frac{a^2d(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^5\sqrt{\frac{ad-bc}{b}}}+\frac{\left(c+dx^3\right)^{\frac{7}{2}}}{21bd}+\frac{\left(c+dx^3\right)^{\frac{5}{2}}(-ad-bc)}{15b^2d}+\frac{\sqrt{c+dx^3}\left(-a^3d^2+a^2bcd\right)}{3b^4}\right)}{d} & \\ c^{\frac{3}{2}}\left(\frac{a^2\left(\begin{cases}\frac{x^3}{a} & \text{for}\: b=0\\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right)}{3b^2}-\frac{ax^3}{3b^2}+\frac{x^6}{6b}\right) & \end{cases}$$

3.368. $\int \frac{x^8\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx$

input
```
integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

output
```
Piecewise((2*(a**2*d*(c + d*x**3)**(3/2)/(9*b**3) + a**2*d*(a*d - b*c)**2*
atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**5*sqrt((a*d - b*c)/b)) +
(c + d*x**3)**(7/2)/(21*b*d) + (c + d*x**3)**(5/2)*(-a*d - b*c)/(15*b**2*d
) + sqrt(c + d*x**3)*(-a**3*d**2 + a**2*b*c*d)/(3*b**4))/d, Ne(d, 0)), (c*
*(3/2)*(a**2*Piecewise((x**3/a, Eq(b, 0)), (log(a + b*x**3)/b, True))/(3*b
**2) - a*x**3/(3*b**2) + x**6/(6*b)), True))
```

3.368.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.368.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.25

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^4} + \frac{2\left(15\,(dx^3+c)^{\frac{7}{2}}b^6d^{12}-21\,(dx^3+c)^{\frac{5}{2}}b^6cd^{12}-21\,(dx^3+c)^{\frac{5}{2}}ab^5d^{13}+35\,(dx^3+c)^{\frac{3}{2}}a^2b^4d^{14}+105\,\sqrt{dx^3+c}\right.}{315\,b^7d^{14}}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

output

```
2/3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b
^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) + 2/315*(15*(d*x^3 + c)^(7/2)*b^
6*d^12 - 21*(d*x^3 + c)^(5/2)*b^6*c*d^12 - 21*(d*x^3 + c)^(5/2)*a*b^5*d^13
 + 35*(d*x^3 + c)^(3/2)*a^2*b^4*d^14 + 105*sqrt(d*x^3 + c)*a^2*b^4*c*d^14
- 105*sqrt(d*x^3 + c)*a^3*b^3*d^15)/(b^7*d^14)
```

3.368.9 Mupad [B] (verification not implemented)

Time = 10.39 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.14

$$
\begin{aligned}
\int \frac{x^8 (c+dx^3)^{3/2}}{a+bx^3}\,dx &= \frac{2\,d\,x^9\,\sqrt{d\,x^3+c}}{21\,b} \\
&- \frac{\left(\frac{2\,a\,\left(\frac{c^2}{b}+\frac{a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}\right)}{b}+\frac{2\,c\,\left(\frac{2\,c^2}{b}+\frac{2\,a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{4\,c\,\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{5\,d}\right)}{3\,d}\right)\,\sqrt{d\,x^3+c}}{3\,d} \\
&+ \frac{x^3\,\sqrt{d\,x^3+c}\,\left(\frac{2\,c^2}{b}+\frac{2\,a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{4\,c\,\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{5\,d}\right)}{9\,d} - \frac{x^6\,\sqrt{d\,x^3+c}\,\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{15\,d} \\
&+ \frac{a^2\,\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,{\left(a\,d-b\,c\right)}^{3/2}\,1{}\mathrm{i}}{3\,b^{9/2}}
\end{aligned}
$$

input

```
int((x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

output

```
(2*d*x^9*(c + d*x^3)^(1/2))/(21*b) - (((2*a*(c^2/b + (a*((a*d^2)/b^2 - (2*
c*d)/b))/b))/b + (2*c*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (4*
c*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(5*d)))/(3*d))*(c + d*x^3)^(1/2))/(3*d
) + (x^3*(c + d*x^3)^(1/2)*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b
+ (4*c*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(5*d)))/(9*d) - (x^6*(c + d*x^3)^
(1/2)*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(15*d) + (a^2*log((a^2*d^2 + 2*b^2
*c^2 - b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*
c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*b^(9/2))
```

3.369 $\int \frac{x^5\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx$

3.369.1 Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{x^5\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx=-\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3}-\frac{2a\left(c+dx^3\right)^{3/2}}{9b^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd}+\frac{2a(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

output
```
-2/9*a*(d*x^3+c)^(3/2)/b^2+2/15*(d*x^3+c)^(5/2)/b/d+2/3*a*(-a*d+b*c)^(3/2)
*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)-2/3*a*(-a*d+b*c
)*(d*x^3+c)^(1/2)/b^3
```

3.369.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \frac{x^5\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx=\frac{2\sqrt{c+dx^3}\left(15a^2d^2+3b^2\left(c+dx^3\right)^2-5abd\left(4c+dx^3\right)\right)}{45b^3d}-\frac{2a(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}}$$

input
```
Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
(2*Sqrt[c + d*x^3]*(15*a^2*d^2 + 3*b^2*(c + d*x^3)^2 - 5*a*b*d*(4*c + d*x^
3)))/(45*b^3*d) - (2*a*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3
])/Sqrt[-(b*c) + a*d]])/(3*b^(7/2))
```

3.369.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(c+d x^3\right)^{3/2}}{a+b x^3} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3} \int \frac{x^3 \left(d x^3+c\right)^{3/2}}{b x^3+a}dx^3$$

$$\Big\downarrow \; 90$$

$$\frac{1}{3} \left(\frac{2 \left(c+d x^3\right)^{5/2}}{5 b d}-\frac{a \int \frac{\left(d x^3+c\right)^{3/2}}{b x^3+a}dx^3}{b}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3} \left(\frac{2 \left(c+d x^3\right)^{5/2}}{5 b d}-\frac{a \left(\frac{(b c-a d) \int \frac{\sqrt{d x^3+c}}{b x^3+a}dx^3}{b}+\frac{2 \left(c+d x^3\right)^{3/2}}{3 b}\right)}{b}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3} \left(\frac{2 \left(c+d x^3\right)^{5/2}}{5 b d}-\frac{a \left(\frac{(b c-a d) \left(\frac{(b c-a d) \int \frac{1}{\left(b x^3+a\right) \sqrt{d x^3+c}}dx^3}{b}+\frac{2 \sqrt{c+d x^3}}{b}\right)}{b}+\frac{2 \left(c+d x^3\right)^{3/2}}{3 b}\right)}{b}\right)$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{5/2}}{5bd}-\frac{a\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)}{b}\right)$$

$$\Big\downarrow \textbf{221}$$

$$\frac{1}{3}\left(\frac{2\left(c+dx^3\right)^{5/2}}{5bd}-\frac{a\left(\frac{(bc-ad)\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)}{b}\right)$$

input
```
Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
((2*(c + d*x^3)^(5/2))/(5*b*d) - (a*((2*(c + d*x^3)^(3/2))/(3*b) + ((b*c -
 a*d)*((2*Sqrt[c + d*x^3])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^3])/Sqrt[b*c - a*d]])/b^(3/2)))/b))/b)/3
```

3.369.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

3.369. $\int \frac{x^5\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.369.4 Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.94

method	result
default	$\frac{2(dx^3+c)^{\frac{5}{2}}}{15bd}+\frac{2a\left(-(ad-bc)^2\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{(-dx^3-4c)b}{3}+ad\right)\sqrt{dx^3+c}\sqrt{(ad-bc)b}\right)}{3b^3\sqrt{(ad-bc)b}}$
pseudoelliptic	$\frac{\frac{2\left(\frac{(dx^3+c)^2b^2}{5}-\frac{4\left(\frac{dx^3}{4}+c\right)dab}{3}+a^2d^2\right)\sqrt{(ad-bc)b}\sqrt{dx^3+c}}{3}-\frac{2ad(ad-bc)^2\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3}}{db^3\sqrt{(ad-bc)b}}$
risch	$\frac{2(3b^2d^2x^6-5x^3abd^2+6x^3b^2cd+15a^2d^2-20abcd+3b^2c^2)\sqrt{dx^3+c}}{45db^3}-\frac{2a(a^2d^2-2abcd+b^2c^2)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b^3\sqrt{(ad-bc)b}}$
elliptic	$\frac{2dx^6\sqrt{dx^3+c}}{15b}+\frac{2\left(-\frac{d(ad-2bc)}{b^2}-\frac{4cd}{5b}\right)x^3\sqrt{dx^3+c}}{9d}+\frac{2\left(\frac{a^2d^2-2abcd+b^2c^2}{b^3}-\frac{2\left(-\frac{d(ad-2bc)}{b^2}-\frac{4cd}{5b}\right)c}{3d}\right)\sqrt{dx^3+c}}{3d}+$ $ia\sqrt{2}$

input
```
int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/15*(d*x^3+c)^(5/2)/b/d+2/3*a/b^3*(-(a*d-b*c)^2*arctan(b*(d*x^3+c)^(1/2)/
((a*d-b*c)*b)^(1/2))+(1/3*(-d*x^3-4*c)*b+a*d)*(d*x^3+c)^(1/2)*((a*d-b*c)*b
)^(1/2))/((a*d-b*c)*b)^(1/2)
```

3.369.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.48

$$\int\frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx=\Bigg[-\frac{15\,(abcd-a^2d^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\,(3\,b^2d^2x^6+3\,b^2c^2}{45\,b^3d}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/45*(15*(a*b*c*d - a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c -
a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(3*b^2*d^2
*x^6 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*
sqrt(d*x^3 + c))/(b^3*d), 2/45*(15*(a*b*c*d - a^2*d^2)*sqrt(-(b*c - a*d)/b
)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (3*b^2*d^2
*x^6 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*
sqrt(d*x^3 + c))/(b^3*d)]
```

3.369.6 Sympy [A] (verification not implemented)

Time = 16.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.28

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = \begin{cases} \frac{2\left(-\frac{ad\left(c+dx^3\right)^{\frac{3}{2}}}{9b^2}-\frac{ad(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4\sqrt{\frac{ad-bc}{b}}}+\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15b}+\frac{\sqrt{c+dx^3}\left(a^2d^2-abcd\right)}{3b^3}\right)}{d} & \text{for } d \neq 0 \\ c^{\frac{3}{2}}\left(-\frac{a\left(\begin{cases}\frac{x^3}{a} & \text{for } b=0\\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right)}{3b}+\frac{x^3}{3b}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

output
```
Piecewise((2*(-a*d*(c + d*x**3)**(3/2)/(9*b**2) - a*d*(a*d - b*c)**2*atan(
sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**4*sqrt((a*d - b*c)/b)) + (c +
d*x**3)**(5/2)/(15*b) + sqrt(c + d*x**3)*(a**2*d**2 - a*b*c*d)/(3*b**3))/d
, Ne(d, 0)), (c**(3/2)*(-a*Piecewise((x**3/a, Eq(b, 0)), (log(a + b*x**3)/
b, True))/(3*b) + x**3/(3*b)), True))
```

3.369.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.369.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26

$$\int \frac{x^5\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx = -\frac{2\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^3} + \frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}b^4d^4-5\left(dx^3+c\right)^{\frac{3}{2}}ab^3d^5-15\,\sqrt{dx^3+c}ab^3cd^5+15\,\sqrt{dx^3+c}a^2b^2d^6\right)}{45\,b^5d^5}$$

input

```
integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

output

```
-2/3*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^
2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 2/45*(3*(d*x^3 + c)^(5/2)*b^4*d
^4 - 5*(d*x^3 + c)^(3/2)*a*b^3*d^5 - 15*sqrt(d*x^3 + c)*a*b^3*c*d^5 + 15*s
qrt(d*x^3 + c)*a^2*b^2*d^6)/(b^5*d^5)
```

3.369.9 Mupad [B] (verification not implemented)

Time = 10.36 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.79

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,c^2}{b}+\frac{2\,a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{2\,c\,\left(\frac{2\,a\,d^2}{b^2}-\frac{12\,c\,d}{5\,b}\right)}{3\,d}\right)}{3\,d} + \frac{2\,d\,x^6\,\sqrt{d\,x^3+c}}{15\,b} - \frac{x^3\,\sqrt{d\,x^3+c}\,\left(\frac{2\,a\,d^2}{b^2}-\frac{12\,c\,d}{5\,b}\right)}{9\,d} + \frac{a\,\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d+\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,{\left(a\,d-b\,c\right)}^{3/2}\,1{}\mathrm{i}}{3\,b^{7/2}}$$

input

```
int((x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

output

```
((c + d*x^3)^(1/2)*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (2*c*(
(2*a*d^2)/b^2 - (12*c*d)/(5*b)))/(3*d)))/(3*d) + (2*d*x^6*(c + d*x^3)^(1/2
))/(15*b) - (x^3*(c + d*x^3)^(1/2)*((2*a*d^2)/b^2 - (12*c*d)/(5*b)))/(9*d)
 + (a*log((a^2*d^2 + 2*b^2*c^2 + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/
2)*2i - a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3
/2)*1i)/(3*b^(7/2))
```

3.370 $\int \frac{x^2\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx$

3.370.1 Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx=\frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2}+\frac{2(c+dx^3)^{3/2}}{9b}-\frac{2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

output
```
2/9*(d*x^3+c)^(3/2)/b-2/3*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)
/(-a*d+b*c)^(1/2))/b^(5/2)+2/3*(-a*d+b*c)*(d*x^3+c)^(1/2)/b^2
```

3.370.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx=\frac{2\sqrt{c+dx^3}(4bc-3ad+bdx^3)}{9b^2}+\frac{2(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}}$$

input
```
Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
(2*Sqrt[c + d*x^3]*(4*b*c - 3*a*d + b*d*x^3))/(9*b^2) + (2*(-(b*c) + a*d)^
(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2))
```

3.370.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {946, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(c+d x^3\right)^{3/2}}{a+b x^3} \, dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3} \int \frac{\left(d x^3+c\right)^{3/2}}{b x^3+a}dx^3$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3} \left(\frac{(b c-a d) \int \frac{\sqrt{d x^3+c}}{b x^3+a}dx^3}{b}+\frac{2 \left(c+d x^3\right)^{3/2}}{3 b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3} \left(\frac{(b c-a d) \left(\frac{(b c-a d) \int \frac{1}{\left(b x^3+a\right) \sqrt{d x^3+c}}dx^3}{b}+\frac{2 \sqrt{c+d x^3}}{b}\right)}{b}+\frac{2 \left(c+d x^3\right)^{3/2}}{3 b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3} \left(\frac{(b c-a d) \left(\frac{2 (b c-a d) \int \frac{1}{\frac{b x^6}{d}+a-\frac{b c}{d}}d\sqrt{d x^3+c}}{b d}+\frac{2 \sqrt{c+d x^3}}{b}\right)}{b}+\frac{2 \left(c+d x^3\right)^{3/2}}{3 b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3} \left(\frac{(b c-a d) \left(\frac{2 \sqrt{c+d x^3}}{b}-\frac{2 \sqrt{b c-a d} \text{arctanh}\left(\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right)}{b^{3/2}}\right)}{b}+\frac{2 \left(c+d x^3\right)^{3/2}}{3 b}\right)$$

input

```
Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

---

output
```
((2*(c + d*x^3)^(3/2))/(3*b) + ((b*c - a*d)*((2*Sqrt[c + d*x^3])/b - (2*Sq
rt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2))
)/b)/3
```

3.370.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.370. $\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx$

3.370.4 Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

method	result
default	$-\frac{2\left(-(ad-bc)^2\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{\left(-dx^3-4c\right)b}{3}+ad\right)\sqrt{dx^3+c}\,\sqrt{(ad-bc)b}\right)}{3\sqrt{(ad-bc)b}\,b^2}$
risch	$-\frac{2\sqrt{dx^3+c}\,(-bdx^3+3ad-4bc)}{9b^2}+\frac{2(a^2d^2-2abcd+b^2c^2)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b^2\sqrt{(ad-bc)b}}$
pseudoelliptic	$-\frac{2\left(-(ad-bc)^2\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{\left(-dx^3-4c\right)b}{3}+ad\right)\sqrt{dx^3+c}\,\sqrt{(ad-bc)b}\right)}{3\sqrt{(ad-bc)b}\,b^2}$
elliptic	$\frac{2dx^3\sqrt{dx^3+c}}{9b}+\frac{2\left(-\frac{d(ad-2bc)}{b^2}-\frac{2cd}{3b}\right)\sqrt{dx^3+c}}{3d}+$ $i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\left(-cd^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x\right.}{}}}{}\right.$

input
```
int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(a*d-b*c)^2*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(1/3*(-d*
x^3-4*c)*b+a*d)*(d*x^3+c)^(1/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b
^2
```

3.370.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.12

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\,(bdx^3+4\,bc-3\,ad)\sqrt{dx}}{9\,b^2}\right.$$

$$\left.-\frac{2\left(3\,(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(bdx^3+4\,bc-3\,ad)\sqrt{dx^3+c}\right)}{9\,b^2}\right]$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/9*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sq
rt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(b*d*x^3 + 4*b*c - 3
*a*d)*sqrt(d*x^3 + c))/b^2, -2/9*(3*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arcta
n(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (b*d*x^3 + 4*b*c
- 3*a*d)*sqrt(d*x^3 + c))/b^2]
```

3.370.6 Sympy [A] (verification not implemented)

Time = 7.95 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.29

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \begin{cases} \frac{2\left(\frac{d\left(c+dx^3\right)^{\frac{3}{2}}}{9b}+\frac{\sqrt{c+dx^3}\left(-ad^2+bcd\right)}{3b^2}+\frac{d(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for } d\neq 0 \\ c^{\frac{3}{2}}\left(\begin{cases}\frac{x^3}{3a} & \text{for } b=0\\ \frac{\log\left(3a+3bx^3\right)}{3b} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

output
```
Piecewise((2*(d*(c + d*x**3)**(3/2)/(9*b) + sqrt(c + d*x**3)*(-a*d**2 + b*
c*d)/(3*b**2) + d*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b)
)/(3*b**3*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (c**(3/2)*Piecewise((x**3/(3
*a), Eq(b, 0)), (log(3*a + 3*b*x**3)/(3*b), True)), True))
```

3.370.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.370.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^2} + \frac{2\left((dx^3+c)^{\frac{3}{2}}b^2+3\,\sqrt{dx^3+c}b^2c-3\,\sqrt{dx^3+c}abd\right)}{9\,b^3}$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c +
 a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2 + 3*sqrt(
d*x^3 + c)*b^2*c - 3*sqrt(d*x^3 + c)*a*b*d)/b^3
```

3.370.9 Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.49

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\,d\,x^3\,\sqrt{d\,x^3+c}}{9\,b} - \frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,a\,d^2}{b^2}-\frac{8\,c\,d}{3\,b}\right)}{3\,d} + \frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,{\left(a\,d-b\,c\right)}^{3/2}\,1{}\mathrm{i}}{3\,b^{5/2}}$$

---

input
```
int((x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

output
```
(log((a^2*d^2 + 2*b^2*c^2 - b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i
 - a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1
i)/(3*b^(5/2)) - ((c + d*x^3)^(1/2)*((2*a*d^2)/b^2 - (8*c*d)/(3*b)))/(3*d)
 + (2*d*x^3*(c + d*x^3)^(1/2))/(9*b)
```

3.370. $\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx$

3.371 $\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)}\,dx$

3.371.1 Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)}\,dx=\frac{2d\sqrt{c+dx^3}}{3b}-\frac{2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}$$

output
```
-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a+2/3*(-a*d+b*c)^(3/2)*arcta
nh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a/b^(3/2)+2/3*d*(d*x^3+c)^(1/
2)/b
```

3.371.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)}\,dx=\frac{2\left(a\sqrt{b}d\sqrt{c+dx^3}-(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)-b^{3/2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)\right)}{3ab^{3/2}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]
```

output
```
(2*(a*Sqrt[b]*d*Sqrt[c + d*x^3] - (-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqr
t[c + d*x^3])/Sqrt[-(b*c) + a*d]] - b^(3/2)*c^(3/2)*ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]]))/(3*a*b^(3/2))
```

3.371.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {948, 95, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\left(dx^3+c\right)^{3/2}}{x^3\left(bx^3+a\right)}dx^3$$

$$\Big\downarrow \text{95}$$

$$\frac{1}{3}\left(\frac{\int \frac{d(2bc-ad)x^3+bc^2}{x^3(bx^3+a)\sqrt{dx^3+c}}dx^3}{b}+\frac{2d\sqrt{c+dx^3}}{b}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{\frac{bc^2\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{(bc-ad)^2\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{b}+\frac{2d\sqrt{c+dx^3}}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\frac{2bc^2\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2(bc-ad)^2\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{b}+\frac{2d\sqrt{c+dx^3}}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\frac{2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2bc^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a}}{b}+\frac{2d\sqrt{c+dx^3}}{b}\right)$$

input

```
Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]
```

output
```
((2*d*Sqrt[c + d*x^3])/b + ((-2*b*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]
)/a + (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*
d]])/(a*Sqrt[b]))/b)/3
```

3.371.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.371. $\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx$

3.371.4 Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{-\frac{2(ad-bc)^{2}\arctan\left(\frac{b\sqrt{dx^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3}+\frac{2\left(-\operatorname{arctanh}\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{c}}\right)c^{\frac{3}{2}}b+ad\sqrt{dx^{3}+c}\right)\sqrt{(ad-bc)b}}{3}}{ab\sqrt{(ad-bc)b}}$
default	$\frac{\frac{2dx^{3}\sqrt{dx^{3}+c}}{9}+\frac{8c\sqrt{dx^{3}+c}}{9}-\frac{2c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{c}}\right)}{3}}{a}+\frac{-\frac{2(ad-bc)^{2}\arctan\left(\frac{b\sqrt{dx^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3}+\frac{2\left(\frac{\left(-dx^{3}-4c\right)b}{3}+ad\right)\sqrt{dx^{3}+c}\sqrt{(ad}{3}}{ab\sqrt{(ad-bc)b}}$
elliptic	Expression too large to display

input

```
int((d*x^3+c)^(3/2)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output

```
2/3/((a*d-b*c)*b)^(1/2)*(-(a*d-b*c)^2*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*
b)^(1/2))+(-arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(3/2)*b+a*d*(d*x^3+c)^(1/2)
)*((a*d-b*c)*b)^(1/2))/b/a
```

3.371.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 486, normalized size of antiderivative = 4.67

$$\int\frac{\left(c+dx^{3}\right)^{3/2}}{x\left(a+bx^{3}\right)}\,dx=\left[\frac{bc^{\frac{3}{2}}\log\left(\frac{dx^{3}-2\sqrt{dx^{3}+c}\sqrt{c}+2c}{x^{3}}\right)+2\sqrt{dx^{3}+c}ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^{3}+2bc-ad-2\sqrt{d}{bx^{3}+a}\right.}{3ab}\right.$$

input

```
integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/3*(b*c^(3/2)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqr
t(d*x^3 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c -
a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)))/(a*b), 1/3*(b
*c^(3/2)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3
 + c)*a*d + 2*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*s
qrt(-(b*c - a*d)/b)/(b*c - a*d)))/(a*b), 1/3*(2*b*sqrt(-c)*c*arctan(sqrt(d
*x^3 + c)*sqrt(-c)/c) + 2*sqrt(d*x^3 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*
d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b)
)/(b*x^3 + a)))/(a*b), 2/3*(b*sqrt(-c)*c*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c
) + sqrt(d*x^3 + c)*a*d + (b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*
x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))/(a*b)]
```

3.371.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(90) = 180$.

Time = 6.57 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.82

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx = \begin{cases} \frac{2\left(\frac{d^2\sqrt{c+dx^3}}{3b}+\frac{c^2d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}-\frac{d(ad-bc)^2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3ab^2\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{fo} \\ c^{\frac{3}{2}}\left(-\frac{2b\left(\begin{cases}\frac{\frac{a}{2b}+x^3}{a} & \text{for}\ b=0\\ -\frac{\log\left(a-2b\left(\frac{a}{2b}+x^3\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{3a}-\frac{2b\left(\begin{cases}\frac{\frac{a}{2b}+x^3}{a} & \text{for}\ b=0\\ \frac{\log\left(a+2b\left(\frac{a}{2b}+x^3\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{3a}\right) & \text{o} \end{cases}$$

input
```
integrate((d*x**3+c)**(3/2)/x/(b*x**3+a),x)
```

output
```
Piecewise((2*(d**2*sqrt(c + d*x**3)/(3*b) + c**2*d*atan(sqrt(c + d*x**3)/s
qrt(-c))/(3*a*sqrt(-c)) - d*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d
 - b*c)/b))/(3*a*b**2*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (c**(3/2)*(-2*b*
Piecewise(((a/(2*b) + x**3)/a, Eq(b, 0)), (-log(a - 2*b*(a/(2*b) + x**3))/
(2*b), True))/(3*a) - 2*b*Piecewise(((a/(2*b) + x**3)/a, Eq(b, 0)), (log(a
 + 2*b*(a/(2*b) + x**3))/(2*b), True))/(3*a)), True))
```

3.371.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x), x)
```

3.371.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx = \frac{2\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}} + \frac{2\,\sqrt{dx^3+c}d}{3\,b} - \frac{2\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}ab}$$

input
```
integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*c^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)) + 2/3*sqrt(d*x^3 + c
)*d/b - 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(
-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a*b)
```

3.371.9 Mupad [B] (verification not implemented)

Time = 12.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.49

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx = \frac{c^{3/2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a} + \frac{2\,d\,\sqrt{d\,x^3+c}}{3\,b} + \frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d+\sqrt{b}\,\sqrt{d\,x^3+c}\,(a\,d-b\,c)^{3/2}\,2\mathrm{i}}{b\,x^3+a}\right)\,(a\,d-b\,c)^{3/2}\,1\mathrm{i}}{3\,a\,b^{3/2}}$$

input
```
int((c + d*x^3)^(3/2)/(x*(a + b*x^3)),x)
```

output
```
(c^(3/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)
))/x^6))/(3*a) + (2*d*(c + d*x^3)^(1/2))/(3*b) + (log((a^2*d^2 + 2*b^2*c^2
 + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*c*d*
x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*a*b^(3/2))
```

3.371. $\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)} \, dx$

3.372 $\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx$

3.372.1 Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = -\frac{c\sqrt{c+dx^3}}{3ax^3}+\frac{\sqrt{c}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} - \frac{2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}}$$

output
```
-2/3*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^
2/b^(1/2)+1/3*(-3*a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a^2-
1/3*c*(d*x^3+c)^(1/2)/a/x^3
```

3.372.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = \frac{-\frac{ac\sqrt{c+dx^3}}{x^3}+\frac{2(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}}+\sqrt{c}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x]
```

output
```
(-((a*c*Sqrt[c + d*x^3])/x^3) + (2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sq
rt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqrt[b] + Sqrt[c]*(2*b*c - 3*a*d)*ArcT
anh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)
```

3.372.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\left(dx^3+c\right)^{3/2}}{x^6\left(bx^3+a\right)}dx^3$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3}\left(-\frac{\int \frac{d(bc-2ad)x^3+c(2bc-3ad)}{2x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}-\frac{c\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{d(bc-2ad)x^3+c(2bc-3ad)}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(-\frac{\frac{c(2bc-3ad)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{2(bc-ad)^2\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{\frac{2c(2bc-3ad)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{4(bc-ad)^2\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{\frac{4(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2\sqrt{c}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a}}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x]
```

output
```
(-((c*Sqrt[c + d*x^3])/(a*x^3)) - ((-2*Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]])/a + (4*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/(2*a))/3
```

3.372.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.372.4 Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{\frac{2(ad-bc)^2 \arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)x^3}{3}+\frac{2\left(x^3\left(c^{\frac{3}{2}}b-\frac{3ad\sqrt{c}}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)-\frac{\sqrt{d x^3+c}\,ca}{2}\right)\sqrt{(ad-bc)b}}{3}}{a^2\sqrt{(ad-bc)b}\,x^3}$
risch	$-\frac{c\sqrt{d x^3+c}}{3a\,x^3}+\frac{-\frac{2\sqrt{c}\,(3ad-2bc)\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3a}+\frac{4\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3a\sqrt{(ad-bc)b}}}{2a}$
default	$\frac{-\frac{c\sqrt{d x^3+c}}{3x^3}+\frac{2d\sqrt{d x^3+c}}{3}-\sqrt{c}\,d\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{a}-\frac{b\left(\frac{2d\,x^3\sqrt{d x^3+c}}{9}+\frac{8c\sqrt{d x^3+c}}{9}-\frac{2c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3}\right)}{a^2}-$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/3/((a*d-b*c)*b)^(1/2)*((a*d-b*c)^2*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b
)^(1/2))*x^3+(x^3*(c^(3/2)*b-3/2*a*d*c^(1/2))*arctanh((d*x^3+c)^(1/2)/c^(1
/2))-1/2*(d*x^3+c)^(1/2)*c*a)*((a*d-b*c)*b)^(1/2))/a^2/x^3
```

3.372.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 538, normalized size of antiderivative = 4.64

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = \Bigg[-\frac{2\,(bc-ad)x^3\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+(2\,bc-3\,ad)\sqrt{c}x^3\log}{6\,a^2x^3}$$

$$-\frac{4\,(bc-ad)x^3\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+(2\,bc-3\,ad)\sqrt{c}x^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+2\,\sqrt{dx^3}}{6\,a^2x^3}$$

$$-\frac{(2\,bc-3\,ad)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+(bc-ad)x^3\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+\sqrt{dx^3}}{3\,a^2x^3}$$

$$-\frac{2\,(bc-ad)x^3\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+(2\,bc-3\,ad)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\sqrt{dx^3+c}}{3\,a^2x^3}$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/6*(2*(b*c - a*d)*x^3*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d +
2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + (2*b*c - 3*a*d)*sq
rt(c)*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^
3 + c)*a*c)/(a^2*x^3), -1/6*(4*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/b)*arctan
(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (2*b*c - 3*a*d)*sq
rt(c)*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^
3 + c)*a*c)/(a^2*x^3), -1/3*((2*b*c - 3*a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^
3 + c)*sqrt(-c)/c) + (b*c - a*d)*x^3*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*
b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + sqrt(d
*x^3 + c)*a*c)/(a^2*x^3), -1/3*(2*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/b)*arc
tan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (2*b*c - 3*a*d)
*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^3 + c)*a*c)/(a
^2*x^3)]
```

3.372.6 Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx=\int \frac{(c+dx^3)^{\frac{3}{2}}}{x^4(a+bx^3)}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a),x)
```

output
```
Integral((c + d*x**3)**(3/2)/(x**4*(a + b*x**3)), x)
```

3.372.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^4}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^4), x)
```

3.372.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx=\frac{2\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^2}-\frac{(2\,bc^2-3\,acd)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}-\frac{\sqrt{dx^3+c}c}{3\,ax^3}$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c +
 a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) - 1/3*(2*b*c^2 - 3*a*c*d)*arctan(sqrt(
d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/3*sqrt(d*x^3 + c)*c/(a*x^3)
```

3.372.9 Mupad [B] (verification not implemented)

Time = 13.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.44

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)}\,dx = \frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)\left(3\,a\,d-2\,b\,c\right)}{6\,a^2}-\frac{c\,\sqrt{d\,x^3+c}}{3\,a\,x^3}+\frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)^{3/2}\,2\mathrm{i}}{b\,x^3+a}\right)\left(a\,d-b\,c\right)^{3/2}\,1\mathrm{i}}{3\,a^2\,\sqrt{b}}$$

input

```
int((c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x)
```

output

```
(c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)
))/x^6)*(3*a*d - 2*b*c))/(6*a^2) - (c*(c + d*x^3)^(1/2))/(3*a*x^3) + (log(
(a^2*d^2 + 2*b^2*c^2 - b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*
b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3
*a^2*b^(1/2))
```

3.373 $\int \frac{x^3\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx$

3.373.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{x^3\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx=\frac{cx^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},1,-\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/4*c*x^4*AppellF1(4/3,1,-3/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+
d*x^3/c)^(1/2)
```

3.373.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 280 vs. $2(65) = 130$.

Time = 8.32 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.31

$$\int \frac{x^3\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx=\frac{x\left(8\left(c+dx^3\right)\left(14bc-11ad+5bdx^3\right)+\frac{\left(27b^2c^2-88abcd+55a^2d^2\right)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\right.}{a}\right.}{}$$

input
```
Integrate[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
(x*(8*(c + d*x^3)*(14*b*c - 11*a*d + 5*b*d*x^3) + ((27*b^2*c^2 - 88*a*b*c*
d + 55*a^2*d^2)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^
3)/c), -((b*x^3)/a)])/a - (64*a^2*c^2*(-14*b*c + 11*a*d)*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3,
1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2,
 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d
*x^3)/c), -((b*x^3)/a)])))))/(220*b^2*Sqrt[c + d*x^3])
```

3.373.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(c+dx^3\right)^{3/2}}{a+bx^3} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{c\sqrt{c+dx^3} \int \frac{x^3 \left(\frac{dx^3}{c}+1\right)^{3/2}}{bx^3+a}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{cx^4\sqrt{c+dx^3} \operatorname{AppellF1}\left(\frac{4}{3},1,-\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
(c*x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -3/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a*Sqrt[1 + (d*x^3)/c])
```

3.373.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.373.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.24 (sec) , antiderivative size = 800, normalized size of antiderivative = 12.31

method	result	size
risch	Expression too large to display	800
elliptic	Expression too large to display	846
default	Expression too large to display	1101

input
```
int(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-2/55*x*(-5*b*d*x^3+11*a*d-14*b*c)*(d*x^3+c)^(1/2)/b^2+1/55/b^2*(-2/3*I*(5
5*a^2*d^2-88*a*b*c*d+27*b^2*c^2)/b*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2))+55/3*I*a*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d^2*
2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^
2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2
*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+
2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*
d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*
d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.373.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx=\text{Timed out}$$

input
```
integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.373.6 Sympy [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx=\int \frac{x^3(c+dx^3)^{\frac{3}{2}}}{a+bx^3}\,dx$$

input
```
integrate(x**3*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

output
```
Integral(x**3*(c + d*x**3)**(3/2)/(a + b*x**3), x)
```

3.373.7 Maxima [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}x^3}{bx^3+a}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a), x)
```

3.373.8 Giac [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}x^3}{bx^3+a}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a), x)
```

3.373.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx=\int \frac{x^3\,\left(d\,x^3+c\right)^{3/2}}{b\,x^3+a}\,dx$$

input
```
int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

output
```
int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3), x)
```

3.374 $\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx$

3.374.1 Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{cx^2\sqrt{c+dx^3}\operatorname{AppellF1}\left(\frac{2}{3},1,-\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}}$$

output

```
1/2*c*x^2*AppellF1(2/3,1,-3/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+
d*x^3/c)^(1/2)
```

3.374.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 149 vs. 2(65) = 130.

Time = 10.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.29

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{x^2\left(20ad(c+dx^3)+5c(7bc-4ad)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2d(1}{70ab\sqrt{c+dx^3}}$$

input

```
Integrate[(x*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output

```
(x^2*(20*a*d*(c + d*x^3) + 5*c*(7*b*c - 4*a*d)*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*d*(10*b*c - 7*a*d)*x^3
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)
]))/(70*a*b*Sqrt[c + d*x^3])
```

3.374.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx^3\right)^{3/2}}{a+bx^3}dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{c\sqrt{c+dx^3}\int \frac{x\left(\frac{dx^3}{c}+1\right)^{3/2}}{bx^3+a}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{cx^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},1,-\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

output
```
(c*x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 1, -3/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a*Sqrt[1 + (d*x^3)/c])
```

3.374.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.374.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.57 (sec) , antiderivative size = 921, normalized size of antiderivative = 14.17

method	result	size
risch	Expression too large to display	921
default	Expression too large to display	930
elliptic	Expression too large to display	930

input
```
int(x*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/7*d/b*x^2*(d*x^3+c)^(1/2)-1/7/b*(-2/3*I*(7*a*d-10*b*c)/b*3^(1/2)*(-c*d^2
)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+7/3*I*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d
^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d
^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)
+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*...
```

3.374.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.374.6 Sympy [F]

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx=\int \frac{x(c+dx^3)^{\frac{3}{2}}}{a+bx^3}\,dx$$

input
```
integrate(x*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

output
```
Integral(x*(c + d*x**3)**(3/2)/(a + b*x**3), x)
```

3.374.7 Maxima [F]

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}x}{bx^3+a}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a), x)
```

3.374.8 Giac [F]

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}x}{bx^3+a}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a), x)
```

3.374.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx=\int \frac{x\,\left(d\,x^3+c\right)^{3/2}}{b\,x^3+a}\,dx$$

input
```
int((x*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

output
```
int((x*(c + d*x^3)^(3/2))/(a + b*x^3), x)
```

3.375 $\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx$

3.375.1 Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{cx\sqrt{c+dx^3}\operatorname{AppellF1}\left(\frac{1}{3},1,-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}$$

output
```
c*x*AppellF1(1/3,1,-3/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*x^3/
c)^(1/2)
```

3.375.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 351 vs. 2(60) = 120.

Time = 10.33 (sec) , antiderivative size = 351, normalized size of antiderivative = 5.85

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{x\Bigg(\frac{d(8bc-5ad)x^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\Big(-4ac(2ad^2x^3+b(5c^2+2cdx^3+2d^2x^6))\operatorname{AppellF1}\Big(}{(a+bx^3)\Big(-8ac\operatorname{AppellF1}\Big(\frac{1}{3},\frac{1}{2}}}{20}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(a + b*x^3),x]
```

output
```
(x*((d*(8*b*c - 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3,
-((d*x^3)/c), -((b*x^3)/a)])/a + (8*(-4*a*c*(2*a*d^2*x^3 + b*(5*c^2 + 2*c*
d*x^3 + 2*d^2*x^6))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)]
 + 3*d*x^3*(a + b*x^3)*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*
x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b
*x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((
b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
))/(20*b*Sqrt[c + d*x^3])
```

3.375.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{a+bx^3}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{c\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{3/2}}{bx^3+a}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{cx\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},1,-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(3/2)/(a + b*x^3),x]
```

output
```
(c*x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*Sqrt[1 + (d*x^3)/c])
```

3.375.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.375.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.17 (sec) , antiderivative size = 769, normalized size of antiderivative = 12.82

method	result	size
risch	Expression too large to display	769
default	Expression too large to display	776
elliptic	Expression too large to display	776

input
```
int((d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output

```
2/5*d/b*x*(d*x^3+c)^(1/2)-1/5/b*(-2/3*I*(5*a*d-8*b*c)/b*3^(1/2)*(-c*d^2)^(
1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/3*I*(-5*a^2*d^2+10*a*b
*c*d-5*b^2*c^2)/b/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2
*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^
(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3
)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-
c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3
^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(
1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(
2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.375.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx = \text{Timed out}$$

input

```
integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

output

```
Timed out
```

3.375.6 Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{a+bx^3}\,dx$$

input

```
integrate((d*x**3+c)**(3/2)/(b*x**3+a),x)
```

output
```
Integral((c + d*x**3)**(3/2)/(a + b*x**3), x)
```

3.375.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{bx^3+a}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/(b*x^3 + a), x)
```

3.375.8 Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{bx^3+a}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/(b*x^3 + a), x)
```

3.375.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{b\,x^3+a}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(a + b*x^3),x)
```

output
```
int((c + d*x^3)^(3/2)/(a + b*x^3), x)
```

3.376 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(a+bx^3\right)}\,dx$

3.376.1 Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(a+bx^3\right)}\,dx=-\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},1,-\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-c*AppellF1(-1/3,1,-3/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x/(1+d*x^
3/c)^(1/2)
```

3.376.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 148 vs. 2(63) = 126.

Time = 10.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.35

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(a+bx^3\right)}\,dx=\frac{-20ac\left(c+dx^3\right)+5c(-2bc+5ad)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2d(}{20a^2x\sqrt{c+dx^3}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x]
```

output
```
(-20*a*c*(c + d*x^3) + 5*c*(-2*b*c + 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*Appell
F1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*d*(b*c + 2*a*d)*x^6*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])
/(20*a^2*x*Sqrt[c + d*x^3])
```

3.376.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}dx$$
$$\Big\downarrow \text{1013}$$
$$\frac{c\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{3/2}}{x^2(bx^3+a)}dx}{\sqrt{\frac{dx^3}{c}+1}}$$
$$\Big\downarrow \text{1012}$$
$$-\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},1,-\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x]
```

output
```
-((c*Sqrt[c + d*x^3]*AppellF1[-1/3, 1, -3/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*x*Sqrt[1 + (d*x^3)/c]))
```

3.376.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.376.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.70 (sec) , antiderivative size = 920, normalized size of antiderivative = 14.60

method	result	size
risch	Expression too large to display	920
elliptic	Expression too large to display	924
default	Expression too large to display	1404

input
```
int((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-c/a*(d*x^3+c)^(1/2)/x+1/2/a*(-2/3*I*(2*a*d+b*c)/b*3^(1/2)*(-c*d^2)^(1/3)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)))+2/3*I*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d^2*2^(1/
2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3
))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)...
```

3.376.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.376.6 Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x^2(a+bx^3)}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**2/(b*x**3+a),x)
```

output
```
Integral((c + d*x**3)**(3/2)/(x**2*(a + b*x**3)), x)
```

3.376.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^2), x)
```

3.376.8 Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^2), x)
```

3.376.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx=\int \frac{(d\,x^3+c)^{3/2}}{x^2\,(b\,x^3+a)}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)), x)
```

3.377 $\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx$

3.377.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx=-\frac{c\sqrt{c+dx^3}\operatorname{AppellF1}\left(-\frac{2}{3},1,-\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-1/2*c*AppellF1(-2/3,1,-3/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x^2/(
1+d*x^3/c)^(1/2)
```

3.377.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 343 vs. $2(65) = 130$.

Time = 10.35 (sec) , antiderivative size = 343, normalized size of antiderivative = 5.28

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx=$$

$$-\frac{d(bc-4ad)x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8ac\Bigl(-4ac(2ac+6bcx^3-5adx^3+2bdx^6)\operatorname{AppellF1}\bigl(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-}{(a+bx^3)\Bigl(-8ac\operatorname{AppellF1}\bigl(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c}}}{16a^2x^2\sqrt{c+dx^3}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x]
```

output
```
-1/16*(d*(b*c - 4*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3,
-((d*x^3)/c), -((b*x^3)/a)] + (8*a*c*(-4*a*c*(2*a*c + 6*b*c*x^3 - 5*a*d*x^
3 + 2*b*d*x^6)*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*
x^3*(a + b*x^3)*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c)
, -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a
)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x
^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/
a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(a^2*
x^2*Sqrt[c + d*x^3])
```

3.377.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^3\left(a+bx^3\right)}dx$$

$$\Big\downarrow \textcolor{blue}{1013}$$

$$\frac{c\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{3/2}}{x^3\left(bx^3+a\right)}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \textcolor{blue}{1012}$$

$$-\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x]
```

output
```
-1/2*(c*Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -3/2, 1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(a*x^2*Sqrt[1 + (d*x^3)/c])
```

3.377.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.377.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.26 (sec) , antiderivative size = 771, normalized size of antiderivative = 11.86

method	result	size
risch	Expression too large to display	771
elliptic	Expression too large to display	772
default	Expression too large to display	1096

input
```
int((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c/a*(d*x^3+c)^(1/2)/x^2+1/4/a*(-2/3*I*(4*a*d-b*c)/b*3^(1/2)*(-c*d^2)^
(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/3*I*(-4*a^2*d^2+8*a*b
*c*d-4*b^2*c^2)/b/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2
*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^
(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3
)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-
c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3
^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(
1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(
2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.377.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.377.6 Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x^3(a+bx^3)}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**3/(b*x**3+a),x)
```

output
```
Integral((c + d*x**3)**(3/2)/(x**3*(a + b*x**3)), x)
```

3.377.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^3\,(a+bx^3)}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^3}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^3), x)
```

3.377.8 Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^3\,(a+bx^3)}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^3}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^3), x)
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^3\,(a+bx^3)}\,dx=\int \frac{(d\,x^3+c)^{3/2}}{x^3\,(b\,x^3+a)}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)), x)
```

3.378 $\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

3.378.1 Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx=-\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2(c+dx^3)^{3/2}}{9bd^2}-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

output
```
2/9*(d*x^3+c)^(3/2)/b/d^2-2/3*a^2*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*
c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-2/3*(a*d+b*c)*(d*x^3+c)^(1/2)/b^2/d^2
```

3.378.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\frac{2\sqrt{c+dx^3}(-2bc-3ad+bdx^3)}{9b^2d^2}+\frac{2a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}\sqrt{-bc+ad}}$$

input
```
Integrate[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(2*Sqrt[c + d*x^3]*(-2*b*c - 3*a*d + b*d*x^3))/(9*b^2*d^2) + (2*a^2*ArcTan
[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2)*Sqrt[-(b*c) + a
*d])
```

3.378.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^3+a)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{a^2}{b^2(bx^3+a)\sqrt{dx^3+c}}+\frac{\sqrt{dx^3+c}}{bd}+\frac{-bc-ad}{b^2d\sqrt{dx^3+c}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^3}(ad+bc)}{b^2d^2}+\frac{2(c+dx^3)^{3/2}}{3bd^2}\right)$$

input
```
Int[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
((-2*(b*c + a*d)*Sqrt[c + d*x^3])/(b^2*d^2) + (2*(c + d*x^3)^(3/2))/(3*b*d
^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b^(5/2)*
Sqrt[b*c - a*d]))/3
```

3.378.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.378.4 Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{2\left(-bd\,x^{3}+3ad+2bc\right)\sqrt{d\,x^{3}+c}}{9d^{2}b^{2}}+\frac{2a^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{3b^{2}\sqrt{\left(ad-bc\right)b}}$
pseudoelliptic	$\frac{\frac{2\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)a^{2}d^{2}}{3}-\frac{2\left(\left(-\frac{b\,x^{3}}{3}+a\right)d+\frac{2bc}{3}\right)\sqrt{d\,x^{3}+c}\,\sqrt{\left(ad-bc\right)b}}{3}}{b^{2}d^{2}\sqrt{\left(ad-bc\right)b}}$
default	$\frac{\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d}-\frac{4c\sqrt{d\,x^{3}+c}}{9d^{2}}}{b}-\frac{2a\sqrt{d\,x^{3}+c}}{3b^{2}d}+\frac{2a^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{3b^{2}\sqrt{\left(ad-bc\right)b}}$
elliptic	$\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9bd}+\frac{2\left(-\frac{a}{b^{2}}-\frac{2c}{3db}\right)\sqrt{d\,x^{3}+c}}{3d}-\frac{ia^{2}\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right)}{}$

input
```
int(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(-b*d*x^3+3*a*d+2*b*c)*(d*x^3+c)^(1/2)/d^2/b^2+2/3*a^2/b^2/((a*d-b*c)
*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

3.378.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.78

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{3\sqrt{b^2c-abd}a^2d^2\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,(2\,b^3c^2+ab^2cd-3\,a^2bd^2-(b^3cd-ab^2d^2)x^3)\sqrt{c}}{9\,(b^4cd^2-ab^3d^3)}\right.$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/9*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*
x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*b^3*c^2 + a*b^2*c*d - 3*
a^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(b^4*c*d^2 - a*b^3
*d^3), 2/9*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^3 + c)*sqrt(-b^
2*c + a*b*d)/(b*d*x^3 + b*c)) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^
3*c*d - a*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

3.378.6 Sympy [F]

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**8/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**8/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.378.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.378.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\,a^2\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^2} + \frac{2\left((dx^3+c)^{\frac{3}{2}}b^2d^4-3\,\sqrt{dx^3+c}b^2cd^4-3\,\sqrt{dx^3+c}abd^5\right)}{9\,b^3d^6}$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
2/3*a^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^3 + c)*b^2*c*d^4 - 3
*sqrt(d*x^3 + c)*a*b*d^5)/(b^3*d^6)
```

3.378.9 Mupad [B] (verification not implemented)

Time = 9.69 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,b\,d} - \frac{\left(\frac{2\,a}{b^2}+\frac{4\,c}{3\,b\,d}\right)\,\sqrt{d\,x^3+c}}{3\,d} + \frac{a^2\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^8/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
(2*x^3*(c + d*x^3)^(1/2))/(9*b*d) - (((2*a)/b^2 + (4*c)/(3*b*d))*(c + d*x^
3)^(1/2))/(3*d) + (a^2*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d -
 b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*b^(5/2)*(a*d - b*c)^(1/2))
```

3.379 $\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

3.379.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{c+dx^3}}{3bd} + \frac{2a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

output

```
2/3*a*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)
^(1/2)+2/3*(d*x^3+c)^(1/2)/b/d
```

3.379.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{d} - \frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}\right)}{3b^{3/2}}$$

input

```
Integrate[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output

```
(2*((Sqrt[b]*Sqrt[c + d*x^3])/d - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt
[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d]))/(3*b^(3/2))
```

3.379.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(bx^3+a)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{bd}-\frac{a\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{bd}-\frac{2a\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^3}}{bd}\right)$$

input
```
Int[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
((2*Sqrt[c + d*x^3])/(b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b
*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/3
```

3.379.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.379.4 Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result
default	$\frac{2\sqrt{d\,x^3+c}}{3bd}-\frac{2a\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b\sqrt{(ad-bc)b}}$
risch	$\frac{2\sqrt{d\,x^3+c}}{3bd}-\frac{2a\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b\sqrt{(ad-bc)b}}$
pseudoelliptic	$-\frac{2\left(\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)ad-\sqrt{d\,x^3+c}\,\sqrt{(ad-bc)b}\right)}{3bd\sqrt{(ad-bc)b}}$
elliptic	$\frac{2\sqrt{d\,x^3+c}}{3bd}+\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)}}}{}\right.}{}$

input
```
int(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(d*x^3+c)^(1/2)/b/d-2/3*a/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/
2)/((a*d-b*c)*b)^(1/2))
```

3.379.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.77

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}(b^2c-abd)}{3\,(b^3cd-ab^2d^2)}, -\frac{2\left(\sqrt{-b^2c+abd}ad\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-\sqrt{dx^3+c}(b^2c-abd)\right)}{3\,(b^3cd-ab^2d^2)}\right]$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 +
c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(b^2*c - a*b*d))/
(b^3*c*d - a*b^2*d^2), -2/3*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^3 +
c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - sqrt(d*x^3 + c)*(b^2*c - a*b*d)
)/(b^3*c*d - a*b^2*d^2)]
```

3.379.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**5/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.379.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.379.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(\frac{ad\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b}-\frac{\sqrt{dx^3+c}}{b}\right)}{3\,d}$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-2/3*(a*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*
b*d)*b) - sqrt(d*x^3 + c)/b)/d
```

3.379.9 Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,b\,d}+\frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^5/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*b*d) + (a*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^
(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*b^(3/2)*(a*d - b
*c)^(1/2))
```

3.380 $\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

3.380.1 Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

output
```
-2/3*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^
(1/2)
```

3.380.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3\sqrt{b}\sqrt{-bc+ad}}$$

input
```
Integrate[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*Sqrt[b]*Sqrt[-
(b*c) + a*d])
```

3.380.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {946, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3}\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \; 73$$

$$\frac{2\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{3d}$$

$$\Big\downarrow \; 221$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

input

```
Int[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output

```
(-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*Sqrt[b*
c - a*d])
```

3.380.3.1 Defintions of rubi rules used

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.380.4 Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result
default	$\frac{2\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}$
pseudoelliptic	$\frac{2\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\right.}{}}}{}\right.}{}$

input
```
int(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

3.380.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.55

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)}{3\,\sqrt{b^2c-abd}}, \frac{2\,\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)}{3\,(b^2c-abd)}\right]$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(
b*x^3 + a))/sqrt(b^2*c - a*b*d), 2/3*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^
3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c))/(b^2*c - a*b*d)]
```

3.380.6 Sympy [A] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.73

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}} & \text{for } d\neq 0 \\ \begin{cases} \frac{x^3}{3a\sqrt{c}} & \text{for } b=0 \\ \tilde{\infty}x^3 & \text{for } \sqrt{c}=0 \\ \frac{\log\left(3a\sqrt{c}+3b\sqrt{c}x^3\right)}{3b\sqrt{c}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b
*c)/b)), Ne(d, 0)), (Piecewise((x**3/(3*a*sqrt(c)), Eq(b, 0)), (zoo*x**3,
Eq(sqrt(c), 0)), (log(3*a*sqrt(c) + 3*b*sqrt(c)*x**3)/(3*b*sqrt(c)), True)
), True))
```

3.380.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.380.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}}$$

input

```
integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output

```
2/3*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

3.380.9 Mupad [B] (verification not implemented)

Time = 10.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{\ln\left(\frac{a\,d\,1\mathrm{i}-b\,c\,2\mathrm{i}+2\,\sqrt{d\,x^3+c}\,\sqrt{a\,b\,d-b^2\,c}-b\,d\,x^3\,1\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,\sqrt{a\,b\,d-b^2\,c}}$$

input

```
int(x^2/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output

```
(log((a*d*1i - b*c*2i + 2*(c + d*x^3)^(1/2)*(a*b*d - b^2*c)^(1/2) - b*d*x^
3*1i)/(a + b*x^3))*1i)/(3*(a*b*d - b^2*c)^(1/2))
```

3.381 $\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}}\,dx$

3.381.1 Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}$$

output
```
-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a/c^(1/2)+2/3*arctanh(b^(1/2)*(d*x^3
+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

3.381.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a}$$

input
```
Integrate[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/3*((2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqr
t[-(b*c) + a*d] + (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c])/a
```

3.381.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3(bx^3+a)\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{b\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2b\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}\right)$$

input
```
Int[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
((-2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/3
```

3.381.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.381.4 Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}-\frac{2b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3a\sqrt{(ad-bc)b}}$	66
pseudoelliptic	$-\frac{2\left(b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{c}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\sqrt{(ad-bc)b}\right)}{3a\sqrt{c}\,\sqrt{(ad-bc)b}}$	78
elliptic	Expression too large to display	1598

input
```
int(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a/c^(1/2)-2/3*b/a/((a*d-b*c)*b)^(1/2
)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

3.381.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.07

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{3\,ac}, \frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\right.}{}\right.$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(c*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)
*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + sqrt(c)*log((d*x^3 - 2*sq
rt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), 1/3*(2*c*sqrt(-b/(b*c - a*d))*ar
ctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) +
sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), 1/3*(c*
sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c -
a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^3 + c)
*sqrt(-c)/c))/(a*c), 2/3*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(
b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + sqrt(-c)*arctan(sqrt(d*
x^3 + c)*sqrt(-c)/c))/(a*c)]
```

3.381.6 Sympy [A] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\left(-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}\right)}{d} & \text{for}\ d\neq 0 \\ \frac{2\operatorname{atan}\left(\frac{2\left(\frac{a}{2b}+x^3\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{3b\sqrt{c}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Piecewise((2*(-d*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d
 - b*c)/b)) + d*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c)))/d, Ne(d, 0
)), (2*atan(2*(a/(2*b) + x**3)/sqrt(-a**2/b**2))/(3*b*sqrt(c)*sqrt(-a**2/b
**2)), True))
```

3.381.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=\int \frac{1}{(bx^3+a)\sqrt{dx^3+cx}}\,dx$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x), x)
```

3.381.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{2\,b\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}a}+\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-2/3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c))
```

3.381.9 Mupad [B] (verification not implemented)

Time = 11.77 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=\frac{\ln\left(\frac{{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a\,\sqrt{c}}+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,a\,\sqrt{a\,d-b\,c}}$$

input
```
int(1/(x*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(
3*a*c^(1/2)) + (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d
- b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*a*(a*d - b*c)^(1/2))
```

3.381. $\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}}\,dx$

3.382 $\int \frac{1}{x^4(a+bx^3)\sqrt{c+dx^3}}\,dx$

3.382.1 Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{1}{x^4(a+bx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{3acx^3}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} -\frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}$$

output
```
1/3*(a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-2/3*b^(3/2)*a
rctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/3*
(d*x^3+c)^(1/2)/a/c/x^3
```

3.382.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{-\frac{a\sqrt{c+dx^3}}{cx^3}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{3/2}}}{3a^2}$$

input
```
Integrate[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-((a*Sqrt[c + d*x^3])/(c*x^3)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
3])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/c^(3/2))/(3*a^2)
```

3.382.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+b x^3\right) \sqrt{c+d x^3}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^6 \left(b x^3+a\right) \sqrt{d x^3+c}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3} \left(-\frac{\int \frac{b d x^3+2 b c+a d}{2 x^3 \left(b x^3+a\right) \sqrt{d x^3+c}}dx^3}{a c}-\frac{\sqrt{c+d x^3}}{a c x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{\int \frac{b d x^3+2 b c+a d}{x^3 \left(b x^3+a\right) \sqrt{d x^3+c}}dx^3}{2 a c}-\frac{\sqrt{c+d x^3}}{a c x^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3} \left(-\frac{\frac{(a d+2 b c) \int \frac{1}{x^3 \sqrt{d x^3+c}}dx^3}{a}-\frac{2 b^2 c \int \frac{1}{\left(b x^3+a\right) \sqrt{d x^3+c}}dx^3}{a}}{2 a c}-\frac{\sqrt{c+d x^3}}{a c x^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3} \left(-\frac{\frac{2 (a d+2 b c) \int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{d x^3+c}}{a d}-\frac{4 b^2 c \int \frac{1}{\frac{b x^6}{d}+a-\frac{b c}{d}}d\sqrt{d x^3+c}}{a d}}{2 a c}-\frac{\sqrt{c+d x^3}}{a c x^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{\frac{4b^{3/2}c\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(ad+2bc)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2ac}-\frac{\sqrt{c+dx^3}}{acx^3}\right)$$

input
```
Int[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-(Sqrt[c + d*x^3]/(a*c*x^3)) - ((-2*(2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]
/Sqrt[c]])/(a*Sqrt[c]) + (4*b^(3/2)*c*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sq
rt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*c))/3
```

3.382.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.382.4 Maple [A] (verified)

Time = 4.61 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{\frac{2b^2 \arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{a\sqrt{d x^3+c}}{c x^3}+\frac{(ad+2bc)\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{3a^2}$	92
risch	$-\frac{\sqrt{d x^3+c}}{3ac x^3}-\frac{-\frac{2(ad+2bc)\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}-\frac{4b^2 c \arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3a\sqrt{(ad-bc)b}}}{2ac}$	106
default	$\frac{-\frac{\sqrt{d x^3+c}}{3c x^3}+\frac{d \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{a}+\frac{2b \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{2b^2 \arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3a^2\sqrt{(ad-bc)b}}$	111
elliptic	Expression too large to display	1652

input
```
int(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*(2*b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^
(1/2))-a/c*(d*x^3+c)^(1/2)/x^3+(a*d+2*b*c)/c^(3/2)*arctanh((d*x^3+c)^(1/2)
/c^(1/2)))
```

3.382.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 565, normalized size of antiderivative = 4.83

$$\int \frac{1}{x^4 (a+bx^3) \sqrt{c+dx^3}}\,dx$$

$$= \left[\frac{2\,bc^2x^3\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right) + (2\,bc+ad)\sqrt{c}x^3\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right) - 2}{6\,a^2c^2x^3} \right.$$

$$-\frac{4\,bc^2x^3\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right) - (2\,bc+ad)\sqrt{c}x^3\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right) + 2\,\sqrt{dx^3}}{6\,a^2c^2x^3}$$

$$-\frac{2\,bc^2x^3\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right) + (2\,bc+ad)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + \sqrt{dx^3+}}{3\,a^2c^2x^3}$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(2*b*c^2*x^3*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(
d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + (2*b*c + a*d)*s
qrt(c)*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x
^3 + c)*a*c)/(a^2*c^2*x^3), -1/6*(4*b*c^2*x^3*sqrt(-b/(b*c - a*d))*arctan(
-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - (2*b*
c + a*d)*sqrt(c)*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) +
2*sqrt(d*x^3 + c)*a*c)/(a^2*c^2*x^3), 1/3*(b*c^2*x^3*sqrt(b/(b*c - a*d))*l
og((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*
d)))/(b*x^3 + a)) - (2*b*c + a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt
(-c)/c) - sqrt(d*x^3 + c)*a*c)/(a^2*c^2*x^3), -1/3*(2*b*c^2*x^3*sqrt(-b/(b
*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x
^3 + b*c)) + (2*b*c + a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c)
 + sqrt(d*x^3 + c)*a*c)/(a^2*c^2*x^3)]
```

3.382.6 Sympy [F]

$$\int \frac{1}{x^4 (a+bx^3) \sqrt{c+dx^3}} \, dx = \int \frac{1}{x^4 (a+bx^3) \sqrt{c+dx^3}} \, dx$$

input
```
integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.382.7 Maxima [F]

$$\int \frac{1}{x^4 (a+bx^3) \sqrt{c+dx^3}} \, dx = \int \frac{1}{(bx^3+a)\sqrt{dx^3+c}x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^4), x)
```

3.382.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^4 (a+bx^3) \sqrt{c+dx^3}} \, dx = \frac{2 \, b^2 \arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3 \, \sqrt{-b^2c+abd}a^2} - \frac{(2 \, bc+ad) \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3 \, a^2\sqrt{-c}c} - \frac{\sqrt{dx^3+c}}{3 \, acx^3}$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
2/3*b^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*a^2) - 1/3*(2*b*c + a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)
*c) - 1/3*sqrt(d*x^3 + c)/(a*c*x^3)
```

3.382.9 Mupad [B] (verification not implemented)

Time = 12.83 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^4 (a+bx^3) \sqrt{c+dx^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)^3}{x^6}\right)\,\left(a\,d+2\,b\,c\right)}{6\,a^2\,c^{3/2}} - \frac{\sqrt{d\,x^3+c}}{3\,a\,c\,x^3} + \frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,a^2\,\sqrt{a\,d-b\,c}}$$

input

```
int(1/(x^4*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output

```
(log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*
(a*d + 2*b*c))/(6*a^2*c^(3/2)) - (c + d*x^3)^(1/2)/(3*a*c*x^3) + (b^(3/2)*
log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^
3)/(a + b*x^3))*1i)/(3*a^2*(a*d - b*c)^(1/2))
```

3.383 $\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

3.383.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}$$

output
```
1/4*x^4*AppellF1(4/3,1,1/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/(d*x
^3+c)^(1/2)
```

3.383.2 Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\frac{x^4\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{4a\sqrt{c+dx^3}}$$

input
```
Integrate[x^3/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3
)/a)])/(4*a*Sqrt[c + d*x^3])
```

3.383.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{(bx^3+a)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}$$

input
```
Int[x^3/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*a*Sqrt[c + d*x^3])
```

3.383.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.383.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.28 (sec) , antiderivative size = 719, normalized size of antiderivative = 11.23

method	result	size
default	Expression too large to display	719
elliptic	Expression too large to display	719

input

```
int(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2/3*I/b*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2))+1/3*I*a/b/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))
^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1
/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2
)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3
)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.383.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input

```
integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output

```
Timed out
```

3.383.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**3/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.383.7 Maxima [F]

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.383.8 Giac [F]

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.383.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(b\,x^3+a)\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(x^3/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

3.384 $\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

3.384.1 Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\frac{x^2\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}$$

output

```
1/2*x^2*AppellF1(2/3,1,1/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/(d*x
^3+c)^(1/2)
```

3.384.2 Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\frac{x^2\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{2a\sqrt{c+dx^3}}$$

input

```
Integrate[x/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output

```
(x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3
)/a)])/(2*a*Sqrt[c + d*x^3])
```

3.384.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$
$$\Big\downarrow \text{1013}$$
$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x}{(bx^3+a)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$
$$\Big\downarrow \text{1012}$$
$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}$$

input
```
Int[x/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1, 1/2, 5/3, -((b*x^3)/a), -((d*x^3
)/c)])/(2*a*Sqrt[c + d*x^3])
```

3.384.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.384.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.26 (sec) , antiderivative size = 429, normalized size of antiderivative = 6.70

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\,d\right.}{}\right.}{2\left(-c\right.}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\,d\right.}{}\right.}{2\left(-c\right.}}}{}\right.}{}$

input
```
int(x/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/d^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d
*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1
/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1
/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^
2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2
*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3
*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.384.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.384.6 Sympy [F]

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.384.7 Maxima [F]

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.384.8 Giac [F]

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.384.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x}{(b\,x^3+a)\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(x/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
int(x/((a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

3.385 $\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

3.385.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}$$

output
```
x*AppellF1(1/3,1,1/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/(d*x^3+c)^
(1/2)
```

3.385.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 10.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx=$$

$$-\frac{8acx\,\mathrm{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\sqrt{c+dx^3}\,\big(-8ac\,\mathrm{AppellF1}\big(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\big)+3x^3\big(2bc\,\mathrm{AppellF1}\big(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\big)}$$

input
```
Integrate[1/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-8*a*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*
x^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b
*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3
)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

3.385.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(bx^3+a)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}$$

input
```
Int[1/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*Sqrt[c + d*x^3])
```

3.385.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.385.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.42 (sec) , antiderivative size = 429, normalized size of antiderivative = 7.27

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\,d^{2}\right.}{}\right.}{2\left(-c\right.}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\,d^{2}\right.}{}\right.}{2\left(-c\right.}}}{}\right.}{}$

input

```
int(1/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

3.385. $\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

output
```
-1/3*I/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x
-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*
d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha
-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.385.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.385.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.385.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.385.8 Giac [F]

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.385.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\int \frac{1}{(b\,x^3+a)\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(1/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/((a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

3.386 $\int \frac{1}{x^2(a+bx^3)\sqrt{c+dx^3}}\,dx$

3.386.1 Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

output
```
-AppellF1(-1/3,1,1/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/x/(d*x^3+c
)^(1/2)
```

3.386.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(62) = 124.

Time = 10.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$
$$=\frac{-20a(c+dx^3)+5(-2bc+ad)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bdx^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}}{20a^2cx\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^2*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-20*a*(c + d*x^3) + 5*(-2*b*c + a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3
, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^6*Sqrt[1 + (d*x^3)/c]
*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(20*a^2*c*x*Sqrt[
c + d*x^3])
```

3.386.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3) \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^2(bx^3+a)\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

input
```
Int[1/(x^2*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1, 1/2, 2/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*x*Sqrt[c + d*x^3]))
```

3.386.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.386.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.93 (sec) , antiderivative size = 890, normalized size of antiderivative = 14.35

method	result	size
default	Expression too large to display	890
elliptic	Expression too large to display	891
risch	Expression too large to display	892

input
```
int(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1
/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2))))+1/3*I*b/a/d^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(
1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(
1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))
/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-
I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*...
```

3.386.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.386.6 Sympy [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.386.7 Maxima [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

3.386.8 Giac [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

3.386.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3) \sqrt{c+dx^3}} \, dx = \int \frac{1}{x^2 \left(b\,x^3+a\right) \sqrt{d\,x^3+c}} \, dx$$

input
```
int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

3.387 $\int \frac{1}{x^3(a+bx^3)\sqrt{c+dx^3}}\,dx$

3.387.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}$$

output
```
-1/2*AppellF1(-2/3,1,1/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/x^2/(d
*x^3+c)^(1/2)
```

3.387.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 339 vs. 2(64) = 128.

Time = 10.30 (sec) , antiderivative size = 339, normalized size of antiderivative = 5.30

$$\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$
$$=\frac{-bdx^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8a\left(-4ac(2ac+6bcx^3+3adx^3+2bdx^6)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right.}{(a+bx^3)\left(8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3a\right.}}{16a^2cx^2\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^3*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
(-(b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)]) + (8*a*(-4*a*c*(2*a*c + 6*b*c*x^3 + 3*a*d*x^3 + 2*b*d*x^6)*Ap
pellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(a + b*x^3)*(
c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] +
 a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3
)*(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*x^3*(2
*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1
[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(16*a^2*c*x^2*Sqrt[c +
d*x^3])
```

3.387.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+bx^3\right) \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{x^3 (bx^3+a) \sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1} \operatorname{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

output
```
-1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -((b*x^3)/a), -((d*x
^3)/c)])/(a*x^2*Sqrt[c + d*x^3])
```

3.387.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.387.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.88 (sec) , antiderivative size = 738, normalized size of antiderivative = 11.53

method	result	size
default	Expression too large to display	738
elliptic	Expression too large to display	739
risch	Expression too large to display	740

input
```
int(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/2*(d*x^3+c)^(1/2)/c/x^2+1/6*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I*b/a/d^2*2^(1/2)*sum(1/_alpha^2/(a
*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1
/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)
^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(
1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*
b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*b+a))
```

3.387.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.387.6 Sympy [F]

$$\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.387.7 Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3) \sqrt{c+dx^3}}\, dx = \int \frac{1}{(bx^3+a)\sqrt{dx^3+c}x^3}\, dx$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

3.387.8 Giac [F]

$$\int \frac{1}{x^3 (a+bx^3) \sqrt{c+dx^3}}\, dx = \int \frac{1}{(bx^3+a)\sqrt{dx^3+c}x^3}\, dx$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

3.387.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3) \sqrt{c+dx^3}}\, dx = \int \frac{1}{x^3\, (b\,x^3+a)\, \sqrt{d\,x^3+c}}\, dx$$

input
```
int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

3.388 $\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

3.388.1 Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx=\frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{3bd^2}-\frac{2a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

output
```
-2/3*a^2*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b
*c)^(3/2)+2/3*c^2/d^2/(-a*d+b*c)/(d*x^3+c)^(1/2)+2/3*(d*x^3+c)^(1/2)/b/d^2
```

3.388.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx=\frac{2\left(\frac{\sqrt{b}(ad(c+dx^3)-bc(2c+dx^3))}{d^2(-bc+ad)\sqrt{c+dx^3}}-\frac{a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}\right)}{3b^{3/2}}$$

input
```
Integrate[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2*((Sqrt[b]*(a*d*(c + d*x^3) - b*c*(2*c + d*x^3)))/(d^2*(-(b*c) + a*d)*Sq
rt[c + d*x^3]) - (a^2*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]]
)/(-(b*c) + a*d)^(3/2)))/(3*b^(3/2))
```

3.388.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^3+a)\left(dx^3+c\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{98}$$

$$\frac{1}{3}\int \left(\frac{a^2}{b(bc-ad)\left(bx^3+a\right)\sqrt{dx^3+c}}+\frac{1}{bd\sqrt{dx^3+c}}+\frac{c^2}{d(ad-bc)\left(dx^3+c\right)^{3/2}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}(bc-ad)^{3/2}}+\frac{2c^2}{d^2\sqrt{c+dx^3}(bc-ad)}+\frac{2\sqrt{c+dx^3}}{bd^2}\right)$$

input
```
Int[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((2*c^2)/(d^2*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*Sqrt[c + d*x^3])/(b*d^2) -
 (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b^(3/2)*(b*c
- a*d)^(3/2)))/3
```

3.388.3.1 Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.388.4 Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

method	result
risch	$\frac{2\sqrt{dx^3+c}}{3bd^2}-\frac{2a^2\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b(ad-bc)\sqrt{(ad-bc)b}}-\frac{2c^2}{3d^2(ad-bc)\sqrt{dx^3+c}}$
pseudoelliptic	$\frac{2\sqrt{dx^3+c}}{3bd^2}-\frac{2a^2\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b(ad-bc)\sqrt{(ad-bc)b}}-\frac{2c^2}{3d^2(ad-bc)\sqrt{dx^3+c}}$
default	$\frac{\frac{2c}{3d^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2\sqrt{dx^3+c}}{3d^2}}{b}+\frac{2a}{3b^2d\sqrt{dx^3+c}}-\frac{2a^2\left(b\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{dx^3+c}+\sqrt{(ad-bc)b}\right)}{b^2\sqrt{(ad-bc)b}\sqrt{dx^3+c}\,(3ad-3bc)}$
elliptic	$-\frac{2c^2}{3d^2(ad-bc)\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2\sqrt{dx^3+c}}{3bd^2}+\frac{ia^2\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-cd^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)}{d}\right)}{\left(-cd^2\right)^{\frac{1}{3}}}}}{}\right)}{}$

input
```
int(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(d*x^3+c)^(1/2)/b/d^2-2/3/b*a^2/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan(b
*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))-2/3/d^2*c^2/(a*d-b*c)/(d*x^3+c)^(1/2
)
```

3.388.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(87) = 174$.

Time = 0.39 (sec) , antiderivative size = 440, normalized size of antiderivative = 4.11

$$\int \frac{x^8}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \Bigg[-\frac{(a^2d^3x^3+a^2cd^2)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,(2\,b^3c^3}{3\,(b^4c^3d^2-2\,ab^3c^2d^3+a^2b^2cd^4+(b^4c^2}$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/3*((a^2*d^3*x^3 + a^2*c*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c
- a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*b^3*c^3
 - 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x
^3)*sqrt(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3 + a^2*b^2*c*d^4 + (b^4
*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3), 2/3*((a^2*d^3*x^3 + a^2*c*d^
2)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x
^3 + b*c)) + (2*b^3*c^3 - 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b
^2*c*d^2 + a^2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3
 + a^2*b^2*c*d^4 + (b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3)]
```

3.388.6 Sympy [F]

$$\int \frac{x^8}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^8}{(a+bx^3)\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**8/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**8/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.388.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.388.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

$$\int \frac{x^8}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx = \frac{2\,a^2\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,(b^2c-abd)\sqrt{-b^2c+abd}} + \frac{2\,c^2}{3\,(bcd^2-ad^3)\sqrt{dx^3+c}} + \frac{2\sqrt{dx^3+c}}{3\,bd^2}$$

input
```
integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
2/3*a^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*d)*sq
rt(-b^2*c + a*b*d)) + 2/3*c^2/((b*c*d^2 - a*d^3)*sqrt(d*x^3 + c)) + 2/3*sq
rt(d*x^3 + c)/(b*d^2)
```

3.388.9 Mupad [B] (verification not implemented)

Time = 10.81 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,b\,d^2} - \frac{2\,c^2}{3\,d^2\sqrt{d\,x^3+c}\,(a\,d-b\,c)} + \frac{a^2\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,b^{3/2}\,(a\,d-b\,c)^{3/2}}$$

input
```
int(x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*b*d^2) - (2*c^2)/(3*d^2*(c + d*x^3)^(1/2)*(a*d -
b*c)) + (a^2*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2
)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*b^(3/2)*(a*d - b*c)^(3/2))
```

3.388. $\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

3.389 $\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

3.389.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}}+\frac{2a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}$$

output
```
2/3*a*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b
^(1/2)-2/3*c/d/(-a*d+b*c)/(d*x^3+c)^(1/2)
```

3.389.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{3}\left(\frac{c}{d(-bc+ad)\sqrt{c+dx^3}}+\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

input
```
Integrate[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2*(c/(d*(-(b*c) + a*d)*Sqrt[c + d*x^3]) + (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x
^3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2))))/3
```

3.389.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(bx^3+a)\left(dx^3+c\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(-\frac{a\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{bc-ad}-\frac{2c}{d\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{2a\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{d(bc-ad)}-\frac{2c}{d\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{2c}{d\sqrt{c+dx^3}(bc-ad)}\right)$$

input

```
Int[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output

```
((-2*c)/(d*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2)))/3
```

3.389.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.389.4 Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{\frac{2a\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)d\sqrt{d\,x^3+c}}{3}+\frac{2c\sqrt{(ad-bc)b}}{3}}{(ad-bc)\sqrt{(ad-bc)b}\,d\sqrt{d\,x^3+c}}$
default	$-\frac{2}{3bd\sqrt{d\,x^3+c}}+\frac{2a\left(b\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{d\,x^3+c}+\sqrt{(ad-bc)b}\right)}{b\sqrt{(ad-bc)b}\,\sqrt{d\,x^3+c}\,(3ad-3bc)}$
elliptic	$\frac{2c}{3d(ad-bc)\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{(-}{}\right.}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+}}}{}\right.}{}$

input
```
int(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))*d*(d*x^3+c)^(1/2)+c*(
(a*d-b*c)*b)^(1/2))/(a*d-b*c)/((a*d-b*c)*b)^(1/2)/d/(d*x^3+c)^(1/2)
```

3.389.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(66) = 132$.

Time = 0.34 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.98

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \left[-\frac{(ad^2x^3+acd)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,(b^2c^2-ab}{3\,(b^3c^3d-2\,ab^2c^2d^2+a^2bcd^3+(b^3c^2d^2-2\,ab^2cd^3+a^2bd^4)x^3}\right.$$

$$\left.-\frac{2\left((ad^2x^3+acd)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+(b^2c^2-abcd)\sqrt{dx^3+c}\right)}{3\,(b^3c^3d-2\,ab^2c^2d^2+a^2bcd^3+(b^3c^2d^2-2\,ab^2cd^3+a^2bd^4)x^3)}\right]$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/3*((a*d^2*x^3 + a*c*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d
- 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(b^2*c^2 - a*b*c
*d)*sqrt(d*x^3 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^3 + (b^3*c^2
*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3), -2/3*((a*d^2*x^3 + a*c*d)*sqrt(-b^
2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c))
+ (b^2*c^2 - a*b*c*d)*sqrt(d*x^3 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*
b*c*d^3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)]
```

3.389.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx = \int \frac{x^5}{(a+bx^3)\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**5/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.389.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.389.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = -\frac{2\left(\frac{ad\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)}+\frac{c}{\sqrt{dx^3+c}(bc-ad)}\right)}{3\,d}$$

input
```
integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-2/3*(a*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*
b*d)*(b*c - a*d)) + c/(sqrt(d*x^3 + c)*(b*c - a*d)))/d
```

3.389.9 Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{2\,c}{3\,d\,\sqrt{d\,x^3+c}\,(a\,d-b\,c)} + \frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,\sqrt{b}\,{(a\,d-b\,c)}^{3/2}}$$

input
```
int(x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
(2*c)/(3*d*(c + d*x^3)^(1/2)*(a*d - b*c)) + (a*log((2*b*c - a*d + b^(1/2)*
(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*b^(1
/2)*(a*d - b*c)^(3/2))
```

3.390 $\int \frac{x^2}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx$

3.390.1 Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{x^2}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx=\frac{2}{3(bc-ad)\sqrt{c+dx^3}}-\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

output
```
-2/3*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^
(3/2)+2/3/(-a*d+b*c)/(d*x^3+c)^(1/2)
```

3.390.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx=\frac{2}{(3bc-3ad)\sqrt{c+dx^3}}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3(-bc+ad)^{3/2}}$$

input
```
Integrate[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
2/((3*b*c - 3*a*d)*Sqrt[c + d*x^3]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c +
d*x^3])/Sqrt[-(b*c) + a*d]])/(3*(-(b*c) + a*d)^(3/2))
```

3.390.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {946, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{(bx^3+a)(dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(\frac{b\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{bc-ad}+\frac{2}{\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2b\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{d(bc-ad)}+\frac{2}{\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{2}{\sqrt{c+dx^3}(bc-ad)}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\right)$$

input
```
Int[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2/((b*c - a*d)*Sqrt[c + d*x^3]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/3
```

3.390. $\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

3.390.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.390.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

method	result
default	$-\frac{2\left(b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{d\,x^{3}+c}+\sqrt{(ad-bc)b}\right)}{\sqrt{(ad-bc)b}\,\sqrt{d\,x^{3}+c}\,(3ad-3bc)}$
pseudoelliptic	$-\frac{2\left(b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{d\,x^{3}+c}+\sqrt{(ad-bc)b}\right)}{\sqrt{(ad-bc)b}\,\sqrt{d\,x^{3}+c}\,(3ad-3bc)}$
elliptic	$-\frac{2}{3(ad-bc)\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{ib\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\right.}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$

input
```
int(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))*(d*x^3+c)^(1/2)+((a*d-
b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/(d*x^3+c)^(1/2)/(3*a*d-3*b*c)
```

3.390.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.06

$$\int \frac{x^{2}}{(a+bx^{3})\,(c+dx^{3})^{3/2}}\,dx=\left[-\frac{(dx^{3}+c)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^{3}+2\,bc-ad+2\,\sqrt{dx^{3}+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^{3}+a}\right)-2\,\sqrt{dx^{3}+c}}{3\,((bcd-ad^{2})x^{3}+bc^{2}-acd)},\right.$$
$$\left.-\frac{2\left((dx^{3}+c)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^{3}+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^{3}+bc}\right)-\sqrt{dx^{3}+c}\right)}{3\,((bcd-ad^{2})x^{3}+bc^{2}-acd)}\right]$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/3*((d*x^3 + c)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt
(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) - 2*sqrt(d*x^3 +
 c))/((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d), -2/3*((d*x^3 + c)*sqrt(-b/(b*c
 - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3
 + b*c)) - sqrt(d*x^3 + c))/((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)]
```

3.390.6 Sympy [A] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.57

$$\int \frac{x^2}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{d}{3\sqrt{c+dx^3}(ad-bc)}-\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3\sqrt{\frac{ad-bc}{b}}(ad-bc)}\right)}{d} & \text{for } d\neq 0 \\ \begin{cases} \frac{x^3}{3ac^{\frac{3}{2}}} & \text{for } b=0 \\ \tilde{\infty}x^3 & \text{for } c^{\frac{3}{2}}=0 \\ \frac{\log\left(3ac^{\frac{3}{2}}+3bc^{\frac{3}{2}}x^3\right)}{3bc^{\frac{3}{2}}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Piecewise((2*(-d/(3*sqrt(c + d*x**3)*(a*d - b*c)) - d*atan(sqrt(c + d*x**3
)/sqrt((a*d - b*c)/b))/(3*sqrt((a*d - b*c)/b)*(a*d - b*c)))/d, Ne(d, 0)),
(Piecewise((x**3/(3*a*c**(3/2)), Eq(b, 0)), (zoo*x**3, Eq(c**(3/2), 0)), (
log(3*a*c**(3/2) + 3*b*c**(3/2)*x**3)/(3*b*c**(3/2)), True)), True))
```

3.390.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.390.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\,b\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}(bc-ad)} + \frac{2}{3\sqrt{dx^3+c}(bc-ad)}$$

input
```
integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
2/3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) + 2/3/(sqrt(d*x^3 + c)*(b*c - a*d))
```

3.390.9 Mupad [B] (verification not implemented)

Time = 10.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2}{3\sqrt{d\,x^3+c}\,(a\,d-b\,c)} + \frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,(a\,d-b\,c)^{3/2}}$$

input
```
int(x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
(b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i
 - b*d*x^3)/(a + b*x^3))*1i)/(3*(a*d - b*c)^(3/2)) - 2/(3*(c + d*x^3)^(1/2
)*(a*d - b*c))
```

3.391 $\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx$

3.391.1 Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} - \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}} + \frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}}$$

output
```
-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a/c^(3/2)+2/3*b^(3/2)*arctanh(b^(1/2
)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a/(-a*d+b*c)^(3/2)-2/3*d/c/(-a*d+b*c)/
(d*x^3+c)^(1/2)
```

3.391.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{3}\left(\frac{d}{c(-bc+ad)\sqrt{c+dx^3}} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{a(-bc+ad)^{3/2}} - \frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{ac^{3/2}}\right)$$

input
```
Integrate[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(2*(d/(c*(-(b*c) + a*d)*Sqrt[c + d*x^3]) + (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[-(b*c) + a*d]])/(a*(-(b*c) + a*d)^(3/2)) - ArcTanh[Sqrt[c
+ d*x^3]/Sqrt[c]]/(a*c^(3/2))))/3
```

3.391.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {948, 96, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(bx^3+a\right)\left(dx^3+c\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{96}$$

$$\frac{1}{3}\left(\frac{\int \frac{-bdx^3+bc-ad}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{c(bc-ad)}-\frac{2d}{c\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{\frac{(bc-ad)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{b^2c\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{c(bc-ad)}-\frac{2d}{c\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\frac{2(bc-ad)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2b^2c\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{c(bc-ad)}-\frac{2d}{c\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\frac{2b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)}-\frac{2d}{c\sqrt{c+dx^3}(bc-ad)}\right)$$

input
```
Int[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((-2*d)/(c*(b*c - a*d)*Sqrt[c + d*x^3]) + ((-2*(b*c - a*d)*ArcTanh[Sqrt[c
+ d*x^3]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(3/2)*c*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(c*(b*c - a*d)))/3
```

3.391.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.391.4 Maple [A] (verified)

Time = 4.91 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

method	result	size
pseudoelliptic	$\frac{2b^{2} \arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3(ad-bc)a\sqrt{(ad-bc)b}}+\frac{2d}{3(ad-bc)c\sqrt{d\,x^{3}+c}}-\frac{2\ \mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a\,c^{\frac{3}{2}}}$	103
default	$\frac{\frac{2}{3c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{2\ \mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{a}+\frac{2b\left(b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{d\,x^{3}+c}+\sqrt{(ad-bc)b}\right)}{a\sqrt{(ad-bc)b}\,\sqrt{d\,x^{3}+c}\,(3ad-3bc)}$	130
elliptic	Expression too large to display	1637

input
```
int(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(a*d-b*c)*b^2/a/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c
)*b)^(1/2))+2/3*d/(a*d-b*c)/c/(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/
c^(1/2))/a/c^(3/2)
```

3.391.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(90) = 180.

Time = 0.38 (sec) , antiderivative size = 790, normalized size of antiderivative = 6.93

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \left[-\frac{2\sqrt{dx^3+c}acd+\left(bc^2dx^3+bc^3\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}(bc-ad)\sqrt{}}{bx^3+a}\right.}{3\left(abc^4-a^2c^3d+\left(abc\right.\right.}\right.$$

$$-\frac{2\sqrt{dx^3+c}acd-2\left(bc^2dx^3+bc^3\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)-\left((bcd-ad^2)x^3+bc^2-acd\right.}{3\left(abc^4-a^2c^3d+\left(abc^3d-a^2c^2d^2\right)x^3\right)}$$

$$-\frac{2\sqrt{dx^3+c}acd-2\left((bcd-ad^2)x^3+bc^2-acd\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\left(bc^2dx^3+bc^3\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx}{}\right.}{3\left(abc^4-a^2c^3d+\left(abc^3d-a^2c^2d^2\right)x^3\right)}$$

$$-\frac{2\left(\sqrt{dx^3+c}acd-\left(bc^2dx^3+bc^3\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)-\left((bcd-ad^2)x^3+bc^2-acd\right.\right.}{3\left(abc^4-a^2c^3d+\left(abc^3d-a^2c^2d^2\right)x^3\right)}$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/3*(2*sqrt(d*x^3 + c)*a*c*d + (b*c^2*d*x^3 + b*c^3)*sqrt(b/(b*c - a*d))
*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c -
a*d)))/(b*x^3 + a)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt(c)*log((d
*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*b*c^4 - a^2*c^3*d + (a*b*
c^3*d - a^2*c^2*d^2)*x^3), -1/3*(2*sqrt(d*x^3 + c)*a*c*d - 2*(b*c^2*d*x^3
+ b*c^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/
(b*c - a*d))/(b*d*x^3 + b*c)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt
(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*b*c^4 - a^2*c^3
*d + (a*b*c^3*d - a^2*c^2*d^2)*x^3), -1/3*(2*sqrt(d*x^3 + c)*a*c*d - 2*((b
*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c
)/c) + (b*c^2*d*x^3 + b*c^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*
d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)))/(a*b*
c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^3), -2/3*(sqrt(d*x^3 + c)*a*
c*d - (b*c^2*d*x^3 + b*c^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(
b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((b*c*d - a*d^2)*x^3 +
b*c^2 - a*c*d)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/(a*b*c^4 - a^2
*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^3)]
```

3.391. $\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx$

3.391.6 Sympy [A] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{d^2}{3c\sqrt{c+dx^3}(ad-bc)} + \frac{bd\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}(ad-bc)} + \frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3ac\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{2\operatorname{atan}\left(\frac{2\left(\frac{a}{2b}+x^3\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{3bc^{\frac{3}{2}}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Piecewise((2*(d**2/(3*c*sqrt(c + d*x**3)*(a*d - b*c)) + b*d*atan(sqrt(c +
d*x**3)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)*(a*d - b*c)) + d*ata
n(sqrt(c + d*x**3)/sqrt(-c))/(3*a*c*sqrt(-c)))/d, Ne(d, 0)), (2*atan(2*(a/
(2*b) + x**3)/sqrt(-a**2/b**2))/(3*b*c**(3/2)*sqrt(-a**2/b**2)), True))
```

3.391.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x), x)
```

3.391.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = -\frac{2\,b^2\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,(abc-a^2d)\sqrt{-b^2c+abd}} - \frac{2\,d}{3\,\sqrt{dx^3+c}(bc^2-acd)} + \frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}c}$$

input
```
integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-2/3*b^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b*c - a^2*d)*s
qrt(-b^2*c + a*b*d)) - 2/3*d/(sqrt(d*x^3 + c)*(b*c^2 - a*c*d)) + 2/3*arcta
n(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)*c)
```

3.391.9 Mupad [B] (verification not implemented)

Time = 12.98 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a\,c^{3/2}} + \frac{2\,d}{3\,c\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)} + \frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,a\,\left(a\,d-b\,c\right)^{3/2}}$$

input
```
int(1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(
3*a*c^(3/2)) + (2*d)/(3*c*(c + d*x^3)^(1/2)*(a*d - b*c)) + (b^(3/2)*log((2
*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a
+ b*x^3))*1i)/(3*a*(a*d - b*c)^(3/2))
```

3.392 $\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

3.392.1 Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=-\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}}-\frac{1}{3acx^3\sqrt{c+dx^3}}+\frac{(2bc+3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}}-\frac{2b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}$$

output
```
1/3*(3*a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(5/2)-2/3*b^(5/2)
*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(3/2)-1/
3*d*(-3*a*d+b*c)/a/c^2/(-a*d+b*c)/(d*x^3+c)^(1/2)-1/3/a/c/x^3/(d*x^3+c)^(1
/2)
```

3.392.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{\frac{a\left(-bc\left(c+dx^3\right)+ad\left(c+3dx^3\right)\right)}{c^2(bc-ad)x^3\sqrt{c+dx^3}}-\frac{2b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}+\frac{(2bc+3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{5/2}}}{3a^2}$$

input
```
Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
((a*(-(b*c*(c + d*x^3)) + a*d*(c + 3*d*x^3)))/(c^2*(b*c - a*d)*x^3*Sqrt[c
+ d*x^3]) - (2*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]
])/(-(b*c) + a*d)^(3/2) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]
])/c^(5/2))/(3*a^2)
```

3.392.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 114, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^6 (bx^3+a) (dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3} \left(-\frac{\int \frac{3bdx^3+2bc+3ad}{2x^3(bx^3+a)(dx^3+c)^{3/2}}dx^3}{ac}-\frac{1}{acx^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{\int \frac{3bdx^3+2bc+3ad}{x^3(bx^3+a)(dx^3+c)^{3/2}}dx^3}{2ac}-\frac{1}{acx^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{3} \left(-\frac{\frac{2d(bc-3ad)}{c\sqrt{c+dx^3}(bc-ad)}-\frac{2\int -\frac{bd(bc-3ad)x^3+(bc-ad)(2bc+3ad)}{2x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{c(bc-ad)}}{2ac}-\frac{1}{acx^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{\frac{\int \frac{bd(bc-3ad)x^3+(bc-ad)(2bc+3ad)}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{c(bc-ad)}+\frac{2d(bc-3ad)}{c\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{1}{acx^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{(bc-ad)(3ad+2bc)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{2b^3c^2\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{c(bc-ad)}+\frac{2d(bc-3ad)}{c\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{1}{acx^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{2(bc-ad)(3ad+2bc)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{4b^3c^2\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{c(bc-ad)}+\frac{2d(bc-3ad)}{c\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{1}{acx^3\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{4b^{5/2}c^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)(3ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)}+\frac{2d(bc-3ad)}{c\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{1}{acx^3\sqrt{c+dx^3}}\right)$$

input
```
Int[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(-(1/(a*c*x^3*Sqrt[c + d*x^3])) - ((2*d*(b*c - 3*a*d))/(c*(b*c - a*d)*Sqrt
[c + d*x^3]) + ((-2*(b*c - a*d)*(2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sq
rt[c]])/(a*Sqrt[c]) + (4*b^(5/2)*c^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqr
t[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(c*(b*c - a*d)))/(2*a*c))/3
```

3.392.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.392. $\int \frac{1}{x^4(a+bx^3)(c+dx^3)^{3/2}}\,dx$

3.392.4 Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}}{3c^{2}a\,x^{3}}-\frac{-\frac{2(3ad+2bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{4a\,d^{2}}{3(ad-bc)\sqrt{d\,x^{3}+c}}+\frac{4b^{3}c^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3(ad-bc)a\sqrt{(ad-bc)b}}}{2a\,c^{2}}$
pseudoelliptic	$\frac{d^{2}\left(-\frac{2b^{3}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)a^{2}d^{2}\sqrt{(ad-bc)b}}-\frac{-3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)ad\,x^{3}-2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)bc\,x^{3}+\sqrt{d\,x^{3}+c}\,a\sqrt{c}}{x^{3}c^{\frac{5}{2}}a^{2}d^{2}}-\frac{2}{(ad-bc)c^{2}\sqrt{d\,x^{3}+c}}\right)}{3}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3c^{2}x^{3}}-\frac{2d}{3c^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{a}-\frac{b\left(\frac{2}{3c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{a^{2}}-\frac{2b^{2}\left(b\arctan\left(\frac{b\sqrt{d\,x^{3}+}}{\sqrt{(ad-bc}}\right.\right.}{a^{2}\sqrt{(ad-bc)b}}$
elliptic	Expression too large to display

input
```
int(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/c^2/a*(d*x^3+c)^(1/2)/x^3-1/2/a/c^2*(-2/3*(3*a*d+2*b*c)/a*arctanh((d*
x^3+c)^(1/2)/c^(1/2))/c^(1/2)+4/3*a*d^2/(a*d-b*c)/(d*x^3+c)^(1/2)+4/3*b^3*
c^2/(a*d-b*c)/a/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)
^(1/2)))
```

3.392.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(130) = 260$.

Time = 0.45 (sec) , antiderivative size = 1120, normalized size of antiderivative = 7.09

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \left[-\frac{2\left(b^2c^3dx^6+b^2c^4x^3\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-(}{}\right.$$

$$-\frac{4\left(b^2c^3dx^6+b^2c^4x^3\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)-((2\,b^2c^2d+abcd^2-3\,a^2d^3)x^6+(2\,b^2c^3+}{6\left((a^2bc^4d-a^3c^3d^2)x^6+\right.}$$

$$-\frac{((2\,b^2c^2d+abcd^2-3\,a^2d^3)x^6+(2\,b^2c^3+abc^2d-3\,a^2cd^2)x^3)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+(b^2c^3dx^6+b^2c^4x}{3\left((a^2bc^4d-a^3c^3d^2)x^6+(\right.}$$

$$-\frac{2\left(b^2c^3dx^6+b^2c^4x^3\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)+((2\,b^2c^2d+abcd^2-3\,a^2d^3)x^6+(2\,b^2c^3+}{3\left((a^2bc^4d-a^3c^3d^2)x^6+(a\right.}$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/6*(2*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 +
2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 +
a)) - ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d
- 3*a^2*c*d^2)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/
x^3) + 2*(a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3
+ c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), -1/
6*(4*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3
 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((2*b^2*c^2*d +
a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sq
rt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a*b*c^3 - a^
2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))/((a^2*b*c^4*d -
a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), -1/3*(((2*b^2*c^2*d + a*b
*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(
-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqr
t(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d
)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + (a*b*c^3 - a^2*c^2*d + (a*b*c^2*d -
3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2
*b*c^5 - a^3*c^4*d)*x^3), -1/3*(2*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(-b/(b
*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x
^3 + b*c)) + ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + ...
```

3.392. $\int \frac{1}{x^4(a+bx^3)(c+dx^3)^{3/2}}\,dx$

3.392.6 Sympy [F]

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.392.7 Maxima [F]

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}} x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^4), x)
```

3.392.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{3/2}}\,dx = {} & \frac{2\,b^3 \arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,(a^2bc-a^3d)\sqrt{-b^2c+abd}} \\ & - \frac{(dx^3+c)bcd-3\,(dx^3+c)ad^2+2\,acd^2}{3\,(abc^3-a^2c^2d)\left((dx^3+c)^{\frac{3}{2}}-\sqrt{dx^3+c}c\right)} - \frac{(2\,bc+3\,ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}c^2}\end{aligned}$$

input
```
integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
2/3*b^3*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b*c - a^3*d)*
sqrt(-b^2*c + a*b*d)) - 1/3*((d*x^3 + c)*b*c*d - 3*(d*x^3 + c)*a*d^2 + 2*a
*c*d^2)/((a*b*c^3 - a^2*c^2*d)*((d*x^3 + c)^(3/2) - sqrt(d*x^3 + c)*c)) -
1/3*(2*b*c + 3*a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c^2)
```

3.392.9 Mupad [B] (verification not implemented)

Time = 15.21 (sec) , antiderivative size = 597, normalized size of antiderivative = 3.78

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)^3}{x^6}\right)\left(3\,a\,d+2\,b\,c\right)}{6\,a^2\,c^{5/2}} - \frac{\sqrt{d\,x^3+c}}{3\,a\,c^2\,x^3}$$

$$-\frac{c\left(\frac{c\left(\frac{c\left(\frac{3\,a^2\,d^4+24\,a\,b\,c\,d^3+15\,b^2\,c^2\,d^2}{8\,a^3\,c^5}+\frac{c\left(\frac{c\left(\frac{3\,b^2\,d^4}{8\,a^3\,c^5}+\frac{b^2\,d^4\,\left(5\,a\,d-3\,b\,c\right)}{8\,a^3\,c^4\,\left(b\,c^2-a\,c\,d\right)}-\frac{b\,d^4\,\left(a\,d+2\,b\,c\right)\left(5\,a\,d-3\,b\,c\right)}{4\,a^3\,c^5\,\left(b\,c^2-a\,c\,d\right)}\right)}{d}-\frac{3\,b\,d^3\,\left(a\,d+2\,b\,c\right)}{4\,a^3\,c^5}+\frac{d\,\left(5\,a\,d-3\,b\,c\right)\left(3\,a^2\,d^4+24\,a}{24\,a^3\,c^5\,\left(b\,c^2-\right.}\right.}{d}\right.}{d}\right.}{d}\right.}{}$$

$$+\frac{b^{5/2}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,a^2\,\left(a\,d-b\,c\right)^{3/2}}$$

input

```
int(1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output

```
(log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*
(3*a*d + 2*b*c))/(6*a^2*c^(5/2)) - (c + d*x^3)^(1/2)/(3*a*c^2*x^3) - ((c*(
(c*((c*((3*a^2*d^4 + 15*b^2*c^2*d^2 + 24*a*b*c*d^3)/(8*a^3*c^5) + (c*((c*(
(3*b^2*d^4)/(8*a^3*c^5) + (b^2*d^4*(5*a*d - 3*b*c))/(8*a^3*c^4*(b*c^2 - a*
c*d)) - (b*d^4*(a*d + 2*b*c)*(5*a*d - 3*b*c))/(4*a^3*c^5*(b*c^2 - a*c*d)))
)/d - (3*b*d^3*(a*d + 2*b*c))/(4*a^3*c^5) + (d*(5*a*d - 3*b*c)*(3*a^2*d^4
+ 15*b^2*c^2*d^2 + 24*a*b*c*d^3))/(24*a^3*c^5*(b*c^2 - a*c*d))))/d - (d^2*
(5*a*d - 3*b*c)*(6*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(12*a^3*c^4*(b*c^2 -
 a*c*d))))/d - (d*(6*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(4*a^3*c^4) + (d^2
*(5*a*d - 3*b*c)*(13*a*d + 18*b*c))/(24*a^2*c^3*(b*c^2 - a*c*d))))/d + (d*
(13*a*d + 18*b*c))/(8*a^2*c^3) - (d*(3*a*d + 2*b*c)*(5*a*d - 3*b*c))/(6*a^
2*c^2*(b*c^2 - a*c*d))))/d - (3*a*d + 2*b*c)/(2*a^2*c^2))/(c + d*x^3)^(1/2
) + (b^(5/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2
)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*a^2*(a*d - b*c)^(3/2))
```

3.392. $\int \frac{1}{x^4(a+bx^3)(c+dx^3)^{3/2}}\,dx$

3.393 $\int \frac{x^3}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

3.393.1 Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^3}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

output
```
1/4*x^4*AppellF1(4/3,1,3/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/(d
*x^3+c)^(1/2)
```

3.393.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 231 vs. $2(67) = 134$.

Time = 10.19 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.45

$$\int \frac{x^3}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{x\Bigg(-8-\frac{bx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}-\frac{}{\left(a+bx^3\right)\Big(-8ac\,\text{AppellF1}\Big(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c}}}{12(-bc+ad)\sqrt{}}$$

input
```
Integrate[x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x*(-8 - (b*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c
), -((b*x^3)/a)])/a - (64*a^2*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -
((b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c)
, -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))
)))/(12*(-(b*c) + a*d)*Sqrt[c + d*x^3])
```

3.393.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{(bx^3+a)\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

input
```
Int[x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 3/2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*a*c*Sqrt[c + d*x^3])
```

3.393.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.393.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.45 (sec) , antiderivative size = 749, normalized size of antiderivative = 11.18

method	result	size
elliptic	Expression too large to display	749
default	Expression too large to display	1069

input
```
int(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x/(a*d-b*c)/((x^3+c/d)*d)^(1/2)+2/9*I/(a*d-b*c)*3^(1/2)/d*(-c*d^2)^(1
/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/3*I*a/d^2*2^(1/2)*sum(1
/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(
-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*
d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*
3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.393.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Not
integrable (provided residues have no relations)
```

3.393.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(a+bx^3)(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**3/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.393.7 Maxima [F]

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

3.393.8 Giac [F]

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

3.393.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(b\,x^3+a)\,\left(d\,x^3+c\right)^{3/2}}\,dx$$

input
```
int(x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
int(x^3/((a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

3.394 $\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

3.394.1 Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}$$

output
```
1/2*x^2*AppellF1(2/3,1,3/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/(d
*x^3+c)^(1/2)
```

3.394.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 142 vs. 2(67) = 134.

Time = 10.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.12

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{x^2\left(-20ad+5(3bc+ad)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bdx^3}{30ac(bc-ad)\sqrt{c+dx^3}}$$

input
```
Integrate[x/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x^2*(-20*a*d + 5*(3*b*c + a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[
5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)]))/(30*a*c*(b*c - a*d)*Sqrt[c
 + d*x^3])
```

3.394.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 1013$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x}{(bx^3+a)\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 1012$$

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}$$

input
```
Int[x/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1, 3/2, 5/3, -((b*x^3)/a), -((d*x^3
)/c)])/(2*a*c*Sqrt[c + d*x^3])
```

3.394.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.394.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.43 (sec) , antiderivative size = 907, normalized size of antiderivative = 13.54

method	result	size
default	Expression too large to display	907
elliptic	Expression too large to display	907

input

```
int(x/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/3*d*x^2/c/(a*d-b*c)/((x^3+c/d)*d)^(1/2)+2/9*I/c/(a*d-b*c)*3^(1/2)*(-c*d^
2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I*b/d^2*2^(1/2)*sum(1/(a*d-b*c)^
2/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(
1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^
(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2
*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_a...
```

3.394.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Not
integrable (provided residues have no relations)
```

3.394.6 Sympy [F]

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(a+bx^3)(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.394.7 Maxima [F]

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

3.394.8 Giac [F]

$$\int \frac{x}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

3.394.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(b\,x^3+a)\,\left(d\,x^3+c\right)^{3/2}}\,dx$$

input
```
int(x/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
int(x/((a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

3.395 $\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

3.395.1 Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{1}{3},1,\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}$$

output
```
x*AppellF1(1/3,1,3/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/(d*x^3+c
)^(1/2)
```

3.395.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 338 vs. 2(62) = 124.

Time = 10.32 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.45

$$\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{x\left(\frac{bdx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a(-bc+ad)}+\frac{32ac\left(-3bc+3ad+2bdx^3\right)\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-d\right.}{(bc-ad)\left(a+bx^3\right)\left(-8ac\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3}\right.\right.}\right.}{12c\sqrt{}}$$

input
```
Integrate[1/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x*((b*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
-((b*x^3)/a)])/(a*(-(b*c) + a*d)) + (32*a*c*(-3*b*c + 3*a*d + 2*b*d*x^3)*A
ppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*d*x^3*(a + b*x^
3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*App
ellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))/((b*c - a*d)*(a + b*
x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^
3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*Appe
llF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(12*c*Sqrt[c + d*x
^3])
```

3.395.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{937, 936\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(bx^3+a)\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\operatorname{AppellF1}\left(\frac{1}{3},1,\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}$$

input
```
Int[1/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 3/2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*c*Sqrt[c + d*x^3])
```

3.395.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.395.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.26 (sec) , antiderivative size = 753, normalized size of antiderivative = 12.15

method	result	size
default	Expression too large to display	753
elliptic	Expression too large to display	753

input
```
int(1/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*d*x/c/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-2/9*I/c/(a*d-b*c)*3^(1/2)*(-c*d^2)
^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I*b/d^2*2^(1/2)*su
m(1/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3)
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha
^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/
3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.395.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.395.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(a+bx^3)(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.395.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

3.395.8 Giac [F]

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

3.395.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(b\,x^3+a)\,\left(d\,x^3+c\right)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
int(1/((a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

3.396 $\int \frac{1}{x^2(a+bx^3)(c+dx^3)^{3/2}}\,dx$

3.396.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{1}{x^2(a+bx^3)(c+dx^3)^{3/2}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{1}{3},1,\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}$$

output
```
-AppellF1(-1/3,1,3/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/x/(d*x^3
+c)^(1/2)
```

3.396.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(65) = 130.

Time = 10.21 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.97

$$\int \frac{1}{x^2(a+bx^3)(c+dx^3)^{3/2}}\,dx=\frac{20a(-3bc(c+dx^3)+ad(3c+5dx^3))-5(6b^2c^2-3abcd+5a^2d^2)x^3\sqrt{1+}}{}$$

input
```
Integrate[1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(20*a*(-3*b*c*(c + d*x^3) + a*d*(3*c + 5*d*x^3)) - 5*(6*b^2*c^2 - 3*a*b*c*
d + 5*a^2*d^2)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3
)/c), -((b*x^3)/a)] + 2*b*d*(3*b*c - 5*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*Appell
F1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^2*c^2*(b*c - a*d)*
x*Sqrt[c + d*x^3])
```

3.396.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3) (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{x^2(bx^3+a)\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{1}{3},1,\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}$$

input
```
Int[1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
-((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1, 3/2, 2/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c*x*Sqrt[c + d*x^3]))
```

3.396.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.396.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.86 (sec) , antiderivative size = 952, normalized size of antiderivative = 14.65

method	result	size
elliptic	Expression too large to display	952
risch	Expression too large to display	1382
default	Expression too large to display	1392

input
```
int(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*d^2*x^2/c^2/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/c^2/a*(d*x^3+c)^(1/2)/x-2
/3*I*(1/3*d^2/c^2/(a*d-b*c)+1/2/a/c^2*d)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)))-1/3*I*b^2/a/d^2*2^(1/2)*sum(1/(a*d-b*c)^2/_alpha*(-c*
d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(
-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2
)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alph
a*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*
d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*...
```

3.396.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.396.6 Sympy [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.396.7 Maxima [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^2), x)
```

3.396.8 Giac [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^2), x)
```

3.396.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\,(a+bx^3)\,(c+dx^3)^{3/2}}\,dx=\int \frac{1}{x^2\,\left(b\,x^3+a\right)\,\left(d\,x^3+c\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

3.397 $\int \frac{1}{x^3(a+bx^3)(c+dx^3)^{3/2}}\,dx$

3.397.1 Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{1}{x^3(a+bx^3)(c+dx^3)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(-\frac{2}{3},1,\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}$$

output
```
-1/2*AppellF1(-2/3,1,3/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/x^2/
(d*x^3+c)^(1/2)
```

3.397.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 408 vs. 2(67) = 134.

Time = 10.58 (sec) , antiderivative size = 408, normalized size of antiderivative = 6.09

$$\int \frac{1}{x^3(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{bd(3bc-7ad)x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8a(-4ac(-6b^2}{}}{}$$

input
```
Integrate[1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
(b*d*(3*b*c - 7*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -(
(d*x^3)/c), -((b*x^3)/a)] + (8*a*(-4*a*c*(-6*b^2*c*x^3*(3*c + d*x^3) + 3*a
^2*d*(2*c + 7*d*x^3) + a*b*(-6*c^2 - 3*c*d*x^3 + 14*d^2*x^6))*AppellF1[1/3
, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(a + b*x^3)*(-3*b*c*(c
+ d*x^3) + a*d*(3*c + 7*d*x^3))*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3
)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^
3)/a)])))/((a + b*x^3)*(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((
b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^
3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(4
8*a^2*c^2*(-(b*c) + a*d)*x^2*Sqrt[c + d*x^3])
```

3.397.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+bx^3\right) \left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{x^3 \left(bx^3+a\right) \left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1} \,\text{AppellF1}\left(-\frac{2}{3},1,\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

output
```
-1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 3/2, 1/3, -((b*x^3)/a), -((d*x
^3)/c)])/(a*c*x^2*Sqrt[c + d*x^3])
```

3.397.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.397.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.05 (sec) , antiderivative size = 798, normalized size of antiderivative = 11.91

method	result	size
elliptic	Expression too large to display	798
risch	Expression too large to display	1076
default	Expression too large to display	1084

input
```
int(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*d^2*x/c^2/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/2/c^2/a*(d*x^3+c)^(1/2)/x^2
-2/3*I*(-1/3*d^2/c^2/(a*d-b*c)-1/4/a/c^2*d)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/3*I*b^2/a/d^2*2^(1/2)*sum(1/(a*d
-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-
c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(
-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.397.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.397.6 Sympy [F]

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.397.7 Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^3), x)
```

3.397.8 Giac [F]

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^3), x)
```

3.397.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^3\,\left(b\,x^3+a\right)\,\left(d\,x^3+c\right)^{3/2}} \,dx$$

input
```
int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

output
```
int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

3.398 $\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

3.398.1 Optimal result

Integrand size = 27, antiderivative size = 117

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\,(8c-dx^3)} + \frac{2c\sqrt{c+dx^3}(1146c+47dx^3)}{15d^4} - \frac{3968c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}$$

output
```
-3968/9*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4+7/15*x^6*(d*x^3+c
)^(1/2)/d^2+1/3*x^9*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)+2/15*c*(47*d*x^3+1146*c
)*(d*x^3+c)^(1/2)/d^4
```

3.398.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{2\left(\frac{3\sqrt{c+dx^3}(-9168c^3+770c^2dx^3+19cd^2x^6+d^3x^9)}{-8c+dx^3} - 9920c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{45d^4}$$

input
```
Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
(2*((3*Sqrt[c + d*x^3]*(-9168*c^3 + 770*c^2*d*x^3 + 19*c*d^2*x^6 + d^3*x^9
))/(-8*c + d*x^3) - 9920*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(4
5*d^4)
```

3.398.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 108, 27, 170, 25, 27, 164, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^9\sqrt{dx^3+c}}{(8c-dx^3)^2}dx^3$$

$$\Big\downarrow \text{ 108}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d(8c-dx^3)}-\frac{\int \frac{x^6(7dx^3+6c)}{2(8c-dx^3)\sqrt{dx^3+c}}dx^3}{d}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d(8c-dx^3)}-\frac{\int \frac{x^6(7dx^3+6c)}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{2d}\right)$$

$$\Big\downarrow \text{ 170}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d(8c-dx^3)}-\frac{-\frac{2\int -\frac{cdx^3(141dx^3+112c)}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{5d^2}-\frac{14x^6\sqrt{c+dx^3}}{5d}}{2d}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d(8c-dx^3)}-\frac{\frac{2\int \frac{cdx^3(141dx^3+112c)}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{5d^2}-\frac{14x^6\sqrt{c+dx^3}}{5d}}{2d}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d(8c-dx^3)}-\frac{\frac{2c\int \frac{x^3(141dx^3+112c)}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{5d}-\frac{14x^6\sqrt{c+dx^3}}{5d}}{2d}\right)$$

$$\Big\downarrow \color{blue}{164}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d\left(8c-dx^3\right)}-\frac{\frac{2c\left(\frac{9920c^2\int\frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{d}-\frac{2\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{d^2}\right)}{5d}-\frac{14x^6\sqrt{c+dx^3}}{5d}}{2d}\right)$$

$$\Big\downarrow \color{blue}{73}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d\left(8c-dx^3\right)}-\frac{\frac{2c\left(\frac{19840c^2\int\frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d^2}-\frac{2\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{d^2}\right)}{5d}-\frac{14x^6\sqrt{c+dx^3}}{5d}}{2d}\right)$$

$$\Big\downarrow \color{blue}{219}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{c+dx^3}}{d\left(8c-dx^3\right)}-\frac{\frac{2c\left(\frac{19840c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3d^2}-\frac{2\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{d^2}\right)}{5d}-\frac{14x^6\sqrt{c+dx^3}}{5d}}{2d}\right)$$

input

```
Int[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output

```
((x^9*Sqrt[c + d*x^3])/(d*(8*c - d*x^3)) - ((-14*x^6*Sqrt[c + d*x^3])/(5*d
) + (2*c*((-2*Sqrt[c + d*x^3]*(1146*c + 47*d*x^3))/d^2 + (19840*c^(3/2)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^2)))/(5*d))/(2*d))/3
```

3.398.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.398. $\int \frac{x^{11}\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.398. $\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.398.4 Maple [A] (verified)

Time = 4.41 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$-\frac{158720\left(c^{3}\left(c-\frac{d\,x^{3}}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)+\frac{3\sqrt{d\,x^{3}+c}\left(\sqrt{c}\,d^{3}x^{9}+19c^{\frac{3}{2}}d^{2}x^{6}+770c^{\frac{5}{2}}d\,x^{3}-9168c^{\frac{7}{2}}\right)}{79360}\right)}{\sqrt{c}\left(-45d^{5}x^{3}+360c\,d^{4}\right)}$
risch	$\frac{2\left(d^{2}x^{6}+27cd\,x^{3}+986c^{2}\right)\sqrt{d\,x^{3}+c}}{15d^{4}}+\frac{64c^{3}\left(-\frac{70\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}+\frac{8c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)}{d^{3}}$
default	$\frac{d\left(\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15}+\frac{2c\,x^{3}\sqrt{d\,x^{3}+c}}{45d}-\frac{4c^{2}\sqrt{d\,x^{3}+c}}{45d^{2}}\right)+\frac{32c\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9d}}{d^{3}}-\frac{64c^{2}\left(-2\sqrt{d\,x^{3}+c}+6\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}\right)}{d^{4}}+$ $512c^{3}$
elliptic	$\frac{512c^{3}\sqrt{d\,x^{3}+c}}{3d^{4}\left(-d\,x^{3}+8c\right)}+\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15d^{2}}+\frac{18c\,x^{3}\sqrt{d\,x^{3}+c}}{5d^{3}}+\frac{1972c^{2}\sqrt{d\,x^{3}+c}}{15d^{4}}+$ $1984ic^{2}\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\right.$ $\left(-c\,d^{2}\right)^{\frac{1}{3}}$

input
```
int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
-158720*(c^3*(c-1/8*d*x^3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+3/79360*(d
*x^3+c)^(1/2)*(c^(1/2)*d^3*x^9+19*c^(3/2)*d^2*x^6+770*c^(5/2)*d*x^3-9168*c
^(7/2)))/c^(1/2)/(-45*d^5*x^3+360*c*d^4)
```

3.398.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.87

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$
$$= \left[\frac{2\left(4960\,(c^2dx^3-8\,c^3)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,(d^3x^9+19\,cd^2x^6+770\,c^2dx^3-9168\,c^3)\sqrt{dx^3+c}\right.}{45\,(d^5x^3-8\,cd^4)}\right.$$

input
```
integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[2/45*(4960*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + 3*(d^3*x^9 + 19*c*d^2*x^6 + 770*c^2*d*x^3 -
9168*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/45*(9920*(c^2*d*x^3 - 8*
c^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(d^3*x^9 + 19*c*d
^2*x^6 + 770*c^2*d*x^3 - 9168*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

3.398.6 Sympy [F]

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^{11}\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**11*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

3.398.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$$
$$=\frac{2\left(4960\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+3\,(dx^3+c)^{\frac{5}{2}}+75\,(dx^3+c)^{\frac{3}{2}}c+2880\,\sqrt{dx^3+c}c^2-\frac{3840\,\sqrt{dx^3+c}c^3}{dx^3-8\,c}\right)}{45\,d^4}$$

input

```
integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output

```
2/45*(4960*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 75*(d*x^3 + c)^(3/2)*c + 2880*sqrt(d*x^3
 + c)*c^2 - 3840*sqrt(d*x^3 + c)*c^3/(d*x^3 - 8*c))/d^4
```

3.398.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94

$$\int \frac{x^{11}\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx=\frac{3968\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{9\,\sqrt{-c}d^4}-\frac{512\,\sqrt{dx^3+c}c^3}{3\,(dx^3-8\,c)d^4}$$
$$+\frac{2\left((dx^3+c)^{\frac{5}{2}}d^{16}+25\,(dx^3+c)^{\frac{3}{2}}cd^{16}+960\,\sqrt{dx^3+c}c^2d^{16}\right)}{15\,d^{20}}$$

input

```
integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output

```
3968/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 512/3*sqr
t(d*x^3 + c)*c^3/((d*x^3 - 8*c)*d^4) + 2/15*((d*x^3 + c)^(5/2)*d^16 + 25*(
d*x^3 + c)^(3/2)*c*d^16 + 960*sqrt(d*x^3 + c)*c^2*d^16)/d^20
```

3.398.9 Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{1984\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^4}+\frac{1972\,c^2\,\sqrt{d\,x^3+c}}{15\,d^4}+\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d^2}+\frac{18\,c\,x^3\,\sqrt{d\,x^3+c}}{5\,d^3}+\frac{512\,c^3\,\sqrt{d\,x^3+c}}{3\,d^4\,\left(8\,c-d\,x^3\right)}$$

input
```
int((x^11*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

output
```
(1984*c^(5/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^
3)))/(9*d^4) + (1972*c^2*(c + d*x^3)^(1/2))/(15*d^4) + (2*x^6*(c + d*x^3)^
(1/2))/(15*d^2) + (18*c*x^3*(c + d*x^3)^(1/2))/(5*d^3) + (512*c^3*(c + d*x
^3)^(1/2))/(3*d^4*(8*c - d*x^3))
```

3.399 $\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

3.399.1 Optimal result

Integrand size = 27, antiderivative size = 102

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{352c\sqrt{c+dx^3}}{27d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{64c(c+dx^3)^{3/2}}{27d^3(8c-dx^3)}-\frac{352c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

output
```
2/9*(d*x^3+c)^(3/2)/d^3+64/27*c*(d*x^3+c)^(3/2)/d^3/(-d*x^3+8*c)-352/9*c^(
3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3+352/27*c*(d*x^3+c)^(1/2)/d^3
```

3.399.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{2\left(\frac{\sqrt{c+dx^3}(-488c^2+41cdx^3+d^2x^6)}{-8c+dx^3}-176c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{9d^3}$$

input
```
Integrate[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
(2*((Sqrt[c + d*x^3]*(-488*c^2 + 41*c*d*x^3 + d^2*x^6))/(-8*c + d*x^3) - 1
76*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(9*d^3)
```

3.399.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {948, 100, 27, 90, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 \sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6 \sqrt{dx^3+c}}{\left(8c-dx^3\right)^2}dx^3$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{3/2}}{9d^3\left(8c-dx^3\right)}-\frac{\int \frac{cd\sqrt{dx^3+c}\left(9dx^3+104c\right)}{8c-dx^3}dx^3}{9cd^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{3/2}}{9d^3\left(8c-dx^3\right)}-\frac{\int \frac{\sqrt{dx^3+c}\left(9dx^3+104c\right)}{8c-dx^3}dx^3}{9d^2}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{3/2}}{9d^3\left(8c-dx^3\right)}-\frac{176c\int \frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3-\frac{6\left(c+dx^3\right)^{3/2}}{d}}{9d^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{3/2}}{9d^3\left(8c-dx^3\right)}-\frac{176c\left(9c\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{6\left(c+dx^3\right)^{3/2}}{d}}{9d^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{3/2}}{9d^3\left(8c-dx^3\right)}-\frac{176c\left(\frac{18c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{6\left(c+dx^3\right)^{3/2}}{d}}{9d^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{3/2}}{9d^3\left(8c-dx^3\right)}-\frac{176c\left(\frac{6\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{6\left(c+dx^3\right)^{3/2}}{d}}{9d^2}\right)$$

input
```
Int[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
((64*c*(c + d*x^3)^(3/2))/(9*d^3*(8*c - d*x^3)) - ((-6*(c + d*x^3)^(3/2))/
d + 176*c*((-2*Sqrt[c + d*x^3])/d + (6*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*
Sqrt[c])])/d))/(9*d^2))/3
```

3.399.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

3.399. $\int \frac{x^8\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.399.4 Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{\frac{2\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9}+\frac{32c\sqrt{d\,x^{3}+c}}{3}+\frac{32c^{2}\left(\frac{2\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{11\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{3}}{d^{3}}$
default	$\frac{2(d\,x^{3}+c)^{\frac{3}{2}}}{9d^{3}}-\frac{16c\left(-2\sqrt{d\,x^{3}+c}+6\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}\right)}{3d^{3}}+\frac{64c^{2}\left(\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{3d^{3}}$
risch	$\frac{2(d\,x^{3}+49c)\sqrt{d\,x^{3}+c}}{9d^{3}}+\frac{16c^{2}\left(-\frac{26\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}+\frac{4c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)}{d^{2}}$
elliptic	$\frac{64c^{2}\sqrt{d\,x^{3}+c}}{3d^{3}(-d\,x^{3}+8c)}+\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d^{2}}+\frac{98c\sqrt{d\,x^{3}+c}}{9d^{3}}+\frac{176ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)}{}\right)}{\left(-c\,d^{2}\right)}}}{}\right)}{}$

input
```
int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(1/3*(d*x^3+c)^(3/2)+16*c*(d*x^3+c)^(1/2)+16*c^2*(2*(d*x^3+c)^(1/2)/(-
d*x^3+8*c)-11/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d^3
```

3.399.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.87

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$$

$$=\left[\frac{2\left(88\left(cdx^3-8\,c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+\left(d^2x^6+41\,cdx^3-488\,c^2\right)\sqrt{dx^3+c}\right)}{9\left(d^4x^3-8\,cd^3\right)},\ \frac{2\left(176\left(cdx^3-8\right.\right.}{}\right.$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[2/9*(88*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c)
+ 10*c)/(d*x^3 - 8*c)) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))
/(d^4*x^3 - 8*c*d^3), 2/9*(176*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(
d*x^3 + c)*sqrt(-c)/c) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))
/(d^4*x^3 - 8*c*d^3)]
```

3.399.6 Sympy [F]

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx=\int \frac{x^8\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

input
```
integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**8*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

3.399.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx=\frac{2\left(88\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{3}{2}}+48\,\sqrt{dx^3+c}\,c-\frac{96\,\sqrt{dx^3+c}\,c^2}{dx^3-8\,c}\right)}{9\,d^3}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
2/9*(88*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + (d*x^3 + c)^(3/2) + 48*sqrt(d*x^3 + c)*c - 96*sqrt(d*x^3 + c)*c^2
/(d*x^3 - 8*c))/d^3
```

3.399.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{352\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d^3} - \frac{64\sqrt{dx^3+c}c^2}{3\,(dx^3-8\,c)d^3} + \frac{2\left((dx^3+c)^{\frac{3}{2}}d^6+48\sqrt{dx^3+c}cd^6\right)}{9\,d^9}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
352/9*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 64/3*sqrt(
d*x^3 + c)*c^2/((d*x^3 - 8*c)*d^3) + 2/9*((d*x^3 + c)^(3/2)*d^6 + 48*sqrt(
d*x^3 + c)*c*d^6)/d^9
```

3.399.9 Mupad [B] (verification not implemented)

Time = 8.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{98\,c\sqrt{d\,x^3+c}}{9\,d^3} + \frac{176\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^3} + \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2} + \frac{64\,c^2\,\sqrt{d\,x^3+c}}{3\,d^3\,\left(8\,c-d\,x^3\right)}$$

input
```
int((x^8*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

output
```
(98*c*(c + d*x^3)^(1/2))/(9*d^3) + (176*c^(3/2)*log((10*c + d*x^3 - 6*c^(1
/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^3) + (2*x^3*(c + d*x^3)^(1/2))
/(9*d^2) + (64*c^2*(c + d*x^3)^(1/2))/(3*d^3*(8*c - d*x^3))
```

3.400 $\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

3.400.1 Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{26\sqrt{c+dx^3}}{27d^2}+\frac{8(c+dx^3)^{3/2}}{27d^2(8c-dx^3)}-\frac{26\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

output
```
8/27*(d*x^3+c)^(3/2)/d^2/(-d*x^3+8*c)-26/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(
1/2))*c^(1/2)/d^2+26/27*(d*x^3+c)^(1/2)/d^2
```

3.400.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{2\left(\frac{3(-12c+dx^3)\sqrt{c+dx^3}}{-8c+dx^3}-13\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{9d^2}$$

input
```
Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
(2*((3*(-12*c + d*x^3)*Sqrt[c + d*x^3])/(-8*c + d*x^3) - 13*Sqrt[c]*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(9*d^2)
```

3.400.3 Rubi [A] (verified)

Time $=0.21$ (sec) , antiderivative size $=90$, normalized size of antiderivative $=1.10$, number of steps used $=6$, number of rules used $=5$, $\frac{\text{number of rules}}{\text{integrand size}}=0.185$, Rules used $=\{948, 87, 60, 73, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3\sqrt{dx^3+c}}{(8c-dx^3)^2}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{8\left(c+dx^3\right)^{3/2}}{9d^2\left(8c-dx^3\right)}-\frac{13\int \frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3}{9d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{8\left(c+dx^3\right)^{3/2}}{9d^2\left(8c-dx^3\right)}-\frac{13\left(9c\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3-\frac{2\sqrt{c+dx^3}}{d}\right)}{9d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{8\left(c+dx^3\right)^{3/2}}{9d^2\left(8c-dx^3\right)}-\frac{13\left(\frac{18c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)}{9d}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{8\left(c+dx^3\right)^{3/2}}{9d^2\left(8c-dx^3\right)}-\frac{13\left(\frac{6\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)}{9d}\right)$$

input
```
Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

3.400. $\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}dx$

output
```
((8*(c + d*x^3)^(3/2))/(9*d^2*(8*c - d*x^3)) - (13*((-2*Sqrt[c + d*x^3])/d
 + (6*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d))/(9*d))/3
```

3.400.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.400. $\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

3.400.4 Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\frac{2\sqrt{d x^{3}+c}}{3}+\frac{2 c \left(\frac{4 \sqrt{d x^{3}+c}}{-d x^{3}+8 c}-\frac{13\,\mathrm{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right)}{3 \sqrt{c}}\right)}{3}}{d^{2}}$
default	$-\frac{-2 \sqrt{d x^{3}+c}+6\,\mathrm{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right) \sqrt{c}}{3 d^{2}}+\frac{8 c \left(\frac{\sqrt{d x^{3}+c}}{-d x^{3}+8 c}-\frac{\mathrm{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right)}{3 \sqrt{c}}\right)}{3 d^{2}}$
risch	$\frac{2 \sqrt{d x^{3}+c}}{3 d^{2}}+\frac{c \left(-\frac{34\,\mathrm{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right)}{9 d \sqrt{c}}+\frac{8 c \left(-\frac{\sqrt{d x^{3}+c}}{c \left(d x^{3}-8 c\right)}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right)}{3 c^{\frac{3}{2}}}\right)}{3 d}\right)}{d}$
elliptic	$\frac{8 c \sqrt{d x^{3}+c}}{3 d^{2} \left(-d x^{3}+8 c \right)}+\frac{2 \sqrt{d x^{3}+c}}{3 d^{2}}+$ $13 i \sqrt{2}\, \sum_{\_\alpha =\mathrm{RootOf}\left(d \_Z^{3}-8 c \right)} \left(-c\,d^{2}\right)^{\frac{1}{3}} \sqrt{2}\, \sqrt{\frac{i d \left(2 x +\frac{-i \sqrt{3}\, \left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}$ $\sqrt{-3(}$

input
```
int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*((d*x^3+c)^(1/2)+c*(4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-13/3*arctanh(1/3*(d
*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d^2
```

3.400.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.01

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$$
$$= \left[\frac{13\,(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+6\,\sqrt{dx^3+c}(dx^3-12\,c)}{9\,(d^3x^3-8\,cd^2)}, \frac{2\left(13\,(dx^3-8\,c)\sqrt{-c}\,\mathrm{arctan}\left(\frac{\sqrt{dx^3}}{}\right.\right.}{9\,(d^3x^3}\right.$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/9*(13*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + 6*sqrt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2)
, 2/9*(13*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) +
3*sqrt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2)]
```

3.400.6 Sympy [F]

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \int \frac{x^5\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

input
```
integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**5*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

3.400.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{13\,\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+6\,\sqrt{dx^3+c}-\frac{24\,\sqrt{dx^3+c}c}{dx^3-8\,c}}{9\,d^2}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
1/9*(13*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 6*sqrt(d*x^3 + c) - 24*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d^2
```

3.400.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{26\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{9\,\sqrt{-c}d^2} + \frac{2\,\sqrt{dx^3+c}}{3\,d^2} - \frac{8\,\sqrt{dx^3+c}c}{3\,(dx^3-8\,c)d^2}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
26/9*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) + 2/3*sqrt(d*x^
3 + c)/d^2 - 8/3*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d^2)
```

3.400.9 Mupad [B] (verification not implemented)

Time = 8.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{2\,\sqrt{d\,x^3+c}}{3\,d^2} + \frac{13\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^2} + \frac{8\,c\,\sqrt{d\,x^3+c}}{3\,d^2\,\left(8\,c-d\,x^3\right)}$$

input
```
int((x^5*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*d^2) + (13*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*
(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^2) + (8*c*(c + d*x^3)^(1/2))/(3*d^
2*(8*c - d*x^3))
```

3.401 $\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

3.401.1 Optimal result

Integrand size = 27, antiderivative size = 64

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\frac{\sqrt{c+dx^3}}{3d(8c-dx^3)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

output
```
-1/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d/c^(1/2)+1/3*(d*x^3+c)^(1/2)/d/
(-d*x^3+8*c)
```

3.401.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\frac{\frac{3\sqrt{c+dx^3}}{8c-dx^3}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}}{9d}$$

input
```
Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
((3*Sqrt[c + d*x^3])/(8*c - d*x^3) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/
Sqrt[c])/(9*d)
```

3.401.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {946, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{(8c-dx^3)^2}dx^3$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}}{d(8c-dx^3)}-\frac{1}{2}\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}}{d(8c-dx^3)}-\frac{\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}}{d(8c-dx^3)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3\sqrt{c}d}\right)$$

input
```
Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
(Sqrt[c + d*x^3]/(d*(8*c - d*x^3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/
(3*Sqrt[c]*d))/3
```

3.401.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.401.4 Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

method	result
default	$\frac{\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}}{3d}$
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}}{3d}$
elliptic	$\frac{\sqrt{d\,x^{3}+c}}{3d\left(-d\,x^{3}+8c\right)}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,c\right.}}}{}\right.}{}$

input
```
int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*((d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))
/c^(1/2))/d
```

3.401.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.33

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$$
$$=\left[\frac{(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,\sqrt{dx^3+c}c}{18\,(cd^2x^3-8\,c^2d)},\frac{(dx^3-8\,c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)-3\,\sqrt{dx^3+}}{9\,(cd^2x^3-8\,c^2d)}\right.$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/18*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c*d^2*x^3 - 8*c^2*d), 1/9*((d*x^3
 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(d*x^3 + c
)*c)/(c*d^2*x^3 - 8*c^2*d)]
```

3.401.6 Sympy [F]

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\int \frac{x^2\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**2*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

3.401.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\frac{\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}}-\frac{6\sqrt{dx^3+c}}{dx^3-8c}}{18\,d}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
1/18*(log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/sqr
t(c) - 6*sqrt(d*x^3 + c)/(d*x^3 - 8*c))/d
```

3.401.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d}-\frac{\sqrt{dx^3+c}}{3\,(dx^3-8\,c)d}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
1/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 1/3*sqrt(d*x^3 + c
)/((d*x^3 - 8*c)*d)
```

3.401.9 Mupad [B] (verification not implemented)

Time = 8.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx=\frac{\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{18\,\sqrt{c}\,d}+\frac{\sqrt{d\,x^3+c}}{3\,d\,\left(8\,c-d\,x^3\right)}$$

input
```
int((x^2*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

output
```
log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(18*c^(1/2
)*d) + (c + d*x^3)^(1/2)/(3*d*(8*c - d*x^3))
```

3.402 $\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx$

3.402.1 Optimal result

Integrand size = 27, antiderivative size = 88

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx=\frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}+\frac{5\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

output
```
5/288*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/96*arctanh((d*x^3+c)^
(1/2)/c^(1/2))/c^(3/2)+1/24*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)
```

3.402.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx=\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3}+5\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-3\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{288c^{3/2}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]
```

output
```
((12*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + 5*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])] - 3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(288*c^(3/2))
```

3.402.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {948, 110, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^3\left(8c-dx^3\right)^2}dx^3$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}}{8c\left(8c-dx^3\right)}-\frac{\int -\frac{dx^3+2c}{2x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{8c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{dx^3+2c}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{16c}+\frac{\sqrt{c+dx^3}}{8c\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{\frac{1}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{5}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{16c}+\frac{\sqrt{c+dx^3}}{8c\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\frac{5}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{16c}+\frac{\sqrt{c+dx^3}}{8c\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{5\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{16c}+\frac{\sqrt{c+dx^3}}{8c\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\frac{5\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{16c}+\frac{\sqrt{c+dx^3}}{8c\,(8c-dx^3)}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]
```

output
```
(Sqrt[c + d*x^3]/(8*c*(8*c - d*x^3)) + ((5*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt
[c])])/(6*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(2*Sqrt[c]))/(16*c))
/3
```

3.402.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.402.4 Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{96 c^{\frac{3}{2}}}+\frac{\frac{5 \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right) \left(d\,x^{3}-8 c \right)}{\sqrt{c}}-12 \sqrt{d\,x^{3}+c}}{288 \left(d\,x^{3}-8 c \right) c}$	78
default	$\frac{\frac{2 \sqrt{d\,x^{3}+c}}{3}-\frac{2 \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right) \sqrt{c}}{3}}{64 c^{2}}+\frac{\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8 c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{3 \sqrt{c}}}{24 c}+\frac{-2 \sqrt{d\,x^{3}+c}+6 \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right) \sqrt{c}}{192 c^{2}}$	123
elliptic	Expression too large to display	1534

input

```
int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)+1/288*(5*arctanh(1/3*(d*x^3
+c)^(1/2)/c^(1/2))/c^(1/2)*(d*x^3-8*c)-12*(d*x^3+c)^(1/2))/(d*x^3-8*c)/c
```

3.402.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.57

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx$$
$$= \left[\frac{5\left(dx^3-8\,c\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\left(dx^3-8\,c\right)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\,\sqrt{dx^3+c}c}{576\left(c^2dx^3-8\,c^3\right)},\ 3\,(\right.$$

input
```
integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/576*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 1
0*c)/(d*x^3 - 8*c)) + 3*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 +
c)*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^2*d*x^3 - 8*c^3), 1/288*
(3*(d*x^3 - 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 5*(d*x^3 -
8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x^3 + c)*
c)/(c^2*d*x^3 - 8*c^3)]
```

3.402.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx = \int \frac{\sqrt{c+dx^3}}{x\left(-8c+dx^3\right)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(x*(-8*c + d*x**3)**2), x)
```

3.402.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x), x)
```

3.402.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}c} - \frac{5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{288\sqrt{-c}c} - \frac{\sqrt{dx^3+c}}{24\left(dx^3-8c\right)c}$$

input
```
integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 5/288*arctan(1/3*sqrt
(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/24*sqrt(d*x^3 + c)/((d*x^3 - 8*c)*c
)
```

3.402.9 Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx = \frac{5\,\mathrm{atanh}\left(\frac{c\sqrt{d\,x^3+c}}{3\sqrt{c^3}}\right)}{288\sqrt{c^3}} - \frac{\mathrm{atanh}\left(\frac{c\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{96\sqrt{c^3}} + \frac{\sqrt{d\,x^3+c}}{8\,c\,\left(24\,c-3\,d\,x^3\right)}$$

input
```
int((c + d*x^3)^(1/2)/(x*(8*c - d*x^3)^2),x)
```

output
```
(5*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2))))/(288*(c^3)^(1/2)) - atanh
((c*(c + d*x^3)^(1/2))/(c^3)^(1/2))/(96*(c^3)^(1/2)) + (c + d*x^3)^(1/2)/(
8*c*(24*c - 3*d*x^3))
```

3.403 $\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx$

3.403.1 Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx = \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} + \frac{7d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}} - \frac{d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}$$

output
```
7/1152*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/128*d*arctanh((d*x
^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/96*d*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)-1/24*
(d*x^3+c)^(1/2)/c/x^3/(-d*x^3+8*c)
```

3.403.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx = \frac{\frac{12\sqrt{c}\left(4c-dx^3\right)\sqrt{c+dx^3}}{-8cx^3+dx^6} + 7d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 9d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{1152c^{5/2}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2),x]
```

output
```
((12*Sqrt[c]*(4*c - d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 7*d*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(1
152*c^(5/2))
```

3.403.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 110, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^4 \left(8c-dx^3\right)^2} \, dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^6 \left(8c-dx^3\right)^2}dx^3$$

$$\Big\downarrow \ 110$$

$$\frac{1}{3}\left(\frac{\int \frac{3d\left(dx^3+4c\right)}{2x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{8c}-\frac{\sqrt{c+dx^3}}{8cx^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\frac{3d\int \frac{dx^3+4c}{x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \ 168$$

$$\frac{1}{3}\left(\frac{3d\left(\frac{\sqrt{c+dx^3}}{6c\left(8c-dx^3\right)}-\frac{\int -\frac{6cd\left(dx^3+6c\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{72c^2d}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\frac{3d\left(\frac{\int \frac{dx^3+6c}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{12c}+\frac{\sqrt{c+dx^3}}{6c\left(8c-dx^3\right)}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \ 174$$

$$\frac{1}{3}\left(\frac{3d\left(\frac{\frac{3}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{7}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{12c}+\frac{\sqrt{c+dx^3}}{6c(8c-dx^3)}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{3}\left(\frac{3d\left(\frac{\frac{7}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{3\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{12c}+\frac{\sqrt{c+dx^3}}{6c(8c-dx^3)}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow 219$$

$$\frac{1}{3}\left(\frac{3d\left(\frac{\frac{3\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{12c}+\frac{\sqrt{c+dx^3}}{6c(8c-dx^3)}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{3}\left(\frac{3d\left(\frac{\frac{7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{12c}+\frac{\sqrt{c+dx^3}}{6c(8c-dx^3)}\right)}{16c}-\frac{\sqrt{c+dx^3}}{8cx^3\left(8c-dx^3\right)}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2),x]
```

output
```
(-1/8*Sqrt[c + d*x^3]/(c*x^3*(8*c - d*x^3)) + (3*d*(Sqrt[c + d*x^3]/(6*c*(
8*c - d*x^3)) + ((7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*Sqrt[c]) - (3
*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(12*c)))/(16*c))/3
```

3.403. $\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx$

3.403.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.403. $\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)^2}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.403.4 Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{d\left(-\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d\,x^{3}+2\sqrt{d\,x^{3}+c}\,\sqrt{c}}{2d\,x^{3}c^{\frac{5}{2}}}+\frac{\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{7\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{c^{2}}\right)}{192}$
risch	$-\frac{\sqrt{d\,x^{3}+c}}{192c^{2}x^{3}}-\frac{d\left(\frac{\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9\sqrt{c}}-\frac{2c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3}\right)}{128c^{2}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3x^{3}}-\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}}{64c^{2}}+\frac{d\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\sqrt{c}}{3}\right)}{256c^{3}}+\frac{d\left(\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{192c^{2}}+$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192*d*(-1/2*(3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*d*x^3+2*(d*x^3+c)^(1/2)*
c^(1/2))/d/x^3/c^(5/2)+((d*x^3+c)^(1/2)/(-d*x^3+8*c)+7/6*arctanh(1/3*(d*x^
3+c)^(1/2)/c^(1/2))/c^(1/2))/c^2)
```

3.403.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.24

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx$$

$$= \left[\frac{7\,(d^2x^6-8\,cdx^3)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+9\,(d^2x^6-8\,cdx^3)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\,(cdx^3}{2304\,(c^3dx^6-8\,c^4x^3)}\right.$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/2304*(7*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sq
rt(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3
- 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(c*d*x^3 - 4*c^2)*sqrt(d*x^3
+ c))/(c^3*d*x^6 - 8*c^4*x^3), 1/1152*(9*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*ar
ctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 7*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan
(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*(c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(
c^3*d*x^6 - 8*c^4*x^3)]
```

3.403.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx = \int \frac{\sqrt{c+dx^3}}{x^4\left(-8c+dx^3\right)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**4*(-8*c + d*x**3)**2), x)
```

3.403.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^4}\,dx$$

input

```
integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output

```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^4), x)
```

3.403.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx=\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{128\sqrt{-c}c^2}-\frac{7\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{1152\sqrt{-c}c^2}-\frac{(dx^3+c)^{\frac{3}{2}}d-5\sqrt{dx^3+c}cd}{96\left((dx^3+c)^2-10\,(dx^3+c)c+9\,c^2\right)c^2}$$

input

```
integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output

```
1/128*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 7/1152*d*arctan(
1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/96*((d*x^3 + c)^(3/2)*d -
 5*sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c^2)
```

3.403.9 Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx=\frac{\frac{5\,d\sqrt{d\,x^3+c}}{32\,c}-\frac{d\,\left(d\,x^3+c\right)^{3/2}}{32\,c^2}}{3\,\left(d\,x^3+c\right)^2-30\,c\,\left(d\,x^3+c\right)+27\,c^2}+\frac{d\,\left(\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)\,7{}\mathrm{i}}{9}\right)\,1{}\mathrm{i}}{128\,\sqrt{c^5}}$$

input
```
int((c + d*x^3)^(1/2)/(x^4*(8*c - d*x^3)^2),x)
```

output
```
((5*d*(c + d*x^3)^(1/2))/(32*c) - (d*(c + d*x^3)^(3/2))/(32*c^2))/(3*(c +
d*x^3)^2 - 30*c*(c + d*x^3) + 27*c^2) + (d*(atanh((c^2*(c + d*x^3)^(1/2))/
(c^5)^(1/2))*1i - (atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))*7i)/9)*1
i)/(128*(c^5)^(1/2))
```

3.403. $\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx$

3.404 $\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx$

3.404.1 Optimal result

Integrand size = 27, antiderivative size = 164

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx = \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} + \frac{23d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}} - \frac{d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}}$$

output

```
23/18432*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-1/2048*d^2*arcta
nh((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+5/1536*d^2*(d*x^3+c)^(1/2)/c^3/(-d*x^3
+8*c)-1/48*(d*x^3+c)^(1/2)/c/x^6/(-d*x^3+8*c)-7/384*d*(d*x^3+c)^(1/2)/c^2/
x^3/(-d*x^3+8*c)
```

3.404.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx = \frac{\frac{12\sqrt{c}\sqrt{c+dx^3}\left(32c^2+28cdx^3-5d^2x^6\right)}{-8cx^6+dx^9}+23d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-9d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{18432c^{7/2}}$$

input

```
Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2),x]
```

output
```
((12*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 28*c*d*x^3 - 5*d^2*x^6))/(-8*c*x^6
+ d*x^9) + 23*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d^2*ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]])/(18432*c^(7/2))
```

3.404.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {948, 110, 27, 168, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^7 \left(8c-dx^3\right)^2} \, dx$$

$$\Big\downarrow 948$$

$$\frac{1}{3} \int \frac{\sqrt{dx^3+c}}{x^9 \left(8c-dx^3\right)^2} dx^3$$

$$\Big\downarrow 110$$

$$\frac{1}{3} \left(\frac{\int \frac{d\left(5dx^3+14c\right)}{2x^6\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{16c} - \frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)} \right)$$

$$\Big\downarrow 27$$

$$\frac{1}{3} \left(\frac{d\int \frac{5dx^3+14c}{x^6\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{32c} - \frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)} \right)$$

$$\Big\downarrow 168$$

$$\frac{1}{3} \left(\frac{d\left(-\frac{\int -\frac{3cd\left(7dx^3+4c\right)}{x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{8c^2} - \frac{7\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)}{32c} - \frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)} \right)$$

$$\Big\downarrow 27$$

$$\frac{1}{3}\left(\frac{d\left(\frac{3d\int \frac{7dx^3+4c}{x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{8c}-\frac{7\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(\frac{d\left(\frac{3d\left(\frac{5\sqrt{c+dx^3}}{6c\left(8c-dx^3\right)}-\frac{\int -\frac{6cd\left(5dx^3+6c\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{72c^2d}\right)}{8c}-\frac{7\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{d\left(\frac{3d\left(\frac{\int \frac{5dx^3+6c}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{12c}+\frac{5\sqrt{c+dx^3}}{6c\left(8c-dx^3\right)}\right)}{8c}-\frac{7\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{174}$$

3.404. $\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)^2}\,dx$

$$\frac{1}{3}\left(\frac{d\left(\frac{3d\left(\frac{\frac{3}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{23}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{12c}+\frac{5\sqrt{c+dx^3}}{6c\left(8c-dx^3\right)}\right)}{8c}-\frac{7\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{d\left(\frac{3d\left(\frac{\frac{23}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{3\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{12c}+\frac{5\sqrt{c+dx^3}}{6c\left(8c-dx^3\right)}\right)}{8c}-\frac{7\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{d\left(\frac{3d\left(\frac{\frac{3\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{23\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{12c}+\frac{5\sqrt{c+dx^3}}{6c\left(8c-dx^3\right)}\right)}{8c}-\frac{7\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \; 221$$

3.404. $\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx$

$$\frac{1}{3}\left(\frac{d\left(\frac{3d\left(\frac{\frac{23\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{12c}+\frac{5\sqrt{c+dx^3}}{6c\left(8c-dx^3\right)}\right)}{8c}-\frac{7\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c}-\frac{\sqrt{c+dx^3}}{16cx^6\left(8c-dx^3\right)}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2),x]
```

output
```
(-1/16*Sqrt[c + d*x^3]/(c*x^6*(8*c - d*x^3)) + (d*((-7*Sqrt[c + d*x^3])/(4
*c*x^3*(8*c - d*x^3)) + (3*d*((5*Sqrt[c + d*x^3])/(6*c*(8*c - d*x^3)) + ((
23*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*Sqrt[c]) - (3*ArcTanh[Sqrt[c +
 d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(12*c)))/(8*c)))/(32*c))/3
```

3.404.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.404. $\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)^2}\,dx$

3.404.4 Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{(d x^3+c)^{\frac{3}{2}}}{384c^3x^6}-\frac{d^2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{8\sqrt{c}}-\frac{19\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{72\sqrt{c}}-\frac{c\left(-\frac{\sqrt{d x^3+c}}{c(d x^3-8c)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{6}\right)}{256c^3}$
pseudoelliptic	$-\frac{14\sqrt{d x^3+c}\,c^{\frac{3}{2}}d x^3+16\sqrt{d x^3+c}\,c^{\frac{5}{2}}+\frac{\left(-5\sqrt{d x^3+c}\sqrt{c}+\frac{(-d x^3+8c)\left(9\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)-23\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)\right)}{12}\right)d^2x^6}{2}}{768c^{\frac{7}{2}}(-d x^9+8c x^6)}$
default	$\frac{-\frac{\sqrt{d x^3+c}}{6x^6}-\frac{d\sqrt{d x^3+c}}{12c x^3}+\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{12c^{\frac{3}{2}}}}{64c^2}+\frac{d\left(-\frac{\sqrt{d x^3+c}}{3x^3}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3\sqrt{c}}\right)}{256c^3}+\frac{3d^2\left(\frac{2\sqrt{d x^3+c}}{3}-\frac{2\operatorname{arctanh}\left(\sqrt{\right.}\right.}{3}\right.}{4096c^4}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/384*(d*x^3+c)^(3/2)/c^3/x^6-1/256/c^3*d^2*(1/8*arctanh((d*x^3+c)^(1/2)/
c^(1/2))/c^(1/2)-19/72*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/6*c*
(-(d*x^3+c)^(1/2)/c/(d*x^3-8*c)+1/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c
^(3/2)))
```

3.404.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx$$
$$=\left[\frac{23\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+9\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\,(5\,c}{36864\left(c^4dx^9-8\,c^5x^6\right)}\right.$$

input
```
integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/36864*(23*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((
d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d^2*x^6 - 28*c^2*d
*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6), 1/18432*(9*(d^3*x
^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 23*(d^3*x^
9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*(5*c
*d^2*x^6 - 28*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6)
]
```

3.404.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c)**2,x)
```

output
```
Timed out
```

3.404.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^7}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^7), x)
```

3.404.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx = \frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\,\sqrt{-c}c^3} - \frac{23\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{18432\,\sqrt{-c}c^3} - \frac{\sqrt{dx^3+c}d^2}{1536\,(dx^3-8\,c)c^3} - \frac{(dx^3+c)^{\frac{3}{2}}}{384\,c^3x^6}$$

input
```
integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
1/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 23/18432*d^2*
arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/1536*sqrt(d*x^3 +
c)*d^2/((d*x^3 - 8*c)*c^3) - 1/384*(d*x^3 + c)^(3/2)/(c^3*x^6)
```

3.404.9 Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx = \frac{\frac{d^2\,\sqrt{d\,x^3+c}}{512\,c} - \frac{19\,d^2\,\left(d\,x^3+c\right)^{3/2}}{256\,c^2} + \frac{5\,d^2\,\left(d\,x^3+c\right)^{5/2}}{512\,c^3}}{33\,c\,\left(d\,x^3+c\right)^2 - 57\,c^2\,\left(d\,x^3+c\right) - 3\,\left(d\,x^3+c\right)^3 + 27\,c^3} + \frac{d^2\left(\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1\mathrm{i} - \frac{\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)\,23\mathrm{i}}{9}\right)\,1\mathrm{i}}{2048\,\sqrt{c^7}}$$

input
```
int((c + d*x^3)^(1/2)/(x^7*(8*c - d*x^3)^2),x)
```

output
```
((d^2*(c + d*x^3)^(1/2))/(512*c) - (19*d^2*(c + d*x^3)^(3/2))/(256*c^2) +
(5*d^2*(c + d*x^3)^(5/2))/(512*c^3))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x
^3) - 3*(c + d*x^3)^3 + 27*c^3) + (d^2*(atanh((c^3*(c + d*x^3)^(1/2))/(c^7
)^(1/2))*1i - (atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2)))*23i)/9)*1i)/
(2048*(c^7)^(1/2))
```

3.405 $\int \frac{x^7\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

3.405.1 Optimal result

Integrand size = 27, antiderivative size = 663

$$\begin{aligned}
\int \frac{x^7\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx &= \frac{13x^2\sqrt{c+dx^3}}{21d^2}+\frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&+\frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}+\frac{76c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}} \\
&-\frac{76c^{7/6}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}+\frac{76c^{7/6}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}} \\
&-\frac{373\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+\frac{746\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{21\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output

```
-76/9*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d
^(8/3)+76/9*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)+76/9*c^(7
/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)*3^
(1/2)+13/21*x^2*(d*x^3+c)^(1/2)/d^2+1/3*x^5*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)
+746/21*c*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+746/63*c
^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3
)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(8/3)/
(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)
))^2)^(1/2)-373/21*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*
x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2
*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^
(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.405.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.27

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx =$$

$$-\frac{80(52c^2x^2+49cdx^5-3d^2x^8)+520cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+373dx}{840d^2\left(-8c+dx^3\right)\sqrt{c+dx^3}}$$

input

```
Integrate[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output

```
-1/840*(80*(52*c^2*x^2 + 49*c*d*x^5 - 3*d^2*x^8) + 520*c*x^2*(-8*c + d*x^3
)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*
c)] + 373*d*x^5*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8
/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(d^2*(-8*c + d*x^3)*Sqrt[c + d*x^3])
```

3.405.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {967, 27, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}dx$$

$$\Big\downarrow \text{967}$$

$$\frac{x^5\sqrt{c+dx^3}}{3d(8c-dx^3)}-\frac{\int \frac{x^4(13dx^3+10c)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^5\sqrt{c+dx^3}}{3d(8c-dx^3)}-\frac{\int \frac{x^4(13dx^3+10c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{6d}$$

$$\Big\downarrow \text{1052}$$

$$\frac{x^5\sqrt{c+dx^3}}{3d(8c-dx^3)}-\frac{\frac{2\int \frac{cdx\left(373dx^3+208c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d^2}-\frac{26x^2\sqrt{c+dx^3}}{7d}}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^5\sqrt{c+dx^3}}{3d(8c-dx^3)}-\frac{\frac{2c\int \frac{x\left(373dx^3+208c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d}-\frac{26x^2\sqrt{c+dx^3}}{7d}}{6d}$$

$$\Big\downarrow \text{1054}$$

$$\frac{x^5\sqrt{c+dx^3}}{3d(8c-dx^3)}-\frac{\frac{2c\int \left(\frac{3192cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}-\frac{373x}{\sqrt{dx^3+c}}\right)dx}{7d}-\frac{26x^2\sqrt{c+dx^3}}{7d}}{6d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-$$

$$2c\left(-\frac{746\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{373\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{2/3}}\right.$$

input
```
Int[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
(x^5*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)) - ((-26*x^2*Sqrt[c + d*x^3])/(7*
d) + (2*c*((-746*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)) - (532*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c
 + d*x^3]])/(Sqrt[3]*d^(2/3)) + (532*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)
^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)) - (532*c^(1/6)*ArcTanh[Sqrt[c
 + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) + (373*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/
3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(
1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(
d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*Sqrt[c + d*x^3]) - (746*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
d*x^3])))/(7*d))/(6*d)
```

3.405. $\int \frac{x^7\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

3.405.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 967

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1052

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.405.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.54 (sec) , antiderivative size = 897, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	897
risch	Expression too large to display	1758
default	Expression too large to display	2199

input
```
int(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
8/3*x^2*c/d^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/7*x^2*(d*x^3+c)^(1/2)/d^2-746
/63*I/d^3*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3)
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+152/27*I*c
/d^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3
))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)...
```

3.405.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.83 (sec) , antiderivative size = 2568, normalized size of antiderivative = 3.87

$$\int \frac{x^7\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/189*(6714*(c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstr
assPInverse(0, -4*c/d, x)) + 133*(d^4*x^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 -
8*c*d^3))*(c^7/d^16)^(1/6)*log(2535525376/3*((d^16*x^9 + 318*c*d^15*x^6 +
1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1
200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^7/d^16)^(5/6) + 6*(2*c^6*d^2*x^7 + 16
0*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2 - sqrt(-3)*(
5*c^2*d^12*x^5 + 32*c^3*d^11*x^2))*(c^7/d^16)^(2/3) - (7*c^4*d^7*x^6 + 152
*c^5*d^6*x^3 + 64*c^6*d^5 + sqrt(-3)*(7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64
*c^6*d^5))*(c^7/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^10*x^7 + 64*c^4
*d^9*x^4 + 32*c^5*d^8*x)*sqrt(c^7/d^16) + 18*(c^5*d^5*x^8 + 38*c^6*d^4*x^5
 + 64*c^7*d^3*x^2 - sqrt(-3)*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3*x^
2))*(c^7/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
- 133*(d^4*x^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(c^7/d^16)^(1/6)*
log(-2535525376/3*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^
3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3
*d^13))*(c^7/d^16)^(5/6) - 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x -
6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2 - sqrt(-3)*(5*c^2*d^12*x^5 + 32*c^3*d^
11*x^2))*(c^7/d^16)^(2/3) - (7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64*c^6*d^5
+ sqrt(-3)*(7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64*c^6*d^5))*(c^7/d^16)^(1/3
))*sqrt(d*x^3 + c) - 36*(5*c^3*d^10*x^7 + 64*c^4*d^9*x^4 + 32*c^5*d^8*x...
```

3.405.6 Sympy [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\int \frac{x^7\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x**7*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**7*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

3.405.7 Maxima [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\int \frac{\sqrt{dx^3+c}x^7}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c)^2, x)
```

3.405.8 Giac [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\int \frac{\sqrt{dx^3+c}x^7}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c)^2, x)
```

3.405.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\int \frac{x^7\,\sqrt{d\,x^3+c}}{(8\,c-d\,x^3)^2}\,dx$$

input
```
int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

output
```
int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2, x)
```

3.406 $\int \frac{x^4\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

3.406.1 Optimal result

Integrand size = 27, antiderivative size = 641

$$
\begin{aligned}
&\int \frac{x^4\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx \\
&= \frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}+\frac{5\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} \\
&\quad-\frac{5\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{5/3}}+\frac{5\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{5/3}} \\
&\quad-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad+\frac{7\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

output
```
-5/9*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^
(5/3)+5/9*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)+5/9*c^(1/6)
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(5/3)*3^(1/
2)+1/3*x^2*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)+7/3*(d*x^3+c)^(1/2)/d^(5/3)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))+7/9*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(
1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)
*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2)))^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)
*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-7/6*3^(1/4)*c^(1/3)*(c^(1/3)+
d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1
/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3
+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

3.406.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.26

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$
$$=\frac{80cx^2(c+dx^3)+10cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+7dx^5(-8c+dx^3)\sqrt{1+}}{240cd\,(8c-dx^3)\,\sqrt{c+dx^3}}$$

input
```
Integrate[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
(80*c*x^2*(c + d*x^3) + 10*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appell
F1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 7*d*x^5*(-8*c + d*x^3)
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c
)])/(240*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

3.406.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {967, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \text{967}$$

$$\frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{\int \frac{x\left(7dx^3+4c\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{\int \frac{x\left(7dx^3+4c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{6d}$$

$$\Big\downarrow \text{1054}$$

$$\frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{\int \left(\frac{60cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}-\frac{7x}{\sqrt{dx^3+c}}\right)dx}{6d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-$$

$$-\frac{\frac{14\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{7\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{}}{}$$

input
```
Int[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output

```
(x^2*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)) - ((-14*Sqrt[c + d*x^3])/(d^(2/3
)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (10*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/
6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) + (10*c^(1/6
)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)
) - (10*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) + (7*3^(
1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipti
cE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (14*Sqrt[2]*c^(1/
3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(
1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/(6*d)
```

3.406.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 967

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.406. $\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

3.406.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.57 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	877
default	Expression too large to display	1741

input
```
int(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^2*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)-7/9*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)))+10/27*I/d^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)
*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)
^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*
d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3
)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-...
```

3.406.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.73 (sec) , antiderivative size = 2397, normalized size of antiderivative = 3.74

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \text{Too large to display}$$

input

```
integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output

```
-1/108*(36*sqrt(d*x^3 + c)*d*x^2 + 252*(d*x^3 - 8*c)*sqrt(d)*weierstrassZe
ta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 5*(d^3*x^3 - 8*c*d^2 -
sqrt(-3)*(d^3*x^3 - 8*c*d^2))*(c/d^10)^(1/6)*log(3125/3*((d^11*x^9 + 318*c
*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^
10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8))*(c/d^10)^(5/6) + 6*(2*c*d^2*x^7
+ 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2 - sqrt(-3)*(
5*c*d^8*x^5 + 32*c^2*d^7*x^2))*(c/d^10)^(2/3) - (7*c*d^5*x^6 + 152*c^2*d^4
*x^3 + 64*c^3*d^3 + sqrt(-3)*(7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3))
*(c/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c
^3*d^5*x)*sqrt(c/d^10) + 18*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2 -
 sqrt(-3)*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2))*(c/d^10)^(1/6))/(
d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 5*(d^3*x^3 - 8*c*d^2
- sqrt(-3)*(d^3*x^3 - 8*c*d^2))*(c/d^10)^(1/6)*log(-3125/3*((d^11*x^9 + 31
8*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c
*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8))*(c/d^10)^(5/6) - 6*(2*c*d^2*x
^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2 - sqrt(-3
)*(5*c*d^8*x^5 + 32*c^2*d^7*x^2))*(c/d^10)^(2/3) - (7*c*d^5*x^6 + 152*c^2*
d^4*x^3 + 64*c^3*d^3 + sqrt(-3)*(7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^
3))*(c/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 3
2*c^3*d^5*x)*sqrt(c/d^10) + 18*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2...
```

3.406.6 Sympy [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^4\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

input

```
integrate(x**4*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

output

```
Integral(x**4*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

3.406.7 Maxima [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x^4}{(dx^3-8c)^2}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c)^2, x)
```

3.406.8 Giac [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x^4}{(dx^3-8c)^2}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c)^2, x)
```

3.406.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^4\,\sqrt{d\,x^3+c}}{(8\,c-d\,x^3)^2}\,dx$$

input
```
int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

output
```
int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2, x)
```

3.407 $\int \frac{x\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

3.407.1 Optimal result

Integrand size = 25, antiderivative size = 644

$$\begin{aligned}
\int \frac{x\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx &= \frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} \\
&+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{5/6}d^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{5/6}d^{2/3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{5/6}d^{2/3}} \\
&-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)\middle|-7-4\sqrt{3}\right)}{16\,3^{3/4}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
-1/144*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/
d^(2/3)+1/144*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(2/3)+1/144*a
rctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(2/3)
*3^(1/2)+1/24*x^2*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)+1/24*(d*x^3+c)^(1/2)/c/d^
(2/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/72*(c^(1/3)+d^(1/3)*x)*EllipticF((
d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)
))^2)^(1/2)*3^(3/4)/c^(2/3)/d^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1
/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/48*(c^(1/3)+d^(1
/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*
x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(2/3)/d^
(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

3.407.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.25

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$
$$= \frac{80cx^2(c+dx^3)+5cx^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^5(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,A}{1920c^2\,(8c-dx^3)\sqrt{c+dx^3}}$$

input
```
Integrate[(x*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

output
```
(80*c*x^2*(c + d*x^3) + 5*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1
[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/
(1920*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

3.407.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 640, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {969, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}dx$$

$$\Big\downarrow 969$$

$$\frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)}-\frac{\int -\frac{x(2c-dx^3)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{24c}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{x(2c-dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{48c}+\frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)}$$

$$\Big\downarrow 1054$$

$$\frac{\int \left(\frac{x}{\sqrt{dx^3+c}}-\frac{6cx}{(8c-dx^3)\sqrt{dx^3+c}}\right)dx}{48c}+\frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)}$$

$$\Big\downarrow 2009$$

$$\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^2}{(}}}{d^{2/3}}$$

$$\frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)}$$

input
```
Int[(x*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

---

output
```
(x^2*Sqrt[c + d*x^3])/(24*c*(8*c - d*x^3)) + ((2*Sqrt[c + d*x^3])/(d^(2/3)
*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(
c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) - (c^(1/6)*ArcTa
nh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)) + (c^
(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) - (3^(1/4)*Sqrt[2
- Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((
1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7
 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3
)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/
3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x
)^2]*Sqrt[c + d*x^3]))/(48*c)
```

3.407.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 969
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[(e*x)^m*(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
 m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.407.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.49 (sec) , antiderivative size = 883, normalized size of antiderivative = 1.37

method	result	size
default	Expression too large to display	883
elliptic	Expression too large to display	883

input
```
int(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/24*x^2*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)-1/72*I/c*3^(1/2)/d*(-c*d^2)^(1/3)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)))+1/216*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(
1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*
d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1
/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-...
```

3.407.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 2496, normalized size of antiderivative = 3.88

$$\int \frac{x\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/1728*(72*sqrt(d*x^3 + c)*d*x^2 + 72*(d*x^3 - 8*c)*sqrt(d)*weierstrassZe
ta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + (c*d^2*x^3 - 8*c^2*d +
sqrt(-3)*(c*d^2*x^3 - 8*c^2*d))*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d
^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32
*c^6*d^3*x + sqrt(-3)*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x))*(1/
(c^5*d^4))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2 -
sqrt(-3)*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2))*(1/(c^5*d^4))^(5/6) - 2*(7*c^3*
d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 8
0*c^2*d^2*x^4 + 160*c^3*d*x + sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c
^3*d*x))*(1/(c^5*d^4))^(1/6)) - 9*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d
^2*x^2 - sqrt(-3)*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2))*(1/(c^5
*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c*d^2
*x^3 - 8*c^2*d + sqrt(-3)*(c*d^2*x^3 - 8*c^2*d))*(1/(c^5*d^4))^(1/6)*log((
d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^5*x^7 + 64
*c^5*d^4*x^4 + 32*c^6*d^3*x + sqrt(-3)*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 3
2*c^6*d^3*x))*(1/(c^5*d^4))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 +
32*c^6*d^4*x^2 - sqrt(-3)*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2))*(1/(c^5*d^4))^
(5/6) - 2*(7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4))
 + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x + sqrt(-3)*(c*d^3*x^7 + 80*c^
2*d^2*x^4 + 160*c^3*d*x))*(1/(c^5*d^4))^(1/6)) - 9*(c^2*d^4*x^8 + 38*c^...
```

3.407.6 Sympy [F]

$$\int \frac{x\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \int \frac{x\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

input
```
integrate(x*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

3.407.7 Maxima [F]

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\int \frac{\sqrt{dx^3+cx}}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c)^2, x)
```

3.407.8 Giac [F]

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\int \frac{\sqrt{dx^3+cx}}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c)^2, x)
```

3.407.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\int \frac{x\,\sqrt{d\,x^3+c}}{(8\,c-d\,x^3)^2}\,dx$$

input
```
int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

output
```
int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2, x)
```

3.408 $\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx$

3.408.1 Optimal result

Integrand size = 27, antiderivative size = 665

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx &= -\frac{\sqrt{c+dx^3}}{48c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}\\
&-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}}+\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{11/6}}-\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{11/6}}\\
&-\frac{\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32\,3^{3/4}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\\
&+\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{24\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\end{aligned}$$

output
```
1/144*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(11/6)-1/144*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-1/144*
d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/
6)*3^(1/2)-1/48*(d*x^3+c)^(1/2)/c^2/x+1/24*(d*x^3+c)^(1/2)/c/x/(-d*x^3+8*c
)+1/48*d^(1/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/144*d
^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2
/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/
(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)
))^2)^(1/2)-1/96*3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+
c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6
^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1
/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.408.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)^2}\,dx$$

$$= \frac{-80c(6c^2+5cdx^3-d^2x^6)+50cdx^3(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+d^2x^6(-8c+d}{3840c^3\sqrt{c+dx^3}\,(8cx-dx^4)}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)^2),x]
```

output
```
(-80*c*(6*c^2 + 5*c*d*x^3 - d^2*x^6) + 50*c*d*x^3*(8*c - d*x^3)*Sqrt[1 + (
d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d^2*x^
6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/
c), (d*x^3)/(8*c)])/(3840*c^3*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

3.408.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {969, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^2 \left(8c-dx^3\right)^2} \, dx$$

$$\Big\downarrow \text{969}$$

$$\frac{\sqrt{c+dx^3}}{24cx \left(8c-dx^3\right)}-\frac{\int -\frac{5dx^3+8c}{2x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{24c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5dx^3+8c}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{48c}+\frac{\sqrt{c+dx^3}}{24cx \left(8c-dx^3\right)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int -\frac{4cdx\left(20c-dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{\sqrt{c+dx^3}}{cx}}{48c}+\frac{\sqrt{c+dx^3}}{24cx \left(8c-dx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{d\int \frac{x\left(20c-dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{2c}-\frac{\sqrt{c+dx^3}}{cx}}{48c}+\frac{\sqrt{c+dx^3}}{24cx \left(8c-dx^3\right)}$$

$$\Big\downarrow \text{1054}$$

$$\frac{\frac{d\int \left(\frac{12cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{x}{\sqrt{dx^3+c}}\right)dx}{2c}-\frac{\sqrt{c+dx^3}}{cx}}{48c}+\frac{\sqrt{c+dx^3}}{24cx \left(8c-dx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$d\left(\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}}{\left(\left(1+\sqrt{3}\right)\right.}}}{d^{2/3}\sqrt{\frac{\sqrt[3]{}}{\left(\left(1+\right.\right.}}}\right.$$

$$\frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}$$

input
```
Int[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)^2),x]
```

output
```
Sqrt[c + d*x^3]/(24*c*x*(8*c - d*x^3)) + (-(Sqrt[c + d*x^3]/(c*x)) + (d*((
2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (2*c^(1
/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt
[3]*d^(2/3)) + (2*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[
c + d*x^3])])/(3*d^(2/3)) - (2*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/(3*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*
Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]
) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/
3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(2*c))/(48*c)
```

3.408.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 969
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[(e*x)^m*(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
 m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.408.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.67 (sec) , antiderivative size = 898, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	898
risch	Expression too large to display	1758
default	Expression too large to display	2194

input
```
int((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192*x^2/c^2*d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/64*(d*x^3+c)^(1/2)/c^2/x-1/
144*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/216*I/d^2/
c^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3)
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha
^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^...
```

3.408.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 2390, normalized size of antiderivative = 3.59

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output

```
-1/1728*(36*(d*x^4 - 8*c*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrass
PInverse(0, -4*c/d, x)) - (c^2*d*x^4 - 8*c^3*x + sqrt(-3)*(c^2*d*x^4 - 8*c
^3*x))*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 +
640*c^3*d - 9*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x + sqrt(-3)*(5*c^8*
d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x))*(d^2/c^11)^(2/3) + 3*sqrt(d*x^3 + c)*
(6*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d
^2/c^11)^(5/6) - 2*(7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11)
 + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 8
0*c^3*d^2*x^4 + 160*c^4*d*x))*(d^2/c^11)^(1/6)) - 9*(c^4*d^3*x^8 + 38*c^5*
d^2*x^5 + 64*c^6*d*x^2 - sqrt(-3)*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d
*x^2))*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)) + (c^2*d*x^4 - 8*c^3*x + sqrt(-3)*(c^2*d*x^4 - 8*c^3*x))*(d^2/c^11)^(1/
6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^8*
d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x + sqrt(-3)*(5*c^8*d^2*x^7 + 64*c^9*d*x^
4 + 32*c^10*x))*(d^2/c^11)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32
*c^11*x^2 - sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^2/c^11)^(5/6) - 2*(7
*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) + (c^2*d^3*x^7 + 80*
c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c
^4*d*x))*(d^2/c^11)^(1/6)) - 9*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^
2 - sqrt(-3)*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2))*(d^2/c^11)^...
```

3.408.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{c+dx^3}}{x^2\left(-8c+dx^3\right)^2}\,dx$$

input

```
integrate((d*x**3+c)**(1/2)/x**2/(-d*x**3+8*c)**2,x)
```

output

```
Integral(sqrt(c + d*x**3)/(x**2*(-8*c + d*x**3)**2), x)
```

3.408.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^2), x)
```

3.408.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^2), x)
```

3.408.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^2\,\left(8\,c-d\,x^3\right)^2}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)^2),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)^2), x)
```

3.409 $\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx$

3.409.1 Optimal result

Integrand size = 27, antiderivative size = 687

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx &= -\frac{7\sqrt{c+dx^3}}{768c^2x^4}-\frac{d\sqrt{c+dx^3}}{96c^3x}+\frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&+\frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}-\frac{17d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{17/6}} \\
&+\frac{17d^{4/3}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{17/6}}-\frac{17d^{4/3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9216c^{17/6}} \\
&-\frac{\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{64\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{48\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
17/9216*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))
/c^(17/6)-17/9216*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-17
/9216*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/
c^(17/6)*3^(1/2)-7/768*(d*x^3+c)^(1/2)/c^2/x^4-1/96*d*(d*x^3+c)^(1/2)/c^3/
x+1/24*(d*x^3+c)^(1/2)/c/x^4/(-d*x^3+8*c)+1/96*d^(4/3)*(d*x^3+c)^(1/2)/c^3
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/288*d^(4/3)*(c^(1/3)+d^(1/3)*x)*Ellipti
cF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/
2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+
d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/192*3^(1/4)*d^(4/3)*
(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(
1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(8/
3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

3.409.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.29

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx &= \sqrt{c+dx^3}\left(-\frac{1}{256c^2x^4}-\frac{5d}{512c^3x}-\frac{d^2x^2}{1536c^3\left(-8c+dx^3\right)}\right) \\
&\quad+\frac{115d^2x^2\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{24576c^3\sqrt{c+dx^3}} \\
&\quad-\frac{d^3x^5\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{7680c^4\sqrt{c+dx^3}}
\end{aligned}
$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)^2),x]
```

output
```
Sqrt[c + d*x^3]*(-1/256*1/(c^2*x^4) - (5*d)/(512*c^3*x) - (d^2*x^2)/(1536*
c^3*(-8*c + d*x^3))) + (115*d^2*x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2,
 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(24576*c^3*Sqrt[c + d*x^3]) - (d^3*
x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(7680*c^4*Sqrt[c + d*x^3])
```

3.409.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {969, 27, 1053, 25, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^5 \left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \text{ 969}$$

$$\frac{\sqrt{c+dx^3}}{24cx^4 \left(8c-dx^3\right)}-\frac{\int -\frac{11dx^3+14c}{2x^5 \left(8c-dx^3\right) \sqrt{dx^3+c}}dx}{24c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{11dx^3+14c}{x^5 \left(8c-dx^3\right) \sqrt{dx^3+c}}dx}{48c}+\frac{\sqrt{c+dx^3}}{24cx^4 \left(8c-dx^3\right)}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{-\frac{\int -\frac{cd\left(35dx^3+128c\right)}{x^2 \left(8c-dx^3\right) \sqrt{dx^3+c}}dx}{32c^2}-\frac{7\sqrt{c+dx^3}}{16cx^4}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^4 \left(8c-dx^3\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{cd\left(35dx^3+128c\right)}{x^2 \left(8c-dx^3\right) \sqrt{dx^3+c}}dx}{32c^2}-\frac{7\sqrt{c+dx^3}}{16cx^4}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^4 \left(8c-dx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{d\int \frac{35dx^3+128c}{x^2 \left(8c-dx^3\right) \sqrt{dx^3+c}}dx}{32c}-\frac{7\sqrt{c+dx^3}}{16cx^4}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^4 \left(8c-dx^3\right)}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{\frac{d \left(-\frac{\int -\frac{8cdx\left(115c-8dx^3\right)}{\left(8c-dx^3\right) \sqrt{dx^3+c}}dx}{8c^2}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{7\sqrt{c+dx^3}}{16cx^4}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^4 \left(8c-dx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{d\left(\frac{d\int \frac{x\left(115c-8dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{7\sqrt{c+dx^3}}{16cx^4}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{1054}$$

$$\frac{\frac{d\left(\frac{d\int \left(\frac{51cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{8x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{7\sqrt{c+dx^3}}{16cx^4}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$d\left(\frac{d\left(\frac{16\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{8\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}}{((1}}}{d^{2/3}}\right)}{}\right)$$

$$\frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}$$

input
```
Int[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)^2),x]
```

3.409. $\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx$

output
```
Sqrt[c + d*x^3]/(24*c*x^4*(8*c - d*x^3)) + ((-7*Sqrt[c + d*x^3])/(16*c*x^4
) + (d*((-16*Sqrt[c + d*x^3])/(c*x) + (d*((16*Sqrt[c + d*x^3])/(d^(2/3)*((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (17*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(
c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*Sqrt[3]*d^(2/3)) + (17*c^(1/6)*
ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/3))
- (17*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*d^(2/3)) - (8*3^(1/
4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)
*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (16*Sqrt[2]*c^(1/3)
*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/
3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3)
+ d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/c))/(32*c))/(48*c)
```

3.409.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 969
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[(e*x)^m*(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
 m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.409.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.65 (sec) , antiderivative size = 919, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	919
risch	Expression too large to display	1770
default	Expression too large to display	2672

input
```
int((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/1536*d^2*x^2/c^3*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/256*(d*x^3+c)^(1/2)/c^2/
x^4-5/512*d*(d*x^3+c)^(1/2)/c^3/x-1/288*I*d/c^3*3^(1/2)*(-c*d^2)^(1/3)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)))-17/13824*I/d/c^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(
1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*
d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1
/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^...
```

3.409.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.43 (sec) , antiderivative size = 2549, normalized size of antiderivative = 3.71

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/110592*(1152*(d^2*x^7 - 8*c*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, w
eierstrassPInverse(0, -4*c/d, x)) - 17*(c^3*d*x^7 - 8*c^4*x^4 + sqrt(-3)*(
c^3*d*x^7 - 8*c^4*x^4))*(d^8/c^17)^(1/6)*log(1419857*(d^9*x^9 + 318*c*d^8*
x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4
 + 32*c^14*d*x + sqrt(-3)*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x)
)*(d^8/c^17)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sq
rt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^8/c^17)^(5/6) - 2*(7*c^9*d^4*x^6 +
 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) + (c^3*d^7*x^7 + 80*c^4*d^
6*x^4 + 160*c^5*d^5*x + sqrt(-3)*(c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d
^5*x))*(d^8/c^17)^(1/6)) - 9*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^
2 - sqrt(-3)*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2))*(d^8/c^17)^(
1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 17*(c^3*d*x^7
- 8*c^4*x^4 + sqrt(-3)*(c^3*d*x^7 - 8*c^4*x^4))*(d^8/c^17)^(1/6)*log(14198
57*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^12*d
^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x + sqrt(-3)*(5*c^12*d^3*x^7 + 64*c^1
3*d^2*x^4 + 32*c^14*d*x))*(d^8/c^17)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^15*
d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^8/c^17)^(5
/6) - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) +
(c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d^5*x + sqrt(-3)*(c^3*d^7*x^7 + 80
*c^4*d^6*x^4 + 160*c^5*d^5*x))*(d^8/c^17)^(1/6)) - 9*(c^6*d^6*x^8 + 38*...
```

3.409.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{c+dx^3}}{x^5\left(-8c+dx^3\right)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**5/(-d*x**3+8*c)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**5*(-8*c + d*x**3)**2), x)
```

3.409.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5 \left(8c-dx^3\right)^2} \, dx = \int \frac{\sqrt{dx^3+c}}{{\left(dx^3-8\,c\right)}^2 x^5} \, dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^5), x)
```

3.409.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5 \left(8c-dx^3\right)^2} \, dx = \int \frac{\sqrt{dx^3+c}}{{\left(dx^3-8\,c\right)}^2 x^5} \, dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^5), x)
```

3.409.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^5 \left(8c-dx^3\right)^2} \, dx = \int \frac{\sqrt{d\,x^3+c}}{x^5\,{\left(8\,c-d\,x^3\right)}^2} \, dx$$

input
```
int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)^2),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)^2), x)
```

3.410 $\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx$

3.410.1 Optimal result

Integrand size = 27, antiderivative size = 711

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx &= -\frac{5\sqrt{c+dx^3}}{672c^2x^7}-\frac{53d\sqrt{c+dx^3}}{21504c^3x^4}-\frac{d^2\sqrt{c+dx^3}}{5376c^4x} \\
&+\frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)}-\frac{13d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12288\sqrt{3}c^{23/6}} \\
&+\frac{13d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{36864c^{23/6}}-\frac{13d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36864c^{23/6}} \\
&-\frac{\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{3584\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+\frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{2688\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
13/36864*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2)
)/c^(23/6)-13/36864*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-
13/36864*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2
))/c^(23/6)*3^(1/2)-5/672*(d*x^3+c)^(1/2)/c^2/x^7-53/21504*d*(d*x^3+c)^(1/
2)/c^3/x^4-1/5376*d^2*(d*x^3+c)^(1/2)/c^4/x+1/24*(d*x^3+c)^(1/2)/c/x^7/(-d
*x^3+8*c)+1/5376*d^(7/3)*(d*x^3+c)^(1/2)/c^4/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)
))+1/16128*3^(3/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3
)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^
(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(1
1/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/
3)*(1+3^(1/2)))^2)^(1/2)-1/10752*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*Ellip
ticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^
2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(11/3)/(d*x^3+c)^(1/2)/(c^(1
/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.410.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx$$

$$=\frac{1525cd^3x^9(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-8\bigg(20c(384c^4+648c^3dx^3+243c^2d^2x^6-}{3440640c^5x^7\left(8c-dx^3\right)\sqrt{c-}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)^2),x]
```

output
```
(1525*c*d^3*x^9*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/
3, -((d*x^3)/c), (d*x^3)/(8*c)] - 8*(20*c*(384*c^4 + 648*c^3*d*x^3 + 243*c
^2*d^2*x^6 - 25*c*d^3*x^9 - 4*d^4*x^12) + d^4*x^12*(8*c - d*x^3)*Sqrt[1 +
(d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(3440
640*c^5*x^7*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

3.410.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 728, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {969, 27, 1053, 27, 1053, 25, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^8 \left(8c-dx^3\right)^2} \, dx$$

$$\Big\downarrow \text{969}$$

$$\frac{\sqrt{c+dx^3}}{24cx^7 \left(8c-dx^3\right)}-\frac{\int -\frac{17dx^3+20c}{2x^8\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{24c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{17dx^3+20c}{x^8\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{48c}+\frac{\sqrt{c+dx^3}}{24cx^7 \left(8c-dx^3\right)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int -\frac{2cd\left(55dx^3+106c\right)}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{56c^2}-\frac{5\sqrt{c+dx^3}}{14cx^7}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^7 \left(8c-dx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{d\int \frac{55dx^3+106c}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{28c}-\frac{5\sqrt{c+dx^3}}{14cx^7}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^7 \left(8c-dx^3\right)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{\frac{d\left(-\frac{\int -\frac{cd\left(265dx^3+64c\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{53\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{5\sqrt{c+dx^3}}{14cx^7}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^7 \left(8c-dx^3\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{d\left(\frac{\int \frac{cd\left(265dx^3+64c\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{53\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{5\sqrt{c+dx^3}}{14cx^7}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^7 \left(8c-dx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{d\left(\frac{d\int \frac{265dx^3+64c}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{53\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{5\sqrt{c+dx^3}}{14cx^7}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 1053$$

$$\frac{\frac{d\left(\frac{d\left(-\frac{\int -\frac{8cdx\left(305c-4dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{8\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{53\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{5\sqrt{c+dx^3}}{14cx^7}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{d\left(\frac{d\left(\frac{d\int \frac{x\left(305c-4dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{8\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{53\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{5\sqrt{c+dx^3}}{14cx^7}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 1054$$

$$\frac{\frac{d\left(\frac{d\left(\frac{d\int \left(\frac{273cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{4x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{8\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{53\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{5\sqrt{c+dx^3}}{14cx^7}}{48c}+\frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 2009$$

3.410. $\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx$

$$d\left(d\left(d\left(\frac{8\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/}}{\left(\left(\right.\right.}}}{d^{2/3}}\right.\right.\right.$$

$$\frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)}$$

input
```
Int[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)^2),x]
```

3.410. $\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx$

output

```
Sqrt[c + d*x^3]/(24*c*x^7*(8*c - d*x^3)) + ((-5*Sqrt[c + d*x^3])/(14*c*x^7
) + (d*((-53*Sqrt[c + d*x^3])/(16*c*x^4) + (d*((-8*Sqrt[c + d*x^3])/(c*x)
+ (d*((8*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) -
(91*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]
])/(2*Sqrt[3]*d^(2/3)) + (91*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/3)) - (91*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]
/(3*Sqrt[c])])/(6*d^(2/3)) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3)
 + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1
/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqr
t[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*S
qrt[c + d*x^3]) + (8*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) -
 c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) +
 d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/c))
/(32*c)))/(28*c))/(48*c)
```

3.410.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 969

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[(e*x)^m*(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
 m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.410.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.51 (sec) , antiderivative size = 938, normalized size of antiderivative = 1.32

method	result	size
elliptic	Expression too large to display	938
risch	Expression too large to display	1781
default	Expression too large to display	3170

input
```
int((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12288*d^3*x^2/c^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/448*(d*x^3+c)^(1/2)/c^2
/x^7-13/7168*d*(d*x^3+c)^(1/2)/c^3/x^4-3/28672*d^2*(d*x^3+c)^(1/2)/c^4/x-1
/16128*I*d^2/c^4*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^
(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(
x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c
*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-13/55
296*I/c^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)
^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x
+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*
_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/...
```

3.410.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.94 (sec) , antiderivative size = 2582, normalized size of antiderivative = 3.63

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output

```
-1/3096576*(576*(d^3*x^10 - 8*c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4*c/d
, weierstrassPInverse(0, -4*c/d, x)) - 91*(c^4*d*x^10 - 8*c^5*x^7 + sqrt(-
3)*(c^4*d*x^10 - 8*c^5*x^7))*(d^14/c^23)^(1/6)*log(371293*(d^14*x^9 + 318*
c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^16*d^4*x^7 + 64*c^1
7*d^3*x^4 + 32*c^18*d^2*x + sqrt(-3)*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 3
2*c^18*d^2*x))*(d^14/c^23)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32
*c^21*x^2 - sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^14/c^23)^(5/6) - 2*(
7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*d^4)*sqrt(d^14/c^23) + (c^4*d^
11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x + sqrt(-3)*(c^4*d^11*x^7 + 80*c^5
*d^10*x^4 + 160*c^6*d^9*x))*(d^14/c^23)^(1/6)) - 9*(c^8*d^9*x^8 + 38*c^9*d
^8*x^5 + 64*c^10*d^7*x^2 - sqrt(-3)*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^1
0*d^7*x^2))*(d^14/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 5
12*c^3)) + 91*(c^4*d*x^10 - 8*c^5*x^7 + sqrt(-3)*(c^4*d*x^10 - 8*c^5*x^7))
*(d^14/c^23)^(1/6)*log(371293*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x
^3 + 640*c^3*d^11 - 9*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x +
sqrt(-3)*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x))*(d^14/c^23)^(
2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqrt(-3)*(5*c^20
*d*x^5 + 32*c^21*x^2))*(d^14/c^23)^(5/6) - 2*(7*c^12*d^6*x^6 + 152*c^13*d^
5*x^3 + 64*c^14*d^4)*sqrt(d^14/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 1
60*c^6*d^9*x + sqrt(-3)*(c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x...
```

3.410.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx=\text{Timed out}$$

input

```
integrate((d*x**3+c)**(1/2)/x**8/(-d*x**3+8*c)**2,x)
```

output

```
Timed out
```

3.410.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^8}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^8), x)
```

3.410.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^8}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^8), x)
```

3.410.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^8\,\left(8\,c-d\,x^3\right)^2}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)^2),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)^2), x)
```

3.411 $\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

3.411.1 Optimal result

Integrand size = 27, antiderivative size = 134

$$\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx=\frac{1664c^3\sqrt{c+dx^3}}{d^4}+\frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2}+\frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}+\frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{21d^4}-\frac{4992c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

output

```
3/7*x^6*(d*x^3+c)^(3/2)/d^2+1/3*x^9*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)+2/21*c*
(d*x^3+c)^(3/2)*(51*d*x^3+694*c)/d^4-4992*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1
/2)/c^(1/2))/d^4+1664*c^3*(d*x^3+c)^(1/2)/d^4
```

3.411.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx=\frac{2\sqrt{c+dx^3}\left(-145328c^4+12206c^3dx^3+301c^2d^2x^6+16cd^3x^9+d^4x^{12}\right)}{21d^4\left(-8c+dx^3\right)}-\frac{4992c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

input

```
Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output

```
(2*Sqrt[c + d*x^3]*(-145328*c^4 + 12206*c^3*d*x^3 + 301*c^2*d^2*x^6 + 16*c
*d^3*x^9 + d^4*x^12))/(21*d^4*(-8*c + d*x^3)) - (4992*c^(7/2)*ArcTanh[Sqrt
[c + d*x^3]/(3*Sqrt[c])])/d^4
```

3.411.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {948, 108, 27, 170, 25, 27, 164, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11} \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^9 \left(dx^3+c\right)^{3/2}}{\left(8c-dx^3\right)^2}dx^3$$

$$\Big\downarrow \text{108}$$

$$\frac{1}{3} \left(\frac{x^9 \left(c+dx^3\right)^{3/2}}{d \left(8c-dx^3\right)}-\frac{\int \frac{3x^6 \sqrt{dx^3+c} \left(3dx^3+2c\right)}{2 \left(8c-dx^3\right)}dx^3}{d}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(\frac{x^9 \left(c+dx^3\right)^{3/2}}{d \left(8c-dx^3\right)}-\frac{3 \int \frac{x^6 \sqrt{dx^3+c} \left(3dx^3+2c\right)}{8c-dx^3}dx^3}{2d}\right)$$

$$\Big\downarrow \text{170}$$

$$\frac{1}{3} \left(\frac{x^9 \left(c+dx^3\right)^{3/2}}{d \left(8c-dx^3\right)}-\frac{3 \left(-\frac{2 \int -\frac{cdx^3 \sqrt{dx^3+c} \left(85dx^3+48c\right)}{8c-dx^3}dx^3}{7d^2}-\frac{6x^6 \left(c+dx^3\right)^{3/2}}{7d}\right)}{2d}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{x^9\left(c+dx^3\right)^{3/2}}{d\left(8c-dx^3\right)}-\frac{3\left(\frac{2\int\frac{cdx^3\sqrt{dx^3+c}\left(85dx^3+48c\right)}{8c-dx^3}dx^3}{7d^2}-\frac{6x^6\left(c+dx^3\right)^{3/2}}{7d}\right)}{2d}\right)$$

$$\Big\downarrow 27$$

$$\frac{1}{3}\left(\frac{x^9\left(c+dx^3\right)^{3/2}}{d\left(8c-dx^3\right)}-\frac{3\left(\frac{2c\int\frac{x^3\sqrt{dx^3+c}\left(85dx^3+48c\right)}{8c-dx^3}dx^3}{7d}-\frac{6x^6\left(c+dx^3\right)^{3/2}}{7d}\right)}{2d}\right)$$

$$\Big\downarrow 164$$

$$\frac{1}{3}\left(\frac{x^9\left(c+dx^3\right)^{3/2}}{d\left(8c-dx^3\right)}-\frac{3\left(\frac{2c\left(\frac{5824c^2\int\frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3}{d}-\frac{2\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{3d^2}\right)}{7d}-\frac{6x^6\left(c+dx^3\right)^{3/2}}{7d}\right)}{2d}\right)$$

$$\Big\downarrow 60$$

$$\frac{1}{3}\left(\frac{x^9\left(c+dx^3\right)^{3/2}}{d\left(8c-dx^3\right)}-\frac{3\left(\frac{2c\left(\frac{5824c^2\left(9c\int\frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3-\frac{2\sqrt{c+dx^3}}{d}\right)}{d}-\frac{2\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{3d^2}\right)}{7d}-\frac{6x^6\left(c+dx^3\right)^{3/2}}{7d}\right)}{2d}\right)$$

$$\Big\downarrow 73$$

3.411. $\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

$$\frac{1}{3}\left(\frac{x^9\left(c+dx^3\right)^{3/2}}{d\left(8c-dx^3\right)}-\frac{3\left(\frac{2c\left(\frac{5824c^2\left(\frac{18c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)}{d}-\frac{2\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{3d^2}\right)}{7d}-\frac{6x^6\left(c+dx^3\right)^{3/2}}{7d}\right)}{2d}\right)$$

$$\Big\downarrow 219$$

$$\frac{1}{3}\left(\frac{x^9\left(c+dx^3\right)^{3/2}}{d\left(8c-dx^3\right)}-\frac{3\left(\frac{2c\left(\frac{5824c^2\left(\frac{6\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)}{d}-\frac{2\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{3d^2}\right)}{7d}-\frac{6x^6\left(c+dx^3\right)^{3/2}}{7d}\right)}{2d}\right)$$

input
```
Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
((x^9*(c + d*x^3)^(3/2))/(d*(8*c - d*x^3)) - (3*((-6*x^6*(c + d*x^3)^(3/2)
)/(7*d) + (2*c*((-2*(c + d*x^3)^(3/2)*(694*c + 51*d*x^3))/(3*d^2) + (5824*
c^2*((-2*Sqrt[c + d*x^3])/d + (6*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
])])/d))/d))/(7*d)))/(2*d))/3
```

3.411. $\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

3.411.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

3.411. $\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.411.4 Maple [A] (verified)

Time = 4.78 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{39936\left(c^{4}\left(c-\frac{d\,x^{3}}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)+\frac{\left(\sqrt{c}\,d^{4}x^{12}+16c^{\frac{3}{2}}d^{3}x^{9}+301c^{\frac{5}{2}}d^{2}x^{6}+12206c^{\frac{7}{2}}d\,x^{3}-145328c^{\frac{9}{2}}\right)\sqrt{d\,x^{3}+c}}{419328}\right)}{\sqrt{c}\,(-d^{5}x^{3}+8c\,d^{4})}$
risch	$\frac{2(d^{3}x^{9}+24c\,d^{2}x^{6}+493c^{2}d\,x^{3}+16150c^{3})\sqrt{d\,x^{3}+c}}{21d^{4}}+$ $576c^{4}\Bigg(-\frac{86\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}+\frac{8c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right.}{3d}$ d^{3}
default	$\frac{d\left(\frac{2d\,x^{9}\sqrt{d\,x^{3}+c}}{21}+\frac{16c\,x^{6}\sqrt{d\,x^{3}+c}}{105}+\frac{2c^{2}x^{3}\sqrt{d\,x^{3}+c}}{105d}-\frac{4c^{3}\sqrt{d\,x^{3}+c}}{105d^{2}}\right)+\frac{32c\left(d\,x^{3}+c\right)^{\frac{5}{2}}}{15d}}{d^{3}}-$ $128c^{2}\left(81c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-(d\,x\right.$ $3d^{4}$
elliptic	$\frac{1536c^{4}\sqrt{d\,x^{3}+c}}{d^{4}(-d\,x^{3}+8c)}+\frac{2x^{9}\sqrt{d\,x^{3}+c}}{21d}+\frac{16c\,x^{6}\sqrt{d\,x^{3}+c}}{7d^{2}}+\frac{986c^{2}x^{3}\sqrt{d\,x^{3}+c}}{21d^{3}}+\frac{32300c^{3}\sqrt{d\,x^{3}+c}}{21d^{4}}+$ $832ic^{3}\sqrt{2}\Bigg(\_\_\alpha=\mathrm{RootO}$

input
```
int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
-39936/c^(1/2)*(c^4*(c-1/8*d*x^3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+1/4
19328*(c^(1/2)*d^4*x^12+16*c^(3/2)*d^3*x^9+301*c^(5/2)*d^2*x^6+12206*c^(7/
2)*d*x^3-145328*c^(9/2))*(d*x^3+c)^(1/2))/(-d^5*x^3+8*c*d^4)
```

3.411. $\int \frac{x^{11}\left(c+dx^{3}\right)^{3/2}}{\left(8c-dx^{3}\right)^{2}}\,dx$

3.411.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.78

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \Bigg[\frac{2\left(26208\,(c^3dx^3-8\,c^4)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+(d^4x^{12}+16\,cd^3x^9+301\,c^2d}{21\,(d^5x^3-8\,cd^4)}$$

input

```
integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output

```
[2/21*(26208*(c^3*d*x^3 - 8*c^4)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sq
rt(c) + 10*c)/(d*x^3 - 8*c)) + (d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6
+ 12206*c^3*d*x^3 - 145328*c^4)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/21
*(52416*(c^3*d*x^3 - 8*c^4)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c
) + (d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6 + 12206*c^3*d*x^3 - 145328*
c^4)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

3.411.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \text{Timed out}$$

input

```
integrate(x**11*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

output

```
Timed out
```

3.411.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{2\left(26208\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+(dx^3+c)^{\frac{7}{2}}+21\,(dx^3+c)^{\frac{5}{2}}c+448\,(dx^3+c)^{\frac{3}{2}}c^2+}{21\,d^4}$$

input

```
integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
2/21*(26208*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3
*sqrt(c))) + (d*x^3 + c)^(7/2) + 21*(d*x^3 + c)^(5/2)*c + 448*(d*x^3 + c)^
(3/2)*c^2 + 15680*sqrt(d*x^3 + c)*c^3 - 16128*sqrt(d*x^3 + c)*c^4/(d*x^3 -
 8*c))/d^4
```

3.411.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{4992\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^4} - \frac{1536\sqrt{dx^3+c}c^4}{(dx^3-8\,c)d^4} + \frac{2\left((dx^3+c)^{\frac{7}{2}}d^{24}+21\,(dx^3+c)^{\frac{5}{2}}cd^{24}+448\,(dx^3+c)^{\frac{3}{2}}c^2d^{24}+15680\sqrt{dx^3+c}c^3d^{24}\right)}{21\,d^{28}}$$

input
```
integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
4992*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 1536*sqrt(d
*x^3 + c)*c^4/((d*x^3 - 8*c)*d^4) + 2/21*((d*x^3 + c)^(7/2)*d^24 + 21*(d*x
^3 + c)^(5/2)*c*d^24 + 448*(d*x^3 + c)^(3/2)*c^2*d^24 + 15680*sqrt(d*x^3 +
 c)*c^3*d^24)/d^28
```

3.411.9 Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.10

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{2496\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4} + \frac{32300\,c^3\,\sqrt{d\,x^3+c}}{21\,d^4} + \frac{2\,x^9\,\sqrt{d\,x^3+c}}{21\,d} + \frac{16\,c\,x^6\,\sqrt{d\,x^3+c}}{7\,d^2} + \frac{986\,c^2\,x^3\,\sqrt{d\,x^3+c}}{21\,d^3} + \frac{1536\,c^4\,\sqrt{d\,x^3+c}}{d^4\,\left(8\,c-d\,x^3\right)}$$

input
```
int((x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

output
```
(2496*c^(7/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^
3)))/d^4 + (32300*c^3*(c + d*x^3)^(1/2))/(21*d^4) + (2*x^9*(c + d*x^3)^(1/
2))/(21*d) + (16*c*x^6*(c + d*x^3)^(1/2))/(7*d^2) + (986*c^2*x^3*(c + d*x^
3)^(1/2))/(21*d^3) + (1536*c^4*(c + d*x^3)^(1/2))/(d^4*(8*c - d*x^3))
```

3.412 $\int \frac{x^8\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

3.412.1 Optimal result

Integrand size = 27, antiderivative size = 119

$$\int \frac{x^8\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx=\frac{160c^2\sqrt{c+dx^3}}{d^3}+\frac{160c\left(c+dx^3\right)^{3/2}}{27d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}+\frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)}-\frac{480c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

output
```
160/27*c*(d*x^3+c)^(3/2)/d^3+2/15*(d*x^3+c)^(5/2)/d^3+64/27*c*(d*x^3+c)^(5
/2)/d^3/(-d*x^3+8*c)-480*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3+
160*c^2*(d*x^3+c)^(1/2)/d^3
```

3.412.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78

$$\int \frac{x^8\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx=\frac{2\sqrt{c+dx^3}\left(-29944c^3+2515c^2dx^3+62cd^2x^6+3d^3x^9\right)}{45d^3\left(-8c+dx^3\right)}-\frac{480c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

input
```
Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
(2*Sqrt[c + d*x^3]*(-29944*c^3 + 2515*c^2*d*x^3 + 62*c*d^2*x^6 + 3*d^3*x^9
))/(45*d^3*(-8*c + d*x^3)) - (480*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/d^3
```

3.412.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {948, 100, 27, 90, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^6 \left(dx^3+c\right)^{3/2}}{\left(8c-dx^3\right)^2}dx^3$$

$$\Big\downarrow \ 100$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{5/2}}{9d^3\left(8c-dx^3\right)}-\frac{\int \frac{3cd\left(dx^3+c\right)^{3/2}\left(3dx^3+56c\right)}{8c-dx^3}dx^3}{9cd^3}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{5/2}}{9d^3\left(8c-dx^3\right)}-\frac{\int \frac{\left(dx^3+c\right)^{3/2}\left(3dx^3+56c\right)}{8c-dx^3}dx^3}{3d^2}\right)$$

$$\Big\downarrow \ 90$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{5/2}}{9d^3\left(8c-dx^3\right)}-\frac{80c\int \frac{\left(dx^3+c\right)^{3/2}}{8c-dx^3}dx^3-\frac{6\left(c+dx^3\right)^{5/2}}{5d}}{3d^2}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{5/2}}{9d^3\left(8c-dx^3\right)}-\frac{80c\left(9c\int \frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)-\frac{6\left(c+dx^3\right)^{5/2}}{5d}}{3d^2}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{5/2}}{9d^3\left(8c-dx^3\right)}-\frac{80c\left(9c\left(9c\int\frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)-\frac{6\left(c+dx^3\right)^{5/2}}{5d}}{3d^2}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{5/2}}{9d^3\left(8c-dx^3\right)}-\frac{80c\left(9c\left(\frac{18c\int\frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)-\frac{6\left(c+dx^3\right)^{5/2}}{5d}}{3d^2}\right)$$

$$\Big\downarrow 219$$

$$\frac{1}{3}\left(\frac{64c\left(c+dx^3\right)^{5/2}}{9d^3\left(8c-dx^3\right)}-\frac{80c\left(9c\left(\frac{6\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)-\frac{6\left(c+dx^3\right)^{5/2}}{5d}}{3d^2}\right)$$

input
```
Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
((64*c*(c + d*x^3)^(5/2))/(9*d^3*(8*c - d*x^3)) - ((-6*(c + d*x^3)^(5/2))/
(5*d) + 80*c*((-2*(c + d*x^3)^(3/2))/(3*d) + 9*c*((-2*Sqrt[c + d*x^3])/d +
 (6*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d)))/(3*d^2))/3
```

3.412.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.412. $\int \frac{x^8 \left(c+d x^3\right)^{3/2}}{\left(8 c-d x^3\right)^2} \, dx$

3.412.4 Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$-\frac{3840\left(c^{3}\left(c-\frac{d\,x^{3}}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)+\frac{\sqrt{d\,x^{3}+c}\,\left(\sqrt{c}\,d^{3}x^{9}+\frac{62c^{\frac{3}{2}}d^{2}x^{6}}{3}+\frac{2515c^{\frac{5}{2}}d\,x^{3}}{3}-\frac{29944c^{\frac{7}{2}}}{3}\right)}{28800}\right)}{\sqrt{c}\,(-d^{4}x^{3}+8d^{3}c)}$
risch	$\frac{2\left(3d^{2}x^{6}+86cd\,x^{3}+3203c^{2}\right)\sqrt{d\,x^{3}+c}}{45d^{3}}+\frac{144c^{3}\left(-\frac{34\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}+\frac{4c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)}{d^{2}}$
default	$\frac{2\left(d\,x^{3}+c\right)^{\frac{5}{2}}}{15d^{3}}-\frac{32c\left(81c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-\left(d\,x^{3}+28c\right)\sqrt{d\,x^{3}+c}\right)}{9d^{3}}+\frac{64c^{2}\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-3c\left(-\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{\operatorname{arctanh}\left(\right.}{\sqrt{c}}\right.\right.}{d^{3}}$
elliptic	$\frac{192c^{3}\sqrt{d\,x^{3}+c}}{d^{3}(-d\,x^{3}+8c)}+\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15d}+\frac{172c\,x^{3}\sqrt{d\,x^{3}+c}}{45d^{2}}+\frac{6406c^{2}\sqrt{d\,x^{3}+c}}{45d^{3}}+\frac{80ic^{2}\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{}}{}\right.}{}$

input
```
int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
-3840*(c^3*(c-1/8*d*x^3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+1/28800*(d*x
^3+c)^(1/2)*(c^(1/2)*d^3*x^9+62/3*c^(3/2)*d^2*x^6+2515/3*c^(5/2)*d*x^3-299
44/3*c^(7/2)))/c^(1/2)/(-d^4*x^3+8*c*d^3)
```

3.412.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.84

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \left[\frac{2\left(5400\,(c^2dx^3-8\,c^3)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+(3\,d^3x^9+62\,cd^2x^6+2515\,c^2d}{45\,(d^4x^3-8\,cd^3)}\right.$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[2/45*(5400*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + (3*d^3*x^9 + 62*c*d^2*x^6 + 2515*c^2*d*x^3 -
 29944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3), 2/45*(10800*(c^2*d*x^3 -
 8*c^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^3*x^9 + 62*
c*d^2*x^6 + 2515*c^2*d*x^3 - 29944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^
3)]
```

3.412.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

output
```
Timed out
```

3.412.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.90

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{2\left(5400\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+3\,(dx^3+c)^{\frac{5}{2}}+80\,(dx^3+c)^{\frac{3}{2}}c+3120\,\sqrt{dx^3+c}c^2-\right.}{45\,d^3}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
2/45*(5400*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 80*(d*x^3 + c)^(3/2)*c + 3120*sqrt(d*x^3
 + c)*c^2 - 4320*sqrt(d*x^3 + c)*c^3/(d*x^3 - 8*c))/d^3
```

3.412.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{480\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^3} - \frac{192\,\sqrt{dx^3+c}c^3}{(dx^3-8\,c)d^3} + \frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}d^{12}+80\,(dx^3+c)^{\frac{3}{2}}cd^{12}+3120\,\sqrt{dx^3+c}c^2d^{12}\right)}{45\,d^{15}}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
480*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 192*sqrt(d*x
^3 + c)*c^3/((d*x^3 - 8*c)*d^3) + 2/45*(3*(d*x^3 + c)^(5/2)*d^12 + 80*(d*x
^3 + c)^(3/2)*c*d^12 + 3120*sqrt(d*x^3 + c)*c^2*d^12)/d^15
```

3.412.9 Mupad [B] (verification not implemented)

Time = 8.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{240\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3} + \frac{6406\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3} + \frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d} + \frac{172\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2} + \frac{192\,c^3\,\sqrt{d\,x^3+c}}{d^3\,\left(8\,c-d\,x^3\right)}$$

input
```
int((x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

output
```
(240*c^(5/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/d^3 + (6406*c^2*(c + d*x^3)^(1/2))/(45*d^3) + (2*x^6*(c + d*x^3)^(1/2)
)/(15*d) + (172*c*x^3*(c + d*x^3)^(1/2))/(45*d^2) + (192*c^3*(c + d*x^3)^(
1/2))/(d^3*(8*c - d*x^3))
```

3.413 $\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

3.413.1 Optimal result

Integrand size = 27, antiderivative size = 97

$$\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx=\frac{14c\sqrt{c+dx^3}}{d^2}+\frac{14\left(c+dx^3\right)^{3/2}}{27d^2}+\frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)}-\frac{42c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

output
```
14/27*(d*x^3+c)^(3/2)/d^2+8/27*(d*x^3+c)^(5/2)/d^2/(-d*x^3+8*c)-42*c^(3/2)
*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2+14*c*(d*x^3+c)^(1/2)/d^2
```

3.413.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx=\frac{2\sqrt{c+dx^3}\left(-524c^2+44cdx^3+d^2x^6\right)}{9d^2\left(-8c+dx^3\right)}-\frac{42c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

input
```
Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
(2*Sqrt[c + d*x^3]*(-524*c^2 + 44*c*d*x^3 + d^2*x^6))/(9*d^2*(-8*c + d*x^3
)) - (42*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2
```

3.413.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {948, 87, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3 \left(dx^3+c\right)^{3/2}}{\left(8c-dx^3\right)^2}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{8\left(c+dx^3\right)^{5/2}}{9d^2\left(8c-dx^3\right)}-\frac{7\int \frac{\left(dx^3+c\right)^{3/2}}{8c-dx^3}dx^3}{3d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{8\left(c+dx^3\right)^{5/2}}{9d^2\left(8c-dx^3\right)}-\frac{7\left(9c\int \frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)}{3d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{8\left(c+dx^3\right)^{5/2}}{9d^2\left(8c-dx^3\right)}-\frac{7\left(9c\left(9c\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)}{3d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{8\left(c+dx^3\right)^{5/2}}{9d^2\left(8c-dx^3\right)}-\frac{7\left(9c\left(\frac{18c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)}{3d}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{8\left(c+dx^3\right)^{5/2}}{9d^2\left(8c-dx^3\right)}-\frac{7\left(9c\left(\frac{6\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{d}\right)-\frac{2\left(c+dx^3\right)^{3/2}}{3d}\right)}{3d}\right)$$

input
```
Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
((8*(c + d*x^3)^(5/2))/(9*d^2*(8*c - d*x^3)) - (7*((-2*(c + d*x^3)^(3/2))/
(3*d) + 9*c*((-2*Sqrt[c + d*x^3])/d + (6*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])])/d)))/(3*d))/3
```

3.413.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

3.413. $\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.413.4 Maple [A] (verified)

Time = 4.41 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{\frac{2\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9}+\frac{34c\sqrt{d\,x^{3}+c}}{3}+6c^{2}\left(\frac{4\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{7\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{d^{2}}$
risch	$\frac{2\left(d\,x^{3}+52c\right)\sqrt{d\,x^{3}+c}}{9d^{2}}+\frac{9c^{2}\left(-\frac{50\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}+\frac{8c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)}{d}$
default	$-\frac{2\left(81c^{\frac{3}{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-\left(d\,x^{3}+28c\right)\sqrt{d\,x^{3}+c}\right)}{9d^{2}}+\frac{8c\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-3c\left(-\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)\right)}{d^{2}}$
elliptic	$\frac{24c^{2}\sqrt{d\,x^{3}+c}}{d^{2}\left(-d\,x^{3}+8c\right)}+\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d}+\frac{104c\sqrt{d\,x^{3}+c}}{9d^{2}}+\frac{7ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right.}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$

input
```
int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(1/9*(d*x^3+c)^(3/2)+17/3*c*(d*x^3+c)^(1/2)+3*c^2*(4*(d*x^3+c)^(1/2)/(-d
*x^3+8*c)-7*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d^2
```

3.413.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.98

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \left[\frac{189\,(cdx^3-8\,c^2)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+2\,(d^2x^6+44\,cdx^3-524\,c^2)\sqrt{dx^3+}}{9\,(d^3x^3-8\,cd^2)}\right.$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/9*(189*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c)
 + 10*c)/(d*x^3 - 8*c)) + 2*(d^2*x^6 + 44*c*d*x^3 - 524*c^2)*sqrt(d*x^3 +
c))/(d^3*x^3 - 8*c*d^2), 2/9*(189*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sq
rt(d*x^3 + c)*sqrt(-c)/c) + (d^2*x^6 + 44*c*d*x^3 - 524*c^2)*sqrt(d*x^3 +
c))/(d^3*x^3 - 8*c*d^2)]
```

3.413.6 Sympy [F]

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x^5(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**5*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

3.413.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{189\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+2\,(dx^3+c)^{\frac{3}{2}}+102\,\sqrt{dx^3+c}c-\frac{216\,\sqrt{dx^3+c}c^2}{dx^3-8\,c}}{9\,d^2}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
1/9*(189*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + 2*(d*x^3 + c)^(3/2) + 102*sqrt(d*x^3 + c)*c - 216*sqrt(d*x^3 + c
)*c^2/(d*x^3 - 8*c))/d^2
```

3.413.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{42\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d^2} - \frac{24\,\sqrt{dx^3+c}c^2}{(dx^3-8\,c)d^2} + \frac{2\left((dx^3+c)^{\frac{3}{2}}d^4+51\,\sqrt{dx^3+c}cd^4\right)}{9\,d^6}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
42*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 24*sqrt(d*x^3
 + c)*c^2/((d*x^3 - 8*c)*d^2) + 2/9*((d*x^3 + c)^(3/2)*d^4 + 51*sqrt(d*x^3
 + c)*c*d^4)/d^6
```

3.413.9 Mupad [B] (verification not implemented)

Time = 8.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{104\,c\,\sqrt{d\,x^3+c}}{9\,d^2} + \frac{21\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2} + \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d} + \frac{24\,c^2\,\sqrt{d\,x^3+c}}{d^2\,(8\,c-d\,x^3)}$$

input
```
int((x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

output
```
(104*c*(c + d*x^3)^(1/2))/(9*d^2) + (21*c^(3/2)*log((10*c + d*x^3 - 6*c^(1
/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/d^2 + (2*x^3*(c + d*x^3)^(1/2))/(9*
d) + (24*c^2*(c + d*x^3)^(1/2))/(d^2*(8*c - d*x^3))
```

3.413. $\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

3.414 $\int \frac{x^2\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

3.414.1 Optimal result

Integrand size = 27, antiderivative size = 77

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx=\frac{\sqrt{c+dx^3}}{d}+\frac{(c+dx^3)^{3/2}}{3d\,(8c-dx^3)}-\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

output
```
1/3*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)-3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*
c^(1/2)/d+(d*x^3+c)^(1/2)/d
```

3.414.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx=\frac{(25c-2dx^3)\sqrt{c+dx^3}}{3d\,(8c-dx^3)}-\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

input
```
Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
((25*c - 2*d*x^3)*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)) - (3*Sqrt[c]*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

3.414.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {946, 51, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2} \, dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3} \int \frac{\left(dx^3+c\right)^{3/2}}{\left(8c-dx^3\right)^2}dx^3$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3} \left(\frac{\left(c+dx^3\right)^{3/2}}{d \left(8c-dx^3\right)}-\frac{3}{2} \int \frac{\sqrt{dx^3+c}}{8c-dx^3}dx^3\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3} \left(\frac{\left(c+dx^3\right)^{3/2}}{d \left(8c-dx^3\right)}-\frac{3}{2} \left(9c \int \frac{1}{\left(8c-dx^3\right) \sqrt{dx^3+c}}dx^3-\frac{2 \sqrt{c+dx^3}}{d}\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3} \left(\frac{\left(c+dx^3\right)^{3/2}}{d \left(8c-dx^3\right)}-\frac{3}{2} \left(\frac{18c \int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{2 \sqrt{c+dx^3}}{d}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3} \left(\frac{\left(c+dx^3\right)^{3/2}}{d \left(8c-dx^3\right)}-\frac{3}{2} \left(\frac{6 \sqrt{c} \text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3 \sqrt{c}}\right)}{d}-\frac{2 \sqrt{c+dx^3}}{d}\right)\right)$$

input
```
Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
((c + d*x^3)^(3/2)/(d*(8*c - d*x^3)) - (3*((-2*Sqrt[c + d*x^3])/d + (6*Sqr
t[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d))/2)/3
```

3.414. $\int \frac{x^2 \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2} \, dx$

3.414.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.414. $\int \frac{x^2\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

3.414.4 Maple [A] (verified)

Time = 4.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

method	result
default	$\frac{\frac{2\sqrt{d x^3+c}}{3}-3c\left(-\frac{\sqrt{d x^3+c}}{-d x^3+8c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{d}$
pseudoelliptic	$\frac{\frac{2\sqrt{d x^3+c}}{3}+3c\left(\frac{\sqrt{d x^3+c}}{-d x^3+8c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{d}$
risch	$\frac{2\sqrt{d x^3+c}}{3d}+9c\left(-\frac{4\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}+\frac{c\left(-\frac{\sqrt{d x^3+c}}{c\left(d x^3-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)$
elliptic	$\frac{3c\sqrt{d x^3+c}}{d(-d x^3+8c)}+\frac{2\sqrt{d x^3+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x\right.}{-3\left(-c\,d^2\right.}}}{}\right.}{}$

input
```
int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
(2/3*(d*x^3+c)^(1/2)-3*c*(-(d*x^3+c)^(1/2)/(-d*x^3+8*c)+arctanh(1/3*(d*x^3
+c)^(1/2)/c^(1/2))/c^(1/2)))/d
```

3.414.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.10

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \left[\frac{9\,(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+2\,(2\,dx^3-25\,c)\sqrt{dx^3+c}}{6\,(d^2x^3-8\,cd)},\ 9\,(dx^3-8\,c)\right.$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/6*(9*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) + 2*(2*d*x^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d),
 1/3*(9*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (2
*d*x^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d)]
```

3.414.6 Sympy [F]

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x^2(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**2*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

3.414.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{9\,\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+4\,\sqrt{dx^3+c}-\frac{18\,\sqrt{dx^3+c}c}{dx^3-8\,c}}{6\,d}$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
1/6*(9*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + 4*sqrt(d*x^3 + c) - 18*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d
```

3.414.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{3\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d} + \frac{2\,\sqrt{dx^3+c}}{3\,d} - \frac{3\,\sqrt{dx^3+c}c}{(dx^3-8\,c)d}$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
3*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 2/3*sqrt(d*x^3 + c
)/d - 3*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d)
```

3.414.9 Mupad [B] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{2\,\sqrt{d\,x^3+c}}{3\,d} + \frac{3\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{2\,d} + \frac{3\,c\,\sqrt{d\,x^3+c}}{d\,\left(8\,c-d\,x^3\right)}$$

input
```
int((x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*d) + (3*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c
+ d*x^3)^(1/2))/(8*c - d*x^3)))/(2*d) + (3*c*(c + d*x^3)^(1/2))/(d*(8*c -
d*x^3))
```

3.415 $\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(8c-dx^3\right)^2} \, dx$

3.415.1 Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(8c-dx^3\right)^2} \, dx=\frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}-\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

output
```
-3/32*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/96*arctanh((d*x^3+c)^
(1/2)/c^(1/2))/c^(1/2)+3/8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)
```

3.415.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(8c-dx^3\right)^2} \, dx=\frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}-\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x]
```

output
```
(3*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) - (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt
[c])])/(32*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*Sqrt[c])
```

3.415.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {948, 109, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{\left(dx^3+c\right)^{3/2}}{x^3\left(8c-dx^3\right)^2}dx^3$$

$$\Big\downarrow \ 109$$

$$\frac{1}{3}\left(\frac{9\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}-\frac{\int -\frac{cd\left(2c-7dx^3\right)}{2x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{8cd}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\frac{1}{16}\int \frac{2c-7dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3+\frac{9\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \ 174$$

$$\frac{1}{3}\left(\frac{1}{16}\left(\frac{1}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3-\frac{27}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3\right)+\frac{9\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{3}\left(\frac{1}{16}\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{27}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}\right)+\frac{9\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{3}\left(\frac{1}{16}\left(\frac{\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{9\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)+\frac{9\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{3}\left(\frac{1}{16}\left(-\frac{9\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)+\frac{9\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}\right)$$

input

```
Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x]
```

output

```
((9*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) + ((-9*ArcTanh[Sqrt[c + d*x^3]/(3*S
qrt[c])])/(2*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(2*Sqrt[c]))/16)/
3
```

3.415.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.415.4 Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{3 \operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right)}{32 \sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{\sqrt{c}}\right)}{96 \sqrt{c}}+\frac{3 \sqrt{d x^{3}+c}}{-8 d x^{3}+64 c}$
default	$\frac{\frac{2 d x^{3} \sqrt{d x^{3}+c}}{9}+\frac{8 c \sqrt{d x^{3}+c}}{9}-\frac{2 c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{\sqrt{c}}\right)}{3}}{64 c^{2}}+\frac{\frac{2 \sqrt{d x^{3}+c}}{3}-3 c \left(-\frac{\sqrt{d x^{3}+c}}{-d x^{3}+8 c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right)}{\sqrt{c}}\right)}{8 c}+ \frac{81 c^{\frac{3}{2}} \operatorname{arctanh}}{}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/32*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/96*arctanh((d*x^3+c)^
(1/2)/c^(1/2))/c^(1/2)+3*(d*x^3+c)^(1/2)/(-8*d*x^3+64*c)
```

3.415.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.59

$$\int \frac{(c+dx^3)^{3/2}}{x\left(8c-dx^3\right)^2}\,dx = \left[\frac{9\,(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-}{192\,(cdx^3-8\,c^2)}\right.$$

input
```
integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/192*(9*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 1
0*c)/(d*x^3 - 8*c)) + (d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)
*sqrt(c) + 2*c)/x^3) - 72*sqrt(d*x^3 + c)*c)/(c*d*x^3 - 8*c^2), 1/96*((d*x
^3 - 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 9*(d*x^3 - 8*c)*sq
rt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 36*sqrt(d*x^3 + c)*c)/(c*d
*x^3 - 8*c^2)]
```

3.415.6 Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x(-8c+dx^3)^2}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c)**2,x)
```

output
```
Integral((c + d*x**3)**(3/2)/(x*(-8*c + d*x**3)**2), x)
```

3.415.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x), x)
```

3.415.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}} + \frac{3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{32\sqrt{-c}} - \frac{3\sqrt{dx^3+c}}{8(dx^3-8\,c)}$$

---

input
```
integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) + 3/32*arctan(1/3*sqrt(d*x^
3 + c)/sqrt(-c))/sqrt(-c) - 3/8*sqrt(d*x^3 + c)/(d*x^3 - 8*c)
```

3.415.9 Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \frac{3\sqrt{d\,x^3+c}}{8\,(8\,c-d\,x^3)} + \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\,\left(10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}\right)^9}{x^6\,(8\,c-d\,x^3)^9}\right)}{192\,\sqrt{c}}$$

input
```
int((c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x)
```

output
```
(3*(c + d*x^3)^(1/2))/(8*(8*c - d*x^3)) + log((((c + d*x^3)^(1/2) - c^(1/2
))^3*((c + d*x^3)^(1/2) + c^(1/2))*(10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(
1/2))^9)/(x^6*(8*c - d*x^3)^9))/(192*c^(1/2))
```

3.416 $\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx$

3.416.1 Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \frac{5d\sqrt{c+dx^3}}{96c(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24x^3(8c-dx^3)} + \frac{3d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}} - \frac{7d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}$$

output
```
3/128*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-7/384*d*arctanh((d*x^
3+c)^(1/2)/c^(1/2))/c^(3/2)+5/96*d*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)-1/24*(d*
x^3+c)^(1/2)/x^3/(-d*x^3+8*c)
```

3.416.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.80

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \frac{\frac{4\sqrt{c}(4c-5dx^3)\sqrt{c+dx^3}}{-8cx^3+dx^6} + 9d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 7d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x]
```

output
```
((4*Sqrt[c]*(4*c - 5*d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 9*d*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 7*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(
384*c^(3/2))
```

3.416.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 109, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\left(dx^3+c\right)^{3/2}}{x^6\left(8c-dx^3\right)^2}dx^3$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{cd\left(19dx^3+28c\right)}{2x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{8c}-\frac{\sqrt{c+dx^3}}{8x^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{16}d\int \frac{19dx^3+28c}{x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3-\frac{\sqrt{c+dx^3}}{8x^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{5\sqrt{c+dx^3}}{2c\left(8c-dx^3\right)}-\frac{\int -\frac{18cd\left(5dx^3+14c\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{72c^2d}\right)-\frac{\sqrt{c+dx^3}}{8x^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{\int \frac{5dx^3+14c}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{4c}+\frac{5\sqrt{c+dx^3}}{2c\left(8c-dx^3\right)}\right)-\frac{\sqrt{c+dx^3}}{8x^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{\frac{7}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{27}{4}d\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{4c}+\frac{5\sqrt{c+dx^3}}{2c\,(8c-dx^3)}\right)-\frac{\sqrt{c+dx^3}}{8x^3\,(8c-dx^3)}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{\frac{27}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{7\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{4c}+\frac{5\sqrt{c+dx^3}}{2c\,(8c-dx^3)}\right)-\frac{\sqrt{c+dx^3}}{8x^3\,(8c-dx^3)}\right)$$

$$\Big\downarrow 219$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{\frac{7\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{9\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}}{4c}+\frac{5\sqrt{c+dx^3}}{2c\,(8c-dx^3)}\right)-\frac{\sqrt{c+dx^3}}{8x^3\,(8c-dx^3)}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{3}\left(\frac{1}{16}d\left(\frac{\frac{9\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}-\frac{7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{4c}+\frac{5\sqrt{c+dx^3}}{2c\,(8c-dx^3)}\right)-\frac{\sqrt{c+dx^3}}{8x^3\,(8c-dx^3)}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x]
```

output
```
(-1/8*Sqrt[c + d*x^3]/(x^3*(8*c - d*x^3)) + (d*((5*Sqrt[c + d*x^3])/(2*c*(
8*c - d*x^3)) + ((9*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*Sqrt[c]) - (7
*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(4*c)))/16)/3
```

3.416.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.416. $\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(8c-dx^3\right)^2}\,dx$

3.416.4 Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{d\left(-\frac{7\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d\,x^{3}+2\sqrt{d\,x^{3}+c}\,\sqrt{c}}{2d\,x^{3}c^{\frac{3}{2}}}+\frac{\frac{9\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{9\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{2\sqrt{c}}}{c}\right)}{192}$
risch	$-\frac{\sqrt{d\,x^{3}+c}}{192c\,x^{3}}-\frac{d\left(\frac{7\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{\sqrt{c}}-6c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)\right)}{128c}$
default	$\frac{-\frac{c\sqrt{d\,x^{3}+c}}{3x^{3}}+\frac{2d\sqrt{d\,x^{3}+c}}{3}-\sqrt{c}\,d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{64c^{2}}+\frac{d\left(\frac{2d\,x^{3}\sqrt{d\,x^{3}+c}}{9}+\frac{8c\sqrt{d\,x^{3}+c}}{9}-\frac{2c^{\frac{3}{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}\right)}{256c^{3}}+\frac{d\left(2\sqrt{}\right.}{}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192*d*(-1/2*(7*arctanh((d*x^3+c)^(1/2)/c^(1/2))*d*x^3+2*(d*x^3+c)^(1/2)*
c^(1/2))/d/x^3/c^(3/2)+9*((d*x^3+c)^(1/2)/(-d*x^3+8*c)+1/2*arctanh(1/3*(d*
x^3+c)^(1/2)/c^(1/2))/c^(1/2))/c)
```

3.416.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.31

$$\int \frac{(c+dx^3)^{3/2}}{x^4\,(8c-dx^3)^2}\,dx = \left[\frac{9\,(d^2x^6-8\,cdx^3)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+7\,(d^2x^6-8\,cdx^3)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3}}{x^3}\right.}{768\,(c^2dx^6-8\,c^3x^3)}\right.$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/768*(9*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + 7*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 -
 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3
+ c))/(c^2*d*x^6 - 8*c^3*x^3), 1/384*(7*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arc
tan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 9*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(
1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(
c^2*d*x^6 - 8*c^3*x^3)]
```

3.416.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c)**2,x)
```

output
```
Timed out
```

3.416.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^4}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^4), x)
```

3.416.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \frac{7\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}c} - \frac{3\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{128\,\sqrt{-c}c} - \frac{5\,(dx^3+c)^{\frac{3}{2}}d-9\,\sqrt{dx^3+c}cd}{96\left((dx^3+c)^2-10\,(dx^3+c)c+9\,c^2\right)c}$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
7/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 3/128*d*arctan(1/3
*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/96*(5*(d*x^3 + c)^(3/2)*d - 9*
sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c)
```

3.416.9 Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \frac{\frac{9\,d\,\sqrt{d\,x^3+c}}{32}-\frac{5\,d\,{\left(d\,x^3+c\right)}^{3/2}}{32\,c}}{3\,{\left(d\,x^3+c\right)}^2-30\,c\,\left(d\,x^3+c\right)+27\,c^2} + \frac{d\,\left(\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)\,9{}\mathrm{i}}{7}\right)\,7{}\mathrm{i}}{384\,\sqrt{c^3}}$$

input
```
int((c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x)
```

output
```
((9*d*(c + d*x^3)^(1/2))/32 - (5*d*(c + d*x^3)^(3/2))/(32*c))/(3*(c + d*x^
3)^2 - 30*c*(c + d*x^3) + 27*c^2) + (d*(atanh((c*(c + d*x^3)^(1/2))/(c^3)^
(1/2))*1i - (atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2)))*9i)/7)*7i)/(384*
(c^3)^(1/2))
```

3.417 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^7\left(8c-dx^3\right)^2}\,dx$

3.417.1 Optimal result

Integrand size = 27, antiderivative size = 161

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^7\left(8c-dx^3\right)^2}\,dx=\frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)}-\frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)}+\frac{15d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}}-\frac{17d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}}$$

output
```
15/2048*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-17/2048*d^2*arcta
nh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+7/512*d^2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+
8*c)-1/48*(d*x^3+c)^(1/2)/x^6/(-d*x^3+8*c)-23/384*d*(d*x^3+c)^(1/2)/c/x^3/
(-d*x^3+8*c)
```

3.417.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^7\left(8c-dx^3\right)^2}\,dx=\frac{\frac{4\sqrt{c}\sqrt{c+dx^3}\left(32c^2+92cdx^3-21d^2x^6\right)}{-8cx^6+dx^9}+45d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-51d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{5/2}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x]
```

output
```
((4*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 92*c*d*x^3 - 21*d^2*x^6))/(-8*c*x^6
+ d*x^9) + 45*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 51*d^2*ArcTanh[Sq
rt[c + d*x^3]/Sqrt[c]])/(6144*c^(5/2))
```

3.417.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {948, 109, 27, 168, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^7 \left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\left(dx^3+c\right)^{3/2}}{x^9 \left(8c-dx^3\right)^2}dx^3$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{cd\left(37dx^3+46c\right)}{2x^6\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{16c}-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{32}d\int \frac{37dx^3+46c}{x^6\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(-\frac{\int -\frac{3cd\left(23dx^3+68c\right)}{x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{8c^2}-\frac{23\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{3d\int \frac{23dx^3+68c}{x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{8c}-\frac{23\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{3d\left(\frac{7\sqrt{c+dx^3}}{2c(8c-dx^3)}-\frac{\int -\frac{18cd\left(7dx^3+34c\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{72c^2d}\right)}{8c}-\frac{23\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{3d\left(\frac{\int \frac{7dx^3+34c}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{4c}+\frac{7\sqrt{c+dx^3}}{2c(8c-dx^3)}\right)}{8c}-\frac{23\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{3d\left(\frac{\frac{17}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{45}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{4c}+\frac{7\sqrt{c+dx^3}}{2c(8c-dx^3)}\right)}{8c}-\frac{23\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{3d\left(\frac{\frac{45}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{17\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{4c}+\frac{7\sqrt{c+dx^3}}{2c(8c-dx^3)}\right)}{8c}-\frac{23\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{3d\left(\frac{\frac{17\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{15\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}}{4c}+\frac{7\sqrt{c+dx^3}}{2c(8c-dx^3)}\right)}{8c}-\frac{23\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \; 221$$

3.417. $\int \frac{\left(c+dx^3\right)^{3/2}}{x^7\left(8c-dx^3\right)^2}\,dx$

$$\frac{1}{3}\left(\frac{1}{32}d\left(\frac{3d\left(\frac{\frac{15\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2\sqrt{c}}-\frac{17\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{4c}+\frac{7\sqrt{c+dx^3}}{2c(8c-dx^3)}\right)}{8c}-\frac{23\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)-\frac{\sqrt{c+dx^3}}{16x^6\left(8c-dx^3\right)}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x]
```

output
```
(-1/16*Sqrt[c + d*x^3]/(x^6*(8*c - d*x^3)) + (d*((-23*Sqrt[c + d*x^3])/(4*
c*x^3*(8*c - d*x^3)) + (3*d*((7*Sqrt[c + d*x^3])/(2*c*(8*c - d*x^3)) + ((1
5*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*Sqrt[c]) - (17*ArcTanh[Sqrt[c +
 d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(4*c)))/(8*c)))/32)/3
```

3.417.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.417.4 Maple [A] (verified)

Time = 4.65 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$-\frac{408\left(-\frac{15\left(c-\frac{d\,x^{3}}{8}\right)d^{2}x^{6}\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{17}+\left(c-\frac{d\,x^{3}}{8}\right)d^{2}x^{6}\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)-\frac{7\left(d^{2}x^{6}\sqrt{c}-\frac{92d\,x^{3}c^{\frac{3}{2}}}{21}-\frac{32c^{\frac{5}{2}}}{21}\right)\sqrt{d\,x^{3}+c}}{34}\right.}{c^{\frac{5}{2}}\left(-6144d\,x^{9}+49152c\,x^{6}\right)}$
risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(3d\,x^{3}+c\right)}{384c^{2}x^{6}}-\frac{3d^{2}\left(\frac{17\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{24\sqrt{c}}-\frac{11\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{24\sqrt{c}}-\frac{c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{2}\right)}{256c^{2}}$
default	$\frac{-\frac{c\sqrt{d\,x^{3}+c}}{6x^{6}}-\frac{5d\sqrt{d\,x^{3}+c}}{12x^{3}}-\frac{d^{2}\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4\sqrt{c}}}{64c^{2}}+\frac{d\left(-\frac{c\sqrt{d\,x^{3}+c}}{3x^{3}}+\frac{2d\sqrt{d\,x^{3}+c}}{3}-\sqrt{c}\,d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\right)}{256c^{3}}+\frac{3d^{2}\left(\frac{2d\,x^{3}}{}\right.}{}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
-408/c^(5/2)*(-15/17*(c-1/8*d*x^3)*d^2*x^6*arctanh(1/3*(d*x^3+c)^(1/2)/c^(
1/2))+(c-1/8*d*x^3)*d^2*x^6*arctanh((d*x^3+c)^(1/2)/c^(1/2))-7/34*(d^2*x^6
*c^(1/2)-92/21*d*x^3*c^(3/2)-32/21*c^(5/2))*(d*x^3+c)^(1/2))/(-6144*d*x^9+
49152*c*x^6)
```

3.417.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.93

$$\int \frac{\left(c+dx^{3}\right)^{3/2}}{x^{7}\left(8c-dx^{3}\right)^{2}}\,dx=\left[\frac{45\,\left(d^{3}x^{9}-8\,cd^{2}x^{6}\right)\sqrt{c}\log\left(\frac{dx^{3}+6\,\sqrt{dx^{3}+c}\sqrt{c}+10\,c}{dx^{3}-8\,c}\right)+51\,\left(d^{3}x^{9}-8\,cd^{2}x^{6}\right)\sqrt{c}\log\left(\frac{dx^{3}-2}{}\right.}{12288\,\left(c^{3}dx^{9}-8\,c^{4}x^{6}\right)}\right.$$

input
```
integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
[1/12288*(45*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 51*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log(
(d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(21*c*d^2*x^6 - 92*c^2*
d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6), 1/6144*(51*(d^3*
x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 45*(d^3*x
^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(21*
c*d^2*x^6 - 92*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6
)]
```

3.417.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^7\left(8c-dx^3\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c)**2,x)
```

output
```
Timed out
```

3.417.7 Maxima [F]

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^7\left(8c-dx^3\right)^2}\,dx=\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2x^7}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^7), x)
```

3.417.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.80

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx = \frac{17\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\,\sqrt{-c}c^2} - \frac{15\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{2048\,\sqrt{-c}c^2} - \frac{3\,\sqrt{dx^3+c}d^2}{512\,(dx^3-8\,c)c^2} - \frac{3\,(dx^3+c)^{\frac{3}{2}}d^2-2\,\sqrt{dx^3+c}cd^2}{384\,c^2d^2x^6}$$

input
```
integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
17/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 15/2048*d^2*
arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 3/512*sqrt(d*x^3 + c
)*d^2/((d*x^3 - 8*c)*c^2) - 1/384*(3*(d*x^3 + c)^(3/2)*d^2 - 2*sqrt(d*x^3
+ c)*c*d^2)/(c^2*d^2*x^6)
```

3.417.9 Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx = \frac{\frac{81\,d^2\,\sqrt{d\,x^3+c}}{512} - \frac{67\,d^2\,\left(d\,x^3+c\right)^{3/2}}{256\,c} + \frac{21\,d^2\,\left(d\,x^3+c\right)^{5/2}}{512\,c^2}}{33\,c\,\left(d\,x^3+c\right)^2 - 57\,c^2\,\left(d\,x^3+c\right) - 3\,\left(d\,x^3+c\right)^3 + 27\,c^3} + \frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)\,1\mathrm{i} - \frac{\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)\,15\mathrm{i}}{17}\right)\,17\mathrm{i}}{2048\,\sqrt{c^5}}$$

input
```
int((c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x)
```

output
```
((81*d^2*(c + d*x^3)^(1/2))/512 - (67*d^2*(c + d*x^3)^(3/2))/(256*c) + (21
*d^2*(c + d*x^3)^(5/2))/(512*c^2))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3
) - 3*(c + d*x^3)^3 + 27*c^3) + (d^2*(atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^
(1/2))*1i - (atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))*15i)/17)*17i)/
(2048*(c^5)^(1/2))
```

3.418 $\int \frac{x^7\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

3.418.1 Optimal result

Integrand size = 27, antiderivative size = 681

$$\int \frac{x^7\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx=\frac{103cx^2\sqrt{c+dx^3}}{13d^2}+\frac{19x^5\sqrt{c+dx^3}}{39d}+\frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+\frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}+\frac{108\sqrt{3}c^{13/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}$$

$$-\frac{108c^{13/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{108c^{13/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}$$

$$-\frac{2953\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{13d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{5906\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{13\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
1/3*x^5*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)-108*c^(13/6)*arctanh(1/3*(c^(1/3)+d
^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(8/3)+108*c^(13/6)*arctanh(1/3*(d*x
^3+c)^(1/2)/c^(1/2))/d^(8/3)+108*c^(13/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*
x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(8/3)+103/13*c*x^2*(d*x^3+c)^(1/2)/d
^2+19/39*x^5*(d*x^3+c)^(1/2)/d+5906/13*c^2*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))+5906/39*c^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^
(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I
)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2)))^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3
)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-2953/13*3^(1/4)*c^(7/3)*(c^(
1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)
*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(
d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

3.418.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.44 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.28

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{80x^2(-412c^3-388c^2dx^3+25cd^2x^6+d^3x^9)+4120c^2x^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{Appell}}{520d^2\,(-8c}$$

input
```
Integrate[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
(80*x^2*(-412*c^3 - 388*c^2*d*x^3 + 25*c*d^2*x^6 + d^3*x^9) + 4120*c^2*x^2
*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, (d*x^3)/(8*c)] + 2953*c*d*x^5*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1
[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(520*d^2*(-8*c + d*x^3)*S
qrt[c + d*x^3])
```

3.418.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {967, 27, 1051, 27, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \text{ 967}$$

$$\frac{x^5 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{\int \frac{x^4\sqrt{dx^3+c}\left(19dx^3+10c\right)}{2\left(8c-dx^3\right)}dx}{3d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^5 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{\int \frac{x^4\sqrt{dx^3+c}\left(19dx^3+10c\right)}{8c-dx^3}dx}{6d}$$

$$\Big\downarrow \text{ 1051}$$

$$\frac{x^5 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{-\frac{2\int -\frac{3cdx^4\left(721dx^3+550c\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{13d}-\frac{38}{13}x^5\sqrt{c+dx^3}}{6d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^5 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{\frac{3}{13}c\int \frac{x^4\left(721dx^3+550c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx-\frac{38}{13}x^5\sqrt{c+dx^3}}{6d}$$

$$\Big\downarrow \text{ 1052}$$

$$\frac{x^5 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{\frac{3}{13}c\left(\frac{2\int \frac{7cdx\left(2953dx^3+1648c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d^2}-\frac{206x^2\sqrt{c+dx^3}}{d}\right)-\frac{38}{13}x^5\sqrt{c+dx^3}}{6d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^5 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{\frac{3}{13}c\left(\frac{2c\int \frac{x\left(2953dx^3+1648c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{d}-\frac{206x^2\sqrt{c+dx^3}}{d}\right)-\frac{38}{13}x^5\sqrt{c+dx^3}}{6d}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{x^5(c+dx^3)^{3/2}}{3d(8c-dx^3)}-\frac{\frac{3}{13}c\left(\frac{2c\int\left(\frac{25272cx}{(8c-dx^3)\sqrt{dx^3+c}}-\frac{2953x}{\sqrt{dx^3+c}}\right)dx}{d}-\frac{206x^2\sqrt{c+dx^3}}{d}\right)-\frac{38}{13}x^5\sqrt{c+dx^3}}{6d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^5(c+dx^3)^{3/2}}{3d(8c-dx^3)}-$$

$$\frac{3}{13}c\Bigg(2c\Bigg(-\frac{5906\sqrt{2}\sqrt[3]{c}(\sqrt[3]{c}+\sqrt[3]{d}x)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+(1-\sqrt{3})\sqrt[3]{c}}{\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}(\sqrt[3]{c}+\sqrt[3]{d}x)}{((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x)^2}}\sqrt{c+dx^3}}+2953\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}(\sqrt[3]{c}+$$

input
```
Int[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
(x^5*(c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) - ((-38*x^5*Sqrt[c + d*x^3])/1
3 + (3*c*((-206*x^2*Sqrt[c + d*x^3])/d + (2*c*((-5906*Sqrt[c + d*x^3])/(d^
(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (1404*Sqrt[3]*c^(1/6)*ArcTan[
(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(2/3) + (1404*
c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(2
/3) - (1404*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(2/3) + (2953*
3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^
(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elli
pticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x)
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (5906*Sqrt[2]*
c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*
x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3]
)*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3
]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/d))/13)/(6*d)
```

3.418. $\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

3.418.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 967
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1051
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Simp[1/(
b*(m + n*(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*
Simp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
 f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simpl
erQ[e + f*x^n, c + d*x^n])
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.418. $\int \frac{x^7\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

3.418.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.51 (sec) , antiderivative size = 921, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	921
risch	Expression too large to display	1769
default	Expression too large to display	2224

input

```
int(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output

```
24*c^2/d^2*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/13*x^5*(d*x^3+c)^(1/2)/d+64/
13*c*x^2*(d*x^3+c)^(1/2)/d^2-5906/39*I*c^2/d^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)))+72*I*c^2/d^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*
(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/
3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^
(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
)/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d
-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^
(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1...
```

3.418.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 17.75 (sec) , antiderivative size = 2580, normalized size of antiderivative = 3.79

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/13*(5906*(c^2*d*x^3 - 8*c^3)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierst
rassPInverse(0, -4*c/d, x)) + 117*(d^4*x^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 -
 8*c*d^3))*(c^13/d^16)^(1/6)*log(14693280768*((d^16*x^9 + 318*c*d^15*x^6 +
 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 +
1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^13/d^16)^(5/6) + 6*(2*c^11*d^2*x^7 +
 160*c^12*d*x^4 + 320*c^13*x - 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2 - sqrt(
-3)*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2))*(c^13/d^16)^(2/3) - (7*c^7*d^7*x^6
 + 152*c^8*d^6*x^3 + 64*c^9*d^5 + sqrt(-3)*(7*c^7*d^7*x^6 + 152*c^8*d^6*x^
3 + 64*c^9*d^5))*(c^13/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^5*d^10*x^7 +
 64*c^6*d^9*x^4 + 32*c^7*d^8*x)*sqrt(c^13/d^16) + 18*(c^9*d^5*x^8 + 38*c^1
0*d^4*x^5 + 64*c^11*d^3*x^2 - sqrt(-3)*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64
*c^11*d^3*x^2))*(c^13/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3
 - 512*c^3)) - 117*(d^4*x^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(c^1
3/d^16)^(1/6)*log(-14693280768*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14
*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*
x^3 + 640*c^3*d^13))*(c^13/d^16)^(5/6) - 6*(2*c^11*d^2*x^7 + 160*c^12*d*x^
4 + 320*c^13*x - 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2 - sqrt(-3)*(5*c^3*d^1
2*x^5 + 32*c^4*d^11*x^2))*(c^13/d^16)^(2/3) - (7*c^7*d^7*x^6 + 152*c^8*d^6
*x^3 + 64*c^9*d^5 + sqrt(-3)*(7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5
))*(c^13/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^5*d^10*x^7 + 64*c^6*d^9...
```

3.418.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

output
```
Timed out
```

3.418.7 Maxima [F]

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^7}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c)^2, x)
```

3.418.8 Giac [F]

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^7}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c)^2, x)
```

3.418.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x^7\,(d\,x^3+c)^{3/2}}{(8\,c-d\,x^3)^2}\,dx$$

input
```
int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

output
```
int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2, x)
```

3.419 $\int \frac{x^4 \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

3.419.1 Optimal result

Integrand size = 27, antiderivative size = 657

$$
\begin{aligned}
\int \frac{x^4 (c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx &= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&+ \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{9\sqrt{3}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} \\
&- \frac{9c^{7/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{9c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}} \\
&- \frac{265\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{14d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{265\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

output
```
1/3*x^2*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)-9*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1
/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5/3)+9*c^(7/6)*arctanh(1/3*(d*x^3+c)^
(1/2)/c^(1/2))/d^(5/3)+9*c^(7/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2
)/(d*x^3+c)^(1/2))*3^(1/2)/d^(5/3)+13/21*x^2*(d*x^3+c)^(1/2)/d+265/7*c*(d*
x^3+c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+265/21*c^(4/3)*(c^(1/
3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x
^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/
2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-2
65/14*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-
3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*
2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.419.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.27

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx =$$

$$-\frac{16x^2(37c^2+35cdx^3-2d^2x^6)+74cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+53dx^5(-8c}{112d\left(-8c+dx^3\right)\sqrt{c+dx^3}}$$

input
```
Integrate[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
-1/112*(16*x^2*(37*c^2 + 35*c*d*x^3 - 2*d^2*x^6) + 74*c*x^2*(-8*c + d*x^3)
*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c
)] + 53*d*x^5*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3
, -((d*x^3)/c), (d*x^3)/(8*c)])/(d*(-8*c + d*x^3)*Sqrt[c + d*x^3])
```

3.419.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 661, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {967, 27, 1051, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \text{967}$$

$$\frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{\int \frac{x\sqrt{dx^3+c}\left(13dx^3+4c\right)}{2\left(8c-dx^3\right)}dx}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{\int \frac{x\sqrt{dx^3+c}\left(13dx^3+4c\right)}{8c-dx^3}dx}{6d}$$

$$\Big\downarrow \text{1051}$$

$$\frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{-\frac{2\int -\frac{3cdx\left(265dx^3+148c\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{7d}-\frac{26}{7}x^2\sqrt{c+dx^3}}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{\frac{3}{7}c\int \frac{x\left(265dx^3+148c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx-\frac{26}{7}x^2\sqrt{c+dx^3}}{6d}$$

$$\Big\downarrow \text{1054}$$

$$\frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{\frac{3}{7}c\int \left(\frac{2268cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}-\frac{265x}{\sqrt{dx^3+c}}\right)dx-\frac{26}{7}x^2\sqrt{c+dx^3}}{6d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{\frac{3}{7}c\left(-\frac{530\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{265\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}\right.}{}\right.}{}$$

input
```
Int[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
(x^2*(c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) - ((-26*x^2*Sqrt[c + d*x^3])/7
 + (3*c*((-530*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)) - (126*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/
Sqrt[c + d*x^3]])/d^(2/3) + (126*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(
3*c^(1/6)*Sqrt[c + d*x^3])])/d^(2/3) - (126*c^(1/6)*ArcTanh[Sqrt[c + d*x^3
]/(3*Sqrt[c])])/d^(2/3) + (265*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt
[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sq
rt[c + d*x^3]) - (530*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3)
+ d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/7)
/(6*d)
```

3.419.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 967
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1051
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Simp[1/(
b*(m + n*(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*
Simp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
 f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simpl
erQ[e + f*x^n, c + d*x^n])
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.419.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.52 (sec) , antiderivative size = 897, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	897
default	Expression too large to display	1748
risch	Expression too large to display	1758

input
```
int(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
3*c/d*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/7*x^2*(d*x^3+c)^(1/2)/d-265/21*I*
c/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+6*I*c/d^4*2^(1/2
)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(
-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c
*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1...
```

3.419.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.79 (sec) , antiderivative size = 2568, normalized size of antiderivative = 3.91

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/28*(1060*(c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstra
ssPInverse(0, -4*c/d, x)) + 21*(d^3*x^3 - 8*c*d^2 - sqrt(-3)*(d^3*x^3 - 8*
c*d^2))*(c^7/d^10)^(1/6)*log(59049*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*
d^9*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9
*x^3 + 640*c^3*d^8))*(c^7/d^10)^(5/6) + 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 +
 320*c^8*x - 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2 - sqrt(-3)*(5*c^2*d^8*x^5 +
 32*c^3*d^7*x^2))*(c^7/d^10)^(2/3) - (7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64
*c^6*d^3 + sqrt(-3)*(7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3))*(c^7/d
^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^7*x^7 + 64*c^4*d^6*x^4 + 32*c^5*
d^5*x)*sqrt(c^7/d^10) + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2
- sqrt(-3)*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2))*(c^7/d^10)^(1/
6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 21*(d^3*x^3 - 8*
c*d^2 - sqrt(-3)*(d^3*x^3 - 8*c*d^2))*(c^7/d^10)^(1/6)*log(-59049*((d^11*x
^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9
+ 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8))*(c^7/d^10)^(5/6) - 6*(
2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*
x^2 - sqrt(-3)*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2))*(c^7/d^10)^(2/3) - (7*c^4
*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3 + sqrt(-3)*(7*c^4*d^5*x^6 + 152*c^
5*d^4*x^3 + 64*c^6*d^3))*(c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^7
*x^7 + 64*c^4*d^6*x^4 + 32*c^5*d^5*x)*sqrt(c^7/d^10) + 18*(c^5*d^4*x^8 ...
```

3.419.6 Sympy [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx=\int \frac{x^4(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x**4*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x**4*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

3.419.7 Maxima [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^4}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c)^2, x)
```

3.419.8 Giac [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^4}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c)^2, x)
```

3.419.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x^4\,(d\,x^3+c)^{3/2}}{(8\,c-d\,x^3)^2}\,dx$$

input
```
int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

output
```
int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2, x)
```

3.420 $\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

3.420.1 Optimal result

Integrand size = 25, antiderivative size = 638

$$
\begin{aligned}
\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx &= \frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&+ \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} + \frac{9\sqrt{3}\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16d^{2/3}} \\
&- \frac{9\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16d^{2/3}} + \frac{9\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{16d^{2/3}} \\
&- \frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{19\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

output
```
-9/16*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d
^(2/3)+9/16*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(2/3)+9/16*c^(1
/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^
(2/3)+3/8*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+19/8*(d*x^3+c)^(1/2)/d^(2/3)/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2)))+19/24*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF(
(d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(
1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-19/16*3^(1/4)*c^(1/3)*(c^
(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3
)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(2/3)/
(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)
))^2)^(1/2)
```

3.420.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.22

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{x^2\left(\frac{240(c+dx^3)}{8c-dx^3}-25\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-\frac{19dx^3\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{5}{3},\right.}{c}\right.}{640\sqrt{c+dx^3}}$$

input
```
Integrate[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output
```
(x^2*((240*(c + d*x^3))/(8*c - d*x^3) - 25*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - (19*d*x^3*Sqrt[1 + (d*x^3)/
c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c))/(640*Sqrt[
c + d*x^3])
```

3.420.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 631, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {968, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \text{968}$$

$$\frac{\int -\frac{3cdx\left(19dx^3+10c\right)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{24cd}+\frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}-\frac{1}{16}\int \frac{x\left(19dx^3+10c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx$$

$$\Big\downarrow \text{1054}$$

$$\frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}-\frac{1}{16}\int \left(\frac{162cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}-\frac{19x}{\sqrt{dx^3+c}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(\frac{38\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{19\sqrt[4]{3}\sqrt{}}{}\right.$$
$$\frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}$$

input
```
Int[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

output

```
(3*x^2*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) + ((38*Sqrt[c + d*x^3])/(d^(2/3)
*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (9*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]
*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(2/3) - (9*c^(1/6)*Arc
Tanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/d^(2/3) + (9*c^
(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(2/3) - (19*3^(1/4)*Sqrt[2 -
 Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
+ d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1
 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7
- 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (38*Sqrt[2]*c^(1/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3
)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/
3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x
)^2]*Sqrt[c + d*x^3]))/16
```

3.420.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 968

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.420. $\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

3.420.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.49 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.37

method	result	size
default	Expression too large to display	874
elliptic	Expression too large to display	874

input

```
int(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output

```
3/8*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-19/24*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/
2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))
^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/
3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(
-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*
3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^...
```

3.420.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.99 (sec) , antiderivative size = 2335, normalized size of antiderivative = 3.66

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/64*(24*sqrt(d*x^3 + c)*d*x^2 + 152*(d*x^3 - 8*c)*sqrt(d)*weierstrassZet
a(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 3*(d^2*x^3 - 8*c*d - sqr
t(-3)*(d^2*x^3 - 8*c*d))*(c/d^4)^(1/6)*log(59049/4*((d^6*x^9 + 318*c*d^5*x
^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + sqrt(-3)*(d^6*x^9 + 318*c*d^5*x^6 +
1200*c^2*d^4*x^3 + 640*c^3*d^3))*(c/d^4)^(5/6) + 6*(2*c*d^2*x^7 + 160*c^2*
d*x^4 + 320*c^3*x - 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 - sqrt(-3)*(5*c*d^4*x^
5 + 32*c^2*d^3*x^2))*(c/d^4)^(2/3) - (7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c
^3*d + sqrt(-3)*(7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d))*(c/d^4)^(1/3))
*sqrt(d*x^3 + c) - 36*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*x)*sqrt(c
/d^4) + 18*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2 - sqrt(-3)*(c*d^3*x^
8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2))*(c/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6
 + 192*c^2*d*x^3 - 512*c^3)) - 3*(d^2*x^3 - 8*c*d - sqrt(-3)*(d^2*x^3 - 8*
c*d))*(c/d^4)^(1/6)*log(-59049/4*((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*
x^3 + 640*c^3*d^3 + sqrt(-3)*(d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 +
 640*c^3*d^3))*(c/d^4)^(5/6) - 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x
- 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 - sqrt(-3)*(5*c*d^4*x^5 + 32*c^2*d^3*x^2
))*(c/d^4)^(2/3) - (7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d + sqrt(-3)*(7
*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d))*(c/d^4)^(1/3))*sqrt(d*x^3 + c) -
 36*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*x)*sqrt(c/d^4) + 18*(c*d^3*
x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2 - sqrt(-3)*(c*d^3*x^8 + 38*c^2*d^2*...
```

3.420.6 Sympy [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

input
```
integrate(x*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

output
```
Integral(x*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

3.420.7 Maxima [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}x}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c)^2, x)
```

3.420.8 Giac [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}x}{(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c)^2, x)
```

3.420.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx=\int \frac{x\,(d\,x^3+c)^{3/2}}{(8\,c-d\,x^3)^2}\,dx$$

input
```
int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

output
```
int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2, x)
```

3.421 $\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx$

3.421.1 Optimal result

Integrand size = 27, antiderivative size = 522

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = -\frac{\sqrt{c+dx^3}}{16cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3\sqrt{c+dx^3}}{8x(8c-dx^3)}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
-1/16*(d*x^3+c)^(1/2)/c/x+3/8*(d*x^3+c)^(1/2)/x/(-d*x^3+8*c)+1/16*d^(1/3)*
(d*x^3+c)^(1/2)/c/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/48*d^(1/3)*(c^(1/3)+d^
(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c
^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/32*3
^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)
))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2)
)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
^2)^(1/2)/c^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.421.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.74 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.46

$$\int \frac{(c+dx^3)^{3/2}}{x^2\left(8c-dx^3\right)^2}\,dx = \frac{(2c-dx^3)\sqrt{c+dx^3}}{16cx\left(-8c+dx^3\right)} - \frac{\sqrt[6]{-1}\sqrt[3]{-d}\sqrt{(-1)^{5/6}\left(-1+\frac{\sqrt[3]{-d}x}{\sqrt[3]{c}}\right)}\sqrt{1+\frac{\sqrt[3]{-d}x}{\sqrt[3]{c}}+\frac{(-d)^{2/3}x^2}{c^{2/3}}}\left(-i\sqrt{3}E\left(\arcsin\left(\frac{\sqrt{-(-1)^{5/6}-\frac{i\sqrt[3]{-d}x}{\sqrt[3]{c}}}}{\sqrt[4]{3}}\right)\right|\right.\right.}{16\sqrt[4]{3}\sqrt[3]{c}\sqrt{c+dx^3}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x]
```

output
```
((2*c - d*x^3)*Sqrt[c + d*x^3])/(16*c*x*(-8*c + d*x^3)) - ((-1)^(1/6)*(-d)
^(1/3)*Sqrt[(-1)^(5/6)*(-1 + ((-d)^(1/3)*x)/c^(1/3))]*Sqrt[1 + ((-d)^(1/3)
*x)/c^(1/3) + ((-d)^(2/3)*x^2)/c^(2/3)]*((-I)*Sqrt[3]*EllipticE[ArcSin[Sqr
t[-(-1)^(5/6) - (I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)] + (-1)^(1/
3)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)],
 (-1)^(1/3)]))/(16*3^(1/4)*c^(1/3)*Sqrt[c + d*x^3])
```

3.421.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {968, 27, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \; 968$$

$$\frac{\int \frac{3cd}{2x^2\sqrt{dx^3+c}}dx}{24cd}+\frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}\int \frac{1}{x^2\sqrt{dx^3+c}}dx+\frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 847$$

$$\frac{1}{16}\left(\frac{d\int \frac{x}{\sqrt{dx^3+c}}dx}{2c}-\frac{\sqrt{c+dx^3}}{cx}\right)+\frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 832$$

$$\frac{1}{16}\left(\frac{d\left(\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}\int \frac{1}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}\right)}{2c}-\frac{\sqrt{c+dx^3}}{cx}\right)+\frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 759$$

$$\frac{1}{16}\Bigg(\frac{d\Bigg(\frac{\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\Bigg(\arcsin\Bigg(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\Bigg)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{2c}$$

$$\frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}$$

$$\Big\downarrow 2416$$

$$\frac{1}{16}\Bigg(\frac{d\Bigg(\frac{\frac{2\sqrt{c+dx^3}}{\sqrt[3]{d}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}}{\sqrt[3]{d}}-\frac{2\Big(}{}}{2c}$$

$$\frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}$$

3.421. $\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)^2}\,dx$

input

```
Int[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x]
```

output

```
(3*Sqrt[c + d*x^3])/(8*x*(8*c - d*x^3)) + (-(Sqrt[c + d*x^3]/(c*x)) + (d*(
((2*Sqrt[c + d*x^3])/(d^(1/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)], -7 - 4*Sqrt[3]])/(d^(1/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/d^(1/3) - (2*(1 - S
qrt[3])*Sqrt[2 + Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^
(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^
(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(2*c))
/16
```

3.421.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.421.4 Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.93

3.421. $\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx$

method	result
	$i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-c\right.}{}\right.}{}}$
elliptic	$\frac{3d\,x^{2}\sqrt{d\,x^{3}+c}}{64c\left(-d\,x^{3}+8c\right)}-\frac{\sqrt{d\,x^{3}+c}}{64cx}-$
risch	Expression too large to display
default	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/64*d*x^2/c*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/64*(d*x^3+c)^(1/2)/c/x-1/48*I/
c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))
```

3.421. $\int \frac{\left(c+dx^{3}\right)^{3/2}}{x^{2}\left(8c-dx^{3}\right)^{2}}\,dx$

3.421.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.13

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = -\frac{(dx^4-8\,cx)\sqrt{d}\mathrm{weierstrassZeta}\left(0,-\frac{4\,c}{d},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,c}{d},x\right)\right)+\sqrt{dx^3+c}(dx^3-2\,c)}{16\,(cdx^4-8\,c^2x)}$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/16*((d*x^4 - 8*c*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInve
rse(0, -4*c/d, x)) + sqrt(d*x^3 + c)*(d*x^3 - 2*c))/(c*d*x^4 - 8*c^2*x)
```

3.421.6 Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x^2(-8c+dx^3)^2}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**2/(-d*x**3+8*c)**2,x)
```

output
```
Integral((c + d*x**3)**(3/2)/(x**2*(-8*c + d*x**3)**2), x)
```

3.421.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^2), x)
```

3.421.8 Giac [F]

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)^2}\,dx=\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^2), x)
```

3.421.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)^2}\,dx=\int \frac{\left(d\,x^3+c\right)^{3/2}}{x^2\left(8\,c-d\,x^3\right)^2}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2), x)
```

3.422 $\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx$

3.422.1 Optimal result

Integrand size = 27, antiderivative size = 684

$$
\begin{aligned}
&\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = -\frac{13\sqrt{c+dx^3}}{256cx^4}-\frac{d\sqrt{c+dx^3}}{32c^2x}+\frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&+\frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}-\frac{9\sqrt{3}d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}} \\
&+\frac{9d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}-\frac{9d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{11/6}} \\
&-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{64c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{16\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

output
```
9/1024*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
c^(11/6)-9/1024*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-9/10
24*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(
1/2)/c^(11/6)-13/256*(d*x^3+c)^(1/2)/c/x^4-1/32*d*(d*x^3+c)^(1/2)/c^2/x+3/
8*(d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)+1/32*d^(4/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))+1/96*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1
/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2
)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*
x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/64*3^(1/4)*d^(4/3)*(c^(1/3)+
d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1
/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(5/3)/(d*x^3
+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

3.422.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.29

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = \sqrt{c+dx^3}\left(-\frac{1}{256cx^4}-\frac{13d}{512c^2x}-\frac{3d^2x^2}{512c^2(-8c+dx^3)}\right) + \frac{145d^2x^2\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{8192c^2\sqrt{c+dx^3}} - \frac{d^3x^5\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{2560c^3\sqrt{c+dx^3}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x]
```

output
```
Sqrt[c + d*x^3]*(-1/256*1/(c*x^4) - (13*d)/(512*c^2*x) - (3*d^2*x^2)/(512*
c^2*(-8*c + d*x^3))) + (145*d^2*x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2,
 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(8192*c^2*Sqrt[c + d*x^3]) - (d^3*x
^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8
*c)])/(2560*c^3*Sqrt[c + d*x^3])
```

3.422.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {968, 27, 1053, 25, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{x^5 (8c-dx^3)^2} \, dx$$

$$\Big\downarrow \text{968}$$

$$\frac{\int \frac{3cd(17dx^3+26c)}{2x^5(8c-dx^3)\sqrt{dx^3+c}}dx}{24cd}+\frac{3\sqrt{c+dx^3}}{8x^4 (8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{17dx^3+26c}{x^5 (8c-dx^3) \sqrt{dx^3+c}}dx+\frac{3\sqrt{c+dx^3}}{8x^4 (8c-dx^3)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{16} \left(-\frac{\int -\frac{cd(65dx^3+128c)}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c^2}-\frac{13\sqrt{c+dx^3}}{16cx^4}\right)+\frac{3\sqrt{c+dx^3}}{8x^4 (8c-dx^3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{16} \left(\frac{\int \frac{cd(65dx^3+128c)}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c^2}-\frac{13\sqrt{c+dx^3}}{16cx^4}\right)+\frac{3\sqrt{c+dx^3}}{8x^4 (8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16} \left(\frac{d\int \frac{65dx^3+128c}{x^2(8c-dx^3)\sqrt{dx^3+c}}dx}{32c}-\frac{13\sqrt{c+dx^3}}{16cx^4}\right)+\frac{3\sqrt{c+dx^3}}{8x^4 (8c-dx^3)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{16} \left(\frac{d\left(-\frac{\int -\frac{8cdx\left(145c-8dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{13\sqrt{c+dx^3}}{16cx^4}\right)+\frac{3\sqrt{c+dx^3}}{8x^4 (8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{d\left(\frac{d\int \frac{x\left(145c-8dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{13\sqrt{c+dx^3}}{16cx^4}\right)+\frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{1054}$$

$$\frac{1}{16}\left(\frac{d\left(\frac{d\int \left(\frac{81cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{8x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{13\sqrt{c+dx^3}}{16cx^4}\right)+\frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{2009}$$

3.422. $\int \frac{\left(c+dx^3\right)^{3/2}}{x^5\left(8c-dx^3\right)^2}\,dx$

$$\frac{1}{16}\left(\frac{d\left(\frac{d\left(\frac{16\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{8\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{}\right.\right.}{}\right.\right.}{}\right.$$

$$\frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)}$$

input
```
Int[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x]
```

3.422. $\int \frac{\left(c+dx^3\right)^{3/2}}{x^5\left(8c-dx^3\right)^2}\,dx$

output
```
(3*Sqrt[c + d*x^3])/(8*x^4*(8*c - d*x^3)) + ((-13*Sqrt[c + d*x^3])/(16*c*x
^4) + (d*((-16*Sqrt[c + d*x^3])/(c*x) + (d*((16*Sqrt[c + d*x^3])/(d^(2/3)*
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*
c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*d^(2/3)) + (9*c^(1/6)*
ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(2*d^(2/3))
- (9*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*d^(2/3)) - (8*3^(1/4
)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (16*Sqrt[2]*c^(1/3)*
(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3
) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(
1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/c))/(32*c))/16
```

3.422.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

3.422. $\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx$

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.422.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.62 (sec) , antiderivative size = 919, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	919
risch	Expression too large to display	1770
default	Expression too large to display	2691

input
```
int((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/512/c^2*x^2*d^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/256*(d*x^3+c)^(1/2)/c/x^4
-13/512*d*(d*x^3+c)^(1/2)/c^2/x-1/96*I*d/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)))-3/512*I/d/c^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(
1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(
1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))
/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-
I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/...
```

3.422.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 2549, normalized size of antiderivative = 3.73

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^5\left(8c-dx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output

```
-1/4096*(128*(d^2*x^7 - 8*c*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, weie
rstrassPInverse(0, -4*c/d, x)) - 3*(c^2*d*x^7 - 8*c^3*x^4 + sqrt(-3)*(c^2*
d*x^7 - 8*c^3*x^4))*(d^8/c^11)^(1/6)*log(6561*(d^9*x^9 + 318*c*d^8*x^6 + 1
200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^1
0*d*x + sqrt(-3)*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x))*(d^8/c^11
)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt(-3)*(5*c
^10*d*x^5 + 32*c^11*x^2))*(d^8/c^11)^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d^
3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c
^4*d^5*x + sqrt(-3)*(c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x))*(d^8/c
^11)^(1/6)) - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2 - sqrt(-3)*
(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2))*(d^8/c^11)^(1/3))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 3*(c^2*d*x^7 - 8*c^3*x^4 +
sqrt(-3)*(c^2*d*x^7 - 8*c^3*x^4))*(d^8/c^11)^(1/6)*log(6561*(d^9*x^9 + 318
*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^8*d^3*x^7 + 64*c^9*d^
2*x^4 + 32*c^10*d*x + sqrt(-3)*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d
*x))*(d^8/c^11)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 -
 sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^8/c^11)^(5/6) - 2*(7*c^6*d^4*x^
6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d
^6*x^4 + 160*c^4*d^5*x + sqrt(-3)*(c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*
d^5*x))*(d^8/c^11)^(1/6)) - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^...
```

3.422.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^5\left(8c-dx^3\right)^2}\,dx=\text{Timed out}$$

input

```
integrate((d*x**3+c)**(3/2)/x**5/(-d*x**3+8*c)**2,x)
```

output

```
Timed out
```

3.422.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^5}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^5), x)
```

3.422.8 Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^5}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^5), x)
```

3.422.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^5\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2), x)
```

3.423 $\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx$

3.423.1 Optimal result

Integrand size = 27, antiderivative size = 708

$$\begin{aligned}\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx &= -\frac{11\sqrt{c+dx^3}}{224cx^7}-\frac{83d\sqrt{c+dx^3}}{7168c^2x^4}-\frac{19d^2\sqrt{c+dx^3}}{1792c^3x}\\
&+\frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{3\sqrt{c+dx^3}}{8x^7(8c-dx^3)}-\frac{9\sqrt{3}d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4096c^{17/6}}\\
&+\frac{9d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{4096c^{17/6}}-\frac{9d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4096c^{17/6}}\\
&-\frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{3584c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\\
&+\frac{19d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{896\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\end{aligned}$$

output
```
9/4096*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
c^(17/6)-9/4096*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-9/40
96*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(
1/2)/c^(17/6)-11/224*(d*x^3+c)^(1/2)/c/x^7-83/7168*d*(d*x^3+c)^(1/2)/c^2/x
^4-19/1792*d^2*(d*x^3+c)^(1/2)/c^3/x+3/8*(d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)+
19/1792*d^(7/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+19/537
6*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d
^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/
2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)-19/3584*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1
/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3
)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.423.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.30

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx = \sqrt{c+dx^3}\left(-\frac{1}{448cx^7}-\frac{41d}{7168c^2x^4}-\frac{283d^2}{28672c^3x}-\frac{3d^3x^2}{4096c^3(-8c+dx^3)}\right)+\frac{1175d^3x^2\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{229376c^3\sqrt{c+dx^3}}-\frac{19d^4x^5\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{143360c^4\sqrt{c+dx^3}}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x]
```

output
```
Sqrt[c + d*x^3]*(-1/448*1/(c*x^7) - (41*d)/(7168*c^2*x^4) - (283*d^2)/(286
72*c^3*x) - (3*d^3*x^2)/(4096*c^3*(-8*c + d*x^3))) + (1175*d^3*x^2*Sqrt[(c
 + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(229
376*c^3*Sqrt[c + d*x^3]) - (19*d^4*x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1
/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(143360*c^4*Sqrt[c + d*x^3])
```

3.423.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 722, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {968, 27, 1053, 27, 1053, 25, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)^2}dx$$

$$\Big\downarrow \; 968$$

$$\frac{\int \frac{3cd\left(35dx^3+44c\right)}{2x^8\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{24cd}+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}\int \frac{35dx^3+44c}{x^8\left(8c-dx^3\right)\sqrt{dx^3+c}}dx+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 1053$$

$$\frac{1}{16}\left(-\frac{\int -\frac{2cd\left(121dx^3+166c\right)}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{56c^2}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}\left(\frac{d\int \frac{121dx^3+166c}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{28c}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 1053$$

$$\frac{1}{16}\left(\frac{d\left(-\frac{\int -\frac{cd\left(415dx^3+1216c\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{83\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{16}\left(\frac{d\left(\frac{\int \frac{cd\left(415dx^3+1216c\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{83\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow 27$$

$$\frac{1}{16}\left(\frac{d\left(\frac{d\int \frac{415dx^3+1216c}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{83\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow 1053$$

$$\frac{1}{16}\left(\frac{d\left(\frac{d\left(-\frac{\int -\frac{8cdx\left(1175c-76dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{152\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{83\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow 27$$

$$\frac{1}{16}\left(\frac{d\left(\frac{d\left(\frac{d\int \frac{x\left(1175c-76dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{152\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{83\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

3.423. $\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)^2}\,dx$

$$\Big\downarrow 1054$$

$$\frac{1}{16}\left(\frac{d\left(\frac{d\left(\frac{d\int\left(\frac{567cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{76x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{152\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{83\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{11\sqrt{c+dx^3}}{14cx^7}\right)+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow 2009$$

3.423. $\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)^2}\,dx$

$$\frac{1}{16}\Bigg(\frac{d}{\Bigg(\frac{d}{\Bigg(d\Bigg(\frac{152\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{76\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\Big(\sqrt[3]{c}+}{}}{}}{}}$$

3.423. $\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)^2}\,dx$

input
```
Int[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x]
```

output
```
(3*Sqrt[c + d*x^3])/(8*x^7*(8*c - d*x^3)) + ((-11*Sqrt[c + d*x^3])/(14*c*x
^7) + (d*((-83*Sqrt[c + d*x^3])/(16*c*x^4) + (d*((-152*Sqrt[c + d*x^3])/(c
*x) + (d*((152*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)) - (63*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/S
qrt[c + d*x^3]])/(2*d^(2/3)) + (63*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2
/(3*c^(1/6)*Sqrt[c + d*x^3])])/(2*d^(2/3)) - (63*c^(1/6)*ArcTanh[Sqrt[c +
d*x^3]/(3*Sqrt[c])])/(2*d^(2/3)) - (76*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3)
 + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2
/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3]) + (152*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(
c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^
3])))/c))/(32*c)))/(28*c))/16
```

3.423.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

3.423. $\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx$

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.423.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.84 (sec) , antiderivative size = 938, normalized size of antiderivative = 1.32

method	result	size
elliptic	Expression too large to display	938
risch	Expression too large to display	1781
default	Expression too large to display	3187

input
```
int((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/4096/c^3*x^2*d^3*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/448*(d*x^3+c)^(1/2)/c/x^
7-41/7168*d*(d*x^3+c)^(1/2)/c^2/x^4-283/28672*d^2*(d*x^3+c)^(1/2)/c^3/x-19
/5376*I/c^3*d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(
1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x
+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/
3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*
d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-3/2048
*I/c^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1
/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_al
pha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c...
```

3.423.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.95 (sec) , antiderivative size = 2582, normalized size of antiderivative = 3.65

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

output
```
-1/114688*(1216*(d^3*x^10 - 8*c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4*c/d
, weierstrassPInverse(0, -4*c/d, x)) - 21*(c^3*d*x^10 - 8*c^4*x^7 + sqrt(-
3)*(c^3*d*x^10 - 8*c^4*x^7))*(d^14/c^17)^(1/6)*log(6561*(d^14*x^9 + 318*c*
d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^12*d^4*x^7 + 64*c^13*
d^3*x^4 + 32*c^14*d^2*x + sqrt(-3)*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*
c^14*d^2*x))*(d^14/c^17)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c
^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^14/c^17)^(5/6) - 2*(7*
c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*sqrt(d^14/c^17) + (c^3*d^11*
x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x + sqrt(-3)*(c^3*d^11*x^7 + 80*c^4*d^
10*x^4 + 160*c^5*d^9*x))*(d^14/c^17)^(1/6)) - 9*(c^6*d^9*x^8 + 38*c^7*d^8*
x^5 + 64*c^8*d^7*x^2 - sqrt(-3)*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7
*x^2))*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^
3)) + 21*(c^3*d*x^10 - 8*c^4*x^7 + sqrt(-3)*(c^3*d*x^10 - 8*c^4*x^7))*(d^1
4/c^17)^(1/6)*log(6561*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 64
0*c^3*d^11 - 9*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x + sqrt(-3
)*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x))*(d^14/c^17)^(2/3) -
3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5
+ 32*c^16*x^2))*(d^14/c^17)^(5/6) - 2*(7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 +
64*c^11*d^4)*sqrt(d^14/c^17) + (c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d
^9*x + sqrt(-3)*(c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x))*(d^14...
```

3.423.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^8 (8c-dx^3)^2} \, dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(3/2)/x**8/(-d*x**3+8*c)**2,x)
```

output
```
Timed out
```

3.423.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8 (8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2 x^8}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^8), x)
```

3.423.8 Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8 (8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2 x^8}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^8), x)
```

3.423.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^8 (8c-dx^3)^2}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^8\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x)
```

output
```
int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2), x)
```

3.424 $\int \frac{x^{11}}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

3.424.1 Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{x^{11}}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}+\frac{2\sqrt{c+dx^3}\left(170c+7dx^3\right)}{27d^4}-\frac{2944c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}$$

output
```
-2944/81*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4+8/27*x^6*(d*x^3+
c)^(1/2)/d^2/(-d*x^3+8*c)+2/27*(7*d*x^3+170*c)*(d*x^3+c)^(1/2)/d^4
```

3.424.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{2\left(\frac{3\sqrt{c+dx^3}\left(-1360c^2+114cdx^3+3d^2x^6\right)}{-8c+dx^3}-1472c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^4}$$

input
```
Integrate[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(2*((3*Sqrt[c + d*x^3]*(-1360*c^2 + 114*c*d*x^3 + 3*d^2*x^6))/(-8*c + d*x^
3) - 1472*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(81*d^4)
```

3.424.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {948, 109, 27, 164, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9}{(8c-dx^3)^2\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3}\left(\frac{8x^6\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{\int \frac{cx^3(21dx^3+16c)}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9cd^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{8x^6\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{\int \frac{x^3(21dx^3+16c)}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9d^2}\right)$$

$$\Big\downarrow \text{164}$$

$$\frac{1}{3}\left(\frac{8x^6\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{\frac{1472c^2\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{d}-\frac{2\sqrt{c+dx^3}(170c+7dx^3)}{d^2}}{9d^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{8x^6\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{\frac{2944c^2\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d^2}-\frac{2\sqrt{c+dx^3}(170c+7dx^3)}{d^2}}{9d^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{8x^6\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{\frac{2944c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3d^2}-\frac{2\sqrt{c+dx^3}(170c+7dx^3)}{d^2}}{9d^2}\right)$$

input
```
Int[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((8*x^6*Sqrt[c + d*x^3])/(9*d^2*(8*c - d*x^3)) - ((-2*Sqrt[c + d*x^3]*(170
*c + 7*d*x^3))/d^2 + (2944*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(
3*d^2))/(9*d^2))/3
```

3.424.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.424.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{\frac{2\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9}+10c\sqrt{d\,x^{3}+c}+\frac{128c^{2}\left(\frac{4\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{23\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{27}}{d^{4}}$
risch	$\frac{2\left(d\,x^{3}+46c\right)\sqrt{d\,x^{3}+c}}{9d^{4}}+\frac{64c^{2}\left(-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3d\sqrt{c}}+\frac{8c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d}\right)}{d^{3}}$
default	$\frac{d\left(\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d}-\frac{4c\sqrt{d\,x^{3}+c}}{9d^{2}}\right)+\frac{32c\sqrt{d\,x^{3}+c}}{3d}}{d^{3}}-\frac{128c^{\frac{3}{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3d^{4}}+\frac{512c^{3}\left(\frac{\sqrt{d\,x^{3}+c}}{c\left(-d\,x^{3}+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d^{4}}$
elliptic	$\frac{512c^{2}\sqrt{d\,x^{3}+c}}{27d^{4}\left(-d\,x^{3}+8c\right)}+\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d^{3}}+\frac{92c\sqrt{d\,x^{3}+c}}{9d^{4}}+\frac{1472ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\right.}{}\right.}{\left(-c\right.}}}{}\right.}{}$

3.424. $\int \frac{x^{11}}{\left(8c-dx^{3}\right)^{2}\sqrt{c+dx^{3}}}\,dx$

input
```
int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1/9*(d*x^3+c)^(3/2)+5*c*(d*x^3+c)^(1/2)+64/27*c^2*(4*(d*x^3+c)^(1/2)/(-
d*x^3+8*c)-23/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d^4
```

3.424.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.05

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{2\left(736\,(cdx^3-8\,c^2)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,(3\,d^2x^6+114\,cdx^3-1360\,c^2)\sqrt{dx^3+c}\right)}{81\,(d^5x^3-8\,cd^4)}, \frac{2\left(1472\,(\right.}{}\right.$$

input
```
integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[2/81*(736*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c
) + 10*c)/(d*x^3 - 8*c)) + 3*(3*d^2*x^6 + 114*c*d*x^3 - 1360*c^2)*sqrt(d*x
^3 + c))/(d^5*x^3 - 8*c*d^4), 2/81*(1472*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan
(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(3*d^2*x^6 + 114*c*d*x^3 - 1360*c^2)*
sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

3.424.6 Sympy [F]

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^{11}}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**11/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.424.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\left(736\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+9\,(dx^3+c)^{\frac{3}{2}}+405\,\sqrt{dx^3+c}c-\frac{768\,\sqrt{dx^3+c}c^2}{dx^3-8\,c}\right)}{81\,d^4}$$

input
```
integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
2/81*(736*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 9*(d*x^3 + c)^(3/2) + 405*sqrt(d*x^3 + c)*c - 768*sqrt(d*x^3 +
c)*c^2/(d*x^3 - 8*c))/d^4
```

3.424.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2944\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\,\sqrt{-c}d^4} - \frac{512\,\sqrt{dx^3+c}c^2}{27\,(dx^3-8\,c)d^4} + \frac{2\left((dx^3+c)^{\frac{3}{2}}d^8+45\,\sqrt{dx^3+c}cd^8\right)}{9\,d^{12}}$$

input
```
integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
2944/81*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 512/27*s
qrt(d*x^3 + c)*c^2/((d*x^3 - 8*c)*d^4) + 2/9*((d*x^3 + c)^(3/2)*d^8 + 45*s
qrt(d*x^3 + c)*c*d^8)/d^12
```

3.424.9 Mupad [B] (verification not implemented)

Time = 8.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{92\,c\,\sqrt{d\,x^3+c}}{9\,d^4}+\frac{1472\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^4}+\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^3}+\frac{512\,c^2\,\sqrt{d\,x^3+c}}{27\,d^4\,\left(8\,c-d\,x^3\right)}$$

input
```
int(x^11/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
(92*c*(c + d*x^3)^(1/2))/(9*d^4) + (1472*c^(3/2)*log((10*c + d*x^3 - 6*c^(
1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*d^4) + (2*x^3*(c + d*x^3)^(1/2
))/(9*d^3) + (512*c^2*(c + d*x^3)^(1/2))/(27*d^4*(8*c - d*x^3))
```

3.425 $\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

3.425.1 Optimal result

Integrand size = 27, antiderivative size = 83

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{64c\sqrt{c+dx^3}}{27d^3(8c-dx^3)}-\frac{224\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

output
```
-224/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^3+2/3*(d*x^3+c)^(1/
2)/d^3+64/27*c*(d*x^3+c)^(1/2)/d^3/(-d*x^3+8*c)
```

3.425.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{2\left(\frac{3\sqrt{c+dx^3}(-104c+9dx^3)}{-8c+dx^3}-112\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^3}$$

input
```
Integrate[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(2*((3*Sqrt[c + d*x^3]*(-104*c + 9*d*x^3))/(-8*c + d*x^3) - 112*Sqrt[c]*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(81*d^3)
```

3.425.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {948, 100, 27, 90, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\left(8c-dx^3\right)^2 \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{\left(8c-dx^3\right)^2 \sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{3}\left(\frac{64c\sqrt{c+dx^3}}{9d^3\left(8c-dx^3\right)}-\frac{\int \frac{cd\left(9dx^3+40c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9cd^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{64c\sqrt{c+dx^3}}{9d^3\left(8c-dx^3\right)}-\frac{\int \frac{9dx^3+40c}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9d^2}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{64c\sqrt{c+dx^3}}{9d^3\left(8c-dx^3\right)}-\frac{112c\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3-\frac{18\sqrt{c+dx^3}}{d}}{9d^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{64c\sqrt{c+dx^3}}{9d^3\left(8c-dx^3\right)}-\frac{\frac{224c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{d}-\frac{18\sqrt{c+dx^3}}{d}}{9d^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{64c\sqrt{c+dx^3}}{9d^3\left(8c-dx^3\right)}-\frac{\frac{224\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3d}-\frac{18\sqrt{c+dx^3}}{d}}{9d^2}\right)$$

input

```
Int[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((64*c*Sqrt[c + d*x^3])/(9*d^3*(8*c - d*x^3)) - ((-18*Sqrt[c + d*x^3])/d +
 (224*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d))/(9*d^2))/3
```

3.425.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.425. $\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

3.425.4 Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\frac{2\sqrt{dx^3+c}}{3}+\frac{32c\left(\frac{2\sqrt{dx^3+c}}{-dx^3+8c}-\frac{7\,\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{27}}{d^3}$
default	$\frac{2\sqrt{dx^3+c}}{3d^3}-\frac{32\,\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)\sqrt{c}}{9d^3}+\frac{64c^2\left(\frac{\sqrt{dx^3+c}}{c\left(-dx^3+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d^3}$
risch	$\frac{2\sqrt{dx^3+c}}{3d^3}+\frac{16c\left(-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}+\frac{4c\left(-\frac{\sqrt{dx^3+c}}{c\left(dx^3-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d}\right)}{d^2}$
elliptic	$\frac{64c\sqrt{dx^3+c}}{27d^3\left(-dx^3+8c\right)}+\frac{2\sqrt{dx^3+c}}{3d^3}+\frac{112i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-cd^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{}}{}\right.}{}$

input
```
int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((d*x^3+c)^(1/2)+16/9*c*(2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-7/3*arctanh(1/
3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d^3
```

3.425.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.01

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \left[\frac{2\left(56\,(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,(9\,dx^3-104\,c)\sqrt{dx^3+c}\right)}{81\,(d^4x^3-8\,cd^3)}, \frac{2\left(112\,(dx^3-8\,c)\sqrt{-c}\,\mathrm{arct}\right.}{}\right.$$

input
```
integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[2/81*(56*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 1
0*c)/(d*x^3 - 8*c)) + 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*
d^3), 2/81*(112*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)
/c) + 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]
```

3.425.6 Sympy [F]

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^8}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**8/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.425.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\left(56\,\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+27\,\sqrt{dx^3+c}-\frac{96\,\sqrt{dx^3+c}c}{dx^3-8\,c}\right)}{81\,d^3}$$

input
```
integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
2/81*(56*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + 27*sqrt(d*x^3 + c) - 96*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d^3
```

3.425.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{224\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}d^3} + \frac{2\sqrt{dx^3+c}}{3\,d^3} - \frac{64\sqrt{dx^3+c}c}{27\,(dx^3-8\,c)d^3}$$

input
```
integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
224/81*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) + 2/3*sqrt(d*
x^3 + c)/d^3 - 64/27*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d^3)
```

3.425.9 Mupad [B] (verification not implemented)

Time = 8.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,d^3} + \frac{112\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\sqrt{c}\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^3} + \frac{64\,c\sqrt{d\,x^3+c}}{27\,d^3\,\left(8\,c-d\,x^3\right)}$$

input
```
int(x^8/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*d^3) + (112*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)
*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*d^3) + (64*c*(c + d*x^3)^(1/2))/(2
7*d^3*(8*c - d*x^3))
```

3.426 $\int \frac{x^5}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

3.426.1 Optimal result

Integrand size = 27, antiderivative size = 64

$$\int \frac{x^5}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}-\frac{10\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

output
```
-10/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2/c^(1/2)+8/27*(d*x^3+c)^(1/
2)/d^2/(-d*x^3+8*c)
```

3.426.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = -\frac{8\sqrt{c+dx^3}}{27d^2\left(-8c+dx^3\right)}-\frac{10\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

input
```
Integrate[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-8*Sqrt[c + d*x^3])/(27*d^2*(-8*c + d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]
/(3*Sqrt[c])])/(81*Sqrt[c]*d^2)
```

3.426.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {948, 87, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(8c-dx^3)^2\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{8\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{5\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{8\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{10\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9d^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{8\sqrt{c+dx^3}}{9d^2(8c-dx^3)}-\frac{10\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27\sqrt{c}d^2}\right)$$

input
```
Int[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((8*Sqrt[c + d*x^3])/(9*d^2*(8*c - d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])])/(27*Sqrt[c]*d^2))/3
```

3.426.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.426.4 Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\frac{8 \sqrt{d\,x^{3}+c}}{27 \left(-d\,x^{3}+8 c \right)}-\frac{10 \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{81 \sqrt{c}}}{d^{2}}$
default	$-\frac{2 \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{9 d^{2} \sqrt{c}}+\frac{8 c \left(\frac{\sqrt{d\,x^{3}+c}}{c \left(-d\,x^{3}+8 c \right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{3 c^{\frac{3}{2}}}\right)}{27 d^{2}}$
elliptic	$\frac{8 \sqrt{d\,x^{3}+c}}{27 d^{2} \left(-d\,x^{3}+8 c \right)}+\frac{5 i \sqrt{2}\, \left(\sum_{\_\alpha =\operatorname{RootOf}\left(d\,\_Z^{3}-8 c \right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}} \sqrt{2}\, \sqrt{\frac{i d \left(2 x +\frac{-i \sqrt{3}\, \left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{d \left(x -\frac{\left(-c\,d^{2}\right)}{d}\right.}{-3 \left(-c\,d^{2}\right)^{\frac{1}{3}}+i \sqrt{3}\, \left(\right.}}}{}\right.}{}$

input
```
int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/27*(4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-5/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/
2))/c^(1/2))/d^2
```

3.426.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.42

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{5\,(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-24\,\sqrt{dx^3+c}c}{81\,(cd^3x^3-8\,c^2d^2)}, \frac{2\left(5\,(dx^3-8\,c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)-12\right.}{81\,(cd^3x^3-8\,c^2d^2)}\right.$$

input
```
integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/81*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) - 24*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2), 2/81*(
5*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(
d*x^3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2)]
```

3.426.6 Sympy [F]

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^5}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**5/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.426.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{5\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}} - \frac{24\sqrt{dx^3+c}}{dx^3-8c}}{81d^2}$$

input
```
integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
1/81*(5*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/s
qrt(c) - 24*sqrt(d*x^3 + c)/(d*x^3 - 8*c))/d^2
```

3.426.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\left(\frac{5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d} - \frac{12\sqrt{dx^3+c}}{(dx^3-8c)d}\right)}{81d}$$

input
```
integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
2/81*(5*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 12*sqrt(d*x^3
+ c)/((d*x^3 - 8*c)*d))/d
```

3.426.9 Mupad [B] (verification not implemented)

Time = 7.96 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{5\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,\sqrt{c}\,d^2}+\frac{8\,\sqrt{d\,x^3+c}}{27\,d^2\,\left(8\,c-d\,x^3\right)}$$

input
```
int(x^5/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
(5*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*c^
(1/2)*d^2) + (8*(c + d*x^3)^(1/2))/(27*d^2*(8*c - d*x^3))
```

3.427 $\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

3.427.1 Optimal result

Integrand size = 27, antiderivative size = 67

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

output
```
1/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)/d+1/27*(d*x^3+c)^(1/2)/c
/d/(-d*x^3+8*c)
```

3.427.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{\frac{3\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3}+\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

input
```
Integrate[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((3*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/(81*c^(3/2)*d)
```

3.427.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {946, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{(8c-dx^3)^2\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{18c}+\frac{\sqrt{c+dx^3}}{9cd\,(8c-dx^3)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9cd}+\frac{\sqrt{c+dx^3}}{9cd\,(8c-dx^3)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27c^{3/2}d}+\frac{\sqrt{c+dx^3}}{9cd\,(8c-dx^3)}\right)$$

input
```
Int[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(Sqrt[c + d*x^3]/(9*c*d*(8*c - d*x^3)) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
])]/(27*c^(3/2)*d))/3
```

3.427.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.427.4 Maple [A] (verified)

Time = 4.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result
default	$\frac{\frac{\sqrt{d\,x^{3}+c}}{c\left(-d\,x^{3}+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{27d}$
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^{3}+c}}{c\left(-d\,x^{3}+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{27d}$
elliptic	$\frac{\sqrt{d\,x^{3}+c}}{27cd(-d\,x^{3}+8c)}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right.}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-\right.}}}{}\right.}{}$

input
```
int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/27*((d*x^3+c)^(1/2)/c/(-d*x^3+8*c)+1/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/
2))/c^(3/2))/d
```

3.427.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.28

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \left[\frac{(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,\sqrt{dx^3+c}c}{162\,(c^2d^2x^3-8\,c^3d)},\; -\frac{(dx^3-8\,c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\,\sqrt{dx^3+c}c}{81\,(c^2d^2x^3-8\,c^3d)}\right]$$

input
```
integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/162*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 - 8*c^3*d), -1/81*((
d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*sqrt(d*x^
3 + c)*c)/(c^2*d^2*x^3 - 8*c^3*d)]
```

3.427.6 Sympy [F]

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^2}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**2/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.427.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=-\frac{\frac{6\sqrt{dx^3+c}}{(dx^3+c)c-9c^2}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{162\,d}$$

input
```
integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/162*(6*sqrt(d*x^3 + c)/((d*x^3 + c)*c - 9*c^2) + log((sqrt(d*x^3 + c) -
 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(3/2))/d
```

3.427.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}cd}-\frac{\sqrt{dx^3+c}}{27\,(dx^3-8\,c)cd}$$

input
```
integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-1/81*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) - 1/27*sqrt(d*x^
3 + c)/((d*x^3 - 8*c)*c*d)
```

3.427.9 Mupad [B] (verification not implemented)

Time = 7.92 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{162\,c^{3/2}\,d}+\frac{\sqrt{d\,x^3+c}}{27\,c\,d\,\left(8\,c-d\,x^3\right)}$$

input
```
int(x^2/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(162*c^(3/
2)*d) + (c + d*x^3)^(1/2)/(27*c*d*(8*c - d*x^3))
```

3.428 $\int \frac{1}{x(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

3.428.1 Optimal result

Integrand size = 27, antiderivative size = 88

$$\int \frac{1}{x(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{\sqrt{c+dx^3}}{216c^2(8c-dx^3)}+\frac{13\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}$$

output
```
13/2592*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/96*arctanh((d*x^3+c
)^(1/2)/c^(1/2))/c^(5/2)+1/216*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)
```

3.428.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3}+13\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-27\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2592c^{5/2}}$$

input
```
Integrate[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((12*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + 13*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])] - 27*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2592*c^(5/2))
```

3.428.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {948, 114, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}}{72c^2\left(8c-dx^3\right)}-\frac{\int -\frac{d\left(dx^3+18c\right)}{2x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{72c^2d}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{dx^3+18c}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{144c^2}+\frac{\sqrt{c+dx^3}}{72c^2\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{\frac{9}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{13}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{144c^2}+\frac{\sqrt{c+dx^3}}{72c^2\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\frac{13}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{9\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{144c^2}+\frac{\sqrt{c+dx^3}}{72c^2\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\frac{9\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{13\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{144c^2}+\frac{\sqrt{c+dx^3}}{72c^2\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\frac{13\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{9\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{144c^2}+\frac{\sqrt{c+dx^3}}{72c^2\left(8c-dx^3\right)}\right)$$

input

```
Int[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output

```
(Sqrt[c + d*x^3]/(72*c^2*(8*c - d*x^3)) + ((13*ArcTanh[Sqrt[c + d*x^3]/(3*
Sqrt[c])])/(6*Sqrt[c]) - (9*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c]))
/(144*c^2))/3
```

3.428.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.428.4 Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{96 c^{\frac{5}{2}}}+\frac{\frac{13\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right) \left(d\,x^{3}-8 c\right)}{\sqrt{c}}-12 \sqrt{d\,x^{3}+c}}{2592 \left(d\,x^{3}-8 c\right) c^{2}}$	78
default	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{96 c^{\frac{5}{2}}}+\frac{\frac{\sqrt{d\,x^{3}+c}}{c \left(-d\,x^{3}+8 c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{3 c^{\frac{3}{2}}}}{216 c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3 \sqrt{c}}\right)}{288 c^{\frac{5}{2}}}$	92
elliptic	Expression too large to display	1534

input
```
int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/2592*(13*arctanh(1/3*(d*x
^3+c)^(1/2)/c^(1/2))/c^(1/2)*(d*x^3-8*c)-12*(d*x^3+c)^(1/2))/(d*x^3-8*c)/c
^2
```

3.428.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.57

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

$$= \left[\frac{13\left(dx^3-8\,c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+27\left(dx^3-8\,c\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\sqrt{dx^3+c}c}{5184\left(c^3dx^3-8\,c^4\right)},\right.$$

input
```
integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/5184*(13*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) +
 10*c)/(d*x^3 - 8*c)) + 27*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3
 + c)*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 - 8*c^4), 1/2
592*(27*(d*x^3 - 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 13*(d*
x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x^3
 + c)*c)/(c^3*d*x^3 - 8*c^4)]
```

3.428.6 Sympy [F]

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{x\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.428.7 Maxima [F]

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x}\,dx$$

input
```
integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x), x)
```

3.428.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}c^2}-\frac{13\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{2592\sqrt{-c}c^2}-\frac{\sqrt{dx^3+c}}{216\left(dx^3-8c\right)c^2}$$

input
```
integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 13/2592*arctan(1/3*
sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/216*sqrt(d*x^3 + c)/((d*x^3 -
 8*c)*c^2)
```

3.428.9 Mupad [B] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx\\=\frac{13\,\mathrm{atanh}\left(\frac{c^2\sqrt{d\,x^3+c}}{3\sqrt{c^5}}\right)}{2592\sqrt{c^5}}-\frac{\mathrm{atanh}\left(\frac{c^2\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{96\sqrt{c^5}}+\frac{\sqrt{d\,x^3+c}}{72\,c^2\,\left(24\,c-3\,d\,x^3\right)}\end{aligned}$$

input
```
int(1/(x*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
(13*atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2))))/(2592*(c^5)^(1/2)) - a
tanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2))/(96*(c^5)^(1/2)) + (c + d*x^3)^(
1/2)/(72*c^2*(24*c - 3*d*x^3))
```

3.429 $\int \frac{1}{x^4\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

3.429.1 Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} + \frac{11d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}}$$

output

```
11/10368*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+1/384*d*arctanh((d
*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+5/864*d*(d*x^3+c)^(1/2)/c^3/(-d*x^3+8*c)-1/
24*(d*x^3+c)^(1/2)/c^2/x^3/(-d*x^3+8*c)
```

3.429.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{12\sqrt{c}\left(36c-5dx^3\right)\sqrt{c+dx^3}}{-8cx^3+dx^6} + 11d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 27d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{10368c^{7/2}}$$

input

```
Integrate[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((12*Sqrt[c]*(36*c - 5*d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 11*d*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + 27*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]
])/(10368*c^(7/2))
```

3.429.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 114, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^6 \left(8c-dx^3\right)^2 \sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{d\left(4c-3dx^3\right)}{2x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{8c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\int \frac{4c-3dx^3}{x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{\int -\frac{2cd\left(18c-5dx^3\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{72c^2d}-\frac{5\sqrt{c+dx^3}}{18c\left(8c-dx^3\right)}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{\int \frac{18c-5dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{36c}-\frac{5\sqrt{c+dx^3}}{18c(8c-dx^3)}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \ 174$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{\frac{9}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3-\frac{11}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{36c}-\frac{5\sqrt{c+dx^3}}{18c(8c-dx^3)}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{\frac{9\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{11}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{36c}-\frac{5\sqrt{c+dx^3}}{18c(8c-dx^3)}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{\frac{9\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{11\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{36c}-\frac{5\sqrt{c+dx^3}}{18c(8c-dx^3)}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{-\frac{11\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{9\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{36c}-\frac{5\sqrt{c+dx^3}}{18c(8c-dx^3)}\right)}{16c^2}-\frac{\sqrt{c+dx^3}}{8c^2x^3\left(8c-dx^3\right)}\right)$$

input
```
Int[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

3.429. $\int \frac{1}{x^4(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

output
```
(-1/8*Sqrt[c + d*x^3]/(c^2*x^3*(8*c - d*x^3)) - (d*((-5*Sqrt[c + d*x^3])/(
18*c*(8*c - d*x^3)) + ((-11*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*Sqrt[
c]) - (9*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(36*c)))/(16*c^2))
/3
```

3.429.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

3.429. $\int \frac{1}{x^4(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.429.4 Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{d\left(-\frac{-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d\,x^{3}+2\sqrt{d\,x^{3}+c}\,\sqrt{c}}{2d\,x^{3}c^{\frac{7}{2}}}+\frac{\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{11\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{9c^{3}}\right)}{192}$
risch	$-\frac{\sqrt{d\,x^{3}+c}}{192c^{3}x^{3}}-\frac{d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}-\frac{2c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9\sqrt{c}}\right)}{128c^{3}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3c\,x^{3}}+\frac{d\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{64c^{2}}-\frac{d\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{384c^{\frac{7}{2}}}+\frac{d\left(\frac{\sqrt{d\,x^{3}+c}}{c\left(-d\,x^{3}+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{1728c^{2}}+\frac{d\ \operatorname{arctanh}\left(\frac{\sqrt{d}}{3}\right.}{1152c^{\frac{7}{2}}}$
elliptic	Expression too large to display

input
```
int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*d*(-1/2*(-arctanh((d*x^3+c)^(1/2)/c^(1/2))*d*x^3+2*(d*x^3+c)^(1/2)*c
^(1/2))/d/x^3/c^(7/2)+1/9*((d*x^3+c)^(1/2)/(-d*x^3+8*c)+11/6*arctanh(1/3*(
d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))/c^3)
```

3.429.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.26

$$\int \frac{1}{x^4 (8c - dx^3)^2 \sqrt{c + dx^3}}\, dx$$
$$= \left[\frac{11 \left(d^2x^6 - 8\, cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right) + 27 \left(d^2x^6 - 8\, cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right) - 24\,(5\,c}{20736\,(c^4dx^6 - 8\,c^5x^3)}\right.$$
$$-\frac{27 \left(d^2x^6 - 8\, cdx^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + 11 \left(d^2x^6 - 8\, cdx^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right) + 12\,(5\,cdx^3}{10368\,(c^4dx^6 - 8\,c^5x^3)}$$

input
```
integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/20736*(11*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*
sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 27*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x
^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d*x^3 - 36*c^2)*sqrt(
d*x^3 + c))/(c^4*d*x^6 - 8*c^5*x^3), -1/10368*(27*(d^2*x^6 - 8*c*d*x^3)*sq
rt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 11*(d^2*x^6 - 8*c*d*x^3)*sqrt(
-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(5*c*d*x^3 - 36*c^2)*sqrt(
d*x^3 + c))/(c^4*d*x^6 - 8*c^5*x^3)]
```

3.429.6 Sympy [F]

$$\int \frac{1}{x^4 (8c - dx^3)^2 \sqrt{c + dx^3}}\, dx = \int \frac{1}{x^4 (-8c + dx^3)^2 \sqrt{c + dx^3}}\, dx$$

input
```
integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.429.7 Maxima [F]

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{dx^3+c}{\left(dx^3-8\,c\right)}^2x^4}\,dx$$

input
```
integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^4), x)
```

3.429.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}c^3} - \frac{11\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{10368\,\sqrt{-c}c^3} - \frac{5\,{\left(dx^3+c\right)}^{\frac{3}{2}}d-41\,\sqrt{dx^3+c}cd}{864\,\left({\left(dx^3+c\right)}^2-10\,{\left(dx^3+c\right)}c+9\,c^2\right)c^3}$$

input
```
integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-1/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 11/10368*d*arct
an(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/864*(5*(d*x^3 + c)^(3/
2)*d - 41*sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)
*c^3)
```

3.429.9 Mupad [B] (verification not implemented)

Time = 8.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{41\,d\,\sqrt{d\,x^3+c}}{288\,c^2}-\frac{5\,d\,{\left(d\,x^3+c\right)}^{3/2}}{288\,c^3}}{3\,{\left(d\,x^3+c\right)}^2-30\,c\,\left(d\,x^3+c\right)+27\,c^2} - \frac{d\,\left(\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1\mathrm{i}+\frac{\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)\,11\mathrm{i}}{27}\right)\,1\mathrm{i}}{384\,\sqrt{c^7}}$$

input
```
int(1/(x^4*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
((41*d*(c + d*x^3)^(1/2))/(288*c^2) - (5*d*(c + d*x^3)^(3/2))/(288*c^3))/(
3*(c + d*x^3)^2 - 30*c*(c + d*x^3) + 27*c^2) - (d*(atanh((c^3*(c + d*x^3)^
(1/2))/(c^7)^(1/2))*1i + (atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2)))*1
1i)/27)*1i)/(384*(c^7)^(1/2))
```

3.429. $\int \frac{1}{x^4 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx$

3.430 $\int \frac{1}{x^7(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

3.430.1 Optimal result

Integrand size = 27, antiderivative size = 164

$$\begin{aligned}\int \frac{1}{x^7(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = &-\frac{35d^2\sqrt{c+dx^3}}{13824c^4(8c-dx^3)}-\frac{\sqrt{c+dx^3}}{48c^2x^6(8c-dx^3)}\\ &+\frac{3d\sqrt{c+dx^3}}{128c^3x^3(8c-dx^3)}+\frac{31d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}}\\ &-\frac{19d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}}\end{aligned}$$

output
```
31/165888*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)-19/6144*d^2*arc
tanh((d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)-35/13824*d^2*(d*x^3+c)^(1/2)/c^4/(-d
*x^3+8*c)-1/48*(d*x^3+c)^(1/2)/c^2/x^6/(-d*x^3+8*c)+3/128*d*(d*x^3+c)^(1/2
)/c^3/x^3/(-d*x^3+8*c)
```

3.430.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.68

$$\begin{aligned}&\int \frac{1}{x^7(8c-dx^3)^2\sqrt{c+dx^3}}\,dx\\ &=\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}(288c^2-324cdx^3+35d^2x^6)}{-8cx^6+dx^9}+31d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-513d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{165888c^{9/2}}\end{aligned}$$

input
```
Integrate[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((12*Sqrt[c]*Sqrt[c + d*x^3]*(288*c^2 - 324*c*d*x^3 + 35*d^2*x^6))/(-8*c*x
^6 + d*x^9) + 31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 513*d^2*ArcTan
h[Sqrt[c + d*x^3]/Sqrt[c]])/(165888*c^(9/2))
```

3.430.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {948, 114, 27, 168, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^9 \left(8c-dx^3\right)^2 \sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{d\left(18c-5dx^3\right)}{2x^6\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{16c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\int \frac{18c-5dx^3}{x^6\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{\int \frac{cd\left(76c-27dx^3\right)}{x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{8c^2}-\frac{9\sqrt{c+dx^3}}{4cx^3\left(8c-dx^3\right)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\int \frac{76c-27dx^3}{x^3\left(8c-dx^3\right)^2\sqrt{dx^3+c}}dx^3}{8c}-\frac{9\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(-\frac{\int -\frac{2cd\left(342c-35dx^3\right)}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{72c^2d}-\frac{35\sqrt{c+dx^3}}{18c\left(8c-dx^3\right)}\right)}{8c}-\frac{9\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\int \frac{342c-35dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{36c}-\frac{35\sqrt{c+dx^3}}{18c\left(8c-dx^3\right)}\right)}{8c}-\frac{9\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow \text{174}$$

3.430. $\int \frac{1}{x^7(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{171}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{31}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{36c}-\frac{35\sqrt{c+dx^3}}{18c\left(8c-dx^3\right)}\right)}{8c}-\frac{9\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{31}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{171\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{36c}-\frac{35\sqrt{c+dx^3}}{18c\left(8c-dx^3\right)}\right)}{8c}-\frac{9\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6\left(8c-dx^3\right)}\right)$$

$$\Big\downarrow 219$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{171\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{31\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{36c}-\frac{35\sqrt{c+dx^3}}{18c\left(8c-dx^3\right)}\right)}{8c}-\frac{9\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6\left(8c-dx^3\right)}\right)$$

3.430. $\int \frac{1}{x^7\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{31\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{171\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{36c}-\frac{35\sqrt{c+dx^3}}{18c\left(8c-dx^3\right)}\right)}{8c}-\frac{9\sqrt{c+dx^3}}{4cx^3(8c-dx^3)}\right)}{32c^2}-\frac{\sqrt{c+dx^3}}{16c^2x^6\left(8c-dx^3\right)}\right)$$

input
```
Int[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-1/16*Sqrt[c + d*x^3]/(c^2*x^6*(8*c - d*x^3)) - (d*((-9*Sqrt[c + d*x^3])/
(4*c*x^3*(8*c - d*x^3)) - (d*((-35*Sqrt[c + d*x^3])/(18*c*(8*c - d*x^3)) +
 ((31*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*Sqrt[c]) - (171*ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(36*c)))/(8*c)))/(32*c^2))/3
```

3.430.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.430. $\int \frac{1}{x^7(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

3.430.4 Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{4104\left(-\frac{31\left(c-\frac{d\,x^{3}}{8}\right)d^{2}x^{6}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{513}+\left(c-\frac{d\,x^{3}}{8}\right)d^{2}x^{6}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)+\frac{35\left(d^{2}x^{6}\sqrt{c}-\frac{324d\,x^{3}c^{\frac{3}{2}}}{35}+\frac{288c^{\frac{5}{2}}}{35}\right)\sqrt{d\,x}}{342}\right.}{c^{\frac{9}{2}}\left(-165888d\,x^{9}+1327104c\,x^{6}\right)}$
risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(-d\,x^{3}+c\right)}{384c^{4}x^{6}}+\frac{d^{2}\left(-\frac{19\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{24\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{24\sqrt{c}}+\frac{c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{54}\right)}{256c^{4}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{6c\,x^{6}}+\frac{d\sqrt{d\,x^{3}+c}}{4c^{2}x^{3}}-\frac{d^{2}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4c^{\frac{5}{2}}}}{64c^{2}}+\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{3c\,x^{3}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{256c^{3}}-\frac{d^{2}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{2048c^{\frac{9}{2}}}+\frac{d^{2}}{}$
elliptic	Expression too large to display

input

```
int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-4104*(-31/513*(c-1/8*d*x^3)*d^2*x^6*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+
(c-1/8*d*x^3)*d^2*x^6*arctanh((d*x^3+c)^(1/2)/c^(1/2))+35/342*(d^2*x^6*c^(
1/2)-324/35*d*x^3*c^(3/2)+288/35*c^(5/2))*(d*x^3+c)^(1/2))/c^(9/2)/(-16588
8*d*x^9+1327104*c*x^6)
```

3.430.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^7\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$$
$$=\left[\frac{31\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+513\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+24\,(}{331776\left(c^5dx^9-8\,c^6x^6\right)}\right.$$

input

```
integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/331776*(31*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 +
c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 513*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*lo
g((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(35*c*d^2*x^6 - 324*
c^2*d*x^3 + 288*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 - 8*c^6*x^6), 1/165888*(5
13*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 3
1*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c)
+ 12*(35*c*d^2*x^6 - 324*c^2*d*x^3 + 288*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9
- 8*c^6*x^6)]
```

3.430.6 Sympy [F]

$$\int \frac{1}{x^7 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^7 (-8c+dx^3)^2 \sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**7*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.430.7 Maxima [F]

$$\int \frac{1}{x^7 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2 x^7}\,dx$$

input
```
integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^7), x)
```

3.430.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{19\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6144\,\sqrt{-c}c^4} - \frac{31\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{165888\,\sqrt{-c}c^4} - \frac{\sqrt{dx^3+c}d^2}{13824\,(dx^3-8\,c)c^4} + \frac{(dx^3+c)^{\frac{3}{2}}d^2 - 2\,\sqrt{dx^3+c}cd^2}{384\,c^4d^2x^6}$$

input
```
integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
19/6144*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 31/165888*d^
2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/13824*sqrt(d*x^3
 + c)*d^2/((d*x^3 - 8*c)*c^4) + 1/384*((d*x^3 + c)^(3/2)*d^2 - 2*sqrt(d*x^
3 + c)*c*d^2)/(c^4*d^2*x^6)
```

3.430.9 Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^7 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = -\frac{\frac{647\,d^2\,\sqrt{d\,x^3+c}}{4608\,c^2} - \frac{197\,d^2\,{(d\,x^3+c)}^{3/2}}{2304\,c^3} + \frac{35\,d^2\,{(d\,x^3+c)}^{5/2}}{4608\,c^4}}{33\,c\,{(d\,x^3+c)}^2 - 57\,c^2\,(d\,x^3+c) - 3\,{(d\,x^3+c)}^3 + 27\,c^3} + \frac{d^2\left(\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)\,1\mathrm{i} - \frac{\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)\,31\mathrm{i}}{513}\right)\,19\mathrm{i}}{6144\,\sqrt{c^9}}$$

input
```
int(1/(x^7*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
(d^2*(atanh((c^4*(c + d*x^3)^(1/2))/(c^9)^(1/2))*1i - (atanh((c^4*(c + d*x
^3)^(1/2))/(3*(c^9)^(1/2)))*31i)/513)*19i)/(6144*(c^9)^(1/2)) - ((647*d^2*
(c + d*x^3)^(1/2))/(4608*c^2) - (197*d^2*(c + d*x^3)^(3/2))/(2304*c^3) + (
35*d^2*(c + d*x^3)^(5/2))/(4608*c^4))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*
x^3) - 3*(c + d*x^3)^3 + 27*c^3)
```

3.431 $\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

3.431.1 Optimal result

Integrand size = 27, antiderivative size = 641

$$\begin{aligned}
&\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx \\
&= \frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{44\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}} \\
&\quad - \frac{44\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81d^{8/3}} + \frac{44\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^{8/3}} \\
&\quad - \frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad + \frac{62\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
-44/81*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
d^(8/3)+44/81*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)+44/81*c
^(1/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)
*3^(1/2)+8/27*x^2*(d*x^3+c)^(1/2)/d^2/(-d*x^3+8*c)+62/27*(d*x^3+c)^(1/2)/d
^(8/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+62/81*c^(1/3)*(c^(1/3)+d^(1/3)*x)*E
llipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I
*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^
(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-31/27*c^(1/3)*(c
^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/
3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)
/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2)))^2)^(1/2)
```

3.431.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.26

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{320cx^2(c+dx^3)+40cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+31dx^5(-8c+dx^3)\sqrt{1+}}{1080cd^2(8c-dx^3)\sqrt{c+dx^3}}$$

input
```
Integrate[x^7/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(320*c*x^2*(c + d*x^3) + 40*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appel
lF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 31*d*x^5*(-8*c + d*x^
3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8
*c)])/(1080*c*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

3.431.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {970, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{ 970}$$

$$\frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)}-\frac{\int \frac{cx(31dx^3+16c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{27cd^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)}-\frac{\int \frac{x(31dx^3+16c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{27d^2}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)}-\frac{\int \left(\frac{264cx}{(8c-dx^3)\sqrt{dx^3+c}}-\frac{31x}{\sqrt{dx^3+c}}\right)dx}{27d^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)}-$$

$$-\frac{62\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{31\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{}$$

input
```
Int[x^7/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(8*x^2*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) - ((-62*Sqrt[c + d*x^3])/(d
^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (44*c^(1/6)*ArcTan[(Sqrt[3]*
c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) + (44*c
^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(3*d^
(2/3)) - (44*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/(3*d^(2/3)) + (
31*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) -
 c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)
*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (62*Sqrt[2]
*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3
])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[
3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/(27*d^2)
```

3.431.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.431. $\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

3.431.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.69 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	877
default	Expression too large to display	1738

input

```
int(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
8/27*x^2*(d*x^3+c)^(1/2)/d^2/(-d*x^3+8*c)-62/81*I/d^3*3^(1/2)*(-c*d^2)^(1/
3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+88/243*I/d^5*2^(1/2)*sum(1/_alpha*(-c*d^2)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^
(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*...
```

3.431.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.67 (sec) , antiderivative size = 2397, normalized size of antiderivative = 3.74

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/243*(72*sqrt(d*x^3 + c)*d*x^2 + 558*(d*x^3 - 8*c)*sqrt(d)*weierstrassZe
ta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 11*(d^4*x^3 - 8*c*d^3 -
 sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(c/d^16)^(1/6)*log(164916224/3*((d^16*x^9 +
 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 +
318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d^16)^(5/6) + 6*(2*
c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5 + 32*c^2*d^11*x^2
- sqrt(-3)*(5*c*d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2/3) - (7*c*d^7*x^6
 + 152*c^2*d^6*x^3 + 64*c^3*d^5 + sqrt(-3)*(7*c*d^7*x^6 + 152*c^2*d^6*x^3
+ 64*c^3*d^5))*(c/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^10*x^7 + 64*c^2
*d^9*x^4 + 32*c^3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 6
4*c^3*d^3*x^2 - sqrt(-3)*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2))*(c
/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 11*(d^
4*x^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(c/d^16)^(1/6)*log(-164916
224/3*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqr
t(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d
^16)^(5/6) - 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5
+ 32*c^2*d^11*x^2 - sqrt(-3)*(5*c*d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2
/3) - (7*c*d^7*x^6 + 152*c^2*d^6*x^3 + 64*c^3*d^5 + sqrt(-3)*(7*c*d^7*x^6
+ 152*c^2*d^6*x^3 + 64*c^3*d^5))*(c/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c
*d^10*x^7 + 64*c^2*d^9*x^4 + 32*c^3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8...
```

3.431.6 Sympy [F]

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^7}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**7/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.431.7 Maxima [F]

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^7}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.431.8 Giac [F]

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^7}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.431.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^7}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.432 $\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

3.432.1 Optimal result

Integrand size = 27, antiderivative size = 647

$$\begin{aligned}\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}\\
&+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}}-\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}}\\
&-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{18\ 3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\\
&+\frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\end{aligned}$$

output
```
-1/81*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d
^(5/3)+1/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(5/3)+1/81*arct
an(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(5/3)*3^
(1/2)+1/27*x^2*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)+1/27*(d*x^3+c)^(1/2)/c/d^(
5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/81*(c^(1/3)+d^(1/3)*x)*EllipticF((d
^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*
I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2)))^2)^(1/2)*3^(3/4)/c^(2/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/
3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/54*(c^(1/3)+d^(1/
3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x
+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(2/3)/d^(
5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2)))^2)^(1/2)
```

3.432.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.26

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$

$$= \frac{80cx^2(c+dx^3)+10cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^5(-8c+dx^3)\sqrt{1+}}{2160c^2d\,(8c-dx^3)\sqrt{c+dx^3}}$$

input
```
Integrate[x^4/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(80*c*x^2*(c + d*x^3) + 10*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appell
F1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]
)/(2160*c^2*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

3.432.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {971, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{971}$$

$$\frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}-\frac{\int \frac{x(dx^3+4c)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{27cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}-\frac{\int \frac{x(dx^3+4c)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{54cd}$$

$$\Big\downarrow \text{1054}$$

$$\frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}-\frac{\int \left(\frac{12cx}{(8c-dx^3)\sqrt{dx^3+c}}-\frac{x}{\sqrt{dx^3+c}}\right)dx}{54cd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}-$$

$$\frac{-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{}}{d^2}}{}$$

input
```
Int[x^4/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*Sqrt[c + d*x^3])/(27*c*d*(8*c - d*x^3)) - ((-2*Sqrt[c + d*x^3])/(d^(2
/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (2*c^(1/6)*ArcTan[(Sqrt[3]*c^(1
/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) + (2*c^(1/6
)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)
) - (2*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) + (3^(1/4
)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (2*Sqrt[2]*c^(1/3)*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3)
 + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1
/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/(54*c*d)
```

3.432.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.432.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.89 (sec) , antiderivative size = 886, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	886
default	Expression too large to display	1305

input

```
int(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/27*x^2*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)-1/81*I/d^2/c*3^(1/2)*(-c*d^2)^(1
/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+2/243*I/d^4/c*2^(1/2)*sum(1/_alpha*(-c*d^
2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c
*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*
(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*
3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-
(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*...
```

3.432.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 2538, normalized size of antiderivative = 3.92

$$\int \frac{x^4}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/972*(36*sqrt(d*x^3 + c)*d*x^2 + 36*(d*x^3 - 8*c)*sqrt(d)*weierstrassZet
a(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + (c*d^3*x^3 - 8*c^2*d^2 +
 sqrt(-3)*(c*d^3*x^3 - 8*c^2*d^2))*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 + 318
*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4
+ 32*c^6*d^7*x + sqrt(-3)*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x))
*(1/(c^5*d^10))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*
x^2 - sqrt(-3)*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2))*(1/(c^5*d^10))^(5/6) - 2
*(7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c^5*d^10)) + (c*d^
4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x + sqrt(-3)*(c*d^4*x^7 + 80*c^2*d^3*
x^4 + 160*c^3*d^2*x))*(1/(c^5*d^10))^(1/6)) - 9*(c^2*d^6*x^8 + 38*c^3*d^5*
x^5 + 64*c^4*d^4*x^2 - sqrt(-3)*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4
*x^2))*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512
*c^3)) - (c*d^3*x^3 - 8*c^2*d^2 + sqrt(-3)*(c*d^3*x^3 - 8*c^2*d^2))*(1/(c^
5*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9
*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x + sqrt(-3)*(5*c^4*d^9*x^7
+ 64*c^5*d^8*x^4 + 32*c^6*d^7*x))*(1/(c^5*d^10))^(2/3) - 3*sqrt(d*x^3 + c)
*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2 - sqrt(-3)*(5*c^5*d^10*x^5 + 32*c^6*d
^9*x^2))*(1/(c^5*d^10))^(5/6) - 2*(7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^
5*d^5)*sqrt(1/(c^5*d^10)) + (c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x +
sqrt(-3)*(c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x))*(1/(c^5*d^10))^(...
```

3.432.6 Sympy [F]

$$\int \frac{x^4}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**4/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.432.7 Maxima [F]

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.432.8 Giac [F]

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.432.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.433 $\int \frac{x}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

3.433.1 Optimal result

Integrand size = 25, antiderivative size = 644

$$\begin{aligned}
\int \frac{x}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} \\
&\quad -\frac{7\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{11/6}d^{2/3}}+\frac{7\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1296c^{11/6}d^{2/3}}-\frac{7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1296c^{11/6}d^{2/3}} \\
&\quad -\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{144\ 3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad +\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output

```
7/1296*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)
/d^(2/3)-7/1296*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)/d^(2/3)-7/12
96*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)/d^
(2/3)*3^(1/2)+1/216*x^2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)+1/216*(d*x^3+c)^(
1/2)/c^2/d^(2/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/648*(c^(1/3)+d^(1/3)*x)
*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)/d^(2/3)/(d*x^3+c)^(1/2)/(c
^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/432*
(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(
1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/
4)/c^(5/3)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x
+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.433.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.25

$$\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{80cx^2(c+dx^3)+125cx^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^5(-8c+dx^3)\sqrt{1+\frac{dx}{c}}}{17280c^3\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

input

```
Integrate[x/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output

```
(80*c*x^2*(c + d*x^3) + 125*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*Appell
F1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]
)/(17280*c^3*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

3.433.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {972, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \, 972$$

$$\frac{\int \frac{dx(50c-dx^3)}{2(8c-dx^3)\sqrt{dx^3+c}}dx}{216c^2d}+\frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)}$$

$$\Big\downarrow \, 27$$

$$\frac{\int \frac{x(50c-dx^3)}{(8c-dx^3)\sqrt{dx^3+c}}dx}{432c^2}+\frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)}$$

$$\Big\downarrow \, 1054$$

$$\frac{\int \left(\frac{42cx}{(8c-dx^3)\sqrt{dx^3+c}}+\frac{x}{\sqrt{dx^3+c}}\right)dx}{432c^2}+\frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)}$$

$$\Big\downarrow \, 2009$$

$$\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{2/3}}$$

$$\frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)}$$

input
```
Int[x/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*Sqrt[c + d*x^3])/(216*c^2*(8*c - d*x^3)) + ((2*Sqrt[c + d*x^3])/(d^(2
/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (7*c^(1/6)*ArcTan[(Sqrt[3]*c^(1
/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) + (7*c^(1/6
)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)
) - (7*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3)
 + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1
/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/(432*c^2)
```

3.433.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.433.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.32 (sec) , antiderivative size = 883, normalized size of antiderivative = 1.37

method	result	size
default	Expression too large to display	883
elliptic	Expression too large to display	883

input

```
int(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/216*x^2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)-1/648*I/c^2*3^(1/2)/d*(-c*d^2)^
(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(
-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
)/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_a
lpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alp
ha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(...
```

3.433.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 2540, normalized size of antiderivative = 3.94

$$\int \frac{x}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input

```
integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output

```
-1/15552*(72*sqrt(d*x^3 + c)*d*x^2 + 72*(d*x^3 - 8*c)*sqrt(d)*weierstrassZ
eta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 7*(c^2*d^2*x^3 - 8*c^3
*d + sqrt(-3)*(c^2*d^2*x^3 - 8*c^3*d))*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 +
 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^8*d^5*x^7 + 64*c^9*d^4*
x^4 + 32*c^10*d^3*x + sqrt(-3)*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d
^3*x))*(1/(c^11*d^4))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^
11*d^4*x^2 - sqrt(-3)*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2))*(1/(c^11*d^4))^(
5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^11*d^4))
 + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 8
0*c^3*d^2*x^4 + 160*c^4*d*x))*(1/(c^11*d^4))^(1/6)) - 9*(c^4*d^4*x^8 + 38*
c^5*d^3*x^5 + 64*c^6*d^2*x^2 - sqrt(-3)*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64
*c^6*d^2*x^2))*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x
^3 - 512*c^3)) + 7*(c^2*d^2*x^3 - 8*c^3*d + sqrt(-3)*(c^2*d^2*x^3 - 8*c^3*
d))*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 6
40*c^3 - 9*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x + sqrt(-3)*(5*c
^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x))*(1/(c^11*d^4))^(2/3) - 3*sqr
t(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2 - sqrt(-3)*(5*c^10*d^5*x
^5 + 32*c^11*d^4*x^2))*(1/(c^11*d^4))^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d
^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^11*d^4)) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 +
160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x))*(1...
```

3.433.6 Sympy [F]

$$\int \frac{x}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \int \frac{x}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

input

```
integrate(x/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output

```
Integral(x/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.433.7 Maxima [F]

$$\int \frac{x}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{x}{\sqrt{dx^3+c}\,(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.433.8 Giac [F]

$$\int \frac{x}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{x}{\sqrt{dx^3+c}\,(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.433.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{x}{\sqrt{d\,x^3+c}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

input
```
int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.434 $\int \frac{1}{x^2(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

3.434.1 Optimal result

Integrand size = 27, antiderivative size = 665

$$
\begin{aligned}
&\int \frac{1}{x^2\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx \\
&= -\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{7\sqrt[3]{d}\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} \\
&\quad-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{216\sqrt{3}c^{17/6}}+\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{648c^{17/6}}-\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{648c^{17/6}} \\
&\quad-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)\middle|-7-4\sqrt{3}\right)}{288\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad+\frac{7\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

output
```
1/648*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(17/6)-1/648*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/648*
d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/
6)*3^(1/2)-7/432*(d*x^3+c)^(1/2)/c^3/x+1/216*(d*x^3+c)^(1/2)/c^2/x/(-d*x^3
+8*c)+7/432*d^(1/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+7/
1296*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*
x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^
(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2)))^2)^(1/2)-7/864*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c
^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^
(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(
1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/
3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.434.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$$
$$=\frac{-80c(54c^2+47cdx^3-7d^2x^6)+200cdx^3(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+7d^2x^6(-8}{34560c^4\sqrt{c+dx^3}\,(8cx-dx^4)}$$

input
```
Integrate[1/(x^2*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-80*c*(54*c^2 + 47*c*d*x^3 - 7*d^2*x^6) + 200*c*d*x^3*(8*c - d*x^3)*Sqrt[
1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 7
*d^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((
d*x^3)/c), (d*x^3)/(8*c)])/(34560*c^4*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

3.434.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {972, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \textit{972}$$

$$\frac{\int \frac{d\left(5dx^3+56c\right)}{2x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{216c^2d}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}$$

$$\Big\downarrow \textit{27}$$

$$\frac{\int \frac{5dx^3+56c}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}$$

$$\Big\downarrow \textit{1053}$$

$$\frac{-\frac{\int -\frac{4cdx\left(80c-7dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{7\sqrt{c+dx^3}}{cx}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}$$

$$\Big\downarrow \textit{27}$$

$$\frac{\frac{d\int \frac{x\left(80c-7dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{2c}-\frac{7\sqrt{c+dx^3}}{cx}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}$$

$$\Big\downarrow \textit{1054}$$

$$\frac{\frac{d\int \left(\frac{24cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{7x}{\sqrt{dx^3+c}}\right)dx}{2c}-\frac{7\sqrt{c+dx^3}}{cx}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}$$

$$\Big\downarrow \textit{2009}$$

$$d\left(\frac{14\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{7\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{}}{\left(\left(1+\sqrt{3}\right.\right.}}}{d^{2/3}\sqrt{\frac{\sqrt[3]{}}{\left(\left(1\right.\right.}}}\right.$$

$$\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}$$

input

```
Int[1/(x^2*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output

```
Sqrt[c + d*x^3]/(216*c^2*x*(8*c - d*x^3)) + ((-7*Sqrt[c + d*x^3])/(c*x) +
(d*((14*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (
4*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])
/(Sqrt[3]*d^(2/3)) + (4*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)
*Sqrt[c + d*x^3])])/(3*d^(2/3)) - (4*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/(3*d^(2/3)) - (7*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(
1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)
/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(
1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c
+ d*x^3]) + (14*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1
/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1
/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/(2*c))/(
432*c^2)
```

3.434. $\int \frac{1}{x^2(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

3.434.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.434.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.49 (sec) , antiderivative size = 898, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	898
risch	Expression too large to display	1758
default	Expression too large to display	1762

input
```
int(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1728*x^2/c^3*d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/64*(d*x^3+c)^(1/2)/c^3/x-7
/1296*I/c^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3)
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/972*I/c^
3/d^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/
3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d
*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alp
ha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2...
```

3.434.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 2391, normalized size of antiderivative = 3.60

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output

```
-1/7776*(126*(d*x^4 - 8*c*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstras
sPInverse(0, -4*c/d, x)) - (c^3*d*x^4 - 8*c^4*x + sqrt(-3)*(c^3*d*x^4 - 8*
c^4*x))*(d^2/c^17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 +
 640*c^3*d - 9*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x + sqrt(-3)*(5*c
^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x))*(d^2/c^17)^(2/3) + 3*sqrt(d*x^3
+ c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2
))*(d^2/c^17)^(5/6) - 2*(7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^
2/c^17) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x + sqrt(-3)*(c^3*d^3*
x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x))*(d^2/c^17)^(1/6)) - 9*(c^6*d^3*x^8 +
38*c^7*d^2*x^5 + 64*c^8*d*x^2 - sqrt(-3)*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 6
4*c^8*d*x^2))*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
512*c^3)) + (c^3*d*x^4 - 8*c^4*x + sqrt(-3)*(c^3*d*x^4 - 8*c^4*x))*(d^2/c^
17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*
(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x + sqrt(-3)*(5*c^12*d^2*x^7 + 6
4*c^13*d*x^4 + 32*c^14*x))*(d^2/c^17)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^15
*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^2/c^17)^(
5/6) - 2*(7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^17) + (c^3*
d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x + sqrt(-3)*(c^3*d^3*x^7 + 80*c^4*d^
2*x^4 + 160*c^5*d*x))*(d^2/c^17)^(1/6)) - 9*(c^6*d^3*x^8 + 38*c^7*d^2*x^5
+ 64*c^8*d*x^2 - sqrt(-3)*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2)...
```

3.434.6 Sympy [F]

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{x^2\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

input

```
integrate(1/x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output

```
Integral(1/(x**2*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.434.7 Maxima [F]

$$\int \frac{1}{x^2 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^2), x)
```

3.434.8 Giac [F]

$$\int \frac{1}{x^2 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^2), x)
```

3.434.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}} \, dx = \int \frac{1}{x^2\,\sqrt{d\,x^3+c}\,{\left(8\,c-d\,x^3\right)}^2} \, dx$$

input
```
int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.435 $\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}} \, dx$

3.435.1 Optimal result

Integrand size = 27, antiderivative size = 687

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4}+\frac{5d\sqrt{c+dx^3}}{864c^4x}-\frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}-\frac{25d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{23/6}}$$

$$+\frac{25d^{4/3}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{23/6}}-\frac{25d^{4/3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{82944c^{23/6}}$$

$$+\frac{5\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{576\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{5d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{432\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
25/82944*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2)
)/c^(23/6)-25/82944*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-
25/82944*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2
))/c^(23/6)*3^(1/2)-31/6912*(d*x^3+c)^(1/2)/c^3/x^4+5/864*d*(d*x^3+c)^(1/2
)/c^4/x+1/216*(d*x^3+c)^(1/2)/c^2/x^4/(-d*x^3+8*c)-5/864*d^(4/3)*(d*x^3+c)
^(1/2)/c^4/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-5/2592*d^(4/3)*(c^(1/3)+d^(1/3)
*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2
))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(
1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/
3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+5/1728*d^(
4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3
)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*
3^(1/4)/c^(11/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.435.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx$$

$$= \frac{245cd^2x^6(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-16\bigg(2c(216c^3-135c^2dx^3-311cd^2x^6+}{221184c^5x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^5*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(245*c*d^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/
3, -((d*x^3)/c), (d*x^3)/(8*c)] - 16*(2*c*(216*c^3 - 135*c^2*d*x^3 - 311*c
*d^2*x^6 + 40*d^3*x^9) + d^3*x^9*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appell
F1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(221184*c^5*x^4*(8*c -
 d*x^3)*Sqrt[c + d*x^3])
```

3.435.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {972, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{\int \frac{d\left(11dx^3+62c\right)}{2x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{216c^2d}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{11dx^3+62c}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int \frac{5cd\left(128c-31dx^3\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{31\sqrt{c+dx^3}}{16cx^4}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{5d\int \frac{128c-31dx^3}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{31\sqrt{c+dx^3}}{16cx^4}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{5d\left(-\frac{\int -\frac{8cdx\left(49c-8dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{31\sqrt{c+dx^3}}{16cx^4}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{5d\left(\frac{d\int \frac{x\left(49c-8dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{31\sqrt{c+dx^3}}{16cx^4}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 1054$$

$$\frac{-\frac{5d\left(\frac{d\int\left(\frac{8x}{\sqrt{dx^3+c}}-\frac{15cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}\right)dx}{c}-\frac{16\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{31\sqrt{c+dx^3}}{16cx^4}}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 2009$$

$$-\frac{5d\left(\frac{d\left(\frac{16\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{8\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{2/3}}\right.}{}\right.}{}$$

$$\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}$$

input `Int[1/(x^5*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]`

3.435. $\int \frac{1}{x^5\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

output
```
Sqrt[c + d*x^3]/(216*c^2*x^4*(8*c - d*x^3)) + ((-31*Sqrt[c + d*x^3])/(16*c
*x^4) - (5*d*((-16*Sqrt[c + d*x^3])/(c*x) + (d*((16*Sqrt[c + d*x^3])/(d^(2
/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (5*c^(1/6)*ArcTan[(Sqrt[3]*c^(1
/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*Sqrt[3]*d^(2/3)) - (5*c^(1
/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/
3)) + (5*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*d^(2/3)) - (8*3^
(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1
/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (16*Sqrt[2]*c^(1
/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)
/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^
(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/
(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/c))/(32*c))/(432*c^2)
```

3.435.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.435.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.60 (sec) , antiderivative size = 919, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	919
risch	Expression too large to display	1770
default	Expression too large to display	2241

input
```
int(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/13824*d^2*x^2/c^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/256*(d*x^3+c)^(1/2)/c^3
/x^4+3/512*d*(d*x^3+c)^(1/2)/c^4/x+5/2592*I*d/c^4*3^(1/2)*(-c*d^2)^(1/3)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)))-25/124416*I/d/c^4*2^(1/2)*sum(1/_alpha*(-c*d^2
)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3
^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c...
```

3.435.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.07 (sec) , antiderivative size = 2549, normalized size of antiderivative = 3.71

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output

```
1/995328*(5760*(d^2*x^7 - 8*c*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, we
ierstrassPInverse(0, -4*c/d, x)) + 25*(c^4*d*x^7 - 8*c^5*x^4 + sqrt(-3)*(c
^4*d*x^7 - 8*c^5*x^4))*(d^8/c^23)^(1/6)*log(9765625*(d^9*x^9 + 318*c*d^8*x
^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4
+ 32*c^18*d*x + sqrt(-3)*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x))
*(d^8/c^23)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqr
t(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^8/c^23)^(5/6) - 2*(7*c^12*d^4*x^6 +
 152*c^13*d^3*x^3 + 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^
6*x^4 + 160*c^6*d^5*x + sqrt(-3)*(c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d
^5*x))*(d^8/c^23)^(1/6)) - 9*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x
^2 - sqrt(-3)*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2))*(d^8/c^23)
^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 25*(c^4*d*x^
7 - 8*c^5*x^4 + sqrt(-3)*(c^4*d*x^7 - 8*c^5*x^4))*(d^8/c^23)^(1/6)*log(976
5625*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^16
*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x + sqrt(-3)*(5*c^16*d^3*x^7 + 64*c
^17*d^2*x^4 + 32*c^18*d*x))*(d^8/c^23)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^2
0*d*x^5 + 32*c^21*x^2 - sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^8/c^23)^
(5/6) - 2*(7*c^12*d^4*x^6 + 152*c^13*d^3*x^3 + 64*c^14*d^2)*sqrt(d^8/c^23)
 + (c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x + sqrt(-3)*(c^4*d^7*x^7 +
 80*c^5*d^6*x^4 + 160*c^6*d^5*x))*(d^8/c^23)^(1/6)) - 9*(c^8*d^6*x^8 + ...
```

3.435.6 Sympy [F]

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx=\int \frac{1}{x^5 \left(-8c+dx^3\right)^2 \sqrt{c+dx^3}}\,dx$$

input

```
integrate(1/x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output

```
Integral(1/(x**5*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.435.7 Maxima [F]

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^5}\,dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^5), x)
```

3.435.8 Giac [F]

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^5}\,dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^5), x)
```

3.435.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx=\int \frac{1}{x^5\,\sqrt{d\,x^3+c}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

input
```
int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.436 $\int \frac{1}{x^8\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

3.436.1 Optimal result

Integrand size = 27, antiderivative size = 711

$$\begin{aligned}
&\int \frac{1}{x^8 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx \\
&= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&\quad + \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} - \frac{17d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{110592\sqrt{3}c^{29/6}} \\
&\quad + \frac{17d^{7/3}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{331776c^{29/6}} - \frac{17d^{7/3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{331776c^{29/6}} \\
&\quad - \frac{289\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32256\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad + \frac{289d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{24192\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
17/331776*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2
))/c^(29/6)-17/331776*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(29/6
)-17/331776*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(
1/2))/c^(29/6)*3^(1/2)-17/6048*(d*x^3+c)^(1/2)/c^3/x^7+391/193536*d*(d*x^3
+c)^(1/2)/c^4/x^4-289/48384*d^2*(d*x^3+c)^(1/2)/c^5/x+1/216*(d*x^3+c)^(1/2
)/c^2/x^7/(-d*x^3+8*c)+289/48384*d^(7/3)*(d*x^3+c)^(1/2)/c^5/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))+289/145152*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/
3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(
(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(3/4)/c^(14/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*
x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-289/96768*d^(7/3)*(c^(1/3)+d^(
1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)
*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(14/3)/
(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)
))^2)^(1/2)
```

3.436.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.30

$$\begin{aligned}\int \frac{1}{x^8 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx &= \sqrt{c+dx^3}\left(-\frac{1}{448c^3x^7}+\frac{15d}{7168c^4x^4}-\frac{171d^2}{28672c^5x}\right.\\ &\qquad\left.-\frac{d^3x^2}{110592c^5(-8c+dx^3)}\right)\\ &\quad+\frac{9605d^3x^2\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{6193152c^5\sqrt{c+dx^3}}\\ &\quad-\frac{289d^4x^5\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3870720c^6\sqrt{c+dx^3}}\end{aligned}$$

input
```
Integrate[1/(x^8*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output

```
Sqrt[c + d*x^3]*(-1/448*1/(c^3*x^7) + (15*d)/(7168*c^4*x^4) - (171*d^2)/(2
8672*c^5*x) - (d^3*x^2)/(110592*c^5*(-8*c + d*x^3))) + (9605*d^3*x^2*Sqrt[
(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(6
193152*c^5*Sqrt[c + d*x^3]) - (289*d^4*x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(3870720*c^6*Sqrt[c + d*x^3]
)
```

3.436.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 728, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {972, 27, 1053, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (8c-dx^3)^2 \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{\int \frac{17d(dx^3+4c)}{2x^8(8c-dx^3)\sqrt{dx^3+c}}dx}{216c^2d}+\frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{17\int \frac{dx^3+4c}{x^8(8c-dx^3)\sqrt{dx^3+c}}dx}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{17\left(-\frac{\int \frac{2cd\left(46c-11dx^3\right)}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{56c^2}-\frac{\sqrt{c+dx^3}}{14cx^7}\right)}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{17\left(-\frac{d\int \frac{46c-11dx^3}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{28c}-\frac{\sqrt{c+dx^3}}{14cx^7}\right)}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{17\left(-\frac{d\left(-\frac{\int \frac{cd\left(1088c-115dx^3\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{\sqrt{c+dx^3}}{14cx^7}\right)}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow 27$$

$$\frac{17\left(-\frac{d\left(-\frac{d\int \frac{1088c-115dx^3}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{\sqrt{c+dx^3}}{14cx^7}\right)}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow 1053$$

$$\frac{17\left(-\frac{d\left(-\frac{d\left(-\frac{\int -\frac{8cdx\left(565c-68dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{136\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{\sqrt{c+dx^3}}{14cx^7}\right)}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow 27$$

3.436. $\int \frac{1}{x^8\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

$$\frac{17\left(-\frac{d\left(-\frac{d\left(\frac{d\int \frac{x\left(565c-68dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{136\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{\sqrt{c+dx^3}}{14cx^7}\right)}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 1054$$

$$\frac{17\left(-\frac{d\left(-\frac{d\left(\frac{d\int \left(\frac{21cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{68x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{136\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{\sqrt{c+dx^3}}{14cx^7}\right)}{432c^2}+\frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)}$$

$$\Big\downarrow \; 2009$$

3.436. $\int \frac{1}{x^8\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

$$17\left(-\frac{d\left(-\frac{d\left(\frac{d\left(\frac{136\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-68\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\right.\right.}{}\right.}{}\right.}{}\right.$$

3.436. $\int \frac{1}{x^8\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

input

```
Int[1/(x^8*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output

```
Sqrt[c + d*x^3]/(216*c^2*x^7*(8*c - d*x^3)) + (17*(-1/14*Sqrt[c + d*x^3]/(
c*x^7) - (d*((-23*Sqrt[c + d*x^3])/(16*c*x^4) - (d*((-136*Sqrt[c + d*x^3])
/(c*x) + (d*((136*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)) - (7*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c
+ d*x^3]])/(2*Sqrt[3]*d^(2/3)) + (7*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^
2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/3)) - (7*c^(1/6)*ArcTanh[Sqrt[c +
d*x^3]/(3*Sqrt[c])])/(6*d^(2/3)) - (68*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3)
 + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2
/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3]) + (136*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(
c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^
3])))/c))/(32*c)))/(28*c)))/(432*c^2)
```

3.436.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.436.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.80 (sec) , antiderivative size = 938, normalized size of antiderivative = 1.32

method	result	size
elliptic	Expression too large to display	938
risch	Expression too large to display	1781
default	Expression too large to display	2739

input
```
int(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/110592*d^3*x^2/c^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/448*(d*x^3+c)^(1/2)/c^
3/x^7+15/7168*d*(d*x^3+c)^(1/2)/c^4/x^4-171/28672*d^2*(d*x^3+c)^(1/2)/c^5/
x-289/145152*I*d^2/c^5*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c
*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1
/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))
-17/497664*I/c^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(...
```

3.436.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.29 (sec) , antiderivative size = 2582, normalized size of antiderivative = 3.63

$$\int \frac{1}{x^8 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/27869184*(166464*(d^3*x^10 - 8*c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4
*c/d, weierstrassPInverse(0, -4*c/d, x)) - 119*(c^5*d*x^10 - 8*c^6*x^7 + s
qrt(-3)*(c^5*d*x^10 - 8*c^6*x^7))*(d^14/c^29)^(1/6)*log(1419857*(d^14*x^9
+ 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^20*d^4*x^7 +
64*c^21*d^3*x^4 + 32*c^22*d^2*x + sqrt(-3)*(5*c^20*d^4*x^7 + 64*c^21*d^3*x
^4 + 32*c^22*d^2*x))*(d^14/c^29)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^
5 + 32*c^26*x^2 - sqrt(-3)*(5*c^25*d*x^5 + 32*c^26*x^2))*(d^14/c^29)^(5/6)
 - 2*(7*c^15*d^6*x^6 + 152*c^16*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29) + (
c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x + sqrt(-3)*(c^5*d^11*x^7 +
80*c^6*d^10*x^4 + 160*c^7*d^9*x))*(d^14/c^29)^(1/6)) - 9*(c^10*d^9*x^8 + 3
8*c^11*d^8*x^5 + 64*c^12*d^7*x^2 - sqrt(-3)*(c^10*d^9*x^8 + 38*c^11*d^8*x^
5 + 64*c^12*d^7*x^2))*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2
*d*x^3 - 512*c^3)) + 119*(c^5*d*x^10 - 8*c^6*x^7 + sqrt(-3)*(c^5*d*x^10 -
8*c^6*x^7))*(d^14/c^29)^(1/6)*log(1419857*(d^14*x^9 + 318*c*d^13*x^6 + 120
0*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c
^22*d^2*x + sqrt(-3)*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x))*(
d^14/c^29)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2 - sqrt
(-3)*(5*c^25*d*x^5 + 32*c^26*x^2))*(d^14/c^29)^(5/6) - 2*(7*c^15*d^6*x^6 +
 152*c^16*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 80*c^6*
d^10*x^4 + 160*c^7*d^9*x + sqrt(-3)*(c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 1...
```

3.436.6 Sympy [F]

$$\int \frac{1}{x^8 \left(8c - dx^3\right)^2 \sqrt{c + dx^3}}\, dx = \int \frac{1}{x^8 \left(-8c + dx^3\right)^2 \sqrt{c + dx^3}}\, dx$$

input
```
integrate(1/x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**8*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.436.7 Maxima [F]

$$\int \frac{1}{x^8 (8c-dx^3)^2 \sqrt{c+dx^3}}\, dx = \int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2 x^8}\, dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^8), x)
```

3.436.8 Giac [F]

$$\int \frac{1}{x^8 (8c-dx^3)^2 \sqrt{c+dx^3}}\, dx = \int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2 x^8}\, dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^8), x)
```

3.436.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (8c-dx^3)^2 \sqrt{c+dx^3}}\, dx = \int \frac{1}{x^8\,\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\, dx$$

input
```
int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.437 $\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

3.437.1 Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{x^7\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{7}{3},2,\frac{1}{2},\frac{10}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}$$

output
```
1/448*x^7*AppellF1(7/3,1/2,2,10/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/
c^2/(d*x^3+c)^(1/2)
```

3.437.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 239 vs. $2(66) = 132$.

Time = 10.33 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.62

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$=\frac{x\Bigg(-\frac{23dx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c}+\frac{256\Bigg(c+dx^3-\frac{32c^2\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32c\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\Big(\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}}{8c-dx^3}}{864d^2\sqrt{c+dx^3}}$$

input
```
Integrate[x^6/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x*((-23*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c)
, (d*x^3)/(8*c)])/c + (256*(c + d*x^3 - (32*c^2*AppellF1[1/3, 1/2, 1, 4/3,
 -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/
c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))
)/(8*c - d*x^3)))/(864*d^2*Sqrt[c + d*x^3])
```

3.437.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^6}{(8c-dx^3)^2\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^7\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{7}{3},2,\frac{1}{2},\frac{10}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}$$

input
```
Int[x^6/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x^7*Sqrt[1 + (d*x^3)/c]*AppellF1[7/3, 2, 1/2, 10/3, (d*x^3)/(8*c), -((d*x
^3)/c)])/(448*c^2*Sqrt[c + d*x^3])
```

3.437.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.437.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.50 (sec) , antiderivative size = 723, normalized size of antiderivative = 10.95

method	result	size
elliptic	Expression too large to display	723
default	Expression too large to display	1432

input
```
int(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
8/27*x*(d*x^3+c)^(1/2)/d^2/(-d*x^3+8*c)-46/81*I/d^3*3^(1/2)*(-c*d^2)^(1/3)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+64/243*I/d^5*2^(1/2)*sum(1/_
alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1
/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)
^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(
1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/1
8/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Ro
otOf(_Z^3*d-8*c))
```

3.437.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2425 vs. $2(52) = 104$.

Time = 0.72 (sec) , antiderivative size = 2425, normalized size of antiderivative = 36.74

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/243*(36*sqrt(d*x^3 + c)*d*x - 63*(d*x^3 - 8*c)*sqrt(d)*weierstrassPInve
rse(0, -4*c/d, x) + 2*(d^4*x^3 - 8*c*d^3 + sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(
1/(c*d^14))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3
- 9*(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2 + sqrt(-3)*(c*d^12*x^8
 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2))*(1/(c*d^14))^(2/3) + 3*sqrt(d*x^3 +
 c)*((c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x - sqrt(-3)*(c*d^14*x^7
 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x))*(1/(c*d^14))^(5/6) - 2*(7*c*d^9*x^6
+ 152*c^2*d^8*x^3 + 64*c^3*d^7)*sqrt(1/(c*d^14)) + 6*(5*c*d^4*x^5 + 32*c^2
*d^3*x^2 + sqrt(-3)*(5*c*d^4*x^5 + 32*c^2*d^3*x^2))*(1/(c*d^14))^(1/6)) -
9*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x - sqrt(-3)*(5*c*d^7*x^7 + 6
4*c^2*d^6*x^4 + 32*c^3*d^5*x))*(1/(c*d^14))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
 + 192*c^2*d*x^3 - 512*c^3)) - 2*(d^4*x^3 - 8*c*d^3 + sqrt(-3)*(d^4*x^3 -
8*c*d^3))*(1/(c*d^14))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3
 + 640*c^3 - 9*(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2 + sqrt(-3)*
(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2))*(1/(c*d^14))^(2/3) - 3*s
qrt(d*x^3 + c)*((c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x - sqrt(-3)*
(c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x))*(1/(c*d^14))^(5/6) - 2*(7
*c*d^9*x^6 + 152*c^2*d^8*x^3 + 64*c^3*d^7)*sqrt(1/(c*d^14)) + 6*(5*c*d^4*x
^5 + 32*c^2*d^3*x^2 + sqrt(-3)*(5*c*d^4*x^5 + 32*c^2*d^3*x^2))*(1/(c*d^14)
)^(1/6)) - 9*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x - sqrt(-3)*(5...
```

3.437.6 Sympy [F]

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^6}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**6/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.437.7 Maxima [F]

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^6}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.437.8 Giac [F]

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^6}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.437.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^6}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x^6/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^6/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.438 $\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

3.438.1 Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{x^4\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},2,\frac{1}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}$$

output
```
1/256*x^4*AppellF1(4/3,1/2,2,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c
^2/(d*x^3+c)^(1/2)
```

3.438.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 237 vs. $2(66)=132$.

Time = 10.22 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.59

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$=\frac{x\left(x^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-\frac{64c\left(c+dx^3-\frac{32c^2\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\right.}{32c\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\right.\right.}\right.}{d(-8c+dx^3)}\right.}{1728c^2\sqrt{c+dx^3}}$$

input
```
Integrate[x^3/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x*(x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^
3)/(8*c)] - (64*c*(c + d*x^3 - (32*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3
)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^
3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c
)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(d*(-8*
c + d*x^3))))/(1728*c^2*Sqrt[c + d*x^3])
```

3.438.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{(8c-dx^3)^2\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{1}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}$$

input
```
Int[x^3/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 1/2, 7/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(256*c^2*Sqrt[c + d*x^3])
```

3.438.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.438.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.58 (sec) , antiderivative size = 732, normalized size of antiderivative = 11.09

method	result	size
elliptic	Expression too large to display	732
default	Expression too large to display	1151

input
```
int(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/27*x*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)+1/81*I/d^2/c*3^(1/2)*(-c*d^2)^(1/3
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/243*I/d^4/c*2^(1/2)*sum(1
/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^
2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)
^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1
/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=
RootOf(_Z^3*d-8*c))
```

3.438.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2548 vs. $2(52) = 104$.

Time = 0.90 (sec) , antiderivative size = 2548, normalized size of antiderivative = 38.61

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output

```
-1/3888*(144*sqrt(d*x^3 + c)*d*x + 72*(d*x^3 - 8*c)*sqrt(d)*weierstrassPIn
verse(0, -4*c/d, x) - (c*d^3*x^3 - 8*c^2*d^2 + sqrt(-3)*(c*d^3*x^3 - 8*c^2
*d^2))*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 +
 640*c^3 - 9*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2 + sqrt(-3)*(c^
5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2))*(1/(c^7*d^8))^(2/3) + 3*sqrt
(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x - sqrt(-3)*(c^6
*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x))*(1/(c^7*d^8))^(5/6) - 2*(7*c^4
*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)) + 6*(5*c^2*d^3*
x^5 + 32*c^3*d^2*x^2 + sqrt(-3)*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2))*(1/(c^7*
d^8))^(1/6)) - 9*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x - sqrt(-3)
*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x))*(1/(c^7*d^8))^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c*d^3*x^3 - 8*c^2*d^2
 + sqrt(-3)*(c*d^3*x^3 - 8*c^2*d^2))*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318
*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 +
64*c^7*d^6*x^2 + sqrt(-3)*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2))
*(1/(c^7*d^8))^(2/3) - 3*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 +
160*c^8*d^7*x - sqrt(-3)*(c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x))*(
1/(c^7*d^8))^(5/6) - 2*(7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt
(1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2 + sqrt(-3)*(5*c^2*d^3*x^
5 + 32*c^3*d^2*x^2))*(1/(c^7*d^8))^(1/6)) - 9*(5*c^3*d^5*x^7 + 64*c^4*d...
```

3.438.6 Sympy [F]

$$\int \frac{x^3}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

input

```
integrate(x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output

```
Integral(x**3/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.438.7 Maxima [F]

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.438.8 Giac [F]

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.438.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.439 $\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

3.439.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}$$

output
```
1/64*x*AppellF1(1/3,1/2,2,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/
(d*x^3+c)^(1/2)
```

3.439.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 237 vs. $2(64) = 128$.

Time = 10.20 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.70

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx =\frac{x\left(\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{64\left(\frac{c+dx^3}{c^2}+\frac{832\,\mathrm{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32c\,\mathrm{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\mathrm{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\,\mathrm{App}\right.}\right.}{8c-dx^3}\right.}{13824\sqrt{c+dx^3}}$$

input
```
Integrate[1/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d
*x^3)/(8*c)])/c^3 + (64*((c + d*x^3)/c^2 + (832*AppellF1[1/3, 1/2, 1, 4/3,
 -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/
c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))
)/(8*c - d*x^3)))/(13824*Sqrt[c + d*x^3])
```

3.439.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(8c-dx^3)^2\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}$$

input
```
Int[1/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 1/2, 4/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(64*c^2*Sqrt[c + d*x^3])
```

3.439.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.439.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.48 (sec) , antiderivative size = 729, normalized size of antiderivative = 11.39

method	result	size
default	Expression too large to display	729
elliptic	Expression too large to display	729

input
```
int(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/216*x*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)+1/648*I/c^2*3^(1/2)/d*(-c*d^2)^(1
/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-5/972*I/c^2/d^3*2^(1/2)*s
um(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d
^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-
c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*
d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_al
pha=RootOf(_Z^3*d-8*c))
```

3.439.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2498 vs. $2(50) = 100$.

Time = 0.87 (sec) , antiderivative size = 2498, normalized size of antiderivative = 39.03

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/15552*(72*sqrt(d*x^3 + c)*d*x - 288*(d*x^3 - 8*c)*sqrt(d)*weierstrassPI
nverse(0, -4*c/d, x) - 5*(c^2*d^2*x^3 - 8*c^3*d + sqrt(-3)*(c^2*d^2*x^3 -
8*c^3*d))*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x
^3 + 640*c^3 - 9*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2 + sqrt(-
3)*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2))*(1/(c^13*d^2))^(2/3)
 + 3*sqrt(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x - s
qrt(-3)*(c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x))*(1/(c^13*d^2))^
(5/6) - 2*(7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(1/(c^13*d^2))
+ 6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2 + sqrt(-3)*(5*c^3*d^2*x^5 + 32*c^4*d*x^2
))*(1/(c^13*d^2))^(1/6)) - 9*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x
- sqrt(-3)*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x))*(1/(c^13*d^2))^(
1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 5*(c^2*d^2*x^3
 - 8*c^3*d + sqrt(-3)*(c^2*d^2*x^3 - 8*c^3*d))*(1/(c^13*d^2))^(1/6)*log((d
^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^4*x^8 + 38*c^
10*d^3*x^5 + 64*c^11*d^2*x^2 + sqrt(-3)*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 6
4*c^11*d^2*x^2))*(1/(c^13*d^2))^(2/3) - 3*sqrt(d*x^3 + c)*((c^11*d^4*x^7 +
 80*c^12*d^3*x^4 + 160*c^13*d^2*x - sqrt(-3)*(c^11*d^4*x^7 + 80*c^12*d^3*x
^4 + 160*c^13*d^2*x))*(1/(c^13*d^2))^(5/6) - 2*(7*c^7*d^3*x^6 + 152*c^8*d^
2*x^3 + 64*c^9*d)*sqrt(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2 + s
qrt(-3)*(5*c^3*d^2*x^5 + 32*c^4*d*x^2))*(1/(c^13*d^2))^(1/6)) - 9*(5*c^...
```

3.439.6 Sympy [F]

$$\int \frac{1}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.439.7 Maxima [F]

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.439.8 Giac [F]

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.439.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.440 $\int \frac{1}{x^3\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

3.440.1 Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^3\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^2x^2\sqrt{c+dx^3}}$$

output
```
-1/128*AppellF1(-2/3,1/2,2,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2
/x^2/(d*x^3+c)^(1/2)
```

3.440.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 266 vs. 2(66) = 132.

Time = 10.21 (sec) , antiderivative size = 266, normalized size of antiderivative = 4.03

$$\int \frac{1}{x^3\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{-\frac{64(c+dx^3)(-216c+29dx^3)}{c^3x^2(-8c+dx^3)}+\frac{29d^2x^4\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^4}-\frac{4096dx\,\mathrm{Appe}}{c(8c-dx^3)\left(32c\,\mathrm{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\right.\right.}}{221184\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^3*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output

```
((-64*(c + d*x^3)*(-216*c + 29*d*x^3))/(c^3*x^2*(-8*c + d*x^3)) + (29*d^2*
x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/c^4 - (4096*d*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8
*c)])/(c*(8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x
^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*
c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(22118
4*Sqrt[c + d*x^3])
```

3.440.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^3 \left(8c-dx^3\right)^2 \sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\, \text{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128 c^2 x^2 \sqrt{c+dx^3}}$$

input

```
Int[1/(x^3*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output

```
-1/128*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 1/2, 1/3, (d*x^3)/(8*c), -((
d*x^3)/c)])/(c^2*x^2*Sqrt[c + d*x^3])
```

3.440.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.440.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.56 (sec) , antiderivative size = 744, normalized size of antiderivative = 11.27

method	result	size
elliptic	Expression too large to display	744
default	Expression too large to display	1456
risch	Expression too large to display	1457

input
```
int(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1728*x/c^3*d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/128*(d*x^3+c)^(1/2)/c^3/x^2+
29/10368*I/c^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1
/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3
)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
)^(1/2))-19/15552*I/c^3/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^
(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^
2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/
2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)
*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

3.440.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (8c - dx^3)^2 \sqrt{c + dx^3}}\, dx = \text{Timed out}$$

input
```
integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.440.6 Sympy [F]

$$\int \frac{1}{x^3 (8c - dx^3)^2 \sqrt{c + dx^3}}\, dx = \int \frac{1}{x^3 (-8c + dx^3)^2 \sqrt{c + dx^3}}\, dx$$

input
```
integrate(1/x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.440.7 Maxima [F]

$$\int \frac{1}{x^3 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2 x^3}\,dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^3), x)
```

3.440.8 Giac [F]

$$\int \frac{1}{x^3 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2 x^3}\,dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^3), x)
```

3.440.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^3\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

input
```
int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.441 $\int \frac{1}{x^6\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

3.441.1 Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^6\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{5}{3},2,\frac{1}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}$$

output
```
-1/320*AppellF1(-5/3,1/2,2,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^
2/x^5/(d*x^3+c)^(1/2)
```

3.441.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 279 vs. $2(66)=132$.

Time = 10.22 (sec) , antiderivative size = 279, normalized size of antiderivative = 4.23

$$\int \frac{1}{x^6\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

$$=\frac{\frac{64(c+dx^3)(864c^2-1080cdx^3+119d^2x^6)}{c^4x^5(-8c+dx^3)}-\frac{119d^3x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^5}+\frac{140}{c^2(8c-dx^3)\Bigl(32c\,\text{AppellF1}\bigl(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},}}{2211840\sqrt{c+dx^3}}$$

input
```
Integrate[1/(x^6*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((64*(c + d*x^3)*(864*c^2 - 1080*c*d*x^3 + 119*d^2*x^6))/(c^4*x^5*(-8*c +
d*x^3)) - (119*d^3*x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d
*x^3)/c), (d*x^3)/(8*c)])/c^5 + (1404928*d^2*x*AppellF1[1/3, 1/2, 1, 4/3,
-((d*x^3)/c), (d*x^3)/(8*c)])/(c^2*(8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3,
 -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)]))))/(2211840*Sqrt[c + d*x^3])
```

3.441.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \; 1013$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^6\left(8c-dx^3\right)^2\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 1012$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{5}{3},2,\frac{1}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}$$

input
```
Int[1/(x^6*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
-1/320*(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 2, 1/2, -2/3, (d*x^3)/(8*c), -(
(d*x^3)/c)])/(c^2*x^5*Sqrt[c + d*x^3])
```

3.441.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.441.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.56 (sec) , antiderivative size = 765, normalized size of antiderivative = 11.59

method	result	size
elliptic	Expression too large to display	765
risch	Expression too large to display	1464
default	Expression too large to display	1783

input
```
int(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/13824*d^2*x/c^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/320*(d*x^3+c)^(1/2)/c^3/x
^5+9/2560*d*(d*x^3+c)^(1/2)/c^4/x^2-119/103680*I*d/c^4*3^(1/2)*(-c*d^2)^(1
/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-7/31104*I/d/c^4*2^(1/2)*s
um(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d
^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-
c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*
d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_al
pha=RootOf(_Z^3*d-8*c))
```

3.441.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2561 vs. $2(52) = 104$.

Time = 4.25 (sec) , antiderivative size = 2561, normalized size of antiderivative = 38.80

$$\int \frac{1}{x^6 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\, dx = \text{Too large to display}$$

input
```
integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
1/2488320*(11088*(d^2*x^8 - 8*c*d*x^5)*sqrt(d)*weierstrassPInverse(0, -4*c
/d, x) + 35*(c^4*d*x^8 - 8*c^5*x^5 + sqrt(-3)*(c^4*d*x^8 - 8*c^5*x^5))*(d^
10/c^25)^(1/6)*log(16807*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 6
40*c^3*d^8 - 9*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2 + sqrt(-3
)*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2))*(d^10/c^25)^(2/3) +
3*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x - sqrt(-3)*(
c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x))*(d^10/c^25)^(5/6) - 2*(7*c^13*
d^5*x^6 + 152*c^14*d^4*x^3 + 64*c^15*d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x
^5 + 32*c^6*d^7*x^2 + sqrt(-3)*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2))*(d^10/c^2
5)^(1/6)) - 9*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x - sqrt(-3)*
(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x))*(d^10/c^25)^(1/3))/(d^3
*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 35*(c^4*d*x^8 - 8*c^5*x^
5 + sqrt(-3)*(c^4*d*x^8 - 8*c^5*x^5))*(d^10/c^25)^(1/6)*log(16807*(d^11*x^
9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^17*d^4*x^8 + 38
*c^18*d^3*x^5 + 64*c^19*d^2*x^2 + sqrt(-3)*(c^17*d^4*x^8 + 38*c^18*d^3*x^5
 + 64*c^19*d^2*x^2))*(d^10/c^25)^(2/3) - 3*sqrt(d*x^3 + c)*((c^21*d^2*x^7
+ 80*c^22*d*x^4 + 160*c^23*x - sqrt(-3)*(c^21*d^2*x^7 + 80*c^22*d*x^4 + 16
0*c^23*x))*(d^10/c^25)^(5/6) - 2*(7*c^13*d^5*x^6 + 152*c^14*d^4*x^3 + 64*c
^15*d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2 + sqrt(-3)*(5
*c^5*d^8*x^5 + 32*c^6*d^7*x^2))*(d^10/c^25)^(1/6)) - 9*(5*c^9*d^7*x^7 +...
```

3.441.6 Sympy [F]

$$\int \frac{1}{x^6 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^6 \left(-8c+dx^3\right)^2 \sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**6*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.441.7 Maxima [F]

$$\int \frac{1}{x^6 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2 x^6}\,dx$$

input
```
integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^6), x)
```

3.441.8 Giac [F]

$$\int \frac{1}{x^6 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2 x^6}\,dx$$

input
```
integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^6), x)
```

3.441.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^6\,\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.442 $\int \frac{x^{11}}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.442.1 Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{x^{11}}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{2(38c+39dx^3)}{81d^4\sqrt{c+dx^3}}-\frac{640\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}$$

output
```
-640/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^4+8/27*x^6/d^2/(-d
*x^3+8*c)/(d*x^3+c)^(1/2)+2/81*(39*d*x^3+38*c)/d^4/(d*x^3+c)^(1/2)
```

3.442.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{x^{11}}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=-\frac{2\left(912c^2+822cdx^3-81d^2x^6-320\sqrt{c}(8c-dx^3)\sqrt{c+dx^3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{243d^4\left(-8c+dx^3\right)\sqrt{c+dx^3}}$$

input
```
Integrate[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-2*(912*c^2 + 822*c*d*x^3 - 81*d^2*x^6 - 320*Sqrt[c]*(8*c - d*x^3)*Sqrt[c
 + d*x^3]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(243*d^4*(-8*c + d*x^3)*S
qrt[c + d*x^3])
```

3.442.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {948, 109, 27, 163, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^9}{\left(8c-dx^3\right)^2\left(dx^3+c\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{ 109}$$

$$\frac{1}{3}\left(\frac{8x^6}{9d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{\int \frac{cx^3\left(13dx^3+16c\right)}{\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx^3}{9cd^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{8x^6}{9d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{\int \frac{x^3\left(13dx^3+16c\right)}{\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx^3}{9d^2}\right)$$

$$\Big\downarrow \text{ 163}$$

$$\frac{1}{3}\left(\frac{8x^6}{9d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{\frac{320c\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{3d}-\frac{2\left(38c+39dx^3\right)}{3d^2\sqrt{c+dx^3}}}{9d^2}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{8x^6}{9d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{\frac{640c\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{3d^2}-\frac{2\left(38c+39dx^3\right)}{3d^2\sqrt{c+dx^3}}}{9d^2}\right)$$

$$\Big\downarrow 219$$

$$\frac{1}{3}\left(\frac{8x^6}{9d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{\frac{640\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}-\frac{2\left(38c+39dx^3\right)}{3d^2\sqrt{c+dx^3}}}{9d^2}\right)$$

input

```
Int[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output

```
((8*x^6)/(9*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) - ((-2*(38*c + 39*d*x^3))/(
3*d^2*Sqrt[c + d*x^3]) + (640*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]
)/(9*d^2))/(9*d^2))/3
```

3.442.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.442.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{\frac{2\sqrt{d x^3+c}}{3}+\frac{128c\left(\frac{4\sqrt{d x^3+c}}{-d x^3+8c}-\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{243}+\frac{2c}{243\sqrt{d x^3+c}}}{d^4}$
risch	$\frac{2\sqrt{d x^3+c}}{3d^4}+\frac{c\left(\frac{2}{243d\sqrt{d x^3+c}}-\frac{2432\,\mathrm{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{729d\sqrt{c}}+\frac{512c\left(-\frac{\sqrt{d x^3+c}}{c\left(d x^3-8c\right)}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{243d}\right)}{d^3}$
default	$\frac{d\left(\frac{2c}{3d^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2\sqrt{d x^3+c}}{3d^2}\right)-\frac{32c}{3d\sqrt{d x^3+c}}}{d^3}+\frac{128\sqrt{c}\left(-\frac{\mathrm{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)\sqrt{d x^3+c}}{3}+\sqrt{c}\right)}{9d^4\sqrt{d x^3+c}}+\frac{512c\left(-\frac{2}{\sqrt{d x^3+c}}+\frac{\sqrt{d x^3}}{-d x^3}\right.}{24}$
elliptic	$\frac{512c\sqrt{d x^3+c}}{243d^4(-d x^3+8c)}+\frac{2c}{243d^4\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2\sqrt{d x^3+c}}{3d^4}+\frac{320i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-\right.}{}\right.}{\left(-\right.}}}{}\right.}{}$

input
```
int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((d*x^3+c)^(1/2)+64/81*c*(4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-5*arctanh(1/3
*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+1/81*c/(d*x^3+c)^(1/2))/d^4
```

3.442. $\int \frac{x^{11}}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.442.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.45

$$\int \frac{x^{11}}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \left[\frac{2\left(160\,\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,\left(27\,d^2x^6-2\right.\right.}{243\,\left(d^6x^6-7\,cd^5x^3-8\,c^2d^4\right)}\right.$$

input
```
integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[2/243*(160*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^
3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 3*(27*d^2*x^6 - 274*c*d*x^3 - 304*
c^2)*sqrt(d*x^3 + c))/(d^6*x^6 - 7*c*d^5*x^3 - 8*c^2*d^4), 2/243*(320*(d^2
*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c)
+ 3*(27*d^2*x^6 - 274*c*d*x^3 - 304*c^2)*sqrt(d*x^3 + c))/(d^6*x^6 - 7*c*d
^5*x^3 - 8*c^2*d^4)]
```

3.442.6 Sympy [F]

$$\int \frac{x^{11}}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \int \frac{x^{11}}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**11/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.442.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{2\left(160\,\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+81\,\sqrt{dx^3+c}-\frac{3\,\left(85\,\left(dx^3+c\right)c+3\,c^2\right)}{\left(dx^3+c\right)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}c}\right)}{243\,d^4}$$

input
```
integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
2/243*(160*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 81*sqrt(d*x^3 + c) - 3*(85*(d*x^3 + c)*c + 3*c^2)/((d*x^3 + c)
^(3/2) - 9*sqrt(d*x^3 + c)*c))/d^4
```

3.442.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{x^{11}}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{640\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}d^4} + \frac{2\sqrt{dx^3+c}}{3\,d^4} - \frac{2\,(85\,(dx^3+c)c+3\,c^2)}{81\left((dx^3+c)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)d^4}$$

input
```
integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
640/243*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) + 2/3*sqrt(d
*x^3 + c)/d^4 - 2/81*(85*(d*x^3 + c)*c + 3*c^2)/(((d*x^3 + c)^(3/2) - 9*sq
rt(d*x^3 + c)*c)*d^4)
```

3.442.9 Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,d^4} + \frac{320\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\sqrt{c}\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,d^4} + \frac{\sqrt{d\,x^3+c}\left(\frac{176\,c^2}{81\,d^4}+\frac{170\,c\,x^3}{81\,d^3}\right)}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

input
```
int(x^11/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*d^4) + (320*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)
*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(243*d^4) + ((c + d*x^3)^(1/2)*((176*c
^2)/(81*d^4) + (170*c*x^3)/(81*d^3)))/(8*c^2 - d^2*x^6 + 7*c*d*x^3)
```

3.443 $\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.443.1 Optimal result

Integrand size = 27, antiderivative size = 83

$$\begin{aligned}\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = & -\frac{22}{81d^3\sqrt{c+dx^3}} \\ & + \frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{32\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}d^3}\end{aligned}$$

output
```
-32/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3/c^(1/2)-22/81/d^3/(d*x^3+
c)^(1/2)+64/27*c/d^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

3.443.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{2\left(\frac{3(8c+11dx^3)}{(8c-dx^3)\sqrt{c+dx^3}} - \frac{16\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{243d^3}$$

input
```
Integrate[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(2*((3*(8*c + 11*d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) - (16*ArcTanh[Sqr
t[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))/(243*d^3)
```

3.443.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {948, 100, 27, 87, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^6}{(8c-dx^3)^2 (dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{ 100}$$

$$\frac{1}{3}\left(\frac{64c}{9d^3(8c-dx^3)\sqrt{c+dx^3}}-\frac{\int -\frac{3cd(8c-3dx^3)}{(8c-dx^3)(dx^3+c)^{3/2}}dx^3}{9cd^3}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{\int \frac{8c-3dx^3}{(8c-dx^3)(dx^3+c)^{3/2}}dx^3}{3d^2}+\frac{64c}{9d^3(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{3}\left(\frac{-\frac{16}{9}\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3-\frac{22}{9d\sqrt{c+dx^3}}}{3d^2}+\frac{64c}{9d^3(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{-\frac{32\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9d}-\frac{22}{9d\sqrt{c+dx^3}}}{3d^2}+\frac{64c}{9d^3(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{-\frac{32\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27\sqrt{c}d}-\frac{22}{9d\sqrt{c+dx^3}}}{3d^2}+\frac{64c}{9d^3(8c-dx^3)\sqrt{c+dx^3}}\right)$$

input
```
Int[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((64*c)/(9*d^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (-22/(9*d*Sqrt[c + d*x^3])
 - (32*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(27*Sqrt[c]*d))/(3*d^2))/3
```

3.443.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.443. $\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.443.4 Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{\frac{64\sqrt{d\,x^{3}+c}}{243\left(-d\,x^{3}+8c\right)}-\frac{32\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{243\sqrt{c}}-\frac{2}{243\sqrt{d\,x^{3}+c}}}{d^{3}}$
default	$-\frac{2}{3d^{3}\sqrt{d\,x^{3}+c}}+\frac{-\frac{32\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{d\,x^{3}+c}}{81}+\frac{32\sqrt{c}}{27}}{d^{3}\sqrt{c}\,\sqrt{d\,x^{3}+c}}+\frac{-\frac{128}{243\sqrt{d\,x^{3}+c}}+\frac{64\sqrt{d\,x^{3}+c}}{243\left(-d\,x^{3}+8c\right)}+\frac{64\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{243\sqrt{c}}}{d^{3}}$
elliptic	$\frac{64\sqrt{d\,x^{3}+c}}{243d^{3}(-d\,x^{3}+8c)}-\frac{2}{243d^{3}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{16i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$

input
```
int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/243*(32*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-16*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1
/2))/c^(1/2)-1/(d*x^3+c)^(1/2))/d^3
```

3.443.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.69

$$\int \frac{x^8}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \left[\frac{2\left(8\,(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\,(11\,cdx^3+8\,c\right.}{243\,(cd^5x^6-7\,c^2d^4x^3-8\,c^3d^3)}\right.$$

input
```
integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[2/243*(8*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3
+ c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 3*(11*c*d*x^3 + 8*c^2)*sqrt(d*x^3 +
c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3), 2/243*(16*(d^2*x^6 - 7*c*d*x^
3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*(11*c*d*x^3
 + 8*c^2)*sqrt(d*x^3 + c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)]
```

3.443.6 Sympy [F]

$$\int \frac{x^8}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^8}{(-8c+dx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**8/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.443.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \frac{x^8}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{2\left(\frac{8\,\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)}{\sqrt{c}}-\frac{3\,(11\,dx^3+8\,c)}{(dx^3+c)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}c}\right)}{243\,d^3}$$

input
```
integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
2/243*(8*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/
sqrt(c) - 3*(11*d*x^3 + 8*c)/((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c))/d^
3
```

3.443.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{32\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}d^3} - \frac{2\,(11\,dx^3+8\,c)}{81\left((dx^3+c)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)d^3}$$

input
```
integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
32/243*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/81*(11*d*x^
3 + 8*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*d^3)
```

3.443.9 Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{16\ln\left(\frac{10\,c+d\,x^3-6\sqrt{c}\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\sqrt{c}\,d^3} + \frac{\sqrt{d\,x^3+c}\left(\frac{16\,c}{81\,d^3}+\frac{22\,x^3}{81\,d^2}\right)}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

input
```
int(x^8/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
(16*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(243*
c^(1/2)*d^3) + ((c + d*x^3)^(1/2)*((16*c)/(81*d^3) + (22*x^3)/(81*d^2)))/(
8*c^2 - d^2*x^6 + 7*c*d*x^3)
```

3.444 $\int \frac{x^5}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.444.1 Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{x^5}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=-\frac{2}{81cd^2\sqrt{c+dx^3}}+\frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}$$

output
```
2/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)/d^2-2/81/c/d^2/(d*x^3+c
)^(1/2)+8/27/d^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

3.444.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{2\left(\frac{3\sqrt{c}(4c+dx^3)}{(8c-dx^3)\sqrt{c+dx^3}}+\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{243c^{3/2}d^2}$$

input
```
Integrate[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(2*((3*Sqrt[c]*(4*c + d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) + ArcTanh[Sq
rt[c + d*x^3]/(3*Sqrt[c])]))/(243*c^(3/2)*d^2)
```

3.444.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {948, 87, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(8c-dx^3)^2 (dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{(8c-dx^3)(dx^3+c)^{3/2}}dx^3}{3d}+\frac{8}{9d^2(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(\frac{\frac{\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9c}-\frac{2}{9cd\sqrt{c+dx^3}}}{3d}+\frac{8}{9d^2(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\frac{2\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9cd}-\frac{2}{9cd\sqrt{c+dx^3}}}{3d}+\frac{8}{9d^2(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27c^{3/2}d}-\frac{2}{9cd\sqrt{c+dx^3}}}{3d}+\frac{8}{9d^2(8c-dx^3)\sqrt{c+dx^3}}\right)$$

input
```
Int[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(8/(9*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (-2/(9*c*d*Sqrt[c + d*x^3]) + (
2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(27*c^(3/2)*d))/(3*d))/3
```

3.444.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.444. $\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.444.4 Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{\frac{8\sqrt{d\,x^{3}+c}}{243\left(-d\,x^{3}+8c\right)}+\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{243\sqrt{c}}+\frac{2}{243\sqrt{d\,x^{3}+c}}}{c\,d^{2}}$
default	$\frac{-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{d\,x^{3}+c}}{81}+\frac{2\sqrt{c}}{27}}{d^{2}\sqrt{d\,x^{3}+c}\,c^{\frac{3}{2}}}+\frac{-\frac{16}{243\sqrt{d\,x^{3}+c}}+\frac{8\sqrt{d\,x^{3}+c}}{243\left(-d\,x^{3}+8c\right)}+\frac{8\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{243\sqrt{c}}}{c\,d^{2}}$
elliptic	$\frac{8\sqrt{d\,x^{3}+c}}{243d^{2}c(-d\,x^{3}+8c)}+\frac{2}{243d^{2}c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)}{d}\right.}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$

input
```
int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/243/c*(4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2
))/c^(1/2)+1/(d*x^3+c)^(1/2))/d^2
```

3.444.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.62

$$\int \frac{x^5}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \left[\frac{(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,(cdx^3+4\,c^2)\sqrt{dx^3}}{243\,(c^2d^4x^6-7\,c^3d^3x^3-8\,c^4d^2)}\right.$$

$$\left.-\frac{2\left((d^2x^6-7\,cdx^3-8\,c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\,(cdx^3+4\,c^2)\sqrt{dx^3+c}\right)}{243\,(c^2d^4x^6-7\,c^3d^3x^3-8\,c^4d^2)}\right]$$

input
```
integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 +
c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(
c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2), -2/243*((d^2*x^6 - 7*c*d*x^3 - 8
*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(c*d*x^3 + 4*c^2
)*sqrt(d*x^3 + c))/(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)]
```

3.444.6 Sympy [F]

$$\int \frac{x^5}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^5}{(-8c+dx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**5/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.444.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = -\frac{\frac{6\,(dx^3+4\,c)}{(dx^3+c)^{\frac{3}{2}}\,c-9\,\sqrt{dx^3+c}c^2}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{243\,d^2}$$

input
```
integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/243*(6*(d*x^3 + 4*c)/((d*x^3 + c)^(3/2)*c - 9*sqrt(d*x^3 + c)*c^2) + lo
g((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(3/2))/d^
2
```

3.444.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = -\frac{2\left(\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}cd}+\frac{3\left(dx^3+4c\right)}{\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)cd}\right)}{243\,d}$$

input
```
integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-2/243*(arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) + 3*(d*x^3 + 4
*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c*d))/d
```

3.444.9 Mupad [B] (verification not implemented)

Time = 8.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13

$$\int \frac{x^5}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\left(\frac{8}{81\,d^2}+\frac{2\,x^3}{81\,c\,d}\right)\sqrt{d\,x^3+c}}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}+\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,c^{3/2}\,d^2}$$

input
```
int(x^5/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
((8/(81*d^2) + (2*x^3)/(81*c*d))*(c + d*x^3)^(1/2))/(8*c^2 - d^2*x^6 + 7*c
*d*x^3) + log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/
(243*c^(3/2)*d^2)
```

3.445 $\int \frac{x^2}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.445.1 Optimal result

Integrand size = 27, antiderivative size = 88

$$\int \frac{x^2}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=-\frac{1}{81c^2d\sqrt{c+dx^3}}+\frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}$$

output
```
1/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)/d-1/81/c^2/d/(d*x^3+c)^
(1/2)+1/27/c/d/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

3.445.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{\frac{3\sqrt{c}\left(-5c+dx^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}+\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}$$

input
```
Integrate[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((3*Sqrt[c]*(-5*c + d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) + ArcTanh[Sqrt
[c + d*x^3]/(3*Sqrt[c])])/(243*c^(5/2)*d)
```

3.445.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {946, 52, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(8c-dx^3)^2 (c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3}\int \frac{1}{(8c-dx^3)^2 (dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \; 52$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{(8c-dx^3)(dx^3+c)^{3/2}}dx^3}{6c}+\frac{1}{9cd(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{3}\left(\frac{\frac{\int \frac{1}{(8c-dx^3)\sqrt{dx^3+c}}dx^3}{9c}-\frac{2}{9cd\sqrt{c+dx^3}}}{6c}+\frac{1}{9cd(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{\frac{2\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9cd}-\frac{2}{9cd\sqrt{c+dx^3}}}{6c}+\frac{1}{9cd(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{27c^{3/2}d}-\frac{2}{9cd\sqrt{c+dx^3}}}{6c}+\frac{1}{9cd(8c-dx^3)\sqrt{c+dx^3}}\right)$$

input
```
Int[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(1/(9*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (-2/(9*c*d*Sqrt[c + d*x^3]) + (
2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(27*c^(3/2)*d))/(6*c))/3
```

3.445.3.1 Defintions of rubi rules used

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.445. $\int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.445.4 Maple [A] (verified)

Time = 4.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

method	result
default	$\frac{-\frac{2}{\sqrt{d x^3+c}}+\frac{\sqrt{d x^3+c}}{-d x^3+8c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{\sqrt{c}}}{243c^2 d}$
pseudoelliptic	$\frac{-\frac{2}{\sqrt{d x^3+c}}+\frac{\sqrt{d x^3+c}}{-d x^3+8c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{\sqrt{c}}}{243c^2 d}$
elliptic	$\frac{\sqrt{d x^3+c}}{243c^2 d(-d x^3+8c)}-\frac{2}{243d c^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)}{d}\right.}{\left(-c d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$

input
```
int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/243/c^2*(-2/(d*x^3+c)^(1/2)+(d*x^3+c)^(1/2)/(-d*x^3+8*c)+arctanh(1/3*(d*
x^3+c)^(1/2)/c^(1/2))/c^(1/2))/d
```

3.445.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.49

$$\int \frac{x^2}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \left[\frac{(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,(cdx^3-5\,c^2)\sqrt{dx^3}}{486\,(c^3d^3x^6-7\,c^4d^2x^3-8\,c^5d)}\right.$$

$$\left.-\frac{(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\,(cdx^3-5\,c^2)\sqrt{dx^3+c}}{243\,(c^3d^3x^6-7\,c^4d^2x^3-8\,c^5d)}\right]$$

input
```
integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/486*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 +
c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 - 5*c^2)*sqrt(d*x^3 + c))/(
c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d), -1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c
^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(c*d*x^3 - 5*c^2)*
sqrt(d*x^3 + c))/(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)]
```

3.445.6 Sympy [F]

$$\int \frac{x^2}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx=\int \frac{x^2}{(-8c+dx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**2/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.445.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx=-\frac{\frac{6\,(dx^3-5\,c)}{(dx^3+c)^{\frac{3}{2}}c^2-9\,\sqrt{dx^3+c}c^3}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{486\,d}$$

input
```
integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/486*(6*(d*x^3 - 5*c)/((d*x^3 + c)^(3/2)*c^2 - 9*sqrt(d*x^3 + c)*c^3) +
log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(5/2))/
d
```

3.445.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}c^2d} - \frac{dx^3-5c}{81\left((dx^3+c)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)c^2d}$$

input
```
integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-1/243*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2*d) - 1/81*(d*x^3
 - 5*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^2*d)
```

3.445.9 Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{486\,c^{5/2}\,d} - \frac{\left(\frac{5}{81\,c\,d}-\frac{x^3}{81\,c^2}\right)\sqrt{d\,x^3+c}}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

input
```
int(x^2/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(486*c^(5/
2)*d) - ((5/(81*c*d) - x^3/(81*c^2))*(c + d*x^3)^(1/2))/(8*c^2 - d^2*x^6 +
 7*c*d*x^3)
```

3.446 $\int \frac{1}{x(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.446.1 Optimal result

Integrand size = 27, antiderivative size = 106

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{5}{648c^3\sqrt{c+dx^3}}+\frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

output
```
7/7776*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-1/96*arctanh((d*x^3+c)
^(1/2)/c^(1/2))/c^(7/2)+5/648/c^3/(d*x^3+c)^(1/2)+1/216/c^2/(-d*x^3+8*c)/(
d*x^3+c)^(1/2)
```

3.446.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{\frac{12\sqrt{c}\left(43c-5dx^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}+7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-81\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{7776c^{7/2}}$$

input
```
Integrate[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((12*Sqrt[c]*(43*c - 5*d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) + 7*ArcTanh
[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 81*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(7776
*c^(7/2))
```

3.446.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 114, 27, 169, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(8c-dx^3\right)^2\left(dx^3+c\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(\frac{1}{72c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{\int -\frac{3d\left(dx^3+6c\right)}{2x^3\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx^3}{72c^2d}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{dx^3+6c}{x^3\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx^3}{48c^2}+\frac{1}{72c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{3}\left(\frac{\frac{2\int \frac{cd\left(54c-5dx^3\right)}{2x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c^2d}+\frac{10}{9c\sqrt{c+dx^3}}}{48c^2}+\frac{1}{72c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\frac{\int \frac{54c-5dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{10}{9c\sqrt{c+dx^3}}}{48c^2}+\frac{1}{72c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{\frac{\frac{27}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{7}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{10}{9c\sqrt{c+dx^3}}}{48c^2}+\frac{1}{72c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{3}\left(\frac{\frac{\frac{7}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{27\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{9c}+\frac{10}{9c\sqrt{c+dx^3}}}{48c^2}+\frac{1}{72c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow 219$$

$$\frac{1}{3}\left(\frac{\frac{\frac{27\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{9c}+\frac{10}{9c\sqrt{c+dx^3}}}{48c^2}+\frac{1}{72c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{3}\left(\frac{\frac{\frac{7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{9c}+\frac{10}{9c\sqrt{c+dx^3}}}{48c^2}+\frac{1}{72c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

input
```
Int[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(1/(72*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (10/(9*c*Sqrt[c + d*x^3]) + ((
7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*Sqrt[c]) - (27*ArcTanh[Sqrt[c +
 d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(9*c))/(48*c^2))/3
```

3.446.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.446. $\int \frac{1}{x(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.446.4 Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{96c^{\frac{7}{2}}}+\frac{\frac{7\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\left(d\,x^{3}-8c\right)}{\sqrt{c}}-4\sqrt{d\,x^{3}+c}}{7776(d\,x^{3}-8c)c^{3}}+\frac{2}{243c^{3}\sqrt{d\,x^{3}+c}}$
default	$\frac{\frac{2}{3c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{64c^{2}}+\frac{-\frac{2}{\sqrt{d\,x^{3}+c}}+\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{\sqrt{c}}}{1944c^{3}}-\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{d\,x^{3}+c}}{3}+\sqrt{c}}{864c^{\frac{7}{2}}\sqrt{d\,x^{3}+c}}$
elliptic	Expression too large to display

input
```
int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+1/7776*(7*arctanh(1/3*(d*x^
3+c)^(1/2)/c^(1/2))/c^(1/2)*(d*x^3-8*c)-4*(d*x^3+c)^(1/2))/(d*x^3-8*c)/c^3
+2/243/c^3/(d*x^3+c)^(1/2)
```

3.446.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.98

$$\int \frac{1}{x\left(8c-dx^{3}\right)^{2}\left(c+dx^{3}\right)^{3/2}}\,dx=\left[\frac{7\left(d^{2}x^{6}-7\,cdx^{3}-8\,c^{2}\right)\sqrt{c}\log\left(\frac{dx^{3}+6\,\sqrt{dx^{3}+c}\sqrt{c}+10\,c}{dx^{3}-8\,c}\right)+81\left(d^{2}x^{6}-7\,cdx^{3}\right.}{15552\left(c^{4}d^{2}x^{6}-7\right.}\right.$$

input
```
integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/15552*(7*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^
3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 81*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*s
qrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(5*c*d*x^3
- 43*c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^6 - 7*c^5*d*x^3 - 8*c^6), 1/7776*(81
*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c)
 - 7*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqr
t(-c)/c) + 12*(5*c*d*x^3 - 43*c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^6 - 7*c^5*d
*x^3 - 8*c^6)]
```

3.446.6 Sympy [F]

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.446.7 Maxima [F]

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x}\,dx$$

input
```
integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x), x)
```

3.446.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}c^3}-\frac{7\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{7776\sqrt{-c}c^3}+\frac{5\,dx^3-43\,c}{648\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)c^3}$$

input
```
integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 7/7776*arctan(1/3*s
qrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 1/648*(5*d*x^3 - 43*c)/(((d*x^3
+ c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^3)
```

3.446.9 Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=-\frac{\frac{5\,\left(d\,x^3+c\right)}{216\,c^3}-\frac{2}{9\,c^2}}{27\,c\,\sqrt{d\,x^3+c}-3\,{\left(d\,x^3+c\right)}^{3/2}}+\frac{\left(\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)\,7{}\mathrm{i}}{81}\right)\,1{}\mathrm{i}}{96\,\sqrt{c^7}}$$

input
```
int(1/(x*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
((atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2))*1i - (atanh((c^3*(c + d*x^3)^
(1/2))/(3*(c^7)^(1/2)))*7i)/81)*1i)/(96*(c^7)^(1/2)) - ((5*(c + d*x^3))/(2
16*c^3) - 2/(9*c^2))/(27*c*(c + d*x^3)^(1/2) - 3*(c + d*x^3)^(3/2))
```

3.447 $\int \frac{1}{x^4(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.447.1 Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{1}{x^4(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{35d}{2592c^4\sqrt{c+dx^3}}+\frac{5d}{864c^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3(8c-dx^3)\sqrt{c+dx^3}}+\frac{5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}}+\frac{5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}}$$

output
```
5/31104*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)+5/384*d*arctanh((d*
x^3+c)^(1/2)/c^(1/2))/c^(9/2)-35/2592*d/c^4/(d*x^3+c)^(1/2)+5/864*d/c^3/(-
d*x^3+8*c)/(d*x^3+c)^(1/2)-1/24/c^2/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

3.447.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{-\frac{12\sqrt{c}(108c^2+265cdx^3-35d^2x^6)}{x^3(8c-dx^3)\sqrt{c+dx^3}}+5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+405d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right.}{31104c^{9/2}}$$

input
```
Integrate[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((-12*Sqrt[c]*(108*c^2 + 265*c*d*x^3 - 35*d^2*x^6))/(x^3*(8*c - d*x^3)*Sqr
t[c + d*x^3]) + 5*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + 405*d*ArcTanh[S
qrt[c + d*x^3]/Sqrt[c]])/(31104*c^(9/2))
```

3.447.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {948, 114, 27, 168, 27, 169, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^6 (8c-dx^3)^2 (dx^3+c)^{3/2}} dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3} \left(-\frac{\int \frac{5d(4c-dx^3)}{2x^3(8c-dx^3)^2(dx^3+c)^{3/2}}dx^3}{8c^2} - \frac{1}{8c^2x^3(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{5d\int \frac{4c-dx^3}{x^3(8c-dx^3)^2(dx^3+c)^{3/2}}dx^3}{16c^2} - \frac{1}{8c^2x^3(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3} \left(-\frac{5d\left(-\frac{\int -\frac{6cd\left(6c-dx^3\right)}{x^3\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx^3}{72c^2d} - \frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2} - \frac{1}{8c^2x^3(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{5d\left(\frac{\int \frac{6c-dx^3}{x^3\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx^3}{12c} - \frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2} - \frac{1}{8c^2x^3(8c-dx^3)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{3}\left(-\frac{5d\left(\frac{\frac{2\int \frac{cd\left(54c-7dx^3\right)}{2x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c^2d}+\frac{14}{9c\sqrt{c+dx^3}}}{12c}-\frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow\ 27$$

$$\frac{1}{3}\left(-\frac{5d\left(\frac{\frac{\int \frac{54c-7dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{14}{9c\sqrt{c+dx^3}}}{12c}-\frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow\ 174$$

$$\frac{1}{3}\left(-\frac{5d\left(\frac{\frac{\frac{27}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3-\frac{1}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{14}{9c\sqrt{c+dx^3}}}{12c}-\frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow\ 73$$

$$\frac{1}{3}\left(-\frac{5d\left(\frac{\frac{\frac{27\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{1}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}}{9c}+\frac{14}{9c\sqrt{c+dx^3}}}{12c}-\frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow\ 219$$

3.447. $\int \frac{1}{x^4\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

$$\frac{1}{3}\left(-\frac{5d\left(\frac{\frac{\frac{27\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{9c}+\frac{14}{9c\sqrt{c+dx^3}}}{12c}-\frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{3}\left(-\frac{5d\left(\frac{\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{9c}+\frac{14}{9c\sqrt{c+dx^3}}}{12c}-\frac{1}{18c(8c-dx^3)\sqrt{c+dx^3}}\right)}{16c^2}-\frac{1}{8c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

input
```
Int[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-1/8*1/(c^2*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) - (5*d*(-1/18*1/(c*(8*c -
d*x^3)*Sqrt[c + d*x^3]) + (14/(9*c*Sqrt[c + d*x^3]) + (-1/6*ArcTanh[Sqrt[c
 + d*x^3]/(3*Sqrt[c])]/Sqrt[c] - (27*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*
Sqrt[c]))/(9*c))/(12*c)))/(16*c^2))/3
```

3.447. $\int \frac{1}{x^4(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.447.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.447.4 Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{d\left(-\frac{-5\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d\,x^{3}+2\sqrt{d\,x^{3}+c}\,\sqrt{c}}{128d\,x^{3}c^{\frac{9}{2}}}+\frac{\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{2\sqrt{c}}}{5184c^{4}}-\frac{2}{81c^{4}\sqrt{d\,x^{3}+c}}\right)}{3}$
risch	$-\frac{\sqrt{d\,x^{3}+c}}{192c^{4}x^{3}}-\frac{d\left(-\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}+\frac{256}{243\sqrt{d\,x^{3}+c}}-\frac{13\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{729\sqrt{c}}-\frac{2c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{243}\right)}{128c^{4}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3c^{2}x^{3}}-\frac{2d}{3c^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{64c^{2}}+\frac{d\left(\frac{2}{3c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{256c^{3}}+\frac{d\left(-\frac{2}{\sqrt{d\,x^{3}+c}}+\frac{\sqrt{d\,x^{3}+}}{-d\,x^{3}+}\right.}{155}$
elliptic	Expression too large to display

input
```
int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*d*(-1/128*(-5*arctanh((d*x^3+c)^(1/2)/c^(1/2))*d*x^3+2*(d*x^3+c)^(1/2)
*c^(1/2))/d/x^3/c^(9/2)+1/5184*((d*x^3+c)^(1/2)/(-d*x^3+8*c)+5/2*arctanh(1
/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))/c^4-2/81/c^4/(d*x^3+c)^(1/2))
```

3.447.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.57

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \left[\frac{5\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+405\left(d^3x^9-\right.}{62208}\right.$$

$$-\frac{405\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+5\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+}}{3}\right.}{31104\left(c^5d^2x^9-7\,c^6dx^6-8\,c^7x^3\right)}$$

input
```
integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/62208*(5*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x^3)*sqrt(c)*log((d*x^3 + 6*s
qrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 405*(d^3*x^9 - 7*c*d^2*x^6
 - 8*c^2*d*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3)
 - 24*(35*c*d^2*x^6 - 265*c^2*d*x^3 - 108*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x
^9 - 7*c^6*d*x^6 - 8*c^7*x^3), -1/31104*(405*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^
2*d*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 5*(d^3*x^9 - 7*c*d^
2*x^6 - 8*c^2*d*x^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*
(35*c*d^2*x^6 - 265*c^2*d*x^3 - 108*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x^9 - 7
*c^6*d*x^6 - 8*c^7*x^3)]
```

3.447.6 Sympy [F]

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \int \frac{1}{x^4\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**4*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.447.7 Maxima [F]

$$\int \frac{1}{x^4 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2 x^4}\,dx$$

input
```
integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^4), x)
```

3.447.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = -\frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}c^4} - \frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{31104\,\sqrt{-c}c^4} - \frac{35\,\left(dx^3+c\right)^2 d - 335\,\left(dx^3+c\right)cd + 192\,c^2 d}{2592\,\left(\left(dx^3+c\right)^{\frac{5}{2}} - 10\,\left(dx^3+c\right)^{\frac{3}{2}}c + 9\,\sqrt{dx^3+c}c^2\right)c^4}$$

input
```
integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-5/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 5/31104*d*arcta
n(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/2592*(35*(d*x^3 + c)^2*
d - 335*(d*x^3 + c)*c*d + 192*c^2*d)/(((d*x^3 + c)^(5/2) - 10*(d*x^3 + c)^
(3/2)*c + 9*sqrt(d*x^3 + c)*c^2)*c^4)
```

3.447.9 Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = -\frac{\frac{2\,d}{9\,c^2} + \frac{35\,d\,\left(d\,x^3+c\right)^2}{864\,c^4} - \frac{335\,d\,\left(d\,x^3+c\right)}{864\,c^3}}{3\,\left(d\,x^3+c\right)^{5/2} - 30\,c\,\left(d\,x^3+c\right)^{3/2} + 27\,c^2\,\sqrt{d\,x^3+c}} - \frac{d\,\left(\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)\,1\mathrm{i} + \frac{\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)\,1\mathrm{i}}{81}\right)\,5\mathrm{i}}{384\,\sqrt{c^9}}$$

input
```
int(1/(x^4*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
- ((2*d)/(9*c^2) + (35*d*(c + d*x^3)^2)/(864*c^4) - (335*d*(c + d*x^3))/(8
64*c^3))/(3*(c + d*x^3)^(5/2) - 30*c*(c + d*x^3)^(3/2) + 27*c^2*(c + d*x^3
)^(1/2)) - (d*(atanh((c^4*(c + d*x^3)^(1/2))/(c^9)^(1/2))*1i + (atanh((c^4
*(c + d*x^3)^(1/2))/(3*(c^9)^(1/2)))*1i)/81)*5i)/(384*(c^9)^(1/2))
```

3.447. $\int \frac{1}{x^4(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.448 $\int \frac{1}{x^7\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.448.1 Optimal result

Integrand size = 27, antiderivative size = 185

$$\begin{aligned}\int \frac{1}{x^7\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx &= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} \\ &- \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} \\ &+ \frac{17d}{384c^3x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{13d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{497664c^{11/2}} - \frac{33d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{11/2}}\end{aligned}$$

output
```
13/497664*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/2)-33/2048*d^2*ar
ctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(11/2)+665/41472*d^2/c^5/(d*x^3+c)^(1/2)-
71/13824*d^2/c^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2)-1/48/c^2/x^6/(-d*x^3+8*c)/(d
*x^3+c)^(1/2)+17/384*d/c^3/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

3.448.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^7\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\frac{12\sqrt{c}\left(864c^3-1836c^2dx^3-5107cd^2x^6+665d^3x^9\right)}{x^6\left(-8c+dx^3\right)\sqrt{c+dx^3}} + 13d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 8019d^2\mathrm{ar}}{497664c^{11/2}}$$

input
```
Integrate[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((12*Sqrt[c]*(864*c^3 - 1836*c^2*d*x^3 - 5107*c*d^2*x^6 + 665*d^3*x^9))/(x
^6*(-8*c + d*x^3)*Sqrt[c + d*x^3]) + 13*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqr
t[c])] - 8019*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(497664*c^(11/2))
```

3.448.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {948, 114, 27, 168, 27, 168, 27, 169, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^9 \left(8c-dx^3\right)^2 \left(dx^3+c\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{d\left(34c-7dx^3\right)}{2x^6\left(8c-dx^3\right)^2\left(dx^3+c\right)^{3/2}}dx^3}{16c^2}-\frac{1}{16c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\int \frac{34c-7dx^3}{x^6\left(8c-dx^3\right)^2\left(dx^3+c\right)^{3/2}}dx^3}{32c^2}-\frac{1}{16c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{\int \frac{cd\left(396c-85dx^3\right)}{x^3\left(8c-dx^3\right)^2\left(dx^3+c\right)^{3/2}}dx^3}{8c^2}-\frac{17}{4cx^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\int \frac{396c-85dx^3}{x^3\left(8c-dx^3\right)^2\left(dx^3+c\right)^{3/2}}dx^3}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(-\frac{\int -\frac{6cd\left(594c-71dx^3\right)}{x^3\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx^3}{72c^2d}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\int \frac{594c-71dx^3}{x^3\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx^3}{12c}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{169}$$

3.448. $\int \frac{1}{x^7(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{2\int \frac{cd\left(5346c-665dx^3\right)}{2x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c^2d}+\frac{1330}{9c\sqrt{c+dx^3}}}{12c}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\left(8c-dx^3\right)\sqrt{c+}}\right.$$

$$\Big\downarrow 27$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{\int \frac{5346c-665dx^3}{x^3\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{1330}{9c\sqrt{c+dx^3}}}{12c}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{1}{16c^2x^6\left(8c-dx^3\right)\sqrt{c+}}\right.$$

$$\Big\downarrow 174$$

3.448. $\int \frac{1}{x^7\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{\frac{2673}{4}\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3+\frac{13}{4}d\int \frac{1}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx^3}{9c}+\frac{1330}{9c\sqrt{c+dx^3}}}{12c}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{}{16c^2x}\right.$$

$$\Big\downarrow \ 73$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{\frac{13}{2}\int \frac{1}{9c-x^6}d\sqrt{dx^3+c}+\frac{2673\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}}{9c}+\frac{1330}{9c\sqrt{c+dx^3}}}{12c}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{}{16c^2x^6\,(8}\right.$$

$$\Big\downarrow \ 219$$

3.448. $\int \frac{1}{x^7\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{\frac{2673\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{2d}+\frac{13\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{9c}+\frac{1330}{9c\sqrt{c+dx^3}}}{12c}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{}{16c^2x^6}\right.$$

$$\Big\downarrow 221$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{d\left(\frac{\frac{\frac{13\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{6\sqrt{c}}-\frac{2673\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2\sqrt{c}}}{9c}+\frac{1330}{9c\sqrt{c+dx^3}}}{12c}-\frac{71}{18c\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{8c}-\frac{17}{4cx^3(8c-dx^3)\sqrt{c+dx^3}}\right)}{32c^2}-\frac{}{16c^2}\right.$$

input
```
Int[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

3.448. $\int \frac{1}{x^7(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

output
```
(-1/16*1/(c^2*x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]) - (d*(-17/(4*c*x^3*(8*c -
 d*x^3)*Sqrt[c + d*x^3]) - (d*(-71/(18*c*(8*c - d*x^3)*Sqrt[c + d*x^3]) +
(1330/(9*c*Sqrt[c + d*x^3]) + ((13*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(
6*Sqrt[c]) - (2673*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2*Sqrt[c]))/(9*c))/(
12*c)))/(8*c)))/(32*c^2))/3
```

3.448.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.448. $\int \frac{1}{x^7(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.448.4 Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(-3d\,x^{3}+c\right)}{384c^{5}x^{6}}+\frac{d^{2}\left(-\frac{33\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{8\sqrt{c}}+\frac{512}{243\sqrt{d\,x^{3}+c}}+\frac{35\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{5832\sqrt{c}}+\frac{c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\sqrt{\ }\right.}{3c}\right.}{486}\right.}{256c^{5}}$
pseudoelliptic	$\frac{d^{2}\left(-\frac{99\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d^{2}x^{6}-48d\,x^{3}\sqrt{d\,x^{3}+c}\,\sqrt{c}+16\sqrt{d\,x^{3}+c}\,c^{\frac{3}{2}}}{2048d^{2}x^{6}c^{\frac{11}{2}}}+\frac{\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{13\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{4\sqrt{c}}}{41472c^{5}}+\frac{2}{81c^{5}\sqrt{d\,x^{3}+c}}\right)}{3}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{6c^{2}x^{6}}+\frac{7d\sqrt{d\,x^{3}+c}}{12c^{3}x^{3}}+\frac{2d^{2}}{3c^{3}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{5d^{2}\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4c^{\frac{7}{2}}}}{64c^{2}}+\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{3c^{2}x^{3}}-\frac{2d}{3c^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{d\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)}{256c^{3}}$
elliptic	Expression too large to display

input
```
int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384*(d*x^3+c)^(1/2)*(-3*d*x^3+c)/c^5/x^6+1/256/c^5*d^2*(-33/8*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+512/243/(d*x^3+c)^(1/2)+35/5832*arctanh(1/
3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+1/486*c*(-(d*x^3+c)^(1/2)/c/(d*x^3-8*c)
+1/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)))
```

3.448.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.15

$$\int \frac{1}{x^{7}\left(8c-dx^{3}\right)^{2}\left(c+dx^{3}\right)^{3/2}}\,dx=\left[\frac{13\left(d^{4}x^{12}-7\,cd^{3}x^{9}-8\,c^{2}d^{2}x^{6}\right)\sqrt{c}\log\left(\frac{dx^{3}+6\sqrt{dx^{3}+c}\sqrt{c}+10\,c}{dx^{3}-8\,c}\right)+8019\left(d^{4}\right.}{}\right.$$

input
```
integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output

```
[1/995328*(13*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(c)*log((d*x^3
+ 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 8019*(d^4*x^12 - 7*c*
d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) +
2*c)/x^3) + 24*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 - 1836*c^3*d*x^3 + 864*c^
4)*sqrt(d*x^3 + c))/(c^6*d^2*x^12 - 7*c^7*d*x^9 - 8*c^8*x^6), 1/497664*(80
19*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c
)*sqrt(-c)/c) - 13*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(-c)*arcta
n(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 -
 1836*c^3*d*x^3 + 864*c^4)*sqrt(d*x^3 + c))/(c^6*d^2*x^12 - 7*c^7*d*x^9 -
8*c^8*x^6)]
```

3.448.6 Sympy [F]

$$\int \frac{1}{x^7\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^7\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output

```
Integral(1/(x**7*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.448.7 Maxima [F]

$$\int \frac{1}{x^7\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^7}\,dx$$

input

```
integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output

```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^7), x)
```

3.448.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^7 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{33\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\sqrt{-c}c^5} - \frac{13\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{497664\sqrt{-c}c^5} + \frac{341\,(dx^3+c)d^2-3072\,cd^2}{41472\left((dx^3+c)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)c^5} + \frac{3\,(dx^3+c)^{\frac{3}{2}}d^2-4\sqrt{dx^3+c}cd^2}{384\,c^5d^2x^6}$$

input
```
integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
33/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) - 13/497664*d^
2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) + 1/41472*(341*(d*x^
3 + c)*d^2 - 3072*c*d^2)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^5) +
 1/384*(3*(d*x^3 + c)^(3/2)*d^2 - 4*sqrt(d*x^3 + c)*c*d^2)/(c^5*d^2*x^6)
```

3.448.9 Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^7 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{\frac{2\,d^2}{9\,c^2}-\frac{10373\,d^2\,\left(d\,x^3+c\right)}{13824\,c^3}+\frac{3551\,d^2\,{\left(d\,x^3+c\right)}^2}{6912\,c^4}-\frac{665\,d^2\,{\left(d\,x^3+c\right)}^3}{13824\,c^5}}{33\,c\,{\left(d\,x^3+c\right)}^{5/2}-3\,{\left(d\,x^3+c\right)}^{7/2}+27\,c^3\,\sqrt{d\,x^3+c}-57\,c^2\,{\left(d\,x^3+c\right)}^{3/2}} + \frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^5\,\sqrt{d\,x^3+c}}{\sqrt{c^{11}}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^5\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^{11}}}\right)\,13{}\mathrm{i}}{8019}\right)\,33{}\mathrm{i}}{2048\,\sqrt{c^{11}}}$$

input
```
int(1/(x^7*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
((2*d^2)/(9*c^2) - (10373*d^2*(c + d*x^3))/(13824*c^3) + (3551*d^2*(c + d*
x^3)^2)/(6912*c^4) - (665*d^2*(c + d*x^3)^3)/(13824*c^5))/(33*c*(c + d*x^3
)^(5/2) - 3*(c + d*x^3)^(7/2) + 27*c^3*(c + d*x^3)^(1/2) - 57*c^2*(c + d*x
^3)^(3/2)) + (d^2*(atanh((c^5*(c + d*x^3)^(1/2))/(c^11)^(1/2))*1i - (atanh
((c^5*(c + d*x^3)^(1/2))/(3*(c^11)^(1/2)))*13i)/8019)*33i)/(2048*(c^11)^(1
/2))
```

3.449 $\int \frac{x^7}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.449.1 Optimal result

Integrand size = 27, antiderivative size = 668

$$
\begin{aligned}
\int \frac{x^7}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx &= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}}+\frac{8x^2}{27d^2\,(8c-dx^3)\,\sqrt{c+dx^3}} \\
&+\frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{4\arctan\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{5/6}d^{8/3}} \\
&-\frac{4\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{5/6}d^{8/3}}+\frac{4\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/6}d^{8/3}} \\
&-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\ 3^{3/4}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{81\sqrt[4]{3}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

output
```
-4/243*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/
d^(8/3)+4/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(8/3)+4/243*a
rctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(8/3)
*3^(1/2)-2/81*x^2/c/d^2/(d*x^3+c)^(1/2)+8/27*x^2/d^2/(-d*x^3+8*c)/(d*x^3+c
)^(1/2)+2/81*(d*x^3+c)^(1/2)/c/d^(8/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+2/2
43*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+
d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(2/3)/d^(8
/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2)))^2)^(1/2)-1/81*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3
^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2)))^2)^(1/2)*3^(1/4)/c^(2/3)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+
d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.449.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.25

$$\int \frac{x^7}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{80cx^2(4c+dx^3)+40cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\right.}{3240c^2d^2\,(8c-dx^3)}$$

input
```
Integrate[x^7/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(80*c*x^2*(4*c + d*x^3) + 40*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appe
llF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c
)])/(3240*c^2*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

3.449.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {970, 27, 1049, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 970$$

$$\frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}}-\frac{\int \frac{cx(7dx^3+16c)}{(8c-dx^3)(dx^3+c)^{3/2}}dx}{27cd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}}-\frac{\int \frac{x(7dx^3+16c)}{(8c-dx^3)(dx^3+c)^{3/2}}dx}{27d^2}$$

$$\Big\downarrow \; 1049$$

$$\frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}}-\frac{\frac{2x^2}{3c\sqrt{c+dx^3}}-\frac{2\int -\frac{9cdx\left(dx^3+16c\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27c^2d}}{27d^2}$$

$$\Big\downarrow \; 27$$

$$\frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}}-\frac{\frac{\int \frac{x\left(dx^3+16c\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{3c}+\frac{2x^2}{3c\sqrt{c+dx^3}}}{27d^2}$$

$$\Big\downarrow \; 1054$$

$$\frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}}-\frac{\frac{\int \left(\frac{24cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}-\frac{x}{\sqrt{dx^3+c}}\right)dx}{3c}+\frac{2x^2}{3c\sqrt{c+dx^3}}}{27d^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} -$$

$$-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}}{\left(\left(1+\sqrt{3}\right)\right.}}}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}}{\left(\left(1+\right.\right.}}}$$

input
```
Int[x^7/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(8*x^2)/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) - ((2*x^2)/(3*c*Sqrt[c + d*
x^3]) + ((-2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
) - (4*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x
^3]])/(Sqrt[3]*d^(2/3)) + (4*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)) - (4*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(3*d^(2/3)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) +
d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(
c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3]) - (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^
(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^
(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/(3*c))/
(27*d^2)
```

3.449. $\int \frac{x^7}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.449.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.449.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.59 (sec) , antiderivative size = 910, normalized size of antiderivative = 1.36

method	result	size
elliptic	Expression too large to display	910
default	Expression too large to display	2256

3.449. $\int \frac{x^7}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

input
```
int(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
8/243*x^2/c/d^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/243/d^2*x^2/c/((x^3+c/d)*d)
^(1/2)-2/243*I/d^3/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d
^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(
-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d
*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+8
/729*I/d^5/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d
^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)
+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*...
```

3.449.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 2681, normalized size of antiderivative = 4.01

$$\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/729*(18*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/d
, weierstrassPInverse(0, -4*c/d, x)) + (c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*
d^3 + sqrt(-3)*(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3))*(1/(c^5*d^16))^(1/
6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^13
*x^7 + 64*c^5*d^12*x^4 + 32*c^6*d^11*x + sqrt(-3)*(5*c^4*d^13*x^7 + 64*c^5
*d^12*x^4 + 32*c^6*d^11*x))*(1/(c^5*d^16))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5
*c^5*d^15*x^5 + 32*c^6*d^14*x^2 - sqrt(-3)*(5*c^5*d^15*x^5 + 32*c^6*d^14*x
^2))*(1/(c^5*d^16))^(5/6) - 2*(7*c^3*d^10*x^6 + 152*c^4*d^9*x^3 + 64*c^5*d
^8)*sqrt(1/(c^5*d^16)) + (c*d^5*x^7 + 80*c^2*d^4*x^4 + 160*c^3*d^3*x + sqr
t(-3)*(c*d^5*x^7 + 80*c^2*d^4*x^4 + 160*c^3*d^3*x))*(1/(c^5*d^16))^(1/6))
- 9*(c^2*d^8*x^8 + 38*c^3*d^7*x^5 + 64*c^4*d^6*x^2 - sqrt(-3)*(c^2*d^8*x^8
 + 38*c^3*d^7*x^5 + 64*c^4*d^6*x^2))*(1/(c^5*d^16))^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*
d^3 + sqrt(-3)*(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3))*(1/(c^5*d^16))^(1/
6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^13
*x^7 + 64*c^5*d^12*x^4 + 32*c^6*d^11*x + sqrt(-3)*(5*c^4*d^13*x^7 + 64*c^5
*d^12*x^4 + 32*c^6*d^11*x))*(1/(c^5*d^16))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5
*c^5*d^15*x^5 + 32*c^6*d^14*x^2 - sqrt(-3)*(5*c^5*d^15*x^5 + 32*c^6*d^14*x
^2))*(1/(c^5*d^16))^(5/6) - 2*(7*c^3*d^10*x^6 + 152*c^4*d^9*x^3 + 64*c^5*d
^8)*sqrt(1/(c^5*d^16)) + (c*d^5*x^7 + 80*c^2*d^4*x^4 + 160*c^3*d^3*x + ...
```

3.449.6 Sympy [F]

$$\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^7}{(-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**7/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.449.7 Maxima [F]

$$\int \frac{x^7}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{x^7}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.449.8 Giac [F]

$$\int \frac{x^7}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{x^7}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

input
```
integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.449.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{x^7}{\left(d\,x^3+c\right)^{3/2}\left(8\,c-d\,x^3\right)^2}\,dx$$

input
```
int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.450 $\int \frac{x^4}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.450.1 Optimal result

Integrand size = 27, antiderivative size = 671

$$\begin{aligned}
\int \frac{x^4}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx &= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} \\
&+ \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&- \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{11/6}d^{5/3}} + \frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{11/6}d^{5/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{11/6}d^{5/3}} \\
&- \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)\middle|-7-4\sqrt{3}\right)}{54\,3^{3/4}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{81\sqrt[4]{3}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

output
```
1/243*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)/
d^(5/3)-1/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)/d^(5/3)-1/243*
arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)/d^(5/
3)*3^(1/2)-1/81*x^2/c^2/d/(d*x^3+c)^(1/2)+1/27*x^2/c/d/(-d*x^3+8*c)/(d*x^3
+c)^(1/2)+1/81*(d*x^3+c)^(1/2)/c^2/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
+1/243*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3
)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)/
d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2)))^2)^(1/2)-1/162*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)
*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-
1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(5/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(
1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.450.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.25

$$\int \frac{x^4}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{80cx^2(-5c+dx^3)+50cx^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\right.}{6480c^3d\,(8c-dx^3)}$$

input
```
Integrate[x^4/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(80*c*x^2*(-5*c + d*x^3) + 50*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*Appe
llF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c
)])/(6480*c^3*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

3.450.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {971, 27, 1049, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 971$$

$$\frac{x^2}{27cd\,(8c-dx^3)\,\sqrt{c+dx^3}}-\frac{\int \frac{x(4c-5dx^3)}{2(8c-dx^3)(dx^3+c)^{3/2}}dx}{27cd}$$

$$\Big\downarrow \; 27$$

$$\frac{x^2}{27cd\,(8c-dx^3)\,\sqrt{c+dx^3}}-\frac{\int \frac{x(4c-5dx^3)}{(8c-dx^3)(dx^3+c)^{3/2}}dx}{54cd}$$

$$\Big\downarrow \; 1049$$

$$\frac{x^2}{27cd\,(8c-dx^3)\,\sqrt{c+dx^3}}-\frac{\frac{2x^2}{3c\sqrt{c+dx^3}}-\frac{2\int \frac{9cdx\left(20c-dx^3\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27c^2d}}{54cd}$$

$$\Big\downarrow \; 27$$

$$\frac{x^2}{27cd\,(8c-dx^3)\,\sqrt{c+dx^3}}-\frac{\frac{2x^2}{3c\sqrt{c+dx^3}}-\frac{\int \frac{x\left(20c-dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{3c}}{54cd}$$

$$\Big\downarrow \; 1054$$

$$\frac{x^2}{27cd\,(8c-dx^3)\,\sqrt{c+dx^3}}-\frac{\frac{2x^2}{3c\sqrt{c+dx^3}}-\frac{\int \left(\frac{12cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{x}{\sqrt{dx^3+c}}\right)dx}{3c}}{54cd}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\frac{2x^2}{3c\sqrt{c+dx^3}} - \frac{\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{}}{}$$

input
```
Int[x^4/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
x^2/(27*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3]) - ((2*x^2)/(3*c*Sqrt[c + d*x^3]
) - ((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (
2*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])
/(Sqrt[3]*d^(2/3)) + (2*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)
*Sqrt[c + d*x^3])])/(3*d^(2/3)) - (2*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/(3*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/
3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)
*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF
[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)
*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/(3*c))/(54*c
*d)
```

3.450. $\int \frac{x^4}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.450.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.450.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.62 (sec) , antiderivative size = 910, normalized size of antiderivative = 1.36

method	result	size
elliptic	Expression too large to display	910
default	Expression too large to display	1789

3.450. $\int \frac{x^4}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

input
```
int(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/243*x^2/c^2/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243/d*x^2/c^2/((x^3+c/d)*d)
^(1/2)-1/243*I/c^2/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c
*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1
/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))
-2/729*I/c^2/d^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*...
```

3.450.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.65 (sec) , antiderivative size = 2723, normalized size of antiderivative = 4.06

$$\int \frac{x^4}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/2916*(36*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/
d, weierstrassPInverse(0, -4*c/d, x)) - (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c
^4*d^2 + sqrt(-3)*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2))*(1/(c^11*d^10
))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^
8*d^9*x^7 + 64*c^9*d^8*x^4 + 32*c^10*d^7*x + sqrt(-3)*(5*c^8*d^9*x^7 + 64*
c^9*d^8*x^4 + 32*c^10*d^7*x))*(1/(c^11*d^10))^(2/3) + 3*sqrt(d*x^3 + c)*(6
*(5*c^10*d^10*x^5 + 32*c^11*d^9*x^2 - sqrt(-3)*(5*c^10*d^10*x^5 + 32*c^11*
d^9*x^2))*(1/(c^11*d^10))^(5/6) - 2*(7*c^6*d^7*x^6 + 152*c^7*d^6*x^3 + 64*
c^8*d^5)*sqrt(1/(c^11*d^10)) + (c^2*d^4*x^7 + 80*c^3*d^3*x^4 + 160*c^4*d^2
*x + sqrt(-3)*(c^2*d^4*x^7 + 80*c^3*d^3*x^4 + 160*c^4*d^2*x))*(1/(c^11*d^1
0))^(1/6)) - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2 - sqrt(-3)*(
c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2))*(1/(c^11*d^10))^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^2*d^4*x^6 - 7*c^3*d^
3*x^3 - 8*c^4*d^2 + sqrt(-3)*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2))*(1
/(c^11*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^
3 - 9*(5*c^8*d^9*x^7 + 64*c^9*d^8*x^4 + 32*c^10*d^7*x + sqrt(-3)*(5*c^8*d^
9*x^7 + 64*c^9*d^8*x^4 + 32*c^10*d^7*x))*(1/(c^11*d^10))^(2/3) - 3*sqrt(d*
x^3 + c)*(6*(5*c^10*d^10*x^5 + 32*c^11*d^9*x^2 - sqrt(-3)*(5*c^10*d^10*x^5
 + 32*c^11*d^9*x^2))*(1/(c^11*d^10))^(5/6) - 2*(7*c^6*d^7*x^6 + 152*c^7*d^
6*x^3 + 64*c^8*d^5)*sqrt(1/(c^11*d^10)) + (c^2*d^4*x^7 + 80*c^3*d^3*x^4...
```

3.450.6 Sympy [F]

$$\int \frac{x^4}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{x^4}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**4/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.450.7 Maxima [F]

$$\int \frac{x^4}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\int \frac{x^4}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.450.8 Giac [F]

$$\int \frac{x^4}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\int \frac{x^4}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.450.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\int \frac{x^4}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.451 $\int \frac{x}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.451.1 Optimal result

Integrand size = 25, antiderivative size = 665

$$
\begin{aligned}
\int \frac{x}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx &= \frac{5x^2}{648c^3\sqrt{c+dx^3}}+\frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} \\
&- \frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{5\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}c^{17/6}d^{2/3}} \\
&+ \frac{5\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{17/6}d^{2/3}}-\frac{5\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3888c^{17/6}d^{2/3}} \\
&+ \frac{5\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{432\ 3^{3/4}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&- \frac{5\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{324\sqrt{2}\sqrt[4]{3}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

output
```
5/3888*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(17/6)
/d^(2/3)-5/3888*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)/d^(2/3)-5/38
88*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/6)/d^
(2/3)*3^(1/2)+5/648*x^2/c^3/(d*x^3+c)^(1/2)+1/216*x^2/c^2/(-d*x^3+8*c)/(d*
x^3+c)^(1/2)-5/648*(d*x^3+c)^(1/2)/c^3/d^(2/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/
2)))-5/1944*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/
(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x
+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)/d^(
2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/
3)*(1+3^(1/2)))^2)^(1/2)+5/1296*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c
^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^
(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(
1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(8/3)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3
)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.451.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.25

$$\int \frac{x}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{16cx^2(43c-5dx^3)+5cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\Big(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c}}{10368c^4\,(8c-dx^3)}$$

input
```
Integrate[x/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(16*c*x^2*(43*c - 5*d*x^3) + 5*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Ap
pellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(8*c - d*x^3
)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*
c)])/(10368*c^4*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

3.451.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {972, 27, 1049, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(8c-dx^3)^2 (c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{ 972}$$

$$\frac{\int \frac{5dx(dx^3+10c)}{2(8c-dx^3)(dx^3+c)^{3/2}}dx}{216c^2d}+\frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5\int \frac{x(dx^3+10c)}{(8c-dx^3)(dx^3+c)^{3/2}}dx}{432c^2}+\frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1049}$$

$$\frac{5\left(\frac{2x^2}{3c\sqrt{c+dx^3}}-\frac{2\int \frac{9cdx\left(2c-dx^3\right)}{2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27c^2d}\right)}{432c^2}+\frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5\left(\frac{2x^2}{3c\sqrt{c+dx^3}}-\frac{\int \frac{x\left(2c-dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{3c}\right)}{432c^2}+\frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{5\left(\frac{2x^2}{3c\sqrt{c+dx^3}}-\frac{\int \left(\frac{x}{\sqrt{dx^3+c}}-\frac{6cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}\right)dx}{3c}\right)}{432c^2}+\frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 2009}$$

$$5\left(\frac{2x^2}{3c\sqrt{c+dx^3}}-\frac{\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{}\right.}{}}{}\right.$$

$$\frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

input
```
Int[x/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
x^2/(216*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (5*((2*x^2)/(3*c*Sqrt[c + d*
x^3]) - ((2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x))
 + (c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]
])/(Sqrt[3]*d^(2/3)) - (c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)
*Sqrt[c + d*x^3])]])/(3*d^(2/3)) + (c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt
[c])]])/(3*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)
*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)
*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*
x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[A
rcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x
))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/(3*c)))/(432*c
^2)
```

3.451. $\int \frac{x}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.451.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.451.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.53 (sec) , antiderivative size = 904, normalized size of antiderivative = 1.36

method	result	size
default	Expression too large to display	904
elliptic	Expression too large to display	904

input
```
int(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1944/c^3*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/243*x^2/c^3/((x^3+c/d)*d)^(1
/2)+5/1944*I/c^3*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2
)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I
*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^
(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(
-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-5/5
832*I/c^3/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*
d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*
(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3
)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I...
```

3.451.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 2684, normalized size of antiderivative = 4.04

$$\int \frac{x}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
1/46656*(360*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c
/d, weierstrassPInverse(0, -4*c/d, x)) + 5*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 -
8*c^5*d + sqrt(-3)*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d))*(1/(c^17*d^4))
^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^12
*d^5*x^7 + 64*c^13*d^4*x^4 + 32*c^14*d^3*x + sqrt(-3)*(5*c^12*d^5*x^7 + 64
*c^13*d^4*x^4 + 32*c^14*d^3*x))*(1/(c^17*d^4))^(2/3) + 3*sqrt(d*x^3 + c)*(
6*(5*c^15*d^5*x^5 + 32*c^16*d^4*x^2 - sqrt(-3)*(5*c^15*d^5*x^5 + 32*c^16*d
^4*x^2))*(1/(c^17*d^4))^(5/6) - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c
^11*d^2)*sqrt(1/(c^17*d^4)) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x
+ sqrt(-3)*(c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x))*(1/(c^17*d^4))^(1
/6)) - 9*(c^6*d^4*x^8 + 38*c^7*d^3*x^5 + 64*c^8*d^2*x^2 - sqrt(-3)*(c^6*d^
4*x^8 + 38*c^7*d^3*x^5 + 64*c^8*d^2*x^2))*(1/(c^17*d^4))^(1/3))/(d^3*x^9 -
 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 5*(c^3*d^3*x^6 - 7*c^4*d^2*x^3
 - 8*c^5*d + sqrt(-3)*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d))*(1/(c^17*d^
4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c
^12*d^5*x^7 + 64*c^13*d^4*x^4 + 32*c^14*d^3*x + sqrt(-3)*(5*c^12*d^5*x^7 +
 64*c^13*d^4*x^4 + 32*c^14*d^3*x))*(1/(c^17*d^4))^(2/3) - 3*sqrt(d*x^3 + c
)*(6*(5*c^15*d^5*x^5 + 32*c^16*d^4*x^2 - sqrt(-3)*(5*c^15*d^5*x^5 + 32*c^1
6*d^4*x^2))*(1/(c^17*d^4))^(5/6) - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 6
4*c^11*d^2)*sqrt(1/(c^17*d^4)) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^...
```

3.451.6 Sympy [F]

$$\int \frac{x}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{x}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.451.7 Maxima [F]

$$\int \frac{x}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\int \frac{x}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.451.8 Giac [F]

$$\int \frac{x}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\int \frac{x}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.451.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\int \frac{x}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.452 $\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.452.1 Optimal result

Integrand size = 27, antiderivative size = 686

$$
\begin{aligned}
\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx &= \frac{5}{648c^3x\sqrt{c+dx^3}}+\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} \\
&-\frac{31\sqrt{c+dx^3}}{1296c^4x}+\frac{31\sqrt[3]{d}\sqrt{c+dx^3}}{1296c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}c^{23/6}} \\
&+\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{23/6}}-\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3888c^{23/6}} \\
&-\frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{864\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+\frac{31\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{648\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

output
```
1/3888*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
c^(23/6)-1/3888*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-1/38
88*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(
23/6)*3^(1/2)+5/648/c^3/x/(d*x^3+c)^(1/2)+1/216/c^2/x/(-d*x^3+8*c)/(d*x^3+
c)^(1/2)-31/1296*(d*x^3+c)^(1/2)/c^4/x+31/1296*d^(1/3)*(d*x^3+c)^(1/2)/c^4
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+31/3888*d^(1/3)*(c^(1/3)+d^(1/3)*x)*Ellip
ticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2)))^2)^(1/2)*3^(3/4)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/
3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-31/2592*d^(1/3)*(c^
(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3
)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/
c^(11/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2)))^2)^(1/2)
```

3.452.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^2(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{-80c(162c^2+227cdx^3-31d^2x^6)+650cdx^3(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{Appell}}{103680}$$

input
```
Integrate[1/(x^2*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-80*c*(162*c^2 + 227*c*d*x^3 - 31*d^2*x^6) + 650*c*d*x^3*(8*c - d*x^3)*Sq
rt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)]
+ 31*d^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3,
 -((d*x^3)/c), (d*x^3)/(8*c)])/(103680*c^5*Sqrt[c + d*x^3]*(8*c*x - d*x^4)
)
```

3.452.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {972, 27, 1049, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{\int \frac{d\left(11dx^3+56c\right)}{2x^2\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx}{216c^2d}+\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{11dx^3+56c}{x^2\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx}{432c^2}+\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1049}$$

$$\frac{\frac{10}{3cx\sqrt{c+dx^3}}-\frac{2\int -\frac{9cd\left(248c-25dx^3\right)}{2x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27c^2d}}{432c^2}+\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{248c-25dx^3}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{3c}+\frac{10}{3cx\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{\frac{-\frac{\int -\frac{4cdx\left(260c-31dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{31\sqrt{c+dx^3}}{cx}}{3c}+\frac{10}{3cx\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{d\int \frac{x\left(260c-31dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{2c}-\frac{31\sqrt{c+dx^3}}{cx}}{3c}+\frac{10}{3cx\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1054}$$

$$\frac{\frac{\frac{d\int\left(\frac{12cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{31x}{\sqrt{dx^3+c}}\right)dx}{2c}-\frac{31\sqrt{c+dx^3}}{cx}}{3c}+\frac{10}{3cx\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{d\left(\frac{62\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{31\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{2/3}}\right.}{}}{}$$

$$\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

input
```
Int[1/(x^2*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
1/(216*c^2*x*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (10/(3*c*x*Sqrt[c + d*x^3])
+ ((-31*Sqrt[c + d*x^3])/(c*x) + (d*((62*Sqrt[c + d*x^3])/(d^(2/3)*((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)) - (2*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3
) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(2/3)) + (2*c^(1/6)*ArcTanh[(
c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3*d^(2/3)) - (2*c^(1
/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3*d^(2/3)) - (31*3^(1/4)*Sqrt[2
 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[(
(1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (62*Sqrt[2]*c^(1/3)*(c^(1/3)
 + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1
/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(
2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*Sqrt[c + d*x^3])))/(2*c))/(3*c))/(432*c^2)
```

3.452. $\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.452.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.452. $\int \frac{1}{x^2(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.452.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.33 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	920
risch	Expression too large to display	2220
default	Expression too large to display	2270

input

```
int(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/15552*d*x^2/c^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243*d*x^2/c^4/((x^3+c/d)*
d)^(1/2)-1/64*(d*x^3+c)^(1/2)/c^4/x-31/3888*I/c^4*3^(1/2)*(-c*d^2)^(1/3)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)))-1/5832*I/c^4/d^2*2^(1/2)*sum(1/_alpha*(-c*d^2)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^
(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*...
```

3.452.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.91 (sec) , antiderivative size = 2534, normalized size of antiderivative = 3.69

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/46656*(1116*(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)*sqrt(d)*weierstrassZeta(0,
-4*c/d, weierstrassPInverse(0, -4*c/d, x)) - (c^4*d^2*x^7 - 7*c^5*d*x^4 -
8*c^6*x + sqrt(-3)*(c^4*d^2*x^7 - 7*c^5*d*x^4 - 8*c^6*x))*(d^2/c^23)^(1/6)
*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^16*d
^2*x^7 + 64*c^17*d*x^4 + 32*c^18*x + sqrt(-3)*(5*c^16*d^2*x^7 + 64*c^17*d*
x^4 + 32*c^18*x))*(d^2/c^23)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 +
32*c^21*x^2 - sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^2/c^23)^(5/6) - 2*
(7*c^12*d^2*x^6 + 152*c^13*d*x^3 + 64*c^14)*sqrt(d^2/c^23) + (c^4*d^3*x^7
+ 80*c^5*d^2*x^4 + 160*c^6*d*x + sqrt(-3)*(c^4*d^3*x^7 + 80*c^5*d^2*x^4 +
160*c^6*d*x))*(d^2/c^23)^(1/6)) - 9*(c^8*d^3*x^8 + 38*c^9*d^2*x^5 + 64*c^1
0*d*x^2 - sqrt(-3)*(c^8*d^3*x^8 + 38*c^9*d^2*x^5 + 64*c^10*d*x^2))*(d^2/c^
23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^4*d^2*
x^7 - 7*c^5*d*x^4 - 8*c^6*x + sqrt(-3)*(c^4*d^2*x^7 - 7*c^5*d*x^4 - 8*c^6*
x))*(d^2/c^23)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640
*c^3*d - 9*(5*c^16*d^2*x^7 + 64*c^17*d*x^4 + 32*c^18*x + sqrt(-3)*(5*c^16*
d^2*x^7 + 64*c^17*d*x^4 + 32*c^18*x))*(d^2/c^23)^(2/3) - 3*sqrt(d*x^3 + c)
*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(
d^2/c^23)^(5/6) - 2*(7*c^12*d^2*x^6 + 152*c^13*d*x^3 + 64*c^14)*sqrt(d^2/c
^23) + (c^4*d^3*x^7 + 80*c^5*d^2*x^4 + 160*c^6*d*x + sqrt(-3)*(c^4*d^3*x^7
 + 80*c^5*d^2*x^4 + 160*c^6*d*x))*(d^2/c^23)^(1/6)) - 9*(c^8*d^3*x^8 + ...
```

3.452.6 Sympy [F]

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**2*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.452.7 Maxima [F]

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^2), x)
```

3.452.8 Giac [F]

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^2), x)
```

3.452.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^2\,\left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

input
```
int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.453 $\int \frac{1}{x^5(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.453.1 Optimal result

Integrand size = 27, antiderivative size = 708

$$\int \frac{1}{x^5\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{5}{648c^3x^4\sqrt{c+dx^3}}$$

$$+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{253\sqrt{c+dx^3}}{20736c^4x^4}+\frac{77d\sqrt{c+dx^3}}{2592c^5x}$$

$$-\frac{77d^{4/3}\sqrt{c+dx^3}}{2592c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{11d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{82944\sqrt{3}c^{29/6}}$$

$$+\frac{11d^{4/3}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{248832c^{29/6}}-\frac{11d^{4/3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{248832c^{29/6}}$$

$$+\frac{77\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{1728\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{77d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{1296\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
11/248832*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2
))/c^(29/6)-11/248832*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(29/6
)-11/248832*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(
1/2))/c^(29/6)*3^(1/2)+5/648/c^3/x^4/(d*x^3+c)^(1/2)+1/216/c^2/x^4/(-d*x^3
+8*c)/(d*x^3+c)^(1/2)-253/20736*(d*x^3+c)^(1/2)/c^4/x^4+77/2592*d*(d*x^3+c
)^(1/2)/c^5/x-77/2592*d^(4/3)*(d*x^3+c)^(1/2)/c^5/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2)))-77/7776*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(
1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/
3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)
/c^(14/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))^2)^(1/2)+77/5184*d^(4/3)*(c^(1/3)+d^(1/3)*x)*Elliptic
E((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/
(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(14/3)/(d*x^3+c)^(1/2)/
(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.453.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^5 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{-24475cd^2x^6(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-16(}{}$$

input
```
Integrate[1/(x^5*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-24475*c*d^2*x^6*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 16*(10*c*(648*c^3 - 2997*c^2*d*x^3 - 4
565*c*d^2*x^6 + 616*d^3*x^9) + 77*d^3*x^9*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/
c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(3317760*c^6*
x^4*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

3.453.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {972, 27, 1049, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{\int \frac{d\left(17dx^3+62c\right)}{2x^5\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx}{216c^2d}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{17dx^3+62c}{x^5\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1049}$$

$$\frac{\frac{10}{3cx^4\sqrt{c+dx^3}}-\frac{2\int -\frac{99cd\left(46c-5dx^3\right)}{2x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27c^2d}}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{11\int \frac{46c-5dx^3}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{3c}+\frac{10}{3cx^4\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{\frac{11\left(-\frac{\int \frac{cd\left(896c-115dx^3\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{3c}+\frac{10}{3cx^4\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{11\left(-\frac{d\int \frac{896c-115dx^3}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{3c}+\frac{10}{3cx^4\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{\frac{11\left(-\frac{d\left(-\frac{\int -\frac{8cdx\left(445c-56dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{112\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{3c}+\frac{10}{3cx^4\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{11\left(-\frac{d\left(\frac{d\int \frac{x\left(445c-56dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{112\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{3c}+\frac{10}{3cx^4\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1054}$$

$$\frac{\frac{11\left(-\frac{d\left(\frac{d\int \left(\frac{56x}{\sqrt{dx^3+c}}-\frac{3cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}\right)dx}{c}-\frac{112\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{23\sqrt{c+dx^3}}{16cx^4}\right)}{3c}+\frac{10}{3cx^4\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{2009}$$

3.453. $\int \frac{1}{x^5\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

$$11\left(-\frac{d\left(d\left(\frac{112\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{56\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{}\right.\right.}{}\right.$$

$$\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

input
```
Int[1/(x^5*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

3.453. $\int \frac{1}{x^5\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

output
```
1/(216*c^2*x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (10/(3*c*x^4*Sqrt[c + d*x^
3]) + (11*((-23*Sqrt[c + d*x^3])/(16*c*x^4) - (d*((-112*Sqrt[c + d*x^3])/(
c*x) + (d*((112*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)) + (c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*
x^3]])/(2*Sqrt[3]*d^(2/3)) - (c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c
^(1/6)*Sqrt[c + d*x^3])])/(6*d^(2/3)) + (c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])])/(6*d^(2/3)) - (56*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt
[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sq
rt[c + d*x^3]) + (112*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3)
+ d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]))/c)
)/(32*c)))/(3*c))/(432*c^2)
```

3.453.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.453.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.31 (sec) , antiderivative size = 943, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	943
risch	Expression too large to display	2232
default	Expression too large to display	2775

input
```
int(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/124416*d^2*x^2/c^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/243*d^2*x^2/c^5/((x^3+
c/d)*d)^(1/2)-1/256*(d*x^3+c)^(1/2)/c^4/x^4+11/512*d*(d*x^3+c)^(1/2)/c^5/x
+77/7776*I*d/c^5*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^
(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(
x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c
*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-11/37
3248*I/c^5/d*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d
^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)
+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)...
```

3.453.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.80 (sec) , antiderivative size = 2692, normalized size of antiderivative = 3.80

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
1/2985984*(88704*(d^3*x^10 - 7*c*d^2*x^7 - 8*c^2*d*x^4)*sqrt(d)*weierstras
sZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 11*(c^5*d^2*x^10 - 7
*c^6*d*x^7 - 8*c^7*x^4 + sqrt(-3)*(c^5*d^2*x^10 - 7*c^6*d*x^7 - 8*c^7*x^4)
)*(d^8/c^29)^(1/6)*log(161051*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3
+ 640*c^3*d^6 - 9*(5*c^20*d^3*x^7 + 64*c^21*d^2*x^4 + 32*c^22*d*x + sqrt(-
3)*(5*c^20*d^3*x^7 + 64*c^21*d^2*x^4 + 32*c^22*d*x))*(d^8/c^29)^(2/3) + 3*
sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2 - sqrt(-3)*(5*c^25*d*x^5 +
32*c^26*x^2))*(d^8/c^29)^(5/6) - 2*(7*c^15*d^4*x^6 + 152*c^16*d^3*x^3 + 64
*c^17*d^2)*sqrt(d^8/c^29) + (c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^7*d^5*x
+ sqrt(-3)*(c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^7*d^5*x))*(d^8/c^29)^(1/6
)) - 9*(c^10*d^6*x^8 + 38*c^11*d^5*x^5 + 64*c^12*d^4*x^2 - sqrt(-3)*(c^10*
d^6*x^8 + 38*c^11*d^5*x^5 + 64*c^12*d^4*x^2))*(d^8/c^29)^(1/3))/(d^3*x^9 -
 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 11*(c^5*d^2*x^10 - 7*c^6*d*x^7
 - 8*c^7*x^4 + sqrt(-3)*(c^5*d^2*x^10 - 7*c^6*d*x^7 - 8*c^7*x^4))*(d^8/c^2
9)^(1/6)*log(161051*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*
d^6 - 9*(5*c^20*d^3*x^7 + 64*c^21*d^2*x^4 + 32*c^22*d*x + sqrt(-3)*(5*c^20
*d^3*x^7 + 64*c^21*d^2*x^4 + 32*c^22*d*x))*(d^8/c^29)^(2/3) - 3*sqrt(d*x^3
 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2 - sqrt(-3)*(5*c^25*d*x^5 + 32*c^26*x^
2))*(d^8/c^29)^(5/6) - 2*(7*c^15*d^4*x^6 + 152*c^16*d^3*x^3 + 64*c^17*d^2)
*sqrt(d^8/c^29) + (c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^7*d^5*x + sqrt(...
```

3.453.6 Sympy [F]

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^5 \left(-8c+dx^3\right)^2 \left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**5*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.453.7 Maxima [F]

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^5}\,dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^5), x)
```

3.453.8 Giac [F]

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^5}\,dx$$

input
```
integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^5), x)
```

3.453.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^5\,\left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

input
```
int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.454 $\int \frac{1}{x^8\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.454.1 Optimal result

Integrand size = 27, antiderivative size = 732

$$\int \frac{1}{x^8 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}}$$

$$-\frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} - \frac{5179d^2\sqrt{c+dx^3}}{145152c^6x}$$

$$+\frac{5179d^{7/3}\sqrt{c+dx^3}}{145152c^6\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{7d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{331776\sqrt{3}c^{35/6}}$$

$$+\frac{7d^{7/3}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{995328c^{35/6}} - \frac{7d^{7/3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{995328c^{35/6}}$$

$$-\frac{5179\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{96768\ 3^{3/4}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{5179d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{72576\sqrt{2}\sqrt[4]{3}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output

```
7/995328*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2)
)/c^(35/6)-7/995328*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(35/6)-
7/995328*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2
))/c^(35/6)*3^(1/2)+5/648/c^3/x^7/(d*x^3+c)^(1/2)+1/216/c^2/x^7/(-d*x^3+8*
c)/(d*x^3+c)^(1/2)-191/18144*(d*x^3+c)^(1/2)/c^4/x^7+8257/580608*d*(d*x^3+
c)^(1/2)/c^5/x^4-5179/145152*d^2*(d*x^3+c)^(1/2)/c^6/x+5179/145152*d^(7/3)
*(d*x^3+c)^(1/2)/c^6/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+5179/435456*d^(7/3)*(
c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(17/3)*2^(1/2)/(d*x^3+
c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(
1/2)-5179/290304*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*
(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1
/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(17/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(
1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.454.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^8 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{829375cd^3x^9(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-8\Bigg(}{}$$

input

```
Integrate[1/(x^8*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output

```
(829375*c*d^3*x^9*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 8*(20*c*(10368*c^4 - 18792*c^3*d*x^3 +
 101817*c^2*d^2*x^6 + 153269*c*d^3*x^9 - 20716*d^4*x^12) + 5179*d^4*x^12*(
8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c),
(d*x^3)/(8*c)]))/(92897280*c^7*x^7*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

3.454.3 Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 757, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {972, 27, 1049, 27, 1053, 27, 1053, 27, 1053, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{\int \frac{d\left(23dx^3+68c\right)}{2x^8\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx}{216c^2d}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{23dx^3+68c}{x^8\left(8c-dx^3\right)\left(dx^3+c\right)^{3/2}}dx}{432c^2}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1049}$$

$$\frac{\frac{10}{3cx^7\sqrt{c+dx^3}}-\frac{2\int -\frac{9cd\left(764c-85dx^3\right)}{2x^8\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{27c^2d}}{432c^2}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{764c-85dx^3}{x^8\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{\frac{-\frac{\int \frac{2cd\left(16514c-2101dx^3\right)}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{56c^2}-\frac{191\sqrt{c+dx^3}}{14cx^7}}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{-\frac{d\int \frac{16514c-2101dx^3}{x^5\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{28c}-\frac{191\sqrt{c+dx^3}}{14cx^7}}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{\frac{-\frac{d\left(-\frac{\int \frac{cd\left(331456c-41285dx^3\right)}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c^2}-\frac{8257\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{191\sqrt{c+dx^3}}{14cx^7}}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{-\frac{d\left(-\frac{d\int \frac{331456c-41285dx^3}{x^2\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{32c}-\frac{8257\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{191\sqrt{c+dx^3}}{14cx^7}}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow 1053$$

$$\frac{\frac{-\frac{d\left(-\frac{d\left(-\frac{\int -\frac{8cdx\left(165875c-20716dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{8c^2}-\frac{41432\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{8257\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{191\sqrt{c+dx^3}}{14cx^7}}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{-\frac{d\left(-\frac{d\left(\frac{d\int \frac{x\left(165875c-20716dx^3\right)}{\left(8c-dx^3\right)\sqrt{dx^3+c}}dx}{c}-\frac{41432\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{8257\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{191\sqrt{c+dx^3}}{14cx^7}}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow 1054$$

3.454. $\int \frac{1}{x^8\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

$$\frac{\frac{-\frac{d\left(-\frac{d\left(\frac{d\int\left(\frac{147cx}{\left(8c-dx^3\right)\sqrt{dx^3+c}}+\frac{20716x}{\sqrt{dx^3+c}}\right)dx}{c}-\frac{41432\sqrt{c+dx^3}}{cx}\right)}{32c}-\frac{8257\sqrt{c+dx^3}}{16cx^4}\right)}{28c}-\frac{191\sqrt{c+dx^3}}{14cx^7}}{3c}+\frac{10}{3cx^7\sqrt{c+dx^3}}}{432c^2}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{d\Bigg(-\frac{d\Bigg(\frac{d\Bigg(\frac{41432\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{20716\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\Big(\sqrt[3]{c}+}{}}{}}{}}{}$$

$$\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

input
```
Int[1/(x^8*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

3.454. $\int \frac{1}{x^8\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

output

```
1/(216*c^2*x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (10/(3*c*x^7*Sqrt[c + d*x^
3]) + ((-191*Sqrt[c + d*x^3])/(14*c*x^7) - (d*((-8257*Sqrt[c + d*x^3])/(16
*c*x^4) - (d*((-41432*Sqrt[c + d*x^3])/(c*x) + (d*((41432*Sqrt[c + d*x^3])
/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (49*c^(1/6)*ArcTan[(Sqrt[
3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*Sqrt[3]*d^(2/3)) +
(49*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/
(6*d^(2/3)) - (49*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(6*d^(2/3)
) - (20716*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) +
 d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (41
432*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/c))/(32*c)))/(28*c))
/(3*c))/(432*c^2)
```

3.454.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))    Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.454.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.67 (sec) , antiderivative size = 962, normalized size of antiderivative = 1.31

method	result	size
elliptic	Expression too large to display	962
risch	Expression too large to display	2243
default	Expression too large to display	3300

input
```
int(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/995328*d^3*x^2/c^6*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243*d^3*x^2/c^6/((x^3+
c/d)*d)^(1/2)-1/448*(d*x^3+c)^(1/2)/c^4/x^7+43/7168*d*(d*x^3+c)^(1/2)/c^5/
x^4-787/28672*d^2*(d*x^3+c)^(1/2)/c^6/x-5179/435456*I/c^6*d^2*3^(1/2)*(-c*
d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-7/1492992*I/c^6*2^(1/2)*sum(1/_alp
ha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I
*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*
_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1...
```

3.454.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.57 (sec) , antiderivative size = 2725, normalized size of antiderivative = 3.72

$$\int \frac{1}{x^8 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/83607552*(2983104*(d^4*x^13 - 7*c*d^3*x^10 - 8*c^2*d^2*x^7)*sqrt(d)*wei
erstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 49*(c^6*d^2*x
^13 - 7*c^7*d*x^10 - 8*c^8*x^7 + sqrt(-3)*(c^6*d^2*x^13 - 7*c^7*d*x^10 - 8
*c^8*x^7))*(d^14/c^35)^(1/6)*log(16807*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c
^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26
*d^2*x + sqrt(-3)*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26*d^2*x))*(d^1
4/c^35)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^30*d*x^5 + 32*c^31*x^2 - sqrt(-3
)*(5*c^30*d*x^5 + 32*c^31*x^2))*(d^14/c^35)^(5/6) - 2*(7*c^18*d^6*x^6 + 15
2*c^19*d^5*x^3 + 64*c^20*d^4)*sqrt(d^14/c^35) + (c^6*d^11*x^7 + 80*c^7*d^1
0*x^4 + 160*c^8*d^9*x + sqrt(-3)*(c^6*d^11*x^7 + 80*c^7*d^10*x^4 + 160*c^8
*d^9*x))*(d^14/c^35)^(1/6)) - 9*(c^12*d^9*x^8 + 38*c^13*d^8*x^5 + 64*c^14*
d^7*x^2 - sqrt(-3)*(c^12*d^9*x^8 + 38*c^13*d^8*x^5 + 64*c^14*d^7*x^2))*(d^
14/c^35)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 49*(
c^6*d^2*x^13 - 7*c^7*d*x^10 - 8*c^8*x^7 + sqrt(-3)*(c^6*d^2*x^13 - 7*c^7*d
*x^10 - 8*c^8*x^7))*(d^14/c^35)^(1/6)*log(16807*(d^14*x^9 + 318*c*d^13*x^6
 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4
+ 32*c^26*d^2*x + sqrt(-3)*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26*d^2
*x))*(d^14/c^35)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^30*d*x^5 + 32*c^31*x^2
- sqrt(-3)*(5*c^30*d*x^5 + 32*c^31*x^2))*(d^14/c^35)^(5/6) - 2*(7*c^18*d^6
*x^6 + 152*c^19*d^5*x^3 + 64*c^20*d^4)*sqrt(d^14/c^35) + (c^6*d^11*x^7 ...
```

3.454.6 Sympy [F]

$$\int \frac{1}{x^8\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^8\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**8*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.454.7 Maxima [F]

$$\int \frac{1}{x^8 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^8}\,dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^8), x)
```

3.454.8 Giac [F]

$$\int \frac{1}{x^8 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^8}\,dx$$

input
```
integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^8), x)
```

3.454.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^8\,\left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

input
```
int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.455 $\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}} \, dx$

3.455.1 Optimal result

Integrand size $=27$, antiderivative size $=256$

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}} \, dx = \frac{2x(4c+dx^3)}{81cd^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{81\sqrt[4]{3}cd^{7/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

output
```
2/81*x*(d*x^3+4*c)/c/d^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2)-2/243*(c^(1/3)+d^(1/
3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x
+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c/d^(7/3)/(
d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

3.455.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.56 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.74

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx =$$

$$-\frac{6\sqrt[3]{-d}x(4c+dx^3)+2i3^{3/4}\sqrt[3]{c}\sqrt{\frac{(-1)^{5/6}\left(-\sqrt[3]{c}+\sqrt[3]{-d}x\right)}{\sqrt[3]{c}}}\sqrt{1+\frac{\sqrt[3]{-d}x}{\sqrt[3]{c}}+\frac{(-d)^{2/3}x^2}{c^{2/3}}}(-8c+dx^3)\,\text{EllipticF}\left(\text{arc}\right.}{243c(-d)^{7/3}(-8c+dx^3)\sqrt{c+dx^3}}$$

input
```
Integrate[x^6/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
-1/243*(6*(-d)^(1/3)*x*(4*c + d*x^3) + (2*I)*3^(3/4)*c^(1/3)*Sqrt[((-1)^(5
/6)*(-c^(1/3) + (-d)^(1/3)*x))/c^(1/3)]*Sqrt[1 + ((-d)^(1/3)*x)/c^(1/3) +
((-d)^(2/3)*x^2)/c^(2/3)]*(-8*c + d*x^3)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6)
 - (I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)])/(c*(-d)^(7/3)*(-8*c +
d*x^3)*Sqrt[c + d*x^3])
```

3.455.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^6}{(8c-dx^3)^2\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^7\sqrt{\frac{dx^3}{c}+1}\,\mathrm{AppellF1}\left(\frac{7}{3},2,\frac{3}{2},\frac{10}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^3\sqrt{c+dx^3}}$$

input
```
Int[x^6/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x^7*Sqrt[1 + (d*x^3)/c]*AppellF1[7/3, 2, 3/2, 10/3, (d*x^3)/(8*c), -((d*x
^3)/c)])/(448*c^3*Sqrt[c + d*x^3])
```

3.455.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.455.4 Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.32

method	result
elliptic	$\frac{8x\sqrt{dx^3+c}}{243cd^2(-dx^3+8c)}+\frac{2x}{243d^2c\sqrt{(x^3+\frac{c}{d})d}}+\frac{2i\sqrt{3}\,(-cd^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-cd^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-cd^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-cd^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-cd^2)^{\frac{1}{3}}}{d}}{-\frac{3(-cd^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,(-cd^2)^{\frac{1}{3}}}{2d}}}}{}$
default	Expression too large to display

input
```
int(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
8/243*x/c/d^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/243/d^2*x/c/((x^3+c/d)*d)^(1/
2)+2/243*I/d^3/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^
(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(
x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2))
```

3.455.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.35

$$\int \frac{x^6}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = -\frac{2\left((d^2x^6-7\,cdx^3-8\,c^2)\sqrt{d}\mathrm{weierstrassPInverse}\left(0,-\frac{4\,c}{d},x\right)+(d^2x^4+4\,cdx)\sqrt{dx^3+c}\right)}{81\,(cd^5x^6-7\,c^2d^4x^3-8\,c^3d^3)}$$

input
```
integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/81*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassPInverse(0, -4*c/d
, x) + (d^2*x^4 + 4*c*d*x)*sqrt(d*x^3 + c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8
*c^3*d^3)
```

3.455.6 Sympy [F]

$$\int \frac{x^6}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^6}{(-8c+dx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**6/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.455.7 Maxima [F]

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\int \frac{x^6}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^6/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.455.8 Giac [F]

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\int \frac{x^6}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^6/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.455.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\int \frac{x^6}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x^6/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^6/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.456 $\int \frac{x^3}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.456.1 Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{x^3}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},2,\frac{3}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3\sqrt{c+dx^3}}$$

output
```
1/256*x^4*AppellF1(4/3,3/2,2,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c
^3/(d*x^3+c)^(1/2)
```

3.456.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 242 vs. 2(66) = 132.

Time = 10.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.67

$$\int \frac{x^3}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{x\Bigg(\frac{3x^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{192\Big(\frac{-5c+dx^3}{c^2}+\frac{}{32c\operatorname{AppellF1}\big(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}}}{}}{15552\sqrt{c+dx^3}}$$

input
```
Integrate[x^3/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x*((3*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d
*x^3)/(8*c)])/c^3 + (192*((-5*c + d*x^3)/c^2 + (160*AppellF1[1/3, 1/2, 1,
4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x
^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)
]))))/(d*(8*c - d*x^3))))/(15552*Sqrt[c + d*x^3])
```

3.456.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{(8c-dx^3)^2\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{3}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3\sqrt{c+dx^3}}$$

input
```
Int[x^3/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 3/2, 7/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(256*c^3*Sqrt[c + d*x^3])
```

3.456.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.456.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.42 (sec) , antiderivative size = 754, normalized size of antiderivative = 11.42

method	result	size
elliptic	Expression too large to display	754
default	Expression too large to display	1479

input
```
int(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/243*x/c^2/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243/d*x/c^2/((x^3+c/d)*d)^(1/
2)+1/243*I/c^2/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2
)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I
*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2))-1/243*I/c^2/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))
^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1
/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2
)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3
)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

3.456.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2713 vs. $2(52) = 104$.

Time = 1.07 (sec) , antiderivative size = 2713, normalized size of antiderivative = 41.11

$$\int \frac{x^3}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output

```
1/3888*(24*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassPInverse(0, -4
*c/d, x) + (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2 + sqrt(-3)*(c^2*d^4*x^
6 - 7*c^3*d^3*x^3 - 8*c^4*d^2))*(1/(c^13*d^8))^(1/6)*log((d^3*x^9 + 318*c*
d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64
*c^11*d^6*x^2 + sqrt(-3)*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64*c^11*d^6*x^2)
)*(1/(c^13*d^8))^(2/3) + 3*sqrt(d*x^3 + c)*((c^11*d^9*x^7 + 80*c^12*d^8*x^
4 + 160*c^13*d^7*x - sqrt(-3)*(c^11*d^9*x^7 + 80*c^12*d^8*x^4 + 160*c^13*d
^7*x))*(1/(c^13*d^8))^(5/6) - 2*(7*c^7*d^6*x^6 + 152*c^8*d^5*x^3 + 64*c^9*
d^4)*sqrt(1/(c^13*d^8)) + 6*(5*c^3*d^3*x^5 + 32*c^4*d^2*x^2 + sqrt(-3)*(5*
c^3*d^3*x^5 + 32*c^4*d^2*x^2))*(1/(c^13*d^8))^(1/6)) - 9*(5*c^5*d^5*x^7 +
64*c^6*d^4*x^4 + 32*c^7*d^3*x - sqrt(-3)*(5*c^5*d^5*x^7 + 64*c^6*d^4*x^4 +
 32*c^7*d^3*x))*(1/(c^13*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*
x^3 - 512*c^3)) - (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2 + sqrt(-3)*(c^2
*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2))*(1/(c^13*d^8))^(1/6)*log((d^3*x^9 +
 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^8*x^8 + 38*c^10*d^7*x
^5 + 64*c^11*d^6*x^2 + sqrt(-3)*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64*c^11*d
^6*x^2))*(1/(c^13*d^8))^(2/3) - 3*sqrt(d*x^3 + c)*((c^11*d^9*x^7 + 80*c^12
*d^8*x^4 + 160*c^13*d^7*x - sqrt(-3)*(c^11*d^9*x^7 + 80*c^12*d^8*x^4 + 160
*c^13*d^7*x))*(1/(c^13*d^8))^(5/6) - 2*(7*c^7*d^6*x^6 + 152*c^8*d^5*x^3 +
64*c^9*d^4)*sqrt(1/(c^13*d^8)) + 6*(5*c^3*d^3*x^5 + 32*c^4*d^2*x^2 + sq...
```

3.456.6 Sympy [F]

$$\int \frac{x^3}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input

```
integrate(x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output

```
Integral(x**3/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.456.7 Maxima [F]

$$\int \frac{x^3}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.456.8 Giac [F]

$$\int \frac{x^3}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.456.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(d\,x^3+c)^{3/2}(8\,c-d\,x^3)^2}\,dx$$

input
```
int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.457 $\int \frac{1}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.457.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{1}{3},2,\frac{3}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}$$

output
```
1/64*x*AppellF1(1/3,3/2,2,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^3/
(d*x^3+c)^(1/2)
```

3.457.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 253 vs. 2(64) = 128.

Time = 10.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.95

$$\int \frac{1}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{x\left(-15dx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+192c\left(\frac{-43c+5dx^3}{-8c+dx^3}+\right.\right.}{}$$

input
```
Integrate[1/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x*(-15*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 192*c*((-43*c + 5*d*x^3)/(-8*c + d*x^3) + (1216*c^2*Appe
llF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*
AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF
1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1,
 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))))/(124416*c^4*Sqrt[c + d*x^3])
```

3.457.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$$
$$\Big\downarrow \text{937}$$
$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(8c-dx^3)^2\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$
$$\Big\downarrow \text{936}$$
$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{3}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}$$

input
```
Int[1/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 3/2, 4/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(64*c^3*Sqrt[c + d*x^3])
```

3.457.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.457.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.55 (sec) , antiderivative size = 748, normalized size of antiderivative = 11.69

method	result	size
default	Expression too large to display	748
elliptic	Expression too large to display	748

input
```
int(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1944/c^3*x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/243*x/c^3/((x^3+c/d)*d)^(1/2)-
5/1944*I/c^3*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1
/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3
)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
)^(1/2))-1/972*I/c^3/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2
*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(
d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/
2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*
(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_a
lpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

3.457.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2640 vs. $2(50) = 100$.

Time = 0.97 (sec) , antiderivative size = 2640, normalized size of antiderivative = 41.25

$$\int \frac{1}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output

```
1/15552*(192*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassPInverse(0,
-4*c/d, x) + (c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d + sqrt(-3)*(c^3*d^3*x^
6 - 7*c^4*d^2*x^3 - 8*c^5*d))*(1/(c^19*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^
2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*
c^15*d^2*x^2 + sqrt(-3)*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2)
)*(1/(c^19*d^2))^(2/3) + 3*sqrt(d*x^3 + c)*((c^16*d^4*x^7 + 80*c^17*d^3*x^
4 + 160*c^18*d^2*x - sqrt(-3)*(c^16*d^4*x^7 + 80*c^17*d^3*x^4 + 160*c^18*d
^2*x))*(1/(c^19*d^2))^(5/6) - 2*(7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^
12*d)*sqrt(1/(c^19*d^2)) + 6*(5*c^4*d^2*x^5 + 32*c^5*d*x^2 + sqrt(-3)*(5*c
^4*d^2*x^5 + 32*c^5*d*x^2))*(1/(c^19*d^2))^(1/6)) - 9*(5*c^7*d^3*x^7 + 64*
c^8*d^2*x^4 + 32*c^9*d*x - sqrt(-3)*(5*c^7*d^3*x^7 + 64*c^8*d^2*x^4 + 32*c
^9*d*x))*(1/(c^19*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 5
12*c^3)) - (c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d + sqrt(-3)*(c^3*d^3*x^6
- 7*c^4*d^2*x^3 - 8*c^5*d))*(1/(c^19*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*
x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^
15*d^2*x^2 + sqrt(-3)*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2))*
(1/(c^19*d^2))^(2/3) - 3*sqrt(d*x^3 + c)*((c^16*d^4*x^7 + 80*c^17*d^3*x^4
+ 160*c^18*d^2*x - sqrt(-3)*(c^16*d^4*x^7 + 80*c^17*d^3*x^4 + 160*c^18*d^2
*x))*(1/(c^19*d^2))^(5/6) - 2*(7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^12
*d)*sqrt(1/(c^19*d^2)) + 6*(5*c^4*d^2*x^5 + 32*c^5*d*x^2 + sqrt(-3)*(5*...
```

3.457.6 Sympy [F]

$$\int \frac{1}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(1/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output

```
Integral(1/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.457.7 Maxima [F]

$$\int \frac{1}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.457.8 Giac [F]

$$\int \frac{1}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

input
```
integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.457.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.458 $\int \frac{1}{x^3(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

3.458.1 Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^3\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}$$

output
```
-1/128*AppellF1(-2/3,3/2,2,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^3
/x^2/(d*x^3+c)^(1/2)
```

3.458.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(66) = 132.

Time = 10.25 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.92

$$\int \frac{1}{x^3\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{167d^2x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+\frac{64c\left(-648c^2-1249cdx^3+\right.}{}}{663}$$

input
```
Integrate[1/(x^3*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output

```
(167*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
(d*x^3)/(8*c)] + (64*c*(-648*c^2 - 1249*c*d*x^3 + 167*d^2*x^6 - (19648*c^2
*d*x^3*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*Appe
llF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/
3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3
, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(8*c - d*x^3))/(663552*c^5*x^2*Sqrt[c +
 d*x^3])
```

3.458.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{x^3 (8c-dx^3)^2 \left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1} \operatorname{AppellF1}\left(-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}$$

input

```
Int[1/(x^3*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output

```
-1/128*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 3/2, 1/3, (d*x^3)/(8*c), -((
d*x^3)/c)])/(c^3*x^2*Sqrt[c + d*x^3])
```

3.458.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.458.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.04 (sec) , antiderivative size = 764, normalized size of antiderivative = 11.58

method	result	size
elliptic	Expression too large to display	764
risch	Expression too large to display	1760
default	Expression too large to display	1806

input
```
int(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15552*d*x/c^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243*d*x/c^4/((x^3+c/d)*d)^(
1/2)-1/128*(d*x^3+c)^(1/2)/c^4/x^2+167/31104*I/c^4*3^(1/2)*(-c*d^2)^(1/3)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/5184*I/c^4/d^2*2^(1/2)*sum(
1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d
^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2
)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-
1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_al
pha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha
=RootOf(_Z^3*d-8*c))
```

3.458.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2650 vs. $2(52) = 104$.

Time = 2.07 (sec) , antiderivative size = 2650, normalized size of antiderivative = 40.15

$$\int \frac{1}{x^3 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/82944*(1264*(d^2*x^8 - 7*c*d*x^5 - 8*c^2*x^2)*sqrt(d)*weierstrassPInver
se(0, -4*c/d, x) - (c^4*d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2 + sqrt(-3)*(c^4*
d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2))*(d^4/c^25)^(1/6)*log((d^6*x^9 + 318*c*
d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - 9*(c^17*d^3*x^8 + 38*c^18*d^2*x
^5 + 64*c^19*d*x^2 + sqrt(-3)*(c^17*d^3*x^8 + 38*c^18*d^2*x^5 + 64*c^19*d*
x^2))*(d^4/c^25)^(2/3) + 3*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4
+ 160*c^23*x - sqrt(-3)*(c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x))*(d^4/
c^25)^(5/6) - 2*(7*c^13*d^3*x^6 + 152*c^14*d^2*x^3 + 64*c^15*d)*sqrt(d^4/c
^25) + 6*(5*c^5*d^4*x^5 + 32*c^6*d^3*x^2 + sqrt(-3)*(5*c^5*d^4*x^5 + 32*c^
6*d^3*x^2))*(d^4/c^25)^(1/6)) - 9*(5*c^9*d^4*x^7 + 64*c^10*d^3*x^4 + 32*c^
11*d^2*x - sqrt(-3)*(5*c^9*d^4*x^7 + 64*c^10*d^3*x^4 + 32*c^11*d^2*x))*(d^
4/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^4*
d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2 + sqrt(-3)*(c^4*d^2*x^8 - 7*c^5*d*x^5 -
8*c^6*x^2))*(d^4/c^25)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x
^3 + 640*c^3*d^3 - 9*(c^17*d^3*x^8 + 38*c^18*d^2*x^5 + 64*c^19*d*x^2 + sqr
t(-3)*(c^17*d^3*x^8 + 38*c^18*d^2*x^5 + 64*c^19*d*x^2))*(d^4/c^25)^(2/3) -
 3*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x - sqrt(-3)*
(c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x))*(d^4/c^25)^(5/6) - 2*(7*c^13*
d^3*x^6 + 152*c^14*d^2*x^3 + 64*c^15*d)*sqrt(d^4/c^25) + 6*(5*c^5*d^4*x^5
+ 32*c^6*d^3*x^2 + sqrt(-3)*(5*c^5*d^4*x^5 + 32*c^6*d^3*x^2))*(d^4/c^25...
```

3.458.6 Sympy [F]

$$\int \frac{1}{x^3 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\, dx = \int \frac{1}{x^3 \left(-8c+dx^3\right)^2 \left(c+dx^3\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**3*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.458.7 Maxima [F]

$$\int \frac{1}{x^3 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}} (dx^3-8\,c)^2 x^3}\,dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^3), x)
```

3.458.8 Giac [F]

$$\int \frac{1}{x^3 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}} (dx^3-8\,c)^2 x^3}\,dx$$

input
```
integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^3), x)
```

3.458.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^3\,(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.459 $\int \frac{1}{x^6\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.459.1 Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^6\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(-\frac{5}{3},2,\frac{3}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}$$

output
```
-1/320*AppellF1(-5/3,3/2,2,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^
3/x^5/(d*x^3+c)^(1/2)
```

3.459.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 283 vs. $2(66) = 132$.

Time = 10.24 (sec) , antiderivative size = 283, normalized size of antiderivative = 4.29

$$\int \frac{1}{x^6\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{\frac{64\left(2592c^3-7128c^2dx^3-15373cd^2x^6+2027d^3x^9\right)}{c^5x^5\left(-8c+dx^3\right)}-\frac{2027d^3x^4\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c}}{c^6}}{}$$

input
```
Integrate[1/(x^6*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((64*(2592*c^3 - 7128*c^2*d*x^3 - 15373*c*d^2*x^6 + 2027*d^3*x^9))/(c^5*x^
5*(-8*c + d*x^3)) - (2027*d^3*x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1
, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c^6 + (16789504*d^2*x*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(c^3*(8*c - d*x^3)*(32*c*AppellF
1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3,
1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -
((d*x^3)/c), (d*x^3)/(8*c)]))))/(6635520*Sqrt[c + d*x^3])
```

3.459.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (8c-dx^3)^2 (c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^6(8c-dx^3)^2\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{5}{3},2,\frac{3}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}$$

input
```
Int[1/(x^6*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
-1/320*(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 2, 3/2, -2/3, (d*x^3)/(8*c), -(
(d*x^3)/c)])/(c^3*x^5*Sqrt[c + d*x^3])
```

3.459.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.459.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.21 (sec) , antiderivative size = 787, normalized size of antiderivative = 11.92

method	result	size
elliptic	Expression too large to display	787
risch	Expression too large to display	1772
default	Expression too large to display	2157

input
```
int(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/124416*d^2*x/c^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/243*d^2*x/c^5/((x^3+c/d)
*d)^(1/2)-1/320*(d*x^3+c)^(1/2)/c^4/x^5+29/2560*d*(d*x^3+c)^(1/2)/c^5/x^2-
2027/311040*I*d/c^5*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^
2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-
I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))-1/31104*I/c^5/d*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I
*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))
)^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(
1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/
3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

3.459.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2698 vs. $2(52) = 104$.

Time = 5.32 (sec) , antiderivative size = 2698, normalized size of antiderivative = 40.88

$$\int \frac{1}{x^6 \left(8c - dx^3\right)^2 \left(c + dx^3\right)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
1/2488320*(49008*(d^3*x^11 - 7*c*d^2*x^8 - 8*c^2*d*x^5)*sqrt(d)*weierstras
sPInverse(0, -4*c/d, x) + 5*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5 + sqrt
(-3)*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5))*(d^10/c^31)^(1/6)*log((d^11
*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^21*d^4*x^8 +
 38*c^22*d^3*x^5 + 64*c^23*d^2*x^2 + sqrt(-3)*(c^21*d^4*x^8 + 38*c^22*d^3*
x^5 + 64*c^23*d^2*x^2))*(d^10/c^31)^(2/3) + 3*sqrt(d*x^3 + c)*((c^26*d^2*x
^7 + 80*c^27*d*x^4 + 160*c^28*x - sqrt(-3)*(c^26*d^2*x^7 + 80*c^27*d*x^4 +
 160*c^28*x))*(d^10/c^31)^(5/6) - 2*(7*c^16*d^5*x^6 + 152*c^17*d^4*x^3 + 6
4*c^18*d^3)*sqrt(d^10/c^31) + 6*(5*c^6*d^8*x^5 + 32*c^7*d^7*x^2 + sqrt(-3)
*(5*c^6*d^8*x^5 + 32*c^7*d^7*x^2))*(d^10/c^31)^(1/6)) - 9*(5*c^11*d^7*x^7
+ 64*c^12*d^6*x^4 + 32*c^13*d^5*x - sqrt(-3)*(5*c^11*d^7*x^7 + 64*c^12*d^6
*x^4 + 32*c^13*d^5*x))*(d^10/c^31)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^
2*d*x^3 - 512*c^3)) - 5*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5 + sqrt(-3)
*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5))*(d^10/c^31)^(1/6)*log((d^11*x^9
 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^21*d^4*x^8 + 38*
c^22*d^3*x^5 + 64*c^23*d^2*x^2 + sqrt(-3)*(c^21*d^4*x^8 + 38*c^22*d^3*x^5
+ 64*c^23*d^2*x^2))*(d^10/c^31)^(2/3) - 3*sqrt(d*x^3 + c)*((c^26*d^2*x^7 +
 80*c^27*d*x^4 + 160*c^28*x - sqrt(-3)*(c^26*d^2*x^7 + 80*c^27*d*x^4 + 160
*c^28*x))*(d^10/c^31)^(5/6) - 2*(7*c^16*d^5*x^6 + 152*c^17*d^4*x^3 + 64*c^
18*d^3)*sqrt(d^10/c^31) + 6*(5*c^6*d^8*x^5 + 32*c^7*d^7*x^2 + sqrt(-3)*...
```

3.459.6 Sympy [F]

$$\int \frac{1}{x^6 \left(8c - dx^3\right)^2 \left(c + dx^3\right)^{3/2}}\, dx = \int \frac{1}{x^6 \left(-8c + dx^3\right)^2 \left(c + dx^3\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**6*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.459.7 Maxima [F]

$$\int \frac{1}{x^6 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2 x^6}\,dx$$

input
```
integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^6), x)
```

3.459.8 Giac [F]

$$\int \frac{1}{x^6 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2 x^6}\,dx$$

input
```
integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^6), x)
```

3.459.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^6\,(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

input
```
int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

output
```
int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.460 $\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

3.460.1 Optimal result

Integrand size = 24, antiderivative size = 161

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)}+\frac{2(c+dx^3)^{3/2}}{9b^2d} -\frac{a^2(c+dx^3)^{3/2}}{3b^2(bc-ad)(a+bx^3)}+\frac{a(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}\sqrt{bc-ad}}$$

output
```
2/9*(d*x^3+c)^(3/2)/b^2/d-1/3*a^2*(d*x^3+c)^(3/2)/b^2/(-a*d+b*c)/(b*x^3+a)
+1/3*a*(-5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^
(7/2)/(-a*d+b*c)^(1/2)-1/3*a*(-5*a*d+4*b*c)*(d*x^3+c)^(1/2)/b^3/(-a*d+b*c)
```

3.460.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.78

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{\sqrt{c+dx^3}(-15a^2d+2ab(c-5dx^3)+2b^2x^3(c+dx^3))}{9b^3d(a+bx^3)} +\frac{a(-4bc+5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}\sqrt{-bc+ad}}$$

input
```
Integrate[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
(Sqrt[c + d*x^3]*(-15*a^2*d + 2*a*b*(c - 5*d*x^3) + 2*b^2*x^3*(c + d*x^3))
)/(9*b^3*d*(a + b*x^3)) + (a*(-4*b*c + 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x
^3])/Sqrt[-(b*c) + a*d]])/(3*b^(7/2)*Sqrt[-(b*c) + a*d])
```

3.460.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {948, 100, 27, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6\sqrt{dx^3+c}}{\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{3}\left(\frac{\int -\frac{\sqrt{dx^3+c}(a(2bc-3ad)-2b(bc-ad)x^3)}{2(bx^3+a)}dx^3}{b^2(bc-ad)}-\frac{a^2\left(c+dx^3\right)^{3/2}}{b^2\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{\sqrt{dx^3+c}(a(2bc-3ad)-2b(bc-ad)x^3)}{bx^3+a}dx^3}{2b^2(bc-ad)}-\frac{a^2\left(c+dx^3\right)^{3/2}}{b^2\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(-\frac{a(4bc-5ad)\int \frac{\sqrt{dx^3+c}}{bx^3+a}dx^3-\frac{4\left(c+dx^3\right)^{3/2}(bc-ad)}{3d}}{2b^2(bc-ad)}-\frac{a^2\left(c+dx^3\right)^{3/2}}{b^2\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(-\frac{a(4bc-5ad)\left(\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{b}+\frac{2\sqrt{c+dx^3}}{b}\right)-\frac{4\left(c+dx^3\right)^{3/2}(bc-ad)}{3d}}{2b^2(bc-ad)}-\frac{a^2\left(c+dx^3\right)^{3/2}}{b^2\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{a(4bc-5ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)-\frac{4\left(c+dx^3\right)^{3/2}(bc-ad)}{3d}}{2b^2(bc-ad)}-\frac{a^2\left(c+dx^3\right)^{3/2}}{b^2\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{a^2\left(c+dx^3\right)^{3/2}}{b^2\left(a+bx^3\right)(bc-ad)}-\frac{a(4bc-5ad)\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)-\frac{4\left(c+dx^3\right)^{3/2}(bc-ad)}{3d}}{2b^2(bc-ad)}\right)$$

input
```
Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
(-((a^2*(c + d*x^3)^(3/2))/(b^2*(b*c - a*d)*(a + b*x^3))) - ((-4*(b*c - a*
d)*(c + d*x^3)^(3/2))/(3*d) + a*(4*b*c - 5*a*d)*((2*Sqrt[c + d*x^3])/b - (
2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3
/2)))/(2*b^2*(b*c - a*d)))/3
```

3.460.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

3.460. $\int \frac{x^8\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.460. $\int \frac{x^8\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

3.460.4 Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{5\left(-\left(ad-\frac{4bc}{5}\right)d\left(b\,x^{3}+a\right)a\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\left(-\frac{2x^{3}\left(d\,x^{3}+c\right)b^{2}}{15}-\frac{2a\left(-5d\,x^{3}+c\right)b}{15}+a^{2}d\right)\sqrt{d\,x^{3}+c}\right)}{3\sqrt{(ad-bc)b}\,d\,b^{3}\left(b\,x^{3}+a\right)}$
default	$\frac{2\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9b^{2}d}+\frac{a^{2}\left(-\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3b^{3}}-\frac{4a\left(\sqrt{d\,x^{3}+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3b^{3}}$
risch	$-\frac{2\left(-bd\,x^{3}+6ad-bc\right)\sqrt{d\,x^{3}+c}}{9d\,b^{3}}+\frac{a\left(\frac{2(3ad-2bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}-\frac{a\left(d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)\left(b\,x^{3}+a\right)+\sqrt{d\,x^{3}+c}\,\sqrt{(ad-bc)b}\right.}{3\sqrt{(ad-bc)b}\left(b\,x^{3}+a\right)}\right.}{b^{3}}$
elliptic	$-\frac{a^{2}\sqrt{d\,x^{3}+c}}{3b^{3}\left(b\,x^{3}+a\right)}+\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9b^{2}}+\frac{2\left(-\frac{2ad-bc}{b^{3}}-\frac{2c}{3b^{2}}\right)\sqrt{d\,x^{3}+c}}{3d}-\frac{i a\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(5ad-4bc)\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{}}{}\right.}{}$

input
```
int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/3/((a*d-b*c)*b)^(1/2)*(-(a*d-4/5*b*c)*d*(b*x^3+a)*a*arctan(b*(d*x^3+c)^
(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(-2/15*x^3*(d*x^3+c)*b^2-2/
15*a*(-5*d*x^3+c)*b+a^2*d)*(d*x^3+c)^(1/2))/d/b^3/(b*x^3+a)
```

3.460.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.91

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$

$$= \Bigg[-\frac{3\,(4\,a^2bcd-5\,a^3d^2+(4\,ab^2cd-5\,a^2bd^2)x^3)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,(2\,(b^4cd}{18\,(ab^5cd-a^2b^4d^2+(b^6cd}$$

$$-\frac{3\,(4\,a^2bcd-5\,a^3d^2+(4\,ab^2cd-5\,a^2bd^2)x^3)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-(2\,(b^4cd-ab^3d^2)}{9\,(ab^5cd-a^2b^4d^2+(b^6cd-ab}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/18*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt
(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c
- a*b*d))/(b*x^3 + a)) - 2*(2*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17
*a^2*b^2*c*d + 15*a^3*b*d^2 + 2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^
3)*sqrt(d*x^3 + c))/(a*b^5*c*d - a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3),
 -1/9*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c
)) - (2*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17*a^2*b^2*c*d + 15*a^3*
b*d^2 + 2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a
*b^5*c*d - a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3)]
```

3.460.6 Sympy [F]

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \int \frac{x^8\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$

input
```
integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x**8*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

3.460.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.460.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = -\frac{\sqrt{dx^3+c}a^2d}{3\left((dx^3+c)b-bc+ad\right)b^3} - \frac{(4\,abc-5\,a^2d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^3} + \frac{2\left((dx^3+c)^{\frac{3}{2}}b^4d^2-6\sqrt{dx^3+c}ab^3d^3\right)}{9\,b^6d^3}$$

input
```
integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*sqrt(d*x^3 + c)*a^2*d/(((d*x^3 + c)*b - b*c + a*d)*b^3) - 1/3*(4*a*b*
c - 5*a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c +
 a*b*d)*b^3) + 2/9*((d*x^3 + c)^(3/2)*b^4*d^2 - 6*sqrt(d*x^3 + c)*a*b^3*d^
3)/(b^6*d^3)
```

3.460.9 Mupad [B] (verification not implemented)

Time = 10.88 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,b^2}-\frac{\sqrt{d\,x^3+c}\,\left(\frac{4\,c}{3\,b^2}-\frac{2\,b^2\,c-2\,a\,b\,d}{b^4}+\frac{2\,a\,d}{b^3}\right)}{3\,d}$$
$$+\frac{a^2\,\left(\frac{2\,a\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{2\,b\,c}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)\,\sqrt{d\,x^3+c}}{b^2\,\left(b\,x^3+a\right)}$$
$$+\frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(5\,a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{7/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int((x^8*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

output
```
(2*x^3*(c + d*x^3)^(1/2))/(9*b^2) - ((c + d*x^3)^(1/2)*((4*c)/(3*b^2) - (2
*b^2*c - 2*a*b*d)/b^4 + (2*a*d)/b^3))/(3*d) + (a*log((2*b*c - a*d + b^(1/2
)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5*a*d -
4*b*c)*1i)/(6*b^(7/2)*(a*d - b*c)^(1/2)) + (a^2*((2*a*d)/(3*(2*b^2*c - 2*a
*b*d)) - (2*b*c)/(3*(2*b^2*c - 2*a*b*d)))*(c + d*x^3)^(1/2))/(b^2*(a + b*x
^3))
```

3.461 $\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

3.461.1 Optimal result

Integrand size = 24, antiderivative size = 136

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx=\frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)}+\frac{a\left(c+dx^3\right)^{3/2}}{3b(bc-ad)\left(a+bx^3\right)}-\frac{(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

output
```
1/3*a*(d*x^3+c)^(3/2)/b/(-a*d+b*c)/(b*x^3+a)-1/3*(-3*a*d+2*b*c)*arctanh(b^
(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)+1/3*(-3*a
*d+2*b*c)*(d*x^3+c)^(1/2)/b^2/(-a*d+b*c)
```

3.461.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx=\frac{\frac{\sqrt{b}\left(3a+2bx^3\right)\sqrt{c+dx^3}}{a+bx^3}+\frac{(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}}{3b^{5/2}}$$

input
```
Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
((Sqrt[b]*(3*a + 2*b*x^3)*Sqrt[c + d*x^3])/(a + b*x^3) + ((2*b*c - 3*a*d)*
ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d])/
(3*b^(5/2))
```

3.461.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {948, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^3\sqrt{dx^3+c}}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \; 87$$

$$\frac{1}{3}\left(\frac{(2bc-3ad)\int \frac{\sqrt{dx^3+c}}{bx^3+a}dx^3}{2b(bc-ad)}+\frac{a\left(c+dx^3\right)^{3/2}}{b\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3}\left(\frac{(2bc-3ad)\left(\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{b}+\frac{2\sqrt{c+dx^3}}{b}\right)}{2b(bc-ad)}+\frac{a\left(c+dx^3\right)^{3/2}}{b\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{(2bc-3ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)}{2b(bc-ad)}+\frac{a\left(c+dx^3\right)^{3/2}}{b\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(\frac{(2bc-3ad)\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{2b(bc-ad)}+\frac{a\left(c+dx^3\right)^{3/2}}{b\left(a+bx^3\right)(bc-ad)}\right)$$

input
```
Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
((a*(c + d*x^3)^(3/2))/(b*(b*c - a*d)*(a + b*x^3)) + ((2*b*c - 3*a*d)*((2*
Sqrt[c + d*x^3])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/
Sqrt[b*c - a*d]])/b^(3/2)))/(2*b*(b*c - a*d)))/3
```

3.461.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

---

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.461.4 Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{-\left(ad-\frac{2bc}{3}\right)\left(b\,x^{3}+a\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)+\left(\frac{2b\,x^{3}}{3}+a\right)\sqrt{\left(ad-bc\right)b}\,\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}\,b^{2}\left(b\,x^{3}+a\right)}$
default	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\left(ad-bc\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{3\sqrt{\left(ad-bc\right)b}}}{b^{2}}-\frac{a\left(-\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{\sqrt{\left(ad-bc\right)b}}\right)}{3b^{2}}$
risch	$\frac{2\sqrt{d\,x^{3}+c}}{3b^{2}}-\frac{\frac{2\left(2ad-bc\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{3\sqrt{\left(ad-bc\right)b}}-\frac{a\left(d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)\left(b\,x^{3}+a\right)+\sqrt{d\,x^{3}+c}\,\sqrt{\left(ad-bc\right)b}\right)}{3\sqrt{\left(ad-bc\right)b}\,\left(b\,x^{3}+a\right)}}{b^{2}}$
elliptic	$\frac{a\sqrt{d\,x^{3}+c}}{3b^{2}\left(b\,x^{3}+a\right)}+\frac{2\sqrt{d\,x^{3}+c}}{3b^{2}}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(3ad-2bc\right)\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{}}{}\right.}{}$

input
```
int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/((a*d-b*c)*b)^(1/2)*(-(a*d-2/3*b*c)*(b*x^3+a)*arctan(b*(d*x^3+c)^(1/2)/(
(a*d-b*c)*b)^(1/2))+(2/3*b*x^3+a)*((a*d-b*c)*b)^(1/2)*(d*x^3+c)^(1/2))/b^2
/(b*x^3+a)
```

3.461.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.46

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$
$$= \left[-\frac{((2\,b^2c-3\,abd)x^3+2\,abc-3\,a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,(3\,ab^2c-3\,a^2bd+}{6\,(ab^4c-a^2b^3d+(b^5c-ab^4d)x^3)}\right.$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/6*(((2*b^2*c - 3*a*b*d)*x^3 + 2*a*b*c - 3*a^2*d)*sqrt(b^2*c - a*b*d)*l
og((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3
+ a)) - 2*(3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d)*x^3)*sqrt(d*x^3 + c
))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3), 1/3*(((2*b^2*c - 3*a*b*d
)*x^3 + 2*a*b*c - 3*a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqr
t(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a
*b^2*d)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3
)]
```

3.461.6 Sympy [F]

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$

input
```
integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x**5*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

3.461.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.461.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.75

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \frac{\sqrt{dx^3+c}ad}{3\left((dx^3+c)b-bc+ad\right)b^2} + \frac{(2\,bc-3\,ad)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^2} + \frac{2\,\sqrt{dx^3+c}}{3\,b^2}$$

input
```
integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*sqrt(d*x^3 + c)*a*d/((((d*x^3 + c)*b - b*c + a*d)*b^2) + 1/3*(2*b*c - 3
*a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*b^2) + 2/3*sqrt(d*x^3 + c)/b^2
```

3.461.9 Mupad [B] (verification not implemented)

Time = 10.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.12

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \frac{2\,\sqrt{d\,x^3+c}}{3\,b^2} - \frac{a\,\left(\frac{2\,a\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{2\,b\,c}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)\,\sqrt{d\,x^3+c}}{b\,\left(b\,x^3+a\right)} + \frac{\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(3\,a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int((x^5*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

output
```
(2*(c + d*x^3)^(1/2))/(3*b^2) + (log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1
/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(3*a*d - 2*b*c)*1i)/(6*b^
(5/2)*(a*d - b*c)^(1/2)) - (a*((2*a*d)/(3*(2*b^2*c - 2*a*b*d)) - (2*b*c)/(
3*(2*b^2*c - 2*a*b*d)))*(c + d*x^3)^(1/2))/(b*(a + b*x^3))
```

3.461. $\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

3.462 $\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

3.462.1 Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = -\frac{\sqrt{c+dx^3}}{3b\,(a+bx^3)} - \frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

output
```
-1/3*d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c
)^(1/2)-1/3*(d*x^3+c)^(1/2)/b/(b*x^3+a)
```

3.462.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = -\frac{\sqrt{c+dx^3}}{3b\,(a+bx^3)} + \frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{3/2}\sqrt{-bc+ad}}$$

input
```
Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
-1/3*Sqrt[c + d*x^3]/(b*(a + b*x^3)) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[-(b*c) + a*d]])/(3*b^(3/2)*Sqrt[-(b*c) + a*d])
```

3.462.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {946, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \textbf{946}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \textbf{51}$$

$$\frac{1}{3}\left(\frac{d\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{2b}-\frac{\sqrt{c+dx^3}}{b\,(a+bx^3)}\right)$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{b}-\frac{\sqrt{c+dx^3}}{b\,(a+bx^3)}\right)$$

$$\Big\downarrow \textbf{221}$$

$$\frac{1}{3}\left(-\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^3}}{b\,(a+bx^3)}\right)$$

input
```
Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
(-(Sqrt[c + d*x^3]/(b*(a + b*x^3))) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[b*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/3
```

3.462.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.462.4 Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

method	result
default	$\frac{-\frac{\sqrt{d x^3+c}}{b x^3+a}+\frac{d \arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{3b}$
pseudoelliptic	$\frac{-\frac{\sqrt{d x^3+c}}{b x^3+a}+\frac{d \arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{3b}$
elliptic	$-\frac{\sqrt{d x^3+c}}{3b(b x^3+a)}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right.}}}{}\right.}{}$

input
```
int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/b*(-(d*x^3+c)^(1/2)/(b*x^3+a)+d/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)
^(1/2)/((a*d-b*c)*b)^(1/2)))
```

3.462.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.19

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$
$$=\left[\frac{(bdx^3+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,\sqrt{dx^3+c}(b^2c-abd)}{6\left(ab^3c-a^2b^2d+(b^4c-ab^3d)x^3\right)},\ \frac{(bdx^3+ad)\sqrt{-b^2c}}{}\right.$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/6*((b*d*x^3 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*s
qrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*sqrt(d*x^3 + c)*(b^2*
c - a*b*d))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^3), 1/3*((b*d*x^3 +
 a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*
d*x^3 + b*c)) - sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(a*b^3*c - a^2*b^2*d + (b
^4*c - a*b^3*d)*x^3)]
```

3.462.6 Sympy [F]

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

input
```
integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x**2*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

3.462.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.462.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{d\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}db} - \frac{\sqrt{dx^3+c}d}{3\left((dx^3+c)b-bc+ad\right)b}$$

input

```
integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output

```
1/3*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*b) - 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*b)
```

3.462.9 Mupad [B] (verification not implemented)

Time = 9.67 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.56

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{\left(\frac{2\,a\,d}{3\,(2\,b^2\,c-2\,a\,b\,d)}-\frac{2\,b\,c}{3\,(2\,b^2\,c-2\,a\,b\,d)}\right)\,\sqrt{d\,x^3+c}}{b\,x^3+a} + \frac{d\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{6\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

input

```
int((x^2*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

output

```
(((2*a*d)/(3*(2*b^2*c - 2*a*b*d)) - (2*b*c)/(3*(2*b^2*c - 2*a*b*d)))*(c +
d*x^3)^(1/2))/(a + b*x^3) + (d*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2
)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(6*b^(3/2)*(a*d - b*c)^
(1/2))
```

3.463 $\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx$

3.463.1 Optimal result

Integrand size = 24, antiderivative size = 121

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} - \frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\sqrt{bc-ad}}$$

output
```
-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a^2+1/3*(-a*d+2*b*c)*arctanh
(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/b^(1/2)/(-a*d+b*c)^(1/2)+1/
3*(d*x^3+c)^(1/2)/a/(b*x^3+a)
```

3.463.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx = \frac{\frac{a\sqrt{c+dx^3}}{a+bx^3}+\frac{(-2bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\sqrt{-bc+ad}}-2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

output
```
((a*Sqrt[c + d*x^3])/(a + b*x^3) + ((-2*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c
+ d*x^3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*Sqrt[-(b*c) + a*d]) - 2*Sqrt[c]*Ar
cTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)
```

3.463.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^3\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}}{a\left(a+bx^3\right)}-\frac{\int -\frac{dx^3+2c}{2x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{dx^3+2c}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{2a}+\frac{\sqrt{c+dx^3}}{a\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{\frac{2c\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{(2bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{2a}+\frac{\sqrt{c+dx^3}}{a\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\frac{4c\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2(2bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{2a}+\frac{\sqrt{c+dx^3}}{a\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\frac{2(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}\sqrt{bc-ad}}-\frac{4\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a}}{2a}+\frac{\sqrt{c+dx^3}}{a\left(a+bx^3\right)}\right)$$

input
```
Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

output
```
(Sqrt[c + d*x^3]/(a*(a + b*x^3)) + ((-4*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sq
rt[c]])/a + (2*(2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c -
a*d]])/(a*Sqrt[b]*Sqrt[b*c - a*d]))/(2*a))/3
```

3.463.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.463.4 Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{\left(b\,x^{3}+a\right)\left(ad-2bc\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)-2\left(\sqrt{c}\,\left(b\,x^{3}+a\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)-\frac{\sqrt{d\,x^{3}+c}\,a}{2}\right)\sqrt{\left(ad-bc\right)b}}{3\sqrt{\left(ad-bc\right)b}\,a^{2}\left(b\,x^{3}+a\right)}$
default	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\sqrt{c}}{3}}{a^{2}}-\frac{2\left(\sqrt{d\,x^{3}+c}-\frac{\left(ad-bc\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{\sqrt{\left(ad-bc\right)b}}\right)}{3a^{2}}-\frac{-\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{\sqrt{\left(ad-bc\right)b}}}{3a}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)*b)^(1/2)*((b*x^3+a)*(a*d-2*b*c)*arctan(b*(d*x^3+c)^(1/2)/((
a*d-b*c)*b)^(1/2))-2*(c^(1/2)*(b*x^3+a)*arctanh((d*x^3+c)^(1/2)/c^(1/2))-1
/2*(d*x^3+c)^(1/2)*a)*((a*d-b*c)*b)^(1/2))/a^2/(b*x^3+a)
```

3.463.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(97) = 194.

Time = 0.39 (sec) , antiderivative size = 856, normalized size of antiderivative = 7.07

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx$$

$$= \Bigg[-\frac{((2\,b^2c-abd)x^3+2\,abc-a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,(ab^2c-a^2bd+(b^3c}{6\,(a^3b^2c-a^4bd+(a^2b^3c-a^3b^2d)x^3)}$$

$$-\frac{((2\,b^2c-abd)x^3+2\,abc-a^2d)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-(ab^2c-a^2bd+(b^3c-ab^2d)x}{3\,(a^3b^2c-a^4bd+(a^2b^3c-a^3b^2d)x^3)}$$

$$-\frac{((2\,b^2c-abd)x^3+2\,abc-a^2d)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-2\,(ab^2c-a^2bd+(b^3c-ab^2d)}{3\,(a^3b^2c-a^4bd+(a^2b^3c-a^3b^2d)x^3)}$$

---

input
```
integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((
b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)
) - 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*s
qrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c)
)/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*(((2*b^2*c - a
*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*s
qrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (a*b^2*c - a^2*b*d + (b^3*c - a*b^2
*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - (a*b
^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b
^2*d)*x^3), 1/6*(4*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-c)*ar
ctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - ((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*
d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt
(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a^
3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*(((2*b^2*c - a*b*d)
*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-
b^2*c + a*b*d)/(b*d*x^3 + b*c)) - 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)
*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (a*b^2*c - a^2*b*d)*sq
rt(d*x^3 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3)]
```

3.463.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x/(b*x**3+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(x*(a + b*x**3)**2), x)
```

3.463.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2x}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x), x)
```

3.463.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx=\frac{\sqrt{dx^3+c}d}{3\left((dx^3+c)b-bc+ad\right)a}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^2}+\frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

input
```
integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*a) - 1/3*(2*b*c - a*d)*
arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2)
+ 2/3*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c))
```

3.463.9 Mupad [B] (verification not implemented)

Time = 12.81 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx=\frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2}-\frac{\left(\frac{b\,d}{3\,\left(b^2\,c-a\,b\,d\right)}-\frac{b^2\,c}{3\,a\,\left(b^2\,c-a\,b\,d\right)}\right)\,\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{d\,x^3+c}\,\sqrt{a\,b\,d-b^2\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,a^2\,\sqrt{a\,b\,d-b^2\,c}}$$

input
```
int((c + d*x^3)^(1/2)/(x*(a + b*x^3)^2),x)
```

output
```
(c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)
))/x^6))/(3*a^2) - (((b*d)/(3*(b^2*c - a*b*d)) - (b^2*c)/(3*a*(b^2*c - a*b
*d)))*(c + d*x^3)^(1/2))/(a + b*x^3) + (log((2*b*c - a*d + (c + d*x^3)^(1/
2)*(a*b*d - b^2*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - 2*b*c)*1i)/(6*a
^2*(a*b*d - b^2*c)^(1/2))
```

3.463. $\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2}\,dx$

3.464 $\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx$

3.464.1 Optimal result

Integrand size = 24, antiderivative size = 161

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx = -\frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{(4bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}} - \frac{\sqrt{b}(4bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{bc-ad}}$$

output
```
1/3*(-a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^3/c^(1/2)-1/3*(-3*a*d+
4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a^3/(-a*d
+b*c)^(1/2)-2/3*b*(d*x^3+c)^(1/2)/a^2/(b*x^3+a)-1/3*(d*x^3+c)^(1/2)/a/x^3/
(b*x^3+a)
```

3.464.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx = \frac{-\frac{a(a+2bx^3)\sqrt{c+dx^3}}{x^3(a+bx^3)} + \frac{\sqrt{b}(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{(4bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^3}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]
```

output
```
(-((a*(a + 2*b*x^3)*Sqrt[c + d*x^3])/(x^3*(a + b*x^3))) + (Sqrt[b]*(4*b*c
- 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c)
 + a*d] + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c])/(3*a^3
)
```

3.464.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 110, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt{dx^3+c}}{x^6\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\left(\frac{\int -\frac{3bdx^3+4bc-ad}{2x^3(bx^3+a)^2\sqrt{dx^3+c}}dx^3}{a}-\frac{\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{3bdx^3+4bc-ad}{x^3(bx^3+a)^2\sqrt{dx^3+c}}dx^3}{2a}-\frac{\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(-\frac{\frac{\int \frac{(bc-ad)\left(2bdx^3+4bc-ad\right)}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a(bc-ad)}+\frac{4b\sqrt{c+dx^3}}{a(a+bx^3)}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\frac{\int \frac{2bdx^3+4bc-ad}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}+\frac{4b\sqrt{c+dx^3}}{a(a+bx^3)}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow 174$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{(4bc-ad)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{b(4bc-3ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{a}+\frac{4b\sqrt{c+dx^3}}{a(a+bx^3)}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{2(4bc-ad)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2b(4bc-3ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{a}+\frac{4b\sqrt{c+dx^3}}{a(a+bx^3)}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{2\sqrt{b}(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(4bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{a}+\frac{4b\sqrt{c+dx^3}}{a(a+bx^3)}}{2a}-\frac{\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

input

```
Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]
```

output

```
(-(Sqrt[c + d*x^3]/(a*x^3*(a + b*x^3))) - ((4*b*Sqrt[c + d*x^3])/(a*(a + b
*x^3)) + ((-2*(4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(a*Sqrt[c])
+ (2*Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c -
a*d]])/(a*Sqrt[b*c - a*d]))/a)/(2*a))/3
```

3.464. $\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx$

3.464.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.464. $\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx$

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.464.4 Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$-\frac{-4x^{3}b\sqrt{c}\,(b\,x^{3}+a)\left(bc-\frac{3ad}{4}\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\,\left(x^{3}(b\,x^{3}+a)(ad-4bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)+(2b\,x^{3}+a)\right.}{3\sqrt{c}\,\sqrt{(ad-bc)b}\,a^{3}(b\,x^{3}+a)x^{3}}$
risch	$-\frac{\sqrt{d\,x^{3}+c}}{3a^{2}x^{3}}-\frac{-\frac{2(-ad+4bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{2b\left(d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)\left(b\,x^{3}+a\right)+\sqrt{d\,x^{3}+c}\,\sqrt{(ad-bc)b}\right)}{3\sqrt{(ad-bc)b}\,\left(b\,x^{3}+a\right)}+\frac{4b(ad-2bc)\arctan}{3a\sqrt{(ad-}}}{2a^{2}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3x^{3}}-\frac{d\ \mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}}{a^{2}}-\frac{2b\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\ \mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\sqrt{c}}{3}\right)}{a^{3}}+\frac{b\left(-\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3a^{2}}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/c^(1/2)*(-4*x^3*b*c^(1/2)*(b*x^3+a)*(b*c-3/4*a*d)*arctan(b*(d*x^3+c)^
(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(x^3*(b*x^3+a)*(a*d-4*b*c)*
arctanh((d*x^3+c)^(1/2)/c^(1/2))+(2*b*x^3+a)*(d*x^3+c)^(1/2)*c^(1/2)*a))/(
(a*d-b*c)*b)^(1/2)/a^3/(b*x^3+a)/x^3
```

3.464.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 870, normalized size of antiderivative = 5.40

$$\int \frac{\sqrt{c+dx^3}}{x^4 \left(a+bx^3\right)^2}\,dx$$

$$= \Bigg[-\frac{((4\,b^2c^2-3\,abcd)x^6+(4\,abc^2-3\,a^2cd)x^3)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+((4\,b^2c-}{6\,(a^3bcx^6+a^4cx^3)}$$

$$-\frac{2\,((4\,b^2c^2-3\,abcd)x^6+(4\,abc^2-3\,a^2cd)x^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)+((4\,b^2c-abd}{6\,(a^3bcx^6+a^4cx^3)}$$

$$-\frac{2\,((4\,b^2c-abd)x^6+(4\,abc-a^2d)x^3)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+((4\,b^2c^2-3\,abcd)x^6+(4\,abc^2-3\,a^2c}{6\,(a^3bcx^6+a^4cx^3)}$$

$$-\frac{((4\,b^2c^2-3\,abcd)x^6+(4\,abc^2-3\,a^2cd)x^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)+((4\,b^2c-abd)x}{3\,(a^3bcx^6+a^4cx^3)}$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/6*(((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(b/
(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sq
rt(b/(b*c - a*d)))/(b*x^3 + a)) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*
d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(2*
a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^4*c*x^3), -1/6*(2*((4
*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(-b/(b*c - a*
d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*
c)) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(c)*log((d*x^3 -
 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(2*a*b*c*x^3 + a^2*c)*sqrt(d*x^
3 + c))/(a^3*b*c*x^6 + a^4*c*x^3), -1/6*(2*((4*b^2*c - a*b*d)*x^6 + (4*a*b
*c - a^2*d)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((4*b^2*c^2
 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((
b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))
/(b*x^3 + a)) + 2*(2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^
4*c*x^3), -1/3*(((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3
)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c -
a*d))/(b*d*x^3 + b*c)) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*s
qrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (2*a*b*c*x^3 + a^2*c)*sqrt(d*
x^3 + c))/(a^3*b*c*x^6 + a^4*c*x^3)]
```

3.464.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**4*(a + b*x**3)**2), x)
```

3.464.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2x^4}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^4), x)
```

3.464.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx = \frac{\left(4\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}a^3}-\frac{\left(4\,bc-ad\right)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}}-\frac{2\,\left(dx^3+c\right)^{\frac{3}{2}}bd-2\,\sqrt{dx^3+c}bcd+\sqrt{dx^3+c}ad^2}{3\,\left(\left(dx^3+c\right)^2b-2\,\left(dx^3+c\right)bc+bc^2+\left(dx^3+c\right)ad-acd\right)a^2}$$

input
```
integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*(4*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sq
rt(-b^2*c + a*b*d)*a^3) - 1/3*(4*b*c - a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c
))/(a^3*sqrt(-c)) - 1/3*(2*(d*x^3 + c)^(3/2)*b*d - 2*sqrt(d*x^3 + c)*b*c*d
 + sqrt(d*x^3 + c)*a*d^2)/(((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 +
(d*x^3 + c)*a*d - a*c*d)*a^2)
```

3.464.9 Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.72

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx$$

$$= \frac{\left(\frac{a\left(\frac{a\left(\frac{a\left(\frac{b^2\,d^2}{2\,a^3\,c^2}-\frac{b^2\,d^2\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^2\,c^2\,\left(a^2\,d-a\,b\,c\right)}+\frac{b^2\,d\,\left(2\,a\,d-b\,c\right)\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}-\frac{b\,d\,\left(2\,a\,d-b\,c\right)}{2\,a^3\,c^2}+\frac{b\,\left(3\,a\,d-4\,b\,c\right)\,\left(-a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+2\,b}{2\,a^3\,c^2}}{b}\right.}{b\,x^3+a}$$

$$-\frac{\sqrt{d\,x^3+c}}{3\,a^2\,x^3}+\frac{\ln\left(\frac{{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)\,\left(a\,d-4\,b\,c\right)}{6\,a^3\,\sqrt{c}}$$
$$+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(3\,a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{6\,a^3\,\sqrt{a\,d-b\,c}}$$

input
```
int((c + d*x^3)^(1/2)/(x^4*(a + b*x^3)^2),x)
```

output
```
(((a*((a*((a*((b^2*d^2)/(2*a^3*c^2) - (b^2*d^2*(3*a*d - 4*b*c))/(6*a^2*c^2
*(a^2*d - a*b*c)) + (b^2*d*(2*a*d - b*c)*(3*a*d - 4*b*c))/(6*a^3*c^2*(a^2*
d - a*b*c))))/b - (b*d*(2*a*d - b*c))/(2*a^3*c^2) + (b*(3*a*d - 4*b*c)*(2*
b^2*c^2 - a^2*d^2 + 2*a*b*c*d))/(6*a^3*c^2*(a^2*d - a*b*c))))/b - (2*b^2*c
^2 - a^2*d^2 + 2*a*b*c*d)/(2*a^3*c^2) + (b*(a*d - 4*b*c)*(3*a*d - 4*b*c))/
(6*a^2*c*(a^2*d - a*b*c))))/b - (a*d - 4*b*c)/(2*a^2*c))*(c + d*x^3)^(1/2)
)/(a + b*x^3) - (c + d*x^3)^(1/2)/(3*a^2*x^3) + (log((((c + d*x^3)^(1/2) -
 c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)*(a*d - 4*b*c))/(6*a^3*c^(1
/2)) + (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(
1/2)*2i - b*d*x^3)/(a + b*x^3))*(3*a*d - 4*b*c)*1i)/(6*a^3*(a*d - b*c)^(1/
2))
```

3.465 $\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

3.465.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx=\frac{x^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},2,-\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/4*x^4*AppellF1(4/3,2,-1/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+
d*x^3/c)^(1/2)
```

3.465.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 235 vs. $2(64) = 128$.

Time = 10.22 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.67

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$
$$=\frac{x\Bigg(\frac{5dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\Big(-c-dx^3+\frac{8ac^2\,\text{AppellF1}\big(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}}{8ac\,\text{AppellF1}\big(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\big)-3x^3\big(2bc\,\text{AppellF1}\big(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}}}{a+bx^3}}{24b\sqrt{c+dx^3}}$$

input
```
Integrate[(x^3*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
(x*((5*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
-((b*x^3)/a)])/a + (8*(-c - d*x^3 + (8*a*c^2*AppellF1[1/3, 1/2, 1, 4/3, -(
(d*x^3)/c), -((b*x^3)/a)])/(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 -((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((
b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
)/(a + b*x^3)))/(24*b*Sqrt[c + d*x^3])
```

3.465.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{\frac{dx^3}{c}+1}}{(bx^3+a)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},2,-\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x^3*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
(x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 2, -1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)])/(4*a^2*Sqrt[1 + (d*x^3)/c])
```

3.465.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.465.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.74 (sec) , antiderivative size = 748, normalized size of antiderivative = 11.69

method	result	size
elliptic	Expression too large to display	748
default	Expression too large to display	1468

input
```
int(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x/b*(d*x^3+c)^(1/2)/(b*x^3+a)-5/9*I/b^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/b^2/d^2*2^(1/2)*sum((5*a*d-2
*b*c)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3)
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha
^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/
3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.465.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.465.6 Sympy [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

input
```
integrate(x**3*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x**3*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

3.465.7 Maxima [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a)^2, x)
```

3.465.8 Giac [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a)^2, x)
```

3.465.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x^3\,\sqrt{d\,x^3+c}}{{(b\,x^3+a)}^2}\,dx$$

input
```
int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

output
```
int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3)^2, x)
```

3.466 $\int \frac{x\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

3.466.1 Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx=\frac{x^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},2,-\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/2*x^2*AppellF1(2/3,2,-1/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+
d*x^3/c)^(1/2)
```

3.466.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 153 vs. 2(64) = 128.

Time = 10.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.39

$$\int \frac{x\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$
$$=\frac{10ax^2(c+dx^3)+5cx^2(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-dx^5(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{Ap}}{30a^2\left(a+bx^3\right)\sqrt{c+dx^3}}$$

input
```
Integrate[(x*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
(10*a*x^2*(c + d*x^3) + 5*c*x^2*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2
/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - d*x^5*(a + b*x^3)*Sqrt[1 +
(d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(30*a^2
*(a + b*x^3)*Sqrt[c + d*x^3])
```

3.466.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{c+dx^3}\int \frac{x\sqrt{\frac{dx^3}{c}+1}}{(bx^3+a)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},2,-\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

output
```
(x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 2, -1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a^2*Sqrt[1 + (d*x^3)/c])
```

3.466.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.466.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.56 (sec) , antiderivative size = 908, normalized size of antiderivative = 14.19

method	result	size
default	Expression too large to display	908
elliptic	Expression too large to display	908

input
```
int(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^2/a*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/a/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)))+1/18*I/a/b/d^2*2^(1/2)*sum((-a*d-2*b*c)/_alpha/(a*
d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)
^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1
/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b
/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp...
```

3.466.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.466.6 Sympy [F]

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

input
```
integrate(x*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

3.466.7 Maxima [F]

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}\,x}{(bx^3+a)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a)^2, x)
```

3.466.8 Giac [F]

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}\,x}{(bx^3+a)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a)^2, x)
```

3.466.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx=\int \frac{x\,\sqrt{d\,x^3+c}}{{\left(b\,x^3+a\right)}^2}\,dx$$

input
```
int((x*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

output
```
int((x*(c + d*x^3)^(1/2))/(a + b*x^3)^2, x)
```

3.467 $\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

3.467.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx=\frac{x\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},2,-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
x*AppellF1(1/3,2,-1/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+d*x^3/
c)^(1/2)
```

3.467.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 232 vs. $2(59) = 118$.

Time = 10.23 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.93

$$\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$
$$=\frac{x\left(\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a^2}+\frac{8\left(\frac{c+dx^3}{a}+\frac{16c^2\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bc\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right.}\right.}{a+bx^3}\right.}{24\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(a + b*x^3)^2,x]
```

output
```
(x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)])/a^2 + (8*((c + d*x^3)/a + (16*c^2*AppellF1[1/3, 1/2, 1, 4/3,
-((d*x^3)/c), -((b*x^3)/a)])/(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c
), -((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -
((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])
)))/(a + b*x^3)))/(24*Sqrt[c + d*x^3])
```

3.467.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{c+dx^3}\int \frac{\sqrt{\frac{dx^3}{c}+1}}{(bx^3+a)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},2,-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(a + b*x^3)^2,x]
```

output
```
(x*Sqrt[c + d*x^3]*AppellF1[1/3, 2, -1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(a^2*Sqrt[1 + (d*x^3)/c])
```

3.467.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.467.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.40 (sec) , antiderivative size = 753, normalized size of antiderivative = 12.76

method	result	size
default	Expression too large to display	753
elliptic	Expression too large to display	753

input
```
int((d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x/a*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/a/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a/b/d^2*2^(1/2)*sum((a*d-4*b*
c)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(
-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*
d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*
3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.467.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.467.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

3.467.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/(b*x^3 + a)^2, x)
```

3.467.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/(b*x^3 + a)^2, x)
```

3.467.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{\left(b\,x^3+a\right)^2}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(a + b*x^3)^2,x)
```

output
```
int((c + d*x^3)^(1/2)/(a + b*x^3)^2, x)
```

3.468 $\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx$

3.468.1 Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx=-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},2,-\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-AppellF1(-1/3,2,-1/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x/(1+d*x^
3/c)^(1/2)
```

3.468.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(62) = 124.

Time = 10.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.77

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx$$
$$=\frac{-20a(3a+4bx^3)\left(c+dx^3\right)+5(-8bc+9ad)x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+8b}{60a^3x\left(a+bx^3\right)\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)^2),x]
```

output
```
(-20*a*(3*a + 4*b*x^3)*(c + d*x^3) + 5*(-8*b*c + 9*a*d)*x^3*(a + b*x^3)*Sq
rt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] +
 8*b*d*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d
*x^3)/c), -((b*x^3)/a)])/(60*a^3*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

3.468.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^2 \left(a+bx^3\right)^2} \, dx$$

$$\Big\downarrow \ 1013$$

$$\frac{\sqrt{c+dx^3} \int \frac{\sqrt{\frac{dx^3}{c}+1}}{x^2 (bx^3+a)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \ 1012$$

$$-\frac{\sqrt{c+dx^3} \operatorname{AppellF1}\left(-\frac{1}{3},2,-\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2 x \sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)^2),x]
```

output
```
-((Sqrt[c + d*x^3]*AppellF1[-1/3, 2, -1/2, 2/3, -((b*x^3)/a), -((d*x^3)/c)
])/(a^2*x*Sqrt[1 + (d*x^3)/c]))
```

3.468.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.468.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.39 (sec) , antiderivative size = 920, normalized size of antiderivative = 14.84

method	result	size
elliptic	Expression too large to display	920
risch	Expression too large to display	1819
default	Expression too large to display	2227

input
```
int((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b/a^2*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/a^2*(d*x^3+c)^(1/2)/x-4/9*I/a^2
*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a^2/d^2*2^(1/2
)*sum((-5*a*d+8*b*c)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*...
```

3.468.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.468.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**2/(b*x**3+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**2*(a + b*x**3)**2), x)
```

3.468.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^2), x)
```

3.468.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2x^2}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^2), x)
```

3.468.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^2\,{\left(b\,x^3+a\right)}^2}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)^2),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)^2), x)
```

3.469 $\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx$

3.469.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx=-\frac{\sqrt{c+dx^3}\,\operatorname{AppellF1}\left(-\frac{2}{3},2,-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-1/2*AppellF1(-2/3,2,-1/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x^2/(
1+d*x^3/c)^(1/2)
```

3.469.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 338 vs. 2(64) = 128.

Time = 10.32 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.28

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx$$
$$=\frac{-5bdx^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\Bigl(32ac\left(6ac+30bcx^3-3adx^3+10bdx^6\right)\operatorname{AppellF1}\bigl(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}}{\left(a+bx^3\right)\Bigl(-8ac\operatorname{AppellF1}\bigl(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\bigr)}}{48a^3x^2\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)^2),x]
```

output
```
(-5*b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -
((b*x^3)/a)] + (a*(32*a*c*(6*a*c + 30*b*c*x^3 - 3*a*d*x^3 + 10*b*d*x^6)*Ap
pellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(3*a + 5*b*x
^3)*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/
a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a +
b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*
x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*Ap
pellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(48*a^3*x^2*Sqrt[
c + d*x^3])
```

3.469.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{c+dx^3}\int \frac{\sqrt{\frac{dx^3}{c}+1}}{x^3\left(bx^3+a\right)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{c+dx^3}\operatorname{AppellF1}\left(-\frac{2}{3},2,-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)^2),x]
```

output
```
-1/2*(Sqrt[c + d*x^3]*AppellF1[-2/3, 2, -1/2, 1/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*x^2*Sqrt[1 + (d*x^3)/c])
```

3.469.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.469.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.58 (sec) , antiderivative size = 766, normalized size of antiderivative = 11.97

method	result	size
elliptic	Expression too large to display	766
risch	Expression too large to display	1513
default	Expression too large to display	1768

input
```
int((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b/a^2*x*(d*x^3+c)^(1/2)/(b*x^3+a)-1/2/a^2*(d*x^3+c)^(1/2)/x^2+5/18*I/
a^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/
18*I/a^2/d^2*2^(1/2)*sum((-7*a*d+10*b*c)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)
*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)
^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*
d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3
)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d
^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)
)
```

3.469.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.469.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/x**3/(b*x**3+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**3)/(x**3*(a + b*x**3)**2), x)
```

3.469.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2x^3}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^3), x)
```

3.469.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2x^3}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^3), x)
```

3.469.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^3\,(a+bx^3)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^3\,(b\,x^3+a)^2}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)^2),x)
```

output
```
int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)^2), x)
```

3.470 $\int \frac{x^8 \left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2} \, dx$

3.470.1 Optimal result

Integrand size = 24, antiderivative size = 189

$$\begin{aligned}\int \frac{x^8 \left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2} \, dx &= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4}-\frac{a(4bc-7ad)\left(c+dx^3\right)^{3/2}}{9b^3(bc-ad)} \\ &+\frac{2\left(c+dx^3\right)^{5/2}}{15b^2d}-\frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2(bc-ad)\left(a+bx^3\right)}+\frac{a(4bc-7ad)\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}\end{aligned}$$

output
```
-1/9*a*(-7*a*d+4*b*c)*(d*x^3+c)^(3/2)/b^3/(-a*d+b*c)+2/15*(d*x^3+c)^(5/2)/
b^2/d-1/3*a^2*(d*x^3+c)^(5/2)/b^2/(-a*d+b*c)/(b*x^3+a)+1/3*a*(-7*a*d+4*b*c
)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(9/
2)-1/3*a*(-7*a*d+4*b*c)*(d*x^3+c)^(1/2)/b^4
```

3.470.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \frac{x^8 \left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2} \, dx &= \frac{\sqrt{c+dx^3} (105a^3d^2+6b^3x^3\left(c+dx^3\right)^2+5a^2bd\left(-19c+14dx^3\right)+2ab^2(3c^2-34cdx^3}{45b^4d\left(a+bx^3\right)} \\ &+\frac{a(4bc-7ad)\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{9/2}}\end{aligned}$$

input
```
Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(Sqrt[c + d*x^3]*(105*a^3*d^2 + 6*b^3*x^3*(c + d*x^3)^2 + 5*a^2*b*d*(-19*c
 + 14*d*x^3) + 2*a*b^2*(3*c^2 - 34*c*d*x^3 - 7*d^2*x^6)))/(45*b^4*d*(a + b
*x^3)) + (a*(4*b*c - 7*a*d)*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(9/2))
```

3.470.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 100, 27, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 \left(c+d x^3\right)^{3/2}}{\left(a+b x^3\right)^2} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^6 \left(d x^3+c\right)^{3/2}}{\left(b x^3+a\right)^2}dx^3$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{3} \left(\frac{\int -\frac{\left(d x^3+c\right)^{3/2} \left(a (2 b c-5 a d)-2 b (b c-a d) x^3\right)}{2 \left(b x^3+a\right)}dx^3}{b^2 (b c-a d)}-\frac{a^2 \left(c+d x^3\right)^{5/2}}{b^2 \left(a+b x^3\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{\int \frac{\left(d x^3+c\right)^{3/2} \left(a (2 b c-5 a d)-2 b (b c-a d) x^3\right)}{b x^3+a}dx^3}{2 b^2 (b c-a d)}-\frac{a^2 \left(c+d x^3\right)^{5/2}}{b^2 \left(a+b x^3\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3} \left(-\frac{a (4 b c-7 a d) \int \frac{\left(d x^3+c\right)^{3/2}}{b x^3+a}dx^3-\frac{4 \left(c+d x^3\right)^{5/2} (b c-a d)}{5 d}}{2 b^2 (b c-a d)}-\frac{a^2 \left(c+d x^3\right)^{5/2}}{b^2 \left(a+b x^3\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(-\frac{a(4bc-7ad)\left(\frac{(bc-ad)\int\frac{\sqrt{dx^3+c}}{bx^3+a}dx^3}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)-\frac{4\left(c+dx^3\right)^{5/2}(bc-ad)}{5d}}{2b^2(bc-ad)}-\frac{a^2\left(c+dx^3\right)^{5/2}}{b^2\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow 60$$

$$\frac{1}{3}\left(-\frac{a(4bc-7ad)\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int\frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{b}+\frac{2\sqrt{c+dx^3}}{b}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)-\frac{4\left(c+dx^3\right)^{5/2}(bc-ad)}{5d}}{2b^2(bc-ad)}-\frac{a^2(c+d}{b^2\left(a+bx^3\right)}\right.$$

$$\Big\downarrow 73$$

$$\frac{1}{3}\left(-\frac{a(4bc-7ad)\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int\frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)-\frac{4\left(c+dx^3\right)^{5/2}(bc-ad)}{5d}}{2b^2(bc-ad)}-\frac{a^2(c+}{b^2(a+bx^3}\right.$$

$$\Big\downarrow 221$$

$$\frac{1}{3}\left(-\frac{a^2\left(c+dx^3\right)^{5/2}}{b^2\left(a+bx^3\right)(bc-ad)}-\frac{a(4bc-7ad)\left(\frac{(bc-ad)\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)-\frac{4(c+d}{}}{2b^2(bc-ad)}\right.$$

3.470. $\int\frac{x^8\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}dx$

input
```
Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(-((a^2*(c + d*x^3)^(5/2))/(b^2*(b*c - a*d)*(a + b*x^3))) - ((-4*(b*c - a*
d)*(c + d*x^3)^(5/2))/(5*d) + a*(4*b*c - 7*a*d)*((2*(c + d*x^3)^(3/2))/(3*
b) + ((b*c - a*d)*((2*Sqrt[c + d*x^3])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2)))/b))/(2*b^2*(b*c - a*d)))
/3
```

3.470.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.470.4 Maple [A] (verified)

Time = 4.65 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{7\left(\left(ad-\frac{4bc}{7}\right)d\left(bx^3+a\right)a(ad-bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)-\sqrt{dx^3+c}\left(\frac{2x^3\left(dx^3+c\right)^2b^3}{35}+\frac{2\left(-\frac{7}{3}d^2x^6-\frac{34}{3}cdx^3+c^2\right)ab^2}{35}-19da^2\right.\right.}{3\sqrt{(ad-bc)b}\,b^4d\left(bx^3+a\right)}$
default	$\frac{2\left(dx^3+c\right)^{\frac{5}{2}}}{15b^2d}+\frac{a^2\left(-d\left(bx^3+a\right)(ad-bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{\left(2dx^3-c\right)b}{3}+ad\right)\sqrt{dx^3+c}\,\sqrt{(ad-bc)b}\right)}{b^4\sqrt{(ad-bc)b}\,\left(bx^3+a\right)}+4a\left(-(ad-b\right.$
risch	$\frac{2\left(3b^2d^2x^6-10x^3abd^2+6x^3b^2cd+45a^2d^2-40abcd+3b^2c^2\right)\sqrt{dx^3+c}}{45db^4}-a\left(\frac{2\left(4a^2d^2-6abcd+2b^2c^2\right)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}-a\left(a\right.\right.$
elliptic	$\frac{a^2(ad-bc)\sqrt{dx^3+c}}{3b^4\left(bx^3+a\right)}+\frac{2dx^6\sqrt{dx^3+c}}{15b^2}+\frac{2\left(-\frac{2(ad-bc)d}{b^3}-\frac{4cd}{5b^2}\right)x^3\sqrt{dx^3+c}}{9d}+\frac{2\left(\frac{3a^2d^2-4abcd+b^2c^2}{b^4}-\frac{2\left(-\frac{2(ad-bc)d}{b^3}-\frac{4cd}{5b^2}\right.}{3d}\right.}{3d}$

input
```
int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-7/3*((a*d-4/7*b*c)*d*(b*x^3+a)*a*(a*d-b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d
-b*c)*b)^(1/2))-(d*x^3+c)^(1/2)*(2/35*x^3*(d*x^3+c)^2*b^3+2/35*(-7/3*d^2*x
^6-34/3*c*d*x^3+c^2)*a*b^2-19/21*d*a^2*(-14/19*d*x^3+c)*b+a^3*d^2)*((a*d-b
*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b^4/d/(b*x^3+a)
```

3.470.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.34

$$\int\frac{x^8\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx=\left[-\frac{15\left(4a^2bcd-7a^3d^2+\left(4ab^2cd-7a^2bd^2\right)x^3\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}b\sqrt{bc}}{bx^3+a}\right.}{}\right.$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/90*(15*(4*a^2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqr
t((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*
c - a*d)/b))/(b*x^3 + a)) - 2*(6*b^3*d^2*x^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)
*x^6 + 6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*
c*d + 35*a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^5*d*x^3 + a*b^4*d), 1/45*(15*
(4*a^2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqrt(-(b*c - a
*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (6*b^
3*d^2*x^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)*x^6 + 6*a*b^2*c^2 - 95*a^2*b*c*d +
 105*a^3*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*c*d + 35*a^2*b*d^2)*x^3)*sqrt(d*x^3
 + c))/(b^5*d*x^3 + a*b^4*d)]
```

3.470.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.470.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.470.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.12

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = -\frac{(4\,ab^2c^2-11\,a^2bcd+7\,a^3d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^4} - \frac{\sqrt{dx^3+c}a^2bcd-\sqrt{dx^3+c}a^3d^2}{3\left((dx^3+c)b-bc+ad\right)b^4} + \frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}b^8d^4-10\,(dx^3+c)^{\frac{3}{2}}ab^7d^5-30\sqrt{dx^3+c}ab^7cd^5+45\sqrt{dx^3+c}a^2b^6d^6\right)}{45\,b^{10}d^5}$$

input
```
integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*(4*a*b^2*c^2 - 11*a^2*b*c*d + 7*a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqr
t(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) - 1/3*(sqrt(d*x^3 + c)*a^2*b
*c*d - sqrt(d*x^3 + c)*a^3*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^4) + 2/45*(
3*(d*x^3 + c)^(5/2)*b^8*d^4 - 10*(d*x^3 + c)^(3/2)*a*b^7*d^5 - 30*sqrt(d*x
^3 + c)*a*b^7*c*d^5 + 45*sqrt(d*x^3 + c)*a^2*b^6*d^6)/(b^10*d^5)
```

3.470.9 Mupad [B] (verification not implemented)

Time = 12.04 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.75

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{\sqrt{d\,x^3+c}\left(\frac{2\,(a\,d-b\,c)^2}{b^4}+\frac{2\,c\left(\frac{2\,d\,(a\,d-2\,b\,c)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{8\,c\,d}{5\,b^2}\right)}{3\,d}+\frac{2\,a\left(\frac{d\,(a\,d-2\,b\,c)}{b^3}+\frac{a\,d^2}{b^3}\right)}{b}\right)}{3\,d} + \frac{2\,d\,x^6\sqrt{d\,x^3+c}}{15\,b^2} - \frac{x^3\sqrt{d\,x^3+c}\left(\frac{2\,d\,(a\,d-2\,b\,c)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{8\,c\,d}{5\,b^2}\right)}{9\,d} - \frac{a^2\left(\frac{2\,b\,c^2}{3\,(2\,b^2\,c-2\,a\,b\,d)}+\frac{a\left(\frac{2\,a\,d^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\sqrt{d\,x^3+c}}{b^2\,\left(b\,x^3+a\right)} + \frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\sqrt{a\,d-b\,c}\,(7\,a\,d-4\,b\,c)\,1{}\mathrm{i}}{6\,b^{9/2}}$$

input
```
int((x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

output

```
((c + d*x^3)^(1/2)*((2*(a*d - b*c)^2)/b^4 + (2*c*((2*d*(a*d - 2*b*c))/b^3
+ (2*a*d^2)/b^3 + (8*c*d)/(5*b^2)))/(3*d) + (2*a*((d*(a*d - 2*b*c))/b^3 +
(a*d^2)/b^3))/b))/(3*d) + (2*d*x^6*(c + d*x^3)^(1/2))/(15*b^2) - (x^3*(c +
 d*x^3)^(1/2)*((2*d*(a*d - 2*b*c))/b^3 + (2*a*d^2)/b^3 + (8*c*d)/(5*b^2)))
/(9*d) + (a*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)
*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(7*a*d - 4*b*c)*1i)/(6*b^(9/
2)) - (a^2*((2*b*c^2)/(3*(2*b^2*c - 2*a*b*d)) + (a*((2*a*d^2)/(3*(2*b^2*c
- 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*a*b*d))))/b)*(c + d*x^3)^(1/2))/(b
^2*(a + b*x^3))
```

3.470. $\int \frac{x^8 (c+dx^3)^{3/2}}{(a+bx^3)^2} \, dx$

3.471 $\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx$

3.471.1 Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx = \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)\left(c+dx^3\right)^{3/2}}{9b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} - \frac{(2bc-5ad)\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

output
```
1/9*(-5*a*d+2*b*c)*(d*x^3+c)^(3/2)/b^2/(-a*d+b*c)+1/3*a*(d*x^3+c)^(5/2)/b/
(-a*d+b*c)/(b*x^3+a)-1/3*(-5*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-
a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(7/2)+1/3*(-5*a*d+2*b*c)*(d*x^3+c)^(1/2
)/b^3
```

3.471.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.77

$$\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}\left(-15a^2d+ab\left(11c-10dx^3\right)+2b^2x^3\left(4c+dx^3\right)\right)}{9b^3\left(a+bx^3\right)} - \frac{(2bc-5ad)\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}}$$

input
```
Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(Sqrt[c + d*x^3]*(-15*a^2*d + a*b*(11*c - 10*d*x^3) + 2*b^2*x^3*(4*c + d*x
^3)))/(9*b^3*(a + b*x^3)) - ((2*b*c - 5*a*d)*Sqrt[-(b*c) + a*d]*ArcTan[(Sq
rt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(7/2))
```

3.471.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3 \left(dx^3+c\right)^{3/2}}{\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{(2bc-5ad)\int \frac{\left(dx^3+c\right)^{3/2}}{bx^3+a}dx^3}{2b(bc-ad)}+\frac{a\left(c+dx^3\right)^{5/2}}{b\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{(2bc-5ad)\left(\frac{(bc-ad)\int \frac{\sqrt{dx^3+c}}{bx^3+a}dx^3}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)}{2b(bc-ad)}+\frac{a\left(c+dx^3\right)^{5/2}}{b\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{(2bc-5ad)\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{b}+\frac{2\sqrt{c+dx^3}}{b}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)}{2b(bc-ad)}+\frac{a\left(c+dx^3\right)^{5/2}}{b\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{(2bc-5ad)\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)}{2b(bc-ad)}+\frac{a\left(c+dx^3\right)^{5/2}}{b\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{(2bc-5ad)\left(\frac{(bc-ad)\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}+\frac{2\left(c+dx^3\right)^{3/2}}{3b}\right)}{2b(bc-ad)}+\frac{a\left(c+dx^3\right)^{5/2}}{b\left(a+bx^3\right)(bc-ad)}\right)$$

input
```
Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
((a*(c + d*x^3)^(5/2))/(b*(b*c - a*d)*(a + b*x^3)) + ((2*b*c - 5*a*d)*((2*
(c + d*x^3)^(3/2))/(3*b) + ((b*c - a*d)*((2*Sqrt[c + d*x^3])/b - (2*Sqrt[b
*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2)))/b)
)/(2*b*(b*c - a*d)))/3
```

3.471. $\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}dx$

3.471.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.471. $\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$

3.471.4 Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$-\frac{5(-(b x^3+a)(ad-\frac{2bc}{5})(ad-bc)\arctan(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}})+\sqrt{(ad-bc)b}(-\frac{8x^3(\frac{d x^3}{4}+c)b^2}{15}-\frac{11(-\frac{10d x^3}{11}+c)ab}{15}+a^2d)\sqrt{d x^3}}{3\sqrt{(ad-bc)b}\,b^3(b x^3+a)}$
default	$-\frac{2(-(ad-bc)^2\arctan(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}})+\sqrt{d x^3+c}(\frac{(-d x^3-4c)b}{3}+ad)\sqrt{(ad-bc)b})}{3b^3\sqrt{(ad-bc)b}}-\frac{a(-d(b x^3+a)(ad-bc)\arctan(\frac{b\sqrt{}}{\sqrt{(}}}{b^3}$
risch	$-\frac{2(-bd x^3+6ad-4bc)\sqrt{d x^3+c}}{9b^3}+\frac{\frac{2(3a^2d^2-4abcd+b^2c^2)\arctan(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}})}{3\sqrt{(ad-bc)b}}-\frac{a(a^2d^2-2abcd+b^2c^2)(d\arctan(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}})(}{3\sqrt{(ad-bc)b}\,(ad-bc)(b}}{b^3}$
elliptic	$-\frac{(ad-bc)a\sqrt{d x^3+c}}{3b^3(b x^3+a)}+\frac{2d x^3\sqrt{d x^3+c}}{9b^2}+\frac{2(-\frac{2(ad-bc)d}{b^3}-\frac{2cd}{3b^2})\sqrt{d x^3+c}}{3d}+\frac{i\sqrt{2}\,(\sum_{\_\alpha=\mathrm{RootOf}(b\_Z^3+a)}\frac{(-5a^2d^2+7abcd}{}}{}$

input
```
int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/3/((a*d-b*c)*b)^(1/2)*(-(b*x^3+a)*(a*d-2/5*b*c)*(a*d-b*c)*arctan(b*(d*x
^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(-8/15*x^3*(1/4*d*x^3
+c)*b^2-11/15*(-10/11*d*x^3+c)*a*b+a^2*d)*(d*x^3+c)^(1/2))/b^3/(b*x^3+a)
```

3.471.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.93

$$\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx = \left[-\frac{3\left((2\,b^2c-5\,abd)x^3+2\,abc-5\,a^2d\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-}{18\left(b^4x^3+ab^3\right)}\right.$$

$$-\frac{3\left((2\,b^2c-5\,abd)x^3+2\,abc-5\,a^2d\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(2\,b^2dx^6+2\left(4\,b^2c-5\,abd\right)x^3\right.}{9\left(b^4x^3+ab^3\right)}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/18*(3*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*sqrt((b*c - a*d)/b
)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b
*x^3 + a)) - 2*(2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 11*a*b*c - 15*a^
2*d)*sqrt(d*x^3 + c))/(b^4*x^3 + a*b^3), -1/9*(3*((2*b^2*c - 5*a*b*d)*x^3
+ 2*a*b*c - 5*a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-
(b*c - a*d)/b)/(b*c - a*d)) - (2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 1
1*a*b*c - 15*a^2*d)*sqrt(d*x^3 + c))/(b^4*x^3 + a*b^3)]
```

3.471.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.471.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5 (c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.471.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.06

$$\int \frac{x^5 (c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{(2\,b^2c^2 - 7\,abcd + 5\,a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^3} + \frac{\sqrt{dx^3+c}abcd - \sqrt{dx^3+c}a^2d^2}{3\,((dx^3+c)b - bc + ad)b^3} + \frac{2\,\left((dx^3+c)^{\frac{3}{2}}b^4 + 3\,\sqrt{dx^3+c}b^4c - 6\,\sqrt{dx^3+c}ab^3d\right)}{9\,b^6}$$

input
```
integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*(2*b^2*c^2 - 7*a*b*c*d + 5*a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2
*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/3*(sqrt(d*x^3 + c)*a*b*c*d - s
qrt(d*x^3 + c)*a^2*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^3) + 2/9*((d*x^3 +
c)^(3/2)*b^4 + 3*sqrt(d*x^3 + c)*b^4*c - 6*sqrt(d*x^3 + c)*a*b^3*d)/b^6
```

3.471.9 Mupad [B] (verification not implemented)

Time = 11.58 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.40

$$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{2\,d\,x^3\,\sqrt{d\,x^3+c}}{9\,b^2} - \frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,d\,(a\,d-2\,b\,c)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{4\,c\,d}{3\,b^2}\right)}{3\,d} + \frac{a\,\left(\frac{2\,b\,c^2}{3\,(2\,b^2\,c-2\,a\,b\,d)}+\frac{a\,\left(\frac{2\,a\,d^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\,\sqrt{d\,x^3+c}}{b\,\left(b\,x^3+a\right)} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,(5\,a\,d-2\,b\,c)\,1\mathrm{i}}{6\,b^{7/2}}$$

input

```
int((x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

output

```
(2*d*x^3*(c + d*x^3)^(1/2))/(9*b^2) - ((c + d*x^3)^(1/2)*((2*d*(a*d - 2*b*
c))/b^3 + (2*a*d^2)/b^3 + (4*c*d)/(3*b^2)))/(3*d) + (log((2*b*c - a*d + b^
(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d
- b*c)^(1/2)*(5*a*d - 2*b*c)*1i)/(6*b^(7/2)) + (a*((2*b*c^2)/(3*(2*b^2*c -
 2*a*b*d)) + (a*((2*a*d^2)/(3*(2*b^2*c - 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c
 - 2*a*b*d))))/b)*(c + d*x^3)^(1/2))/(b*(a + b*x^3))
```

3.472 $\int \frac{x^2\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx$

3.472.1 Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\frac{d\sqrt{c+dx^3}}{b^2}-\frac{(c+dx^3)^{3/2}}{3b\,(a+bx^3)}-\frac{d\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

output
```
-1/3*(d*x^3+c)^(3/2)/b/(b*x^3+a)-d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b
*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(5/2)+d*(d*x^3+c)^(1/2)/b^2
```

3.472.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\frac{\sqrt{c+dx^3}(-bc+3ad+2bdx^3)}{3b^2\,(a+bx^3)}-\frac{d\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{b^{5/2}}$$

input
```
Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(Sqrt[c + d*x^3]*(-(b*c) + 3*a*d + 2*b*d*x^3))/(3*b^2*(a + b*x^3)) - (d*Sq
rt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/b^(
5/2)
```

3.472.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {946, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2} \, dx$$

$$\Big\downarrow \; \color{blue}{946}$$

$$\frac{1}{3} \int \frac{\left(dx^3+c\right)^{3/2}}{\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \; \color{blue}{51}$$

$$\frac{1}{3} \left(\frac{3d \int \frac{\sqrt{dx^3+c}}{bx^3+a}dx^3}{2b}-\frac{\left(c+dx^3\right)^{3/2}}{b\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \; \color{blue}{60}$$

$$\frac{1}{3} \left(\frac{3d \left(\frac{(bc-ad) \int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{b}+\frac{2\sqrt{c+dx^3}}{b}\right)}{2b}-\frac{\left(c+dx^3\right)^{3/2}}{b\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \; \color{blue}{73}$$

$$\frac{1}{3} \left(\frac{3d \left(\frac{2(bc-ad) \int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd}+\frac{2\sqrt{c+dx^3}}{b}\right)}{2b}-\frac{\left(c+dx^3\right)^{3/2}}{b\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \; \color{blue}{221}$$

$$\frac{1}{3} \left(\frac{3d \left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{2b}-\frac{\left(c+dx^3\right)^{3/2}}{b\left(a+bx^3\right)}\right)$$

input
```
Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(-((c + d*x^3)^(3/2)/(b*(a + b*x^3))) + (3*d*((2*Sqrt[c + d*x^3])/b - (2*S
qrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2)
))/(2*b))/3
```

3.472.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.472.4 Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

method	result
default	$\frac{-d(bx^3+a)(ad-bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{(2dx^3-c)b}{3}+ad\right)\sqrt{dx^3+c}\,\sqrt{(ad-bc)b}}{\sqrt{(ad-bc)b}\,b^2(bx^3+a)}$
pseudoelliptic	$\frac{-d(bx^3+a)(ad-bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{(2dx^3-c)b}{3}+ad\right)\sqrt{dx^3+c}\,\sqrt{(ad-bc)b}}{\sqrt{(ad-bc)b}\,b^2(bx^3+a)}$
risch	$\frac{2d\sqrt{dx^3+c}}{3b^2}-\frac{\frac{4(ad-bc)d\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}+\frac{(-a^2d^2+2abcd-b^2c^2)\left(d\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)(bx^3+a)+\sqrt{dx^3+c}\,\sqrt{(ad-bc)b}\right)}{3\sqrt{(ad-bc)b}\,(ad-bc)(bx^3+a)}}{b^2}$
elliptic	$\frac{(ad-bc)\sqrt{dx^3+c}}{3b^2(bx^3+a)}+\frac{2d\sqrt{dx^3+c}}{3b^2}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-cd^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{}{-3(}}}{}\right.}{}$

input
```
int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/((a*d-b*c)*b)^(1/2)*(-d*(b*x^3+a)*(a*d-b*c)*arctan(b*(d*x^3+c)^(1/2)/((a
*d-b*c)*b)^(1/2))+(1/3*(2*d*x^3-c)*b+a*d)*(d*x^3+c)^(1/2)*((a*d-b*c)*b)^(1
/2))/b^2/(b*x^3+a)
```

3.472.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.49

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \left[\frac{3\left(bdx^3+ad\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left(2\,bdx^3-bc+3\,ad\right)\sqrt{d}}{6\left(b^3x^3+ab^2\right)}\right.$$

$$\left.-\frac{3\left(bdx^3+ad\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(2\,bdx^3-bc+3\,ad\right)\sqrt{dx^3+c}}{3\left(b^3x^3+ab^2\right)}\right]$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/6*(3*(b*d*x^3 + a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2
*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(2*b*d*x^3 - b*c
+ 3*a*d)*sqrt(d*x^3 + c))/(b^3*x^3 + a*b^2), -1/3*(3*(b*d*x^3 + a*d)*sqrt(
-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)
) - (2*b*d*x^3 - b*c + 3*a*d)*sqrt(d*x^3 + c))/(b^3*x^3 + a*b^2)]
```

3.472.6 Sympy [F]

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{x^2(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

input
```
integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x**2*(c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

3.472.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.472.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{2\sqrt{dx^3+c}d}{3\,b^2} + \frac{(bcd-ad^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2} - \frac{\sqrt{dx^3+c}bcd-\sqrt{dx^3+c}ad^2}{3\left((dx^3+c)b-bc+ad\right)b^2}$$

input
```
integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
2/3*sqrt(d*x^3 + c)*d/b^2 + (b*c*d - a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(
-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) - 1/3*(sqrt(d*x^3 + c)*b*c*d -
 sqrt(d*x^3 + c)*a*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^2)
```

3.472.9 Mupad [B] (verification not implemented)

Time = 11.37 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.81

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{2\,d\,\sqrt{d\,x^3+c}}{3\,b^2} - \frac{\left(\frac{2\,b\,c^2}{3\,(2\,b^2\,c-2\,a\,b\,d)}+\frac{a\,\left(\frac{2\,a\,d^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\,\sqrt{d\,x^3+c}}{b\,x^3+a} + \frac{d\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,1\mathrm{i}}{2\,b^{5/2}}$$

input
```
int((x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

output
```
(2*d*(c + d*x^3)^(1/2))/(3*b^2) - (((2*b*c^2)/(3*(2*b^2*c - 2*a*b*d)) + (a
*((2*a*d^2)/(3*(2*b^2*c - 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*a*b*d))))/
b)*(c + d*x^3)^(1/2))/(a + b*x^3) + (d*log((a*d - 2*b*c + b^(1/2)*(c + d*x
^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1
i)/(2*b^(5/2))
```

3.473 $\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx$

3.473.1 Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx = \frac{(bc-ad)\sqrt{c+dx^3}}{3ab(a+bx^3)} - \frac{2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{\sqrt{bc-ad}(2bc+ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}}$$

output
```
-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2+1/3*(a*d+2*b*c)*arctanh(
b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a^2/b^(3/2)+1/3
*(-a*d+b*c)*(d*x^3+c)^(1/2)/a/b/(b*x^3+a)
```

3.473.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx = \frac{\frac{a(bc-ad)\sqrt{c+dx^3}}{b(a+bx^3)} + \frac{\sqrt{-bc+ad}(2bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{b^{3/2}} - 2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x]
```

output
```
((a*(b*c - a*d)*Sqrt[c + d*x^3])/(b*(a + b*x^3)) + (Sqrt[-(b*c) + a*d]*(2*
b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/b^(3/2) -
 2*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)
```

3.473.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \textcolor{blue}{948}$$

$$\frac{1}{3}\int \frac{\left(dx^3+c\right)^{3/2}}{x^3\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \textcolor{blue}{109}$$

$$\frac{1}{3}\left(\frac{\int \frac{d(bc+ad)x^3+2bc^2}{2x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{ab}+\frac{\sqrt{c+dx^3}(bc-ad)}{ab\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{d(bc+ad)x^3+2bc^2}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{2ab}+\frac{\sqrt{c+dx^3}(bc-ad)}{ab\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \textcolor{blue}{174}$$

$$\frac{1}{3}\left(\frac{\frac{2bc^2\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{(bc-ad)(ad+2bc)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{2ab}+\frac{\sqrt{c+dx^3}(bc-ad)}{ab\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \textcolor{blue}{73}$$

$$\frac{1}{3}\left(\frac{\frac{4bc^2\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2(bc-ad)(ad+2bc)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{2ab}+\frac{\sqrt{c+dx^3}(bc-ad)}{ab\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$\frac{1}{3}\left(\frac{\frac{2\sqrt{bc-ad}(ad+2bc)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{4bc^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a}}{2ab}+\frac{\sqrt{c+dx^3}(bc-ad)}{ab\left(a+bx^3\right)}\right)$$

input

```
Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x]
```

output

```
(((b*c - a*d)*Sqrt[c + d*x^3])/(a*b*(a + b*x^3)) + ((-4*b*c^(3/2)*ArcTanh[
Sqrt[c + d*x^3]/Sqrt[c]])/a + (2*Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/(2*a*b))/3
```

3.473.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.473.4 Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{-\left(b\,x^{3}+a\right)\left(a d+2 b c\right)\left(a d-b c\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d-b c\right) b}}\right)+\sqrt{\left(a d-b c\right) b}\,\left(2 b\,c^{\frac{3}{2}}\left(b\,x^{3}+a\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)+a\sqrt{d\,x^{3}+c}\,\left(a d-b c\right)}{3\sqrt{\left(a d-b c\right) b}\,a^{2} b\left(b\,x^{3}+a\right)}$
default	$\frac{\frac{2 d\,x^{3}\sqrt{d\,x^{3}+c}}{9}+\frac{8 c\sqrt{d\,x^{3}+c}}{9}-\frac{2 c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}}{a^{2}}+\frac{-\frac{2\left(a d-b c\right)^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(a d-b c\right) b}}\right)}{3}+\frac{2\sqrt{d\,x^{3}+c}\,\left(\frac{\left(-d\,x^{3}-4 c\right) b}{3}+a d\right)\sqrt{(ad}{3}}{b\,a^{2}\sqrt{\left(a d-b c\right) b}}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-(b*x^3+a)*(a*d+2*b*c)*(a*d-b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)
*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(2*b*c^(3/2)*(b*x^3+a)*arctanh((d*x^3+c)^(1
/2)/c^(1/2))+a*(d*x^3+c)^(1/2)*(a*d-b*c)))/((a*d-b*c)*b)^(1/2)/a^2/b/(b*x^
3+a)
```

3.473.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 686, normalized size of antiderivative = 5.24

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)^2}\,dx=\left[\frac{\left(\left(2\,b^2c+abd\right)x^3+2\,abc+a^2d\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left(b^2cx^3+\right.}{6\left(a^2b^2x^3+a^3b\right)}\right.$$

input
```
integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

output

```
[1/6*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt((b*c - a*d)/b)*log((b
*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a
)) + 2*(b^2*c*x^3 + a*b*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c)
+ 2*c)/x^3) + 2*sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/
3*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-
sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b^2*c*x^3 + a*b*c)*
sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + sqrt(d*x^3 +
c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/6*(4*(b^2*c*x^3 + a*b*c)*sqrt
(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((2*b^2*c + a*b*d)*x^3 + 2*a*b*c
 + a^2*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 +
c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(a*b*c - a^2*d)
)/(a^2*b^2*x^3 + a^3*b), 1/3*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sq
rt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a
*d)) + 2*(b^2*c*x^3 + a*b*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) +
 sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b)]
```

3.473.6 Sympy [F]

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)^2}\,dx=\int \frac{\left(c+dx^3\right)^{\frac{3}{2}}}{x\left(a+bx^3\right)^2}\,dx$$

input

```
integrate((d*x**3+c)**(3/2)/x/(b*x**3+a)**2,x)
```

output

```
Integral((c + d*x**3)**(3/2)/(x*(a + b*x**3)**2), x)
```

3.473.7 Maxima [F]

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)^2}\,dx=\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2x}\,dx$$

input

```
integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

output

```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x), x)
```

3.473.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.18

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)^2}\,dx = \frac{2\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}} - \frac{\left(2\,b^2c^2-abcd-a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^2b} + \frac{\sqrt{dx^3+c}bcd-\sqrt{dx^3+c}ad^2}{3\left((dx^3+c)b-bc+ad\right)ab}$$

input
```
integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
2/3*c^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/3*(2*b^2*c^2 -
 a*b*c*d - a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*a^2*b) + 1/3*(sqrt(d*x^3 + c)*b*c*d - sqrt(d*x^3 + c)*a*d^2
)/(((d*x^3 + c)*b - b*c + a*d)*a*b)
```

3.473.9 Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.63

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)^2}\,dx = \frac{c^{3/2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2} + \frac{\sqrt{d\,x^3+c}\left(\frac{a\left(\frac{b\,d^2}{3\left(b^2\,c-a\,b\,d\right)}-\frac{2\,b^2\,c\,d}{3\,a\left(b^2\,c-a\,b\,d\right)}\right)}{b}+\frac{b^2\,c^2}{3\,a\,(b^2\,c-a\,b\,d)}\right)}{b\,x^3+a} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,(a\,d+2\,b\,c)\,1\mathrm{i}}{6\,a^2\,b^{3/2}}$$

input
```
int((c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x)
```

output
```
(c^(3/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)
))/x^6))/(3*a^2) + ((c + d*x^3)^(1/2)*((a*((b*d^2)/(3*(b^2*c - a*b*d)) - (
2*b^2*c*d)/(3*a*(b^2*c - a*b*d))))/b + (b^2*c^2)/(3*a*(b^2*c - a*b*d))))/(
a + b*x^3) + (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/
2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(a*d + 2*b*c)*1i)/(6*a^2*b
^(3/2))
```

3.474 $\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx$

3.474.1 Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx = -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{c\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{\sqrt{c}(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3} - \frac{\sqrt{bc-ad}(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{b}}$$

output
```
1/3*(-3*a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a^3-1/3*(-a*d+
4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/
a^3/b^(1/2)-1/3*(-a*d+2*b*c)*(d*x^3+c)^(1/2)/a^2/(b*x^3+a)-1/3*c*(d*x^3+c)
^(1/2)/a/x^3/(b*x^3+a)
```

3.474.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx = \frac{\frac{a\sqrt{c+dx^3}(-ac-2bcx^3+adx^3)}{x^3(a+bx^3)} + \frac{(4b^2c^2-5abcd+a^2d^2)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\sqrt{-bc+ad}} + \sqrt{c}(4bc-3ad)\text{arctanh}\Big(}{3a^3}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]
```

output
```
((a*Sqrt[c + d*x^3]*(-(a*c) - 2*b*c*x^3 + a*d*x^3))/(x^3*(a + b*x^3)) + ((
4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b
*c) + a*d]])/(Sqrt[b]*Sqrt[-(b*c) + a*d]) + Sqrt[c]*(4*b*c - 3*a*d)*ArcTan
h[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3)
```

3.474.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {948, 109, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4 \left(a+bx^3\right)^2} \, dx$$

$$\Big\downarrow \textbf{948}$$

$$\frac{1}{3} \int \frac{\left(dx^3+c\right)^{3/2}}{x^6 \left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \textbf{109}$$

$$\frac{1}{3} \left(-\frac{\int \frac{d(3bc-2ad)x^3+c(4bc-3ad)}{2x^3 (bx^3+a)^2 \sqrt{dx^3+c}}dx^3}{a}-\frac{c\sqrt{c+dx^3}}{ax^3 \left(a+bx^3\right)}\right)$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{3} \left(-\frac{\int \frac{d(3bc-2ad)x^3+c(4bc-3ad)}{x^3 (bx^3+a)^2 \sqrt{dx^3+c}}dx^3}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3 \left(a+bx^3\right)}\right)$$

$$\Big\downarrow \textbf{168}$$

$$\frac{1}{3} \left(-\frac{\frac{\int \frac{d(bc-ad)(2bc-ad)x^3+c(4bc-3ad)(bc-ad)}{x^3 \left(bx^3+a\right) \sqrt{dx^3+c}}dx^3}{a(bc-ad)}+\frac{2\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)}}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3 \left(a+bx^3\right)}\right)$$

$$\Big\downarrow \textbf{174}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{c(4bc-3ad)(bc-ad)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{(bc-ad)^2(4bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{a(bc-ad)}+\frac{2\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)}}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{2c(4bc-3ad)(bc-ad)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2(bc-ad)^2(4bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{a(bc-ad)}+\frac{2\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)}}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{2(bc-ad)^{3/2}(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2\sqrt{c}(4bc-3ad)(bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a}}{a(bc-ad)}+\frac{2\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)}}{2a}-\frac{c\sqrt{c+dx^3}}{ax^3\left(a+bx^3\right)}\right)$$

input
```
Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]
```

output
```
(-((c*Sqrt[c + d*x^3])/(a*x^3*(a + b*x^3))) - ((2*(2*b*c - a*d)*Sqrt[c + d
*x^3])/(a*(a + b*x^3)) + ((-2*Sqrt[c]*(4*b*c - 3*a*d)*(b*c - a*d)*ArcTanh[
Sqrt[c + d*x^3]/Sqrt[c]])/a + (2*(b*c - a*d)^(3/2)*(4*b*c - a*d)*ArcTanh[(
Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/(a*(b*c - a*d)))/(
2*a))/3
```

3.474.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.474. $\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx$

3.474.4 Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{\frac{x^{3}\left(b\,x^{3}+a\right)(ad-bc)(ad-4bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3}+\frac{4\sqrt{(ad-bc)b}\,\left(x^{3}\left(c^{\frac{3}{2}}b-\frac{3ad\sqrt{c}}{4}\right)\left(b\,x^{3}+a\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)-\frac{\left(2bc\,x^{3}+a\left(-d\,x^{3}\right.\right.}{4}\right.}{3}}{x^{3}\sqrt{(ad-bc)b}\,(b\,x^{3}+a)a^{3}}$
risch	$-\frac{c\sqrt{d\,x^{3}+c}}{3a^{2}x^{3}}-\frac{\frac{2\sqrt{c}\,(3ad-4bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a}+\frac{8bc(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3a\sqrt{(ad-bc)b}}+\frac{\left(-2a^{2}d^{2}+4abcd-2b^{2}c^{2}\right)\left(d\arctan\left(\frac{b\sqrt{d\,x}}{\sqrt{(ad-}}\right.\right.}{3\sqrt{(ad-bc)b}\,(ad-}}{2a^{2}}$
default	$\frac{-\frac{c\sqrt{d\,x^{3}+c}}{3x^{3}}+\frac{2d\sqrt{d\,x^{3}+c}}{3}-\sqrt{c}\,d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{a^{2}}-\frac{2b\left(\frac{2d\,x^{3}\sqrt{d\,x^{3}+c}}{9}+\frac{8c\sqrt{d\,x^{3}+c}}{9}-\frac{2c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}\right)}{a^{3}}+\frac{-d(b}{}$
elliptic	Expression too large to display

input
```
int((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
4/3*(1/4*x^3*(b*x^3+a)*(a*d-b*c)*(a*d-4*b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*
d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(x^3*(c^(3/2)*b-3/4*a*d*c^(1/2))*(b*x
^3+a)*arctanh((d*x^3+c)^(1/2)/c^(1/2))-1/4*(2*b*c*x^3+a*(-d*x^3+c))*a*(d*x
^3+c)^(1/2)))/((a*d-b*c)*b)^(1/2)/x^3/(b*x^3+a)/a^3
```

3.474.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 838, normalized size of antiderivative = 4.93

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)^2}\,dx = \left[-\frac{\left((4\,b^2c-abd)x^6+(4\,abc-a^2d)x^3\right)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+((4}{}\right.$$

$$-\frac{2\left((4\,b^2c-abd)x^6+(4\,abc-a^2d)x^3\right)\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+((4\,b^2c-3\,abd)x^6+(4\,abc-}{6\,(a^3bx^6+a^4x^3)}$$

$$-\frac{2\left((4\,b^2c-3\,abd)x^6+(4\,abc-3\,a^2d)x^3\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\left((4\,b^2c-abd)x^6+(4\,abc-a^2d)x^3\right)\sqrt{}}{6\,(a^3bx^6+a^4x^3)}$$

$$-\frac{\left((4\,b^2c-abd)x^6+(4\,abc-a^2d)x^3\right)\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+((4\,b^2c-3\,abd)x^6+(4\,abc-3}{3\,(a^3bx^6+a^4x^3)}$$

input

```
integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

output

```
[-1/6*(((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt((b*c - a*d)/b)
*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*
x^3 + a)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(c)*lo
g((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*((2*a*b*c - a^2*d)*x^
3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1/6*(2*((4*b^2*c - a*b
*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 +
 c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*
b*c - 3*a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/
x^3) + 2*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4
*x^3), -1/6*(2*((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(-c
)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c -
 a^2*d)*x^3)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3
 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*((2*a*b*c - a^2*d)*x^3 + a^2
*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1/3*(((4*b^2*c - a*b*d)*x^6 +
 (4*a*b*c - a^2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqr
t(-(b*c - a*d)/b)/(b*c - a*d)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a
^2*d)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((2*a*b*c - a^2*d
)*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3)]
```

3.474.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.474.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2x^4}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^4), x)
```

3.474.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx &= \frac{(4\,b^2c^2-5\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}a^3} \\ &\quad - \frac{(4\,bc^2-3\,acd)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}} \\ &\quad - \frac{2\,(dx^3+c)^{\frac{3}{2}}bcd-2\,\sqrt{dx^3+c}bc^2d-(dx^3+c)^{\frac{3}{2}}ad^2+2\,\sqrt{dx^3+c}acd^2}{3\,\left((dx^3+c)^2b-2\,(dx^3+c)bc+bc^2+(dx^3+c)ad-acd\right)a^2}\end{aligned}$$

input
```
integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*(4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c
 + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3) - 1/3*(4*b*c^2 - 3*a*c*d)*arctan(sqr
t(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)) - 1/3*(2*(d*x^3 + c)^(3/2)*b*c*d - 2
*sqrt(d*x^3 + c)*b*c^2*d - (d*x^3 + c)^(3/2)*a*d^2 + 2*sqrt(d*x^3 + c)*a*c
*d^2)/(((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*
c*d)*a^2)
```

3.474.9 Mupad [B] (verification not implemented)

Time = 15.79 (sec) , antiderivative size = 531, normalized size of antiderivative = 3.12

$$\int \frac{(c+dx^3)^{3/2}}{x^4\,(a+bx^3)^2}\,dx = \frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)\,(3\,a\,d-4\,b\,c)}{6\,a^3}-\frac{c\,\sqrt{d\,x^3+c}}{3\,a^2\,x^3}$$

$$-\frac{\sqrt{d\,x^3+c}\,\left(\frac{3\,a\,d-4\,b\,c}{2\,a^2}-\frac{a\,\left(\frac{a\,\left(\frac{a\,\left(\frac{b\,d^2\,(a\,d+b\,c)}{a^3\,c^2}-\frac{a\,\left(\frac{b^2\,d^3}{2\,a^3\,c^2}-\frac{b^2\,d^3\,(3\,a\,d-4\,b\,c)}{6\,a^2\,c^2\,\left(a^2\,d-a\,b\,c\right)}+\frac{b^2\,d^2\,(a\,d+b\,c)\,(3\,a\,d-4\,b\,c)}{3\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}+\frac{b\,(3\,a\,d-4\,b\,c)\,\left(a^2\,d^3+4\,a\,b\,c\,d^2\right.}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right.}{b}\right.}{b}\right.}{}\right.}{b\,x^3+a}$$

$$+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,(a\,d-4\,b\,c)\,1{}\mathrm{i}}{6\,a^3\,\sqrt{b}}$$

input
```
int((c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x)
```

output
```
(c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)
))/x^6)*(3*a*d - 4*b*c))/(6*a^3) - (c*(c + d*x^3)^(1/2))/(3*a^2*x^3) - ((c
 + d*x^3)^(1/2)*((3*a*d - 4*b*c)/(2*a^2) - (a*((a*((a*((b*d^2*(a*d + b*c))
/(a^3*c^2) - (a*((b^2*d^3)/(2*a^3*c^2) - (b^2*d^3*(3*a*d - 4*b*c))/(6*a^2*
c^2*(a^2*d - a*b*c)) + (b^2*d^2*(a*d + b*c)*(3*a*d - 4*b*c))/(3*a^3*c^2*(a
^2*d - a*b*c))))/b + (b*(3*a*d - 4*b*c)*(a^2*d^3 - b^2*c^2*d + 4*a*b*c*d^2
))/(6*a^3*c^2*(a^2*d - a*b*c))))/b - (a^2*d^3 - b^2*c^2*d + 4*a*b*c*d^2)/(
2*a^3*c^2) + (b*(3*a*d - 4*b*c)*(2*b^2*c^3 - 4*a^2*c*d^2 + 2*a*b*c^2*d))/(
6*a^3*c^2*(a^2*d - a*b*c))))/b - (2*b^2*c^3 - 4*a^2*c*d^2 + 2*a*b*c^2*d)/(
2*a^3*c^2) + (b*(3*a*d - 4*b*c)^2)/(6*a^2*(a^2*d - a*b*c))))/b))/(a + b*x^
3) + (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i +
b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(a*d - 4*b*c)*1i)/(6*a^3*b^(1/2))
```

3.474. $\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)^2}\,dx$

3.475 $\int \frac{x^3\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx$

3.475.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{x^3\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx=\frac{cx^4\sqrt{c+dx^3}\,\operatorname{AppellF1}\left(\frac{4}{3},2,-\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/4*c*x^4*AppellF1(4/3,2,-3/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(
1+d*x^3/c)^(1/2)
```

3.475.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 338 vs. 2(65) = 130.

Time = 10.47 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.20

$$\int \frac{x^3\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx=\frac{x^4\left(\frac{d(43bc-55ad)\sqrt{1+\frac{dx^3}{c}}\,\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\left(-8acd\left(11ad+b\left(c+6dx^3\right)\right)\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3}\right.\right.}{\left(a+bx^3\right)\left(-8ac\operatorname{AppellF1}\left(\frac{1}{3}\right.\right.}\right.}{120}$$

input
```
Integrate[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(x^4*((d*(43*b*c - 55*a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3,
-((d*x^3)/c), -((b*x^3)/a)])/a + (8*(-8*a*c*d*(11*a*d + b*(c + 6*d*x^3))*A
ppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*(c + d*x^3)*(5*b
*c - 11*a*d - 6*b*d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -
((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])
))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)
/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)]
 + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(120*b^
2*Sqrt[c + d*x^3])
```

3.475.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{c\sqrt{c+dx^3}\int \frac{x^3\left(\frac{dx^3}{c}+1\right)^{3/2}}{\left(bx^3+a\right)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{cx^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},2,-\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(c*x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 2, -3/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a^2*Sqrt[1 + (d*x^3)/c])
```

3.475.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.475.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.48 (sec) , antiderivative size = 808, normalized size of antiderivative = 12.43

method	result	size
elliptic	Expression too large to display	808
risch	Expression too large to display	1564
default	Expression too large to display	1587

input
```
int(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*d-b*c)*x/b^2*(d*x^3+c)^(1/2)/(b*x^3+a)+2/5*x/b^2*d*(d*x^3+c)^(1/2)-
2/3*I*(-11/6*(a*d-b*c)*d/b^3-2/5*c*d/b^2)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/b^3/d^2*2^(1/2)*sum((-11*a^2*
d^2+13*a*b*c*d-2*b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(
x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*
(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c
*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alph
a-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.475.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3 (c+dx^3)^{3/2}}{(a+bx^3)^2}\, dx = \text{Timed out}$$

input
```
integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.475.6 Sympy [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\int \frac{x^3(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

input
```
integrate(x**3*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x**3*(c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

3.475.7 Maxima [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}x^3}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a)^2, x)
```

3.475.8 Giac [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}x^3}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a)^2, x)
```

3.475.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{x^3\,(d\,x^3+c)^{3/2}}{(b\,x^3+a)^2}\,dx$$

input
```
int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

output
```
int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2, x)
```

3.476 $\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx$

3.476.1 Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \frac{x(c+dx^3)^{3/2}}{\left(a+bx^3\right)^2}\,dx=\frac{cx^2\sqrt{c+dx^3}\,\operatorname{AppellF1}\left(\frac{2}{3},2,-\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
1/2*c*x^2*AppellF1(2/3,2,-3/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(
1+d*x^3/c)^(1/2)
```

3.476.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(65) = 130.

Time = 10.20 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.72

$$\int \frac{x(c+dx^3)^{3/2}}{\left(a+bx^3\right)^2}\,dx=\frac{x^2\left(-10a(-bc+ad)\left(c+dx^3\right)+5c(bc+2ad)\left(a+bx^3\right)\sqrt{1+\frac{dx^3}{c}}\,\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\right.\right.}{30a^2b\left(a+b\right.}$$

input
```
Integrate[(x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(x^2*(-10*a*(-(b*c) + a*d)*(c + d*x^3) + 5*c*(b*c + 2*a*d)*(a + b*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] -
d*(b*c - 7*a*d)*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1,
8/3, -((d*x^3)/c), -((b*x^3)/a)]))/(30*a^2*b*(a + b*x^3)*Sqrt[c + d*x^3])
```

3.476.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{c\sqrt{c+dx^3}\int \frac{x\left(\frac{dx^3}{c}+1\right)^{3/2}}{\left(bx^3+a\right)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{cx^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},2,-\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

output
```
(c*x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 2, -3/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a^2*Sqrt[1 + (d*x^3)/c])
```

3.476.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.476.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.77 (sec) , antiderivative size = 955, normalized size of antiderivative = 14.69

method	result	size
default	Expression too large to display	955
elliptic	Expression too large to display	955

input
```
int(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a*d-b*c)/b/a*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2+1/6/b^2*d*
(a*d-b*c)/a)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1
/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/
3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/
a/b^2/d^2*2^(1/2)*sum((7*a^2*d^2-5*a*b*c*d-2*b^2*c^2)/_alpha/(a*d-b*c)*(-c
*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/
2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha
*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I...
```

3.476.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\text{Timed out}$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.476.6 Sympy [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{x(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

input
```
integrate(x*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

output
```
Integral(x*(c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

3.476.7 Maxima [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x}{(bx^3+a)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a)^2, x)
```

3.476.8 Giac [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x}{(bx^3+a)^2}\,dx$$

input
```
integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a)^2, x)
```

3.476.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{x\,(d\,x^3+c)^{3/2}}{(b\,x^3+a)^2}\,dx$$

input
```
int((x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

output
```
int((x*(c + d*x^3)^(3/2))/(a + b*x^3)^2, x)
```

3.477 $\int \frac{\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx$

3.477.1 Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx=\frac{cx\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},2,-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
c*x*AppellF1(1/3,2,-3/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+d*x^
3/c)^(1/2)
```

3.477.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 339 vs. 2(60) = 120.

Time = 10.32 (sec) , antiderivative size = 339, normalized size of antiderivative = 5.65

$$\int \frac{\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx=\frac{x\Bigl(d(bc+5ad)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\bigl(-64ac(-ad^2x^3+bc(3c+dx^3)}{(a+bx^3)\bigl(-8a}}{}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(a + b*x^3)^2,x]
```

output
```
(x*(d*(b*c + 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)] + (a*(-64*a*c*(-(a*d^2*x^3) + b*c*(3*c + d*x^3))*
AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 24*(b*c - a*d)*x^
3*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)
] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*
x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^
3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*Appe
llF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(24*a^2*b*Sqrt[c +
 d*x^3])
```

3.477.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{c\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{3/2}}{\left(bx^3+a\right)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{cx\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},2,-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(3/2)/(a + b*x^3)^2,x]
```

output
```
(c*x*Sqrt[c + d*x^3]*AppellF1[1/3, 2, -3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*Sqrt[1 + (d*x^3)/c])
```

3.477.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.477.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.57 (sec) , antiderivative size = 801, normalized size of antiderivative = 13.35

method	result	size
default	Expression too large to display	801
elliptic	Expression too large to display	801

input
```
int((d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a*d-b*c)/b/a*x*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2-1/6/b^2*d*(a
*d-b*c)/a)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3
))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2))+1/18*I/a/b^2/d^2*2^(1/2)*sum((5*a^2*d^2-a*b*c*d-4*b^2*c^2)/_alpha^2
/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d
^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2
)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1
/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_al
pha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*b+a))
```

3.477.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.477.6 Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

output
```
Integral((c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

3.477.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/(b*x^3 + a)^2, x)
```

3.477.8 Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/(b*x^3 + a)^2, x)
```

3.477.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx=\int \frac{{\left(d\,x^3+c\right)}^{3/2}}{{\left(b\,x^3+a\right)}^2} \,d x$$

input
```
int((c + d*x^3)^(3/2)/(a + b*x^3)^2,x)
```

output
```
int((c + d*x^3)^(3/2)/(a + b*x^3)^2, x)
```

3.478 $\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx$

3.478.1 Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = -\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},2,-\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-c*AppellF1(-1/3,2,-3/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x/(1+d*
x^3/c)^(1/2)
```

3.478.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(63) = 126.

Time = 10.25 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.02

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = \frac{-20a(c+dx^3)(3ac+4bcx^3-adx^3)+5c(-8bc+11ad)x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{Appell}}{60a^3x\,(}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x]
```

output
```
(-20*a*(c + d*x^3)*(3*a*c + 4*b*c*x^3 - a*d*x^3) + 5*c*(-8*b*c + 11*a*d)*x
^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, -((b*x^3)/a)] + 2*d*(4*b*c - a*d)*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*Ap
pellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^3*x*(a + b*x^3
)*Sqrt[c + d*x^3])
```

3.478.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{c\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{3/2}}{x^2\left(bx^3+a\right)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{c\sqrt{c+dx^3}\operatorname{AppellF1}\left(-\frac{1}{3},2,-\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x]
```

output
```
-((c*Sqrt[c + d*x^3]*AppellF1[-1/3, 2, -3/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*x*Sqrt[1 + (d*x^3)/c]))
```

3.478.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.478.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.32 (sec) , antiderivative size = 970, normalized size of antiderivative = 15.40

method	result	size
elliptic	Expression too large to display	970
risch	Expression too large to display	1854
default	Expression too large to display	2364

input
```
int((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*d-b*c)/a^2*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-c/a^2*(d*x^3+c)^(1/2)/x-2/
3*I*(-1/6*d*(a*d-b*c)/a^2/b+1/2*d*c/a^2)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)))+1/18*I/a^2/b/d^2*2^(1/2)*sum((-a^2*d^2-7*a*b*c*d+8*b^
2*c^2)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))
/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^
2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2...
```

3.478.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.478.6 Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx=\int \frac{(c+dx^3)^{\frac{3}{2}}}{x^2(a+bx^3)^2}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/x**2/(b*x**3+a)**2,x)
```

output
```
Integral((c + d*x**3)**(3/2)/(x**2*(a + b*x**3)**2), x)
```

3.478.7 Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^2), x)
```

3.478.8 Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2x^2}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^2), x)
```

3.478.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^2\,(b\,x^3+a)^2}\,dx$$

input

```
int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x)
```

output

```
int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2), x)
```

3.479 $\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx$

3.479.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(c+dx^3)^{3/2}}{x^3\,(a+bx^3)^2}\,dx=-\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},2,-\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}}$$

output
```
-1/2*c*AppellF1(-2/3,2,-3/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x^2
/(1+d*x^3/c)^(1/2)
```

3.479.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 370 vs. 2(65) = 130.

Time = 10.41 (sec) , antiderivative size = 370, normalized size of antiderivative = 5.69

$$\int \frac{(c+dx^3)^{3/2}}{x^3\,(a+bx^3)^2}\,dx=\frac{-d(5bc-2ad)x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8a\big(4ac(10bcx^3(3c+dx^3)+a(6}{}}{}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x]
```

output

```
(-(d*(5*b*c - 2*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -(
(d*x^3)/c), -((b*x^3)/a)]) + (8*a*(4*a*c*(10*b*c*x^3*(3*c + d*x^3) + a*(6*
c^2 - 15*c*d*x^3 - 4*d^2*x^6))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -(
(b*x^3)/a)] - 3*x^3*(c + d*x^3)*(3*a*c + 5*b*c*x^3 - 2*a*d*x^3)*(2*b*c*App
ellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/
2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/
3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1
/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -
((d*x^3)/c), -((b*x^3)/a)]))))/(48*a^3*x^2*Sqrt[c + d*x^3])
```

3.479.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^3\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{c\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{3/2}}{x^3\left(bx^3+a\right)^2}dx}{\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},2,-\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

input

```
Int[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x]
```

output

```
-1/2*(c*Sqrt[c + d*x^3]*AppellF1[-2/3, 2, -3/2, 1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(a^2*x^2*Sqrt[1 + (d*x^3)/c])
```

3.479.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.479.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.81 (sec) , antiderivative size = 815, normalized size of antiderivative = 12.54

method	result	size
elliptic	Expression too large to display	815
risch	Expression too large to display	1549
default	Expression too large to display	1902

input
```
int((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*d-b*c)/a^2*x*(d*x^3+c)^(1/2)/(b*x^3+a)-1/2*c/a^2*(d*x^3+c)^(1/2)/x^
2-2/3*I*(1/6*d*(a*d-b*c)/a^2/b-1/4*d*c/a^2)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a^2/b/d^2*2^(1/2)*sum((a^2*
d^2-11*a*b*c*d+10*b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*
(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alp
ha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alp
ha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.479.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.479.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^3\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*x**3+c)**(3/2)/x**3/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.479.7 Maxima [F]

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^3\left(a+bx^3\right)^2}\,dx=\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2x^3}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^3), x)
```

3.479.8 Giac [F]

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^3\left(a+bx^3\right)^2}\,dx=\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2x^3}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^3), x)
```

3.479.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx=\int \frac{(d\,x^3+c)^{3/2}}{x^3\,(b\,x^3+a)^2}\,dx$$

input

```
int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x)
```

output

```
int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2), x)
```

3.480 $\int \frac{x^8}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

3.480.1 Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^8}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{c+dx^3}}{3b^2d} - \frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)} + \frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}}$$

output
```
1/3*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(
5/2)/(-a*d+b*c)^(3/2)+2/3*(d*x^3+c)^(1/2)/b^2/d-1/3*a^2*(d*x^3+c)^(1/2)/b^
2/(-a*d+b*c)/(b*x^3+a)
```

3.480.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \frac{x^8}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{\sqrt{b}\sqrt{c+dx^3}\left(-3a^2d+2b^2cx^3+2ab\left(c-dx^3\right)\right)}{d(bc-ad)(a+bx^3)} + \frac{a(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{3b^{5/2}}$$

input
```
Integrate[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((Sqrt[b]*Sqrt[c + d*x^3]*(-3*a^2*d + 2*b^2*c*x^3 + 2*a*b*(c - d*x^3)))/(d
*(b*c - a*d)*(a + b*x^3)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(3*b^(5/2))
```

3.480.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 100, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\left(a+bx^3\right)^2 \sqrt{c+dx^3}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{\left(bx^3+a\right)^2 \sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{3}\left(\frac{\int -\frac{a(2bc-ad)-2b(bc-ad)x^3}{2\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^3}}{b^2\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{a(2bc-ad)-2b(bc-ad)x^3}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^3}}{b^2\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(-\frac{a(4bc-3ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3-\frac{4\sqrt{c+dx^3}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^3}}{b^2\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{\frac{2a(4bc-3ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{d}-\frac{4\sqrt{c+dx^3}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^3}}{b^2\left(a+bx^3\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{a^2\sqrt{c+dx^3}}{b^2\left(a+bx^3\right)(bc-ad)}-\frac{-\frac{2a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}-\frac{4\sqrt{c+dx^3}(bc-ad)}{d}}{2b^2(bc-ad)}\right)$$

input
```
Int[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-((a^2*Sqrt[c + d*x^3])/(b^2*(b*c - a*d)*(a + b*x^3))) - ((-4*(b*c - a*d)
*Sqrt[c + d*x^3])/d - (2*a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3
])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a*d]))/(2*b^2*(b*c - a*d)))/3
```

3.480.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.480.4 Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{-d(b x^3+a)a\left(ad-\frac{4bc}{3}\right)\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{d x^3+c}\left(-\frac{2b^2 c x^3}{3}-\frac{2a\left(-d x^3+c\right)b}{3}+a^2 d\right)\sqrt{(ad-bc)b}}{\sqrt{(ad-bc)b}\, d b^2(ad-bc)(b x^3+a)}$
risch	$\frac{2\sqrt{d x^3+c}}{3b^2 d}-\frac{a\left(\frac{4\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}-\frac{a\left(d\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)\left(b x^3+a\right)+\sqrt{d x^3+c}\,\sqrt{(ad-bc)b}\right)}{3\sqrt{(ad-bc)b}\,(ad-bc)\left(b x^3+a\right)}\right)}{b^2}$
default	$\frac{2\sqrt{d x^3+c}}{3b^2 d}+\frac{a^2\left(d\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)(b x^3+a)+\sqrt{d x^3+c}\,\sqrt{(ad-bc)b}\right)}{3b^2\sqrt{(ad-bc)b}\,(ad-bc)(b x^3+a)}-\frac{4a\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b^2\sqrt{(ad-bc)b}}$
elliptic	$\frac{a^2\sqrt{d x^3+c}}{3(ad-bc)b^2(b x^3+a)}+\frac{2\sqrt{d x^3+c}}{3b^2 d}+\frac{i a\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(3ad-4bc)\left(-c d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right.}{d}\right.}{\left(-c d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$

input
```
int(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((a*d-b*c)*b)^(1/2)*(-d*(b*x^3+a)*a*(a*d-4/3*b*c)*arctan(b*(d*x^3+c)^(1/
2)/((a*d-b*c)*b)^(1/2))+(d*x^3+c)^(1/2)*(-2/3*b^2*c*x^3-2/3*a*(-d*x^3+c)*b
+a^2*d)*((a*d-b*c)*b)^(1/2))/d/b^2/(a*d-b*c)/(b*x^3+a)
```

3.480.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(103) = 206$.

Time = 0.29 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.86

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

$$= \Bigg[\frac{(4\,a^2bcd-3\,a^3d^2+(4\,ab^2cd-3\,a^2bd^2)x^3)\sqrt{b^2c-abd}\log\Big(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\Big)+2\,(2\,ab^3c^2-5}{6\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a}$$

$$-\frac{(4\,a^2bcd-3\,a^3d^2+(4\,ab^2cd-3\,a^2bd^2)x^3)\sqrt{-b^2c+abd}\arctan\Big(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\Big)-(2\,ab^3c^2-5\,a^2b^2cd}{3\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a^2b^4}$$

input
```
integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(b^2
*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*
b*d))/(b*x^3 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4
*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^2*d - 2*a
^2*b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^3
), -1/3*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c
)) - (2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d
 + a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3
*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^3)]
```

3.480.6 Sympy [F]

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**8/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.480.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.480.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{dx^3+c}a^2d}{3\,(b^3c-ab^2d)((dx^3+c)b-bc+ad)}-\frac{(4\,abc-3\,a^2d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,(b^3c-ab^2d)\sqrt{-b^2c+abd}}+\frac{2\,\sqrt{dx^3+c}}{3\,b^2d}$$

input
```
integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(d*x^3 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^3 + c)*b - b*c + a*d))
 - 1/3*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/
((b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 2/3*sqrt(d*x^3 + c)/(b^2*d)
```

3.480.9 Mupad [B] (verification not implemented)

Time = 11.61 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.30

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{d\,x^3+c}\,(2\,b^2\,c-2\,a\,b\,d)}{3\,d\,(2\,b^4\,c-2\,a\,b^3\,d)} - \frac{2\,a^2\,\sqrt{d\,x^3+c}}{3\,b\,(b\,x^3+a)\,(2\,b^2\,c-2\,a\,b\,d)} + \frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,(3\,a\,d-4\,b\,c)\,1\mathrm{i}}{6\,b^{5/2}\,(a\,d-b\,c)^{3/2}}$$

input
```
int(x^8/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
(2*(c + d*x^3)^(1/2)*(2*b^2*c - 2*a*b*d))/(3*d*(2*b^4*c - 2*a*b^3*d)) - (2
*a^2*(c + d*x^3)^(1/2))/(3*b*(a + b*x^3)*(2*b^2*c - 2*a*b*d)) + (a*log((a*
d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a +
 b*x^3))*(3*a*d - 4*b*c)*1i)/(6*b^(5/2)*(a*d - b*c)^(3/2))
```

3.481 $\int \frac{x^5}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

3.481.1 Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{x^5}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

output
```
-1/3*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2
)/(-a*d+b*c)^(3/2)+1/3*a*(d*x^3+c)^(1/2)/b/(-a*d+b*c)/(b*x^3+a)
```

3.481.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{x^5}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{\frac{a\sqrt{b}\sqrt{c+dx^3}}{(bc-ad)(a+bx^3)}-\frac{(2bc-ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{3b^{3/2}}$$

input
```
Integrate[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((a*Sqrt[b]*Sqrt[c + d*x^3])/((b*c - a*d)*(a + b*x^3)) - ((2*b*c - a*d)*Ar
cTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/
(3*b^(3/2))
```

3.481.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(bx^3+a)^2\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{(2bc-ad)\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{2b(bc-ad)}+\frac{a\sqrt{c+dx^3}}{b\,(a+bx^3)\,(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{(2bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bd(bc-ad)}+\frac{a\sqrt{c+dx^3}}{b\,(a+bx^3)\,(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{a\sqrt{c+dx^3}}{b\,(a+bx^3)\,(bc-ad)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}(bc-ad)^{3/2}}\right)$$

input
```
Int[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((a*Sqrt[c + d*x^3])/(b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b^(3/2)*(b*c - a*d)^(3/2)))/3
```

3.481.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.481.4 Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-\frac{a\sqrt{d x^3+c}}{b x^3+a}+\frac{(ad-2bc)\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{3(ad-bc)b}$
default	$\frac{2\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b\sqrt{(ad-bc)b}}-\frac{a\left(d\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)(b x^3+a)+\sqrt{d x^3+c}\,\sqrt{(ad-bc)b}\right)}{3b\sqrt{(ad-bc)b}\,(ad-bc)(b x^3+a)}$
elliptic	$-\frac{a\sqrt{d x^3+c}}{3(ad-bc)b(b x^3+a)}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(b\_Z^3+a\right)}\frac{(-ad+2bc)\left(-c d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{a}{-3(-}}}{}\right.}{}$

input
```
int(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(a*d-b*c)/b*(-a*(d*x^3+c)^(1/2)/(b*x^3+a)+(a*d-2*b*c)/((a*d-b*c)*b)^(1
/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

3.481.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.52

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{((2\,b^2c-abd)x^3+2\,abc-a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,(ab^2c-a^2bd)\sqrt{dx^3+c}}{6\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^3)}\right.$$

input
```
integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a))
 + 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3
*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^3), 1/3*(((2*b^2*c - a*
b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sq
rt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))
/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b
^3*d^2)*x^3)]
```

3.481.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.481.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.481.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{\sqrt{dx^3+c}ad^2}{(b^2c-abd)((dx^3+c)b-bc+ad)} + \frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{3\,d}$$

input
```
integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*(sqrt(d*x^3 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^3 + c)*b - b*c + a*d)) +
 (2*b*c*d - a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c
- a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

3.481.9 Mupad [B] (verification not implemented)

Time = 11.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.12

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\,a\sqrt{d\,x^3+c}}{3\,\left(b\,x^3+a\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,\left(a\,d-2\,b\,c\right)\,1\mathrm{i}}{6\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

input
```
int(x^5/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
(log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x
^3)/(a + b*x^3))*(a*d - 2*b*c)*1i)/(6*b^(3/2)*(a*d - b*c)^(3/2)) + (2*a*(c
 + d*x^3)^(1/2))/(3*(a + b*x^3)*(2*b^2*c - 2*a*b*d))
```

3.482 $\int \frac{x^2}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

3.482.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^2}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{c+dx^3}}{3(bc-ad)\left(a+bx^3\right)}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}$$

output

```
1/3*d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b
^(1/2)-1/3*(d*x^3+c)^(1/2)/(-a*d+b*c)/(b*x^3+a)
```

3.482.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{1}{3}\left(-\frac{\sqrt{c+dx^3}}{(bc-ad)\left(a+bx^3\right)}+\frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

input

```
Integrate[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output

```
(-(Sqrt[c + d*x^3]/((b*c - a*d)*(a + b*x^3))) + (d*ArcTan[(Sqrt[b]*Sqrt[c
+ d*x^3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/3
```

3.482.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {946, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{(bx^3+a)^2\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{d\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{2(bc-ad)}-\frac{\sqrt{c+dx^3}}{(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{bc-ad}-\frac{\sqrt{c+dx^3}}{(a+bx^3)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^3}}{(a+bx^3)(bc-ad)}\right)$$

input
```
Int[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-(Sqrt[c + d*x^3]/((b*c - a*d)*(a + b*x^3))) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2)))/3
```

3.482.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.482.4 Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

method	result
default	$\frac{d \arctan\left(\frac{b \sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)(b x^3+a)+\sqrt{d x^3+c}\,\sqrt{(ad-bc)b}}{3\sqrt{(ad-bc)b}\,(ad-bc)(b x^3+a)}$
pseudoelliptic	$\frac{d \arctan\left(\frac{b \sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)(b x^3+a)+\sqrt{d x^3+c}\,\sqrt{(ad-bc)b}}{3\sqrt{(ad-bc)b}\,(ad-bc)(b x^3+a)}$
elliptic	$\frac{\sqrt{d x^3+c}}{3(ad-bc)(b x^3+a)} - \frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(b\_Z^3+a\right)} \frac{\left(-c d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(\right.}}}{}\right.}{}$

input
```
int(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)*b)^(1/2)*(d*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))*(
b*x^3+a)+(d*x^3+c)^(1/2)*((a*d-b*c)*b)^(1/2))/(a*d-b*c)/(b*x^3+a)
```

3.482.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(71) = 142$.

Time = 0.27 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.47

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \left[-\frac{(bdx^3+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}(b^2c-abd)}{6\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^3)}, -\frac{(bdx^3+ad)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+\sqrt{dx^3+c}(b^2c-abd)}{3\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^3)}\right]$$

input
```
integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/6*((b*d*x^3 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*
sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(b^2
*c - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c
*d + a^2*b^2*d^2)*x^3), -1/3*((b*d*x^3 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(
sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + sqrt(d*x^3 + c)*(b
^2*c - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3
*c*d + a^2*b^2*d^2)*x^3)]
```

3.482.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.482.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.482.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^3+c}d}{3\left((dx^3+c)b-bc+ad\right)(bc-ad)}$$

input
```
integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-1/3*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*(b*c - a*d)) - 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*(b*c -
 a*d))
```

3.482.9 Mupad [B] (verification not implemented)

Time = 10.66 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = &-\frac{2\,b\,\sqrt{d\,x^3+c}}{3\,\left(b\,x^3+a\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\\ &+\frac{d\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{6\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}\end{aligned}$$

input
```
int(x^2/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
(d*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d
*x^3)/(a + b*x^3))*1i)/(6*b^(1/2)*(a*d - b*c)^(3/2)) - (2*b*(c + d*x^3)^(1
/2))/(3*(a + b*x^3)*(2*b^2*c - 2*a*b*d))
```

3.483 $\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

3.483.1 Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}$$

output
```
1/3*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/
2)/a^2/(-a*d+b*c)^(3/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1
/3*b*(d*x^3+c)^(1/2)/a/(-a*d+b*c)/(b*x^3+a)
```

3.483.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{-\frac{ab\sqrt{c+dx^3}}{(-bc+ad)(a+bx^3)}+\frac{\sqrt{b}(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^2}$$

input
```
Integrate[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(-((a*b*Sqrt[c + d*x^3])/((-(b*c) + a*d)*(a + b*x^3))) + (Sqrt[b]*(2*b*c -
 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*
d)^(3/2) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c])/(3*a^2)
```

3.483.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \left(a+b x^3\right)^2 \sqrt{c+d x^3}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^3 \left(b x^3+a\right)^2 \sqrt{d x^3+c}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3} \left(\frac{\int \frac{b d x^3+2 b c-2 a d}{2 x^3 \left(b x^3+a\right) \sqrt{d x^3+c}}dx^3}{a (b c-a d)}+\frac{b \sqrt{c+d x^3}}{a \left(a+b x^3\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(\frac{\int \frac{b d x^3+2 (b c-a d)}{x^3 \left(b x^3+a\right) \sqrt{d x^3+c}}dx^3}{2 a (b c-a d)}+\frac{b \sqrt{c+d x^3}}{a \left(a+b x^3\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3} \left(\frac{\frac{2 (b c-a d) \int \frac{1}{x^3 \sqrt{d x^3+c}}dx^3}{a}-\frac{b (2 b c-3 a d) \int \frac{1}{\left(b x^3+a\right) \sqrt{d x^3+c}}dx^3}{a}}{2 a (b c-a d)}+\frac{b \sqrt{c+d x^3}}{a \left(a+b x^3\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3} \left(\frac{\frac{4 (b c-a d) \int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{d x^3+c}}{a d}-\frac{2 b (2 b c-3 a d) \int \frac{1}{\frac{b x^6}{d}+a-\frac{b c}{d}}d\sqrt{d x^3+c}}{a d}}{2 a (b c-a d)}+\frac{b \sqrt{c+d x^3}}{a \left(a+b x^3\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\frac{2\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{4(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^3}}{a\left(a+bx^3\right)(bc-ad)}\right)$$

input
```
Int[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((b*Sqrt[c + d*x^3])/(a*(b*c - a*d)*(a + b*x^3)) + ((-4*(b*c - a*d)*ArcTan
h[Sqrt[c + d*x^3]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*(2*b*c - 3*a*d)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*(
b*c - a*d)))/3
```

3.483.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.483.4 Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{-2b\sqrt{c}\left(bc-\frac{3ad}{2}\right)\left(b\,x^{3}+a\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)+\left(2\left(b\,x^{3}+a\right)(ad-bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)+\sqrt{d\,x^{3}+c}\,\sqrt{c}\,ab\right)\sqrt{(ad-bc)b}}{3\sqrt{c}\,\sqrt{(ad-bc)b}\,a^{2}(ad-bc)\left(b\,x^{3}+a\right)}$
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a^{2}\sqrt{c}}-\frac{2b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3a^{2}\sqrt{(ad-bc)b}}-\frac{b\left(d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)\left(b\,x^{3}+a\right)+\sqrt{d\,x^{3}+c}\,\sqrt{(ad-bc)b}\right)}{3a\sqrt{(ad-bc)b}\,(ad-bc)\left(b\,x^{3}+a\right)}$
elliptic	Expression too large to display

input
```
int(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/c^(1/2)*(-2*b*c^(1/2)*(b*c-3/2*a*d)*(b*x^3+a)*arctan(b*(d*x^3+c)^(1/2
)/((a*d-b*c)*b)^(1/2))+(2*(b*x^3+a)*(a*d-b*c)*arctanh((d*x^3+c)^(1/2)/c^(1
/2))+(d*x^3+c)^(1/2)*c^(1/2)*a*b)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)
/a^2/(a*d-b*c)/(b*x^3+a)
```

3.483.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 862, normalized size of antiderivative = 6.53

$$\int \frac{1}{x\left(a+bx^{3}\right)^{2}\sqrt{c+dx^{3}}}\,dx$$

$$= \left[\frac{2\sqrt{dx^{3}+c}abc+\left(2\,abc^{2}-3\,a^{2}cd+\left(2\,b^{2}c^{2}-3\,abcd\right)x^{3}\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^{3}+2\,bc-ad+2\,\sqrt{dx^{3}+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^{3}+a}\right)-}{6\left(a^{3}bc^{2}-a^{4}cd+\left(a^{2}b^{2}c^{2}-a^{3}bcd\right)x^{3}\right)}\right.$$

input

```
integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output

```
[1/6*(2*sqrt(d*x^3 + c)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*
b*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3
+ c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*((b^2*c - a*b*d)*x^
3 + a*b*c - a^2*d)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x
^3))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/3*(sqrt(d*x^
3 + c)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(
-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(
b*d*x^3 + b*c)) + ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*sqrt(c)*log((d*x^3
 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*
c^2 - a^3*b*c*d)*x^3), 1/6*(2*sqrt(d*x^3 + c)*a*b*c + 4*((b^2*c - a*b*d)*x
^3 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (2*a*b*c
^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d
*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b
*x^3 + a)))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/3*(sq
rt(d*x^3 + c)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3
)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c -
a*d))/(b*d*x^3 + b*c)) + 2*((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*sqrt(-c)*
arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 -
a^3*b*c*d)*x^3)]
```

3.483.6 Sympy [F]

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$

input

```
integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output

```
Integral(1/(x*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.483.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}\,x}\,dx$$

input
```
integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x), x)
```

3.483.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{\sqrt{dx^3+c}bd}{3\left(abc-a^2d\right)\left((dx^3+c)b-bc+ad\right)}-\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(d*x^3 + c)*b*d/((a*b*c - a^2*d)*((d*x^3 + c)*b - b*c + a*d)) - 1/
3*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2
*b*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))
/(a^2*sqrt(-c))
```

3.483.9 Mupad [B] (verification not implemented)

Time = 14.69 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2\,\sqrt{c}}+\frac{b^2\,\sqrt{d\,x^3+c}}{3\,a\,\left(b\,x^3+a\right)\,\left(b^2\,c-a\,b\,d\right)}+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(3\,a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,a^2\,{\left(a\,d-b\,c\right)}^{3/2}}$$

input
```
int(1/(x*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(
3*a^2*c^(1/2)) + (b^2*(c + d*x^3)^(1/2))/(3*a*(a + b*x^3)*(b^2*c - a*b*d))
 + (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)
*2i - b*d*x^3)/(a + b*x^3))*(3*a*d - 2*b*c)*1i)/(6*a^2*(a*d - b*c)^(3/2))
```

3.483. $\int \frac{1}{x(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

3.484 $\int \frac{1}{x^4(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

3.484.1 Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{1}{x^4\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=-\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)\left(a+bx^3\right)}-\frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)}+\frac{(4bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}}-\frac{b^{3/2}(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{3/2}}$$

output
```
1/3*(a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/3*b^(3/2)*(
-5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+
b*c)^(3/2)-1/3*b*(-a*d+2*b*c)*(d*x^3+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^3+a)-1
/3*(d*x^3+c)^(1/2)/a/c/x^3/(b*x^3+a)
```

3.484.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{\frac{a\sqrt{c+dx^3}\left(-a^2d+2b^2cx^3+ab\left(c-dx^3\right)\right)}{c(-bc+ad)x^3\left(a+bx^3\right)}-\frac{b^{3/2}(4bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}+\frac{(4bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{3/2}}}{3a^3}$$

---

input
```
Integrate[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
((a*Sqrt[c + d*x^3]*(-(a^2*d) + 2*b^2*c*x^3 + a*b*(c - d*x^3)))/(c*(-(b*c)
 + a*d)*x^3*(a + b*x^3)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcT
anh[Sqrt[c + d*x^3]/Sqrt[c]])/c^(3/2))/(3*a^3)
```

3.484.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {948, 114, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+bx^3\right)^2 \sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^6 \left(bx^3+a\right)^2 \sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{3bdx^3+4bc+ad}{2x^3\left(bx^3+a\right)^2\sqrt{dx^3+c}}dx^3}{ac}-\frac{\sqrt{c+dx^3}}{acx^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{3bdx^3+4bc+ad}{x^3\left(bx^3+a\right)^2\sqrt{dx^3+c}}dx^3}{2ac}-\frac{\sqrt{c+dx^3}}{acx^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(-\frac{\frac{\int \frac{bd(2bc-ad)x^3+(bc-ad)(4bc+ad)}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a(bc-ad)}+\frac{2b\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^3}}{acx^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{(bc-ad)(ad+4bc)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{b^2c(4bc-5ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^3}}{acx^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{2(bc-ad)(ad+4bc)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2b^2c(4bc-5ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^3}}{acx^3\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{2b^{3/2}c(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)(ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^3}(2bc-ad)}{a(a+bx^3)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^3}}{acx^3\left(a+bx^3\right)}\right)$$

input

```
Int[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output

```
(-(Sqrt[c + d*x^3]/(a*c*x^3*(a + b*x^3))) - ((2*b*(2*b*c - a*d)*Sqrt[c + d
*x^3])/(a*(b*c - a*d)*(a + b*x^3)) + ((-2*(b*c - a*d)*(4*b*c + a*d)*ArcTan
h[Sqrt[c + d*x^3]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(3/2)*c*(4*b*c - 5*a*d)*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(a*(
b*c - a*d)))/(2*a*c))/3
```

3.484.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.484. $\int \frac{1}{x^4(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

3.484.4 Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-4x^3c^{\frac{5}{2}}\left(bc-\frac{5ad}{4}\right)b^2(bx^3+a)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)+(cx^3(bx^3+a)(ad+4bc)(ad-bc)\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)+c^{\frac{3}{2}}(2b^2cx^3+a(}{3\sqrt{(ad-bc)b}\,a^3(ad-bc)(bx^3+a)c^{\frac{5}{2}}x^3}$
risch	$-\frac{\sqrt{dx^3+c}}{3ca^2x^3}-\frac{-\frac{2(ad+4bc)\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}-\frac{2b^2c\left(d\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)(bx^3+a)+\sqrt{dx^3+c}\sqrt{(ad-bc)b}\right)}{3\sqrt{(ad-bc)b}\,(ad-bc)(bx^3+a)}-\frac{8b^2c\arctan(\frac{b\sqrt{d}}{\sqrt{(a}}}{3a\sqrt{(ad-bc}}}{2ca^2}$
default	$\frac{-\frac{\sqrt{dx^3+c}}{3cx^3}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{a^2}+\frac{4b\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3a^3\sqrt{c}}+\frac{b^2(d\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)(bx^3+a)+\sqrt{dx^3+c}\sqrt{(ad-bc)b}}{3a^2\sqrt{(ad-bc)b}\,(ad-bc)(bx^3+a)}$
elliptic	Expression too large to display

input
```
int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)*b)^(1/2)*(-4*x^3*c^(5/2)*(b*c-5/4*a*d)*b^2*(b*x^3+a)*arctan
(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(c*x^3*(b*x^3+a)*(a*d+4*b*c)*(a*d-
b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))+c^(3/2)*(2*b^2*c*x^3+a*(-d*x^3+c)*b-
a^2*d)*a*(d*x^3+c)^(1/2))*((a*d-b*c)*b)^(1/2))/a^3/(a*d-b*c)/(b*x^3+a)/c^(
5/2)/x^3
```

3.484.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 1236, normalized size of antiderivative = 6.68

$$\int \frac{1}{x^4\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3
)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c
- a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2
*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(c)*log((d*x^
3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*
b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^
6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/6*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6
 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*
x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((4*b^3*c^2 -
 3*a*b^2*c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)
*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a^2*b*c^2
 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3
 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/6*(2*((4*b^3*c^2 -
3*a*b^2*c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*
sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)
*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3
 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3
 + a)) + 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^
3 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -
1/3*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x...
```

3.484.6 Sympy [F]

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.484.7 Maxima [F]

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^2 \sqrt{dx^3+c}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^4), x)
```

3.484.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{(4\,b^3c-5\,ab^2d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,(a^3bc-a^4d)\sqrt{-b^2c+abd}} - \frac{2\,(dx^3+c)^{\frac{3}{2}}b^2cd-2\,\sqrt{dx^3+c}b^2c^2d-(dx^3+c)^{\frac{3}{2}}abd^2+2\,\sqrt{dx^3+c}abcd^2-\sqrt{dx^3+c}a^2d^3}{3\,(a^2bc^2-a^3cd)\left((dx^3+c)^2b-2\,(dx^3+c)bc+bc^2+(dx^3+c)ad-acd\right)} - \frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}c}$$

input
```
integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3 + c)^(3/2)*b^2*c*d
 - 2*sqrt(d*x^3 + c)*b^2*c^2*d - (d*x^3 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^3
+ c)*a*b*c*d^2 - sqrt(d*x^3 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^3 +
 c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*c*d)) - 1/3*(4*b
*c + a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

3.484.9 Mupad [B] (verification not implemented)

Time = 15.99 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.92

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx$$
$$= \frac{\sqrt{d\,x^3+c}\left(\frac{d\,a^2+4\,b\,c\,a}{2\,a^3\,c^2}-\frac{a\left(\frac{2\,c\,b^2+2\,a\,d\,b}{2\,a^3\,c^2}-\frac{a\left(\frac{b^2\,d}{2\,a^3\,c^2}+\frac{b\left(2\,c\,b^2+2\,a\,d\,b\right)\,(3\,a\,d-4\,b\,c)}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}-\frac{b^2\,d\,(3\,a\,d-4\,b\,c)}{6\,a^2\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}+\frac{b\left(d\,a^2+4\,b\,c\,a\right)\,(3\,a\,d-4\,b\,c)}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}\right)}{b\,x^3+a}$$
$$-\frac{\sqrt{d\,x^3+c}}{3\,a^2\,c\,x^3}+\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\,{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}^3}{x^6}\right)\,(a\,d+4\,b\,c)}{6\,a^3\,c^{3/2}}$$
$$+\frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,(5\,a\,d-4\,b\,c)\,1{}\mathrm{i}}{6\,a^3\,{(a\,d-b\,c)}^{3/2}}$$

input
```
int(1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
((c + d*x^3)^(1/2)*((a^2*d + 4*a*b*c)/(2*a^3*c^2) - (a*((2*b^2*c + 2*a*b*d
)/(2*a^3*c^2) - (a*((b^2*d)/(2*a^3*c^2) + (b*(2*b^2*c + 2*a*b*d)*(3*a*d -
4*b*c))/(6*a^3*c^2*(a^2*d - a*b*c)) - (b^2*d*(3*a*d - 4*b*c))/(6*a^2*c^2*(
a^2*d - a*b*c))))/b + (b*(a^2*d + 4*a*b*c)*(3*a*d - 4*b*c))/(6*a^3*c^2*(a^
2*d - a*b*c))))/b))/(a + b*x^3) - (c + d*x^3)^(1/2)/(3*a^2*c*x^3) + (log((
((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(a*d +
 4*b*c))/(6*a^3*c^(3/2)) + (b^(3/2)*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)
^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5*a*d - 4*b*c)*1i)/(6
*a^3*(a*d - b*c)^(3/2))
```

3.485 $\int \frac{x^3}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

3.485.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}$$

output
```
1/4*x^4*AppellF1(4/3,2,1/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/(d
*x^3+c)^(1/2)
```

3.485.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 238 vs. 2(64) = 128.

Time = 10.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.72

$$\int \frac{x^3}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$
$$=\frac{x\left(\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\left(c+dx^3+\frac{8ac^2\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bc\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right.\right.}\right.}{a+bx^3}\right.}{24(-bc+ad)\sqrt{c+dx^3}}$$

input
```
Integrate[x^3/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)])/a + (8*(c + d*x^3 + (8*a*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*
x^3)/c), -((b*x^3)/a)])/(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -
((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*
x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/
(a + b*x^3)))/(24*(-(b*c) + a*d)*Sqrt[c + d*x^3])
```

3.485.3 Rubi [A] (verified)

Time $= 0.21$ (sec) , antiderivative size $= 64$, normalized size of antiderivative $= 1.00$, number of steps used $= 2$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used $= \{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{(bx^3+a)^2\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}$$

input
```
Int[x^3/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*a^2*Sqrt[c + d*x^3])
```

3.485.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.485.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.51 (sec) , antiderivative size = 764, normalized size of antiderivative = 11.94

method	result	size
elliptic	Expression too large to display	764
default	Expression too large to display	1207

input
```
int(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(a*d-b*c)*x*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/(a*d-b*c)/b*3^(1/2)*(-c*d^
2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/b/d^2*2^(1/2)
*sum((a*d+2*b*c)/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d
)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.485.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.485.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**3/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.485.7 Maxima [F]

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{(bx^3+a)^2\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

3.485.8 Giac [F]

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{(bx^3+a)^2\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

3.485.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{{\left(b\,x^3+a\right)}^2\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(x^3/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

3.486 $\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

3.486.1 Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\frac{x^2\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},2,\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}$$

output
```
1/2*x^2*AppellF1(2/3,2,1/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/(d
*x^3+c)^(1/2)
```

3.486.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(64) = 128.

Time = 10.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.69

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$
$$=\frac{10abx^2(c+dx^3)+5(bc-3ad)x^2(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-bdx^5(a+bx^3)\sqrt{}}{30a^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}$$

input
```
Integrate[x/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(10*a*b*x^2*(c + d*x^3) + 5*(b*c - 3*a*d)*x^2*(a + b*x^3)*Sqrt[1 + (d*x^3)
/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - b*d*x^5*(a +
b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x
^3)/a)])/(30*a^2*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3])
```

3.486.3 Rubi [A] (verified)

Time $=0.20$ (sec) , antiderivative size $=64$, normalized size of antiderivative $=1.00$, number of steps used $=2$, number of rules used $=2$, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used $=\{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x}{(bx^3+a)^2\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\operatorname{AppellF1}\left(\frac{2}{3},2,\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}$$

input
```
Int[x/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 2, 1/2, 5/3, -((b*x^3)/a), -((d*x^3
)/c)])/(2*a^2*Sqrt[c + d*x^3])
```

3.486.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.486.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.56 (sec) , antiderivative size = 923, normalized size of antiderivative = 14.42

method	result	size
default	Expression too large to display	923
elliptic	Expression too large to display	923

input
```
int(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b/(a*d-b*c)/a*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/(a*d-b*c)/a*3^(1/2)
*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a/d^2*2^(1/2)*sum((-5*
a*d+2*b*c)/(a*d-b*c)^2/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^
(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+
1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_
alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*...
```

3.486.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.486.6 Sympy [F]

$$\int \frac{x}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{x}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.486.7 Maxima [F]

$$\int \frac{x}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{x}{\left(bx^3+a\right)^2\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

3.486.8 Giac [F]

$$\int \frac{x}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{x}{\left(bx^3+a\right)^2\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

3.486.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x}{\left(b\,x^3+a\right)^2\,\sqrt{d\,x^3+c}}\,dx$$

input

```
int(x/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output

```
int(x/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

3.487 $\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

3.487.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}$$

output
```
x*AppellF1(1/3,2,1/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/(d*x^3+c
)^(1/2)
```

3.487.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 392 vs. $2(59) = 118$.

Time = 10.28 (sec) , antiderivative size = 392, normalized size of antiderivative = 6.64

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$
$$=\frac{-8acx\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\Bigg(8a(3bc-3ad+bdx^3)+bdx^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\bigg(\frac{4}{3},}{24a^2(bc-ad)\,(a+bx^3)\sqrt{c+dx^3}\,\big(-8ac}$$

input
```
Integrate[1/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output

```
(-8*a*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)]*(8*a*(3*b
*c - 3*a*d + b*d*x^3) + b*d*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4
/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]) + 3*b*x^4*(8*a*(c + d*x^3) +
 d*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3
)/c), -((b*x^3)/a)])*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*
x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))/(2
4*a^2*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1
, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/
3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/
c), -((b*x^3)/a)])))
```

3.487.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \; 937$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(bx^3+a)^2\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 936$$

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}$$

input

```
Int[1/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output

```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*Sqrt[c + d*x^3])
```

3.487.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.487.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.67 (sec) , antiderivative size = 769, normalized size of antiderivative = 13.03

method	result	size
default	Expression too large to display	769
elliptic	Expression too large to display	769

input
```
int(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b/(a*d-b*c)/a*x*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/(a*d-b*c)/a*3^(1/2)*(
-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a/d^2*2^
(1/2)*sum((-7*a*d+4*b*c)/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*
(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alp
ha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alp
ha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.487.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.487.6 Sympy [F]

$$\int \frac{1}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.487.7 Maxima [F]

$$\int \frac{1}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

3.487.8 Giac [F]

$$\int \frac{1}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

3.487.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^2\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(1/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

3.488 $\int \frac{1}{x^2(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

3.488.1 Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{1}{3},2,\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{c+dx^3}}$$

output
```
-AppellF1(-1/3,2,1/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/x/(d*x^3
+c)^(1/2)
```

3.488.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 226 vs. $2(62) = 124$.

Time = 10.26 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{20a(c+dx^3)\left(3a^2d-4b^2cx^3-3ab(c-dx^3)\right)-5\left(8b^2c^2-15abcd+3a^2d^2\right)x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF}}{60a^3c(bc-ad)x\,(a+}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(20*a*(c + d*x^3)*(3*a^2*d - 4*b^2*c*x^3 - 3*a*b*(c - d*x^3)) - 5*(8*b^2*c
^2 - 15*a*b*c*d + 3*a^2*d^2)*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[
2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*(4*b*c - 3*a*d)*x^6*
(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -
((b*x^3)/a)])/(60*a^3*c*(b*c - a*d)*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

3.488.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3)^2 \sqrt{c+dx^3}} dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1} \int \frac{1}{x^2 (bx^3+a)^2 \sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1} \text{AppellF1}\left(-\frac{1}{3},2,\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2 x \sqrt{c+dx^3}}$$

input
```
Int[1/(x^2*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
-((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 2, 1/2, 2/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*x*Sqrt[c + d*x^3]))
```

3.488.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.488.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.40 (sec) , antiderivative size = 963, normalized size of antiderivative = 15.53

method	result	size
elliptic	Expression too large to display	963
default	Expression too large to display	1818
risch	Expression too large to display	1819

input
```
int(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(a*d-b*c)/a^2*b^2*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/c/a^2*(d*x^3+c)^(1/2
)/x-2/3*I*(-1/6*b*d/(a*d-b*c)/a^2+1/2*d/c/a^2)*3^(1/2)/d*(-c*d^2)^(1/3)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I*b/a^2/d^2*2^(1/2)*sum((11*a*d-8*b*c)/(a*
d-b*c)^2/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c
*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*
(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-
c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))...
```

3.488.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a+bx^3\right)^2 \sqrt{c+dx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.488.6 Sympy [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.488.7 Maxima [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}\,x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^2), x)
```

3.488.8 Giac [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}\,x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^2), x)
```

3.488.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^2 \left(b\,x^3+a\right)^2 \sqrt{d\,x^3+c}}\,dx$$

input
```
int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

3.489 $\int \frac{1}{x^3(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

3.489.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^3\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{c+dx^3}}$$

output
```
-1/2*AppellF1(-2/3,2,1/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/x^2/
(d*x^3+c)^(1/2)
```

3.489.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 411 vs. 2(64) = 128.

Time = 10.63 (sec) , antiderivative size = 411, normalized size of antiderivative = 6.42

$$\int \frac{1}{x^3\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{bd(5bc-3ad)x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a(32ac(-10b^2cx^3(3c+dx^3)+3a^2d(2c+3dx^3)+3ab(-2c^2+}{(a+bx^3}}{48}$$

input
```
Integrate[1/(x^3*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
(b*d*(5*b*c - 3*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -(
(d*x^3)/c), -((b*x^3)/a)] + (a*(32*a*c*(-10*b^2*c*x^3*(3*c + d*x^3) + 3*a^
2*d*(2*c + 3*d*x^3) + 3*a*b*(-2*c^2 + 7*c*d*x^3 + 2*d^2*x^6))*AppellF1[1/3
, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 24*x^3*(c + d*x^3)*(-3*a^2*d
+ 5*b^2*c*x^3 + 3*a*b*(c - d*x^3))*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*
x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b
*x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((
b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))
)/(48*a^3*c*(-(b*c) + a*d)*x^2*Sqrt[c + d*x^3])
```

3.489.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b x^3\right)^2 \sqrt{c+d x^3}} \, dx$$

$$\Big\downarrow \textcolor{blue}{1013}$$

$$\frac{\sqrt{\frac{d x^3}{c}+1} \int \frac{1}{x^3 \left(b x^3+a\right)^2 \sqrt{\frac{d x^3}{c}+1}}dx}{\sqrt{c+d x^3}}$$

$$\Big\downarrow \textcolor{blue}{1012}$$

$$-\frac{\sqrt{\frac{d x^3}{c}+1} \text{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right)}{2 a^2 x^2 \sqrt{c+d x^3}}$$

input
```
Int[1/(x^3*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

output
```
-1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 1/2, 1/3, -((b*x^3)/a), -((d*x
^3)/c)])/(a^2*x^2*Sqrt[c + d*x^3])
```

3.489.3.1 Defintions of rubi rules used

rule 1012

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.489.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.31 (sec) , antiderivative size = 809, normalized size of antiderivative = 12.64

method	result	size
elliptic	Expression too large to display	809
default	Expression too large to display	1512
risch	Expression too large to display	1513

input

```
int(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(a*d-b*c)/a^2*b^2*x*(d*x^3+c)^(1/2)/(b*x^3+a)-1/2/c/a^2*(d*x^3+c)^(1/2
)/x^2-2/3*I*(1/6*b*d/(a*d-b*c)/a^2-1/4*d/c/a^2)*3^(1/2)/d*(-c*d^2)^(1/3)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I*b/a^2/d^2*2^(1/2)*sum((
13*a*d-10*b*c)/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c
*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d
*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/
3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(
-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/
(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

3.489.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+bx^3\right)^2 \sqrt{c+dx^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.489.6 Sympy [F]

$$\int \frac{1}{x^3 \left(a+bx^3\right)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^3 \left(a+bx^3\right)^2 \sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.489.7 Maxima [F]

$$\int \frac{1}{x^3 \left(a+bx^3\right)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{\left(bx^3+a\right)^2 \sqrt{dx^3+c}\,x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^3), x)
```

3.489.8 Giac [F]

$$\int \frac{1}{x^3 \left(a+bx^3\right)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{\left(bx^3+a\right)^2 \sqrt{dx^3+c}\,x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^3), x)
```

3.489.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{x^3\left(b\,x^3+a\right)^2\sqrt{d\,x^3+c}}\,dx$$

input
```
int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

3.490 $\int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

3.490.1 Optimal result

Integrand size = 24, antiderivative size = 150

$$\int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{-2b^2c^2-a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)(a+bx^3)\sqrt{c+dx^3}} + \frac{a(4bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{5/2}}$$

output
```
1/3*a*(-a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/
2)/(-a*d+b*c)^(5/2)+1/3*(-a^2*d^2-2*b^2*c^2)/b^2/d/(-a*d+b*c)^2/(d*x^3+c)^
(1/2)-1/3*a^2/b^2/(-a*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)
```

3.490.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{-\frac{\sqrt{b}(2abc^2+2b^2c^2x^3+a^2d(c+dx^3))}{d(bc-ad)^2(a+bx^3)\sqrt{c+dx^3}} + \frac{a(-4bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}}{3b^{3/2}}$$

input
```
Integrate[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-((Sqrt[b]*(2*a*b*c^2 + 2*b^2*c^2*x^3 + a^2*d*(c + d*x^3)))/(d*(b*c - a*d
)^2*(a + b*x^3)*Sqrt[c + d*x^3])) + (a*(-4*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt
[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2))/(3*b^(3/2))
```

3.490.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \textcolor{blue}{948}$$

$$\frac{1}{3}\int \frac{x^6}{\left(bx^3+a\right)^2 \left(dx^3+c\right)^{3/2}}dx^3$$

$$\Big\downarrow \textcolor{blue}{100}$$

$$\frac{1}{3}\left(\frac{\int -\frac{a(2bc+ad)-2b(bc-ad)x^3}{2\left(bx^3+a\right)\left(dx^3+c\right)^{3/2}}dx^3}{b^2(bc-ad)}-\frac{a^2}{b^2\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{a(2bc+ad)-2b(bc-ad)x^3}{\left(bx^3+a\right)\left(dx^3+c\right)^{3/2}}dx^3}{2b^2(bc-ad)}-\frac{a^2}{b^2\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \textcolor{blue}{87}$$

$$\frac{1}{3}\left(-\frac{\frac{ab(4bc-ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{bc-ad}+\frac{2\left(a^2d^2+2b^2c^2\right)}{d\sqrt{c+dx^3}(bc-ad)}}{2b^2(bc-ad)}-\frac{a^2}{b^2\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \textcolor{blue}{73}$$

$$\frac{1}{3}\left(-\frac{\frac{2ab(4bc-ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{d(bc-ad)}+\frac{2\left(a^2d^2+2b^2c^2\right)}{d\sqrt{c+dx^3}(bc-ad)}}{2b^2(bc-ad)}-\frac{a^2}{b^2\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{3}\left(-\frac{\frac{2(a^2d^2+2b^2c^2)}{d\sqrt{c+dx^3}(bc-ad)}-\frac{2a\sqrt{b}(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{2b^2(bc-ad)}-\frac{a^2}{b^2\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

input
```
Int[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-(a^2/(b^2*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3])) - ((2*(2*b^2*c^2 + a
^2*d^2))/(d*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*a*Sqrt[b]*(4*b*c - a*d)*ArcT
anh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(2*b^2*
(b*c - a*d)))/3
```

3.490.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.490.4 Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$-\frac{-ad\sqrt{dx^3+c}\,(bx^3+a)(ad-4bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\,(2b^2c^2x^3+2bc^2a+da^2(dx^3+c))}{3\sqrt{dx^3+c}\,\sqrt{(ad-bc)b}\,db(bx^3+a)(ad-bc)^2}$
default	$-\frac{2}{3b^2d\sqrt{dx^3+c}}+\frac{a^2d\left(-\frac{2}{\sqrt{dx^3+c}}-\frac{b\sqrt{dx^3+c}}{d(bx^3+a)}-\frac{3\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)b}{\sqrt{(ad-bc)b}}\right)}{3b^2(ad-bc)^2}+\frac{4a\left(b\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{dx^3+c}+\sqrt{(ad-\right.}{b^2\sqrt{(ad-bc)b}\,\sqrt{dx^3+c}\,(3ad-3bc)}$
elliptic	$-\frac{2c^2}{3d(ad-bc)^2\sqrt{(x^3+\frac{c}{d})d}}-\frac{a^2\sqrt{dx^3+c}}{3(ad-bc)^2b(bx^3+a)}-\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}(b\_Z^3+a)}\frac{(ad-4bc)(-cd^2)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}(}{}\right.}{(}}}{}\right.}{}$

input
```
int(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-a*d*(d*x^3+c)^(1/2)*(b*x^3+a)*(a*d-4*b*c)*arctan(b*(d*x^3+c)^(1/2)/
((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(2*b^2*c^2*x^3+2*b*c^2*a+d*a^2*(d
*x^3+c)))/(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)/d/b/(b*x^3+a)/(a*d-b*c)^2
```

3.490.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(129) = 258$.

Time = 0.35 (sec) , antiderivative size = 746, normalized size of antiderivative = 4.97

$$\int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \left[-\frac{((4ab^2cd^2-a^2bd^3)x^6+4a^2bc^2d-a^3cd^2+(4ab^2c^2d+3a^2bcd^2-a^3d^3)x}{6(ab^5c^4d-3a^2b^4c^3d^2+3a^3b^3c^2d^3-a}\right.$$

$$-\frac{((4ab^2cd^2-a^2bd^3)x^6+4a^2bc^2d-a^3cd^2+(4ab^2c^2d+3a^2bcd^2-a^3d^3)x^3)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-}}{bdx^3-}\right.}{3(ab^5c^4d-3a^2b^4c^3d^2+3a^3b^3c^2d^3-a^4b^2cd^4+(b^6c^3d^2-3ab^5c^2d^3+3a^2b^4}$$

input
```
integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/6*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^
3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*
(2*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*a*b^3*c^2*d +
a^2*b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^4*d - 3*a^2*b^4*
c^3*d^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 - 3*a*b^5*c^2*d
^3 + 3*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^6 + (b^6*c^4*d - 2*a*b^5*c^3*d^2 + 2
*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^3), -1/3*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6
 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x
^3)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*
x^3 + b*c)) + (2*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*
a*b^3*c^2*d + a^2*b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^4*
d - 3*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 -
 3*a*b^5*c^2*d^3 + 3*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^6 + (b^6*c^4*d - 2*a*b
^5*c^3*d^2 + 2*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^3)]
```

3.490.6 Sympy [F]

$$\int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx=\int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**8/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.490.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.490.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.30

$$\int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{(4\,abc-a^2d)\arctan\left(\frac{\sqrt{dx^3+cb}}{\sqrt{-b^2c+abd}}\right)}{3\,(b^3c^2-2\,ab^2cd+a^2bd^2)\sqrt{-b^2c+abd}} - \frac{2\,(dx^3+c)b^2c^2-2\,b^2c^3+2\,abc^2d+(dx^3+c)a^2d^2}{3\,(b^3c^2d-2\,ab^2cd^2+a^2bd^3)\left((dx^3+c)^{\frac{3}{2}}b-\sqrt{dx^3+c}bc+\sqrt{dx^3+c}ad\right)}$$

input
```
integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-1/3*(4*a*b*c - a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3 + c
)*b^2*c^2 - 2*b^2*c^3 + 2*a*b*c^2*d + (d*x^3 + c)*a^2*d^2)/((b^3*c^2*d - 2
*a*b^2*c*d^2 + a^2*b*d^3)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + sqr
t(d*x^3 + c)*a*d))
```

3.490.9 Mupad [B] (verification not implemented)

Time = 12.27 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.45

$$\int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{d\,x^3+c}\,\left(x^3\,\left(\frac{\left(\frac{3\,b\,d\,(a\,d+b\,c)-b\,d\,(a\,d+2\,b\,c)}{3\,\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}-\frac{b\,d\,(a\,d+b\,c)}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)\,(a\,d+b\,c)}{b\,d}+\frac{}{a^2\,b\,d^3-}\right.\right.}{b\,d\,x^6+(a\,d+b\,c)} + \frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,(a\,d-4\,b\,c)\,1{}\mathrm{i}}{6\,b^{3/2}\,{(a\,d-b\,c)}^{5/2}}$$

input
```
int(x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output

```
((c + d*x^3)^(1/2)*(x^3*((((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2
*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*
c^2*d - 2*a*b^2*c*d^2))*(a*d + b*c))/(b*d) + (a*b*c*d)/(a^2*b*d^3 + b^3*c^
2*d - 2*a*b^2*c*d^2)) + (a*c*((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(
a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b
^3*c^2*d - 2*a*b^2*c*d^2)))/(b*d)))/(a*c + x^3*(a*d + b*c) + b*d*x^6) + (a
*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x
^3)/(a + b*x^3))*(a*d - 4*b*c)*1i)/(6*b^(3/2)*(a*d - b*c)^(5/2))
```

3.490. $\int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

3.491 $\int \frac{x^5}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.491.1 Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{x^5}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}}+\frac{a}{3b(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}-\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{5/2}}$$

output

```
-1/3*(a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b
*c)^(5/2)/b^(1/2)+1/3*(a*d+2*b*c)/b/(-a*d+b*c)^2/(d*x^3+c)^(1/2)+1/3*a/b/(
-a*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)
```

3.491.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{1}{3}\left(\frac{3ac+2bcx^3+adx^3}{(bc-ad)^2\left(a+bx^3\right)\sqrt{c+dx^3}}+\frac{(2bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{5/2}}\right)$$

input
```
Integrate[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((3*a*c + 2*b*c*x^3 + a*d*x^3)/((b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3])
 + ((2*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(S
qrt[b]*(-(b*c) + a*d)^(5/2)))/3
```

3.491.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {948, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(bx^3+a)^2 (dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{(ad+2bc)\int \frac{1}{(bx^3+a)(dx^3+c)^{3/2}}dx^3}{2b(bc-ad)}+\frac{a}{b(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(\frac{(ad+2bc)\left(\frac{b\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{bc-ad}+\frac{2}{\sqrt{c+dx^3}(bc-ad)}\right)}{2b(bc-ad)}+\frac{a}{b(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{(ad+2bc)\left(\frac{2b\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{d(bc-ad)}+\frac{2}{\sqrt{c+dx^3}(bc-ad)}\right)}{2b(bc-ad)}+\frac{a}{b(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{(ad+2bc)\left(\frac{2}{\sqrt{c+dx^3}(bc-ad)}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\right)}{2b(bc-ad)}+\frac{a}{b\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

input

```
Int[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output

```
(a/(b*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) + ((2*b*c + a*d)*(2/((b*c -
 a*d)*Sqrt[c + d*x^3]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt
[b*c - a*d]])/(b*c - a*d)^(3/2)))/(2*b*(b*c - a*d)))/3
```

3.491.3.1 Defintions of rubi rules used

rule 61

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.491.4 Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^3+c}\,\left(b\,x^3+a\right)(ad+2bc)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3}+\left(\frac{2bc\,x^3}{3}+a\left(\frac{d\,x^3}{3}+c\right)\right)\sqrt{(ad-bc)b}}{\sqrt{d\,x^3+c}\,\sqrt{(ad-bc)b}\,(ad-bc)^2\,(b\,x^3+a)}$
default	$-\frac{2\left(b\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{d\,x^3+c}+\sqrt{(ad-bc)b}\right)}{b\sqrt{(ad-bc)b}\,\sqrt{d\,x^3+c}\,(3ad-3bc)}-\frac{ad\left(-\frac{2}{\sqrt{d\,x^3+c}}-\frac{b\sqrt{d\,x^3+c}}{d\left(b\,x^3+a\right)}-\frac{3\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)b}{\sqrt{(ad-bc)b}}\right)}{3b(ad-bc)^2}$
elliptic	$\frac{2c}{3(ad-bc)^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{a\sqrt{d\,x^3+c}}{3(ad-bc)^2(b\,x^3+a)}+$ $i\sqrt{2}$ $\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}$ $(-ad-2bc)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}$ $id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right.}{\left(-c\,d^2\right.}\right.$

input
```
int(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(1/3*(d*x^3+c)^(1/2)*(b*x^3+a)*(a*d+2*b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d-
b*c)*b)^(1/2))+(2/3*b*c*x^3+a*(1/3*d*x^3+c))*((a*d-b*c)*b)^(1/2))/(d*x^3+c
)^(1/2)/((a*d-b*c)*b)^(1/2)/(a*d-b*c)^2/(b*x^3+a)
```

3.491.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(114) = 228$.

Time = 0.38 (sec) , antiderivative size = 630, normalized size of antiderivative = 4.70

$$\int \frac{x^5}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \left[\frac{((2\,b^2cd+abd^2)x^6+2\,abc^2+a^2cd+(2\,b^2c^2+3\,abcd+a^2d^2)x^3)\sqrt{b^2c-ab}}{6\,(ab^4c^4-3\,a^2b^3c^3d+3\,a^3b^2c^2d^2-a^4bcd^3+(b^5c^3d-}\right.$$

input

```
integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output

```
[1/6*(((2*b^2*c*d + a*b*d^2)*x^6 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*
b*c*d + a^2*d^2)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*s
qrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(3*a*b^2*c^2 - 3*a^2*
b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c
^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^5*c^3*d - 3*a*
b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (b^5*c^4 - 2*a*b^4*c^3*
d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^3), 1/3*(((2*b^2*c*d + a*b*d^2)*x^6 + 2
*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^3)*sqrt(-b^2*c +
a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a
*b^2*c^2 - 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^3)*sqrt(d*x
^3 + c))/(a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 +
(b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (b^5*c
^4 - 2*a*b^4*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^3)]
```

3.491.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^5}{(a+bx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

input

```
integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output

```
Integral(x**5/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.491.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.491.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.35

$$\int \frac{x^5}{\left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \frac{\frac{\left(2\,bcd+ad^2\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}} + \frac{2\,\left(dx^3+c\right)bcd-2\,bc^2d+\left(dx^3+c\right)ad^2+2\,acd^2}{\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\left(dx^3+c\right)^{\frac{3}{2}}b-\sqrt{dx^3+c}bc+\sqrt{dx^3+c}ad\right)}}{3\,d}$$

input
```
integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
1/3*((2*b*c*d + a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (2*(d*x^3 + c)*b*c*d
- 2*b*c^2*d + (d*x^3 + c)*a*d^2 + 2*a*c*d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + sqrt(d*x^3 + c)*a*d)))/d
```

3.491.9 Mupad [B] (verification not implemented)

Time = 11.73 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.84

$$\int \frac{x^5}{\left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = -\frac{\sqrt{d\,x^3+c}\,\left(x^3\,\left(\frac{3\,b\,d\,\left(a\,d+b\,c\right)-b\,d\,\left(a\,d+2\,b\,c\right)}{3\,\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}-\frac{b\,d\,\left(a\,d+b\,c\right)}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)-\frac{a\,b\,c\,d}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)}{b\,d\,x^6+\left(a\,d+b\,c\right)\,x^3+a\,c} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(a\,d+2\,b\,c\right)\,1{}\mathrm{i}}{6\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{5/2}}$$

input
```
int(x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
(log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x
^3)/(a + b*x^3))*(a*d + 2*b*c)*1i)/(6*b^(1/2)*(a*d - b*c)^(5/2)) - ((c + d
*x^3)^(1/2)*(x^3*((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 +
b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^2*d - 2
*a*b^2*c*d^2)) - (a*b*c*d)/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)))/(a*c
+ x^3*(a*d + b*c) + b*d*x^6)
```

3.491. $\int \frac{x^5}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

3.492 $\int \frac{x^2}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.492.1 Optimal result

Integrand size = 24, antiderivative size = 108

$$\int \frac{x^2}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=-\frac{d}{(bc-ad)^2\sqrt{c+dx^3}}-\frac{1}{3(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}+\frac{\sqrt{b}d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

output
```
d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(5/
2)-d/(-a*d+b*c)^2/(d*x^3+c)^(1/2)-1/3/(-a*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)
```

3.492.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{-2ad-b\left(c+3dx^3\right)}{3(bc-ad)^2\left(a+bx^3\right)\sqrt{c+dx^3}}-\frac{\sqrt{b}d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}$$

input
```
Integrate[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-2*a*d - b*(c + 3*d*x^3))/(3*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3]) -
 (Sqrt[b]*d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c)
+ a*d)^(5/2)
```

3.492.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {946, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$$

$$\Big\downarrow \textcolor{blue}{946}$$

$$\frac{1}{3}\int \frac{1}{(bx^3+a)^2(dx^3+c)^{3/2}}dx^3$$

$$\Big\downarrow \textcolor{blue}{52}$$

$$\frac{1}{3}\left(-\frac{3d\int \frac{1}{(bx^3+a)(dx^3+c)^{3/2}}dx^3}{2(bc-ad)}-\frac{1}{(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \textcolor{blue}{61}$$

$$\frac{1}{3}\left(-\frac{3d\left(\frac{b\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}dx^3}{bc-ad}+\frac{2}{\sqrt{c+dx^3}(bc-ad)}\right)}{2(bc-ad)}-\frac{1}{(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \textcolor{blue}{73}$$

$$\frac{1}{3}\left(-\frac{3d\left(\frac{2b\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{d(bc-ad)}+\frac{2}{\sqrt{c+dx^3}(bc-ad)}\right)}{2(bc-ad)}-\frac{1}{(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$\frac{1}{3}\left(-\frac{3d\left(\frac{2}{\sqrt{c+dx^3}(bc-ad)}-\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\right)}{2(bc-ad)}-\frac{1}{(a+bx^3)\sqrt{c+dx^3}(bc-ad)}\right)$$

input
```
Int[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-(1/((b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3])) - (3*d*(2/((b*c - a*d)*Sqr
t[c + d*x^3]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*
d]])/(b*c - a*d)^(3/2)))/(2*(b*c - a*d)))/3
```

3.492.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.492.4 Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{-\frac{b\sqrt{d\,x^{3}+c}}{3\left(b\,x^{3}+a\right)}-\frac{db\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{2d}{3\sqrt{d\,x^{3}+c}}}{(ad-bc)^{2}}$
default	$\frac{d\left(-\frac{2}{\sqrt{d\,x^{3}+c}}-\frac{b\sqrt{d\,x^{3}+c}}{d\left(b\,x^{3}+a\right)}-\frac{3\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)b}{\sqrt{(ad-bc)b}}\right)}{3(ad-bc)^{2}}$
elliptic	$-\frac{2d}{3(ad-bc)^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{b\sqrt{d\,x^{3}+c}}{3(ad-bc)^{2}(b\,x^{3}+a)}+\frac{ib\sqrt{2}\,\Bigg(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\Big(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+(}{d}}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}}{}$

input
```
int(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)^2*(-1/3*b*(d*x^3+c)^(1/2)/(b*x^3+a)-d*b/((a*d-b*c)*b)^(1/2)*ar
ctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))-2/3*d/(d*x^3+c)^(1/2))
```

3.492.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(92) = 184.

Time = 0.33 (sec) , antiderivative size = 450, normalized size of antiderivative = 4.17

$$\int \frac{x^2}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \Bigg[\frac{3\,(bd^2x^6+(bcd+ad^2)x^3+acd)\sqrt{\frac{b}{bc-ad}}\log\Bigg(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{}{bc-}}}{bx^3+a}}{6\,((b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^6+ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^3}$$

input
```
integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(b*d^2*x^6 + (b*c*d + a*d^2)*x^3 + a*c*d)*sqrt(b/(b*c - a*d))*log(
(b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d))
)/(b*x^3 + a)) - 2*(3*b*d*x^3 + b*c + 2*a*d)*sqrt(d*x^3 + c))/((b^3*c^2*d
- 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^3), 1/3*(3*(b*d^2*x^6 +
 (b*c*d + a*d^2)*x^3 + a*c*d)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)
*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - (3*b*d*x^3 + b*c + 2*
a*d)*sqrt(d*x^3 + c))/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 +
a^3*d^3)*x^3)]
```

3.492.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x^2}{(a+bx^3)^2(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x**2/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.492.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.492.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.42

$$\int \frac{x^2}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{bd\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}} - \frac{3\,(dx^3+c)bd-2\,bcd+2\,ad^2}{3\,(b^2c^2-2\,abcd+a^2d^2)\left((dx^3+c)^{\frac{3}{2}}b-\sqrt{dx^3+c}bc+\sqrt{dx^3+c}ad\right)}$$

input
```
integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-b*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(3*(d*x^3 + c)*b*d - 2*b*c*d + 2*a*
d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 +
c)*b*c + sqrt(d*x^3 + c)*a*d))
```

3.492.9 Mupad [B] (verification not implemented)

Time = 11.31 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.84

$$\int \frac{x^2}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{\left(\frac{3\,b\,d\,(a\,d+b\,c)-b\,d\,(a\,d+2\,b\,c)}{3\,(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d)}+\frac{b^2\,d^2\,x^3}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)\sqrt{d\,x^3+c}}{b\,d\,x^6+(a\,d+b\,c)\,x^3+a\,c} + \frac{\sqrt{b}\,d\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)1{}\mathrm{i}}{2\,(a\,d-b\,c)^{5/2}}$$

input
```
int(x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
(b^(1/2)*d*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*
2i - b*d*x^3)/(a + b*x^3))*1i)/(2*(a*d - b*c)^(5/2)) - (((3*b*d*(a*d + b*c
) - b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (b^2*
d^2*x^3)/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2))*(c + d*x^3)^(1/2))/(a*c
+ x^3*(a*d + b*c) + b*d*x^6)
```

3.493 $\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.493.1 Optimal result

Integrand size = 24, antiderivative size = 172

$$\begin{aligned}\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx &= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} \\ &+ \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} \\ &+ \frac{b^{3/2}(2bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{5/2}}\end{aligned}$$

output
```
-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(3/2)+1/3*b^(3/2)*(-5*a*d+2*b*
c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(5/2)+
1/3*d*(2*a*d+b*c)/a/c/(-a*d+b*c)^2/(d*x^3+c)^(1/2)+1/3*b/a/(-a*d+b*c)/(b*x
^3+a)/(d*x^3+c)^(1/2)
```

3.493.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\frac{a\left(2a^2d^2+2abd^2x^3+b^2c\left(c+dx^3\right)\right)}{c(bc-ad)^2\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{b^{3/2}(2bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{3/2}}}{3a^2}$$

input
```
Integrate[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
((a*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3)))/(c*(b*c - a*d)^2*(a +
 b*x^3)*Sqrt[c + d*x^3]) - (b^(3/2)*(2*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2) - (2*ArcTanh[Sqrt[c +
 d*x^3]/Sqrt[c]])/c^(3/2))/(3*a^2)
```

3.493.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 114, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^3 (bx^3+a)^2 (dx^3+c)^{3/2}} dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3} \left(\frac{\int \frac{3bdx^3+2bc-2ad}{2x^3(bx^3+a)(dx^3+c)^{3/2}}dx^3}{a(bc-ad)} + \frac{b}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(\frac{\int \frac{3bdx^3+2(bc-ad)}{x^3(bx^3+a)(dx^3+c)^{3/2}}dx^3}{2a(bc-ad)} + \frac{b}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)} \right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{3} \left(\frac{\frac{2d(2ad+bc)}{c\sqrt{c+dx^3}(bc-ad)} - \frac{2\int -\frac{bd(bc+2ad)x^3+2(bc-ad)^2}{2x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{c(bc-ad)}}{2a(bc-ad)} + \frac{b}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\frac{\int \frac{bd(bc+2ad)x^3+2(bc-ad)^2}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{c(bc-ad)}+\frac{2d(2ad+bc)}{c\sqrt{c+dx^3}(bc-ad)}}{2a(bc-ad)}+\frac{b}{a\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(\frac{\frac{\frac{2(bc-ad)^2\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{b^2c(2bc-5ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{c(bc-ad)}+\frac{2d(2ad+bc)}{c\sqrt{c+dx^3}(bc-ad)}}{2a(bc-ad)}+\frac{b}{a\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{\frac{\frac{4(bc-ad)^2\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2b^2c(2bc-5ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{c(bc-ad)}+\frac{2d(2ad+bc)}{c\sqrt{c+dx^3}(bc-ad)}}{2a(bc-ad)}+\frac{b}{a\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(\frac{\frac{\frac{2b^{3/2}c(2bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{4(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)}+\frac{2d(2ad+bc)}{c\sqrt{c+dx^3}(bc-ad)}}{2a(bc-ad)}+\frac{b}{a\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}\right)$$

input
```
Int[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(b/(a*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) + ((2*d*(b*c + 2*a*d))/(c*(
b*c - a*d)*Sqrt[c + d*x^3]) + ((-4*(b*c - a*d)^2*ArcTanh[Sqrt[c + d*x^3]/S
qrt[c]])/(a*Sqrt[c]) + (2*b^(3/2)*c*(2*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(c*(b*c - a*d)))/(2*a*(
b*c - a*d)))/3
```

3.493. $\int \frac{1}{x(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

3.493.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.493. $\int \frac{1}{x(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.493.4 Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{-2\left(-\frac{5ad}{2}+bc\right)c^{\frac{5}{2}}\sqrt{d\,x^{3}+c}\,b^{2}\left(b\,x^{3}+a\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)+\left(-2c\sqrt{d\,x^{3}+c}\left(ad-bc\right)^{2}\left(b\,x^{3}+a\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)+c^{\frac{3}{2}}a(\right.}{3\sqrt{\left(ad-bc\right)b}\,\sqrt{d\,x^{3}+c}\,a^{2}\left(b\,x^{3}+a\right)\left(ad-bc\right)^{2}c^{\frac{5}{2}}}$
default	$\frac{\frac{2}{3c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{a^{2}}+\frac{2b\left(b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)\sqrt{d\,x^{3}+c}+\sqrt{\left(ad-bc\right)b}\right)}{a^{2}\sqrt{\left(ad-bc\right)b}\,\sqrt{d\,x^{3}+c}\left(3ad-3bc\right)}-\frac{bd\left(-\frac{2}{\sqrt{d\,x^{3}+c}}-\frac{b\sqrt{d\,x^{3}+c}}{d\left(b\,x^{3}+a\right)}-\right.}{3a(ad-}$
elliptic	Expression too large to display

input
```
int(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-2*(-5/2*a*d+b*c)*c^(5/2)*(d*x^3+c)^(1/2)*b^2*(b*x^3+a)*arctan(b*(d*x
^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(-2*c*(d*x^3+c)^(1/2)*(a*d-b*c)^2*(b*x^3+
a)*arctanh((d*x^3+c)^(1/2)/c^(1/2))+c^(3/2)*a*(c*(d*x^3+c)*b^2+2*x^3*a*b*d
^2+2*a^2*d^2))*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/(d*x^3+c)^(1/2)/a^
2/(b*x^3+a)/(a*d-b*c)^2/c^(5/2)
```

3.493.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(144) = 288.

Time = 0.60 (sec) , antiderivative size = 1819, normalized size of antiderivative = 10.58

$$\int \frac{1}{x\left(a+bx^{3}\right)^{2}\left(c+dx^{3}\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/6*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^6
+ (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(b/(b*c - a*d))*l
og((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*
d)))/(b*x^3 + a)) - 2*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 +
a^3*d^3)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) -
 2*(a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^
3 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^
3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*
c^3*d^2 + a^5*c^2*d^3)*x^3), 1/3*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^
3*d - 5*a*b^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)
*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*
c - a*d))/(b*d*x^3 + b*c)) + ((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6
+ a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c
*d^2 + a^3*d^3)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)
/x^3) + (a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt
(d*x^3 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d -
 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a
^4*b*c^3*d^2 + a^5*c^2*d^3)*x^3), 1/6*(4*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2
*b*d^3)*x^6 + a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*...
```

3.493.6 Sympy [F]

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.493.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{(bx^3+a)^2(dx^3+c)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x), x)
```

3.493.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=&-\frac{(2\,b^3c-5\,ab^2d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)\sqrt{-b^2c+abd}}\\&+\frac{(dx^3+c)b^2cd+2\,(dx^3+c)abd^2-2\,abcd^2+2\,a^2d^3}{3\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2)\left((dx^3+c)^{\frac{3}{2}}b-\sqrt{dx^3+c}bc+\sqrt{dx^3+c}ad\right)}\\&+\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}c}\end{aligned}$$

input
```
integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
-1/3*(2*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/
((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b^2*c + a*b*d)) + 1/3*((d*x^3
 + c)*b^2*c*d + 2*(d*x^3 + c)*a*b*d^2 - 2*a*b*c*d^2 + 2*a^2*d^3)/((a*b^2*c
^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c
 + sqrt(d*x^3 + c)*a*d)) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(
-c)*c)
```

3.493.9 Mupad [B] (verification not implemented)

Time = 16.41 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2\,c^{3/2}}+\frac{\left(\frac{{\left(2\,a\,d+b\,c\right)}^4+{\left(2\,a\,d+b\,c\right)}^2\,\left(\left(a\,d+2\,b\,c\right)\,\left(2\,a\,d+b\,c\right)-9\,a\,b\,c\,d\right)}{9\,a\,c\,{\left(2\,a\,d+b\,c\right)}^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{b\,d\,x^3\,\left(2\,a\,d+b\,c\right)}{3\,a\,c\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,\sqrt{d\,x^3+c}}{b\,d\,x^6+\left(a\,d+b\,c\right)\,x^3+a\,c}+\frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(5\,a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,a^2\,{\left(a\,d-b\,c\right)}^{5/2}}$$

input
```
int(1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(
3*a^2*c^(3/2)) + ((((2*a*d + b*c)^4 + (2*a*d + b*c)^2*((a*d + 2*b*c)*(2*a*
d + b*c) - 9*a*b*c*d))/(9*a*c*(2*a*d + b*c)^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d)) + (b*d*x^3*(2*a*d + b*c))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(c
 + d*x^3)^(1/2))/(a*c + x^3*(a*d + b*c) + b*d*x^6) + (b^(3/2)*log((2*b*c -
 a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^
3))*(5*a*d - 2*b*c)*1i)/(6*a^2*(a*d - b*c)^(5/2))
```

3.494 $\int \frac{1}{x^4\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.494.1 Optimal result

Integrand size = 24, antiderivative size = 241

$$\int \frac{1}{x^4\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = -\frac{d(2b^2c^2-2abcd+3a^2d^2)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{(4bc+3ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}} - \frac{b^{5/2}(4bc-7ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{5/2}}$$

output
```
1/3*(3*a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^3/c^(5/2)-1/3*b^(5/2)
*(-7*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*
d+b*c)^(5/2)-1/3*d*(3*a^2*d^2-2*a*b*c*d+2*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(d
*x^3+c)^(1/2)-1/3*b*(-a*d+2*b*c)/a^2/c/(-a*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2
)-1/3/a/c/x^3/(b*x^3+a)/(d*x^3+c)^(1/2)
```

3.494.2 Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4 \left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \frac{-\frac{a\left(2b^3c^2x^3\left(c+dx^3\right)+a^3d^2\left(c+3dx^3\right)+ab^2c\left(c^2-cdx^3-2d^2x^6\right)+a^2bd\left(-2c^2-cdx^3+3d^2x^6\right)\right)}{c^2(bc-ad)^2x^3\left(a+bx^3\right)\sqrt{c+dx^3}}+}{3a^3}$$

input
```
Integrate[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-((a*(2*b^3*c^2*x^3*(c + d*x^3) + a^3*d^2*(c + 3*d*x^3) + a*b^2*c*(c^2 -
c*d*x^3 - 2*d^2*x^6) + a^2*b*d*(-2*c^2 - c*d*x^3 + 3*d^2*x^6)))/(c^2*(b*c
- a*d)^2*x^3*(a + b*x^3)*Sqrt[c + d*x^3])) + (b^(5/2)*(4*b*c - 7*a*d)*ArcT
an[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2) + (
(4*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/c^(5/2))/(3*a^3)
```

3.494.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {948, 114, 27, 168, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^6 \left(bx^3+a\right)^2 \left(dx^3+c\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{5bdx^3+4bc+3ad}{2x^3\left(bx^3+a\right)^2\left(dx^3+c\right)^{3/2}}dx^3}{ac}-\frac{1}{acx^3\left(a+bx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{5bdx^3+4bc+3ad}{x^3\left(bx^3+a\right)^2\left(dx^3+c\right)^{3/2}}dx^3}{2ac}-\frac{1}{acx^3\left(a+bx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(-\frac{\frac{\int \frac{3bd(2bc-ad)x^3+(bc-ad)(4bc+3ad)}{x^3\left(bx^3+a\right)\left(dx^3+c\right)^{3/2}}dx^3}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{1}{acx^3\left(a+bx^3\right)\sqrt{c+dx^3}}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{2d\left(3a^2d^2-2abcd+2b^2c^2\right)}{c\sqrt{c+dx^3}(bc-ad)}-\frac{2\int -\frac{bd\left(2b^2c^2-2abdc+3a^2d^2\right)x^3+(bc-ad)^2(4bc+3ad)}{2x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{c(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{1}{acx^3\left(a+bx^3\right)\sqrt{c}}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{\int \frac{bd\left(2b^2c^2-2abdc+3a^2d^2\right)x^3+(bc-ad)^2(4bc+3ad)}{x^3\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{c(bc-ad)}+\frac{2d\left(3a^2d^2-2abcd+2b^2c^2\right)}{c\sqrt{c+dx^3}(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{1}{acx^3\left(a+bx^3\right)\sqrt{c+}}\right.$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{\frac{(bc-ad)^2(3ad+4bc)\int \frac{1}{x^3\sqrt{dx^3+c}}dx^3}{a}-\frac{b^3c^2(4bc-7ad)\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}dx^3}{a}}{c(bc-ad)}+\frac{2d\left(3a^2d^2-2abcd+2b^2c^2\right)}{c\sqrt{c+dx^3}(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}}{2ac}-\frac{}{acx}\right.$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{\frac{2(bc-ad)^2(3ad+4bc)\int \frac{1}{\frac{x^6}{d}-\frac{c}{d}}d\sqrt{dx^3+c}}{ad}-\frac{2b^3c^2(4bc-7ad)\int \frac{1}{\frac{bx^6}{d}+a-\frac{bc}{d}}d\sqrt{dx^3+c}}{ad}}{c(bc-ad)}+\frac{2d\left(3a^2d^2-2abcd+2b^2c^2\right)}{c\sqrt{c+dx^3}(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^3)\sqrt{c+dx^3}(bc-ad)}}{2ac}-\right.$$

$$\Big\downarrow \text{221}$$

3.494. $\int \frac{1}{x^4(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

$$\frac{1}{3}\Bigg(-\frac{\frac{\frac{2d\left(3a^2d^2-2abcd+2b^2c^2\right)}{c\sqrt{c+dx^3}(bc-ad)}+\frac{\frac{2b^{5/2}c^2(4bc-7ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)^2(3ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^3)\sqrt{c+dx^3}(bc-a}}{2ac}$$

input
```
Int[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-(1/(a*c*x^3*(a + b*x^3)*Sqrt[c + d*x^3])) - ((2*b*(2*b*c - a*d))/(a*(b*c
 - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) + ((2*d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^
2*d^2))/(c*(b*c - a*d)*Sqrt[c + d*x^3]) + ((-2*(b*c - a*d)^2*(4*b*c + 3*a*
d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(5/2)*c^2*(4*b*c -
 7*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c -
a*d]))/(c*(b*c - a*d)))/(a*(b*c - a*d)))/(2*a*c))/3
```

3.494.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.494. $\int \frac{1}{x^4(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

3.494.4 Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{\sqrt{d x^3+c}}{3c^2a^2x^3}-\frac{-\frac{2(3ad+4bc)\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{4a^2d^3}{3(ad-bc)^2\sqrt{d x^3+c}}+\frac{2b^3c^2\left(d\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)\left(b x^3+a\right)+\sqrt{d x^3+c}\sqrt{(ad-b}\right.}{3(ad-bc)^2\sqrt{(ad-bc)b}\left(b x^3+a\right)}}{2a^2c^2}$
pseudoelliptic	$-\frac{-4x^3c^{\frac{9}{2}}b^3\sqrt{d x^3+c}\left(bc-\frac{7ad}{4}\right)\left(b x^3+a\right)\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\left(-3\left(ad+\frac{4bc}{3}\right)x^3\sqrt{d x^3+c}\left(b x^3+a\right)c^2(ad-bc)\right.}{3\sqrt{d x^3+c}\sqrt{(ad-bc)}}$
default	$\frac{-\frac{\sqrt{d x^3+c}}{3c^2x^3}-\frac{2d}{3c^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{d\ \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{a^2}-\frac{2b\left(\frac{2}{3c\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{2\ \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{a^3}+\frac{b^2d\left(-\frac{2}{\sqrt{d x^3+c}}-\frac{b\sqrt{\ }}{d\left(b\right.}\right.}{3}$
elliptic	Expression too large to display

input
```
int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/c^2/a^2*(d*x^3+c)^(1/2)/x^3-1/2/a^2/c^2*(-2/3/a*(3*a*d+4*b*c)*arctanh
((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+4/3*a^2*d^3/(a*d-b*c)^2/(d*x^3+c)^(1/2)+
2/3*b^3*c^2/(a*d-b*c)^2*(d*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))*(
b*x^3+a)+(d*x^3+c)^(1/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/(b*x^3+a
)+4/3*b^3*c^2*(3*a*d-2*b*c)/a/(a*d-b*c)^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*
x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

3.494.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(209) = 418.

Time = 0.72 (sec) , antiderivative size = 2384, normalized size of antiderivative = 9.89

$$\int \frac{1}{x^4\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/6*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3*c^4*d -
 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt(b/(b*c
 - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b
/(b*c - a*d)))/(b*x^3 + a)) - ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*
c*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 +
a^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^
2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) +
 2*c)/x^3) + 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d
 - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d -
 a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^
4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*
c^4*d^2 + a^6*c^3*d^3)*x^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x
^3), -1/6*(2*((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3*c
^4*d - 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt(
-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(
b*d*x^3 + b*c)) - ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^
3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3
+ 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^
4*c*d^3)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) +
 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*...
```

3.494.6 Sympy [F]

$$\int \frac{1}{x^4\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^4\left(a+bx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.494.7 Maxima [F]

$$\int \frac{1}{x^4\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^4), x)
```

3.494.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^4\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{\left(4\,b^4c-7\,ab^3d\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\left(a^3b^2c^2-2\,a^4bcd+a^5d^2\right)\sqrt{-b^2c+abd}}$$
$$-\frac{2\left(dx^3+c\right)^2b^3c^2d-2\left(dx^3+c\right)b^3c^3d-2\left(dx^3+c\right)^2ab^2cd^2+3\left(dx^3+c\right)ab^2c^2d^2+3\left(dx^3+c\right)^2a^2bd^3-7\,(d}{3\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2\right)\Bigl(\left(dx^3+c\right)^{\frac{5}{2}}b-2\left(dx^3+c\right)^{\frac{3}{2}}bc+\sqrt{dx^3+c}bc^2+(d}$$
$$-\frac{\left(4\,bc+3\,ad\right)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}c^2}$$

input
```
integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
1/3*(4*b^4*c - 7*a*b^3*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^
3 + c)^2*b^3*c^2*d - 2*(d*x^3 + c)*b^3*c^3*d - 2*(d*x^3 + c)^2*a*b^2*c*d^2
 + 3*(d*x^3 + c)*a*b^2*c^2*d^2 + 3*(d*x^3 + c)^2*a^2*b*d^3 - 7*(d*x^3 + c)
*a^2*b*c*d^3 + 2*a^2*b*c^2*d^3 + 3*(d*x^3 + c)*a^3*d^4 - 2*a^3*c*d^4)/((a^
2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*((d*x^3 + c)^(5/2)*b - 2*(d*x^3 +
 c)^(3/2)*b*c + sqrt(d*x^3 + c)*b*c^2 + (d*x^3 + c)^(3/2)*a*d - sqrt(d*x^3
 + c)*a*c*d)) - 1/3*(4*b*c + 3*a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*
sqrt(-c)*c^2)
```

3.494.9 Mupad [B] (verification not implemented)

Time = 23.90 (sec) , antiderivative size = 18847, normalized size of antiderivative = 78.20

$$\int \frac{1}{x^4 \left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
int(1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output

```
(2*b*log(1/x^6))/(3*a^3*c^(3/2)) - (c + d*x^3)^(1/2)/(3*a^2*c^2*x^3) + (d*
log(1/x^6))/(2*a^2*c^(5/2)) + (2*b*log(c^(3/2)*(c + d*x^3)^(1/2) - c^(1/2)
*(c + d*x^3)^(3/2) + d^2*x^6 + 2*c*d*x^3 + 3*c^(1/2)*d*x^3*(c + d*x^3)^(1/
2)))/(3*a^3*c^(3/2)) + (d*log(c^(3/2)*(c + d*x^3)^(1/2) - c^(1/2)*(c + d*x
^3)^(3/2) + d^2*x^6 + 2*c*d*x^3 + 3*c^(1/2)*d*x^3*(c + d*x^3)^(1/2)))/(2*a
^2*c^(5/2)) - (b^7*c^9*x^4*(c + d*x^3)^(1/2))/(2*(2*a^9*c^6*d^5*x + 2*a^9*
c^5*d^6*x^4 + a^5*b^4*c^9*d^2*x^4 + a^6*b^3*c^8*d^3*x^4 - 3*a^7*b^2*c^7*d^
4*x^4 + a^5*b^4*c^8*d^3*x^7 - 3*a^7*b^2*c^6*d^5*x^7 - 3*a^8*b*c^7*d^4*x +
a^6*b^3*c^9*d^2*x - a^8*b*c^6*d^5*x^4 + 2*a^8*b*c^5*d^6*x^7)) - (5*a^9*d^7
*x^4*(c + d*x^3)^(1/2))/(4*(a^6*b^5*c^9*x + a^5*b^6*c^9*x^4 - 3*a^7*b^4*c^
7*d^2*x^4 - a^8*b^3*c^6*d^3*x^4 + 2*a^9*b^2*c^5*d^4*x^4 - 3*a^7*b^4*c^6*d^
3*x^7 + 2*a^8*b^3*c^5*d^4*x^7 - 3*a^8*b^3*c^7*d^2*x + 2*a^9*b^2*c^6*d^3*x
+ a^6*b^5*c^8*d*x^4 + a^5*b^6*c^8*d*x^7)) + (3*a^2*d^2*x*(c + d*x^3)^(1/2)
)/(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a
^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7) + (2*b^2*c^2*x*(
c + d*x^3)^(1/2))/(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a
^4*c^3*d*x + 3*a^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7)
- (b^(7/2)*c*log((a^6*b^(15/2)*c^10*36i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d
 - b*c)^(1/2)) - (a^7*b^(13/2)*c^9*d*198i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a
*d - b*c)^(1/2)) + (a^12*b^(3/2)*c^4*d^6*18i)/(a*(a*d - b*c)^(1/2) + b*...
```

3.495 $\int \frac{x^3}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

3.495.1 Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^3}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}$$

output
```
1/4*x^4*AppellF1(4/3,2,3/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/
(d*x^3+c)^(1/2)
```

3.495.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 381 vs. 2(67) = 134.

Time = 10.32 (sec) , antiderivative size = 381, normalized size of antiderivative = 5.69

$$\int \frac{x^3}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=$$
$$-\frac{x^4\left(-8abcd\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\left(8a+\left(a+bx^3\right)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right.\right.}{8a(bc-ad)^2\left(a+bx^3\right)\sqrt{c+dx^3}\left(-8ac\,\text{Appe}\right.}$$

input
```
Integrate[x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
-1/8*(x^4*(-8*a*b*c*d*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a
)]*(8*a + (a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*
x^3)/c), -((b*x^3)/a)]) + (8*a*(2*a*d + b*(c + 3*d*x^3)) + 3*b*d*x^3*(a +
b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x
^3)/a)])*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a
*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/(a*(b*c - a*d
)^2*(a + b*x^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^
3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c
), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/
a)])))
```

3.495.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{\left(bx^3+a\right)^2\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}$$

input
```
Int[x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*a^2*c*Sqrt[c + d*x^3])
```

3.495.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.495.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.44 (sec) , antiderivative size = 787, normalized size of antiderivative = 11.75

method	result	size
elliptic	Expression too large to display	787
default	Expression too large to display	1593

input
```
int(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*d*x/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)-1/3*b/(a*d-b*c)^2*x*(d*x^3+c)^(1/
2)/(b*x^3+a)+1/3*I/(a*d-b*c)^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x
-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2))+1/18*I/d^2*2^(1/2)*sum((-7*a*d-2*b*c)/(a*d-b*c)^3
/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^
2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)
^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/
2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*b+a))
```

3.495.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.495.6 Sympy [F]

$$\int \frac{x^3}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{x^3}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(x**3/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output

```
Integral(x**3/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.495.7 Maxima [F]

$$\int \frac{x^3}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{x^3}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output

```
integrate(x^3/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

3.495.8 Giac [F]

$$\int \frac{x^3}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{x^3}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output

```
integrate(x^3/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

3.495.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(b\,x^3+a)^2\,(d\,x^3+c)^{3/2}}\,dx$$

input
```
int(x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
int(x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

3.496 $\int \frac{x}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.496.1 Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},2,\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2c\sqrt{c+dx^3}}$$

output
```
1/2*x^2*AppellF1(2/3,2,3/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/
(d*x^3+c)^(1/2)
```

3.496.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 216 vs. 2(67) = 134.

Time = 10.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.22

$$\int \frac{x}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=$$

$$-\frac{x^2\Bigg(-10a(2a^2d^2+2abd^2x^3+b^2c(c+dx^3))+5(-b^2c^2+6abcd+a^2d^2)\left(a+bx^3\right)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\right.}{30a^2c(bc-ad)^2\left(a+bx^3\right.}$$

input
```
Integrate[x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
-1/30*(x^2*(-10*a*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3)) + 5*(-(b
^2*c^2) + 6*a*b*c*d + a^2*d^2)*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + b*d*(b*c + 2*a*d)*x^3*(a + b
*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^
3)/a)]))/(a^2*c*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3])
```

3.496.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{x}{(bx^3+a)^2\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},2,\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2c\sqrt{c+dx^3}}$$

input
```
Int[x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 2, 3/2, 5/3, -((b*x^3)/a), -((d*x^3
)/c)])/(2*a^2*c*Sqrt[c + d*x^3])
```

3.496.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.496.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.38 (sec) , antiderivative size = 986, normalized size of antiderivative = 14.72

method	result	size
default	Expression too large to display	986
elliptic	Expression too large to display	986

input
```
int(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*d^2*x^2/c/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)+1/3*b^2/(a*d-b*c)^2/a*x^2*(d
*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(-1/3*d^2/(a*d-b*c)^2/c-1/6*b*d/(a*d-b*c)^2/
a)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a/d^2*b*2^
(1/2)*sum((11*a*d-2*b*c)/(a*d-b*c)^3/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x
-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*
d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3...
```

3.496.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.496.6 Sympy [F]

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x}{(a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(x/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.496.7 Maxima [F]

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+a)^2 (dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

3.496.8 Giac [F]

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+a)^2 (dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

3.496.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{x}{\left(b\,x^3+a\right)^2\,\left(d\,x^3+c\right)^{3/2}} \,d x$$

input
```
int(x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
int(x/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

3.497 $\int \frac{1}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

3.497.1 Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\sqrt{c+dx^3}}$$

output
```
x*AppellF1(1/3,2,3/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/(d*x^3
+c)^(1/2)
```

3.497.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 381 vs. 2(62) = 124.

Time = 10.52 (sec) , antiderivative size = 381, normalized size of antiderivative = 6.15

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{x\Bigg(bd(bc+2ad)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\big(64ac(3a^2d^2+}{}}{}$$

input
```
Integrate[1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x*(b*d*(b*c + 2*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -
((d*x^3)/c), -((b*x^3)/a)] + (a*(64*a*c*(3*a^2*d^2 + 2*a*b*d*(-3*c + d*x^3
) + b^2*c*(3*c + d*x^3))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3
)/a)] - 24*x^3*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3))*(2*b*c*Appe
llF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2
, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(8*a*c*AppellF1[1/3,
 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2
, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)])))))/(24*a^2*c*(b*c - a*d)^2*Sqrt[c + d*x^3])
```

3.497.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{(bx^3+a)^2\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\sqrt{c+dx^3}}$$

input
```
Int[1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 3/2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*c*Sqrt[c + d*x^3])
```

3.497.3.1 Defintions of rubi rules used

rule 936

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.497.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.41 (sec) , antiderivative size = 830, normalized size of antiderivative = 13.39

method	result	size
default	Expression too large to display	830
elliptic	Expression too large to display	830

input

```
int(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/3*d^2*x/c/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)+1/3*b^2/(a*d-b*c)^2/a*x*(d*x^3
+c)^(1/2)/(b*x^3+a)-2/3*I*(1/3*d^2/(a*d-b*c)^2/c+1/6*b*d/(a*d-b*c)^2/a)*3^
(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I
/a/d^2*b*2^(1/2)*sum((13*a*d-4*b*c)/(a*d-b*c)^3/_alpha^2*(-c*d^2)^(1/3)*(1
/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3
^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)
^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)...
```

3.497.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \text{Timed out}$$

input

```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output

```
Timed out
```

3.497.6 Sympy [F]

$$\int \frac{1}{\left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(a+bx^3\right)^2 \left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.497.7 Maxima [F]

$$\int \frac{1}{\left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

3.497.8 Giac [F]

$$\int \frac{1}{\left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

3.497.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(b\,x^3+a)^2\,(d\,x^3+c)^{3/2}}\,dx$$

input

```
int(1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output

```
int(1/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

3.498 $\int \frac{1}{x^2\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

3.498.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(-\frac{1}{3},2,\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2cx\sqrt{c+dx^3}}$$

output
```
-AppellF1(-1/3,2,3/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/x/(d*x
^3+c)^(1/2)
```

3.498.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 308 vs. 2(65) = 130.

Time = 10.42 (sec) , antiderivative size = 308, normalized size of antiderivative = 4.74

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{-20a(4b^3c^2x^3(c+dx^3)+a^3d^2(3c+5dx^3)+3ab^2c(c^2-cdx^3-2d^2x^6)+}{}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-20*a*(4*b^3*c^2*x^3*(c + d*x^3) + a^3*d^2*(3*c + 5*d*x^3) + 3*a*b^2*c*(c
^2 - c*d*x^3 - 2*d^2*x^6) + a^2*b*d*(-6*c^2 - 3*c*d*x^3 + 5*d^2*x^6)) + 5*
(-8*b^3*c^3 + 21*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 5*a^3*d^3)*x^3*(a + b*x^3)*
Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)]
 + 2*b*d*(4*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*x^6*(a + b*x^3)*Sqrt[1 + (d*x
^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^3*c^2
*(b*c - a*d)^2*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

3.498.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^2(bx^3+a)^2\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{1}{3},2,\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2cx\sqrt{c+dx^3}}$$

input
```
Int[1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
-((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 2, 3/2, 2/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c*x*Sqrt[c + d*x^3]))
```

3.498.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.498.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 7.53 (sec) , antiderivative size = 1019, normalized size of antiderivative = 15.68

method	result	size
elliptic	Expression too large to display	1019
risch	Expression too large to display	2334
default	Expression too large to display	2383

input
```
int(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*d^3*x^2/c^2/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)-1/3/(a*d-b*c)^2/a^2*b^3*x
^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/c^2/a^2*(d*x^3+c)^(1/2)/x-2/3*I*(1/3*d^3/c^
2/(a*d-b*c)^2+1/6*b^2*d/a^2/(a*d-b*c)^2+1/2*d/c^2/a^2)*3^(1/2)/d*(-c*d^2)^
(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/18*I*b^2/a^2/d^2*2^(1/2)*sum((17*a*d-
8*b*c)/(a*d-b*c)^3/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3
))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*...
```

3.498.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.498.6 Sympy [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(a+bx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.498.7 Maxima [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^2), x)
```

3.498.8 Giac [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^2), x)
```

3.498.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^2\,\left(b\,x^3+a\right)^2\,\left(d\,x^3+c\right)^{3/2}} \,d x$$

input
```
int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

3.499 $\int \frac{1}{x^3(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

3.499.1 Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{1}{x^3\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}$$

output
```
-1/2*AppellF1(-2/3,2,3/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/x^
2/(d*x^3+c)^(1/2)
```

3.499.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 515 vs. 2(67) = 134.

Time = 10.92 (sec) , antiderivative size = 515, normalized size of antiderivative = 7.69

$$\int \frac{1}{x^3\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{-bd(5b^2c^2-6abcd+7a^2d^2)\,x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{}$$

input
```
Integrate[1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
(-(b*d*(5*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF
1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]) + (a*(32*a*c*(10*b^3*c^2*
x^3*(3*c + d*x^3) + 3*a^3*d^2*(2*c + 7*d*x^3) + 3*a*b^2*c*(2*c^2 - 13*c*d*
x^3 - 4*d^2*x^6) + 2*a^2*b*d*(-6*c^2 - 6*c*d*x^3 + 7*d^2*x^6))*AppellF1[1/
3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(5*b^3*c^2*x^3*(c + d
*x^3) + a^3*d^2*(3*c + 7*d*x^3) + 3*a*b^2*c*(c^2 - c*d*x^3 - 2*d^2*x^6) +
a^2*b*d*(-6*c^2 - 3*c*d*x^3 + 7*d^2*x^6))*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3
, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c
), -((b*x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x
^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/
c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)
/a)]))))/(48*a^3*c^2*(b*c - a*d)^2*x^2*Sqrt[c + d*x^3])
```

3.499.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1013$$

$$\frac{\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^3\left(bx^3+a\right)^2\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$\Big\downarrow \; 1012$$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

output
```
-1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 3/2, 1/3, -((b*x^3)/a), -((d*x
^3)/c)])/(a^2*c*x^2*Sqrt[c + d*x^3])
```

3.499.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.499.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 7.34 (sec) , antiderivative size = 863, normalized size of antiderivative = 12.88

method	result	size
elliptic	Expression too large to display	863
risch	Expression too large to display	1874
default	Expression too large to display	1919

input
```
int(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*d^3*x/c^2/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)-1/3/(a*d-b*c)^2/a^2*b^3*x*(
d*x^3+c)^(1/2)/(b*x^3+a)-1/2/c^2/a^2*(d*x^3+c)^(1/2)/x^2-2/3*I*(-1/3*d^3/c
^2/(a*d-b*c)^2-1/6*b^2*d/a^2/(a*d-b*c)^2-1/4*d/c^2/a^2)*3^(1/2)/d*(-c*d^2)
^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/18*I*b^2/a^2/d^2*2^(
1/2)*sum((19*a*d-10*b*c)/(a*d-b*c)^3/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*
(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alp
ha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alp
ha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+...
```

3.499.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.499.6 Sympy [F]

$$\int \frac{1}{x^3 \left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^3 \left(a+bx^3\right)^2 \left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.499.7 Maxima [F]

$$\int \frac{1}{x^3 \left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{(bx^3+a)^2(dx^3+c)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^3), x)
```

3.499.8 Giac [F]

$$\int \frac{1}{x^3 \left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{(bx^3+a)^2(dx^3+c)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^3), x)
```

3.499.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^3 \left(b\,x^3+a\right)^2 \left(d\,x^3+c\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

output
```
int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

3.500 $\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx$

3.500.1 Optimal result

Integrand size = 24, antiderivative size = 134

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{2B(ex)^{1+m} \left(a+bx^3\right)^{7/2}}{be(23+2m)} - \frac{a^2(2aB(1+m)-Ab(23+2m))(ex)^{1+m}\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{be(1+m)(23+2m)\sqrt{1+\frac{bx^3}{a}}}$$

output
```
2*B*(e*x)^(1+m)*(b*x^3+a)^(7/2)/b/e/(23+2*m)-a^2*(2*a*B*(1+m)-A*b*(23+2*m)
)*(e*x)^(1+m)*hypergeom([-5/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(b*x^3+a)^
(1/2)/b/e/(1+m)/(23+2*m)/(1+b*x^3/a)^(1/2)
```

3.500.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{a^2x(ex)^m\sqrt{a+bx^3}\left(A(4+m)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x^3\,\text{Hype}\right.}{(1+m)(4+m)\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^m*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(a^2*x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-5/2, (1 + m)/
3, (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-5/2, (4 + m
)/3, (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

3.500.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) (ex)^m \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{2B\left(a+bx^3\right)^{7/2}(ex)^{m+1}}{be(2m+23)}-\frac{(2aB(m+1)-Ab(2m+23))\int (ex)^m \left(bx^3+a\right)^{5/2}dx}{b(2m+23)}$$

$$\Big\downarrow \text{889}$$

$$\frac{2B\left(a+bx^3\right)^{7/2}(ex)^{m+1}}{be(2m+23)}-\frac{a^2\sqrt{a+bx^3}(2aB(m+1)-Ab(2m+23))\int (ex)^m \left(\frac{bx^3}{a}+1\right)^{5/2}dx}{b(2m+23)\sqrt{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2B\left(a+bx^3\right)^{7/2}(ex)^{m+1}}{be(2m+23)}-\frac{a^2\sqrt{a+bx^3}(ex)^{m+1}(2aB(m+1)-Ab(2m+23))\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{be(m+1)(2m+23)\sqrt{\frac{bx^3}{a}+1}}$$

input
```
Int[(e*x)^m*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(2*B*(e*x)^(1 + m)*(a + b*x^3)^(7/2))/(b*e*(23 + 2*m)) - (a^2*(2*a*B*(1 +
m) - A*b*(23 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-5/2,
 (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(23 + 2*m)*Sqrt[1 + (b*
x^3)/a])
```

3.500.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.500.4 Maple [F]

$$\int (ex)^m \left(b\,x^3+a\right)^{\frac{5}{2}} \left(x^3B+A\right) dx$$

input
```
int((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

output
```
int((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

3.500.5 Fricas [F]

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^
2)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

3.500.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.96 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.83

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right)\,dx = \frac{Aa^{\frac{5}{2}}e^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{2Aa^{\frac{3}{2}}be^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{A\sqrt{a}b^2e^m x^{m+7}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{7}{3}\\ \frac{m}{3}+\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{10}{3}\right)} + \frac{Ba^{\frac{5}{2}}e^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{2Ba^{\frac{3}{2}}be^m x^{m+7}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{7}{3}\\ \frac{m}{3}+\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{10}{3}\right)} + \frac{B\sqrt{a}b^2e^m x^{m+10}\Gamma\left(\frac{m}{3}+\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{10}{3}\\ \frac{m}{3}+\frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{13}{3}\right)}$$

input

```
integrate((e*x)**m*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

output
```
A*a**(5/2)*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3
+ 4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + 2*A*a**(3/2)*b*e
**m*x**(m + 4)*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 7/3)) + A*sqrt(a)*b**2*e**m*x**(m +
7)*gamma(m/3 + 7/3)*hyper((-1/2, m/3 + 7/3), (m/3 + 10/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(m/3 + 10/3)) + B*a**(5/2)*e**m*x**(m + 4)*gamma(m/3 +
 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(m/3 + 7/3)) + 2*B*a**(3/2)*b*e**m*x**(m + 7)*gamma(m/3 + 7/3)*hyper(
(-1/2, m/3 + 7/3), (m/3 + 10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 +
 10/3)) + B*sqrt(a)*b**2*e**m*x**(m + 10)*gamma(m/3 + 10/3)*hyper((-1/2, m
/3 + 10/3), (m/3 + 13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 13/3))
```

3.500.7 Maxima [F]

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right)\, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^m\, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^m, x)
```

3.500.8 Giac [F]

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right)\, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^m\, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^m, x)
```

3.500.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^m\,{\left(b\,x^3+a\right)}^{5/2} \,d x$$

input
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(5/2),x)
```

output
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(5/2), x)
```

3.501 $\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

3.501.1 Optimal result

Integrand size = 24, antiderivative size = 132

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{5/2}}{be(17+2m)} - \frac{a(2aB(1+m)-Ab(17+2m))(ex)^{1+m}\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{be(1+m)(17+2m)\sqrt{1+\frac{bx^3}{a}}}$$

output
```
2*B*(e*x)^(1+m)*(b*x^3+a)^(5/2)/b/e/(17+2*m)-a*(2*a*B*(1+m)-A*b*(17+2*m))*
(e*x)^(1+m)*hypergeom([-3/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(b*x^3+a)^(1
/2)/b/e/(1+m)/(17+2*m)/(1+b*x^3/a)^(1/2)
```

3.501.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{ax(ex)^m\sqrt{a+bx^3}\left(A(4+m)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x^3\,\text{Hyper}\right.}{(1+m)(4+m)\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^m*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(a*x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-3/2, (1 + m)/3,
 (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-3/2, (4 + m)/
3, (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

3.501.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) (ex)^m \, dx$$

$$\Big\downarrow \text{ 959}$$

$$\frac{2B\left(a+bx^3\right)^{5/2} (ex)^{m+1}}{be(2m+17)}-\frac{(2aB(m+1)-Ab(2m+17)) \int (ex)^m \left(bx^3+a\right)^{3/2}dx}{b(2m+17)}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{2B\left(a+bx^3\right)^{5/2} (ex)^{m+1}}{be(2m+17)}-\frac{a\sqrt{a+bx^3}(2aB(m+1)-Ab(2m+17)) \int (ex)^m \left(\frac{bx^3}{a}+1\right)^{3/2}dx}{b(2m+17)\sqrt{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2B\left(a+bx^3\right)^{5/2} (ex)^{m+1}}{be(2m+17)}-\frac{a\sqrt{a+bx^3}(ex)^{m+1}(2aB(m+1)-Ab(2m+17)) \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{be(m+1)(2m+17)\sqrt{\frac{bx^3}{a}+1}}$$

input
```
Int[(e*x)^m*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(2*B*(e*x)^(1 + m)*(a + b*x^3)^(5/2))/(b*e*(17 + 2*m)) - (a*(2*a*B*(1 + m)
 - A*b*(17 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, (
1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(17 + 2*m)*Sqrt[1 + (b*x^
3)/a])
```

3.501.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.501.4 Maple [F]

$$\int (ex)^{m} \left(b\,x^{3}+a\right)^{\frac{3}{2}} \left(x^{3} B+A\right) dx$$

input
```
int((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

output
```
int((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

3.501.5 Fricas [F]

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^m\, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

3.501.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.89 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.86

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx = \frac{Aa^{\frac{3}{2}}e^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{A\sqrt{a}be^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{Ba^{\frac{3}{2}}e^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{B\sqrt{a}be^m x^{m+7}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{7}{3}\\ \frac{m}{3}+\frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{10}{3}\right)}$$

input
```
integrate((e*x)**m*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
A*a**(3/2)*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3
+ 4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + A*sqrt(a)*b*e**m
*x**(m + 4)*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(m/3 + 7/3)) + B*a**(3/2)*e**m*x**(m + 4)*gamm
a(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(m/3 + 7/3)) + B*sqrt(a)*b*e**m*x**(m + 7)*gamma(m/3 + 7/3)*hy
per((-1/2, m/3 + 7/3), (m/3 + 10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m
/3 + 10/3))
```

3.501.7 Maxima [F]

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^m, x)
```

3.501.8 Giac [F]

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^m, x)
```

3.501.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^m\,{\left(b\,x^3+a\right)}^{3/2} \,d x$$

input
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(3/2),x)
```

output
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(3/2), x)
```

3.502 $\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

3.502.1 Optimal result

Integrand size = 24, antiderivative size = 131

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2B(ex)^{1+m}\left(a+bx^3\right)^{3/2}}{be(11+2m)}-\frac{(2aB(1+m)-Ab(11+2m))(ex)^{1+m}\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{be(1+m)(11+2m)\sqrt{1+\frac{bx^3}{a}}}$$

output
```
2*B*(e*x)^(1+m)*(b*x^3+a)^(3/2)/b/e/(11+2*m)-(2*a*B*(1+m)-A*b*(11+2*m))*(e
*x)^(1+m)*hypergeom([-1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(b*x^3+a)^(1/2
)/b/e/(1+m)/(11+2*m)/(1+b*x^3/a)^(1/2)
```

3.502.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{x(ex)^m\sqrt{a+bx^3}\left(A(4+m)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x^3\,\text{Hypergeometric2}\right.}{(1+m)(4+m)\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^m*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-1/2, (1 + m)/3, (
4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-1/2, (4 + m)/3,
 (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

3.502.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)(ex)^m\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{2B\left(a+bx^3\right)^{3/2}(ex)^{m+1}}{be(2m+11)}-\frac{(2aB(m+1)-Ab(2m+11))\int (ex)^m\sqrt{bx^3+a}dx}{b(2m+11)}$$

$$\Big\downarrow \text{889}$$

$$\frac{2B\left(a+bx^3\right)^{3/2}(ex)^{m+1}}{be(2m+11)}-\frac{\sqrt{a+bx^3}(2aB(m+1)-Ab(2m+11))\int (ex)^m\sqrt{\frac{bx^3}{a}+1}dx}{b(2m+11)\sqrt{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2B\left(a+bx^3\right)^{3/2}(ex)^{m+1}}{be(2m+11)}-\frac{\sqrt{a+bx^3}(ex)^{m+1}(2aB(m+1)-Ab(2m+11))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{be(m+1)(2m+11)\sqrt{\frac{bx^3}{a}+1}}$$

input
```
Int[(e*x)^m*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(2*B*(e*x)^(1 + m)*(a + b*x^3)^(3/2))/(b*e*(11 + 2*m)) - ((2*a*B*(1 + m) -
 A*b*(11 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, (1
+ m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(11 + 2*m)*Sqrt[1 + (b*x^3)
/a])
```

3.502.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.502.4 Maple [F]

$$\int (ex)^m \sqrt{b\,x^3+a}\,\left(x^3B+A\right)dx$$

input
```
int((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

output
```
int((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

3.502.5 Fricas [F]

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

3.502.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{A\sqrt{a}e^mx^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{B\sqrt{a}e^mx^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

input
```
integrate((e*x)**m*(b*x**3+a)**(1/2)*(B*x**3+A),x)
```

output
```
A*sqrt(a)*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3 +
 4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + B*sqrt(a)*e**m*x*
*(m + 4)*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*gamma(m/3 + 7/3))
```

3.502.7 Maxima [F]

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

3.502.8 Giac [F]

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

3.502.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^m\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(1/2),x)
```

output
```
int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(1/2), x)
```

3.503 $\int \frac{(ex)^m(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

3.503.1 Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(ex)^m(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)} - \frac{(2aB(1+m)-Ab(5+2m))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{be(1+m)(5+2m)\sqrt{a+bx^3}}$$

output
```
2*B*(e*x)^(1+m)*(b*x^3+a)^(1/2)/b/e/(5+2*m)-(2*a*B*(1+m)-A*b*(5+2*m))*(e*x
)^(1+m)*hypergeom([1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(1+b*x^3/a)^(1/2)
/b/e/(1+m)/(5+2*m)/(b*x^3+a)^(1/2)
```

3.503.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

$$\int \frac{(ex)^m(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{x(ex)^m\sqrt{1+\frac{bx^3}{a}}\Bigl(A(4+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x^3\,\text{Hypergeometric2F1}}{(1+m)(4+m)\sqrt{a+bx^3}}$$

input
```
Integrate[((e*x)^m*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(A*(4 + m)*Hypergeometric2F1[1/2, (1 + m)/3
, (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[1/2, (4 + m)/
3, (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[a + b*x^3])
```

3.503.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^3)(ex)^m}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{2B\sqrt{a+bx^3}(ex)^{m+1}}{be(2m+5)}-\frac{(2aB(m+1)-Ab(2m+5))\int \frac{(ex)^m}{\sqrt{bx^3+a}}dx}{b(2m+5)}$$

$$\Big\downarrow \text{889}$$

$$\frac{2B\sqrt{a+bx^3}(ex)^{m+1}}{be(2m+5)}-\frac{\sqrt{\frac{bx^3}{a}+1}(2aB(m+1)-Ab(2m+5))\int \frac{(ex)^m}{\sqrt{\frac{bx^3}{a}+1}}dx}{b(2m+5)\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2B\sqrt{a+bx^3}(ex)^{m+1}}{be(2m+5)}-\frac{\sqrt{\frac{bx^3}{a}+1}(ex)^{m+1}(2aB(m+1)-Ab(2m+5))\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{be(m+1)(2m+5)\sqrt{a+bx^3}}$$

input
```
Int[((e*x)^m*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(2*B*(e*x)^(1 + m)*Sqrt[a + b*x^3])/(b*e*(5 + 2*m)) - ((2*a*B*(1 + m) - A*
b*(5 + 2*m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1 +
 m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(5 + 2*m)*Sqrt[a + b*x^3])
```

3.503.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.503.4 Maple [F]

$$\int \frac{(ex)^m (x^3B+A)}{\sqrt{b\,x^3+a}}dx$$

input
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

output
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

3.503.5 Fricas [F]

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*(e*x)^m/sqrt(b*x^3 + a), x)
```

3.503.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{Ae^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{Be^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

input
```
integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
A*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((1/2, m/3 + 1/3), (m/3 + 4/3,), b
*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(m/3 + 4/3)) + B*e**m*x**(m + 4)*
gamma(m/3 + 4/3)*hyper((1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*
pi)/a)/(3*sqrt(a)*gamma(m/3 + 7/3))
```

3.503.7 Maxima [F]

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/sqrt(b*x^3 + a), x)
```

3.503.8 Giac [F]

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^m}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/sqrt(b*x^3 + a), x)
```

3.503.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(B\,x^3+A)\,(e\,x)^m}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(1/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(1/2), x)
```

3.504 $\int \frac{(ex)^m(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.504.1 Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)(ex)^{1+m}}{3abe\sqrt{a+bx^3}} + \frac{(2aB(1+m)+A(b-2bm))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{3abe(1+m)\sqrt{a+bx^3}}$$

output
```
2/3*(A*b-B*a)*(e*x)^(1+m)/a/b/e/(b*x^3+a)^(1/2)+1/3*(2*a*B*(1+m)+A*(-2*b*m
+b))*(e*x)^(1+m)*hypergeom([1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(1+b*x^3
/a)^(1/2)/a/b/e/(1+m)/(b*x^3+a)^(1/2)
```

3.504.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{x(ex)^m\sqrt{1+\frac{bx^3}{a}}\left(A(4+m)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m\right.}{a(1+m)(4+m)\sqrt{a+bx^3}}$$

input
```
Integrate[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(A*(4 + m)*Hypergeometric2F1[3/2, (1 + m)/3
, (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[3/2, (4 + m)/
3, (7 + m)/3, -((b*x^3)/a)]))/(a*(1 + m)*(4 + m)*Sqrt[a + b*x^3])
```

3.504.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {957, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^3)(ex)^m}{(a+bx^3)^{3/2}}dx$$

$$\Big\downarrow 957$$

$$\frac{(2aB(m+1)+A(b-2bm))\int \frac{(ex)^m}{\sqrt{bx^3+a}}dx}{3ab}+\frac{2(ex)^{m+1}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow 889$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}(2aB(m+1)+A(b-2bm))\int \frac{(ex)^m}{\sqrt{\frac{bx^3}{a}+1}}dx}{3ab\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow 888$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}(ex)^{m+1}(2aB(m+1)+A(b-2bm))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{3abe(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

input
```
Int[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*(A*b - a*B)*(e*x)^(1 + m))/(3*a*b*e*Sqrt[a + b*x^3]) + ((2*a*B*(1 + m)
+ A*(b - 2*b*m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2,
(1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(3*a*b*e*(1 + m)*Sqrt[a + b*x^3])
```

3.504.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.504.4 Maple [F]

$$\int \frac{(ex)^m (x^3 B + A)}{(b\,x^3 + a)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

output
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

3.504.5 Fricas [F]

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{3/2}}\,dx = \int \frac{(Bx^3 + A)(ex)^m}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2),
x)
```

3.504.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.54 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.88

$$\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{Ae^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{Be^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

input
```
integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
A*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((3/2, m/3 + 1/3), (m/3 + 4/3,), b
*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(m/3 + 4/3)) + B*e**m*x**(m + 4)
*gamma(m/3 + 4/3)*hyper((3/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I
*pi)/a)/(3*a**(3/2)*gamma(m/3 + 7/3))
```

3.504.7 Maxima [F]

$$\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(3/2), x)
```

3.504.8 Giac [F]

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(3/2), x)
```

3.504.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(B\,x^3+A)\,(e\,x)^m}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(3/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(3/2), x)
```

3.505 $\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

3.505.1 Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\frac{2(Ab-aB)(ex)^{1+m}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{(Ab(7-2m)+2aB(1+m))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{9a^2be(1+m)\sqrt{a+bx^3}}$$

output
```
2/9*(A*b-B*a)*(e*x)^(1+m)/a/b/e/(b*x^3+a)^(3/2)+1/9*(A*b*(7-2*m)+2*a*B*(1+
m))*(e*x)^(1+m)*hypergeom([3/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(1+b*x^3/
a)^(1/2)/a^2/b/e/(1+m)/(b*x^3+a)^(1/2)
```

3.505.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

$$\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\frac{x(ex)^m\sqrt{1+\frac{bx^3}{a}}\left(A(4+m)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m\right.}{a^2(1+m)(4+m)\sqrt{a+bx^3}}$$

input
```
Integrate[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(A*(4 + m)*Hypergeometric2F1[5/2, (1 + m)/3
, (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[5/2, (4 + m)/
3, (7 + m)/3, -((b*x^3)/a)]))/(a^2*(1 + m)*(4 + m)*Sqrt[a + b*x^3])
```

3.505.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {957, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^3)(ex)^m}{(a+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(2aB(m+1)+Ab(7-2m))\int \frac{(ex)^m}{(bx^3+a)^{3/2}}dx}{9ab}+\frac{2(ex)^{m+1}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}(2aB(m+1)+Ab(7-2m))\int \frac{(ex)^m}{\left(\frac{bx^3}{a}+1\right)^{3/2}}dx}{9a^2b\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}(ex)^{m+1}(2aB(m+1)+Ab(7-2m))\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{9a^2be(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

input
```
Int[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(A*b - a*B)*(e*x)^(1 + m))/(9*a*b*e*(a + b*x^3)^(3/2)) + ((A*b*(7 - 2*m
) + 2*a*B*(1 + m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2
, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(9*a^2*b*e*(1 + m)*Sqrt[a + b*x^3])
```

3.505.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.505.4 Maple [F]

$$\int \frac{(ex)^m (x^3B + A)}{(b\,x^3 + a)^{\frac{5}{2}}}dx$$

input
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

output
```
int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

3.505.5 Fricas [F]

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{5/2}}\,dx = \int \frac{(Bx^3 + A)(ex)^m}{(bx^3 + a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^
2*b*x^3 + a^3), x)
```

3.505.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

3.505.7 Maxima [F]

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{5/2}}\, dx = \int \frac{(Bx^3 + A)(ex)^m}{(bx^3 + a)^{\frac{5}{2}}}\, dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(5/2), x)
```

3.505.8 Giac [F]

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{5/2}}\, dx = \int \frac{(Bx^3 + A)(ex)^m}{(bx^3 + a)^{\frac{5}{2}}}\, dx$$

input
```
integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(5/2), x)
```

3.505.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(B\,x^3+A)\,(e\,x)^m}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(5/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(5/2), x)
```

3.506 $\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

3.506.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}$$

output
```
-1/3*(a*d+b*c)*arctanh(d^(1/2)*(b*x^3+a)^(1/2)/b^(1/2)/(d*x^3+c)^(1/2))/b^
(3/2)/d^(3/2)+1/3*(b*x^3+a)^(1/2)*(d*x^3+c)^(1/2)/b/d
```

3.506.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{d}\sqrt{a+bx^3}}\right)}{3b^{3/2}d^{3/2}}$$

input
```
Integrate[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[b]*
Sqrt[c + d*x^3])/(Sqrt[d]*Sqrt[a + b*x^3])])/(3*b^(3/2)*d^(3/2))
```

3.506.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {948, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt{bx^3+a}\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{bd}-\frac{(ad+bc)\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}dx^3}{2bd}\right)$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{3}\left(\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{bd}-\frac{(ad+bc)\int \frac{1}{b-dx^6}d\frac{\sqrt{bx^3+a}}{\sqrt{dx^3+c}}}{bd}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{bd}-\frac{(ad+bc)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{b^{3/2}d^{3/2}}\right)$$

input
```
Int[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
((Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(b^(3/2)*d^(3/2)))/3
```

3.506.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.506.4 Maple [F]

$$\int \frac{x^5}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

input
```
int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

3.506.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.91

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{4\sqrt{bx^3+a}\sqrt{dx^3+c}bd + (bc+ad)\sqrt{bd}\log\big(8\,b^2d^2x^6 + b^2c^2 + 6\,abcd + a^2d^2 + 8\,(b^2cd+abd^2)x^3 - 4\,(2}{12\,b^2d^2}\right.$$

input
```
integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(4*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8
*b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^3 -
 4*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(b*d)))/(b^
2*d^2), 1/6*(2*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*b*d + (b*c + a*d)*sqrt(-b*d
)*arctan(1/2*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(
-b*d)/(b^2*d^2*x^6 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^3)))/(b^2*d^2)]
```

3.506.6 Sympy [F]

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**5/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**5/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.506.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.506.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\frac{(bc+ad)\log\left(\left|-\sqrt{bx^3+a}\sqrt{bd}+\sqrt{b^2c+(bx^3+a)bd-abd}\right|\right)}{\sqrt{bd}d}+\frac{\sqrt{bx^3+a}\sqrt{b^2c+(bx^3+a)bd-abd}}{bd}}{3\,|b|}$$

input
```
integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*((b*c + a*d)*log(abs(-sqrt(b*x^3 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^3
+ a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b*x^3 + a)*sqrt(b^2*c + (b*x^3 +
a)*b*d - a*b*d)/(b*d))/abs(b)
```

3.506.9 Mupad [B] (verification not implemented)

Time = 13.12 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.22

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\frac{2\,a\,d}{3}+\frac{2\,b\,c}{3}\right)}{d^3\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}+\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\left(\frac{2\,a\,d}{3}+\frac{2\,b\,c}{3}\right)}{b\,d^2\,{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}^3}-\frac{8\,\sqrt{a}\,\sqrt{c}\,{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^2}{3\,d^2\,{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}^2}}{\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^4}{{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}^4}+\frac{b^2}{d^2}-\frac{2\,b\,{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^2}{d\,{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}^2}} - \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}\right)\,\left(a\,d+b\,c\right)}{3\,b^{3/2}\,d^{3/2}}$$

input
```
int(x^5/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
((((a + b*x^3)^(1/2) - a^(1/2))*((2*a*d)/3 + (2*b*c)/3))/(d^3*((c + d*x^3)
^(1/2) - c^(1/2))) + (((a + b*x^3)^(1/2) - a^(1/2))^3*((2*a*d)/3 + (2*b*c)
/3))/(b*d^2*((c + d*x^3)^(1/2) - c^(1/2))^3) - (8*a^(1/2)*c^(1/2)*((a + b*
x^3)^(1/2) - a^(1/2))^2)/(3*d^2*((c + d*x^3)^(1/2) - c^(1/2))^2))/(((a + b
*x^3)^(1/2) - a^(1/2))^4/((c + d*x^3)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*
((a + b*x^3)^(1/2) - a^(1/2))^2)/(d*((c + d*x^3)^(1/2) - c^(1/2))^2)) - (2
*atanh((d^(1/2)*((a + b*x^3)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^3)^(1/2)
 - c^(1/2))))*(a*d + b*c))/(3*b^(3/2)*d^(3/2))
```

3.506. $\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

3.507 $\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

3.507.1 Optimal result

Integrand size = 26, antiderivative size = 48

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

output
```
2/3*arctanh(d^(1/2)*(b*x^3+a)^(1/2)/b^(1/2)/(d*x^3+c)^(1/2))/b^(1/2)/d^(1/
2)
```

3.507.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{d}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

input
```
Integrate[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/(Sqrt[d]*Sqrt[a + b*x^3])])/(3*Sqrt[b
]*Sqrt[d])
```

3.507.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {946, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3}\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \; 66$$

$$\frac{2}{3}\int \frac{1}{b-dx^6}d\frac{\sqrt{bx^3+a}}{\sqrt{dx^3+c}}$$

$$\Big\downarrow \; 221$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

input
```
Int[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*Sqrt[b
]*Sqrt[d])
```

3.507.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.507.4 Maple [F]

$$\int \frac{x^2}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

input

```
int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output

```
int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

3.507.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(34) = 68$.

Time $= 0.28$ (sec) , antiderivative size $= 194$, normalized size of antiderivative $= 4.04$

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^6+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^3+4\,(2\,bdx^3+bc+ad)\sqrt{bx^3+a}\sqrt{dx^3+c}\right.}{6\,bd} - \frac{\sqrt{-bd}\arctan\left(\frac{(2\,bdx^3+bc+ad)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{-bd}}{2\,(b^2d^2x^6+abcd+(b^2cd+abd^2)x^3)}\right)}{3\,bd}\right]$$

input

```
integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output

```
[1/6*sqrt(b*d)*log(8*b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*
c*d + a*b*d^2)*x^3 + 4*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3
+ c)*sqrt(b*d))/(b*d), -1/3*sqrt(-b*d)*arctan(1/2*(2*b*d*x^3 + b*c + a*d)*
sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-b*d)/(b^2*d^2*x^6 + a*b*c*d + (b^2*c
*d + a*b*d^2)*x^3))/(b*d)]
```

3.507.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**2/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.507.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.507.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=-\frac{2\,b\log\left(\left|-\sqrt{bx^3+a}\sqrt{bd}+\sqrt{b^2c+(bx^3+a)bd-abd}\right|\right)}{3\,\sqrt{bd}|b|}$$

input
```
integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-2/3*b*log(abs(-sqrt(b*x^3 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^3 + a)*b*d -
 a*b*d)))/(sqrt(b*d)*abs(b))
```

3.507.9 Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}{\sqrt{-b\,d}\,\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}\right)}{3\,\sqrt{-b\,d}}$$

input
```
int(x^2/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
-(4*atan((b*((c + d*x^3)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a + b*x^3)^(1/2
) - a^(1/2)))))/(3*(-b*d)^(1/2))
```

3.508 $\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

3.508.1 Optimal result

Integrand size = 26, antiderivative size = 48

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

output
```
-2/3*arctanh(c^(1/2)*(b*x^3+a)^(1/2)/a^(1/2)/(d*x^3+c)^(1/2))/a^(1/2)/c^(1
/2)
```

3.508.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c+dx^3}}{\sqrt{c}\sqrt{a+bx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^3])/(Sqrt[c]*Sqrt[a + b*x^3])])/(3*Sqrt[
a]*Sqrt[c])
```

3.508.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3\sqrt{bx^3+a}\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{104}$$

$$\frac{2}{3}\int \frac{1}{cx^6-a}d\frac{\sqrt{bx^3+a}}{\sqrt{dx^3+c}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

input
```
Int[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*Sqrt[
a]*Sqrt[c])
```

3.508.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.508.4 Maple [F]

$$\int \frac{1}{x\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}dx$$

input
```
int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

3.508.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(34) = 68.

Time = 0.29 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.25

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \left[\frac{\sqrt{ac}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^6+8\,a^2c^2+8\,(abc^2+a^2cd)x^3-4\,((bc+ad)x^3+2\,ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{ac}}{x^6}\right)}{6\,ac}, \frac{\sqrt{-ac}\,\arctan\left(\frac{((bc+ad)x}{2\,(abc}\right.}{3}\right.$$

input
```
integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*sqrt(a*c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 8*a^2*c^2 + 8*(a
*b*c^2 + a^2*c*d)*x^3 - 4*((b*c + a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d
*x^3 + c)*sqrt(a*c))/x^6)/(a*c), 1/3*sqrt(-a*c)*arctan(1/2*((b*c + a*d)*x^
3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-a*c)/(a*b*c*d*x^6 + a^2*c
^2 + (a*b*c^2 + a^2*c*d)*x^3))/(a*c)]
```

3.508.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.508.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.508.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(34) = 68.

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.85

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{2\sqrt{bd}\,b\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2}{2\sqrt{-abcd}\,b}\right)}{3\sqrt{-abcd}\,|b|}$$

input
```
integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(b*d)*b*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^3 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*
c*d)*abs(b))
```

3.508.9 Mupad [B] (verification not implemented)

Time = 11.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.83

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{\ln\left(\frac{\sqrt{b\,x^3+a}-\sqrt{a}}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)-\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^3+a}-\sqrt{a}\,\sqrt{d\,x^3+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\,\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)}{3\,\sqrt{a}\,\sqrt{c}}$$

input

```
int(1/(x*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output

```
-(log(((a + b*x^3)^(1/2) - a^(1/2))/((c + d*x^3)^(1/2) - c^(1/2))) - log((
(c^(1/2)*(a + b*x^3)^(1/2) - a^(1/2)*(c + d*x^3)^(1/2))*(b*c^(1/2) - (a^(1
/2)*d*((a + b*x^3)^(1/2) - a^(1/2)))/((c + d*x^3)^(1/2) - c^(1/2))))/((c +
 d*x^3)^(1/2) - c^(1/2))))/(3*a^(1/2)*c^(1/2))
```

3.509 $\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

3.509.1 Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3}+\frac{(bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}$$

output
```
1/3*(a*d+b*c)*arctanh(c^(1/2)*(b*x^3+a)^(1/2)/a^(1/2)/(d*x^3+c)^(1/2))/a^(
3/2)/c^(3/2)-1/3*(b*x^3+a)^(1/2)*(d*x^3+c)^(1/2)/a/c/x^3
```

3.509.2 Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3}+\frac{(bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
-1/3*(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(a*c*x^3) + ((b*c + a*d)*ArcTanh[(S
qrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*a^(3/2)*c^(3/2))
```

3.509.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {948, 107, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^6\sqrt{bx^3+a}\sqrt{dx^3+c}}dx^3$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{3}\left(-\frac{(ad+bc)\int \frac{1}{x^3\sqrt{bx^3+a}\sqrt{dx^3+c}}dx^3}{2ac}-\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{acx^3}\right)$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{3}\left(-\frac{(ad+bc)\int \frac{1}{cx^6-a}d\frac{\sqrt{bx^3+a}}{\sqrt{dx^3+c}}}{ac}-\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{acx^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{(ad+bc)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{acx^3}\right)$$

input
```
Int[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(-((Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(a*c*x^3)) + ((b*c + a*d)*ArcTanh[(Sq
rt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(a^(3/2)*c^(3/2)))/3
```

3.509.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.509.4 Maple [F]

$$\int \frac{1}{x^4\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

input
```
int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

3.509.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.05

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{\sqrt{ac}(bc+ad)x^3\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^6+8\,a^2c^2+8\,(abc^2+a^2cd)x^3+4\,((bc+ad)x^3+2\,ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{ac}}{x^6}\right)-4\,\sqrt{bx^3+}}{12\,a^2c^2x^3}\right.$$
$$\left.-\frac{\sqrt{-ac}(bc+ad)x^3\arctan\left(\frac{((bc+ad)x^3+2\,ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{-ac}}{2\,(abcdx^6+a^2c^2+(abc^2+a^2cd)x^3)}\right)+2\,\sqrt{bx^3+a}\sqrt{dx^3+c}ac}{6\,a^2c^2x^3}\right]$$

input

```
integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output

```
[1/12*(sqrt(a*c)*(b*c + a*d)*x^3*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6
+ 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^3 + 4*((b*c + a*d)*x^3 + 2*a*c)*sqrt
(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(a*c))/x^6) - 4*sqrt(b*x^3 + a)*sqrt(d*x^3
 + c)*a*c)/(a^2*c^2*x^3), -1/6*(sqrt(-a*c)*(b*c + a*d)*x^3*arctan(1/2*((b*
c + a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-a*c)/(a*b*c*d*
x^6 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^3)) + 2*sqrt(b*x^3 + a)*sqrt(d*x^3 +
 c)*a*c)/(a^2*c^2*x^3)]
```

3.509.6 Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input

```
integrate(1/x**4/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output

```
Integral(1/(x**4*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.509.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.509.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(71) = 142$.

Time = 0.29 (sec) , antiderivative size = 413, normalized size of antiderivative = 4.54

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

$$= \frac{\sqrt{bd}b^4d\left(\frac{(bc+ad)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^3+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}ab^3cd} - \frac{2\left(b^3c^2-2ab^2cd+a^2bd^2-\left(\sqrt{b\right.\right.}{\left(b^4c^2-2ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+c\right.\right.}\right.}{3|b|}$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(b*d)*b^4*d*((b*c + a*d)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^3
+ a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)
*b))/(sqrt(-a*b*c*d)*a*b^3*c*d) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - (
sqrt(b*x^3 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*b*c -
 (sqrt(b*x^3 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*a*d
)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^3 + a)*sqrt(b*d) - s
qrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^3 + a)*sqrt(b*
d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^3 + a)*sqr
t(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^4)*a*b^2*c*d))/abs(b)
```

3.509.9 Mupad [B] (verification not implemented)

Time = 15.12 (sec) , antiderivative size = 481, normalized size of antiderivative = 5.29

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\frac{c\,b^2}{12}+\frac{a\,d\,b}{12}\right)}{a^{3/2}\,c^{3/2}\,d\,\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}-\frac{b^2}{12\,a\,c\,d}+\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2\left(\frac{a^2\,d^2}{12}-\frac{a\,b\,c\,d}{4}+\frac{b^2\,c^2}{12}\right)}{a^2\,c^2\,d\,\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3}{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3}+\frac{b\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}-\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2(a\,d+b\,c)}{\sqrt{a}\,\sqrt{c}\,d\,\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}} + \frac{\ln\left(\frac{\sqrt{b\,x^3+a}-\sqrt{a}}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}+a^{3/2}\,\sqrt{c}\,d\right)}{6\,a^2\,c^2} - \frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^3+a}-\sqrt{a}\,\sqrt{d\,x^3+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}+a^{3/2}\,\sqrt{c}\,d\right)}{6\,a^2\,c^2} - \frac{d\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{12\,a\,c\,\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}$$

input
```
int(1/(x^4*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
((((a + b*x^3)^(1/2) - a^(1/2))*((b^2*c)/12 + (a*b*d)/12))/(a^(3/2)*c^(3/2
)*d*((c + d*x^3)^(1/2) - c^(1/2))) - b^2/(12*a*c*d) + (((a + b*x^3)^(1/2)
- a^(1/2))^2*((a^2*d^2)/12 + (b^2*c^2)/12 - (a*b*c*d)/4))/(a^2*c^2*d*((c +
 d*x^3)^(1/2) - c^(1/2))^2))/((((a + b*x^3)^(1/2) - a^(1/2))^3/((c + d*x^3)
^(1/2) - c^(1/2))^3 + (b*((a + b*x^3)^(1/2) - a^(1/2)))/(d*((c + d*x^3)^(1
/2) - c^(1/2))) - (((a + b*x^3)^(1/2) - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c
^(1/2)*d*((c + d*x^3)^(1/2) - c^(1/2))^2)) + (log(((a + b*x^3)^(1/2) - a^(
1/2))/((c + d*x^3)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2)*
d))/(6*a^2*c^2) - (log(((c^(1/2)*(a + b*x^3)^(1/2) - a^(1/2)*(c + d*x^3)^(
1/2))*(b*c^(1/2) - (a^(1/2)*d*((a + b*x^3)^(1/2) - a^(1/2)))/((c + d*x^3)^
(1/2) - c^(1/2))))/((c + d*x^3)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(
3/2)*c^(1/2)*d))/(6*a^2*c^2) - (d*((a + b*x^3)^(1/2) - a^(1/2)))/(12*a*c*(
(c + d*x^3)^(1/2) - c^(1/2)))
```

3.510 $\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

3.510.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{x^5\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output
```
1/5*x^5*AppellF1(5/3,1/2,1/2,8/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d
*x^3/c)^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

3.510.2 Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{x^5\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Integrate[x^4/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(x^5*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3,
-((b*x^3)/a), -((d*x^3)/c)])/(5*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

3.510.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \textit{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{x^4}{\sqrt{\frac{bx^3}{a}+1}\sqrt{dx^3+c}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \textit{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{x^4}{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$\Big\downarrow \textit{1012}$$

$$\frac{x^5\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[x^4/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(x^5*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3,
-((b*x^3)/a), -((d*x^3)/c)])/(5*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

3.510.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.510.4 Maple [F]

$$\int \frac{x^4}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

input
```
int(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

3.510.5 Fricas [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^4/(b*d*x^6 + (b*c + a*d)*x^3 +
a*c), x)
```

3.510.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x**4/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**4/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.510.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x^4}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.510.8 Giac [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x^4}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.510.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x^4}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(x^4/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.511 $\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

3.511.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{x^4\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output
```
1/4*x^4*AppellF1(4/3,1/2,1/2,7/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d
*x^3/c)^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

3.511.2 Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{x^4\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Integrate[x^3/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/3,
-((b*x^3)/a), -((d*x^3)/c)])/(4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

3.511.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{x^3}{\sqrt{\frac{bx^3}{a}+1}\sqrt{dx^3+c}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{x^3}{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[x^3/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(x^4*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/3,
-((b*x^3)/a), -((d*x^3)/c)])/(4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

3.511.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.511.4 Maple [F]

$$\int \frac{x^{3}}{\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}dx$$

input
```
int(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

3.511.5 Fricas [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x^{3}}{\sqrt{bx^{3}+a}\sqrt{dx^{3}+c}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3/(b*d*x^6 + (b*c + a*d)*x^3 +
a*c), x)
```

3.511.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x^{3}}{\sqrt{a+bx^{3}}\sqrt{c+dx^{3}}}\,dx$$

input
```
integrate(x**3/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.511.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.511.8 Giac [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.511.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(x^3/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.512 $\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

3.512.1 Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{x^2\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output
```
1/2*x^2*AppellF1(2/3,1/2,1/2,5/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d
*x^3/c)^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

3.512.2 Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{x^2\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Integrate[x/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3,
-((b*x^3)/a), -((d*x^3)/c)])/(2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

3.512.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{x}{\sqrt{\frac{bx^3}{a}+1}\sqrt{dx^3+c}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{x}{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[x/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3,
-((b*x^3)/a), -((d*x^3)/c)])/(2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

3.512.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.512.4 Maple [F]

$$\int \frac{x}{\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}dx$$

input
```
int(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

3.512.5 Fricas [F]

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x/(b*d*x^6 + (b*c + a*d)*x^3 + a*
c), x)
```

3.512.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(x/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(x/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.512.7 Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.512.8 Giac [F]

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.512.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(x/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
int(x/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.513 $\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

3.513.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{x\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output
```
x*AppellF1(1/3,1/2,1/2,4/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*x^3/c
)^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

3.513.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 170 vs. 2(83) = 166.

Time = 2.40 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=$$

$$-\frac{8acx\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}\,\bigl(-8ac\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\bigl(ad\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}$$

input
```
Integrate[1/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(-8*a*c*x*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(Sqrt[
a + b*x^3]*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/
a), -((d*x^3)/c)] + 3*x^3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)])))
```

3.513.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}dx$$

$$\Big\downarrow 937$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{\sqrt{\frac{bx^3}{a}+1}\sqrt{dx^3+c}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow 937$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$\Big\downarrow 936$$

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[1/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1/2, 1/2, 4/3, -(
(b*x^3)/a), -((d*x^3)/c)])/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

3.513.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.513.4 Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}dx$$

input
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

3.513.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^6 + (b*c + a*d)*x^3 + a*c)
, x)
```

3.513.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.513.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.513.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.513.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

input

```
int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output

```
int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.514 $\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

3.514.1 Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output

```
-AppellF1(-1/3,1/2,1/2,2/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*x^3/c
)^(1/2)/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

3.514.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(86) = 172.

Time = 2.50 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$
$$=\frac{-20(a+bx^3)\,(c+dx^3)+5(bc+ad)x^3\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+8bdx^6\sqrt{1}}{20acx\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input

```
Integrate[1/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(-20*(a + b*x^3)*(c + d*x^3) + 5*(b*c + a*d)*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[
1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)] +
8*b*d*x^6*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1/2,
8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*a*c*x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3
])
```

3.514.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{x^2\sqrt{\frac{bx^3}{a}+1}\sqrt{dx^3+c}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^2\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[1/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
-((Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1/2, 1/2, 2/3, -
((b*x^3)/a), -((d*x^3)/c)])/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]))
```

3.514.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.514.4 Maple [F]

$$\int \frac{1}{x^{2} \sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}dx$$

input
```
int(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

3.514.5 Fricas [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^8 + (b*c + a*d)*x^5 + a*c*
x^2), x)
```

3.514.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.514.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

3.514.8 Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

3.514.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{x^2\,\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(1/(x^2*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.515 $\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

3.515.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output
```
-1/2*AppellF1(-2/3,1/2,1/2,1/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*x
^3/c)^(1/2)/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

3.515.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(88) = 176.

Time = 2.59 (sec) , antiderivative size = 365, normalized size of antiderivative = 4.15

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{bdx^6\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4\left(-4ac(2ac+3bcx^3+3adx^3+2bdx^6)\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx}{a}\right.\right.}{8ac\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx}{c}\right.}}{8acx^2\sqrt{a+bx^3}\sqrt{c+dx}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(b*d*x^6*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7
/3, -((b*x^3)/a), -((d*x^3)/c)] + (4*(-4*a*c*(2*a*c + 3*b*c*x^3 + 3*a*d*x^
3 + 2*b*d*x^6)*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] +
3*x^3*(a + b*x^3)*(c + d*x^3)*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/
a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])))/(8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)]
- 3*x^3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*
c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/(8*a*c*x^2*S
qrt[a + b*x^3]*Sqrt[c + d*x^3])
```

3.515.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = $\{1013, 1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{x^3\sqrt{\frac{bx^3}{a}+1}\sqrt{dx^3+c}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{x^3\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[1/(x^3*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
-1/2*(Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1/2, 1/2, 1/3
, -((b*x^3)/a), -((d*x^3)/c)])/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

3.515.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.515.4 Maple [F]

$$\int \frac{1}{x^{3} \sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}dx$$

input
```
int(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

3.515.5 Fricas [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^9 + (b*c + a*d)*x^6 + a*c*
x^3), x)
```

3.515.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.515.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

3.515.8 Giac [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

3.515.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{x^3\,\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

input

```
int(1/(x^3*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output

```
int(1/(x^3*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.516 $\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

3.516.1 Optimal result

Integrand size = 26, antiderivative size = 161

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2}+\frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}-\frac{a^2(2Ab-aB)e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{5/2}}$$

output
```
1/9*B*(e*x)^(9/2)*(b*x^3+a)^(3/2)/b/e-1/24*a^2*(2*A*b-B*a)*e^(7/2)*arctanh
((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)+1/24*a*(2*A*b-B*a)*e
^2*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b^2+1/12*(2*A*b-B*a)*(e*x)^(9/2)*(b*x^3+a)^
(1/2)/b/e
```

3.516.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.76

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{(ex)^{7/2}\sqrt{a+bx^3}\left(6aAb-3a^2B+12Ab^2x^3+2abBx^3+8b^2Bx^6\right)}{72b^2x^2}+\frac{a^2(-2Ab+aB)(ex)^{7/2}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{24b^{5/2}x^{7/2}}$$

input
```
Integrate[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
((e*x)^(7/2)*Sqrt[a + b*x^3]*(6*a*A*b - 3*a^2*B + 12*A*b^2*x^3 + 2*a*b*B*x
^3 + 8*b^2*B*x^6))/(72*b^2*x^2) + (a^2*(-2*A*b + a*B)*(e*x)^(7/2)*Log[Sqrt
[b]*x^(3/2) + Sqrt[a + b*x^3]])/(24*b^(5/2)*x^(7/2))
```

3.516.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {959, 811, 843, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(2Ab-aB)\int (ex)^{7/2}\sqrt{bx^3+a}dx}{2b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\int \frac{(ex)^{7/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)}{2b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

$$\Big\downarrow \text{843}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^3\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{2b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)}{2b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)}{2b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

$$\Big\downarrow \text{807}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)}{2b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

$$\Big\downarrow \textcolor{blue}{224}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int\frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)}{2b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)}{2b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

input

```
Int[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output

```
(B*(e*x)^(9/2)*(a + b*x^3)^(3/2))/(9*b*e) + ((2*A*b - a*B)*(((e*x)^(9/2)*S
qrt[a + b*x^3])/(6*e) + (a*((e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*b) - (a*e
^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*b^
(3/2))))/4))/(2*b)
```

3.516.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 807

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.516.4 Maple [A] (verified)

Time = 5.65 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.86

method	result
risch	$\frac{x^{2}\left(8b^{2}B\,x^{6}+12A\,b^{2}x^{3}+2Bab\,x^{3}+6abA-3a^{2}B\right)\sqrt{b\,x^{3}+a}\,e^{4}}{72b^{2}\sqrt{ex}}-\frac{a^{2}(2Ab-Ba)\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)e^{4}\sqrt{(b\,x^{3}+a)ex}}{24b^{2}\sqrt{be}\,\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$
default	$-\frac{e^{3}\sqrt{ex}\,\sqrt{b\,x^{3}+a}\,\Bigg(-8B\sqrt{(b\,x^{3}+a)ex}\,\sqrt{be}\,b^{2}x^{7}-12A\sqrt{(b\,x^{3}+a)ex}\,\sqrt{be}\,b^{2}x^{4}-2B\sqrt{(b\,x^{3}+a)ex}\,\sqrt{be}\,ab\,x^{4}+6A\operatorname{arctanh}\Bigg(\frac{\sqrt{\left(b\,x^{3}+a\right.}}{x^{2}\sqrt{be}}}{72\sqrt{(b\,x^{3}+a)ex}\,b^{2}\sqrt{be}}$
elliptic	Expression too large to display

input
```
int((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/72*x^2*(8*B*b^2*x^6+12*A*b^2*x^3+2*B*a*b*x^3+6*A*a*b-3*B*a^2)*(b*x^3+a)^
(1/2)/b^2*e^4/(e*x)^(1/2)-1/24*a^2*(2*A*b-B*a)/b^2/(b*e)^(1/2)*arctanh(((b
*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e^4*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/
(b*x^3+a)^(1/2)
```

3.516.5 Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.83

$$\int (ex)^{7/2}\sqrt{a+bx^3}(A+Bx^3)\,dx = \left[-\frac{3\,(Ba^3-2\,Aa^2b)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e+4\,(2\,b^2x^4+abx)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)}{288\,b^2}\right.$$

$$-\frac{3\,(Ba^3-2\,Aa^2b)e^3\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{ex}bx\sqrt{-\frac{e}{b}}}{2\,bex^3+ae}\right)-2\,(8\,Bb^2e^3x^7+2\,(Bab+6\,Ab^2)e^3x^4-3\,(Ba^2-2\,A}{144\,b^2}$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/288*(3*(B*a^3 - 2*A*a^2*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^
3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4
*(8*B*b^2*e^3*x^7 + 2*(B*a*b + 6*A*b^2)*e^3*x^4 - 3*(B*a^2 - 2*A*a*b)*e^3*
x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/144*(3*(B*a^3 - 2*A*a^2*b)*e^3*sqrt(
-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e))
 - 2*(8*B*b^2*e^3*x^7 + 2*(B*a*b + 6*A*b^2)*e^3*x^4 - 3*(B*a^2 - 2*A*a*b)*
e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]
```

3.516.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(141) = 282$.

Time = 18.80 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.85

$$\int (ex)^{7/2}\sqrt{a+bx^3}(A+Bx^3)\,dx = \begin{cases} \frac{2\left(\begin{cases}\text{NaN} \\ \frac{Ae^3\left(\begin{cases} -\frac{a^2e^3\left(\begin{cases}\frac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3}+2\sqrt{\frac{b}{e^3}}\sqrt{a+bx^3}\right)}{\sqrt{\frac{b}{e^3}}} & \text{for } a\neq 0 \\ \frac{(ex)^{\frac{3}{2}}\log\left((ex)^{\frac{3}{2}}\right)}{\sqrt{bx^3}} & \text{otherwise}\end{cases}\right)}{8b} + \sqrt{a+bx^3}\left(\frac{ae^3(ex)^{\frac{3}{2}}}{8b}+\frac{(ex)^{\frac{9}{2}}}{4}\right) & \text{for} \\ \frac{\sqrt{a}(ex)^{\frac{9}{2}}}{3} & \text{otl}\end{cases}\right)}{}\end{cases}\right)}{} \\ 0 \end{cases}$$

input
```
integrate((e*x)**(7/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*e**3*Piecewise((-a**2*e**3*
Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sq
rt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True)
)/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2)/(8*b) + (e*x)**(9/2)/4), N
e(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True)) + B*Piecewise((a**3*e**6*Pi
ecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt
(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/
(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*(e*x)**(3/2)/(16*b**2) + a*e**3*(
e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(15
/2)/5, True)))/(3*e**3), True))/e, Ne(e, 0)), (0, True))
```

3.516.7 Maxima [F]

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{7}{2}}\,dx$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(7/2), x)
```

3.516.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(126)=252$.

Time = 0.43 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.03

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{1}{12}\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2\,x^3}{e}+\frac{a}{be}\right)Ax|e|^2$$
$$+\frac{1}{72}\sqrt{be^4x^3+ae^4}\left(2\,e^3x^3\left(\frac{4\,x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3\,a^2}{b^2e}\right)\sqrt{ex}Bx|e|^2$$
$$-\frac{\left(B^2a^6e-4\,ABa^5be+4\,A^2a^4b^2e\right)^2e^5\log\left(\left|-\left(\sqrt{ex}Ba^3e^2x-2\,\sqrt{ex}Aa^2be^2x\right)\sqrt{be}+\sqrt{B^2a^7e^6-4\,ABa^6be^6}\right.\right.}{24\,\sqrt{be}b^2|B^2a^6e-4\,ABa^5be+4\,A^2a^4b^2e||-Ba^3+2\,Aa^2}$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))*A*x*abs(e)^2 +
1/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b
^2*e))*sqrt(e*x)*B*x*abs(e)^2 - 1/24*(B^2*a^6*e - 4*A*B*a^5*b*e + 4*A^2*a^
4*b^2*e)^2*e^5*log(abs(-(sqrt(e*x)*B*a^3*e^2*x - 2*sqrt(e*x)*A*a^2*b*e^2*x
)*sqrt(b*e) + sqrt(B^2*a^7*e^6 - 4*A*B*a^6*b*e^6 + 4*A^2*a^5*b^2*e^6 + (sq
rt(e*x)*B*a^3*e^2*x - 2*sqrt(e*x)*A*a^2*b*e^2*x)^2*b*e)))/(sqrt(b*e)*b^2*a
bs(B^2*a^6*e - 4*A*B*a^5*b*e + 4*A^2*a^4*b^2*e)*abs(-B*a^3 + 2*A*a^2*b)*ab
s(e)^2)
```

3.516.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(1/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(1/2), x)
```

3.517 $\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

3.517.1 Optimal result

Integrand size = 26, antiderivative size = 324

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{3a(16Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{320b^2}$$

$$+\frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

$$-\frac{3^{3/4}a^{5/3}(16Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Bigg(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}}{640b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/8*B*(e*x)^(7/2)*(b*x^3+a)^(3/2)/b/e+1/80*(16*A*b-7*B*a)*(e*x)^(7/2)*(b*x
^3+a)^(1/2)/b/e+3/320*a*(16*A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2
-1/640*3^(3/4)*a^(5/3)*(16*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3
)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^
(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a
^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(
1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.517.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

$$\int (ex)^{5/2}\sqrt{a+bx^3}(A+Bx^3)\,dx=\frac{e^2\sqrt{ex}\sqrt{a+bx^3}\bigg(-\bigg((a+bx^3)\sqrt{1+\frac{bx^3}{a}}(-16Ab+7aB-10bBx^3)\bigg)+a(-16Ab+7aB)\,\mathrm{Hype}}{80b^2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^(5/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(-16*A*b
 + 7*a*B - 10*b*B*x^3)) + a*(-16*A*b + 7*a*B)*Hypergeometric2F1[-1/2, 1/6,
 7/6, -((b*x^3)/a)]))/(80*b^2*Sqrt[1 + (b*x^3)/a])
```

3.517.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {959, 811, 843, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(16Ab-7aB)\int (ex)^{5/2}\sqrt{bx^3+a}dx}{16b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(16Ab-7aB)\left(\frac{3}{10}a\int\frac{(ex)^{5/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)}{16b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

$$\Big\downarrow \text{843}$$

$$\frac{(16Ab-7aB)\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^3\int\frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{4b}\right)+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)}{16b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

$$\Big\downarrow 851$$

$$\frac{(16Ab-7aB)\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b}\right)+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)}{16b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

$$\Big\downarrow 766$$

$$\frac{(16Ab-7aB)\Bigg(\frac{3}{10}a\Bigg(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{a^{2/3}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+}\right.\right.}{4\sqrt[4]{3}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{16b}$$

$$\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

input
```
Int[(e*x)^(5/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(B*(e*x)^(7/2)*(a + b*x^3)^(3/2))/(8*b*e) + ((16*A*b - 7*a*B)*(((e*x)^(7/2
)*Sqrt[a + b*x^3])/(5*e) + (3*a*((e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(2*b) - (
a^(2/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*
b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2
]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1
 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*e*
x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sq
rt[a + b*x^3]))/10))/(16*b)
```

3.517.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

3.517. $\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.517.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.04 (sec) , antiderivative size = 777, normalized size of antiderivative = 2.40

method	result
risch	$\frac{(40b^2Bx^6+64Ab^2x^3+12Babx^3+48abA-21a^2B)x\sqrt{bx^3+a}\,e^3}{320b^2\sqrt{ex}} - \frac{3a^2(16Ab-7Ba)\left(\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-a}{}\right.}{\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}+\right.}}}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/320*(40*B*b^2*x^6+64*A*b^2*x^3+12*B*a*b*x^3+48*A*a*b-21*B*a^2)*x*(b*x^3+
a)^(1/2)/b^2*e^3/(e*x)^(1/2)-3/320*a^2*(16*A*b-7*B*a)/b*(1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3
)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*
(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1
/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(
-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3
)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))*e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

3.517. $\int (ex)^{5/2}\sqrt{a+bx^3}(A+Bx^3)\,dx$

3.517.5 Fricas [F]

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

3.517.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.30

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{A\sqrt{a}e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{6}\right)}+\frac{B\sqrt{a}e^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}$$

input
```
integrate((e*x)**(5/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

output
```
A*sqrt(a)*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(13/6)) + B*sqrt(a)*e**(5/2)*x**(13/2)*gamma(13
/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(19/6))
```

3.517.7 Maxima [F]

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(5/2), x)
```

3.517.8 Giac [F]

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(5/2), x)
```

3.517.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(1/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(1/2), x)
```

3.518 $\int (ex)^{3/2}\sqrt{a+bx^3}(A+Bx^3)\,dx$

3.518.1 Optimal result

Integrand size = 26, antiderivative size = 581

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be}$$

$$+\frac{3\left(1+\sqrt{3}\right)a(14Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{112b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

$$-\frac{3\sqrt[4]{3}a^{4/3}(14Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\Bigg(\arccos\Bigg(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\Bigg)|\frac{1}{4}\left(2+\sqrt{3}\right.}{112b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}(14Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\Bigg(\arccos\Bigg(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)}}{224b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/7*B*(e*x)^(5/2)*(b*x^3+a)^(3/2)/b/e+1/56*(14*A*b-5*B*a)*(e*x)^(5/2)*(b*x
^3+a)^(1/2)/b/e+3/112*a*(14*A*b-5*B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)
^(1/2)/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-3/112*3^(1/4)*a^(4/3)*(14*A
*b-5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3
)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^
(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^
2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b
^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/224*3^(3/4)*a^(4/3)*(14*A*b-5*B*a)*e*(a^(
1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^
(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*
(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/
3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.518.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.16

$$\int (ex)^{3/2}\sqrt{a+bx^3}(A+Bx^3)\,dx = \frac{x(ex)^{3/2}\sqrt{a+bx^3}\left(5B(a+bx^3)\sqrt{1+\frac{bx^3}{a}}+(14Ab-5aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{11}{6},-\right.\right.}{35b\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^(3/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(5*B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a] + (14*
A*b - 5*a*B)*Hypergeometric2F1[-1/2, 5/6, 11/6, -((b*x^3)/a)]))/(35*b*Sqrt
[1 + (b*x^3)/a])
```

3.518.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {959, 811, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(14Ab-5aB)\int (ex)^{3/2}\sqrt{bx^3+a}dx}{14b}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(14Ab-5aB)\left(\frac{3}{8}a\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{14b}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(14Ab-5aB)\left(\frac{3a\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{14b}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

$$\Big\downarrow \text{837}$$

$$\frac{(14Ab-5aB)\left(\frac{3a\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{14b}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

$$\Big\downarrow \text{25}$$

$$\frac{(14Ab-5aB)\left(\frac{3a\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{14b}+$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

$$\Big\downarrow \text{766}$$

$$(14Ab-5aB)\Bigg(\frac{3a\Bigg(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\Bigg(\arccos\Bigg(\Bigg(}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{4e}}{14b}$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

$$\Big\downarrow \text{2420}$$

3.518. $\int (ex)^{3/2}\sqrt{a+bx^3}(A+Bx^3)\,dx$

$$(14Ab-5aB)\left(3a\left(\frac{\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right.\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}}\right.\right.$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

input

```
Int[(e*x)^(3/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

3.518. $\int (ex)^{3/2}\sqrt{a+bx^3}(A+Bx^3)\ dx$

output

```
(B*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(7*b*e) + ((14*A*b - 5*a*B)*(((e*x)^(5/2
)*Sqrt[a + b*x^3])/(4*e) + (3*a*((((1 + Sqrt[3])*e^3*Sqrt[e*x]*Sqrt[a + b*
x^3])/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) - (3^(1/4)*a^(1/3)*e*Sqrt[e*
x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b
^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticE[ArcCo
s[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/
3)*e*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/
(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) -
 ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3
)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3]
)*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)
/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^
(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[
3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(4*e)))/(14*b)
```

3.518.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 837

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.518.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 1140, normalized size of antiderivative = 1.96

method	result	size
risch	Expression too large to display	1140
elliptic	Expression too large to display	1184
default	Expression too large to display	5358

input
```
int((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/56*x^3*(8*B*b*x^3+14*A*b+3*B*a)*(b*x^3+a)^(1/2)/b*e^2/(e*x)^(1/2)+3/112*
a*(14*A*b-5*B*a)/b*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^
(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(
1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1
/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a
*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x...
```

3.518.5 Fricas [F]

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*e*x^4 + A*e*x)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

3.518.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.92 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{A\sqrt{a}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{6}\\\frac{11}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)}+\frac{B\sqrt{a}e^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{11}{6}\\\frac{17}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)}$$

input
```
integrate((e*x)**(3/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

output
```
A*sqrt(a)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(11/6)) + B*sqrt(a)*e**(3/2)*x**(11/2)*gamma(11
/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6))
```

3.518.7 Maxima [F]

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(3/2), x)
```

3.518.8 Giac [F]

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(3/2), x)
```

3.518.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(1/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(1/2), x)
```

3.519 $\int \sqrt{ex}\sqrt{a+bx^3}(A+Bx^3)\,dx$

3.519.1 Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}+\frac{a(4Ab-aB)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}}$$

output
```
1/6*B*(e*x)^(3/2)*(b*x^3+a)^(3/2)/b/e+1/12*a*(4*A*b-B*a)*arctanh((e*x)^(3/
2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/2)/b^(3/2)+1/12*(4*A*b-B*a)*(e*x)
^(3/2)*(b*x^3+a)^(1/2)/b/e
```

3.519.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{x\sqrt{ex}\sqrt{a+bx^3}(4Ab+aB+2bBx^3)}{12b}-\frac{a(-4Ab+aB)\sqrt{ex}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{12b^{3/2}\sqrt{x}}$$

input
```
Integrate[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(x*Sqrt[e*x]*Sqrt[a + b*x^3]*(4*A*b + a*B + 2*b*B*x^3))/(12*b) - (a*(-4*A*
b + a*B)*Sqrt[e*x]*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(12*b^(3/2)*Sqr
t[x])
```

3.519.3 Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {959, 811, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(4Ab-aB)\int \sqrt{ex}\sqrt{bx^3+a}dx}{4b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(4Ab-aB)\left(\frac{1}{2}a\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{4b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(4Ab-aB)\left(\frac{a\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{4b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

$$\Big\downarrow \text{807}$$

$$\frac{(4Ab-aB)\left(\frac{a\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{4b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

$$\Big\downarrow \text{224}$$

$$\frac{(4Ab-aB)\left(\frac{a\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{4b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

$$\Big\downarrow \text{219}$$

$$\frac{(4Ab-aB)\left(\frac{a\sqrt{e}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3\sqrt{b}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{4b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

input
```
Int[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

output
```
(B*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(6*b*e) + ((4*A*b - a*B)*(((e*x)^(3/2)*S
qrt[a + b*x^3])/(3*e) + (a*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*
Sqrt[a + (b*x)/e^2])])/(3*Sqrt[b])))/(4*b)
```

3.519.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.519.4 Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.90

method	result
risch	$\frac{x^{2} \left(2 b B \,x^{3}+4 A b +B a \right) \sqrt{b \,x^{3}+a}\, e}{12 b \sqrt{e x}}+\frac{a \left(4 A b -B a \right) \operatorname{arctanh}\left(\frac{\sqrt{\left(b \,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) e \sqrt{\left(b \,x^{3}+a \right) e x}}{12 b \sqrt{b e}\, \sqrt{e x}\, \sqrt{b \,x^{3}+a}}$
default	$\frac{\sqrt{e x}\, \sqrt{b \,x^{3}+a}\, (2 B \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, b \,x^{4}+4 A \operatorname{arctanh}\left(\frac{\sqrt{\left(b \,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) a b e +4 A \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, b x -B \operatorname{arctanh}\left(\frac{\sqrt{\left(b \,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) a}{12 \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, b}$
elliptic	Expression too large to display

input
```
int((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2*(2*B*b*x^3+4*A*b+B*a)*(b*x^3+a)^(1/2)/b*e/(e*x)^(1/2)+1/12*a*(4*A
*b-B*a)/b/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e*((b
*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

3.519.5 Fricas [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.83

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$= \left[-\frac{\left(Ba^{2} - 4\,Aab\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^{2}ex^{6} - 8\,abex^{3} - a^{2}e - 4\,\left(2\,b^{2}x^{4} + abx\right)\sqrt{bx^{3} + a}\sqrt{ex}\sqrt{\frac{e}{b}}\right) - 4\,(2\,Bbx^{4} + (}{48\,b}\right.$$

input
```
integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output

```
[-1/48*((B*a^2 - 4*A*a*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e
 - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(2*B*b*x
^4 + (B*a + 4*A*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b, 1/24*((B*a^2 - 4*A*a*b
)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3
+ a*e)) + 2*(2*B*b*x^4 + (B*a + 4*A*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b]
```

3.519.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(104) = 208$.

Time = 1.55 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.09

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$= \begin{cases} \frac{2\left(\begin{cases} \text{NaN} \\ \frac{Ae^3\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3}+2\sqrt{\frac{b}{e^3}}\sqrt{a+bx^3}\right)}{\sqrt{\frac{b}{e^3}}} & \text{for } a\neq 0 \\ \frac{(ex)^{\frac{3}{2}}\log\left((ex)^{\frac{3}{2}}\right)}{\sqrt{bx^3}} & \text{otherwise} \end{cases}\right)}{2}+\frac{(ex)^{\frac{3}{2}}\sqrt{a+bx^3}}{2} & \text{for } \frac{b}{e^3}\neq 0 \\ \sqrt{a}(ex)^{\frac{3}{2}} & \text{otherwise} \end{cases}\right)+B\left(\begin{cases} -\frac{a^2e^3\left(\begin{cases} \frac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3}\right.}{} \\ \frac{(ex)^{\frac{3}{2}}\log\left(\right.}{\sqrt{bx^3}} \end{cases}\right.}{} \\ \frac{\sqrt{a}(ex)^{\frac{9}{2}}}{3} \end{cases}\right.}{3e^3} \end{cases}\right.}{e} \\ 0 \end{cases}$$

input

```
integrate((B*x**3+A)*(e*x)**(1/2)*(b*x**3+a)**(1/2),x)
```

output

```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*e**3*Piecewise((a*Piecewise
((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3
), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/2 + (e*
x)**(3/2)*sqrt(a + b*x**3)/2, Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(3/2), True)
) + B*Piecewise((-a**2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(
b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)
**(3/2))/sqrt(b*x**3), True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2
)/(8*b) + (e*x)**(9/2)/4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True))
)/(3*e**3), True))/e, Ne(e, 0)), (0, True))
```

3.519.7 Maxima [F]

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}\sqrt{ex}\,dx$$

input
```
integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

3.519.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.46

$$\begin{aligned}
&\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx\\
&=\frac{Ba^2e\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{12\sqrt{be}b}+\frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2x^3}{e}+\frac{a}{be}\right)Bx|e|^2}{12e^3}\\
&\quad-\frac{\left(\frac{ae^4\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{\sqrt{be}}-\sqrt{be^4x^3+ae^4}\sqrt{ex}ex\right)A|e|^2}{3e^5}
\end{aligned}$$

input
```
integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/12*B*a^2*e*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/
(sqrt(b*e)*b) + 1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))
*B*x*abs(e)^2/e^3 - 1/3*(a*e^4*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e
^4*x^3 + a*e^4)))/sqrt(b*e) - sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*e*x)*A*abs
(e)^2/e^5
```

3.519.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\sqrt{e\,x}\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(1/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(1/2), x)
```

3.520 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx$

3.520.1 Optimal result

Integrand size = 26, antiderivative size = 286

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx=\frac{(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{20be}+\frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be}$$
$$+\frac{3^{3/4}a^{2/3}(10Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\right.}{40be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/5*B*(b*x^3+a)^(3/2)*(e*x)^(1/2)/b/e+1/20*(10*A*b-B*a)*(e*x)^(1/2)*(b*x^3
+a)^(1/2)/b/e+1/40*3^(3/4)*a^(2/3)*(10*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1
/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(
1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(
a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b/e/(b*x^3+a)^(1/2)/(b^(1/3)*
x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.520.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{x\sqrt{a+bx^3}\left(B(a+bx^3)\sqrt{1+\frac{bx^3}{a}}+(10Ab-aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{5b\sqrt{ex}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/Sqrt[e*x],x]
```

output
```
(x*Sqrt[a + b*x^3]*(B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a] + (10*A*b - a*B)*Hyp
ergeometric2F1[-1/2, 1/6, 7/6, -((b*x^3)/a)]))/(5*b*Sqrt[e*x]*Sqrt[1 + (b*
x^3)/a])
```

3.520.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {959, 811, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx$$

$$\Big\downarrow \ 959$$

$$\frac{(10Ab-aB)\int \frac{\sqrt{bx^3+a}}{\sqrt{ex}}dx}{10b}+\frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be}$$

$$\Big\downarrow \ 811$$

$$\frac{(10Ab-aB)\left(\frac{3}{4}a\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)}{10b}+\frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be}$$

$$\Big\downarrow \ 851$$

$$\frac{(10Ab-aB)\left(\frac{3a\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2e}+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)}{10b}+\frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be}$$

$$\Big\downarrow 766$$

$$\frac{(10Ab-aB)\left(\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}+\right.}{10b}$$

$$\frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/Sqrt[e*x],x]
```

output
```
(B*Sqrt[e*x]*(a + b*x^3)^(3/2))/(5*b*e) + ((10*A*b - a*B)*((Sqrt[e*x]*Sqrt
[a + b*x^3])/(2*e) + (3^(3/4)*a^(2/3)*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*
Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e +
(1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*
b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(
4*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[
3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(10*b)
```

3.520.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

3.520. $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx$

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.520.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.98 (sec) , antiderivative size = 744, normalized size of antiderivative = 2.60

method	result
risch	$\frac{(4bBx^3+10Ab+3Ba)x\sqrt{bx^3+a}}{20b\sqrt{ex}}+\frac{3a(10Ab-Ba)\left(\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)}}\bigl(}{}$
elliptic	$\frac{\sqrt{(bx^3+a)ex}\,\Biggl(\frac{Bx^3\sqrt{bex^4+aex}}{5e}+\frac{\left(Ab+\frac{3Ba}{10}\right)\sqrt{bex^4+aex}}{2be}+\frac{2\left(Aa-\frac{a\left(Ab+\frac{3Ba}{10}\right)}{4b}\right)\left(\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\bigl(-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}}{\bigl(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-}{}}}}{}}{}$
default	Expression too large to display

input
```
int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*(4*B*b*x^3+10*A*b+3*B*a)*x*(b*x^3+a)^(1/2)/b/(e*x)^(1/2)+3/20*a*(10*A
*b-B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(
1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(
b*x^3+a)^(1/2)
```

3.520. $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{\sqrt{ex}}\,dx$

3.520.5 Fricas [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx=\int \frac{(Bx^3+A)\sqrt{bx^3+a}}{\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(e*x), x)
```

3.520.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx=\frac{A\sqrt{a}\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\\frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{7}{6}\right)}+\frac{B\sqrt{a}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\\frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(1/2),x)
```

output
```
A*sqrt(a)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I
*pi)/a)/(3*sqrt(e)*gamma(7/6)) + B*sqrt(a)*x**(7/2)*gamma(7/6)*hyper((-1/2
, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6))
```

3.520.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx=\int \frac{(Bx^3+A)\sqrt{bx^3+a}}{\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/sqrt(e*x), x)
```

3.520.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx=\int \frac{(Bx^3+A)\sqrt{bx^3+a}}{\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/sqrt(e*x), x)
```

3.520.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx=\int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{\sqrt{e\,x}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(1/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(1/2), x)
```

3.521 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx$

3.521.1 Optimal result

Integrand size = 26, antiderivative size = 580

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx=\frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4}$$

$$+\frac{3\left(1+\sqrt{3}\right)(8Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{8b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}-\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

$$-\frac{3\sqrt[4]{3}\sqrt[3]{a}(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{a}(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right.\right.}{16b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-2*A*(b*x^3+a)^(3/2)/a/e/(e*x)^(1/2)+1/4*(8*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)
^(1/2)/a/e^4+3/8*(8*A*b+B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/
3)/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-3/8*3^(1/4)*a^(1/3)*(8*A*b+B*a)*(a^
(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+
3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3
)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1
+3^(1/2)))^2)^(1/2)-1/16*3^(3/4)*a^(1/3)*(8*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*(
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF
((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/
(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))^2)^(1/2)
```

3.521.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = -\frac{2Ax(a+bx^3)^{3/2}}{a(ex)^{3/2}} - \frac{4\left(-4Ab-\frac{aB}{2}\right)x^4\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{5a(ex)^{3/2}\sqrt{1+\frac{bx^3}{a}}}$$

input

```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(3/2),x]
```

output

```
(-2*A*x*(a + b*x^3)^(3/2))/(a*(e*x)^(3/2)) - (4*(-4*A*b - (a*B)/2)*x^4*Sqr
t[a + b*x^3]*Hypergeometric2F1[-1/2, 5/6, 11/6, -((b*x^3)/a)])/(5*a*(e*x)^
(3/2)*Sqrt[1 + (b*x^3)/a])
```

3.521.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {955, 811, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{(aB+8Ab)\int (ex)^{3/2}\sqrt{bx^3+a}dx}{ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{(aB+8Ab)\left(\frac{3}{8}a\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(aB+8Ab)\left(\frac{3a\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 837}$$

$$\frac{(aB+8Ab)\left(\frac{3a\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(aB+8Ab)\left(\frac{3a\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \; 766$$

$$(aB+8Ab)\Bigg(\frac{3a\Bigg(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\,\text{EllipticF}\Bigg(\arccos\Bigg(\frac{(1-}{(1+}}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{4e}$$

$$ae^3$$

$$\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \; 2420$$

3.521. $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx$

$$(aB+8Ab)\left(\frac{3a\left(\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right)|\frac{1}{4}\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.}{2b^{2/3}}\right.$$

$$\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(3/2),x]
```

3.521. $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx$

output
```
(-2*A*(a + b*x^3)^(3/2))/(a*e*Sqrt[e*x]) + ((8*A*b + a*B)*(((e*x)^(5/2)*Sq
rt[a + b*x^3])/(4*e) + (3*a*((((1 + Sqrt[3])*e^3*Sqrt[e*x]*Sqrt[a + b*x^3]
)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) - (3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(
a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/
3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticE[ArcCos[(a
^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e
*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(
1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1
 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^
2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^
(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^
(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3
)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*
b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(4*e)))/(a*e^3)
```

3.521.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

3.521. $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx$

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.521.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.83 (sec) , antiderivative size = 1123, normalized size of antiderivative = 1.94

method	result	size
risch	Expression too large to display	1123
elliptic	Expression too large to display	1161
default	Expression too large to display	5736

input
```
int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/4*(b*x^3+a)^(1/2)*(-B*x^3+8*A)/e/(e*x)^(1/2)+(3*A*b+3/8*B*a)*(x*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^
(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/
3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE
(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),(...
```

3.521.5 Fricas [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{3}{2}}}\,dx$$

input

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")
```

output

```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^2*x^2), x)
```

3.521.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)} + \frac{B\sqrt{a}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(3/2),x)
```

output
```
A*sqrt(a)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + B*sqrt(a)*x**(5/2)*gamma(5/6)*hyper((-
1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6))
```

3.521.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(3/2), x)
```

3.521.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx=\int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(3/2), x)
```

3.521.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx=\int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{(e\,x)^{3/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(3/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(3/2), x)
```

3.522 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx$

3.522.1 Optimal result

Integrand size = 26, antiderivative size = 118

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}}$$

output
```
-2/3*A*(b*x^3+a)^(3/2)/a/e/(e*x)^(3/2)+1/3*(2*A*b+B*a)*arctanh((e*x)^(3/2)
*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/e^(5/2)/b^(1/2)+1/3*(2*A*b+B*a)*(e*x)^(3
/2)*(b*x^3+a)^(1/2)/a/e^4
```

3.522.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = \frac{x\left(\sqrt{b}\sqrt{a+bx^3}(-2A+Bx^3)+(2Ab+aB)x^{3/2}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{3\sqrt{b}(ex)^{5/2}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2),x]
```

output
```
(x*(Sqrt[b]*Sqrt[a + b*x^3]*(-2*A + B*x^3) + (2*A*b + a*B)*x^(3/2)*Log[Sqr
t[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(3*Sqrt[b]*(e*x)^(5/2))
```

3.522.3 Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {955, 811, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{5/2}}dx$$

$$\Big\downarrow \textcolor{blue}{955}$$

$$\frac{(aB+2Ab)\int \sqrt{ex}\sqrt{bx^3+a}dx}{ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{811}$$

$$\frac{(aB+2Ab)\left(\frac{1}{2}a\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{851}$$

$$\frac{(aB+2Ab)\left(\frac{a\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{807}$$

$$\frac{(aB+2Ab)\left(\frac{a\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{224}$$

$$\frac{(aB+2Ab)\left(\frac{a\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$\frac{(aB+2Ab)\left(\frac{a\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3\sqrt{b}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)}{ae^3}-\frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2),x]
```

output
```
(-2*A*(a + b*x^3)^(3/2))/(3*a*e*(e*x)^(3/2)) + ((2*A*b + a*B)*(((e*x)^(3/2
)*Sqrt[a + b*x^3])/(3*e) + (a*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/
2)*Sqrt[a + (b*x)/e^2])])/(3*Sqrt[b])))/(a*e^3)
```

3.522.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.522.4 Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-x^{3}B+2A\right)}{3x\,e^{2}\sqrt{ex}}+\frac{2\left(Ab+\frac{Ba}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)\sqrt{(b\,x^{3}+a)ex}}{3\sqrt{be}\,e^{2}\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$
default	$\frac{\sqrt{b\,x^{3}+a}\,\left(2A\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)be\,x^{2}+B\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)ae\,x^{2}+B\sqrt{(b\,x^{3}+a)ex}\,\sqrt{be}\,x^{3}-2A\sqrt{(b\,x^{3}+a)ex}\,\sqrt{be}\right)}{3x\,e^{2}\sqrt{ex}\,\sqrt{(b\,x^{3}+a)ex}\,\sqrt{be}}$
elliptic	Expression too large to display

input

```
int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-1/3*(b*x^3+a)^(1/2)*(-B*x^3+2*A)/x/e^2/(e*x)^(1/2)+2/3*(A*b+1/2*B*a)/(b*e
)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))/e^2*((b*x^3+a)*e*x)
^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

3.522.5 Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx=\left[\frac{(Ba+2\,Ab)\sqrt{be}x^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\right.}{12\,be^3x^2}\right.$$

$$\left.-\frac{(Ba+2\,Ab)\sqrt{-be}x^2\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{-be}\sqrt{ex}x}{2\,bex^3+ae}\right)-2\,(Bbx^3-2\,Ab)\sqrt{bx^3+a}\sqrt{ex}}{6\,be^3x^2}\right]$$

input

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*((B*a + 2*A*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e
- 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(B*b*x^3 - 2*
A*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2), -1/6*((B*a + 2*A*b)*sqrt(-b*e
)*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) -
 2*(B*b*x^3 - 2*A*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

3.522.6 Sympy [A] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = -\frac{2A\sqrt{a}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{2A\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3e^{\frac{5}{2}}} - \frac{2Abx^{\frac{3}{2}}}{3\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{B\sqrt{a}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} + \frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}e^{\frac{5}{2}}}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(5/2),x)
```

output
```
-2*A*sqrt(a)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*A*sqrt(b)*asinh(
sqrt(b)*x**(3/2)/sqrt(a))/(3*e**(5/2)) - 2*A*b*x**(3/2)/(3*sqrt(a)*e**(5/2
)*sqrt(1 + b*x**3/a)) + B*sqrt(a)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2))
 + B*a*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(3*sqrt(b)*e**(5/2))
```

3.522.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(5/2), x)
```

3.522.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: Recu
rsive assumption sageVARa>=(-sageVARb*sageVARe/(sageVARe^4*t_nostep^6)) ig
nored2/sageVARe^3*sageVARB/6/sageVARe^3*sqrt(sageVARe*sageVARx)*sqrt(sageV
ARe*sageVARx)*sqrt(
```

3.522.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{(e\,x)^{5/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(5/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(5/2), x)
```

3.523 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx$

3.523.1 Optimal result

Integrand size = 26, antiderivative size = 283

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx=\frac{(4Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{10ae^4}-\frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}}$$
$$+\frac{3^{3/4}(4Ab+5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+\sqrt{}}{20\sqrt[3]{a}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/5*A*(b*x^3+a)^(3/2)/a/e/(e*x)^(5/2)+1/10*(4*A*b+5*B*a)*(e*x)^(1/2)*(b*x
^3+a)^(1/2)/a/e^4+1/20*3^(3/4)*(4*A*b+5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)
+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3
)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(
1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4
*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(1/3)/e^4/(b*x^3+a)^(1/2)/(b^(
1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.523.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{2x\sqrt{a+bx^3}\left(-A(a+bx^3)\sqrt{1+\frac{bx^3}{a}}+(4Ab+5aB)x^3\,\text{Hypergeometric2F1}\left(-\right.\right.}{5a(ex)^{7/2}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(7/2),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]) + (4*A*b + 5*a*
B)*x^3*Hypergeometric2F1[-1/2, 1/6, 7/6, -((b*x^3)/a)]))/(5*a*(e*x)^(7/2)*
Sqrt[1 + (b*x^3)/a])
```

3.523.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {955, 811, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{(5aB+4Ab)\int \frac{\sqrt{bx^3+a}}{\sqrt{ex}}dx}{5ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{(5aB+4Ab)\left(\frac{3}{4}a\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)}{5ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(5aB+4Ab)\left(\frac{3a\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2e}+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)}{5ae^3}-\frac{2A\left(a+bx^3\right)^{3/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow 766$$

$$\frac{(5aB+4Ab)\left(\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}+\right.}{5ae^3}$$

$$\frac{2A\left(a+bx^3\right)^{3/2}}{5ae(ex)^{5/2}}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(7/2),x]
```

output
```
(-2*A*(a + b*x^3)^(3/2))/(5*a*e*(e*x)^(5/2)) + ((4*A*b + 5*a*B)*((Sqrt[e*x
]*Sqrt[a + b*x^3])/(2*e) + (3^(3/4)*a^(2/3)*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)
*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3
)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqr
t[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])
/4])/(4*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 +
 Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(5*a*e^3)
```

3.523.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.523.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.70 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.63

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-5x^{3}B+4A\right)}{10x^{2}e^{3}\sqrt{ex}}+\frac{2\left(\frac{3Ab}{5}+\frac{3Ba}{4}\right)\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\,\left(x-\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*(b*x^3+a)^(1/2)*(-5*B*x^3+4*A)/x^2/e^3/(e*x)^(1/2)+2*(3/5*A*b+3/4*B*
a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))
^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/
(b*x^3+a)^(1/2)
```

3.523.5 Fricas [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^4*x^4), x)
```

3.523.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{5}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{6}, -\frac{1}{2} \\ \frac{1}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)} + \frac{B\sqrt{a}\sqrt{x}\Gamma\left(\frac{1}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{6} \\ \frac{7}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(7/2),x)
```

output
```
A*sqrt(a)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(7/2)*x**(5/2)*gamma(1/6)) + B*sqrt(a)*sqrt(x)*gamma(1/6)*hyper((-
1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6))
```

3.523.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(7/2), x)
```

3.523.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx=\int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(7/2), x)
```

3.523.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx=\int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{{(e\,x)}^{7/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(7/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(7/2), x)
```

3.524 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{9/2}}\,dx$

3.524.1 Optimal result

Integrand size = 24, antiderivative size = 564

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx &= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} \\
&+ \frac{3\left(1+\sqrt{3}\right)\sqrt[3]{b}(2Ab+7aB)\sqrt{x}\sqrt{a+bx^3}}{7a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}} \\
&- \frac{3\sqrt[4]{3}\sqrt[3]{b}(2Ab+7aB)\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} \\
&- \frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{b}(2Ab+7aB)\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right.\right.}{14a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}
$$

output
```
-2/7*A*(b*x^3+a)^(3/2)/a/x^(7/2)-2/7*(2*A*b+7*B*a)*(b*x^3+a)^(1/2)/a/x^(1/
2)+3/7*b^(1/3)*(2*A*b+7*B*a)*(1+3^(1/2))*x^(1/2)*(b*x^3+a)^(1/2)/a/(a^(1/3
)+b^(1/3)*x*(1+3^(1/2)))-3/7*3^(1/4)*b^(1/3)*(2*A*b+7*B*a)*(a^(1/3)+b^(1/3
)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Ell
ipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*x^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2
)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-
1/14*3^(3/4)*b^(1/3)*(2*A*b+7*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x
*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*
x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1
/4*2^(1/2))*(1-3^(1/2))*x^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x
*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.524.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \frac{2\sqrt{a+bx^3}\left(-A(a+bx^3)-\frac{(2Ab+7aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{7ax^{7/2}}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(9/2),x]
```

output
```
(2*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) - ((2*A*b + 7*a*B)*x^3*Hypergeometric
2F1[-1/2, -1/6, 5/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(7*a*x^(7/2))
```

3.524.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {955, 809, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{9/2}}dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{(7aB+2Ab)\int \frac{\sqrt{bx^3+a}}{x^{3/2}}dx}{7a}-\frac{2A\left(a+bx^3\right)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 809}$$

$$\frac{(7aB+2Ab)\left(3b\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}}{\sqrt{x}}\right)}{7a}-\frac{2A\left(a+bx^3\right)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(7aB+2Ab)\left(6b\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}-\frac{2\sqrt{a+bx^3}}{\sqrt{x}}\right)}{7a}-\frac{2A\left(a+bx^3\right)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 837}$$

$$\frac{(7aB+2Ab)\left(6b\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)-\frac{2\sqrt{a+bx^3}}{\sqrt{x}}\right)}{7a}-\frac{2A\left(a+bx^3\right)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(7aB+2Ab)\left(6b\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)-\frac{2\sqrt{a+bx^3}}{\sqrt{x}}\right)}{7a}-\frac{2A\left(a+bx^3\right)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{(7aB+2Ab)\left(6b\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.}{7a}$$

$$\frac{2A\left(a+bx^3\right)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \; 2420$$

$$\frac{(7aB+2Ab)\left(6b\left(\frac{\frac{\left(1+\sqrt{3}\right)\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2b^{2/3}}-\right.\right.}{7a}$$

$$\frac{2A\left(a+bx^3\right)^{3/2}}{7ax^{7/2}}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(9/2),x]
```

3.524. $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{9/2}}\,dx$

output
```
(-2*A*(a + b*x^3)^(3/2))/(7*a*x^(7/2)) + ((2*A*b + 7*a*B)*((-2*Sqrt[a + b*
x^3])/Sqrt[x] + 6*b*((((1 + Sqrt[3])*Sqrt[x]*Sqrt[a + b*x^3])/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x) - (3^(1/4)*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
 + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*x*(a^(1/3)
+ b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2*
b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*
(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x
^3]))))/(7*a)
```

3.524.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

3.524. $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^{9/2}}\,dx$

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.524.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.98 (sec) , antiderivative size = 1127, normalized size of antiderivative = 2.00

method	result	size
risch	Expression too large to display	1127
elliptic	Expression too large to display	1177
default	Expression too large to display	5911

input
```
int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*(b*x^3+a)^(1/2)*(3*A*b*x^3+7*B*a*x^3+A*a)/x^(7/2)/a+3/7*b*(2*A*b+7*B*
a)/a*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(
1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*((
(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/
b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*
(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/
(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^...
```

3.524.5 Fricas [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx=\int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)
```

3.524.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.85 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{7}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{6}, -\frac{1}{2} \\ -\frac{1}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^{\frac{7}{2}}\Gamma\left(-\frac{1}{6}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{1}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{6} \\ \frac{5}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{x}\Gamma\left(\frac{5}{6}\right)}$$

input

```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(9/2),x)
```

output

```
A*sqrt(a)*gamma(-7/6)*hyper((-7/6, -1/2), (-1/6,), b*x**3*exp_polar(I*pi)/
a)/(3*x**(7/2)*gamma(-1/6)) + B*sqrt(a)*gamma(-1/6)*hyper((-1/2, -1/6), (5
/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(x)*gamma(5/6))
```

3.524.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{9}{2}}}\,dx$$

input

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

output

```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)
```

3.524.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx=\int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)
```

3.524.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx=\int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^{9/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(9/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(9/2), x)
```

3.525 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx$

3.525.1 Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx=-\frac{2B\sqrt{a+bx^3}}{3x^{3/2}}-\frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}}+\frac{2}{3}\sqrt{b}B\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)$$

output
```
-2/9*A*(b*x^3+a)^(3/2)/a/x^(9/2)+2/3*B*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a)^(
1/2))*b^(1/2)-2/3*B*(b*x^3+a)^(1/2)/x^(3/2)
```

3.525.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx=-\frac{2\sqrt{a+bx^3}(aA+Abx^3+3aBx^3)}{9ax^{9/2}}+\frac{2}{3}\sqrt{b}B\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)$$

input
```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2),x]
```

output
```
(-2*Sqrt[a + b*x^3]*(a*A + A*b*x^3 + 3*a*B*x^3))/(9*a*x^(9/2)) + (2*Sqrt[b
]*B*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/3
```

3.525.3 Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {953, 809, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11/2}}dx$$

$$\Big\downarrow \; 953$$

$$B\int \frac{\sqrt{bx^3+a}}{x^{5/2}}dx-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \; 809$$

$$B\left(b\int \frac{\sqrt{x}}{\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}}{3x^{3/2}}\right)-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \; 851$$

$$B\left(2b\int \frac{x}{\sqrt{bx^3+a}}d\sqrt{x}-\frac{2\sqrt{a+bx^3}}{3x^{3/2}}\right)-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \; 807$$

$$B\left(\frac{2}{3}b\int \frac{1}{\sqrt{a+bx}}dx^{3/2}-\frac{2\sqrt{a+bx^3}}{3x^{3/2}}\right)-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \; 224$$

$$B\left(\frac{2}{3}b\int \frac{1}{1-bx}d\frac{x^{3/2}}{\sqrt{a+bx}}-\frac{2\sqrt{a+bx^3}}{3x^{3/2}}\right)-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \; 219$$

$$B\left(\frac{2}{3}\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx^3}}{3x^{3/2}}\right)-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}$$

input

```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2),x]
```

output

```
(-2*A*(a + b*x^3)^(3/2))/(9*a*x^(9/2)) + B*((-2*Sqrt[a + b*x^3])/(3*x^(3/2
)) + (2*Sqrt[b]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x]])/3)
```

3.525. $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11/2}}dx$

3.525.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 953
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[d/e^n   Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] &&
(IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && G
tQ[m + n, -1]))
```

3.525. $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11/2}}\,dx$

3.525.4 Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

method	result	size
risch	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(A b\,x^{3}+3 B a\,x^{3}+A a\right)}{9 x^{\frac{9}{2}} a}+\frac{2 B \sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{x \left(b\,x^{3}+a\right)}}{x^{2} \sqrt{b}}\right) \sqrt{x \left(b\,x^{3}+a\right)}}{3 \sqrt{x}\,\sqrt{b\,x^{3}+a}}$	84
default	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-3 B\,\operatorname{arctanh}\left(\frac{\sqrt{x \left(b\,x^{3}+a\right)}}{x^{2} \sqrt{b}}\right) \sqrt{b}\,a\,x^{5}+A \sqrt{x \left(b\,x^{3}+a\right)}\,b\,x^{3}+3 B \sqrt{x \left(b\,x^{3}+a\right)}\,a\,x^{3}+A \sqrt{x \left(b\,x^{3}+a\right)}\,a\right)}{9 x^{\frac{9}{2}} \sqrt{x \left(b\,x^{3}+a\right)}\,a}$	108
elliptic	Expression too large to display	1051

input
```
int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(b*x^3+a)^(1/2)*(A*b*x^3+3*B*a*x^3+A*a)/x^(9/2)/a+2/3*B*b^(1/2)*arcta
nh(1/x^2*(x*(b*x^3+a))^(1/2)/b^(1/2))*(x*(b*x^3+a))^(1/2)/x^(1/2)/(b*x^3+a
)^(1/2)
```

3.525.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.28

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = \Bigg[\frac{3\,Ba\sqrt{b}x^5\log\left(-8\,b^2x^6-8\,abx^3-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\sqrt{b}\sqrt{x}-a^2\right)-4\,(}{18\,ax^5}$$

$$-\frac{3\,Ba\sqrt{-b}x^5\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{-b}x^{\frac{3}{2}}}{2\,bx^3+a}\right)+2\,((3\,Ba+Ab)x^3+Aa)\sqrt{bx^3+a}\sqrt{x}}{9\,ax^5}\Bigg]$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="fricas")
```

output
```
[1/18*(3*B*a*sqrt(b)*x^5*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(2*b*x^4 + a*x)*sq
rt(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*
x^3 + a)*sqrt(x))/(a*x^5), -1/9*(3*B*a*sqrt(-b)*x^5*arctan(2*sqrt(b*x^3 +
a)*sqrt(-b)*x^(3/2)/(2*b*x^3 + a)) + 2*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*x^
3 + a)*sqrt(x))/(a*x^5)]
```

3.525.6 Sympy [A] (verification not implemented)

Time = 26.87 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.66

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = -\frac{2A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{9x^3} - \frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{9a} - \frac{2B\sqrt{a}}{3x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{2B\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3} - \frac{2Bbx^{\frac{3}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(11/2),x)
```

output
```
-2*A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(9*x**3) - 2*A*b**(3/2)*sqrt(a/(b*x**3)
+ 1)/(9*a) - 2*B*sqrt(a)/(3*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*B*sqrt(b)*asi
nh(sqrt(b)*x**(3/2)/sqrt(a))/3 - 2*B*b*x**(3/2)/(3*sqrt(a)*sqrt(1 + b*x**3
/a))
```

3.525.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = -\frac{1}{3}\left(\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)+\frac{2\sqrt{bx^3+a}}{x^{\frac{3}{2}}}\right)B - \frac{2\,(bx^3+a)^{\frac{3}{2}}A}{9\,ax^{\frac{9}{2}}}$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="maxima")
```

output
```
-1/3*(sqrt(b)*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x
^3 + a)/x^(3/2))) + 2*sqrt(b*x^3 + a)/x^(3/2))*B - 2/9*(b*x^3 + a)^(3/2)*A
/(a*x^(9/2))
```

3.525.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = -\frac{2\,Bb\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}} + \frac{2\left(3\,Bab\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+3\,Ba\sqrt{-b}\sqrt{b}+A\sqrt{-b}b^{\frac{3}{2}}\right)}{9\,a\sqrt{-b}} - \frac{2\left(3\,Ba^3\sqrt{b+\frac{a}{x^3}}+Aa^2\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}\right)}{9\,a^3}$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="giac")
```

output
```
-2/3*B*b*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b) + 2/9*(3*B*a*b*arctan(s
qrt(b)/sqrt(-b)) + 3*B*a*sqrt(-b)*sqrt(b) + A*sqrt(-b)*b^(3/2))/(a*sqrt(-b
)) - 2/9*(3*B*a^3*sqrt(b + a/x^3) + A*a^2*(b + a/x^3)^(3/2))/a^3
```

3.525.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = \int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{x^{11/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(11/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(11/2), x)
```

3.526 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{13/2}}\,dx$

3.526.1 Optimal result

Integrand size = 24, antiderivative size = 269

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}} - \frac{2A(a+bx^3)^{3/2}}{11ax^{11/2}}$$

$$-\frac{3^{3/4}b(2Ab-11aB)\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+\right.}{55a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/11*A*(b*x^3+a)^(3/2)/a/x^(11/2)+2/55*(2*A*b-11*B*a)*(b*x^3+a)^(1/2)/a/x
^(5/2)-1/55*3^(3/4)*b*(2*A*b-11*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3
)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)
+1/4*2^(1/2))*x^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(4/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+
b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.526.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \frac{2\sqrt{a+bx^3}\left(-5A(a+bx^3)+\frac{(2Ab-11aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{55ax^{11/2}}$$

input

```
Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(13/2),x]
```

output

```
(2*Sqrt[a + b*x^3]*(-5*A*(a + b*x^3) + ((2*A*b - 11*a*B)*x^3*Hypergeometri
c2F1[-5/6, -1/2, 1/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(55*a*x^(11/2))
```

3.526.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {955, 809, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(2Ab-11aB)\int \frac{\sqrt{bx^3+a}}{x^{7/2}}dx}{11a}-\frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{809}$$

$$-\frac{(2Ab-11aB)\left(\frac{3}{5}b\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}}{5x^{5/2}}\right)}{11a}-\frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{851}$$

$$-\frac{(2Ab-11aB)\left(\frac{6}{5}b\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}-\frac{2\sqrt{a+bx^3}}{5x^{5/2}}\right)}{11a}-\frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{766}$$

$$-\frac{(2Ab-11aB)\left(\frac{3^{3/4}b\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{a+bx^3}}{5x^{5/2}}\right)}{11a}$$

$$\frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}}$$

input
```
Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(13/2),x]
```

output
```
(-2*A*(a + b*x^3)^(3/2))/(11*a*x^(11/2)) - ((2*A*b - 11*a*B)*((-2*Sqrt[a +
 b*x^3])/(5*x^(5/2)) + (3^(3/4)*b*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3)
 + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(
11*a)
```

3.526.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.526.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.52 (sec) , antiderivative size = 745, normalized size of antiderivative = 2.77

method	result
risch	$-\frac{2\sqrt{b\,x^3+a}\,(3Ab\,x^3+11Ba\,x^3+5Aa)}{55x^{\frac{11}{2}}\,a}-\frac{6b^2(2Ab-11Ba)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-a\,b}{b}\right.}}}{}$
elliptic	$\frac{\sqrt{x(b\,x^3+a)}\left(-\frac{2A\sqrt{b\,x^4+ax}}{11x^6}-\frac{2(3Ab+11Ba)\sqrt{b\,x^4+ax}}{55a\,x^3}+\frac{2\left(Bb-\frac{2b(3Ab+11Ba)}{55a}\right)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\right.}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b\right.}{2b}\right.}}}{}\right.}{}$
default	Expression too large to display

input
```
int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/55*(b*x^3+a)^(1/2)*(3*A*b*x^3+11*B*a*x^3+5*A*a)/x^(11/2)/a-6/55*b^2*(2*
A*b-11*B*a)/a*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*
b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-
1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*(x*(b*x^3+a))^(1/2)/x^(1/2)/(b*
x^3+a)^(1/2)
```

3.526.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = -\frac{2\left(3\left(11\,Bab-2\,Ab^2\right)\sqrt{a}x^6\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)+\left(\left(11\,Ba^2+3\,Aab\right)x^3+5\,Aa^2\right)\sqrt{bx^3+a}\sqrt{x}\right)}{55\,a^2x^6}$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="fricas")
```

output
```
-2/55*(3*(11*B*a*b - 2*A*b^2)*sqrt(a)*x^6*weierstrassPInverse(0, -4*b/a, 1
/x) + ((11*B*a^2 + 3*A*a*b)*x^3 + 5*A*a^2)*sqrt(b*x^3 + a)*sqrt(x))/(a^2*x
^6)
```

3.526.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 67.78 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{11}{6}\right)\,{}_2F_1\left(\begin{matrix}-\frac{11}{6},-\frac{1}{2}\\ -\frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{\frac{11}{2}}\Gamma\left(-\frac{5}{6}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{5}{6}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}$$

input
```
integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(13/2),x)
```

output
```
A*sqrt(a)*gamma(-11/6)*hyper((-11/6, -1/2), (-5/6,), b*x**3*exp_polar(I*pi
)/a)/(3*x**(11/2)*gamma(-5/6)) + B*sqrt(a)*gamma(-5/6)*hyper((-5/6, -1/2),
 (1/6,), b*x**3*exp_polar(I*pi)/a)/(3*x**(5/2)*gamma(1/6))
```

3.526.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(13/2), x)
```

3.526.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx=\int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(13/2), x)
```

3.526.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx=\int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^{13/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(13/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(13/2), x)
```

3.527 $\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

3.527.1 Optimal result

Integrand size = 26, antiderivative size = 201

$$\begin{aligned}\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx &= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} \\ &+ \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{72be} \\ &+ \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be} - \frac{a^3(8Ab-3aB)e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{5/2}}\end{aligned}$$

output
```
1/72*(8*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(3/2)/b/e+1/12*B*(e*x)^(9/2)*(b*x
^3+a)^(5/2)/b/e-1/192*a^3*(8*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2
)/e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)+1/192*a^2*(8*A*b-3*B*a)*e^2*(e*x)^(3/2)
*(b*x^3+a)^(1/2)/b^2+1/96*a*(8*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(1/2)/b/e
```

3.527.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.72

$$\begin{aligned}&\int (ex)^{7/2} (a \\ &+bx^3)^{3/2} \left(A+Bx^3\right) \, dx = \frac{e^3\sqrt{ex}\left(\sqrt{b}x^{3/2}\sqrt{a+bx^3}(-9a^3B+6a^2b(4A+Bx^3)+16b^3x^6(4A+3Bx^3)+8ab^2\right.}{576b^{5/2}\sqrt{x}}\end{aligned}$$

input
```
Integrate[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(-9*a^3*B + 6*a^2*b*(4*A +
 B*x^3) + 16*b^3*x^6*(4*A + 3*B*x^3) + 8*a*b^2*x^3*(14*A + 9*B*x^3)) + 3*a
^3*(-8*A*b + 3*a*B)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(576*b^(5/2)*
Sqrt[x])
```

3.527.3 Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {959, 811, 811, 843, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{7/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(8Ab-3aB)\int (ex)^{7/2}\left(bx^3+a\right)^{3/2}dx}{8b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\int (ex)^{7/2}\sqrt{bx^3+a}dx+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)}{8b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\int\frac{(ex)^{7/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)}{8b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

$$\Big\downarrow \text{843}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^3\int\frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{2b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)}{8b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int\frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)}{8b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

$$\Big\downarrow \ 807$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int\frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)}{8b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

$$\Big\downarrow \ 224$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int\frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)}{8b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

$$\Big\downarrow \ 219$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)}{8b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

input

```
Int[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output

```
(B*(e*x)^(9/2)*(a + b*x^3)^(5/2))/(12*b*e) + ((8*A*b - 3*a*B)*(((e*x)^(9/2
)*(a + b*x^3)^(3/2))/(9*e) + (a*(((e*x)^(9/2)*Sqrt[a + b*x^3])/(6*e) + (a*
((e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*b) - (a*e^(7/2)*ArcTanh[(Sqrt[b]*(e*
x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*b^(3/2))))/4))/2))/(8*b)
```

3.527. $\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

3.527.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.527. $\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

3.527.4 Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.81

method	result
risch	$\frac{x^{2}\left(48 b^{3} B\,x^{9}+64 x^{6} b^{3} A+72 B\,x^{6} a\,b^{2}+112 a A\,b^{2} x^{3}+6 B\,a^{2} b\,x^{3}+24 a^{2} b A-9 a^{3} B\right) \sqrt{b\,x^{3}+a}\,e^{4}}{576 b^{2} \sqrt{e x}}-\frac{a^{3}\left(8 A b-3 B a\right) \operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right) e}}{x^{2} \sqrt{b e}}\right.}{192 b^{2} \sqrt{b e}\,\sqrt{e x}\,\sqrt{b\,x^{3}}}$
default	$-\frac{e^{3} \sqrt{e x}\,\sqrt{b\,x^{3}+a}\,\left(-48 B \sqrt{\left(b\,x^{3}+a\right) e x}\,\sqrt{b e}\,b^{3} x^{10}-64 A \sqrt{\left(b\,x^{3}+a\right) e x}\,\sqrt{b e}\,b^{3} x^{7}-72 B \sqrt{\left(b\,x^{3}+a\right) e x}\,\sqrt{b e}\,a\,b^{2} x^{7}-112 A \sqrt{\left(b\,x^{3}+a\right) e x}\right.}{}$
elliptic	Expression too large to display

input
```
int((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/576/b^2*x^2*(48*B*b^3*x^9+64*A*b^3*x^6+72*B*a*b^2*x^6+112*A*a*b^2*x^3+6*
B*a^2*b*x^3+24*A*a^2*b-9*B*a^3)*(b*x^3+a)^(1/2)*e^4/(e*x)^(1/2)-1/192*a^3/
b^2*(8*A*b-3*B*a)/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2
))*e^4*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

3.527.5 Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.77

$$\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx = \left[-\frac{3\,(3\,Ba^4-8\,Aa^3b)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e+4\,(2\,b^2x^4+abx)\sqrt{bx^3}\right.}{}\right.$$

input
```
integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
[-1/2304*(3*(3*B*a^4 - 8*A*a^3*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e
*x^3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b))
- 4*(48*B*b^3*e^3*x^10 + 8*(9*B*a*b^2 + 8*A*b^3)*e^3*x^7 + 2*(3*B*a^2*b +
56*A*a*b^2)*e^3*x^4 - 3*(3*B*a^3 - 8*A*a^2*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(
e*x))/b^2, -1/1152*(3*(3*B*a^4 - 8*A*a^3*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b
*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(48*B*b^3*e^3*x^
10 + 8*(9*B*a*b^2 + 8*A*b^3)*e^3*x^7 + 2*(3*B*a^2*b + 56*A*a*b^2)*e^3*x^4
- 3*(3*B*a^3 - 8*A*a^2*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]
```

3.527.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(180) = 360$.

Time = 31.07 (sec) , antiderivative size = 634, normalized size of antiderivative = 3.15

$$\int (ex)^{7/2} \left(a + bx^3\right)^{3/2} \left(A + Bx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**(7/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*a*e**3*Piecewise((-a**2*e**
3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/
sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), Tru
e))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2)/(8*b) + (e*x)**(9/2)/4),
 Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True)) + A*b*Piecewise((a**3*e**
6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/
sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), Tru
e))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*(e*x)**(3/2)/(16*b**2) + a*e*
*3*(e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6), Ne(b/e**3, 0)), (sqrt(a)*(e*x)*
*(15/2)/5, True)) + B*a*Piecewise((a**3*e**6*Piecewise((log(2*b*(e*x)**(3/
2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)
**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/(16*b**2) + sqrt(a + b*x**3
)*(-a**2*e**6*(e*x)**(3/2)/(16*b**2) + a*e**3*(e*x)**(9/2)/(24*b) + (e*x)*
*(15/2)/6), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(15/2)/5, True)) + B*b*Piecewi
se((-5*a**4*e**9*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqr
t(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sq
rt(b*x**3), True))/(128*b**3) + sqrt(a + b*x**3)*(5*a**3*e**9*(e*x)**(3/2)
/(128*b**3) - 5*a**2*e**6*(e*x)**(9/2)/(192*b**2) + a*e**3*(e*x)**(15/2)/(
48*b) + (e*x)**(21/2)/8), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(21/2)/7, True))
/e**3)/(3*e**3), True))/e, Ne(e, 0)), (0, True))
```

3.527.7 Maxima [F]

$$\int (ex)^{7/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}\,dx$$

input
```
integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(7/2), x)
```

3.527.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(163) = 326.

Time = 0.51 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.43

$$\int (ex)^{7/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{1}{12}\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2\,x^3}{e}+\frac{a}{be}\right)Aax|e|^2$$
$$+\frac{1}{72}\sqrt{be^4x^3+ae^4}\left(2\,e^3x^3\left(\frac{4\,x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3\,a^2}{b^2e}\right)\sqrt{ex}Bax|e|^2$$
$$+\frac{1}{72}\sqrt{be^4x^3+ae^4}\left(2\,e^3x^3\left(\frac{4\,x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3\,a^2}{b^2e}\right)\sqrt{ex}Abx|e|^2$$
$$+\frac{1}{576}\sqrt{be^4x^3+ae^4}\left(2\left(4\,e^3x^3\left(\frac{6\,x^3}{e^7}+\frac{a}{be^7}\right)-\frac{5\,a^2}{b^2e^4}\right)e^3x^3+\frac{15\,a^3}{b^3e}\right)\sqrt{ex}Bbx|e|^2$$
$$-\frac{(9\,B^2a^8e-48\,ABa^7be+64\,A^2a^6b^2e)^2e^5\log\left(\left|-(3\,\sqrt{ex}Ba^4e^2x-8\,\sqrt{ex}Aa^3be^2x)\sqrt{be}+\sqrt{9\,B^2a^9e^6-48}\right.\right.}{192\,\sqrt{be}b^2|9\,B^2a^8e-48\,ABa^7be+64\,A^2a^6b^2e||-3\,Ba^4}$$

input
```
integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))*A*a*x*abs(e)^2
+ 1/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/
(b^2*e))*sqrt(e*x)*B*a*x*abs(e)^2 + 1/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^
3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b^2*e))*sqrt(e*x)*A*b*x*abs(e)^2 + 1/57
6*sqrt(b*e^4*x^3 + a*e^4)*(2*(4*e^3*x^3*(6*x^3/e^7 + a/(b*e^7)) - 5*a^2/(b
^2*e^4))*e^3*x^3 + 15*a^3/(b^3*e))*sqrt(e*x)*B*b*x*abs(e)^2 - 1/192*(9*B^2
*a^8*e - 48*A*B*a^7*b*e + 64*A^2*a^6*b^2*e)^2*e^5*log(abs(-(3*sqrt(e*x)*B*
a^4*e^2*x - 8*sqrt(e*x)*A*a^3*b*e^2*x)*sqrt(b*e) + sqrt(9*B^2*a^9*e^6 - 48
*A*B*a^8*b*e^6 + 64*A^2*a^7*b^2*e^6 + (3*sqrt(e*x)*B*a^4*e^2*x - 8*sqrt(e*
x)*A*a^3*b*e^2*x)^2*b*e)))/(sqrt(b*e)*b^2*abs(9*B^2*a^8*e - 48*A*B*a^7*b*e
 + 64*A^2*a^6*b^2*e)*abs(-3*B*a^4 + 8*A*a^3*b)*abs(e)^2)
```

3.527.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{7/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\left(e\,x\right)^{7/2}\,{\left(b\,x^3+a\right)}^{3/2}\,dx$$

input
```
int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(3/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(3/2), x)
```

3.528 $\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

3.528.1 Optimal result

Integrand size = 26, antiderivative size = 364

$$\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{7040b^2} + \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be} + \frac{(22Ab-7aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{176be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be} - \frac{9\,3^{3/4}a^{8/3}(22Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)\right.}{14080b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/176*(22*A*b-7*B*a)*(e*x)^(7/2)*(b*x^3+a)^(3/2)/b/e+1/11*B*(e*x)^(7/2)*(b
*x^3+a)^(5/2)/b/e+9/1760*a*(22*A*b-7*B*a)*(e*x)^(7/2)*(b*x^3+a)^(1/2)/b/e+
27/7040*a^2*(22*A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2-9/14080*3^(
3/4)*a^(8/3)*(22*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1
-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(
1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*
x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*
2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/
3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.528.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.32

$$\int (ex)^{5/2} (a+bx^3)^{3/2} (A+Bx^3)\, dx = \frac{e^2\sqrt{ex}\sqrt{a+bx^3}\left(-(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}(-22Ab+7aB-16bBx^3)+a^2(-22Ab+\right.}{176b^2\sqrt{1+\frac{bx^3}{a}}}$$

input

```
Integrate[(e*x)^(5/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output

```
(e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(-22*A
*b + 7*a*B - 16*b*B*x^3)) + a^2*(-22*A*b + 7*a*B)*Hypergeometric2F1[-3/2,
1/6, 7/6, -((b*x^3)/a)]))/(176*b^2*Sqrt[1 + (b*x^3)/a])
```

3.528.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {959, 811, 811, 843, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{5/2} (a+bx^3)^{3/2} (A+Bx^3)\, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(22Ab-7aB)\int (ex)^{5/2}(bx^3+a)^{3/2}dx}{22b}+\frac{B(ex)^{7/2}(a+bx^3)^{5/2}}{11be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(22Ab-7aB)\left(\frac{9}{16}a\int (ex)^{5/2}\sqrt{bx^3+a}dx+\frac{(ex)^{7/2}(a+bx^3)^{3/2}}{8e}\right)}{22b}+\frac{B(ex)^{7/2}(a+bx^3)^{5/2}}{11be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(22Ab-7aB)\left(\frac{9}{16}a\left(\frac{3}{10}a\int\frac{(ex)^{5/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)+\frac{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8e}\right)}{22b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}$$

$$\Big\downarrow \text{843}$$

$$\frac{(22Ab-7aB)\left(\frac{9}{16}a\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^3\int\frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{4b}\right)+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)+\frac{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8e}\right)}{22b}+$$
$$\frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(22Ab-7aB)\left(\frac{9}{16}a\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b}\right)+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)+\frac{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8e}\right)}{22b}+$$
$$\frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}$$

$$\Big\downarrow \text{766}$$

$$\frac{(22Ab-7aB)\left(\frac{9}{16}a\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{a^{2/3}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{4\sqrt[4]{3}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.\right.\right.}{22b}$$
$$\frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}$$

input
```
Int[(e*x)^(5/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(B*(e*x)^(7/2)*(a + b*x^3)^(5/2))/(11*b*e) + ((22*A*b - 7*a*B)*(((e*x)^(7/
2)*(a + b*x^3)^(3/2))/(8*e) + (9*a*(((e*x)^(7/2)*Sqrt[a + b*x^3])/(5*e) +
(3*a*((e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(2*b) - (a^(2/3)*e*Sqrt[e*x]*(a^(1/3
)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2
*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)
*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)],
(2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x)
)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/10))/16))/
(22*b)
```

3.528. $\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx$

3.528.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.528. $\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

3.528.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.53 (sec) , antiderivative size = 801, normalized size of antiderivative = 2.20

method	result	size
risch	Expression too large to display	801
elliptic	Expression too large to display	976
default	Expression too large to display	4619

input
```
int((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/7040/b^2*(640*B*b^3*x^9+880*A*b^3*x^6+1000*B*a*b^2*x^6+1672*A*a*b^2*x^3+
108*B*a^2*b*x^3+594*A*a^2*b-189*B*a^3)*x*(b*x^3+a)^(1/2)*e^3/(e*x)^(1/2)-2
7/7040*a^3/b*(22*A*b-7*B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/
2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(
1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*e^3*((b*x^3+a)*
e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

3.528.5 Fricas [F]

$$\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*b*e^2*x^8 + (B*a + A*b)*e^2*x^5 + A*a*e^2*x^2)*sqrt(b*x^3 + a)
*sqrt(e*x), x)
```

3.528.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 63.41 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.55

$$\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{Aa^{\frac{3}{2}}e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{6}\right)}+\frac{A\sqrt{a}be^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}+\frac{Ba^{\frac{3}{2}}e^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}+\frac{B\sqrt{a}be^{\frac{5}{2}}x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{19}{6}\\ \frac{25}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{25}{6}\right)}$$

input
```
integrate((e*x)**(5/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
A*a**(3/2)*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(13/6)) + A*sqrt(a)*b*e**(5/2)*x**(13/2)*gamma
(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(19/
6)) + B*a**(3/2)*e**(5/2)*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(19/6)) + B*sqrt(a)*b*e**(5/2)*x**(19
/2)*gamma(19/6)*hyper((-1/2, 19/6), (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(25/6))
```

3.528.7 Maxima [F]

$$\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(5/2), x)
```

3.528.8 Giac [F]

$$\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(5/2), x)
```

3.528.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}\,{\left(b\,x^3+a\right)}^{3/2}\,dx$$

input
```
int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(3/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(3/2), x)
```

3.529 $\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

3.529.1 Optimal result

Integrand size = 26, antiderivative size = 621

$$\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be}$$

$$+ \frac{27\left(1+\sqrt{3}\right)a^2(4Ab-aB)e\sqrt{ex}\sqrt{a+bx^3}}{448b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$+ \frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be}$$

$$- \frac{27\sqrt[4]{3}a^{7/3}(4Ab-aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{448b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{9\ 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}(4Ab-aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)}\right.\right.}{896b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
1/28*(4*A*b-B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/b/e+1/10*B*(e*x)^(5/2)*(b*x^3
+a)^(5/2)/b/e+9/224*a*(4*A*b-B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+27/448*a
^2*(4*A*b-B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))-27/448*3^(1/4)*a^(7/3)*(4*A*b-B*a)*e*(a^(1/3)+b^(1/
3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2
)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*El
lipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)
^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2)-9/896*3^(3/4)*a^(7/3)*(4*A*b-B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)
+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(
1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/
(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.529.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.15

$$\int (ex)^{3/2} (a+bx^3)^{3/2} (A+Bx^3) \, dx = \frac{x(ex)^{3/2}\sqrt{a+bx^3}\left(B(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}+a(4Ab-aB)\,\text{Hypergeometric2F1}\left(-\right.\right.}{10b\sqrt{1+\frac{bx^3}{a}}}$$

input

```
Integrate[(e*x)^(3/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output

```
(x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a] + a*(4
*A*b - a*B)*Hypergeometric2F1[-3/2, 5/6, 11/6, -((b*x^3)/a)]))/(10*b*Sqrt[
1 + (b*x^3)/a])
```

3.529.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {959, 811, 811, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(4Ab-aB)\int (ex)^{3/2} \left(bx^3+a\right)^{3/2}dx}{4b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{5/2}}{10be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(4Ab-aB)\left(\frac{9}{14}a\int (ex)^{3/2}\sqrt{bx^3+a}dx+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)}{4b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{5/2}}{10be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(4Ab-aB)\left(\frac{9}{14}a\left(\frac{3}{8}a\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)}{4b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{5/2}}{10be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(4Ab-aB)\left(\frac{9}{14}a\left(\frac{3a\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)}{4b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{5/2}}{10be}$$

$$\Big\downarrow \text{837}$$

$$\frac{(4Ab-aB)\left(\frac{9}{14}a\left(\frac{3a\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/}}{7e}}{4b}$$

$$\frac{B(ex)^{5/2} \left(a+bx^3\right)^{5/2}}{10be}$$

$$\Big\downarrow \text{25}$$

$$\frac{(4Ab-aB)\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)}{4b}$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be}$$

$$\Big\downarrow 766$$

$$(4Ab-aB)\Bigg(\frac{9}{14}a\Bigg(3a\Bigg(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\Bigg(\text{arcco}}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)}}}$$

$$4e$$

$$4b$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be}$$

$$\Big\downarrow 2420$$

3.529. $\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$

$$\frac{(4Ab-aB)\Bigg(\frac{9}{14}a\Bigg(\frac{3a\Bigg(\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\Bigg(\arccos\Bigg(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+}}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}}}{}}{}$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be}$$

input
```
Int[(e*x)^(3/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

3.529. $\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$

output

```
(B*(e*x)^(5/2)*(a + b*x^3)^(5/2))/(10*b*e) + ((4*A*b - a*B)*(((e*x)^(5/2)*
(a + b*x^3)^(3/2))/(7*e) + (9*a*(((e*x)^(5/2)*Sqrt[a + b*x^3])/(4*e) + (3*
a*((((1 + Sqrt[3])*e^3*Sqrt[e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3
])*b^(1/3)*e*x) - (3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*S
qrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (
1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b
^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(S
qrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(
1/3)*e*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt
[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x
+ b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[Ar
cCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^
(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/
3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b
*x^3])))/(4*e)))/14))/(4*b)
```

3.529.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 837

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

3.529. $\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.529.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.99 (sec) , antiderivative size = 1164, normalized size of antiderivative = 1.87

method	result	size
risch	Expression too large to display	1164
elliptic	Expression too large to display	1279
default	Expression too large to display	5790

input
```
int((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output

```
1/1120/b*x^3*(112*B*b^2*x^6+160*A*b^2*x^3+184*B*a*b*x^3+340*A*a*b+27*B*a^2
)*(b*x^3+a)^(1/2)*e^2/(e*x)^(1/2)+27/448*a^2/b*(4*A*b-B*a)*(x*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)
*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*El
lipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/
2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^...
```

3.529.5 Fricas [F]

$$\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}\,dx$$

input

```
integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

output

```
integral((B*b*e*x^7 + (B*a + A*b)*e*x^4 + A*a*e*x)*sqrt(b*x^3 + a)*sqrt(e*
x), x)
```

3.529.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.76 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.32

$$\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{Aa^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\\frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)}+\frac{A\sqrt{a}be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\\frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)}+\frac{Ba^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\\frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)}+\frac{B\sqrt{a}be^{\frac{3}{2}}x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{17}{6}\\\frac{23}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{23}{6}\right)}$$

input
```
integrate((e*x)**(3/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

output
```
A*a**(3/2)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(11/6)) + A*sqrt(a)*b*e**(3/2)*x**(11/2)*gamma
(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/
6)) + B*a**(3/2)*e**(3/2)*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6)) + B*sqrt(a)*b*e**(3/2)*x**(17
/2)*gamma(17/6)*hyper((-1/2, 17/6), (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(23/6))
```

3.529.7 Maxima [F]

$$\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(3/2), x)
```

3.529.8 Giac [F]

$$\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(3/2), x)
```

3.529.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}\,{\left(b\,x^3+a\right)}^{3/2} \,d x$$

input
```
int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(3/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(3/2), x)
```

3.530 $\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$

3.530.1 Optimal result

Integrand size = 26, antiderivative size = 161

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be}+\frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}+\frac{a^2(6Ab-aB)\sqrt{e}\,\mathrm{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{3/2}}$$

output
```
1/36*(6*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/b/e+1/9*B*(e*x)^(3/2)*(b*x^3+
a)^(5/2)/b/e+1/24*a^2*(6*A*b-B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x
^3+a)^(1/2))*e^(1/2)/b^(3/2)+1/24*a*(6*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2
)/b/e
```

3.530.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{x\sqrt{ex}\sqrt{a+bx^3}\left(30aAb+3a^2B+12Ab^2x^3+14abBx^3+8b^2Bx^6\right)}{72b}-\frac{a^2(-6Ab+aB)\sqrt{ex}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{24b^{3/2}\sqrt{x}}$$

input
```
Integrate[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(x*Sqrt[e*x]*Sqrt[a + b*x^3]*(30*a*A*b + 3*a^2*B + 12*A*b^2*x^3 + 14*a*b*B
*x^3 + 8*b^2*B*x^6))/(72*b) - (a^2*(-6*A*b + a*B)*Sqrt[e*x]*Log[Sqrt[b]*x^
(3/2) + Sqrt[a + b*x^3]])/(24*b^(3/2)*Sqrt[x])
```

3.530.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {959, 811, 811, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(6Ab-aB)\int \sqrt{ex}\left(bx^3+a\right)^{3/2}dx}{6b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\int \sqrt{ex}\sqrt{bx^3+a}dx+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)}{6b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)}{6b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{a\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)}{6b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$\Big\downarrow \text{807}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{a\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)}{6b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$\Big\downarrow \text{224}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{a\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)}{6b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$\Big\downarrow \text{219}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{a\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3\sqrt{b}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)}{6b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

input
```
Int[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

output
```
(B*(e*x)^(3/2)*(a + b*x^3)^(5/2))/(9*b*e) + ((6*A*b - a*B)*(((e*x)^(3/2)*(
a + b*x^3)^(3/2))/(6*e) + (3*a*(((e*x)^(3/2)*Sqrt[a + b*x^3])/(3*e) + (a*S
qrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*Sq
rt[b])))/4))/(6*b)
```

3.530.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.530.4 Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

method	result
risch	$\frac{x^{2} \left(8 b^{2} B \,x^{6}+12 A \,b^{2} x^{3}+14 B a b \,x^{3}+30 a b A +3 a^{2} B \right) \sqrt{b \,x^{3}+a}\, e}{72 b \sqrt{e x}}+\frac{a^{2} \left(6 A b -B a \right) \operatorname{arctanh}\left(\frac{\sqrt{\left(b \,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) e \sqrt{\left(b \,x^{3}+a \right) e x}}{24 b \sqrt{b e}\, \sqrt{e x}\, \sqrt{b \,x^{3}+a}}$
default	$\frac{\sqrt{b \,x^{3}+a}\, \sqrt{e x}\, \left(8 B \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, b^{2} x^{7}+12 A \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, b^{2} x^{4}+14 B \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, a b \,x^{4}+30 A \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, a b x +1\right.}{72 \sqrt{\left(b \,x^{3}+a \right) e x}\, b \sqrt{b e}}$
elliptic	Expression too large to display

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/72/b*x^2*(8*B*b^2*x^6+12*A*b^2*x^3+14*B*a*b*x^3+30*A*a*b+3*B*a^2)*(b*x^3
+a)^(1/2)*e/(e*x)^(1/2)+1/24*a^2/b*(6*A*b-B*a)/(b*e)^(1/2)*arctanh(((b*x^3
+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3
+a)^(1/2)
```

3.530.5 Fricas [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.70

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \left[-\frac{3\left(Ba^3-6\,Aa^2b\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-}{288\,b}\right.$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/288*(3*(B*a^3 - 6*A*a^2*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 -
a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(8*
B*b^2*x^7 + 2*(7*B*a*b + 6*A*b^2)*x^4 + 3*(B*a^2 + 10*A*a*b)*x)*sqrt(b*x^3
 + a)*sqrt(e*x))/b, 1/144*(3*(B*a^3 - 6*A*a^2*b)*sqrt(-e/b)*arctan(2*sqrt(
b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(8*B*b^2*x^7 +
2*(7*B*a*b + 6*A*b^2)*x^4 + 3*(B*a^2 + 10*A*a*b)*x)*sqrt(b*x^3 + a)*sqrt(e
*x))/b]
```

3.530.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(138) = 276$.

Time = 3.44 (sec) , antiderivative size = 546, normalized size of antiderivative = 3.39

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\begin{cases}2\left(\begin{cases}\text{NaN} & \\ \dfrac{Aae^3\left(\begin{cases}\dfrac{a\left(\begin{cases}\dfrac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3}+2\sqrt{\frac{b}{e^3}}\sqrt{a+bx^3}\right)}{\sqrt{\frac{b}{e^3}}} & \text{for } a\neq 0\\ \dfrac{(ex)^{\frac{3}{2}}\log\left((ex)^{\frac{3}{2}}\right)}{\sqrt{bx^3}} & \text{otherwise}\end{cases}\right)}{2}+\dfrac{(ex)^{\frac{3}{2}}\sqrt{a+bx^3}}{2} & \text{for } \frac{b}{e^3}\neq 0\\ \sqrt{a}(ex)^{\frac{3}{2}} & \text{otherwise}\end{cases}\right)+Ab\left(\begin{cases} a^2e^3 \\ -\text{—} \\ \frac{\sqrt{a}(ex}{3}\end{cases}\right.}{}\end{cases}\right.}{} \\ 0 & \end{cases}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)*(e*x)**(1/2),x)
```

output
```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*a*e**3*Piecewise((a*Piecewi
se((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e*
*3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/2 + (
e*x)**(3/2)*sqrt(a + b*x**3)/2, Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(3/2), Tru
e)) + A*b*Piecewise((-a**2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*s
qrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((
e*x)**(3/2))/sqrt(b*x**3), True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**
(3/2)/(8*b) + (e*x)**(9/2)/4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, Tr
ue)) + B*a*Piecewise((-a**2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*
sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log(
(e*x)**(3/2))/sqrt(b*x**3), True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)*
*(3/2)/(8*b) + (e*x)**(9/2)/4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, T
rue)) + B*b*Piecewise((a**3*e**6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*
sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log(
(e*x)**(3/2))/sqrt(b*x**3), True))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**
6*(e*x)**(3/2)/(16*b**2) + a*e**3*(e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6),
Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(15/2)/5, True))/e**3)/(3*e**3), True))/e,
 Ne(e, 0)), (0, True))
```

3.530. $\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$

3.530.7 Maxima [F]

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}\sqrt{ex}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*sqrt(e*x), x)
```

3.530.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(126) = 252$.

Time = 0.45 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.58

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2\,x^3}{e}+\frac{a}{be}\right)Bax|e|^2}{12\,e^3}$$
$$+\frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2\,x^3}{e}+\frac{a}{be}\right)Abx|e|^2}{12\,e^3}$$
$$+\frac{\sqrt{be^4x^3+ae^4}\left(2\,e^3x^3\left(\frac{4\,x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3\,a^2}{b^2e}\right)\sqrt{ex}Bbx|e|^2}{72\,e^3}$$
$$-\frac{(B^2a^6+4\,ABa^5b+4\,A^2a^4b^2)e^4\log\left(\left|\left(\sqrt{ex}Ba^3x+2\,\sqrt{ex}Aa^2bx\right)\sqrt{be}+\sqrt{B^2a^7e^2+4\,ABa^6be^2+4\,A^2a^5}\right.\right.}{24\,\sqrt{be}b|Ba^3e+2\,Aa^2be||e|^2}$$
$$-\frac{\left(\frac{ae^4\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{\sqrt{be}}-\sqrt{be^4x^3+ae^4}\sqrt{ex}ex\right)Aa|e|^2}{3\,e^5}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))*B*a*x*abs(e)^2/
e^3 + 1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))*A*b*x*abs
(e)^2/e^3 + 1/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)
) - 3*a^2/(b^2*e))*sqrt(e*x)*B*b*x*abs(e)^2/e^3 - 1/24*(B^2*a^6 + 4*A*B*a^
5*b + 4*A^2*a^4*b^2)*e^4*log(abs((sqrt(e*x)*B*a^3*x + 2*sqrt(e*x)*A*a^2*b*
x)*sqrt(b*e) + sqrt(B^2*a^7*e^2 + 4*A*B*a^6*b*e^2 + 4*A^2*a^5*b^2*e^2 + (s
qrt(e*x)*B*a^3*x + 2*sqrt(e*x)*A*a^2*b*x)^2*b*e)))/(sqrt(b*e)*b*abs(B*a^3*
e + 2*A*a^2*b*e)*abs(e)^2) - 1/3*(a*e^4*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x +
 sqrt(b*e^4*x^3 + a*e^4)))/sqrt(b*e) - sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*e
*x)*A*a*abs(e)^2/e^5
```

3.530.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\sqrt{e\,x}\,{\left(b\,x^3+a\right)}^{3/2}\,dx$$

input
```
int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(3/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(3/2), x)
```

3.531 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx$

3.531.1 Optimal result

Integrand size = 26, antiderivative size = 324

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{9a(16Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{320be} + \frac{(16Ab-aB)\sqrt{ex}(a+bx^3)^{3/2}}{80be} + \frac{B\sqrt{ex}(a+bx^3)^{5/2}}{8be}$$

$$+ \frac{9\,3^{3/4}a^{5/3}(16Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Bigg(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right),\frac{1}{4}(}{640be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/80*(16*A*b-B*a)*(b*x^3+a)^(3/2)*(e*x)^(1/2)/b/e+1/8*B*(b*x^3+a)^(5/2)*(e
*x)^(1/2)/b/e+9/320*a*(16*A*b-B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+9/640*3
^(3/4)*a^(5/3)*(16*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(
1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^
(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1
-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1
/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))^2)^(1/2)/b/e/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x
)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.531.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{x\sqrt{a+bx^3}\left(B(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}+a(16Ab-aB)\,\text{Hypergeometric2F1}\left(-\right.\right.}{8b\sqrt{ex}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/Sqrt[e*x],x]
```

output
```
(x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a] + a*(16*A*b - a*B)
*Hypergeometric2F1[-3/2, 1/6, 7/6, -((b*x^3)/a)]))/(8*b*Sqrt[e*x]*Sqrt[1 +
 (b*x^3)/a])
```

3.531.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {959, 811, 811, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}dx$$

$$\Big\downarrow \; 959$$

$$\frac{(16Ab-aB)\int \frac{(bx^3+a)^{3/2}}{\sqrt{ex}}dx}{16b}+\frac{B\sqrt{ex}(a+bx^3)^{5/2}}{8be}$$

$$\Big\downarrow \; 811$$

$$\frac{(16Ab-aB)\left(\frac{9}{10}a\int \frac{\sqrt{bx^3+a}}{\sqrt{ex}}dx+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)}{16b}+\frac{B\sqrt{ex}(a+bx^3)^{5/2}}{8be}$$

$$\Big\downarrow \; 811$$

$$\frac{(16Ab-aB)\left(\frac{9}{10}a\left(\frac{3}{4}a\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)}{16b}+\frac{B\sqrt{ex}(a+bx^3)^{5/2}}{8be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(16Ab-aB)\left(\frac{9}{10}a\left(\frac{3a\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2e}+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5e}\right)}{16b}+\frac{B\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8be}$$

$$\Big\downarrow \text{766}$$

$$\frac{(16Ab-aB)\left(\frac{9}{10}a\left(\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right.\right.\right.}{4e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{16b}$$

$$\frac{B\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8be}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/Sqrt[e*x],x]
```

output
```
(B*Sqrt[e*x]*(a + b*x^3)^(5/2))/(8*b*e) + ((16*A*b - a*B)*((Sqrt[e*x]*(a +
 b*x^3)^(3/2))/(5*e) + (9*a*((Sqrt[e*x]*Sqrt[a + b*x^3])/(2*e) + (3^(3/4)*
a^(2/3)*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^
(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*
EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 +
 Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/
3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b
*x^3])))/10))/(16*b)
```

3.531.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

3.531. $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx$

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.531.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.80 (sec) , antiderivative size = 768, normalized size of antiderivative = 2.37

method	result
risch	$\frac{\left(40b^2Bx^6+64Ab^2x^3+76Babx^3+208abA+27a^2B\right)x\sqrt{bx^3+a}}{320b\sqrt{ex}}+\frac{27a^2(16Ab-Ba)\left(\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}\right)}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/320/b*(40*B*b^2*x^6+64*A*b^2*x^3+76*B*a*b*x^3+208*A*a*b+27*B*a^2)*x*(b*x
^3+a)^(1/2)/(e*x)^(1/2)+27/320*a^2*(16*A*b-B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*
(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)
^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e
*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*Ell
ipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2
),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

3.531.5 Fricas [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(e*x)
, x)
```

3.531.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{Aa^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{1}{6}\right) {}_2F_1\left(\begin{array}{c|c} -\frac{1}{2},\frac{1}{6} & \frac{bx^3e^{i\pi}}{a} \\ \frac{7}{6} & \end{array}\right)}{3\sqrt{e}\Gamma\left(\frac{7}{6}\right)} + \frac{A\sqrt{a}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right) {}_2F_1\left(\begin{array}{c|c} -\frac{1}{2},\frac{7}{6} & \frac{bx^3e^{i\pi}}{a} \\ \frac{13}{6} & \end{array}\right)}{3\sqrt{e}\Gamma\left(\frac{13}{6}\right)} + \frac{Ba^{\frac{3}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right) {}_2F_1\left(\begin{array}{c|c} -\frac{1}{2},\frac{7}{6} & \frac{bx^3e^{i\pi}}{a} \\ \frac{13}{6} & \end{array}\right)}{3\sqrt{e}\Gamma\left(\frac{13}{6}\right)} + \frac{B\sqrt{a}bx^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right) {}_2F_1\left(\begin{array}{c|c} -\frac{1}{2},\frac{13}{6} & \frac{bx^3e^{i\pi}}{a} \\ \frac{19}{6} & \end{array}\right)}{3\sqrt{e}\Gamma\left(\frac{19}{6}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(1/2),x)
```

output
```
A*a**(3/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(e)*gamma(7/6)) + A*sqrt(a)*b*x**(7/2)*gamma(7/6)*hyper((-
1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6)) + B*
a**(3/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(e)*gamma(13/6)) + B*sqrt(a)*b*x**(13/2)*gamma(13/6)*hyper
((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6))
```

3.531.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/sqrt(e*x), x)
```

3.531.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/sqrt(e*x), x)
```

3.531.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{\sqrt{e\,x}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(1/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(1/2), x)
```

3.532 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx$

3.532.1 Optimal result

Integrand size = 26, antiderivative size = 614

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx=\frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4}$$

$$+\frac{27\left(1+\sqrt{3}\right)a(14Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{112b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$+\frac{(14Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7ae^4}-\frac{2A\left(a+bx^3\right)^{5/2}}{ae\sqrt{ex}}$$

$$-\frac{27\sqrt[4]{3}a^{4/3}(14Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\Bigg(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)}{112b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{9\ 3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}(14Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Bigg(\arccos\Bigg(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)}}{224b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
1/7*(14*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/a/e^4-2*A*(b*x^3+a)^(5/2)/a/e
/(e*x)^(1/2)+9/56*(14*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/e^4+27/112*a*(1
4*A*b+B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/3)/e^2/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))-27/112*3^(1/4)*a^(4/3)*(14*A*b+B*a)*(a^(1/3)+b^(1/3)*
x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Ellip
ticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^
2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a
)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^
(1/2)-9/224*3^(3/4)*a^(4/3)*(14*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)
+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(
1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1
/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2
)
```

3.532.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.14

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{2x\sqrt{a+bx^3}\left(-\frac{5A(a+bx^3)^2}{a}+\frac{(14Ab+aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5(ex)^{3/2}}$$

input

```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

output

```
(2*x*Sqrt[a + b*x^3]*((-5*A*(a + b*x^3)^2)/a + ((14*A*b + a*B)*x^3*Hyperge
ometric2F1[-3/2, 5/6, 11/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*(e*x)^
(3/2))
```

3.532.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {955, 811, 811, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(aB+14Ab)\int (ex)^{3/2}(bx^3+a)^{3/2}dx}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{811}$$

$$\frac{(aB+14Ab)\left(\frac{9}{14}a\int (ex)^{3/2}\sqrt{bx^3+a}dx+\frac{(ex)^{5/2}(a+bx^3)^{3/2}}{7e}\right)}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{811}$$

$$\frac{(aB+14Ab)\left(\frac{9}{14}a\left(\frac{3}{8}a\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(a+bx^3)^{3/2}}{7e}\right)}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{851}$$

$$\frac{(aB+14Ab)\left(\frac{9}{14}a\left(\frac{3a\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(a+bx^3)^{3/2}}{7e}\right)}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{837}$$

$$\frac{(aB+14Ab)\left(\frac{9}{14}a\left(\frac{3a\left(-\frac{(1-\sqrt{3})a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+(1-\sqrt{3})a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(a+bx^3)}{7e}}{ae^3}$$

$$\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{25}$$

$$\frac{(aB+14Ab)\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right.}{ae^3}$$

$$\frac{2A\left(a+bx^3\right)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \; 766$$

$$\frac{(aB+14Ab)\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\text{arc}\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right.}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right.}}}\right.\right.}{4e}\right.\right.}{ae^3}$$

$$\frac{2A\left(a+bx^3\right)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \; 2420$$

3.532. $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx$

$$\frac{(aB+14Ab)\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+}\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}}\right.}{}\right.\right.}{}$$

$$\frac{2A\left(a+bx^3\right)^{5/2}}{ae\sqrt{ex}}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

3.532. $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx$

output
```
(-2*A*(a + b*x^3)^(5/2))/(a*e*Sqrt[e*x]) + ((14*A*b + a*B)*(((e*x)^(5/2)*(
a + b*x^3)^(3/2))/(7*e) + (9*a*(((e*x)^(5/2)*Sqrt[a + b*x^3])/(4*e) + (3*a
*((((1 + Sqrt[3])*e^3*Sqrt[e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3]
)*b^(1/3)*e*x) - (3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sq
rt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1
 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^
(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(Sq
rt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1
/3)*e*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[
e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x +
 b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[Arc
Cos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(
1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3
)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*
x^3])))/(4*e)))/14))/(a*e^3)
```

3.532.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.532.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.05 (sec) , antiderivative size = 1140, normalized size of antiderivative = 1.86

method	result	size
risch	Expression too large to display	1140
elliptic	Expression too large to display	1233
default	Expression too large to display	6142

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/56*(b*x^3+a)^(1/2)*(-8*B*b*x^6-14*A*b*x^3-17*B*a*x^3+112*A*a)/e/(e*x)^(
1/2)+27/112*a*(14*A*b+B*a)*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(
-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))
^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a
*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(...
```

3.532.5 Fricas [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^2*
x^2), x)
```

3.532.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.87 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.33

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)} + \frac{A\sqrt{a}bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)} + \frac{Ba^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)} + \frac{B\sqrt{a}bx^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(3/2),x)
```

output
```
A*a**(3/2)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/
a)/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + A*sqrt(a)*b*x**(5/2)*gamma(5/6)*hyper
((-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6))
+ B*a**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*exp_po
lar(I*pi)/a)/(3*e**(3/2)*gamma(11/6)) + B*sqrt(a)*b*x**(11/2)*gamma(11/6)*
hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(1
7/6))
```

3.532.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(3/2), x)
```

3.532.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(3/2), x)
```

3.532.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx=\int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{(e\,x)^{3/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(3/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(3/2), x)
```

3.533 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx$

3.533.1 Optimal result

Integrand size = 26, antiderivative size = 152

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx = \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}} + \frac{a(4Ab+aB)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{4\sqrt{b}e^{5/2}}$$

output
```
1/6*(4*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/a/e^4-2/3*A*(b*x^3+a)^(5/2)/a/
e/(e*x)^(3/2)+1/4*a*(4*A*b+B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3
+a)^(1/2))/e^(5/2)/b^(1/2)+1/4*(4*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/e^4
```

3.533.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.66

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx = \frac{x\Big(\sqrt{b}\sqrt{a+bx^3}\left(-8aA+4Abx^3+5aBx^3+2bBx^6\right)+3a(4Ab+aB)x^{3/2}\log}{12\sqrt{b}(ex)^{5/2}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

output
```
(x*(Sqrt[b]*Sqrt[a + b*x^3]*(-8*a*A + 4*A*b*x^3 + 5*a*B*x^3 + 2*b*B*x^6) +
 3*a*(4*A*b + a*B)*x^(3/2)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(12*Sq
rt[b]*(e*x)^(5/2))
```

3.533.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {955, 811, 811, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{5/2}}dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{(aB+4Ab)\int \sqrt{ex}\left(bx^3+a\right)^{3/2}dx}{ae^3}-\frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\int \sqrt{ex}\sqrt{bx^3+a}dx+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(\frac{a\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(\frac{a\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(\frac{a\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow 219$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(\frac{a\sqrt{e}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3\sqrt{b}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}(a+bx^3)^{3/2}}{6e}\right)}{ae^3}-\frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}}$$

input
```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

output
```
(-2*A*(a + b*x^3)^(5/2))/(3*a*e*(e*x)^(3/2)) + ((4*A*b + a*B)*(((e*x)^(3/2
)*(a + b*x^3)^(3/2))/(6*e) + (3*a*(((e*x)^(3/2)*Sqrt[a + b*x^3])/(3*e) + (
a*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3
*Sqrt[b])))/4))/(a*e^3)
```

3.533.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.533.4 Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-2 b B\,x^{6}-4 A b\,x^{3}-5 B a\,x^{3}+8 A a\right)}{12 x\,e^{2} \sqrt{e x}}+\frac{a \left(4 A b +B a \right) \operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) \sqrt{\left(b\,x^{3}+a \right) e x}}{4 \sqrt{b e}\,e^{2} \sqrt{e x}\,\sqrt{b\,x^{3}+a}}$
default	$\frac{\sqrt{b\,x^{3}+a}\,\left(2 B \sqrt{\left(b\,x^{3}+a \right) e x}\,\sqrt{b e}\,b\,x^{6}+12 A \operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) a b e\,x^{2}+4 A \sqrt{\left(b\,x^{3}+a \right) e x}\,\sqrt{b e}\,b\,x^{3}+3 B \operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right)\right.}{12 x\,e^{2} \sqrt{e x}\,\sqrt{\left(b\,x^{3}+a \right) e x}\,\sqrt{b e}}$
elliptic	Expression too large to display

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x^3+a)^(1/2)*(-2*B*b*x^6-4*A*b*x^3-5*B*a*x^3+8*A*a)/x/e^2/(e*x)^(
1/2)+1/4*a*(4*A*b+B*a)/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)
^(1/2))/e^2*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

3.533.5 Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \Bigg[\frac{3\,(Ba^2+4\,Aab)\sqrt{be}\,x^2\log\Big(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,bx^4+ax)\sqrt{bx}}{48\,be}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(B*a^2 + 4*A*a*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 -
a^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(2*B*b^
2*x^6 + (5*B*a*b + 4*A*b^2)*x^3 - 8*A*a*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e
^3*x^2), -1/24*(3*(B*a^2 + 4*A*a*b)*sqrt(-b*e)*x^2*arctan(2*sqrt(b*x^3 + a
)*sqrt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) - 2*(2*B*b^2*x^6 + (5*B*a*b +
4*A*b^2)*x^3 - 8*A*a*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

3.533.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(138) = 276.

Time = 17.82 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.90

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = -\frac{2Aa^{\frac{3}{2}}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{A\sqrt{a}bx^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} - \frac{2A\sqrt{a}bx^{\frac{3}{2}}}{3e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{Aa\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{e^{\frac{5}{2}}} + \frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} + \frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}}{12e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{B\sqrt{a}bx^{\frac{9}{2}}}{4e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{4\sqrt{b}e^{\frac{5}{2}}} + \frac{Bb^2x^{\frac{15}{2}}}{6\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(5/2),x)
```

output
```
-2*A*a**(3/2)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b*x**(3
/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) - 2*A*sqrt(a)*b*x**(3/2)/(3*e**(5/2)*s
qrt(1 + b*x**3/a)) + A*a*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/e**(5/2)
+ B*a**(3/2)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a**(3/2)*x**(3/2
)/(12*e**(5/2)*sqrt(1 + b*x**3/a)) + B*sqrt(a)*b*x**(9/2)/(4*e**(5/2)*sqrt
(1 + b*x**3/a)) + B*a**2*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*sqrt(b)*e**(5/
2)) + B*b**2*x**(15/2)/(6*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a))
```

3.533.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(5/2), x)
```

3.533.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx=\text{Exception raised: NotImplementedError}$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: Recu
rsive assumption sageVARa>=(-sageVARb*sageVARe/(sageVARe^4*t_nostep^6)) ig
nored2/sageVARe^3*(120*sageVARb^5*sageVARe^3*sageVARB/1440/sageVARb^4/sage
VARe^9*sqrt(sageVAR
```

3.533.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{(e\,x)^{5/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(5/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(5/2), x)
```

3.534 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx$

3.534.1 Optimal result

Integrand size = 26, antiderivative size = 314

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{9(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{20e^4}$$

$$+\frac{(2Ab+aB)\sqrt{ex}(a+bx^3)^{3/2}}{5ae^4}-\frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}}$$

$$+\frac{9\,3^{3/4}a^{2/3}(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2}{40e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/5*A*(b*x^3+a)^(5/2)/a/e/(e*x)^(5/2)+1/5*(2*A*b+B*a)*(b*x^3+a)^(3/2)*(e*
x)^(1/2)/a/e^4+9/20*(2*A*b+B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/e^4+9/40*3^(3/
4)*a^(2/3)*(2*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2))
)^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)
))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1
/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*
(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1
+3^(1/2)))^2)^(1/2)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^
(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.534.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.27

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{7/2}}\,dx=\frac{2x\sqrt{a+bx^3}\left(-\frac{A\left(a+bx^3\right)^2}{a}+\frac{5(2Ab+aB)x^3\operatorname{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{6},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5(ex)^{7/2}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(-((A*(a + b*x^3)^2)/a) + (5*(2*A*b + a*B)*x^3*Hyperg
eometric2F1[-3/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*(e*x)^
(7/2))
```

3.534.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {955, 811, 811, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{7/2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(aB+2Ab)\int \frac{\left(bx^3+a\right)^{3/2}}{\sqrt{ex}}dx}{ae^3}-\frac{2A\left(a+bx^3\right)^{5/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{811}$$

$$\frac{(aB+2Ab)\left(\frac{9}{10}a\int \frac{\sqrt{bx^3+a}}{\sqrt{ex}}dx+\frac{\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{5/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{811}$$

$$\frac{(aB+2Ab)\left(\frac{9}{10}a\left(\frac{3}{4}a\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{5/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow 851$$

$$\frac{(aB+2Ab)\left(\frac{9}{10}a\left(\frac{3a\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2e}+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{5/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow 766$$

$$\frac{(aB+2Ab)\left(\frac{9}{10}a\left(\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.\right.}{ae^3}$$

$$\frac{2A\left(a+bx^3\right)^{5/2}}{5ae(ex)^{5/2}}$$

input

```
Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

output

```
(-2*A*(a + b*x^3)^(5/2))/(5*a*e*(e*x)^(5/2)) + ((2*A*b + a*B)*((Sqrt[e*x]*
(a + b*x^3)^(3/2))/(5*e) + (9*a*((Sqrt[e*x]*Sqrt[a + b*x^3])/(2*e) + (3^(3
/4)*a^(2/3)*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3
)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)
^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e +
(1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*e^2*Sqrt[(b^(1/3)*e*x*(a
^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a
 + b*x^3])))/10))/(a*e^3)
```

3.534.3.1 Defintions of rubi rules used

rule 766

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

3.534. $\int\frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{7/2}}\,dx$

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.534.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.77 (sec) , antiderivative size = 759, normalized size of antiderivative = 2.42

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-4 b B\,x^{6}-10 A b\,x^{3}-13 B a\,x^{3}+8 A a\right)}{20 x^{2} e^{3} \sqrt{e x}}+\frac{27 a \left(2 A b+B a\right) \left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{\frac{\left(-\frac{3 \left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a\,b\right.}{2 b}\right.}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \left(\right.}}}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
-1/20*(b*x^3+a)^(1/2)*(-4*B*b*x^6-10*A*b*x^3-13*B*a*x^3+8*A*a)/x^2/e^3/(e*
x)^(1/2)+27/20*a*(2*A*b+B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1
/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2
)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/e^3*((b*x^3+
a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

3.534.5 Fricas [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input

```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="fricas")
```

output

```
integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^4*
x^4), x)
```

3.534.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 23.15 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)} + \frac{A\sqrt{a}b\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)} + \frac{Ba^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)} + \frac{B\sqrt{a}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(7/2),x)
```

output
```
A*a**(3/2)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/
a)/(3*e**(7/2)*x**(5/2)*gamma(1/6)) + A*sqrt(a)*b*sqrt(x)*gamma(1/6)*hyper
((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6)) +
B*a**(3/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(
I*pi)/a)/(3*e**(7/2)*gamma(7/6)) + B*sqrt(a)*b*x**(7/2)*gamma(7/6)*hyper((
-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(13/6))
```

3.534.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(7/2), x)
```

3.534.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(7/2), x)
```

3.534.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx=\int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{(e\,x)^{7/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(7/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(7/2), x)
```

3.535 $\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx$

3.535.1 Optimal result

Integrand size = 26, antiderivative size = 241

$$\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be}+\frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be}+\frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}-\frac{a^4(10Ab-3aB)e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{384b^{5/2}}$$

output

```
1/144*a*(10*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(3/2)/b/e+1/120*(10*A*b-3*B*a
)*(e*x)^(9/2)*(b*x^3+a)^(5/2)/b/e+1/15*B*(e*x)^(9/2)*(b*x^3+a)^(7/2)/b/e-1
/384*a^4*(10*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3
+a)^(1/2))/b^(5/2)+1/384*a^3*(10*A*b-3*B*a)*e^2*(e*x)^(3/2)*(b*x^3+a)^(1/2
)/b^2+1/192*a^2*(10*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(1/2)/b/e
```

3.535.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.68

$$\int (ex)^{7/2} (a+bx^3)^{5/2} \left(A+Bx^3\right) dx = \frac{e^3\sqrt{ex}\left(\sqrt{b}x^{3/2}\sqrt{a+bx^3}(-45a^4B+30a^3b(5A+Bx^3)+96b^4x^9(5A+4Bx^3)+16\right.}{}$$

input
```
Integrate[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(-45*a^4*B + 30*a^3*b*(5*A
 + B*x^3) + 96*b^4*x^9*(5*A + 4*B*x^3) + 16*a*b^3*x^6*(85*A + 63*B*x^3) +
4*a^2*b^2*x^3*(295*A + 186*B*x^3)) + 15*a^4*(-10*A*b + 3*a*B)*Log[Sqrt[b]*
x^(3/2) + Sqrt[a + b*x^3]]))/(5760*b^(5/2)*Sqrt[x])
```

3.535.3 Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {959, 811, 811, 811, 843, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{7/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(10Ab-3aB)\int (ex)^{7/2} \left(bx^3+a\right)^{5/2}dx}{10b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\int (ex)^{7/2}\left(bx^3+a\right)^{3/2}dx+\frac{(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12e}\right)}{10b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\int (ex)^{7/2}\sqrt{bx^3+a}dx+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12e}\right)}{10b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\int \frac{(ex)^{7/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12e}\right)}{10b}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

$$\Big\downarrow \text{843}$$

$$\frac{(10Ab-3aB)\Bigg(\frac{5}{8}a\Bigg(\frac{1}{2}a\Bigg(\frac{1}{4}a\Bigg(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^3\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{2b}\Bigg)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\Bigg)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\Bigg)+\frac{(ex)^{9/2}(a+}{12e}}{10b}$$

$$\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(10Ab-3aB)\Bigg(\frac{5}{8}a\Bigg(\frac{1}{2}a\Bigg(\frac{1}{4}a\Bigg(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{b}\Bigg)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\Bigg)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\Bigg)+\frac{(ex)^{9/2}(a}{12}}{10b}$$

$$\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

$$\Big\downarrow \text{807}$$

$$\frac{(10Ab-3aB)\Bigg(\frac{5}{8}a\Bigg(\frac{1}{2}a\Bigg(\frac{1}{4}a\Bigg(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3b}\Bigg)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\Bigg)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\Bigg)+(ex)}{10b}$$

$$\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

$$\Big\downarrow \text{224}$$

3.535. $\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx$

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int\frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3b}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)+}{10b}$$

$$\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

$$\Big\downarrow \; 219$$

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}\right)+\frac{(ex)^{9/2}\sqrt{a+bx^3}}{6e}\right)+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9e}\right)}{10b}$$

$$\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

input
```
Int[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(B*(e*x)^(9/2)*(a + b*x^3)^(7/2))/(15*b*e) + ((10*A*b - 3*a*B)*(((e*x)^(9/
2)*(a + b*x^3)^(5/2))/(12*e) + (5*a*(((e*x)^(9/2)*(a + b*x^3)^(3/2))/(9*e)
 + (a*(((e*x)^(9/2)*Sqrt[a + b*x^3])/(6*e) + (a*((e^2*(e*x)^(3/2)*Sqrt[a +
 b*x^3])/(3*b) - (a*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a
+ (b*x)/e^2])])/(3*b^(3/2))))/4))/2))/8))/(10*b)
```

3.535.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

3.535. $\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx$

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.535.4 Maple [A] (verified)

Time = 4.98 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.77

method	result
risch	$\frac{x^{2}\left(384 B\,b^{4} x^{12}+480 A\,b^{4} x^{9}+1008 B a\,b^{3} x^{9}+1360 A a\,b^{3} x^{6}+744 B\,a^{2} b^{2} x^{6}+1180 A\,a^{2} b^{2} x^{3}+30 B\,a^{3} b\,x^{3}+150 A\,a^{3} b-45 B\,a^{4}\right) \sqrt{b\,x^{3}+a}\,e^{4}}{5760 b^{2} \sqrt{e x}}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/5760/b^2*x^2*(384*B*b^4*x^12+480*A*b^4*x^9+1008*B*a*b^3*x^9+1360*A*a*b^3
*x^6+744*B*a^2*b^2*x^6+1180*A*a^2*b^2*x^3+30*B*a^3*b*x^3+150*A*a^3*b-45*B*
a^4)*(b*x^3+a)^(1/2)*e^4/(e*x)^(1/2)-1/384*a^4/b^2*(10*A*b-3*B*a)/(b*e)^(1
/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e^4*((b*x^3+a)*e*x)^(1/
2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

3.535.5 Fricas [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.70

$$\int (ex)^{7/2} (a+bx^3)^{5/2} (A+Bx^3)\, dx = \left[-\frac{15\,(3\,Ba^5 - 10\,Aa^4b)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6 - 8\,abex^3 - a^2e + 4\,(2\,b^2x^4 + abx)\sqrt{b}\right.}{}\right.$$

input
```
integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
[-1/23040*(15*(3*B*a^5 - 10*A*a^4*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*
b*e*x^3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b
)) - 4*(384*B*b^4*e^3*x^13 + 48*(21*B*a*b^3 + 10*A*b^4)*e^3*x^10 + 8*(93*B
*a^2*b^2 + 170*A*a*b^3)*e^3*x^7 + 10*(3*B*a^3*b + 118*A*a^2*b^2)*e^3*x^4 -
 15*(3*B*a^4 - 10*A*a^3*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/11520
*(15*(3*B*a^5 - 10*A*a^4*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e
*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(384*B*b^4*e^3*x^13 + 48*(21*B*a
*b^3 + 10*A*b^4)*e^3*x^10 + 8*(93*B*a^2*b^2 + 170*A*a*b^3)*e^3*x^7 + 10*(3
*B*a^3*b + 118*A*a^2*b^2)*e^3*x^4 - 15*(3*B*a^4 - 10*A*a^3*b)*e^3*x)*sqrt(
b*x^3 + a)*sqrt(e*x))/b^2]
```

3.535.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(216) = 432$.

Time = 49.13 (sec) , antiderivative size = 1028, normalized size of antiderivative = 4.27

$$\int (ex)^{7/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right)\, dx = \text{Too large to display}$$

input
```
integrate((e*x)**(7/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

output
```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*a**2*e**3*Piecewise((-a**2*
e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3
))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3),
True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2)/(8*b) + (e*x)**(9/2)/
4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True)) + 2*A*a*b*Piecewise((a
**3*e**6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*
x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**
3), True))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*(e*x)**(3/2)/(16*b**2)
 + a*e**3*(e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6), Ne(b/e**3, 0)), (sqrt(a)
*(e*x)**(15/2)/5, True)) + A*b**2*Piecewise((-5*a**4*e**9*Piecewise((log(2
*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a
, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/(128*b**3) + s
qrt(a + b*x**3)*(5*a**3*e**9*(e*x)**(3/2)/(128*b**3) - 5*a**2*e**6*(e*x)**
(9/2)/(192*b**2) + a*e**3*(e*x)**(15/2)/(48*b) + (e*x)**(21/2)/8), Ne(b/e*
*3, 0)), (sqrt(a)*(e*x)**(21/2)/7, True))/e**3 + B*a**2*Piecewise((a**3*e*
*6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))
/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), Tr
ue))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*(e*x)**(3/2)/(16*b**2) + a*e
**3*(e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6), Ne(b/e**3, 0)), (sqrt(a)*(e*x)
**(15/2)/5, True)) + 2*B*a*b*Piecewise((-5*a**4*e**9*Piecewise((log(2*b...
```

3.535.7 Maxima [F]

$$\int (ex)^{7/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right) \left(bx^3+a\right)^{\frac{5}{2}} (ex)^{\frac{7}{2}} \, dx$$

input
```
integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(7/2), x)
```

3.535.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(197) = 394$.

Time = 0.60 (sec) , antiderivative size = 692, normalized size of antiderivative = 2.87

$$\begin{aligned}
\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx &= \frac{1}{12}\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2\,x^3}{e}+\frac{a}{be}\right)Aa^2x|e|^2 \\
&+\frac{1}{72}\sqrt{be^4x^3+ae^4}\left(2\,e^3x^3\left(\frac{4\,x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3\,a^2}{b^2e}\right)\sqrt{ex}Ba^2x|e|^2 \\
&+\frac{1}{36}\sqrt{be^4x^3+ae^4}\left(2\,e^3x^3\left(\frac{4\,x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3\,a^2}{b^2e}\right)\sqrt{ex}Aabx|e|^2 \\
&+\frac{1}{288}\sqrt{be^4x^3+ae^4}\left(2\left(4\,e^3x^3\left(\frac{6\,x^3}{e^7}+\frac{a}{be^7}\right)-\frac{5\,a^2}{b^2e^4}\right)e^3x^3+\frac{15\,a^3}{b^3e}\right)\sqrt{ex}Babx|e|^2 \\
&+\frac{1}{576}\sqrt{be^4x^3+ae^4}\left(2\left(4\,e^3x^3\left(\frac{6\,x^3}{e^7}+\frac{a}{be^7}\right)-\frac{5\,a^2}{b^2e^4}\right)e^3x^3+\frac{15\,a^3}{b^3e}\right)\sqrt{ex}Ab^2x|e|^2 \\
&+\frac{1}{5760}\sqrt{be^4x^3+ae^4}\left(2\left(4\left(6\,e^3x^3\left(\frac{8\,x^3}{e^{10}}+\frac{a}{be^{10}}\right)-\frac{7\,a^2}{b^2e^7}\right)e^3x^3+\frac{35\,a^3}{b^3e^4}\right)e^3x^3-\frac{105\,a^4}{b^4e}\right)\sqrt{ex}Bb^2x|e|^2 \\
&-\frac{\left(9\,B^2a^{10}e-60\,ABa^9be+100\,A^2a^8b^2e\right)^2e^5\log\left(\left|-\left(3\,\sqrt{ex}Ba^5e^2x-10\,\sqrt{ex}Aa^4be^2x\right)\sqrt{be}+\sqrt{9\,B^2a^{11}e^6-}\right.\right.}{384\,\sqrt{be}b^2|9\,B^2a^{10}e-60\,ABa^9be+100\,A^2a^8b^2e||-3}
\end{aligned}$$

input
```
integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))*A*a^2*x*abs(e)^
2 + 1/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^
2/(b^2*e))*sqrt(e*x)*B*a^2*x*abs(e)^2 + 1/36*sqrt(b*e^4*x^3 + a*e^4)*(2*e^
3*x^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b^2*e))*sqrt(e*x)*A*a*b*x*abs(e)^2
+ 1/288*sqrt(b*e^4*x^3 + a*e^4)*(2*(4*e^3*x^3*(6*x^3/e^7 + a/(b*e^7)) - 5*
a^2/(b^2*e^4))*e^3*x^3 + 15*a^3/(b^3*e))*sqrt(e*x)*B*a*b*x*abs(e)^2 + 1/57
6*sqrt(b*e^4*x^3 + a*e^4)*(2*(4*e^3*x^3*(6*x^3/e^7 + a/(b*e^7)) - 5*a^2/(b
^2*e^4))*e^3*x^3 + 15*a^3/(b^3*e))*sqrt(e*x)*A*b^2*x*abs(e)^2 + 1/5760*sqr
t(b*e^4*x^3 + a*e^4)*(2*(4*(6*e^3*x^3*(8*x^3/e^10 + a/(b*e^10)) - 7*a^2/(b
^2*e^7))*e^3*x^3 + 35*a^3/(b^3*e^4))*e^3*x^3 - 105*a^4/(b^4*e))*sqrt(e*x)*
B*b^2*x*abs(e)^2 - 1/384*(9*B^2*a^10*e - 60*A*B*a^9*b*e + 100*A^2*a^8*b^2*
e)^2*e^5*log(abs(-(3*sqrt(e*x)*B*a^5*e^2*x - 10*sqrt(e*x)*A*a^4*b*e^2*x)*s
qrt(b*e) + sqrt(9*B^2*a^11*e^6 - 60*A*B*a^10*b*e^6 + 100*A^2*a^9*b^2*e^6 +
 (3*sqrt(e*x)*B*a^5*e^2*x - 10*sqrt(e*x)*A*a^4*b*e^2*x)^2*b*e)))/(sqrt(b*e
)*b^2*abs(9*B^2*a^10*e - 60*A*B*a^9*b*e + 100*A^2*a^8*b^2*e)*abs(-3*B*a^5
+ 10*A*a^4*b)*abs(e)^2)
```

3.535.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{7/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}\,{\left(b\,x^3+a\right)}^{5/2} \,dx$$

input

```
int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(5/2),x)
```

output

```
int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(5/2), x)
```

3.536 $\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx$

3.536.1 Optimal result

Integrand size = 26, antiderivative size = 404

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{81a^3(4Ab-aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{5632b^2}$$
$$+\frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be}+\frac{15a(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{704be}$$
$$+\frac{(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{44be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$
$$-\frac{27\,3^{3/4}a^{11/3}(4Ab-aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\ \text{EllipticF}\Bigg(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),}{11264b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
15/704*a*(4*A*b-B*a)*(e*x)^(7/2)*(b*x^3+a)^(3/2)/b/e+1/44*(4*A*b-B*a)*(e*x
)^(7/2)*(b*x^3+a)^(5/2)/b/e+1/14*B*(e*x)^(7/2)*(b*x^3+a)^(7/2)/b/e+27/1408
*a^2*(4*A*b-B*a)*(e*x)^(7/2)*(b*x^3+a)^(1/2)/b/e+81/5632*a^3*(4*A*b-B*a)*e
^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2-27/11264*3^(3/4)*a^(11/3)*(4*A*b-B*a)*e
^2*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*
x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/
b^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2)
```

3.536.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.23 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.29

$$\int (ex)^{5/2} (a +bx^3)^{5/2} (A+Bx^3)\ dx = \frac{e^2\sqrt{ex}\sqrt{a+bx^3}\left(-(a+bx^3)^3\sqrt{1+\frac{bx^3}{a}}(-28Ab+7aB-22bBx^3)+7a^3(-4Ab+\right.}{308b^2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^(5/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)^3*Sqrt[1 + (b*x^3)/a]*(-28*A
*b + 7*a*B - 22*b*B*x^3)) + 7*a^3*(-4*A*b + a*B)*Hypergeometric2F1[-5/2, 1
/6, 7/6, -((b*x^3)/a)]))/(308*b^2*Sqrt[1 + (b*x^3)/a])
```

3.536.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {959, 811, 811, 811, 843, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx$$

$$\Big\downarrow \ 959$$

$$\frac{(4Ab-aB)\int (ex)^{5/2}\left(bx^3+a\right)^{5/2}dx}{4b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

$$\Big\downarrow \ 811$$

$$\frac{(4Ab-aB)\left(\frac{15}{22}a\int (ex)^{5/2}\left(bx^3+a\right)^{3/2}dx+\frac{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11e}\right)}{4b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

$$\Big\downarrow \ 811$$

$$\frac{(4Ab-aB)\left(\frac{15}{22}a\left(\frac{9}{16}a\int (ex)^{5/2}\sqrt{bx^3+a}dx+\frac{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8e}\right)+\frac{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11e}\right)}{4b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

$$\Big\downarrow \; 811$$

$$\frac{(4Ab-aB)\left(\frac{15}{22}a\left(\frac{9}{16}a\left(\frac{3}{10}a\int \frac{(ex)^{5/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)+\frac{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8e}\right)+\frac{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11e}\right)}{4b}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

$$\Big\downarrow \; 843$$

$$\frac{(4Ab-aB)\left(\frac{15}{22}a\left(\frac{9}{16}a\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^3\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{4b}\right)+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)+\frac{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8e}\right)+\frac{(ex)^{7/2}(a+}{11e}}{4b}$$

$$\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

$$\Big\downarrow \; 851$$

$$\frac{(4Ab-aB)\left(\frac{15}{22}a\left(\frac{9}{16}a\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b}\right)+\frac{(ex)^{7/2}\sqrt{a+bx^3}}{5e}\right)+\frac{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8e}\right)+\frac{(ex)^{7/2}(a+b}{11e}}{4b}$$

$$\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

$$\Big\downarrow \; 766$$

$$\frac{(4Ab-aB)\left(\frac{15}{22}a\left(\frac{9}{16}a\left(\frac{3}{10}a\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{a^{2/3}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{(1-\sqrt{}}{(1+\sqrt{}}\right.\right.}{4\sqrt[4]{3}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.\right.\right.\right.}{4b}$$

$$\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

input
```
Int[(e*x)^(5/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

3.536. $\int (ex)^{5/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)dx$

output

```
(B*(e*x)^(7/2)*(a + b*x^3)^(7/2))/(14*b*e) + ((4*A*b - a*B)*(((e*x)^(7/2)*
(a + b*x^3)^(5/2))/(11*e) + (15*a*(((e*x)^(7/2)*(a + b*x^3)^(3/2))/(8*e) +
 (9*a*(((e*x)^(7/2)*Sqrt[a + b*x^3])/(5*e) + (3*a*((e^2*Sqrt[e*x]*Sqrt[a +
 b*x^3])/(2*b) - (a^(2/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2
/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[
3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*
x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*
b*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*
b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/10))/16))/22))/(4*b)
```

3.536.3.1 Defintions of rubi rules used

rule 766

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

3.536. $\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx$

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.536.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.89 (sec) , antiderivative size = 825, normalized size of antiderivative = 2.04

method	result	size
risch	Expression too large to display	825
elliptic	Expression too large to display	1134
default	Expression too large to display	5063

input
```
int((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/39424/b^2*(2816*B*b^4*x^12+3584*A*b^4*x^9+7552*B*a*b^3*x^9+10528*A*a*b^3
*x^6+5816*B*a^2*b^2*x^6+9968*A*a^2*b^2*x^3+324*B*a^3*b*x^3+2268*A*a^3*b-56
7*B*a^4)*x*(b*x^3+a)^(1/2)*e^3/(e*x)^(1/2)-81/5632*a^4/b*(4*A*b-B*a)*(1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*
(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*
(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))*e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^
(1/2)
```

3.536.5 Fricas [F]

$$\int (ex)^{5/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}\,dx$$

input

```
integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

output

```
integral((B*b^2*e^2*x^11 + (2*B*a*b + A*b^2)*e^2*x^8 + (B*a^2 + 2*A*a*b)*e
^2*x^5 + A*a^2*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

3.536.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 150.12 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.76

$$\begin{aligned}\int (ex)^{5/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx &= \frac{Aa^{\frac{5}{2}}e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{6}\right)}\\ &+\frac{2Aa^{\frac{3}{2}}be^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}+\frac{A\sqrt{a}b^2e^{\frac{5}{2}}x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{19}{6}\\ \frac{25}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{25}{6}\right)}\\ &+\frac{Ba^{\frac{5}{2}}e^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}+\frac{2Ba^{\frac{3}{2}}be^{\frac{5}{2}}x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{19}{6}\\ \frac{25}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{25}{6}\right)}\\ &+\frac{B\sqrt{a}b^2e^{\frac{5}{2}}x^{\frac{25}{2}}\Gamma\left(\frac{25}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{25}{6}\\ \frac{31}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{31}{6}\right)}\end{aligned}$$

input

```
integrate((e*x)**(5/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

output
```
A*a**(5/2)*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(13/6)) + 2*A*a**(3/2)*b*e**(5/2)*x**(13/2)*ga
mma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(
19/6)) + A*sqrt(a)*b**2*e**(5/2)*x**(19/2)*gamma(19/6)*hyper((-1/2, 19/6),
 (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(25/6)) + B*a**(5/2)*e**(5/2)*
x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(19/6)) + 2*B*a**(3/2)*b*e**(5/2)*x**(19/2)*gamma(19/6)*hyper((
-1/2, 19/6), (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(25/6)) + B*sqrt(a
)*b**2*e**(5/2)*x**(25/2)*gamma(25/6)*hyper((-1/2, 25/6), (31/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(31/6))
```

3.536.7 Maxima [F]

$$\int (ex)^{5/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(5/2), x)
```

3.536.8 Giac [F]

$$\int (ex)^{5/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(5/2), x)
```

3.536.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}\,{\left(b\,x^3+a\right)}^{5/2} \,dx$$

input
```
int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(5/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(5/2), x)
```

3.537 $\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx$

3.537.1 Optimal result

Integrand size = 26, antiderivative size = 661

$$\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be}$$

$$+\frac{81\left(1+\sqrt{3}\right)a^3(26Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{11648b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be}$$

$$+\frac{(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{260be}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be}$$

$$-\frac{81\sqrt[4]{3}a^{10/3}(26Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\right.}{11648b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\ 3^{3/4}\left(1-\sqrt{3}\right)a^{10/3}(26Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+(1-}{\sqrt[3]{a}+(1-}\right.\right.}{23296b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
3/728*a*(26*A*b-5*B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/b/e+1/260*(26*A*b-5*B*a
)*(e*x)^(5/2)*(b*x^3+a)^(5/2)/b/e+1/13*B*(e*x)^(5/2)*(b*x^3+a)^(7/2)/b/e+2
7/5824*a^2*(26*A*b-5*B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+81/11648*a^3*(26
*A*b-5*B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))-81/11648*3^(1/4)*a^(10/3)*(26*A*b-5*B*a)*e*(a^(1/3)+b^
(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))
*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1
/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3
+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2
)^(1/2)-27/23296*3^(3/4)*a^(10/3)*(26*A*b-5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a
^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(
a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((
1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3
+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2
)^(1/2)
```

3.537.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.15

$$\int (ex)^{3/2} (a+bx^3)^{5/2} (A+Bx^3)\, dx = \frac{x(ex)^{3/2}\sqrt{a+bx^3}\left(5B(a+bx^3)^3\sqrt{1+\frac{bx^3}{a}}+a^2(26Ab-5aB)\,\text{Hypergeometric2F}\right.}{65b\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(e*x)^(3/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(5*B*(a + b*x^3)^3*Sqrt[1 + (b*x^3)/a] + a^
2*(26*A*b - 5*a*B)*Hypergeometric2F1[-5/2, 5/6, 11/6, -((b*x^3)/a)]))/(65*
b*Sqrt[1 + (b*x^3)/a])
```

3.537.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {959, 811, 811, 811, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(26Ab-5aB)\int (ex)^{3/2} \left(bx^3+a\right)^{5/2}dx}{26b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{7/2}}{13be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\int (ex)^{3/2} \left(bx^3+a\right)^{3/2}dx+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10e}\right)}{26b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{7/2}}{13be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\left(\frac{9}{14}a\int (ex)^{3/2}\sqrt{bx^3+a}dx+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10e}\right)}{26b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{7/2}}{13be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3}{8}a\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10e}\right)}{26b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{7/2}}{13be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7e}\right)+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10e}\right)}{26b}+\frac{B(ex)^{5/2} \left(a+bx^3\right)^{7/2}}{13be}$$

$$\Big\downarrow \text{837}$$

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int-\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(}{}\right.\right.}{26b}$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be}$$

$$\Big\downarrow 25$$

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(a+b}{7e}\right.\right.}{26b}$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be}$$

$$\Big\downarrow 766$$

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{Ellipt}}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex(\sqrt[3]{a}e-}{(\sqrt[3]{a}e+(1+\sqrt{3}}}}\right.}{4e}\right.\right.\right.}{26b}$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be}$$

$$\Big\downarrow 2420$$

3.537. $\int(ex)^{3/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx$

$$\frac{(26Ab-5aB)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}}\right.}{}\right.\right.\right.}{}$$

$$\frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be}$$

input

```
Int[(e*x)^(3/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

3.537. $\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx$

output
```
(B*(e*x)^(5/2)*(a + b*x^3)^(7/2))/(13*b*e) + ((26*A*b - 5*a*B)*(((e*x)^(5/
2)*(a + b*x^3)^(5/2))/(10*e) + (3*a*(((e*x)^(5/2)*(a + b*x^3)^(3/2))/(7*e)
 + (9*a*(((e*x)^(5/2)*Sqrt[a + b*x^3])/(4*e) + (3*a*((((1 + Sqrt[3])*e^3*S
qrt[e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) - (3^(1/
4)*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/
3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x
)^2]*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e +
 (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*(a^(1/3
)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*
x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/
3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1
/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - S
qrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3
])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^
(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(4*e)))/14))/4)
)/(26*b)
```

3.537.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

---

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.537.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.82 (sec) , antiderivative size = 1188, normalized size of antiderivative = 1.80

method	result	size
risch	Expression too large to display	1188
elliptic	Expression too large to display	1410
default	Expression too large to display	6202

input
```
int((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/29120/b*x^3*(2240*B*b^3*x^9+2912*A*b^3*x^6+6160*B*a*b^2*x^6+8944*A*a*b^2
*x^3+5000*B*a^2*b*x^3+9542*A*a^2*b+405*B*a^3)*(b*x^3+a)^(1/2)*e^2/(e*x)^(1
/2)+81/11648*a^3/b*(26*A*b-5*B*a)*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2
*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^
(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-
1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/...
```

3.537.5 Fricas [F]

$$\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

output
```
integral((B*b^2*e*x^10 + (2*B*a*b + A*b^2)*e*x^7 + (B*a^2 + 2*A*a*b)*e*x^4
 + A*a^2*e*x)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

3.537.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 52.73 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.47

$$\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx = \frac{Aa^{\frac{5}{2}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)} + \frac{2Aa^{\frac{3}{2}}be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)} + \frac{A\sqrt{a}b^2e^{\frac{3}{2}}x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{17}{6}\\ \frac{23}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{23}{6}\right)} + \frac{Ba^{\frac{5}{2}}e^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)} + \frac{2Ba^{\frac{3}{2}}be^{\frac{3}{2}}x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{17}{6}\\ \frac{23}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{23}{6}\right)} + \frac{B\sqrt{a}b^2e^{\frac{3}{2}}x^{\frac{23}{2}}\Gamma\left(\frac{23}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{23}{6}\\ \frac{29}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{29}{6}\right)}$$

input
```
integrate((e*x)**(3/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

output
```
A*a**(5/2)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(11/6)) + 2*A*a**(3/2)*b*e**(3/2)*x**(11/2)*ga
mma(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(
17/6)) + A*sqrt(a)*b**2*e**(3/2)*x**(17/2)*gamma(17/6)*hyper((-1/2, 17/6),
 (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(23/6)) + B*a**(5/2)*e**(3/2)*
x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(17/6)) + 2*B*a**(3/2)*b*e**(3/2)*x**(17/2)*gamma(17/6)*hyper((
-1/2, 17/6), (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(23/6)) + B*sqrt(a
)*b**2*e**(3/2)*x**(23/2)*gamma(23/6)*hyper((-1/2, 23/6), (29/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(29/6))
```

3.537.7 Maxima [F]

$$\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(3/2), x)
```

3.537.8 Giac [F]

$$\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(3/2), x)
```

3.537.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}\,{\left(b\,x^3+a\right)}^{5/2}\,dx$$

input
```
int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(5/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(5/2), x)
```

3.538 $\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx$

3.538.1 Optimal result

Integrand size = 26, antiderivative size = 201

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be}+\frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be}+\frac{(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{72be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}+\frac{5a^3(8Ab-aB)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{3/2}}$$

output
```
5/288*a*(8*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/b/e+1/72*(8*A*b-B*a)*(e*x)
^(3/2)*(b*x^3+a)^(5/2)/b/e+1/12*B*(e*x)^(3/2)*(b*x^3+a)^(7/2)/b/e+5/192*a^
3*(8*A*b-B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/2)
/b^(3/2)+5/192*a^2*(8*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b/e
```

3.538.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.70

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}(A+Bx^3)\,dx=\frac{\sqrt{ex}\Big(\sqrt{b}x^{3/2}\sqrt{a+bx^3}(15a^3B+16b^3x^6(4A+3Bx^3)+8ab^2x^3(26A+17Bx^3)+2a^2b(132A+5}{576b^{3/2}\sqrt{x}}$$

input
```
Integrate[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output
```
(Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(15*a^3*B + 16*b^3*x^6*(4*A +
3*B*x^3) + 8*a*b^2*x^3*(26*A + 17*B*x^3) + 2*a^2*b*(132*A + 59*B*x^3)) - 1
5*a^3*(-8*A*b + a*B)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(576*b^(3/2)
*Sqrt[x])
```

3.538.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {959, 811, 811, 811, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(8Ab-aB)\int \sqrt{ex}\left(bx^3+a\right)^{5/2}dx}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\int \sqrt{ex}\left(bx^3+a\right)^{3/2}dx+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{ex}\sqrt{bx^3+a}dx+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

$$\Big\downarrow \ 807$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

$$\Big\downarrow \ 224$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

$$\Big\downarrow \ 219$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\sqrt{e}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3\sqrt{b}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{8b}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

input

```
Int[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

output

```
(B*(e*x)^(3/2)*(a + b*x^3)^(7/2))/(12*b*e) + ((8*A*b - a*B)*(((e*x)^(3/2)*
(a + b*x^3)^(5/2))/(9*e) + (5*a*(((e*x)^(3/2)*(a + b*x^3)^(3/2))/(6*e) + (
3*a*(((e*x)^(3/2)*Sqrt[a + b*x^3])/(3*e) + (a*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*
x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*Sqrt[b])))/4))/6))/(8*b)
```

3.538. $\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx$

3.538.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.538. $\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx$

3.538.4 Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.79

method	result
risch	$\frac{x^2\left(48b^3Bx^9+64x^6b^3A+136Bx^6ab^2+208aAb^2x^3+118Ba^2bx^3+264a^2bA+15a^3B\right)\sqrt{bx^3+a}\,e}{576b\sqrt{ex}}+\frac{5a^3(8Ab-Ba)\operatorname{arctanh}\left(\frac{\sqrt{(bx^3}}{x^2\sqrt{}}\right.}{192b\sqrt{be}\sqrt{ex}\sqrt{}}$
default	$\frac{\sqrt{bx^3+a}\sqrt{ex}\left(48B\sqrt{(bx^3+a)ex}\sqrt{be}\,b^3x^{10}+64A\sqrt{(bx^3+a)ex}\sqrt{be}\,b^3x^7+136B\sqrt{(bx^3+a)ex}\sqrt{be}\,ab^2x^7+208A\sqrt{(bx^3+a)ex}\sqrt{be}\,a\right.}{}$
elliptic	Expression too large to display

input
```
int((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/576/b*x^2*(48*B*b^3*x^9+64*A*b^3*x^6+136*B*a*b^2*x^6+208*A*a*b^2*x^3+118
*B*a^2*b*x^3+264*A*a^2*b+15*B*a^3)*(b*x^3+a)^(1/2)*e/(e*x)^(1/2)+5/192*a^3
/b*(8*A*b-B*a)/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*
e*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

3.538.5 Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.61

$$\int\sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\left[-\frac{15\left(Ba^4-8\,Aa^3b\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-}{}\right.$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="fricas")
```

output

```
[-1/2304*(15*(B*a^4 - 8*A*a^3*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3
- a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(
48*B*b^3*x^10 + 8*(17*B*a*b^2 + 8*A*b^3)*x^7 + 2*(59*B*a^2*b + 104*A*a*b^2
)*x^4 + 3*(5*B*a^3 + 88*A*a^2*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b, 1/1152*(
15*(B*a^4 - 8*A*a^3*b)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*s
qrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(48*B*b^3*x^10 + 8*(17*B*a*b^2 + 8*A*b^3)
*x^7 + 2*(59*B*a^2*b + 104*A*a*b^2)*x^4 + 3*(5*B*a^3 + 88*A*a^2*b)*x)*sqrt
(b*x^3 + a)*sqrt(e*x))/b]
```

3.538.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(177) = 354$.

Time = 7.75 (sec) , antiderivative size = 896, normalized size of antiderivative = 4.46

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx = \text{Too large to display}$$

input

```
integrate((b*x**3+a)**(5/2)*(B*x**3+A)*(e*x)**(1/2),x)
```

output

```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*a**2*e**3*Piecewise((a*Piec
ewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b
/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/2
+ (e*x)**(3/2)*sqrt(a + b*x**3)/2, Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(3/2),
True)) + 2*A*a*b*Piecewise((-a**2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**
3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2
)*log((e*x)**(3/2))/sqrt(b*x**3), True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*
(e*x)**(3/2)/(8*b) + (e*x)**(9/2)/4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2
)/3, True)) + A*b**2*Piecewise((a**3*e**6*Piecewise((log(2*b*(e*x)**(3/2)/
e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(
3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/(16*b**2) + sqrt(a + b*x**3)*(
-a**2*e**6*(e*x)**(3/2)/(16*b**2) + a*e**3*(e*x)**(9/2)/(24*b) + (e*x)**(1
5/2)/6), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(15/2)/5, True))/e**3 + B*a**2*Pi
ecewise((-a**2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*
sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))
/sqrt(b*x**3), True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2)/(8*b)
+ (e*x)**(9/2)/4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True)) + 2*B*a
*b*Piecewise((a**3*e**6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e*
*3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3
/2))/sqrt(b*x**3), True))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*(e*x...
```

3.538.7 Maxima [F]

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}\sqrt{ex}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*sqrt(e*x), x)
```

3.538.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(159) = 318$.

Time = 0.52 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.94

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2\,x^3}{e}+\frac{a}{be}\right)Ba^2x|e|^2}{12\,e^3}$$
$$+\frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2\,x^3}{e}+\frac{a}{be}\right)Aabx|e|^2}{6\,e^3}$$
$$+\frac{\sqrt{be^4x^3+ae^4}\left(2\,e^3x^3\left(\frac{4\,x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3\,a^2}{b^2e}\right)\sqrt{ex}Babx|e|^2}{36\,e^3}$$
$$+\frac{\sqrt{be^4x^3+ae^4}\left(2\,e^3x^3\left(\frac{4\,x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3\,a^2}{b^2e}\right)\sqrt{ex}Ab^2x|e|^2}{72\,e^3}$$
$$+\frac{\sqrt{be^4x^3+ae^4}\left(2\left(4\,e^3x^3\left(\frac{6\,x^3}{e^7}+\frac{a}{be^7}\right)-\frac{5\,a^2}{b^2e^4}\right)e^3x^3+\frac{15\,a^3}{b^3e}\right)\sqrt{ex}Bb^2x|e|^2}{576\,e^3}$$
$$-\frac{(25\,B^2a^8+240\,ABa^7b+576\,A^2a^6b^2)e^4\log\left(\left|\left(5\,\sqrt{ex}Ba^4x+24\,\sqrt{ex}Aa^3bx\right)\sqrt{be}+\sqrt{25\,B^2a^9e^2+240\,AB}\right.\right.}{192\,\sqrt{be}b|5\,Ba^4e+24\,Aa^3be||e|^2}$$
$$-\frac{\left(\frac{ae^4\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{\sqrt{be}}-\sqrt{be^4x^3+ae^4}\sqrt{ex}ex\right)Aa^2|e|^2}{3\,e^5}$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))*B*a^2*x*abs(e)^
2/e^3 + 1/6*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))*A*a*b*x*
abs(e)^2/e^3 + 1/36*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e
^4)) - 3*a^2/(b^2*e))*sqrt(e*x)*B*a*b*x*abs(e)^2/e^3 + 1/72*sqrt(b*e^4*x^3
 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b^2*e))*sqrt(e*x)*A*
b^2*x*abs(e)^2/e^3 + 1/576*sqrt(b*e^4*x^3 + a*e^4)*(2*(4*e^3*x^3*(6*x^3/e^
7 + a/(b*e^7)) - 5*a^2/(b^2*e^4))*e^3*x^3 + 15*a^3/(b^3*e))*sqrt(e*x)*B*b^
2*x*abs(e)^2/e^3 - 1/192*(25*B^2*a^8 + 240*A*B*a^7*b + 576*A^2*a^6*b^2)*e^
4*log(abs((5*sqrt(e*x)*B*a^4*x + 24*sqrt(e*x)*A*a^3*b*x)*sqrt(b*e) + sqrt(
25*B^2*a^9*e^2 + 240*A*B*a^8*b*e^2 + 576*A^2*a^7*b^2*e^2 + (5*sqrt(e*x)*B*
a^4*x + 24*sqrt(e*x)*A*a^3*b*x)^2*b*e)))/(sqrt(b*e)*b*abs(5*B*a^4*e + 24*A
*a^3*b*e)*abs(e)^2) - 1/3*(a*e^4*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b
*e^4*x^3 + a*e^4)))/sqrt(b*e) - sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*e*x)*A*a
^2*abs(e)^2/e^5
```

3.538.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\sqrt{e\,x}\,\left(b\,x^3+a\right)^{5/2}\,dx$$

input
```
int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(5/2),x)
```

output
```
int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(5/2), x)
```

3.539 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx$

3.539.1 Optimal result

Integrand size = 26, antiderivative size = 364

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{27a^2(22Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{1408be}$$

$$+\frac{3a(22Ab-aB)\sqrt{ex}(a+bx^3)^{3/2}}{352be}+\frac{(22Ab-aB)\sqrt{ex}(a+bx^3)^{5/2}}{176be}+\frac{B\sqrt{ex}(a+bx^3)^{7/2}}{11be}$$

$$+\frac{27\,3^{3/4}a^{8/3}(22Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Bigg(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}}{2816be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
3/352*a*(22*A*b-B*a)*(b*x^3+a)^(3/2)*(e*x)^(1/2)/b/e+1/176*(22*A*b-B*a)*(b
*x^3+a)^(5/2)*(e*x)^(1/2)/b/e+1/11*B*(b*x^3+a)^(7/2)*(e*x)^(1/2)/b/e+27/14
08*a^2*(22*A*b-B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+27/2816*3^(3/4)*a^(8/3
)*(22*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1
/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/
(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1
/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2)/b/e/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.539.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.23

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{x\sqrt{a+bx^3}\left(B(a+bx^3)^3-\frac{a^2(-22Ab+aB)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{6},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{11b\sqrt{ex}}$$

input
```
Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/Sqrt[e*x],x]
```

output
```
(x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^3 - (a^2*(-22*A*b + a*B)*Hypergeometric2
F1[-5/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(11*b*Sqrt[e*x])
```

3.539.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {959, 811, 811, 811, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(22Ab-aB)\int \frac{(bx^3+a)^{5/2}}{\sqrt{ex}}dx}{22b}+\frac{B\sqrt{ex}(a+bx^3)^{7/2}}{11be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(22Ab-aB)\left(\frac{15}{16}a\int \frac{(bx^3+a)^{3/2}}{\sqrt{ex}}dx+\frac{\sqrt{ex}(a+bx^3)^{5/2}}{8e}\right)}{22b}+\frac{B\sqrt{ex}(a+bx^3)^{7/2}}{11be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(22Ab-aB)\left(\frac{15}{16}a\left(\frac{9}{10}a\int \frac{\sqrt{bx^3+a}}{\sqrt{ex}}dx+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)+\frac{\sqrt{ex}(a+bx^3)^{5/2}}{8e}\right)}{22b}+\frac{B\sqrt{ex}(a+bx^3)^{7/2}}{11be}$$

$$\Big\downarrow \text{811}$$

$$\frac{(22Ab-aB)\left(\frac{15}{16}a\left(\frac{9}{10}a\left(\frac{3}{4}a\int\frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5e}\right)+\frac{\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8e}\right)}{22b}+\frac{B\sqrt{ex}\left(a+bx^3\right)^{7/2}}{11be}$$

$$\Big\downarrow \color{blue}{851}$$

$$\frac{(22Ab-aB)\left(\frac{15}{16}a\left(\frac{9}{10}a\left(\frac{3a\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2e}+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5e}\right)+\frac{\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8e}\right)}{22b}+\frac{B\sqrt{ex}\left(a+bx^3\right)^{7/2}}{11be}$$

$$\Big\downarrow \color{blue}{766}$$

$$\frac{(22Ab-aB)\left(\frac{15}{16}a\left(\frac{9}{10}a\left(\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(\right.\right.}{4e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.\right.\right.}{22b}$$

$$\frac{B\sqrt{ex}\left(a+bx^3\right)^{7/2}}{11be}$$

input
```
Int[((a + b*x^3)^(5/2)*(A + B*x^3))/Sqrt[e*x],x]
```

output
```
(B*Sqrt[e*x]*(a + b*x^3)^(7/2))/(11*b*e) + ((22*A*b - a*B)*((Sqrt[e*x]*(a
+ b*x^3)^(5/2))/(8*e) + (15*a*((Sqrt[e*x]*(a + b*x^3)^(3/2))/(5*e) + (9*a*
((Sqrt[e*x]*Sqrt[a + b*x^3])/(2*e) + (3^(3/4)*a^(2/3)*Sqrt[e*x]*(a^(1/3)*e
 + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^
2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e
+ (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2
+ Sqrt[3])/4])/(4*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3
)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/10))/16))/(22*b)
```

3.539. $\int\frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx$

3.539.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.539.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.84 (sec) , antiderivative size = 792, normalized size of antiderivative = 2.18

method	result
risch	$\frac{\left(128 b^{3} B\,x^{9}+176 x^{6} b^{3} A+376 B\,x^{6} a\,b^{2}+616 a A\,b^{2} x^{3}+356 B\,a^{2} b\,x^{3}+1034 a^{2} b A+81 a^{3} B\right) x \sqrt{b\,x^{3}+a}}{1408 b \sqrt{e x}}+\frac{81 a^{3} \left(22 A b-B a\right) \left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1408/b*(128*B*b^3*x^9+176*A*b^3*x^6+376*B*a*b^2*x^6+616*A*a*b^2*x^3+356*
B*a^2*b*x^3+1034*A*a^2*b+81*B*a^3)*x*(b*x^3+a)^(1/2)/(e*x)^(1/2)+81/1408*a
^3*(22*A*b-B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-
a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*((b*x^3+a)*e*x)^(1/2)/(e*x)
^(1/2)/(b*x^3+a)^(1/2)
```

3.539. $\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx$

3.539.5 Fricas [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^
2)*sqrt(b*x^3 + a)*sqrt(e*x)/(e*x), x)
```

3.539.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.85 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx &= \frac{Aa^{\frac{5}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{7}{6}\right)}\\ &+\frac{2Aa^{\frac{3}{2}}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)}+\frac{A\sqrt{a}b^2x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{19}{6}\right)}\\ &+\frac{Ba^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)}+\frac{2Ba^{\frac{3}{2}}bx^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{19}{6}\right)}\\ &+\frac{B\sqrt{a}b^2x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{19}{6}\\ \frac{25}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{25}{6}\right)}\end{aligned}$$

input
```
integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(1/2),x)
```

output
```
A*a**(5/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(e)*gamma(7/6)) + 2*A*a**(3/2)*b*x**(7/2)*gamma(7/6)*hyper
((-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6)) +
 A*sqrt(a)*b**2*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*
exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6)) + B*a**(5/2)*x**(7/2)*gamma(7/6
)*hyper((-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(1
3/6)) + 2*B*a**(3/2)*b*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,),
b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6)) + B*sqrt(a)*b**2*x**(19/
2)*gamma(19/6)*hyper((-1/2, 19/6), (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*s
qrt(e)*gamma(25/6))
```

3.539.7 Maxima [F]

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/sqrt(e*x), x)
```

3.539.8 Giac [F]

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/sqrt(e*x), x)
```

3.539.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{5/2}}{\sqrt{e\,x}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(1/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(1/2), x)
```

3.540 $\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx$

3.540.1 Optimal result

Integrand size = 26, antiderivative size = 650

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx=\frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4}$$

$$+\frac{81\left(1+\sqrt{3}\right)a^2(20Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{448b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{3(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28e^4}$$

$$+\frac{(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10ae^4}-\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

$$-\frac{81\sqrt[4]{3}a^{7/3}(20Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{448b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\ 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}(20Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right.}{\sqrt[3]{a}+\left(1+\sqrt{3}\right.}\right.\right.}{896b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
3/28*(20*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/e^4+1/10*(20*A*b+B*a)*(e*x)^
(5/2)*(b*x^3+a)^(5/2)/a/e^4-2*A*(b*x^3+a)^(7/2)/a/e/(e*x)^(1/2)+27/224*a*(
20*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/e^4+81/448*a^2*(20*A*b+B*a)*(1+3^(
1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/3)/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2
)))-81/448*3^(1/4)*a^(7/3)*(20*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1
/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+
b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1
/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*
x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-27/896*3^(3
/4)*a^(7/3)*(20*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^
(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2)
)*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3
)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(
a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.540.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.13

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{2x\sqrt{a+bx^3}\left(-5A(a+bx^3)^3+\frac{a^2(20Ab+aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5a(ex)^{3/2}}$$

input

```
Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

output

```
(2*x*Sqrt[a + b*x^3]*(-5*A*(a + b*x^3)^3 + (a^2*(20*A*b + a*B)*x^3*Hyperge
ometric2F1[-5/2, 5/6, 11/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*a*(e*x
)^(3/2))
```

3.540.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {955, 811, 811, 811, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{(aB+20Ab)\int (ex)^{3/2}(bx^3+a)^{5/2}dx}{ae^3}-\frac{2A(a+bx^3)^{7/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{(aB+20Ab)\left(\frac{3}{4}a\int (ex)^{3/2}(bx^3+a)^{3/2}dx+\frac{(ex)^{5/2}(a+bx^3)^{5/2}}{10e}\right)}{ae^3}-\frac{2A(a+bx^3)^{7/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{(aB+20Ab)\left(\frac{3}{4}a\left(\frac{9}{14}a\int (ex)^{3/2}\sqrt{bx^3+a}dx+\frac{(ex)^{5/2}(a+bx^3)^{3/2}}{7e}\right)+\frac{(ex)^{5/2}(a+bx^3)^{5/2}}{10e}\right)}{ae^3}-\frac{2A(a+bx^3)^{7/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{(aB+20Ab)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3}{8}a\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(a+bx^3)^{3/2}}{7e}\right)+\frac{(ex)^{5/2}(a+bx^3)^{5/2}}{10e}\right)}{ae^3}-\frac{2A(a+bx^3)^{7/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(aB+20Ab)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(a+bx^3)^{3/2}}{7e}\right)+\frac{(ex)^{5/2}(a+bx^3)^{5/2}}{10e}\right)}{ae^3}-\frac{2A(a+bx^3)^{7/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 837}$$

$$\frac{(aB+20Ab)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int-\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(a}{7}\right.\right.\right.}{ae^3}$$

$$\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(aB+20Ab)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4e}+\frac{(ex)^{5/2}\sqrt{a+bx^3}}{4e}\right)+\frac{(ex)^{5/2}(a+bx}{7e}\right.\right.}{ae^3}$$

$$\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{(aB+20Ab)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\,\text{Elliptic}}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\right.}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\right.}}}\right.}{4e}\right.\right.\right.}{ae^3}$$

$$\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 2420}$$

3.540. $\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx$

$$\frac{(aB+20Ab)\left(\frac{3}{4}a\left(\frac{9}{14}a\left(\frac{3a\left(\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.}{2b^{2/3}}\right.\right.\right.}{}$$

$$\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

input
```
Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

3.540. $\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx$

output
```
(-2*A*(a + b*x^3)^(7/2))/(a*e*Sqrt[e*x]) + ((20*A*b + a*B)*(((e*x)^(5/2)*(
a + b*x^3)^(5/2))/(10*e) + (3*a*(((e*x)^(5/2)*(a + b*x^3)^(3/2))/(7*e) + (
9*a*(((e*x)^(5/2)*Sqrt[a + b*x^3])/(4*e) + (3*a*((((1 + Sqrt[3])*e^3*Sqrt[
e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) - (3^(1/4)*a
^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b
^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]
*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1
+ Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*(a^(1/3)*e
+ b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3]
))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e
*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*
e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[
3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4
])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3
)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(4*e)))/14))/4))/(a
*e^3)
```

3.540.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.540.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.94 (sec) , antiderivative size = 1166, normalized size of antiderivative = 1.79

method	result	size
risch	Expression too large to display	1166
elliptic	Expression too large to display	1341
default	Expression too large to display	6530

input
```
int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/1120*(b*x^3+a)^(1/2)*(-112*B*b^2*x^9-160*A*b^2*x^6-344*B*a*b*x^6-620*A*
a*b*x^3-367*B*a^2*x^3+2240*A*a^2)/e/(e*x)^(1/2)+81/448*a^2*(20*A*b+B*a)*(x
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^
(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a
*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*
b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*E
llipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/...
```

3.540.5 Fricas [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input

```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="fricas")
```

output

```
integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^
2)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^2*x^2), x)
```

3.540.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.62 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.48

$$
\begin{aligned}
\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx &= \frac{Aa^{\frac{5}{2}}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)} \\
&+ \frac{2Aa^{\frac{3}{2}}bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)} + \frac{A\sqrt{a}b^2x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)} \\
&+ \frac{Ba^{\frac{5}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)} + \frac{2Ba^{\frac{3}{2}}bx^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)} \\
&+ \frac{B\sqrt{a}b^2x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{17}{6}\\ \frac{23}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{23}{6}\right)}
\end{aligned}
$$

input

```
integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(3/2),x)
```

output

```
A*a**(5/2)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/
a)/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + 2*A*a**(3/2)*b*x**(5/2)*gamma(5/6)*hy
per((-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6
)) + A*sqrt(a)*b**2*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b*x
**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6)) + B*a**(5/2)*x**(5/2)*gamm
a(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*g
amma(11/6)) + 2*B*a**(3/2)*b*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17
/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6)) + B*sqrt(a)*b**2*
x**(17/2)*gamma(17/6)*hyper((-1/2, 17/6), (23/6,), b*x**3*exp_polar(I*pi)/
a)/(3*e**(3/2)*gamma(23/6))
```

3.540.7 Maxima [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(3/2), x)
```

3.540.8 Giac [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(3/2), x)
```

3.540.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx=\int \frac{(B\,x^3+A)\,\left(b\,x^3+a\right)^{5/2}}{(e\,x)^{3/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(3/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(3/2), x)
```

3.541 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx$

3.541.1 Optimal result

Integrand size = 26, antiderivative size = 188

$$\begin{aligned}\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx &= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4}\\ &+ \frac{5(6Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}(a+bx^3)^{5/2}}{9ae^4}\\ &- \frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}} + \frac{5a^2(6Ab+aB)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24\sqrt{b}e^{5/2}}\end{aligned}$$

output

```
5/36*(6*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/e^4+1/9*(6*A*b+B*a)*(e*x)^(3/
2)*(b*x^3+a)^(5/2)/a/e^4-2/3*A*(b*x^3+a)^(7/2)/a/e/(e*x)^(3/2)+5/24*a^2*(6
*A*b+B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/e^(5/2)/b^(
1/2)+5/24*a*(6*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/e^4
```

3.541.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \frac{x\Big(\sqrt{b}\sqrt{a+bx^3}(4b^2x^6(3A+2Bx^3)+a^2(-48A+33Bx^3)+a(54Abx^3+26}{72\sqrt{b}(ex)^{5/2}}$$

input
```
Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

output
```
(x*(Sqrt[b]*Sqrt[a + b*x^3]*(4*b^2*x^6*(3*A + 2*B*x^3) + a^2*(-48*A + 33*B
*x^3) + a*(54*A*b*x^3 + 26*b*B*x^6)) + 15*a^2*(6*A*b + a*B)*x^(3/2)*Log[Sq
rt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(72*Sqrt[b]*(e*x)^(5/2))
```

3.541.3 Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {955, 811, 811, 811, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{5/2}}dx$$

$$\Big\downarrow \ 955$$

$$\frac{(aB+6Ab)\int \sqrt{ex}\left(bx^3+a\right)^{5/2}dx}{ae^3}-\frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \ 811$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\int \sqrt{ex}\left(bx^3+a\right)^{3/2}dx+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \ 811$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{ex}\sqrt{bx^3+a}dx+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \ 811$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \ 851$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \ 807$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \ 224$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3e}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3\sqrt{b}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}}{3e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6e}\right)+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9e}\right)}{ae^3}-\frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}}$$

input

```
Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

output

```
(-2*A*(a + b*x^3)^(7/2))/(3*a*e*(e*x)^(3/2)) + ((6*A*b + a*B)*(((e*x)^(3/2
)*(a + b*x^3)^(5/2))/(9*e) + (5*a*(((e*x)^(3/2)*(a + b*x^3)^(3/2))/(6*e) +
 (3*a*(((e*x)^(3/2)*Sqrt[a + b*x^3])/(3*e) + (a*Sqrt[e]*ArcTanh[(Sqrt[b]*(
e*x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*Sqrt[b])))/4))/6))/(a*e^3)
```

3.541. $\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx$

3.541.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.541. $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx$

3.541.4 Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-8 b^{2} B\,x^{9}-12 A\,b^{2} x^{6}-26 B\,x^{6} a b-54 a A b\,x^{3}-33 a^{2} B\,x^{3}+48 a^{2} A\right)}{72 x\,e^{2} \sqrt{e x}}+\frac{5 a^{2} \left(6 A b+B a\right) \operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right) e x}}{x^{2} \sqrt{b e}}\right) \sqrt{(b\,x^{3}+a}}{24 \sqrt{b e}\,e^{2} \sqrt{e x}\,\sqrt{b\,x^{3}+a}}$
default	$\frac{\sqrt{b\,x^{3}+a}\,\left(8 B \sqrt{(b\,x^{3}+a) e x}\,\sqrt{b e}\,b^{2} x^{9}+12 A \sqrt{(b\,x^{3}+a) e x}\,\sqrt{b e}\,b^{2} x^{6}+26 B \sqrt{(b\,x^{3}+a) e x}\,\sqrt{b e}\,a b\,x^{6}+90 A \operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right) e x}}{x^{2} \sqrt{b e}}\right) a^{2} b\right.}{72 x\,e^{2} \sqrt{e x}\,\sqrt{(b}}$
elliptic	Expression too large to display

input
```
int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/72*(b*x^3+a)^(1/2)*(-8*B*b^2*x^9-12*A*b^2*x^6-26*B*a*b*x^6-54*A*a*b*x^3
-33*B*a^2*x^3+48*A*a^2)/x/e^2/(e*x)^(1/2)+5/24*a^2*(6*A*b+B*a)/(b*e)^(1/2)
*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))/e^2*((b*x^3+a)*e*x)^(1/2)/
(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

3.541.5 Fricas [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.64

$$\int \frac{\left(a+bx^{3}\right)^{5/2}\left(A+Bx^{3}\right)}{(ex)^{5/2}}\,dx=\left[\frac{15\,\left(Ba^{3}+6\,Aa^{2}b\right)\sqrt{be}x^{2}\log\left(-8\,b^{2}ex^{6}-8\,abex^{3}-a^{2}e-4\,\left(2\,bx^{4}+ax\right)\sqrt{}\right.}{}\right.$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/288*(15*(B*a^3 + 6*A*a^2*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^
3 - a^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(8*
B*b^3*x^9 + 2*(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A*a^2*b + 3*(11*B*a^2*b + 18
*A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2), -1/144*(15*(B*a^3 +
 6*A*a^2*b)*sqrt(-b*e)*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x
/(2*b*e*x^3 + a*e)) - 2*(8*B*b^3*x^9 + 2*(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A
*a^2*b + 3*(11*B*a^2*b + 18*A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^
3*x^2)]
```

3.541.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(180) = 360$.

Time = 40.09 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.14

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = -\frac{2Aa^{\frac{5}{2}}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{2Aa^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} - \frac{7Aa^{\frac{3}{2}}bx^{\frac{3}{2}}}{12e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{A\sqrt{a}b^2x^{\frac{9}{2}}}{4e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{5Aa^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{4e^{\frac{5}{2}}} + \frac{Ab^3x^{\frac{15}{2}}}{6\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{Ba^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} + \frac{Ba^{\frac{5}{2}}x^{\frac{3}{2}}}{8e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{35Ba^{\frac{3}{2}}bx^{\frac{9}{2}}}{72e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{17B\sqrt{a}b^2x^{\frac{15}{2}}}{36e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{5Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{24\sqrt{b}e^{\frac{5}{2}}} + \frac{Bb^3x^{\frac{21}{2}}}{9\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}$$

input

```
integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(5/2),x)
```

output

```
-2*A*a**(5/2)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*A*a**(3/2)*b*x*
*(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) - 7*A*a**(3/2)*b*x**(3/2)/(12*e**(5
/2)*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b**2*x**(9/2)/(4*e**(5/2)*sqrt(1 + b*x
**3/a)) + 5*A*a**2*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*e**(5/2)) +
A*b**3*x**(15/2)/(6*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a)) + B*a**(5/2)*x**(
3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a**(5/2)*x**(3/2)/(8*e**(5/2)*sqr
t(1 + b*x**3/a)) + 35*B*a**(3/2)*b*x**(9/2)/(72*e**(5/2)*sqrt(1 + b*x**3/a
)) + 17*B*sqrt(a)*b**2*x**(15/2)/(36*e**(5/2)*sqrt(1 + b*x**3/a)) + 5*B*a*
*3*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(24*sqrt(b)*e**(5/2)) + B*b**3*x**(21/2
)/(9*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a))
```

3.541.7 Maxima [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(5/2), x)
```

3.541.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx=\text{Exception raised: NotImplementedError}$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: Recu
rsive assumption sageVARa>=(-sageVARb*sageVARe/(sageVARe^4*t_nostep^6)) ig
nored2/sageVARe^3*((8870400*sageVARb^12*sageVARe^9*sageVARB/159667200/sage
VARb^10/sageVARe^18
```

3.541.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx=\int \frac{(B\,x^3+A)\,(b\,x^3+a)^{5/2}}{(e\,x)^{5/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(5/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(5/2), x)
```

3.542 $\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{7/2}}\,dx$

3.542.1 Optimal result

Integrand size = 26, antiderivative size = 352

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{7/2}}\,dx=\frac{27a(16Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{320e^4}$$

$$+\frac{3(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80e^4}+\frac{(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{5/2}}{40ae^4}-\frac{2A\left(a+bx^3\right)^{7/2}}{5ae(ex)^{5/2}}$$

$$+\frac{27\ 3^{3/4}a^{5/3}(16Ab+5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\right.}{640e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/5*A*(b*x^3+a)^(7/2)/a/e/(e*x)^(5/2)+3/80*(16*A*b+5*B*a)*(b*x^3+a)^(3/2)
*(e*x)^(1/2)/e^4+1/40*(16*A*b+5*B*a)*(b*x^3+a)^(5/2)*(e*x)^(1/2)/a/e^4+27/
320*a*(16*A*b+5*B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/e^4+27/640*3^(3/4)*a^(5/3
)*(16*A*b+5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a
^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^
(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^
2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^
(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))^2)^(1/2)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.542.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.25

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx=\frac{2x\sqrt{a+bx^3}\left(-A(a+bx^3)^3+\frac{a^2(16Ab+5aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{6},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5a(ex)^{7/2}}$$

input
```
Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)^3) + (a^2*(16*A*b + 5*a*B)*x^3*Hyper
geometric2F1[-5/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*a*(e*
x)^(7/2))
```

3.542.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {955, 811, 811, 811, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}dx$$

$$\Big\downarrow \ 955$$

$$\frac{(5aB+16Ab)\int \frac{(bx^3+a)^{5/2}}{\sqrt{ex}}dx}{5ae^3}-\frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \ 811$$

$$\frac{(5aB+16Ab)\left(\frac{15}{16}a\int \frac{(bx^3+a)^{3/2}}{\sqrt{ex}}dx+\frac{\sqrt{ex}(a+bx^3)^{5/2}}{8e}\right)}{5ae^3}-\frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \ 811$$

$$\frac{(5aB+16Ab)\left(\frac{15}{16}a\left(\frac{9}{10}a\int \frac{\sqrt{bx^3+a}}{\sqrt{ex}}dx+\frac{\sqrt{ex}(a+bx^3)^{3/2}}{5e}\right)+\frac{\sqrt{ex}(a+bx^3)^{5/2}}{8e}\right)}{5ae^3}-\frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \textcolor{blue}{811}$$

$$\frac{(5aB+16Ab)\left(\frac{15}{16}a\left(\frac{9}{10}a\left(\frac{3}{4}a\int\frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5e}\right)+\frac{\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8e}\right)}{5ae^3}-\frac{2A\left(a+bx^3\right)^{7/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \textcolor{blue}{851}$$

$$\frac{(5aB+16Ab)\left(\frac{15}{16}a\left(\frac{9}{10}a\left(\frac{3a\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2e}+\frac{\sqrt{ex}\sqrt{a+bx^3}}{2e}\right)+\frac{\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5e}\right)+\frac{\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8e}\right)}{5ae^3}-\frac{2A\left(a+bx^3\right)^{7/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \textcolor{blue}{766}$$

$$\frac{(5aB+16Ab)\left(\frac{15}{16}a\left(\frac{9}{10}a\left(\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\right.\right.\right.\right.}{4e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{5ae^3}$$

$$\frac{2A\left(a+bx^3\right)^{7/2}}{5ae(ex)^{5/2}}$$

input
```
Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

output
```
(-2*A*(a + b*x^3)^(7/2))/(5*a*e*(e*x)^(5/2)) + ((16*A*b + 5*a*B)*((Sqrt[e*
x]*(a + b*x^3)^(5/2))/(8*e) + (15*a*((Sqrt[e*x]*(a + b*x^3)^(3/2))/(5*e) +
 (9*a*((Sqrt[e*x]*Sqrt[a + b*x^3])/(2*e) + (3^(3/4)*a^(2/3)*Sqrt[e*x]*(a^(
1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*
e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1
/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)
], (2 + Sqrt[3])/4])/(4*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(
a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3]))/10))/16))/(5*
a*e^3)
```

3.542. $\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{7/2}}\,dx$

3.542.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.542.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.86 (sec) , antiderivative size = 786, normalized size of antiderivative = 2.23

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-40b^{2}B\,x^{9}-64A\,b^{2}x^{6}-140B\,x^{6}ab-368aAb\,x^{3}-235a^{2}B\,x^{3}+128a^{2}A\right)}{320x^{2}e^{3}\sqrt{ex}}+\frac{81a^{2}(16Ab+5Ba)\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/320*(b*x^3+a)^(1/2)*(-40*B*b^2*x^9-64*A*b^2*x^6-140*B*a*b*x^6-368*A*a*b
*x^3-235*B*a^2*x^3+128*A*a^2)/x^2/e^3/(e*x)^(1/2)+81/320*a^2*(16*A*b+5*B*a
)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^
2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)
^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x
-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(
b*x^3+a)^(1/2)
```

3.542. $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx$

3.542.5 Fricas [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input

```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="fricas")
```

output

```
integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^
2)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^4*x^4), x)
```

3.542.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 44.17 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx &= \frac{Aa^{\frac{5}{2}}\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)} \\ &+ \frac{2Aa^{\frac{3}{2}}b\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)} + \frac{A\sqrt{a}b^2x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{13}{6}\right)} \\ &+ \frac{Ba^{\frac{5}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)} + \frac{2Ba^{\frac{3}{2}}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{13}{6}\right)} \\ &+ \frac{B\sqrt{a}b^2x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{19}{6}\right)}\end{aligned}$$

input

```
integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(7/2),x)
```

output
```
A*a**(5/2)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/
a)/(3*e**(7/2)*x**(5/2)*gamma(1/6)) + 2*A*a**(3/2)*b*sqrt(x)*gamma(1/6)*hy
per((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6))
 + A*sqrt(a)*b**2*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*e
xp_polar(I*pi)/a)/(3*e**(7/2)*gamma(13/6)) + B*a**(5/2)*sqrt(x)*gamma(1/6)
*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/
6)) + 2*B*a**(3/2)*b*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**
3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(13/6)) + B*sqrt(a)*b**2*x**(13/2)*g
amma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7
/2)*gamma(19/6))
```

3.542.7 Maxima [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(7/2), x)
```

3.542.8 Giac [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(7/2), x)
```

3.542.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/2}\,(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{5/2}}{(e\,x)^{7/2}}\,dx$$

input
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(7/2),x)
```

output
```
int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(7/2), x)
```

3.543 $\int \frac{(ex)^{7/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

3.543.1 Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}-\frac{a(4Ab-3aB)e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}}$$

output
```
-1/12*a*(4*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a
)^(1/2))/b^(5/2)+1/12*(4*A*b-3*B*a)*e^2*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b^2+1/
6*B*(e*x)^(9/2)*(b*x^3+a)^(1/2)/b/e
```

3.543.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\frac{(ex)^{7/2}\sqrt{a+bx^3}(4Ab-3aB+2bBx^3)}{12b^2x^2}+\frac{a(-4Ab+3aB)(ex)^{7/2}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{12b^{5/2}x^{7/2}}$$

input
```
Integrate[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
((e*x)^(7/2)*Sqrt[a + b*x^3]*(4*A*b - 3*a*B + 2*b*B*x^3))/(12*b^2*x^2) + (
a*(-4*A*b + 3*a*B)*(e*x)^(7/2)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(12
*b^(5/2)*x^(7/2))
```

3.543.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {959, 843, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(4Ab-3aB)\int \frac{(ex)^{7/2}}{\sqrt{bx^3+a}}dx}{4b}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

$$\Big\downarrow \text{843}$$

$$\frac{(4Ab-3aB)\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^3\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{2b}\right)}{4b}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(4Ab-3aB)\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{b}\right)}{4b}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

$$\Big\downarrow \text{807}$$

$$\frac{(4Ab-3aB)\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3b}\right)}{4b}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

$$\Big\downarrow \text{224}$$

$$\frac{(4Ab-3aB)\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^2\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3b}\right)}{4b}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(4Ab-3aB)\left(\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ae^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}\right)}{4b}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

input

```
Int[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output

```
(B*(e*x)^(9/2)*Sqrt[a + b*x^3])/(6*b*e) + ((4*A*b - 3*a*B)*((e^2*(e*x)^(3/
2)*Sqrt[a + b*x^3])/(3*b) - (a*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3
/2)*Sqrt[a + (b*x)/e^2])])/(3*b^(3/2))))/(4*b)
```

3.543.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

3.543. $\int \frac{(ex)^{7/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.543.4 Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

method	result
risch	$\frac{x^{2} \left(2 b B \,x^{3}+4 A b -3 B a \right) \sqrt{b \,x^{3}+a}\, e^{4}}{12 b^{2} \sqrt{e x}}-\frac{a \left(4 A b -3 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{\left(b \,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) e^{4} \sqrt{\left(b \,x^{3}+a \right) e x}}{12 b^{2} \sqrt{b e}\, \sqrt{e x}\, \sqrt{b \,x^{3}+a}}$
default	$-\frac{e^{3} \sqrt{e x}\, \sqrt{b \,x^{3}+a}\, \left(-2 B \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, b \,x^{4}+4 A \,\operatorname{arctanh}\left(\frac{\sqrt{\left(b \,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) a b e -4 A \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, b x -3 B \,\operatorname{arctanh}\left(\right.\right.}{12 \sqrt{\left(b \,x^{3}+a \right) e x}\, b^{2} \sqrt{b e}}$
elliptic	Expression too large to display

input
```
int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2*(2*B*b*x^3+4*A*b-3*B*a)*(b*x^3+a)^(1/2)/b^2*e^4/(e*x)^(1/2)-1/12*
a*(4*A*b-3*B*a)/b^2/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1
/2))*e^4*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

3.543.5 Fricas [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \frac{(ex)^{7/2}\,(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \left[-\frac{(3\,Ba^2-4\,Aab)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e+4\,(2\,b^2x^4+abx)\sqrt{bx^3}\right.}{48\,b^2}\right.$$

$$\left.-\frac{(3\,Ba^2-4\,Aab)e^3\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\,\sqrt{bx^3+a}\sqrt{ex}bx\sqrt{-\frac{e}{b}}}{2\,bex^3+ae}\right)-2\,(2\,Bbe^3x^4-(3\,Ba-4\,Ab)e^3x)\sqrt{bx^3+a}\sqrt{ex}}{24\,b^2}\right]$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*((3*B*a^2 - 4*A*a*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 -
 a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(2
*B*b*e^3*x^4 - (3*B*a - 4*A*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/2
4*((3*B*a^2 - 4*A*a*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b
*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(2*B*b*e^3*x^4 - (3*B*a - 4*A*b)*e^3*
x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]
```

3.543.6 Sympy [A] (verification not implemented)

Time = 13.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.60

$$\int \frac{(ex)^{7/2}\,(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{2\left(\begin{cases} \text{NaN} \\ \frac{\begin{cases} -\frac{ae^3\left(Ae^3-\frac{3Bae^3}{4b}\right)\left(\begin{cases} \frac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3}+2\sqrt{\frac{b}{e^3}}\sqrt{a+bx^3}\right)}{\sqrt{\frac{b}{e^3}}} & \text{for } a\neq 0 \\ \frac{(ex)^{\frac{3}{2}}\log\left((ex)^{\frac{3}{2}}\right)}{\sqrt{bx^3}} & \text{otherwise} \end{cases}\right)}{2b}+\sqrt{a+bx^3}\left(\frac{Be}{}\right. \\ \frac{\frac{Ae^3(ex)^{\frac{9}{2}}}{3}+\frac{B(ex)^{\frac{15}{2}}}{5}}{\sqrt{a}} \end{cases}}{3e^3} \end{cases}\right.}{e} \\ 0 \end{cases}$$

input
```
integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), (Piecewise((-a*e**3*(A*e**3 - 3
*B*a*e**3/(4*b))*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqr
t(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sq
rt(b*x**3), True))/(2*b) + sqrt(a + b*x**3)*(B*e**3*(e*x)**(9/2)/(4*b) + e
**3*(e*x)**(3/2)*(A*e**3 - 3*B*a*e**3/(4*b))/(2*b)), Ne(b/e**3, 0)), ((A*e
**3*(e*x)**(9/2)/3 + B*(e*x)**(15/2)/5)/sqrt(a), True))/(3*e**3), True))/e
, Ne(e, 0)), (0, True))
```

3.543.7 Maxima [F]

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{7}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(7/2)/sqrt(b*x^3 + a), x)
```

3.543.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.21

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}e^5x\left(\frac{2\,Bx^3}{be^2}-\frac{3\,Bab^3e^5-4\,Ab^4e^5}{b^5e^7}\right)}{12\,|e|^2}-\frac{(3\,Ba^2b^3e^9-4\,Aab^4e^9)\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{12\,\sqrt{be}b^5e|e|^4}$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*e^5*x*(2*B*x^3/(b*e^2) - (3*B*a*b^3
*e^5 - 4*A*b^4*e^5)/(b^5*e^7))/abs(e)^2 - 1/12*(3*B*a^2*b^3*e^9 - 4*A*a*b^
4*e^9)*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(sqrt(
b*e)*b^5*e*abs(e)^4)
```

3.543.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(1/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(1/2), x)
```

3.544 $\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

3.544.1 Optimal result

Integrand size = 26, antiderivative size = 286

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\frac{(10Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{20b^2}+\frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be}-\frac{a^{2/3}(10Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2-}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/5*B*(e*x)^(7/2)*(b*x^3+a)^(1/2)/b/e+1/20*(10*A*b-7*B*a)*e^2*(e*x)^(1/2)*
(b*x^3+a)^(1/2)/b^2-1/120*a^(2/3)*(10*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*(
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF
((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^2/(b*x^3+a)^(1
/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2
)
```

3.544.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.34

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\frac{e^2\sqrt{ex}\left(-\left(\left(a+bx^3\right)\left(-10Ab+7aB-4bBx^3\right)\right)+a(-10Ab+7aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hy}\right.}{20b^2\sqrt{a+bx^3}}$$

input

```
Integrate[((e*x)^(5/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output

```
(e^2*Sqrt[e*x]*(-((a + b*x^3)*(-10*A*b + 7*a*B - 4*b*B*x^3)) + a*(-10*A*b
+ 7*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)
]))/(20*b^2*Sqrt[a + b*x^3])
```

3.544.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {959, 843, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(10Ab-7aB)\int \frac{(ex)^{5/2}}{\sqrt{bx^3+a}}dx}{10b}+\frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be}$$

$$\Big\downarrow \text{843}$$

$$\frac{(10Ab-7aB)\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^3\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{4b}\right)}{10b}+\frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(10Ab-7aB)\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{ae^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b}\right)}{10b}+\frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be}$$

$$\Big\downarrow \text{766}$$

$$\frac{(10Ab-7aB)\left(\frac{e^2\sqrt{ex}\sqrt{a+bx^3}}{2b}-\frac{a^{2/3}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right)\right.}{4\sqrt[4]{3}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.}{10b}$$

$$\frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be}$$

input
```
Int[((e*x)^(5/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(B*(e*x)^(7/2)*Sqrt[a + b*x^3])/(5*b*e) + ((10*A*b - 7*a*B)*((e^2*Sqrt[e*x
]*Sqrt[a + b*x^3])/(2*b) - (a^(2/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*
Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e +
(1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*
b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(
4*3^(1/4)*b*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 +
 Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(10*b)
```

3.544.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

3.544. $\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.544.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.90 (sec) , antiderivative size = 753, normalized size of antiderivative = 2.63

method	result
risch	$\frac{\left(4bB\,x^{3}+10Ab-7Ba\right)x\sqrt{b\,x^{3}+a}\,e^{3}}{20b^{2}\sqrt{ex}}-\frac{a(10Ab-7Ba)\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right.}}}{}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{\left(b\,x^{3}+a\right)ex}\left(\frac{B\,e^{2}x^{3}\sqrt{be\,x^{4}+aex}}{5b}+\frac{\left(A\,e^{3}-\frac{7B\,e^{3}a}{10b}\right)\sqrt{be\,x^{4}+aex}}{2be}-\frac{\left(A\,e^{3}-\frac{7B\,e^{3}a}{10b}\right)a\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-\right.}{}\right.}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\right.}}}{}\right.}{}$
default	Expression too large to display

input
```
int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*(4*B*b*x^3+10*A*b-7*B*a)*x*(b*x^3+a)^(1/2)/b^2*e^3/(e*x)^(1/2)-1/20*a
*(10*A*b-7*B*a)/b*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*
(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*e^3*((b*x^3+a)*e*x)^(1/2)
/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

3.544.5 Fricas [F]

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

3.544.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.70 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.33

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\frac{Ae^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{13}{6}\right)}+\frac{Be^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{19}{6}\right)}$$

input
```
integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
A*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((1/2, 7/6), (13/6,), b*x**3*exp_polar
(I*pi)/a)/(3*sqrt(a)*gamma(13/6)) + B*e**(5/2)*x**(13/2)*gamma(13/6)*hyper
((1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(19/6))
```

3.544.7 Maxima [F]

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(5/2)/sqrt(b*x^3 + a), x)
```

3.544.8 Giac [F]

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^(5/2)/sqrt(b*x^3 + a), x)
```

3.544.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(1/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(1/2), x)
```

3.545 $\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

3.545.1 Optimal result

Integrand size = 26, antiderivative size = 543

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be} + \frac{\left(1+\sqrt{3}\right)(8Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$
$$- \frac{\sqrt[4]{3}\sqrt[3]{a}(8Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$- \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}(8Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)\right.}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
1/4*B*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+1/8*(8*A*b-5*B*a)*e*(1+3^(1/2))*(e*x
)^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-1/8*3^(1/4
)*a^(1/3)*(8*A*b-5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1
/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2
))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)
*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/48*a^(1/3)*(8*A*b-5*B*a)*e*
(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*
(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(
1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3
)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^
2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a
^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.545.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{(ex)^{3/2}\,(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{x(ex)^{3/2}\left(5B(a+bx^3)+(8Ab-5aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\right.\right.}{20b\sqrt{a+bx^3}}$$

input

```
Integrate[((e*x)^(3/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output

```
(x*(e*x)^(3/2)*(5*B*(a + b*x^3) + (8*A*b - 5*a*B)*Sqrt[1 + (b*x^3)/a]*Hype
rgeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]))/(20*b*Sqrt[a + b*x^3])
```

3.545.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {959, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2} \left(A+Bx^3\right)}{\sqrt{a+bx^3}} \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(8Ab-5aB) \int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx}{8b}+\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}$$

$$\Big\downarrow \text{851}$$

$$\frac{(8Ab-5aB) \int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{4be}+\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}$$

$$\Big\downarrow \text{837}$$

$$\frac{(8Ab-5aB) \left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2 \int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4be}+\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}$$

$$\Big\downarrow \text{25}$$

$$\frac{(8Ab-5aB) \left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2 \int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{4be}+\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}$$

$$\Big\downarrow \text{766}$$

$$\frac{(8Ab-5aB) \Bigg(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\Bigg(\arccos}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{4be}$$

$$\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}$$

$$\Big\downarrow \text{2420}$$

3.545. $\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}} \, dx$

$$\frac{(8Ab-5aB)\left(\frac{\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right)|\frac{1}{4}\left(2\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}}\right.}{4be}$$

$$\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}$$

input
```
Int[((e*x)^(3/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(B*(e*x)^(5/2)*Sqrt[a + b*x^3])/(4*b*e) + ((8*A*b - 5*a*B)*((((1 + Sqrt[3]
)*e^3*Sqrt[e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) -
 (3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2
- a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1
/3)*e*x)^2]*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1
/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*
(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt
[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e
+ b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2
)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e +
 (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 +
 Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*
x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(4*b*e)
```

3.545. $\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

3.545.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.545. $\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

3.545.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.81 (sec) , antiderivative size = 1124, normalized size of antiderivative = 2.07

method	result	size
risch	Expression too large to display	1124
elliptic	Expression too large to display	1128
default	Expression too large to display	4914

input
```
int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*B*x^3/b*(b*x^3+a)^(1/2)*e^2/(e*x)^(1/2)+1/8*(8*A*b-5*B*a)/b*(x*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^
(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/
3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE
(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),(...
```

3.545.5 Fricas [F]

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*e*x^4 + A*e*x)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

3.545.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.78 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.17

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{6}\\\frac{11}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{11}{6}\right)}+\frac{Be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{11}{6}\\\frac{17}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{17}{6}\right)}$$

input
```
integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

output
```
A*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((1/2, 5/6), (11/6,), b*x**3*exp_polar
(I*pi)/a)/(3*sqrt(a)*gamma(11/6)) + B*e**(3/2)*x**(11/2)*gamma(11/6)*hyper
((1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(17/6))
```

3.545.7 Maxima [F]

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(3/2)/sqrt(b*x^3 + a), x)
```

3.545.8 Giac [F]

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^(3/2)/sqrt(b*x^3 + a), x)
```

3.545.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(1/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(1/2), x)
```

3.546 $\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

3.546.1 Optimal result

Integrand size = 26, antiderivative size = 83

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}+\frac{(2Ab-aB)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

output
```
1/3*(2*A*b-B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/
2)/b^(3/2)+1/3*B*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b/e
```

3.546.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{\sqrt{ex}\left(\sqrt{b}Bx^{3/2}\sqrt{a+bx^3}+(2Ab-aB)\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{3b^{3/2}\sqrt{x}}$$

input
```
Integrate[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(Sqrt[e*x]*(Sqrt[b]*B*x^(3/2)*Sqrt[a + b*x^3] + (2*A*b - a*B)*Log[Sqrt[b]*
x^(3/2) + Sqrt[a + b*x^3]]))/(3*b^(3/2)*Sqrt[x])
```

3.546.3 Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {959, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(2Ab-aB)\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{2b}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

$$\Big\downarrow \; 851$$

$$\frac{(2Ab-aB)\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{be}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

$$\Big\downarrow \; 807$$

$$\frac{(2Ab-aB)\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3be}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

$$\Big\downarrow \; 224$$

$$\frac{(2Ab-aB)\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3be}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{e}(2Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

input
```
Int[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

output
```
(B*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*b*e) + ((2*A*b - a*B)*Sqrt[e]*ArcTanh[(
Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*b^(3/2))
```

3.546.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.546.4 Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

method	result	size
risch	$\frac{B\,x^{2}\sqrt{b\,x^{3}+a}\,e}{3b\sqrt{ex}}+\frac{\left(2Ab-Ba\right)\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)e\sqrt{\left(b\,x^{3}+a\right)ex}}{3b\sqrt{be}\,\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$	94
default	$\frac{\sqrt{ex}\,\sqrt{b\,x^{3}+a}\,\left(2A\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)be+B\sqrt{\left(b\,x^{3}+a\right)ex}\,\sqrt{be}\,x-B\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)ae\right)}{3\sqrt{\left(b\,x^{3}+a\right)ex}\,b\sqrt{be}}$	112
elliptic	Expression too large to display	1046

3.546. $\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

input
```
int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*B*x^2/b*(b*x^3+a)^(1/2)*e/(e*x)^(1/2)+1/3*(2*A*b-B*a)/b/(b*e)^(1/2)*ar
ctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e*((b*x^3+a)*e*x)^(1/2)/(e*x)
^(1/2)/(b*x^3+a)^(1/2)
```

3.546.5 Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$$

$$= \left[\frac{4\sqrt{bx^3+a}\sqrt{ex}Bx-(Ba-2\,Ab)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,b^2x^4+abx)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right.}{12\,b}\right.$$

input
```
integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(4*sqrt(b*x^3 + a)*sqrt(e*x)*B*x - (B*a - 2*A*b)*sqrt(e/b)*log(-8*b^
2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt
(e*x)*sqrt(e/b)))/b, 1/6*(2*sqrt(b*x^3 + a)*sqrt(e*x)*B*x + (B*a - 2*A*b)*
sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 +
a*e)))/b]
```

3.546.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(73) = 146$.

Time = 1.51 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$$
$$= \begin{cases} \frac{2\left(\begin{cases}\text{NaN} & \text{for } e^3 = \\ \frac{\begin{cases} \frac{Be^3(ex)^{\frac{3}{2}}\sqrt{a+bx^3}}{2b} + \left(Ae^3 - \frac{Bae^3}{2b}\right)\left(\begin{cases} \frac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3}+2\sqrt{\frac{b}{e^3}}\sqrt{a+bx^3}\right)}{\sqrt{\frac{b}{e^3}}} & \text{for } a\neq 0 \\ \frac{(ex)^{\frac{3}{2}}\log\left((ex)^{\frac{3}{2}}\right)}{\sqrt{bx^3}} & \text{otherwise}\end{cases}\right) & \text{for } \frac{b}{e^3}\neq 0 \\ \frac{Ae^3(ex)^{\frac{3}{2}}+\frac{B(ex)^{\frac{9}{2}}}{3}}{\sqrt{a}} & \text{otherwise}\end{cases}}{3e^3} & \text{otherwis}\end{cases}\right)}{e} \\ 0 \end{cases}$$

input
```
integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((2*Piecewise((nan, Eq(e**3, 0)), (Piecewise((B*e**3*(e*x)**(3/2)
*sqrt(a + b*x**3)/(2*b) + (A*e**3 - B*a*e**3/(2*b))*Piecewise((log(2*b*(e*
x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)),
 ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True)), Ne(b/e**3, 0)), ((A
*e**3*(e*x)**(3/2) + B*(e*x)**(9/2)/3)/sqrt(a), True))/(3*e**3), True))/e,
 Ne(e, 0)), (0, True))
```

3.546.7 Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)\sqrt{ex}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

3.546.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}Bx}{3\,b|e|^2} + \frac{(Bae^5-2\,Abe^5)\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{3\,\sqrt{be}b|e|^4}$$

input

```
integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output

```
1/3*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*B*x/(b*abs(e)^2) + 1/3*(B*a*e^5 - 2*
A*b*e^5)*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(sqr
t(b*e)*b*abs(e)^4)
```

3.546.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{e\,x}}{\sqrt{b\,x^3+a}}\,dx$$

input

```
int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(1/2),x)
```

output

```
int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(1/2), x)
```

3.547 $\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx$

3.547.1 Optimal result

Integrand size = 26, antiderivative size = 249

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be}+\frac{(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{4\sqrt[4]{3}\sqrt[3]{a}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
1/2*B*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+1/12*(4*A*b-B*a)*(a^(1/3)+b^(1/3)*x)
*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Ellipti
cF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b/e/(b
*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2)
```

3.547.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx$$
$$=\frac{Bx(a+bx^3)+(4Ab-aB)x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)}{2b\sqrt{ex}\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/(Sqrt[e*x]*Sqrt[a + b*x^3]),x]
```

output
```
(B*x*(a + b*x^3) + (4*A*b - a*B)*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1
/6, 1/2, 7/6, -((b*x^3)/a)])/(2*b*Sqrt[e*x]*Sqrt[a + b*x^3])
```

3.547.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {959, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx$$
$$\Big\downarrow \text{959}$$
$$\frac{(4Ab-aB)\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{4b}+\frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be}$$
$$\Big\downarrow \text{851}$$
$$\frac{(4Ab-aB)\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2be}+\frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be}$$
$$\Big\downarrow \text{766}$$

$$\frac{\sqrt{ex}(4Ab-aB)\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}(2+\sqrt{3})\right.}{4\sqrt[4]{3}\sqrt[3]{a}be^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}$$

$$\frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be}$$

input
```
Int[(A + B*x^3)/(Sqrt[e*x]*Sqrt[a + b*x^3]),x]
```

output
```
(B*Sqrt[e*x]*Sqrt[a + b*x^3])/(2*b*e) + ((4*A*b - a*B)*Sqrt[e*x]*(a^(1/3)*
e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x
^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e
 + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2
 + Sqrt[3])/4])/(4*3^(1/4)*a^(1/3)*b*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^
(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])
```

3.547.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.547.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.70 (sec) , antiderivative size = 728, normalized size of antiderivative = 2.92

method	result
risch	$\frac{Bx\sqrt{b\,x^{3}+a}}{2b\sqrt{ex}}+\frac{(4Ab-Ba)\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{(}{b(-}}}{2\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\right.}$
elliptic	$\frac{\sqrt{(b\,x^{3}+a)ex}\left(\frac{B\sqrt{be\,x^{4}+aex}}{2be}+\frac{2\left(A-\frac{aB}{4b}\right)\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)}{b}\right.}{(}\right.}{}$
default	Expression too large to display

input
```
int((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*B/b*x*(b*x^3+a)^(1/2)/(e*x)^(1/2)+1/2*(4*A*b-B*a)*(1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-
a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^
(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3
)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

3.547.5 Fricas [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b*e*x^4 + a*e*x), x)
```

3.547.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.38

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\frac{A\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{1}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\sqrt{e}\Gamma\left(\frac{7}{6}\right)}+\frac{Bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\sqrt{e}\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((B*x**3+A)/(e*x)**(1/2)/(b*x**3+a)**(1/2),x)
```

output
```
A*sqrt(x)*gamma(1/6)*hyper((1/6, 1/2), (7/6,), b*x**3*exp_polar(I*pi)/a)/(
3*sqrt(a)*sqrt(e)*gamma(7/6)) + B*x**(7/2)*gamma(7/6)*hyper((1/2, 7/6), (1
3/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*sqrt(e)*gamma(13/6))
```

3.547.7 Maxima [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*sqrt(e*x)), x)
```

3.547.8 Giac [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*sqrt(e*x)), x)
```

3.547.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{\sqrt{e\,x}\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(1/2)), x)
```

3.548 $\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx$

3.548.1 Optimal result

Integrand size = 26, antiderivative size = 542

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx=-\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}+\frac{\left(1+\sqrt{3}\right)(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{ab^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\left(1-\sqrt{3}\right)(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Bigg(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2}{2\sqrt[4]{3}a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(1/2)+(2*A*b+B*a)*(1+3^(1/2))*(e*x)^(1/2)*(
b*x^3+a)^(1/2)/a/b^(2/3)/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-3^(1/4)*(2*A*
b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1
/2)/a^(2/3)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^
(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/6*(2*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*
((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2
)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Elliptic
F((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(
2/3)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b
^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.548.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx = \frac{x\left(-10A(a+bx^3)+2(2Ab+aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right.\right.}{5a(ex)^{3/2}\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(3/2)*Sqrt[a + b*x^3]),x]
```

output
```
(x*(-10*A*(a + b*x^3) + 2*(2*A*b + a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeome
tric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]))/(5*a*(e*x)^(3/2)*Sqrt[a + b*x^3])
```

3.548.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {955, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(aB+2Ab)\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx}{ae^3}-\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{851}$$

$$\frac{2(aB+2Ab)\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{ae^4}-\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{837}$$

$$\frac{2(aB+2Ab)\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{ae^4}-\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2(aB+2Ab)\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{ae^4}-\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{766}$$

$$\frac{2(aB+2Ab)\Bigg(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\Bigg(\text{arccos}}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{ae^4}$$

$$\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{2420}$$

$$\frac{2(aB+2Ab)\left(\frac{\frac{(1+\sqrt{3})e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+(1+\sqrt{3})\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+(1+\sqrt{3})\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}xe+\sqrt[3]{a}e}{(1+\sqrt{3})\sqrt[3]{b}xe+\sqrt[3]{a}e}\right)|\frac{1}{4}(2\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+(1+\sqrt{3})\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}}\right.}{ae^4}$$

$$\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}$$

input
```
Int[(A + B*x^3)/((e*x)^(3/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*A*Sqrt[a + b*x^3])/(a*e*Sqrt[e*x]) + (2*(2*A*b + a*B)*((((1 + Sqrt[3])
*e^3*Sqrt[e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) -
(3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 -
 a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/
3)*e*x)^2]*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/
3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*(
a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[
a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e +
 b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)
/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e +
(1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 +
Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x
))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(a*e^4)
```

3.548. $\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx$

3.548.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.548. $\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx$

3.548.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.96 (sec) , antiderivative size = 1119, normalized size of antiderivative = 2.06

method	result	size
risch	Expression too large to display	1119
elliptic	Expression too large to display	1132
default	Expression too large to display	5385

input

```
int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(1/2)+(2*A*b+B*a)/a*(x*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*
(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(
((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^...
```

3.548.5 Fricas [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b*e^2*x^5 + a*e^2*x^2), x)
```

3.548.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.18

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx=\frac{A\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{6},\frac{1}{2}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)}+\frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

input
```
integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(1/2),x)
```

output
```
A*gamma(-1/6)*hyper((-1/6, 1/2), (5/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*e**(3/2)*sqrt(x)*gamma(5/6)) + B*x**(5/2)*gamma(5/6)*hyper((1/2, 5/6),
 (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*e**(3/2)*gamma(11/6))
```

3.548.7 Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(3/2)), x)
```

3.548.8 Giac [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(3/2)), x)
```

3.548.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx = \int \frac{B\,x^3+A}{(e\,x)^{3/2}\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(1/2)), x)
```

3.549 $\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx$

3.549.1 Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx=-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{2B\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}}$$

output
```
2/3*B*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/e^(5/2)/b^(1/2)
-2/3*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(3/2)
```

3.549.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx=\frac{2x\left(-\frac{A\sqrt{a+bx^3}}{a}+\frac{Bx^{3/2}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{\sqrt{b}}\right)}{3(ex)^{5/2}}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

output
```
(2*x*(-((A*Sqrt[a + b*x^3])/a) + (B*x^(3/2)*Log[Sqrt[b]*x^(3/2) + Sqrt[a +
 b*x^3]])/Sqrt[b]))/(3*(e*x)^(5/2))
```

3.549.3 Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {953, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \; 953$$

$$\frac{B\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{e^3}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \; 851$$

$$\frac{2B\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{e^4}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \; 807$$

$$\frac{2B\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3e^4}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \; 224$$

$$\frac{2B\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3e^4}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{2B\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3\sqrt{b}e^{5/2}}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

input
```
Int[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*A*Sqrt[a + b*x^3])/(3*a*e*(e*x)^(3/2)) + (2*B*ArcTanh[(Sqrt[b]*(e*x)^(
3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*Sqrt[b]*e^(5/2))
```

3.549.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 953

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[d/e^n   Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] &&
(IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && G
tQ[m + n, -1]))
```

3.549.4 Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

method	result	size
risch	$-\frac{2A\sqrt{b\,x^3+a}}{3ax\,e^2\sqrt{ex}}+\frac{2B\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^3+a\right)ex}}{x^2\sqrt{be}}\right)\sqrt{\left(b\,x^3+a\right)ex}}{3\sqrt{be}\,e^2\sqrt{ex}\,\sqrt{b\,x^3+a}}$	87
default	$-\frac{2\sqrt{b\,x^3+a}\left(-B\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^3+a\right)ex}}{x^2\sqrt{be}}\right)ae\,x^2+A\sqrt{\left(b\,x^3+a\right)ex}\,\sqrt{be}\right)}{3x\,e^2\sqrt{ex}\,\sqrt{\left(b\,x^3+a\right)ex}\,a\sqrt{be}}$	93
elliptic	Expression too large to display	1037

input
```
int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/a*A*(b*x^3+a)^(1/2)/x/e^2/(e*x)^(1/2)+2/3*B/(b*e)^(1/2)*arctanh(((b*x
^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))/e^2*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b
*x^3+a)^(1/2)
```

3.549.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.44

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx = \left[\frac{\sqrt{be}Bax^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\sqrt{be}\sqrt{ex}\right)-4}{6\,abe^3x^2}\right.$$

$$\left.-\frac{\sqrt{-be}Bax^2\arctan\left(\frac{2\,\sqrt{bx^3+a}\sqrt{-be}\sqrt{ex}x}{2\,bex^3+ae}\right)+2\,\sqrt{bx^3+a}\sqrt{ex}Ab}{3\,abe^3x^2}\right]$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(sqrt(b*e)*B*a*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b*x^
4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) - 4*sqrt(b*x^3 + a)*sqrt(e*x
)*A*b)/(a*b*e^3*x^2), -1/3*(sqrt(-b*e)*B*a*x^2*arctan(2*sqrt(b*x^3 + a)*sq
rt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) + 2*sqrt(b*x^3 + a)*sqrt(e*x)*A*b)
/(a*b*e^3*x^2)]
```

3.549.6 Sympy [A] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ae^{\frac{5}{2}}}+\frac{2B\,\mathrm{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}e^{\frac{5}{2}}}$$

input
```
integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(1/2),x)
```

output
```
-2*A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*e**(5/2)) + 2*B*asinh(sqrt(b)*x**(3
/2)/sqrt(a))/(3*sqrt(b)*e**(5/2))
```

3.549.7 Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
B*integrate(sqrt(x)/sqrt(b*x^3 + a), x)/e^(5/2) - 2/3*(b*sqrt(e)*x^4 + a*s
qrt(e)*x)*A/(sqrt(b*x^3 + a)*a*e^3*x^(5/2))
```

3.549.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx=-\frac{2\,e\left(\frac{\frac{B\arctan\left(\frac{\sqrt{be+\frac{ae}{x^3}}}{\sqrt{-be}}\right)}{\sqrt{-be}}+\frac{\sqrt{be+\frac{ae}{x^3}}\,A}{ae}}{e}-\frac{Bae\arctan\left(\frac{\sqrt{be}}{\sqrt{-be}}\right)+\sqrt{be}\sqrt{-be}A}{\sqrt{-be}ae^2}\right)}{3\,|e|^2}$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
-2/3*e*((B*arctan(sqrt(b*e + a*e/x^3)/sqrt(-b*e))/sqrt(-b*e) + sqrt(b*e +
a*e/x^3)*A/(a*e))/e - (B*a*e*arctan(sqrt(b*e)/sqrt(-b*e)) + sqrt(b*e)*sqrt
(-b*e)*A)/(sqrt(-b*e)*a*e^2))/abs(e)^2
```

3.549.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{{(e\,x)}^{5/2}\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(1/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(1/2)), x)
```

3.550 $\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx$

3.550.1 Optimal result

Integrand size = 26, antiderivative size = 246

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = -\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}} - \frac{(2Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/5*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(5/2)-1/15*(2*A*b-5*B*a)*(a^(1/3)+b^(1/3)
*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^
(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Elli
pticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))
^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/e^4
/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1
/2)))^2)^(1/2)
```

3.550.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.33

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = -\frac{2x\left(A(a+bx^3)+(2Ab-5aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{5a(ex)^{7/2}\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(7/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*x*(A*(a + b*x^3) + (2*A*b - 5*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeomet
ric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(5*a*(e*x)^(7/2)*Sqrt[a + b*x^3])
```

3.550.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {955, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(2Ab-5aB)\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{5ae^3}-\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{851}$$

$$-\frac{2(2Ab-5aB)\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{5ae^4}-\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{766}$$

$$-\frac{\sqrt{ex}(2Ab-5aB)\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}(2+\sqrt{\ }\right.}{5\sqrt[4]{3}a^{4/3}e^5\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}$$

$$\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}$$

input
```
Int[(A + B*x^3)/((e*x)^(7/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*A*Sqrt[a + b*x^3])/(5*a*e*(e*x)^(5/2)) - ((2*A*b - 5*a*B)*Sqrt[e*x]*(a
^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3
)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^
(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*
x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*e^5*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e
 + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3
])
```

3.550.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.550.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.94 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.01

method	result
risch	$-\frac{2A\sqrt{b\,x^{3}+a}}{5a\,x^{2}e^{3}\sqrt{ex}}-\frac{2(2Ab-5Ba)\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}}{5a\left(-\frac{3\left(-a\,b^{2}\right)}{2b}\right)}$
elliptic	$\frac{\sqrt{(b\,x^{3}+a)ex}\left(-\frac{2A\sqrt{be\,x^{4}+aex}}{5e^{4}a\,x^{3}}+\frac{2\left(\frac{B}{e^{3}}-\frac{2bA}{5a\,e^{3}}\right)\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\right)}{}\right)}{}$
default	Expression too large to display

input
```
int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/a*A*(b*x^3+a)^(1/2)/x^2/e^3/(e*x)^(1/2)-2/5*(2*A*b-5*B*a)/a*(1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*
b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1
/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^
2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/
(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2))/e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1
/2)
```

3.550.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.24

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = -\frac{2\left((5\,Ba-2\,Ab)\sqrt{ae}x^3\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)+\sqrt{bx^3+a}\sqrt{ex}Aa\right)}{5\,a^2e^4x^3}$$

input
```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*((5*B*a - 2*A*b)*sqrt(a*e)*x^3*weierstrassPInverse(0, -4*b/a, 1/x) +
sqrt(b*x^3 + a)*sqrt(e*x)*A*a)/(a^2*e^4*x^3)
```

3.550.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx=\frac{A\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}+\frac{B\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{1}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}$$

input

```
integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(1/2),x)
```

output

```
A*gamma(-5/6)*hyper((-5/6, 1/2), (1/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*e**(7/2)*x**(5/2)*gamma(1/6)) + B*sqrt(x)*gamma(1/6)*hyper((1/6, 1/2),
 (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*e**(7/2)*gamma(7/6))
```

3.550.7 Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{7}{2}}}\,dx$$

input

```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output

```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(7/2)), x)
```

3.550.8 Giac [F]

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{7}{2}}}\,dx$$

input

```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output

```
integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(7/2)), x)
```

3.550.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{{\left(e\,x\right)}^{7/2}\,\sqrt{b\,x^3+a}}\,dx$$

input

```
int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(1/2)),x)
```

output

```
int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(1/2)), x)
```

3.551 $\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.551.1 Optimal result

Integrand size = 26, antiderivative size = 120

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=-\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}+\frac{(2Ab-3aB)e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

output

```
1/3*(2*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1
/2))/b^(5/2)-1/3*(2*A*b-3*B*a)*e^2*(e*x)^(3/2)/b^2/(b*x^3+a)^(1/2)+1/3*B*(
e*x)^(9/2)/b/e/(b*x^3+a)^(1/2)
```

3.551.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{(ex)^{7/2}\left(\frac{\sqrt{b}x^{3/2}(-2Ab+3aB+bBx^3)}{\sqrt{a+bx^3}}+(2Ab-3aB)\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{3b^{5/2}x^{7/2}}$$

input

```
Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
((e*x)^(7/2)*((Sqrt[b]*x^(3/2)*(-2*A*b + 3*a*B + b*B*x^3))/Sqrt[a + b*x^3]
 + (2*A*b - 3*a*B)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(3*b^(5/2)*x^(
7/2))
```

3.551.3 Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {959, 817, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(2Ab-3aB)\int \frac{(ex)^{7/2}}{\left(bx^3+a\right)^{3/2}}dx}{2b}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(2Ab-3aB)\left(\frac{e^3\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{2b}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{851}$$

$$\frac{(2Ab-3aB)\left(\frac{2e^2\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{2b}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{807}$$

$$\frac{(2Ab-3aB)\left(\frac{2e^2\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{2b}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{224}$$

$$\frac{(2Ab-3aB)\left(\frac{2e^2\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{2b}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(2Ab-3aB)\left(\frac{2e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{2b}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

input
```
Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(B*(e*x)^(9/2))/(3*b*e*Sqrt[a + b*x^3]) + ((2*A*b - 3*a*B)*((-2*e^2*(e*x)^
(3/2))/(3*b*Sqrt[a + b*x^3]) + (2*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e
^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*b^(3/2))))/(2*b)
```

3.551.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.551.4 Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08

method	result
risch	$\frac{B x^{2} \sqrt{b x^{3}+a}\, e^{4}}{3 b^{2} \sqrt{e x}}+\frac{\left(\frac{2 \left(2 A b -3 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{\left(b \,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right)}{3 \sqrt{b e}}-\frac{4 \left(A b -B a \right) x^{2}}{3 \sqrt{\left(x^{3}+\frac{a}{b}\right) b e x}}\right) e^{4} \sqrt{\left(b \,x^{3}+a \right) e x}}{2 b^{2} \sqrt{e x}\, \sqrt{b \,x^{3}+a}}$
default	$\frac{e^{3} \sqrt{e x}\, \left(B \sqrt{b e}\, b \,x^{5}-2 A \sqrt{b e}\, b \,x^{2}+3 B \sqrt{b e}\, a \,x^{2}+2 A \sqrt{\left(b \,x^{3}+a \right) e x}\, \operatorname{arctanh}\left(\frac{\sqrt{\left(b \,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) b -3 B \sqrt{\left(b \,x^{3}+a \right) e x}\, \operatorname{arctanh}\left(\frac{\sqrt{\left(b \,x^{3}+a \right) e}}{x^{2} \sqrt{b e}}\right.\right.}{3 x \sqrt{b \,x^{3}+a}\, b^{2} \sqrt{b e}}$
elliptic	Expression too large to display

input
```
int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*B*x^2/b^2*(b*x^3+a)^(1/2)*e^4/(e*x)^(1/2)+1/2/b^2*(2/3*(2*A*b-3*B*a)/(
b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))-4/3*(A*b-B*a)*x^
2/((x^3+a/b)*b*e*x)^(1/2))*e^4*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)
^(1/2)
```

3.551.5 Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.56

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\left[-\frac{\left((3\,Bab-2\,Ab^2)e^3x^3+(3\,Ba^2-2\,Aab)e^3\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e\right.}{12\,(}\right.$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(((3*B*a*b - 2*A*b^2)*e^3*x^3 + (3*B*a^2 - 2*A*a*b)*e^3)*sqrt(e/b)*
log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3
+ a)*sqrt(e*x)*sqrt(e/b)) - 4*(B*b*e^3*x^4 + (3*B*a - 2*A*b)*e^3*x)*sqrt(b
*x^3 + a)*sqrt(e*x))/(b^3*x^3 + a*b^2), 1/6*(((3*B*a*b - 2*A*b^2)*e^3*x^3
+ (3*B*a^2 - 2*A*a*b)*e^3)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b
*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(B*b*e^3*x^4 + (3*B*a - 2*A*b)*e^3*x)
*sqrt(b*x^3 + a)*sqrt(e*x))/(b^3*x^3 + a*b^2)]
```

3.551.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
Timed out
```

3.551.7 Maxima [F]

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{7}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(7/2)/(b*x^3 + a)^(3/2), x)
```

3.551.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{\left(\frac{Be^4x^3}{b}+\frac{3\,Bab^3e^4-2\,Ab^4e^4}{b^5}\right)\sqrt{ex}ex}{3\,\sqrt{be^4x^3+ae^4}}+\frac{\left(3\,Bab^3e^4-2\,Ab^4e^4\right)e^2\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{3\,\sqrt{be}b^5|e|^2}$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
1/3*(B*e^4*x^3/b + (3*B*a*b^3*e^4 - 2*A*b^4*e^4)/b^5)*sqrt(e*x)*e*x/sqrt(b
*e^4*x^3 + a*e^4) + 1/3*(3*B*a*b^3*e^4 - 2*A*b^4*e^4)*e^2*log(abs(-sqrt(b*
e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(sqrt(b*e)*b^5*abs(e)^2)
```

3.551.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}}{{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(3/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(3/2), x)
```

3.552 $\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.552.1 Optimal result

Integrand size = 26, antiderivative size = 286

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=-\frac{(4Ab-7aB)e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}}+\frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}}+\frac{(4Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{12\sqrt[4]{3}\sqrt[3]{a}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/2*B*(e*x)^(7/2)/b/e/(b*x^3+a)^(1/2)-1/6*(4*A*b-7*B*a)*e^2*(e*x)^(1/2)/b^
2/(b*x^3+a)^(1/2)+1/36*(4*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)
+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(
1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^
(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^2/(b*x^3+a)^(1/2)/
(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.552.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.30

$$\int \frac{(ex)^{5/2} (A + Bx^3)}{(a + bx^3)^{3/2}}\, dx = \frac{e^2\sqrt{ex}\Big(-4Ab + 7aB + 3bBx^3 + (4Ab - 7aB)\sqrt{1 + \frac{bx^3}{a}}\, \text{Hypergeometric2F1}\left(\frac{1}{6}\right.}{6b^2\sqrt{a + bx^3}}$$

input
```
Integrate[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(e^2*Sqrt[e*x]*(-4*A*b + 7*a*B + 3*b*B*x^3 + (4*A*b - 7*a*B)*Sqrt[1 + (b*x
^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(6*b^2*Sqrt[a + b*
x^3])
```

3.552.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {959, 817, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2} (A + Bx^3)}{(a + bx^3)^{3/2}}\, dx$$

$$\Big\downarrow \ 959$$

$$\frac{(4Ab - 7aB)\int \frac{(ex)^{5/2}}{(bx^3+a)^{3/2}}dx}{4b} + \frac{B(ex)^{7/2}}{2be\sqrt{a + bx^3}}$$

$$\Big\downarrow \ 817$$

$$\frac{(4Ab - 7aB)\left(\frac{e^3\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{3b} - \frac{2e^2\sqrt{ex}}{3b\sqrt{a+bx^3}}\right)}{4b} + \frac{B(ex)^{7/2}}{2be\sqrt{a + bx^3}}$$

$$\Big\downarrow \ 851$$

$$\frac{(4Ab - 7aB)\left(\frac{2e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{3b} - \frac{2e^2\sqrt{ex}}{3b\sqrt{a+bx^3}}\right)}{4b} + \frac{B(ex)^{7/2}}{2be\sqrt{a + bx^3}}$$

$$\Big\downarrow 766$$

$$\frac{(4Ab-7aB)\left(\frac{e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}\sqrt[3]{a}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}-\frac{2e^2\sqrt{e}}{3b\sqrt{a+b}}}{4b}$$

$$\frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}}$$

input
```
Int[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(B*(e*x)^(7/2))/(2*b*e*Sqrt[a + b*x^3]) + ((4*A*b - 7*a*B)*((-2*e^2*Sqrt[e
*x])/(3*b*Sqrt[a + b*x^3]) + (e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(
a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + S
qrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3
)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(3*3^(1
/4)*a^(1/3)*b*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1
 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(4*b)
```

3.552.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.552.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.91 (sec) , antiderivative size = 787, normalized size of antiderivative = 2.75

method	result	size
elliptic	Expression too large to display	787
risch	Expression too large to display	2115
default	Expression too large to display	3760

input
```
int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(b*x^3+a)^(1/2)*((b*x^3+a)*e*x)^(1/2)*(-2/3/b^2*e^3*x*(A
*b-B*a)/((x^3+a/b)*b*e*x)^(1/2)+1/2*B*e^2/b^2*(b*e*x^4+a*e*x)^(1/2)+2*(1/3
*(A*b-B*a)*e^3/b^2-1/4*B/b^2*e^3*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/
(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/
3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*
(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.552.5 Fricas [F]

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*x^6 + 2*a*
b*x^3 + a^2), x)
```

3.552.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
Timed out
```

3.552.7 Maxima [F]

$$\int \frac{(ex)^{5/2} (A+Bx^3)}{(a+bx^3)^{3/2}}\, dx = \int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{(bx^3+a)^{\frac{3}{2}}}\, dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(3/2), x)
```

3.552.8 Giac [F]

$$\int \frac{(ex)^{5/2} (A+Bx^3)}{(a+bx^3)^{3/2}}\, dx = \int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{(bx^3+a)^{\frac{3}{2}}}\, dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(3/2), x)
```

3.552.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2} (A+Bx^3)}{(a+bx^3)^{3/2}}\, dx = \int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}}{{\left(b\,x^3+a\right)}^{3/2}}\, dx$$

input
```
int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(3/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(3/2), x)
```

3.553 $\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.553.1 Optimal result

Integrand size $= 26$, antiderivative size $= 553$

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} - \frac{\left(1+\sqrt{3}\right)(2Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{3ab^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$
$$+\frac{(2Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{\left(1-\sqrt{3}\right)(2Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Bigg(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}}{6\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*(A*b-B*a)*(e*x)^(5/2)/a/b/e/(b*x^3+a)^(1/2)-1/3*(2*A*b-5*B*a)*e*(1+3^(
1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)+1/3*(2*A*b-5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))
^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2))
)*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+
3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(
1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+1/18*(2*A*b-5*B*a
)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3
)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(
1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2
)))^2)^(1/2)*3^(3/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b
^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.553.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.14

$$\int \frac{(ex)^{3/2}\,(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{x(ex)^{3/2}\left(5aB+(2Ab-5aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{5ab\sqrt{a+bx^3}}$$

input
```
Integrate[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(x*(e*x)^(3/2)*(5*a*B + (2*A*b - 5*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[5/6, 3/2, 11/6, -((b*x^3)/a)]))/(5*a*b*Sqrt[a + b*x^3])
```

3.553.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {957, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \textbf{957}$$

$$\frac{2(ex)^{5/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}-\frac{(2Ab-5aB)\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx}{3ab}$$

$$\Big\downarrow \textbf{851}$$

$$\frac{2(ex)^{5/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}-\frac{2(2Ab-5aB)\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{3abe}$$

$$\Big\downarrow \textbf{837}$$

$$\frac{2(ex)^{5/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}-\frac{2(2Ab-5aB)\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3abe}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{2(ex)^{5/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}-\frac{2(2Ab-5aB)\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3abe}$$

$$\Big\downarrow \textbf{766}$$

$$\frac{2(ex)^{5/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}-$$

$$\frac{2(2Ab-5aB)\Bigg(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\Bigg(\text{arcco}}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{3abe}$$

$$\Big\downarrow \textbf{2420}$$

$$\frac{2(ex)^{5/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}-$$

$$\frac{2(2Ab-5aB)\left(\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right)|\frac{1}{4}\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}}\right.}{3ab}$$

input
```
Int[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output
```
(2*(A*b - a*B)*(e*x)^(5/2))/(3*a*b*e*Sqrt[a + b*x^3]) - (2*(2*A*b - 5*a*B)
*((((1 + Sqrt[3])*e^3*Sqrt[e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3]
)*b^(1/3)*e*x) - (3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sq
rt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1
 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^
(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(Sq
rt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1
/3)*e*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[
e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x +
 b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[Arc
Cos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(
1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3
)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*
x^3])))/(3*a*b*e)
```

3.553. $\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.553.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.553. $\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

3.553.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 1154, normalized size of antiderivative = 2.09

method	result	size
elliptic	Expression too large to display	1154
default	Expression too large to display	5392

input

```
int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/e/x*(e*x)^(1/2)/(b*x^3+a)^(1/2)*((b*x^3+a)*e*x)^(1/2)*(2/3/b*e^2*x^3/a*(
A*b-B*a)/((x^3+a/b)*b*e*x)^(1/2)+(B*e^2/b-2/3*(A*b-B*a)/a/b*e^2)*(x*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))
^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(
2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1
/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)
))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Elliptic
E(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a...
```

3.553.5 Fricas [F]

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((B*e*x^4 + A*e*x)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*x^6 + 2*a*b*x^3
+ a^2), x)
```

3.553.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 77.73 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.17

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{6},\frac{3}{2}\\ \frac{11}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}+\frac{Be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)}$$

input
```
integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

output
```
A*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((5/6, 3/2), (11/6,), b*x**3*exp_polar
(I*pi)/a)/(3*a**(3/2)*gamma(11/6)) + B*e**(3/2)*x**(11/2)*gamma(11/6)*hype
r((3/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(17/6))
```

3.553.7 Maxima [F]

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(3/2), x)
```

3.553.8 Giac [F]

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(3/2), x)
```

3.553.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}}{{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(3/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(3/2), x)
```

3.554 $\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

3.554.1 Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}} + \frac{2B\sqrt{e}\,\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

output

```
2/3*B*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/2)/b^(3/2)
+2/3*(A*b-B*a)*(e*x)^(3/2)/a/b/e/(b*x^3+a)^(1/2)
```

3.554.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{ex}\left(\frac{\sqrt{b}(Ab-aB)x^{3/2}}{a\sqrt{a+bx^3}} + B\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{3b^{3/2}\sqrt{x}}$$

input

```
Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output

```
(2*Sqrt[e*x]*((Sqrt[b]*(A*b - a*B)*x^(3/2))/(a*Sqrt[a + b*x^3]) + B*Log[Sq
rt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(3*b^(3/2)*Sqrt[x])
```

3.554.3 Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {954, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{954}$$

$$\frac{B\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{b}+\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{851}$$

$$\frac{2B\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{be}+\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{807}$$

$$\frac{2B\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3be}+\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{224}$$

$$\frac{2B\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3be}+\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}+\frac{2B\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}$$

input

```
Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

output

```
(2*(A*b - a*B)*(e*x)^(3/2))/(3*a*b*e*Sqrt[a + b*x^3]) + (2*B*Sqrt[e]*ArcTa
nh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*b^(3/2))
```

3.554.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

3.554.4 Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2\sqrt{ex}\left(A\sqrt{be}\,b\,x^{2}-B\sqrt{be}\,a\,x^{2}+B\sqrt{\left(b\,x^{3}+a\right)ex}\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)a\right)}{3\sqrt{b\,x^{3}+a}\,x b\sqrt{be}\,a}$	92
elliptic	Expression too large to display	1050

input
```
int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(e*x)^(1/2)/(b*x^3+a)^(1/2)*(A*(b*e)^(1/2)*b*x^2-B*(b*e)^(1/2)*a*x^2+B
*((b*x^3+a)*e*x)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*a)/x
/b/(b*e)^(1/2)/a
```

3.554.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.75

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \left[-\frac{4\sqrt{bx^3+a}(Ba-Ab)\sqrt{ex}x-(Babx^3+Ba^2)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e\right.}{6\,(ab^2x^3+a^2b)}\right.$$

$$\left.-\frac{2\sqrt{bx^3+a}(Ba-Ab)\sqrt{ex}x+(Babx^3+Ba^2)\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{ex}bx\sqrt{-\frac{e}{b}}}{2\,bex^3+ae}\right)}{3\,(ab^2x^3+a^2b)}\right]$$

input
```
integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/6*(4*sqrt(b*x^3 + a)*(B*a - A*b)*sqrt(e*x)*x - (B*a*b*x^3 + B*a^2)*sqr
t(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt
(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)))/(a*b^2*x^3 + a^2*b), -1/3*(2*sqrt(b*x^3
+ a)*(B*a - A*b)*sqrt(e*x)*x + (B*a*b*x^3 + B*a^2)*sqrt(-e/b)*arctan(2*sqr
t(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)))/(a*b^2*x^3 + a^2
*b)]
```

3.554.6 Sympy [A] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2A\sqrt{e}x^{\frac{3}{2}}}{3a^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+B\left(\frac{2\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}}-\frac{2\sqrt{e}x^{\frac{3}{2}}}{3\sqrt{a}b\sqrt{1+\frac{bx^3}{a}}}\right)$$

input
```
integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(3/2),x)
```

output
```
2*A*sqrt(e)*x**(3/2)/(3*a**(3/2)*sqrt(1 + b*x**3/a)) + B*(2*sqrt(e)*asinh(
sqrt(b)*x**(3/2)/sqrt(a))/(3*b**(3/2)) - 2*sqrt(e)*x**(3/2)/(3*sqrt(a)*b*s
qrt(1 + b*x**3/a)))
```

3.554.7 Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{ex}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(e*x)/(b*x^3 + a)^(3/2), x)
```

3.554.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2\,Be^3\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{3\,\sqrt{be}b|e|^2} - \frac{2\,(Bae-Abe)\sqrt{ex}ex}{3\,\sqrt{be^4x^3+ae^4}ab}$$

input
```
integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
-2/3*B*e^3*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(s
qrt(b*e)*b*abs(e)^2) - 2/3*(B*a*e - A*b*e)*sqrt(e*x)*e*x/(sqrt(b*e^4*x^3 +
 a*e^4)*a*b)
```

3.554.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{e\,x}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(3/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(3/2), x)
```

3.555 $\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{3/2}}\,dx$

3.555.1 Optimal result

Integrand size = 26, antiderivative size = 258

$$\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{3/2}}\,dx=\frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}}+\frac{(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*(A*b-B*a)*(e*x)^(1/2)/a/b/e/(b*x^3+a)^(1/2)+1/9*(2*A*b+B*a)*(a^(1/3)+b
^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(
1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/
3)/b/e/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(
1+3^(1/2)))^2)^(1/2)
```

3.555.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx=\frac{2x\left(Ab-aB+(2Ab+aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{3ab\sqrt{ex}\sqrt{a+bx^3}}$$

input

```
Integrate[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(3/2)),x]
```

output

```
(2*x*(A*b - a*B + (2*A*b + a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6,
 1/2, 7/6, -((b*x^3)/a)]))/(3*a*b*Sqrt[e*x]*Sqrt[a + b*x^3])
```

3.555.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {957, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB+2Ab)\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{3ab}+\frac{2\sqrt{ex}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{851}$$

$$\frac{2(aB+2Ab)\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{3abe}+\frac{2\sqrt{ex}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{766}$$

$$\frac{\sqrt{ex}(aB+2Ab)\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}(2+\sqrt{3})}{3\sqrt[4]{3}a^{4/3}be^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}$$

$$\frac{2\sqrt{ex}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

input

```
Int[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(3/2)),x]
```

output

```
(2*(A*b - a*B)*Sqrt[e*x])/(3*a*b*e*Sqrt[a + b*x^3]) + ((2*A*b + a*B)*Sqrt[
e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x +
 b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[Arc
Cos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(
1/3)*e*x)], (2 + Sqrt[3])/4])/(3*3^(1/4)*a^(4/3)*b*e^2*Sqrt[(b^(1/3)*e*x*(
a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[
a + b*x^3])
```

3.555.3.1 Defintions of rubi rules used

rule 766

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.555.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.79 (sec) , antiderivative size = 754, normalized size of antiderivative = 2.92

method	result
elliptic	$\frac{\sqrt{(b x^3+a)ex}\left(\frac{2x(Ab-Ba)}{3ba\sqrt{\left(x^3+\frac{a}{b}\right)bex}}+\frac{2\left(\frac{B}{b}+\frac{\frac{2Ab}{3}-\frac{2Ba}{3}}{ab}\right)\left(\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(2/3/b*x/a*(A*b-B*a)/((x
^3+a/b)*b*e*x)^(1/2)+2*(B/b+2/3*(A*b-B*a)/a/b)*(1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(
-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^
(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*
e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*El
lipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/
2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)))
```

3.555.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx =$$
$$-\frac{2\left(((Bab+2\,Ab^2)x^3+Ba^2+2\,Aab)\sqrt{ae}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)+\sqrt{bx^3+a}(Ba^2-Aab)\sqrt{ex}\right)}{3\,(a^2b^2ex^3+a^3be)}$$

input

```
integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

output

```
-2/3*(((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*sqrt(a*e)*weierstrassPInve
rse(0, -4*b/a, 1/x) + sqrt(b*x^3 + a)*(B*a^2 - A*a*b)*sqrt(e*x))/(a^2*b^2*
e*x^3 + a^3*b*e)
```

3.555.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 22.61 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx = \frac{A\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{3}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\sqrt{e}\Gamma\left(\frac{7}{6}\right)} + \frac{Bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{7}{6},\frac{3}{2}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\sqrt{e}\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((B*x**3+A)/(b*x**3+a)**(3/2)/(e*x)**(1/2),x)
```

output
```
A*sqrt(x)*gamma(1/6)*hyper((1/6, 3/2), (7/6,), b*x**3*exp_polar(I*pi)/a)/(
3*a**(3/2)*sqrt(e)*gamma(7/6)) + B*x**(7/2)*gamma(7/6)*hyper((7/6, 3/2), (
13/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*sqrt(e)*gamma(13/6))
```

3.555.7 Maxima [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*sqrt(e*x)), x)
```

3.555.8 Giac [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*sqrt(e*x)), x)
```

3.555.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx=\int \frac{B\,x^3+A}{\sqrt{e\,x}\,(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(3/2)), x)
```

3.556 $\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx$

3.556.1 Optimal result

Integrand size = 26, antiderivative size = 585

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

$$-\frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}}+\frac{2\left(1+\sqrt{3}\right)(4Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{3a^2b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$-\frac{2(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3^{3/4}a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\left(1-\sqrt{3}\right)(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\Bigg(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2}{3\sqrt[4]{3}a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-2/3*(4*A*b-B*a)*(e*x)^(5/2)/a^2/e^4/(b*x^3+a)^(1/2)-2*A/a/e/(e*x)^(1/2)/(
b*x^3+a)^(1/2)+2/3*(4*A*b-B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a^2
/b^(2/3)/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-2/3*(4*A*b-B*a)*(a^(1/3)+b^(1
/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^
2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*E
llipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2
)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5/3)/
b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3
)*x*(1+3^(1/2)))^2)^(1/2)-1/9*(4*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3
)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^
(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b^(2/3)
/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+
3^(1/2)))^2)^(1/2)
```

3.556.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx=\frac{x\left(-10aA+2(-4Ab+aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{5a^2(ex)^{3/2}\sqrt{a+bx^3}}$$

input

```
Integrate[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x]
```

output

```
(x*(-10*a*A + 2*(-4*A*b + a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[5
/6, 3/2, 11/6, -((b*x^3)/a)]))/(5*a^2*(e*x)^(3/2)*Sqrt[a + b*x^3])
```

3.556.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {955, 819, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(4Ab-aB)\int \frac{(ex)^{3/2}}{\left(bx^3+a\right)^{3/2}}dx}{ae^3}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(4Ab-aB)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{2\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx}{3a}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{851}$$

$$-\frac{(4Ab-aB)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{3ae}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{837}$$

$$-\frac{(4Ab-aB)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3ae}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{(4Ab-aB)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3ae}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 766$$

$$-\frac{(4Ab-aB)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\int \frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{Elli}}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}}{\left(\sqrt[3]{a}e+\left(1+\sqrt{}\right.\right.}}}\right.}{3ae}\right.}{ae^3}$$

$$\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 2420$$

3.556. $\int \frac{A+Bx^3}{(ex)^{3/2}(a+bx^3)^{3/2}}\,dx$

$$-\frac{(4Ab-aB)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}}\right)}{}\right)}{}$$

$$\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

input
```
Int[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x]
```

3.556. $\int \frac{A+Bx^3}{(ex)^{3/2}(a+bx^3)^{3/2}}\,dx$

output
```
(-2*A)/(a*e*Sqrt[e*x]*Sqrt[a + b*x^3]) - ((4*A*b - a*B)*((2*(e*x)^(5/2))/(
3*a*e*Sqrt[a + b*x^3]) - (4*((((1 + Sqrt[3])*e^3*Sqrt[e*x]*Sqrt[a + b*x^3]
)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) - (3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(
a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/
3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticE[ArcCos[(a
^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e
*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(
1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1
 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^
2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^
(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^
(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3
)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*
b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(3*a*e)))/(a*e^3)
```

3.556.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.556.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.20 (sec) , antiderivative size = 1177, normalized size of antiderivative = 2.01

method	result	size
elliptic	Expression too large to display	1177
risch	Expression too large to display	2209
default	Expression too large to display	5563

input
```
int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(-2*(b*e*x^3+a*e)/e^2/a^
2*A/(x*(b*e*x^3+a*e))^(1/2)-2/3/e*x^3/a^2*(A*b-B*a)/((x^3+a/b)*b*e*x)^(1/2
)+(2*b/a^2/e*A+2/3/a^2*(A*b-B*a)/e)*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))
^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+...
```

3.556.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.18

$$\int \frac{A+Bx^3}{(ex)^{3/2}\,(a+bx^3)^{3/2}}\,dx =$$

$$-\frac{2\left(\left((Bab-4\,Ab^2)x^4+(Ba^2-4\,Aab)x\right)\sqrt{ae}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,b}{a},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)\right)+\right.}{3\left(a^2b^2e^2x^4+a^3be^2x\right)}$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*sqrt(a*e)*weierstrassZ
eta(0, -4*b/a, weierstrassPInverse(0, -4*b/a, 1/x)) + sqrt(b*x^3 + a)*(B*a
^2 - A*a*b)*sqrt(e*x))/(a^2*b^2*e^2*x^4 + a^3*b*e^2*x)
```

3.556.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 42.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{6},\frac{3}{2}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)} + \frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}\frac{5}{6},\frac{3}{2}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

input
```
integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(3/2),x)
```

output
```
A*gamma(-1/6)*hyper((-1/6, 3/2), (5/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*e**(3/2)*sqrt(x)*gamma(5/6)) + B*x**(5/2)*gamma(5/6)*hyper((5/6, 3/2)
, (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*e**(3/2)*gamma(11/6))
```

3.556.7 Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(3/2)), x)
```

3.556.8 Giac [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(3/2)), x)
```

3.556.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{B\,x^3+A}{(e\,x)^{3/2}\,(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)), x)
```

3.557 $\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx$

3.557.1 Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}-\frac{2(2Ab-aB)(ex)^{3/2}}{3a^2e^4\sqrt{a+bx^3}}$$

output
```
-2/3*A/a/e/(e*x)^(3/2)/(b*x^3+a)^(1/2)-2/3*(2*A*b-B*a)*(e*x)^(3/2)/a^2/e^4
/(b*x^3+a)^(1/2)
```

3.557.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx=\frac{2x(-aA-2Abx^3+aBx^3)}{3a^2(ex)^{5/2}\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(2*x*(-(a*A) - 2*A*b*x^3 + a*B*x^3))/(3*a^2*(e*x)^(5/2)*Sqrt[a + b*x^3])
```

3.557.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {955, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(2Ab-aB)\int \frac{\sqrt{ex}}{\left(bx^3+a\right)^{3/2}}dx}{ae^3}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{2(ex)^{3/2}(2Ab-aB)}{3a^2e^4\sqrt{a+bx^3}}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}$$

input
```
Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*A)/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^3]) - (2*(2*A*b - a*B)*(e*x)^(3/2))
/(3*a^2*e^4*Sqrt[a + b*x^3])
```

3.557.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.557.4 Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{2x\left(2Ab\,x^{3}-Ba\,x^{3}+Aa\right)}{3\sqrt{b\,x^{3}+a}\,a^{2}\left(ex\right)^{\frac{5}{2}}}$	39
default	$-\frac{2\left(2Ab\,x^{3}-Ba\,x^{3}+Aa\right)}{3x\sqrt{b\,x^{3}+a}\,a^{2}e^{2}\sqrt{ex}}$	44
risch	$-\frac{2A\sqrt{b\,x^{3}+a}}{3a^{2}x\,e^{2}\sqrt{ex}}-\frac{2(Ab-Ba)x^{2}}{3a^{2}e^{2}\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$	61
elliptic	$\frac{\sqrt{\left(b\,x^{3}+a\right)ex}\,\left(-\frac{2x^{2}\left(Ab-Ba\right)}{3e^{2}a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)bex}}-\frac{2A\sqrt{be\,x^{4}+aex}}{3e^{3}a^{2}x^{2}}\right)}{\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$	88

input
```
int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(2*A*b*x^3-B*a*x^3+A*a)/(b*x^3+a)^(1/2)/a^2/(e*x)^(5/2)
```

3.557.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx=\frac{2\left((Ba-2\,Ab)x^3-Aa\right)\sqrt{bx^3+a}\sqrt{ex}}{3\left(a^2be^3x^5+a^3e^3x^2\right)}$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*((B*a - 2*A*b)*x^3 - A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(a^2*b*e^3*x^5 + a
^3*e^3*x^2)
```

3.557.6 Sympy [A] (verification not implemented)

Time = 83.59 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx = A\left(-\frac{2}{3a\sqrt{b}e^{\frac{5}{2}}x^3\sqrt{\frac{a}{bx^3}+1}}-\frac{4\sqrt{b}}{3a^2e^{\frac{5}{2}}\sqrt{\frac{a}{bx^3}+1}}\right)+\frac{2B}{3a\sqrt{b}e^{\frac{5}{2}}\sqrt{\frac{a}{bx^3}+1}}$$

input
```
integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(3/2),x)
```

output
```
A*(-2/(3*a*sqrt(b)*e**(5/2)*x**3*sqrt(a/(b*x**3) + 1)) - 4*sqrt(b)/(3*a**2
*e**(5/2)*sqrt(a/(b*x**3) + 1))) + 2*B/(3*a*sqrt(b)*e**(5/2)*sqrt(a/(b*x**
3) + 1))
```

3.557.7 Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(5/2)), x)
```

3.557.8 Giac [B] (verification not implemented)

Error detected during grading. Assigning place holder grade for now.

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx = Recursiveaassumption \geq -\frac{2\,Ae\left(\frac{\sqrt{be+\frac{ae}{x^3}}}{ae^2}-\frac{\sqrt{be}}{ae^2}\right)}{3\,a|e|^2}+\frac{2\,(Ba-Ab)\sqrt{ex}x}{3\,\sqrt{be^4x^3+ae^4}a^2e}-\frac{bignored}{e^3t_{nostep}^6}$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
Recursive*a*assumption >= -2/3*A*e*(sqrt(b*e + a*e/x^3)/(a*e^2) - sqrt(b*e
)/(a*e^2))/(a*abs(e)^2) + 2/3*(B*a - A*b)*sqrt(e*x)*x/(sqrt(b*e^4*x^3 + a*
e^4)*a^2*e) - b*ignored/(e^3*t_nostep^6)
```

3.557.9 Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx = -\frac{\left(\frac{2\,A}{3\,a\,b\,e^2}+\frac{x^3\,\left(4\,A\,b-2\,B\,a\right)}{3\,a^2\,b\,e^2}\right)\,\sqrt{b\,x^3+a}}{x^4\,\sqrt{e\,x}+\frac{a\,x\,\sqrt{e\,x}}{b}}$$

input
```
int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x)
```

output
```
-(((2*A)/(3*a*b*e^2) + (x^3*(4*A*b - 2*B*a))/(3*a^2*b*e^2))*(a + b*x^3)^(1
/2))/(x^4*(e*x)^(1/2) + (a*x*(e*x)^(1/2))/b)
```

3.558 $\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx$

3.558.1 Optimal result

Integrand size = 26, antiderivative size = 283

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}-\frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}}$$

$$-\frac{2(8Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{15\sqrt[4]{3}a^{7/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/5*A/a/e/(e*x)^(5/2)/(b*x^3+a)^(1/2)-2/15*(8*A*b-5*B*a)*(e*x)^(1/2)/a^2/
e^4/(b*x^3+a)^(1/2)-2/45*(8*A*b-5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1
/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/
2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1
/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(7/3)/e^4/(b*x^3+a)^(1/2)/(b
^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.558.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.34

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx = \frac{x\left(-2(3aA+8Abx^3-5aBx^3)+4(-8Ab+5aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2}\right.}{15a^2(ex)^{7/2}\sqrt{a+bx^3}}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(x*(-2*(3*a*A + 8*A*b*x^3 - 5*a*B*x^3) + 4*(-8*A*b + 5*a*B)*x^3*Sqrt[1 + (
b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(15*a^2*(e*x)^(
7/2)*Sqrt[a + b*x^3])
```

3.558.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {955, 819, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(8Ab-5aB)\int \frac{1}{\sqrt{ex}\left(bx^3+a\right)^{3/2}}dx}{5ae^3}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 819$$

$$-\frac{(8Ab-5aB)\left(\frac{2\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{3a}+\frac{2\sqrt{ex}}{3ae\sqrt{a+bx^3}}\right)}{5ae^3}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 851$$

$$-\frac{(8Ab-5aB)\left(\frac{4\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{3ae}+\frac{2\sqrt{ex}}{3ae\sqrt{a+bx^3}}\right)}{5ae^3}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 766$$

$$-\frac{(8Ab-5aB)\left(\frac{2\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}+\frac{2\sqrt{}}{3ae\sqrt{}}\right.}{5ae^3}$$

$$\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}$$

input
```
Int[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*A)/(5*a*e*(e*x)^(5/2)*Sqrt[a + b*x^3]) - ((8*A*b - 5*a*B)*((2*Sqrt[e*x
])/(3*a*e*Sqrt[a + b*x^3]) + (2*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(
a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + S
qrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3
)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(3*3^(1
/4)*a^(4/3)*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e +
(1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(5*a*e^3)
```

3.558.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.558.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.02 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.77

method	result	size
elliptic	Expression too large to display	784
risch	Expression too large to display	1444
default	Expression too large to display	3783

input
```
int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(-2/3/e^3*x/a^2*(A*b-B*a
)/((x^3+a/b)*b*e*x)^(1/2)-2/5/e^4/a^2*A*(b*e*x^4+a*e*x)^(1/2)/x^3+2*(-2/3/
a^2*(A*b-B*a)/e^3-2/5*b/a^2/e^3*A)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(
-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3
)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(
-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.558.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.41

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx =$$

$$-\frac{2\left(2\left((5\,Bab-8\,Ab^2)x^6+(5\,Ba^2-8\,Aab)x^3\right)\sqrt{ae}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)-\left((5\,Ba^2-8\,Aab)x^3-\right.\right.}{15\left(a^3be^4x^6+a^4e^4x^3\right)}$$

input

```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output

```
-2/15*(2*((5*B*a*b - 8*A*b^2)*x^6 + (5*B*a^2 - 8*A*a*b)*x^3)*sqrt(a*e)*wei
erstrassPInverse(0, -4*b/a, 1/x) - ((5*B*a^2 - 8*A*a*b)*x^3 - 3*A*a^2)*sqr
t(b*x^3 + a)*sqrt(e*x))/(a^3*b*e^4*x^6 + a^4*e^4*x^3)
```

3.558.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 152.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.34

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{5}{6}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{6},\frac{3}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)} + \frac{B\sqrt{x}\,\Gamma\left(\frac{1}{6}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{3}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}$$

input
```
integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(3/2),x)
```

output
```
A*gamma(-5/6)*hyper((-5/6, 3/2), (1/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*e**(7/2)*x**(5/2)*gamma(1/6)) + B*sqrt(x)*gamma(1/6)*hyper((1/6, 3/2)
, (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*e**(7/2)*gamma(7/6))
```

3.558.7 Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(7/2)), x)
```

3.558.8 Giac [F]

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(7/2)), x)
```

3.558.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{B\,x^3+A}{{\left(e\,x\right)}^{7/2}\,{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

input
```
int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)), x)
```

3.559 $\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

3.559.1 Optimal result

Integrand size = 26, antiderivative size = 114

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{2Be^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

output
```
2/9*(A*b-B*a)*(e*x)^(9/2)/a/b/e/(b*x^3+a)^(3/2)+2/3*B*e^(7/2)*arctanh((e*x
)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)-2/3*B*e^2*(e*x)^(3/2)/b^2
/(b*x^3+a)^(1/2)
```

3.559.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2e^3\sqrt{ex}\left(\frac{\sqrt{b}x^{3/2}\left(-3a^2B+Ab^2x^3-4abBx^3\right)}{a\left(a+bx^3\right)^{3/2}} + 3B\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{9b^{5/2}\sqrt{x}}$$

input
```
Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*e^3*Sqrt[e*x]*((Sqrt[b]*x^(3/2)*(-3*a^2*B + A*b^2*x^3 - 4*a*b*B*x^3))/(
a*(a + b*x^3)^(3/2)) + 3*B*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(9*b^(
5/2)*Sqrt[x])
```

3.559.3 Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {954, 817, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}dx$$

$$\Big\downarrow \textrm{954}$$

$$\frac{B\int \frac{(ex)^{7/2}}{\left(bx^3+a\right)^{3/2}}dx}{b}+\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \textrm{817}$$

$$\frac{B\left(\frac{e^3\int \frac{\sqrt{ex}}{\sqrt{bx^3+a}}dx}{b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{b}+\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \textrm{851}$$

$$\frac{B\left(\frac{2e^2\int \frac{ex}{\sqrt{bx^3+a}}d\sqrt{ex}}{b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{b}+\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \textrm{807}$$

$$\frac{B\left(\frac{2e^2\int \frac{1}{\sqrt{a+\frac{bx}{e^2}}}d(ex)^{3/2}}{3b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{b}+\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \textrm{224}$$

$$\frac{B\left(\frac{2e^2\int \frac{1}{1-\frac{bx}{e^2}}d\frac{(ex)^{3/2}}{\sqrt{a+\frac{bx}{e^2}}}}{3b}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{b}+\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \textrm{219}$$

$$\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}+\frac{B\left(\frac{2e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+\frac{bx}{e^2}}}\right)}{3b^{3/2}}-\frac{2e^2(ex)^{3/2}}{3b\sqrt{a+bx^3}}\right)}{b}$$

input
```
Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(A*b - a*B)*(e*x)^(9/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (B*((-2*e^2*(e*x
)^(3/2))/(3*b*Sqrt[a + b*x^3]) + (2*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/
(e^(3/2)*Sqrt[a + (b*x)/e^2])])/(3*b^(3/2))))/b
```

3.559.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

3.559.4 Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.38

method	result
default	$\frac{2\left(A\,b^{2}x^{5}\sqrt{be}-4Bab\,x^{5}\sqrt{be}+3B\ \operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)ab\,x^{3}\sqrt{(b\,x^{3}+a)ex}-3B\,a^{2}x^{2}\sqrt{be}+3B\ \operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)a^{2}\sqrt{(b\,x^{3}}}{9\sqrt{be}\,b^{2}a(b\,x^{3}+a)^{\frac{3}{2}}x}$
elliptic	Expression too large to display

input
```
int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(A*b^2*x^5*(b*e)^(1/2)-4*B*a*b*x^5*(b*e)^(1/2)+3*B*arctanh(((b*x^3+a)*
e*x)^(1/2)/x^2/(b*e)^(1/2))*a*b*x^3*((b*x^3+a)*e*x)^(1/2)-3*B*a^2*x^2*(b*e
)^(1/2)+3*B*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*a^2*((b*x^3+a)*
e*x)^(1/2))*(e*x)^(1/2)*e^3/(b*e)^(1/2)/b^2/a/(b*x^3+a)^(3/2)/x
```

3.559.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.03

$$\int \frac{(ex)^{7/2}\,(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \Bigg[\frac{3\,(Bab^2e^3x^6+2\,Ba^2be^3x^3+Ba^3e^3)\sqrt{\frac{e}{b}}\log\,(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,b^2}{18\,(ab^4x^6+2}$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/18*(3*(B*a*b^2*e^3*x^6 + 2*B*a^2*b*e^3*x^3 + B*a^3*e^3)*sqrt(e/b)*log(-
8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*
sqrt(e*x)*sqrt(e/b)) - 4*((4*B*a*b - A*b^2)*e^3*x^4 + 3*B*a^2*e^3*x)*sqrt(
b*x^3 + a)*sqrt(e*x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2), -1/9*(3*(B*a*
b^2*e^3*x^6 + 2*B*a^2*b*e^3*x^3 + B*a^3*e^3)*sqrt(-e/b)*arctan(2*sqrt(b*x^
3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*((4*B*a*b - A*b^2)*
e^3*x^4 + 3*B*a^2*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/(a*b^4*x^6 + 2*a^2*b^3
*x^3 + a^3*b^2)]
```

3.559.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

3.559.7 Maxima [F]

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{7}{2}}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(7/2)/(b*x^3 + a)^(5/2), x)
```

3.559.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.11

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2\,Be^6\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{3\,\sqrt{be}b^2|e|^2} - \frac{2\left(\frac{3\,Bae^8}{b^2}+\frac{(4\,Ba^5b^6e^{24}-Aa^4b^7e^{24})x^3}{a^5b^7e^{16}}\right)\sqrt{ex}ex}{9\,(be^4x^3+ae^4)^{\frac{3}{2}}}$$

input

```
integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output

```
-2/3*B*e^6*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(s
qrt(b*e)*b^2*abs(e)^2) - 2/9*(3*B*a*e^8/b^2 + (4*B*a^5*b^6*e^24 - A*a^4*b^
7*e^24)*x^3/(a^5*b^7*e^16))*sqrt(e*x)*e*x/(b*e^4*x^3 + a*e^4)^(3/2)
```

3.559.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}}{{\left(b\,x^3+a\right)}^{5/2}} \,dx$$

input

```
int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(5/2),x)
```

output

```
int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(5/2), x)
```

3.560 $\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

3.560.1 Optimal result

Integrand size = 26, antiderivative size = 299

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)(ex)^{7/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}}$$

$$+\frac{(2Ab+7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Bigg(\arccos\Bigg(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\Bigg),\frac{1}{4}\left(2+\sqrt{3}\right)}{27\sqrt[4]{3}a^{4/3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*(A*b-B*a)*(e*x)^(7/2)/a/b/e/(b*x^3+a)^(3/2)-2/27*(2*A*b+7*B*a)*e^2*(e*
x)^(1/2)/a/b^2/(b*x^3+a)^(1/2)+1/81*(2*A*b+7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*
((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2
)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Elliptic
F((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/b^2/(b*
x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)^2)^(1/2)
```

3.560.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.36

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\frac{2e^2\sqrt{ex}\Bigg(-7a^2B+Ab^2x^3-2ab(A+5Bx^3)+(2Ab+7aB)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}}{27ab^2\left(a+bx^3\right)^{3/2}}$$

input

```
Integrate[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output

```
(2*e^2*Sqrt[e*x]*(-7*a^2*B + A*b^2*x^3 - 2*a*b*(A + 5*B*x^3) + (2*A*b + 7*
a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b
*x^3)/a)]))/(27*a*b^2*(a + b*x^3)^(3/2))
```

3.560.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {957, 817, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(7aB+2Ab)\int \frac{(ex)^{5/2}}{\left(bx^3+a\right)^{3/2}}dx}{9ab}+\frac{2(ex)^{7/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(7aB+2Ab)\left(\frac{e^3\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{3b}-\frac{2e^2\sqrt{ex}}{3b\sqrt{a+bx^3}}\right)}{9ab}+\frac{2(ex)^{7/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{(7aB+2Ab)\left(\frac{2e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{3b}-\frac{2e^2\sqrt{ex}}{3b\sqrt{a+bx^3}}\right)}{9ab}+\frac{2(ex)^{7/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{766}$$

$$\frac{(7aB+2Ab)\left(\frac{e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}\sqrt[3]{a}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}-\frac{2e^2\sqrt{e}}{3b\sqrt{a+b}}\right.}{9ab}$$

$$\frac{2(ex)^{7/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

input
```
Int[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(A*b - a*B)*(e*x)^(7/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + ((2*A*b + 7*a*B)
*((-2*e^2*Sqrt[e*x])/(3*b*Sqrt[a + b*x^3]) + (e*Sqrt[e*x]*(a^(1/3)*e + b^(
1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^
(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 -
 Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt
[3])/4])/(3*3^(1/4)*a^(1/3)*b*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))
/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3]))/(9*a*b)
```

3.560.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.560.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.60 (sec) , antiderivative size = 809, normalized size of antiderivative = 2.71

method	result	size
elliptic	Expression too large to display	809
default	Expression too large to display	7083

input
```
int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(b*x^3+a)^(1/2)*((b*x^3+a)*e*x)^(1/2)*(-2/9*e^2/b^4*(A*b
-B*a)*(b*e*x^4+a*e*x)^(1/2)/(x^3+a/b)^2+2/27/b^2*e^3*x/a*(A*b-10*B*a)/((x^
3+a/b)*b*e*x)^(1/2)+2*(B*e^3/b^2+2/27/b^2/a*e^3*(A*b-10*B*a))*(1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2
)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)
*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^
(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a
*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)))
```

3.560.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.57

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx =$$

$$-\frac{2\left(\left(\left(7\,Bab^2+2\,Ab^3\right)e^2x^6+2\left(7\,Ba^2b+2\,Aab^2\right)e^2x^3+\left(7\,Ba^3+2\,Aa^2b\right)e^2\right)\sqrt{ae}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4}{a}\right.}{27\left(a^2b^4x^6+2\,a^3b^3x^3+a^4b^2\right)}$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/27*(((7*B*a*b^2 + 2*A*b^3)*e^2*x^6 + 2*(7*B*a^2*b + 2*A*a*b^2)*e^2*x^3
+ (7*B*a^3 + 2*A*a^2*b)*e^2)*sqrt(a*e)*weierstrassPInverse(0, -4*b/a, 1/x)
 + ((10*B*a^2*b - A*a*b^2)*e^2*x^3 + (7*B*a^3 + 2*A*a^2*b)*e^2)*sqrt(b*x^3
 + a)*sqrt(e*x))/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)
```

3.560.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

3.560.7 Maxima [F]

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(5/2), x)
```

3.560.8 Giac [F]

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(5/2), x)
```

3.560.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}}{{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(5/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(5/2), x)
```

3.561 $\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

3.561.1 Optimal result

Integrand size = 26, antiderivative size = 596

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}}$$

$$+\frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}}-\frac{2\left(1+\sqrt{3}\right)(4Ab+5aB)e\sqrt{ex}\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$+\frac{2(4Ab+5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\ 3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\left(1-\sqrt{3}\right)(4Ab+5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\Bigg(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
2/9*(A*b-B*a)*(e*x)^(5/2)/a/b/e/(b*x^3+a)^(3/2)+2/27*(4*A*b+5*B*a)*(e*x)^(
5/2)/a^2/b/e/(b*x^3+a)^(1/2)-2/27*(4*A*b+5*B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*
(b*x^3+a)^(1/2)/a^2/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))+2/27*(4*A*b+5*
B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1
/2)*3^(1/4)/a^(5/3)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)
/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+1/81*(4*A*b+5*B*a)*e*(a^(1/3)+b^
(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))
*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1
/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^
(3/4)/a^(5/3)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1
/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.561.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.14

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{x(ex)^{3/2}\left(-5a^2B+\left(4Ab+5aB\right)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{2},\right.\right.}{10a^2b\left(a+bx^3\right)^{3/2}}$$

input

```
Integrate[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output

```
(x*(e*x)^(3/2)*(-5*a^2*B + (4*A*b + 5*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]
*Hypergeometric2F1[5/6, 5/2, 11/6, -((b*x^3)/a)]))/(10*a^2*b*(a + b*x^3)^(
3/2))
```

3.561.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {957, 819, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2} \left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}} \, dx$$

$$\Big\downarrow \; 957$$

$$\frac{(5aB+4Ab) \int \frac{(ex)^{3/2}}{\left(bx^3+a\right)^{3/2}}dx}{9ab}+\frac{2(ex)^{5/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 819$$

$$\frac{(5aB+4Ab) \left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{2\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx}{3a}\right)}{9ab}+\frac{2(ex)^{5/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 851$$

$$\frac{(5aB+4Ab) \left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{3ae}\right)}{9ab}+\frac{2(ex)^{5/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 837$$

$$\frac{(5aB+4Ab) \left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3ae}\right)}{9ab}+\frac{2(ex)^{5/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{(5aB+4Ab)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3ae}\right)}{9ab}+\frac{2(ex)^{5/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{(5aB+4Ab)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\,\text{Ellip}}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e\right.}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right.\right.}}}\right.}{3ae}\right.}{9ab}$$

$$\frac{2(ex)^{5/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 2420}$$

3.561. $\int\frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

$$\frac{(5aB+4Ab)\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}}{2b^{2/3}}\right.}{}\right.}{}$$

$$\frac{2(ex)^{5/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

input
```
Int[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

3.561. $\int\frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

output

```
(2*(A*b - a*B)*(e*x)^(5/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + ((4*A*b + 5*a*B)
*((2*(e*x)^(5/2))/(3*a*e*Sqrt[a + b*x^3]) - (4*((((1 + Sqrt[3])*e^3*Sqrt[e
*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x) - (3^(1/4)*a^
(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^
(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*
EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 +
 Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*e*x*(a^(1/3)*e +
 b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])
)/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*
x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e
 + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3
])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4]
)/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)
*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3]))/(3*a*e)))/(9*a*b)
```

3.561.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

3.561. $\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.561.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.63 (sec) , antiderivative size = 1190, normalized size of antiderivative = 2.00

method	result	size
elliptic	Expression too large to display	1190
default	Expression too large to display	10786

input
```
int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/e/x*(e*x)^(1/2)/(b*x^3+a)^(1/2)*((b*x^3+a)*e*x)^(1/2)*(2/9*e/a/b^3*x^2*(
A*b-B*a)*(b*e*x^4+a*e*x)^(1/2)/(x^3+a/b)^2+2/27/b*e^2*x^3/a^2*(4*A*b+5*B*a
)/((x^3+a/b)*b*e*x)^(1/2)-2/27/b/a^2*e^2*(4*A*b+5*B*a)*(x*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-
1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*Ellipt
icF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),(
(3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/...
```

3.561.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.28

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx =$$

$$-\frac{2\left(\left((5\,Bab^2+4\,Ab^3)ex^7+2\,(5\,Ba^2b+4\,Aab^2)ex^4+(5\,Ba^3+4\,Aa^2b)ex\right)\sqrt{ae}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,b}{a},\mathrm{wei}\right.\right.}{27\left(a^2b^4x^7+2\,a^3b^3x^4+\right.}$$

input

```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output

```
-2/27*(((5*B*a*b^2 + 4*A*b^3)*e*x^7 + 2*(5*B*a^2*b + 4*A*a*b^2)*e*x^4 + (5
*B*a^3 + 4*A*a^2*b)*e*x)*sqrt(a*e)*weierstrassZeta(0, -4*b/a, weierstrassP
Inverse(0, -4*b/a, 1/x)) + ((8*B*a^2*b + A*a*b^2)*e*x^3 + (5*B*a^3 + 4*A*a
^2*b)*e)*sqrt(b*x^3 + a)*sqrt(e*x))/(a^2*b^4*x^7 + 2*a^3*b^3*x^4 + a^4*b^2
*x)
```

3.561.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

3.561.7 Maxima [F]

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(5/2), x)
```

3.561.8 Giac [F]

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(5/2), x)
```

3.561.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}}{{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

input
```
int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(5/2),x)
```

output
```
int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(5/2), x)
```

3.562 $\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

3.562.1 Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\frac{2(Ab-aB)(ex)^{3/2}}{9abe\,(a+bx^3)^{3/2}}+\frac{2(2Ab+aB)(ex)^{3/2}}{9a^2be\sqrt{a+bx^3}}$$

output
```
2/9*(A*b-B*a)*(e*x)^(3/2)/a/b/e/(b*x^3+a)^(3/2)+2/9*(2*A*b+B*a)*(e*x)^(3/2
)/a^2/b/e/(b*x^3+a)^(1/2)
```

3.562.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\frac{2x\sqrt{ex}(3aA+2Abx^3+aBx^3)}{9a^2\,(a+bx^3)^{3/2}}$$

input
```
Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*x*Sqrt[e*x]*(3*a*A + 2*A*b*x^3 + a*B*x^3))/(9*a^2*(a + b*x^3)^(3/2))
```

3.562.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {957, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB+2Ab)\int \frac{\sqrt{ex}}{\left(bx^3+a\right)^{3/2}}dx}{3ab}+\frac{2(ex)^{3/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{796}$$

$$\frac{2(ex)^{3/2}(aB+2Ab)}{9a^2be\sqrt{a+bx^3}}+\frac{2(ex)^{3/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

output
```
(2*(A*b - a*B)*(e*x)^(3/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(2*A*b + a*B)
*(e*x)^(3/2))/(9*a^2*b*e*Sqrt[a + b*x^3])
```

3.562.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.562.4 Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{2x\left(2Ab\,x^{3}+Ba\,x^{3}+3Aa\right)\sqrt{ex}}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{2}}$	39
default	$\frac{2x\left(2Ab\,x^{3}+Ba\,x^{3}+3Aa\right)\sqrt{ex}}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{2}}$	39
elliptic	$\frac{\sqrt{ex}\,\sqrt{\left(b\,x^{3}+a\right)ex}\,\left(\frac{2x\left(Ab-Ba\right)\sqrt{be\,x^{4}+aex}}{9a\,b^{3}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2e\,x^{2}\left(2Ab+Ba\right)}{9b\,a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)bex}}\right)}{ex\sqrt{b\,x^{3}+a}}$	111

input
```
int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x*(2*A*b*x^3+B*a*x^3+3*A*a)*(e*x)^(1/2)/(b*x^3+a)^(3/2)/a^2
```

3.562.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\left((Ba+2\,Ab)x^4+3\,Aax\right)\sqrt{bx^3+a}\sqrt{ex}}{9\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}$$

input
```
integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/9*((B*a + 2*A*b)*x^4 + 3*A*a*x)*sqrt(b*x^3 + a)*sqrt(e*x)/(a^2*b^2*x^6 +
 2*a^3*b*x^3 + a^4)
```

3.562.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

3.562.7 Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\int \frac{(Bx^3+A)\sqrt{ex}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)*sqrt(e*x)/(b*x^3 + a)^(5/2), x)
```

3.562.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\frac{2\left(\frac{3\,Ae^5}{a}+\frac{(Ba^5b^5e^{21}+2\,Aa^4b^6e^{21})x^3}{a^6b^5e^{16}}\right)\sqrt{ex}ex}{9\,(be^4x^3+ae^4)^{\frac{3}{2}}}$$

input
```
integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
2/9*(3*A*e^5/a + (B*a^5*b^5*e^21 + 2*A*a^4*b^6*e^21)*x^3/(a^6*b^5*e^16))*s
qrt(e*x)*e*x/(b*e^4*x^3 + a*e^4)^(3/2)
```

3.562.9 Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\frac{\left(\frac{2\,A\,x\,\sqrt{e\,x}}{3\,a\,b^2}+\frac{x^4\,\sqrt{e\,x}\,(4\,A\,b+2\,B\,a)}{9\,a^2\,b^2}\right)\,\sqrt{b\,x^3+a}}{x^6+\frac{a^2}{b^2}+\frac{2\,a\,x^3}{b}}$$

input
```
int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(5/2),x)
```

output
```
(((2*A*x*(e*x)^(1/2))/(3*a*b^2) + (x^4*(e*x)^(1/2)*(4*A*b + 2*B*a))/(9*a^2
*b^2))*(a + b*x^3)^(1/2))/(x^6 + a^2/b^2 + (2*a*x^3)/b)
```

3.563 $\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx$

3.563.1 Optimal result

Integrand size = 26, antiderivative size = 297

$$\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx=\frac{2(Ab-aB)\sqrt{ex}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}}$$

$$+\frac{2(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{7/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*(A*b-B*a)*(e*x)^(1/2)/a/b/e/(b*x^3+a)^(3/2)+2/27*(8*A*b+B*a)*(e*x)^(1/
2)/a^2/b/e/(b*x^3+a)^(1/2)+2/81*(8*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+
b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)
+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1
/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*
6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(7/3)/b/e/(b*x^3+a)^(1/
2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.563.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx=\frac{2x\Big(-2a^2B+8Ab^2x^3+ab(11A+Bx^3)+2(8Ab+aB)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\,\text{Hyper}}{27a^2b\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

input
```
Integrate[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(5/2)),x]
```

output
```
(2*x*(-2*a^2*B + 8*A*b^2*x^3 + a*b*(11*A + B*x^3) + 2*(8*A*b + a*B)*(a + b
*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))
/(27*a^2*b*Sqrt[e*x]*(a + b*x^3)^(3/2))
```

3.563.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {957, 819, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{(aB+8Ab)\int \frac{1}{\sqrt{ex}\left(bx^3+a\right)^{3/2}}dx}{9ab}+\frac{2\sqrt{ex}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{(aB+8Ab)\left(\frac{2\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{3a}+\frac{2\sqrt{ex}}{3ae\sqrt{a+bx^3}}\right)}{9ab}+\frac{2\sqrt{ex}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{(aB+8Ab)\left(\frac{4\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{3ae}+\frac{2\sqrt{ex}}{3ae\sqrt{a+bx^3}}\right)}{9ab}+\frac{2\sqrt{ex}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{766}$$

$$\frac{(aB+8Ab)\left(\frac{2\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}+\frac{2\sqrt{ex}}{3ae\sqrt{a+b}}\right.}{9ab}$$

$$\frac{2\sqrt{ex}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(5/2)),x]
```

output
```
(2*(A*b - a*B)*Sqrt[e*x])/(9*a*b*e*(a + b*x^3)^(3/2)) + ((8*A*b + a*B)*((2
*Sqrt[e*x])/(3*a*e*Sqrt[a + b*x^3]) + (2*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*
x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e
 + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3
])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4]
)/(3*3^(1/4)*a^(4/3)*e^2*Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(
1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a + b*x^3])))/(9*a*b)
```

3.563.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.563.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.54 (sec) , antiderivative size = 785, normalized size of antiderivative = 2.64

method	result	size
elliptic	Expression too large to display	785
default	Expression too large to display	7077

input
```
int((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(2/9/e/a/b^3*(A*b-B*a)*(
b*e*x^4+a*e*x)^(1/2)/(x^3+a/b)^2+2/27/b*x/a^2*(8*A*b+B*a)/((x^3+a/b)*b*e*x
)^(1/2)+4/27/a^2*(8*A*b+B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1
/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^
(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.563.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2\left(2\left((Bab^2+8\,Ab^3)x^6+Ba^3+8\,Aa^2b+2\left(Ba^2b+8\,Aab^2\right)x^3\right)\sqrt{ae}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)+(2\,}{27\left(a^3b^3ex^6+2\,a^4b^2ex^3+a^5be\right)}$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/27*(2*((B*a*b^2 + 8*A*b^3)*x^6 + B*a^3 + 8*A*a^2*b + 2*(B*a^2*b + 8*A*a
*b^2)*x^3)*sqrt(a*e)*weierstrassPInverse(0, -4*b/a, 1/x) + (2*B*a^3 - 11*A
*a^2*b - (B*a^2*b + 8*A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(a^3*b^3*e*
x^6 + 2*a^4*b^2*e*x^3 + a^5*b*e)
```

3.563.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/(b*x**3+a)**(5/2)/(e*x)**(1/2),x)
```

output
```
Timed out
```

3.563.7 Maxima [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*sqrt(e*x)), x)
```

3.563.8 Giac [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*sqrt(e*x)), x)
```

3.563.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx=\int \frac{B\,x^3+A}{\sqrt{e\,x}\,{(b\,x^3+a)}^{5/2}}\,dx$$

input

```
int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(5/2)),x)
```

output

```
int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(5/2)), x)
```

3.564 $\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx$

3.564.1 Optimal result

Integrand size = 26, antiderivative size = 624

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx=-\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}-\frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}}$$

$$-\frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}}+\frac{8\left(1+\sqrt{3}\right)(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{27a^3b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$-\frac{8(10Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\ 3^{3/4}a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{4\left(1-\sqrt{3}\right)(10Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}}{27\sqrt[4]{3}a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/9*(10*A*b-B*a)*(e*x)^(5/2)/a^2/e^4/(b*x^3+a)^(3/2)-2*A/a/e/(b*x^3+a)^(3
/2)/(e*x)^(1/2)-8/27*(10*A*b-B*a)*(e*x)^(5/2)/a^3/e^4/(b*x^3+a)^(1/2)+8/27
*(10*A*b-B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a^3/b^(2/3)/e^2/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))-8/27*(10*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3
)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/
3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^
(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/
4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)/b^(2/3)/e^2/(b*
x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)^2)^(1/2)-4/81*(10*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^
(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(
1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(
1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^
(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)/b^(2/3)/e^2/(b*x^3+a
)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^
(1/2)
```

3.564.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx = \frac{2x\left(-5a^2A+(-10Ab+aB)x^3(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{2},\right.\right.}{5a^3(ex)^{3/2}\left(a+bx^3\right)^{3/2}}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x]
```

output
```
(2*x*(-5*a^2*A + (-10*A*b + a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyper
geometric2F1[5/6, 5/2, 11/6, -((b*x^3)/a)]))/(5*a^3*(e*x)^(3/2)*(a + b*x^3
)^(3/2))
```

3.564.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {955, 819, 819, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{3/2} \left(a+bx^3\right)^{5/2}} \, dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(10Ab-aB)\int \frac{(ex)^{3/2}}{\left(bx^3+a\right)^{5/2}}dx}{ae^3}-\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \ 819$$

$$-\frac{(10Ab-aB)\left(\frac{4\int \frac{(ex)^{3/2}}{\left(bx^3+a\right)^{3/2}}dx}{9a}+\frac{2(ex)^{5/2}}{9ae\left(a+bx^3\right)^{3/2}}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \ 819$$

$$-\frac{(10Ab-aB)\left(\frac{4\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{2\int \frac{(ex)^{3/2}}{\sqrt{bx^3+a}}dx}{3a}\right)}{9a}+\frac{2(ex)^{5/2}}{9ae\left(a+bx^3\right)^{3/2}}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \ 851$$

$$-\frac{(10Ab-aB)\left(\frac{4\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\int \frac{e^2x^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{3ae}\right)}{9a}+\frac{2(ex)^{5/2}}{9ae\left(a+bx^3\right)^{3/2}}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \ 837$$

$$-\frac{(10Ab-aB)\left(\frac{4\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\int-\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3ae}\right)}{9a}+\frac{2(ex)^{5/2}}{9ae(a+bx^3)^{3/2}}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{(10Ab-aB)\left(\frac{4\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}e^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}\right)}{3ae}\right)}{9a}+\frac{2(ex)^{5/2}}{9ae(a+bx^3)^{3/2}}\right)}{ae^3}-\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{766}$$

3.564. $\int\frac{A+Bx^3}{(ex)^{3/2}(a+bx^3)^{5/2}}\,dx$

$$-\frac{(10Ab-aB)\left(\frac{4\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\int\frac{2b^{2/3}x^2e^2+\left(1-\sqrt{3}\right)a^{2/3}e^2}{\sqrt{bx^3+a}}d\sqrt{ex}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{El}}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{}\right.}{\left(\sqrt[3]{a}e+\left(1+\right.\right.}}}\right.}{3ae}\right.}{9a}\right.}{ae^3}$$

$$\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 2420$$

3.564. $\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx$

$$-\frac{(10Ab-aB)\left(\frac{4\left(\frac{2(ex)^{5/2}}{3ae\sqrt{a+bx^3}}-\frac{4\left(\frac{\left(1+\sqrt{3}\right)e^3\sqrt{ex}\sqrt{a+bx^3}}{\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right.}{\left(1+\sqrt{3}\right.}\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}\right.}{2b^{2/3}}\right.}{}\right.}{}$$

3.564. $\int \frac{A+Bx^3}{(ex)^{3/2}(a+bx^3)^{5/2}}\,dx$ $\quad \frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$

input
```
Int[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x]
```

output
```
(-2*A)/(a*e*Sqrt[e*x]*(a + b*x^3)^(3/2)) - ((10*A*b - a*B)*((2*(e*x)^(5/2)
)/(9*a*e*(a + b*x^3)^(3/2)) + (4*((2*(e*x)^(5/2))/(3*a*e*Sqrt[a + b*x^3])
- (4*((((1 + Sqrt[3])*e^3*Sqrt[e*x]*Sqrt[a + b*x^3])/(a^(1/3)*e + (1 + Sqr
t[3])*b^(1/3)*e*x) - (3^(1/4)*a^(1/3)*e*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x
)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e
+ (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticE[ArcCos[(a^(1/3)*e + (1 - Sqrt[3]
)*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])
/(Sqrt[(b^(1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*
b^(1/3)*e*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*e*S
qrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(1/3)*e^2
*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*EllipticF
[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 + Sqrt[3])
*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*e*x*(a^
(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*Sqrt[a
+ b*x^3])))/(3*a*e)))/(9*a)))/(a*e^3)
```

3.564.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.564.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.03 (sec) , antiderivative size = 1225, normalized size of antiderivative = 1.96

method	result	size
elliptic	Expression too large to display	1225
risch	Expression too large to display	3336
default	Expression too large to display	10961

input
```
int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(-2*(b*e*x^3+a*e)/e^2/a^
3*A/(x*(b*e*x^3+a*e))^(1/2)-2/9/e^2/a^2/b^2*x^2*(A*b-B*a)*(b*e*x^4+a*e*x)^
(1/2)/(x^3+a/b)^2-2/27/e*x^3/a^3*(13*A*b-4*B*a)/((x^3+a/b)*b*e*x)^(1/2)+(2
*b/a^3/e*A+2/27/a^3*(13*A*b-4*B*a)/e)*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3
))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b...
```

3.564.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.27

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx =$$
$$-\frac{2\left(4\left(\left(Bab^2-10\,Ab^3\right)x^7+2\left(Ba^2b-10\,Aab^2\right)x^4+\left(Ba^3-10\,Aa^2b\right)x\right)\sqrt{ae}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,b}{a},\mathrm{weierst}\right.}{27\left(a^3b^3e^2x^7+2\,a^4b^2e^2x^4+\right.}$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/27*(4*((B*a*b^2 - 10*A*b^3)*x^7 + 2*(B*a^2*b - 10*A*a*b^2)*x^4 + (B*a^3
 - 10*A*a^2*b)*x)*sqrt(a*e)*weierstrassZeta(0, -4*b/a, weierstrassPInverse
(0, -4*b/a, 1/x)) + (4*B*a^3 - 13*A*a^2*b + (B*a^2*b - 10*A*a*b^2)*x^3)*sq
rt(b*x^3 + a)*sqrt(e*x))/(a^3*b^3*e^2*x^7 + 2*a^4*b^2*e^2*x^4 + a^5*b*e^2*
x)
```

3.564.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

3.564.7 Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(3/2)), x)
```

3.564.8 Giac [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(3/2)), x)
```

3.564.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{B\,x^3+A}{{\left(e\,x\right)}^{3/2}\,{\left(b\,x^3+a\right)}^{5/2}} \,d x$$

input
```
int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)), x)
```

3.565 $\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx$

3.565.1 Optimal result

Integrand size = 26, antiderivative size = 104

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx=-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}-\frac{2(4Ab-aB)(ex)^{3/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{4(4Ab-aB)(ex)^{3/2}}{9a^3e^4\sqrt{a+bx^3}}$$

output
```
-2/3*A/a/e/(e*x)^(3/2)/(b*x^3+a)^(3/2)-2/9*(4*A*b-B*a)*(e*x)^(3/2)/a^2/e^4
/(b*x^3+a)^(3/2)-4/9*(4*A*b-B*a)*(e*x)^(3/2)/a^3/e^4/(b*x^3+a)^(1/2)
```

3.565.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx=\frac{2x\left(-3a^2A-12aAbx^3+3a^2Bx^3-8Ab^2x^6+2abBx^6\right)}{9a^3(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]
```

output
```
(2*x*(-3*a^2*A - 12*a*A*b*x^3 + 3*a^2*B*x^3 - 8*A*b^2*x^6 + 2*a*b*B*x^6))/
(9*a^3*(e*x)^(5/2)*(a + b*x^3)^(3/2))
```

3.565.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {955, 805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(4Ab-aB)\int \frac{\sqrt{ex}}{\left(bx^3+a\right)^{5/2}}dx}{ae^3}-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 805}$$

$$-\frac{(4Ab-aB)\left(\frac{2\int \frac{\sqrt{ex}}{\left(bx^3+a\right)^{3/2}}dx}{3a}+\frac{2(ex)^{3/2}}{9ae\left(a+bx^3\right)^{3/2}}\right)}{ae^3}-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{(4Ab-aB)\left(\frac{4(ex)^{3/2}}{9a^2e\sqrt{a+bx^3}}+\frac{2(ex)^{3/2}}{9ae\left(a+bx^3\right)^{3/2}}\right)}{ae^3}-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]
```

output
```
(-2*A)/(3*a*e*(e*x)^(3/2)*(a + b*x^3)^(3/2)) - ((4*A*b - a*B)*((2*(e*x)^(3
/2))/(9*a*e*(a + b*x^3)^(3/2)) + (4*(e*x)^(3/2))/(9*a^2*e*Sqrt[a + b*x^3])
))/(a*e^3)
```

3.565.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.565.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2x\left(8A\,b^{2}x^{6}-2B\,x^{6}ab+12aAb\,x^{3}-3a^{2}B\,x^{3}+3a^{2}A\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{3}\left(ex\right)^{\frac{5}{2}}}$	62
default	$-\frac{2\left(8A\,b^{2}x^{6}-2B\,x^{6}ab+12aAb\,x^{3}-3a^{2}B\,x^{3}+3a^{2}A\right)}{9a^{3}\sqrt{ex}\,e^{2}\left(b\,x^{3}+a\right)^{\frac{3}{2}}x}$	67
risch	$-\frac{2A\sqrt{b\,x^{3}+a}}{3a^{3}x\,e^{2}\sqrt{ex}}-\frac{2\left(5A\,b^{2}x^{3}-2Bab\,x^{3}+6abA-3a^{2}B\right)x^{2}}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{3}e^{2}\sqrt{ex}}$	82
elliptic	$\frac{\sqrt{\left(b\,x^{3}+a\right)ex}\,\left(-\frac{2x\left(Ab-Ba\right)\sqrt{be\,x^{4}+aex}}{9e^{3}a^{2}b^{2}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2x^{2}\left(5Ab-2Ba\right)}{9e^{2}a^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)bex}}-\frac{2A\sqrt{be\,x^{4}+aex}}{3e^{3}a^{3}x^{2}}\right)}{\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$	133

input
```
int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*x*(8*A*b^2*x^6-2*B*a*b*x^6+12*A*a*b*x^3-3*B*a^2*x^3+3*A*a^2)/(b*x^3+a
)^(3/2)/a^3/(e*x)^(5/2)
```

3.565.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx=\frac{2\left(2\left(Bab-4\,Ab^2\right)x^6+3\left(Ba^2-4\,Aab\right)x^3-3\,Aa^2\right)\sqrt{bx^3+a}\sqrt{ex}}{9\left(a^3b^2e^3x^8+2\,a^4be^3x^5+a^5e^3x^2\right)}$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/9*(2*(B*a*b - 4*A*b^2)*x^6 + 3*(B*a^2 - 4*A*a*b)*x^3 - 3*A*a^2)*sqrt(b*x
^3 + a)*sqrt(e*x)/(a^3*b^2*e^3*x^8 + 2*a^4*b*e^3*x^5 + a^5*e^3*x^2)
```

3.565.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

3.565.7 Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(5/2)), x)
```

3.565.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx = \text{Exception raised: NotImplementedError}$$

input

```
integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output

```
Exception raised: NotImplementedError >> unable to parse Giac output: Recu
rsive assumption sageVARa>=(-sageVARb*sageVARe/(sageVARe^4*t_nostep^6)) ig
nored2*(-(23914845*sageVARb^7*sageVARe^18*sageVARa^6*sageVARA-9565938*sage
VARb^6*sageVARe^18*
```

3.565.9 Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx = -\frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,A}{3\,a\,b^2\,e^2}-\frac{x^3\,\left(6\,B\,a^2-24\,A\,a\,b\right)}{9\,a^3\,b^2\,e^2}+\frac{x^6\,\left(16\,A\,b^2-4\,B\,a\,b\right)}{9\,a^3\,b^2\,e^2}\right)}{x^7\,\sqrt{e\,x}+\frac{a^2\,x\,\sqrt{e\,x}}{b^2}+\frac{2\,a\,x^4\,\sqrt{e\,x}}{b}}$$

input

```
int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x)
```

output

```
-((a + b*x^3)^(1/2)*((2*A)/(3*a*b^2*e^2) - (x^3*(6*B*a^2 - 24*A*a*b))/(9*a
^3*b^2*e^2) + (x^6*(16*A*b^2 - 4*B*a*b))/(9*a^3*b^2*e^2)))/(x^7*(e*x)^(1/2
) + (a^2*x*(e*x)^(1/2))/b^2 + (2*a*x^4*(e*x)^(1/2))/b)
```

3.566 $\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx$

3.566.1 Optimal result

Integrand size = 26, antiderivative size = 320

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx=-\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}}-\frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}}-\frac{16(14Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Bigg(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+\sqrt{}}{135\sqrt[4]{3}a^{10/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/5*A/a/e/(e*x)^(5/2)/(b*x^3+a)^(3/2)-2/45*(14*A*b-5*B*a)*(e*x)^(1/2)/a^2
/e^4/(b*x^3+a)^(3/2)-16/135*(14*A*b-5*B*a)*(e*x)^(1/2)/a^3/e^4/(b*x^3+a)^(
1/2)-16/405*(14*A*b-5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1
/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(
1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-
3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/
2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*
x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(10/3)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a
^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.566.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.38

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx = \frac{x\bigg(-224Ab^2x^6+a^2(-54A+110Bx^3)+a(-308Abx^3+80bBx^6)+32(-14Ab}{135a^3(ex)^{7/2}\,(a+b}$$

input
```
Integrate[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x]
```

output
```
(x*(-224*A*b^2*x^6 + a^2*(-54*A + 110*B*x^3) + a*(-308*A*b*x^3 + 80*b*B*x^
6) + 32*(-14*A*b + 5*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometr
ic2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(135*a^3*(e*x)^(7/2)*(a + b*x^3)^(3/2
))
```

3.566.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {955, 819, 819, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(14Ab-5aB)\int \frac{1}{\sqrt{ex}\,(bx^3+a)^{5/2}}dx}{5ae^3}-\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 819$$

$$-\frac{(14Ab-5aB)\left(\frac{8\int \frac{1}{\sqrt{ex}\left(bx^3+a\right)^{3/2}}dx}{9a}+\frac{2\sqrt{ex}}{9ae(a+bx^3)^{3/2}}\right)}{5ae^3}-\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 819$$

$$-\frac{(14Ab-5aB)\left(\frac{8\left(\frac{2\int \frac{1}{\sqrt{ex}\sqrt{bx^3+a}}dx}{3a}+\frac{2\sqrt{ex}}{3ae\sqrt{a+bx^3}}\right)}{9a}+\frac{2\sqrt{ex}}{9ae(a+bx^3)^{3/2}}\right)}{5ae^3}-\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{851}$$

$$-\frac{(14Ab-5aB)\left(\frac{8\left(\frac{4\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{ex}}{3ae}+\frac{2\sqrt{ex}}{3ae\sqrt{a+bx^3}}\right)}{9a}+\frac{2\sqrt{ex}}{9ae(a+bx^3)^{3/2}}\right)}{5ae^3}-\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{766}$$

$$-\frac{(14Ab-5aB)\left(\frac{8\left(\frac{2\sqrt{ex}\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)\sqrt{\frac{a^{2/3}e^2-\sqrt[3]{a}\sqrt[3]{b}e^2x+b^{2/3}e^2x^2}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xe+\sqrt[3]{a}e}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}e^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}ex\left(\sqrt[3]{a}e+\sqrt[3]{b}ex\right)}{\left(\sqrt[3]{a}e+\left(1+\sqrt{3}\right)\sqrt[3]{b}ex\right)^2}}}+\frac{2\sqrt{ex}}{3ae\sqrt{a+bx}}\right.}{9a}\right.}{5ae^3}$$

$$\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x]
```

output
```
(-2*A)/(5*a*e*(e*x)^(5/2)*(a + b*x^3)^(3/2)) - ((14*A*b - 5*a*B)*((2*Sqrt[
e*x])/(9*a*e*(a + b*x^3)^(3/2)) + (8*((2*Sqrt[e*x])/(3*a*e*Sqrt[a + b*x^3]
) + (2*Sqrt[e*x]*(a^(1/3)*e + b^(1/3)*e*x)*Sqrt[(a^(2/3)*e^2 - a^(1/3)*b^(
1/3)*e^2*x + b^(2/3)*e^2*x^2)/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x)^2]*E
llipticF[ArcCos[(a^(1/3)*e + (1 - Sqrt[3])*b^(1/3)*e*x)/(a^(1/3)*e + (1 +
Sqrt[3])*b^(1/3)*e*x)], (2 + Sqrt[3])/4])/(3*3^(1/4)*a^(4/3)*e^2*Sqrt[(b^(
1/3)*e*x*(a^(1/3)*e + b^(1/3)*e*x))/(a^(1/3)*e + (1 + Sqrt[3])*b^(1/3)*e*x
)^2]*Sqrt[a + b*x^3])))/(9*a)))/(5*a*e^3)
```

3.566. $\int \frac{A+Bx^3}{(ex)^{7/2}(a+bx^3)^{5/2}}\,dx$

3.566.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.566.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.98 (sec) , antiderivative size = 829, normalized size of antiderivative = 2.59

method	result	size
elliptic	Expression too large to display	829
risch	Expression too large to display	2182
default	Expression too large to display	7299

input
```
int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(-2/9/e^4/a^2/b^2*(A*b-B
*a)*(b*e*x^4+a*e*x)^(1/2)/(x^3+a/b)^2-2/27/e^3*x/a^3*(17*A*b-8*B*a)/((x^3+
a/b)*b*e*x)^(1/2)-2/5/e^4/a^3*A*(b*e*x^4+a*e*x)^(1/2)/x^3+2*(-2/27/a^3*(17
*A*b-8*B*a)/e^3-2/5*b/a^3/e^3*A)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a
*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.566.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.56

$$\int \frac{A+Bx^3}{(ex)^{7/2}\,(a+bx^3)^{5/2}}\,dx =$$
$$-\frac{2\left(16\left((5\,Bab^2-14\,Ab^3)x^9+2\,(5\,Ba^2b-14\,Aab^2)x^6+(5\,Ba^3-14\,Aa^2b)x^3\right)\sqrt{ae}\,\mathrm{weierstrassPInverse}(0,-}{135\,(a^4b^2e^4x^9+2\,a^5be^4x^6+}$$

input
```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/135*(16*((5*B*a*b^2 - 14*A*b^3)*x^9 + 2*(5*B*a^2*b - 14*A*a*b^2)*x^6 +
(5*B*a^3 - 14*A*a^2*b)*x^3)*sqrt(a*e)*weierstrassPInverse(0, -4*b/a, 1/x)
- (8*(5*B*a^2*b - 14*A*a*b^2)*x^6 - 27*A*a^3 + 11*(5*B*a^3 - 14*A*a^2*b)*x
^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(a^4*b^2*e^4*x^9 + 2*a^5*b*e^4*x^6 + a^6*e^
4*x^3)
```

3.566.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(5/2),x)
```

output
```
Timed out
```

3.566.7 Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(7/2)), x)
```

3.566.8 Giac [F]

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(7/2)), x)
```

3.566.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{B\,x^3+A}{\left(e\,x\right)^{7/2}\,\left(b\,x^3+a\right)^{5/2}}\,dx$$

input
```
int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x)
```

output
```
int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)), x)
```

3.567 $\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

3.567.1 Optimal result

Integrand size = 28, antiderivative size = 220

$$\begin{aligned}\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx &= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2(a+bx^3)^{4/3}}{4b^4d}+\frac{a(a+bx^3)^{7/3}}{7b^4d}\\ &\quad-\frac{(a+bx^3)^{10/3}}{10b^4d}+\frac{\sqrt[3]{2}a^{10/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^4d}\\ &\quad+\frac{a^{10/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^4d}-\frac{a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4d}\end{aligned}$$

output
```
-a^3*(b*x^3+a)^(1/3)/b^4/d-1/4*a^2*(b*x^3+a)^(4/3)/b^4/d+1/7*a*(b*x^3+a)^(
7/3)/b^4/d-1/10*(b*x^3+a)^(10/3)/b^4/d+1/6*a^(10/3)*ln(-b*x^3+a)*2^(1/3)/b
^4/d-1/2*a^(10/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b^4/d+1/3*2^
(1/3)*a^(10/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2
))/b^4/d*3^(1/2)
```

3.567.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.92

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$

$$= \frac{-3\sqrt[3]{a+bx^3}(169a^3+37a^2bx^3+22ab^2x^6+14b^3x^9)+140\sqrt[3]{2}\sqrt{3}a^{10/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-140\sqrt[3]{}}{420b^4d}$$

input
```
Integrate[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
(-3*(a + b*x^3)^(1/3)*(169*a^3 + 37*a^2*b*x^3 + 22*a*b^2*x^6 + 14*b^3*x^9)
 + 140*2^(1/3)*Sqrt[3]*a^(10/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^
(1/3))/Sqrt[3]] - 140*2^(1/3)*a^(10/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3
)^(1/3)] + 70*2^(1/3)*a^(10/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)
^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(420*b^4*d)
```

3.567.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {948, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9\sqrt[3]{bx^3+a}}{d\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^9\sqrt[3]{bx^3+a}}{a-bx^3}dx^3}{3d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{\sqrt[3]{bx^3+a}\,a^3}{b^3(a-bx^3)}-\frac{\sqrt[3]{bx^3+a}\,a^2}{b^3}+\frac{(bx^3+a)^{4/3}a}{b^3}-\frac{(bx^3+a)^{7/3}}{b^3}\right)dx^3}{3d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\sqrt[3]{2}\sqrt{3}a^{10/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^4}+\frac{a^{10/3}\log(a-bx^3)}{2^{2/3}b^4}-\frac{3a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4}-\frac{3a^3\sqrt[3]{a+bx^3}}{b^4}-\frac{3a^2(a+b}{4b}}{3d}$$

input
```
Int[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
((-3*a^3*(a + b*x^3)^(1/3))/b^4 - (3*a^2*(a + b*x^3)^(4/3))/(4*b^4) + (3*a
*(a + b*x^3)^(7/3))/(7*b^4) - (3*(a + b*x^3)^(10/3))/(10*b^4) + (2^(1/3)*S
qrt[3]*a^(10/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/b^4 + (a^(10/3)*Log[a - b*x^3])/(2^(2/3)*b^4) - (3*a^(10/3)*Log[2^(
1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^4))/(3*d)
```

3.567.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.567.4 Maple [A] (verified)

Time = 6.64 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{\left(-42 b^{3} x^{9}-66 a \,b^{2} x^{6}-111 a^{2} b \,x^{3}-507 a^{3}\right) \left(b \,x^{3}+a \right)^{\frac{1}{3}}+70\, 2^{\frac{1}{3}} a^{\frac{10}{3}} (2 \arctan \left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}} \left(b \,x^{3}+a \right)^{\frac{1}{3}}\right) \sqrt{3}}{3 a^{\frac{1}{3}}}\right) \sqrt{3}+\ln (\left(b \,x^{3}+a \right)^{\frac{2}{3}}+}{420 b^{4} d}$

input
```
int(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/420*((-42*b^3*x^9-66*a*b^2*x^6-111*a^2*b*x^3-507*a^3)*(b*x^3+a)^(1/3)+70
*2^(1/3)*a^(10/3)*(2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*
3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3
)*a^(2/3))-2*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))))/b^4/d
```

3.567.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.85

$$\int \frac{x^{11} \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{140 \sqrt{3} 2^{\frac{1}{3}} (-a)^{\frac{1}{3}} a^{3} \arctan \left(\frac{\sqrt{3} 2^{\frac{2}{3}} (b x^{3} + a)^{\frac{1}{3}} (-a)^{\frac{2}{3}} + \sqrt{3} a}{3 \, a}\right) + 70 \cdot 2^{\frac{1}{3}} (-a)^{\frac{1}{3}} a^{3} \log (2^{\frac{2}{3}} (-a)^{\frac{2}{3}} - 2^{\frac{1}{3}} (b x^{3} + a)^{\frac{1}{3}} (-}{}$$

input
```
integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/420*(140*sqrt(3)*2^(1/3)*(-a)^(1/3)*a^3*arctan(1/3*(sqrt(3)*2^(2/3)*(b*
x^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 70*2^(1/3)*(-a)^(1/3)*a^3*log(
2^(2/3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2
/3)) - 140*2^(1/3)*(-a)^(1/3)*a^3*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/
3)) + 3*(14*b^3*x^9 + 22*a*b^2*x^6 + 37*a^2*b*x^3 + 169*a^3)*(b*x^3 + a)^(
1/3))/(b^4*d)
```

3.567.6 Sympy [F]

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^{11}\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**11*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**11*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.567.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$
$$= \frac{\frac{140\sqrt{3}2^{\frac{1}{3}}a^{\frac{10}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d} + \frac{70\cdot 2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d} - \frac{140\cdot 2^{\frac{1}{3}}a^{\frac{10}{3}}\log\bigl(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(b}{d}}{420\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
1/420*(140*sqrt(3)*2^(1/3)*a^(10/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^
(1/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 70*2^(1/3)*a^(10/3)*log(2^(2/3)*
a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 140*2
^(1/3)*a^(10/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 3*(14*(b*x^3
 + a)^(10/3) - 20*(b*x^3 + a)^(7/3)*a + 35*(b*x^3 + a)^(4/3)*a^2 + 140*(b*
x^3 + a)^(1/3)*a^3)/d)/b^4
```

3.567.8 Giac [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt{3}2^{\frac{1}{3}}a^{\frac{10}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,b^4d} + \frac{2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^4d} - \frac{2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^4d} - \frac{14\,(bx^3+a)^{\frac{10}{3}}b^{36}d^9-20\,(bx^3+a)^{\frac{7}{3}}ab^{36}d^9+35\,(bx^3+a)^{\frac{4}{3}}a^2b^{36}d^9+140\,(bx^3+a)^{\frac{1}{3}}a^3b^{36}d^9}{140\,b^{40}d^{10}}$$

input
```
integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*2^(1/3)*a^(10/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
 2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^4*d) + 1/6*2^(1/3)*a^(10/3)*log(2^(2/3)*
a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^4*d) -
 1/3*2^(1/3)*a^(10/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^4*
d) - 1/140*(14*(b*x^3 + a)^(10/3)*b^36*d^9 - 20*(b*x^3 + a)^(7/3)*a*b^36*d
^9 + 35*(b*x^3 + a)^(4/3)*a^2*b^36*d^9 + 140*(b*x^3 + a)^(1/3)*a^3*b^36*d^
9)/(b^40*d^10)
```

3.567.9 Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{a\,{\left(b\,x^3+a\right)}^{7/3}}{7\,b^4\,d} - \frac{a^3\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d} - \frac{a^2\,{\left(b\,x^3+a\right)}^{4/3}}{4\,b^4\,d} - \frac{{\left(b\,x^3+a\right)}^{10/3}}{10\,b^4\,d} - \frac{2^{1/3}\,a^{10/3}\,\ln\left({\left(b\,x^3+a\right)}^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^4\,d} - \frac{2^{1/3}\,a^{10/3}\,\ln\left(\frac{6\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d}-\frac{6\,2^{1/3}\,a^{13/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^4\,d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^4\,d} + \frac{2^{1/3}\,a^{10/3}\,\ln\left(\frac{6\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d}+\frac{18\,2^{1/3}\,a^{13/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^4\,d}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^4\,d}$$

input
```
int((x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output

```
(a*(a + b*x^3)^(7/3))/(7*b^4*d) - (a^3*(a + b*x^3)^(1/3))/(b^4*d) - (a^2*(
a + b*x^3)^(4/3))/(4*b^4*d) - (a + b*x^3)^(10/3)/(10*b^4*d) - (2^(1/3)*a^(
10/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*a^(1/3)))/(3*b^4*d) - (2^(1/3)*a^(10
/3)*log((6*a^4*(a + b*x^3)^(1/3))/(b^4*d) - (6*2^(1/3)*a^(13/3)*((3^(1/2)*
1i)/2 - 1/2))/(b^4*d))*((3^(1/2)*1i)/2 - 1/2))/(3*b^4*d) + (2^(1/3)*a^(10/
3)*log((6*a^4*(a + b*x^3)^(1/3))/(b^4*d) + (18*2^(1/3)*a^(13/3)*((3^(1/2)*
1i)/6 + 1/6))/(b^4*d))*((3^(1/2)*1i)/6 + 1/6))/(b^4*d)
```

3.567. $\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

3.568 $\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

3.568.1 Optimal result

Integrand size = 28, antiderivative size = 174

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}+\frac{\sqrt[3]{2}a^{7/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^3d}$$
$$+\frac{a^{7/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^3d}-\frac{a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3d}$$

output
```
-a^2*(b*x^3+a)^(1/3)/b^3/d-1/7*(b*x^3+a)^(7/3)/b^3/d+1/6*a^(7/3)*ln(-b*x^3
+a)*2^(1/3)/b^3/d-1/2*a^(7/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/
b^3/d+1/3*2^(1/3)*a^(7/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(
1/3)*3^(1/2))/b^3/d*3^(1/2)
```

3.568.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.21

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =$$
$$-\frac{48a^2\sqrt[3]{a+bx^3}+12abx^3\sqrt[3]{a+bx^3}+6b^2x^6\sqrt[3]{a+bx^3}-14\sqrt[3]{2}\sqrt{3}a^{7/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+14\sqrt[3]{2}}{42b^3d}$$

input
```
Integrate[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
-1/42*(48*a^2*(a + b*x^3)^(1/3) + 12*a*b*x^3*(a + b*x^3)^(1/3) + 6*b^2*x^6
*(a + b*x^3)^(1/3) - 14*2^(1/3)*Sqrt[3]*a^(7/3)*ArcTan[(1 + (2^(2/3)*(a +
b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 14*2^(1/3)*a^(7/3)*Log[-2*a^(1/3) + 2^(2
/3)*(a + b*x^3)^(1/3)] - 7*2^(1/3)*a^(7/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)
*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^3*d)
```

3.568.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {948, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{ad-bdx^3}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^6\sqrt[3]{bx^3+a}}{d\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{x^6\sqrt[3]{bx^3+a}}{a-bx^3}dx^3}{3d}$$

$$\Big\downarrow \ 99$$

$$\frac{\int \left(\frac{a^2\sqrt[3]{bx^3+a}}{b^2(a-bx^3)}-\frac{(bx^3+a)^{4/3}}{b^2}\right)dx^3}{3d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\sqrt[3]{2}\sqrt{3}a^{7/3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{b^3}+\frac{a^{7/3}\log(a-bx^3)}{2^{2/3}b^3}-\frac{3a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3}-\frac{3a^2\sqrt[3]{a+bx^3}}{b^3}-\frac{3(a+bx^3)^{7/}}{7b^3}}{3d}$$

input
```
Int[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

3.568. $\int \frac{x^8\sqrt[3]{a+bx^3}}{ad-bdx^3}dx$

output
```
((-3*a^2*(a + b*x^3)^(1/3))/b^3 - (3*(a + b*x^3)^(7/3))/(7*b^3) + (2^(1/3)
*Sqrt[3]*a^(7/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(
1/3))])/b^3 + (a^(7/3)*Log[a - b*x^3])/(2^(2/3)*b^3) - (3*a^(7/3)*Log[2^(1
/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^3))/(3*d)
```

3.568.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.568.4 Maple [A] (verified)

Time = 5.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\left(-6 b^{2} x^{6}-12 a b \,x^{3}-48 a^{2}\right) \left(b \,x^{3}+a \right)^{\frac{1}{3}}+7\, 2^{\frac{1}{3}} a^{\frac{7}{3}} \Bigg(2 \arctan \Bigg(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}} \left(b \,x^{3}+a \right)^{\frac{1}{3}}\right) \sqrt{3}}{3 a^{\frac{1}{3}}}\Bigg) \sqrt{3}+\ln \Bigg(\left(b \,x^{3}+a \right)^{\frac{2}{3}}+2^{\frac{1}{3}} a^{\frac{1}{3}} \left(b \,x^{3}+a \right)^{\frac{1}{3}}}{42 b^{3} d}$

input
```
int(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/42*((-6*b^2*x^6-12*a*b*x^3-48*a^2)*(b*x^3+a)^(1/3)+7*2^(1/3)*a^(7/3)*(2*
arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((
b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))-2*ln((b*x^
3+a)^(1/3)-2^(1/3)*a^(1/3))))/b^3/d
```

3.568.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =$$

$$-\frac{14\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^2\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+7\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^2\log\Big(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-a)}{42\,b}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/42*(14*sqrt(3)*2^(1/3)*(-a)^(1/3)*a^2*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^
3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 7*2^(1/3)*(-a)^(1/3)*a^2*log(2^(
2/3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)
) - 14*2^(1/3)*(-a)^(1/3)*a^2*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3))
+ 6*(b^2*x^6 + 2*a*b*x^3 + 8*a^2)*(b*x^3 + a)^(1/3))/(b^3*d)
```

3.568.6 Sympy [F]

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^8\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**8*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**8*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.568.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$
$$= \frac{\frac{14\,\sqrt{3}2^{\frac{1}{3}}a^{\frac{7}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d} + \frac{7\cdot 2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d} - \frac{14\cdot 2^{\frac{1}{3}}a^{\frac{7}{3}}\log\Big(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)}{d}}{42\,b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
1/42*(14*sqrt(3)*2^(1/3)*a^(7/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/
3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 7*2^(1/3)*a^(7/3)*log(2^(2/3)*a^(2/
3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 14*2^(1/3)
*a^(7/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 6*((b*x^3 + a)^(7/3
) + 7*(b*x^3 + a)^(1/3)*a^2)/d)/b^3
```

3.568.8 Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =& \frac{\sqrt{3}2^{\frac{1}{3}}a^{\frac{7}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,b^3d} \\ &+ \frac{2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^3d} \\ &- \frac{2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^3d} \\ &- \frac{(bx^3+a)^{\frac{7}{3}}b^{18}d^6+7\,(bx^3+a)^{\frac{1}{3}}a^2b^{18}d^6}{7\,b^{21}d^7}\end{aligned}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*2^(1/3)*a^(7/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^3*d) + 1/6*2^(1/3)*a^(7/3)*log(2^(2/3)*a^
(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^3*d) - 1
/3*2^(1/3)*a^(7/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^3*d)
- 1/7*((b*x^3 + a)^(7/3)*b^18*d^6 + 7*(b*x^3 + a)^(1/3)*a^2*b^18*d^6)/(b^2
1*d^7)
```

3.568.9 Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.26

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{2^{1/3}\,(-a)^{7/3}\,\ln\left(6\,a^3\,(b\,x^3+a)^{1/3}-6\,2^{1/3}\,(-a)^{10/3}\right)}{3\,b^3\,d}-\frac{a^2\,(b\,x^3+a)^{1/3}}{b^3\,d}-\frac{(b\,x^3+a)^{7/3}}{7\,b^3\,d}-\frac{2^{1/3}\,(-a)^{7/3}\,\ln\left(\frac{6\,a^3\,(b\,x^3+a)^{1/3}}{b^3\,d}+\frac{6\,2^{1/3}\,(-a)^{10/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^3\,d}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^3\,d}+\frac{2^{1/3}\,(-a)^{7/3}\,\ln\left(\frac{6\,a^3\,(b\,x^3+a)^{1/3}}{b^3\,d}-\frac{18\,2^{1/3}\,(-a)^{10/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^3\,d}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^3\,d}$$

input
```
int((x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
(2^(1/3)*(-a)^(7/3)*log(6*a^3*(a + b*x^3)^(1/3) - 6*2^(1/3)*(-a)^(10/3)))/
(3*b^3*d) - (a^2*(a + b*x^3)^(1/3))/(b^3*d) - (a + b*x^3)^(7/3)/(7*b^3*d)
- (2^(1/3)*(-a)^(7/3)*log((6*a^3*(a + b*x^3)^(1/3))/(b^3*d) + (6*2^(1/3)*(
-a)^(10/3)*((3^(1/2)*1i)/2 + 1/2))/(b^3*d))*((3^(1/2)*1i)/2 + 1/2))/(3*b^3
*d) + (2^(1/3)*(-a)^(7/3)*log((6*a^3*(a + b*x^3)^(1/3))/(b^3*d) - (18*2^(1
/3)*(-a)^(10/3)*((3^(1/2)*1i)/6 - 1/6))/(b^3*d))*((3^(1/2)*1i)/6 - 1/6))/(
b^3*d)
```

3.569 $\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

3.569.1 Optimal result

Integrand size = 28, antiderivative size = 172

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{a\sqrt[3]{a+bx^3}}{b^2d}-\frac{\left(a+bx^3\right)^{4/3}}{4b^2d}+\frac{\sqrt[3]{2}a^{4/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d}+\frac{a^{4/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^2d}-\frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d}$$

output
```
-a*(b*x^3+a)^(1/3)/b^2/d-1/4*(b*x^3+a)^(4/3)/b^2/d+1/6*a^(4/3)*ln(-b*x^3+a
)*2^(1/3)/b^2/d-1/2*a^(4/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b^
2/d+1/3*2^(1/3)*a^(4/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/
3)*3^(1/2))/b^2/d*3^(1/2)
```

3.569.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.09

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =$$

$$-\frac{15a\sqrt[3]{a+bx^3}+3bx^3\sqrt[3]{a+bx^3}-4\sqrt[3]{2}\sqrt{3}a^{4/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{2}a^{4/3}\log\left(-2\sqrt[3]{a}+2^{2/3}\right.}{12b^2d}$$

input
```
Integrate[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
-1/12*(15*a*(a + b*x^3)^(1/3) + 3*b*x^3*(a + b*x^3)^(1/3) - 4*2^(1/3)*Sqrt
[3]*a^(4/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 4*
2^(1/3)*a^(4/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 2*2^(1/3)*a^
(4/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x
^3)^(2/3)])/(b^2*d)
```

3.569.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {948, 27, 90, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^3 \sqrt[3]{bx^3+a}}{d\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^3 \sqrt[3]{bx^3+a}}{a-bx^3}dx^3}{3d}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{a\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx^3}{b}-\frac{3\left(a+bx^3\right)^{4/3}}{4b^2}}{3d}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{a\left(2a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx^3-\frac{3\sqrt[3]{a+bx^3}}{b}\right)}{b}-\frac{3\left(a+bx^3\right)^{4/3}}{4b^2}}{3d}$$

$$\Big\downarrow \text{69}$$

$$\frac{\frac{a\left(2a\left(\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\ 2^{2/3}a^{2/3}b}+\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\ 2^{2/3}a^{2/3}b}\right)-\frac{3\sqrt[3]{a+bx^3}}{b}\right)}{b}-}{3d}$$

$$\Big\downarrow \color{blue}{16}$$

$$\frac{\frac{a\left(2a\left(\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)-\frac{3\sqrt[3]{a+bx^3}}{b}\right)}{b}-\frac{3\left(a+bx^3\right)^4}{4b^2}}{3d}$$

$$\Big\downarrow \color{blue}{1082}$$

$$\frac{\frac{a\left(2a\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{2^{2/3}a^{2/3}b}+\frac{\log\left(a-bx^3\right)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)-\frac{3\sqrt[3]{a+bx^3}}{b}\right)}{b}-\frac{3\left(a+bx^3\right)^{4/3}}{4b^2}}{3d}$$

$$\Big\downarrow \color{blue}{217}$$

$$\frac{\frac{a\left(2a\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}b}+\frac{\log\left(a-bx^3\right)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)-\frac{3\sqrt[3]{a+bx^3}}{b}\right)}{b}-\frac{3\left(a+bx^3\right)^{4/3}}{4b^2}}{3d}$$

input
```
Int[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
((-3*(a + b*x^3)^(4/3))/(4*b^2) + (a*((-3*(a + b*x^3)^(1/3))/b + 2*a*((Sqr
t[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/(2^(2/3)*a
^(2/3)*b) + Log[a - b*x^3]/(2*2^(2/3)*a^(2/3)*b) - (3*Log[2^(1/3)*a^(1/3)
- (a + b*x^3)^(1/3)])/(2*2^(2/3)*a^(2/3)*b))))/b)/(3*d)
```

3.569. $\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

3.569.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.569. $\int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.569.4 Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{(-3b x^3-15a)(b x^3+a)^{\frac{1}{3}}+2\,2^{\frac{1}{3}}a^{\frac{4}{3}}\left(2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}(b x^3+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left((b x^3+a)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(b x^3+a)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)-}{12b^2d}$

input
```
int(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/12*((-3*b*x^3-15*a)*(b*x^3+a)^(1/3)+2*2^(1/3)*a^(4/3)*(2*arctan(1/3*(a^(
1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+
2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))-2*ln((b*x^3+a)^(1/3)-2^(1
/3)*a^(1/3))))/b^2/d
```

3.569.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{4\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\,a\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right)+2\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\,a\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+\right.}{12\,b^2d}$$

input
```
integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/12*(4*sqrt(3)*2^(1/3)*(-a)^(1/3)*a*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 +
 a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 2*2^(1/3)*(-a)^(1/3)*a*log(2^(2/3)*
(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) - 4
*2^(1/3)*(-a)^(1/3)*a*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 3*(b*x
^3 + 5*a)*(b*x^3 + a)^(1/3))/(b^2*d)
```

3.569.6 Sympy [F]

$$\int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^5 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**5*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**5*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.569.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

$$\int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\frac{4\sqrt{3}2^{\frac{1}{3}}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d} + \frac{2\cdot 2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d} - \frac{4\cdot 2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right.}{d}}{12\,b^2}$$

input
```
integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
1/12*(4*sqrt(3)*2^(1/3)*a^(4/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 2*2^(1/3)*a^(4/3)*log(2^(2/3)*a^(2/3
) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 4*2^(1/3)*a
^(4/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 3*((b*x^3 + a)^(4/3)
+ 4*(b*x^3 + a)^(1/3)*a)/d)/b^2
```

3.569.8 Giac [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.01

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt{3}2^{\frac{1}{3}}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,b^2d} + \frac{2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^2d} - \frac{2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^2d} - \frac{(bx^3+a)^{\frac{4}{3}}b^6d^3+4\,(bx^3+a)^{\frac{1}{3}}ab^6d^3}{4\,b^8d^4}$$

input

```
integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output

```
1/3*sqrt(3)*2^(1/3)*a^(4/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^2*d) + 1/6*2^(1/3)*a^(4/3)*log(2^(2/3)*a^
(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^2*d) - 1
/3*2^(1/3)*a^(4/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^2*d)
- 1/4*((b*x^3 + a)^(4/3)*b^6*d^3 + 4*(b*x^3 + a)^(1/3)*a*b^6*d^3)/(b^8*d^4
)
```

3.569.9 Mupad [B] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.16

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{{\left(b\,x^3+a\right)}^{4/3}}{4\,b^2\,d}-\frac{a\,{\left(b\,x^3+a\right)}^{1/3}}{b^2\,d}-\frac{2^{1/3}\,a^{4/3}\,\ln\left({\left(b\,x^3+a\right)}^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^2\,d} - \frac{2^{1/3}\,a^{4/3}\,\ln\left(\frac{6\,a^2\,{\left(b\,x^3+a\right)}^{1/3}}{b^2\,d}-\frac{6\,2^{1/3}\,a^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^2\,d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^2\,d} + \frac{2^{1/3}\,a^{4/3}\,\ln\left(\frac{6\,a^2\,{\left(b\,x^3+a\right)}^{1/3}}{b^2\,d}+\frac{18\,2^{1/3}\,a^{7/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^2\,d}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^2\,d}$$

input

```
int((x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
(2^(1/3)*a^(4/3)*log((6*a^2*(a + b*x^3)^(1/3))/(b^2*d) + (18*2^(1/3)*a^(7/
3)*((3^(1/2)*1i)/6 + 1/6))/(b^2*d))*((3^(1/2)*1i)/6 + 1/6))/(b^2*d) - (a*(
a + b*x^3)^(1/3))/(b^2*d) - (2^(1/3)*a^(4/3)*log((a + b*x^3)^(1/3) - 2^(1/
3)*a^(1/3)))/(3*b^2*d) - (2^(1/3)*a^(4/3)*log((6*a^2*(a + b*x^3)^(1/3))/(b
^2*d) - (6*2^(1/3)*a^(7/3)*((3^(1/2)*1i)/2 - 1/2))/(b^2*d))*((3^(1/2)*1i)/
2 - 1/2))/(3*b^2*d) - (a + b*x^3)^(4/3)/(4*b^2*d)
```

3.569. $\int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

3.570 $\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

3.570.1 Optimal result

Integrand size = 28, antiderivative size = 150

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{\sqrt[3]{2}\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}bd}+\frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\ 2^{2/3}bd}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}$$

output
```
-(b*x^3+a)^(1/3)/b/d+1/6*a^(1/3)*ln(-b*x^3+a)*2^(1/3)/b/d-1/2*a^(1/3)*ln(2
^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b/d+1/3*2^(1/3)*a^(1/3)*arctan(1/3
*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/b/d*3^(1/2)
```

3.570.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$

$$= \frac{-6\sqrt[3]{a+bx^3}+2\sqrt[3]{2}\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{2}\sqrt[3]{a}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right)+\sqrt[3]{2}\sqrt[3]{a}\,\mathrm{l}}{6bd}$$

input
```
Integrate[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
(-6*(a + b*x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2^(2/3)*(a
+ b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(1/3)*a^(1/3)*Log[-2*a^(1/3) + 2^(
2/3)*(a + b*x^3)^(1/3)] + 2^(1/3)*a^(1/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*
(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(6*b*d)
```

3.570.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {946, 27, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}}{d\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx^3}{3d}$$

$$\Big\downarrow \text{60}$$

$$\frac{2a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx^3-\frac{3\sqrt[3]{a+bx^3}}{b}}{3d}$$

$$\Big\downarrow \text{69}$$

$$\frac{2a\left(\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\ 2^{2/3}a^{2/3}b}+\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\ 2^{2/3}a^{2/3}b}\right)-\frac{3\sqrt[3]{a+bx^3}}{b}}{3d}$$

$$\Big\downarrow \text{16}$$

3.570. $\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

$$\frac{2a\left(\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log(a-bx^3)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)-\frac{3\sqrt[3]{a+bx^3}}{b}}{3d}$$

$$\Big\downarrow \textcolor{blue}{1082}$$

$$\frac{2a\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{2^{2/3}a^{2/3}b}+\frac{\log(a-bx^3)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)-\frac{3\sqrt[3]{a+bx^3}}{b}}{3d}$$

$$\Big\downarrow \textcolor{blue}{217}$$

$$\frac{2a\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}b}+\frac{\log(a-bx^3)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)-\frac{3\sqrt[3]{a+bx^3}}{b}}{3d}$$

input
```
Int[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
((-3*(a + b*x^3)^(1/3))/b + 2*a*((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)
^(1/3))/a^(1/3))/Sqrt[3]])/(2^(2/3)*a^(2/3)*b) + Log[a - b*x^3]/(2*2^(2/3)
*a^(2/3)*b) - (3*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(2/3)*a^(2
/3)*b)))/(3*d)
```

3.570.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.570. $\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.570.4 Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{2a^{\frac{1}{3}}2^{\frac{1}{3}}\sqrt{3}\,\arctan\bigg(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\bigg)-2a^{\frac{1}{3}}2^{\frac{1}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)+a^{\frac{1}{3}}2^{\frac{1}{3}}\ln\bigg(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+}{6bd}$

input
```
int(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^(1/3)*2^(1/3)*3^(1/2)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3)
)/a^(1/3)*3^(1/2))-2*a^(1/3)*2^(1/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))+a
^(1/3)*2^(1/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*
a^(2/3))-6*(b*x^3+a)^(1/3))/b/d
```

3.570.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.96

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{2\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(bx^3\right.}{6\,bd}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*2^(1/3)*(-a)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 + a)
^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 2^(1/3)*(-a)^(1/3)*log(2^(2/3)*(-a)^(2
/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) - 2*2^(1/3
)*(-a)^(1/3)*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 6*(b*x^3 + a)^(
1/3))/(b*d)
```

3.570.6 Sympy [F]

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^2\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**2*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**2*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.570.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$
$$=\frac{\frac{2\sqrt{3}2^{\frac{1}{3}}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}+\frac{2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}-\frac{2\cdot 2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}}{6b}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
1/6*(2*sqrt(3)*2^(1/3)*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
 + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 2^(1/3)*a^(1/3)*log(2^(2/3)*a^(2/3) +
 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 2*2^(1/3)*a^(1
/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 6*(b*x^3 + a)^(1/3)/d)/b
```

3.570.8 Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx &= \frac{\sqrt{3}2^{\frac{1}{3}}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{3bd} \\ &\quad + \frac{2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6bd} \\ &\quad - \frac{2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3bd} - \frac{(bx^3+a)^{\frac{1}{3}}}{bd}\end{aligned}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*2^(1/3)*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
2*(b*x^3 + a)^(1/3))/a^(1/3))/(b*d) + 1/6*2^(1/3)*a^(1/3)*log(2^(2/3)*a^(2
/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b*d) - 1/3*2
^(1/3)*a^(1/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b*d) - (b*x
^3 + a)^(1/3)/(b*d)
```

3.570.9 Mupad [B] (verification not implemented)

Time = 8.66 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.29

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\frac{2^{1/3}\,(-a)^{1/3}\,\ln\left(6\,a\,(b\,x^3+a)^{1/3}-6\,2^{1/3}\,(-a)^{4/3}\right)}{3\,b\,d}-\frac{(b\,x^3+a)^{1/3}}{b\,d}+\frac{2^{1/3}\,(-a)^{1/3}\,\ln\left(\frac{6\,a\,(b\,x^3+a)^{1/3}}{b\,d}-\frac{6\,2^{1/3}\,(-a)^{4/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b\,d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b\,d}-\frac{2^{1/3}\,(-a)^{1/3}\,\ln\left(\frac{6\,a\,(b\,x^3+a)^{1/3}}{b\,d}+\frac{6\,2^{1/3}\,(-a)^{4/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b\,d}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b\,d}$$

input
```
int((x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
(2^(1/3)*(-a)^(1/3)*log(6*a*(a + b*x^3)^(1/3) - 6*2^(1/3)*(-a)^(4/3)))/(3*
b*d) - (a + b*x^3)^(1/3)/(b*d) + (2^(1/3)*(-a)^(1/3)*log((6*a*(a + b*x^3)^
(1/3))/(b*d) - (6*2^(1/3)*(-a)^(4/3)*((3^(1/2)*1i)/2 - 1/2))/(b*d))*((3^(1
/2)*1i)/2 - 1/2))/(3*b*d) - (2^(1/3)*(-a)^(1/3)*log((6*a*(a + b*x^3)^(1/3)
)/(b*d) + (6*2^(1/3)*(-a)^(4/3)*((3^(1/2)*1i)/2 + 1/2))/(b*d))*((3^(1/2)*1
i)/2 + 1/2))/(3*b*d)
```

3.571 $\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx$

3.571.1 Optimal result

Integrand size = 28, antiderivative size = 214

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx = &-\frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}d}\\ &+\frac{\sqrt[3]{2}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}d}-\frac{\log(x)}{2a^{2/3}d}+\frac{\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{2/3}d}\\ &+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d}\end{aligned}$$

output
```
-1/2*ln(x)/a^(2/3)/d+1/6*ln(-b*x^3+a)*2^(1/3)/a^(2/3)/d+1/2*ln(a^(1/3)-(b*
x^3+a)^(1/3))/a^(2/3)/d-1/2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/a^
(2/3)/d-1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(2/3
)/d*3^(1/2)+1/3*2^(1/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/
3)*3^(1/2))/a^(2/3)/d*3^(1/2)
```

3.571.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx =$$

$$-\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)+2}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^
(1/3)*Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] -
2*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + 2*2^(1/3)*Log[-2*a^(1/3) + 2^(2/3)*(
a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^
(2/3)] - 2^(1/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/
3)*(a + b*x^3)^(2/3)])/(a^(2/3)*d)
```

3.571.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 27, 94, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}}{dx^3\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt[3]{bx^3+a}}{x^3\left(a-bx^3\right)}dx^3}{3d}$$

$$\Big\downarrow \text{94}$$

$$\frac{\int \frac{1}{x^3(bx^3+a)^{2/3}}dx^3+2b\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx^3}{3d}$$

$$\Big\downarrow \text{69}$$

$$\frac{-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+2b\left(\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\ 2^{2/3}a^{2/3}b}+\right.}{3d}$$

$$\Big\downarrow \text{16}$$

$$\frac{-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+2b\left(\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log(a-bx^3)}{2\ 2^{2/3}a^{2/3}b}-\right.}{3d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+2b\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{2^{2/3}a^{2/3}b}+\frac{\log(a-bx^3)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)+}{3d}$$

$$\Big\downarrow \text{217}$$

$$\frac{-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+2b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}b}+\frac{\log(a-bx^3)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}a^{2/3}b}\right)+}{3d}$$

input
```
Int[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x]
```

3.571. $\int \frac{\sqrt[3]{a+bx^3}}{x(ad-bdx^3)}\,dx$

output
```
(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3))
- Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3))
+ 2*b*((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])
/(2^(2/3)*a^(2/3)*b) + Log[a - b*x^3]/(2*2^(2/3)*a^(2/3)*b) - (3*Log[2^(1/
3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(2/3)*a^(2/3)*b)))/(3*d)
```

3.571.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.571. $\int \frac{\sqrt[3]{a+bx^3}}{x(ad-bdx^3)}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.571.4 Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$\frac{2\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)-2\,2^{\frac{1}{3}}\,\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)+2^{\frac{1}{3}}\,\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{6d\,a^{\frac{2}{3}}}$

input
```
int((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*2^(1/3)*3^(1/2)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3
)*3^(1/2))-2*2^(1/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))+2^(1/3)*ln((b*x^3
+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))-2*arctan(1/3*(a
^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-a^
(1/3))-ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3)))/d/a^(2/3)
```

3.571.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(161) = 322$.

Time = 0.26 (sec) , antiderivative size = 634, normalized size of antiderivative = 2.96

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(ad-bdx^3)}\,dx =$$

$$-\frac{1}{6}\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\left(-\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}+1}{a^2d^3}\right)^{\frac{1}{3}}\log\left(-\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\sqrt{-3}a^3d^4+a^3d^4\right)\left(-\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}+1}{a^2d^3}\right)^{\frac{1}{3}}\right.$$
$$\left.+2\left(bx^3+a\right)^{\frac{1}{3}}\right)$$

$$+\frac{1}{6}\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\left(-\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}+1}{a^2d^3}\right)^{\frac{1}{3}}\log\left(\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\sqrt{-3}a^3d^4-a^3d^4\right)\left(-\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}+1}{a^2d^3}\right)^{\frac{1}{3}}\right.$$
$$\left.+2\left(bx^3+a\right)^{\frac{1}{3}}\right)$$

$$-\frac{1}{6}\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\left(\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}-1}{a^2d^3}\right)^{\frac{1}{3}}\log\left(\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\sqrt{-3}a^3d^4+a^3d^4\right)\left(\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}-1}{a^2d^3}\right)^{\frac{1}{3}}\right.$$
$$\left.+2\left(bx^3+a\right)^{\frac{1}{3}}\right)$$

$$+\frac{1}{6}\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\left(\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}-1}{a^2d^3}\right)^{\frac{1}{3}}\log\left(-\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\sqrt{-3}a^3d^4-a^3d^4\right)\left(\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}-1}{a^2d^3}\right)^{\frac{1}{3}}\right.$$
$$\left.+2\left(bx^3+a\right)^{\frac{1}{3}}\right)$$

$$+\frac{1}{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(-\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}+1}{a^2d^3}\right)^{\frac{1}{3}}\log\left(\left(\frac{1}{2}\right)^{\frac{1}{3}}a^3d^4\left(-\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}+1}{a^2d^3}\right)^{\frac{1}{3}}\sqrt{\frac{1}{a^4d^6}}\right.$$
$$\left.+\left(bx^3+a\right)^{\frac{1}{3}}\right)$$

3.571. $\int \frac{\sqrt[3]{a+bx^3}}{x\,(ad-bdx^3)}\,dx$

$$+\frac{1}{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}-1}{a^2d^3}\right)^{\frac{1}{3}}\log\left(-\left(\frac{1}{2}\right)^{\frac{1}{3}}a^3d^4\left(\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}-1}{a^2d^3}\right)^{\frac{1}{3}}\sqrt{\frac{1}{a^4d^6}}\right.$$

input

```
integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output

```
-1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d
^3))^(1/3)*log(-(1/2)^(1/3)*(sqrt(-3)*a^3*d^4 + a^3*d^4)*(-(3*a^2*d^3*sqrt
(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 2*(b*x^3 + a)^(1/3
)) + 1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a
^2*d^3))^(1/3)*log((1/2)^(1/3)*(sqrt(-3)*a^3*d^4 - a^3*d^4)*(-(3*a^2*d^3*s
qrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 2*(b*x^3 + a)^(
1/3)) - 1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/
(a^2*d^3))^(1/3)*log((1/2)^(1/3)*(sqrt(-3)*a^3*d^4 + a^3*d^4)*((3*a^2*d^3*
sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 2*(b*x^3 + a)^
(1/3)) + 1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)
/(a^2*d^3))^(1/3)*log(-(1/2)^(1/3)*(sqrt(-3)*a^3*d^4 - a^3*d^4)*((3*a^2*d^
3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 2*(b*x^3 + a
)^(1/3)) + 1/3*(1/2)^(1/3)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^
(1/3)*log((1/2)^(1/3)*a^3*d^4*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3
))^(1/3)*sqrt(1/(a^4*d^6)) + (b*x^3 + a)^(1/3)) + 1/3*(1/2)^(1/3)*((3*a^2*
d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*log(-(1/2)^(1/3)*a^3*d^4*((3*a
^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + (b*x^3
+ a)^(1/3))
```

3.571.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(ad-bdx^3)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax+bx^4}\,dx}{d}$$

input

```
integrate((b*x**3+a)**(1/3)/x/(-b*d*x**3+a*d),x)
```

output

```
-Integral((a + b*x**3)**(1/3)/(-a*x + b*x**4), x)/d
```

3.571.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x), x)
```

3.571.8 Giac [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx &= \frac{\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}d} \\ &\quad - \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}d} \\ &\quad + \frac{2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}d} \\ &\quad - \frac{2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{2}{3}}d} \\ &\quad - \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}d} + \frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{2}{3}}d}\end{aligned}$$

input
```
integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*x^3
 + a)^(1/3))/a^(1/3))/(a^(2/3)*d) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x
^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(2/3)*d) + 1/6*2^(1/3)*log(2^(2/3)*a^
(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(a^(2/3)*d)
 - 1/3*2^(1/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(a^(2/3)*d)
- 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3
)*d) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*d)
```

3.571.9 Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(ad-bdx^3)}\,dx = \ln\left(\left(b\,x^3+a\right)^{1/3}-a\,d\,\left(\frac{1}{a^2\,d^3}\right)^{1/3}\right)\,\left(\frac{1}{27\,a^2\,d^3}\right)^{1/3}$$
$$+\ln\left(\left(b\,x^3+a\right)^{1/3}+2^{1/3}\,a\,d\,\left(-\frac{1}{a^2\,d^3}\right)^{1/3}\right)\,\left(-\frac{2}{27\,a^2\,d^3}\right)^{1/3}-\ln\left(2^{1/3}\,a\,d\,\left(-\frac{1}{a^2\,d^3}\right)^{1/3}-2\,\left(b\,x^3+a\right)^{1/3}\right.$$

input
```
int((a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x)
```

output
```
log((a + b*x^3)^(1/3) - a*d*(1/(a^2*d^3))^(1/3))*(1/(27*a^2*d^3))^(1/3) +
log((a + b*x^3)^(1/3) + 2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3))*(-2/(27*a^2*d^3)
)^(1/3) - log(2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3) - 2*(a + b*x^3)^(1/3) + 2^(
1/3)*3^(1/2)*a*d*(-1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(-2/(27*a
^2*d^3))^(1/3) + log(2*(a + b*x^3)^(1/3) - 2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3
) + 2^(1/3)*3^(1/2)*a*d*(-1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(-
2/(27*a^2*d^3))^(1/3) + log(2*(a + b*x^3)^(1/3) + a*d*(1/(a^2*d^3))^(1/3)
- 3^(1/2)*a*d*(1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^2*d^
3))^(1/3) - log(2*(a + b*x^3)^(1/3) + a*d*(1/(a^2*d^3))^(1/3) + 3^(1/2)*a*
d*(1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^2*d^3))^(1/3)
```

3.572 $\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx$

3.572.1 Optimal result

Integrand size = 28, antiderivative size = 268

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx &= \frac{b\sqrt[3]{a+bx^3}}{3a^2d}-\frac{(a+bx^3)^{4/3}}{3a^2dx^3}-\frac{4b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}d}\\&\quad+\frac{\sqrt[3]{2}b\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}d}-\frac{2b\log(x)}{3a^{5/3}d}+\frac{b\log(a-bx^3)}{3\ 2^{2/3}a^{5/3}d}\\&\quad+\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{5/3}d}\end{aligned}$$

output
```
1/3*b*(b*x^3+a)^(1/3)/a^2/d-1/3*(b*x^3+a)^(4/3)/a^2/d/x^3-2/3*b*ln(x)/a^(5
/3)/d+1/6*b*ln(-b*x^3+a)*2^(1/3)/a^(5/3)/d+2/3*b*ln(a^(1/3)-(b*x^3+a)^(1/3
))/a^(5/3)/d-1/2*b*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/a^(5/3)/d-4
/9*b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(5/3)/d*3^(
1/2)+1/3*2^(1/3)*b*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^
(1/2))/a^(5/3)/d*3^(1/2)
```

3.572.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx =$$

$$-\frac{6a^{2/3}\sqrt[3]{a+bx^3}+8\sqrt{3}bx^3\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-6\sqrt[3]{2}\sqrt{3}bx^3\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-8bx^3\log}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]
```

output
```
-1/18*(6*a^(2/3)*(a + b*x^3)^(1/3) + 8*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b
*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 6*2^(1/3)*Sqrt[3]*b*x^3*ArcTan[(1 + (2^(2
/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 8*b*x^3*Log[-a^(1/3) + (a + b*x
^3)^(1/3)] + 6*2^(1/3)*b*x^3*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] +
 4*b*x^3*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 3*
2^(1/3)*b*x^3*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*
(a + b*x^3)^(2/3)])/(a^(5/3)*d*x^3)
```

3.572.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {948, 27, 114, 27, 174, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}}{dx^6\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt[3]{bx^3+a}}{x^6\left(a-bx^3\right)}dx^3}{3d}$$

$$\Big\downarrow \ 114$$

$$\frac{-\frac{\int -\frac{b\left(4a-bx^3\right)\sqrt[3]{bx^3+a}}{3x^3\left(a-bx^3\right)}dx^3}{a^2}-\frac{\left(a+bx^3\right)^{4/3}}{a^2x^3}}{3d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{b\int \frac{\left(4a-bx^3\right)\sqrt[3]{bx^3+a}}{x^3\left(a-bx^3\right)}dx^3}{3a^2}-\frac{\left(a+bx^3\right)^{4/3}}{a^2x^3}}{3d}$$

$$\Big\downarrow \ 174$$

$$\frac{\frac{b\left(4\int \frac{\sqrt[3]{bx^3+a}}{x^3}dx^3+3b\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx^3\right)}{3a^2}-\frac{\left(a+bx^3\right)^{4/3}}{a^2x^3}}{3d}$$

$$\Big\downarrow \ 60$$

$$\frac{\frac{b\left(4\left(a\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3+3\sqrt[3]{a+bx^3}\right)+3b\left(2a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx^3-\frac{3\sqrt[3]{a+bx^3}}{b}\right)\right)}{3a^2}-\frac{\left(a+bx^3\right)^{4/3}}{a^2x^3}}{3d}$$

$$\Big\downarrow \ 69$$

$$\frac{\frac{b\left(4\left(a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+3b\left(2a\left(\frac{3\int}{\sqrt[3]{2}}\right.\right.\right.}{3a^2}}{3d}$$

$$\Big\downarrow \ 16$$

$$\frac{\frac{b\left(4\left(a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+3b\left(2a\left(\frac{3\int \frac{}{x^6+2^{2/3}a^{2/3}+}}{}\right.\right.\right.}{3a^2}}{3d}$$

$$\Big\downarrow \ 1082$$

3.572. $\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}dx$

$$\frac{\frac{b\Bigg(4\Bigg(a\Bigg(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\Bigg)+3\sqrt[3]{a+bx^3}\Bigg)+3b\Bigg(2a\Bigg(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+\right.}{2^{2/3}a^{2/3}b}}{3a^2}}{3d}$$

$$\Big\downarrow \color{blue}{217}$$

$$\frac{\frac{b\Bigg(4\Bigg(a\Bigg(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\Bigg)+3\sqrt[3]{a+bx^3}\Bigg)+3b\Bigg(2a\Bigg(\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}b}}{3a^2}}{3d}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]
```

output
```
(-((a + b*x^3)^(4/3)/(a^2*x^3)) + (b*(4*(3*(a + b*x^3)^(1/3) + a*(-((Sqrt[
3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3
]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3)))) + 3*b*(
(-3*(a + b*x^3)^(1/3))/b + 2*a*((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^
(1/3))/a^(1/3))/Sqrt[3]])/(2^(2/3)*a^(2/3)*b) + Log[a - b*x^3]/(2*2^(2/3)*
a^(2/3)*b) - (3*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(2/3)*a^(2/
3)*b)))))/(3*a^2))/(3*d)
```

3.572.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.572. $\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx$

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.572. $\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.572.4 Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{-2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)b\,x^{3}+2^{\frac{1}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)b\,x^{3}+\frac{4\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}\,b\,x}{3}}{}$

input
```
int((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-2^(1/3)*3^(1/2)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3
)*3^(1/2))*b*x^3+2^(1/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))*b*x^3+4/3*arc
tan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*b*x^3-1/2*2^(
1/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))*b
*x^3-4/3*ln((b*x^3+a)^(1/3)-a^(1/3))*b*x^3+2/3*ln((b*x^3+a)^(2/3)+a^(1/3)*
(b*x^3+a)^(1/3)+a^(2/3))*b*x^3+(b*x^3+a)^(1/3)*a^(2/3))/a^(5/3)/x^3/d
```

3.572.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx =$$

$$-\frac{6\sqrt{3}2^{\frac{1}{3}}a^{2}bx^{3}\left(-\frac{1}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}2^{\frac{2}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}a\left(-\frac{1}{a^{2}}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)+3\cdot 2^{\frac{1}{3}}a^{2}bx^{3}\left(-\frac{1}{a^{2}}\right)^{\frac{1}{3}}\log\Bigl(2^{\frac{2}{3}}a^{2}\left(-\frac{1}{a^{2}}\right)}{}$$

input
```
integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/18*(6*sqrt(3)*2^(1/3)*a^2*b*x^3*(-1/a^2)^(1/3)*arctan(1/3*sqrt(3)*2^(2/
3)*(b*x^3 + a)^(1/3)*a*(-1/a^2)^(2/3) + 1/3*sqrt(3)) + 3*2^(1/3)*a^2*b*x^3
*(-1/a^2)^(1/3)*log(2^(2/3)*a^2*(-1/a^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)
*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(2/3)) - 6*2^(1/3)*a^2*b*x^3*(-1/a^2)^(1/3
)*log(2^(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(1/3)) + 8*sqrt(3)*(a^2)^(1/6
)*a*b*x^3*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3
 + a)^(1/3)*(a^2)^(2/3))/a^2) + 4*(a^2)^(2/3)*b*x^3*log((b*x^3 + a)^(2/3)*
a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 8*(a^2)^(2/3)*b*x^3*l
og((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)*a^2)/(a^3*d*x^
3)
```

3.572.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx=-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^4+bx^7}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**4/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**4 + b*x**7), x)/d
```

3.572.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx=\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^4}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^4), x)
```

3.572.8 Giac [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx = \frac{\sqrt{3}2^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{3a^{\frac{5}{3}}d} - \frac{4\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{5}{3}}d} + \frac{2^{\frac{1}{3}}b\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6a^{\frac{5}{3}}d} - \frac{2^{\frac{1}{3}}b\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3a^{\frac{5}{3}}d} - \frac{2b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{5}{3}}d} + \frac{4b\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9a^{\frac{5}{3}}d} - \frac{(bx^3+a)^{\frac{1}{3}}}{3adx^3}$$

input
```
integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*2^(1/3)*b*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*x
^3 + a)^(1/3))/a^(1/3))/(a^(5/3)*d) - 4/9*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*
(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(5/3)*d) + 1/6*2^(1/3)*b*log(2^(2
/3)*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(a^(5
/3)*d) - 1/3*2^(1/3)*b*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(a^(
5/3)*d) - 2/9*b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3
))/(a^(5/3)*d) + 4/9*b*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*d) -
 1/3*(b*x^3 + a)^(1/3)/(a*d*x^3)
```

3.572.9 Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx = \frac{4\,\ln\left(b\left(b\,x^3+a\right)^{1/3}-a^2\,d\left(\frac{b^3}{a^5\,d^3}\right)^{1/3}\right)\left(\frac{b^3}{a^5\,d^3}\right)^{1/3}}{9} + \ln\left(b\left(b\,x^3+a\right)^{1/3}+2^{1/3}\,a^2\,d\left(-\frac{b^3}{a^5\,d^3}\right)^{1/3}\right)\left(-\frac{2\,b^3}{27\,a^5\,d^3}\right)^{1/3} + \ln\left(2\,b\left(b\,x^3+a\right)^{1/3}+a^2\,d\left(\frac{b^3}{a^5\,d^3}\right)^{1/3} - \right.$$

input

```
int((a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x)
```

output

```
(4*log(b*(a + b*x^3)^(1/3) - a^2*d*(b^3/(a^5*d^3))^(1/3))*(b^3/(a^5*d^3))^
(1/3))/9 + log(b*(a + b*x^3)^(1/3) + 2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(1/3))
*(-(2*b^3)/(27*a^5*d^3))^(1/3) + log(2*b*(a + b*x^3)^(1/3) + a^2*d*(b^3/(a
^5*d^3))^(1/3) - 3^(1/2)*a^2*d*(b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 -
 1/2)*((64*b^3)/(729*a^5*d^3))^(1/3) - log(2*b*(a + b*x^3)^(1/3) + a^2*d*(
b^3/(a^5*d^3))^(1/3) + 3^(1/2)*a^2*d*(b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1
i)/2 + 1/2)*((64*b^3)/(729*a^5*d^3))^(1/3) - log(2^(1/3)*a^2*d*(-b^3/(a^5*
d^3))^(1/3) - 2*b*(a + b*x^3)^(1/3) + 2^(1/3)*3^(1/2)*a^2*d*(-b^3/(a^5*d^3
))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(-(2*b^3)/(27*a^5*d^3))^(1/3) + log(2*
b*(a + b*x^3)^(1/3) - 2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(1/3) + 2^(1/3)*3^(1/
2)*a^2*d*(-b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(-(2*b^3)/(27*a
^5*d^3))^(1/3) - (b*(a + b*x^3)^(1/3))/(3*a*(d*(a + b*x^3) - a*d))
```

3.573 $\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)}\,dx$

3.573.1 Optimal result

Integrand size = 28, antiderivative size = 283

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)}\,dx = &-\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3}-\frac{(a+bx^3)^{4/3}}{6a^2dx^6}-\frac{11b^2\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}d}\\&+\frac{\sqrt[3]{2}b^2\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}d}-\frac{11b^2\log(x)}{18a^{8/3}d}+\frac{b^2\log(a-bx^3)}{3\ 2^{2/3}a^{8/3}d}\\&+\frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{8/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{8/3}d}\end{aligned}$$

output
```
-2/9*b*(b*x^3+a)^(1/3)/a^2/d/x^3-1/6*(b*x^3+a)^(4/3)/a^2/d/x^6-11/18*b^2*l
n(x)/a^(8/3)/d+1/6*b^2*ln(-b*x^3+a)*2^(1/3)/a^(8/3)/d+11/18*b^2*ln(a^(1/3)
-(b*x^3+a)^(1/3))/a^(8/3)/d-1/2*b^2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^
(1/3)/a^(8/3)/d-11/27*b^2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3
^(1/2))/a^(8/3)/d*3^(1/2)+1/3*2^(1/3)*b^2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x
^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(8/3)/d*3^(1/2)
```

3.573.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx =$$

$$-\frac{9a^{5/3}\sqrt[3]{a+bx^3}+21a^{2/3}bx^3\sqrt[3]{a+bx^3}+22\sqrt{3}b^2x^6\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-18\sqrt[3]{2}\sqrt{3}b^2x^6\arctan\left(\frac{1+}{}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x]
```

output
```
-1/54*(9*a^(5/3)*(a + b*x^3)^(1/3) + 21*a^(2/3)*b*x^3*(a + b*x^3)^(1/3) +
22*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 1
8*2^(1/3)*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))
/Sqrt[3]] - 22*b^2*x^6*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + 18*2^(1/3)*b^2*
x^6*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] + 11*b^2*x^6*Log[a^(2/3) +
 a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 9*2^(1/3)*b^2*x^6*Log[2*
a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/
(a^(8/3)*d*x^6)
```

3.573.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {948, 27, 114, 27, 166, 27, 174, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}}{dx^9\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt[3]{bx^3+a}}{x^9(a-bx^3)}dx^3}{3d}$$

$$\Big\downarrow \ 114$$

$$\frac{-\frac{\int -\frac{2b\sqrt[3]{bx^3+a}\left(bx^3+2a\right)}{3x^6\left(a-bx^3\right)}dx^3}{2a^2}-\frac{\left(a+bx^3\right)^{4/3}}{2a^2x^6}}{3d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{b\int \frac{\sqrt[3]{bx^3+a}\left(bx^3+2a\right)}{x^6\left(a-bx^3\right)}dx^3}{3a^2}-\frac{\left(a+bx^3\right)^{4/3}}{2a^2x^6}}{3d}$$

$$\Big\downarrow \ 166$$

$$\frac{\frac{b\left(\frac{\int \frac{ab\left(7bx^3+11a\right)}{3x^3\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx^3}{a}-\frac{2\sqrt[3]{a+bx^3}}{x^3}\right)}{3a^2}-\frac{\left(a+bx^3\right)^{4/3}}{2a^2x^6}}{3d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{b\left(\frac{1}{3}b\int \frac{7bx^3+11a}{x^3\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx^3-\frac{2\sqrt[3]{a+bx^3}}{x^3}\right)}{3a^2}-\frac{\left(a+bx^3\right)^{4/3}}{2a^2x^6}}{3d}$$

$$\Big\downarrow \ 174$$

$$\frac{\frac{b\left(\frac{1}{3}b\left(11\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3+18b\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx^3\right)-\frac{2\sqrt[3]{a+bx^3}}{x^3}\right)}{3a^2}-\frac{\left(a+bx^3\right)^{4/3}}{2a^2x^6}}{3d}$$

$$\Big\downarrow \ 69$$

$$b\Bigg(\frac{1}{3}b\Bigg(11\Bigg(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\Bigg)+18b\Bigg(\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{bx^3+a}}}{2\ 2^{2/3}a^{2/3}b}$$

$$3a^2$$

$$3d$$

$$\Big\downarrow \ 16$$

3.573. $\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)}\,dx$

$$\frac{b\left(\frac{1}{3}b\left(11\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+18b\left(\frac{3\int\frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+}}}{2\sqrt[3]{2}\sqrt[3]{a}b}\right.\right.\right.}{3a^2}$$
$$\overline{\qquad 3d \qquad}$$

$$\Big\downarrow \color{blue}{1082}$$

$$\frac{b\left(\frac{1}{3}b\left(11\left(\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+18b\left(-\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{2^{2/3}a^{2/3}b}+\frac{\log\left(a-bx^3\right)}{2\ 2^{2/3}a^{2/3}b}-\right.\right.\right.}{3a^2}$$
$$\overline{\qquad 3d \qquad}$$

$$\Big\downarrow \color{blue}{217}$$

$$\frac{b\left(\frac{1}{3}b\left(11\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+18b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}b}+\frac{\log\left(a-bx^3\right)}{2\ 2^{2/3}a^{2/3}b}-\frac{3\,\mathrm{lo}}{}\right.\right.\right.}{3a^2}$$
$$\overline{\qquad 3d \qquad}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x]
```

output
```
(-1/2*(a + b*x^3)^(4/3)/(a^2*x^6) + (b*((-2*(a + b*x^3)^(1/3))/x^3 + (b*(1
1*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)
) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3)
)) + 18*b*((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[
3]])/(2^(2/3)*a^(2/3)*b) + Log[a - b*x^3]/(2*2^(2/3)*a^(2/3)*b) - (3*Log[2
^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(2/3)*a^(2/3)*b))))/3))/(3*a^2))
/(3*d)
```

3.573. $\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)}\,dx$

3.573.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

3.573. $\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.573.4 Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$18\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)b^{2}x^{6}-18\,2^{\frac{1}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)b^{2}x^{6}+9\,2^{\frac{1}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right.\right.$

input
```
int((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/54/a^(8/3)*(18*2^(1/3)*3^(1/2)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/
3))/a^(1/3)*3^(1/2))*b^2*x^6-18*2^(1/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3)
)*b^2*x^6+9*2^(1/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(
2/3)*a^(2/3))*b^2*x^6-22*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^
(1/2))*3^(1/2)*b^2*x^6+22*ln((b*x^3+a)^(1/3)-a^(1/3))*b^2*x^6-11*ln((b*x^3
+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))*b^2*x^6-21*b*x^3*a^(2/3)*(b*x^3
+a)^(1/3)-9*(b*x^3+a)^(1/3)*a^(5/3))/x^6/d
```

3.573.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx =$$

$$-\frac{18\sqrt{3}2^{\frac{1}{3}}a^2b^2x^6\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)+9\cdot2^{\frac{1}{3}}a^2b^2x^6\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^2\left(-\right.\right.}{}$$

input
```
integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/54*(18*sqrt(3)*2^(1/3)*a^2*b^2*x^6*(-1/a^2)^(1/3)*arctan(1/3*sqrt(3)*2^
(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a^2)^(2/3) + 1/3*sqrt(3)) + 9*2^(1/3)*a^2*b^
2*x^6*(-1/a^2)^(1/3)*log(2^(2/3)*a^2*(-1/a^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^
(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(2/3)) - 18*2^(1/3)*a^2*b^2*x^6*(-1/a
^2)^(1/3)*log(2^(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(1/3)) + 22*sqrt(3)*(
a^2)^(1/6)*a*b^2*x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqr
t(3)*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) + 11*(a^2)^(2/3)*b^2*x^6*log((b*x
^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 22*(a^2
)^(2/3)*b^2*x^6*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*a^2*b*x^3 +
3*a^3)*(b*x^3 + a)^(1/3))/(a^4*d*x^6)
```

3.573.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^7+bx^{10}}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**7/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**7 + b*x**10), x)/d
```

3.573.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx = \int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^7}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^7), x)
```

3.573.8 Giac [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx &= \frac{\sqrt{3}2^{\frac{1}{3}}b^2\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{8}{3}}d} \\ &\quad - \frac{11\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{8}{3}}d} \\ &\quad + \frac{2^{\frac{1}{3}}b^2\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{6\,a^{\frac{8}{3}}d} \\ &\quad - \frac{2^{\frac{1}{3}}b^2\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{8}{3}}d} \\ &\quad - \frac{11\,b^2\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54\,a^{\frac{8}{3}}d} \\ &\quad + \frac{11\,b^2\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{8}{3}}d} - \frac{7\left(bx^3+a\right)^{\frac{4}{3}}b^2-4\left(bx^3+a\right)^{\frac{1}{3}}ab^2}{18\,a^2b^2dx^6}\end{aligned}$$

input
```
integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*2^(1/3)*b^2*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b
*x^3 + a)^(1/3))/a^(1/3))/(a^(8/3)*d) - 11/27*sqrt(3)*b^2*arctan(1/3*sqrt(
3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(8/3)*d) + 1/6*2^(1/3)*b^2*
log(2^(2/3)*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3
))/(a^(8/3)*d) - 1/3*2^(1/3)*b^2*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1
/3)))/(a^(8/3)*d) - 11/54*b^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^
(1/3) + a^(2/3))/(a^(8/3)*d) + 11/27*b^2*log(abs((b*x^3 + a)^(1/3) - a^(1/
3)))/(a^(8/3)*d) - 1/18*(7*(b*x^3 + a)^(4/3)*b^2 - 4*(b*x^3 + a)^(1/3)*a*b
^2)/(a^2*b^2*d*x^6)
```

3.573.9 Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\,(ad-bdx^3)}\,dx = \frac{\frac{2\,b^2\,\left(b\,x^3+a\right)^{1/3}}{9\,a}-\frac{7\,b^2\,\left(b\,x^3+a\right)^{4/3}}{18\,a^2}}{d\,\left(b\,x^3+a\right)^2+a^2\,d-2\,a\,d\,\left(b\,x^3+a\right)} + \frac{11\,\ln\left(b^2\,\left(b\,x^3+a\right)^{1/3}-a^3\,d\,\left(\frac{b^6}{a^8\,d^3}\right)^{1/3}\right)\,\left(\frac{b^6}{a^8\,d^3}\right)^{1/3}}{27} + \ln\left(b^2\,\left(b\,x^3+a\right)^{1/3}+2^{1/3}\,a^3\,d\,\left(-\frac{b^6}{a^8\,d^3}\right)^{1/3}\right)\,\left(-\frac{2\,b^6}{27\,a^8\,d^3}\right)^{1/3} - \ln\left(2^{1/3}\,a^3\,d\,\left(-\frac{b^6}{a^8\,d^3}\right)^{1/3}-2\,b^2\,\left(b\,x^3+\right.\right.$$

input
```
int((a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x)
```

output
```
((2*b^2*(a + b*x^3)^(1/3))/(9*a) - (7*b^2*(a + b*x^3)^(4/3))/(18*a^2))/(d*
(a + b*x^3)^2 + a^2*d - 2*a*d*(a + b*x^3)) + (11*log(b^2*(a + b*x^3)^(1/3)
 - a^3*d*(b^6/(a^8*d^3))^(1/3))*(b^6/(a^8*d^3))^(1/3))/27 + log(b^2*(a + b
*x^3)^(1/3) + 2^(1/3)*a^3*d*(-b^6/(a^8*d^3))^(1/3))*(-(2*b^6)/(27*a^8*d^3)
)^(1/3) - log(2^(1/3)*a^3*d*(-b^6/(a^8*d^3))^(1/3) - 2*b^2*(a + b*x^3)^(1/
3) + 2^(1/3)*3^(1/2)*a^3*d*(-b^6/(a^8*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/
2)*(-(2*b^6)/(27*a^8*d^3))^(1/3) + log(2*b^2*(a + b*x^3)^(1/3) - 2^(1/3)*a
^3*d*(-b^6/(a^8*d^3))^(1/3) + 2^(1/3)*3^(1/2)*a^3*d*(-b^6/(a^8*d^3))^(1/3)
*1i)*((3^(1/2)*1i)/2 - 1/2)*(-(2*b^6)/(27*a^8*d^3))^(1/3) + (11*log(2*b^2*
(a + b*x^3)^(1/3) + a^3*d*(b^6/(a^8*d^3))^(1/3) - 3^(1/2)*a^3*d*(b^6/(a^8*
d^3))^(1/3)*1i)*(3^(1/2)*1i - 1)*(b^6/(a^8*d^3))^(1/3))/54 - (11*log(2*b^2
*(a + b*x^3)^(1/3) + a^3*d*(b^6/(a^8*d^3))^(1/3) + 3^(1/2)*a^3*d*(b^6/(a^8
*d^3))^(1/3)*1i)*(3^(1/2)*1i + 1)*(b^6/(a^8*d^3))^(1/3))/54
```

3.574 $\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

3.574.1 Optimal result

Integrand size = 28, antiderivative size = 268

$$\begin{aligned}\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = & -\frac{7ax^2\sqrt[3]{a+bx^3}}{18b^2d}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}+\frac{11a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{8/3}d}\\ & -\frac{\sqrt[3]{2}a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{8/3}d}+\frac{a^2\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{8/3}d}\\ & +\frac{11a^2\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{8/3}d}-\frac{a^2\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{8/3}d}\end{aligned}$$

output
```
-7/18*a*x^2*(b*x^3+a)^(1/3)/b^2/d-1/6*x^5*(b*x^3+a)^(1/3)/b/d+1/6*a^2*ln(-
b*d*x^3+a*d)*2^(1/3)/b^(8/3)/d+11/18*a^2*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(
8/3)/d-1/2*a^2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/b^(8/3)/d+11/
27*a^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(8/3)/d*3^(1/
2)-1/3*2^(1/3)*a^2*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1
/2))/b^(8/3)/d*3^(1/2)
```

3.574.2 Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.22

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =$$

$$-\frac{21ab^{2/3}x^2\sqrt[3]{a+bx^3}+9b^{5/3}x^5\sqrt[3]{a+bx^3}-22\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+18\sqrt[3]{2}\sqrt{3}a^2\arctan\Bigg(}{}$$

input
```
Integrate[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
-1/54*(21*a*b^(2/3)*x^2*(a + b*x^3)^(1/3) + 9*b^(5/3)*x^5*(a + b*x^3)^(1/3
) - 22*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(
1/3))] + 18*2^(1/3)*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^
(2/3)*(a + b*x^3)^(1/3))] - 22*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] +
 18*2^(1/3)*a^2*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + 11*a^2*Log
[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 9*2^(1/3
)*a^2*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a
 + b*x^3)^(2/3)])/(b^(8/3)*d)
```

3.574.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {978, 27, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$

$$\Big\downarrow \text{978}$$

$$\frac{\int \frac{ax^4(7bx^3+5a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{6bd}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{x^4(7bx^3+5a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{6bd}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}$$

$$\Big\downarrow 1052$$

$$\frac{a\left(\frac{\int \frac{2abx(11bx^3+7a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{3b^2}-\frac{7x^2\sqrt[3]{a+bx^3}}{3b}\right)}{6bd}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}$$

$$\Big\downarrow 27$$

$$\frac{a\left(\frac{2a\int \frac{x(11bx^3+7a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{3b}-\frac{7x^2\sqrt[3]{a+bx^3}}{3b}\right)}{6bd}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}$$

$$\Big\downarrow 1054$$

$$\frac{a\left(\frac{2a\int \left(\frac{18ax}{(a-bx^3)(bx^3+a)^{2/3}}-\frac{11x}{(bx^3+a)^{2/3}}\right)dx}{3b}-\frac{7x^2\sqrt[3]{a+bx^3}}{3b}\right)}{6bd}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}$$

$$\Big\downarrow 2009$$

$$\frac{a\left(\frac{2a\left(\frac{11\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{3\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{b^{2/3}}+\frac{3\log(a-bx^3)}{2^{2/3}b^{2/3}}+\frac{11\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}-\frac{9\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a}\right.}{2^{2/3}b^{2/3}}\right.}{3b}\right.}{6bd}$$

$$\frac{x^5\sqrt[3]{a+bx^3}}{6bd}$$

input
```
Int[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

3.574. $\int \frac{x^7\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

output
```
-1/6*(x^5*(a + b*x^3)^(1/3))/(b*d) + (a*((-7*x^2*(a + b*x^3)^(1/3))/(3*b)
+ (2*a*((11*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3
]*b^(2/3)) - (3*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x
^3)^(1/3))/Sqrt[3]])/b^(2/3) + (3*Log[a - b*x^3])/(2^(2/3)*b^(2/3)) + (11*
Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*b^(2/3)) - (9*Log[2^(1/3)*b^(1/3)*x
 - (a + b*x^3)^(1/3)])/(2^(2/3)*b^(2/3))))/(3*b)))/(6*b*d)
```

3.574.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.574. $\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

3.574.4 Maple [A] (verified)

Time = 7.07 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$-9x^{5}\left(b\,x^{3}+a\right)^{\frac{1}{3}} b^{\frac{5}{3}}-21a\,x^{2}\left(b\,x^{3}+a\right)^{\frac{1}{3}} b^{\frac{2}{3}}+18\,2^{\frac{1}{3}} \sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2^{\frac{2}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}} x\right)}{3 b^{\frac{1}{3}} x}\right) a^{2}-18\,2^{\frac{1}{3}} \ln\Bigg(\frac{-2^{\frac{1}{3}} b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}$

input
```
int(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/54/b^(8/3)*(-9*x^5*(b*x^3+a)^(1/3)*b^(5/3)-21*a*x^2*(b*x^3+a)^(1/3)*b^(2
/3)+18*2^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)
*x)/b^(1/3)/x)*a^2-18*2^(1/3)*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a
^2+9*2^(1/3)*ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^(1/3)*x+(b*
x^3+a)^(2/3))/x^2)*a^2-22*a^2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x
^3+a)^(1/3))/b^(1/3)/x)+22*a^2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-11*a^2*l
n((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))/d
```

3.574.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.35

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =$$

$$-\frac{18\,\sqrt{3}2^{\frac{1}{3}}a^{2}b^{2}\left(-\frac{1}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}b\left(-\frac{1}{b^{2}}\right)^{\frac{2}{3}}+\sqrt{3}x}{3\,x}\right)-18\cdot 2^{\frac{1}{3}}a^{2}b^{2}\left(-\frac{1}{b^{2}}\right)^{\frac{1}{3}}\log\Bigg(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^{2}}\right)^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}}{}$$

input
```
integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/54*(18*sqrt(3)*2^(1/3)*a^2*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/
3)*(b*x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 18*2^(1/3)*a^2*b^2
*(-1/b^2)^(1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) +
9*2^(1/3)*a^2*b^2*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(
1/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 22*s
qrt(3)*a^2*(b^2)^(1/6)*b*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 22*a^2*(b^2)^(2/3)*lo
g(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 11*a^2*(b^2)^(2/3)*log(((b^2
)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2
) + 3*(3*b^3*x^5 + 7*a*b^2*x^2)*(b*x^3 + a)^(1/3))/(b^4*d)
```

3.574.6 Sympy [F]

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^7 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**7*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**7*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.574.7 Maxima [F]

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^7}{bdx^3-ad}\,dx$$

input
```
integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)
```

3.574.8 Giac [F]

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx = \int -\frac{(bx^3+a)^{\frac{1}{3}} x^7}{bdx^3-ad} \, dx$$

input
```
integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)
```

3.574.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx = \int \frac{x^7 \left(b\,x^3+a\right)^{1/3}}{a\,d-b\,d\,x^3} \, dx$$

input
```
int((x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

3.575 $\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

3.575.1 Optimal result

Integrand size = 28, antiderivative size = 233

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=-\frac{x^2\sqrt[3]{a+bx^3}}{3bd}+\frac{4a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}d}-\frac{\sqrt[3]{2}a\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{5/3}d}+\frac{a\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{5/3}d}+\frac{2a\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}d}-\frac{a\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{5/3}d}$$

output
```
-1/3*x^2*(b*x^3+a)^(1/3)/b/d+1/6*a*ln(-b*d*x^3+a*d)*2^(1/3)/b^(5/3)/d+2/3*
a*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/d-1/2*a*ln(2^(1/3)*b^(1/3)*x-(b*x^
3+a)^(1/3))*2^(1/3)/b^(5/3)/d+4/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1
/3))*3^(1/2))/b^(5/3)/d*3^(1/2)-1/3*2^(1/3)*a*arctan(1/3*(1+2*2^(1/3)*b^(1
/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)/d*3^(1/2)
```

3.575.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.26

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =$$

$$-\frac{6b^{2/3}x^2\sqrt[3]{a+bx^3}-8\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+6\sqrt[3]{2}\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-8a\log}{}$$

input
```
Integrate[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
-1/18*(6*b^(2/3)*x^2*(a + b*x^3)^(1/3) - 8*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(1/
3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 6*2^(1/3)*Sqrt[3]*a*ArcTan[(Sqr
t[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] - 8*a*Log[-(b^(1/
3)*x) + (a + b*x^3)^(1/3)] + 6*2^(1/3)*a*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b
*x^3)^(1/3)] + 4*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*
x^3)^(2/3)] - 3*2^(1/3)*a*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3
)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^(5/3)*d)
```

3.575.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {978, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}dx$$

$$\Big\downarrow \; 978$$

$$\frac{\int \frac{2ax(2bx^3+a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{3bd}-\frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

$$\Big\downarrow \; 27$$

$$\frac{2a\int \frac{x(2bx^3+a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{3bd}-\frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

$$\Big\downarrow \text{1054}$$

$$\frac{2a\int\left(\frac{3ax}{(a-bx^3)(bx^3+a)^{2/3}}-\frac{2x}{(bx^3+a)^{2/3}}\right)dx}{3bd}-\frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a\left(\frac{2\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}b^{2/3}}+\frac{\log(a-bx^3)}{2\ 2^{2/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a}\right.}{2\ 2^{2/3}b^{2/3}}\right.}{3bd}$$

$$\frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

input
```
Int[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
-1/3*(x^2*(a + b*x^3)^(1/3))/(b*d) + (2*a*((2*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*2^(
1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(2/3)*b^(2/3)) + Log[a - b
*x^3]/(2*2^(2/3)*b^(2/3)) + Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/b^(2/3) - (
3*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*2^(2/3)*b^(2/3))))/(3*b*d
)
```

3.575.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

3.575. $\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.575.4 Maple [A] (verified)

Time = 4.75 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{-6\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{2} b^{\frac{2}{3}}+6\,2^{\frac{1}{3}} \sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}} x\right)}{3 b^{\frac{1}{3}} x}\right) a-6\,2^{\frac{1}{3}}\,\ln\left(\frac{-2^{\frac{1}{3}} b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) a+3\,2^{\frac{1}{3}}\,\ln\left(\frac{2^{\frac{2}{3}} b^{\frac{2}{3}} x^{2}+2}{}\right.}{}$

input
```
int(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/18/b^(5/3)*(-6*(b*x^3+a)^(1/3)*x^2*b^(2/3)+6*2^(1/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)*x)/b^(1/3)/x)*a-6*2^(1/3)*ln((-2^
(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a+3*2^(1/3)*ln((2^(2/3)*b^(2/3)*x^2+2^
(1/3)*b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a-8*3^(1/2)*arctan(1
/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a+8*ln((-b^(1/3)*x+(b*
x^3+a)^(1/3))/x)*a-4*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(
2/3))/x^2)*a)/d
```

3.575.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.45

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =$$

$$-\frac{6\sqrt{3}2^{\frac{1}{3}}ab^{2}\left(-\frac{1}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}b\left(-\frac{1}{b^{2}}\right)^{\frac{2}{3}}+\sqrt{3}x}{3\,x}\right)-6\cdot 2^{\frac{1}{3}}ab^{2}\left(-\frac{1}{b^{2}}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^{2}}\right)^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)+}{}$$

input
```
integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/18*(6*sqrt(3)*2^(1/3)*a*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*
(b*x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 6*2^(1/3)*a*b^2*(-1/b
^2)^(1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 3*2^(1
/3)*a*b^2*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)*(b*
x^3 + a)^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 6*(b*x^3 + a
)^(1/3)*b^2*x^2 + 8*sqrt(3)*a*(b^2)^(1/6)*b*arctan(1/3*(sqrt(3)*(b^2)^(1/3
)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 8*
a*(b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 4*a*(b^2)^(2
/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)
^(2/3)*b)/x^2))/(b^3*d)
```

3.575.6 Sympy [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^4 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**4*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**4*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.575.7 Maxima [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^4}{bdx^3-ad}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)
```

3.575.8 Giac [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^4}{bdx^3-ad}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)
```

3.575.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int \frac{x^4\,{\left(b\,x^3+a\right)}^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

3.576 $\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

3.576.1 Optimal result

Integrand size = 26, antiderivative size = 201

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d}-\frac{\sqrt[3]{2}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d}+\frac{\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{2/3}d}+\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{2/3}d}$$

output

```
1/6*ln(-b*d*x^3+a*d)*2^(1/3)/b^(2/3)/d+1/2*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b
^(2/3)/d-1/2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/b^(2/3)/d+1/3*a
rctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)/d*3^(1/2)-1/3*2
^(1/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)
/d*3^(1/2)
```

3.576.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.32

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)-}{}$$

input
```
Integrate[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] -
 2*2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*
x^3)^(1/3))] + 2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - 2*2^(1/3)*Log[-2*
b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] - Log[b^(2/3)*x^2 + b^(1/3)*x*(a +
b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + 2^(1/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^
(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(6*b^(2/3)*d)
```

3.576.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {984, 27, 853, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$

$$\Big\downarrow \text{984}$$

$$2a\int \frac{x}{d\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{\int \frac{x}{\left(bx^3+a\right)^{2/3}}dx}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\int \frac{x}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{d}-\frac{\int \frac{x}{\left(bx^3+a\right)^{2/3}}dx}{d}$$

$$\Big\downarrow \text{853}$$

$$\frac{2a\int \frac{x}{(a-bx^3)(bx^3+a)^{2/3}}dx}{d}-\frac{-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}}{d}$$

$$\Big\downarrow \text{992}$$

$$\frac{2a\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}ab^{2/3}}+\frac{\log(a-bx^3)}{6\ 2^{2/3}ab^{2/3}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}ab^{2/3}}\right)}{d}-\frac{-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}}{d}$$

input

```
Int[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output

```
-((-(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3
))) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3)))/d) + (2*a*(-(ArcTan[
(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]*a*
b^(2/3))) + Log[a - b*x^3]/(6*2^(2/3)*a*b^(2/3)) - Log[2^(1/3)*b^(1/3)*x -
 (a + b*x^3)^(1/3)]/(2*2^(2/3)*a*b^(2/3))))/d
```

3.576.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 853

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

3.576.4 Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.09

method	result
pseudoelliptic	$\frac{2\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)-2\,2^{\frac{1}{3}}\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)+2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^{2}+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+(b\,x^{3}+a}{x^{2}}\right.}{6d\,b^{\frac{2}{3}}}$

input
```
int(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*2^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)
*x)/b^(1/3)/x)-2*2^(1/3)*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)+2^(1/3
)*ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3
))/x^2)-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)
/x)+2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)
^(1/3)*x+(b*x^3+a)^(2/3))/x^2))/d/b^(2/3)
```

3.576.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(156) = 312$.

Time = 0.27 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.56

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =$$

$$-\frac{2\sqrt{3}2^{\frac{1}{3}}b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}x}{3\,x}\right)-2\cdot 2^{\frac{1}{3}}b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+2}{}$$

input
```
integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*2^(1/3)*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(b*
x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 2*2^(1/3)*b^2*(-1/b^2)^(
1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 2^(1/3)*b^2
*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^
(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 2*sqrt(3)*(b^2)^(1/6)
*b*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)
^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*(b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3
+ a)^(1/3)*b)/x) + (b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*
(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d)
```

3.576.6 Sympy [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.576.7 Maxima [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}x}{bdx^3-ad}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)
```

3.576.8 Giac [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}x}{bdx^3-ad}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)
```

3.576.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\int \frac{x\,{\left(b\,x^3+a\right)}^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
int((x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

3.577 $\int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)}\,dx$

3.577.1 Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx=-\frac{\sqrt[3]{a+bx^3}}{adx}-\frac{\sqrt[3]{2}\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}ad}+\frac{\sqrt[3]{b}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}ad}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}ad}$$

output
```
-(b*x^3+a)^(1/3)/a/d/x+1/6*b^(1/3)*ln(-b*d*x^3+a*d)*2^(1/3)/a/d-1/2*b^(1/3
)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/a/d-1/3*2^(1/3)*b^(1/3)*ar
ctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a/d*3^(1/2)
```

3.577.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx=$$

$$-\frac{6\sqrt[3]{a+bx^3}+2\sqrt[3]{2}\sqrt{3}\sqrt[3]{b}x\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+2\sqrt[3]{2}\sqrt[3]{b}x\log\left(-2\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}\right)-}{6adx}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x]
```

output
```
-1/6*(6*(a + b*x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*b^(1/3)*x*ArcTan[(Sqrt[3]*b^
(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] + 2*2^(1/3)*b^(1/3)*x*Lo
g[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] - 2^(1/3)*b^(1/3)*x*Log[2*b^(2
/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)]
)/(a*d*x)
```

3.577.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {975, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2 \left(ad-bdx^3\right)} \, dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{2abx}{(a-bx^3)(bx^3+a)^{2/3}}dx}{ad}-\frac{\sqrt[3]{a+bx^3}}{adx}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{x}{(a-bx^3)(bx^3+a)^{2/3}}dx}{d}-\frac{\sqrt[3]{a+bx^3}}{adx}$$

$$\Big\downarrow \text{992}$$

$$\frac{2b\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}ab^{2/3}}+\frac{\log(a-bx^3)}{6\ 2^{2/3}ab^{2/3}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}ab^{2/3}}\right)}{d}-\frac{\sqrt[3]{a+bx^3}}{adx}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x]
```

output
```
-((a + b*x^3)^(1/3)/(a*d*x)) + (2*b*(-(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(
a + b*x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]*a*b^(2/3))) + Log[a - b*x^3]/(
6*2^(2/3)*a*b^(2/3)) - Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(2/
3)*a*b^(2/3))))/d
```

3.577.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

3.577.4 Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{2b^{\frac{1}{3}}2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)x-2b^{\frac{1}{3}}2^{\frac{1}{3}}\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)x+b^{\frac{1}{3}}2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^{2}+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{2}}\right.}{6adx}$

input
```
int((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*b^(1/3)*2^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)
+b^(1/3)*x)/b^(1/3)/x)*x-2*b^(1/3)*2^(1/3)*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a
)^(1/3))/x)*x+b^(1/3)*2^(1/3)*ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x
^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*x-6*(b*x^3+a)^(1/3))/a/d/x
```

3.577.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(125) = 250$.

Time = 84.46 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.53

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\,(ad-bdx^3)}\,dx =$$
$$-\frac{2\sqrt{3}2^{\frac{1}{3}}(-b)^{\frac{1}{3}}\,x\arctan\left(\frac{6\sqrt{3}2^{\frac{2}{3}}\,(19\,b^2x^8+16\,abx^5+a^2x^2)\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}+6\sqrt{3}2^{\frac{1}{3}}\,(5\,b^2x^7-4\,abx^4-a^2x)\,(bx^3+a)^{\frac{2}{3}}(-b)^{\frac{1}{3}}+\sqrt{3}(71}{3\,(109\,b^3x^9+105\,ab^2x^6+3\,a^2bx^3-a^3)}\right.}{}$$

input
```
integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/18*(2*sqrt(3)*2^(1/3)*(-b)^(1/3)*x*arctan(1/3*(6*sqrt(3)*2^(2/3)*(19*b^
2*x^8 + 16*a*b*x^5 + a^2*x^2)*(b*x^3 + a)^(1/3)*(-b)^(2/3) + 6*sqrt(3)*2^(
1/3)*(5*b^2*x^7 - 4*a*b*x^4 - a^2*x)*(b*x^3 + a)^(2/3)*(-b)^(1/3) + sqrt(3
)*(71*b^3*x^9 + 111*a*b^2*x^6 + 33*a^2*b*x^3 + a^3))/(109*b^3*x^9 + 105*a*
b^2*x^6 + 3*a^2*b*x^3 - a^3)) - 2*2^(1/3)*(-b)^(1/3)*x*log(-(6*2^(1/3)*(b*
x^3 + a)^(1/3)*(-b)^(1/3)*b*x^2 + 6*(b*x^3 + a)^(2/3)*b*x + 2^(2/3)*(b*x^3
 - a)*(-b)^(2/3))/(b*x^3 - a)) + 2^(1/3)*(-b)^(1/3)*x*log((3*2^(2/3)*(5*b*
x^4 + a*x)*(b*x^3 + a)^(2/3)*(-b)^(2/3) - 2^(1/3)*(19*b^2*x^6 + 16*a*b*x^3
 + a^2)*(-b)^(1/3) + 12*(2*b^2*x^5 + a*b*x^2)*(b*x^3 + a)^(1/3))/(b^2*x^6
- 2*a*b*x^3 + a^2)) + 18*(b*x^3 + a)^(1/3))/(a*d*x)
```

3.577.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\,(ad-bdx^3)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^2+bx^5}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**2/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**2 + b*x**5), x)/d
```

3.577.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx=\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{(bdx^3-ad)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)
```

3.577.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx=\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{(bdx^3-ad)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)
```

3.577.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^2\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)), x)
```

3.578 $\int \frac{\sqrt[3]{a+bx^3}}{x^5(ad-bdx^3)}\,dx$

3.578.1 Optimal result

Integrand size $=28$, antiderivative size $=183$

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx=-\frac{\sqrt[3]{a+bx^3}}{4adx^4}-\frac{5b\sqrt[3]{a+bx^3}}{4a^2dx}-\frac{\sqrt[3]{2}b^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^2d}+\frac{b^{4/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^2d}-\frac{b^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^2d}$$

output
```
-1/4*(b*x^3+a)^(1/3)/a/d/x^4-5/4*b*(b*x^3+a)^(1/3)/a^2/d/x+1/6*b^(4/3)*ln(
-b*d*x^3+a*d)*2^(1/3)/a^2/d-1/2*b^(4/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/
3))*2^(1/3)/a^2/d-1/3*2^(1/3)*b^(4/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b
*x^3+a)^(1/3))*3^(1/2))/a^2/d*3^(1/2)
```

3.578.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx = -\frac{\sqrt[3]{a+bx^3}\left(a+5bx^3\right)}{4a^2dx^4}-\frac{\sqrt[3]{2}b^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}a^2d}-\frac{\sqrt[3]{2}b^{4/3}\log\left(-2\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}\right)}{3a^2d}+\frac{b^{4/3}\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{3\,2^{2/3}a^2d}$$

input

```
Integrate[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]
```

output

```
-1/4*((a + b*x^3)^(1/3)*(a + 5*b*x^3))/(a^2*d*x^4) - (2^(1/3)*b^(4/3)*ArcT
an[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[3]*
a^2*d) - (2^(1/3)*b^(4/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(
3*a^2*d) + (b^(4/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3
) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(2/3)*a^2*d)
```

3.578.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {975, 27, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{b\left(3bx^3+5a\right)}{x^2\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{4ad}-\frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{3bx^3+5a}{x^2\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{4ad}-\frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

$$\Big\downarrow \color{blue}{1053}$$

$$\frac{b\left(-\frac{\int -\frac{8a^2bx}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{a^2}-\frac{5\sqrt[3]{a+bx^3}}{ax}\right)}{4ad}-\frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{b\left(8b\int \frac{x}{(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{5\sqrt[3]{a+bx^3}}{ax}\right)}{4ad}-\frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

$$\Big\downarrow \color{blue}{992}$$

$$\frac{b\left(8b\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}ab^{2/3}}+\frac{\log\left(a-bx^3\right)}{6\ 2^{2/3}ab^{2/3}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}ab^{2/3}}\right)-\frac{5\sqrt[3]{a+bx^3}}{ax}\right)}{4ad}-\frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]
```

output
```
-1/4*(a + b*x^3)^(1/3)/(a*d*x^4) + (b*((-5*(a + b*x^3)^(1/3))/(a*x) + 8*b*
(-(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*
Sqrt[3]*a*b^(2/3))) + Log[a - b*x^3]/(6*2^(2/3)*a*b^(2/3)) - Log[2^(1/3)*b
^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(2/3)*a*b^(2/3)))))/(4*a*d)
```

3.578. $\int \frac{\sqrt[3]{a+bx^3}}{x^5(ad-bdx^3)}\,dx$

3.578.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.578.4 Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{\left(-15 b\,x^{3}-3 a \right) \left(b\,x^{3}+a \right)^{\frac{1}{3}}-4\,2^{\frac{1}{3}} b^{\frac{4}{3}} x^{4} \left(-\arctan \left(\frac{\sqrt{3}\,\left(2^{\frac{2}{3}} \left(b\,x^{3}+a \right)^{\frac{1}{3}}+b^{\frac{1}{3}} x \right)}{3 b^{\frac{1}{3}} x}\right) \sqrt{3}+\ln \left(\frac{-2^{\frac{1}{3}} b^{\frac{1}{3}} x+\left(b\,x^{3}+a \right)^{\frac{1}{3}}}{x}\right)-\frac{\ln \left(2^{\frac{2}{3}} b^{\frac{2}{3}} x^{2}\right.}{}\right.}{12 x^{4} a^{2} d}$

3.578. $\int \frac{\sqrt[3]{a+bx^3}}{x^5(ad-bdx^3)}\,dx$

input
```
int((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/12*((-15*b*x^3-3*a)*(b*x^3+a)^(1/3)-4*2^(1/3)*b^(4/3)*x^4*(-arctan(1/3*3
^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)*x)/b^(1/3)/x)*3^(1/2)+ln((-2^(1/3)
*b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)
*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/x^4/a^2/d
```

3.578.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.578.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx=-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^5+bx^8}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**5/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**5 + b*x**8), x)/d
```

3.578.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx=\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)
```

3.578.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx=\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)
```

3.578.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^5\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)), x)
```

3.579 $\int \frac{\sqrt[3]{a+bx^3}}{x^8(ad-bdx^3)}\,dx$

3.579.1 Optimal result

Integrand size = 28, antiderivative size = 210

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx = & -\frac{\sqrt[3]{a+bx^3}}{7adx^7}-\frac{2b\sqrt[3]{a+bx^3}}{7a^2dx^4}-\frac{8b^2\sqrt[3]{a+bx^3}}{7a^3dx} \\ & -\frac{\sqrt[3]{2}b^{7/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^3d} \\ & +\frac{b^{7/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^3d}-\frac{b^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^3d}\end{aligned}$$

output

```
-1/7*(b*x^3+a)^(1/3)/a/d/x^7-2/7*b*(b*x^3+a)^(1/3)/a^2/d/x^4-8/7*b^2*(b*x^
3+a)^(1/3)/a^3/d/x+1/6*b^(7/3)*ln(-b*d*x^3+a*d)*2^(1/3)/a^3/d-1/2*b^(7/3)*
ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/a^3/d-1/3*2^(1/3)*b^(7/3)*ar
ctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a^3/d*3^(1/2)
```

3.579.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx =$$
$$-\frac{\frac{6\sqrt[3]{a+bx^3}\left(a^2+2abx^3+8b^2x^6\right)}{x^7}+14\sqrt[3]{2}\sqrt{3}b^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+14\sqrt[3]{2}b^{7/3}\log\left(-2\sqrt[3]{b}x+2^{2/3}\right.}{42a^3d}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x]
```

output
```
-1/42*((6*(a + b*x^3)^(1/3)*(a^2 + 2*a*b*x^3 + 8*b^2*x^6))/x^7 + 14*2^(1/3
)*Sqrt[3]*b^(7/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x
^3)^(1/3))] + 14*2^(1/3)*b^(7/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1
/3)] - 7*2^(1/3)*b^(7/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)
^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(a^3*d)
```

3.579.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {975, 27, 1053, 27, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx$$
$$\Big\downarrow \text{975}$$
$$\frac{\int \frac{2b\left(3bx^3+4a\right)}{x^5\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{7ad}-\frac{\sqrt[3]{a+bx^3}}{7adx^7}$$
$$\Big\downarrow \text{27}$$
$$\frac{2b\int \frac{3bx^3+4a}{x^5\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{7ad}-\frac{\sqrt[3]{a+bx^3}}{7adx^7}$$
$$\Big\downarrow \text{1053}$$

$$\frac{2b\left(-\frac{\int -\frac{4ab\left(3bx^3+4a\right)}{x^2\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{4a^2}-\frac{\sqrt[3]{a+bx^3}}{ax^4}\right)}{7ad}-\frac{\sqrt[3]{a+bx^3}}{7adx^7}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\left(\frac{b\int \frac{3bx^3+4a}{x^2\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{a}-\frac{\sqrt[3]{a+bx^3}}{ax^4}\right)}{7ad}-\frac{\sqrt[3]{a+bx^3}}{7adx^7}$$

$$\Big\downarrow \; 1053$$

$$\frac{2b\left(\frac{b\left(-\frac{\int -\frac{7a^2bx}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{a^2}-\frac{4\sqrt[3]{a+bx^3}}{ax}\right)}{a}-\frac{\sqrt[3]{a+bx^3}}{ax^4}\right)}{7ad}-\frac{\sqrt[3]{a+bx^3}}{7adx^7}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\left(\frac{b\left(7b\int \frac{x}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{4\sqrt[3]{a+bx^3}}{ax}\right)}{a}-\frac{\sqrt[3]{a+bx^3}}{ax^4}\right)}{7ad}-\frac{\sqrt[3]{a+bx^3}}{7adx^7}$$

$$\Big\downarrow \; 992$$

3.579. $\int \frac{\sqrt[3]{a+bx^3}}{x^8(ad-bdx^3)}\,dx$

$$\frac{2b\left(\frac{b\left(7b\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}ab^{2/3}}+\frac{\log\left(a-bx^3\right)}{6\ 2^{2/3}ab^{2/3}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}ab^{2/3}}\right)-\frac{4\sqrt[3]{a+bx^3}}{ax}\right)}{a}-\frac{\sqrt[3]{a+bx^3}}{ax^4}\right)}{7ad}-\frac{\sqrt[3]{a+bx^3}}{7adx^7}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x]
```

output
```
-1/7*(a + b*x^3)^(1/3)/(a*d*x^7) + (2*b*(-((a + b*x^3)^(1/3)/(a*x^4)) + (b
*((-4*(a + b*x^3)^(1/3))/(a*x) + 7*b*(-(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/
(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]*a*b^(2/3))) + Log[a - b*x^3]/
(6*2^(2/3)*a*b^(2/3)) - Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(2
/3)*a*b^(2/3)))))/a))/(7*a*d)
```

3.579.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

3.579. $\int \frac{\sqrt[3]{a+bx^3}}{x^8(ad-bdx^3)}\,dx$

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.579.4 Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\left(-48 b^{2} x^{6}-12 a b \,x^{3}-6 a^{2}\right) \left(b \,x^{3}+a \right)^{\frac{1}{3}}+7 \,2^{\frac{1}{3}} b^{\frac{7}{3}} x^{7} \left(2 \arctan \left(\frac{\sqrt{3}\, \left(2^{\frac{2}{3}} \left(b \,x^{3}+a \right)^{\frac{1}{3}}+b^{\frac{1}{3}} x \right)}{3 b^{\frac{1}{3}} x}\right) \sqrt{3}+\ln \left(\frac{2^{\frac{2}{3}} b^{\frac{2}{3}} x^{2}+2^{\frac{1}{3}} b^{\frac{1}{3}} \left(b \,x^{3}+a \right)^{\frac{1}{3}}}{x^{2}}\right.\right.}{42 x^{7} a^{3} d}$

input
```
int((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/42*((-48*b^2*x^6-12*a*b*x^3-6*a^2)*(b*x^3+a)^(1/3)+7*2^(1/3)*b^(7/3)*x^7
*(2*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)*x)/b^(1/3)/x)*3^(1
/2)+ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2
/3))/x^2)-2*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)))/x^7/a^3/d
```

3.579.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.579.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx=-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^8+bx^{11}}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**8/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**8 + b*x**11), x)/d
```

3.579.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx=\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^8}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)
```

3.579.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx=\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^8}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)
```

3.579.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^8\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)), x)
```

3.580 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)}\,dx$

3.580.1 Optimal result

Integrand size = 28, antiderivative size = 237

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx=-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}-\frac{11b\sqrt[3]{a+bx^3}}{70a^2dx^7}-\frac{37b^2\sqrt[3]{a+bx^3}}{140a^3dx^4}-\frac{169b^3\sqrt[3]{a+bx^3}}{140a^4dx}-\frac{\sqrt[3]{2}b^{10/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^4d}+\frac{b^{10/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^4d}-\frac{b^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^4d}$$

output
```
-1/10*(b*x^3+a)^(1/3)/a/d/x^10-11/70*b*(b*x^3+a)^(1/3)/a^2/d/x^7-37/140*b^
2*(b*x^3+a)^(1/3)/a^3/d/x^4-169/140*b^3*(b*x^3+a)^(1/3)/a^4/d/x+1/6*b^(10/
3)*ln(-b*d*x^3+a*d)*2^(1/3)/a^4/d-1/2*b^(10/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3
+a)^(1/3))*2^(1/3)/a^4/d-1/3*2^(1/3)*b^(10/3)*arctan(1/3*(1+2*2^(1/3)*b^(1
/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a^4/d*3^(1/2)
```

3.580.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx =$$
$$-\frac{\frac{3\sqrt[3]{a+bx^3}(14a^3+22a^2bx^3+37ab^2x^6+169b^3x^9)}{x^{10}}+140\sqrt[3]{2}\sqrt{3}b^{10/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+140\sqrt[3]{2}b^{10/3}\log}{420a^4d}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]
```

output
```
-1/420*((3*(a + b*x^3)^(1/3)*(14*a^3 + 22*a^2*b*x^3 + 37*a*b^2*x^6 + 169*b
^3*x^9))/x^10 + 140*2^(1/3)*Sqrt[3]*b^(10/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b
^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] + 140*2^(1/3)*b^(10/3)*Log[-2*b^(1/
3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] - 70*2^(1/3)*b^(10/3)*Log[2*b^(2/3)*x^2
+ 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(a^4*d
)
```

3.580.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {975, 27, 1053, 27, 1053, 25, 27, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{b(9bx^3+11a)}{x^8(a-bx^3)(bx^3+a)^{2/3}}dx}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{9bx^3+11a}{x^8(a-bx^3)(bx^3+a)^{2/3}}dx}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{b\left(-\frac{\int -\frac{2ab\left(33bx^3+37a\right)}{x^5\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{7a^2}-\frac{11\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$$\Big\downarrow \ 27$$

$$\frac{b\left(\frac{2b\int \frac{33bx^3+37a}{x^5\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{7a}-\frac{11\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$$\Big\downarrow \ 1053$$

$$\frac{b\left(\frac{2b\left(-\frac{\int -\frac{ab\left(111bx^3+169a\right)}{x^2\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{4a^2}-\frac{37\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{11\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$$\Big\downarrow \ 25$$

$$\frac{b\left(\frac{2b\left(\frac{\int \frac{ab\left(111bx^3+169a\right)}{x^2\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{4a^2}-\frac{37\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{11\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$$\Big\downarrow \ 27$$

$$\frac{b\left(\frac{2b\left(\frac{b\int \frac{111bx^3+169a}{x^2\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{4a}-\frac{37\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{11\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$$\Big\downarrow \ 1053$$

3.580. $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)}\,dx$

$$\frac{b\left(\frac{2b\left(\frac{b\left(-\frac{\int -\frac{280a^2bx}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{a^2}-\frac{169\sqrt[3]{a+bx^3}}{ax}\right)}{4a}-\frac{37\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{11\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{b\left(\frac{2b\left(\frac{b\left(280b\int \frac{x}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{169\sqrt[3]{a+bx^3}}{ax}\right)}{4a}-\frac{37\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{11\sqrt[3]{a+bx^3}}{7ax^7}\right)}{10ad}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

$$\Big\downarrow \color{blue}{992}$$

3.580. $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)}\,dx$

$$\frac{b\left(\frac{2b\left(\frac{b\left(280b\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}ab^{2/3}}+\frac{\log\left(a-bx^3\right)}{6\ 2^{2/3}ab^{2/3}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\ 2^{2/3}ab^{2/3}}\right)-\frac{169\sqrt[3]{a+bx^3}}{ax}\right)}{4a}-\frac{37\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{11\sqrt[3]{a}}{7a}\right.}{10ad}$$

$$\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]
```

output
```
-1/10*(a + b*x^3)^(1/3)/(a*d*x^10) + (b*((-11*(a + b*x^3)^(1/3))/(7*a*x^7)
 + (2*b*((-37*(a + b*x^3)^(1/3))/(4*a*x^4) + (b*((-169*(a + b*x^3)^(1/3))/
(a*x) + 280*b*(-(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt
[3]]/(2^(2/3)*Sqrt[3]*a*b^(2/3))) + Log[a - b*x^3]/(6*2^(2/3)*a*b^(2/3)) -
 Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(2/3)*a*b^(2/3)))))/(4*a)
))/(7*a)))/(10*a*d)
```

3.580. $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)}\,dx$

3.580.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.580.4 Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{\left(-507 b^{3} x^{9}-111 a\,b^{2} x^{6}-66 a^{2} b\,x^{3}-42 a^{3}\right) \left(b\,x^{3}+a \right)^{\frac{1}{3}}+70\,2^{\frac{1}{3}} b^{\frac{10}{3}} x^{10} \left(2 \arctan \left(\frac{\sqrt{3}\, \left(2^{\frac{2}{3}} \left(b\,x^{3}+a \right)^{\frac{1}{3}}+b^{\frac{1}{3}} x \right)}{3 b^{\frac{1}{3}} x}\right) \sqrt{3}+\ln \left(\frac{2^{\frac{2}{3}} b^{\frac{2}{3}} x^{2}}{}\right.\right.}{420 x^{10} a^{4} d}$

input
```
int((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/420*((-507*b^3*x^9-111*a*b^2*x^6-66*a^2*b*x^3-42*a^3)*(b*x^3+a)^(1/3)+70
*2^(1/3)*b^(10/3)*x^10*(2*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1
/3)*x)/b^(1/3)/x)*3^(1/2)+ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a
)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/
x)))/x^10/a^4/d
```

3.580.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.580.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^{11}+bx^{14}}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**11/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**11 + b*x**14), x)/d
```

3.580.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)
```

3.580.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)
```

3.580.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx=\int \frac{(b\,x^3+a)^{1/3}}{x^{11}\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)), x)
```

3.581 $\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

3.581.1 Optimal result

Integrand size = 28, antiderivative size = 521

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{3ax\sqrt[3]{a+bx^3}}{5b^2d} - \frac{x^4\sqrt[3]{a+bx^3}}{5bd} - \frac{\sqrt[3]{2}a^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}d}$$

$$- \frac{a^{5/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}b^{7/3}d}$$

$$- \frac{2a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b^2d\left(a+bx^3\right)^{2/3}}$$

$$- \frac{a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d}$$

$$+ \frac{a^{5/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d}$$

$$- \frac{\sqrt[3]{2}a^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{7/3}d}$$

$$+ \frac{a^{5/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}b^{7/3}d}$$

output
```
-3/5*a*x*(b*x^3+a)^(1/3)/b^2/d-1/5*x^4*(b*x^3+a)^(1/3)/b/d-2/5*a^2*x*(1+b*
x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b^2/d/(b*x^3+a)^(2/3)-1/
6*a^(5/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(7/3)
/d+1/6*a^(5/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*
(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(7/3)/d-1/3*2^(1/3)*a^(5/3)
*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(7/3)/d+1/12*a^(5/3)*
ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3
)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(7/3)/d-1/3*2^(1/3)*a^(5/3)*arctan(1/3*(1-
2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)/d*3^(1/2)-
1/6*a^(5/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(
1/2))*2^(1/3)/b^(7/3)/d*3^(1/2)
```

3.581.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 7.81 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.45

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$

$$=\frac{-4(a+bx^3)\left(3ax+bx^4\right)+7abx^4\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{}{(a-bx^3)\left(4a\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\right.\right.}}{20b^2d\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
(-4*(a + b*x^3)*(3*a*x + b*x^4) + 7*a*b*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1
[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a] + (48*a^4*x*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1,
 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*
x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a
]))))/(20*b^2*d*(a + b*x^3)^(2/3))
```

3.581.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.11, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {978, 27, 1052, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}dx$$

$$\Big\downarrow \; 978$$

$$\frac{\int \frac{2ax^3(3bx^3+2a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \; 27$$

$$\frac{2a\int \frac{x^3(3bx^3+2a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \; 1052$$

$$\frac{2a\left(\frac{\int \frac{ab\left(7bx^3+3a\right)}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{2b^2}-\frac{3x\sqrt[3]{a+bx^3}}{2b}\right)}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \; 27$$

$$\frac{2a\left(\frac{a\int \frac{7bx^3+3a}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{2b}-\frac{3x\sqrt[3]{a+bx^3}}{2b}\right)}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \; 1026$$

$$\frac{2a\left(\frac{a\left(10a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-7\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx\right)}{2b}-\frac{3x\sqrt[3]{a+bx^3}}{2b}\right)}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \; 779$$

$$\frac{2a\left(\frac{a\left(10a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{7\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{\left(a+bx^3\right)^{2/3}}\right)}{2b}-\frac{3x\sqrt[3]{a+bx^3}}{2b}\right)}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \; 778$$

$$\frac{2a\left(\frac{a\left(10a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{7x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)}{2b}-\frac{3x\sqrt[3]{a+bx^3}}{2b}\right)}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \; 928$$

$$\frac{2a\left(\frac{a\left(10a\left(\frac{\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)-\frac{7x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)}{2b}-\frac{3x\sqrt[3]{a+bx^3}}{2b}\right)}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \; 779$$

$$\frac{2a\left(\frac{a\left(10a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a\left(a+bx^3\right)^{2/3}}\right)-\frac{7x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)}{2b}-\frac{3x\sqrt[3]{a+bx^3}}{2b}\right)}{5bd}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow \; 778$$

3.581. $\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$\frac{2a\left(\frac{a\left(10a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}\right)-\frac{7x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)}{2b}-3x\sqrt[3]{}}{5bd}$$

$$\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow 927$$

$$\frac{2a\left(\frac{a\left(10a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\right.}{2a\left(a+bx^3\right)^{2/3}}\right.\right.}{2b}\right.}{5bd}$$

$$\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow 982$$

3.581. $\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$\frac{2a\left(\frac{a\left(\frac{10a\left(\frac{9\left(\frac{1}{9}\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+}}\right.}{2a^{2/3}\sqrt[3]{b}}\right.}{2b}\right.}{}\right.}{5bd}$$

$$\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow 821$$

3.581. $\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$2a\Bigg(\frac{a\Big(10a\Big(\frac{9\Big(\frac{2}{9}\Big(\frac{\int \frac{\frac{\sqrt[3]{2}(\sqrt[3]{b}x+\sqrt[3]{a})}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}(\sqrt[3]{b}x+\sqrt[3]{a})^2}{(bx^3+a)^{2/3}}-\frac{\sqrt[3]{2}(\sqrt[3]{b}x+\sqrt[3]{a})}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}(\sqrt[3]{b}x+\sqrt[3]{a})}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\Big)+\frac{1}{9}\Big(\int \frac{}{2^{2/}}}{2a^{2/3}\sqrt[3]{b}}}{}}{}$$

$$\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow 16$$

3.581. $\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$2a\Bigg(a\Bigg(10a\Bigg(\frac{9\Bigg(\frac{2}{9}\Bigg(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\Bigg)+\frac{1}{9}\Bigg(-\frac{\int \frac{2^{2/3}}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-}}{}}{2a^{2/3}\sqrt[3]{b}}$$

$$\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

$$\Big\downarrow 1142$$

3.581. $\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$2a\left(\frac{a\left(10a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}\,d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}\right.\right.}{}\right.$$

3.581. $\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$\Big\downarrow 25$$

3.581. $\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$2a \Bigg| \frac{a\left(10a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\int\frac{}{2^{2/}}}{}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}\right.\right.}{}$$

3.581. $\int\frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$\downarrow$ 27

3.581. $\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$2a\left(\frac{a\left(10a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}\,d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int\right.}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}\right.\right.}{}\right.\right.$$

3.581. $\int\frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$ $\frac{x^4\sqrt[3]{bx^3+a}}{5bd}$

$\downarrow$ 1082

3.581. $\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$2a\left(\frac{a\left(10a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}\,d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}\,d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\right.\right.}{}\right.\right.}{}\right.$$

3.581. $\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

↓ 217

3.581. $\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\, dx$

$$2a \left(a \left(10a \left(9 \left(\frac{2}{9} \left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}\,d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}} - \frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}}{\sqrt[3]{a}}\right.}{3\ 2^{2/}} \right.\right.\right.\right.\right.\right.$$

3.581. $\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$\downarrow$ 1103

3.581. $\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

$$2a\Bigg(\frac{a\Bigg(10a\Bigg(\frac{9\Bigg(\frac{2}{9}\Bigg(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\Bigg)+\frac{1}{9}}{2a^{2/3}\sqrt[3]{}}$$

3.581. $\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$ $\qquad \frac{x^4\sqrt[3]{a+bx^3}}{5bd}$

input

```
Int[(x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output

```
-1/5*(x^4*(a + b*x^3)^(1/3))/(b*d) + (2*a*((-3*x*(a + b*x^3)^(1/3))/(2*b)
+ (a*((-7*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^
3)/a)])/(a + b*x^3)^(2/3) + 10*a*((x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2
F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*(a + b*x^3)^(2/3)) + (9*((2*((-((Sqr
t[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt
[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a +
b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*2^(1/
3)*a^(1/3))))/(3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))
/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-1/3*Log[2^(2/3) - (a^(1/3)
 + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)) - ((Sqrt[3]*ArcTan[(1 +
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/a^(1/3) - Lo
g[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3
) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/(3*2^(2/3)*a^(1/3)))/9))/(
2*a^(2/3)*b^(1/3)))))/(2*b)))/(5*b*d)
```

3.581.3.1 Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

3.581. $\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

3.581. $\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.581.4 Maple [F]

$$\int \frac{x^{6} \left(b\,x^{3}+a \right)^{\frac{1}{3}}}{-bd\,x^{3}+ad}d x$$

input
```
int(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
int(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

3.581.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \text{Timed out}$$

input
```
integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.581.6 Sympy [F]

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^6 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**6*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**6*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.581.7 Maxima [F]

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^6}{bdx^3-ad}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)*x^6/(b*d*x^3 - a*d), x)
```

3.581.8 Giac [F]

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}x^6}{bdx^3-ad}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)*x^6/(b*d*x^3 - a*d), x)
```

3.581.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\int \frac{x^6\,{\left(b\,x^3+a\right)}^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

3.582 $\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

3.582.1 Optimal result

Integrand size = 28, antiderivative size = 494

$$
\begin{aligned}
\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = & -\frac{x\sqrt[3]{a+bx^3}}{2bd}-\frac{\sqrt[3]{2}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d} \\
& -\frac{a^{2/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}b^{4/3}d} \\
& -\frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2bd\left(a+bx^3\right)^{2/3}} \\
& -\frac{a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d} \\
& +\frac{a^{2/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d} \\
& -\frac{\sqrt[3]{2}a^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{4/3}d} \\
& +\frac{a^{2/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}b^{4/3}d}
\end{aligned}
$$

output
```
-1/2*x*(b*x^3+a)^(1/3)/b/d-1/2*a*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],
[4/3],-b*x^3/a)/b/d/(b*x^3+a)^(2/3)-1/6*a^(2/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/
3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(4/3)/d+1/6*a^(2/3)*ln(1+2^(2/3)*(a^(1/3)
+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))
*2^(1/3)/b^(4/3)/d-1/3*2^(1/3)*a^(2/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b
*x^3+a)^(1/3))/b^(4/3)/d+1/12*a^(2/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(
b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(4/3
)/d-1/3*2^(1/3)*a^(2/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/3))*3^(1/2))/b^(4/3)/d*3^(1/2)-1/6*a^(2/3)*arctan(1/3*(1+2^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/b^(4/3)/d*3^(1/2)
```

3.582.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 7.19 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.46

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{x\left(3x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{4\left(-a-bx^3+\frac{4a^3\operatorname{AppellF1}\left(\frac{1}{3},\right.}{\left(a-bx^3\right)\left(4a\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3\operatorname{AppellF}\right.\right.}\right.}{b}\right.}{8d\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
(x*(3*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (
b*x^3)/a] + (4*(-a - b*x^3 + (4*a^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a
), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a),
(b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a]
- 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/b))/(8*d*(a +
 b*x^3)^(2/3))
```

3.582.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.11, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {978, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$$

$$\Big\downarrow \textcolor{blue}{978}$$

$$\frac{\int \frac{a(3bx^3+a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{2bd}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{a\int \frac{3bx^3+a}{(a-bx^3)(bx^3+a)^{2/3}}dx}{2bd}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \textcolor{blue}{1026}$$

$$\frac{a\left(4a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-3\int \frac{1}{(bx^3+a)^{2/3}}dx\right)}{2bd}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \textcolor{blue}{779}$$

$$\frac{a\left(4a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{3\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}\right)}{2bd}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \textcolor{blue}{778}$$

$$\frac{a\left(4a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2bd}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \textcolor{blue}{928}$$

$$\frac{a\left(4a\left(\frac{\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2bd}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \text{779}$$

$$\frac{a\left(4a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a(a+bx^3)^{2/3}}\right)-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2bd}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \text{778}$$

$$\frac{a\left(4a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(a+bx^3)^{2/3}}\right)-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2bd}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \text{927}$$

$$\frac{a\left(4a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\right.}{2a(a+bx^3)^{2/3}}\right.\right.}{2bd}$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \text{982}$$

3.582. $\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$\frac{a\left(4a\left(\frac{9\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}\right)}{2a^{2/3}\sqrt[3]{b}}\right.\right.}{2bd}$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow 821$$

$$a\left(4a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\frac{\int \frac{}{2^{2/3}}\right.\right.}{2a^{2/3}\sqrt[3]{b}}\right.\right.$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow 16$$

3.582. $\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$a\left(4a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}-}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}+}}{}\right.\right.}{2a^{2/3}\sqrt[3]{b}}\right.\right.$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \text{1142}$$

3.582. $\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$a\left(4a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(bx^3+a)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{\int-}{}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}\right.\right.$$

$$\frac{x\sqrt[3]{bx^3+a}}{2bd}$$

$$\Big\downarrow 25$$

3.582. $\int\frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$a\left(4a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}\right.\right.$$

$$\frac{x\sqrt[3]{bx^3+a}}{2bd}$$

$$\Big\downarrow 27$$

3.582. $\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$\frac{a\left(4a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt[3]{}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}\right.\right.}{}$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow \text{1082}$$

3.582. $\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$\frac{a\Bigg(4a\Bigg(9\Bigg(\frac{2}{9}\Bigg(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\Big(}{}}{}}{}$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$$\Big\downarrow 217$$

3.582. $\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$a\Bigg(4a\Bigg(\frac{9\Bigg(\frac{2}{9}\Bigg(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\Bigg(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\right.}{\sqrt[3]{a+b}}}{3\ 2^{2/3}a^{2/}}$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

$\big\downarrow$ 1103

3.582. $\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$a\left(4a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\;2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(\right.\right.}{2a^{2/3}\sqrt[3]{b}}\right.\right.$$

$$\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

input
```
Int[(x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

output
```
-1/2*(x*(a + b*x^3)^(1/3))/(b*d) + (a*((-3*x*x*(1 + (b*x^3)/a)^(2/3)*Hyperge
ometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) + 4*a*((x*(1 +
(b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*(a +
 b*x^3)^(2/3)) + (9*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(
1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3
)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)
*x))/(a + b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3))))/(3*2^(1/3)*a^(1/3)) - Log[1 +
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/
9 + (-1/3*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*
a^(2/3)) - ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^
3)^(1/3))/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a +
 b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1
/3)))/(3*2^(2/3)*a^(1/3)))/9))/(2*a^(2/3)*b^(1/3)))))/(2*b*d)
```

3.582. $\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

3.582.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

3.582. $\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.582. $\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

3.582.4 Maple [F]

$$\int \frac{x^{3} \left(b \,x^{3}+a \right)^{\frac{1}{3}}}{-b d \,x^{3}+a d}d x$$

input
```
int(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
int(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

3.582.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.582.6 Sympy [F]

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^3 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**3*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**3*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.582.7 Maxima [F]

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^3}{bdx^3-ad}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)*x^3/(b*d*x^3 - a*d), x)
```

3.582.8 Giac [F]

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^3}{bdx^3-ad}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)*x^3/(b*d*x^3 - a*d), x)
```

3.582.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int \frac{x^3\,(b\,x^3+a)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

3.583 $\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

3.583.1 Optimal result

Integrand size = 25, antiderivative size = 416

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = &-\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}d}\\
&-\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}d}-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}\\
&+\frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}\\
&-\frac{\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d}\\
&+\frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}\end{aligned}$$

output
```
-1/6*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1
/3)/d+1/6*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1
/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)/d-1/3*2^(1/3)*ln(1
+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(1/3)/b^(1/3)/d+1/12*ln(2*
2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)/d-1/3*2^(1/3)*arctan(1/3*(1-2*2^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(1/3)/b^(1/3)/d*3^(1/
2)-1/6*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))
*2^(1/3)/a^(1/3)/b^(1/3)/d*3^(1/2)
```

3.583.2 Mathematica [A] (verified)

Time = 3.33 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$
$$= \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{-2\sqrt[3]{2}\sqrt[3]{a}-2\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{\sqrt[3]{2}\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right)-4\log\left(\sqrt[3]{2}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(a*d - b*d*x^3),x]
```

output
```
(4*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3)^(1/3))/(-2*2^(1/3)*a^(1/3) - 2*2^(1
/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] + 2*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3
)^(1/3))/(2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] - 4*Lo
g[2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3)] - 2*Log[-(2^(1/
3)*a^(1/3)) - 2^(1/3)*b^(1/3)*x + 2*(a + b*x^3)^(1/3)] + Log[2^(2/3)*a^(2/
3) + 2^(2/3)*b^(2/3)*x^2 + 2*2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 4*(a +
b*x^3)^(2/3) + 2*2^(1/3)*a^(1/3)*(2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))]
+ 2*Log[2^(2/3)*a^(2/3) + 2^(2/3)*b^(2/3)*x^2 - 2^(1/3)*b^(1/3)*x*(a + b*x
^3)^(1/3) + (a + b*x^3)^(2/3) + a^(1/3)*(2*2^(2/3)*b^(1/3)*x - 2^(1/3)*(a
+ b*x^3)^(1/3))])/(6*2^(2/3)*a^(1/3)*b^(1/3)*d)
```

3.583.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$

$$\Big\downarrow \; 927$$

$$\frac{9\sqrt[3]{a}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{\sqrt[3]{b}d}$$

$$\Big\downarrow \; 982$$

$$\frac{9\sqrt[3]{a}\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}\right.}{\sqrt[3]{b}d}$$

$$\Big\downarrow \; 821$$

$$\frac{9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\int \right.\right.}{\sqrt[3]{b}d}$$

$$\Big\downarrow \; 16$$

$$\frac{9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int\frac{2^{2/3}}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}}}{}\right.\right.}{\sqrt[3]{b}d}$$

$$\Big\downarrow \color{blue}{1142}$$

$$\frac{9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{\int-\frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{}}{\sqrt[3]{a}\sqrt[3]{bx^3}}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}$$

$$\Big\downarrow \color{blue}{25}$$

3.583. $\displaystyle\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\int\frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3}}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.$$

$\downarrow$ 27

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+}{\sqrt[3]{a}\sqrt[3]{bx}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.$$

$\downarrow$ 1082

3.583. $\int\frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\right.}{}\right.\right.$$

$$\Big\downarrow 217$$

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\right.}{\sqrt[3]{a+b}}\right.}{3\ 2^{2/3}a^{2/}}\right.\right.$$

$$\Big\downarrow 1103$$

3.583. $\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

$$\frac{9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\right.}{\sqrt[3]{b}d}$$

input
```
Int[(a + b*x^3)^(1/3)/(a*d - b*d*x^3),x]
```

output
```
(9*a^(1/3)*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/
(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3)*(a^(1/3
) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a +
 b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3)))/(3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1/3)
*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-1/3
*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3))
- ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))
/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(
2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/(3*
2^(2/3)*a^(1/3)))/9))/(b^(1/3)*d)
```

3.583.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.583. $\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.583. $\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

3.583.4 Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{-bd\,x^{3}+ad}dx$$

input
```
int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

output
```
int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

3.583.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.583.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.583.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}}{bdx^3-ad}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)
```

3.583.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}}{bdx^3-ad}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)
```

3.583.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\int \frac{(b\,x^3+a)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(a*d - b*d*x^3),x)
```

output
```
int((a + b*x^3)^(1/3)/(a*d - b*d*x^3), x)
```

3.584 $\int \frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx$

3.584.1 Optimal result

Integrand size = 28, antiderivative size = 496

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx = & -\frac{\sqrt[3]{a+bx^3}}{2adx^2}-\frac{\sqrt[3]{2}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}d} \\
& -\frac{b^{2/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^{4/3}d} \\
& +\frac{bx\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2ad\left(a+bx^3\right)^{2/3}} \\
& -\frac{b^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}d} \\
& +\frac{b^{2/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}d} \\
& -\frac{\sqrt[3]{2}b^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{4/3}d} \\
& +\frac{b^{2/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{4/3}d}
\end{aligned}$$

output
```
-1/2*(b*x^3+a)^(1/3)/a/d/x^2+1/2*b*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3
],[4/3],-b*x^3/a)/a/d/(b*x^3+a)^(2/3)-1/6*b^(2/3)*ln(2^(2/3)+(-a^(1/3)-b^(
1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/d+1/6*b^(2/3)*ln(1+2^(2/3)*(a^(1/
3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3
))*2^(1/3)/a^(4/3)/d-1/3*2^(1/3)*b^(2/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/3))/a^(4/3)/d+1/12*b^(2/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2
/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4
/3)/d-1/3*2^(1/3)*b^(2/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x
^3+a)^(1/3))*3^(1/2))/a^(4/3)/d*3^(1/2)-1/6*b^(2/3)*arctan(1/3*(1+2^(1/3)*
(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(4/3)/d*3^(1/2)
```

3.584.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.16 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\,(ad-bdx^3)}\,dx$$
$$= \frac{-4a(a+bx^3)+b^2x^6\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{48a^3bx^3\,\text{A}}{(a-bx^3)\Big(4a\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\Big(}}{8a^2dx^2\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x]
```

output
```
(-4*a*(a + b*x^3) + b^2*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), (b*x^3)/a] + (48*a^3*b*x^3*AppellF1[1/3, 2/3, 1, 4/3, -((
b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x
^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*
x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]))))/(8*a^2
*d*x^2*(a + b*x^3)^(2/3))
```

3.584.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.11, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {975, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{b\left(bx^3+3a\right)}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{bx^3+3a}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{1026}$$

$$\frac{b\left(4a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx\right)}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{779}$$

$$\frac{b\left(4a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{\left(a+bx^3\right)^{2/3}}\right)}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{778}$$

$$\frac{b\left(4a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{928}$$

$$\frac{b\left(4a\left(\frac{\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)-\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{779}$$

$$\frac{b\left(4a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a(a+bx^3)^{2/3}}\right)-\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{778}$$

$$\frac{b\left(4a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(a+bx^3)^{2/3}}\right)-\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)}{2ad}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{927}$$

$$\frac{b\left(4a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\right.}{2a(a+bx^3)^{2/3}}\right.\right.}{2ad}\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{982}$$

3.584. $\int \frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx$

$$\frac{b\left(4a\left(\frac{9\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}\right)}{2a^{2/3}\sqrt[3]{b}}\right.\right.}{2ad}$$

$$\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow 821$$

$$\frac{b\left(4a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\frac{\int \frac{}{2^{2/3}-}\right.\right.}{2a^{2/3}\sqrt[3]{b}}\right.\right.}{}$$

$$\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow 16$$

3.584. $\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx$

$$b\left(4a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}-}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}+}}{}\right.\right.}{2a^{2/3}\sqrt[3]{b}}\right.\right.$$

$$\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \; 1142$$

3.584. $\int \frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx$

$$b\left(4a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(bx^3+a)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{(bx^3+a)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\int--}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}\right.\right.}{}$$

$$\frac{\sqrt[3]{bx^3+a}}{2adx^2}$$

$$\Big\downarrow \textcolor{blue}{25}$$

3.584. $\int\frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx$

$$b\left(4a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}\right.\right.$$

$$\frac{\sqrt[3]{bx^3+a}}{2adx^2}$$

$$\Big\downarrow \text{27}$$

3.584. $\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx$

$$\frac{b\left(4a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt[3]{b}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}\right.\right.}{}$$

$$\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow \text{1082}$$

3.584. $\int \frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx$

$$b\Biggl(4a\Biggl(9\Biggl(\frac{2}{9}\Biggl(\frac{\frac{3\int\frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\log\Bigl($$

$$\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow 217$$

3.584. $\int \frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx$

$$b\left(4a\left(9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{\right.}}{\sqrt[3]{a+b}}\right.}{3\ 2^{2/3}a^{2/}}\right.\right.\right.\right.\right.$$

$$\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

$$\Big\downarrow 1103$$

3.584. $\int\frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx$

$$b\Bigg(4a\Bigg(\frac{9\Bigg(\frac{2}{9}\Bigg(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\Bigg)+\frac{1}{9}\Bigg(}{2a^{2/3}\sqrt[3]{b}}$$

$$\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x]
```

output
```
-1/2*(a + b*x^3)^(1/3)/(a*d*x^2) + (b*(-((x*(1 + (b*x^3)/a)^(2/3)*Hypergeo
metric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3)) + 4*a*((x*(1 +
(b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*(a +
 b*x^3)^(2/3)) + (9*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(
1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3
)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)
*x))/(a + b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3))))/(3*2^(1/3)*a^(1/3)) - Log[1 +
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/
9 + (-1/3*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*
a^(2/3)) - ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^
3)^(1/3))/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a +
 b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1
/3)))/(3*2^(2/3)*a^(1/3)))/9))/(2*a^(2/3)*b^(1/3)))))/(2*a*d)
```

3.584. $\int \frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx$

3.584.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

3.584. $\int \frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx$

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.584. $\int \frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx$

3.584.4 Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{3} \left(-b d\,x^{3}+a d\right)}d x$$

input
```
int((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x)
```

output
```
int((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x)
```

3.584.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\,(ad-bdx^3)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.584.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\,(ad-bdx^3)}\,dx=-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^3+bx^6}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**3/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**3 + b*x**6), x)/d
```

3.584.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^3), x)
```

3.584.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^3), x)
```

3.584.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^3\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)), x)
```

3.585 $\int \frac{\sqrt[3]{a+bx^3}}{x^6(ad-bdx^3)}\,dx$

3.585.1 Optimal result

Integrand size = 28, antiderivative size = 523

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx = &-\frac{\sqrt[3]{a+bx^3}}{5adx^5}-\frac{3b\sqrt[3]{a+bx^3}}{5a^2dx^2}-\frac{\sqrt[3]{2}b^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{7/3}d}\\
&-\frac{b^{5/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^{7/3}d}\\
&+\frac{2b^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5a^2d\left(a+bx^3\right)^{2/3}}\\
&-\frac{b^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{7/3}d}\\
&+\frac{b^{5/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{7/3}d}\\
&-\frac{\sqrt[3]{2}b^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{7/3}d}\\
&+\frac{b^{5/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{7/3}d}\end{aligned}$$

output
```
-1/5*(b*x^3+a)^(1/3)/a/d/x^5-3/5*b*(b*x^3+a)^(1/3)/a^2/d/x^2+2/5*b^2*x*(1+
b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/a^2/d/(b*x^3+a)^(2/3)-
1/6*b^(5/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/
3)/d+1/6*b^(5/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3
)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/d-1/3*2^(1/3)*b^(5/
3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(7/3)/d+1/12*b^(5/3
)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1
/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/d-1/3*2^(1/3)*b^(5/3)*arctan(1/3*(
1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(7/3)/d*3^(1/2
)-1/6*b^(5/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3
^(1/2))*2^(1/3)/a^(7/3)/d*3^(1/2)
```

3.585.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.19 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\,(ad-bdx^3)}\,dx$$
$$= \frac{-\frac{4(a^2+4abx^3+3b^2x^6)}{a^2x^5}+\frac{3b^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\mathrm{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{a^3}+\frac{112b^2x\,\mathrm{AppellF1}(\frac{1}{3}}{(a-bx^3)\left(4a\,\mathrm{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3(3\,\mathrm{AppellF}}}{20d\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x]
```

output
```
((-4*(a^2 + 4*a*b*x^3 + 3*b^2*x^6))/(a^2*x^5) + (3*b^3*x^4*(1 + (b*x^3)/a)
^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])/a^3 + (112*b^2
*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*
AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/
3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -(
(b*x^3)/a), (b*x^3)/a]))))/(20*d*(a + b*x^3)^(2/3))
```

3.585.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.11, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {975, 27, 1053, 25, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{2b(2bx^3+3a)}{x^3(a-bx^3)(bx^3+a)^{2/3}}dx}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{2bx^3+3a}{x^3(a-bx^3)(bx^3+a)^{2/3}}dx}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \text{1053}$$

$$\frac{2b\left(-\frac{\int -\frac{ab\left(3bx^3+7a\right)}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{2a^2}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\left(\frac{\int \frac{ab\left(3bx^3+7a\right)}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{2a^2}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\left(\frac{b\int \frac{3bx^3+7a}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{2a}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \text{1026}$$

$$\frac{2b\left(\frac{b\left(10a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-3\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx\right)}{2a}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \; 779$$

$$\frac{2b\left(\frac{b\left(10a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{3\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{\left(a+bx^3\right)^{2/3}}\right)}{2a}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \; 778$$

$$\frac{2b\left(\frac{b\left(10a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)}{2a}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \; 928$$

$$\frac{2b\left(\frac{b\left(10a\left(\frac{\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)}{2a}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-$$

$$\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \; 779$$

3.585. $\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx$

$$\frac{2b\left(\frac{b\left(10a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a\left(a+bx^3\right)^{2/3}}\right)-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)}{2a}-\frac{3\sqrt[3]{a+bx^3}}{2ax^2}\right)}{5ad}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \; 778$$

$$\frac{2b\left(\frac{b\left(10a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}\right)-\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)}{2a}-\frac{3\sqrt[3]{a}}{2}\right)}{5ad}$$

$$\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \; 927$$

$$\frac{2b\left(\frac{b\left(10a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-b\right)}{2a\left(a+bx^3\right)^{2/3}}\right)\right)}{2a}\right)}{5ad}$$

$$\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

3.585. $\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx$

$$\Big\downarrow \text{982}$$

$$2b\left(\frac{b\left(10a\left(\frac{9\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+}}\right.}{2a^{2/3}\sqrt[3]{b}}\right.\right.}{2a}\right.$$

$$\overline{\qquad\qquad\qquad\qquad 5ad}$$

$$\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \text{821}$$

3.585. $\int \frac{\sqrt[3]{a+bx^3}}{x^6(ad-bdx^3)}\,dx$

$$2b\Bigg(b\Bigg(10a\,\frac{9\Bigg(\frac{2}{9}\Bigg(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\Bigg)+\frac{1}{9}\Bigg(\frac{\int \frac{}{2^{2/3}}}{}}{2a^{2/3}\sqrt[3]{b}}$$

$$\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \; 16$$

3.585. $\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx$

$$\frac{2b\left(\frac{b\left(10a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int\frac{2^{2/3}}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}+}}{}\right.\right.}{2a^{2/3}\sqrt[3]{b}}\right.\right.}{}\right.}{}$$

$$\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

$$\Big\downarrow \text{1142}$$

3.585. $\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx$

$$2b\left(b\left(10a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}\,d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{\int-\frac{}{2^{2}}}{}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.\right.\right.\right.$$

3.585. $\int\frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx$

↓ 25

3.585. $\int \frac{\sqrt[3]{a+bx^3}}{x^6(ad-bdx^3)}\,dx$

$$2b\Bigg(\frac{b\Bigg(10a\Bigg(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\Bigg(\frac{2}{9}\Bigg(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\int\frac{}{2^{2/3}}}{}}{3\sqrt[3]{2}\sqrt[3]{a}}}{}}{}$$

3.585. $\int\frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx$

$\Big\downarrow$ 27

3.585. $\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)} \, dx$

$$\frac{2b\left(\frac{b\left(10a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}\right.\right.}{}\right.}{}$$

3.585. $\int\frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx$ $\frac{\sqrt[3]{bx^3+a}}{5adx^5}$

↓ 1082

3.585. $\int \frac{\sqrt[3]{a+bx^3}}{x^6(ad-bdx^3)}\,dx$

$$2b\left(b\left(10a\left(9\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}\,d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}\,d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\right.\right.\right.\right.\right.\right.$$

3.585. $\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx$

$\downarrow$ 217

3.585. $\int \frac{\sqrt[3]{a+bx^3}}{x^6(ad-bdx^3)}\,dx$

$$2b\left(\frac{b\left(10a\left(\frac{9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}\,d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}\right.}{\sqrt[3]{a+}}\right.}{3\;2^{2/3}}\right.\right.}{}\right.\right.}{}\right.$$

3.585. $\int\frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx$

$$\downarrow \text{1103}$$

3.585. $\int \frac{\sqrt[3]{a+bx^3}}{x^6(ad-bdx^3)}\,dx$

$$\frac{2b}{\Bigg(b \Bigg(10a \Bigg(\frac{9 \Bigg(\frac{2}{9} \Bigg(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}} - \frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}} - \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}} - \frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}} \Bigg) + \frac{1}{9}}{2a^{2/3}\sqrt[3]{}}}}$$

3.585. $\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx$ $\qquad \frac{\sqrt[3]{a+bx^3}}{5adx^5}$

input
```
Int[(a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x]
```

output
```
-1/5*(a + b*x^3)^(1/3)/(a*d*x^5) + (2*b*((-3*(a + b*x^3)^(1/3))/(2*a*x^2)
+ (b*((-3*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^
3)/a)])/(a + b*x^3)^(2/3) + 10*a*((x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2
F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*(a + b*x^3)^(2/3)) + (9*((2*((-((Sqr
t[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt
[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a +
b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*2^(1/
3)*a^(1/3))))/(3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))
/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-1/3*Log[2^(2/3) - (a^(1/3)
 + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)) - ((Sqrt[3]*ArcTan[(1 +
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/a^(1/3) - Lo
g[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3
) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/(3*2^(2/3)*a^(1/3)))/9))/(
2*a^(2/3)*b^(1/3)))))/(2*a)))/(5*a*d)
```

3.585.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

3.585. $\int \frac{\sqrt[3]{a+bx^3}}{x^6(ad-bdx^3)}\,dx$

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.585.4 Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{6}\left(-bd\,x^{3}+ad\right)}dx$$

input
```
int((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x)
```

output
```
int((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x)
```

3.585.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.585.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx=-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^6+bx^9}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(1/3)/x**6/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a*x**6 + b*x**9), x)/d
```

3.585.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^6), x)
```

3.585.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx=\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{(bdx^3-ad)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^6), x)
```

3.585.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^6\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)), x)
```

3.586 $\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

3.586.1 Optimal result

Integrand size = 28, antiderivative size = 223

$$\begin{aligned}\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx = &-\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d}-\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{a\left(a+bx^3\right)^{8/3}}{8b^4d}\\ &-\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}-\frac{2^{2/3}a^{11/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^4d}\\ &+\frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^4d}-\frac{a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^4d}\end{aligned}$$

output

```
-1/2*a^3*(b*x^3+a)^(2/3)/b^4/d-1/5*a^2*(b*x^3+a)^(5/3)/b^4/d+1/8*a*(b*x^3+
a)^(8/3)/b^4/d-1/11*(b*x^3+a)^(11/3)/b^4/d+1/6*a^(11/3)*ln(-b*x^3+a)*2^(2/
3)/b^4/d-1/2*a^(11/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/b^4/d-1/
3*2^(2/3)*a^(11/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^
(1/2))/b^4/d*3^(1/2)
```

3.586.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{3(a+bx^3)^{2/3}\left(293a^3+98a^2bx^3+65ab^2x^6+40b^3x^9\right)+440\,2^{2/3}\sqrt{3}a^{11/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+440}{1320}$$

input
```
Integrate[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/1320*(3*(a + b*x^3)^(2/3)*(293*a^3 + 98*a^2*b*x^3 + 65*a*b^2*x^6 + 40*b
^3*x^9) + 440*2^(2/3)*Sqrt[3]*a^(11/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1
/3))/a^(1/3))/Sqrt[3]] + 440*2^(2/3)*a^(11/3)*Log[-2*a^(1/3) + 2^(2/3)*(a
+ b*x^3)^(1/3)] - 220*2^(2/3)*a^(11/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a
+ b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^4*d)
```

3.586.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {948, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{ad-bdx^3}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^9\left(bx^3+a\right)^{2/3}}{d\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{x^9\left(bx^3+a\right)^{2/3}}{a-bx^3}dx^3}{3d}$$

$$\Big\downarrow \; 99$$

$$\frac{\int \left(\frac{(bx^3+a)^{2/3}a^3}{b^3(a-bx^3)} - \frac{(bx^3+a)^{2/3}a^2}{b^3} + \frac{(bx^3+a)^{5/3}a}{b^3} - \frac{(bx^3+a)^{8/3}}{b^3}\right)dx^3}{3d}$$

$$\Big\downarrow 2009$$

$$\frac{-\frac{2^{2/3}\sqrt{3}a^{11/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^4} + \frac{a^{11/3}\log(a-bx^3)}{\sqrt[3]{2}b^4} - \frac{3a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^4} - \frac{3a^3(a+bx^3)^{2/3}}{2b^4} - \frac{3a^2(a+}{5}}{3d}$$

input
```
Int[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
((-3*a^3*(a + b*x^3)^(2/3))/(2*b^4) - (3*a^2*(a + b*x^3)^(5/3))/(5*b^4) +
(3*a*(a + b*x^3)^(8/3))/(8*b^4) - (3*(a + b*x^3)^(11/3))/(11*b^4) - (2^(2/
3)*Sqrt[3]*a^(11/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*
a^(1/3))])/b^4 + (a^(11/3)*Log[a - b*x^3])/(2^(1/3)*b^4) - (3*a^(11/3)*Log
[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^4))/(3*d)
```

3.586.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.586.4 Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{220\,2^{\frac{2}{3}}\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{1320b^{4}d}$

input
```
int(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/1320*(220*2^(2/3)*(-2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/
3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(
2/3)*a^(2/3))-2*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3)))*a^(11/3)-3*(b*x^3+a)^
(2/3)*(40*b^3*x^9+65*a*b^2*x^6+98*a^2*b*x^3+293*a^3))/b^4/d
```

3.586.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.94

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{440\cdot 4^{\frac{1}{3}}\sqrt{3}(-a^2)^{\frac{1}{3}}\,a^3\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)+220\cdot 4^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}\,a^3\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}+2\right.}{}$$

input
```
integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/1320*(440*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a^3*arctan(1/3*(4^(1/3)*sqrt(3)*
(b*x^3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 220*4^(1/3)*(-a^2)^(1/3)*
a^3*log(4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2
*4^(1/3)*(-a^2)^(1/3)*a) - 440*4^(1/3)*(-a^2)^(1/3)*a^3*log(-4^(2/3)*(-a^2
)^(2/3) + 2*(b*x^3 + a)^(1/3)*a) + 3*(40*b^3*x^9 + 65*a*b^2*x^6 + 98*a^2*b
*x^3 + 293*a^3)*(b*x^3 + a)^(2/3))/(b^4*d)
```

3.586.6 Sympy [F]

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^{11}(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**11*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**11*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.586.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.82

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{\frac{440\sqrt{3}2^{\frac{2}{3}}a^{\frac{11}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}-\frac{220\cdot 2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}+\frac{440\cdot 2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(\right.}{d}}{1320\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-1/1320*(440*sqrt(3)*2^(2/3)*a^(11/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*
a^(1/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 220*2^(2/3)*a^(11/3)*log(2^(2/
3)*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 44
0*2^(2/3)*a^(11/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(40*(b*
x^3 + a)^(11/3) - 55*(b*x^3 + a)^(8/3)*a + 88*(b*x^3 + a)^(5/3)*a^2 + 220*
(b*x^3 + a)^(2/3)*a^3)/d)/b^4
```

3.586.8 Giac [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.96

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}a^{\frac{11}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,b^4d} + \frac{2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^4d} - \frac{2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^4d} - \frac{40\,(bx^3+a)^{\frac{11}{3}}b^{40}d^{10}-55\,(bx^3+a)^{\frac{8}{3}}ab^{40}d^{10}+88\,(bx^3+a)^{\frac{5}{3}}a^2b^{40}d^{10}+220\,(bx^3+a)^{\frac{2}{3}}a^3b^{40}d^{10}}{440\,b^{44}d^{11}}$$

input

```
integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output

```
-1/3*sqrt(3)*2^(2/3)*a^(11/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
+ 2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^4*d) + 1/6*2^(2/3)*a^(11/3)*log(2^(2/3)
*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^4*d)
- 1/3*2^(2/3)*a^(11/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^4
*d) - 1/440*(40*(b*x^3 + a)^(11/3)*b^40*d^10 - 55*(b*x^3 + a)^(8/3)*a*b^40
*d^10 + 88*(b*x^3 + a)^(5/3)*a^2*b^40*d^10 + 220*(b*x^3 + a)^(2/3)*a^3*b^4
0*d^10)/(b^44*d^11)
```

3.586.9 Mupad [B] (verification not implemented)

Time = 8.72 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{a\,(b\,x^3+a)^{8/3}}{8\,b^4\,d} - \frac{a^3\,(b\,x^3+a)^{2/3}}{2\,b^4\,d} - \frac{a^2\,(b\,x^3+a)^{5/3}}{5\,b^4\,d} - \frac{(b\,x^3+a)^{11/3}}{11\,b^4\,d} + \frac{4^{1/3}\,(-a)^{11/3}\,\ln\left(4\,a^8\,(b\,x^3+a)^{1/3}+4\,2^{1/3}\,(-a)^{25/3}\right)}{3\,b^4\,d} - \frac{4^{1/3}\,(-a)^{11/3}\,\ln\left(\frac{4\,a^8\,(b\,x^3+a)^{1/3}}{b^8\,d^2}+\frac{2\,4^{2/3}\,(-a)^{25/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^8\,d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^4\,d} + \frac{4^{1/3}\,(-a)^{11/3}\,\ln\left(\frac{4\,a^8\,(b\,x^3+a)^{1/3}}{b^8\,d^2}+\frac{18\,4^{2/3}\,(-a)^{25/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{b^8\,d^2}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^4\,d}$$

---

input

```
int((x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output

```
(a*(a + b*x^3)^(8/3))/(8*b^4*d) - (a^3*(a + b*x^3)^(2/3))/(2*b^4*d) - (a^2
*(a + b*x^3)^(5/3))/(5*b^4*d) - (a + b*x^3)^(11/3)/(11*b^4*d) + (4^(1/3)*(
-a)^(11/3)*log(4*a^8*(a + b*x^3)^(1/3) + 4*2^(1/3)*(-a)^(25/3)))/(3*b^4*d)
 - (4^(1/3)*(-a)^(11/3)*log((4*a^8*(a + b*x^3)^(1/3))/(b^8*d^2) + (2*4^(2/
3)*(-a)^(25/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(b^8*d^2))*((3^(1/2)*1i)/2 + 1/2)
)/(3*b^4*d) + (4^(1/3)*(-a)^(11/3)*log((4*a^8*(a + b*x^3)^(1/3))/(b^8*d^2)
 + (18*4^(2/3)*(-a)^(25/3)*((3^(1/2)*1i)/6 - 1/6)^2)/(b^8*d^2))*((3^(1/2)*
1i)/6 - 1/6))/(b^4*d)
```

3.586. $\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

3.587 $\int \frac{x^8\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

3.587.1 Optimal result

Integrand size = 28, antiderivative size = 177

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{a^2(a+bx^3)^{2/3}}{2b^3d}-\frac{(a+bx^3)^{8/3}}{8b^3d} - \frac{2^{2/3}a^{8/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^3d} + \frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^3d}-\frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^3d}$$

output
```
-1/2*a^2*(b*x^3+a)^(2/3)/b^3/d-1/8*(b*x^3+a)^(8/3)/b^3/d+1/6*a^(8/3)*ln(-b
*x^3+a)*2^(2/3)/b^3/d-1/2*a^(8/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2
/3)/b^3/d-1/3*2^(2/3)*a^(8/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))
/a^(1/3)*3^(1/2))/b^3/d*3^(1/2)
```

3.587.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.19

$$\int \frac{x^8\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{15a^2\left(a+bx^3\right)^{2/3}+6abx^3\left(a+bx^3\right)^{2/3}+3b^2x^6\left(a+bx^3\right)^{2/3}+8\;2^{2/3}\sqrt{3}a^{8/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+8}{24b^3d}$$

input
```
Integrate[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/24*(15*a^2*(a + b*x^3)^(2/3) + 6*a*b*x^3*(a + b*x^3)^(2/3) + 3*b^2*x^6*
(a + b*x^3)^(2/3) + 8*2^(2/3)*Sqrt[3]*a^(8/3)*ArcTan[(1 + (2^(2/3)*(a + b*
x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 8*2^(2/3)*a^(8/3)*Log[-2*a^(1/3) + 2^(2/3)
*(a + b*x^3)^(1/3)] - 4*2^(2/3)*a^(8/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a
 + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^3*d)
```

3.587.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {948, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^6\left(bx^3+a\right)^{2/3}}{d\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{x^6\left(bx^3+a\right)^{2/3}}{a-bx^3}dx^3}{3d}$$

$$\Big\downarrow \; 99$$

$$\frac{\int \left(\frac{a^2\left(bx^3+a\right)^{2/3}}{b^2\left(a-bx^3\right)}-\frac{\left(bx^3+a\right)^{5/3}}{b^2}\right)dx^3}{3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2^{2/3}\sqrt{3}a^{8/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^3}+\frac{a^{8/3}\log\left(a-bx^3\right)}{\sqrt[3]{2}b^3}-\frac{3a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^3}-\frac{3a^2\left(a+bx^3\right)^{2/3}}{2b^3}-\frac{3\left(a+bx^3\right)}{8b^3}}{3d}$$

input

```
Int[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output

```
((-3*a^2*(a + b*x^3)^(2/3))/(2*b^3) - (3*(a + b*x^3)^(8/3))/(8*b^3) - (2^(
2/3)*Sqrt[3]*a^(8/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]
*a^(1/3))])/b^3 + (a^(8/3)*Log[a - b*x^3])/(2^(1/3)*b^3) - (3*a^(8/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^3))/(3*d)
```

3.587.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.587.4 Maple [A] (verified)

Time = 4.79 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{4\,2^{\frac{2}{3}}\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)\right)}{24b^{3}d}$

input
```
int(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/24*(4*2^(2/3)*(-2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3
^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)
*a^(2/3))-2*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3)))*a^(8/3)-3*(b*x^3+a)^(2/3)
*(b^2*x^6+2*a*b*x^3+5*a^2))/b^3/d
```

3.587.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.11

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{8\cdot 4^{\frac{1}{3}}\sqrt{3}(-a^2)^{\frac{1}{3}}\,a^2\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)+4\cdot 4^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}\,a^2\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}+2\,(bx^3\right.}{}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/24*(8*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a^2*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x
^3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 4*4^(1/3)*(-a^2)^(1/3)*a^2*lo
g(4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/
3)*(-a^2)^(1/3)*a) - 8*4^(1/3)*(-a^2)^(1/3)*a^2*log(-4^(2/3)*(-a^2)^(2/3)
+ 2*(b*x^3 + a)^(1/3)*a) + 3*(b^2*x^6 + 2*a*b*x^3 + 5*a^2)*(b*x^3 + a)^(2/
3))/(b^3*d)
```

3.587.6 Sympy [F]

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^8\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**8*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**8*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.587.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\frac{8\sqrt{3}2^{\frac{2}{3}}a^{\frac{8}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d} - \frac{4\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d} + \frac{8\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right.}{d}}{24\,b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-1/24*(8*sqrt(3)*2^(2/3)*a^(8/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/
3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 4*2^(2/3)*a^(8/3)*log(2^(2/3)*a^(2/
3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 8*2^(2/3)*
a^(8/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*((b*x^3 + a)^(8/3)
 + 4*(b*x^3 + a)^(2/3)*a^2)/d)/b^3
```

3.587.8 Giac [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.99

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}a^{\frac{8}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,b^3d} + \frac{2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^3d} - \frac{2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^3d} - \frac{(bx^3+a)^{\frac{8}{3}}b^{21}d^7+4\,(bx^3+a)^{\frac{2}{3}}a^2b^{21}d^7}{8\,b^{24}d^8}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*2^(2/3)*a^(8/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
 2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^3*d) + 1/6*2^(2/3)*a^(8/3)*log(2^(2/3)*a
^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^3*d) -
1/3*2^(2/3)*a^(8/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^3*d)
 - 1/8*((b*x^3 + a)^(8/3)*b^21*d^7 + 4*(b*x^3 + a)^(2/3)*a^2*b^21*d^7)/(b^
24*d^8)
```

3.587.9 Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.16

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{(b\,x^3+a)^{8/3}}{8\,b^3\,d} - \frac{a^2\,(b\,x^3+a)^{2/3}}{2\,b^3\,d} - \frac{4^{1/3}\,a^{8/3}\,\ln\left((b\,x^3+a)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^3\,d} - \frac{4^{1/3}\,a^{8/3}\,\ln\left(\frac{4\,a^6\,(b\,x^3+a)^{1/3}}{b^6\,d^2}-\frac{2\,4^{2/3}\,a^{19/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^6\,d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^3\,d} + \frac{4^{1/3}\,a^{8/3}\,\ln\left(\frac{4\,a^6\,(b\,x^3+a)^{1/3}}{b^6\,d^2}-\frac{18\,4^{2/3}\,a^{19/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{b^6\,d^2}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^3\,d}$$

input
```
int((x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
(4^(1/3)*a^(8/3)*log((4*a^6*(a + b*x^3)^(1/3))/(b^6*d^2) - (18*4^(2/3)*a^(
19/3)*((3^(1/2)*1i)/6 + 1/6)^2)/(b^6*d^2))*((3^(1/2)*1i)/6 + 1/6))/(b^3*d)
 - (a^2*(a + b*x^3)^(2/3))/(2*b^3*d) - (4^(1/3)*a^(8/3)*log((a + b*x^3)^(1
/3) - 2^(1/3)*a^(1/3)))/(3*b^3*d) - (4^(1/3)*a^(8/3)*log((4*a^6*(a + b*x^3
)^(1/3))/(b^6*d^2) - (2*4^(2/3)*a^(19/3)*((3^(1/2)*1i)/2 - 1/2)^2)/(b^6*d^
2))*((3^(1/2)*1i)/2 - 1/2))/(3*b^3*d) - (a + b*x^3)^(8/3)/(8*b^3*d)
```

3.587. $\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

3.588 $\int \frac{x^5\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

3.588.1 Optimal result

Integrand size = 28, antiderivative size = 175

$$\int \frac{x^5\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx=-\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}-\frac{2^{2/3}a^{5/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d}+\frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^2d}-\frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^2d}$$

output
```
-1/2*a*(b*x^3+a)^(2/3)/b^2/d-1/5*(b*x^3+a)^(5/3)/b^2/d+1/6*a^(5/3)*ln(-b*x
^3+a)*2^(2/3)/b^2/d-1/2*a^(5/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3
)/b^2/d-1/3*2^(2/3)*a^(5/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a
^(1/3)*3^(1/2))/b^2/d*3^(1/2)
```

3.588.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.07

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{21a(a+bx^3)^{2/3}+6bx^3(a+bx^3)^{2/3}+10\ 2^{2/3}\sqrt{3}a^{5/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10\ 2^{2/3}a^{5/3}\log\left(-2\sqrt[3]{a}\right.}{30b^2d}$$

input
```
Integrate[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/30*(21*a*(a + b*x^3)^(2/3) + 6*b*x^3*(a + b*x^3)^(2/3) + 10*2^(2/3)*Sqr
t[3]*a^(5/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 1
0*2^(2/3)*a^(5/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 5*2^(2/3)*
a^(5/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b
*x^3)^(2/3)])/(b^2*d)
```

3.588.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {948, 27, 90, 60, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+bx^3\right)^{2/3}}{ad-bdx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3\left(bx^3+a\right)^{2/3}}{d\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^3\left(bx^3+a\right)^{2/3}}{a-bx^3}dx^3}{3d}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{a\int \frac{\left(bx^3+a\right)^{2/3}}{a-bx^3}dx^3}{b}-\frac{3\left(a+bx^3\right)^{5/3}}{5b^2}}{3d}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{a\left(2a\int \frac{1}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx^3-\frac{3\left(a+bx^3\right)^{2/3}}{2b}\right)}{b}-\frac{3\left(a+bx^3\right)^{5/3}}{5b^2}}{3d}$$

$$\Big\downarrow \text{67}$$

$$\frac{\frac{a\left(2a\left(-\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2b}+\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)-\frac{3\left(a+bx^3\right)^{2/3}}{2b}\right)}{b}-3}{3d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{a\left(2a\left(-\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2b}+\frac{\log\left(a-bx^3\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)-\frac{3\left(a+bx^3\right)^{2/3}}{2b}\right)}{b}-\frac{3\left(a+bx^3\right)^{5}}{5b^2}}{3d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{a\left(2a\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)-\frac{3\left(a+bx^3\right)^{2/3}}{2b}\right)}{b}-\frac{3\left(a+bx^3\right)^{5/3}}{5b^2}}{3d}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{a\left(2a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)-\frac{3\left(a+bx^3\right)^{2/3}}{2b}\right)}{b}-\frac{3\left(a+bx^3\right)^{5/3}}{5b^2}}{3d}$$

3.588. $\int \frac{x^5\left(a+bx^3\right)^{2/3}}{ad-bdx^3}dx$

input
```
Int[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
((-3*(a + b*x^3)^(5/3))/(5*b^2) + (a*((-3*(a + b*x^3)^(2/3))/(2*b) + 2*a*(
-((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/(2^(
1/3)*a^(1/3)*b)) + Log[a - b*x^3]/(2*2^(1/3)*a^(1/3)*b) - (3*Log[2^(1/3)*a
^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(1/3)*a^(1/3)*b))))/b)/(3*d)
```

3.588.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.588.4 Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{-5\,2^{\frac{2}{3}}\left(2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)-\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)\right)}{30b^{2}d}$

input
```
int(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/30*(-5*2^(2/3)*(2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3
^(1/2))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))-ln((b*x^3+a)^(2/3)+2
^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3)))*a^(5/3)-3*(b*x^3+a)^(2/3)
*(2*b*x^3+7*a))/b^2/d
```

3.588.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.03

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{10\cdot 4^{\frac{1}{3}}\sqrt{3}(-a^2)^{\frac{1}{3}}\,a\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)+5\cdot 4^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}\,a\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}+2\,(bx^3\,-}{30}$$

input
```
integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/30*(10*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^
3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 5*4^(1/3)*(-a^2)^(1/3)*a*log(4
^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*
(-a^2)^(1/3)*a) - 10*4^(1/3)*(-a^2)^(1/3)*a*log(-4^(2/3)*(-a^2)^(2/3) + 2*
(b*x^3 + a)^(1/3)*a) + 3*(2*b*x^3 + 7*a)*(b*x^3 + a)^(2/3))/(b^2*d)
```

3.588.6 Sympy [F]

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^5(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**5*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**5*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.588.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{\frac{10\sqrt{3}2^{\frac{2}{3}}a^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}-\frac{5\cdot 2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}+\frac{10\cdot 2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)\right.}{d}}{30\,b^2}$$

input

```
integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output

```
-1/30*(10*sqrt(3)*2^(2/3)*a^(5/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1
/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 5*2^(2/3)*a^(5/3)*log(2^(2/3)*a^(2
/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 10*2^(2/3
)*a^(5/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(2*(b*x^3 + a)^(
5/3) + 5*(b*x^3 + a)^(2/3)*a)/d)/b^2
```

3.588.8 Giac [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}a^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,b^2d} + \frac{2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^2d} - \frac{2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^2d} - \frac{2\,(bx^3+a)^{\frac{5}{3}}b^8d^4+5\,(bx^3+a)^{\frac{2}{3}}ab^8d^4}{10\,b^{10}d^5}$$

input

```
integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output

```
-1/3*sqrt(3)*2^(2/3)*a^(5/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
 2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^2*d) + 1/6*2^(2/3)*a^(5/3)*log(2^(2/3)*a
^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^2*d) -
1/3*2^(2/3)*a^(5/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^2*d)
 - 1/10*(2*(b*x^3 + a)^(5/3)*b^8*d^4 + 5*(b*x^3 + a)^(2/3)*a*b^8*d^4)/(b^1
0*d^5)
```

3.588.9 Mupad [B] (verification not implemented)

Time = 8.66 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.26

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{4^{1/3}\,(-a)^{5/3}\,\ln\left(4\,a^4\,(b\,x^3+a)^{1/3}+4\,2^{1/3}\,(-a)^{13/3}\right)}{3\,b^2\,d} - \frac{a\,(b\,x^3+a)^{2/3}}{2\,b^2\,d} - \frac{(b\,x^3+a)^{5/3}}{5\,b^2\,d} - \frac{4^{1/3}\,(-a)^{5/3}\,\ln\left(\frac{4\,a^4\,(b\,x^3+a)^{1/3}}{b^4\,d^2}+\frac{2\,4^{2/3}\,(-a)^{13/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^4\,d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^2\,d} + \frac{4^{1/3}\,(-a)^{5/3}\,\ln\left(\frac{4\,a^4\,(b\,x^3+a)^{1/3}}{b^4\,d^2}+\frac{18\,4^{2/3}\,(-a)^{13/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{b^4\,d^2}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^2\,d}$$

input
```
int((x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
(4^(1/3)*(-a)^(5/3)*log(4*a^4*(a + b*x^3)^(1/3) + 4*2^(1/3)*(-a)^(13/3)))/
(3*b^2*d) - (a*(a + b*x^3)^(2/3))/(2*b^2*d) - (a + b*x^3)^(5/3)/(5*b^2*d)
- (4^(1/3)*(-a)^(5/3)*log((4*a^4*(a + b*x^3)^(1/3))/(b^4*d^2) + (2*4^(2/3)
*(-a)^(13/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(b^4*d^2))*((3^(1/2)*1i)/2 + 1/2))/
(3*b^2*d) + (4^(1/3)*(-a)^(5/3)*log((4*a^4*(a + b*x^3)^(1/3))/(b^4*d^2) +
(18*4^(2/3)*(-a)^(13/3)*((3^(1/2)*1i)/6 - 1/6)^2)/(b^4*d^2))*((3^(1/2)*1i)
/6 - 1/6))/(b^2*d)
```

3.589 $\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

3.589.1 Optimal result

Integrand size = 28, antiderivative size = 153

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{(a+bx^3)^{2/3}}{2bd} - \frac{2^{2/3}a^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}bd} + \frac{a^{2/3}\log(a-bx^3)}{3\sqrt[3]{2}bd} - \frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}bd}$$

output
```
-1/2*(b*x^3+a)^(2/3)/b/d+1/6*a^(2/3)*ln(-b*x^3+a)*2^(2/3)/b/d-1/2*a^(2/3)*
ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/b/d-1/3*2^(2/3)*a^(2/3)*arctan
(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/b/d*3^(1/2)
```

3.589.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.11

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{3(a+bx^3)^{2/3}+2\ 2^{2/3}\sqrt{3}a^{2/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\ 2^{2/3}a^{2/3}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right)-2^{2/3}}{6bd}$$

input
```
Integrate[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/6*(3*(a + b*x^3)^(2/3) + 2*2^(2/3)*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2^(2/3)
*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*2^(2/3)*a^(2/3)*Log[-2*a^(1/3) +
 2^(2/3)*(a + b*x^3)^(1/3)] - 2^(2/3)*a^(2/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1
/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b*d)
```

3.589.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {946, 27, 60, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3} \int \frac{\left(b x^3+a\right)^{2/3}}{d \left(a-b x^3\right)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(b x^3+a\right)^{2/3}}{a-b x^3}dx^3}{3 d}$$

$$\Big\downarrow \text{60}$$

$$\frac{2 a \int \frac{1}{\left(a-b x^3\right) \sqrt[3]{b x^3+a}}dx^3-\frac{3 \left(a+b x^3\right)^{2/3}}{2 b}}{3 d}$$

$$\Big\downarrow \text{67}$$

$$\frac{2 a \left(-\frac{3 \int \frac{1}{x^6+2^{2/3} a^{2/3}+\sqrt[3]{2} \sqrt[3]{a} \sqrt[3]{b x^3+a}}d\sqrt[3]{b x^3+a}}{2 b}+\frac{3 \int \frac{1}{\sqrt[3]{2} \sqrt[3]{a}-\sqrt[3]{b x^3+a}}d\sqrt[3]{b x^3+a}}{2 \sqrt[3]{2} \sqrt[3]{a} b}+\frac{\log \left(a-b x^3\right)}{2 \sqrt[3]{2} \sqrt[3]{a} b}\right)-\frac{3 \left(a+b x^3\right)^{2/3}}{2 b}}{3 d}$$

$$\Big\downarrow \text{16}$$

$$\frac{2a\left(-\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2b}+\frac{\log\left(a-bx^3\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)-\frac{3\left(a+bx^3\right)^{2/3}}{2b}}{3d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2a\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)-\frac{3\left(a+bx^3\right)^{2/3}}{2b}}{3d}$$

$$\Big\downarrow \text{217}$$

$$\frac{2a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)-\frac{3\left(a+bx^3\right)^{2/3}}{2b}}{3d}$$

input
```
Int[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
((-3*(a + b*x^3)^(2/3))/(2*b) + 2*a*(-((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a +
b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3)*b)) + Log[a - b*x^3]/(2*
2^(1/3)*a^(1/3)*b) - (3*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(1/
3)*a^(1/3)*b)))/(3*d)
```

3.589.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.589.4 Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{-2a^{\frac{2}{3}}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)-2a^{\frac{2}{3}}2^{\frac{2}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)+a^{\frac{2}{3}}2^{\frac{2}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right.}{6bd}$

3.589. $\int \frac{x^2\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

input
```
int(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*a^(2/3)*3^(1/2)*2^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3
))/a^(1/3)*3^(1/2))-2*a^(2/3)*2^(2/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))+
a^(2/3)*2^(2/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)
*a^(2/3))-3*(b*x^3+a)^(2/3))/b/d
```

3.589.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$
$$-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}(-a^2)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)+4^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}+2\,(bx^3+a)^{\frac{2}{3}}a\right.}{6\,bd}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/6*(2*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^3 +
a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 4^(1/3)*(-a^2)^(1/3)*log(4^(2/3)*(
b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*(-a^2)^(
1/3)*a) - 2*4^(1/3)*(-a^2)^(1/3)*log(-4^(2/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)
^(1/3)*a) + 3*(b*x^3 + a)^(2/3))/(b*d)
```

3.589.6 Sympy [F]

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^2(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**2*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**2*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.589.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{\frac{2\sqrt{3}2^{\frac{2}{3}}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}-\frac{2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}+\frac{2\cdot2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}}{6\,b}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*2^(2/3)*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 2^(2/3)*a^(2/3)*log(2^(2/3)*a^(2/3)
+ 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 2*2^(2/3)*a^(
2/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(b*x^3 + a)^(2/3)/d)/
b
```

3.589.8 Giac [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,bd}$$
$$+\frac{2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,bd}$$
$$-\frac{2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,bd}-\frac{(bx^3+a)^{\frac{2}{3}}}{2\,bd}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*2^(2/3)*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
 2*(b*x^3 + a)^(1/3))/a^(1/3))/(b*d) + 1/6*2^(2/3)*a^(2/3)*log(2^(2/3)*a^(
2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b*d) - 1/3*
2^(2/3)*a^(2/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b*d) - 1/2
*(b*x^3 + a)^(2/3)/(b*d)
```

3.589.9 Mupad [B] (verification not implemented)

Time = 8.66 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.22

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{(b\,x^3+a)^{2/3}}{2\,b\,d} - \frac{4^{1/3}\,a^{2/3}\,\ln\left((b\,x^3+a)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b\,d} - \frac{4^{1/3}\,a^{2/3}\,\ln\left(\frac{4\,a^2\,(b\,x^3+a)^{1/3}}{b^2\,d^2}-\frac{2\,4^{2/3}\,a^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^2\,d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b\,d} + \frac{4^{1/3}\,a^{2/3}\,\ln\left(\frac{4\,a^2\,(b\,x^3+a)^{1/3}}{b^2\,d^2}-\frac{18\,4^{2/3}\,a^{7/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{b^2\,d^2}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b\,d}$$

input
```
int((x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
(4^(1/3)*a^(2/3)*log((4*a^2*(a + b*x^3)^(1/3))/(b^2*d^2) - (18*4^(2/3)*a^(
7/3)*((3^(1/2)*1i)/6 + 1/6)^2)/(b^2*d^2))*((3^(1/2)*1i)/6 + 1/6))/(b*d) -
(4^(1/3)*a^(2/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*a^(1/3)))/(3*b*d) - (4^(1
/3)*a^(2/3)*log((4*a^2*(a + b*x^3)^(1/3))/(b^2*d^2) - (2*4^(2/3)*a^(7/3)*(
(3^(1/2)*1i)/2 - 1/2)^2)/(b^2*d^2))*((3^(1/2)*1i)/2 - 1/2))/(3*b*d) - (a +
 b*x^3)^(2/3)/(2*b*d)
```

3.590 $\int \frac{\left(a+bx^3\right)^{2/3}}{x\left(ad-bdx^3\right)}\,dx$

3.590.1 Optimal result

Integrand size = 28, antiderivative size = 214

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x\left(ad-bdx^3\right)}\,dx=\frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}d}-\frac{2^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}d}-\frac{\log(x)}{2\sqrt[3]{a}d}+\frac{\log\left(a-bx^3\right)}{3\sqrt[3]{2}\sqrt[3]{a}d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{a}d}$$

output
```
-1/2*ln(x)/a^(1/3)/d+1/6*ln(-b*x^3+a)*2^(2/3)/a^(1/3)/d+1/2*ln(a^(1/3)-(b*
x^3+a)^(1/3))/a^(1/3)/d-1/2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/a^
(1/3)/d+1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(1/3
)/d*3^(1/2)-1/3*2^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/
3)*3^(1/2))/a^(1/3)/d*3^(1/2)
```

3.590.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x\left(ad-bdx^3\right)}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(-\sqrt[3]{a}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(2/3)
*Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Log
[-a^(1/3) + (a + b*x^3)^(1/3)] - 2*2^(2/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b
*x^3)^(1/3)] - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
] + 2^(2/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a
 + b*x^3)^(2/3)])/(6*a^(1/3)*d)
```

3.590.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 27, 94, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x\left(ad-bdx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)^{2/3}}{dx^3\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(bx^3+a\right)^{2/3}}{x^3\left(a-bx^3\right)}dx^3}{3d}$$

$$\Big\downarrow \text{94}$$

$$\frac{\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3+2b\int \frac{1}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx^3}{3d}$$

$$\Big\downarrow \text{67}$$

$$\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+2b\left(-\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2b}+\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{bx^3+a}}}{2\sqrt[3]{2}\sqrt[3]{a}b}\right.}{3d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+2b\left(-\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2b}+\frac{\log(a-bx^3)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\right.}{2}\right.}{3d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{-\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+2b\left(\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log(a-bx^3)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)+\frac{3\,\mathrm{lo}}{}}{3d}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+2b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log(a-bx^3)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}\right)+\frac{3\log\left(\right.}{}}{3d}$$

input
```
Int[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x]
```

output
```
((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - L
og[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3)) + 2
*b*(-((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/
(2^(1/3)*a^(1/3)*b)) + Log[a - b*x^3]/(2*2^(1/3)*a^(1/3)*b) - (3*Log[2^(1/
3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(1/3)*a^(1/3)*b)))/(3*d)
```

3.590. $\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx$

3.590.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.590. $\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx$

3.590.4 Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$\frac{-2\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)-2\,2^{\frac{2}{3}} \ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}} a^{\frac{1}{3}}\right)+2^{\frac{2}{3}} \ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}} a^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}} a^{\frac{2}{3}}\right.}{6d\,a^{\frac{1}{3}}}$

input
```
int((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*3^(1/2)*2^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/
3)*3^(1/2))-2*2^(2/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))+2^(2/3)*ln((b*x^
3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))+2*arctan(1/3*(
a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-a
^(1/3))-ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3)))/d/a^(1/3)
```

3.590.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.48

$$\int \frac{\left(a+bx^{3}\right)^{2/3}}{x\left(ad-bdx^{3}\right)}\,dx = \left[-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}a\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}(bx^{3}+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-3\sqrt{\frac{1}{3}}a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log}{}\right.$$

$$-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}a\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}(bx^{3}+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+4^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(bx^{3}+a)^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{2}{3}}-2\right.}{}$$

input
```
integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
[-1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3
+ a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*lo
g((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*
a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) +
4^(1/3)*a*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^
(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log
(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) + a^(2/3)*log((b*x^3 + a)^
(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(
1/3) - a^(1/3)))/(a*d), -1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*
4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 4^(1/3)*a*
(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-
1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log(-4^(2/3)*
a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/
3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3
) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3)
 - a^(1/3)))/(a*d)]
```

3.590.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x\,(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax+bx^4}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x + b*x**4), x)/d
```

3.590.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x\,(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x), x)
```

3.590.8 Giac [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}d} + \frac{2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}d} - \frac{2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{1}{3}}d} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}d} - \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}d} + \frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{1}{3}}d}$$

input
```
integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*x^
3 + a)^(1/3))/a^(1/3))/(a^(1/3)*d) + 1/6*2^(2/3)*log(2^(2/3)*a^(2/3) + 2^(
1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(a^(1/3)*d) - 1/3*2^(2
/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(a^(1/3)*d) + 1/3*sqrt(
3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(1/3)*d)
 - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(1/
3)*d) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(1/3)*d)
```

3.590.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.72

$$\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx = \ln\left(2\left(b\,x^3+a\right)^{1/3} - 2\,2^{1/3}\,a\,d^2\left(-\frac{1}{a\,d^3}\right)^{2/3}\right)\left(-\frac{4}{27\,a\,d^3}\right)^{1/3} + \ln\left(\left(b\,x^3+a\right)^{1/3} - a\,d^2\left(\frac{1}{a\,d^3}\right)^{2/3}\right)\left(\frac{1}{27\,a\,d^3}\right)^{1/3} - \ln\left(4\left(b\,x^3\right.\right.$$

input
```
int((a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x)
```

output

```
log(2*(a + b*x^3)^(1/3) - 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3))*(-4/(27*a*d^
3))^(1/3) + log((a + b*x^3)^(1/3) - a*d^2*(1/(a*d^3))^(2/3))*(1/(27*a*d^3)
)^(1/3) - log(4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3) - 2
^(1/3)*3^(1/2)*a*d^2*(-1/(a*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 + 1/2)*(-4/(27
*a*d^3))^(1/3) + log(4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2
/3) + 2^(1/3)*3^(1/2)*a*d^2*(-1/(a*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*
(-4/(27*a*d^3))^(1/3) - log(2*(a + b*x^3)^(1/3) + a*d^2*(1/(a*d^3))^(2/3)
- 3^(1/2)*a*d^2*(1/(a*d^3))^(2/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a*d^3)
)^(1/3) + log(2*(a + b*x^3)^(1/3) + a*d^2*(1/(a*d^3))^(2/3) + 3^(1/2)*a*d^
2*(1/(a*d^3))^(2/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a*d^3))^(1/3)
```

3.590. $\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx$

3.591 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(ad-bdx^3\right)}\,dx$

3.591.1 Optimal result

Integrand size = 28, antiderivative size = 269

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(ad-bdx^3\right)}\,dx = \frac{b\left(a+bx^3\right)^{2/3}}{3a^2d}-\frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3}+\frac{5b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}d}-\frac{2^{2/3}b\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}d}-\frac{5b\log(x)}{6a^{4/3}d}+\frac{b\log\left(a-bx^3\right)}{3\sqrt[3]{2}a^{4/3}d}+\frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{4/3}d}$$

output
```
1/3*b*(b*x^3+a)^(2/3)/a^2/d-1/3*(b*x^3+a)^(5/3)/a^2/d/x^3-5/6*b*ln(x)/a^(4
/3)/d+1/6*b*ln(-b*x^3+a)*2^(2/3)/a^(4/3)/d+5/6*b*ln(a^(1/3)-(b*x^3+a)^(1/3
))/a^(4/3)/d-1/2*b*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/a^(4/3)/d+5
/9*b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)/d*3^(
1/2)-1/3*2^(2/3)*b*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^
(1/2))/a^(4/3)/d*3^(1/2)
```

3.591.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^{2/3}}{x^4\,(ad-bdx^3)}\,dx = \frac{-6\sqrt[3]{a}(a+bx^3)^{2/3}+10\sqrt{3}bx^3\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-6\;2^{2/3}\sqrt{3}bx^3\arctan\left(\frac{1+\frac{2^{2}}{}}{}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]
```

output
```
(-6*a^(1/3)*(a + b*x^3)^(2/3) + 10*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*x^3
)^(1/3))/a^(1/3))/Sqrt[3]] - 6*2^(2/3)*Sqrt[3]*b*x^3*ArcTan[(1 + (2^(2/3)*
(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 10*b*x^3*Log[-a^(1/3) + (a + b*x^3)
^(1/3)] - 6*2^(2/3)*b*x^3*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 5*
b*x^3*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + 3*2^(
2/3)*b*x^3*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a
+ b*x^3)^(2/3)])/(18*a^(4/3)*d*x^3)
```

3.591.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {948, 27, 114, 27, 174, 60, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^4\,(ad-bdx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{2/3}}{dx^6\,(a-bx^3)}dx^3$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(bx^3+a)^{2/3}}{x^6(a-bx^3)}dx^3}{3d}$$

$$\Big\downarrow \text{114}$$

$$\frac{-\frac{\int -\frac{b\left(5a-2bx^3\right)\left(bx^3+a\right)^{2/3}}{3x^3\left(a-bx^3\right)}dx^3}{a^2}-\frac{\left(a+bx^3\right)^{5/3}}{a^2x^3}}{3d}$$

$\downarrow$ 27

$$\frac{\frac{b\int \frac{\left(5a-2bx^3\right)\left(bx^3+a\right)^{2/3}}{x^3\left(a-bx^3\right)}dx^3}{3a^2}-\frac{\left(a+bx^3\right)^{5/3}}{a^2x^3}}{3d}$$

$\downarrow$ 174

$$\frac{\frac{b\left(5\int \frac{\left(bx^3+a\right)^{2/3}}{x^3}dx^3+3b\int \frac{\left(bx^3+a\right)^{2/3}}{a-bx^3}dx^3\right)}{3a^2}-\frac{\left(a+bx^3\right)^{5/3}}{a^2x^3}}{3d}$$

$\downarrow$ 60

$$\frac{\frac{b\left(5\left(a\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)+3b\left(2a\int \frac{1}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx^3-\frac{3\left(a+bx^3\right)^{2/3}}{2b}\right)\right)}{3a^2}-\frac{\left(a+bx^3\right)^{5/3}}{a^2x^3}}{3d}$$

$\downarrow$ 67

$$\frac{\frac{b\left(5\left(a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)+3b\left(2a\left(-\frac{3\int}{x^6+2}\right.\right.\right.}{3a^2}}{3d}$$

$\downarrow$ 16

$$\frac{\frac{b\left(5\left(a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)+3b\left(2a\left(-\frac{3\int}{x^6+2^{2/3}a^{2/3}+}\right.\right.\right.}{3a^2}}{3d}$$

$\downarrow$ 1082

$$\frac{\frac{b\left(5\left(a\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)+3b\left(2a\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+\right.}{\sqrt[3]{2}\sqrt[3]{a}b}\right.\right.\right.}{3a^2}}{3d}$$

3.591. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(ad-bdx^3\right)}dx$

$$\Big\downarrow 217$$

$$\frac{\frac{b\Bigg(5\Bigg(a\Bigg(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\Bigg)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\Bigg)+3b\Bigg(2a\Bigg(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}b}}{3a^2}}{3d}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]
```

output
```
(-((a + b*x^3)^(5/3)/(a^2*x^3)) + (b*(5*((3*(a + b*x^3)^(2/3))/2 + a*((Sqr
t[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^
3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3)))) + 3*b*
((-3*(a + b*x^3)^(2/3))/(2*b) + 2*a*(-((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a +
b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3)*b)) + Log[a - b*x^3]/(2*
2^(1/3)*a^(1/3)*b) - (3*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(1/
3)*a^(1/3)*b)))))/(3*a^2))/(3*d)
```

3.591.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

---

rule 67

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.591. $\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx$

3.591.4 Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$-\frac{2^{\frac{2}{3}} \sqrt{3}\, \arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}\right) \sqrt{3}}{3 a^{\frac{1}{3}}}\right) b\,x^{3}+2^{\frac{2}{3}} \ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}} a^{\frac{1}{3}}\right) b\,x^{3}-\frac{5 \arctan\left(\frac{\left(a^{\frac{1}{3}}+2 \left(b\,x^{3}+a\right)^{\frac{1}{3}}\right) \sqrt{3}}{3 a^{\frac{1}{3}}}\right) \sqrt{3}\, b\,x^{3}}{3}}{}$

input
```
int((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2^(2/3)*3^(1/2)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)
*3^(1/2))*b*x^3+2^(2/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))*b*x^3-5/3*arct
an(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*b*x^3-1/2*2^(2
/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))*b*
x^3-5/3*ln((b*x^3+a)^(1/3)-a^(1/3))*b*x^3+5/6*ln((b*x^3+a)^(2/3)+a^(1/3)*(
b*x^3+a)^(1/3)+a^(2/3))*b*x^3+(b*x^3+a)^(2/3)*a^(1/3))/a^(4/3)/x^3/d
```

3.591.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 612, normalized size of antiderivative = 2.28

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(ad-bdx^3\right)}\,dx= \Bigg[-\frac{6\cdot 4^{\frac{1}{3}}\sqrt{3}abx^3\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-15\sqrt{\frac{1}{3}}abx^3\sqrt{}}{}$$

$$-\frac{6\cdot 4^{\frac{1}{3}}\sqrt{3}abx^3\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+3\cdot 4^{\frac{1}{3}}abx^3\left(-\frac{1}{a}\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}a(\right.}{}$$

input
```
integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
[-1/18*(6*4^(1/3)*sqrt(3)*a*b*x^3*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*
(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 15*sqrt(1/3)*a*b*x^3*sqrt(
-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x
^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3)
+ 3*a)/x^3) + 3*4^(1/3)*a*b*x^3*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)
*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 6*4^(1
/3)*a*b*x^3*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)
) + 5*a^(2/3)*b*x^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^
(2/3)) - 10*a^(2/3)*b*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*(b*x^3 + a)
^(2/3)*a)/(a^2*d*x^3), -1/18*(6*4^(1/3)*sqrt(3)*a*b*x^3*(-1/a)^(1/3)*arcta
n(1/3*4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 3*4^
(1/3)*a*b*x^3*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) -
2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 6*4^(1/3)*a*b*x^3*(-1/a)
^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) - 30*sqrt(1/3)*a
^(2/3)*b*x^3*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) + 5
*a^(2/3)*b*x^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)
) - 10*a^(2/3)*b*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*(b*x^3 + a)^(2/3
)*a)/(a^2*d*x^3)]
```

3.591.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^4+bx^7}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x**4/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x**4 + b*x**7), x)/d
```

3.591.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^4}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^4), x)
```

3.591.8 Giac [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}b\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}d} + \frac{2^{\frac{2}{3}}b\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}d} - \frac{2^{\frac{2}{3}}b\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{4}{3}}d} + \frac{5\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{4}{3}}d} - \frac{5\,b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{4}{3}}d} + \frac{5\,b\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{4}{3}}d} - \frac{(bx^3+a)^{\frac{2}{3}}}{3\,adx^3}$$

input
```
integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*2^(2/3)*b*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*
x^3 + a)^(1/3))/a^(1/3))/(a^(4/3)*d) + 1/6*2^(2/3)*b*log(2^(2/3)*a^(2/3) +
 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(a^(4/3)*d) - 1/3*
2^(2/3)*b*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(a^(4/3)*d) + 5/9
*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^
(4/3)*d) - 5/18*b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2
/3))/(a^(4/3)*d) + 5/9*b*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(4/3)*d)
 - 1/3*(b*x^3 + a)^(2/3)/(a*d*x^3)
```

3.591.9 Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = \ln\left(2\,b^2\,{\left(b\,x^3+a\right)}^{1/3} - 2\,2^{1/3}\,a^3\,d^2\,{\left(-\frac{b^3}{a^4\,d^3}\right)}^{2/3}\right)\,{\left(-\frac{4\,b^3}{27\,a^4\,d^3}\right)}^{1/3} + \frac{5\,\ln\left(b^2\,{\left(b\,x^3+a\right)}^{1/3} - a^3\,d^2\,{\left(\frac{b^3}{a^4\,d^3}\right)}^{2/3}\right)\,{\left(\frac{b^3}{a^4\,d^3}\right)}^{1/3}}{9} - \ln\left(4\right.$$

input

```
int((a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x)
```

output

```
log(2*b^2*(a + b*x^3)^(1/3) - 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3))*(-
(4*b^3)/(27*a^4*d^3))^(1/3) + (5*log(b^2*(a + b*x^3)^(1/3) - a^3*d^2*(b^3/
(a^4*d^3))^(2/3))*(b^3/(a^4*d^3))^(1/3))/9 - log(4*b^2*(a + b*x^3)^(1/3) +
 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3) - 2^(1/3)*3^(1/2)*a^3*d^2*(-b^3/
(a^4*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 + 1/2)*(-(4*b^3)/(27*a^4*d^3))^(1/3)
+ log(4*b^2*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3) +
 2^(1/3)*3^(1/2)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)
*(-(4*b^3)/(27*a^4*d^3))^(1/3) - log(2*b^2*(a + b*x^3)^(1/3) + a^3*d^2*(b^
3/(a^4*d^3))^(2/3) - 3^(1/2)*a^3*d^2*(b^3/(a^4*d^3))^(2/3)*1i)*((3^(1/2)*1
i)/2 + 1/2)*((125*b^3)/(729*a^4*d^3))^(1/3) + log(2*b^2*(a + b*x^3)^(1/3)
+ a^3*d^2*(b^3/(a^4*d^3))^(2/3) + 3^(1/2)*a^3*d^2*(b^3/(a^4*d^3))^(2/3)*1i
)*((3^(1/2)*1i)/2 - 1/2)*((125*b^3)/(729*a^4*d^3))^(1/3) - (b*(a + b*x^3)^
(2/3))/(3*a*(d*(a + b*x^3) - a*d))
```

3.591. $\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx$

3.592 $\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx$

3.592.1 Optimal result

Integrand size = 28, antiderivative size = 284

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = -\frac{5b(a+bx^3)^{2/3}}{18a^2dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2dx^6} + \frac{14b^2\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}d} - \frac{2^{2/3}b^2\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}d} - \frac{7b^2\log(x)}{9a^{7/3}d} + \frac{b^2\log(a-bx^3)}{3\sqrt[3]{2}a^{7/3}d} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{9a^{7/3}d} - \frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{7/3}d}$$

output
```
-5/18*b*(b*x^3+a)^(2/3)/a^2/d/x^3-1/6*(b*x^3+a)^(5/3)/a^2/d/x^6-7/9*b^2*ln
(x)/a^(7/3)/d+1/6*b^2*ln(-b*x^3+a)*2^(2/3)/a^(7/3)/d+7/9*b^2*ln(a^(1/3)-(b
*x^3+a)^(1/3))/a^(7/3)/d-1/2*b^2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/
3)/a^(7/3)/d+14/27*b^2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1
/2))/a^(7/3)/d*3^(1/2)-1/3*2^(2/3)*b^2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+
a)^(1/3))/a^(1/3)*3^(1/2))/a^(7/3)/d*3^(1/2)
```

3.592.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^{2/3}}{x^7\left(ad-bdx^3\right)}\,dx = \frac{-9a^{4/3}(a+bx^3)^{2/3}-24\sqrt[3]{a}bx^3(a+bx^3)^{2/3}+28\sqrt{3}b^2x^6\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]
```

output
```
(-9*a^(4/3)*(a + b*x^3)^(2/3) - 24*a^(1/3)*b*x^3*(a + b*x^3)^(2/3) + 28*Sq
rt[3]*b^2*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 18*2^(
2/3)*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt
[3]] + 28*b^2*x^6*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - 18*2^(2/3)*b^2*x^6*L
og[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 14*b^2*x^6*Log[a^(2/3) + a^(1
/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + 9*2^(2/3)*b^2*x^6*Log[2*a^(2/
3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(54*a
^(7/3)*d*x^6)
```

3.592.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {948, 27, 114, 27, 166, 27, 174, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^7\left(ad-bdx^3\right)}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)^{2/3}}{dx^9\left(a-bx^3\right)}dx^3$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(bx^3+a\right)^{2/3}}{x^9\left(a-bx^3\right)}dx^3}{3d}$$

$$\Big\downarrow \; 114$$

$$\frac{-\frac{\int -\frac{b\left(bx^3+a\right)^{2/3}\left(bx^3+5a\right)}{3x^6\left(a-bx^3\right)}dx^3}{2a^2}-\frac{\left(a+bx^3\right)^{5/3}}{2a^2x^6}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{b\int \frac{\left(bx^3+a\right)^{2/3}\left(bx^3+5a\right)}{x^6\left(a-bx^3\right)}dx^3}{6a^2}-\frac{\left(a+bx^3\right)^{5/3}}{2a^2x^6}}{3d}$$

$$\Big\downarrow \; 166$$

$$\frac{\frac{b\left(\frac{\int \frac{4ab\left(2bx^3+7a\right)}{3x^3\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx^3}{a}-\frac{5\left(a+bx^3\right)^{2/3}}{x^3}\right)}{6a^2}-\frac{\left(a+bx^3\right)^{5/3}}{2a^2x^6}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{b\left(\frac{4}{3}b\int \frac{2bx^3+7a}{x^3\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx^3-\frac{5\left(a+bx^3\right)^{2/3}}{x^3}\right)}{6a^2}-\frac{\left(a+bx^3\right)^{5/3}}{2a^2x^6}}{3d}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{b\left(\frac{4}{3}b\left(7\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3+9b\int \frac{1}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx^3\right)-\frac{5\left(a+bx^3\right)^{2/3}}{x^3}\right)}{6a^2}-\frac{\left(a+bx^3\right)^{5/3}}{2a^2x^6}}{3d}$$

$$\Big\downarrow \; 67$$

$$\frac{\frac{b\Bigg(\frac{4}{3}b\Bigg(7\Bigg(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\Bigg)+9b\Bigg(-\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}}}{2b}}{6a^2}}{3d}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{b\Bigg(\frac{4}{3}b\Bigg(7\Bigg(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\Bigg)+9b\Bigg(-\frac{3\int \frac{1}{x^6+2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2b}}{6a^2}}{3d}$$

3.592. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(ad-bdx^3\right)}\,dx$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{b\left(\frac{4}{3}b\left(7\left(-\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+9b\left(\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2^{2/3}\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\,\mathrm{l}}{}\right.\right.\right.}{6a^2}}{3d}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{b\left(\frac{4}{3}b\left(7\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+9b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}b}+\frac{\log\left(a-bx^3\right)}{2\sqrt[3]{2}\sqrt[3]{a}b}-\frac{3\log\left(\right.}{}\right.\right.\right.}{6a^2}}{3d}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]
```

output
```
(-1/2*(a + b*x^3)^(5/3)/(a^2*x^6) + (b*((-5*(a + b*x^3)^(2/3))/x^3 + (4*b*
(7*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3)
- Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3)))
 + 9*b*(-((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3
]])/(2^(1/3)*a^(1/3)*b)) + Log[a - b*x^3]/(2*2^(1/3)*a^(1/3)*b) - (3*Log[2
^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2*2^(1/3)*a^(1/3)*b))))/3))/(6*a^2))
/(3*d)
```

3.592.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.592. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(ad-bdx^3\right)}\,dx$

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.592. $\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.592.4 Maple [A] (verified)

Time = 4.74 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$\frac{-18\sqrt{3}\, 2^{\frac{2}{3}} \arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right) b^{2}x^{6}-18\, 2^{\frac{2}{3}} \ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}} a^{\frac{1}{3}}\right) b^{2}x^{6}+9\, 2^{\frac{2}{3}} \ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}} a^{\frac{1}{3}} \left(b\,x^{3}+\right.\right.}{}$

input
```
int((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/54*(-18*3^(1/2)*2^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(
1/3)*3^(1/2))*b^2*x^6-18*2^(2/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))*b^2*x
^6+9*2^(2/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^
(2/3))*b^2*x^6+28*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*
3^(1/2)*b^2*x^6+28*ln((b*x^3+a)^(1/3)-a^(1/3))*b^2*x^6-14*ln((b*x^3+a)^(2/
3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))*b^2*x^6-24*b*x^3*a^(1/3)*(b*x^3+a)^(2/
3)-9*(b*x^3+a)^(2/3)*a^(4/3))/a^(7/3)/x^6/d
```

3.592.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 660, normalized size of antiderivative = 2.32

$$\int \frac{(a+bx^3)^{2/3}}{x^7\,(ad-bdx^3)}\,dx = \left[-\frac{18\cdot 4^{\frac{1}{3}}\sqrt{3}ab^2x^6\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-42\sqrt{\frac{1}{3}}ab^2x}{}\right.$$

$$-\frac{18\cdot 4^{\frac{1}{3}}\sqrt{3}ab^2x^6\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+9\cdot 4^{\frac{1}{3}}ab^2x^6\left(-\frac{1}{a}\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}\right.}{}$$

input
```
integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
[-1/54*(18*4^(1/3)*sqrt(3)*a*b^2*x^6*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(
3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 42*sqrt(1/3)*a*b^2*x^6*
sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) -
 (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(
2/3) + 3*a)/x^3) + 9*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a
)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) -
 18*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3
+ a)^(1/3)) + 14*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)
*a^(1/3) + a^(2/3)) - 28*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(1/3) - a^(1/3))
+ 3*(8*a*b*x^3 + 3*a^2)*(b*x^3 + a)^(2/3))/(a^3*d*x^6), -1/54*(18*4^(1/3)*
sqrt(3)*a*b^2*x^6*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3
)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 9*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(4^(2/
3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3
+ a)^(2/3)) - 18*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3
) + 2*(b*x^3 + a)^(1/3)) - 84*sqrt(1/3)*a^(2/3)*b^2*x^6*arctan(sqrt(1/3)*(
2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) + 14*a^(2/3)*b^2*x^6*log((b*x^3 +
a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 28*a^(2/3)*b^2*x^6*log((
b*x^3 + a)^(1/3) - a^(1/3)) + 3*(8*a*b*x^3 + 3*a^2)*(b*x^3 + a)^(2/3))/(a^
3*d*x^6)]
```

3.592.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^7+bx^{10}}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x**7/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x**7 + b*x**10), x)/d
```

3.592.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^7}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^7), x)
```

3.592.8 Giac [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx &= -\frac{\sqrt{3}2^{\frac{2}{3}}b^2\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}d} \\ &+ \frac{2^{\frac{2}{3}}b^2\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,a^{\frac{7}{3}}d} \\ &- \frac{2^{\frac{2}{3}}b^2\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{7}{3}}d} + \frac{14\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{7}{3}}d} \\ &- \frac{7\,b^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27\,a^{\frac{7}{3}}d} \\ &+ \frac{14\,b^2\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{7}{3}}d} - \frac{8\,(bx^3+a)^{\frac{5}{3}}b^2-5\,(bx^3+a)^{\frac{2}{3}}ab^2}{18\,a^2b^2dx^6}\end{aligned}$$

input
```
integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*2^(2/3)*b^2*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(
b*x^3 + a)^(1/3))/a^(1/3))/(a^(7/3)*d) + 1/6*2^(2/3)*b^2*log(2^(2/3)*a^(2/
3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(a^(7/3)*d) -
1/3*2^(2/3)*b^2*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(a^(7/3)*d)
 + 14/27*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^
(1/3))/(a^(7/3)*d) - 7/27*b^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^
(1/3) + a^(2/3))/(a^(7/3)*d) + 14/27*b^2*log(abs((b*x^3 + a)^(1/3) - a^(1/
3)))/(a^(7/3)*d) - 1/18*(8*(b*x^3 + a)^(5/3)*b^2 - 5*(b*x^3 + a)^(2/3)*a*b
^2)/(a^2*b^2*d*x^6)
```

3.592.9 Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^3)^{2/3}}{x^7\,(ad-bdx^3)}\,dx = \frac{\frac{5\,b^2\,{(b\,x^3+a)}^{2/3}}{18\,a}-\frac{4\,b^2\,{(b\,x^3+a)}^{5/3}}{9\,a^2}}{d\,{(b\,x^3+a)}^2+a^2\,d-2\,a\,d\,(b\,x^3+a)}$$
$$+\ln\left(2\,b^4\,{(b\,x^3+a)}^{1/3}-2\,2^{1/3}\,a^5\,d^2\,{\left(-\frac{b^6}{a^7\,d^3}\right)}^{2/3}\right)\,{\left(-\frac{4\,b^6}{27\,a^7\,d^3}\right)}^{1/3}+\frac{14\,\ln\left(b^4\,{(b\,x^3+a)}^{1/3}-a^5\,d^2\,\left(\frac{b^6}{a^7\,d^3}\right)\right.}{27}$$

input
```
int((a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x)
```

output
```
((5*b^2*(a + b*x^3)^(2/3))/(18*a) - (4*b^2*(a + b*x^3)^(5/3))/(9*a^2))/(d*
(a + b*x^3)^2 + a^2*d - 2*a*d*(a + b*x^3)) + log(2*b^4*(a + b*x^3)^(1/3) -
 2*2^(1/3)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3))*(-(4*b^6)/(27*a^7*d^3))^(1/3) +
 (14*log(b^4*(a + b*x^3)^(1/3) - a^5*d^2*(b^6/(a^7*d^3))^(2/3))*(b^6/(a^7*
d^3))^(1/3))/27 - log(4*b^4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^5*d^2*(-b^6/(a
^7*d^3))^(2/3) - 2^(1/3)*3^(1/2)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3)*2i)*((3^(1
/2)*1i)/2 + 1/2)*(-(4*b^6)/(27*a^7*d^3))^(1/3) + log(4*b^4*(a + b*x^3)^(1/
3) + 2*2^(1/3)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3) + 2^(1/3)*3^(1/2)*a^5*d^2*(-
b^6/(a^7*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(-(4*b^6)/(27*a^7*d^3))^(1
/3) - (7*log(2*b^4*(a + b*x^3)^(1/3) + a^5*d^2*(b^6/(a^7*d^3))^(2/3) - 3^(
1/2)*a^5*d^2*(b^6/(a^7*d^3))^(2/3)*1i)*(3^(1/2)*1i + 1)*(b^6/(a^7*d^3))^(1
/3))/27 + (7*log(2*b^4*(a + b*x^3)^(1/3) + a^5*d^2*(b^6/(a^7*d^3))^(2/3) +
 3^(1/2)*a^5*d^2*(b^6/(a^7*d^3))^(2/3)*1i)*(3^(1/2)*1i - 1)*(b^6/(a^7*d^3)
)^(1/3))/27
```

3.593 $\int \frac{x^6 \left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

3.593.1 Optimal result

Integrand size = 28, antiderivative size = 264

$$\begin{aligned}\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx &= -\frac{4ax(a+bx^3)^{2/3}}{9b^2d}-\frac{x^4(a+bx^3)^{2/3}}{6bd}\\ &\quad -\frac{14a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}d}+\frac{2^{2/3}a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}d}\\ &\quad +\frac{a^2\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}b^{7/3}d}-\frac{a^2\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^{7/3}d}+\frac{7a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{9b^{7/3}d}\end{aligned}$$

output
```
-4/9*a*x*(b*x^3+a)^(2/3)/b^2/d-1/6*x^4*(b*x^3+a)^(2/3)/b/d+1/6*a^2*ln(-b*d
*x^3+a*d)*2^(2/3)/b^(7/3)/d-1/2*a^2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*
2^(2/3)/b^(7/3)/d+7/9*a^2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)/d-14/27*a
^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)/d*3^(1/2)+1
/3*2^(2/3)*a^2*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))
/b^(7/3)/d*3^(1/2)
```

3.593.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.23

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{24a\sqrt[3]{b}x(a+bx^3)^{2/3}+9b^{4/3}x^4(a+bx^3)^{2/3}+28\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-18\ 2^{2/3}\sqrt{3}a^2\arctan\Bigg(}{}$$

input

```
Integrate[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output

```
-1/54*(24*a*b^(1/3)*x*(a + b*x^3)^(2/3) + 9*b^(4/3)*x^4*(a + b*x^3)^(2/3)
+ 28*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/
3))] - 18*2^(2/3)*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2
/3)*(a + b*x^3)^(1/3))] - 28*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 1
8*2^(2/3)*a^2*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + 14*a^2*Log[b
^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 9*2^(2/3)*
a^2*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a +
 b*x^3)^(2/3)])/(b^(7/3)*d)
```

3.593.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {978, 27, 1052, 27, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(a+bx^3\right)^{2/3}}{ad-bdx^3}dx$$

$$\Big\downarrow \text{978}$$

$$\frac{\int \frac{4ax^3\left(2bx^3+a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{6bd}-\frac{x^4\left(a+bx^3\right)^{2/3}}{6bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\int \frac{x^3\left(2bx^3+a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{3bd}-\frac{x^4\left(a+bx^3\right)^{2/3}}{6bd}$$

$$\Big\downarrow \text{1052}$$

$$\frac{2a\left(\frac{\int \frac{ab\left(7bx^3+2a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{3b^2}-\frac{2x\left(a+bx^3\right)^{2/3}}{3b}\right)}{3bd}-\frac{x^4\left(a+bx^3\right)^{2/3}}{6bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\left(\frac{a\int \frac{7bx^3+2a}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{3b}-\frac{2x\left(a+bx^3\right)^{2/3}}{3b}\right)}{3bd}-\frac{x^4\left(a+bx^3\right)^{2/3}}{6bd}$$

$$\Big\downarrow \text{1026}$$

$$\frac{2a\left(\frac{a\left(9a\int \frac{1}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx-7\int \frac{1}{\sqrt[3]{bx^3+a}}dx\right)}{3b}-\frac{2x\left(a+bx^3\right)^{2/3}}{3b}\right)}{3bd}-\frac{x^4\left(a+bx^3\right)^{2/3}}{6bd}$$

$$\Big\downarrow \text{769}$$

$$\frac{2a\left(\frac{a\left(9a\int \frac{1}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx-7\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)\right)}{3b}-\frac{2x\left(a+bx^3\right)^{2/3}}{3b}\right)}{3bd}-\frac{x^4\left(a+bx^3\right)^{2/3}}{6bd}$$

$$\Big\downarrow \text{901}$$

3.593. $\int \frac{x^6\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

$$\frac{2a\left(\frac{a\left(9a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\sqrt[3]{b}}+\frac{\log\left(a-bx^3\right)}{6\sqrt[3]{2}a\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}a\sqrt[3]{b}}\right)-7\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\right.}{2\sqrt[3]{b}}\right.\right.}{3b}\right.}{3bd}$$

$$\frac{x^4\left(a+bx^3\right)^{2/3}}{6bd}$$

input
```
Int[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/6*(x^4*(a + b*x^3)^(2/3))/(b*d) + (2*a*((-2*x*(a + b*x^3)^(2/3))/(3*b)
+ (a*(9*a*(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(
2^(1/3)*Sqrt[3]*a*b^(1/3)) + Log[a - b*x^3]/(6*2^(1/3)*a*b^(1/3)) - Log[2^
(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(1/3)*a*b^(1/3))) - 7*(ArcTan[(1
 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(
1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3)))))/(3*b)))/(3*b*d)
```

3.593.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

3.593. $\int \frac{x^6\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

3.593.4 Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-9x^{4}\left(b\,x^{3}+a\right)^{\frac{2}{3}} b^{\frac{4}{3}}-18\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\,\left(2^{\frac{2}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}} x\right)}{3 b^{\frac{1}{3}} x}\right) a^{2}-18\,2^{\frac{2}{3}} \ln\left(\frac{-2^{\frac{1}{3}} b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) a^{2}+9\,2^{\frac{2}{3}} \ln\Bigg(2^{\frac{2}{3}} b^{\frac{2}{3}}$

input
```
int(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output

```
1/54*(-9*x^4*(b*x^3+a)^(2/3)*b^(4/3)-18*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)
*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)*x)/b^(1/3)/x)*a^2-18*2^(2/3)*ln((-2^(1/3
)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2+9*2^(2/3)*ln((2^(2/3)*b^(2/3)*x^2+2^(1
/3)*b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^2-24*a*x*(b*x^3+a)^(
2/3)*b^(1/3)+28*a^2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3
))/b^(1/3)/x)+28*a^2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-14*a^2*ln((b^(2/3)
*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))/d/b^(7/3)
```

3.593.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.66

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \left[-\frac{18\cdot 4^{\frac{1}{3}}\sqrt{3}a^2b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3\,x}\right) - 42\sqrt{\frac{1}{3}}a^2b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\Bigg(}{} \right.$$

$$-\frac{18\cdot 4^{\frac{1}{3}}\sqrt{3}a^2b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3\,x}\right) - 18\cdot 4^{\frac{1}{3}}a^2b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}bx\left(-\frac{1}{b}\right)^{\frac{2}{3}}-2\,(bx^3+a)^{\frac{1}{3}}}{x}\right) +}{}$$

input

```
integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
[-1/54*(18*4^(1/3)*sqrt(3)*a^2*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(
1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 42*sqrt(1/3)*a^2*b*sqrt(
-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b
^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt
(-1/b^(2/3)) + 2*a) - 18*4^(1/3)*a^2*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/
b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 9*4^(1/3)*a^2*b*(-1/b)^(1/3)*log(-(2*
4^(1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) -
2*(b*x^3 + a)^(2/3))/x^2) - 28*a^2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(
1/3))/x) + 14*a^2*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x +
 (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 8*a*b*x)*(b*x^3 + a)^(2/3))/(b^3
*d), -1/54*(18*4^(1/3)*sqrt(3)*a^2*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x -
 4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 18*4^(1/3)*a^2*b*(-1
/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 9*4^(
1/3)*a^2*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^
3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) - 84*sqrt(1/3)*a
^2*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))
 - 28*a^2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + 14*a^2*b^(2/3)
*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2)
+ 3*(3*b^2*x^4 + 8*a*b*x)*(b*x^3 + a)^(2/3))/(b^3*d)]
```

3.593.6 Sympy [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^6(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**6*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**6*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.593.7 Maxima [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x^6}{bdx^3-ad}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)
```

3.593.8 Giac [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{2}{3}}x^6}{bdx^3-ad}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)
```

3.593.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int \frac{x^6\,(b\,x^3+a)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

3.594 $\int \frac{x^3 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$

3.594.1 Optimal result

Integrand size = 28, antiderivative size = 229

$$\int \frac{x^3 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx = -\frac{x \left(a+b x^3\right)^{2/3}}{3 b d}-\frac{5 a \arctan \left(\frac{1+\frac{2 \sqrt[3]{b} x}{\sqrt[3]{a+b x^3}}}{\sqrt{3}}\right)}{3 \sqrt{3} b^{4/3} d}+\frac{2^{2/3} a \arctan \left(\frac{1+\frac{2 \sqrt[3]{2} \sqrt[3]{b} x}{\sqrt[3]{a+b x^3}}}{\sqrt{3}}\right)}{\sqrt{3} b^{4/3} d}+\frac{a \log \left(a d-b d x^3\right)}{3 \sqrt[3]{2} b^{4/3} d}-\frac{a \log \left(\sqrt[3]{2} \sqrt[3]{b} x-\sqrt[3]{a+b x^3}\right)}{\sqrt[3]{2} b^{4/3} d}+\frac{5 a \log \left(-\sqrt[3]{b} x+\sqrt[3]{a+b x^3}\right)}{6 b^{4/3} d}$$

output

```
-1/3*x*(b*x^3+a)^(2/3)/b/d+1/6*a*ln(-b*d*x^3+a*d)*2^(2/3)/b^(4/3)/d-1/2*a*
ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/b^(4/3)/d+5/6*a*ln(-b^(1/3)*
x+(b*x^3+a)^(1/3))/b^(4/3)/d-5/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/
3))*3^(1/2))/b^(4/3)/d*3^(1/2)+1/3*2^(2/3)*a*arctan(1/3*(1+2*2^(1/3)*b^(1/
3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d*3^(1/2)
```

3.594.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.27

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{6\sqrt[3]{b}x(a+bx^3)^{2/3}+10\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-6\;2^{2/3}\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-10a\,\mathrm{l}}{}$$

input
```
Integrate[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/18*(6*b^(1/3)*x*(a + b*x^3)^(2/3) + 10*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(1/3
)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 6*2^(2/3)*Sqrt[3]*a*ArcTan[(Sqrt
[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] - 10*a*Log[-(b^(1/
3)*x) + (a + b*x^3)^(1/3)] + 6*2^(2/3)*a*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b
*x^3)^(1/3)] + 5*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*
x^3)^(2/3)] - 3*2^(2/3)*a*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3
)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^(4/3)*d)
```

3.594.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {978, 27, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+bx^3\right)^{2/3}}{ad-bdx^3}dx$$

$$\Big\downarrow \; 978$$

$$\frac{\int \frac{a\left(5bx^3+a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{3bd}-\frac{x\left(a+bx^3\right)^{2/3}}{3bd}$$

$$\Big\downarrow \; 27$$

$$\frac{a\int \frac{5bx^3+a}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{3bd}-\frac{x\left(a+bx^3\right)^{2/3}}{3bd}$$

$$\Big\downarrow \text{1026}$$

$$\frac{a\left(6a\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx-5\int \frac{1}{\sqrt[3]{bx^3+a}}dx\right)}{3bd}-\frac{x\left(a+bx^3\right)^{2/3}}{3bd}$$

$$\Big\downarrow \text{769}$$

$$\frac{a\left(6a\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx-5\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)\right)}{3bd}-\frac{x\left(a+bx^3\right)^{2/3}}{3bd}$$

$$\Big\downarrow \text{901}$$

$$\frac{a\left(6a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\sqrt[3]{b}}+\frac{\log(a-bx^3)}{6\sqrt[3]{2}a\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}a\sqrt[3]{b}}\right)-5\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+\right.}}{2}\right.}{3bd}$$

$$\frac{x\left(a+bx^3\right)^{2/3}}{3bd}$$

input
```
Int[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/3*(x*(a + b*x^3)^(2/3))/(b*d) + (a*(6*a*(ArcTan[(1 + (2*2^(1/3)*b^(1/3)
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]*a*b^(1/3)) + Log[a - b*x^
3]/(6*2^(1/3)*a*b^(1/3)) - Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2
^(1/3)*a*b^(1/3))) - 5*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[
3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3)))
))/(3*b*d)
```

3.594. $\int \frac{x^3\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

3.594.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.594.4 Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-6\,2^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)a-6\,2^{\frac{2}{3}}\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a+3\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^{2}+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+}{x^{2}}\right.$

input
```
int(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/18*(-6*2^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/
3)*x)/b^(1/3)/x)*a-6*2^(2/3)*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a+
3*2^(2/3)*ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3
+a)^(2/3))/x^2)*a-6*(b*x^3+a)^(2/3)*x*b^(1/3)+10*3^(1/2)*arctan(1/3*3^(1/2
)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a+10*ln((-b^(1/3)*x+(b*x^3+a)^(
1/3))/x)*a-5*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^
2)*a)/d/b^(4/3)
```

3.594.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 653, normalized size of antiderivative = 2.85

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \left[-\frac{6\cdot 4^{\frac{1}{3}}\sqrt{3}ab\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3\,x}\right)-15\,\sqrt{\frac{1}{3}}ab\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx\right.}{}\right.$$

$$-\frac{6\cdot 4^{\frac{1}{3}}\sqrt{3}ab\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3\,x}\right)-6\cdot 4^{\frac{1}{3}}ab\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}bx\left(-\frac{1}{b}\right)^{\frac{2}{3}}-2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)+3\cdot}{}$$

input
```
integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
[-1/18*(6*4^(1/3)*sqrt(3)*a*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3
)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 15*sqrt(1/3)*a*b*sqrt(-1/b^
(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3
)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b
^(2/3)) + 2*a) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3)
 - 2*(b*x^3 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*
x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 +
 a)^(2/3))/x^2) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*log(-(b^(1/3)*x -
 (b*x^3 + a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*
b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d), -1/18*(6*4^(1/3)*sqrt(3)*a*b
*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(
-1/b)^(1/3))/x) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3
) - 2*(b*x^3 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b
*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3
+ a)^(2/3))/x^2) - 30*sqrt(1/3)*a*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*
(b*x^3 + a)^(1/3))/(b^(1/3)*x)) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*l
og(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b
*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d)]
```

3.594.6 Sympy [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^3\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**3*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**3*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.594.7 Maxima [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}x^3}{bdx^3-ad}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)
```

3.594.8 Giac [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x^3}{bdx^3-ad}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)
```

3.594.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int \frac{x^3\,(b\,x^3+a)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

3.595 $\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

3.595.1 Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d}+\frac{2^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d} + \frac{\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\sqrt[3]{b}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{b}d}+\frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}d}$$

output
```
1/6*ln(-b*d*x^3+a*d)*2^(2/3)/b^(1/3)/d-1/2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^
(1/3))*2^(2/3)/b^(1/3)/d+1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)/d-1/3*
arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)/d*3^(1/2)+1/3*
2^(2/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3
)/d*3^(1/2)
```

3.595.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.32

$$\int \frac{\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)+}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(a*d - b*d*x^3),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3
))] - 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a
 + b*x^3)^(1/3))] - 2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 2*2^(2/3)*Lo
g[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + Log[b^(2/3)*x^2 + b^(1/3)*x*
(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 2^(2/3)*Log[2*b^(2/3)*x^2 + 2^(2/
3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^(1/3)*d)
```

3.595.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {916, 27, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{ad-bdx^3}dx$$

$$\Big\downarrow \ 916$$

$$2a\int \frac{1}{d\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx-\frac{\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{2a\int \frac{1}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{d}-\frac{\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}$$

$$\Big\downarrow \ 769$$

$$\frac{2a\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{d}-\frac{\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}}{d}$$

$$\Big\downarrow \; 901$$

$$\frac{2a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\sqrt[3]{b}}+\frac{\log(a-bx^3)}{6\sqrt[3]{2}a\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}a\sqrt[3]{b}}\right)}{d}-\frac{\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}}{d}$$

input
```
Int[(a + b*x^3)^(2/3)/(a*d - b*d*x^3),x]
```

output
```
(2*a*(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(1/
3)*Sqrt[3]*a*b^(1/3)) + Log[a - b*x^3]/(6*2^(1/3)*a*b^(1/3)) - Log[2^(1/3)
*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(1/3)*a*b^(1/3))))/d - (ArcTan[(1 + (
2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)
*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3)))/d
```

3.595.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

3.595. $\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 916
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[b/d   Int[(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*
x^n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c -
a*d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

3.595.4 Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{-2\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}} x\right)}{3 b^{\frac{1}{3}} x}\right)+2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}} b^{\frac{2}{3}} x^{2}+2^{\frac{1}{3}} b^{\frac{1}{3}}\left(b x^{3}+a\right)^{\frac{1}{3}} x+\left(b x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\,2^{\frac{2}{3}}\ln\left(\frac{-2^{\frac{1}{3}} b^{\frac{1}{3}} x+(b x^{3}+}{x}\right.}{6 d b^{\frac{1}{3}}}$

input
```
int((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3
)*x)/b^(1/3)/x)+2^(2/3)*ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^
(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*2^(2/3)*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(
1/3))/x)+2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3
)/x)+2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a
)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))/d/b^(1/3)
```

3.595.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 611, normalized size of antiderivative = 3.06

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \Bigg[-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3\,x}\right)-3\sqrt{\frac{1}{3}}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\right.}{}$$

$$-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3\,x}\right)-2\cdot 4^{\frac{1}{3}}b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}bx\left(-\frac{1}{b}\right)^{\frac{2}{3}}-2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)+4^{\frac{1}{3}}b(-}{}$$

input

```
integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output

```
[-1/6*(2*4^(1/3)*sqrt(3)*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*s
qrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 3*sqrt(1/3)*b*sqrt(-1/b^(2/3))
*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3
+ (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)
) + 2*a) - 2*4^(1/3)*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*
x^3 + a)^(1/3))/x) + 4^(1/3)*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(
1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x
^2) - 2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + b^(2/3)*log((b^(
2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d), -
1/6*(2*4^(1/3)*sqrt(3)*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqr
t(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 2*4^(1/3)*b*(-1/b)^(1/3)*log(-(4
^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 4^(1/3)*b*(-1/b)^(1/3)
*log(-(2*4^(1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)
^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) - 6*sqrt(1/3)*b^(2/3)*arctan(sqrt(1/3)*
(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x)) - 2*b^(2/3)*log(-(b^(1/3)*x
 - (b*x^3 + a)^(1/3))/x) + b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^
(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d)]
```

3.595.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.595.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{bdx^3-ad}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)
```

3.595.8 Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{bdx^3-ad}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)
```

3.595.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

input

```
int((a + b*x^3)^(2/3)/(a*d - b*d*x^3),x)
```

output

```
int((a + b*x^3)^(2/3)/(a*d - b*d*x^3), x)
```

3.596 $\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx$

3.596.1 Optimal result

Integrand size = 28, antiderivative size = 157

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{2adx^2} + \frac{2^{2/3}b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}ad} + \frac{b^{2/3}\log(ad-bdx^3)}{3\sqrt[3]{2}ad} - \frac{b^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}ad}$$

output
```
-1/2*(b*x^3+a)^(2/3)/a/d/x^2+1/6*b^(2/3)*ln(-b*d*x^3+a*d)*2^(2/3)/a/d-1/2*
b^(2/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/a/d+1/3*2^(2/3)*b^(2
/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a/d*3^(1/2
)
```

3.596.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = \frac{-3(a+bx^3)^{2/3} + 2\,2^{2/3}\sqrt{3}b^{2/3}x^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right) - 2\,2^{2/3}b^{2/3}x^2\log\left(-\right.}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x]
```

output
```
(-3*(a + b*x^3)^(2/3) + 2*2^(2/3)*Sqrt[3]*b^(2/3)*x^2*ArcTan[(Sqrt[3]*b^(1
/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] - 2*2^(2/3)*b^(2/3)*x^2*Lo
g[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + 2^(2/3)*b^(2/3)*x^2*Log[2*b^
(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3
)])/(6*a*d*x^2)
```

3.596.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {975, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^3\left(ad-bdx^3\right)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{4ab}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{2ad}-\frac{\left(a+bx^3\right)^{2/3}}{2adx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{1}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{d}-\frac{\left(a+bx^3\right)^{2/3}}{2adx^2}$$

$$\Big\downarrow \text{901}$$

$$\frac{2b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\sqrt[3]{b}}+\frac{\log\left(a-bx^3\right)}{6\sqrt[3]{2}a\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}a\sqrt[3]{b}}\right)}{d}-\frac{\left(a+bx^3\right)^{2/3}}{2adx^2}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x]
```

---

output
```
-1/2*(a + b*x^3)^(2/3)/(a*d*x^2) + (2*b*(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)
/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]*a*b^(1/3)) + Log[a - b*x^3]/
(6*2^(1/3)*a*b^(1/3)) - Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(1
/3)*a*b^(1/3))))/d
```

3.596.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

3.596.4 Maple [A] (verified)

Time = 4.75 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{-2b^{\frac{2}{3}}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)x^{2}+b^{\frac{2}{3}}2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^{2}+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)x^{2}-2b^{\frac{2}{3}}2^{\frac{2}{3}}\ln\Big(-2}{6ad\,x^{2}}$

input
```
int((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*b^(2/3)*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3
)+b^(1/3)*x)/b^(1/3)/x)*x^2+b^(2/3)*2^(2/3)*ln((2^(2/3)*b^(2/3)*x^2+2^(1/3
)*b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*x^2-2*b^(2/3)*2^(2/3)*ln
((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)*x^2-3*(b*x^3+a)^(2/3))/a/d/x^2
```

3.596.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(125) = 250$.

Time = 81.74 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.76

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^3\left(ad-bdx^3\right)}\,dx =$$

$$-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}(-b^2)^{\frac{1}{3}}\,x^2\arctan\left(\frac{3\cdot 4^{\frac{2}{3}}\sqrt{3}(5\,b^2x^7-4\,abx^4-a^2x)\,(bx^3+a)^{\frac{2}{3}}\,(-b^2)^{\frac{2}{3}}+6\cdot 4^{\frac{1}{3}}\sqrt{3}(19\,b^3x^8+16\,ab^2x^5+a^2bx^2)\,(bx^3+a)^{\frac{1}{3}}\,(-b^2)^{\frac{1}{3}}-}{3\,(109\,b^4x^9+105\,ab^3x^6+3\,a^2b^2x^3-a^3b)}\right.}{}$$

input
```
integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
-1/18*(2*4^(1/3)*sqrt(3)*(-b^2)^(1/3)*x^2*arctan(1/3*(3*4^(2/3)*sqrt(3)*(5
*b^2*x^7 - 4*a*b*x^4 - a^2*x)*(b*x^3 + a)^(2/3)*(-b^2)^(2/3) + 6*4^(1/3)*s
qrt(3)*(19*b^3*x^8 + 16*a*b^2*x^5 + a^2*b*x^2)*(b*x^3 + a)^(1/3)*(-b^2)^(1
/3) - sqrt(3)*(71*b^4*x^9 + 111*a*b^3*x^6 + 33*a^2*b^2*x^3 + a^3*b))/(109*
b^4*x^9 + 105*a*b^3*x^6 + 3*a^2*b^2*x^3 - a^3*b)) - 2*4^(1/3)*(-b^2)^(1/3)
*x^2*log((3*4^(2/3)*(b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x^2 - 6*(b*x^3 + a)^(2/
3)*b*x + 4^(1/3)*(b*x^3 - a)*(-b^2)^(1/3))/(b*x^3 - a)) + 4^(1/3)*(-b^2)^(
1/3)*x^2*log(-(6*4^(1/3)*(5*b^2*x^4 + a*b*x)*(b*x^3 + a)^(2/3)*(-b^2)^(1/3
) - 4^(2/3)*(19*b^2*x^6 + 16*a*b*x^3 + a^2)*(-b^2)^(2/3) - 24*(2*b^3*x^5 +
 a*b^2*x^2)*(b*x^3 + a)^(1/3))/(b^2*x^6 - 2*a*b*x^3 + a^2)) + 9*(b*x^3 + a
)^(2/3))/(a*d*x^2)
```

3.596.6 Sympy [F]

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^3\left(ad-bdx^3\right)}\,dx = -\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^3+bx^6}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x**3/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x**3 + b*x**6), x)/d
```

3.596.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)
```

3.596.8 Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)
```

3.596.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^3\,(a\,d-b\,d\,x^3)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)), x)
```

3.597 $\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx$

3.597.1 Optimal result

Integrand size = 28, antiderivative size = 182

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{5adx^5} - \frac{7b(a+bx^3)^{2/3}}{10a^2dx^2} + \frac{2^{2/3}b^{5/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^2d} + \frac{b^{5/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^2d} - \frac{b^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^2d}$$

output
```
-1/5*(b*x^3+a)^(2/3)/a/d/x^5-7/10*b*(b*x^3+a)^(2/3)/a^2/d/x^2+1/6*b^(5/3)*
ln(-b*d*x^3+a*d)*2^(2/3)/a^2/d-1/2*b^(5/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^
(1/3))*2^(2/3)/a^2/d+1/3*2^(2/3)*b^(5/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x
/(b*x^3+a)^(1/3))*3^(1/2))/a^2/d*3^(1/2)
```

3.597.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}(2a+7bx^3)}{10a^2dx^5} + \frac{2^{2/3}b^{5/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}a^2d} - \frac{2^{2/3}b^{5/3}\log\left(-2\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}\right)}{3a^2d} + \frac{b^{5/3}\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}(a+bx^3)^{2/3}\right)}{3\sqrt[3]{2}a^2d}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x]
```

output
```
-1/10*((a + b*x^3)^(2/3)*(2*a + 7*b*x^3))/(a^2*d*x^5) + (2^(2/3)*b^(5/3)*A
rcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[
3]*a^2*d) - (2^(2/3)*b^(5/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)]
)/(3*a^2*d) + (b^(5/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(
1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(1/3)*a^2*d)
```

3.597.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {975, 27, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{b(3bx^3+7a)}{x^3(a-bx^3)\sqrt[3]{bx^3+a}}dx}{5ad} - \frac{(a+bx^3)^{2/3}}{5adx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{3bx^3+7a}{x^3(a-bx^3)\sqrt[3]{bx^3+a}}dx}{5ad} - \frac{(a+bx^3)^{2/3}}{5adx^5}$$

$$\Big\downarrow \color{blue}{1053}$$

$$\frac{b\left(-\frac{\int -\frac{20a^2b}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{2a^2}-\frac{7(a+bx^3)^{2/3}}{2ax^2}\right)}{5ad}-\frac{(a+bx^3)^{2/3}}{5adx^5}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{b\left(10b\int \frac{1}{(a-bx^3)\sqrt[3]{bx^3+a}}dx-\frac{7(a+bx^3)^{2/3}}{2ax^2}\right)}{5ad}-\frac{(a+bx^3)^{2/3}}{5adx^5}$$

$$\Big\downarrow \color{blue}{901}$$

$$\frac{b\left(10b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\sqrt[3]{b}}+\frac{\log(a-bx^3)}{6\sqrt[3]{2}a\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}a\sqrt[3]{b}}\right)-\frac{7(a+bx^3)^{2/3}}{2ax^2}\right)}{5ad}-\frac{(a+bx^3)^{2/3}}{5adx^5}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x]
```

output
```
-1/5*(a + b*x^3)^(2/3)/(a*d*x^5) + (b*((-7*(a + b*x^3)^(2/3))/(2*a*x^2) +
10*b*(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(1/
3)*Sqrt[3]*a*b^(1/3)) + Log[a - b*x^3]/(6*2^(1/3)*a*b^(1/3)) - Log[2^(1/3)
*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(1/3)*a*b^(1/3)))))/(5*a*d)
```

3.597. $\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx$

3.597.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.597.4 Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{5x^{5}2^{\frac{2}{3}}\left(-2\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^{2}+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+(b\,x^{3}+a}{x}\right.\right.}{30x^{5}a^{2}d}$

input
```
int((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/30*(5*x^5*2^(2/3)*(-2*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3
)*x)/b^(1/3)/x)*3^(1/2)+ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^
(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)
)*b^(5/3)-3*(b*x^3+a)^(2/3)*(7*b*x^3+2*a))/x^5/a^2/d
```

3.597.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.597.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^6+bx^9}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x**6/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x**6 + b*x**9), x)/d
```

3.597.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)
```

3.597.8 Giac [F]

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(ad-bdx^3\right)}\,dx=\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)
```

3.597.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(ad-bdx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^6\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)), x)
```

3.598 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(ad-bdx^3\right)}\,dx$

3.598.1 Optimal result

Integrand size = 28, antiderivative size = 209

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(ad-bdx^3\right)}\,dx=-\frac{\left(a+bx^3\right)^{2/3}}{8adx^8}-\frac{b\left(a+bx^3\right)^{2/3}}{4a^2dx^5}-\frac{5b^2\left(a+bx^3\right)^{2/3}}{8a^3dx^2}+\frac{2^{2/3}b^{8/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^3d}+\frac{b^{8/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}a^3d}-\frac{b^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^3d}$$

output
```
-1/8*(b*x^3+a)^(2/3)/a/d/x^8-1/4*b*(b*x^3+a)^(2/3)/a^2/d/x^5-5/8*b^2*(b*x^
3+a)^(2/3)/a^3/d/x^2+1/6*b^(8/3)*ln(-b*d*x^3+a*d)*2^(2/3)/a^3/d-1/2*b^(8/3
)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/a^3/d+1/3*2^(2/3)*b^(8/3)*
arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a^3/d*3^(1/2)
```

3.598.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(ad-bdx^3\right)}\,dx=\frac{-\frac{3\left(a+bx^3\right)^{2/3}\left(a^2+2abx^3+5b^2x^6\right)}{x^8}+8\ 2^{2/3}\sqrt{3}b^{8/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-8\ 2^{2/3}b^{8/3}}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x]
```

output
```
((-3*(a + b*x^3)^(2/3)*(a^2 + 2*a*b*x^3 + 5*b^2*x^6))/x^8 + 8*2^(2/3)*Sqrt
[3]*b^(8/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1
/3))] - 8*2^(2/3)*b^(8/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] +
4*2^(2/3)*b^(8/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3)
+ 2^(1/3)*(a + b*x^3)^(2/3)])/(24*a^3*d)
```

3.598.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {975, 27, 1053, 27, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^9 \left(ad-bdx^3\right)}dx$$

$$\Big\downarrow \textbf{975}$$

$$\frac{\int \frac{2b(3bx^3+5a)}{x^6(a-bx^3)\sqrt[3]{bx^3+a}}dx}{8ad}-\frac{\left(a+bx^3\right)^{2/3}}{8adx^8}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{b\int \frac{3bx^3+5a}{x^6(a-bx^3)\sqrt[3]{bx^3+a}}dx}{4ad}-\frac{\left(a+bx^3\right)^{2/3}}{8adx^8}$$

$$\Big\downarrow \textbf{1053}$$

$$\frac{b\left(-\frac{\int -\frac{5ab\left(3bx^3+5a\right)}{x^3\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{5a^2}-\frac{\left(a+bx^3\right)^{2/3}}{ax^5}\right)}{4ad}-\frac{\left(a+bx^3\right)^{2/3}}{8adx^8}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{b\left(\frac{b\int \frac{3bx^3+5a}{x^3\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{a}-\frac{\left(a+bx^3\right)^{2/3}}{ax^5}\right)}{4ad}-\frac{\left(a+bx^3\right)^{2/3}}{8adx^8}$$

$$\Big\downarrow \text{1053}$$

$$\frac{b\left(\frac{b\left(-\frac{\int -\frac{16a^2b}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{2a^2}-\frac{5\left(a+bx^3\right)^{2/3}}{2ax^2}\right)}{a}-\frac{\left(a+bx^3\right)^{2/3}}{ax^5}\right)}{4ad}-\frac{\left(a+bx^3\right)^{2/3}}{8adx^8}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{b\left(8b\int \frac{1}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx-\frac{5\left(a+bx^3\right)^{2/3}}{2ax^2}\right)}{a}-\frac{\left(a+bx^3\right)^{2/3}}{ax^5}\right)}{4ad}-\frac{\left(a+bx^3\right)^{2/3}}{8adx^8}$$

$$\Big\downarrow \text{901}$$

$$\frac{b\left(\frac{b\left(8b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\sqrt[3]{b}}+\frac{\log\left(a-bx^3\right)}{6\sqrt[3]{2}a\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}a\sqrt[3]{b}}\right)-\frac{5\left(a+bx^3\right)^{2/3}}{2ax^2}\right)}{a}-\frac{\left(a+bx^3\right)^{2/3}}{ax^5}\right)}{4ad}-\frac{\left(a+bx^3\right)^{2/3}}{8adx^8}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x]
```

3.598. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(ad-bdx^3\right)}\,dx$

output

```
-1/8*(a + b*x^3)^(2/3)/(a*d*x^8) + (b*(-((a + b*x^3)^(2/3)/(a*x^5)) + (b*(
(-5*(a + b*x^3)^(2/3))/(2*a*x^2) + 8*b*(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/
(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]*a*b^(1/3)) + Log[a - b*x^3]/(
6*2^(1/3)*a*b^(1/3)) - Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(1/
3)*a*b^(1/3)))))/a))/(4*a*d)
```

3.598.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 975

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.598.4 Maple [A] (verified)

Time = 4.83 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{4x^{8}2^{\frac{2}{3}}\left(-2\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^{2}+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right.}{x}\right.\right.}{24x^{8}a^{3}d}$

input
```
int((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/24*(4*x^8*2^(2/3)*(-2*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3
)*x)/b^(1/3)/x)*3^(1/2)+ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^
(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)
)*b^(8/3)-3*(b*x^3+a)^(2/3)*(5*b^2*x^6+2*a*b*x^3+a^2))/x^8/a^3/d
```

3.598.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(ad-bdx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.598.6 Sympy [F]

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(ad-bdx^3\right)}\,dx=-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^9+bx^{12}}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x**9/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x**9 + b*x**12), x)/d
```

3.598.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^9}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)
```

3.598.8 Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^9}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)
```

3.598.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^9\,(a\,d-b\,d\,x^3)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)), x)
```

3.599 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)} \, dx$

3.599.1 Optimal result

Integrand size = 28, antiderivative size = 236

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)} \, dx = &-\frac{\left(a+bx^3\right)^{2/3}}{11adx^{11}}-\frac{13b\left(a+bx^3\right)^{2/3}}{88a^2dx^8}-\frac{49b^2\left(a+bx^3\right)^{2/3}}{220a^3dx^5}\\ &-\frac{293b^3\left(a+bx^3\right)^{2/3}}{440a^4dx^2}+\frac{2^{2/3}b^{11/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^4d}\\ &+\frac{b^{11/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}a^4d}-\frac{b^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^4d}\end{aligned}$$

output
```
-1/11*(b*x^3+a)^(2/3)/a/d/x^11-13/88*b*(b*x^3+a)^(2/3)/a^2/d/x^8-49/220*b^
2*(b*x^3+a)^(2/3)/a^3/d/x^5-293/440*b^3*(b*x^3+a)^(2/3)/a^4/d/x^2+1/6*b^(1
1/3)*ln(-b*d*x^3+a*d)*2^(2/3)/a^4/d-1/2*b^(11/3)*ln(2^(1/3)*b^(1/3)*x-(b*x
^3+a)^(1/3))*2^(2/3)/a^4/d+1/3*2^(2/3)*b^(11/3)*arctan(1/3*(1+2*2^(1/3)*b^
(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a^4/d*3^(1/2)
```

3.599.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)}\,dx = \frac{-\frac{3\left(a+bx^3\right)^{2/3}\left(40a^3+65a^2bx^3+98ab^2x^6+293b^3x^9\right)}{x^{11}}+440\ 2^{2/3}\sqrt{3}b^{11/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+b}}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x]
```

output
```
((-3*(a + b*x^3)^(2/3)*(40*a^3 + 65*a^2*b*x^3 + 98*a*b^2*x^6 + 293*b^3*x^9
))/x^11 + 440*2^(2/3)*Sqrt[3]*b^(11/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)
*x + 2^(2/3)*(a + b*x^3)^(1/3))] - 440*2^(2/3)*b^(11/3)*Log[-2*b^(1/3)*x +
 2^(2/3)*(a + b*x^3)^(1/3)] + 220*2^(2/3)*b^(11/3)*Log[2*b^(2/3)*x^2 + 2^(
2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(1320*a^4*d
)
```

3.599.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {975, 27, 1053, 27, 1053, 25, 27, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)}dx$$

$$\Big\downarrow \ 975$$

$$\frac{\int \frac{b\left(9bx^3+13a\right)}{x^9\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{11ad}-\frac{\left(a+bx^3\right)^{2/3}}{11adx^{11}}$$

$$\Big\downarrow \ 27$$

$$\frac{b\int \frac{9bx^3+13a}{x^9\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{11ad}-\frac{\left(a+bx^3\right)^{2/3}}{11adx^{11}}$$

$$\Big\downarrow \ 1053$$

$$\frac{b\left(-\frac{\int -\frac{2ab\left(39bx^3+49a\right)}{x^6\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{8a^2}-\frac{13\left(a+bx^3\right)^{2/3}}{8ax^8}\right)}{11ad}-\frac{\left(a+bx^3\right)^{2/3}}{11adx^{11}}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{b\int \frac{39bx^3+49a}{x^6\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{4a}-\frac{13\left(a+bx^3\right)^{2/3}}{8ax^8}\right)}{11ad}-\frac{\left(a+bx^3\right)^{2/3}}{11adx^{11}}$$

$$\Big\downarrow \; 1053$$

$$\frac{b\left(\frac{b\left(-\frac{\int -\frac{ab\left(147bx^3+293a\right)}{x^3\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{5a^2}-\frac{49\left(a+bx^3\right)^{2/3}}{5ax^5}\right)}{4a}-\frac{13\left(a+bx^3\right)^{2/3}}{8ax^8}\right)}{11ad}-\frac{\left(a+bx^3\right)^{2/3}}{11adx^{11}}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{b\left(\frac{\int \frac{ab\left(147bx^3+293a\right)}{x^3\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{5a^2}-\frac{49\left(a+bx^3\right)^{2/3}}{5ax^5}\right)}{4a}-\frac{13\left(a+bx^3\right)^{2/3}}{8ax^8}\right)}{11ad}-\frac{\left(a+bx^3\right)^{2/3}}{11adx^{11}}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{b\left(\frac{b\int \frac{147bx^3+293a}{x^3\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{5a}-\frac{49\left(a+bx^3\right)^{2/3}}{5ax^5}\right)}{4a}-\frac{13\left(a+bx^3\right)^{2/3}}{8ax^8}\right)}{11ad}-\frac{\left(a+bx^3\right)^{2/3}}{11adx^{11}}$$

$$\Big\downarrow \; 1053$$

3.599. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)}\,dx$

$$\frac{b\left(\frac{b\left(\frac{b\left(-\frac{\int -\frac{880a^2b}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{2a^2}-\frac{293\left(a+bx^3\right)^{2/3}}{2ax^2}\right)}{5a}-\frac{49\left(a+bx^3\right)^{2/3}}{5ax^5}\right)}{4a}-\frac{13\left(a+bx^3\right)^{2/3}}{8ax^8}\right)}{11ad}-\frac{\left(a+bx^3\right)^{2/3}}{11adx^{11}}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{b\left(\frac{b\left(\frac{b\left(440b\int \frac{1}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx-\frac{293\left(a+bx^3\right)^{2/3}}{2ax^2}\right)}{5a}-\frac{49\left(a+bx^3\right)^{2/3}}{5ax^5}\right)}{4a}-\frac{13\left(a+bx^3\right)^{2/3}}{8ax^8}\right)}{11ad}-\frac{\left(a+bx^3\right)^{2/3}}{11adx^{11}}$$

$$\Big\downarrow \color{blue}{901}$$

3.599. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)}\,dx$

$$\frac{b\left(\frac{b\left(\frac{b\left(440b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\sqrt[3]{b}}+\frac{\log\left(a-bx^3\right)}{6\sqrt[3]{2}a\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{2}a\sqrt[3]{b}}\right)-\frac{293\left(a+bx^3\right)^{2/3}}{2ax^2}\right)}{5a}-\frac{49\left(a+bx^3\right)^{2/3}}{5ax^5}\right)}{4a}-\frac{13\left(a+bx^3\right)^{2/3}}{8ax^8}\right)}{11ad}$$

$$\frac{\left(a+bx^3\right)^{2/3}}{11adx^{11}}$$

input

```
Int[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x]
```

output

```
-1/11*(a + b*x^3)^(2/3)/(a*d*x^11) + (b*((-13*(a + b*x^3)^(2/3))/(8*a*x^8)
 + (b*((-49*(a + b*x^3)^(2/3))/(5*a*x^5) + (b*((-293*(a + b*x^3)^(2/3))/(2
*a*x^2) + 440*b*(ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt
[3]]/(2^(1/3)*Sqrt[3]*a*b^(1/3)) + Log[a - b*x^3]/(6*2^(1/3)*a*b^(1/3)) -
Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*2^(1/3)*a*b^(1/3)))))/(5*a))
)/(4*a)))/(11*a*d)
```

3.599. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)}\,dx$

3.599.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.599.4 Maple [A] (verified)

Time = 5.07 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{-220x^{11}2^{\frac{2}{3}}\left(2\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)\sqrt{3}+2\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^{2}+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(b x^{3}+a\right)^{\frac{1}{3}}x+\left(b x\right.}{x^{2}}\right.\right.}{1320x^{11}a^{4}d}$

3.599. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)}\,dx$

input
```
int((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/1320*(-220*x^11*2^(2/3)*(2*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b
^(1/3)*x)/b^(1/3)/x)*3^(1/2)+2*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)-
ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))
/x^2))*b^(11/3)-3*(b*x^3+a)^(2/3)*(293*b^3*x^9+98*a*b^2*x^6+65*a^2*b*x^3+4
0*a^3))/x^11/a^4/d
```

3.599.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.599.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^{12}+bx^{15}}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x**12/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x**12 + b*x**15), x)/d
```

3.599.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^{12}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)
```

3.599.8 Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^{12}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)
```

3.599.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^{12}\,(a\,d-b\,d\,x^3)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)), x)
```

3.600 $\int \frac{x^7 \left(a+bx^3\right)^{2/3}}{ad-bdx^3} \, dx$

3.600.1 Optimal result

Integrand size = 28, antiderivative size = 512

$$
\begin{aligned}
\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = &-\frac{9ax^2(a+bx^3)^{2/3}}{28b^2d}-\frac{x^5(a+bx^3)^{2/3}}{7bd} \\
&+\frac{2^{2/3}a^{7/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{8/3}d}+\frac{a^{7/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{8/3}d} \\
&-\frac{19a^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{28b^2d\sqrt[3]{a+bx^3}} \\
&+\frac{a^{7/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{8/3}d} \\
&+\frac{a^{7/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{8/3}d} \\
&-\frac{2^{2/3}a^{7/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{8/3}d} \\
&-\frac{a^{7/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{8/3}d}
\end{aligned}
$$

output
```
-9/28*a*x^2*(b*x^3+a)^(2/3)/b^2/d-1/7*x^5*(b*x^3+a)^(2/3)/b/d-19/28*a^2*x^
2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/b^2/d/(b*x^3+a)^(
1/3)+1/12*a^(7/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)/
b^(8/3)/d+1/6*a^(7/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/b^(8/3)/d-1/3*2^(2/3)*
a^(7/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(8/3)/d-1/4*a^
(7/3)*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/
3)/a^(1/3))*2^(2/3)/b^(8/3)/d+1/3*2^(2/3)*a^(7/3)*arctan(1/3*(1-2*2^(1/3)*
(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(8/3)/d*3^(1/2)+1/6*a^(7/3
)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2
/3)/b^(8/3)/d*3^(1/2)
```

3.600.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 8.54 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.29

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\frac{-5(9a^2x^2+13abx^5+4b^2x^8)+45a^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+3}{140b^2d\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
(-5*(9*a^2*x^2 + 13*a*b*x^5 + 4*b^2*x^8) + 45*a^2*x^2*(1 + (b*x^3)/a)^(1/3
)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a] + 38*a*b*x^5*(1 + (b
*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a])/(140*b
^2*d*(a + b*x^3)^(1/3))
```

3.600.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {978, 27, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \left(a+bx^3\right)^{2/3}}{ad-bdx^3}dx$$

$$\Big\downarrow \ 978$$

$$\frac{\int \frac{ax^4\left(9bx^3+5a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{7bd}-\frac{x^5\left(a+bx^3\right)^{2/3}}{7bd}$$

$$\Big\downarrow \ 27$$

$$\frac{a\int \frac{x^4\left(9bx^3+5a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{7bd}-\frac{x^5\left(a+bx^3\right)^{2/3}}{7bd}$$

$$\Big\downarrow \ 1052$$

$$\frac{a\left(\frac{\int \frac{2abx\left(19bx^3+9a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{4b^2}-\frac{9x^2\left(a+bx^3\right)^{2/3}}{4b}\right)}{7bd}-\frac{x^5\left(a+bx^3\right)^{2/3}}{7bd}$$

$$\Big\downarrow \ 27$$

$$\frac{a\left(\frac{a\int \frac{x\left(19bx^3+9a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{2b}-\frac{9x^2\left(a+bx^3\right)^{2/3}}{4b}\right)}{7bd}-\frac{x^5\left(a+bx^3\right)^{2/3}}{7bd}$$

$$\Big\downarrow \ 1054$$

$$\frac{a\left(\frac{a\int \left(\frac{28ax}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}-\frac{19x}{\sqrt[3]{bx^3+a}}\right)dx}{2b}-\frac{9x^2\left(a+bx^3\right)^{2/3}}{4b}\right)}{7bd}-\frac{x^5\left(a+bx^3\right)^{2/3}}{7bd}$$

$$\Big\downarrow \ 2009$$

3.600. $\int \frac{x^7\left(a+bx^3\right)^{2/3}}{ad-bdx^3}dx$

$$a\Bigg(\frac{a\Bigg(\frac{14\ 2^{2/3}\sqrt[3]{a}\arctan\Bigg(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\Bigg)}{\sqrt{3}b^{2/3}}+\frac{7\ 2^{2/3}\sqrt[3]{a}\arctan\Bigg(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\Bigg)}{\sqrt{3}b^{2/3}}+\frac{7\ 2^{2/3}\sqrt[3]{a}\log\Bigg(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}(}{\sqrt[3]{}}}{3b^{2/3}}}{}$$

$$\frac{x^5\left(a+bx^3\right)^{2/3}}{7bd}$$

input
```
Int[(x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/7*(x^5*(a + b*x^3)^(2/3))/(b*d) + (a*((-9*x^2*(a + b*x^3)^(2/3))/(4*b)
+ (a*((14*2^(2/3)*a^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (7*2^(2/3)*a^(1/3)*ArcTan[(
1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*
b^(2/3)) - (19*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3,
-((b*x^3)/a)])/(2*(a + b*x^3)^(1/3)) + (7*a^(1/3)*Log[((a^(1/3) - b^(1/3)*
x)^2*(a^(1/3) + b^(1/3)*x))/a])/(3*2^(1/3)*b^(2/3)) + (7*2^(2/3)*a^(1/3)*L
og[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(
1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(2/3)) - (14*2^(2/3)*a^(1/3)*L
og[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(2/3)) - (
7*a^(1/3)*Log[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/3)*b^(1/3)*(
a + b*x^3)^(1/3))/a^(1/3)])/(2^(1/3)*b^(2/3))))/(2*b))/(7*b*d)
```

3.600. $\int \frac{x^7\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

3.600.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.600.4 Maple [F]

$$\int \frac{x^{7} \left(b\,x^{3}+a\right)^{\frac{2}{3}}}{-bd\,x^{3}+ad}d x$$

input
```
int(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output
```
int(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

3.600.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \text{Timed out}$$

input
```
integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.600.6 Sympy [F]

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^7(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**7*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**7*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.600.7 Maxima [F]

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x^7}{bdx^3-ad}\,dx$$

input
```
integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)*x^7/(b*d*x^3 - a*d), x)
```

3.600.8 Giac [F]

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{2}{3}}x^7}{bdx^3-ad}\,dx$$

input
```
integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)*x^7/(b*d*x^3 - a*d), x)
```

3.600.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int \frac{x^7\,\left(b\,x^3+a\right)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

3.601 $\int \frac{x^4\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

3.601.1 Optimal result

Integrand size = 28, antiderivative size = 485

$$\begin{aligned}\int \frac{x^4 (a+bx^3)^{2/3}}{ad-bdx^3}\,dx = &-\frac{x^2 (a+bx^3)^{2/3}}{4bd} \\ &+\frac{2^{2/3} a^{4/3} \arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{5/3}d}+\frac{a^{4/3} \arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{5/3}d} \\ &-\frac{3ax^2 \sqrt[3]{1+\frac{bx^3}{a}}\ \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{4bd\sqrt[3]{a+bx^3}} \\ &+\frac{a^{4/3} \log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2 \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{5/3}d} \\ &+\frac{a^{4/3} \log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{5/3}d} \\ &-\frac{2^{2/3} a^{4/3} \log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{5/3}d} \\ &-\frac{a^{4/3} \log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{5/3}d}\end{aligned}$$

output

```
-1/4*x^2*(b*x^3+a)^(2/3)/b/d-3/4*a*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2
/3],[5/3],-b*x^3/a)/b/d/(b*x^3+a)^(1/3)+1/12*a^(4/3)*ln((a^(1/3)-b^(1/3)*x
)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)/b^(5/3)/d+1/6*a^(4/3)*ln(1+2^(2/3)*(a^(
1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1
/3))*2^(2/3)/b^(5/3)/d-1/3*2^(2/3)*a^(4/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x
)/(b*x^3+a)^(1/3))/b^(5/3)/d-1/4*a^(4/3)*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^
(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1/3))*2^(2/3)/b^(5/3)/d+1/3*2^(2/
3)*a^(4/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^
(1/2))/b^(5/3)/d*3^(1/2)+1/6*a^(4/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)/b^(5/3)/d*3^(1/2)
```

3.601.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 8.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.26

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{x^2\left(-5(a+bx^3)+5a\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+6bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{Ap}\right.}{20bd\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
(x^2*(-5*(a + b*x^3) + 5*a*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3
, -((b*x^3)/a), (b*x^3)/a] + 6*b*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1
/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]))/(20*b*d*(a + b*x^3)^(1/3))
```

3.601.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {978, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$$

$$\Big\downarrow \text{978}$$

$$\frac{\int \frac{2ax\left(3bx^3+a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{4bd}-\frac{x^2\left(a+bx^3\right)^{2/3}}{4bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{x\left(3bx^3+a\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{2bd}-\frac{x^2\left(a+bx^3\right)^{2/3}}{4bd}$$

$$\Big\downarrow \text{1054}$$

$$\frac{a\int \left(\frac{4ax}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}-\frac{3x}{\sqrt[3]{bx^3+a}}\right)dx}{2bd}-\frac{x^2\left(a+bx^3\right)^{2/3}}{4bd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(\frac{2\ 2^{2/3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{2^{2/3}\sqrt[3]{a}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{2^{2/3}\sqrt[3]{a}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}(}{\sqrt[3]{}}\right.}{3b^{2/3}}\right.}{}$$

$$\frac{x^2\left(a+bx^3\right)^{2/3}}{4bd}$$

input
```
Int[(x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/4*(x^2*(a + b*x^3)^(2/3))/(b*d) + (a*((2*2^(2/3)*a^(1/3)*ArcTan[(1 - (2
*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2
/3)) + (2^(2/3)*a^(1/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b
*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) - (3*x^2*(1 + (b*x^3)/a)^(1/3)*Hy
pergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(1/3)) + (a^(
1/3)*Log[((a^(1/3) - b^(1/3)*x)^2*(a^(1/3) + b^(1/3)*x))/a])/(3*2^(1/3)*b^
(2/3)) + (2^(2/3)*a^(1/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b
*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]])/(3*b^(2/
3)) - (2*2^(2/3)*a^(1/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^
3)^(1/3)]])/(3*b^(2/3)) - (a^(1/3)*Log[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1
/3) - (2^(2/3)*b^(1/3)*(a + b*x^3)^(1/3))/a^(1/3)]])/(2^(1/3)*b^(2/3))))/(2
*b*d)
```

3.601. $\int \frac{x^4\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

3.601.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.601.4 Maple [F]

$$\int \frac{x^{4} \left(b \,x^{3}+a \right)^{\frac{2}{3}}}{-b d \,x^{3}+a d}d x$$

input
```
int(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output
```
int(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

3.601.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.601.6 Sympy [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^4(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x**4*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x**4*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.601.7 Maxima [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x^4}{bdx^3-ad}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)*x^4/(b*d*x^3 - a*d), x)
```

3.601.8 Giac [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{2}{3}}x^4}{bdx^3-ad}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)*x^4/(b*d*x^3 - a*d), x)
```

3.601.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int \frac{x^4\,(b\,x^3+a)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
int((x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

3.602 $\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

3.602.1 Optimal result

Integrand size = 26, antiderivative size = 457

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{2^{2/3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d}$$

$$+\frac{\sqrt[3]{a}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{2/3}d}$$

$$-\frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$$+\frac{\sqrt[3]{a}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{2/3}d}$$

$$+\frac{\sqrt[3]{a}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{2/3}d}$$

$$-\frac{2^{2/3}\sqrt[3]{a}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d}$$

$$-\frac{\sqrt[3]{a}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{2/3}d}$$

output

```
-1/2*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/d/(b*x^3+a
)^(1/3)+1/12*a^(1/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/
3)/b^(2/3)/d+1/6*a^(1/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3
)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/b^(2/3)/d-1/3*2^(2/
3)*a^(1/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(2/3)/d-1/4
*a^(1/3)*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^
(1/3)/a^(1/3))*2^(2/3)/b^(2/3)/d+1/3*2^(2/3)*a^(1/3)*arctan(1/3*(1-2*2^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)/d*3^(1/2)+1/6*a^(
1/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2
^(2/3)/b^(2/3)/d*3^(1/2)
```

3.602.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.14

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

input

```
Integrate[(x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output

```
(x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), (b*x^
3)/a])/(2*d*(a + b*x^3)^(1/3))
```

3.602.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {984, 27, 889, 888, 991, 27, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$$

$$\Big\downarrow \text{984}$$

$$2a\int \frac{x}{d\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx-\frac{\int \frac{x}{\sqrt[3]{bx^3+a}}dx}{d}$$

$$\Big\downarrow 27$$

$$\frac{2a\int \frac{x}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{d}-\frac{\int \frac{x}{\sqrt[3]{bx^3+a}}dx}{d}$$

$$\Big\downarrow 889$$

$$\frac{2a\int \frac{x}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{d}-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{d\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow 888$$

$$\frac{2a\int \frac{x}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{d}-\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow 991$$

$$\frac{2a\left(\frac{\int \frac{\sqrt[3]{a}}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3a^{2/3}\sqrt[3]{b}}-\frac{\int \frac{1}{\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{\sqrt[3]{a}b^{2/3}}\right)}{d}-\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow 27$$

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{\sqrt[3]{a}b^{2/3}}\right)}{d}-\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow 750$$

3.602. $\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{1}{3}\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d}{\sqrt[3]{a}b^{2/3}}\right.}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \ 16$$

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}\sqrt[3]{a}}}{\sqrt[3]{a}b^{2/3}}\right)}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \ 27$$

3.602. $\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\sqrt[3]{2}\int \frac{2^{2/3}-\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}\sqrt[3]{a}}}{\sqrt[3]{a}b^{2/3}}\right.}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2\sqrt[3]{2}}-\frac{\int -\frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\right.}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}}}{}\right.}{\sqrt[3]{a}b^{2/3}}\right.}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{25}$$

3.602. $\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2\sqrt[3]{2}}+\frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{}}{}\right.}{\int \frac{}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-}}\right.}{\sqrt[3]{a}b^{2/3}}\right.}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \; 27$$

3.602. $\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2\sqrt[3]{2}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(\sqrt[3]{}}{\sqrt[3]{bx}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-}}{}}{\sqrt[3]{a}b^{2/3}}}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1082}$$

3.602. $\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{2^{2/3}\sqrt[3]{a}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{}\right.}{\sqrt[3]{bx^3+a}}}}{2\sqrt[3]{2}}\right.}{\sqrt[3]{a}b^{2/3}}\right.}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \; 217$$

3.602. $\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\sqrt[3]{2}\left(\frac{\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}-\right.}{\sqrt[3]{a+}}}{\sqrt{3}}\right.}{2^{2/3}\sqrt[3]{a}}\right.}{\sqrt[3]{a}b^{2/3}}\right.}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \; 1103$$

3.602. $\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

$$\frac{2a\left(\frac{\int \frac{1}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt[3]{bx^3+a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt[3]{a}}-\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\ 2^{2/3}\sqrt[3]{a}}\right)+}{\sqrt[3]{a}b^{2/3}}\right.}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{2574}$$

3.602. $\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

$$\frac{2a\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}}-\frac{3\log\left(\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}\right)}{4\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\right)}{4\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\frac{1}{3}\sqrt[3]{2}\left(-\right.}{}\right.}{d}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

input
```
Int[(x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

output
```
-1/2*(x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)
/a)])/(d*(a + b*x^3)^(1/3)) + (2*a*(-(((2^(1/3)*(-((Sqrt[3]*ArcTan[(1 - (2
*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(2/3)*a^(1
/3))) - Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(
1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*2^(2/3)*a^(1/3))))/3 + L
og[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(1/3)*a^(1/
3)))/(a^(1/3)*b^(2/3))) + ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3
)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)*a^(1/3)*b^(1/3)) + Log[(a^(1
/3) - b^(1/3)*x)^2*(a^(1/3) + b^(1/3)*x)]/(4*2^(1/3)*a^(1/3)*b^(1/3)) - (3
*Log[b^(1/3)*(a^(1/3) + b^(1/3)*x) - 2^(2/3)*b^(1/3)*(a + b*x^3)^(1/3)])/(
4*2^(1/3)*a^(1/3)*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/d
```

3.602. $\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

3.602.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

3.602. $\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

rule 991
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Simp[-q^2/(3*d)   Int[1/((1 - q*x)*(a + b*x^3)^(1/3)
), x], x] + Simp[q/d   Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x
^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c +
 a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

3.602.4 Maple [F]

$$\int \frac{x(b\,x^{3}+a)^{\frac{2}{3}}}{-bd\,x^{3}+ad}dx$$

input
```
int(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

output
```
int(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

3.602.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\text{Timed out}$$

input
```
integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.602.6 Sympy [F]

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=-\frac{\int \frac{x(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

input
```
integrate(x*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

output
```
-Integral(x*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.602.7 Maxima [F]

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{2}{3}}x}{bdx^3-ad}\,dx$$

input
```
integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)*x/(b*d*x^3 - a*d), x)
```

3.602.8 Giac [F]

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x}{bdx^3-ad}\,dx$$

input
```
integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)*x/(b*d*x^3 - a*d), x)
```

3.602.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int \frac{x\,(b\,x^3+a)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

input
```
int((x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

output
```
int((x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

3.603 $\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx$

3.603.1 Optimal result

Integrand size $=28$, antiderivative size $=483$

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^2\left(ad-bdx^3\right)}\,dx=-\frac{\left(a+bx^3\right)^{2/3}}{adx}+\frac{2^{2/3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}d}+\frac{\sqrt[3]{b}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{2/3}d}$$
$$+\frac{bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2ad\sqrt[3]{a+bx^3}}+\frac{\sqrt[3]{b}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{2/3}d}$$
$$+\frac{\sqrt[3]{b}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}a^{2/3}d}-\frac{2^{2/3}\sqrt[3]{b}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{2/3}d}$$
$$-\frac{\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{2/3}d}$$

output
```
-(b*x^3+a)^(2/3)/a/d/x+1/2*b*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5
/3],-b*x^3/a)/a/d/(b*x^3+a)^(1/3)+1/12*b^(1/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a
^(1/3)+b^(1/3)*x)/a)*2^(2/3)/a^(2/3)/d+1/6*b^(1/3)*ln(1+2^(2/3)*(a^(1/3)+b
^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2
^(2/3)/a^(2/3)/d-1/3*2^(2/3)*b^(1/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x
^3+a)^(1/3))/a^(2/3)/d-1/4*b^(1/3)*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-
2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1/3))*2^(2/3)/a^(2/3)/d+1/3*2^(2/3)*b^(
1/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))
/a^(2/3)/d*3^(1/2)+1/6*b^(1/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(
b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)/a^(2/3)/d*3^(1/2)
```

3.603.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.28

$$\int \frac{(a+bx^3)^{2/3}}{x^2\,(ad-bdx^3)}\,dx = \frac{15abx^3\sqrt[3]{1+\frac{bx^3}{a}}\ \text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2\Bigg(5a(a+bx^3)+b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}}{10a^2dx\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x]
```

output
```
(15*a*b*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a),
 (b*x^3)/a] - 2*(5*a*(a + b*x^3) + b^2*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[
5/3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]))/(10*a^2*d*x*(a + b*x^3)^(1/3)
)
```

3.603.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 467, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {975, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^2\,(ad-bdx^3)}\,dx$$

$$\Big\downarrow \ 975$$

$$\frac{\int \frac{bx(3a-bx^3)}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{ad}-\frac{(a+bx^3)^{2/3}}{adx}$$

$$\Big\downarrow \ 27$$

$$\frac{b\int \frac{x(3a-bx^3)}{(a-bx^3)\sqrt[3]{bx^3+a}}dx}{ad}-\frac{(a+bx^3)^{2/3}}{adx}$$

$$\Big\downarrow \ 1054$$

$$\frac{b\int \left(\frac{2ax}{(a-bx^3)\sqrt[3]{bx^3+a}}+\frac{x}{\sqrt[3]{bx^3+a}}\right)dx}{ad}-\frac{\left(a+bx^3\right)^{2/3}}{adx}$$

$$\Big\downarrow \text{2009}$$

$$b\left(\frac{2^{2/3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{\sqrt[3]{a}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{2/3}}+\frac{\sqrt[3]{a}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{\sqrt[3]{a+bx^3}}\right.}{3\sqrt[3]{2}b^{2/3}}\right.$$

$$\frac{\left(a+bx^3\right)^{2/3}}{adx}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x]
```

output
```
-((a + b*x^3)^(2/3)/(a*d*x)) + (b*((2^(2/3)*a^(1/3)*ArcTan[(1 - (2*2^(1/3)
*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (
a^(1/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqr
t[3]])/(2^(1/3)*Sqrt[3]*b^(2/3)) + (x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometr
ic2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(1/3)) + (a^(1/3)*Log[(
(a^(1/3) - b^(1/3)*x)^2*(a^(1/3) + b^(1/3)*x))/a])/(6*2^(1/3)*b^(2/3)) + (
a^(1/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(
1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(1/3)*b^(2/3)) - (2^(
2/3)*a^(1/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(
3*b^(2/3)) - (a^(1/3)*Log[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/
3)*b^(1/3)*(a + b*x^3)^(1/3))/a^(1/3)])/(2*2^(1/3)*b^(2/3))))/(a*d)
```

3.603. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^2(ad-bdx^3)}\,dx$

3.603.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.603.4 Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{2}{3}}}{x^{2}\left(-bd\,x^{3}+ad\right)}dx$$

input
```
int((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x)
```

output
```
int((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x)
```

3.603.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^2\left(ad-bdx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.603.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2\left(ad-bdx^3\right)}\,dx=-\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^2+bx^5}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x**2/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x**2 + b*x**5), x)/d
```

3.603.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2\left(ad-bdx^3\right)}\,dx=\int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^2), x)
```

3.603.8 Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^2}\,dx$$

input

```
integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="giac")
```

output

```
integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^2), x)
```

3.603.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^2\,(a\,d-b\,d\,x^3)}\,dx$$

input

```
int((a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x)
```

output

```
int((a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)), x)
```

3.604 $\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}\,dx$

3.604.1 Optimal result

Integrand size = 28, antiderivative size = 512

$$
\begin{aligned}
\int \frac{\left(a+bx^3\right)^{2/3}}{x^5\left(ad-bdx^3\right)}\,dx &= -\frac{\left(a+bx^3\right)^{2/3}}{4adx^4}-\frac{3b\left(a+bx^3\right)^{2/3}}{2a^2dx} \\
&+\frac{2^{2/3}b^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{5/3}d}+\frac{b^{4/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{5/3}d} \\
&+\frac{3b^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{4a^2d\sqrt[3]{a+bx^3}} \\
&+\frac{b^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{5/3}d} \\
&+\frac{b^{4/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}a^{5/3}d} \\
&-\frac{2^{2/3}b^{4/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{5/3}d} \\
&-\frac{b^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{5/3}d}
\end{aligned}
$$

output
```
-1/4*(b*x^3+a)^(2/3)/a/d/x^4-3/2*b*(b*x^3+a)^(2/3)/a^2/d/x+3/4*b^2*x^2*(1+
b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/a^2/d/(b*x^3+a)^(1/3)+
1/12*b^(4/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)/a^(5/
3)/d+1/6*b^(4/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3
)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/a^(5/3)/d-1/3*2^(2/3)*b^(4/
3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(5/3)/d-1/4*b^(4/3)
*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^
(1/3))*2^(2/3)/a^(5/3)/d+1/3*2^(2/3)*b^(4/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1
/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(5/3)/d*3^(1/2)+1/6*b^(4/3)*arc
tan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)/a
^(5/3)/d*3^(1/2)
```

3.604.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.29

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(ad-bdx^3)}\,dx = \frac{-5a(a^2+7abx^3+6b^2x^6)+35ab^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\,\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)-6b^3}{20a^3dx^4\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x]
```

output
```
(-5*a*(a^2 + 7*a*b*x^3 + 6*b^2*x^6) + 35*a*b^2*x^6*(1 + (b*x^3)/a)^(1/3)*A
ppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a] - 6*b^3*x^9*(1 + (b*x^3
)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a])/(20*a^3*d*
x^4*(a + b*x^3)^(1/3))
```

3.604.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 496, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {975, 27, 1053, 25, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^5\left(ad-bdx^3\right)}dx$$

$$\Big\downarrow \ 975$$

$$\frac{\int \frac{2b\left(bx^3+3a\right)}{x^2\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{4ad}-\frac{\left(a+bx^3\right)^{2/3}}{4adx^4}$$

$$\Big\downarrow \ 27$$

$$\frac{b\int \frac{bx^3+3a}{x^2\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{2ad}-\frac{\left(a+bx^3\right)^{2/3}}{4adx^4}$$

$$\Big\downarrow \ 1053$$

$$\frac{b\left(-\frac{\int -\frac{abx\left(7a-3bx^3\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{a^2}-\frac{3\left(a+bx^3\right)^{2/3}}{ax}\right)}{2ad}-\frac{\left(a+bx^3\right)^{2/3}}{4adx^4}$$

$$\Big\downarrow \ 25$$

$$\frac{b\left(\frac{\int \frac{abx\left(7a-3bx^3\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{a^2}-\frac{3\left(a+bx^3\right)^{2/3}}{ax}\right)}{2ad}-\frac{\left(a+bx^3\right)^{2/3}}{4adx^4}$$

$$\Big\downarrow \ 27$$

$$\frac{b\left(\frac{b\int \frac{x\left(7a-3bx^3\right)}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}dx}{a}-\frac{3\left(a+bx^3\right)^{2/3}}{ax}\right)}{2ad}-\frac{\left(a+bx^3\right)^{2/3}}{4adx^4}$$

$$\Big\downarrow \ 1054$$

$$\frac{b\left(\frac{b\int \left(\frac{4ax}{\left(a-bx^3\right)\sqrt[3]{bx^3+a}}+\frac{3x}{\sqrt[3]{bx^3+a}}\right)dx}{a}-\frac{3\left(a+bx^3\right)^{2/3}}{ax}\right)}{2ad}-\frac{\left(a+bx^3\right)^{2/3}}{4adx^4}$$

$$\Big\downarrow \ 2009$$

3.604. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^5\left(ad-bdx^3\right)}dx$

$$b\left(b\left(\frac{2\ 2^{2/3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{2^{2/3}\sqrt[3]{a}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{2^{2/3}\sqrt[3]{a}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\right.}{\sqrt[3]{a+b}}\right.}{3b^{2/3}}\right.\right.$$

$$\frac{\left(a+bx^3\right)^{2/3}}{4adx^4}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x]
```

output
```
-1/4*(a + b*x^3)^(2/3)/(a*d*x^4) + (b*((-3*(a + b*x^3)^(2/3))/(a*x) + (b*(
(2*2^(2/3)*a^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^
3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (2^(2/3)*a^(1/3)*ArcTan[(1 + (2^(1
/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3))
+ (3*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/
a)])/(2*(a + b*x^3)^(1/3)) + (a^(1/3)*Log[((a^(1/3) - b^(1/3)*x)^2*(a^(1/3
) + b^(1/3)*x))/a]])/(3*2^(1/3)*b^(2/3)) + (2^(2/3)*a^(1/3)*Log[1 + (2^(2/3
)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)
*x))/(a + b*x^3)^(1/3)]])/(3*b^(2/3)) - (2*2^(2/3)*a^(1/3)*Log[1 + (2^(1/3)
*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]])/(3*b^(2/3)) - (a^(1/3)*Log[(b^
(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/3)*b^(1/3)*(a + b*x^3)^(1/3))
/a^(1/3)]])/(2^(1/3)*b^(2/3))))/a))/(2*a*d)
```

3.604. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^5\left(ad-bdx^3\right)}\,dx$

3.604.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.604. $\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}\,dx$

3.604.4 Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{5} \left(-bd\,x^{3}+ad\right)}d x$$

input
```
int((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x)
```

output
```
int((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x)
```

3.604.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(ad-bdx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="fricas")
```

output
```
Timed out
```

3.604.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(ad-bdx^3)}\,dx = -\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^5+bx^8}\,dx}{d}$$

input
```
integrate((b*x**3+a)**(2/3)/x**5/(-b*d*x**3+a*d),x)
```

output
```
-Integral((a + b*x**3)**(2/3)/(-a*x**5 + b*x**8), x)/d
```

3.604.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^5), x)
```

3.604.8 Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^5), x)
```

3.604.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^5\,(a\,d-b\,d\,x^3)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)), x)
```

3.605 $\int \frac{x^{14}}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.605.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
2/5*(-x^3+1)^(5/3)-1/4*(-x^3+1)^(8/3)+1/11*(-x^3+1)^(11/3)-1/12*ln(x^3+1)*
2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1+2^(2/3)*(
-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.605.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.14

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{220}\left(1-x^3\right)^{2/3}\left(53-38x^3+5x^6-20x^9\right) + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}} - \frac{\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}$$

input
```
Integrate[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((1 - x^3)^(2/3)*(53 - 38*x^3 + 5*x^6 - 20*x^9))/220 + ArcTan[(1 + 2^(2/3)
*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[-2 + 2^(2/3)*(1 - x^3)^
(1/3)]/(3*2^(1/3)) - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(
2/3)]/(6*2^(1/3))
```

3.605.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^{12}}{\sqrt[3]{1-x^3}\,(x^3+1)}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(-\left(1-x^3\right)^{8/3}+2\left(1-x^3\right)^{5/3}-2\left(1-x^3\right)^{2/3}+\frac{1}{(x^3+1)\sqrt[3]{1-x^3}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{3}{11}\left(1-x^3\right)^{11/3}-\frac{3}{4}\left(1-x^3\right)^{8/3}+\frac{6}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\right.}{2}\right.$$

input
```
Int[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((6*(1 - x^3)^(5/3))/5 - (3*(1 - x^3)^(8/3))/4 + (3*(1 - x^3)^(11/3))/11 +
 (Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3) - Log[1 +
 x^3]/(2*2^(1/3)) + (3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(1/3)))/3
```

3.605.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.605.4 Maple [A] (verified)

Time = 9.74 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{\left(2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}-\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)+2\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\right)2^{\frac{2}{3}}}{12}-\frac{\left(-x^{3}+1\right)}{}$
trager	$\left(-\frac{1}{11}x^{9}+\frac{1}{44}x^{6}-\frac{19}{110}x^{3}+\frac{53}{220}\right)\left(-x^{3}+1\right)^{\frac{2}{3}}+\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^{3}-4\right)^{2}+6\_Z\operatorname{RootOf}\right.$
risch	$\frac{\left(20x^{9}-5x^{6}+38x^{3}-53\right)\left(x^{3}-1\right)}{220\left(-x^{3}+1\right)^{\frac{1}{3}}}+\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^{3}-4\right)^{2}+6\_Z\operatorname{RootOf}\left(\_Z^{3}-4\right)+36\_Z\right.$

input
```
int(x^14/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*(2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-ln((-x^3+1)
^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))+2*ln((-x^3+1)^(1/3)-2^(1/3)))*2^(2/
3)-1/220*(-x^3+1)^(2/3)*(20*x^9-5*x^6+38*x^3-53)
```

3.605.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= -\frac{1}{220}\left(20\,x^9-5\,x^6+38\,x^3-53\right)\left(-x^3+1\right)^{\frac{2}{3}} \\ &\quad + \frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) \\ &\quad - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) \\ &\quad + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)\end{aligned}$$

input
```
integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/220*(20*x^9 - 5*x^6 + 38*x^3 - 53)*(-x^3 + 1)^(2/3) + 1/6*sqrt(6)*2^(1/
6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/
12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/
6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

3.605.6 Sympy [F]

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^{14}}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x**14/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**14/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.605.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{11}\left(-x^3+1\right)^{\frac{11}{3}} - \frac{1}{4}\left(-x^3+1\right)^{\frac{8}{3}} + \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}} - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
1/11*(-x^3 + 1)^(11/3) - 1/4*(-x^3 + 1)^(8/3) + 1/6*sqrt(3)*2^(2/3)*arctan
(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 2/5*(-x^3 + 1)^(5/3
) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)
) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

3.605.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{11}\left(x^3-1\right)^3\left(-x^3+1\right)^{\frac{2}{3}} - \frac{1}{4}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}} - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

input
```
integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/11*(x^3 - 1)^3*(-x^3 + 1)^(2/3) - 1/4*(x^3 - 1)^2*(-x^3 + 1)^(2/3) + 1/
6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3)
)) + 2/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^
(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3
)))
```

3.605.9 Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{2\left(1-x^3\right)^{5/3}}{5} - \frac{\left(1-x^3\right)^{8/3}}{4}+\frac{\left(1-x^3\right)^{11/3}}{11} + \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(x^14/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2*(1 - x^3)^(5/3))/5 - (1 -
x^3)^(8/3)/4 + (1 - x^3)^(11/3)/11 + (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/
3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x^3)^(1
/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

3.606 $\int \frac{x^{11}}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.606.1 Optimal result

Integrand size = 22, antiderivative size = 128

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{8}\left(1-x^3\right)^{8/3} -\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
-1/2*(-x^3+1)^(2/3)+1/5*(-x^3+1)^(5/3)-1/8*(-x^3+1)^(8/3)+1/12*ln(x^3+1)*2
^(2/3)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-1/6*arctan(1/3*(1+2^(2/3)*(-
x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.606.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{120}\Bigg(-3\left(1-x^3\right)^{2/3}\left(17-2x^3+5x^6\right) -20\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-20\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+10\,2^{2/3}\log\Big(2+2^{2/3}\sqrt[3]{1-x^3}+$$

input
```
Integrate[x^11/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(-3*(1 - x^3)^(2/3)*(17 - 2*x^3 + 5*x^6) - 20*2^(2/3)*Sqrt[3]*ArcTan[(1 +
2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 20*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^
(1/3)] + 10*2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2
/3)])/120
```

3.606.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9}{\sqrt[3]{1-x^3}\,(x^3+1)}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\left(1-x^3\right)^{5/3}-\left(1-x^3\right)^{2/3}-\frac{1}{(x^3+1)\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{3}{8}\left(1-x^3\right)^{8/3}+\frac{3}{5}\left(1-x^3\right)^{5/3}-\frac{3}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}-\right.}{2\sqrt[3]{}}\right.$$

input
```
Int[x^11/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((-3*(1 - x^3)^(2/3))/2 + (3*(1 - x^3)^(5/3))/5 - (3*(1 - x^3)^(8/3))/8 -
(Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3) + Log[1 +
x^3]/(2*2^(1/3)) - (3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(1/3)))/3
```

3.606.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.606.4 Maple [A] (verified)

Time = 10.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{\left(-2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}+\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)-2\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\right)2^{\frac{2}{3}}}{12}-$ $(-x^3+$
risch	Expression too large to display
trager	Expression too large to display

input
```
int(x^11/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+ln((-x^3+1
)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))-2*ln((-x^3+1)^(1/3)-2^(1/3)))*2^(2
/3)-1/40*(-x^3+1)^(2/3)*(5*x^6-2*x^3+17)
```

3.606.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.07

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{6}\,\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\,\sqrt{6}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\right)\right) - \frac{1}{12} \cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) - \frac{1}{40}\left(5\,x^6-2\,x^3+17\right)\left(-x^3+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/6*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(2*sqrt(6)*(-1)^(1/3)*(
-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) - 1/12*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*
(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6
*2^(2/3)*(-1)^(1/3)*log(-2^(1/3)*(-1)^(2/3) + (-x^3 + 1)^(1/3)) - 1/40*(5*
x^6 - 2*x^3 + 17)*(-x^3 + 1)^(2/3)
```

3.606.6 Sympy [F]

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^{11}}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**11/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.606.7 Maxima [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.93

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{8}\left(-x^3+1\right)^{\frac{8}{3}} - \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}} + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) - \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/8*(-x^3 + 1)^(8/3) - 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^
(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/5*(-x^3 + 1)^(5/3) + 1/12*2^(2/3)*log(2^(
2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(2/3)*log(-2^(
1/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3)
```

3.606.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.99

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{8}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{2}{3}} - \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}} + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) - \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/8*(x^3 - 1)^2*(-x^3 + 1)^(2/3) - 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)
*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/5*(-x^3 + 1)^(5/3) + 1/12*2^(
2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(2
/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/2*(-x^3 + 1)^(2/3)
```

3.606.9 Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04

$$
\begin{aligned}
\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{\left(1-x^3\right)^{5/3}}{5}-\frac{\left(1-x^3\right)^{2/3}}{2} \\
&\quad -\frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}-\frac{\left(1-x^3\right)^{8/3}}{8} \\
&\quad -\frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} \\
&\quad +\frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}
\end{aligned}
$$

input

```
int(x^11/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output

```
(1 - x^3)^(5/3)/5 - (1 - x^3)^(2/3)/2 - (2^(2/3)*log((1 - x^3)^(1/3) - 2^(
1/3)))/6 - (1 - x^3)^(8/3)/8 - (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^
(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 + (2^(2/3)*log((1 - x^3)^(1/3) -
(2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

3.607 $\int \frac{x^8}{\sqrt[3]{1-x^3}(1+x^3)} \, dx$

3.607.1 Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)} \, dx = \frac{1}{5}\left(1-x^3\right)^{5/3} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
1/5*(-x^3+1)^(5/3)-1/12*ln(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2
^(2/3)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.607.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.31

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)} \, dx = \frac{1}{60}\left(12\left(1-x^3\right)^{5/3} + 10\, 2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 10\, 2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - 5\, 2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\right.\right.$$

input
```
Integrate[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(12*(1 - x^3)^(5/3) + 10*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/
3))/Sqrt[3]] + 10*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 5*2^(2/3)*Lo
g[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/60
```

3.607.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{\sqrt[3]{1-x^3}\,(x^3+1)}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}-\left(1-x^3\right)^{2/3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{3}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

input
```
Int[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((3*(1 - x^3)^(5/3))/5 + (Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqr
t[3]])/2^(1/3) - Log[1 + x^3]/(2*2^(1/3)) + (3*Log[2^(1/3) - (1 - x^3)^(1/
3)])/(2*2^(1/3)))/3
```

3.607.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.607.4 Maple [A] (verified)

Time = 9.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{(-x^3+1)^{\frac{2}{3}}x^3}{5}+\frac{(-x^3+1)^{\frac{2}{3}}}{5}+\frac{2^{\frac{2}{3}}\ln\left((-x^3+1)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)}{6}-\frac{2^{\frac{2}{3}}\ln\left((-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{12}+$ arctan
trager	Expression too large to display
risch	Expression too large to display

input
```
int(x^8/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-x^3+1)^(2/3)*x^3+1/5*(-x^3+1)^(2/3)+1/6*2^(2/3)*ln((-x^3+1)^(1/3)-2
^(1/3))-1/12*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))+1/6
*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.607.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = & -\frac{1}{5}\left(x^3-1\right)\left(-x^3+1\right)^{\frac{2}{3}} \\ & + \frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) \\ & - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) \\ & + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)\end{aligned}$$

input
```
integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/5*(x^3 - 1)*(-x^3 + 1)^(2/3) + 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(
sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3)
+ 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3)
+ (-x^3 + 1)^(1/3))
```

3.607.6 Sympy [F]

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^8}{\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**8/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.607.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}} - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1
)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

3.607.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}} - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

input
```
integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1
)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1
/3)))
```

3.607.9 Mupad [B] (verification not implemented)

Time = 8.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{\left(1-x^3\right)^{5/3}}{5} + \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(x^8/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (1 - x^3)^(5/3)/5 + (2^(2/3)*
log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12
 - (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)
*1i + 1))/12
```

3.608 $\int \frac{x^5}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.608.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
-1/2*(-x^3+1)^(2/3)+1/12*ln(x^3+1)*2^(2/3)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*
2^(2/3)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.608.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{12}\Bigg(-6\left(1-x^3\right)^{2/3}$$
$$-2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + 2^{2/3}\log\Big(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1$$

input
```
Integrate[x^5/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(-6*(1 - x^3)^(2/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] - 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(2/3)*Log[2
+ 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/12
```

3.608.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 90, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt[3]{1-x^3}\,(x^3+1)}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(-\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx^3-\frac{3}{2}\left(1-x^3\right)^{2/3}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{3}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{3}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}-\frac{3}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{3}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

input
```
Int[x^5/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((-3*(1 - x^3)^(2/3))/2 - (Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sq
rt[3]])/2^(1/3) + Log[1 + x^3]/(2*2^(1/3)) - (3*Log[2^(1/3) - (1 - x^3)^(1
/3)])/(2*2^(1/3)))/3
```

3.608.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.608.4 Maple [A] (verified)

Time = 9.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$-\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{2}-\frac{2^{\frac{2}{3}} \ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)}{6}+\frac{2^{\frac{2}{3}} \ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{12}-\frac{\arctan\left(\frac{\left(1+2^{\frac{2}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{}}{3}\right.}{6}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(x^5/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-x^3+1)^(2/3)-1/6*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))+1/12*2^(2/3)*ln
((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))-1/6*arctan(1/3*(1+2^(2/3)*
(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.608.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.28

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}(-1)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)-\frac{1}{2}\left(-x^{3}+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/6*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(2*sqrt(6)*(-1)^(1/3)*(
-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) - 1/12*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*
(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6
*2^(2/3)*(-1)^(1/3)*log(-2^(1/3)*(-1)^(2/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^
3 + 1)^(2/3)
```

3.608.6 Sympy [F]

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^5}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x**5/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.608.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx =& -\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\ &+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\ &-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}\end{aligned}$$

input
```
integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) - 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3
)
```

3.608.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) - \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) - 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/2*(-x^3 + 1)
^(2/3)
```

3.608.9 Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6} - \frac{\left(1-x^3\right)^{2/3}}{2} - \frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{12} + \frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}$$

input
```
int(x^5/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i
 + 1))/12 - (1 - x^3)^(2/3)/2 - (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3
^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x^3)^(1/3) -
 2^(1/3)))/6
```

3.609 $\int \frac{x^2}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.609.1 Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output

```
-1/12*ln(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(
1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.609.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$
$$=\frac{2\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+2\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}$$

input

```
Integrate[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output

```
(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 2*Log[-2 + 2^(2
/3)*(1 - x^3)^(1/3)] - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)
^(2/3)])/(6*2^(1/3))
```

3.609.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {946, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3}\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx^3$$

$$\Big\downarrow \text{ 67}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

input

```
Int[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output

```
((Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3) - Log[1 +
 x^3]/(2*2^(1/3)) + (3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(1/3)))/3
```

3.609.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.609.4 Maple [A] (verified)

Time = 4.97 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

method	result	size
pseudoelliptic	$\frac{\left(2 \arctan\left(\frac{\left(1+2^{\frac{2}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}\right) \sqrt{3}}{3}\right) \sqrt{3}-\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)+2 \ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\right) 2^{\frac{2}{3}}}{12}$	80
trager	Expression too large to display	759

input
```
int(x^2/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*(2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-ln((-x^3+1)
^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))+2*ln((-x^3+1)^(1/3)-2^(1/3)))*2^(2/
3)
```

3.609.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)$$

input
```
integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3
+ 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3
 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

3.609.6 Sympy [F]

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^2}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.609.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\&\quad-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\&\quad+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)\end{aligned}$$

input
```
integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

3.609.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\&\quad-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\&\quad+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)\end{aligned}$$

input
```
integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

3.609.9 Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6} + \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(x^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2^(2/3)*log((1 - x^3)^(1/3)
- (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 -
 x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

3.610 $\int \frac{1}{x\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.610.1 Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x)}{2}+\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
-1/2*ln(x)+1/12*ln(x^3+1)*2^(2/3)+1/2*ln(1-(-x^3+1)^(1/3))-1/4*ln(2^(1/3)-
(-x^3+1)^(1/3))*2^(2/3)+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/
2)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.610.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.35

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\frac{1}{12}\left(4\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)\right.$$

$$-2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+4\log\left(-1+\sqrt[3]{1-x^3}\right)-2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-2\log (1$$

input
```
Integrate[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] - 2*2^(2/3)*Sqrt[3]*Arc
Tan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 4*Log[-1 + (1 - x^3)^(1/3)] -
 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2*Log[1 + (1 - x^3)^(1/3) +
 (1 - x^3)^(2/3)] + 2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 -
 x^3)^(2/3)])/12
```

3.610.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {948, 97, 67, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{1-x^3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3\sqrt[3]{1-x^3}\left(x^3+1\right)}dx^3$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{3}\left(\int \frac{1}{x^3\sqrt[3]{1-x^3}}dx^3-\int \frac{1}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx^3\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}+\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{3}{2}\int \frac{}{x^6}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3}{2}\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3}{2}\log\left(1-\right.\right.$$

$$\Big\downarrow \color{blue}{217}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3\right)}{2}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3}{2}\log\left(1-\sqrt[3]{\right.\right.$$

$$\Big\downarrow \color{blue}{1083}$$

$$\frac{1}{3}\left(-3\int \frac{1}{-x^6-3}d\left(2\sqrt[3]{1-x^3}+1\right)-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3\right)}{2}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3}{2}\log\left(1-\sqrt[3]{1\right.\right.$$

$$\Big\downarrow \color{blue}{217}$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3\right)}{2}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3}{2}\log\left(1-\sqrt[3]{1-x\right.\right.$$

input
```
Int[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] - (Sqrt[3]*ArcTan[(1 + 2^
(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3) - Log[x^3]/2 + Log[1 + x^3]/(2*2^
(1/3)) + (3*Log[1 - (1 - x^3)^(1/3)])/2 - (3*Log[2^(1/3) - (1 - x^3)^(1/3)
])/(2*2^(1/3)))/3
```

3.610. $\int \frac{1}{x\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.610.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.610. $\int \frac{1}{x\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.610.4 Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$-\frac{\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)}{6}+\frac{\arctan\left(\frac{\left(1+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln\left(-1+\left(-x^3+1\right)^{\frac{1}{3}}\right)}{3}-\frac{2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}\right.}{6}$

input
```
int(1/x/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3
))*3^(1/2))*3^(1/2)+1/3*ln(-1+(-x^3+1)^(1/3))-1/6*2^(2/3)*ln((-x^3+1)^(1/3
)-2^(1/3))+1/12*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))-
1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.610.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.99

$$
\begin{aligned}
&\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&= \frac{1}{12}\cdot 2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)\log\left(\frac{1}{8}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^3-\frac{3}{4}\right.\\
&\qquad\left.\cdot 2^{\frac{1}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2+3\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\\
&\quad-\frac{1}{24}\left(2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)-2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2}\right)\log\left(\frac{3}{8}\right.\\
&\qquad\cdot 2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+\frac{3}{8}\\
&\qquad\left.\cdot 2^{\frac{1}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2+3\left(-x^3+1\right)^{\frac{1}{3}}\right)\\
&\quad-\frac{1}{24}\left(2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2}\right)\log\left(-\frac{3}{8}\right.\\
&\qquad\cdot 2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+\frac{3}{8}\\
&\quad\left.\cdot 2^{\frac{1}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2+3\left(-x^3+1\right)^{\frac{1}{3}}\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}(-x^3+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)\\
&\quad+\frac{1}{3}\log\left(-\frac{1}{24}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^3+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{4}{3}\right)\\
&\quad-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)
\end{aligned}
$$

input
```
integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

3.610. $\int \frac{1}{x\sqrt[3]{1-x^3}(1+x^3)}\,dx$

output
```
1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(1/8*(I*sqrt(3)*(-1)^(
1/3) - (-1)^(1/3))^3 - 3/4*2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 +
 3*(-x^3 + 1)^(1/3) + 1) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3
)) - 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2))*log
(3/8*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2
)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(3)*(-1)^(1/3)
- (-1)^(1/3))^2 + 3*(-x^3 + 1)^(1/3)) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3
) - (-1)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(
1/3))^2))*log(-3/8*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) -
 (-1)^(1/3))^2)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(
3)*(-1)^(1/3) - (-1)^(1/3))^2 + 3*(-x^3 + 1)^(1/3)) + 1/3*sqrt(3)*arctan(2
/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/3*log(-1/24*(I*sqrt(3)*(-1)
^(1/3) - (-1)^(1/3))^3 + (-x^3 + 1)^(1/3) - 4/3) - 1/6*log((-x^3 + 1)^(2/3
) + (-x^3 + 1)^(1/3) + 1)
```

3.610.6 Sympy [F]

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(1/x/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.610.7 Maxima [F]

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x}\,dx$$

input
```
integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x), x)
```

3.610.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$
\begin{aligned}
\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = {} & -\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) \\
& + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) \\
& - \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) \\
& + \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right) \\
& - \frac{1}{6}\,\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right) \\
& + \frac{1}{3}\,\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)
\end{aligned}
$$

input
```
integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) - 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/3*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 + 1)^(2/3) + (-
x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

3.610.9 Mupad [B] (verification not implemented)

Time = 8.41 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.87

$$
\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\ln\left(6-6\left(1-x^3\right)^{1/3}\right)}{3}
$$

$$
+\ln\left(\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^3\left(1458\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2-135\left(1-x^3\right)^{1/3}\right)-\left(1-x^3\right)^{1/3}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\Bigg(
$$

input
```
int(1/(x*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output

```
log(6 - 6*(1 - x^3)^(1/3))/3 + log(((3^(1/2)*1i)/6 - 1/6)^3*(1458*((3^(1/2
)*1i)/6 - 1/6)^2 - 135*(1 - x^3)^(1/3)) - (1 - x^3)^(1/3))*((3^(1/2)*1i)/6
 - 1/6) - log(- ((3^(1/2)*1i)/6 + 1/6)^3*(1458*((3^(1/2)*1i)/6 + 1/6)^2 -
135*(1 - x^3)^(1/3)) - (1 - x^3)^(1/3))*((3^(1/2)*1i)/6 + 1/6) - (2^(2/3)*
log((3*(1 - x^3)^(1/3))/2 - (3*2^(1/3))/2))/6 + ((-1)^(1/3)*2^(2/3)*log((3
*(1 - x^3)^(1/3))/2 - (3*(-1)^(2/3)*2^(1/3))/2))/6 - ((-1)^(1/3)*2^(2/3)*l
og(- ((3^(1/2)*1i + 1)^3*(135*(1 - x^3)^(1/3) - (81*(-1)^(2/3)*2^(1/3)*(3^
(1/2)*1i + 1)^2)/4))/432 - (1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

3.610. $\int \frac{1}{x\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.611 $\int \frac{1}{x^4 \sqrt[3]{1-x^3}(1+x^3)} \, dx$

3.611.1 Optimal result

Integrand size = 22, antiderivative size = 157

$$\begin{aligned}\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = &-\frac{(1-x^3)^{2/3}}{3x^3}-\frac{2\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} \\ &+\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log(x)}{3}-\frac{\log(1+x^3)}{6\sqrt[3]{2}} \\ &-\frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\end{aligned}$$

output
```
-1/3*(-x^3+1)^(2/3)/x^3+1/3*ln(x)-1/12*ln(x^3+1)*2^(2/3)-1/3*ln(1-(-x^3+1)
^(1/3))+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-2/9*arctan(1/3*(1+2*(-x^3+1
)^(1/3))*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2
))*2^(2/3)*3^(1/2)
```

3.611.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{36}\left(-\frac{12(1-x^3)^{2/3}}{x^3}-8\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)\right.$$
$$+6\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-8\log\left(-1+\sqrt[3]{1-x^3}\right)+6\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+4\log\left(1\right.$$

input
```
Integrate[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((-12*(1 - x^3)^(2/3))/x^3 - 8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt
[3]] + 6*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 8
*Log[-1 + (1 - x^3)^(1/3)] + 6*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] +
 4*Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(2/3)*Log[2 + 2^(2/3)*
(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/36
```

3.611.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {948, 114, 27, 174, 67, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(x^3+1)}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\int \frac{2-x^3}{3x^3\sqrt[3]{1-x^3}\,(x^3+1)}dx^3-\frac{(1-x^3)^{2/3}}{x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{1}{3}\int \frac{2-x^3}{x^3\sqrt[3]{1-x^3}\left(x^3+1\right)}dx^3-\frac{\left(1-x^3\right)^{2/3}}{x^3}\right)$$

$$\Big\downarrow 174$$

$$\frac{1}{3}\left(\frac{1}{3}\left(3\int \frac{1}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx^3-2\int \frac{1}{x^3\sqrt[3]{1-x^3}}dx^3\right)-\frac{\left(1-x^3\right)^{2/3}}{x^3}\right)$$

$$\Big\downarrow 67$$

$$\frac{1}{3}\left(\frac{1}{3}\left(3\left(-\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}\right)-2\left(-\frac{3}{2}\int\right.\right.\right.$$

$$\Big\downarrow 16$$

$$\frac{1}{3}\left(\frac{1}{3}\left(3\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)-2\left(\frac{3}{2}\int \frac{}{x^6+\sqrt[3]{1}}\right.\right.\right.$$

$$\Big\downarrow 1082$$

$$\frac{1}{3}\left(\frac{1}{3}\left(3\left(-\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)-2\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}}\right.\right.\right.$$

$$\Big\downarrow 217$$

$$\frac{1}{3}\left(\frac{1}{3}\left(3\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)-2\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}\right.\right.\right.$$

$$\Big\downarrow 1083$$

$$\frac{1}{3}\left(\frac{1}{3}\left(3\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)-2\left(-3\int \frac{1}{-x^6-3}d\left(2\sqrt[3]{1-}\right.\right.\right.\right.$$

$$\Big\downarrow 217$$

3.611. $\int \frac{1}{x^4\sqrt[3]{1-x^3}(1+x^3)}\,dx$

$$\frac{1}{3}\left(\frac{1}{3}\left(3\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)-2\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}}\right.\right.\right.\right.$$

input
```
Int[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(-((1 - x^3)^(2/3)/x^3) + (-2*(Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt
[3]] - Log[x^3]/2 + (3*Log[1 - (1 - x^3)^(1/3)])/2) + 3*((Sqrt[3]*ArcTan[(
1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3) - Log[1 + x^3]/(2*2^(1/3))
+ (3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(1/3))))/3)/3
```

3.611.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.611.4 Maple [A] (verified)

Time = 6.52 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.34

method	result
pseudoelliptic	$\frac{-6\,2^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)x^{3}-6\,2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)x^{3}+3\,2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)x^{3}}{36\Bigl(\left(-x^{3}+1\right)^{\frac{2}{3}}+(-x^{3}+1)}$

input
```
int(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/36*(-6*2^(2/3)*3^(1/2)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*x^
3-6*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*x^3+3*2^(2/3)*ln((-x^3+1)^(2/3)+2^(
1/3)*(-x^3+1)^(1/3)+2^(2/3))*x^3+8*3^(1/2)*arctan(1/3*(1+2*(-x^3+1)^(1/3))
*3^(1/2))*x^3-4*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)*x^3+8*ln(-1+(-x^3+1)^(
1/3))*x^3+12*(-x^3+1)^(2/3))/((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)/(-1+(-x^3+1
)^(1/3))
```

3.611.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^4 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{6\sqrt{6}2^{\frac{1}{6}}x^3 \arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}} + 2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right) - 3\cdot 2^{\frac{2}{3}}x^3 \log\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}} + (-x^3+1)^{\frac{2}{3}}\right)}{}$$

input
```
integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/36*(6*sqrt(6)*2^(1/6)*x^3*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6
)*(-x^3 + 1)^(1/3))) - 3*2^(2/3)*x^3*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3
) + (-x^3 + 1)^(2/3)) + 6*2^(2/3)*x^3*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 8
*sqrt(3)*x^3*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 4*x^3*lo
g((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 8*x^3*log((-x^3 + 1)^(1/3) -
1) - 12*(-x^3 + 1)^(2/3))/x^3
```

3.611.6 Sympy [F]

$$\int \frac{1}{x^4 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^4 \sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(1/x**4/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.611.7 Maxima [F]

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^4), x)
```

3.611.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}} \arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\ &\quad-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\ &\quad+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)\\ &\quad-\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{3\,x^3}\\ &\quad+\frac{1}{9}\,\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\\ &\quad-\frac{2}{9}\,\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)\end{aligned}$$

input
```
integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 2/9*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/3*(-x^3 + 1)^(2/3)/x^3 + 1/9
*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 2/9*log(abs((-x^3 + 1)^(1/
3) - 1))
```

3.611.9 Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.43

$$\int \frac{1}{x^4 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2^{2/3}\,\ln\left(\frac{2^{1/3}\left(\frac{2^{2/3}\left(81\,2^{1/3}-75\,\left(1-x^3\right)^{1/3}\right)}{6}-\frac{38}{3}\right)}{18}+\frac{16\,\left(1-x^3\right)^{1/3}}{27}\right)}{6} - \frac{\left(1-x^3\right)^{2/3}}{3\,x^3} - \frac{2\,\ln\left(\frac{344\,\left(1-x^3\right)^{1/3}}{243}-\frac{344}{243}\right)}{9}$$

$$+\ln\left(\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2\left(\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)\left(1458\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2-75\,\left(1-x^3\right)^{1/3}\right)-\frac{38}{3}\right)+\frac{16\,\left(1-x^3\right)^{1/3}}{27}\right)$$

input
```
int(1/(x^4*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((2^(1/3)*((2^(2/3)*(81*2^(1/3) - 75*(1 - x^3)^(1/3)))/6 - 38/
3))/18 + (16*(1 - x^3)^(1/3))/27))/6 - (1 - x^3)^(2/3)/(3*x^3) - (2*log((3
44*(1 - x^3)^(1/3))/243 - 344/243))/9 + log(((3^(1/2)*1i)/9 + 1/9)^2*(((3^
(1/2)*1i)/9 + 1/9)*(1458*((3^(1/2)*1i)/9 + 1/9)^2 - 75*(1 - x^3)^(1/3)) -
38/3) + (16*(1 - x^3)^(1/3))/27)*((3^(1/2)*1i)/9 + 1/9) - log((16*(1 - x^3
)^(1/3))/27 - ((3^(1/2)*1i)/9 - 1/9)^2*(((3^(1/2)*1i)/9 - 1/9)*(1458*((3^(
1/2)*1i)/9 - 1/9)^2 - 75*(1 - x^3)^(1/3)) + 38/3))*((3^(1/2)*1i)/9 - 1/9)
+ (2^(2/3)*log((16*(1 - x^3)^(1/3))/27 + (2^(1/3)*(3^(1/2)*1i - 1)^2*((2^(
2/3)*(3^(1/2)*1i - 1)*((81*2^(1/3)*(3^(1/2)*1i - 1)^2)/4 - 75*(1 - x^3)^(1
/3)))/12 - 38/3))/72)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((16*(1 - x^3)^(1
/3))/27 - (2^(1/3)*(3^(1/2)*1i + 1)^2*((2^(2/3)*(3^(1/2)*1i + 1)*((81*2^(1
/3)*(3^(1/2)*1i + 1)^2)/4 - 75*(1 - x^3)^(1/3)))/12 + 38/3))/72)*(3^(1/2)*
1i + 1))/12
```

3.612 $\int \frac{x^6}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.612.1 Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{1}{3}\log\left(x+\sqrt[3]{1-x^3}\right)$$

output
```
-1/3*x*(-x^3+1)^(2/3)-1/12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1
/3))*2^(2/3)-1/3*ln(x+(-x^3+1)^(1/3))+2/9*arctan(1/3*(1-2*x/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*
2^(2/3)*3^(1/2)
```

3.612.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.43

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{36}\Bigg(-12x\left(1-x^3\right)^{2/3}+8\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)$$

$$-6\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-8\log\left(x+\sqrt[3]{1-x^3}\right)+6\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+4\log\big(x^2-$$

input
```
Integrate[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(-12*x*(1 - x^3)^(2/3) + 8*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/
3))] - 6*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))]
 - 8*Log[x + (1 - x^3)^(1/3)] + 6*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3
)] + 4*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(2/3)*Log[-2*x
^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/36
```

3.612.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {979, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \ 979$$

$$\frac{1}{3}\int \frac{1-2x^3}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{1}{3}x\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \ 1026$$

$$\frac{1}{3}\left(3\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-2\int \frac{1}{\sqrt[3]{1-x^3}}dx\right)-\frac{1}{3}x\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \ 769$$

$$\frac{1}{3}\left(3\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-2\left(\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)-\frac{1}{3}x\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow 901$$

$$\frac{1}{3}\left(3\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)-2\left(\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\right.\right.$$
$$\frac{1}{3}x\left(1-x^3\right)^{2/3}$$

input
```
Int[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/3*(x*(1 - x^3)^(2/3)) + (3*(-(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3)
)/Sqrt[3]]/(2^(1/3)*Sqrt[3])) - Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x
) - (1 - x^3)^(1/3)]/(2*2^(1/3))) - 2*(-(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3)
)/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/2))/3
```

3.612.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.612.4 Maple [A] (verified)

Time = 6.40 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.51

method	result
pseudoelliptic	$\frac{6\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)+6\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-3\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)-1}{36\left(\left(-x^{3}+1\right)^{\frac{2}{3}}-\left(-x^{3}+1\right)^{\frac{1}{3}}x+\right.}$

input
```
int(x^6/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/36*(6*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)+
6*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)-3*2^(2/3)*ln((2^(2/3)*x^2-2^(1/
3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)-12*x*(-x^3+1)^(2/3)-8*3^(1/2)*arc
tan(1/3*(-2*(-x^3+1)^(1/3)+x)*3^(1/2)/x)+4*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/
3)*x+x^2)/x^2)-8*ln((x+(-x^3+1)^(1/3))/x))/((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*
x+x^2)/(x+(-x^3+1)^(1/3))
```

3.612.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = {}& -\frac{1}{3}\left(-x^3+1\right)^{\frac{2}{3}}x \\ & -\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(-\frac{2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}x-2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)}{6\,x}\right) \\ & +\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{1}{3}}x+(-x^3+1)^{\frac{1}{3}}}{x}\right)-\frac{1}{12} \\ & \cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right) \\ & +\frac{2}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3\,x}\right) \\ & -\frac{2}{9}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right) \\ & +\frac{1}{9}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)\end{aligned}$$

input

```
integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output

```
-1/3*(-x^3 + 1)^(2/3)*x - 1/6*sqrt(6)*2^(1/6)*arctan(-1/6*2^(1/6)*(sqrt(6)
*2^(1/3)*x - 2*sqrt(6)*(-x^3 + 1)^(1/3))/x) + 1/6*2^(2/3)*log((2^(1/3)*x +
 (-x^3 + 1)^(1/3))/x) - 1/12*2^(2/3)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 + 1)
^(1/3)*x + (-x^3 + 1)^(2/3))/x^2) + 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2
*sqrt(3)*(-x^3 + 1)^(1/3))/x) - 2/9*log((x + (-x^3 + 1)^(1/3))/x) + 1/9*lo
g((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

3.612.6 Sympy [F]

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^6}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x**6/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.612.7 Maxima [F]

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.612.8 Giac [F]

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.612.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{x^6}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(x^6/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(x^6/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.613 $\int \frac{x^3}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.613.1 Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

output

```
1/12*ln(x^3+1)*2^(2/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+1/2*ln(x+
(-x^3+1)^(1/3))-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/6
*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.613.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.52

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\frac{1}{12}\left(-4\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)+2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)+4\log\left(x+\sqrt[3]{1-x^3}\right)-2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)-2\log\left(x^2-\right.\right.$$

input

```
Integrate[x^3/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output

```
(-4*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3))] + 2*2^(2/3)*Sqrt[3
]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] + 4*Log[x + (1 - x^3)^
(1/3)] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] - 2*Log[x^2 - x*(1 -
 x^3)^(1/3) + (1 - x^3)^(2/3)] + 2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^
(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/12
```

3.613.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {983, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \ 983$$

$$\int \frac{1}{\sqrt[3]{1-x^3}}dx-\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \ 769$$

$$-\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

$$\Big\downarrow \ 901$$

$$-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}+\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

input

```
Int[x^3/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^
(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(
1/3)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3)) + Log[x + (1 - x^3
)^(1/3)]/2
```

3.613.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

3.613.4 Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.33

method	result
pseudoelliptic	$-\frac{2^{\frac{2}{3}} \ln \left(\frac{2^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)}{6}+\frac{2^{\frac{2}{3}} \ln \left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)}{12}-\frac{\sqrt{3}\, 2^{\frac{2}{3}} \arctan \left(\frac{\sqrt{3} \left(-2^{\frac{2}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3 x}\right)}{6}$

input
```
int(x^3/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)+1/12*2^(2/3)*ln((2^(2/3)*x^2
-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)-1/6*3^(1/2)*2^(2/3)*arctan(
1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)-1/6*ln(((-x^3+1)^(2/3)-(-x^3+1)
^(1/3)*x+x^2)/x^2)+1/3*3^(1/2)*arctan(1/3*(-2*(-x^3+1)^(1/3)+x)*3^(1/2)/x)
+1/3*ln((x+(-x^3+1)^(1/3))/x)
```

3.613.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 452, normalized size of antiderivative = 3.35

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{12}$$

$$\cdot 2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)\log\left(-\frac{x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^3-6\cdot 2^{\frac{1}{3}}x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2+8\,x-24\,(}{8\,x}\right.$$

$$-\frac{1}{24}\left(2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)-2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2}\right)\log\left(-\frac{3\left(2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(\right.}}{}\right.$$

$$-\frac{1}{24}\left(2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2}\right)\log\left(\frac{3\left(2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-\right.}}{}\right.$$

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3\,x}\right)$$

$$+\frac{1}{3}\log\left(\frac{x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^3+32\,x+24\,(-x^3+1)^{\frac{1}{3}}}{24\,x}\right)$$

$$-\frac{1}{6}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

input
```
integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output

```
1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(-1/8*(x*(I*sqrt(3)*(-
1)^(1/3) - (-1)^(1/3))^3 - 6*2^(1/3)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))
^2 + 8*x - 24*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) -
 (-1)^(1/3)) - 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3
))^2))*log(-3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (
-1)^(1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 2^(1/3)*x*(I*sqrt(3)
*(-1)^(1/3) - (-1)^(1/3))^2 - 8*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sq
rt(3)*(-1)^(1/3) - (-1)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)
^(1/3) - (-1)^(1/3))^2))*log(3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(
3)*(-1)^(1/3) - (-1)^(1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) - 2^(
1/3)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 8*(-x^3 + 1)^(1/3))/x) - 1/
3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/3*lo
g(1/24*(x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^3 + 32*x + 24*(-x^3 + 1)^(1/
3))/x) - 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

3.613.6 Sympy [F]

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^3}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input

```
integrate(x**3/(-x**3+1)**(1/3)/(x**3+1),x)
```

output

```
Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.613.7 Maxima [F]

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input

```
integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output

```
integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.613.8 Giac [F]

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.613.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^3}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(x^3/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(x^3/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.614 $\int \frac{1}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.614.1 Optimal result

Integrand size = 19, antiderivative size = 88

$$\int \frac{1}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx=-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
-1/12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*arct
an(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.614.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx =$$
$$-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+\log\Big(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}(1-x^3)}{6\sqrt[3]{2}}$$

input
```
Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[
2*x + 2^(2/3)*(1 - x^3)^(1/3)] + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) -
2^(1/3)*(1 - x^3)^(2/3)])/2^(1/3)
```

3.614.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{901\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}\,dx$$

$$\Big\downarrow \text{ 901}$$

$$-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}$$

input
```
Int[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
 Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3)
)
```

3.614.3.1 Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

3.614.4 Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$\frac{\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)+\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}\right)2^{\frac{2}{3}}}{6}$	95
trager	Expression too large to display	825

input
```
int(1/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*(3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)+ln((2^(1/3)
*x+(-x^3+1)^(1/3))/x)-1/2*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1
)^(2/3))/x^2))*2^(2/3)
```

3.614.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(67) = 134.

Time = 1.71 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.88

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx =$$

$$-\frac{1}{18}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-\sqrt{6}2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)+12\sqrt{}\right.}{6\,(109x^9-105x^6+3x^3+1)}\right.$$

$$+\frac{1}{18}\cdot 2^{\frac{2}{3}}\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}(x^3+1)+6\,(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)-\frac{1}{36}$$

$$\cdot 2^{\frac{2}{3}}\log\left(\frac{3\cdot 2^{\frac{2}{3}}(5x^4-x)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(19x^6-16x^3+1)-12\,(2x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)$$

input
```
integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/18*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7 + 4*x^4
 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) +
 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 +
 3*x^3 + 1)) + 1/18*2^(2/3)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*
(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/36*2^(2/3)*log((3*2^(2/3)
*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5
- x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

3.614.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(1/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.614.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.614.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.614.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(1/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(1/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.615 $\int \frac{1}{x^3\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.615.1 Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{(1-x^3)^{2/3}}{2x^2}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log(1+x^3)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
-1/2*(-x^3+1)^(2/3)/x^2+1/12*ln(x^3+1)*2^(2/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^
(1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2
/3)*3^(1/2)
```

3.615.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{12}\left(-\frac{6(1-x^3)^{2/3}}{x^2}+2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\right.\right.$$

input
```
Integrate[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((-6*(1 - x^3)^(2/3))/x^2 + 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2
/3)*(1 - x^3)^(1/3))] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(
2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/12
```

3.615.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {980, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \sqrt[3]{1-x^3} \left(x^3+1\right)} \, dx$$

$$\Big\downarrow \, 980$$

$$\frac{1}{2} \int -\frac{2}{\sqrt[3]{1-x^3} \left(x^3+1\right)}dx - \frac{\left(1-x^3\right)^{2/3}}{2x^2}$$

$$\Big\downarrow \, 27$$

$$-\int \frac{1}{\sqrt[3]{1-x^3} \left(x^3+1\right)}dx - \frac{\left(1-x^3\right)^{2/3}}{2x^2}$$

$$\Big\downarrow \, 901$$

$$\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}} - \frac{\left(1-x^3\right)^{2/3}}{2x^2}$$

input
```
Int[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/2*(1 - x^3)^(2/3)/x^2 + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt
[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[-(2^(1/3)*x) - (1
- x^3)^(1/3)]/(2*2^(1/3))
```

3.615.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

3.615.4 Maple [A] (verified)

Time = 22.00 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$\frac{-2\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)x^{2}-2\,2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)x^{2}+2\,2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right.}{12x^{2}}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-2*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)
*x^2-2*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x^2+2^(2/3)*ln((2^(2/3)*x^
2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)*x^2-6*(-x^3+1)^(2/3))/x^2
```

3.615.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(81) = 162$.

Time = 1.69 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.92

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{2\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\,x^2\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}(71x^9-\right.}{6(109x^9-105x^6+3x^3+1)}\right.}{}$$

input
```
integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/36*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*x^2*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2
/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^(1/3
)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111
*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^(1/3
)*x^2*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)^(1/3)*
(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3)*x^2*log(
-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(
19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1
)) + 18*(-x^3 + 1)^(2/3))/x^2
```

3.615.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^3\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(1/x**3/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.615.7 Maxima [F]

$$\int \frac{1}{x^3 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^3}\,dx$$

input
```
integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^3), x)
```

3.615.8 Giac [F]

$$\int \frac{1}{x^3 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^3}\,dx$$

input
```
integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^3), x)
```

3.615.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^3\,(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(1/(x^3*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^3*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.616 $\int \frac{1}{x^6\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.616.1 Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{(1-x^3)^{2/3}}{5x^5}+\frac{(1-x^3)^{2/3}}{5x^2}-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} -\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
-1/5*(-x^3+1)^(2/3)/x^5+1/5*(-x^3+1)^(2/3)/x^2-1/12*ln(x^3+1)*2^(2/3)+1/4*
ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+
1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.616.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{60}\left(-\frac{12(1-x^3)^{5/3}}{x^5}\right. -10\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)+10\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)-5\,2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-}\right.$$

input
```
Integrate[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((-12*(1 - x^3)^(5/3))/x^5 - 10*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^
(2/3)*(1 - x^3)^(1/3))] + 10*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] -
5*2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)
])/60
```

3.616.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {980, 25, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \sqrt[3]{1-x^3} \left(x^3+1\right)} \, dx$$

$$\Big\downarrow \text{980}$$

$$\frac{1}{5} \int -\frac{2-3 x^3}{x^3 \sqrt[3]{1-x^3} \left(x^3+1\right)}dx-\frac{\left(1-x^3\right)^{2/3}}{5 x^5}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{5} \int \frac{2-3 x^3}{x^3 \sqrt[3]{1-x^3} \left(x^3+1\right)}dx-\frac{\left(1-x^3\right)^{2/3}}{5 x^5}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{5} \left(\frac{1}{2} \int \frac{10}{\sqrt[3]{1-x^3} \left(x^3+1\right)}dx+\frac{\left(1-x^3\right)^{2/3}}{x^2}\right)-\frac{\left(1-x^3\right)^{2/3}}{5 x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5} \left(5 \int \frac{1}{\sqrt[3]{1-x^3} \left(x^3+1\right)}dx+\frac{\left(1-x^3\right)^{2/3}}{x^2}\right)-\frac{\left(1-x^3\right)^{2/3}}{5 x^5}$$

$$\Big\downarrow \text{901}$$

$$\frac{1}{5}\left(5\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)+\frac{\left(1-x^3\right)^{2/3}}{x^2}\right)-\frac{\left(1-x^3\right)^{2/3}}{5x^5}$$

input

```
Int[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output

```
-1/5*(1 - x^3)^(2/3)/x^5 + ((1 - x^3)^(2/3)/x^2 + 5*(-(ArcTan[(1 - (2*2^(1
/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) - Log[1 + x^3]/(6*2^(1
/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))))/5
```

3.616.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 980

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.616.4 Maple [A] (verified)

Time = 22.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{10\,2^{\frac{2}{3}} \ln \left(\frac{2^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right) x^{5}+12\left(x^{3}-1\right)\left(-x^{3}+1\right)^{\frac{2}{3}}+5\,2^{\frac{2}{3}} x^{5} \left(2\sqrt{3}\,\arctan \left(\frac{\sqrt{3}\,\left(-2^{\frac{2}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3 x}\right)-\ln \left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}}\right.\right.}{60 x^{5}}$
risch	Expression too large to display
trager	Expression too large to display

input

```
int(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output

```
1/60*(10*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x^5+12*(x^3-1)*(-x^3+1)^
(2/3)+5*2^(2/3)*x^5*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)
+x)/x)-ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)))/x^5
```

3.616.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(95) = 190.

Time = 1.76 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.28

$$\int \frac{1}{x^{6} \sqrt[3]{1-x^{3}}\,(1+x^{3})}\,dx =$$

$$-\frac{10\,\sqrt{6}2^{\frac{1}{6}} x^{5} \arctan \left(\frac{2^{\frac{1}{6}} \left(6\,\sqrt{6}2^{\frac{2}{3}}\left(5\,x^{7}+4\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-\sqrt{6}2^{\frac{1}{3}}\left(71\,x^{9}-111\,x^{6}+33\,x^{3}-1\right)+12\,\sqrt{6}\left(19\,x^{8}-16\,x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}\right)}{6\left(109\,x^{9}-105\,x^{6}+3\,x^{3}+1\right)}\right.}{}$$

input

```
integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output

```
-1/180*(10*sqrt(6)*2^(1/6)*x^5*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^
7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x
^3 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 -
105*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*x^5*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2
 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2/3)*x^5*lo
g((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1)
- 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 36*(x^3 - 1)*(-x
^3 + 1)^(2/3))/x^5
```

3.616.6 Sympy [F]

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^6\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input

```
integrate(1/x**6/(-x**3+1)**(1/3)/(x**3+1),x)
```

output

```
Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.616.7 Maxima [F]

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^6}\,dx$$

input

```
integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output

```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^6), x)
```

3.616.8 Giac [F]

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^6}\,dx$$

input
```
integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^6), x)
```

3.616.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{1}{x^6\,(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(1/(x^6*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^6*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.617 $\int \frac{1}{x^9 \sqrt[3]{1-x^3}(1+x^3)} \, dx$

3.617.1 Optimal result

Integrand size = 22, antiderivative size = 141

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3}\,(1+x^3)} \, dx = -\frac{(1-x^3)^{2/3}}{8x^8} + \frac{(1-x^3)^{2/3}}{20x^5} - \frac{17(1-x^3)^{2/3}}{40x^2} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
-1/8*(-x^3+1)^(2/3)/x^8+1/20*(-x^3+1)^(2/3)/x^5-17/40*(-x^3+1)^(2/3)/x^2+1
/12*ln(x^3+1)*2^(2/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan
(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.617.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^9\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{120}\left(-\frac{3(1-x^3)^{2/3}\,(5-2x^3+17x^6)}{x^8}\right.$$
$$\left.+20\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-20\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+10\,2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-}\right.\right.$$

input
```
Integrate[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((-3*(1 - x^3)^(2/3)*(5 - 2*x^3 + 17*x^6))/x^8 + 20*2^(2/3)*Sqrt[3]*ArcTan
[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 20*2^(2/3)*Log[2*x + 2^(2/3)
*(1 - x^3)^(1/3)] + 10*2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^
(1/3)*(1 - x^3)^(2/3)])/120
```

3.617.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {980, 27, 1053, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9\sqrt[3]{1-x^3}\,(x^3+1)}dx$$
$$\Big\downarrow \ 980$$
$$\frac{1}{8}\int -\frac{2(1-3x^3)}{x^6\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{(1-x^3)^{2/3}}{8x^8}$$
$$\Big\downarrow \ 27$$
$$-\frac{1}{4}\int \frac{1-3x^3}{x^6\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{(1-x^3)^{2/3}}{8x^8}$$
$$\Big\downarrow \ 1053$$
$$\frac{1}{4}\left(\frac{1}{5}\int \frac{17-3x^3}{x^3\sqrt[3]{1-x^3}\,(x^3+1)}dx+\frac{(1-x^3)^{2/3}}{5x^5}\right)-\frac{(1-x^3)^{2/3}}{8x^8}$$

$$\Big\downarrow 1053$$

$$\frac{1}{4}\left(\frac{1}{5}\left(-\frac{1}{2}\int \frac{40}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{17(1-x^3)^{2/3}}{2x^2}\right)+\frac{(1-x^3)^{2/3}}{5x^5}\right)-\frac{(1-x^3)^{2/3}}{8x^8}$$

$$\Big\downarrow 27$$

$$\frac{1}{4}\left(\frac{1}{5}\left(-20\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{17(1-x^3)^{2/3}}{2x^2}\right)+\frac{(1-x^3)^{2/3}}{5x^5}\right)-\frac{(1-x^3)^{2/3}}{8x^8}$$

$$\Big\downarrow 901$$

$$\frac{1}{4}\left(\frac{1}{5}\left(-20\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)-\frac{17(1-x^3)^{2/3}}{2x^2}\right)+\frac{(1-x}{5x}\right.$$

$$\frac{(1-x^3)^{2/3}}{8x^8}$$

input
```
Int[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/8*(1 - x^3)^(2/3)/x^8 + ((1 - x^3)^(2/3)/(5*x^5) + ((-17*(1 - x^3)^(2/3
))/(2*x^2) - 20*(-(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^
(1/3)*Sqrt[3])) - Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^
(1/3)]/(2*2^(1/3))))/5)/4
```

3.617.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.617.4 Maple [A] (verified)

Time = 22.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{-20\,2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right) x^{8}+\left(-51 x^{6}+6 x^{3}-15\right) \left(-x^{3}+1\right)^{\frac{2}{3}}-10\,2^{\frac{2}{3}} x^{8} \left(2 \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\, \left(-2^{\frac{2}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3 x}\right)-\ln(}{120 x^{8}}$
risch	Expression too large to display
trager	Expression too large to display

input
```
int(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/120*(-20*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x^8+(-51*x^6+6*x^3-15)
*(-x^3+1)^(2/3)-10*2^(2/3)*x^8*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x
^3+1)^(1/3)+x)/x)-ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))
/x^2)))/x^8
```

3.617.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(109) = 218$.

Time = 1.70 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx =$$

$$-\frac{20\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}x^8 \arctan\Bigg(\frac{2^{\frac{1}{6}}\Big(6\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}(71x^9}{6(109x^9-105x^6+3x^3+1)}}{}$$

input
```
integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/360*(20*sqrt(6)*2^(1/6)*(-1)^(1/3)*x^8*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^
(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^(1
/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 - 1
11*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 20*2^(2/3)*(-1)^(
1/3)*x^8*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)^(1/
3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 10*2^(2/3)*(-1)^(1/3)*x^
8*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(
2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x
^3 + 1)) + 9*(17*x^6 - 2*x^3 + 5)*(-x^3 + 1)^(2/3))/x^8
```

3.617.6 Sympy [F]

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^9 \sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(1/x**9/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.617.7 Maxima [F]

$$\int \frac{1}{x^9\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^9}\,dx$$

input
```
integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^9), x)
```

3.617.8 Giac [F]

$$\int \frac{1}{x^9\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^9}\,dx$$

input
```
integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^9), x)
```

3.617.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^9\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^9\,(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(1/(x^9*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^9*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.618 $\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

3.618.1 Optimal result

Integrand size = 22, antiderivative size = 271

$$\begin{aligned}\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= -\frac{1}{4}x^2\left(1-x^3\right)^{2/3}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}\\
&\quad+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}\\
&\quad-\frac{1}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\\
&\quad+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}\\
&\quad-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\end{aligned}$$

output
```
-1/4*x^2*(-x^3+1)^(2/3)-1/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/24*ln((1
-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x
)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1
/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*(1-
x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/3)*(1-
x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.618.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.15

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{4}x^2\left(-\left(1-x^3\right)^{2/3} + \text{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right)\right)$$

input
```
Integrate[x^7/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(x^2*(-(1 - x^3)^(2/3) + AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3]))/4
```

3.618.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {979, 27, 984, 888, 991, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(x^3+1)}\,dx$$

$$\Big\downarrow 979$$

$$\frac{1}{4}\int \frac{2x\left(1-x^3\right)^{2/3}}{x^3+1}dx - \frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow 27$$

$$\frac{1}{2}\int \frac{x\left(1-x^3\right)^{2/3}}{x^3+1}dx - \frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \; 984$$

$$\frac{1}{2}\left(2\int \frac{x}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx-\int \frac{x}{\sqrt[3]{1-x^3}}dx\right)-\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \; 888$$

$$\frac{1}{2}\left(2\int \frac{x}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\right)-\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \; 991$$

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\int \frac{1}{\frac{2(1-x)^3}{1-x^3}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}\right)-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\right)-$$
$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \; 750$$

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{3}\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}\right)\right.$$
$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}\right)-\frac{1}{2}x^2\,\text{Hy}\right.$$
$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}\right)-\frac{1}{2}x\right.$$
$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \; 1142$$

3.618. $\int \frac{x^7}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int\frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int\frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\int-\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d}{2\ 2^{2/3}}\right.\right.\right.$$

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int\frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int\frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d}{2\ 2^{2/3}}\right.\right.\right.$$

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int\frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int\frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d}{2\sqrt[3]{2}}\right.\right.\right.$$

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \textbf{1082}$$

3.618. $\int\frac{x^7}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int\frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int\frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}+\frac{\int\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-}}}{2\sqrt[3]{2}}\right.\right.\right.$$

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\sqrt[3]{2}\left(\frac{\int\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}\right)-\frac{1}{3}\int\frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\right.\right.$$

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\int\frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)-\right.\right.$$

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

$$\Big\downarrow \ 2574$$

3.618. $\int\frac{x^7}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

$$\frac{1}{2}\left(2\left(-\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)+\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}-\right.}{\sqrt{3}}\right.}{2\sqrt[3]{2}}\right.\right.\right.$$

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$

input
```
Int[x^7/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/4*(x^2*(1 - x^3)^(2/3)) + (-1/2*(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x
^3]) + 2*(-1/3*(2^(1/3)*(-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x
^3)^(1/3))/Sqrt[3]])/2^(2/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3
) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(2*2^(2/3)))) - Log[1 + (2^(1/3)*(1
 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 -
x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)) + Log[(1 - x)*(1 + x)^2]/(4*2^(
1/3)) - (3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)))/3))/2
```

3.618.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 991
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Simp[-q^2/(3*d)   Int[1/((1 - q*x)*(a + b*x^3)^(1/3)
), x], x] + Simp[q/d   Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x
^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c +
 a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.618. $\int \frac{x^7}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

3.618.4 Maple [F]

$$\int \frac{x^{7}}{\left(-x^{3}+1\right)^{\frac{1}{3}} \left(x^{3}+1\right)}d x$$

input
```
int(x^7/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x^7/(-x^3+1)^(1/3)/(x^3+1),x)
```

3.618.5 Fricas [F]

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^{7}}{{\left(x^{3} + 1\right)} {\left(-x^{3} + 1\right)}^{\frac{1}{3}}}\,{d x}$$

input
```
integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral(-(-x^3 + 1)^(2/3)*x^7/(x^6 - 1), x)
```

3.618.6 Sympy [F]

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{x^7}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x**7/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.618.7 Maxima [F]

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.618.8 Giac [F]

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.618.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^7}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(x^7/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(x^7/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.619 $\int \frac{x^4}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.619.1 Optimal result

Integrand size = 22, antiderivative size = 254

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/24*ln((1-x)*(1+x)^2)*2^(2/3)-1/1
2*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3
)+1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/8*ln(-1+x+2^(2/3)*(-x^3
+1)^(1/3))*2^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/
2))*2^(2/3)*3^(1/2)-1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/
2))*2^(2/3)*3^(1/2)
```

3.619.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.10

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{5}x^5 \operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)$$

input

```
Integrate[x^4/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output

```
(x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/5
```

3.619.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {983, 888, 991, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{983}$$

$$\int \frac{x}{\sqrt[3]{1-x^3}}dx-\int \frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{2}x^2 \operatorname{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)-\int \frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{991}$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\int \frac{1}{\frac{2(1-x)^3}{1-x^3}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{1}{2}x^2 \operatorname{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{1}{3}\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{3}\sqrt[3]{2}\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\int -\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\ 2^{2/3}}\right)+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \ 25$$

3.619. $\int \frac{x^4}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\ 2^{2/3}}\right)+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow\ 27$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right)+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow\ 1082$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right)+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

3.619. $\int \frac{x^4}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

$$\Big\downarrow 217$$

$$\frac{1}{3}\sqrt[3]{2}\left(\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}\right)+$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow 1103$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+$$

$$\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)+$$

$$\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow 2574$$

3.619. $\int \frac{x^4}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

$$\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)+$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}\right)+$$

$$\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

input

```
Int[x^4/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output

```
(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 + (2^(1/3)*(-((Sqrt[3]*ArcTa
n[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3)) - Log[1 + (
2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(2
*2^(2/3))))/3 + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) + (
-1/2*(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(
1/3) - Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + (3*Log[-1 + x + 2^(2/3)*(1 - x
^3)^(1/3)])/(4*2^(1/3)))/3
```

3.619.3.1 Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 991
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Simp[-q^2/(3*d)   Int[1/((1 - q*x)*(a + b*x^3)^(1/3)
), x], x] + Simp[q/d   Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x
^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c +
 a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.619. $\int \frac{x^4}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

3.619.4 Maple [F]

$$\int \frac{x^4}{(-x^3+1)^{\frac{1}{3}}\,(x^3+1)}dx$$

input
```
int(x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

3.619.5 Fricas [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral(-(-x^3 + 1)^(2/3)*x^4/(x^6 - 1), x)
```

3.619.6 Sympy [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^4}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x**4/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.619.7 Maxima [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.619.8 Giac [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.619.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{x^4}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input

```
int(x^4/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output

```
int(x^4/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.620 $\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)} \, dx$

3.620.1 Optimal result

Integrand size = 20, antiderivative size = 233

$$\begin{aligned}\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)} \, dx = {} & \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} \\ & + \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} \\ & - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\end{aligned}$$

output
```
1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^
(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))
*2^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2
^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2
^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.620.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.21

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$
$$= \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{}\right.}{}$$

input
```
Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1 - x
^3)^(1/3))] - 4*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2
^(1/3)*x + (1 - x^3)^(1/3))] - 4*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3
)] - 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] + 2*Log[2^(2/3) - 2*2
^(2/3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)] + L
og[2^(2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*
(1 - x^3)^(2/3)])/(12*2^(1/3))
```

3.620.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {991, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}\,dx$$

$$\Big\downarrow \text{ 991}$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\int \frac{1}{\frac{2(1-x)^3}{1-x^3}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 750}$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{3}\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{16}$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\int -\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\ 2^{2/3}}\right)-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{25}$$

3.620. $\int \frac{x}{\sqrt[3]{1-x^3}\left(1+x^3\right)}dx$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\ 2^{2/3}}\right)-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow\ 27$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right)-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow\ 1082$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right)-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

3.620. $\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{3}\sqrt[3]{2}\left(\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}\right)-$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-$$

$$\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{2574}$$

3.620. $\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

$$-\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)+\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}\right)-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

input
```
Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/3*(2^(1/3)*(-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]])/2^(2/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/
3)*(1 - x))/(1 - x^3)^(1/3)]/(2*2^(2/3)))) - Log[1 + (2^(1/3)*(1 - x))/(1
- x^3)^(1/3)]/(3*2^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x
^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)) + Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) - (3
*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)))/3
```

3.620.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 991
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Simp[-q^2/(3*d)   Int[1/((1 - q*x)*(a + b*x^3)^(1/3)
), x], x] + Simp[q/d   Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x
^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c +
 a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

3.620. $\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.620.4 Maple [F]

$$\int \frac{x}{\left(-x^{3}+1\right)^{\frac{1}{3}} \left(x^{3}+1\right)}dx$$

input
```
int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

3.620.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(171) = 342$.

Time = 1.70 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.60

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx =$$

$$-\frac{1}{36}\,\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}} \arctan \Bigg(\frac{2^{\frac{1}{6}}\Big(24\,\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\,(x^{14}-2\,x^{11}-6\,x^{8}-2\,x^{5}+x^{2})(-x^{3}+1)^{\frac{2}{3}}+12\,\sqrt{6}(-1)^{\frac{1}{3}}\,(}{6\,(x^{18}-}$$

$$-\frac{1}{72}$$

$$\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}} \log \Bigg(-\frac{12\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(x^{8}-4\,x^{5}+x^{2})(-x^{3}+1)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(x^{12}-32\,x^{9}+78\,x^{6}-32\,x^{3}+1)-}{x^{12}+4\,x^{9}+6\,x^{6}+4\,x^{3}+1}$$

$$+\frac{1}{36}$$

$$\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}} \log \left(-\frac{12\,(-x^{3}+1)^{\frac{2}{3}}x^{2}-6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(x^{6}+2\,x^{3}+1)}{x^{6}+2\,x^{3}+1}\right)$$

input
```
integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output

```
-1/36*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(-
1)^(2/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) + 12*sqrt(
6)*(-1)^(1/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1
)^(1/3) + sqrt(6)*2^(1/3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 +
 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 +
1)) - 1/72*2^(2/3)*(-1)^(1/3)*log(-(12*2^(2/3)*(-1)^(1/3)*(x^8 - 4*x^5 + x
^2)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3
 + 1) - 6*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6
*x^6 + 4*x^3 + 1)) + 1/36*2^(2/3)*(-1)^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2
 - 6*2^(1/3)*(-1)^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x
^6 + 2*x^3 + 1))/(x^6 + 2*x^3 + 1))
```

3.620.6 Sympy [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input

```
integrate(x/(-x**3+1)**(1/3)/(x**3+1),x)
```

output

```
Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.620.7 Maxima [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input

```
integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output

```
integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.620.8 Giac [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{x}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.620.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{x}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(x/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(x/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.621 $\int \frac{1}{x^2 \sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.621.1 Optimal result

Integrand size = 22, antiderivative size = 270

$$\begin{aligned}\int \frac{1}{x^2 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx =& -\frac{(1-x^3)^{2/3}}{x} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} \\ &- \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} \\ &- \frac{1}{2}x^2 \,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) \\ &- \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} \\ &+ \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\end{aligned}$$

output
```
-(-x^3+1)^(2/3)/x-1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/24*ln((1-x)*(1
+x)^2)*2^(2/3)-1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^
3+1)^(1/3))*2^(2/3)+1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/8*ln(
-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x
^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x
^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.621.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{(1-x^3)^{2/3}}{x} - x^2\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) - \frac{1}{5}x^5\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)$$

input
```
Integrate[1/(x^2*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-((1 - x^3)^(2/3)/x) - x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - (x^5*Ap
pellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/5
```

3.621.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {980, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(x^3+1)}\,dx$$

$$\Big\downarrow \text{980}$$

$$\int -\frac{x(x^3+2)}{\sqrt[3]{1-x^3}\,(x^3+1)}dx - \frac{(1-x^3)^{2/3}}{x}$$

$$\Big\downarrow \; 25$$

$$-\int \frac{x\left(x^3+2\right)}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx-\frac{\left(1-x^3\right)^{2/3}}{x}$$

$$\Big\downarrow \; 1054$$

$$-\int \left(\frac{x}{\sqrt[3]{1-x^3}}+\frac{x}{\sqrt[3]{1-x^3}\left(x^3+1\right)}\right)dx-\frac{\left(1-x^3\right)^{2/3}}{x}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)-\frac{\left(1-x^3\right)^{2/3}}{x}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}$$

input
```
Int[1/(x^2*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-((1 - x^3)^(2/3)/x) - ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sq
rt[3]]/(2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/
Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])
/2 - Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1
- x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2
^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) + Log[-1 + x + 2^(2/3)*(1 - x
^3)^(1/3)]/(4*2^(1/3))
```

3.621. $\int \frac{1}{x^2\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.621.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.621.4 Maple [F]

$$\int \frac{1}{x^{2} \left(-x^{3}+1\right)^{\frac{1}{3}} \left(x^{3}+1\right)}d x$$

input
```
int(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

3.621.5 Fricas [F]

$$\int \frac{1}{x^2 \sqrt[3]{1-x^3} (1+x^3)} \, dx=\int \frac{1}{(x^{3} + 1)(-x^{3} + 1)^{\frac{1}{3}} x^{2}} \, dx$$

input
```
integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral(-(-x^3 + 1)^(2/3)/(x^8 - x^2), x)
```

3.621.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^2\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(1/x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.621.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^2), x)
```

3.621.8 Giac [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^2), x)
```

3.621.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^2\,(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(1/(x^2*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^2*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.622 $\int \frac{1}{x^5\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.622.1 Optimal result

Integrand size = 22, antiderivative size = 289

$$\begin{aligned}\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= -\frac{(1-x^3)^{2/3}}{4x^4}+\frac{(1-x^3)^{2/3}}{2x} \\ &+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} \\ &+\frac{1}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) \\ &+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} \\ &-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\end{aligned}$$

output

```
-1/4*(-x^3+1)^(2/3)/x^4+1/2*(-x^3+1)^(2/3)/x+1/4*x^2*hypergeom([1/3, 2/3],
[5/3],x^3)+1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+
1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^
3+1)^(1/3))*2^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan
(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arct
an(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.622.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{5(1-x^3)^{2/3}\,(-1+2x^3)+15x^6\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right)+2x^9\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)}{20x^4}$$

input
```
Integrate[1/(x^5*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(5*(1 - x^3)^(2/3)*(-1 + 2*x^3) + 15*x^6*AppellF1[2/3, 1/3, 1, 5/3, x^3, -
x^3] + 2*x^9*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/(20*x^4)
```

3.622.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {980, 27, 975, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(x^3+1)}\,dx$$

$$\Big\downarrow \ 980$$

$$\frac{1}{4}\int -\frac{2\left(1-x^3\right)^{2/3}}{x^2\,(x^3+1)}dx-\frac{\left(1-x^3\right)^{2/3}}{4x^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{2}\int \frac{\left(1-x^3\right)^{2/3}}{x^2\,(x^3+1)}dx-\frac{\left(1-x^3\right)^{2/3}}{4x^4}$$

$$\Big\downarrow \ 975$$

$$\frac{1}{2}\left(\frac{\left(1-x^3\right)^{2/3}}{x}-\int -\frac{x\left(x^3+3\right)}{\sqrt[3]{1-x^3}\,(x^3+1)}dx\right)-\frac{\left(1-x^3\right)^{2/3}}{4x^4}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\int \frac{x\left(x^3+3\right)}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx+\frac{\left(1-x^3\right)^{2/3}}{x}\right)-\frac{\left(1-x^3\right)^{2/3}}{4x^4}$$

$$\Big\downarrow \text{1054}$$

$$\frac{1}{2}\left(\int \left(\frac{x}{\sqrt[3]{1-x^3}}+\frac{2x}{\sqrt[3]{1-x^3}\left(x^3+1\right)}\right)dx+\frac{\left(1-x^3\right)^{2/3}}{x}\right)-\frac{\left(1-x^3\right)^{2/3}}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\left(1-x^3\right)^{2}}{x}\right.$$

$$\frac{\left(1-x^3\right)^{2/3}}{4x^4}$$

input
```
Int[1/(x^5*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/4*(1 - x^3)^(2/3)/x^4 + ((1 - x^3)^(2/3)/x + (2^(2/3)*ArcTan[(1 - (2*2^
(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + ArcTan[(1 + (2^(1/3)*(
1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + (x^2*Hypergeometric2
F1[1/3, 2/3, 5/3, x^3])/2 + Log[(1 - x)*(1 + x)^2]/(6*2^(1/3)) + Log[1 + (
2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3
*2^(1/3)) - (2^(2/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[-
1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(2*2^(1/3)))/2
```

3.622.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 980

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.622.4 Maple [F]

$$\int \frac{1}{x^{5} \left(-x^{3}+1\right)^{\frac{1}{3}} \left(x^{3}+1\right)}dx$$

input

```
int(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

output

```
int(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

3.622.5 Fricas [F]

$$\int \frac{1}{x^5 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral(-(-x^3 + 1)^(2/3)/(x^11 - x^5), x)
```

3.622.6 Sympy [F]

$$\int \frac{1}{x^5 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^5 \sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(1/x**5/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.622.7 Maxima [F]

$$\int \frac{1}{x^5 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^5), x)
```

3.622.8 Giac [F]

$$\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^5), x)
```

3.622.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{1}{x^5\,(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(1/(x^5*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^5*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.623 $\int \frac{x^{11}}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)} \, dx$

3.623.1 Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{x^{11}}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)} \, dx = -\sqrt[3]{1-x^3}+\frac{1}{4}\left(1-x^3\right)^{4/3} -\frac{1}{7}\left(1-x^3\right)^{7/3}+\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-(-x^3+1)^(1/3)+1/4*(-x^3+1)^(4/3)-1/7*(-x^3+1)^(7/3)+1/12*ln(x^3+1)*2^(1/
3)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+
1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.623.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int \frac{x^{11}}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)} \, dx = \frac{1}{84}\left(3\sqrt[3]{1-x^3}\left(-25+x^3-4x^6\right)\right. +14\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-14\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+7\sqrt[3]{2}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-\right.$$

input
```
Integrate[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(3*(1 - x^3)^(1/3)*(-25 + x^3 - 4*x^6) + 14*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^
(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 14*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1
/3)] + 7*2^(1/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)
])/84
```

3.623.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9}{(1-x^3)^{2/3}(x^3+1)}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left((1-x^3)^{4/3}-\sqrt[3]{1-x^3}-\frac{1}{(x^3+1)(1-x^3)^{2/3}}+\frac{1}{(1-x^3)^{2/3}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{3}{7}(1-x^3)^{7/3}+\frac{3}{4}(1-x^3)^{4/3}-3\sqrt[3]{1-x^3}+\frac{\log(x^3+1)}{2\ 2^{2/3}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1\right.}{2\ 2^{2/3}}\right.$$

input
```
Int[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-3*(1 - x^3)^(1/3) + (3*(1 - x^3)^(4/3))/4 - (3*(1 - x^3)^(7/3))/7 + (Sqr
t[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3) + Log[1 + x^3]
/(2*2^(2/3)) - (3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(2/3)))/3
```

3.623.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.623.4 Maple [A] (verified)

Time = 9.50 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{\left(-4 x^{6}+x^{3}-25\right) \left(-x^{3}+1\right)^{\frac{1}{3}}}{28}+\frac{2^{\frac{1}{3}} \left(2 \arctan \left(\frac{\left(1+2^{\frac{2}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}\right) \sqrt{3}}{3}\right) \sqrt{3}+\ln \left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)-2 \ln \left(\left(\right.\right.}{12}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(x^11/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/28*(-4*x^6+x^3-25)*(-x^3+1)^(1/3)+1/12*2^(1/3)*(2*arctan(1/3*(1+2^(2/3)*
(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+
2^(2/3))-2*ln((-x^3+1)^(1/3)-2^(1/3)))
```

3.623.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14

$$
\begin{aligned}
\int \frac{x^{11}}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = {} & -\frac{1}{6} \\
& \cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24} \\
& \cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12} \\
& \cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(-1)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)-\frac{1}{28}\left(4x^6-x^3+25\right)\left(-x^3+1\right)^{\frac{1}{3}}
\end{aligned}
$$

input
```
integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(
2/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4
^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(
2/3)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3
)) - 1/28*(4*x^6 - x^3 + 25)*(-x^3 + 1)^(1/3)
```

3.623.6 Sympy [F]

$$
\int \frac{x^{11}}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \int \frac{x^{11}}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx
$$

input
```
integrate(x**11/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.623.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.95

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{7}\left(-x^3+1\right)^{\frac{7}{3}} + \frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}} + \frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) - \left(-x^3+1\right)^{\frac{1}{3}}$$

input
```
integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/7*(-x^3 + 1)^(7/3) + 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^
(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/4*(-x^3 + 1)^(4/3) + 1/12*2^(1/3)*log(2^(
2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1/3)*log(-2^(
1/3) + (-x^3 + 1)^(1/3)) - (-x^3 + 1)^(1/3)
```

3.623.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{7}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{1}{3}} + \frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}} + \frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) - \left(-x^3+1\right)^{\frac{1}{3}}$$

input
```
integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/7*(x^3 - 1)^2*(-x^3 + 1)^(1/3) + 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)
*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/4*(-x^3 + 1)^(4/3) + 1/12*2^(
1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1
/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - (-x^3 + 1)^(1/3)
```

3.623.9 Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{(1-x^3)^{4/3}}{4} - (1-x^3)^{1/3} - \frac{2^{1/3}\ln\left(3\,2^{1/3} - 3\,(1-x^3)^{1/3}\right)}{6} - \frac{(1-x^3)^{7/3}}{7} - \frac{2^{1/3}\ln\left(3\,(1-x^3)^{1/3} - \frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} + \frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2} + 3\,(1-x^3)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(x^11/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
(1 - x^3)^(4/3)/4 - (1 - x^3)^(1/3) - (2^(1/3)*log(3*2^(1/3) - 3*(1 - x^3)
^(1/3)))/6 - (1 - x^3)^(7/3)/7 - (2^(1/3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/
3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 + (2^(1/3)*log((3*2^(1/3)*(3^
(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

3.624 $\int \frac{x^8}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$

3.624.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{x^8}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx = \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
1/4*(-x^3+1)^(4/3)-1/12*ln(x^3+1)*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2
^(1/3)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.624.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.30

$$\int \frac{x^8}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx = \frac{1}{12}\Bigg(3\left(1-x^3\right)^{4/3} -2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+2\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-\sqrt[3]{2}\log\Big(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3)$$

input
```
Integrate[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(3*(1 - x^3)^(4/3) - 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] + 2*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2^(1/3)*Log[2 +
 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/12
```

3.624.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{1}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}-\sqrt[3]{1-x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}+\frac{3}{4}\left(1-x^3\right)^{4/3}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\right)$$

input
```
Int[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
((3*(1 - x^3)^(4/3))/4 - (Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqr
t[3]])/2^(2/3) - Log[1 + x^3]/(2*2^(2/3)) + (3*Log[2^(1/3) - (1 - x^3)^(1/
3)])/(2*2^(2/3)))/3
```

3.624.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.624.4 Maple [A] (verified)

Time = 9.51 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{\left(-x^{3}+1\right)^{\frac{1}{3}} x^{3}}{4}+\frac{\left(-x^{3}+1\right)^{\frac{1}{3}}}{4}+\frac{2^{\frac{1}{3}} \ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)}{6}-\frac{2^{\frac{1}{3}} \ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{12}-$ arctan
trager	Expression too large to display
risch	Expression too large to display

input
```
int(x^8/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-x^3+1)^(1/3)*x^3+1/4*(-x^3+1)^(1/3)+1/6*2^(1/3)*ln((-x^3+1)^(1/3)-2
^(1/3))-1/12*2^(1/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))-1/6
*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.624.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}(-x^3+1)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right) - \frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{2}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)-\frac{1}{4}\left(x^3-1\right)\left(-x^3+1\right)^{\frac{1}{3}}$$

input
```
integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3)
+ 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3
+ 1)^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3))
- 1/4*(x^3 - 1)*(-x^3 + 1)^(1/3)
```

3.624.6 Sympy [F]

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^8}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**8/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.624.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/4*(-x^3 + 1)^(4/3) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 +
1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)
)
```

3.624.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

input
```
integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/4*(-x^3 + 1)^(4/3) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 +
1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(
1/3)))
```

3.624.9 Mupad [B] (verification not implemented)

Time = 8.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

$$\int \frac{x^8}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{2^{1/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{2}-\frac{2^{1/3}}{2}\right)}{6}+\frac{\left(1-x^3\right)^{4/3}}{4}+\frac{2^{1/3}\ln\left(3\left(1-x^3\right)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\left(1-x^3\right)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input

```
int(x^8/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output

```
(2^(1/3)*log((1 - x^3)^(1/3)/2 - 2^(1/3)/2))/6 + (1 - x^3)^(4/3)/4 + (2^(1
/3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i -
1))/12 - (2^(1/3)*log((3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*
(3^(1/2)*1i + 1))/12
```

3.624. $\int \frac{x^8}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

3.625 $\int \frac{x^5}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)} \, dx$

3.625.1 Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{x^5}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)} \, dx = -\sqrt[3]{1-x^3}+\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-(-x^3+1)^(1/3)+1/12*ln(x^3+1)*2^(1/3)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1
/3)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.625.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.33

$$\int \frac{x^5}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)} \, dx = \frac{1}{12}\left(-12\sqrt[3]{1-x^3} \right. \\ \left. +2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-2\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+\sqrt[3]{2}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3)\right.\right.$$

input
```
Integrate[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-12*(1 - x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/
3))/Sqrt[3]] - 2*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(1/3)*Log[2
 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/12
```

3.625.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 90, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{x^3}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx^3$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{3}\left(-\int \frac{1}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx^3-3\sqrt[3]{1-x^3}\right)$$

$$\Big\downarrow \text{ 69}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\ 2^{2/3}}+\frac{3\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-3\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-3\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{2^{2/3}}-3\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-3\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\right)$$

input
```
Int[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-3*(1 - x^3)^(1/3) + (Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3
]])/2^(2/3) + Log[1 + x^3]/(2*2^(2/3)) - (3*Log[2^(1/3) - (1 - x^3)^(1/3)]
)/(2*2^(2/3)))/3
```

3.625.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.625.4 Maple [A] (verified)

Time = 8.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\left(-x^{3}+1\right)^{\frac{1}{3}}-\frac{2^{\frac{1}{3}} \ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)}{6}+\frac{2^{\frac{1}{3}} \ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{12}+\frac{\arctan\left(\frac{\left(1+2^{\frac{2}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}\right.}{3}\right.}{6}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(x^5/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-(-x^3+1)^(1/3)-1/6*2^(1/3)*ln((-x^3+1)^(1/3)-2^(1/3))+1/12*2^(1/3)*ln((-x
^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))+1/6*arctan(1/3*(1+2^(2/3)*(-x^
3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.625.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.37

$$
\begin{aligned}
\int \frac{x^{5}}{\left(1-x^{3}\right)^{2/3}\left(1+x^{3}\right)}\,dx &= -\frac{1}{6} \\
&\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}} \arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24} \\
&\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}} \log\left(-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+2\left(-x^{3}+1\right)^{\frac{2}{3}}\right) \\
&+\frac{1}{12}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}} \log\left(4^{\frac{2}{3}}(-1)^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)-\left(-x^{3}+1\right)^{\frac{1}{3}}
\end{aligned}
$$

input
```
integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(
2/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4
^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(
2/3)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3
)) - (-x^3 + 1)^(1/3)
```

3.625.6 Sympy [F]

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^5}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**5/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.625.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-\left(-x^3+1\right)^{\frac{1}{3}}$$

input
```
integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - (-x^3 + 1)^(1/3)
```

3.625.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{x^5}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = {}& \frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) \\ & + \frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) \\ & - \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) - \left(-x^3+1\right)^{\frac{1}{3}}\end{aligned}$$

input
```
integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - (-x^3 + 1)^(1/3
)
```

3.625.9 Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{x^5}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = {}& -\frac{2^{1/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{2}-\frac{2^{1/3}}{2}\right)}{6} - \left(1-x^3\right)^{1/3} \\ & - \frac{2^{1/3}\ln\left(3\left(1-x^3\right)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} \\ & + \frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\left(1-x^3\right)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}\end{aligned}$$

input
```
int(x^5/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
(2^(1/3)*log((3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*
1i + 1))/12 - (1 - x^3)^(1/3) - (2^(1/3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3
)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log((1 - x^3)^(1/3)
/2 - 2^(1/3)/2))/6
```

3.626 $\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx$

3.626.1 Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx=-\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(1+x^3)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-1/12*ln(x^3+1)*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan(
1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.626.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-2\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\ 2^{2/3}}$$

input
```
Integrate[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 2*Log[-2 +
 2^(2/3)*(1 - x^3)^(1/3)] + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 -
 x^3)^(2/3)])/2^(2/3)
```

3.626.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {946, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{3}\int \frac{1}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx^3$$

$$\Big\downarrow \text{ 69}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\ 2^{2/3}}-\frac{3\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{2^{2/3}}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\right)$$

input
```
Int[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-((Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3)) - Log[
1 + x^3]/(2*2^(2/3)) + (3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(2/3)))/3
```

3.626.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.626.4 Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$-\frac{2^{\frac{1}{3}}\left(2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}+\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)-2\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\right)}{12}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^{3}-2\right)\ln\left(\frac{-6\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^{3}-2\right)^{2}+6\_Z\operatorname{RootOf}\left(\_Z^{3}-2\right)+36\_Z^{2}\right)\operatorname{RootOf}\left(\_Z^{3}-2\right)^{4}x^{3}-180\operatorname{RootOf}}{}\right.}{}$

input
```
int(x^2/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/12*2^(1/3)*(2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+ln
((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))-2*ln((-x^3+1)^(1/3)-2^(1/3
)))
```

3.626.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

$$\int \frac{x^{2}}{\left(1-x^{3}\right)^{2/3}\left(1+x^{3}\right)}\,dx = -\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right) - \frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{2}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3)
+ 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3
+ 1)^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3))
```

3.626.6 Sympy [F]

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx=\int \frac{x^2}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**2/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.626.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx=-\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) + 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

3.626.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx=-\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

input
```
integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) + 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

3.626.9 Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{2^{1/3}\ln\left(3\,2^{1/3}-3\,(1-x^3)^{1/3}\right)}{6} + \frac{2^{1/3}\ln\left(3\,(1-x^3)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\,(1-x^3)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(x^2/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
(2^(1/3)*log(3*2^(1/3) - 3*(1 - x^3)^(1/3)))/6 + (2^(1/3)*log(3*(1 - x^3)^
(1/3) - (3*2^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*lo
g((3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

3.627 $\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx$

3.627.1 Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log(1+x^3)}{6\ 2^{2/3}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-1/2*ln(x)+1/12*ln(x^3+1)*2^(1/3)+1/2*ln(1-(-x^3+1)^(1/3))-1/4*ln(2^(1/3)-
(-x^3+1)^(1/3))*2^(1/3)-1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/
2)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.627.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.35

$$\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{12}\left(-4\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 4\log\left(-1+\sqrt[3]{1-x^3}\right) - 2\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - 2\log\left(1+\sqrt[3]{1}\right.\right.$$

input
```
Integrate[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Sqrt[3]*Ar
cTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 4*Log[-1 + (1 - x^3)^(1/3)]
- 2*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2*Log[1 + (1 - x^3)^(1/3)
+ (1 - x^3)^(2/3)] + 2^(1/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1
- x^3)^(2/3)])/12
```

3.627.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {948, 97, 69, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx^3$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{3}\left(\int \frac{1}{x^3\left(1-x^3\right)^{2/3}}dx^3-\int \frac{1}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx^3\right)$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{3}\Bigg(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}+\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\ 2^{2/3}}-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\Bigg(-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+\frac{3\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-\frac{1}{2}\log\left(x^3\right)+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3}{2}\log$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{2^{2/3}}-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3}{2}\log\left(1\right.\right.$$

$$\Big\downarrow \color{blue}{217}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^3\right)}{2}+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3}{2}\log\left(1-\right.\right.$$

$$\Big\downarrow \color{blue}{1083}$$

$$\frac{1}{3}\left(3\int \frac{1}{-x^6-3}d\left(2\sqrt[3]{1-x^3}+1\right)+\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^3\right)}{2}+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3}{2}\log\left(1-\sqrt[3]{1-}\right.\right.$$

$$\Big\downarrow \color{blue}{217}$$

$$\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)+\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^3\right)}{2}+\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3}{2}\log\left(1-\sqrt[3]{1-}\right.\right.$$

input

```
Int[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output

```
(-(Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]) + (Sqrt[3]*ArcTan[(1 +
 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3) - Log[x^3]/2 + Log[1 + x^3]/(2
*2^(2/3)) + (3*Log[1 - (1 - x^3)^(1/3)])/2 - (3*Log[2^(1/3) - (1 - x^3)^(1
/3)])/(2*2^(2/3)))/3
```

3.627. $\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx$

3.627.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.627. $\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx$

3.627.4 Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$-\frac{\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)}{6}-\frac{\arctan\left(\frac{\left(1+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln\left(-1+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)}{3}-\frac{2^{\frac{1}{3}}\ln\Big(\big(-x^{3}+1\big)^{\frac{1}{3}}}{6}$

input
```
int(1/x/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)-1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3
))*3^(1/2))*3^(1/2)+1/3*ln(-1+(-x^3+1)^(1/3))-1/6*2^(1/3)*ln((-x^3+1)^(1/3
)-2^(1/3))+1/12*2^(1/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))+
1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.627.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.33

$$\begin{aligned}\int \frac{1}{x\left(1-x^{3}\right)^{2/3}\left(1+x^{3}\right)}\,dx &= -\frac{1}{6}\\ &\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\\ &\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+2\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\\ &\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(-1)^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)\\ &-\frac{1}{6}\log\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-1\right)\end{aligned}$$

input
```
integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(
2/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4
^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(
2/3)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3
)) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*
log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) -
1)
```

3.627.6 Sympy [F]

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{1}{x\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(1/x/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.627.7 Maxima [F]

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}x}\,dx$$

input
```
integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x), x)
```

3.627.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx &= \frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\ &\quad-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{12}\\ &\quad\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)\\ &\quad-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)\end{aligned}$$

input
```
integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/12*2^(
1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1
/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/6*log((-x^3 + 1)^(2/3) + (-x
^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

3.627.9 Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.51

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{\ln\left(5-5\left(1-x^3\right)^{1/3}\right)}{3} - \frac{2^{1/3}\ln\left(6\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(\frac{2^{2/3}\left(243\,2^{1/3}+243\left(1-x^3\right)^{1/3}\right)}{36}+9\right)}{6}\right)}{6}$$

$$+\ln\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\left(243\left(1-x^3\right)^{1/3}+243-\sqrt{3}\,243\mathrm{i}\right)+9\right)+6\left(1-x^3\right)^{1/3}\right)\left(-\frac{1}{6}+\right.$$

input
```
int(1/(x*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
log(5 - 5*(1 - x^3)^(1/3))/3 - (2^(1/3)*log(6*(1 - x^3)^(1/3) - (2^(1/3)*(
(2^(2/3)*(243*2^(1/3) + 243*(1 - x^3)^(1/3)))/36 + 9))/6))/6 + log(((3^(1/
2)*1i)/6 - 1/6)*(((3^(1/2)*1i)/6 - 1/6)^2*(243*(1 - x^3)^(1/3) - 3^(1/2)*2
43i + 243) + 9) + 6*(1 - x^3)^(1/3))*((3^(1/2)*1i)/6 - 1/6) - log(6*(1 - x
^3)^(1/3) - ((3^(1/2)*1i)/6 + 1/6)*(((3^(1/2)*1i)/6 + 1/6)^2*(3^(1/2)*243i
 + 243*(1 - x^3)^(1/3) + 243) + 9))*((3^(1/2)*1i)/6 + 1/6) + ((-1)^(1/3)*2
^(1/3)*log(6*(1 - x^3)^(1/3) - ((-1)^(1/3)*2^(1/3)*(((-1)^(2/3)*2^(2/3)*(2
43*(-1)^(1/3)*2^(1/3) - 243*(1 - x^3)^(1/3)))/36 - 9))/6))/6 - ((-1)^(1/3)
*2^(1/3)*log(6*(1 - x^3)^(1/3) - ((-1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1)*(((-
1)^(2/3)*2^(2/3)*(3^(1/2)*1i + 1)^2*(243*(1 - x^3)^(1/3) + (243*(-1)^(1/3)
*2^(1/3)*(3^(1/2)*1i + 1))/2))/144 + 9))/12)*(3^(1/2)*1i + 1))/12
```

3.628 $\int \frac{1}{x^4\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

3.628.1 Optimal result

Integrand size = 22, antiderivative size = 158

$$\int \frac{1}{x^4\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = -\frac{\sqrt[3]{1-x^3}}{3x^3}+\frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(x)}{6} - \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-1/3*(-x^3+1)^(1/3)/x^3+1/6*ln(x)-1/12*ln(x^3+1)*2^(1/3)-1/6*ln(1-(-x^3+1)
^(1/3))+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)+1/9*arctan(1/3*(1+2*(-x^3+1
)^(1/3))*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2
))*2^(1/3)*3^(1/2)
```

3.628.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^4\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{1}{36}\left(-\frac{12\sqrt[3]{1-x^3}}{x^3}+4\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)\right.$$
$$\left.-6\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-4\log\left(-1+\sqrt[3]{1-x^3}\right)+6\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+2\log\left(1+\sqrt[3]{1}\right.\right.$$

input
```
Integrate[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
((-12*(1 - x^3)^(1/3))/x^3 + 4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt
[3]] - 6*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 4
*Log[-1 + (1 - x^3)^(1/3)] + 6*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] +
 2*Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(1/3)*Log[2 + 2^(2/3)*
(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/36
```

3.628.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {948, 114, 27, 174, 69, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{1}{x^6\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx^3$$

$$\Big\downarrow \text{ 114}$$

$$\frac{1}{3}\left(-\int \frac{1-2x^3}{3x^3\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx^3-\frac{\sqrt[3]{1-x^3}}{x^3}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(-\frac{1}{3}\int \frac{1-2x^3}{x^3\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx^3-\frac{\sqrt[3]{1-x^3}}{x^3}\right)$$

$$\Big\downarrow \ 174$$

$$\frac{1}{3}\left(\frac{1}{3}\left(3\int \frac{1}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx^3-\int \frac{1}{x^3\left(1-x^3\right)^{2/3}}dx^3\right)-\frac{\sqrt[3]{1-x^3}}{x^3}\right)$$

$$\Big\downarrow \ 69$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(-\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\ 2^{2/3}}-\frac{3}{\ }\right.\right.\right.$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(-\frac{3\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}\right.}{2\ }\right.\right.\right.$$

$$\Big\downarrow \ 1082$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{2^{2/3}}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1}\right.}{2\ 2^{2/3}}\right.\right.\right.$$

$$\Big\downarrow \ 217$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1}\right.}{2\ 2^{2/3}}\right.\right.\right.$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{3}\left(\frac{1}{3}\left(-3\int \frac{1}{-x^6-3}d\left(2\sqrt[3]{1-x^3}+1\right)+3\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1}\right.}{2\ 2^{2/3}}\right.\right.\right.$$

$$\Big\downarrow \ 217$$

3.628. $\int \frac{1}{x^4\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

$$\frac{1}{3}\Bigg(\frac{1}{3}\Bigg(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)+3\Bigg(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^3+1\right)}{2\ 2^{2/3}}+\frac{3\log\Big(\sqrt[3]{2}-\sqrt[3]{1-x^3}}{2\ 2^{2/3}}$$

input
```
Int[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-((1 - x^3)^(1/3)/x^3) + (Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]
 + Log[x^3]/2 - (3*Log[1 - (1 - x^3)^(1/3)])/2 + 3*(-((Sqrt[3]*ArcTan[(1 +
 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3)) - Log[1 + x^3]/(2*2^(2/3)) +
(3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(2/3))))/3)/3
```

3.628.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.628.4 Maple [A] (verified)

Time = 7.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.34

method	result
pseudoelliptic	$\frac{6\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)x^{3}-6\,2^{\frac{1}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)x^{3}+3\,2^{\frac{1}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)x^{3}-}{36\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)\right.}$

input
```
int(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/36*(6*2^(1/3)*3^(1/2)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*x^3
-6*2^(1/3)*ln((-x^3+1)^(1/3)-2^(1/3))*x^3+3*2^(1/3)*ln((-x^3+1)^(2/3)+2^(1
/3)*(-x^3+1)^(1/3)+2^(2/3))*x^3-4*3^(1/2)*arctan(1/3*(1+2*(-x^3+1)^(1/3))*
3^(1/2))*x^3-2*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)*x^3+4*ln(-1+(-x^3+1)^(1
/3))*x^3+12*(-x^3+1)^(1/3))/((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)/(-1+(-x^3+1)
^(1/3))
```

3.628.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^4\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx =$$
$$-\frac{12\cdot 4^{\frac{1}{6}}\sqrt{3}x^3\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}(-x^3+1)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)+3\cdot 4^{\frac{2}{3}}x^3\log\left(4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}+\right.}{}$$

input
```
integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/72*(12*4^(1/6)*sqrt(3)*x^3*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 +
1)^(1/3) + 4^(1/3)*sqrt(3))) + 3*4^(2/3)*x^3*log(4^(2/3)*(-x^3 + 1)^(1/3)
+ 2*(-x^3 + 1)^(2/3) + 2*4^(1/3)) - 6*4^(2/3)*x^3*log(-4^(2/3) + 2*(-x^3 +
 1)^(1/3)) - 8*sqrt(3)*x^3*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(
3)) - 4*x^3*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 8*x^3*log((-x^3
 + 1)^(1/3) - 1) + 24*(-x^3 + 1)^(1/3))/x^3
```

3.628.6 Sympy [F]

$$\int \frac{1}{x^4\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \int \frac{1}{x^4\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(1/x**4/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.628.7 Maxima [F]

$$\int \frac{1}{x^4 \left(1-x^3\right)^{2/3} \left(1+x^3\right)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^4), x)
```

3.628.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4 \left(1-x^3\right)^{2/3} \left(1+x^3\right)}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{12} \cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) - \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{3\,x^3} + \frac{1}{18}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right) - \frac{1}{9}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/12*2^
(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(
1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/3*(-x^3 + 1)^(1/3)/x^3 + 1/
18*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 1/9*log(abs((-x^3 + 1)^(
1/3) - 1))
```

3.628.9 Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.33

$$\int \frac{1}{x^4\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{2^{1/3}\ln\left(\frac{10\left(1-x^3\right)^{1/3}}{9}-\frac{2^{1/3}\left(\frac{2^{2/3}\left(243\,2^{1/3}+27\left(1-x^3\right)^{1/3}\right)}{36}-\frac{25}{3}\right)}{6}\right)}{6} - \frac{\left(1-x^3\right)^{1/3}}{3\,x^3}-\frac{\ln\left(\frac{31\left(1-x^3\right)^{1/3}}{243}-\frac{31}{243}\right)}{9}$$

$$-\ln\biggl(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)\biggl(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2\left(27\left(1-x^3\right)^{1/3}+81-\sqrt{3}\,81\mathrm{i}\right)-\frac{25}{3}\biggr)+\frac{10\left(1-x^3\right)^{1/3}}{9}\biggr)\ \biggl(-\frac{1}{18}$$

input
```
int(1/(x^4*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
(2^(1/3)*log((10*(1 - x^3)^(1/3))/9 - (2^(1/3)*((2^(2/3)*(243*2^(1/3) + 27
*(1 - x^3)^(1/3)))/36 - 25/3))/6))/6 - (1 - x^3)^(1/3)/(3*x^3) - log((31*(
1 - x^3)^(1/3))/243 - 31/243)/9 - log(((3^(1/2)*1i)/18 - 1/18)*(((3^(1/2)*
1i)/18 - 1/18)^2*(27*(1 - x^3)^(1/3) - 3^(1/2)*81i + 81) - 25/3) + (10*(1
- x^3)^(1/3))/9)*((3^(1/2)*1i)/18 - 1/18) + log((10*(1 - x^3)^(1/3))/9 - (
(3^(1/2)*1i)/18 + 1/18)*(((3^(1/2)*1i)/18 + 1/18)^2*(3^(1/2)*81i + 27*(1 -
 x^3)^(1/3) + 81) - 25/3))*((3^(1/2)*1i)/18 + 1/18) + (2^(1/3)*log((10*(1
- x^3)^(1/3))/9 - (2^(1/3)*(3^(1/2)*1i - 1)*((2^(2/3)*(3^(1/2)*1i - 1)^2*(
(243*2^(1/3)*(3^(1/2)*1i - 1))/2 + 27*(1 - x^3)^(1/3)))/144 - 25/3))/12)*(
3^(1/2)*1i - 1))/12 - (2^(1/3)*log((10*(1 - x^3)^(1/3))/9 - (2^(1/3)*(3^(1
/2)*1i + 1)*((2^(2/3)*(3^(1/2)*1i + 1)^2*((243*2^(1/3)*(3^(1/2)*1i + 1))/2
 - 27*(1 - x^3)^(1/3)))/144 + 25/3))/12)*(3^(1/2)*1i + 1))/12
```

3.629 $\int \frac{x^7}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

3.629.1 Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{x^7}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{1}{6}\log\left(-x-\sqrt[3]{1-x^3}\right) - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-1/3*x^2*(-x^3+1)^(1/3)+1/12*ln(x^3+1)*2^(1/3)+1/6*ln(-x-(-x^3+1)^(1/3))-1
/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)+1/9*arctan(1/3*(1-2*x/(-x^3+1)^(1
/3))*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2
))*2^(1/3)*3^(1/2)
```

3.629.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.39

$$\int \frac{x^7}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\frac{1}{36}\Biggl(-12x^2\sqrt[3]{1-x^3}+4\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)$$

$$-6\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)+4\log\left(x+\sqrt[3]{1-x^3}\right)-6\sqrt[3]{2}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)-2\log\bigl(x^2-x\sqrt[3]{1}$$

input

```
Integrate[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output

```
(-12*x^2*(1 - x^3)^(1/3) + 4*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(
1/3))] - 6*2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3)
)] + 4*Log[x + (1 - x^3)^(1/3)] - 6*2^(1/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1
/3)] - 2*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] + 3*2^(1/3)*Log[-2
*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/36
```

3.629.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {979, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \ 979$$

$$\frac{1}{3}\int \frac{x\left(2-x^3\right)}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx-\frac{1}{3}x^2\sqrt[3]{1-x^3}$$

$$\Big\downarrow \ 1054$$

$$\frac{1}{3}\int \left(\frac{3x}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}-\frac{x}{\left(1-x^3\right)^{2/3}}\right)dx-\frac{1}{3}x^2\sqrt[3]{1-x^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\log\left(x^3+1\right)}{2\,2^{2/3}}+\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)-\frac{3\log\left(-\sqrt[3]{\ }\right.}{2}\right.$$

$$\frac{1}{3}x^2\sqrt[3]{1-x^3}$$

input
```
Int[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-1/3*(x^2*(1 - x^3)^(1/3)) + (ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/
Sqrt[3] - (Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/2^
(2/3) + Log[1 + x^3]/(2*2^(2/3)) + Log[-x - (1 - x^3)^(1/3)]/2 - (3*Log[-(
2^(1/3)*x) - (1 - x^3)^(1/3)])/(2*2^(2/3)))/3
```

3.629.3.1 Defintions of rubi rules used

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.629.4 Maple [A] (verified)

Time = 6.54 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.46

method	result
pseudoelliptic	$\frac{3\,2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}-\left(-x^{3}+1\right)^{\frac{1}{3}}x+x^{2}}{x^{2}}\right)-6\,2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)+4\ln\Bigg(\frac{x+}{}}{36\left(x+(-x^{3}+1)^{\frac{1}{3}}\right)\left((-x^{3}+1)\right.}$

input
```
int(x^7/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/36*(3*2^(1/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x
^2)-2*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*x+x^2)/x^2)-6*2^(1/3)*ln((2^(1/3)*
x+(-x^3+1)^(1/3))/x)+4*ln((x+(-x^3+1)^(1/3))/x)-12*x^2*(-x^3+1)^(1/3)+(6*a
rctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)*2^(1/3)-4*arctan(1/3*(-2*
(-x^3+1)^(1/3)+x)*3^(1/2)/x))*3^(1/2))/(x+(-x^3+1)^(1/3))/((-x^3+1)^(2/3)+
x*(x-(-x^3+1)^(1/3)))
```

3.629.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{x^{7}}{\left(1-x^{3}\right)^{2/3}\left(1+x^{3}\right)}\,dx &= -\frac{1}{3}\left(-x^{3}+1\right)^{\frac{1}{3}}x^{2}+\frac{1}{6}\\ &\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}x\right)}{6\,x}\right)\\ &+\frac{1}{12}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x-2\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{24}\\ &\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,x^{2}+4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+2\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)\\ &+\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^{3}+1)^{\frac{1}{3}}}{3\,x}\right)+\frac{1}{9}\log\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)\\ &-\frac{1}{18}\log\left(\frac{x^{2}-\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)\end{aligned}$$

input
```
integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/3*(-x^3 + 1)^(1/3)*x^2 + 1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1
/6)*(4^(2/3)*sqrt(3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 4^(1/3)*sqrt(3)*x)/x) +
 1/12*4^(2/3)*(-1)^(1/3)*log(-(4^(2/3)*(-1)^(1/3)*x - 2*(-x^3 + 1)^(1/3))/
x) - 1/24*4^(2/3)*(-1)^(1/3)*log((2*4^(1/3)*(-1)^(2/3)*x^2 + 4^(2/3)*(-1)^
(1/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2) + 1/9*sqrt(3)*arctan(-
1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/9*log((x + (-x^3 + 1)^
(1/3))/x) - 1/18*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

3.629.6 Sympy [F]

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx=\int \frac{x^7}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**7/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.629.7 Maxima [F]

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx=\int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.629.8 Giac [F]

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx=\int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.629.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx=\int \frac{x^7}{(1-x^3)^{2/3}\,(x^3+1)}\,dx$$

input
```
int(x^7/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(x^7/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.630 $\int \frac{x^4}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

3.630.1 Optimal result

Integrand size = 22, antiderivative size = 139

$$\begin{aligned}\int \frac{x^4}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = {} & -\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} \\ & -\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{1}{2}\log\left(-x-\sqrt[3]{1-x^3}\right)+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\end{aligned}$$

output
```
-1/12*ln(x^3+1)*2^(1/3)-1/2*ln(-x-(-x^3+1)^(1/3))+1/4*ln(-2^(1/3)*x-(-x^3+
1)^(1/3))*2^(1/3)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1
/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.630.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.48

$$\begin{aligned}\int \frac{x^4}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = {} & \frac{1}{12}\left(-4\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)\right. \\ & +2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-4\log\left(x+\sqrt[3]{1-x^3}\right)+2\sqrt[3]{2}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+2\log\left(x^2-x\sqrt[3]{1}\right.\end{aligned}$$

input
```
Integrate[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-4*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3))] + 2*2^(1/3)*Sqrt[3
]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 4*Log[x + (1 - x^3)^
(1/3)] + 2*2^(1/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2*Log[x^2 - x*(1 -
 x^3)^(1/3) + (1 - x^3)^(2/3)] - 2^(1/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^
(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/12
```

3.630.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {983, 853, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{983}$$

$$\int \frac{x}{(1-x^3)^{2/3}}dx-\int \frac{x}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{853}$$

$$-\int \frac{x}{(1-x^3)^{2/3}(x^3+1)}dx-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)$$

$$\Big\downarrow \text{992}$$

$$-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}$$

input
```
Int[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^
(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(
2/3)) - Log[-x - (1 - x^3)^(1/3)]/2 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/
(2*2^(2/3))
```

3.630.3.1 Defintions of rubi rules used

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

3.630.4 Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$-\frac{2^{\frac{1}{3}} \ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)}{12}+\frac{\ln\left(\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}-\left(-x^{3}+1\right)^{\frac{1}{3}} x+x^{2}}{x^{2}}\right)}{6}+\frac{2^{\frac{1}{3}} \ln\left(\frac{2^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)}{6}-$

input
```
int(x^4/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/12*2^(1/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2
)+1/6*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*x+x^2)/x^2)+1/6*2^(1/3)*ln((2^(1/3
)*x+(-x^3+1)^(1/3))/x)-1/3*ln((x+(-x^3+1)^(1/3))/x)+1/6*(-arctan(1/3*3^(1/
2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)*2^(1/3)+2*arctan(1/3*(-2*(-x^3+1)^(1/3)+
x)*3^(1/2)/x))*3^(1/2)
```

3.630.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.42

$$
\begin{aligned}
\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx &= \frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(-\frac{4^{\frac{1}{6}}\left(4^{\frac{1}{3}}\sqrt{3}x-4^{\frac{2}{3}}\sqrt{3}(-x^3+1)^{\frac{1}{3}}\right)}{6\,x}\right) \\
&\quad+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}x+2\,(-x^3+1)^{\frac{1}{3}}}{x}\right)-\frac{1}{24} \\
&\quad\cdot 4^{\frac{2}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}x^2-4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}x+2\,(-x^3+1)^{\frac{2}{3}}}{x^2}\right) \\
&\quad-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3\,x}\right) \\
&\quad-\frac{1}{3}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right)+\frac{1}{6}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)
\end{aligned}
$$

input
```
integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/6*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*(4^(1/3)*sqrt(3)*x - 4^(2/3)*sqrt(
3)*(-x^3 + 1)^(1/3))/x) + 1/12*4^(2/3)*log((4^(2/3)*x + 2*(-x^3 + 1)^(1/3)
)/x) - 1/24*4^(2/3)*log((2*4^(1/3)*x^2 - 4^(2/3)*(-x^3 + 1)^(1/3)*x + 2*(-
x^3 + 1)^(2/3))/x^2) - 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^
3 + 1)^(1/3))/x) - 1/3*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^
3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

3.630.6 Sympy [F]

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^4}{\left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**4/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.630.7 Maxima [F]

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.630.8 Giac [F]

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.630.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{x^4}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}\,dx$$

input

```
int(x^4/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output

```
int(x^4/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.631 $\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

3.631.1 Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
1/12*ln(x^3+1)*2^(1/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)-1/6*arcta
n(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.631.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+\log\left(-2x^2+2^{2}\right.}{6\ 2^{2/3}}$$

input
```
Integrate[x/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[2*x
+ 2^(2/3)*(1 - x^3)^(1/3)] + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1
/3)*(1 - x^3)^(2/3)])/(6*2^(2/3))
```

3.631.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-x^3)^{2/3}(x^3+1)}\,dx$$

$$\Big\downarrow \; 992$$

$$-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}$$

input
```
Int[x/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) +
 Log[1 + x^3]/(6*2^(2/3)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(2/3)
)
```

3.631.3.1 Defintions of rubi rules used

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

3.631.4 Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09

method	result	siz
pseudoelliptic	$\frac{2^{\frac{1}{3}} \left(2 \sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(-2^{\frac{2}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}+x \right)}{3 x}\right)-2 \ln \left(\frac{2^{\frac{1}{3}} x +\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)+\ln \left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}} x +\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)\right)}{12}$	96
trager	Expression too large to display	939

input
```
int(x/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*2^(1/3)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)-
2*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)+ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*
x+(-x^3+1)^(2/3))/x^2))
```

3.631.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(67) = 134.

Time = 1.47 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.22

$$\int \frac{x}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)}\, dx = -\frac{1}{18}$$

$$\cdot 4^{\frac{1}{6}} \sqrt{3} (-1)^{\frac{1}{3}} \arctan \left(-\frac{4^{\frac{1}{6}} \left(6 \cdot 4^{\frac{2}{3}} \sqrt{3} (-1)^{\frac{2}{3}} \left(19 \, x^{8} - 16 \, x^{5} + x^{2}\right) (-x^{3} + 1)^{\frac{1}{3}} - 12 \, \sqrt{3} (-1)^{\frac{1}{3}} \left(5 \, x^{7} + 4 \, x^{4} - x\right) \right.}{6 \left(109 \, x^{9} - 105 \, x^{6} + 3 \, x^{3} + 1\right)} \right.$$

$$+ \frac{1}{36} \cdot 4^{\frac{2}{3}} (-1)^{\frac{1}{3}} \log \left(-\frac{3 \cdot 4^{\frac{2}{3}} (-1)^{\frac{1}{3}} \left(-x^{3} + 1\right)^{\frac{1}{3}} x^{2} - 4^{\frac{1}{3}} (-1)^{\frac{2}{3}} \left(x^{3} + 1\right) - 6 \left(-x^{3} + 1\right)^{\frac{2}{3}} x}{x^{3} + 1}\right) - \frac{1}{72}$$

$$\cdot 4^{\frac{2}{3}} (-1)^{\frac{1}{3}} \log \left(\frac{6 \cdot 4^{\frac{1}{3}} (-1)^{\frac{2}{3}} \left(5 \, x^{4} - x\right) (-x^{3} + 1)^{\frac{2}{3}} - 4^{\frac{2}{3}} (-1)^{\frac{1}{3}} \left(19 \, x^{6} - 16 \, x^{3} + 1\right) - 24 \left(2 \, x^{5} - x^{2}\right) (-x^{3} + 1)}{x^{6} + 2 \, x^{3} + 1} \right.$$

input
```
integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/18*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(-
1)^(2/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - 12*sqrt(3)*(-1)^(1/3)*
(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x^6 +
 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) + 1/36*4^(2/3)*(-1)^(1/3)*l
og(-(3*4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3)*x^2 - 4^(1/3)*(-1)^(2/3)*(x^3 +
 1) - 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/72*4^(2/3)*(-1)^(1/3)*log((6*4^
(1/3)*(-1)^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(19*x^6
 - 16*x^3 + 1) - 24*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

3.631.6 Sympy [F]

$$\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{x}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.631.7 Maxima [F]

$$\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{x}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.631.8 Giac [F]

$$\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{x}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.631.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{x}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}\,dx$$

input
```
int(x/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(x/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.632 $\int \frac{1}{x^2(1-x^3)^{2/3}(1+x^3)}\,dx$

3.632.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{1}{x^2(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(1+x^3)}{6\ 2^{2/3}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-(-x^3+1)^(1/3)/x-1/12*ln(x^3+1)*2^(1/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))
*2^(1/3)+1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^
(1/2)
```

3.632.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^2(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\sqrt[3]{1-x^3}}{x} - \frac{\arctan\left(\frac{\sqrt{3}x}{-x+2^{2/3}\sqrt[3]{1-x^3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)}{3\ 2^{2/3}} - \frac{\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\ 2^{2/3}}$$

input
```
Integrate[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-((1 - x^3)^(1/3)/x) - ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/
(2^(2/3)*Sqrt[3]) + Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[-
2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))
```

3.632.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {980, 25, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (1-x^3)^{2/3} (x^3+1)}dx$$

$$\Big\downarrow \text{980}$$

$$\int -\frac{x}{(1-x^3)^{2/3} (x^3+1)}dx-\frac{\sqrt[3]{1-x^3}}{x}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{x}{(1-x^3)^{2/3} (x^3+1)}dx-\frac{\sqrt[3]{1-x^3}}{x}$$

$$\Big\downarrow \text{992}$$

$$\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\sqrt[3]{1-x^3}}{x}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}$$

input
```
Int[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-((1 - x^3)^(1/3)/x) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]
/(2^(2/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(2/3)) + Log[-(2^(1/3)*x) - (1 - x^
3)^(1/3)]/(2*2^(2/3))
```

3.632.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

3.632.4 Maple [A] (verified)

Time = 22.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.17

method	result
pseudoelliptic	$\frac{-2\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)x+2\,2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)x-2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)}{12x}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-2*2^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)
*x+2*2^(1/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x-2^(1/3)*ln((2^(2/3)*x^2-2^
(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)*x-12*(-x^3+1)^(1/3))/x
```

3.632.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(81) = 162$.

Time = 1.42 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.64

$$\int \frac{1}{x^2 \left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx = \frac{4 \cdot 4^{\frac{1}{6}} \sqrt{3} x \arctan \left(\frac{4^{\frac{1}{6}} \left(6 \cdot 4^{\frac{2}{3}} \sqrt{3} (19 x^8 - 16 x^5 + x^2) \left(-x^3+1\right)^{\frac{1}{3}} + 12 \sqrt{3} (5 x^7 + 4 x^4 - x) \left(-x^3+1\right)^{\frac{2}{3}} -\right.}{6 \left(109 x^9 - 105 x^6 + 3 x^3 + 1\right)} \right.}{}$$

input
```
integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/72*(4*4^(1/6)*sqrt(3)*x*arctan(1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(19*x^8 -
16*x^5 + x^2)*(-x^3 + 1)^(1/3) + 12*sqrt(3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)
^(2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x
^6 + 3*x^3 + 1)) + 2*4^(2/3)*x*log((3*4^(2/3)*(-x^3 + 1)^(1/3)*x^2 + 6*(-x
^3 + 1)^(2/3)*x + 4^(1/3)*(x^3 + 1))/(x^3 + 1)) - 4^(2/3)*x*log((6*4^(1/3)
*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 - 16*x^3 + 1) - 24*(2*x^5
- x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 72*(-x^3 + 1)^(1/3))/x
```

3.632.6 Sympy [F]

$$\int \frac{1}{x^2 \left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx = \int \frac{1}{x^2 \left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}} \left(x+1\right)\left(x^2-x+1\right)} \, dx$$

input
```
integrate(1/x**2/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.632.7 Maxima [F]

$$\int \frac{1}{x^2 \left(1-x^3\right)^{2/3} \left(1+x^3\right)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)
```

3.632.8 Giac [F]

$$\int \frac{1}{x^2 \left(1-x^3\right)^{2/3} \left(1+x^3\right)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)
```

3.632.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(1-x^3\right)^{2/3} \left(1+x^3\right)}\,dx = \int \frac{1}{x^2 \left(1-x^3\right)^{2/3} \left(x^3+1\right)}\,dx$$

input
```
int(1/(x^2*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^2*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.633 $\int \frac{1}{x^5(1-x^3)^{2/3}(1+x^3)}\,dx$

3.633.1 Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{1}{x^5\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=-\frac{\sqrt[3]{1-x^3}}{4x^4}+\frac{\sqrt[3]{1-x^3}}{4x}-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-1/4*(-x^3+1)^(1/3)/x^4+1/4*(-x^3+1)^(1/3)/x+1/12*ln(x^3+1)*2^(1/3)-1/4*ln
(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)
^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.633.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^5\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\frac{1}{12}\left(-\frac{3(1-x^3)^{4/3}}{x^4}-2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\sqrt[3]{2}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+\sqrt[3]{2}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}(1\right.\right.$$

input
```
Integrate[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
((-3*(1 - x^3)^(4/3))/x^4 - 2*2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2
/3)*(1 - x^3)^(1/3))] - 2*2^(1/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(
1/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/12
```

3.633.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {980, 25, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(1-x^3\right)^{2/3} \left(x^3+1\right)}dx$$

$$\Big\downarrow \text{980}$$

$$\frac{1}{4} \int -\frac{1-3 x^3}{x^2 \left(1-x^3\right)^{2/3} \left(x^3+1\right)}dx-\frac{\sqrt[3]{1-x^3}}{4 x^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{4} \int \frac{1-3 x^3}{x^2 \left(1-x^3\right)^{2/3} \left(x^3+1\right)}dx-\frac{\sqrt[3]{1-x^3}}{4 x^4}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{4} \left(\int \frac{4 x}{\left(1-x^3\right)^{2/3} \left(x^3+1\right)}dx+\frac{\sqrt[3]{1-x^3}}{x}\right)-\frac{\sqrt[3]{1-x^3}}{4 x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4} \left(4 \int \frac{x}{\left(1-x^3\right)^{2/3} \left(x^3+1\right)}dx+\frac{\sqrt[3]{1-x^3}}{x}\right)-\frac{\sqrt[3]{1-x^3}}{4 x^4}$$

$$\Big\downarrow \text{992}$$

$$\frac{1}{4}\left(4\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}\right)+\frac{\sqrt[3]{1-x^3}}{x}\right)-\frac{\sqrt[3]{1-x^3}}{4x^4}$$

input
```
Int[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-1/4*(1 - x^3)^(1/3)/x^4 + ((1 - x^3)^(1/3)/x + 4*(-(ArcTan[(1 - (2*2^(1/3
)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + Log[1 + x^3]/(6*2^(2/3
)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(2/3))))/4
```

3.633.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.633.4 Maple [A] (verified)

Time = 22.48 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{-2\,2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)x^{4}+\left(3x^{3}-3\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{1}{3}}x^{4}\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)+\ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x\right.}{}\right.\right.}{12x^{4}}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-2*2^(1/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x^4+(3*x^3-3)*(-x^3+1)^(
1/3)+2^(1/3)*x^4*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)
/x)+ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)))/x^4
```

3.633.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(95) = 190.

Time = 1.41 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.52

$$\int \frac{1}{x^5\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx =$$

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\,x^{4}\arctan\left(-\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\left(19\,x^{8}-16\,x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-12\sqrt{3}(-1)^{\frac{1}{3}}\left(5\,x^{7}+4\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}}\sqrt{3}(71\,x^{9}\right.}{6\left(109\,x^{9}-105\,x^{6}+3\,x^{3}+1\right)}\right.}{}$$

input
```
integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/72*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^4*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqr
t(3)*(-1)^(2/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - 12*sqrt(3)*(-1)
^(1/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 11
1*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*4^(2/3)*(-1)^(1/
3)*x^4*log(-(3*4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3)*x^2 - 4^(1/3)*(-1)^(2/3
)*(x^3 + 1) - 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 4^(2/3)*(-1)^(1/3)*x^4*lo
g((6*4^(1/3)*(-1)^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*
(19*x^6 - 16*x^3 + 1) - 24*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 +
1)) - 18*(x^3 - 1)*(-x^3 + 1)^(1/3))/x^4
```

3.633.6 Sympy [F]

$$\int \frac{1}{x^5 \left(1-x^3\right)^{2/3} \left(1+x^3\right)}\, dx = \int \frac{1}{x^5 \left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}} \left(x+1\right)\left(x^2-x+1\right)}\, dx$$

input
```
integrate(1/x**5/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.633.7 Maxima [F]

$$\int \frac{1}{x^5 \left(1-x^3\right)^{2/3} \left(1+x^3\right)}\, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}} x^5}\, dx$$

input
```
integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)
```

3.633.8 Giac [F]

$$\int \frac{1}{x^5 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}} x^5} \, dx$$

input
```
integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)
```

3.633.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{x^5 (1-x^3)^{2/3} (x^3+1)} \, dx$$

input
```
int(1/(x^5*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^5*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.634 $\int \frac{x^6}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$

3.634.1 Optimal result

Integrand size = 22, antiderivative size = 291

$$\int \frac{x^6}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx=-\frac{1}{2} x \sqrt[3]{1-x^3}+\frac{\arctan\left(\frac{1-\frac{2 \sqrt[3]{2} (1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3} \sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2} (1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3} \sqrt{3}}+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}-\frac{\log\left(1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2} (1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}+\frac{\log\left(1+\frac{\sqrt[3]{2} (1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(2 \sqrt[3]{2}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3} (1-x)}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}}$$

output
```
-1/2*x*(-x^3+1)^(1/3)+1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*2^(1/3)-1/12*
ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+
1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/24*ln(2*2^(1/3)+(1-x)^2/(
-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/6*arctan(1/3*(1-2*2^
(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)+1/12*arctan(1/3*(1+2^
(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.634.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.40

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{2}x\sqrt[3]{1-x^3}\Bigg(-1$$

$$-\frac{4\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right)}{(1+x^3)\,(-4\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right)+x^3\,(3\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},x^3,-x^3\right)+\text{AppellF1}(\frac{4}{3},\frac{2}{3},1,}$$

input
```
Integrate[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(x*(1 - x^3)^(1/3)*(-1 - (4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 +
x^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/
3, 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))))/2
```

3.634.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {979, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{979}$$

$$\frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{x^3+1}dx-\frac{1}{2}x\sqrt[3]{1-x^3}$$

$$\Big\downarrow \text{927}$$

$$-\frac{9}{2}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}\sqrt[3]{1-x^3}x$$

$$\Big\downarrow \text{982}$$

$$-\frac{9}{2}\left(\frac{1}{9}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{2}{9}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}\right)-\frac{1}{2}\sqrt[3]{1-x^3}x$$

$$\Big\downarrow \text{821}$$

$$-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}\right)+\frac{1}{9}\left(\frac{\int \frac{1}{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\ 2^{2/3}}-\right.\right.$$

$$\frac{1}{2}\sqrt[3]{1-x^3}x$$

$$\Big\downarrow \text{16}$$

$$-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}}d\frac{1-}{\sqrt[3]{1}}}{3\ 2^{2/3}}\right.\right.$$

$$\frac{1}{2}\sqrt[3]{1-x^3}x$$

$$\Big\downarrow \text{1142}$$

3.634. $\int \frac{x^6}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

$$-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{\int -\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)\right)$$

$$\frac{1}{2}\sqrt[3]{1-x^3}x$$

$$\Big\downarrow \text{25}$$

$$-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\int \frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)\right)+$$

$$\frac{1}{2}\sqrt[3]{1-x^3}x$$

$$\Big\downarrow \text{27}$$

$$-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)\right)$$

$$\frac{1}{2}\sqrt[3]{1-x^3}x$$

3.634. $\int \frac{x^6}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

$$\Big\downarrow \text{1082}$$

$$\frac{-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(\right.\right.}{\frac{1}{2}\sqrt[3]{1-x^3}x}$$

$$\Big\downarrow \text{217}$$

$$\frac{-\frac{9}{2}\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\sqrt{3}}{}\right.\right.}{\frac{1}{2}\sqrt[3]{1-x^3}x}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\sqrt{3}\arctan\left(\right.}{}\right.\right.}{\frac{1}{2}\sqrt[3]{1-x^3}x}$$

3.634. $\int \frac{x^6}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

input
```
Int[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-1/2*(x*(1 - x^3)^(1/3)) - (9*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 -
 x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1
- x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(2*2^(1/3))))/(3*2^(1/3))
 - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3))))/9 + (-1/3*Log[
2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/2^(2/3) - (Sqrt[3]*ArcTan[(1 + (2^(1/3)
*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]] - Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^
(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/2)/(3*2^(2/3)))/9))/2
```

3.634.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.634.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 18.21 (sec) , antiderivative size = 694, normalized size of antiderivative = 2.38

method	result	size
trager	Expression too large to display	694
risch	Expression too large to display	988

input
```
int(x^6/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-x^3+1)^(1/3)*x+1/4*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z
^2)*ln(-(18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_
Z^3-2)^3*x^3+12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf
(_Z^3-2)^4*x^3+3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootO
f(_Z^3-2)*x^6+2*RootOf(_Z^3-2)^2*x^6-18*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-
2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^2*x^2+18*(-x^3+1)^(1/3)*Ro
otOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^4-6*RootOf(RootOf(_Z^3
-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^3-4*RootOf(_Z^3-2)^2*x^
3-12*x^2*(-x^3+1)^(2/3)-18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*Roo
tOf(_Z^3-2)+9*_Z^2)*x+3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2
)*RootOf(_Z^3-2)+2*RootOf(_Z^3-2)^2)/(1+x)^2/(x^2-x+1)^2)+1/12*RootOf(_Z^3
-2)*ln(-(18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_
Z^3-2)^2*x^3+12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf
(_Z^3-2)^3*x^3-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^6-2
*RootOf(_Z^3-2)*x^6+18*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(
_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^2+6*(-x^3+1)^(1/3)*x^4+18*RootOf(RootOf(_
Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^3+12*RootOf(_Z^3-2)*x^3-6*(-x^3+1)^
(1/3)*x-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)-2*RootOf(_Z^
3-2))/(1+x)^2/(x^2-x+1)^2)
```

3.634.5 Fricas [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.22

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{36}$$

$$\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan (- \frac{4^{\frac{1}{6}} (6\cdot 4^{\frac{2}{3}}\sqrt{3}(x^{16}-33\,x^{13}+110\,x^{10}-110\,x^7+33\,x^4-x)(-x^3+1)^{\frac{1}{3}} - 48\,\sqrt{3}(x^{14}-2\,x^{11}}{6\,(x^{18}-102\,x^{15}+447\,x^{12}}$$

$$+\frac{1}{72}\cdot 4^{\frac{2}{3}}\log\left(-\frac{12\,(-x^3+1)^{\frac{2}{3}}x^2-3\cdot 4^{\frac{2}{3}}(x^4-x)(-x^3+1)^{\frac{1}{3}}+4^{\frac{1}{3}}(x^6+2\,x^3+1)}{x^6+2\,x^3+1}\right)-\frac{1}{144}$$

$$\cdot 4^{\frac{2}{3}}\log (\frac{24\cdot 4^{\frac{1}{3}}(x^8-4\,x^5+x^2)(-x^3+1)^{\frac{2}{3}}+4^{\frac{2}{3}}(x^{12}-32\,x^9+78\,x^6-32\,x^3+1)+12\,(x^{10}-11\,x^7+11\,x^4}{x^{12}+4\,x^9+6\,x^6+4\,x^3+1}$$

$$-\frac{1}{2}\,(-x^3+1)^{\frac{1}{3}}x$$

input
```
integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/36*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(x^16 - 33*x^1
3 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 48*sqrt(3)*(x^14 -
 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(x^18 +
42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 4
47*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) + 1/72*4^(2/3)*log(-(12*(-x^3
+ 1)^(2/3)*x^2 - 3*4^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(x^6 + 2*x
^3 + 1))/(x^6 + 2*x^3 + 1)) - 1/144*4^(2/3)*log((24*4^(1/3)*(x^8 - 4*x^5 +
 x^2)*(-x^3 + 1)^(2/3) + 4^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 1
2*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4
*x^3 + 1)) - 1/2*(-x^3 + 1)^(1/3)*x
```

3.634.6 Sympy [F]

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^6}{\left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**6/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.634.7 Maxima [F]

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.634.8 Giac [F]

$$\int \frac{x^6}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.634.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{x^6}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}\,dx$$

input
```
int(x^6/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(x^6/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.635 $\int \frac{x^3}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

3.635.1 Optimal result

Integrand size = 22, antiderivative size = 294

$$\int \frac{x^3}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right) - \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} -$$

output

```
1/2*x*hypergeom([1/3, 2/3],[4/3],x^3)-1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3
))*2^(1/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)
^(1/3))*2^(1/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/24*ln(2*2
^(1/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*ar
ctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)-1/12*
arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.635.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.09

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{4}x^4\,\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},x^3,-x^3\right)$$

input
```
Integrate[x^3/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(x^4*AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])/4
```

3.635.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {983, 778, 928, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{983}$$

$$\int \frac{1}{(1-x^3)^{2/3}}dx-\int \frac{1}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{778}$$

$$x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-\int \frac{1}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{928}$$

$$-\frac{1}{2}\int \frac{1}{(1-x^3)^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{x^3+1}dx+x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$$\Big\downarrow \text{778}$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-\frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{x^3+1}dx$$

$$\Big\downarrow \text{927}$$

$$\frac{9}{2}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$$\Big\downarrow \text{982}$$

$$\frac{9}{2}\left(\frac{1}{9}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{2}{9}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}\right)+ \frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$$\Big\downarrow \text{821}$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}\right)+\frac{1}{9}\left(\frac{\int \frac{1}{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\ 2^{2/3}}-\ - \int\right.\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}}d\frac{1-x}{\sqrt[3]{1-}}}{3\ 2^{2/3}}\right.\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$$\Big\downarrow \text{1142}$$

3.635. $\int \frac{x^3}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{\int -\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$\downarrow$ 25

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\int \frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$\downarrow$ 27

3.635. $\int \frac{x^3}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$$\Big\downarrow 1082$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\right.\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$$\Big\downarrow 217$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\sqrt{3}\,\mathrm{ar}}{}\right.\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

3.635. $\int \frac{x^3}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\sqrt{3}\arctan\left(\right.}{}\right.\right.$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

input

```
Int[x^3/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output

```
(x*Hypergeometric2F1[1/3, 2/3, 4/3, x^3])/2 + (9*((2*((-((Sqrt[3]*ArcTan[(
1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3)) + Log[1 + (2^(
2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(2*2^
(1/3)))/(3*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3
))))/9 + (-1/3*Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/2^(2/3) - (Sqrt[3]*A
rcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]] - Log[2*2^(1/3) + (
1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/2)/(3*2^(2/3
)))/9))/2
```

3.635.3.1 Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.635. $\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

3.635. $\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.635.4 Maple [F]

$$\int \frac{x^3}{\left(-x^3+1\right)^{\frac{2}{3}} \left(x^3+1\right)}dx$$

input
```
int(x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

3.635.5 Fricas [F]

$$\int \frac{x^3}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx=\int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{2}{3}}} \, dx$$

input
```
integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral(-(-x^3 + 1)^(1/3)*x^3/(x^6 - 1), x)
```

3.635.6 Sympy [F]

$$\int \frac{x^3}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{x^3}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**3/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.635.7 Maxima [F]

$$\int \frac{x^3}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.635.8 Giac [F]

$$\int \frac{x^3}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.635.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^3}{(1-x^3)^{2/3}\,(x^3+1)}\,dx$$

input
```
int(x^3/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(x^3/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.636 $\int \frac{1}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

3.636.1 Optimal result

Integrand size = 19, antiderivative size = 293

$$\int \frac{1}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}$$
$$+\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right) + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \log\Bigg($$

output
```
1/2*x*hypergeom([1/3, 2/3],[4/3],x^3)+1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3
))*2^(1/3)-1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)
^(1/3))*2^(1/3)+1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/24*ln(2*2
^(1/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/6*ar
ctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)+1/12*
arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.636.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.38

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx =$$

$$-\frac{4x\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right)}{(1-x^3)^{2/3}(1+x^3)\left(-4\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right)+x^3\left(3\,\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},x^3,-x^3\right)-2\,\text{AppellF1}\right.\right.}$$

input
```
Integrate[1/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
(-4*x*AppellF1[1/3, 2/3, 1, 4/3, x^3, -x^3])/((1 - x^3)^(2/3)*(1 + x^3)*(-
4*AppellF1[1/3, 2/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, 2/3, 2, 7/3
, x^3, -x^3] - 2*AppellF1[4/3, 5/3, 1, 7/3, x^3, -x^3])))
```

3.636.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {928, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x^3)^{2/3}(x^3+1)}dx$$

$$\Big\downarrow \text{928}$$

$$\frac{1}{2}\int \frac{1}{(1-x^3)^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{x^3+1}dx$$

$$\Big\downarrow \text{778}$$

$$\frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{x^3+1}dx+\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)$$

$$\Big\downarrow \text{927}$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-\frac{9}{2}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{982}$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-\frac{9}{2}\left(\frac{1}{9}\int\frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{2}{9}\int\frac{1-x}{\sqrt[3]{1-x^3}\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\int\frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}\right)+\frac{1}{9}\left(\frac{\int\frac{1}{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\ 2^{2/3}}-\int\right.\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int\frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}}d\frac{1-x}{\sqrt[3]{1-}}}{3\ 2^{2/3}}\right.\right.$$

$$\Big\downarrow \text{1142}$$

3.636. $\int \frac{1}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{\int-\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\right.$$

$$\Big\downarrow\ 25$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\int\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\right.$$

$$\Big\downarrow\ 27$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\right.$$

$$\Big\downarrow\ 1082$$

3.636. $\int\frac{1}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-$$
$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3\int\frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\right.\right.$$

$$\Big\downarrow \color{blue}{217}$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-$$
$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\right.\right.$$

$$\Big\downarrow \color{blue}{1103}$$

$$\frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right)-$$
$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\right.\right.$$

input
```
Int[1/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

3.636. $\int\frac{1}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

output

```
(x*Hypergeometric2F1[1/3, 2/3, 4/3, x^3])/2 - (9*((2*((-((Sqrt[3]*ArcTan[(
1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3)) + Log[1 + (2^(
2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(2*2^
(1/3)))/(3*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3
))))/9 + (-1/3*Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/2^(2/3) - (Sqrt[3]*A
rcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]] - Log[2*2^(1/3) + (
1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/2)/(3*2^(2/3
)))/9))/2
```

3.636.3.1 Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 821

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

3.636. $\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx$

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.636.4 Maple [F]

$$\int \frac{1}{\left(-x^{3}+1\right)^{\frac{2}{3}} \left(x^{3}+1\right)}dx$$

input
```
int(1/(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(1/(-x^3+1)^(2/3)/(x^3+1),x)
```

3.636.5 Fricas [F]

$$\int \frac{1}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral(-(-x^3 + 1)^(1/3)/(x^6 - 1), x)
```

3.636.6 Sympy [F]

$$\int \frac{1}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{1}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(1/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(1/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.636.7 Maxima [F]

$$\int \frac{1}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.636.8 Giac [F]

$$\int \frac{1}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.636.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\int \frac{1}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}\,dx$$

input
```
int(1/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(1/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.637 $\int \frac{1}{x^3(1-x^3)^{2/3}(1+x^3)}\,dx$

3.637.1 Optimal result

Integrand size = 22, antiderivative size = 294

$$\int \frac{1}{x^3\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=-\frac{\sqrt[3]{1-x^3}}{2x^2}-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}}$$

output
```
-1/2*(-x^3+1)^(1/3)/x^2-1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*2^(1/3)+1/1
2*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3
)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/24*ln(2*2^(1/3)+(1-x)^2
/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan(1/3*(1-2*
2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)-1/12*arctan(1/3*(1+
2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.637.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 11.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^3\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\frac{\sqrt[3]{1-x^3}\Big(-1+\frac{4x^3\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right)}{(1+x^3)\left(-4\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right)+x^3\left(3\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},x^3,-x^3\right)+\text{Ap}\right.\right.}}{2x^2}$$

input
```
Integrate[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
((1 - x^3)^(1/3)*(-1 + (4*x^3*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1
+ x^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -
1/3, 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))))/(2*x^
2)
```

3.637.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.545$, Rules used = {980, 25, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(1-x^3\right)^{2/3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{980}$$

$$\frac{1}{2}\int -\frac{\sqrt[3]{1-x^3}}{x^3+1}dx-\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{x^3+1}dx-\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$$\Big\downarrow \text{927}$$

$$\frac{9}{2}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$$\Big\downarrow \text{982}$$

$$\frac{9}{2}\left(\frac{1}{9}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{2}{9}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}\right)-\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$$\Big\downarrow \text{821}$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}\right)+\frac{1}{9}\left(\frac{\int \frac{1}{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\ 2^{2/3}}-\;-\right.\right.$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}}d\frac{1-x}{\sqrt[3]{1-}}}{3\ 2^{2/3}}\right.\right.$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$$\Big\downarrow \text{1142}$$

3.637. $\int \frac{1}{x^3\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{\int -\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\right.$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$\downarrow$ 25

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\int \frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\right.$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$\downarrow$ 27

3.637. $\int \frac{1}{x^3\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\right.$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$$\Big\downarrow 1082$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\right.\right.$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

$$\Big\downarrow 217$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\sqrt{3}\,\mathrm{ar}}{\ }\right.\right.$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

3.637. $\int \frac{1}{x^3\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{9}{2}\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\sqrt{3}\arctan\left(\right.}{}\right.\right.$$

$$\frac{\sqrt[3]{1-x^3}}{2x^2}$$

input
```
Int[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

output
```
-1/2*(1 - x^3)^(1/3)/x^2 + (9*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 -
 x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1
- x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(2*2^(1/3)))/(3*2^(1/3))
 - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3))))/9 + (-1/3*Log[
2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/2^(2/3) - (Sqrt[3]*ArcTan[(1 + (2^(1/3)
*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]] - Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^
(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/2)/(3*2^(2/3)))/9))/2
```

3.637.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.637. $\int \frac{1}{x^3(1-x^3)^{2/3}(1+x^3)}\,dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.637.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 47.56 (sec) , antiderivative size = 695, normalized size of antiderivative = 2.36

method	result	size
risch	Expression too large to display	695
trager	Expression too large to display	1729

input
```
int(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^3-1)/x^2/(-x^3+1)^(2/3)+(1/4*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z
^3+2)+9*_Z^2)*ln(-(18*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^
2*RootOf(_Z^3+2)^2*x^3+12*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z
^2)*RootOf(_Z^3+2)^3*x^3-3*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_
Z^2)*x^6-2*RootOf(_Z^3+2)*x^6+9*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+3
*_Z*RootOf(_Z^3+2)+9*_Z^2)*(x^6-2*x^3+1)^(2/3)*x-18*RootOf(_Z^3+2)*(x^6-2*
x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^2-6*Roo
tOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)*x^2+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootO
f(_Z^3+2)+9*_Z^2)*x^3+4*RootOf(_Z^3+2)*x^3-3*RootOf(RootOf(_Z^3+2)^2+3*_Z*
RootOf(_Z^3+2)+9*_Z^2)-2*RootOf(_Z^3+2))/(1+x)^2/(x^2-x+1)^2)+1/12*RootOf(
_Z^3+2)*ln((36*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*RootO
f(_Z^3+2)^2*x^3+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*Root
Of(_Z^3+2)^3*x^3+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^6
+RootOf(_Z^3+2)*x^6+9*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf
(_Z^3+2)+9*_Z^2)*(x^6-2*x^3+1)^(2/3)*x-6*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1
/3)*x^2-36*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^3-6*RootO
f(_Z^3+2)*x^3-6*(x^6-2*x^3+1)^(2/3)*x+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootO
f(_Z^3+2)+9*_Z^2)+RootOf(_Z^3+2))/(1+x)^2/(x^2-x+1)^2))/(-x^3+1)^(2/3)*((x
^3-1)^2)^(1/3)
```

3.637.5 Fricas [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^3\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx =$$

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\,x^2\arctan\left(\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\left(x^{16}-33\,x^{13}+110\,x^{10}-110\,x^{7}+33\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+48\,\sqrt{3}(-1)^{\frac{1}{3}}\left(x^{14}-2\,x^{11}-6\,x^{8}-2\,x\right.\right.}{6\left(x^{18}-102\,x^{15}+447\,x^{12}-628\,x^{9}+447\,x^{6}\right.}\right.}{}$$

input
```
integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/144*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^2*arctan(1/6*4^(1/6)*(6*4^(2/3)*sqr
t(3)*(-1)^(2/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 +
 1)^(1/3) + 48*sqrt(3)*(-1)^(1/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-
x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 41
7*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102
*x^3 + 1)) + 4^(2/3)*(-1)^(1/3)*x^2*log((24*4^(1/3)*(-1)^(2/3)*(x^8 - 4*x^
5 + x^2)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(x^12 - 32*x^9 + 78*x^6 - 3
2*x^3 + 1) + 12*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x
^9 + 6*x^6 + 4*x^3 + 1)) - 2*4^(2/3)*(-1)^(1/3)*x^2*log(-(12*(-x^3 + 1)^(2
/3)*x^2 + 3*4^(2/3)*(-1)^(1/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(-1)^(
2/3)*(x^6 + 2*x^3 + 1))/(x^6 + 2*x^3 + 1)) + 72*(-x^3 + 1)^(1/3))/x^2
```

3.637.6 Sympy [F]

$$\int \frac{1}{x^3\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \int \frac{1}{x^{3}\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

input
```
integrate(1/x**3/(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.637.7 Maxima [F]

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}x^3}\,dx$$

input
```
integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)
```

3.637.8 Giac [F]

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}x^3}\,dx$$

input
```
integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)
```

3.637.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)}\,dx = \int \frac{1}{x^3 (1-x^3)^{2/3} (x^3+1)}\,dx$$

input
```
int(1/(x^3*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^3*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.638 $\int \frac{x^{14}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

3.638.1 Optimal result

Integrand size = 22, antiderivative size = 141

$$\begin{aligned}\int \frac{x^{14}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{1}{2\sqrt[3]{1-x^3}}+\left(1-x^3\right)^{2/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}\\ &\quad+\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\end{aligned}$$

output
```
1/2/(-x^3+1)^(1/3)+(-x^3+1)^(2/3)-2/5*(-x^3+1)^(5/3)+1/8*(-x^3+1)^(8/3)-1/
24*ln(x^3+1)*2^(2/3)+1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/
3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.638.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{x^{14}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{1}{120}\left(-\frac{3\left(-49+23x^3+x^6+5x^9\right)}{\sqrt[3]{1-x^3}}\right.\\ &\quad+10\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+10\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-5\,2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(\right.\end{aligned}$$

input
```
Integrate[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((-3*(-49 + 23*x^3 + x^6 + 5*x^9))/(1 - x^3)^(1/3) + 10*2^(2/3)*Sqrt[3]*Ar
cTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 10*2^(2/3)*Log[-2 + 2^(2/3)*
(1 - x^3)^(1/3)] - 5*2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1
- x^3)^(2/3)])/120
```

3.638.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^{12}}{(1-x^3)^{4/3}(x^3+1)}dx^3$$

$$\Big\downarrow \text{98}$$

$$\frac{1}{3}\int \left(-\frac{x^6}{\sqrt[3]{1-x^3}}-\frac{1}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}(1-x^6)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}+\frac{3}{8}(1-x^3)^{8/3}-\frac{6}{5}(1-x^3)^{5/3}+3(1-x^3)^{2/3}+\frac{3}{2\sqrt[3]{1-x^3}}-\frac{\log(x^3+1)}{4\sqrt[3]{2}}+\right.$$

input
```
Int[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(3/(2*(1 - x^3)^(1/3)) + 3*(1 - x^3)^(2/3) - (6*(1 - x^3)^(5/3))/5 + (3*(1
 - x^3)^(8/3))/8 + (Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])
/(2*2^(1/3)) - Log[1 + x^3]/(4*2^(1/3)) + (3*Log[2^(1/3) - (1 - x^3)^(1/3)
])/(4*2^(1/3)))/3
```

3.638.3.1 Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.638.4 Maple [A] (verified)

Time = 8.93 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$\frac{-15x^{9}-3x^{6}+10\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}+10\,2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-5\,2^{\frac{2}{3}}\ln((-x}{120(-x^{3}+1)^{\frac{1}{3}}}$
trager	$\frac{(5x^{9}+x^{6}+23x^{3}-49)\left(-x^{3}+1\right)^{\frac{2}{3}}}{40x^{3}-40}+\frac{\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^{3}-4\right)^{2}+6\_Z\,\mathrm{RootOf}\left(\_Z^{3}-4\right)+36\_Z^{2}\right)\ln(\frac{15\,\mathrm{RootOf}(\mathrm{Ro}}{}}{}$
risch	Expression too large to display

input
```
int(x^14/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/120*(-15*x^9-3*x^6+10*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(
2/3)*3^(1/2)*(-x^3+1)^(1/3)+10*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)
^(1/3)-5*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-x^3+1
)^(1/3)-69*x^3+147)/(-x^3+1)^(1/3)
```

3.638.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{10\sqrt{6}2^{\frac{1}{6}}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right)-5\cdot 2^{\frac{2}{3}}(x^3-1)\,\mathrm{lo}}{}$$

input
```
integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/120*(10*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) +
2*sqrt(6)*(-x^3 + 1)^(1/3))) - 5*2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(
-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 10*2^(2/3)*(x^3 - 1)*log(-2^(1/3) +
(-x^3 + 1)^(1/3)) + 3*(5*x^9 + x^6 + 23*x^3 - 49)*(-x^3 + 1)^(2/3))/(x^3 -
 1)
```

3.638.6 Sympy [F]

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{14}}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**14/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**14/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.638.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx &= \frac{1}{8}\left(-x^3+1\right)^{\frac{8}{3}} \\ &+ \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}} \\ &- \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12} \\ &\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}\end{aligned}$$

input
```
integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
1/8*(-x^3 + 1)^(8/3) + 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^
(1/3) + 2*(-x^3 + 1)^(1/3))) - 2/5*(-x^3 + 1)^(5/3) - 1/24*2^(2/3)*log(2^(
2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/12*2^(2/3)*log(-2^
(1/3) + (-x^3 + 1)^(1/3)) + (-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

3.638.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx &= \frac{1}{8}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{2}{3}} \\ &+ \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}} \\ &- \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{12} \\ &\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) + \left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}\end{aligned}$$

input
```
integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/8*(x^3 - 1)^2*(-x^3 + 1)^(2/3) + 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)
*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) - 2/5*(-x^3 + 1)^(5/3) - 1/24*2^(
2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/12*2^(
2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + (-x^3 + 1)^(2/3) + 1/2/(-x^3
+ 1)^(1/3)
```

3.638.9 Mupad [B] (verification not implemented)

Time = 8.54 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{1}{2\,(1-x^3)^{1/3}}+(1-x^3)^{2/3}-\frac{2\,(1-x^3)^{5/3}}{5}+\frac{(1-x^3)^{8/3}}{8}+\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24}-\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

input

```
int(x^14/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output

```
(2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) +
(1 - x^3)^(2/3) - (2*(1 - x^3)^(5/3))/5 + (1 - x^3)^(8/3)/8 + (2^(2/3)*log
((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/24
 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1
/2)*1i + 1))/24
```

3.639 $\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx$

3.639.1 Optimal result

Integrand size = 22, antiderivative size = 130

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3} -\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log(1+x^3)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/(-x^3+1)^(1/3)+1/2*(-x^3+1)^(2/3)-1/5*(-x^3+1)^(5/3)+1/24*ln(x^3+1)*2^
(2/3)-1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan(1/3*(1+2^(2/3)*(-
x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.639.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{120}\Bigg(-\frac{12(-8+x^3+2x^6)}{\sqrt[3]{1-x^3}} -10\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-10\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+5\,2^{2/3}\log\Big(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\big($$

input
```
Integrate[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output

```
((-12*(-8 + x^3 + 2*x^6))/(1 - x^3)^(1/3) - 10*2^(2/3)*Sqrt[3]*ArcTan[(1 +
 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 10*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)
^(1/3)] + 5*2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2
/3)])/120
```

3.639.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx^3$$

$$\Big\downarrow \text{98}$$

$$\frac{1}{3}\int \left(-\frac{x^3}{\sqrt[3]{1-x^3}}-\frac{x^3}{\sqrt[3]{1-x^3}\left(x^6-1\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{3}{5}\left(1-x^3\right)^{5/3}+\frac{3}{2}\left(1-x^3\right)^{2/3}+\frac{3}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{4\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}-\right.}{4\sqrt[3]{2}}\right.$$

input

```
Int[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output

```
(3/(2*(1 - x^3)^(1/3)) + (3*(1 - x^3)^(2/3))/2 - (3*(1 - x^3)^(5/3))/5 - (
Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)) + Log[1
 + x^3]/(4*2^(1/3)) - (3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(4*2^(1/3)))/3
```

3.639.3.1 Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.639.4 Maple [A] (verified)

Time = 8.62 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{-24x^{6}-10\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}-10\,2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+5\,2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)\right.}{120(-x^{3}+1)^{\frac{1}{3}}}$
trager	$\frac{(2x^{6}+x^{3}-8)\left(-x^{3}+1\right)^{\frac{2}{3}}}{10x^{3}-10}+\frac{\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^{3}+4\right)^{2}+6\_Z\operatorname{RootOf}\left(\_Z^{3}+4\right)+36\_Z^{2}\right)\ln\left(\frac{15\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_\right.\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(x^11/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/120*(-24*x^6-10*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3
^(1/2)*(-x^3+1)^(1/3)-10*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(1/3)
+5*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-x^3+1)^(1/3
)-12*x^3+96)/(-x^3+1)^(1/3)
```

3.639.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.22

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx =$$

$$-\frac{10\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}(-1)^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\right)\right)+5\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^3-1)\log\left(2^{\frac{1}{3}}(\right.}{}$$

input
```
integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/120*(10*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt
(6)*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 5*2^(2/3)*(-1)^(1/3)
*(x^3 - 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) +
(-x^3 + 1)^(2/3)) - 10*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3
) + (-x^3 + 1)^(1/3)) - 12*(2*x^6 + x^3 - 8)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

3.639.6 Sympy [F]

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{11}}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**11/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.639.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx &= -\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right)\\ &\quad-\frac{1}{5}(-x^3+1)^{\frac{5}{3}}+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)\\ &\quad-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)+\frac{1}{2}(-x^3+1)^{\frac{2}{3}}+\frac{1}{2(-x^3+1)^{\frac{1}{3}}}\end{aligned}$$

input
```
integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(
1/3))) - 1/5*(-x^3 + 1)^(5/3) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 +
 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/
3)) + 1/2*(-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

3.639.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx &= -\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\ &\quad-\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\ &\quad-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}\end{aligned}$$

input
```
integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(
1/3))) - 1/5*(-x^3 + 1)^(5/3) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 +
 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)
^(1/3))) + 1/2*(-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

3.639.9 Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx &= \frac{1}{2\,(1-x^3)^{1/3}}-\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{(1-x^3)^{2/3}}{2}\\ &\quad-\frac{(1-x^3)^{5/3}}{5}-\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24}\\ &\quad+\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}\end{aligned}$$

input
```
int(x^11/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
1/(2*(1 - x^3)^(1/3)) - (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 +
(1 - x^3)^(2/3)/2 - (1 - x^3)^(5/3)/5 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (
2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/24 + (2^(2/3)*log((1 - x
^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*1i + 1))/24
```

3.639. $\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx$

3.640 $\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx$

3.640.1 Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/(-x^3+1)^(1/3)+1/2*(-x^3+1)^(2/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(2^(1/3
)-(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/
2))*2^(2/3)*3^(1/2)
```

3.640.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.15

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{24}\Bigg(-\frac{12(-2+x^3)}{\sqrt[3]{1-x^3}} + 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - 2^{2/3}\log\Big(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x$$

input
```
Integrate[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((-12*(-2 + x^3))/(1 - x^3)^(1/3) + 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*
(1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] -
2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

3.640.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{(1-x^3)^{4/3}(x^3+1)}dx^3$$

$$\Big\downarrow \text{98}$$

$$\frac{1}{3}\int \left(\frac{1}{\sqrt[3]{1-x^3}\,(1-x^6)}-\frac{1}{\sqrt[3]{1-x^3}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}+\frac{3}{2}(1-x^3)^{2/3}+\frac{3}{2\sqrt[3]{1-x^3}}-\frac{\log(x^3+1)}{4\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\right)$$

input
```
Int[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(3/(2*(1 - x^3)^(1/3)) + (3*(1 - x^3)^(2/3))/2 + (Sqrt[3]*ArcTan[(1 + 2^(2
/3)*(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)) - Log[1 + x^3]/(4*2^(1/3)) + (3
*Log[2^(1/3) - (1 - x^3)^(1/3)])/(4*2^(1/3)))/3
```

3.640.3.1 Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.640.4 Maple [A] (verified)

Time = 8.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$-\frac{-2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}+12x^{3}+2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-2\,2}{24\left(-x^{3}+1\right)^{\frac{1}{3}}}$
risch	Expression too large to display
trager	Expression too large to display

input
```
int(x^8/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)*(
-x^3+1)^(1/3)+12*x^3+2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2
/3))*(-x^3+1)^(1/3)-2*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(1/3)-24
)/(-x^3+1)^(1/3)
```

3.640.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.13

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{2\sqrt{6}2^{\frac{1}{6}}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}(x^3-1)\log\left(\right.}{}$$

input
```
integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*
sqrt(6)*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3
 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3
 + 1)^(1/3)) + 12*(x^3 - 2)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

3.640.6 Sympy [F]

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^8}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**8/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.640.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx &= \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\ &\quad-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\\ &\quad\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}\end{aligned}$$

input
```
integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) + 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2*(-x^3 + 1)^(2/
3) + 1/2/(-x^3 + 1)^(1/3)
```

3.640.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) + \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

input
```
integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2*(-x^3 + 1
)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

3.640.9 Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12} + \frac{1}{2(1-x^3)^{1/3}} + \frac{(1-x^3)^{2/3}}{2} + \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24} - \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

input
```
int(x^8/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) +
(1 - x^3)^(2/3)/2 + (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i
- 1)^2)/16)*(3^(1/2)*1i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/
3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*1i + 1))/24
```

3.640. $\int \frac{x^8}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

3.641 $\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx$

3.641.1 Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx=\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log(1+x^3)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/(-x^3+1)^(1/3)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2
^(2/3)-1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.641.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.26

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx=\frac{1}{24}\left(\frac{12}{\sqrt[3]{1-x^3}}\right.$$
$$-2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-\right.$$

input
```
Integrate[x^5/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(12/(1 - x^3)^(1/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] - 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(2/3)*Log[2
+ 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

3.641.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(1-x^3)^{4/3}(x^3+1)}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{3}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}(x^3+1)}dx^3\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}+\frac{\log(x^3+1)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}+\frac{\log(x^3+1)}{2\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}+\frac{\log(x^3+1)}{2\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

input
```
Int[x^5/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(3/(2*(1 - x^3)^(1/3)) + (-((Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/
Sqrt[3]])/2^(1/3)) + Log[1 + x^3]/(2*2^(1/3)) - (3*Log[2^(1/3) - (1 - x^3)
^(1/3)])/(2*2^(1/3)))/2)/3
```

3.641.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.641. $\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx$

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.641.4 Maple [A] (verified)

Time = 8.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$\frac{-2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}-2\,2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x\right.\right.}{24\left(-x^{3}+1\right)^{\frac{1}{3}}}$
trager	$-\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{2\left(x^{3}-1\right)}+\frac{\operatorname{RootOf}\left(\_Z^{3}+4\right)\ln\left(-\frac{6\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^{3}+4\right)^{2}+6\_Z\operatorname{RootOf}\left(\_Z^{3}+4\right)+36\_Z^{2}\right)\operatorname{RootOf}\left(\_Z^{3}\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(x^5/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)*(-
x^3+1)^(1/3)-2*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(1/3)+2^(2/3)*l
n((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-x^3+1)^(1/3)+12)/(-x^3+
1)^(1/3)
```

3.641.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(73) = 146.

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.48

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx =$$

$$-\frac{2\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}(-1)^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\right)\right)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^3-1)\log\left(2^{\frac{1}{3}}(-1)\right.}{24}$$

input
```
integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/24*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(6
)*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 2^(2/3)*(-1)^(1/3)*(x^
3 - 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^
3 + 1)^(2/3)) - 2*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) + (
-x^3 + 1)^(1/3)) + 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

3.641.6 Sympy [F]

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^5}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**5/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.641.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

input
```
integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(
1/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^
(2/3)) - 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2/(-x^3 + 1)^(1
/3)
```

3.641.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

input
```
integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(
1/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^
(2/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2/(-x^3 +
1)^(1/3)
```

3.641.9 Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2(1-x^3)^{1/3}} - \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}$$
$$-\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$
$$+\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

input
```
int(x^5/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
1/(2*(1 - x^3)^(1/3)) - (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 -
(2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)
*1i - 1))/24 + (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^
2)/16)*(3^(1/2)*1i + 1))/24
```

3.642 $\int \frac{x^2}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)} \, dx$

3.642.1 Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{x^2}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)} \, dx = \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/(-x^3+1)^(1/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2
^(2/3)+1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.642.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)} \, dx = \frac{1}{24}\left(\frac{12}{\sqrt[3]{1-x^3}} + 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - 2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-\right.\right.$$

input
```
Integrate[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(12/(1 - x^3)^(1/3) + 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] + 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2^(2/3)*Log[2
+ 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

3.642.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {946, 61, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx^3$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx^3+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

input
```
Int[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(3/(2*(1 - x^3)^(1/3)) + ((Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sq
rt[3]])/2^(1/3) - Log[1 + x^3]/(2*2^(1/3)) + (3*Log[2^(1/3) - (1 - x^3)^(1
/3)])/(2*2^(1/3)))/2)/3
```

3.642.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.642. $\int \frac{x^2}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.642.4 Maple [A] (verified)

Time = 8.42 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$-\frac{-2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-2\,2^{\frac{2}{3}}\ln\left(\left(\right.\right.}{24\left(-x^{3}+1\right)^{\frac{1}{3}}}$
trager	$-\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{2\left(x^{3}-1\right)}+\frac{\operatorname{RootOf}\left(\_Z^{3}-4\right)\ln\left(\frac{-6\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^{3}-4\right)^{2}+6\_Z\operatorname{RootOf}\left(\_Z^{3}-4\right)+36\_Z^{2}\right)\operatorname{RootOf}\left(\_Z^{3}\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(x^2/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)*(
-x^3+1)^(1/3)+2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-
x^3+1)^(1/3)-2*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(1/3)-12)/(-x^3
+1)^(1/3)
```

3.642.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{2\sqrt{6}2^{\frac{1}{6}}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}(x^3-1)\log\left(\right.}{}$$

input
```
integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*
sqrt(6)*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3
 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3
 + 1)^(1/3)) - 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

3.642.6 Sympy [F]

$$\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^2}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**2/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.642.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx &= \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\ &\quad-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\ &\quad+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}\end{aligned}$$

input
```
integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) + 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2/(-x^3 + 1)^(1/
3)
```

3.642.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx &= \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\ &\quad-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\ &\quad+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}\end{aligned}$$

input
```
integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2/(-x^3 + 1
)^(1/3)
```

3.642.9 Mupad [B] (verification not implemented)

Time = 8.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\begin{aligned}\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx &= \frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{1}{2\left(1-x^3\right)^{1/3}}\\ &\quad+\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24}\\ &\quad-\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}\end{aligned}$$

input
```
int(x^2/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) +
(2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)
*1i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^
2)/16)*(3^(1/2)*1i + 1))/24
```

3.642. $\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx$

3.643 $\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

3.643.1 Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output

```
1/2/(-x^3+1)^(1/3)-1/2*ln(x)+1/24*ln(x^3+1)*2^(2/3)+1/2*ln(1-(-x^3+1)^(1/3
))-1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/
3))*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2
^(2/3)*3^(1/2)
```

3.643.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{24}\left(\frac{12}{\sqrt[3]{1-x^3}}+8\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)\right.$$
$$-2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+8\log\left(-1+\sqrt[3]{1-x^3}\right)-2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-4\log\left(1+\right.$$

input
```
Integrate[1/(x*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(12/(1 - x^3)^(1/3) + 8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] -
2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 8*Log[-1
 + (1 - x^3)^(1/3)] - 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 4*Log[
1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] + 2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)
^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

3.643.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {948, 96, 174, 67, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx$$
$$\Big\downarrow \text{948}$$
$$\frac{1}{3}\int \frac{1}{x^3\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx^3$$
$$\Big\downarrow \text{96}$$
$$\frac{1}{3}\left(\frac{1}{2}\int \frac{x^3+2}{x^3\sqrt[3]{1-x^3}\left(x^3+1\right)}dx^3+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$
$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2\int\frac{1}{x^3\sqrt[3]{1-x^3}}dx^3-\int\frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx^3\right)+\frac{3}{2\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \; 67$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3\int\frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}-\frac{3}{2}\int\frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}+2\left(-\frac{3}{2}\int\frac{1}{1-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x}\right.\right.\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{3}{2}\int\frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}+2\left(\frac{3}{2}\int\frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)+\frac{3}{2}\log\left(1\right.\right.\right.\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3\int\frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}+2\left(\frac{3}{2}\int\frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right.\right.\right.\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2\left(\frac{3}{2}\int\frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\right)-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right.}{\sqrt[3]{2}}\right.\right.$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2\left(-3\int\frac{1}{-x^6-3}d\left(2\sqrt[3]{1-x^3}+1\right)-\frac{1}{2}\log\left(x^3\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\right)-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}\right.\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+2\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)-\frac{\log\left(x^3\right)}{2}+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\right)+\right.\right.$$

3.643. $\int\frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx$

input
```
Int[1/(x*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(3/(2*(1 - x^3)^(1/3)) + (-((Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/
Sqrt[3]])/2^(1/3)) + Log[1 + x^3]/(2*2^(1/3)) + 2*(Sqrt[3]*ArcTan[(1 + 2*(
1 - x^3)^(1/3))/Sqrt[3]] - Log[x^3]/2 + (3*Log[1 - (1 - x^3)^(1/3)])/2) -
(3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(1/3)))/2)/3
```

3.643.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.643.4 Maple [A] (verified)

Time = 6.86 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{-2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}-2\,2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x\right.\right.}{}$

input
```
int(1/x/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)*(-
x^3+1)^(1/3)-2*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(1/3)+2^(2/3)*l
n((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-x^3+1)^(1/3)+8*arctan(1
/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*(-x^3+1)^(1/3)-4*ln((-x^3+1)^(2/3
)+(-x^3+1)^(1/3)+1)*(-x^3+1)^(1/3)+8*ln(-1+(-x^3+1)^(1/3))*(-x^3+1)^(1/3)+
12)/(-x^3+1)^(1/3)
```

3.643.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.47

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx =$$

$$-\frac{2\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\right)\right)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\log\Bigl(2^{\frac{1}{3}}(-1)}{}$$

input
```
integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/24*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(6
)*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 2^(2/3)*(-1)^(1/3)*(x^
3 - 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^
3 + 1)^(2/3)) - 2*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) + (
-x^3 + 1)^(1/3)) - 8*sqrt(3)*(x^3 - 1)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3)
 + 1/3*sqrt(3)) + 4*(x^3 - 1)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1)
 - 8*(x^3 - 1)*log((-x^3 + 1)^(1/3) - 1) + 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

3.643.6 Sympy [F]

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \int \frac{1}{x\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(1/x/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.643.7 Maxima [F]

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x}\,dx$$

input
```
integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x), x)
```

3.643.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = -\frac{1}{12}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)$$
$$+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}$$
$$\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)$$
$$+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}-\frac{1}{6}\,\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\,\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(
1/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^
(2/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/3*sqrt(3)*
arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2/(-x^3 + 1)^(1/3) - 1/6*
log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3
) - 1))
```

3.643.9 Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.64

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{\ln\left(\frac{17}{4}-\frac{17\left(1-x^3\right)^{1/3}}{4}\right)}{3}$$

$$+\ln\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(1458\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2-\frac{459\left(1-x^3\right)^{1/3}}{4}\right)-\frac{63}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right.\right.$$

input
```
int(1/(x*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output

```
log(17/4 - (17*(1 - x^3)^(1/3))/4)/3 + log(((3^(1/2)*1i)/6 - 1/6)*(1458*((
3^(1/2)*1i)/6 - 1/6)^2 - (459*(1 - x^3)^(1/3))/4) - 63/4)*((3^(1/2)*1i)/6
- 1/6) - log(((3^(1/2)*1i)/6 + 1/6)*(1458*((3^(1/2)*1i)/6 + 1/6)^2 - (459*
(1 - x^3)^(1/3))/4) + 63/4)*((3^(1/2)*1i)/6 + 1/6) - (2^(2/3)*log((2^(2/3)
*((81*2^(1/3))/4 - (459*(1 - x^3)^(1/3))/4))/12 + 63/4))/12 + 1/(2*(1 - x^
3)^(1/3)) + ((-1)^(1/3)*2^(2/3)*log(((-1)^(1/3)*2^(2/3)*((81*(-1)^(2/3)*2^
(1/3))/4 - (459*(1 - x^3)^(1/3))/4))/12 - 63/4))/12 - ((-1)^(1/3)*2^(2/3)*
log(((-1)^(1/3)*2^(2/3)*(3^(1/2)*1i + 1)*((459*(1 - x^3)^(1/3))/4 - (81*(-
1)^(2/3)*2^(1/3)*(3^(1/2)*1i + 1)^2)/16))/24 - 63/4)*(3^(1/2)*1i + 1))/24
```

3.643. $\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx$

3.644 $\int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)}\,dx$

3.644.1 Optimal result

Integrand size = 22, antiderivative size = 175

$$\begin{aligned}\int \frac{1}{x^4\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}}\\ &+ \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{6}\\ &- \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\end{aligned}$$

output
```
5/6/(-x^3+1)^(1/3)-1/3/x^3/(-x^3+1)^(1/3)-1/6*ln(x)-1/24*ln(x^3+1)*2^(2/3)
+1/6*ln(1-(-x^3+1)^(1/3))+1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/9*arcta
n(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1+2^(2/3)*(-x
^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.644.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^4 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx = \frac{1}{72}\left(\frac{12(-2+5x^3)}{x^3\sqrt[3]{1-x^3}}+8\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)\right.$$

$$+6\, 2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+8\log\left(-1+\sqrt[3]{1-x^3}\right)+6\, 2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-4\log\left(1+\sqrt[3]{}\right.$$

input
```
Integrate[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((12*(-2 + 5*x^3))/(x^3*(1 - x^3)^(1/3)) + 8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^
3)^(1/3))/Sqrt[3]] + 6*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] + 8*Log[-1 + (1 - x^3)^(1/3)] + 6*2^(2/3)*Log[-2 + 2^(2/3)*(1 -
 x^3)^(1/3)] - 4*Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(2/3)*Lo
g[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/72
```

3.644.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {948, 114, 27, 174, 61, 67, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(1-x^3\right)^{4/3} \left(x^3+1\right)} \, dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{1}{x^6 \left(1-x^3\right)^{4/3} \left(x^3+1\right)}dx^3$$

$$\Big\downarrow \ 114$$

$$\frac{1}{3}\left(-\int -\frac{4x^3+1}{3x^3 \left(1-x^3\right)^{4/3} \left(x^3+1\right)}dx^3-\frac{1}{x^3\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\frac{1}{3}\int \frac{4x^3+1}{x^3\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx^3-\frac{1}{x^3\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\int \frac{1}{x^3\left(1-x^3\right)^{4/3}}dx^3+3\int \frac{1}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx^3\right)-\frac{1}{x^3\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\int \frac{1}{x^3\sqrt[3]{1-x^3}}dx^3+3\left(\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx^3+\frac{3}{2\sqrt[3]{1-x^3}}\right)+\frac{3}{\sqrt[3]{1-x^3}}\right)-\frac{1}{x^3\sqrt[3]{1-x^3}}\right)$$

$$\Big\downarrow \; 67$$

$$\frac{1}{3}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(\frac{1}{2}\left(-\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}\right.\right.\right.\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log}{}\right.\right.\right.\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(\frac{1}{2}\left(-\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\right.}{2}\right.\right.\right.\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\left(\frac{1}{2}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{}\right.}{2\sqrt[3]{2}}\right.\right.\right.\right.$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{3}\left(\frac{1}{3}\left(-3\int \frac{1}{-x^6-3}d\left(2\sqrt[3]{1-x^3}+1\right)+3\left(\frac{1}{2}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{}\right.}{2\sqrt[3]{2}}\right.\right.\right.\right.$$

3.644. $\int \frac{1}{x^4\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

$$\Big\downarrow 217$$

$$\frac{1}{3}\Bigg(\frac{1}{3}\Bigg(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)+3\Bigg(\frac{1}{2}\Bigg(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(x^3+1\right)}{2\sqrt[3]{2}}+\frac{3\log\Big(\sqrt[3]{2}-\sqrt[3]{1-}}{2\sqrt[3]{2}}$$

input
```
Int[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(-(1/(x^3*(1 - x^3)^(1/3))) + (3/(1 - x^3)^(1/3) + Sqrt[3]*ArcTan[(1 + 2*(
1 - x^3)^(1/3))/Sqrt[3]] - Log[x^3]/2 + (3*Log[1 - (1 - x^3)^(1/3)])/2 + 3
*(3/(2*(1 - x^3)^(1/3)) + ((Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/S
qrt[3]])/2^(1/3) - Log[1 + x^3]/(2*2^(1/3)) + (3*Log[2^(1/3) - (1 - x^3)^(
1/3)])/(2*2^(1/3)))/2))/3)/3
```

3.644.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.644. $\int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)}\,dx$

3.644.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(131) = 262.

Time = 6.55 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.53

method	result
pseudoelliptic	$-\frac{\frac{3\,2^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)x^{3}\left(-x^{3}+1\right)^{\frac{1}{3}}}{4}+\sqrt{3}\,\arctan\left(\frac{\left(1+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)x^{3}\left(-x^{3}+1\right)^{\frac{1}{3}}-\frac{3\,2^{\frac{2}{3}}\ln((-x^{3}+1}{}}{9(-x^{3}}$

input
```
int(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/9/(-x^3+1)^(1/3)*(3/4*2^(2/3)*3^(1/2)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1
/3))*3^(1/2))*x^3*(-x^3+1)^(1/3)+3^(1/2)*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3
^(1/2))*x^3*(-x^3+1)^(1/3)-3/8*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^
(1/3)+2^(2/3))*x^3*(-x^3+1)^(1/3)+3/4*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*x
^3*(-x^3+1)^(1/3)-1/2*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)*x^3*(-x^3+1)^(1/
3)+ln(-1+(-x^3+1)^(1/3))*x^3*(-x^3+1)^(1/3)+15/2*x^3-3)/((-x^3+1)^(2/3)+(-
x^3+1)^(1/3)+1)/(-1+(-x^3+1)^(1/3))
```

3.644.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^{4}\left(1-x^{3}\right)^{4/3}\left(1+x^{3}\right)}\,dx=\frac{6\sqrt{6}2^{\frac{1}{6}}(x^{6}-x^{3})\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^{3}+1)^{\frac{1}{3}}\right)\right)-3\cdot 2^{\frac{2}{3}}(x^{6}-x}{}$$

input
```
integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/72*(6*sqrt(6)*2^(1/6)*(x^6 - x^3)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) +
2*sqrt(6)*(-x^3 + 1)^(1/3))) - 3*2^(2/3)*(x^6 - x^3)*log(2^(2/3) + 2^(1/3)
*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 6*2^(2/3)*(x^6 - x^3)*log(-2^(1/3)
 + (-x^3 + 1)^(1/3)) + 8*sqrt(3)*(x^6 - x^3)*arctan(2/3*sqrt(3)*(-x^3 + 1)
^(1/3) + 1/3*sqrt(3)) - 4*(x^6 - x^3)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1
/3) + 1) + 8*(x^6 - x^3)*log((-x^3 + 1)^(1/3) - 1) - 12*(5*x^3 - 2)*(-x^3
+ 1)^(2/3))/(x^6 - x^3)
```

3.644.6 Sympy [F]

$$\int \frac{1}{x^4 \left(1-x^3\right)^{4/3} \left(1+x^3\right)}\,dx = \int \frac{1}{x^4 \left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}} \left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(1/x**4/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.644.7 Maxima [F]

$$\int \frac{1}{x^4 \left(1-x^3\right)^{4/3} \left(1+x^3\right)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^4), x)
```

3.644.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{1}{x^4 \left(1-x^3\right)^{4/3} \left(1+x^3\right)}\,dx &= \frac{1}{12}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) \\ &\quad -\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) \\ &\quad +\frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{5\,x^3-2}{6\left(\left(-x^3+1\right)^{\frac{4}{3}}-\left(-x^3+1\right)^{\frac{1}{3}}\right)} \\ &\quad -\frac{1}{18}\,\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{9}\,\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)\end{aligned}$$

input
```
integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(
2/3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/9*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*(5*x^3 - 2)/((-x^3 + 1)^
(4/3) - (-x^3 + 1)^(1/3)) - 1/18*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) +
 1) + 1/9*log(abs((-x^3 + 1)^(1/3) - 1))
```

3.644.9 Mupad [B] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.28

$$\int \frac{1}{x^4\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{\ln\left(\frac{11\left(1-x^3\right)^{1/3}}{972}-\frac{11}{972}\right)}{9} + \frac{2^{2/3}\,\ln\left(\frac{2^{1/3}\left(\frac{2^{2/3}\left(\frac{81\,2^{1/3}}{4}-\frac{75\left(1-x^3\right)^{1/3}}{4}\right)}{12}-\frac{35}{12}\right)}{72}+\frac{\left(1-x^3\right)^{1/3}}{27}\right)}{12}$$

$$+\ln\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)\left(1458\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2-\frac{75\left(1-x^3\right)^{1/3}}{4}\right)-\frac{35}{12}\right)+\frac{\left(1-x^3\right)^{1/}}{27}$$

input
```
int(1/(x^4*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
log((11*(1 - x^3)^(1/3))/972 - 11/972)/9 + (2^(2/3)*log((2^(1/3)*((2^(2/3)
*((81*2^(1/3))/4 - (75*(1 - x^3)^(1/3))/4))/12 - 35/12))/72 + (1 - x^3)^(1
/3)/27))/12 + log(((3^(1/2)*1i)/18 - 1/18)^2*(((3^(1/2)*1i)/18 - 1/18)*(14
58*((3^(1/2)*1i)/18 - 1/18)^2 - (75*(1 - x^3)^(1/3))/4) - 35/12) + (1 - x^
3)^(1/3)/27)*((3^(1/2)*1i)/18 - 1/18) - log((1 - x^3)^(1/3)/27 - ((3^(1/2)
*1i)/18 + 1/18)^2*(((3^(1/2)*1i)/18 + 1/18)*(1458*((3^(1/2)*1i)/18 + 1/18)
^2 - (75*(1 - x^3)^(1/3))/4) + 35/12))*((3^(1/2)*1i)/18 + 1/18) + ((5*x^3)
/6 - 1/3)/((1 - x^3)^(1/3) - (1 - x^3)^(4/3)) + (2^(2/3)*log((1 - x^3)^(1/
3)/27 + (2^(1/3)*(3^(1/2)*1i - 1)^2*((2^(2/3)*(3^(1/2)*1i - 1)*((81*2^(1/3
)*(3^(1/2)*1i - 1)^2)/16 - (75*(1 - x^3)^(1/3))/4))/24 - 35/12))/288)*(3^(
1/2)*1i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/27 - (2^(1/3)*(3^(1/2)*1i
+ 1)^2*((2^(2/3)*(3^(1/2)*1i + 1)*((81*2^(1/3)*(3^(1/2)*1i + 1)^2)/16 - (7
5*(1 - x^3)^(1/3))/4))/24 + 35/12))/288)*(3^(1/2)*1i + 1))/24
```

3.644. $\int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)}\,dx$

3.645 $\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)} \, dx$

3.645.1 Optimal result

Integrand size = 22, antiderivative size = 174

$$\begin{aligned}\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)} \, dx &= \frac{x^4}{2\sqrt[3]{1-x^3}}+\frac{5}{6}x\left(1-x^3\right)^{2/3} \\ &+ \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} \\ &+ \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{1}{6}\log\left(x+\sqrt[3]{1-x^3}\right)\end{aligned}$$

output
```
1/2*x^4/(-x^3+1)^(1/3)+5/6*x*(-x^3+1)^(2/3)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(
-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(x+(-x^3+1)^(1/3))+1/9*arctan(1/3
*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-
x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.645.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.30

$$\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{72}\left(-\frac{12x(-5+2x^3)}{\sqrt[3]{1-x^3}} + 8\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)\right.$$

$$+6\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 8\log\left(x+\sqrt[3]{1-x^3}\right) - 6\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + 4\log\Big(x^2 - x$$

input
```
Integrate[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((-12*x*(-5 + 2*x^3))/(1 - x^3)^(1/3) + 8*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x -
2*(1 - x^3)^(1/3))] + 6*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1
 - x^3)^(1/3))] - 8*Log[x + (1 - x^3)^(1/3)] - 6*2^(2/3)*Log[2*x + 2^(2/3)
*(1 - x^3)^(1/3)] + 4*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] + 3*2
^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/
72
```

3.645.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {970, 1052, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{970}$$

$$\frac{x^4}{2\sqrt[3]{1-x^3}} - \frac{1}{2}\int \frac{x^3\left(5x^3+4\right)}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{1052}$$

$$\frac{1}{2}\left(\frac{5}{3}x\left(1-x^3\right)^{2/3} - \frac{1}{3}\int \frac{2x^3+5}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx\right) + \frac{x^4}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1026}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-2\int \frac{1}{\sqrt[3]{1-x^3}}dx-3\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx\right)+\frac{5}{3}\left(1-x^3\right)^{2/3}x\right)+\frac{x^4}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \ 769$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-3\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-2\left(\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{5}{3}\left(1-x^3\right)^{2/3}x\right)+$$

$$\frac{x^4}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \ 901$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-3\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)-2\left(\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\right.\right.\right.$$

$$\frac{x^4}{2\sqrt[3]{1-x^3}}$$

input
```
Int[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
x^4/(2*(1 - x^3)^(1/3)) + ((5*x*(1 - x^3)^(2/3))/3 + (-3*(-(ArcTan[(1 - (2
*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) - Log[1 + x^3]/(6
*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))) - 2*(-(ArcTan
[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/
2))/3)/2
```

3.645. $\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

3.645.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

3.645. $\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx$

3.645.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(132) = 264$.

Time = 6.65 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.68

method	result
pseudoelliptic	$\frac{-6\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-6\,2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+3\,2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x^{3}\right.}{}\right.}{72}$

input
```
int(x^9/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/72*(-6*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)
*(-x^3+1)^(1/3)-6*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*(-x^3+1)^(1/3)+
3*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)*(-
x^3+1)^(1/3)-24*x^4-8*3^(1/2)*arctan(1/3*(-2*(-x^3+1)^(1/3)+x)*3^(1/2)/x)*
(-x^3+1)^(1/3)+4*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*x+x^2)/x^2)*(-x^3+1)^(1
/3)-8*ln((x+(-x^3+1)^(1/3))/x)*(-x^3+1)^(1/3)+60*x)/((-x^3+1)^(2/3)-(-x^3+
1)^(1/3)*x+x^2)/(x+(-x^3+1)^(1/3))/(-x^3+1)^(1/3)
```

3.645.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(132) = 264$.

Time = 0.32 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.56

$$\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{6\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}x+2\sqrt{6}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6x}\right)+6\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x}{}$$

input
```
integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/72*(6*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2
^(1/3)*x + 2*sqrt(6)*(-1)^(1/3)*(-x^3 + 1)^(1/3))/x) + 6*2^(2/3)*(-1)^(1/3
)*(x^3 - 1)*log((2^(1/3)*(-1)^(2/3)*x + (-x^3 + 1)^(1/3))/x) - 3*2^(2/3)*(
-1)^(1/3)*(x^3 - 1)*log(-(2^(2/3)*(-1)^(1/3)*x^2 + 2^(1/3)*(-1)^(2/3)*(-x^
3 + 1)^(1/3)*x - (-x^3 + 1)^(2/3))/x^2) + 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*
(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) - 8*(x^3 - 1)*log((x + (-x^3 +
 1)^(1/3))/x) + 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/
3))/x^2) + 12*(2*x^4 - 5*x)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

3.645.6 Sympy [F]

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^9}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**9/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**9/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.645.7 Maxima [F]

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^9}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.645.8 Giac [F]

$$\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\int \frac{x^9}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.645.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

input
```
int(x^9/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(x^9/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.646 $\int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)} \, dx$

3.646.1 Optimal result

Integrand size = 22, antiderivative size = 153

$$\int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)} \, dx = \frac{x}{2\sqrt[3]{1-x^3}}+\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

output
```
1/2*x/(-x^3+1)^(1/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/
3))*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))
*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*
2^(2/3)*3^(1/2)
```

3.646.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.44

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{24}\left(\frac{12x}{\sqrt[3]{1-x^3}}+8\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)\right.$$

$$-2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-8\log\left(x+\sqrt[3]{1-x^3}\right)+2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+4\log\left(x^2-x\right.$$

input
```
Integrate[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((12*x)/(1 - x^3)^(1/3) + 8*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1
/3))] - 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))
] - 8*Log[x + (1 - x^3)^(1/3)] + 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/
3)] + 4*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 2^(2/3)*Log[-2*x^
2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/24
```

3.646.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {970, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \text{970}$$

$$\frac{x}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{2x^3+1}{\sqrt[3]{1-x^3}(x^3+1)}dx$$

$$\Big\downarrow \text{1026}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt[3]{1-x^3}(x^3+1)}dx-2\int \frac{1}{\sqrt[3]{1-x^3}}dx\right)+\frac{x}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{769}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-2\left(\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{x}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow 901$$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-2\left(\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{\ }\right.}{}\right.$$

$$\frac{x}{2\sqrt[3]{1-x^3}}$$

input
```
Int[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
x/(2*(1 - x^3)^(1/3)) + (-(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt
[3]]/(2^(1/3)*Sqrt[3])) - Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1
 - x^3)^(1/3)]/(2*2^(1/3)) - 2*(-(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[
3]]/Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/2))/2
```

3.646.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.646.4 Maple [A] (verified)

Time = 5.54 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.43

method	result
pseudoelliptic	$2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right) \left(-x^{3}+1\right)^{\frac{1}{3}}-4 \ln\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right) \left(-x^{3}+1\right)^{\frac{1}{3}}+\left(\left(-\frac{\ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}+\right.\right.$

input
```
int(x^6/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12/(-x^3+1)^(1/3)*(2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*(-x^3+1)^(1/
3)-4*ln((x+(-x^3+1)^(1/3))/x)*(-x^3+1)^(1/3)+((-1/2*ln((2^(2/3)*x^2-2^(1/3
)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)+3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/
3)*(-x^3+1)^(1/3)+x)/x))*2^(2/3)-4*3^(1/2)*arctan(1/3*(-2*(-x^3+1)^(1/3)+x
)*3^(1/2)/x)+2*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*x+x^2)/x^2))*(-x^3+1)^(1/
3)+6*x)
```

3.646.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(118) = 236$.

Time = 0.33 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.56

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx =$$

$$-\frac{2\sqrt{6}2^{\frac{1}{6}}(x^3-1)\arctan\left(-\frac{2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}x-2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)}{6x}\right)-2\cdot 2^{\frac{2}{3}}(x^3-1)\log\left(\frac{2^{\frac{1}{3}}x+(-x^3+1)^{\frac{1}{3}}}{x}\right)+2^{\frac{2}{3}}(x^3-1)\,\mathrm{lo}}{}$$

input
```
integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(-1/6*2^(1/6)*(sqrt(6)*2^(1/3)*x
- 2*sqrt(6)*(-x^3 + 1)^(1/3))/x) - 2*2^(2/3)*(x^3 - 1)*log((2^(1/3)*x + (-
x^3 + 1)^(1/3))/x) + 2^(2/3)*(x^3 - 1)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 +
1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2) - 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*(sqr
t(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 8*(x^3 - 1)*log((x + (-x^3 + 1)^
(1/3))/x) - 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/
x^2) + 12*(-x^3 + 1)^(2/3)*x)/(x^3 - 1)
```

3.646.6 Sympy [F]

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^6}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**6/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.646.7 Maxima [F]

$$\int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.646.8 Giac [F]

$$\int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.646.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

input
```
int(x^6/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(x^6/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.647 $\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx$

3.647.1 Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx=\frac{x}{2\sqrt[3]{1-x^3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log(1+x^3)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2*x/(-x^3+1)^(1/3)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/
3))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3
)*3^(1/2)
```

3.647.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.31

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx=\frac{1}{24}\Bigg(\frac{12x}{\sqrt[3]{1-x^3}}+2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+2^{2/3}\log\Big(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}$$

input
```
Integrate[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((12*x)/(1 - x^3)^(1/3) + 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3
)*(1 - x^3)^(1/3))] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(2/
3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/24
```

3.647.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {971, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \text{971}$$

$$\frac{x}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}(x^3+1)}dx$$

$$\Big\downarrow \text{901}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)+\frac{x}{2\sqrt[3]{1-x^3}}$$

input
```
Int[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
x/(2*(1 - x^3)^(1/3)) + (ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3
]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[-(2^(1/3)*x) - (1 -
x^3)^(1/3)]/(2*2^(1/3)))/2
```

3.647.3.1 Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

3.647.4 Maple [A] (verified)

Time = 7.63 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.33

method	result
pseudoelliptic	$-\frac{\sqrt{3}\, 2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-\frac{2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{2}+2^{\frac{2}{3}} \ln}{12\left(-x^{3}+1\right)^{\frac{1}{3}}}$
risch	Expression too large to display
trager	Expression too large to display

input
```
int(x^3/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/12/(-x^3+1)^(1/3)*(3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1
)^(1/3)+x)/x)*(-x^3+1)^(1/3)-1/2*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^
(1/3)*x+(-x^3+1)^(2/3))/x^2)*(-x^3+1)^(1/3)+2^(2/3)*ln((2^(1/3)*x+(-x^3+1)
^(1/3))/x)*(-x^3+1)^(1/3)-6*x)
```

3.647.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(79) = 158$.

Time = 1.91 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.00

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx =$$

$$-\frac{2\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}(x^3-1)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}(71\right.}{6(109x^9-105x^6+3x^3+1)}\right.}{}$$

input
```
integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/72*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(6*sqrt(6
)*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1
)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9
 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1
)^(1/3)*(x^3 - 1)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)
*(-1)^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1
/3)*(x^3 - 1)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^
(1/3)*(-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3)
)/(x^6 + 2*x^3 + 1)) + 36*(-x^3 + 1)^(2/3)*x)/(x^3 - 1)
```

3.647.6 Sympy [F]

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^3}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**3/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.647.7 Maxima [F]

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.647.8 Giac [F]

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.647.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^3}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

input
```
int(x^3/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(x^3/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.648 $\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx$

3.648.1 Optimal result

Integrand size = 19, antiderivative size = 106

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{x}{2\sqrt[3]{1-x^3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2*x/(-x^3+1)^(1/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/
3))*2^(2/3)-1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3
)*3^(1/2)
```

3.648.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.32

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{24}\left(\frac{12x}{\sqrt[3]{1-x^3}}\right.$$
$$-2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)+2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)-2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\right.$$

input

```
Integrate[1/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output

```
((12*x)/(1 - x^3)^(1/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3
)*(1 - x^3)^(1/3))] + 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] - 2^(2/
3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/24
```

3.648.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {907, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{907}$$

$$\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx+\frac{x}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{901}$$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)+\frac{x}{2\sqrt[3]{1-x^3}}$$

input

```
Int[1/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output

```
x/(2*(1 - x^3)^(1/3)) + (-(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt
[3]]/(2^(1/3)*Sqrt[3])) - Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1
 - x^3)^(1/3)]/(2*2^(1/3)))/2
```

3.648.3.1 Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

3.648.4 Maple [A] (verified)

Time = 6.83 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.33

method	result
pseudoelliptic	$\frac{\sqrt{3}\, 2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-\frac{2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{2}+2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}} x+\cdots}{}\right.}{12\left(-x^{3}+1\right)^{\frac{1}{3}}}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(1/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12/(-x^3+1)^(1/3)*(3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)
^(1/3)+x)/x)*(-x^3+1)^(1/3)-1/2*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(
1/3)*x+(-x^3+1)^(2/3))/x^2)*(-x^3+1)^(1/3)+2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^
(1/3))/x)*(-x^3+1)^(1/3)+6*x)
```

3.648.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(79) = 158$.

Time = 1.77 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.72

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx =$$

$$-\frac{2\sqrt{6}2^{\frac{1}{6}}(x^3-1)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-\sqrt{6}2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)+12\sqrt{6}(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}\right.}{6(109x^9-105x^6+3x^3+1)}\right.}{}$$

input
```
integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/72*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(
5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 +
33*x^3 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^
9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(x^3 - 1)*log((6*2^(1/3)*(-x^3 + 1)^
(1/3)*x^2 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)
*(x^3 - 1)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 -
 16*x^3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 36*
(-x^3 + 1)^(2/3)*x)/(x^3 - 1)
```

3.648.6 Sympy [F]

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(1/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.648.7 Maxima [F]

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.648.8 Giac [F]

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.648.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

input
```
int(1/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(1/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.649 $\int \frac{1}{x^3(1-x^3)^{4/3}(1+x^3)}\,dx$

3.649.1 Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{1}{x^3\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{2x^2\sqrt[3]{1-x^3}}-\frac{\left(1-x^3\right)^{2/3}}{x^2}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/x^2/(-x^3+1)^(1/3)-(-x^3+1)^(2/3)/x^2+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2
^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(
1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.649.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^3\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{24}\Bigg(\frac{12(-1+2x^3)}{x^2\sqrt[3]{1-x^3}}+2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+2^{2/3}\log\Big(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}$$

input
```
Integrate[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((12*(-1 + 2*x^3))/(x^2*(1 - x^3)^(1/3)) + 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[
3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^
3)^(1/3)] + 2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 -
x^3)^(2/3)])/24
```

3.649.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {972, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(1-x^3\right)^{4/3} \left(x^3+1\right)}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{1}{2}\int \frac{3x^3+4}{x^3 \sqrt[3]{1-x^3} \left(x^3+1\right)}dx+\frac{1}{2x^2 \sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{2}{\sqrt[3]{1-x^3} \left(x^3+1\right)}dx-\frac{2\left(1-x^3\right)^{2/3}}{x^2}\right)+\frac{1}{2x^2 \sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\int \frac{1}{\sqrt[3]{1-x^3} \left(x^3+1\right)}dx-\frac{2\left(1-x^3\right)^{2/3}}{x^2}\right)+\frac{1}{2x^2 \sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{901}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{2\left(1-x^3\right)^{2/3}}{x^2}\right)+\frac{1}{2x^2\sqrt[3]{1-x^3}}$$

input

```
Int[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output

```
1/(2*x^2*(1 - x^3)^(1/3)) + ((-2*(1 - x^3)^(2/3))/x^2 + ArcTan[(1 - (2*2^(
1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1
/3)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3)))/2
```

3.649.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 972

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.649.4 Maple [A] (verified)

Time = 21.60 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.26

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3 x}\right) x^{2} \left(-x^{3}+1\right)^{\frac{1}{3}}-\frac{2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right) x^{2} \left(-x^{3}+1\right)^{\frac{1}{3}}}{2}+2^{\frac{2}{3}} \ln}{12\left(-x^{3}+1\right)^{\frac{1}{3}} x^{2}}$
risch	Expression too large to display
trager	Expression too large to display

input
```
int(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/12/(-x^3+1)^(1/3)*(3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1
)^(1/3)+x)/x)*x^2*(-x^3+1)^(1/3)-1/2*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3
+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)*x^2*(-x^3+1)^(1/3)+2^(2/3)*ln((2^(1/3)*x+
(-x^3+1)^(1/3))/x)*x^2*(-x^3+1)^(1/3)-12*x^3+6)/x^2
```

3.649.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(95) = 190.

Time = 1.68 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.74

$$\int \frac{1}{x^3\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx =$$

$$-\frac{2\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^{5}-x^{2}\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5x^{7}+4x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}\left(19x^{8}-16x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}(7\right.}{6\left(109x^{9}-105x^{6}+3x^{3}+1\right)}\right.}{}$$

input
```
integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/72*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^5 - x^2)*arctan(1/6*2^(1/6)*(6*sqrt
(6)*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(
-1)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x
^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(
-1)^(1/3)*(x^5 - x^2)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(
2/3)*(-1)^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1
)^(1/3)*(x^5 - x^2)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3
) - 2^(1/3)*(-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)
^(1/3))/(x^6 + 2*x^3 + 1)) + 36*(2*x^3 - 1)*(-x^3 + 1)^(2/3))/(x^5 - x^2)
```

3.649.6 Sympy [F]

$$\int \frac{1}{x^3 \left(1-x^3\right)^{4/3} \left(1+x^3\right)}\, dx = \int \frac{1}{x^3 \left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}} \left(x+1\right)\left(x^2-x+1\right)}\, dx$$

input
```
integrate(1/x**3/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.649.7 Maxima [F]

$$\int \frac{1}{x^3 \left(1-x^3\right)^{4/3} \left(1+x^3\right)}\, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^3}\, dx$$

input
```
integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)
```

3.649.8 Giac [F]

$$\int \frac{1}{x^3 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}} x^3} \, dx$$

input
```
integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)
```

3.649.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx = \int \frac{1}{x^3 \left(1-x^3\right)^{4/3} \left(x^3+1\right)} \, dx$$

input
```
int(1/(x^3*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^3*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.650 $\int \frac{1}{x^6 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx$

3.650.1 Optimal result

Integrand size = 22, antiderivative size = 144

$$\int \frac{1}{x^6 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx = \frac{1}{2x^5\sqrt[3]{1-x^3}} - \frac{7\left(1-x^3\right)^{2/3}}{10x^5} - \frac{4\left(1-x^3\right)^{2/3}}{5x^2} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/x^5/(-x^3+1)^(1/3)-7/10*(-x^3+1)^(2/3)/x^5-4/5*(-x^3+1)^(2/3)/x^2-1/24
*ln(x^3+1)*2^(2/3)+1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan(1
/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.650.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^6 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx = \frac{1}{120}\Bigg(-\frac{12\left(2+x^3-8x^6\right)}{x^5\sqrt[3]{1-x^3}} - 10\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) + 10\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) - 5\,2^{2/3}\log\Bigg(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}$$

input
```
Integrate[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((-12*(2 + x^3 - 8*x^6))/(x^5*(1 - x^3)^(1/3)) - 10*2^(2/3)*Sqrt[3]*ArcTan
[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] + 10*2^(2/3)*Log[2*x + 2^(2/3)
*(1 - x^3)^(1/3)] - 5*2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(
1/3)*(1 - x^3)^(2/3)])/120
```

3.650.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {972, 1053, 25, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(1-x^3\right)^{4/3} \left(x^3+1\right)} \, dx$$

$$\Big\downarrow \text{972}$$

$$\frac{1}{2} \int \frac{6 x^3+7}{x^6 \sqrt[3]{1-x^3} \left(x^3+1\right)}dx+\frac{1}{2 x^5 \sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{2} \left(-\frac{1}{5} \int -\frac{21 x^3+16}{x^3 \sqrt[3]{1-x^3} \left(x^3+1\right)}dx-\frac{7 \left(1-x^3\right)^{2/3}}{5 x^5}\right)+\frac{1}{2 x^5 \sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(\frac{1}{5} \int \frac{21 x^3+16}{x^3 \sqrt[3]{1-x^3} \left(x^3+1\right)}dx-\frac{7 \left(1-x^3\right)^{2/3}}{5 x^5}\right)+\frac{1}{2 x^5 \sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{2} \left(\frac{1}{5} \left(-\frac{1}{2} \int -\frac{10}{\sqrt[3]{1-x^3} \left(x^3+1\right)}dx-\frac{8 \left(1-x^3\right)^{2/3}}{x^2}\right)-\frac{7 \left(1-x^3\right)^{2/3}}{5 x^5}\right)+\frac{1}{2 x^5 \sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\frac{1}{5} \left(5 \int \frac{1}{\sqrt[3]{1-x^3} \left(x^3+1\right)}dx-\frac{8 \left(1-x^3\right)^{2/3}}{x^2}\right)-\frac{7 \left(1-x^3\right)^{2/3}}{5 x^5}\right)+\frac{1}{2 x^5 \sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{901}$$

$$\frac{1}{2}\left(\frac{1}{5}\left(5\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)-\frac{8\left(1-x^3\right)^{2/3}}{x^2}\right)-\frac{7\left(1-x^3\right)^2}{5x^5}\right.$$

$$\frac{1}{2x^5\sqrt[3]{1-x^3}}$$

input
```
Int[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
1/(2*x^5*(1 - x^3)^(1/3)) + ((-7*(1 - x^3)^(2/3))/(5*x^5) + ((-8*(1 - x^3)
^(2/3))/x^2 + 5*(-(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^
(1/3)*Sqrt[3])) - Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^
(1/3)]/(2*2^(1/3))))/5)/2
```

3.650.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.650.4 Maple [A] (verified)

Time = 22.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right) x^{5}\left(-x^{3}+1\right)^{\frac{1}{3}}-\frac{2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right) x^{5}\left(-x^{3}+1\right)^{\frac{1}{3}}}{2}+2^{\frac{2}{3}} \ln\left(\right.}{12\left(-x^{3}+1\right)^{\frac{1}{3}} x^{5}}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12/(-x^3+1)^(1/3)*(3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)
^(1/3)+x)/x)*x^5*(-x^3+1)^(1/3)-1/2*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+
1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)*x^5*(-x^3+1)^(1/3)+2^(2/3)*ln((2^(1/3)*x+(
-x^3+1)^(1/3))/x)*x^5*(-x^3+1)^(1/3)+48/5*x^6-6/5*x^3-12/5)/x^5
```

3.650.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(109) = 218$.

Time = 1.69 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.19

$$\int \frac{1}{x^6 (1-x^3)^{4/3} (1+x^3)} \, dx =$$

$$-\frac{10\sqrt{6}2^{\frac{1}{6}}(x^8-x^5)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-\sqrt{6}2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)+12\sqrt{6}(19x^8-16x^5+x^2)(-x^3+1}{6(109x^9-105x^6+3x^3+1)}\right.}{}$$

input
```
integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/360*(10*sqrt(6)*2^(1/6)*(x^8 - x^5)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/
3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^
6 + 33*x^3 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(10
9*x^9 - 105*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*(x^8 - x^5)*log((6*2^(1/3)*(-x^
3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) +
5*2^(2/3)*(x^8 - x^5)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3
)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3
+ 1)) + 36*(8*x^6 - x^3 - 2)*(-x^3 + 1)^(2/3))/(x^8 - x^5)
```

3.650.6 Sympy [F]

$$\int \frac{1}{x^6 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{x^6 \left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}} (x+1)(x^2-x+1)} \, dx$$

input
```
integrate(1/x**6/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.650.7 Maxima [F]

$$\int \frac{1}{x^6 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^6} \, dx$$

input
```
integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

3.650.8 Giac [F]

$$\int \frac{1}{x^6 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^6} \, dx$$

input
```
integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

3.650.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx = \int \frac{1}{x^6 \left(1-x^3\right)^{4/3} \left(x^3+1\right)} \, dx$$

input
```
int(1/(x^6*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^6*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.651 $\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

3.651.1 Optimal result

Integrand size = 22, antiderivative size = 162

$$\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{2x^8\sqrt[3]{1-x^3}}-\frac{5\left(1-x^3\right)^{2/3}}{8x^8}-\frac{13\left(1-x^3\right)^{2/3}}{20x^5} -\frac{49\left(1-x^3\right)^{2/3}}{40x^2}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/2/x^8/(-x^3+1)^(1/3)-5/8*(-x^3+1)^(2/3)/x^8-13/20*(-x^3+1)^(2/3)/x^5-49/
40*(-x^3+1)^(2/3)/x^2+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2^(1/3)*x-(-x^3+1)^(1
/3))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/
3)*3^(1/2)
```

3.651.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{120}\left(-\frac{3\left(5+x^3+23x^6-49x^9\right)}{x^8\sqrt[3]{1-x^3}} +10\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-10\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+5\,2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\right.\right.$$

input
```
Integrate[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
((-3*(5 + x^3 + 23*x^6 - 49*x^9))/(x^8*(1 - x^3)^(1/3)) + 10*2^(2/3)*Sqrt[
3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 10*2^(2/3)*Log[2*x
+ 2^(2/3)*(1 - x^3)^(1/3)] + 5*2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1
/3) - 2^(1/3)*(1 - x^3)^(2/3)])/120
```

3.651.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {972, 1053, 27, 1053, 25, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{1}{2}\int \frac{9x^3+10}{x^9\sqrt[3]{1-x^3}\left(x^3+1\right)}dx+\frac{1}{2x^8\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{2}\left(-\frac{1}{8}\int -\frac{4\left(15x^3+13\right)}{x^6\sqrt[3]{1-x^3}\left(x^3+1\right)}dx-\frac{5\left(1-x^3\right)^{2/3}}{4x^8}\right)+\frac{1}{2x^8\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{15x^3+13}{x^6\sqrt[3]{1-x^3}\left(x^3+1\right)}dx-\frac{5\left(1-x^3\right)^{2/3}}{4x^8}\right)+\frac{1}{2x^8\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{5}\int -\frac{39x^3+49}{x^3\sqrt[3]{1-x^3}\left(x^3+1\right)}dx-\frac{13\left(1-x^3\right)^{2/3}}{5x^5}\right)-\frac{5\left(1-x^3\right)^{2/3}}{4x^8}\right)+\frac{1}{2x^8\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{5}\int \frac{39x^3+49}{x^3\sqrt[3]{1-x^3}\left(x^3+1\right)}dx-\frac{13\left(1-x^3\right)^{2/3}}{5x^5}\right)-\frac{5\left(1-x^3\right)^{2/3}}{4x^8}\right)+\frac{1}{2x^8\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{5}\left(-\frac{1}{2}\int \frac{20}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{49(1-x^3)^{2/3}}{2x^2}\right)-\frac{13(1-x^3)^{2/3}}{5x^5}\right)-\frac{5(1-x^3)^{2/3}}{4x^8}\right)+\frac{1}{2x^8\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{5}\left(-10\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{49(1-x^3)^{2/3}}{2x^2}\right)-\frac{13(1-x^3)^{2/3}}{5x^5}\right)-\frac{5(1-x^3)^{2/3}}{4x^8}\right)+\frac{1}{2x^8\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \; 901$$

$$\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(\frac{1}{5}\Bigg(-10\Bigg(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\Bigg)-\frac{49(1-x^3)^{2/3}}{2x^2}\Bigg)-13$$

$$\frac{1}{2x^8\sqrt[3]{1-x^3}}$$

input
```
Int[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
1/(2*x^8*(1 - x^3)^(1/3)) + ((-5*(1 - x^3)^(2/3))/(4*x^8) + ((-13*(1 - x^3
)^(2/3))/(5*x^5) + ((-49*(1 - x^3)^(2/3))/(2*x^2) - 10*(-(ArcTan[(1 - (2*2
^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) - Log[1 + x^3]/(6*2
^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))))/5)/2)/2
```

3.651.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.651.4 Maple [A] (verified)

Time = 21.04 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{-10\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right) x^{8}\left(-x^{3}+1\right)^{\frac{1}{3}}-10\,2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right) x^{8}\left(-x^{3}+1\right)^{\frac{1}{3}}+5\,2^{\frac{2}{3}} \ln\Bigg(\frac{2^{\frac{2}{3}} x^{2}-}{}}{120x^{8}\left(-x^{3}+1\right)^{\frac{1}{3}}}$
risch	Expression too large to display
trager	Expression too large to display

input
```
int(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/120*(-10*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/
x)*x^8*(-x^3+1)^(1/3)-10*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x^8*(-x^
3+1)^(1/3)+5*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/
3))/x^2)*x^8*(-x^3+1)^(1/3)+147*x^9-69*x^6-3*x^3-15)/x^8/(-x^3+1)^(1/3)
```

3.651.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(123) = 246$.

Time = 1.71 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^9 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx =$$

$$-\frac{10 \sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^{11}-x^{8}\right)\arctan\Bigg(\frac{2^{\frac{1}{6}}\Big(6\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5x^{7}+4x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}\left(19x^{8}-16x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}}{6\left(109x^{9}-105x^{6}+3x^{3}+1\right)}}{}$$

input
```
integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/360*(10*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^11 - x^8)*arctan(1/6*2^(1/6)*(6*s
qrt(6)*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6
)*(-1)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(7
1*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 10*2^(2/
3)*(-1)^(1/3)*(x^11 - x^8)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2
 - 2^(2/3)*(-1)^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2
/3)*(-1)^(1/3)*(x^11 - x^8)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 +
 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-
x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 9*(49*x^9 - 23*x^6 - x^3 - 5)*(-x^3 +
 1)^(2/3))/(x^11 - x^8)
```

3.651.6 Sympy [F]

$$\int \frac{1}{x^9 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx = \int \frac{1}{x^{9} \left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{4}{3}} \left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

input
```
integrate(1/x**9/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.651.7 Maxima [F]

$$\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^9}\,dx$$

input
```
integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)
```

3.651.8 Giac [F]

$$\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^9}\,dx$$

input
```
integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)
```

3.651.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

input
```
int(1/(x^9*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^9*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.652 $\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx$

3.652.1 Optimal result

Integrand size = 22, antiderivative size = 292

$$\begin{aligned}\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx &= \frac{x^5}{2\sqrt[3]{1-x^3}}+\frac{3}{4}x^2\left(1-x^3\right)^{2/3}-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}\\
&\quad-\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\\
&\quad-\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}\\
&\quad+\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}\end{aligned}$$

output
```
1/2*x^5/(-x^3+1)^(1/3)+3/4*x^2*(-x^3+1)^(2/3)-1/2*x^2*hypergeom([1/3, 2/3]
,[5/3],x^3)-1/48*ln((1-x)*(1+x)^2)*2^(2/3)-1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3
+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/12*ln(1+2^(1/3)*(1-x)/(-
x^3+1)^(1/3))*2^(2/3)+1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/12*ar
ctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*
arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.652.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.24

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{20}x^2\left(-\frac{5(-3+x^3)}{\sqrt[3]{1-x^3}} - 15\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) - 4x^3\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)\right)$$

input
```
Integrate[x^10/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(x^2*((-5*(-3 + x^3))/(1 - x^3)^(1/3) - 15*AppellF1[2/3, 1/3, 1, 5/3, x^3,
 -x^3] - 4*x^3*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/20
```

3.652.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {970, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \text{970}$$

$$\frac{x^5}{2\sqrt[3]{1-x^3}} - \frac{1}{2}\int \frac{x^4(6x^3+5)}{\sqrt[3]{1-x^3}(x^3+1)}dx$$

$$\Big\downarrow \text{1052}$$

$$\frac{1}{2}\left(\frac{3}{2}x^2(1-x^3)^{2/3} - \frac{1}{4}\int \frac{4x(2x^3+3)}{\sqrt[3]{1-x^3}(x^3+1)}dx\right) + \frac{x^5}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{3}{2}x^2(1-x^3)^{2/3} - \int \frac{x(2x^3+3)}{\sqrt[3]{1-x^3}(x^3+1)}dx\right) + \frac{x^5}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1054}$$

$$\frac{1}{2}\left(\frac{3}{2}x^2\left(1-x^3\right)^{2/3}-\int\left(\frac{2x}{\sqrt[3]{1-x^3}}+\frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}\right)dx\right)+\frac{x^5}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+x^2\left(-\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\right)-\log\Big(\frac{2^{2/3}}{(1-}\right.$$

$$\frac{x^5}{2\sqrt[3]{1-x^3}}$$

input
```
Int[x^10/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
x^5/(2*(1 - x^3)^(1/3)) + ((3*x^2*(1 - x^3)^(2/3))/2 - ArcTan[(1 - (2*2^(1
/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^
(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - x^2*Hyperge
ometric2F1[1/3, 2/3, 5/3, x^3] - Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) - Log
[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/
3)]/(6*2^(1/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) +
 Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(1/3)))/2
```

3.652.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

3.652. $\int \frac{x^{10}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.652.4 Maple [F]

$$\int \frac{x^{10}}{\left(-x^{3}+1\right)^{\frac{4}{3}} \left(x^{3}+1\right)}d x$$

input
```
int(x^10/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
int(x^10/(-x^3+1)^(4/3)/(x^3+1),x)
```

3.652.5 Fricas [F]

$$\int \frac{x^{10}}{(1-x^3)^{4/3} (1+x^3)} \, dx=\int { \frac{x^{10}}{{\left(x^{3} + 1\right)} {\left(-x^{3} + 1\right)}^{\frac{4}{3}}} \,d x }$$

input
```
integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)*x^10/(x^9 - x^6 - x^3 + 1), x)
```

3.652.6 Sympy [F]

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{10}}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**10/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**10/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.652.7 Maxima [F]

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{10}}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^10/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.652.8 Giac [F]

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{10}}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^10/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.652.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{10}}{(1-x^3)^{4/3}\,(x^3+1)}\,dx$$

input
```
int(x^10/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(x^10/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.653 $\int \frac{x^7}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

3.653.1 Optimal result

Integrand size = 22, antiderivative size = 274

$$\begin{aligned}\int \frac{x^7}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{x^2}{2\sqrt[3]{1-x^3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}\\&\quad+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{3}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\\&\quad+\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}\\&\quad-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}\end{aligned}$$

output
```
1/2*x^2/(-x^3+1)^(1/3)-3/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/48*ln((1-
x)*(1+x)^2)*2^(2/3)+1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)
/(-x^3+1)^(1/3))*2^(2/3)-1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1
/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*(
1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/24*arctan(1/3*(1+2^(1/3)*(
1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.653.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.24

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{10}x^2\left(\frac{5}{\sqrt[3]{1-x^3}} - 5\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) - 3x^3\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)\right)$$

input

```
Integrate[x^7/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output

```
(x^2*(5/(1 - x^3)^(1/3) - 5*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - 3*x^3*
AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/10
```

3.653.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {970, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \text{970}$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{1}{2}\int \frac{x(3x^3+2)}{\sqrt[3]{1-x^3}(x^3+1)}dx$$

$$\Big\downarrow \text{1054}$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{1}{2}\int \left(\frac{3x}{\sqrt[3]{1-x^3}} - \frac{x}{\sqrt[3]{1-x^3}(x^3+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{3}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{2^{2/3}(1-x)}{(1-x^3)^{2/3}}\right.}{}\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

input
```
Int[x^7/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
x^2/(2*(1 - x^3)^(1/3)) + (ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3)
)/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/
3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - (3*x^2*Hypergeometric2F1[1/3, 2/3, 5/3,
 x^3])/2 + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^
2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[
1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-1 + x + 2^(2/3)*
(1 - x^3)^(1/3)]/(4*2^(1/3)))/2
```

3.653.3.1 Defintions of rubi rules used

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.653.4 Maple [F]

$$\int \frac{x^7}{(-x^3+1)^{\frac{4}{3}} (x^3+1)}dx$$

input
```
int(x^7/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
int(x^7/(-x^3+1)^(4/3)/(x^3+1),x)
```

3.653.5 Fricas [F]

$$\int \frac{x^7}{(1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{4}{3}}} \, dx$$

input
```
integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)*x^7/(x^9 - x^6 - x^3 + 1), x)
```

3.653.6 Sympy [F]

$$\int \frac{x^7}{(1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{x^7}{(-(x-1)(x^2+x+1))^{\frac{4}{3}} (x+1) (x^2-x+1)} \, dx$$

input
```
integrate(x**7/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.653.7 Maxima [F]

$$\int \frac{x^7}{\left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx=\int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{4}{3}}} \, dx$$

input
```
integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.653.8 Giac [F]

$$\int \frac{x^7}{\left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx=\int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{4}{3}}} \, dx$$

input
```
integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.653.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx=\int \frac{x^7}{\left(1-x^3\right)^{4/3}\,\left(x^3+1\right)} \,d x$$

input
```
int(x^7/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(x^7/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.654 $\int \frac{x^4}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

3.654.1 Optimal result

Integrand size = 22, antiderivative size = 274

$$\begin{aligned}\int \frac{x^4}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}\\ &-\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{4}x^2\operatorname{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\\ &-\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}\\ &+\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}\end{aligned}$$

output

```
1/2*x^2/(-x^3+1)^(1/3)-1/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/48*ln((1-
x)*(1+x)^2)*2^(2/3)-1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)
/(-x^3+1)^(1/3))*2^(2/3)+1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1
/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan(1/3*(1-2*2^(1/3)*(
1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*arctan(1/3*(1+2^(1/3)*(
1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.654.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.24

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{10}x^2\left(\frac{5}{\sqrt[3]{1-x^3}} - 5\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) - x^3\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)\right)$$

input
```
Integrate[x^4/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(x^2*(5/(1 - x^3)^(1/3) - 5*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - x^3*Ap
pellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/10
```

3.654.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {971, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

$$\Big\downarrow \text{971}$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{1}{2}\int \frac{x(x^3+2)}{\sqrt[3]{1-x^3}(x^3+1)}dx$$

$$\Big\downarrow \text{1054}$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{1}{2}\int \left(\frac{x}{\sqrt[3]{1-x^3}} + \frac{x}{\sqrt[3]{1-x^3}(x^3+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)-\frac{\log\left(\frac{2^{2/3}(1-}{(1-x^3)^2}\right.}{}\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

input
```
Int[x^4/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
x^2/(2*(1 - x^3)^(1/3)) + (-(ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/
3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^
(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3
, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) - Log[1 + (2^(2/3)*(1 - x)
^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log
[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) + Log[-1 + x + 2^(2/3)
*(1 - x^3)^(1/3)]/(4*2^(1/3)))/2
```

3.654.3.1 Defintions of rubi rules used

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.654.4 Maple [F]

$$\int \frac{x^4}{(-x^3+1)^{\frac{4}{3}}(x^3+1)}dx$$

input
```
int(x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
int(x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

3.654.5 Fricas [F]

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)*x^4/(x^9 - x^6 - x^3 + 1), x)
```

3.654.6 Sympy [F]

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^4}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x**4/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.654.7 Maxima [F]

$$\int \frac{x^4}{(1-x^3)^{4/3}\,(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.654.8 Giac [F]

$$\int \frac{x^4}{(1-x^3)^{4/3}\,(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.654.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(1-x^3)^{4/3}\,(1+x^3)}\,dx = \int \frac{x^4}{(1-x^3)^{4/3}\,(x^3+1)}\,dx$$

input
```
int(x^4/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(x^4/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.655 $\int \frac{x}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

3.655.1 Optimal result

Integrand size = 20, antiderivative size = 274

$$\begin{aligned}\int \frac{x}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{x^2}{2\sqrt[3]{1-x^3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}\\
&+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\\
&+\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}\\
&-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}\end{aligned}$$

output
```
1/2*x^2/(-x^3+1)^(1/3)-1/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/48*ln((1-
x)*(1+x)^2)*2^(2/3)+1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)
/(-x^3+1)^(1/3))*2^(2/3)-1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1
/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*(
1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/24*arctan(1/3*(1+2^(1/3)*(
1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.655.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.16

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{1}{10}x^5 \operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)$$

input
```
Integrate[x/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
x^2/(2*(1 - x^3)^(1/3)) - (x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/10
```

3.655.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {972, 25, 983, 888, 991, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{1}{2}\int -\frac{x^4}{\sqrt[3]{1-x^3}(x^3+1)}dx + \frac{x^2}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{1}{2}\int \frac{x^4}{\sqrt[3]{1-x^3}(x^3+1)}dx$$

$$\Big\downarrow \text{983}$$

$$\frac{1}{2}\left(\int \frac{x}{\sqrt[3]{1-x^3}(x^3+1)}dx - \int \frac{x}{\sqrt[3]{1-x^3}}dx\right) + \frac{x^2}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{2}\left(\int \frac{x}{\sqrt[3]{1-x^3}(x^3+1)}dx - \frac{1}{2}x^2 \operatorname{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\right) + \frac{x^2}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 991}$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\int \frac{1}{\frac{2(1-x)^3}{1-x^3}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\right)+$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{3}\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}x^2\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x\right.\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\right.\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 1142}$$

3.655. $\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx$

$$\frac{1}{2}\left(-\frac{1}{3}\int\frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int\frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\int-\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-}{\sqrt[3]{1-}}}{2\ 2^{2/3}}\right.\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(-\frac{1}{3}\int\frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int\frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-}}}{2\ 2^{2/3}}\right.\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(-\frac{1}{3}\int\frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int\frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-}}}{2\sqrt[3]{2}}\right.\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \ 1082$$

3.655. $\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right.\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \; 217$$

$$\frac{1}{2}\left(-\frac{1}{3}\sqrt[3]{2}\left(\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}\right)-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{2}x\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)-\frac{1}{2}x\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \; 2574$$

3.655. $\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx$

$$\frac{1}{2}\left(-\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)+\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}\right.\right.$$

$$\frac{x^2}{2\sqrt[3]{1-x^3}}$$

input

```
Int[x/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

output

```
x^2/(2*(1 - x^3)^(1/3)) + (-1/2*(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3]
) - (2^(1/3)*(-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/
Sqrt[3]])/2^(2/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3
)*(1 - x))/(1 - x^3)^(1/3)]/(2*2^(2/3))))/3 - Log[1 + (2^(1/3)*(1 - x))/(1
 - x^3)^(1/3)]/(3*2^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 -
x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)) + Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) - (
3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)))/3)/2
```

3.655.3.1 Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 991
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Simp[-q^2/(3*d)   Int[1/((1 - q*x)*(a + b*x^3)^(1/3)
), x], x] + Simp[q/d   Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x
^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c +
 a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.655. $\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

3.655.4 Maple [F]

$$\int \frac{x}{\left(-x^{3}+1\right)^{\frac{4}{3}} \left(x^{3}+1\right)}dx$$

input
```
int(x/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
int(x/(-x^3+1)^(4/3)/(x^3+1),x)
```

3.655.5 Fricas [F]

$$\int \frac{x}{\left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx=\int \frac{x}{{\left(x^{3}+1\right)}{\left(-x^{3}+1\right)}^{\frac{4}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)*x/(x^9 - x^6 - x^3 + 1), x)
```

3.655.6 Sympy [F]

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx=\int \frac{x}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(x/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(x/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.655.7 Maxima [F]

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx=\int \frac{x}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.655.8 Giac [F]

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx=\int \frac{x}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.655.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx=\int \frac{x}{(1-x^3)^{4/3}\,(x^3+1)}\,dx$$

input

```
int(x/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output

```
int(x/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.656 $\int \frac{1}{x^2\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

3.656.1 Optimal result

Integrand size $=22$, antiderivative size $=292$

$$\begin{aligned}\int \frac{1}{x^2\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{1}{2x\sqrt[3]{1-x^3}}-\frac{3\left(1-x^3\right)^{2/3}}{2x}-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}\\
&-\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{3}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\\
&-\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}\\
&+\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}\end{aligned}$$

output

```
1/2/x/(-x^3+1)^(1/3)-3/2*(-x^3+1)^(2/3)/x-3/4*x^2*hypergeom([1/3, 2/3],[5/
3],x^3)-1/48*ln((1-x)*(1+x)^2)*2^(2/3)-1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^
(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/12*ln(1+2^(1/3)*(1-x)/(-x^3+
1)^(1/3))*2^(2/3)+1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan
(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*arct
an(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.656.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^2(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{-2+3x^3}{2x\sqrt[3]{1-x^3}} - x^2\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) - \frac{3}{10}x^5\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)$$

input
```
Integrate[1/(x^2*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
(-2 + 3*x^3)/(2*x*(1 - x^3)^(1/3)) - x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -
x^3] - (3*x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/10
```

3.656.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {972, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(1-x^3)^{4/3}(x^3+1)}dx$$

$$\Big\downarrow \text{ 972}$$

$$\frac{1}{2}\int \frac{2x^3+3}{x^2\sqrt[3]{1-x^3}(x^3+1)}dx + \frac{1}{2x\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{1}{2}\left(-\int \frac{x(3x^3+4)}{\sqrt[3]{1-x^3}(x^3+1)}dx - \frac{3(1-x^3)^{2/3}}{x}\right) + \frac{1}{2x\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{1}{2}\left(-\int \left(\frac{3x}{\sqrt[3]{1-x^3}} + \frac{x}{\sqrt[3]{1-x^3}(x^3+1)}\right)dx - \frac{3(1-x^3)^{2/3}}{x}\right) + \frac{1}{2x\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{3}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)-\frac{3(1-x^3)^{2/3}}{x}\right.$$

$$\frac{1}{2x\sqrt[3]{1-x^3}}$$

input
```
Int[1/(x^2*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
1/(2*x*(1 - x^3)^(1/3)) + ((-3*(1 - x^3)^(2/3))/x - ArcTan[(1 - (2*2^(1/3)
*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(1/
3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - (3*x^2*Hyperge
ometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) -
Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^
(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)
) + Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(1/3)))/2
```

3.656.3.1 Defintions of rubi rules used

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
 - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.656.4 Maple [F]

$$\int \frac{1}{x^{2} \left(-x^{3}+1\right)^{\frac{4}{3}} \left(x^{3}+1\right)}dx$$

input
```
int(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
int(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

3.656.5 Fricas [F]

$$\int \frac{1}{x^2 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{{\left(x^{3} + 1\right)} {\left(-x^{3} + 1\right)}^{\frac{4}{3}} x^{2}} \,d x$$

input
```
integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)/(x^11 - x^8 - x^5 + x^2), x)
```

3.656.6 Sympy [F]

$$\int \frac{1}{x^2 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{x^{2} \left(- \left(x - 1\right) \left(x^{2} + x + 1\right)\right)^{\frac{4}{3}} \left(x + 1\right) \left(x^{2} - x + 1\right)}\, dx$$

input
```
integrate(1/x**2/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.656.7 Maxima [F]

$$\int \frac{1}{x^2 \left(1-x^3\right)^{4/3} \left(1+x^3\right)}\, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^2}\, dx$$

input
```
integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^2), x)
```

3.656.8 Giac [F]

$$\int \frac{1}{x^2 \left(1-x^3\right)^{4/3} \left(1+x^3\right)}\, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^2}\, dx$$

input
```
integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^2), x)
```

3.656.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(1-x^3\right)^{4/3} \left(1+x^3\right)}\, dx = \int \frac{1}{x^2 \left(1-x^3\right)^{4/3} \left(x^3+1\right)}\, dx$$

input
```
int(1/(x^2*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^2*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.657 $\int \frac{1}{x^5(1-x^3)^{4/3}(1+x^3)}\,dx$

3.657.1 Optimal result

Integrand size = 22, antiderivative size = 308

$$\int \frac{1}{x^5\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{2x^4\sqrt[3]{1-x^3}}-\frac{3\left(1-x^3\right)^{2/3}}{4x^4}-\frac{\left(1-x^3\right)^{2/3}}{x}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}-\log\Big(1$$

output
```
1/2/x^4/(-x^3+1)^(1/3)-3/4*(-x^3+1)^(2/3)/x^4-(-x^3+1)^(2/3)/x-1/2*x^2*hyp
ergeom([1/3, 2/3],[5/3],x^3)+1/48*ln((1-x)*(1+x)^2)*2^(2/3)+1/24*ln(1+2^(2
/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/12*ln(1
+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3)
)*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2
/3)*3^(1/2)+1/24*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2
/3)*3^(1/2)
```

3.657.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^5\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \\ -\frac{\frac{5\left(1+x^3-4x^6\right)}{\sqrt[3]{1-x^3}}+5x^6\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right)+4x^9\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)}{20x^4}$$

input

```
Integrate[1/(x^5*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output

```
-1/20*((5*(1 + x^3 - 4*x^6))/(1 - x^3)^(1/3) + 5*x^6*AppellF1[2/3, 1/3, 1,
 5/3, x^3, -x^3] + 4*x^9*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/x^4
```

3.657.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {972, 1053, 27, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\left(1-x^3\right)^{4/3}\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{1}{2}\int \frac{5x^3+6}{x^5\sqrt[3]{1-x^3}\left(x^3+1\right)}dx+\frac{1}{2x^4\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{2}\left(-\frac{1}{4}\int -\frac{4\left(3x^3+2\right)}{x^2\sqrt[3]{1-x^3}\left(x^3+1\right)}dx-\frac{3\left(1-x^3\right)^{2/3}}{2x^4}\right)+\frac{1}{2x^4\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\int \frac{3x^3+2}{x^2\sqrt[3]{1-x^3}\left(x^3+1\right)}dx-\frac{3\left(1-x^3\right)^{2/3}}{2x^4}\right)+\frac{1}{2x^4\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{2}\left(-\int \frac{x(2x^3+1)}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{2(1-x^3)^{2/3}}{x}-\frac{3(1-x^3)^{2/3}}{2x^4}\right)+\frac{1}{2x^4\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{1054}$$

$$\frac{1}{2}\left(-\int \left(\frac{2x}{\sqrt[3]{1-x^3}}-\frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}\right)dx-\frac{2(1-x^3)^{2/3}}{x}-\frac{3(1-x^3)^{2/3}}{2x^4}\right)+\frac{1}{2x^4\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+x^2\left(-\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\right)-\frac{2(1-x^3)^{2}}{x}\right.$$

$$\frac{1}{2x^4\sqrt[3]{1-x^3}}$$

input
```
Int[1/(x^5*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

output
```
1/(2*x^4*(1 - x^3)^(1/3)) + ((-3*(1 - x^3)^(2/3))/(2*x^4) - (2*(1 - x^3)^(
2/3))/x + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/
3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2
^(1/3)*Sqrt[3]) - x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3] + Log[(1 - x)*
(1 + x)^2]/(12*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2
^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x))/(
1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(
1/3)))/2
```

3.657. $\int \frac{1}{x^5(1-x^3)^{4/3}(1+x^3)}\,dx$

3.657.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.657.4 Maple [F]

$$\int \frac{1}{x^{5} \left(-x^{3}+1\right)^{\frac{4}{3}} \left(x^{3}+1\right)}d x$$

input
```
int(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

output
```
int(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

3.657.5 Fricas [F]

$$\int \frac{1}{x^5 \left(1-x^3\right)^{4/3} \left(1+x^3\right)}\,dx=\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)/(x^14 - x^11 - x^8 + x^5), x)
```

3.657.6 Sympy [F]

$$\int \frac{1}{x^5 \left(1-x^3\right)^{4/3} \left(1+x^3\right)}\,dx=\int \frac{1}{x^5 \left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

input
```
integrate(1/x**5/(-x**3+1)**(4/3)/(x**3+1),x)
```

output
```
Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
 x)
```

3.657.7 Maxima [F]

$$\int \frac{1}{x^5 \left(1-x^3\right)^{4/3} \left(1+x^3\right)}\,dx=\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^5), x)
```

3.657.8 Giac [F]

$$\int \frac{1}{x^5\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^5), x)
```

3.657.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\int \frac{1}{x^5\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

input
```
int(1/(x^5*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

output
```
int(1/(x^5*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.658 $\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

3.658.1 Optimal result

Integrand size = 24, antiderivative size = 264

$$\begin{aligned}\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx =& -\frac{c^3\sqrt[3]{a+bx^3}}{d^4}+\frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{4/3}}{4b^3d^3}\\&-\frac{(bc+2ad)\,(a+bx^3)^{7/3}}{7b^3d^2}+\frac{(a+bx^3)^{10/3}}{10b^3d}\\&-\frac{c^3\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{13/3}}-\frac{c^3\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{13/3}}\\&+\frac{c^3\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}}\end{aligned}$$

output

```
-c^3*(b*x^3+a)^(1/3)/d^4+1/4*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(4/3)/b^3
/d^3-1/7*(2*a*d+b*c)*(b*x^3+a)^(7/3)/b^3/d^2+1/10*(b*x^3+a)^(10/3)/b^3/d-1
/6*c^3*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^(13/3)+1/2*c^3*(-a*d+b*c)^(1/3)*ln((
-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(13/3)-1/3*c^3*(-a*d+b*c)^(1/3)
*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(13/
3)*3^(1/2)
```

3.658.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(9a^3d^3-3a^2bd^2(-5c+dx^3)+ab^2d(35c^2-5cdx^3+2d^2x^6)+b^3(-140c^3+35c^2dx^3-20cd^2x^6+14d^3x^9))}{b^3}-140\sqrt{3}c^3\sqrt[3]{bc-}}{}$$

input
```
Integrate[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
((3*d^(1/3)*(a + b*x^3)^(1/3)*(9*a^3*d^3 - 3*a^2*b*d^2*(-5*c + d*x^3) + a*
b^2*d*(35*c^2 - 5*c*d*x^3 + 2*d^2*x^6) + b^3*(-140*c^3 + 35*c^2*d*x^3 - 20
*c*d^2*x^6 + 14*d^3*x^9)))/b^3 - 140*Sqrt[3]*c^3*(b*c - a*d)^(1/3)*ArcTan[
(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 140*c^3*(
b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - 70*c
^3*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a
+ b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(420*d^(13/3))
```

3.658.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9\sqrt[3]{bx^3+a}}{dx^3+c}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(-\frac{\sqrt[3]{bx^3+a}c^3}{d^3\left(dx^3+c\right)}+\frac{\left(bx^3+a\right)^{7/3}}{b^2d}+\frac{(-bc-2ad)\left(bx^3+a\right)^{4/3}}{b^2d^2}+\frac{\left(b^2c^2+abdc+a^2d^2\right)\sqrt[3]{bx^3+a}}{b^2d^3}\right)dx^3$$

$$\Big\downarrow 2009$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}\left(a^2d^2+abcd+b^2c^2\right)}{4b^3d^3}-\frac{\sqrt{3}c^3\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{13/3}}-\frac{3\left(a+bx^3\right)^{7/3}(2ad+bc}{7b^3d^2}\right.$$

input
```
Int[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
((-3*c^3*(a + b*x^3)^(1/3))/d^4 + (3*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*
x^3)^(4/3))/(4*b^3*d^3) - (3*(b*c + 2*a*d)*(a + b*x^3)^(7/3))/(7*b^3*d^2)
+ (3*(a + b*x^3)^(10/3))/(10*b^3*d) - (Sqrt[3]*c^3*(b*c - a*d)^(1/3)*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/d^(13/3)
 - (c^3*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(2*d^(13/3)) + (3*c^3*(b*c - a*d
)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3)))/
3
```

3.658.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.658. $\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

3.658.4 Maple [A] (verified)

Time = 6.04 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{\frac{27\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}d\left(\frac{\left(14d^{3}x^{9}-20c\,d^{2}x^{6}+35c^{2}d\,x^{3}-140c^{3}\right)b^{3}}{9}+\frac{35\left(\frac{2}{35}d^{2}x^{6}-\frac{1}{7}cd\,x^{3}+c^{2}\right)da\,b^{2}}{9}+\frac{5\left(-\frac{d\,x^{3}}{5}+c\right)d^{2}a^{2}b}{3}+a^{3}d^{3}\right)}{70}+}{}$

input
```
int(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(27/70*(1/d*(a*d-b*c))^(2/3)*(b*x^3+a)^(1/3)*d*(1/9*(14*d^3*x^9-20*c*d
^2*x^6+35*c^2*d*x^3-140*c^3)*b^3+35/9*(2/35*d^2*x^6-1/7*c*d*x^3+c^2)*d*a*b
^2+5/3*(-1/5*d*x^3+c)*d^2*a^2*b+a^3*d^3)+b^3*c^3*(a*d-b*c)*(2*arctan(1/3*3
^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^
(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b
*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/(1/d*(a*d-b*c))^
(2/3)/b^3/d^5
```

3.658.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.23

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx =$$
$$-\frac{140\sqrt{3}b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+70\,b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}-(bx\right.}{}$$

input
```
integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/420*(140*sqrt(3)*b^3*c^3*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(
b*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d
)) + 70*b^3*c^3*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^
(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 140*b^3*c^3*((b*c -
 a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(14*b^3*
d^3*x^9 - 2*(10*b^3*c*d^2 - a*b^2*d^3)*x^6 - 140*b^3*c^3 + 35*a*b^2*c^2*d
+ 15*a^2*b*c*d^2 + 9*a^3*d^3 + (35*b^3*c^2*d - 5*a*b^2*c*d^2 - 3*a^2*b*d^3
)*x^3)*(b*x^3 + a)^(1/3))/(b^3*d^4)
```

3.658.6 Sympy [F]

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x**11*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**11*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.658.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.658.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.44

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = -\frac{(b^{34}c^4d^6 - ab^{33}c^3d^7)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{34}cd^{10} - ab^{33}d^{11})} + \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^5} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^5} - \frac{140\,(bx^3+a)^{\frac{1}{3}}b^{30}c^3d^6 - 35\,(bx^3+a)^{\frac{4}{3}}b^{29}c^2d^7 + 20\,(bx^3+a)^{\frac{7}{3}}b^{28}cd^8 - 35\,(bx^3+a)^{\frac{4}{3}}ab^{28}cd^8 - 14\,(bx^3+}{140\,b^{30}d^{10}}$$

input
```
integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b^34*c^4*d^6 - a*b^33*c^3*d^7)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3
 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^34*c*d^10 - a*b^33*d^11) + 1/3*s
qrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3
) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^5 + 1/6*(-b*c*d^2 +
a*d^3)^(1/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d
)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^5 - 1/140*(140*(b*x^3 + a)^(1/3)*b^30*
c^3*d^6 - 35*(b*x^3 + a)^(4/3)*b^29*c^2*d^7 + 20*(b*x^3 + a)^(7/3)*b^28*c*
d^8 - 35*(b*x^3 + a)^(4/3)*a*b^28*c*d^8 - 14*(b*x^3 + a)^(10/3)*b^27*d^9 +
 40*(b*x^3 + a)^(7/3)*a*b^27*d^9 - 35*(b*x^3 + a)^(4/3)*a^2*b^27*d^9)/(b^3
0*d^10)
```

3.658.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.67

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \left(\frac{3\,a^2}{4\,b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{4\,b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{4/3} - \left(\frac{3\,a}{7\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{7\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{7/3} - {\left(b\,x^3+a\right)}^{1/3}\,\left(\frac{a^3}{b^3\,d}+\frac{\left(\frac{3\,a^2}{b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{b^3\,d}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{b^3\,d}\right)+\frac{{\left(b\,x^3+a\right)}^{10/3}}{10\,b^3\,d}-\frac{c^3\,\ln\Big(\big(a\,d-}{}$$

input
```
int((x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
((3*a^2)/(4*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
 - a*b^3*d))/(4*b^3*d))*(a + b*x^3)^(4/3) - ((3*a)/(7*b^3*d) + (b^4*c - a*
b^3*d)/(7*b^6*d^2))*(a + b*x^3)^(7/3) - (a + b*x^3)^(1/3)*(a^3/(b^3*d) + (
((3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c -
 a*b^3*d))/(b^3*d))*(b^4*c - a*b^3*d))/(b^3*d)) + (a + b*x^3)^(10/3)/(10*b
^3*d) - (c^3*log((a*d - b*c)^(1/3) - d^(1/3)*(a + b*x^3)^(1/3))*(a*d - b*c
)^(1/3))/(3*d^(13/3)) - (c^3*log((3*(a + b*x^3)^(1/3)*(b*c^4 - a*c^3*d))/d
^2 + (3*c^3*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(4/3))/d^(7/3))*((3^(1/2)*1
i)/2 - 1/2)*(a*d - b*c)^(1/3))/(3*d^(13/3)) + (c^3*log((3*(a + b*x^3)^(1/3
)*(b*c^4 - a*c^3*d))/d^2 - (9*c^3*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(4/3)
)/d^(7/3))*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(1/3))/d^(13/3)
```

3.659 $\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

3.659.1 Optimal result

Integrand size = 24, antiderivative size = 220

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{c^2 \sqrt[3]{a+bx^3}}{d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^2d} + \frac{c^2 \sqrt[3]{bc-ad} \arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}} + \frac{c^2 \sqrt[3]{bc-ad} \log\left(c+dx^3\right)}{6d^{10/3}} - \frac{c^2 \sqrt[3]{bc-ad} \log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}$$

output
```
c^2*(b*x^3+a)^(1/3)/d^3-1/4*(a*d+b*c)*(b*x^3+a)^(4/3)/b^2/d^2+1/7*(b*x^3+a
)^(7/3)/b^2/d+1/6*c^2*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^(10/3)-1/2*c^2*(-a*d+
b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(10/3)+1/3*c^2*(
-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*
3^(1/2))/d^(10/3)*3^(1/2)
```

3.659.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.20

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}\left(-3a^2d^2+abd\left(-7c+dx^3\right)+b^2\left(28c^2-7cdx^3+4d^2x^6\right)\right)}{b^2}+28\sqrt{3}c^2\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-}{}$$

input
```
Integrate[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
((3*d^(1/3)*(a + b*x^3)^(1/3)*(-3*a^2*d^2 + a*b*d*(-7*c + d*x^3) + b^2*(28
*c^2 - 7*c*d*x^3 + 4*d^2*x^6)))/b^2 + 28*Sqrt[3]*c^2*(b*c - a*d)^(1/3)*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 28*c^
2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 1
4*c^2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*
(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(84*d^(10/3))
```

3.659.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^6\sqrt[3]{bx^3+a}}{dx^3+c}dx^3$$

$$\Big\downarrow \; 99$$

$$\frac{1}{3}\int \left(\frac{\sqrt[3]{bx^3+a}c^2}{d^2\left(dx^3+c\right)}+\frac{\left(bx^3+a\right)^{4/3}}{bd}+\frac{(-bc-ad)\sqrt[3]{bx^3+a}}{bd^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}c^2\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{10/3}}-\frac{3\left(a+bx^3\right)^{4/3}(ad+bc)}{4b^2d^2}+\frac{3\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{bc-ad}\log(c}{2d^{10/3}}\right.$$

input
```
Int[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
((3*c^2*(a + b*x^3)^(1/3))/d^3 - (3*(b*c + a*d)*(a + b*x^3)^(4/3))/(4*b^2*
d^2) + (3*(a + b*x^3)^(7/3))/(7*b^2*d) + (Sqrt[3]*c^2*(b*c - a*d)^(1/3)*Ar
cTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/d^(10
/3) + (c^2*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(2*d^(10/3)) - (3*c^2*(b*c -
a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3)
))/3
```

3.659.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.659.4 Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{\frac{9\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} d\left(\left(-\frac{4b\,x^{3}}{3}+a\right)\left(b\,x^{3}+a\right)d^{2}+\frac{7\left(b\,x^{3}+a\right)bcd}{3}-\frac{28b^{2}c^{2}}{3}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{14}+b^{2}c^{2}(ad-bc)\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad}{}\right.\right.}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}b^{2}d^{4}}$

input
```
int(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(1/d*(a*d-b*c))^(2/3)*(9/14*(1/d*(a*d-b*c))^(2/3)*d*((-4/3*b*x^3+a)*(
b*x^3+a)*d^2+7/3*(b*x^3+a)*b*c*d-28/3*b^2*c^2)*(b*x^3+a)^(1/3)+b^2*c^2*(a*
d-b*c)*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/
d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^
3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/
3))))/b^2/d^4
```

3.659.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.28

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx =$$

$$-\frac{28\sqrt{3}b^2c^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+14\,b^2c^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\right.}{}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/84*(28*sqrt(3)*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b
*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d
)) + 14*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)
^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 28*b^2*c^2*(-(b*
c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 3*(4*b
^2*d^2*x^6 + 28*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2 - (7*b^2*c*d - a*b*d^2)*x^
3)*(b*x^3 + a)^(1/3))/(b^2*d^3)
```

3.659.6 Sympy [F]

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x**8*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**8*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.659.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.659.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx &= \frac{(b^{17}c^3d^4-ab^{16}c^2d^5)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{17}cd^7-ab^{16}d^8)}\\ &\quad-\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^4}\\ &\quad-\frac{(-bcd^2+ad^3)^{\frac{1}{3}}c^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^4}\\ &\quad+\frac{28\,(bx^3+a)^{\frac{1}{3}}b^{14}c^2d^4-7\,(bx^3+a)^{\frac{4}{3}}b^{13}cd^5+4\,(bx^3+a)^{\frac{7}{3}}b^{12}d^6-7\,(bx^3+a)^{\frac{4}{3}}ab^{12}d^6}{28\,b^{14}d^7}\end{aligned}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*(b^17*c^3*d^4 - a*b^16*c^2*d^5)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^17*c*d^7 - a*b^16*d^8) - 1/3*sqrt
(3)*(-b*c*d^2 + a*d^3)^(1/3)*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
 (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^4 - 1/6*(-b*c*d^2 + a*d
^3)^(1/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(
1/3) + (-(b*c - a*d)/d)^(2/3))/d^4 + 1/28*(28*(b*x^3 + a)^(1/3)*b^14*c^2*d
^4 - 7*(b*x^3 + a)^(4/3)*b^13*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^12*d^6 - 7*(b*
x^3 + a)^(4/3)*a*b^12*d^6)/(b^14*d^7)
```

3.659.9 Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.53

$$
\begin{aligned}
&\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx\\
&= \left(\frac{a^2}{b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{b^2\,d}\right)\,{\left(b\,x^3+a\right)}^{1/3}\\
&\quad-\left(\frac{a}{2\,b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{4\,b^4\,d^2}\right)\,{\left(b\,x^3+a\right)}^{4/3}+\frac{{\left(b\,x^3+a\right)}^{7/3}}{7\,b^2\,d}\\
&\quad+\frac{c^2\,\ln\left({\left(a\,d-b\,c\right)}^{1/3}-d^{1/3}\,{\left(b\,x^3+a\right)}^{1/3}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{3\,d^{10/3}}\\
&\quad-\frac{c^2\,\ln\left(\frac{3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(b\,c^3-a\,c^2\,d\right)}{d}-\frac{3\,c^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{d^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{3\,d^{10/3}}\\
&\quad+\frac{c^2\,\ln\left(\frac{3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(b\,c^3-a\,c^2\,d\right)}{d}+\frac{9\,c^2\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{d^{4/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{10/3}}
\end{aligned}
$$

input
```
int((x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
(a^2/(b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*b
^2*d))/(b^2*d))*(a + b*x^3)^(1/3) - (a/(2*b^2*d) + (b^3*c - a*b^2*d)/(4*b^
4*d^2))*(a + b*x^3)^(4/3) + (a + b*x^3)^(7/3)/(7*b^2*d) + (c^2*log((a*d -
b*c)^(1/3) - d^(1/3)*(a + b*x^3)^(1/3))*(a*d - b*c)^(1/3))/(3*d^(10/3)) -
(c^2*log((3*(a + b*x^3)^(1/3)*(b*c^3 - a*c^2*d))/d - (3*c^2*((3^(1/2)*1i)/
2 + 1/2)*(a*d - b*c)^(4/3))/d^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1
/3))/(3*d^(10/3)) + (c^2*log((3*(a + b*x^3)^(1/3)*(b*c^3 - a*c^2*d))/d + (
9*c^2*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3))/d^(4/3))*((3^(1/2)*1i)/6 -
 1/6)*(a*d - b*c)^(1/3))/d^(10/3)
```

3.660 $\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

3.660.1 Optimal result

Integrand size $= 24$, antiderivative size $= 186$

$$\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}} - \frac{c\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{7/3}} + \frac{c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}$$

output
```
-c*(b*x^3+a)^(1/3)/d^2+1/4*(b*x^3+a)^(4/3)/b/d-1/6*c*(-a*d+b*c)^(1/3)*ln(d
*x^3+c)/d^(7/3)+1/2*c*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+
a)^(1/3))/d^(7/3)-1/3*c*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)
^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(7/3)*3^(1/2)
```

3.660.2 Mathematica [A] (verified)

Time $= 0.38$ (sec), antiderivative size $= 226$, normalized size of antiderivative $= 1.22$

$$\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}\left(-4bc+ad+bdx^3\right)}{b} - 4\sqrt{3}c\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + 4c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}-\right.}{12d^{7/3}}$$

input
```
Integrate[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
((3*d^(1/3)*(a + b*x^3)^(1/3)*(-4*b*c + a*d + b*d*x^3))/b - 4*Sqrt[3]*c*(b
*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]] + 4*c*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b
*x^3)^(1/3)] - 2*c*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c
- a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(12*d^(7/3))
```

3.660.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {948, 90, 60, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3} \int \frac{x^3 \sqrt[3]{bx^3+a}}{dx^3+c}dx^3$$

$$\Big\downarrow \; 90$$

$$\frac{1}{3} \left(\frac{3\left(a+bx^3\right)^{4/3}}{4bd}-\frac{c\int \frac{\sqrt[3]{bx^3+a}}{dx^3+c}dx^3}{d}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3} \left(\frac{3\left(a+bx^3\right)^{4/3}}{4bd}-\frac{c\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{d}\right)}{d}\right)$$

$$\Big\downarrow \; 70$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4bd}-\frac{c\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int\frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int\frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{b}}}{2\sqrt[3]{d}(bc}\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \; 16$$

3.660. $\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4bd}-\frac{c\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log}{}\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \color{blue}{1082}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4bd}-\frac{c\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{}\right.}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \color{blue}{217}$$

3.660. $\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

$$\frac{1}{3}\Bigg(\frac{3\left(a+bx^3\right)^{4/3}}{4bd}-\frac{c\Bigg(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\Bigg(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\Big(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{}}{2\sqrt[3]{d}(bc-ad)^{2/3}}}{d}}{d}$$

input
```
Int[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
((3*(a + b*x^3)^(4/3))/(4*b*d) - (c*((3*(a + b*x^3)^(1/3))/d - ((b*c - a*d
)*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))
/Sqrt[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c -
 a*d)^(2/3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d
^(1/3)*(b*c - a*d)^(2/3))))/d))/d)/3
```

3.660. $\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

3.660.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.660. $\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

3.660.4 Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{\frac{3d\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(d\left(b\,x^{3}+a\right)-4bc\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{2}+bc(ad-bc)\Bigg(2\arctan\Bigg(\frac{\sqrt{3}\,\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\Bigg)\sqrt{3}+\ln\Bigg(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\Big(\frac{ad-}{d}}{6\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}b\,d^{3}}$

input
```
int(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/(1/d*(a*d-b*c))^(2/3)*(3/2*d*(1/d*(a*d-b*c))^(2/3)*(d*(b*x^3+a)-4*b*c)
*(b*x^3+a)^(1/3)+b*c*(a*d-b*c)*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1
/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/
d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(
1/3)-(1/d*(a*d-b*c))^(1/3))))/b/d^3
```

3.660.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.19

$$\int\frac{x^{5}\sqrt[3]{a+bx^{3}}}{c+dx^{3}}\,dx=$$
$$-\frac{4\sqrt{3}bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^{3}+a\right)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+2\,bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\Big(\left(bx^{3}+a\right)^{\frac{2}{3}}-\left(bx^{3}+a\right)^{\frac{1}{3}}}{12\,ba}$$

input
```
integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/12*(4*sqrt(3)*b*c*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 +
 a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*
b*c*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c
- a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 4*b*c*((b*c - a*d)/d)^(1/3)*log
((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(b*d*x^3 - 4*b*c + a*d)*(b
*x^3 + a)^(1/3))/(b*d^2)
```

3.660.6 Sympy [F]

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x**5*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**5*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.660.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.660.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.48

$$\begin{aligned}\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx &= -\frac{(b^6c^2d^2-ab^5cd^3)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^6cd^4-ab^5d^5)}\\ &\quad+\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}c\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^3}\\ &\quad+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}c\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^3}\\ &\quad-\frac{4\,(bx^3+a)^{\frac{1}{3}}b^4cd^2-(bx^3+a)^{\frac{4}{3}}b^3d^3}{4\,b^4d^4}\end{aligned}$$

input

```
integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output

```
-1/3*(b^6*c^2*d^2 - a*b^5*c*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a
)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^6*c*d^4 - a*b^5*d^5) + 1/3*sqrt(3)*(
-b*c*d^2 + a*d^3)^(1/3)*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c
 - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^3 + 1/6*(-b*c*d^2 + a*d^3)^(1/
3)*c*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-
(b*c - a*d)/d)^(2/3))/d^3 - 1/4*(4*(b*x^3 + a)^(1/3)*b^4*c*d^2 - (b*x^3 +
a)^(4/3)*b^3*d^3)/(b^4*d^4)
```

3.660.9 Mupad [B] (verification not implemented)

Time = 8.54 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.60

$$
\begin{aligned}
\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx &= \frac{\left(b\,x^3+a\right)^{4/3}}{4\,b\,d}-\left(b\,x^3+a\right)^{1/3}\left(\frac{a}{b\,d}+\frac{b^2\,c-a\,b\,d}{b^2\,d^2}\right) \\
&\quad -\frac{c\,\ln\left(\left(b\,x^3+a\right)^{1/3}\left(3\,b\,c^2-3\,a\,c\,d\right)+\frac{c\,\left(a\,d-b\,c\right)^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\,\left(a\,d-b\,c\right)^{1/3}}{3\,d^{7/3}} \\
&\quad -\frac{c\,\ln\left(\left(b\,x^3+a\right)^{1/3}\left(3\,b\,c^2-3\,a\,c\,d\right)+\frac{c\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{1/3}}{3\,d^{7/3}} \\
&\quad +\frac{c\,\ln\left(\left(b\,x^3+a\right)^{1/3}\left(3\,b\,c^2-3\,a\,c\,d\right)-\frac{c\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{1/3}}{3\,d^{7/3}}
\end{aligned}
$$

input
```
int((x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
(a + b*x^3)^(4/3)/(4*b*d) - (a + b*x^3)^(1/3)*(a/(b*d) + (b^2*c - a*b*d)/(
b^2*d^2)) - (c*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d) + (c*(a*d - b*c)^
(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*(a*d - b*c)^(1/3))/(3*d^(7/3)) -
 (c*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d) + (c*((3^(1/2)*1i)/2 - 1/2)*
(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*((3^(1/2)*1i)/2 - 1/
2)*(a*d - b*c)^(1/3))/(3*d^(7/3)) + (c*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*
a*c*d) - (c*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2)
)/(3*d^(7/3)))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/(3*d^(7/3))
```

3.661 $\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

3.661.1 Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad} \arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3} d^{4/3}} + \frac{\sqrt[3]{bc-ad} \log\left(c+dx^3\right)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad} \log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}}$$

output
```
(b*x^3+a)^(1/3)/d+1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^(4/3)-1/2*(-a*d+b*c)^
(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(4/3)+1/3*(-a*d+b*c)^
(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d
^(4/3)*3^(1/2)
```

3.661.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.29

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{6\sqrt[3]{d}\sqrt[3]{a+bx^3} + 2\sqrt{3}\sqrt[3]{bc-ad} \arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 2\sqrt[3]{bc-ad} \log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{6d^{4/3}}$$

input
```
Integrate[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(6*d^(1/3)*(a + b*x^3)^(1/3) + 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*
d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/
3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*
Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2
/3)*(a + b*x^3)^(2/3)])/(6*d^(4/3))
```

3.661.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {946, 60, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3} \int \frac{\sqrt[3]{bx^3+a}}{dx^3+c} dx^3$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3} \left(\frac{3\sqrt[3]{a+bx^3}}{d} - \frac{(bc-ad) \int \frac{1}{(bx^3+a)^{2/3}(dx^3+c)} dx^3}{d} \right)$$

$$\Big\downarrow \; 70$$

$$\frac{1}{3} \left(\frac{3\sqrt[3]{a+bx^3}}{d} - \frac{(bc-ad) \left(\frac{3 \int \frac{1}{x^6 + \frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}} d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}} + \frac{3 \int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}} + \sqrt[3]{bx^3+a}} d\sqrt[3]{bx^3}}{2\sqrt[3]{d}(bc-ad)^{2/3}} \right.}{d} \right.$$

3.661. $\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

$$\Big\downarrow \ \color{blue}{16}$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\right.}{2\sqrt[3]{d}(bc}\right.}{d}\right.$$

$$\Big\downarrow \ \color{blue}{1082}$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right)}{d}\right.$$

$$\Big\downarrow \ \color{blue}{217}$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right)}{d}\right.$$

3.661. $\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

input
```
Int[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
((3*(a + b*x^3)^(1/3))/d - ((b*c - a*d)*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)
*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(1/3)*(b*c - a*d)^(2/3
))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)^(2/3)) + (3*Log[(b*c - a*d)^(1
/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3)*(b*c - a*d)^(2/3))))/d)/3
```

3.661.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.661. $\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.661.4 Maple [A] (verified)

Time = 4.71 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{d}+\frac{\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)\left(ad-bc\right)}{3d^{2}\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)\left(ad-bc\right)}{6d^{2}\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}}-$

input
```
int(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
(b*x^3+a)^(1/3)/d+1/3/d^2/(1/d*(a*d-b*c))^(2/3)*ln((b*x^3+a)^(1/3)-(1/d*(a
*d-b*c))^(1/3))*(a*d-b*c)-1/6/d^2/(1/d*(a*d-b*c))^(2/3)*ln((b*x^3+a)^(2/3)
+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))*(a*d-b*c)-1/
3/d^2/(1/d*(a*d-b*c))^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+
(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*(a*d-b*c)
```

3.661.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.30

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx =$$

$$-\frac{2\,\sqrt{3}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\,\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}\left(bc-ad\right)}{3\left(bc-ad\right)}\right)+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right.}{6\,d}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^
(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + (-(b*
c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/
d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 +
 a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 6*(b*x^3 + a)^(1/3))/d
```

3.661.6 Sympy [F]

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x**2*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**2*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.661.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.661.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.40

$$
\begin{aligned}
&\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx \\
&= \frac{(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bcd-ad^2)} \\
&\quad - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^2} + \frac{(bx^3+a)^{\frac{1}{3}}}{d} \\
&\quad - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2}
\end{aligned}
$$

input

```
integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output

```
1/3*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
- a*d)/d)^(1/3)))/(b*c*d - a*d^2) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*a
rctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/d^2 + (b*x^3 + a)^(1/3)/d - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*lo
g((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c -
a*d)/d)^(2/3))/d^2
```

3.661.9 Mupad [B] (verification not implemented)

Time = 8.46 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.57

$$
\begin{aligned}
&\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx \\
&= \frac{(b\,x^3+a)^{1/3}}{d} + \frac{\ln\left((b\,x^3+a)^{1/3}\,(3\,a\,d^2-3\,b\,c\,d)-\frac{(a\,d-b\,c)^{1/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{4/3}}\right)\,(a\,d-b\,c)^{1/3}}{3\,d^{4/3}} \\
&\quad - \frac{\ln\left((b\,x^3+a)^{1/3}\,(3\,a\,d^2-3\,b\,c\,d)+\frac{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{1/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{1/3}}{3\,d^{4/3}} \\
&\quad + \frac{\ln\left((b\,x^3+a)^{1/3}\,(3\,a\,d^2-3\,b\,c\,d)-\frac{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(a\,d-b\,c)^{1/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{d^{4/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(a\,d-b\,c)^{1/3}}{d^{4/3}}
\end{aligned}
$$

input
```
int((x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
(a + b*x^3)^(1/3)/d + (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d) - ((a*d -
 b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(4/3)))*(a*d - b*c)^(1/3))/(3*d^(4
/3)) - (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d) + (((3^(1/2)*1i)/2 + 1/2
)*(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(4/3)))*((3^(1/2)*1i)/2 +
1/2)*(a*d - b*c)^(1/3))/(3*d^(4/3)) + (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*
b*c*d) - (((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/
d^(4/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(1/3))/d^(4/3)
```

3.661. $\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

3.662 $\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx$

3.662.1 Optimal result

Integrand size = 24, antiderivative size = 246

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(c+dx^3\right)}\,dx = -\frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c} - \frac{\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{d}} - \frac{\sqrt[3]{a}\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c\sqrt[3]{d}} + \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} + \frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{d}}$$

output
```
-1/2*a^(1/3)*ln(x)/c-1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c/d^(1/3)+1/2*a^(1/3
)*ln(a^(1/3)-(b*x^3+a)^(1/3))/c+1/2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d
^(1/3)*(b*x^3+a)^(1/3))/c/d^(1/3)-1/3*a^(1/3)*arctan(1/3*(a^(1/3)+2*(b*x^3
+a)^(1/3))/a^(1/3)*3^(1/2))/c*3^(1/2)-1/3*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2
*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/d^(1/3)*3^(1/2)
```

3.662.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(c+dx^3\right)}\,dx =$$

$$-\frac{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{d}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt{3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-2\sqrt[3]{a}\sqrt[3]{d}\log\left(-\sqrt[3]{a}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]
```

output
```
-1/6*(2*Sqrt[3]*a^(1/3)*d^(1/3)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))
/Sqrt[3]] + 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)
^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*a^(1/3)*d^(1/3)*Log[-a^(1/3) + (a
+ b*x^3)^(1/3)] - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a +
 b*x^3)^(1/3)] + a^(1/3)*d^(1/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) +
 (a + b*x^3)^(2/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b
*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(c*d^(1/3)
)
```

3.662.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 94, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}}{x^3\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \text{94}$$

$$\frac{1}{3}\left(\frac{(bc-ad)\int \frac{1}{(bx^3+a)^{2/3}(dx^3+c)}dx^3}{c}+\frac{a\int \frac{1}{x^3(bx^3+a)^{2/3}}dx^3}{c}\right)$$

$$\Big\downarrow 69$$

$$\frac{1}{3}\left(\frac{a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2a^{2/3}}\right)}{c}+\frac{(bc-ad)\int \frac{1}{(bx^3+a)^{2/3}}}{c}\right.$$

$$\Big\downarrow 16$$

$$\frac{1}{3}\left(\frac{a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)}{c}+\frac{(bc-ad)\int \frac{1}{(bx^3+a)^{2/3}(dx^3+c)}d}{c}\right.$$

$$\Big\downarrow 70$$

$$\frac{1}{3}\left(\frac{a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)}{c}+\frac{(bc-ad)\left(\frac{3\int \frac{}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-}}{}\right.}{}\right.$$

$$\Big\downarrow 16$$

3.662. $\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx$

$$\frac{1}{3}\left(\frac{a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)}{c}+\frac{(bc-ad)\left(\frac{3\int \frac{}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-}}{}\right.}{}\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{a\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)}{c}+\frac{(bc-ad)\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right.}{\sqrt[3]{d}(bc-ad)^{2/3}}\right.}{}\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)}{c}+\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right.}{\sqrt[3]{d}(bc-ad)^{2/3}}\right.}{}\right.$$

input
```
Int[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]
```

3.662. $\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx$

output
```
((a*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/
3)) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/
3))))/c + ((b*c - a*d)*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3
))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x
^3]/(2*d^(1/3)*(b*c - a*d)^(2/3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
+ b*x^3)^(1/3)])/(2*d^(1/3)*(b*c - a*d)^(2/3))))/c)/3
```

3.662.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.662. $\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx$

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.662.4 Maple [A] (verified)

Time = 4.83 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} d \left(2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)\right)a^{\frac{1}{3}}-$

input
```
int((b*x^3+a)^(1/3)/x/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/(1/d*(a*d-b*c))^(2/3)*(-(1/d*(a*d-b*c))^(2/3)*d*(2*arctan(1/3*(a^(1/3)
+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b
*x^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*a^(1/3)+(a*d-b*c)*(2
*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*
c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3
)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/c/d
```

3.662.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(c+dx^3)}\,dx =$$

$$-\frac{2\sqrt{3}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+2\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right)+a^{\frac{1}{3}}\,\mathrm{l}}{}$$

input
```
integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(
1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*sqrt(
3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3) + sqrt(3)*a)/a)
 + a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) +
((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a*
d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 2*a^(1/3)*log((b*x^3 + a)^(1/3) - a
^(1/3)) - 2*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^
(1/3)))/c
```

3.662.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(c+dx^3)}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{x\,(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x*(c + d*x**3)), x)
```

3.662.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(c+dx^3)}\,dx=\int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)x}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x), x)
```

3.662.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(c+dx^3)}\,dx = -\frac{(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} - \frac{\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,c} - \frac{a^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,c} + \frac{a^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,c} + \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,cd}$$

input
```
integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b*c^2 - a*c*d) - 1/3*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3
)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/c - 1/6*a^(1/3)*log((b*x^3 + a)
^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/c + 1/3*a^(1/3)*log(abs((b*x
^3 + a)^(1/3) - a^(1/3)))/c + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*arctan(
1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d
)^(1/3))/(c*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x
^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c*d)
```

3.662.9 Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 1607, normalized size of antiderivative = 6.53

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(c+dx^3)}\,dx = \text{Too large to display}$$

input

```
int((a + b*x^3)^(1/3)/(x*(c + d*x^3)),x)
```

output

```
log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4
+ 9*a^2*b^6*c^2*d^3) - (a/(27*c^3))^(1/3)*(((243*a*b^6*c^6*d^3 - 729*a^2*b
^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a/(27*c^3))^(1/3) - (a + b*x^3)^(1/3)*(
81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4))*(a/(27*c^3))^(2/3) - 9*a*b^7*c^4*d
^2 + 27*a^2*b^6*c^3*d^3 - 18*a^3*b^5*c^2*d^4))*(a/(27*c^3))^(1/3) + log((a
 + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^
2*b^6*c^2*d^3) - (((243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*
c^4*d^5)*(-(a*d - b*c)/(27*c^3*d))^(1/3) - (a + b*x^3)^(1/3)*(81*a*b^6*c^5
*d^3 - 81*a^2*b^5*c^4*d^4))*(-(a*d - b*c)/(27*c^3*d))^(2/3) - 9*a*b^7*c^4*
d^2 + 27*a^2*b^6*c^3*d^3 - 18*a^3*b^5*c^2*d^4)*(-(a*d - b*c)/(27*c^3*d))^(
1/3))*(-(a*d - b*c)/(27*c^3*d))^(1/3) + log((a + b*x^3)^(1/3)*(6*a^4*b^4*d
^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) + ((3^(1/2)*1
i)/2 - 1/2)*(-(a*d - b*c)/(27*c^3*d))^(1/3)*(((3^(1/2)*1i)/2 - 1/2)^2*((a
+ b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4) - ((3^(1/2)*1i)/2 -
 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a
*d - b*c)/(27*c^3*d))^(1/3))*(-(a*d - b*c)/(27*c^3*d))^(2/3) + 9*a*b^7*c^4
*d^2 - 27*a^2*b^6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 - 1/2)*(-
(a*d - b*c)/(27*c^3*d))^(1/3) - log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a
*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) - ((3^(1/2)*1i)/2 + 1
/2)*(-(a*d - b*c)/(27*c^3*d))^(1/3)*(((3^(1/2)*1i)/2 + 1/2)^2*((a + b*x...
```

3.663 $\int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)}\,dx$

3.663.1 Optimal result

Integrand size = 24, antiderivative size = 340

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx =& \frac{d\sqrt[3]{a+bx^3}}{c^2}+\frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2}-\frac{\left(a+bx^3\right)^{4/3}}{3acx^3}\\ &-\frac{(bc-3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}c^2}\\ &+\frac{d^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2}-\frac{(bc-3ad)\log(x)}{6a^{2/3}c^2}\\ &+\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^2}+\frac{(bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2}\\ &-\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2}\end{aligned}$$

output

```
d*(b*x^3+a)^(1/3)/c^2+1/3*(-3*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2-1/3*(b*x^3+a)
^(4/3)/a/c/x^3-1/6*(-3*a*d+b*c)*ln(x)/a^(2/3)/c^2+1/6*d^(2/3)*(-a*d+b*c)^(
1/3)*ln(d*x^3+c)/c^2+1/6*(-3*a*d+b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(2/3)/
c^2-1/2*d^(2/3)*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/
3))/c^2-1/9*(-3*a*d+b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^
(1/2))/a^(2/3)/c^2*3^(1/2)+1/3*d^(2/3)*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^
(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2*3^(1/2)
```

3.663.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx$$

$$= \frac{-\frac{6c\sqrt[3]{a+bx^3}}{x^3}+\frac{2\sqrt{3}(-bc+3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+6\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)+\frac{2(bc}{}}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]
```

output
```
((-6*c*(a + b*x^3)^(1/3))/x^3 + (2*Sqrt[3]*(-(b*c) + 3*a*d)*ArcTan[(1 + (2
*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) + 6*Sqrt[3]*d^(2/3)*(b*c -
a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sq
rt[3]] + (2*(b*c - 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(2/3) - 6*d
^(2/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)
] + ((-(b*c) + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3
)^(2/3)])/a^(2/3) + 3*d^(2/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^
(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(1
8*c^2)
```

3.663.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {948, 114, 27, 174, 60, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}}{x^6\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow 114$$

$$\frac{1}{3}\left(-\frac{\int -\frac{\sqrt[3]{bx^3+a}\,(bdx^3+bc-3ad)}{3x^3(dx^3+c)}dx^3}{ac}-\frac{\left(a+bx^3\right)^{4/3}}{acx^3}\right)$$

$$\Big\downarrow 27$$

$$\frac{1}{3}\left(\frac{\int \frac{\sqrt[3]{bx^3+a}\,(bdx^3+bc-3ad)}{x^3(dx^3+c)}dx^3}{3ac}-\frac{\left(a+bx^3\right)^{4/3}}{acx^3}\right)$$

$$\Big\downarrow 174$$

$$\frac{1}{3}\left(\frac{\frac{3ad^2\int \frac{\sqrt[3]{bx^3+a}}{dx^3+c}dx^3}{c}+\frac{(bc-3ad)\int \frac{\sqrt[3]{bx^3+a}}{x^3}dx^3}{c}}{3ac}-\frac{\left(a+bx^3\right)^{4/3}}{acx^3}\right)$$

$$\Big\downarrow 60$$

$$\frac{1}{3}\left(\frac{\frac{3ad^2\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{d}\right)}{c}+\frac{(bc-3ad)\left(a\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3+3\sqrt[3]{a+bx^3}\right)}{c}}{3ac}-\frac{\left(a+bx^3\right)^{4/3}}{acx^3}\right)$$

$$\Big\downarrow 69$$

$$\frac{1}{3}\left(\frac{\frac{(bc-3ad)\left(a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{3ad^2}{}}{3ac}\right.$$

$$\Big\downarrow 16$$

3.663. $\int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)}\,dx$

$$\frac{1}{3}\left(\frac{\frac{(bc-3ad)\left(a\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{3ad^2\left(\frac{3\sqrt[3]{a+}}{d}\right.}{}}{3ac}\right.$$

$$\Big\downarrow \text{70}$$

$$\frac{1}{3}\left(\frac{\frac{(bc-3ad)\left(a\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{3ad^2\left(\frac{3\sqrt[3]{a+}}{d}\right.}{}}{}\right.$$

$$\Big\downarrow \text{16}$$

3.663. $\int\frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(\frac{\frac{(bc-3ad)\left(a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{3ad^2\left(\frac{3\sqrt[3]{a+}}{d}\right.}{}}{3}\right.$$

$$\Big\downarrow \textbf{1082}$$

$$\frac{1}{3}\left(\frac{\frac{(bc-3ad)\left(a\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{3ad^2\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad}{}\right.}{}}{3ac}\right.$$

$$\Big\downarrow \textbf{217}$$

3.663. $\int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)}\,dx$

$$\frac{1}{3}\left(\frac{\frac{(bc-3ad)\left(a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{3ad^2\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad}{}\right.}{3ac}\right.$$

input
```
Int[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]
```

output
```
(-((a + b*x^3)^(4/3)/(a*c*x^3)) + (((b*c - 3*a*d)*(3*(a + b*x^3)^(1/3) + a
*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3))
 - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3))
)))/c + (3*a*d^2*((3*(a + b*x^3)^(1/3))/d - ((b*c - a*d)*(-((Sqrt[3]*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(1/3)
*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)^(2/3)) + (3*L
og[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3)*(b*c - a*d)^
(2/3))))/d))/c)/(3*a*c))/3
```

3.663. $\int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)}\,dx$

3.663.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

3.663. $\int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)}\,dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.663.4 Maple [A] (verified)

Time = 5.23 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-\frac{-\frac{x^{3}\left(a^{\frac{2}{3}} b c-a^{\frac{5}{3}} d\right) \ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{a d-b c}{d}\right)^{\frac{2}{3}}\right)}{2}-x^{3}\sqrt{3}\left(a^{\frac{2}{3}} b c-a^{\frac{5}{3}} d\right) \arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad}{}\right.\right.}{3\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}}\right.}{}$

input
```
int((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-1/2*x^3*(a^(2/3)*b*c-a^(5/3)*d)*ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^
(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-x^3*3^(1/2)*(a^(2/3)*b*c-a^(5
/3)*d)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(
a*d-b*c))^(1/3))-1/2*(1/d*(a*d-b*c))^(2/3)*x^3*(a*d-1/3*b*c)*ln((b*x^3+a)^
(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))+x^3*(a^(2/3)*b*c-a^(5/3)*d)*ln((b*x
^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))+(1/d*(a*d-b*c))^(2/3)*(-x^3*3^(1/2)*(a*
d-1/3*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))+x^3*(a*
d-1/3*b*c)*ln((b*x^3+a)^(1/3)-a^(1/3))+(b*x^3+a)^(1/3)*a^(2/3)*c))/a^(2/3)
/(1/d*(a*d-b*c))^(2/3)/c^2/x^3
```

3.663.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx =$$

$$-\frac{6\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}a^2x^3\arctan\left(-\frac{2\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}-\sqrt{3}(bcd-ad^2)}{3\,(bcd-ad^2)}\right)+3\,(-bcd^2+ad^3)^{\frac{1}{3}}a^2x^3\log((}{}$$

input
```
integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/18*(6*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*arctan(-1/3*(2*sqrt(3)*(
-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d^2))/(b*c*
d - a*d^2)) + 3*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*log((b*x^3 + a)^(2/3)*d^2
 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)
) - 6*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2
 + a*d^3)^(1/3)) + 2*sqrt(3)*(a*b*c - 3*a^2*d)*x^3*sqrt(-(-a^2)^(1/3))*arc
tan(-1/3*(sqrt(3)*(-a^2)^(1/3)*a - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3
))*sqrt(-(-a^2)^(1/3))/a^2) + (-a^2)^(2/3)*(b*c - 3*a*d)*x^3*log((b*x^3 +
a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(
2/3)*(b*c - 3*a*d)*x^3*log((b*x^3 + a)^(1/3)*a - (-a^2)^(2/3)) + 6*(b*x^3
+ a)^(1/3)*a^2*c)/(a^2*c^2*x^3)
```

3.663.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**4/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**4*(c + d*x**3)), x)
```

3.663.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^4}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^4), x)
```

3.663.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.03

$$
\begin{aligned}
\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx
&= \frac{(bcd-ad^2)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)} \\
&\quad-\frac{\sqrt{3}(bc-3\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{2}{3}}c^2} \\
&\quad-\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^2} \\
&\quad-\frac{(bc-3\,ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{2}{3}}c^2} \\
&\quad-\frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^2} \\
&\quad+\frac{(bc-3\,ad)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{2}{3}}c^2}-\frac{(bx^3+a)^{\frac{1}{3}}}{3\,cx^3}
\end{aligned}
$$

input

```
integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

output

```
1/3*(b*c*d - a*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(
b*c - a*d)/d)^(1/3)))/(b*c^3 - a*c^2*d) - 1/9*sqrt(3)*(b*c - 3*a*d)*arctan
(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(2/3)*c^2) - 1/3*
sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
 (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^2 - 1/18*(b*c - 3*a*d)*
log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3)*c^2)
 - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*
(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^2 + 1/9*(b*c - 3*a*d)*l
og(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*c^2) - 1/3*(b*x^3 + a)^(1/3)
/(c*x^3)
```

3.663.9 Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 1917, normalized size of antiderivative = 5.64

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

input

```
int((a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x)
```

output

```
log(- ((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*
d) - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - b*c)^3/
(a^2*c^6))^(1/3))*(-(3*a*d - b*c)^3/(a^2*c^6))^(2/3))/81 - (b^5*d^4*(27*a^
3*d^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*(-(3*a*d - b*c)
^3/(a^2*c^6))^(1/3))/9 - (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*
c^4 + 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*(-(2
7*a^3*d^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^(1/3)
 + log(- ((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b
*c*d) - 81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*
c))/c^6)^(1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 - (b^5*d^4*(27*a^3*d^3 +
b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)
^(1/3))/3 - (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*
b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((a*d^3 - b*c*d^2
)/(27*c^6))^(1/3) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(
27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) - 81*a*
b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2
*(a*d - b*c))/c^6)^(1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 + (b^5*d^4*(27*
a^3*d^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d -
b*c))/c^6)^(1/3))/3 - (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4
 + 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((3^...
```

3.664 $\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)}\,dx$

3.664.1 Optimal result

Integrand size = 24, antiderivative size = 370

$$
\begin{aligned}
\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx = {} & \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} \\
& + \frac{\left(b^2c^2+3abcd-9a^2d^2\right)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}c^3} \\
& - \frac{d^{5/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3} \\
& + \frac{\left(b^2c^2+3abcd-9a^2d^2\right)\log(x)}{18a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^3} \\
& - \frac{\left(b^2c^2+3abcd-9a^2d^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3} \\
& + \frac{d^{5/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}
\end{aligned}
$$

output
```
1/9*(3*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x^3-1/6*(b*x^3+a)^(4/3)/a/c/x^6+1/18
*(-9*a^2*d^2+3*a*b*c*d+b^2*c^2)*ln(x)/a^(5/3)/c^3-1/6*d^(5/3)*(-a*d+b*c)^(
1/3)*ln(d*x^3+c)/c^3-1/18*(-9*a^2*d^2+3*a*b*c*d+b^2*c^2)*ln(a^(1/3)-(b*x^3
+a)^(1/3))/a^(5/3)/c^3+1/2*d^(5/3)*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^
(1/3)*(b*x^3+a)^(1/3))/c^3+1/27*(-9*a^2*d^2+3*a*b*c*d+b^2*c^2)*arctan(1/3*
(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(5/3)/c^3*3^(1/2)-1/3*d^(5/
3)*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/
3))*3^(1/2))/c^3*3^(1/2)
```

3.664.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx = \frac{\frac{3c\sqrt[3]{a+bx^3}(-3ac-bcx^3+6adx^3)}{ax^6}+\frac{2\sqrt{3}(b^2c^2+3abcd-9a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}-18\sqrt{3}d^{5/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-}{}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]
```

output
```
((3*c*(a + b*x^3)^(1/3)*(-3*a*c - b*c*x^3 + 6*a*d*x^3))/(a*x^6) + (2*Sqrt[
3]*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(
1/3))/Sqrt[3]])/a^(5/3) - 18*Sqrt[3]*d^(5/3)*(b*c - a*d)^(1/3)*ArcTan[(1 -
 (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - (2*(b^2*c^2 +
 3*a*b*c*d - 9*a^2*d^2)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(5/3) + 18*d^
(5/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]
 + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1
/3) + (a + b*x^3)^(2/3)])/a^(5/3) - 9*d^(5/3)*(b*c - a*d)^(1/3)*Log[(b*c -
 a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b
*x^3)^(2/3)])/(54*c^3)
```

3.664.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {948, 114, 27, 166, 27, 174, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7 \left(c+dx^3\right)} \, dx$$

$$\Big\downarrow \textbf{948}$$

$$\frac{1}{3} \int \frac{\sqrt[3]{bx^3+a}}{x^9 \left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \textbf{114}$$

$$\frac{1}{3} \left(-\frac{\int \frac{2 \sqrt[3]{bx^3+a} \left(bdx^3+bc+3ad\right)}{3x^6 \left(dx^3+c\right)}dx^3}{2ac}-\frac{\left(a+bx^3\right)^{4/3}}{2acx^6}\right)$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{3} \left(-\frac{\int \frac{\sqrt[3]{bx^3+a} \left(bdx^3+bc+3ad\right)}{x^6 \left(dx^3+c\right)}dx^3}{3ac}-\frac{\left(a+bx^3\right)^{4/3}}{2acx^6}\right)$$

$$\Big\downarrow \textbf{166}$$

$$\frac{1}{3} \left(-\frac{\frac{\int \frac{bd(bc-6ad)x^3+b^2c^2-9a^2d^2+3abcd}{3x^3 \left(bx^3+a\right)^{2/3} \left(dx^3+c\right)}dx^3}{c}-\frac{\sqrt[3]{a+bx^3} (3ad+bc)}{cx^3}}{3ac}-\frac{\left(a+bx^3\right)^{4/3}}{2acx^6}\right)$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{3} \left(-\frac{\frac{\int \frac{bd(bc-6ad)x^3+b^2c^2-9a^2d^2+3abcd}{x^3 \left(bx^3+a\right)^{2/3} \left(dx^3+c\right)}dx^3}{3c}-\frac{\sqrt[3]{a+bx^3} (3ad+bc)}{cx^3}}{3ac}-\frac{\left(a+bx^3\right)^{4/3}}{2acx^6}\right)$$

$$\Big\downarrow \textbf{174}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3}{c}-\frac{9ad^2(bc-ad)\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{c}}{3c}-\frac{\sqrt[3]{a+bx^3}(3ad+bc)}{cx^3}}{3ac}-\frac{\left(a+bx^3\right)^{4/3}}{2acx^6}\right)$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{9ad^2(bc-ad)\int}{}}{3c}}{3ac}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(-\frac{\frac{\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{9ad^2(bc-ad)\int \frac{1}{\left(bx^3+a\right)^{2/3}}}{c}}{3c}}{3ac}\right.$$

$$\Big\downarrow \text{70}$$

$$\frac{1}{3}\left(-\frac{\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{9ad^2(bc-ad)\left(\frac{3\int \frac{}{x^6+\frac{(bc-}{a}}}{}\right.}{}}{3c}\right.$$

3.664. $\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(-\frac{\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{9ad^2(bc-ad)\left(\frac{3\int\frac{}{x^6+\frac{(bc-}{c}}}{}\right.}{3c}}{3a}\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(-\frac{\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\left(\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{9ad^2(bc-ad)\left(\frac{3\int\frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{b}}{\sqrt[3]{bc-}}\right.}{\sqrt[3]{d}(bc-ad)^{2/3}}\right.}{3c}}{3ac}\right.$$

$$\Big\downarrow \text{217}$$

3.664. $\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)}\,dx$

$$\frac{1}{3}\left(-\frac{\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{9ad^2(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a}}{\sqrt[3]{bc-}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}\right.}{3c}}{3ac}\right.$$

input
```
Int[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]
```

output
```
(-1/2*(a + b*x^3)^(4/3)/(a*c*x^6) - (-(((b*c + 3*a*d)*(a + b*x^3)^(1/3))/(
c*x^3)) + (((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(-((Sqrt[3]*ArcTan[(1 + (2*(
a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3]/(2*a^(2/3)) + (3*
Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3))))/c - (9*a*d^2*(b*c - a*d)*(
-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sq
rt[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*
d)^(2/3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1
/3)*(b*c - a*d)^(2/3))))/c)/(3*c))/(3*a*c))/3
```

3.664.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.664. $\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)}\,dx$

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.664.4 Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$-\frac{-\frac{\left(a^{\frac{11}{3}} d-a^{\frac{8}{3}} b c\right) x^{6} d \ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{a d-b c}{d}\right)^{\frac{2}{3}}\right)}{2}-\left(a^{\frac{11}{3}} d-a^{\frac{8}{3}} b c\right) x^{6} \sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\right.}{3\left(\frac{ad-bc}{d}\right.}\right.}{}$

input
```
int((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(1/d*(a*d-b*c))^(2/3)*(-1/2*(a^(11/3)*d-a^(8/3)*b*c)*x^6*d*ln((b*x^3+
a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-(a^(
11/3)*d-a^(8/3)*b*c)*x^6*3^(1/2)*d*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(
1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))+1/2*(1/d*(a*d-b*c))^(2/3)*x^6
*(a^2*d^2-1/3*a*b*c*d-1/9*b^2*c^2)*a*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^
(1/3)+a^(2/3))+(a^(11/3)*d-a^(8/3)*b*c)*x^6*d*ln((b*x^3+a)^(1/3)-(1/d*(a*d
-b*c))^(1/3))-(1/d*(a*d-b*c))^(2/3)*(-x^6*3^(1/2)*(a^2*d^2-1/3*a*b*c*d-1/9
*b^2*c^2)*a*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))+x^6*(a
^2*d^2-1/3*a*b*c*d-1/9*b^2*c^2)*a*ln((b*x^3+a)^(1/3)-a^(1/3))-1/6*c*(b*x^3
+a)^(1/3)*((-6*d*x^3+3*c)*a^(8/3)+a^(5/3)*b*c*x^3)))/a^(8/3)/c^3/x^6
```

3.664.5 Fricas [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx =$$

$$-\frac{18\sqrt{3}(bcd^2-ad^3)^{\frac{1}{3}}a^3dx^6\arctan\left(-\frac{2\sqrt{3}(bcd^2-ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}-\sqrt{3}(bcd-ad^2)}{3(bcd-ad^2)}\right)+9\left(bcd^2-ad^3\right)^{\frac{1}{3}}a^3dx^6\log\Big((b}{}$$

input
```
integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/54*(18*sqrt(3)*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*arctan(-1/3*(2*sqrt(3)
*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d^2))/(b*c
*d - a*d^2)) + 9*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*log((b*x^3 + a)^(2/3)*d
^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)
) - 18*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*log((b*x^3 + a)^(1/3)*d + (b*c*d^
2 - a*d^3)^(1/3)) - 2*sqrt(3)*(a*b^2*c^2 + 3*a^2*b*c*d - 9*a^3*d^2)*(a^2)^
(1/6)*x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3
 + a)^(1/3)*(a^2)^(2/3))/a^2) - (b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(a^2)^(2
/3)*x^6*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^
(2/3)) + 2*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(a^2)^(2/3)*x^6*log((b*x^3 +
a)^(1/3)*a - (a^2)^(2/3)) + 3*(3*a^3*c^2 + (a^2*b*c^2 - 6*a^3*c*d)*x^3)*(b
*x^3 + a)^(1/3))/(a^3*c^3*x^6)
```

3.664.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**7/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**7*(c + d*x**3)), x)
```

3.664.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{(dx^3+c)x^7}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^7), x)
```

3.664.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.23

$$\begin{aligned}
&\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx \\
&= -\frac{(bcd^2-ad^3)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^4-ac^3d\right)} \\
&\quad+\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^3} \\
&\quad+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}d\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^3} \\
&\quad+\frac{\sqrt{3}(b^2c^2+3\,abcd-9\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{5}{3}}c^3} \\
&\quad+\frac{(b^2c^2+3\,abcd-9\,a^2d^2)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54\,a^{\frac{5}{3}}c^3} \\
&\quad-\frac{(b^2c^2+3\,abcd-9\,a^2d^2)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{5}{3}}c^3} \\
&\quad-\frac{(bx^3+a)^{\frac{4}{3}}b^2c+2\,(bx^3+a)^{\frac{1}{3}}ab^2c-6\,(bx^3+a)^{\frac{4}{3}}abd+6\,(bx^3+a)^{\frac{1}{3}}a^2bd}{18\,ab^2c^2x^6}
\end{aligned}$$

input
```
integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b*c*d^2 - a*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) -
(-(b*c - a*d)/d)^(1/3)))/(b*c^4 - a*c^3*d) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3
)^(1/3)*d*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3)
)/(-(b*c - a*d)/d)^(1/3))/c^3 + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*d*log((b*x^3
+ a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(
2/3))/c^3 + 1/27*sqrt(3)*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*arctan(1/3*sqrt
(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(5/3)*c^3) + 1/54*(b^2*c^2
 + 3*a*b*c*d - 9*a^2*d^2)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3
) + a^(2/3))/(a^(5/3)*c^3) - 1/27*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*log(ab
s((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*c^3) - 1/18*((b*x^3 + a)^(4/3)*b^
2*c + 2*(b*x^3 + a)^(1/3)*a*b^2*c - 6*(b*x^3 + a)^(4/3)*a*b*d + 6*(b*x^3 +
 a)^(1/3)*a^2*b*d)/(a*b^2*c^2*x^6)
```

3.664.9 Mupad [B] (verification not implemented)

Time = 16.50 (sec) , antiderivative size = 2767, normalized size of antiderivative = 7.48

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x)
```

output

```
log(((((81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^5*(a*d - b
*c))/c^9)^(1/3) + (9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 +
 a*b^2*c^2*d - 14*a^2*b*c*d^2))/a)*(-(d^5*(a*d - b*c))/c^9)^(2/3))/9 - (b^
5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 8
64*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(d
^5*(a*d - b*c))/c^9)^(1/3))/3 - (b^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 +
 b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4
 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8)
)*(-(a*d^6 - b*c*d^5)/(27*c^9))^(1/3) + log((((((9*b^5*c^2*d^3*(a + b*x^3)
^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a + 9*a*b^4*
c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c
*d)^3/(a^5*c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(
2/3))/729 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*
b^3*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81
*a^3*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))/27 - (b
^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 13
5*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*
c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8))*(-(b^6*c^6 - 729*a^6*d^6 - 135*a
^3*b^3*c^3*d^3 + 9*a*b^5*c^5*d + 729*a^5*b*c*d^5)/(19683*a^5*c^9))^(1/3) -
 (((a + b*x^3)^(1/3)*(b^2*c + 3*a*b*d))/(9*c^2) - (b*(a + b*x^3)^(4/3)*...
```

3.664. $\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)}\,dx$

3.665 $\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

3.665.1 Optimal result

Integrand size = 24, antiderivative size = 336

$$
\begin{aligned}
\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = & -\frac{(6bc-ad)x^2\sqrt[3]{a+bx^3}}{18bd^2} + \frac{x^5\sqrt[3]{a+bx^3}}{6d} \\
& - \frac{(9b^2c^2-3abcd-a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}d^3} \\
& + \frac{c^{5/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3} \\
& - \frac{c^{5/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^3} \\
& - \frac{(9b^2c^2-3abcd-a^2d^2)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}d^3} \\
& + \frac{c^{5/3}\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3}
\end{aligned}
$$

output
```
-1/18*(-a*d+6*b*c)*x^2*(b*x^3+a)^(1/3)/b/d^2+1/6*x^5*(b*x^3+a)^(1/3)/d-1/6
*c^(5/3)*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^3-1/18*(-a^2*d^2-3*a*b*c*d+9*b^2*c
^2)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/d^3+1/2*c^(5/3)*(-a*d+b*c)^(1/3)
*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^3-1/27*(-a^2*d^2-3*a*b*c
*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)/
d^3*3^(1/2)+1/3*c^(5/3)*(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*
x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)
```

3.665.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.45 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.57

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\frac{6dx^2\sqrt[3]{a+bx^3}(-6bc+ad+3bdx^3)}{b} - \frac{4\sqrt{3}(9b^2c^2-3abcd-a^2d^2)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}_x}{\sqrt[3]{b}_{x+2}\sqrt[3]{a+bx^3}}\right)}{b^{5/3}} - 18\sqrt{-6-6i\sqrt{3}}c^{5/3}\sqrt[3]{bc-a}}{}$$

input
```
Integrate[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
((6*d*x^2*(a + b*x^3)^(1/3)*(-6*b*c + a*d + 3*b*d*x^3))/b - (4*Sqrt[3]*(9*
b^2*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(
a + b*x^3)^(1/3))])/b^(5/3) - 18*Sqrt[-6 - (6*I)*Sqrt[3]]*c^(5/3)*(b*c - a
*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3
*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(-9*b^2*c^2 + 3*a*b*c*d + a
^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(5/3) + (18*I)*(I + Sqrt[
3])*c^(5/3)*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*
c^(1/3)*(a + b*x^3)^(1/3)] + (2*(9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*Log[b^(2
/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(5/3) + 9*(1
 - I*Sqrt[3])*c^(5/3)*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1
- I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3
])*c^(2/3)*(a + b*x^3)^(2/3)])/(108*d^3)
```

3.665.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {978, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$\Big\downarrow \ 978$$

$$\frac{x^5\sqrt[3]{a+bx^3}}{6d}-\frac{\int \frac{x^4\left((6bc-ad)x^3+5ac\right)}{(bx^3+a)^{2/3}(dx^3+c)}dx}{6d}$$

$$\Big\downarrow \ 1052$$

$$\frac{x^5\sqrt[3]{a+bx^3}}{6d}-\frac{\frac{x^2\sqrt[3]{a+bx^3}(6bc-ad)}{3bd}-\frac{\int \frac{2x\left(\left(9b^2c^2-3abdc-a^2d^2\right)x^3+ac(6bc-ad)\right)}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{3bd}}{6d}$$

$$\Big\downarrow \ 27$$

$$\frac{x^5\sqrt[3]{a+bx^3}}{6d}-\frac{\frac{x^2\sqrt[3]{a+bx^3}(6bc-ad)}{3bd}-\frac{2\int \frac{x\left(\left(9b^2c^2-3abdc-a^2d^2\right)x^3+ac(6bc-ad)\right)}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{3bd}}{6d}$$

$$\Big\downarrow \ 1054$$

$$\frac{x^5\sqrt[3]{a+bx^3}}{6d}-\frac{\frac{x^2\sqrt[3]{a+bx^3}(6bc-ad)}{3bd}-\frac{2\int \left(\frac{\left(9b^2c^2-3abdc-a^2d^2\right)x}{d\left(bx^3+a\right)^{2/3}}+\frac{9\left(abc^2d-b^2c^3\right)x}{d\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}\right)dx}{3bd}}{6d}$$

$$\Big\downarrow \ 2009$$

$$\frac{x^5\sqrt[3]{a+bx^3}}{6d}-$$

$$\frac{x^2\sqrt[3]{a+bx^3}(6bc-ad)}{3bd}-2\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\left(-a^2d^2-3abcd+9b^2c^2\right)}{\sqrt{3}b^{2/3}d}-\frac{\left(-a^2d^2-3abcd+9b^2c^2\right)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}+3\sqrt{3}bc^{5/3}\right.$$

$$6d$$

input

```
Int[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output

```
(x^5*(a + b*x^3)^(1/3))/(6*d) - (((6*b*c - a*d)*x^2*(a + b*x^3)^(1/3))/(3*
b*d) - (2*(-(((9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTan[(1 + (2*b^(1/3)*x)/
(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)*d)) + (3*Sqrt[3]*b*c^(5/3)*(
b*c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^
(1/3)))/Sqrt[3]])/d - (3*b*c^(5/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(2*d)
 - ((9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/
(2*b^(2/3)*d) + (9*b*c^(5/3)*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c
^(1/3) - (a + b*x^3)^(1/3)])/(2*d)))/(3*b*d))/(6*d)
```

3.665.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 978

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1052

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.665.4 Maple [A] (verified)

Time = 6.38 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.32

method	result
pseudoelliptic	$\dfrac{-\dfrac{\left(b^{\frac{11}{3}}c-b^{\frac{8}{3}}ad\right)c\ln\left(\dfrac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}x+\left(bx^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}-\left(b^{\frac{11}{3}}c-b^{\frac{8}{3}}ad\right)\sqrt{3}\,c\arctan\Bigg(\dfrac{\sqrt{3}\,\Big(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\big(b}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}}{}$

input
```
int(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-1/2*(b^(11/3)*c-b^(8/3)*a*d)*c*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c
)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-(b^(11/3)*c-b^(8/3)*a*d
)*3^(1/2)*c*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/(
(a*d-b*c)/c)^(1/3)/x)-1/18*((a*d-b*c)/c)^(2/3)*b*(a^2*d^2+3*a*b*c*d-9*b^2*
c^2)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(b^(1
1/3)*c-b^(8/3)*a*d)*c*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+1/9*(-
3^(1/2)*b*(a^2*d^2+3*a*b*c*d-9*b^2*c^2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b
*x^3+a)^(1/3))/b^(1/3)/x)+b*(a^2*d^2+3*a*b*c*d-9*b^2*c^2)*ln((-b^(1/3)*x+(
b*x^3+a)^(1/3))/x)+3/2*x^2*((3*d*x^3-6*c)*b^(8/3)+a*b^(5/3)*d)*d*(b*x^3+a)
^(1/3))*((a*d-b*c)/c)^(2/3))/((a*d-b*c)/c)^(2/3)/b^(8/3)/d^3
```

3.665.5 Fricas [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.47

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$=\frac{18\sqrt{3}(bc^3-ac^2d)^{\frac{1}{3}}b^3c\arctan\left(-\frac{\sqrt{3}(bc^2-acd)x+2\sqrt{3}(bc^3-ac^2d)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}}{3\,(bc^2-acd)x}\right)+18\,(bc^3-ac^2d)^{\frac{1}{3}}b^3c\log\Bigg(\frac{(bx^3+a)^{\frac{1}{3}}}{}}{}$$

input
```
integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

---

output

```
1/54*(18*sqrt(3)*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*arctan(-1/3*(sqrt(3)*(b*c^2
 - a*c*d)*x + 2*sqrt(3)*(b*c^3 - a*c^2*d)^(2/3)*(b*x^3 + a)^(1/3))/((b*c^2
 - a*c*d)*x)) + 18*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*log(((b*x^3 + a)^(1/3)*c
- (b*c^3 - a*c^2*d)^(1/3)*x)/x) - 9*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*log(((b*
x^3 + a)^(2/3)*c^2 + (b*c^3 - a*c^2*d)^(1/3)*(b*x^3 + a)^(1/3)*c*x + (b*c^
3 - a*c^2*d)^(2/3)*x^2)/x^2) + 2*sqrt(3)*(9*b^3*c^2 - 3*a*b^2*c*d - a^2*b*
d^2)*(b^2)^(1/6)*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 +
a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*(9*b^2*c^2 - 3*a*b*c*d - a^
2*d^2)*(b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (9*b^2*
c^2 - 3*a*b*c*d - a^2*d^2)*(b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a
)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2) + 3*(3*b^3*d^2*x^5 - (6*
b^3*c*d - a*b^2*d^2)*x^2)*(b*x^3 + a)^(1/3))/(b^3*d^3)
```

3.665.6 Sympy [F]

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input

```
integrate(x**7*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output

```
Integral(x**7*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.665.7 Maxima [F]

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{1}{3}}x^7}{dx^3+c}\,dx$$

input

```
integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output

```
integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)
```

3.665.8 Giac [F]

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \int \frac{(bx^3+a)^{\frac{1}{3}} x^7}{dx^3+c} \, dx$$

input

```
integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output

```
integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)
```

3.665.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \int \frac{x^7 \left(b\,x^3+a\right)^{1/3}}{d\,x^3+c} \, dx$$

input

```
int((x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output

```
int((x^7*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

3.666 $\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

3.666.1 Optimal result

Integrand size = 24, antiderivative size = 276

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\frac{x^2\sqrt[3]{a+bx^3}}{3d}+\frac{(3bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}d^2}-\frac{c^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{c^{2/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^2}+\frac{(3bc-ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}d^2}-\frac{c^{2/3}\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2}$$

output
```
1/3*x^2*(b*x^3+a)^(1/3)/d+1/6*c^(2/3)*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^2+1/6
*(-a*d+3*b*c)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)/d^2-1/2*c^(2/3)*(-a*d+
b*c)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2+1/9*(-a*d+3*
b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)/d^2*3^(1/
2)-1/3*c^(2/3)*(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)
/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)
```

3.666.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.47 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.69

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{12dx^2\sqrt[3]{a+bx^3}+\frac{4\sqrt{3}(3bc-ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{2/3}}+6\sqrt{-6-6i\sqrt{3}}c^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{}{\sqrt{3}\sqrt[3]{bc-ad}}\right.}{}$$

input
```
Integrate[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(12*d*x^2*(a + b*x^3)^(1/3) + (4*Sqrt[3]*(3*b*c - a*d)*ArcTan[(Sqrt[3]*b^(
1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(2/3) + 6*Sqrt[-6 - (6*I)*Sq
rt[3]]*c^(2/3)*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(
b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(3*b
*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(2/3) + 6*(1 - I*Sqrt[3
])*c^(2/3)*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c
^(1/3)*(a + b*x^3)^(1/3)] + (2*(-3*b*c + a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*
(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(2/3) + (3*I)*(I + Sqrt[3])*c^(2
/3)*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/
3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*
x^3)^(2/3)])/(36*d^2)
```

3.666.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {978, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$\Big\downarrow \ 978$$

$$\frac{x^2\sqrt[3]{a+bx^3}}{3d}-\frac{\int \frac{x\left((3bc-ad)x^3+2ac\right)}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{3d}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{x^2\sqrt[3]{a+bx^3}}{3d}-\frac{\int \left(\frac{(3bc-ad)x}{d\left(bx^3+a\right)^{2/3}}+\frac{3\left(acd-bc^2\right)x}{d\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}\right)dx}{3d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2\sqrt[3]{a+bx^3}}{3d}-$$

$$\frac{-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-ad)}{\sqrt{3}b^{2/3}d}+\frac{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{d}-\frac{(3bc-ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-c^{2/3}}{3d}$$

input
```
Int[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(x^2*(a + b*x^3)^(1/3))/(3*d) - (-(((3*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x
)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)*d)) + (Sqrt[3]*c^(2/3)*(b*
c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1
/3)))/Sqrt[3]])/d - (c^(2/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(2*d) - ((3
*b*c - a*d)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*b^(2/3)*d) + (3*c^(2/3)
*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*d))/(3*d)
```

3.666.3.1 Defintions of rubi rules used

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

3.666. $\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.666.4 Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.42

method	result
pseudoelliptic	$-\frac{-\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}(ad-3bc)\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{6}+\left(-ad\,b^{\frac{2}{3}}+b^{\frac{5}{3}}c\right)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\sqrt{3}\left(\frac{ad-bc}{c}\right.}{}}{}$

input
```
int(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^(2/3)/((a*d-b*c)/c)^(2/3)*(-1/6*((a*d-b*c)/c)^(2/3)*(a*d-3*b*c)*ln(
(b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(-a*d*b^(2/3)
+b^(5/3)*c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/3*3^(1/2)*((a*
d-b*c)/c)^(2/3)*(a*d-3*b*c)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3
))/b^(1/3)/x)+1/3*((a*d-b*c)/c)^(2/3)*(a*d-3*b*c)*ln((-b^(1/3)*x+(b*x^3+a)
^(1/3))/x)-(b*x^3+a)^(1/3)*x^2*((a*d-b*c)/c)^(2/3)*d*b^(2/3)+(a*d*b^(2/3)-
b^(5/3)*c)*(arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/(
(a*d-b*c)/c)^(1/3)/x)*3^(1/2)+1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c
)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/d^2
```

3.666.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(222) = 444$.

Time = 0.34 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.64

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{6\,(bx^3+a)^{\frac{1}{3}}b^2dx^2 + 6\,\sqrt{3}(-bc^3+ac^2d)^{\frac{1}{3}}b^2\arctan\left(-\frac{\sqrt{3}\,(bc^2-acd)x+2\,\sqrt{3}\,(-bc^3+ac^2d)^{\frac{2}{3}}\,(bx^3+a)^{\frac{1}{3}}}{3\,(bc^2-acd)x}\right) + 6\,(-bc^3+a}{}$$

input
```
integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/18*(6*(b*x^3 + a)^(1/3)*b^2*d*x^2 + 6*sqrt(3)*(-b*c^3 + a*c^2*d)^(1/3)*b
^2*arctan(-1/3*(sqrt(3)*(b*c^2 - a*c*d)*x + 2*sqrt(3)*(-b*c^3 + a*c^2*d)^(
2/3)*(b*x^3 + a)^(1/3))/((b*c^2 - a*c*d)*x)) + 6*(-b*c^3 + a*c^2*d)^(1/3)*
b^2*log(((b*x^3 + a)^(1/3)*c + (-b*c^3 + a*c^2*d)^(1/3)*x)/x) - 3*(-b*c^3
+ a*c^2*d)^(1/3)*b^2*log(((b*x^3 + a)^(2/3)*c^2 - (-b*c^3 + a*c^2*d)^(1/3)
*(b*x^3 + a)^(1/3)*c*x + (-b*c^3 + a*c^2*d)^(2/3)*x^2)/x^2) - 2*sqrt(3)*(3
*b^2*c - a*b*d)*sqrt(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x
- 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) +
 2*(-b^2)^(2/3)*(3*b*c - a*d)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/
x) - (-b^2)^(2/3)*(3*b*c - a*d)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/
3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d^2)
```

3.666.6 Sympy [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x**4*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**4*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.666.7 Maxima [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^4}{dx^3+c}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)
```

3.666.8 Giac [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^4}{dx^3+c}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)
```

3.666.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^4\,(b\,x^3+a)^{1/3}}{d\,x^3+c}\,dx$$

input
```
int((x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
int((x^4*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

3.667 $\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

3.667.1 Optimal result

Integrand size = 22, antiderivative size = 234

$$\begin{aligned}\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = & -\frac{\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d}+\frac{\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}d} \\ & -\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6\sqrt[3]{c}d}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d} \\ & +\frac{\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}d}\end{aligned}$$

output
```
-1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(1/3)/d-1/2*b^(1/3)*ln(b^(1/3)*x-(b*x^
3+a)^(1/3))/d+1/2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)
^(1/3))/c^(1/3)/d-1/3*b^(1/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3
^(1/2))/d*3^(1/2)+1/3*(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/
c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(1/3)/d*3^(1/2)
```

3.667.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.79 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.81

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$=\frac{-4\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-\frac{2\sqrt{-6-6i\sqrt{3}}\sqrt[3]{bc-ad}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{c}}-4}{}$$

input
```
Integrate[(x*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(-4*Sqrt[3]*b^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^
(1/3))] - (2*Sqrt[-6 - (6*I)*Sqrt[3]]*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a
*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b
*x^3)^(1/3))])/c^(1/3) - 4*b^(1/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] +
 ((2*I)*(I + Sqrt[3])*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I
*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/c^(1/3) + 2*b^(1/3)*Log[b^(2/3)*x^2
+ b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((1 - I*Sqrt[3])*(b*c
 - a*d)^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c
- a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/
3)])/c^(1/3))/(12*d)
```

3.667.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {984, 853, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$\Big\downarrow \text{984}$$

$$\frac{b\int \frac{x}{\left(bx^3+a\right)^{2/3}}dx}{d}-\frac{(bc-ad)\int \frac{x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{d}$$

$$\Big\downarrow 853$$

$$\frac{b\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)}{d}-\frac{(bc-ad)\int \frac{x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{d}$$

$$\Big\downarrow 992$$

$$\frac{b\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)}{d}-\frac{(bc-ad)\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{d}$$

input
```
Int[(x*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(b*(-(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/
3))) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3))))/d - ((b*c - a*d)*(
-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]
]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c -
a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^
(1/3)*(b*c - a*d)^(2/3))))/d
```

3.667. $\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

3.667.3.1 Defintions of rubi rules used

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

3.667.4 Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.44

method	result
pseudoelliptic	$\frac{b^{\frac{1}{3}} \ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right) c \left(\frac{a d-b c}{c}\right)^{\frac{2}{3}}+\left(-2 a d+2 b c\right) \ln\left(\frac{\left(\frac{a d-b c}{c}\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)+2 b^{\frac{1}{3}} \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\right.}{}\right.}{}$

input
```
int(x*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(b^(1/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^
2)*c*((a*d-b*c)/c)^(2/3)+(-2*a*d+2*b*c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a
)^(1/3))/x)+2*b^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1
/3))/b^(1/3)/x)*c*((a*d-b*c)/c)^(2/3)-2*b^(1/3)*ln((-b^(1/3)*x+(b*x^3+a)^(
1/3))/x)*c*((a*d-b*c)/c)^(2/3)+(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*
x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3
)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a*d-b*
c))/((a*d-b*c)/c)^(2/3)/c/d
```

3.667.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.41

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$= \frac{2\sqrt{3}\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}}{3\,(bc-ad)x}\right)-2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}bx+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}}{3\,bx}\right)+}{}$$

input

```
integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output

```
1/6*(2*sqrt(3)*((b*c - a*d)/c)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x +
2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b*c - a*d)/c)^(2/3))/((b*c - a*d)*x)) - 2*
sqrt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(
-b)^(2/3))/(b*x)) + 2*(-b)^(1/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x)
 + 2*((b*c - a*d)/c)^(1/3)*log(-(x*((b*c - a*d)/c)^(1/3) - (b*x^3 + a)^(1/
3))/x) - (-b)^(1/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x +
 (b*x^3 + a)^(2/3))/x^2) - ((b*c - a*d)/c)^(1/3)*log((x^2*((b*c - a*d)/c)^
(2/3) + (b*x^3 + a)^(1/3)*x*((b*c - a*d)/c)^(1/3) + (b*x^3 + a)^(2/3))/x^2
))/d
```

3.667.6 Sympy [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input

```
integrate(x*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output

```
Integral(x*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.667.7 Maxima [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x}{dx^3+c}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)
```

3.667.8 Giac [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x}{dx^3+c}\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)
```

3.667.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x\,\left(b\,x^3+a\right)^{1/3}}{d\,x^3+c}\,dx$$

input
```
int((x*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
int((x*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

3.668 $\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)}\,dx$

3.668.1 Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{cx} - \frac{\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}} + \frac{\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^{4/3}} - \frac{\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}}$$

output
```
-(b*x^3+a)^(1/3)/c/x+1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(4/3)-1/2*(-a*d+b*
c)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(4/3)-1/3*(-a*d+
b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(
1/2))/c^(4/3)*3^(1/2)
```

3.668.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.41 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx$$
$$= \frac{-\frac{12\sqrt[3]{c}\sqrt[3]{a+bx^3}}{x}+2\sqrt{-6-6i\sqrt{3}}\sqrt[3]{bc-ad}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+2(1-i\sqrt{3})}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x]
```

output
```
((-12*c^(1/3)*(a + b*x^3)^(1/3))/x + 2*Sqrt[-6 - (6*I)*Sqrt[3]]*(b*c - a*d
)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I
 + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 2*(1 - I*Sqrt[3])*(b*c - a*d)^(1
/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]
 + I*(I + Sqrt[3])*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I
*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*
c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(4/3))
```

3.668.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {975, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx$$
$$\Big\downarrow \; 975$$
$$\frac{\int \frac{(bc-ad)x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{c}-\frac{\sqrt[3]{a+bx^3}}{cx}$$
$$\Big\downarrow \; 27$$

$$\frac{(bc-ad)\int \frac{x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{c}-\frac{\sqrt[3]{a+bx^3}}{cx}$$

$$\Big\downarrow \; 992$$

$$\frac{(bc-ad)\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c}-\frac{\sqrt[3]{a+bx^3}}{cx}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x]
```

output
```
-((a + b*x^3)^(1/3)/(c*x)) + ((b*c - a*d)*(-(ArcTan[(1 + (2*(b*c - a*d)^(1
/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^
(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c - a*d)^
(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3))))/c
```

3.668.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

3.668.4 Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$-\frac{-\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)(ad-bc)x+3\left(b\,x^{3}+a\right)^{\frac{1}{3}}c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}+x\left(\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+\right.}{3\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x\,c^{2}}$

input

```
int((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output

```
-1/3/((a*d-b*c)/c)^(2/3)*(-ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*(
a*d-b*c)*x+3*(b*x^3+a)^(1/3)*c*((a*d-b*c)/c)^(2/3)+x*(arctan(1/3*3^(1/2)*(
((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+1/
2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3
+a)^(2/3))/x^2))*(a*d-b*c))/x/c^2
```

3.668.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input

```
integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="fricas")
```

output

```
Timed out
```

3.668.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**2/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**2*(c + d*x**3)), x)
```

3.668.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx = \int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)
```

3.668.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx = \int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)
```

3.668.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^2\,\left(d\,x^3+c\right)} \,d x$$

input
```
int((a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^2*(c + d*x^3)), x)
```

3.669 $\int \frac{\sqrt[3]{a+bx^3}}{x^5(c+dx^3)}\,dx$

3.669.1 Optimal result

Integrand size = 24, antiderivative size = 204

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx=-\frac{\sqrt[3]{a+bx^3}}{4cx^4}-\frac{(bc-4ad)\sqrt[3]{a+bx^3}}{4ac^2x}+\frac{d\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}}-\frac{d\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{7/3}}+\frac{d\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}}$$

output
```
-1/4*(b*x^3+a)^(1/3)/c/x^4-1/4*(-4*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x-1/6*d*
(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(7/3)+1/2*d*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^
(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(7/3)+1/3*d*(-a*d+b*c)^(1/3)*arctan(1/3
*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(7/3)*3^(1/2)
```

3.669.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.62 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx = \frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}\left(-ac-bcx^3+4adx^3\right)}{ax^4}-2\sqrt{-6-6i\sqrt{3}}d\sqrt[3]{bc-ad}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)-}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x]
```

output
```
((3*c^(1/3)*(a + b*x^3)^(1/3)*(-(a*c) - b*c*x^3 + 4*a*d*x^3))/(a*x^4) - 2*
Sqrt[-6 - (6*I)*Sqrt[3]]*d*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x
)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)
)] + (2*I)*(I + Sqrt[3])*d*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (
1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + (1 - I*Sqrt[3])*d*(b*c - a*d)^
(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(
1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12
*c^(7/3))
```

3.669.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {975, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx$$

$$\Big\downarrow \; 975$$

$$\frac{\int \frac{-3bdx^3+bc-4ad}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4c}-\frac{\sqrt[3]{a+bx^3}}{4cx^4}$$

$$\Big\downarrow \; 1053$$

$$\frac{-\frac{\int \frac{4ad(bc-ad)x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}(bc-4ad)}{acx}}{4c}-\frac{\sqrt[3]{a+bx^3}}{4cx^4}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{4d(bc-ad)\int \frac{x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{c}-\frac{\sqrt[3]{a+bx^3}(bc-4ad)}{acx}}{4c}-\frac{\sqrt[3]{a+bx^3}}{4cx^4}$$

$$\Big\downarrow \; 992$$

$$\frac{-\frac{4d(bc-ad)\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c}-\frac{\sqrt[3]{a+bx^3}(bc-4ad)}{acx}}{4c}-\frac{\sqrt[3]{a+bx^3}}{4cx^4}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x]
```

output
```
-1/4*(a + b*x^3)^(1/3)/(c*x^4) + (-(((b*c - 4*a*d)*(a + b*x^3)^(1/3))/(a*c
*x)) - (4*d*(b*c - a*d)*(-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a
 + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c +
d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
 (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3))))/c)/(4*c)
```

3.669. $\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx$

3.669.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.669.4 Maple [A] (verified)

Time = 4.81 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$\frac{-2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a(ad-bc)d x^{4}-\frac{3\left((-4ad+bc)x^{3}+ac\right)\left(b x^{3}+a\right)^{\frac{1}{3}}c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}}{2}+x^{4}\left(2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x\right.}{3\left(\frac{ad-b}{c}\right.}\right.\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{4}c^{3}a}$

input
```
int((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/c)^(2/3)*(-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*
a*(a*d-b*c)*d*x^4-3/2*((-4*a*d+b*c)*x^3+a*c)*(b*x^3+a)^(1/3)*c*((a*d-b*c)/
c)^(2/3)+x^4*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3
))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c
)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*d*a*(a*d-b*c))/x^4/c^3/a
```

3.669.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.669.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**5/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**5*(c + d*x**3)), x)
```

3.669.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx=\int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)
```

3.669.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx=\int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)
```

3.669.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^5\,\left(d\,x^3+c\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^5*(c + d*x^3)), x)
```

3.670 $\int \frac{\sqrt[3]{a+bx^3}}{x^8(c+dx^3)}\,dx$

3.670.1 Optimal result

Integrand size = 24, antiderivative size = 258

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx =& -\frac{\sqrt[3]{a+bx^3}}{7cx^7}-\frac{(bc-7ad)\sqrt[3]{a+bx^3}}{28ac^2x^4}\\&+\frac{\left(3b^2c^2+7abcd-28a^2d^2\right)\sqrt[3]{a+bx^3}}{28a^2c^3x}\\&-\frac{d^2\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{10/3}}+\frac{d^2\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{10/3}}\\&-\frac{d^2\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{10/3}}\end{aligned}$$

output

```
-1/7*(b*x^3+a)^(1/3)/c/x^7-1/28*(-7*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x^4+1/2
8*(-28*a^2*d^2+7*a*b*c*d+3*b^2*c^2)*(b*x^3+a)^(1/3)/a^2/c^3/x+1/6*d^2*(-a*
d+b*c)^(1/3)*ln(d*x^3+c)/c^(10/3)-1/2*d^2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(
1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(10/3)-1/3*d^2*(-a*d+b*c)^(1/3)*arctan(1
/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(10/3)*3^(1
/2)
```

3.670.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.04 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx$$

$$= \frac{-\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}\left(-3b^2c^2x^6+abcx^3(c-7dx^3)+a^2(4c^2-7cdx^3+28d^2x^6)\right)}{a^2x^7}+14\sqrt{-6-6i\sqrt{3}}d^2\sqrt[3]{bc-ad}\arctan\left(\frac{}{\sqrt{3}\sqrt[3]{bc-}}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]
```

output
```
((-3*c^(1/3)*(a + b*x^3)^(1/3)*(-3*b^2*c^2*x^6 + a*b*c*x^3*(c - 7*d*x^3) +
 a^2*(4*c^2 - 7*c*d*x^3 + 28*d^2*x^6)))/(a^2*x^7) + 14*Sqrt[-6 - (6*I)*Sqr
t[3]]*d^2*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
 a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 14*(1 - I*Sq
rt[3])*d^2*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c
^(1/3)*(a + b*x^3)^(1/3)] + (7*I)*(I + Sqrt[3])*d^2*(b*c - a*d)^(1/3)*Log[
2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a
+ b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(84*c^(10/3))
```

3.670.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {975, 1053, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{-6bdx^3+bc-7ad}{x^5\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{7c}-\frac{\sqrt[3]{a+bx^3}}{7cx^7}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int \frac{3bd(bc-7ad)x^3+3b^2c^2-28a^2d^2+7abcd}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4ac}-\frac{\sqrt[3]{a+bx^3}(bc-7ad)}{4acx^4}}{7c}-\frac{\sqrt[3]{a+bx^3}}{7cx^7}$$

$$\Big\downarrow 1053$$

$$\frac{-\frac{-\frac{\int \frac{28a^2d^2(bc-ad)x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{3b^2c}{a}-\frac{28ad^2}{c}+7bd\right)}{x}}{4ac}-\frac{\sqrt[3]{a+bx^3}(bc-7ad)}{4acx^4}}{7c}-\frac{\sqrt[3]{a+bx^3}}{7cx^7}$$

$$\Big\downarrow 27$$

$$\frac{-\frac{-\frac{28ad^2(bc-ad)\int \frac{x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{c}-\frac{\sqrt[3]{a+bx^3}\left(\frac{3b^2c}{a}-\frac{28ad^2}{c}+7bd\right)}{x}}{4ac}-\frac{\sqrt[3]{a+bx^3}(bc-7ad)}{4acx^4}}{7c}-\frac{\sqrt[3]{a+bx^3}}{7cx^7}$$

$$\Big\downarrow 992$$

$$-\frac{-\frac{28ad^2(bc-ad)\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c}-\frac{\sqrt[3]{a+bx^3}\left(\frac{3b^2c}{a}-\frac{28ad^2}{c}+7bd\right)}{x}}{4ac}$$

$$7c$$

$$\frac{\sqrt[3]{a+bx^3}}{7cx^7}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]
```

output
```
-1/7*(a + b*x^3)^(1/3)/(c*x^7) + (-1/4*((b*c - 7*a*d)*(a + b*x^3)^(1/3))/(
a*c*x^4) - (-((((3*b^2*c)/a + 7*b*d - (28*a*d^2)/c)*(a + b*x^3)^(1/3))/x)
- (28*a*d^2*(b*c - a*d)*(-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a
 + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c +
d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
 (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3))))/c)/(4*a*c))/(7*c)
```

3.670. $\int \frac{\sqrt[3]{a+bx^3}}{x^8(c+dx^3)}\,dx$

3.670.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.670.4 Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{\frac{6\left(\left(-\frac{3b\,x^{3}}{4}+a\right)\left(b\,x^{3}+a\right)c^{2}-\frac{7\left(b\,x^{3}+a\right)acd\,x^{3}}{4}+7a^{2}d^{2}x^{6}\right)c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{7}+a^{2}d^{2}x^{7}(ad-bc)\Bigg(2\arctan\Bigg(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\right.}{3\left(\frac{ad}{}\right.}}{6\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}}$

input
```
int((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/((a*d-b*c)/c)^(2/3)*(6/7*((-3/4*b*x^3+a)*(b*x^3+a)*c^2-7/4*(b*x^3+a)*
a*c*d*x^3+7*a^2*d^2*x^6)*c*((a*d-b*c)/c)^(2/3)*(b*x^3+a)^(1/3)+a^2*d^2*x^7
*(a*d-b*c)*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))
/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^
(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+
(b*x^3+a)^(1/3))/x)))/x^7/c^4/a^2
```

3.670.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.670.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**8/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**8*(c + d*x**3)), x)
```

3.670.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8(c+dx^3)}\,dx=\int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)x^8}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)
```

3.670.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8(c+dx^3)}\,dx=\int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)x^8}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)
```

3.670.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8(c+dx^3)}\,dx=\int \frac{(b\,x^3+a)^{1/3}}{x^8\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^8*(c + d*x^3)), x)
```

3.671 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(c+dx^3)}\,dx$

3.671.1 Optimal result

Integrand size = 24, antiderivative size = 318

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx = &-\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}-\frac{(bc-10ad)\sqrt[3]{a+bx^3}}{70ac^2x^7} \\ &+\frac{\left(3b^2c^2+5abcd-35a^2d^2\right)\sqrt[3]{a+bx^3}}{140a^2c^3x^4} \\ &-\frac{\left(9b^3c^3+15ab^2c^2d+35a^2bcd^2-140a^3d^3\right)\sqrt[3]{a+bx^3}}{140a^3c^4x} \\ &+\frac{d^3\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{13/3}}-\frac{d^3\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{13/3}} \\ &+\frac{d^3\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{13/3}}\end{aligned}$$

output
```
-1/10*(b*x^3+a)^(1/3)/c/x^10-1/70*(-10*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x^7+
1/140*(-35*a^2*d^2+5*a*b*c*d+3*b^2*c^2)*(b*x^3+a)^(1/3)/a^2/c^3/x^4-1/140*
(-140*a^3*d^3+35*a^2*b*c*d^2+15*a*b^2*c^2*d+9*b^3*c^3)*(b*x^3+a)^(1/3)/a^3
/c^4/x-1/6*d^3*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(13/3)+1/2*d^3*(-a*d+b*c)^(1
/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(13/3)+1/3*d^3*(-a*d+
b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(
1/2))/c^(13/3)*3^(1/2)
```

3.671.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.44 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx$$

$$= \frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}\left(-9b^3c^3x^9+3ab^2c^2x^6\left(c-5dx^3\right)+a^2bcx^3\left(-2c^2+5cdx^3-35d^2x^6\right)+a^3\left(-14c^3+20c^2dx^3-35cd^2x^6+140d^3x^9\right)\right)}{a^3x^{10}}-70\sqrt{-6}}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]
```

output
```
((3*c^(1/3)*(a + b*x^3)^(1/3)*(-9*b^3*c^3*x^9 + 3*a*b^2*c^2*x^6*(c - 5*d*x
^3) + a^2*b*c*x^3*(-2*c^2 + 5*c*d*x^3 - 35*d^2*x^6) + a^3*(-14*c^3 + 20*c^
2*d*x^3 - 35*c*d^2*x^6 + 140*d^3*x^9)))/(a^3*x^10) - 70*Sqrt[-6 - (6*I)*Sq
rt[3]]*d^3*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c
- a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (70*I)*(I +
 Sqrt[3])*d^3*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3]
)*c^(1/3)*(a + b*x^3)^(1/3)] + 35*(1 - I*Sqrt[3])*d^3*(b*c - a*d)^(1/3)*Lo
g[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(
a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(420*c^(13/
3))
```

3.671.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {975, 1053, 27, 1053, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx$$

$$\Big\downarrow \; 975$$

$$\frac{\int \frac{-9bdx^3+bc-10ad}{x^8\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{10c}-\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \ 1053$$

$$-\frac{-\frac{\int \frac{2\left(3bd(bc-10ad)x^3+3b^2c^2-35a^2d^2+5abcd\right)}{x^5\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{7ac}-\frac{\sqrt[3]{a+bx^3}(bc-10ad)}{7acx^7}}{10c}-\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{2\int \frac{3bd(bc-10ad)x^3+3b^2c^2-35a^2d^2+5abcd}{x^5\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{7ac}-\frac{\sqrt[3]{a+bx^3}(bc-10ad)}{7acx^7}}{10c}-\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \ 1053$$

$$-\frac{-\frac{2\left(-\frac{\int \frac{9b^3c^3+15ab^2dc^2+35a^2bd^2c-140a^3d^3+3bd\left(3b^2c^2+5abdc-35a^2d^2\right)x^3}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{3b^2c}{a}-\frac{35ad^2}{c}+5bd\right)}{4x^4}\right)}{7ac}-\frac{\sqrt[3]{a+bx^3}(bc-10ad)}{7acx^7}}{10c}-\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \ 1053$$

$$-\frac{-\frac{2\left(-\frac{-\frac{\int \frac{140a^3d^3(bc-ad)x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}\left(-140a^3d^3+35a^2bcd^2+15ab^2c^2d+9b^3c^3\right)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{3b^2c}{a}-\frac{35ad^2}{c}+5bd\right)}{4x^4}\right)}{7ac}-\frac{\sqrt[3]{a+bx^3}(bc}{7acx^7}}{10c}$$

$$\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{2\left(-\frac{-\frac{140a^2d^3(bc-ad)\int \frac{x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{c}-\frac{\sqrt[3]{a+bx^3}\left(-140a^3d^3+35a^2bcd^2+15ab^2c^2d+9b^3c^3\right)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{3b^2c}{a}-\frac{35ad^2}{c}+5bd\right)}{4x^4}\right)}{7ac}-}{10c}$$

$$\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \ 992$$

3.671. $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(c+dx^3)}\,dx$

$$-\frac{2\left(-\frac{-\frac{140a^2d^3(bc-ad)\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c}}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(-140a^3d^3+35a\right.}{acx}\right.}{7ac}}{10c}$$

$$\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]
```

output
```
-1/10*(a + b*x^3)^(1/3)/(c*x^10) + (-1/7*((b*c - 10*a*d)*(a + b*x^3)^(1/3)
)/(a*c*x^7) - (2*(-1/4*(((3*b^2*c)/a + 5*b*d - (35*a*d^2)/c)*(a + b*x^3)^(
1/3))/x^4 - (-(((9*b^3*c^3 + 15*a*b^2*c^2*d + 35*a^2*b*c*d^2 - 140*a^3*d^3
)*(a + b*x^3)^(1/3))/(a*c*x)) - (140*a^2*d^3*(b*c - a*d)*(-(ArcTan[(1 + (2
*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/
3)*(b*c - a*d)^(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Lo
g[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d
)^(2/3))))/c)/(4*a*c)))/(7*a*c))/(10*c)
```

3.671. $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(c+dx^3)}\,dx$

3.671.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.671.4 Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{-\frac{3\left(\left(\frac{9}{14}b^{2}x^{6}-\frac{6}{7}ab\,x^{3}+a^{2}\right)\left(b\,x^{3}+a\right)c^{3}-\frac{10x^{3}\left(-\frac{3b\,x^{3}}{4}+a\right)d\left(b\,x^{3}+a\right)a\,c^{2}}{7}+\frac{5\left(b\,x^{3}+a\right)a^{2}c\,d^{2}x^{6}}{2}-10a^{3}d^{3}x^{9}\right)\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}c\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{5}}{}$

3.671. $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(c+dx^3)}\,dx$

input
```
int((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/c)^(2/3)*(-3/5*((9/14*b^2*x^6-6/7*a*b*x^3+a^2)*(b*x^3+a)*c^
3-10/7*x^3*(-3/4*b*x^3+a)*d*(b*x^3+a)*a*c^2+5/2*(b*x^3+a)*a^2*c*d^2*x^6-10
*a^3*d^3*x^9)*((a*d-b*c)/c)^(2/3)*c*(b*x^3+a)^(1/3)+a^3*d^3*x^10*(a*d-b*c)
*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c
)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x
^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^
(1/3))/x)))/x^10/c^5/a^3
```

3.671.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.671.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**11/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**11*(c + d*x**3)), x)
```

3.671.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)
```

3.671.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)
```

3.671.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^{11}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^11*(c + d*x^3)), x)
```

3.672 $\int \frac{x^6\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

3.672.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{x^7\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{7}{3},-\frac{1}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/7*x^7*(b*x^3+a)^(1/3)*AppellF1(7/3,-1/3,1,10/3,-b*x^3/a,-d*x^3/c)/c/(1+b
*x^3/a)^(1/3)
```

3.672.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(64) = 128.

Time = 7.70 (sec) , antiderivative size = 281, normalized size of antiderivative = 4.39

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{x\left(4(a+bx^3)\left(-5bc+ad+2bdx^3\right)-\frac{\left(-10b^2c^2+5abcd+a^2d^2\right)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{}{(c+dx^3)\left(-4\right.}\right.}{40bd^2\left(a+bx^3\right)^{2/}}$$

input
```
Integrate[(x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(x*(4*(a + b*x^3)*(-5*b*c + a*d + 2*b*d*x^3) - ((-10*b^2*c^2 + 5*a*b*c*d +
 a^2*d^2)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a
), -((d*x^3)/c)])/c + (16*a^2*c^2*(-5*b*c + a*d)*AppellF1[1/3, 2/3, 1, 4/3
, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1,
4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -
((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a)
, -((d*x^3)/c)]))))/(40*b*d^2*(a + b*x^3)^(2/3))
```

3.672.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$$

$$\Big\downarrow \ 1013$$

$$\frac{\sqrt[3]{a+bx^3} \int \frac{x^6 \sqrt[3]{\frac{bx^3}{a}+1}}{dx^3+c}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \ 1012$$

$$\frac{x^7 \sqrt[3]{a+bx^3} \text{AppellF1}\left(\frac{7}{3},-\frac{1}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -1/3, 1, 10/3, -((b*x^3)/a), -((d*x^3
)/c)])/(7*c*(1 + (b*x^3)/a)^(1/3))
```

3.672.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.672.4 Maple [F]

$$\int \frac{x^{6} \left(b \,x^{3}+a \right)^{\frac{1}{3}}}{d \,x^{3}+c}d x$$

input
```
int(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

3.672.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \text{Timed out}$$

input
```
integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.672.6 Sympy [F]

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x**6*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**6*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.672.7 Maxima [F]

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^6}{dx^3+c}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^6/(d*x^3 + c), x)
```

3.672.8 Giac [F]

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^6}{dx^3+c}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^6/(d*x^3 + c), x)
```

3.672.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^6\,{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c} \,dx$$

input

```
int((x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output

```
int((x^6*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

3.673 $\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

3.673.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{x^4 \sqrt[3]{a+bx^3} \operatorname{AppellF1}\left(\frac{4}{3},-\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/4*x^4*(b*x^3+a)^(1/3)*AppellF1(4/3,-1/3,1,7/3,-b*x^3/a,-d*x^3/c)/c/(1+b*
x^3/a)^(1/3)
```

3.673.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 240 vs. $2(64) = 128$.

Time = 7.11 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.75

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{x\left(\frac{(-2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c} + 4\left(a+bx^3+\frac{4a^2c^2\,\mathrm{Ap}}{(c+dx^3)\left(-4ac\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3(3a\right.}\right.\right.}{8d\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(x*(((-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/c + 4*(a + b*x^3 + (4*a^2*c^2*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3,
 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3,
2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((
b*x^3)/a), -((d*x^3)/c)]))))))/(8*d*(a + b*x^3)^(2/3))
```

3.673.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{a+bx^3}\int \frac{x^3\sqrt[3]{\frac{bx^3}{a}+1}}{dx^3+c}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

output
```
(x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*c*(1 + (b*x^3)/a)^(1/3))
```

3.673. $\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}dx$

3.673.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.673.4 Maple [F]

$$\int \frac{x^{3} \left(b\,x^{3}+a \right)^{\frac{1}{3}}}{d\,x^{3}+c}d x$$

input
```
int(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

3.673.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.673.6 Sympy [F]

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate(x**3*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**3*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.673.7 Maxima [F]

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^3}{dx^3+c}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^3/(d*x^3 + c), x)
```

3.673.8 Giac [F]

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^3}{dx^3+c}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^3/(d*x^3 + c), x)
```

3.673.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^3\,{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c}\,dx$$

input
```
int((x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

output
```
int((x^3*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

3.674 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

3.674.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{x\sqrt[3]{a+bx^3}\,\mathrm{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a)
^(1/3)
```

3.674.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(59) = 118.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} \, dx$$

$$= \frac{4acx\sqrt[3]{a+bx^3}\,\mathrm{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\,(4ac\,\mathrm{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\,(-3ad\,\mathrm{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bc\,\mathrm{App}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

output
```
(4*a*c*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*
x^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d*x
^3)/c)] + b*c*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

3.674.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}dx$$

$$\Big\downarrow \; 937$$

$$\frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{dx^3+c}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \; 936$$

$$\frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(c*(1 + (b*x^3)/a)^(1/3))
```

3.674.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.674.4 Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{d\,x^{3}+c}d x$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

3.674.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.674.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.674.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

3.674.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

3.674.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(1/3)/(c + d*x^3), x)
```

3.675 $\int \frac{\sqrt[3]{a+bx^3}}{x^3(c+dx^3)}\,dx$

3.675.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx=-\frac{\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{1}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/2*(b*x^3+a)^(1/3)*AppellF1(-2/3,-1/3,1,1/3,-b*x^3/a,-d*x^3/c)/c/x^2/(1+
b*x^3/a)^(1/3)
```

3.675.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 327 vs. $2(64) = 128$.

Time = 10.27 (sec) , antiderivative size = 327, normalized size of antiderivative = 5.11

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx$$
$$=\frac{-bdx^6\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{c\left(16ac\left(bdx^6+a\left(c+3dx^3\right)\right)\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-4x^3(c}{\left(c+dx^3\right)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\right.}}{8c^2x^2\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x]
```

output
```
(-(b*d*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
-((d*x^3)/c)]) + (c*(16*a*c*(b*d*x^6 + a*(c + 3*d*x^3))*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 4*x^3*(a + b*x^3)*(c + d*x^3)*(3*a*
d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[
4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(-4*a*c*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4
/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(8*c^2*x^2*(a + b*x^3)^(2/3))
```

3.675.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}dx$$

$$\Big\downarrow \; 1013$$

$$\frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{x^3(dx^3+c)}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \; 1012$$

$$-\frac{\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{1}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x]
```

output
```
-1/2*((a + b*x^3)^(1/3)*AppellF1[-2/3, -1/3, 1, 1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(c*x^2*(1 + (b*x^3)/a)^(1/3))
```

3.675.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.675.4 Maple [F]

$$\int \frac{\left(b \,x^{3}+a \right)^{\frac{1}{3}}}{x^{3} \left(d \,x^{3}+c \right)}d x$$

input
```
int((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x)
```

3.675.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.675.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**3/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**3*(c + d*x**3)), x)
```

3.675.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^3), x)
```

3.675.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^3), x)
```

3.675.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^3\,\left(d\,x^3+c\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^3*(c + d*x^3)), x)
```

3.676 $\int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx$

3.676.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\,(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{5}{3},-\frac{1}{3},1,-\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/5*(b*x^3+a)^(1/3)*AppellF1(-5/3,-1/3,1,-2/3,-b*x^3/a,-d*x^3/c)/c/x^5/(1
+b*x^3/a)^(1/3)
```

3.676.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 289 vs. 2(64) = 128.

Time = 10.38 (sec) , antiderivative size = 289, normalized size of antiderivative = 4.52

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\,(c+dx^3)}\,dx$$

$$= \frac{-\frac{4(a+bx^3)(2ac+bcx^3-5adx^3)}{ac^2x^5}+\frac{bd(-bc+5ad)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3}+\frac{16(}{c(c+dx^3)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx}{c}\right.\right.}}{40\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x]
```

output
```
((-4*(a + b*x^3)*(2*a*c + b*c*x^3 - 5*a*d*x^3))/(a*c^2*x^5) + (b*d*(-(b*c)
 + 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a
), -((d*x^3)/c)])/(a*c^3) + (16*(b^2*c^2 + 5*a*b*c*d - 10*a^2*d^2)*x*Appel
lF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(c + d*x^3)*(-4*a*c*
AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*Appell
F1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3
, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(40*(a + b*x^3)^(2/3))
```

3.676.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{x^6(dx^3+c)}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{5}{3},-\frac{1}{3},1,-\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x]
```

output
```
-1/5*((a + b*x^3)^(1/3)*AppellF1[-5/3, -1/3, 1, -2/3, -((b*x^3)/a), -((d*x
^3)/c)])/(c*x^5*(1 + (b*x^3)/a)^(1/3))
```

3.676. $\int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx$

3.676.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.676.4 Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{6}\left(d\,x^{3}+c\right)}dx$$

input
```
int((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x)
```

3.676.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(c+dx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.676.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(c+dx^3\right)}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/x**6/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(x**6*(c + d*x**3)), x)
```

3.676.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^6), x)
```

3.676.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^6), x)
```

3.676.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx=\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^6\,\left(d\,x^3+c\right)}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(1/3)/(x^6*(c + d*x^3)), x)
```

3.677 $\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

3.677.1 Optimal result

Integrand size = 24, antiderivative size = 266

$$\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = -\frac{c^3\left(a+bx^3\right)^{2/3}}{2d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{\left(a+bx^3\right)^{11/3}}{11b^3d} - \frac{c^3(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{14/3}} + \frac{c^3(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{14/3}} - \frac{c^3(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}}$$

output
```
-1/2*c^3*(b*x^3+a)^(2/3)/d^4+1/5*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(5/3)
/b^3/d^3-1/8*(2*a*d+b*c)*(b*x^3+a)^(8/3)/b^3/d^2+1/11*(b*x^3+a)^(11/3)/b^3
/d+1/6*c^3*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^(14/3)-1/2*c^3*(-a*d+b*c)^(2/3)*
ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(14/3)-1/3*c^3*(-a*d+b*c)^(
2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^
(14/3)*3^(1/2)
```

3.677.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{\frac{3d^{2/3}\left(a+bx^3\right)^{2/3}\left(18a^3d^3+3a^2bd^2\left(11c-4dx^3\right)+2ab^2d\left(44c^2-11cdx^3+5d^2x^6\right)+b^3\left(-220c^3+88c^2dx^3-55cd^2x^6+40\right.\right.}{b^3}}{}$$

input
```
Integrate[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
((3*d^(2/3)*(a + b*x^3)^(2/3)*(18*a^3*d^3 + 3*a^2*b*d^2*(11*c - 4*d*x^3) +
 2*a*b^2*d*(44*c^2 - 11*c*d*x^3 + 5*d^2*x^6) + b^3*(-220*c^3 + 88*c^2*d*x^
3 - 55*c*d^2*x^6 + 40*d^3*x^9)))/b^3 - 440*Sqrt[3]*c^3*(b*c - a*d)^(2/3)*A
rcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 440
*c^3*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]
+ 220*c^3*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1
/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(1320*d^(14/3))
```

3.677.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{c+dx^3}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^9\left(bx^3+a\right)^{2/3}}{dx^3+c}dx^3$$

$$\Big\downarrow \ 99$$

$$\frac{1}{3}\int \left(-\frac{\left(bx^3+a\right)^{2/3}c^3}{d^3\left(dx^3+c\right)}+\frac{\left(bx^3+a\right)^{8/3}}{b^2d}+\frac{(-bc-2ad)\left(bx^3+a\right)^{5/3}}{b^2d^2}+\frac{\left(b^2c^2+abdc+a^2d^2\right)\left(bx^3+a\right)^{2/3}}{b^2d^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{5/3}\left(a^2d^2+abcd+b^2c^2\right)}{5b^3d^3}-\frac{\sqrt{3}c^3(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{14/3}}-\frac{3\left(a+bx^3\right)^{8/3}(2ad+}{8b^3d^2}\right.$$

input
```
Int[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
((-3*c^3*(a + b*x^3)^(2/3))/(2*d^4) + (3*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a
+ b*x^3)^(5/3))/(5*b^3*d^3) - (3*(b*c + 2*a*d)*(a + b*x^3)^(8/3))/(8*b^3*d
^2) + (3*(a + b*x^3)^(11/3))/(11*b^3*d) - (Sqrt[3]*c^3*(b*c - a*d)^(2/3)*A
rcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/d^(1
4/3) + (c^3*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(2*d^(14/3)) - (3*c^3*(b*c -
 a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(14/3
)))/3
```

3.677.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.677. $\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

3.677.4 Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{\frac{27\left(\left(b\,x^{3}+a\right)\left(\frac{20}{9}b^{2}x^{6}-\frac{5}{3}ab\,x^{3}+a^{2}\right)d^{3}+\frac{11\left(-\frac{5b\,x^{3}}{3}+a\right)b\left(b\,x^{3}+a\right)c\,d^{2}}{6}+\frac{44b^{2}c^{2}\left(b\,x^{3}+a\right)d}{9}-\frac{110b^{3}c^{3}}{9}\right)d\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}{110}+b^{3}c^{3}}{}$

input

```
int(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output

```
1/6*(27/110*((b*x^3+a)*(20/9*b^2*x^6-5/3*a*b*x^3+a^2)*d^3+11/6*(-5/3*b*x^3
+a)*b*(b*x^3+a)*c*d^2+44/9*b^2*c^2*(b*x^3+a)*d-110/9*b^3*c^3)*d*(b*x^3+a)^
(2/3)*(1/d*(a*d-b*c))^(1/3)+b^3*c^3*(a*d-b*c)*(-2*arctan(1/3*3^(1/2)*(2*(b
*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*
x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-
2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/(1/d*(a*d-b*c))^(1/3)/b^3/d^
5
```

3.677.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. 2(219) = 438.

Time = 0.52 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.71

$$\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx =$$

$$-\frac{440\,\sqrt{3}b^{3}c^{3}\left(-\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{d^{2}}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\,\sqrt{3}(bx^{3}+a)^{\frac{1}{3}}d\left(-\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{d^{2}}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+220\,b^{3}c^{3}\left(-\frac{b^{2}c^{2}-2\,ab}{d}\right.}{}$$

input

```
integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/1320*(440*sqrt(3)*b^3*c^3*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*
arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/d^2)^(1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 220*b^3*c^3*(-(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d
)*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) - 440*b^3*c^3*(-(b^2*c^2 -
 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/
d^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) - 3*(40*b^3*d^3*x^9 - 5*(11*b^
3*c*d^2 - 2*a*b^2*d^3)*x^6 - 220*b^3*c^3 + 88*a*b^2*c^2*d + 33*a^2*b*c*d^2
 + 18*a^3*d^3 + 2*(44*b^3*c^2*d - 11*a*b^2*c*d^2 - 6*a^2*b*d^3)*x^3)*(b*x^
3 + a)^(2/3))/(b^3*d^4)
```

3.677.6 Sympy [F]

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{x^{11}(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**11*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**11*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.677.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.677.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.54

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx =$$

$$-\frac{\left(b^{37}c^4d^7\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ab^{36}c^3d^8\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{37}cd^{11}-ab^{36}d^{12}\right)}$$

$$-\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^6}$$

$$+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^6}$$

$$-\frac{220\,(bx^3+a)^{\frac{2}{3}}b^{33}c^3d^7-88\,(bx^3+a)^{\frac{5}{3}}b^{32}c^2d^8+55\,(bx^3+a)^{\frac{8}{3}}b^{31}cd^9-88\,(bx^3+a)^{\frac{5}{3}}ab^{31}cd^9-40\,(bx^3+a}{440\,b^{33}d^{11}}$$

input
```
integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b^37*c^4*d^7*(-(b*c - a*d)/d)^(1/3) - a*b^36*c^3*d^8*(-(b*c - a*d)/d
)^(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/
d)^(1/3)))/(b^37*c*d^11 - a*b^36*d^12) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2
/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/
(-(b*c - a*d)/d)^(1/3))/d^6 + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((b*x^3
+ a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(
2/3))/d^6 - 1/440*(220*(b*x^3 + a)^(2/3)*b^33*c^3*d^7 - 88*(b*x^3 + a)^(5/
3)*b^32*c^2*d^8 + 55*(b*x^3 + a)^(8/3)*b^31*c*d^9 - 88*(b*x^3 + a)^(5/3)*a
*b^31*c*d^9 - 40*(b*x^3 + a)^(11/3)*b^30*d^10 + 110*(b*x^3 + a)^(8/3)*a*b^
30*d^10 - 88*(b*x^3 + a)^(5/3)*a^2*b^30*d^10)/(b^33*d^11)
```

3.677.9 Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.84

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx = \left(\frac{3\,a^2}{5\,b^3\,d} + \frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\left(b^4\,c-a\,b^3\,d\right)}{5\,b^3\,d}\right)\left(b\,x^3+a\right)^{5/3}$$

$$-\left(\frac{3\,a}{8\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{8\,b^6\,d^2}\right)\left(b\,x^3+a\right)^{8/3}$$

$$-\left(b\,x^3+a\right)^{2/3}\left(\frac{a^3}{2\,b^3\,d}+\frac{\left(\frac{3\,a^2}{b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\left(b^4\,c-a\,b^3\,d\right)}{b^3\,d}\right)\left(b^4\,c-a\,b^3\,d\right)}{2\,b^3\,d}\right)+\frac{\left(b\,x^3+a\right)^{11/3}}{11\,b^3\,d}-\frac{c^3\,\ln\left(\frac{\left(b\,x^3+a\right)}{}\right.}{}$$

input
```
int((x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
((3*a^2)/(5*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
 - a*b^3*d))/(5*b^3*d))*(a + b*x^3)^(5/3) - ((3*a)/(8*b^3*d) + (b^4*c - a*
b^3*d)/(8*b^6*d^2))*(a + b*x^3)^(8/3) - (a + b*x^3)^(2/3)*(a^3/(2*b^3*d) +
 (((3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
 - a*b^3*d))/(b^3*d))*(b^4*c - a*b^3*d))/(2*b^3*d)) + (a + b*x^3)^(11/3)/(
11*b^3*d) - (c^3*log(((a + b*x^3)^(1/3)*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7
*d))/d^7 - (c^6*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(28/3)))*(a*
d - b*c)^(2/3))/(3*d^(14/3)) - (c^3*log((c^6*((3^(1/2)*1i)/2 + 1/2)*(a*d -
 b*c)^(7/3))/d^(22/3) + (c^6*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^7)*((3^(1/
2)*1i)/2 - 1/2)*(a*d - b*c)^(2/3))/(3*d^(14/3)) + (c^3*log((c^6*(a + b*x^3
)^(1/3)*(a*d - b*c)^2)/d^7 - (c^6*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(7/3))/(4
*d^(22/3)))*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(2/3))/d^(14/3)
```

3.678 $\int \frac{x^8\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

3.678.1 Optimal result

Integrand size = 24, antiderivative size = 223

$$\begin{aligned}\int \frac{x^8\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx &= \frac{c^2\left(a+bx^3\right)^{2/3}}{2d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2} \\ &\quad+\frac{\left(a+bx^3\right)^{8/3}}{8b^2d}+\frac{c^2(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}} \\ &\quad-\frac{c^2(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{11/3}}+\frac{c^2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}}\end{aligned}$$

output

```
1/2*c^2*(b*x^3+a)^(2/3)/d^3-1/5*(a*d+b*c)*(b*x^3+a)^(5/3)/b^2/d^2+1/8*(b*x
^3+a)^(8/3)/b^2/d-1/6*c^2*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^(11/3)+1/2*c^2*(-
a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(11/3)+1/3*c
^2*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/
3))*3^(1/2))/d^(11/3)*3^(1/2)
```

3.678.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.19

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\frac{3d^{2/3}(a+bx^3)^{2/3}\left(-3a^2d^2+2abd(-4c+dx^3)+b^2(20c^2-8cdx^3+5d^2x^6)\right)}{b^2}+40\sqrt{3}c^2(bc-ad)^{2/3}\arctan\left(\right.}{}$$

input
```
Integrate[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
((3*d^(2/3)*(a + b*x^3)^(2/3)*(-3*a^2*d^2 + 2*a*b*d*(-4*c + d*x^3) + b^2*(
20*c^2 - 8*c*d*x^3 + 5*d^2*x^6)))/b^2 + 40*Sqrt[3]*c^2*(b*c - a*d)^(2/3)*A
rcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 40*
c^2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] -
 20*c^2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3
)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(120*d^(11/3))
```

3.678.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6(bx^3+a)^{2/3}}{dx^3+c}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{(bx^3+a)^{2/3}c^2}{d^2(dx^3+c)}+\frac{(bx^3+a)^{5/3}}{bd}+\frac{(-bc-ad)(bx^3+a)^{2/3}}{bd^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}c^2(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{11/3}}-\frac{3\left(a+bx^3\right)^{5/3}(ad+bc)}{5b^2d^2}+\frac{3\left(a+bx^3\right)^{8/3}}{8b^2d}-\frac{c^2(bc-ad)^{2/3}\,\mathrm{lo}}{2d^{11/}}\right.$$

input
```
Int[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
((3*c^2*(a + b*x^3)^(2/3))/(2*d^3) - (3*(b*c + a*d)*(a + b*x^3)^(5/3))/(5*
b^2*d^2) + (3*(a + b*x^3)^(8/3))/(8*b^2*d) + (Sqrt[3]*c^2*(b*c - a*d)^(2/3
)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/d
^(11/3) - (c^2*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(2*d^(11/3)) + (3*c^2*(b*
c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1
1/3)))/3
```

3.678.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.678.4 Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-\frac{\frac{9d\left(\left(-\frac{5b x^3}{3}+a\right)\left(b x^3+a\right)d^2+\frac{8\left(b x^3+a\right)bcd}{3}-\frac{20b^2c^2}{3}\right)\left(b x^3+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}{20}+b^2c^2(ad-bc)\left(-2\arctan\left(\frac{\sqrt{3}\left(2\left(b x^3+a\right)^{\frac{1}{3}}+(\right.}{3\left(\frac{ad-bc}{d}\right)}\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}d^4b}$

input
```
int(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(1/d*(a*d-b*c))^(1/3)*(9/20*d*((-5/3*b*x^3+a)*(b*x^3+a)*d^2+8/3*(b*x^
3+a)*b*c*d-20/3*b^2*c^2)*(b*x^3+a)^(2/3)*(1/d*(a*d-b*c))^(1/3)+b^2*c^2*(a*
d-b*c)*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1
/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x
^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1
/3))))/d^4/b^2
```

3.678.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(181) = 362.

Time = 0.49 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.78

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx=\frac{40\sqrt{3}b^2c^2\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)-20}{}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/120*(40*sqrt(3)*b^2*c^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arct
an(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^
2)^(1/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 20*b^2*c^2*((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + 40*b^2*c^2*((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)/d^2)^(1/3)*log(-d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) -
 (b*x^3 + a)^(1/3)*(b*c - a*d)) + 3*(5*b^2*d^2*x^6 + 20*b^2*c^2 - 8*a*b*c*
d - 3*a^2*d^2 - 2*(4*b^2*c*d - a*b*d^2)*x^3)*(b*x^3 + a)^(2/3))/(b^2*d^3)
```

3.678.6 Sympy [F]

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{x^8(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**8*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**8*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.678.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.678.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.57

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx=\frac{\left(b^{19}c^3d^5\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ab^{18}c^2d^6\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{19}cd^8-ab^{18}d^9)}$$

$$+\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^5}$$

$$-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^5}$$

$$+\frac{20\,(bx^3+a)^{\frac{2}{3}}b^{16}c^2d^5-8\,(bx^3+a)^{\frac{5}{3}}b^{15}cd^6+5\,(bx^3+a)^{\frac{8}{3}}b^{14}d^7-8\,(bx^3+a)^{\frac{5}{3}}ab^{14}d^7}{40\,b^{16}d^8}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*(b^19*c^3*d^5*(-(b*c - a*d)/d)^(1/3) - a*b^18*c^2*d^6*(-(b*c - a*d)/d)
^(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b^19*c*d^8 - a*b^18*d^9) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)
*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(
b*c - a*d)/d)^(1/3))/d^5 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^2*log((b*x^3 + a
)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3
))/d^5 + 1/40*(20*(b*x^3 + a)^(2/3)*b^16*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^1
5*c*d^6 + 5*(b*x^3 + a)^(8/3)*b^14*d^7 - 8*(b*x^3 + a)^(5/3)*a*b^14*d^7)/(
b^16*d^8)
```

3.678.9 Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.73

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \left(\frac{a^2}{2\,b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{2\,b^2\,d}\right)\,{\left(b\,x^3+a\right)}^{2/3} - \left(\frac{2\,a}{5\,b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{5\,b^4\,d^2}\right)\,{\left(b\,x^3+a\right)}^{5/3}+\frac{{\left(b\,x^3+a\right)}^{8/3}}{8\,b^2\,d} + \frac{c^2\,\ln\left(\frac{{\left(b\,x^3+a\right)}^{1/3}\,\left(a^2\,c^4\,d^2-2\,a\,b\,c^5\,d+b^2\,c^6\right)}{d^5}-\frac{c^4\,{\left(a\,d-b\,c\right)}^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{9\,d^{22/3}}\right)\,{\left(a\,d-b\,c\right)}^{2/3}}{3\,d^{11/3}} - \frac{c^2\,\ln\left(\frac{c^4\,{\left(b\,x^3+a\right)}^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d^5}-\frac{c^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{7/3}}{d^{16/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{2/3}}{3\,d^{11/3}} + \frac{c^2\,\ln\left(\frac{c^4\,{\left(b\,x^3+a\right)}^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d^5}-\frac{c^4\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(a\,d-b\,c\right)}^{7/3}}{4\,d^{16/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{2/3}}{d^{11/3}}$$

input
```
int((x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output

```
(a^2/(2*b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a
*b^2*d))/(2*b^2*d))*(a + b*x^3)^(2/3) - ((2*a)/(5*b^2*d) + (b^3*c - a*b^2*
d)/(5*b^4*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^2*d) + (c^2*log
(((a + b*x^3)^(1/3)*(b^2*c^6 + a^2*c^4*d^2 - 2*a*b*c^5*d))/d^5 - (c^4*(a*d
 - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(22/3)))*(a*d - b*c)^(2/3))/(3*d
^(11/3)) - (c^2*log((c^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^5 - (c^4*((3^(
1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(16/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d
 - b*c)^(2/3))/(3*d^(11/3)) + (c^2*log((c^4*(a + b*x^3)^(1/3)*(a*d - b*c)^
2)/d^5 - (c^4*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(7/3))/(4*d^(16/3)))*((3^(1/2
)*1i)/6 - 1/6)*(a*d - b*c)^(2/3))/d^(11/3)
```

3.678. $\int \frac{x^8 (a+bx^3)^{2/3}}{c+dx^3}\, dx$

3.679 $\int \frac{x^5 \left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx$

3.679.1 Optimal result

Integrand size = 24, antiderivative size = 188

$$\begin{aligned}\int \frac{x^5 (a+b x^3)^{2/3}}{c+d x^3} \, dx = & -\frac{c (a+b x^3)^{2/3}}{2 d^2}+\frac{(a+b x^3)^{5/3}}{5 b d} \\ & -\frac{c (b c-a d)^{2/3} \arctan \left(\frac{1-\frac{2 \sqrt[3]{d} \sqrt[3]{a+b x^3}}{\sqrt[3]{b c-a d}}}{\sqrt{3}}\right)}{\sqrt{3} d^{8/3}}+\frac{c (b c-a d)^{2/3} \log \left(c+d x^3\right)}{6 d^{8/3}} \\ & -\frac{c (b c-a d)^{2/3} \log \left(\sqrt[3]{b c-a d}+\sqrt[3]{d} \sqrt[3]{a+b x^3}\right)}{2 d^{8/3}}\end{aligned}$$

output
```
-1/2*c*(b*x^3+a)^(2/3)/d^2+1/5*(b*x^3+a)^(5/3)/b/d+1/6*c*(-a*d+b*c)^(2/3)*
ln(d*x^3+c)/d^(8/3)-1/2*c*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*
x^3+a)^(1/3))/d^(8/3)-1/3*c*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^
3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(8/3)*3^(1/2)
```

3.679.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.21

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\frac{3d^{2/3}(a+bx^3)^{2/3}(-5bc+2ad+2bdx^3)}{b} - 10\sqrt{3}c(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 10}{}$$

input

```
Integrate[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output

```
((3*d^(2/3)*(a + b*x^3)^(2/3)*(-5*b*c + 2*a*d + 2*b*d*x^3))/b - 10*Sqrt[3]
*c*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)
^(1/3))/Sqrt[3]] - 10*c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*
(a + b*x^3)^(1/3)] + 5*c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)
*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(30*d^(
8/3))
```

3.679.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {948, 90, 60, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3(bx^3+a)^{2/3}}{dx^3+c}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{3(a+bx^3)^{5/3}}{5bd}-\frac{c\int \frac{(bx^3+a)^{2/3}}{dx^3+c}dx^3}{d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{5/3}}{5bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\int\frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{68}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{5/3}}{5bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(-\frac{3\int\frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int\frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\,\sqrt[3]{b}}{\sqrt[3]{d}}}}{2d}\right)}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{16}$$

3.679. $\int \frac{x^5\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{5/3}}{5bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(\frac{3\int\frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\,\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log}{}\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{5/3}}{5bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(\frac{3\int\frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\,\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{}\right.}{2d^{2/3}\sqrt[3]{bc-a}}\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \; 217$$

3.679. $\int \frac{x^5\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{5/3}}{5bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{}\right.}{2d^{2/3}\sqrt[3]{bc-ac}}\right.}{d}\right.}{d}\right.$$

input
```
Int[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
((3*(a + b*x^3)^(5/3))/(5*b*d) - (c*((3*(a + b*x^3)^(2/3))/(2*d) - ((b*c -
 a*d)*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1
/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b
*c - a*d)^(1/3)) - (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/
(2*d^(2/3)*(b*c - a*d)^(1/3))))/d))/d)/3
```

3.679. $\int \frac{x^5\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

3.679.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.679. $\int \frac{x^5 \left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx$

3.679.4 Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{\frac{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}} d\left(\left(d\,x^{3}-\frac{5c}{2}\right)b+ad\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{5}+bc(ad-bc)\left(-2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\right.\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}} b\,d^{3}}$

input

```
int(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output

```
1/6/(1/d*(a*d-b*c))^(1/3)*(6/5*(1/d*(a*d-b*c))^(1/3)*d*((d*x^3-5/2*c)*b+a*
d)*(b*x^3+a)^(2/3)+b*c*(a*d-b*c)*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)
+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+
(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a
)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/b/d^3
```

3.679.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(149) = 298.

Time = 0.45 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.88

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx =$$

$$-\frac{10\sqrt{3}bc\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+5\,bc\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right.}{}$$

input

```
integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output

```
-1/30*(10*sqrt(3)*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(
-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)
^(1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 5*b*c*(-(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^2*c^2
- 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) - 10*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^
3 + a)^(1/3)*(b*c - a*d)) - 3*(2*b*d*x^3 - 5*b*c + 2*a*d)*(b*x^3 + a)^(2/3
))/(b*d^2)
```

3.679.6 Sympy [F]

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^5(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input

```
integrate(x**5*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output

```
Integral(x**5*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.679.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.679.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(149) = 298$.

Time = 0.30 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.63

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = -\frac{\left(b^7c^2d^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ab^6cd^4\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^7cd^5-ab^6d^6)} - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}c\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^4} + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^4} - \frac{5\,(bx^3+a)^{\frac{2}{3}}b^5cd^3-2\,(bx^3+a)^{\frac{5}{3}}b^4d^4}{10\,b^5d^5}$$

input
```
integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b^7*c^2*d^3*(-(b*c - a*d)/d)^(1/3) - a*b^6*c*d^4*(-(b*c - a*d)/d)^(1
/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(
1/3)))/(b^7*c*d^5 - a*b^6*d^6) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*c*ar
ctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a
*d)/d)^(1/3))/d^4 + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c*log((b*x^3 + a)^(2/3) +
 (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^4 -
1/10*(5*(b*x^3 + a)^(2/3)*b^5*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^4*d^4)/(b^5*d^
5)
```

3.679.9 Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.61

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{(b\,x^3+a)^{5/3}}{5\,b\,d} - (b\,x^3+a)^{2/3}\left(\frac{a}{2\,b\,d}+\frac{b^2\,c-a\,b\,d}{2\,b^2\,d^2}\right) - \frac{c\,\ln\left(\frac{(b\,x^3+a)^{1/3}\,(a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4)}{d^3} - \frac{c^2\,(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{9\,d^{16/3}}\right)\,(a\,d}{3\,d^{8/3}}$$

input
```
int((x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
(a + b*x^3)^(5/3)/(5*b*d) - (a + b*x^3)^(2/3)*(a/(2*b*d) + (b^2*c - a*b*d)
/(2*b^2*d^2)) - (c*log(((a + b*x^3)^(1/3)*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c
^3*d))/d^3 - (c^2*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(16/3)))*(
a*d - b*c)^(2/3))/(3*d^(8/3)) - (c*log((c^2*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c)^(7/3))/d^(10/3) + (c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^3)*((3^(1/2
)*1i)/2 - 1/2)*(a*d - b*c)^(2/3))/(3*d^(8/3)) + (c*log((c^2*(a + b*x^3)^(1
/3)*(a*d - b*c)^2)/d^3 - (c^2*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^
(10/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(2/3))/(3*d^(8/3))
```

3.679. $\int \frac{x^5 \left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

3.680 $\int \frac{x^2\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

3.680.1 Optimal result

Integrand size = 24, antiderivative size = 162

$$\int \frac{x^2\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx=\frac{\left(a+bx^3\right)^{2/3}}{2d}+\frac{(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}}-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{5/3}}+\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}}$$

output
```
1/2*(b*x^3+a)^(2/3)/d-1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^(5/3)+1/2*(-a*d+b
*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(5/3)+1/3*(-a*d+b
*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2
))/d^(5/3)*3^(1/2)
```

3.680.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.27

$$\int \frac{x^2\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx=\frac{3d^{2/3}\left(a+bx^3\right)^{2/3}+2\sqrt{3}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)+2(bc-ad)^{2/3}\log}{}$$

input
```
Integrate[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(3*d^(2/3)*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*
d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*(b*c - a*d)^(2/
3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - (b*c - a*d)^(2/3)*
Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2
/3)*(a + b*x^3)^(2/3)])/(6*d^(5/3))
```

3.680.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {946, 60, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+bx^3\right)^{2/3}}{c+dx^3}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)^{2/3}}{dx^3+c}dx^3$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{d}\right)$$

$$\Big\downarrow \text{68}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\right.}{2d}\right.}{d}\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\,\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-}\right.}{2d^{2/3}}\right.}{d}\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\,\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right.}{d}\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right.}{2d^{2/3}\sqrt[3]{bc-ad}}\right.}{d}\right.$$

3.680. $\int \frac{x^2\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

input
```
Int[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
((3*(a + b*x^3)^(2/3))/(2*d) - ((b*c - a*d)*(-((Sqrt[3]*ArcTan[(1 - (2*d^(
1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d)^
(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/3)) - (3*Log[(b*c - a*d
)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(1/3))))/d)/3
```

3.680.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.680.4 Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.37

method	result
pseudoelliptic	$\frac{(b\,x^3+a)^{\frac{2}{3}}}{2d}+\frac{\ln\left((b\,x^3+a)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)(ad-bc)}{3d^2\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}-\frac{\ln\left((b\,x^3+a)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}(b\,x^3+a)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)(ad-bc)}{6d^2\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}+$

input
```
int(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^3+a)^(2/3)/d+1/3/d^2/(1/d*(a*d-b*c))^(1/3)*ln((b*x^3+a)^(1/3)-(1/
d*(a*d-b*c))^(1/3))*(a*d-b*c)-1/6/d^2/(1/d*(a*d-b*c))^(1/3)*ln((b*x^3+a)^(
2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))*(a*d-b*c
)+1/3*3^(1/2)/d^2/(1/d*(a*d-b*c))^(1/3)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1
/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*(a*d-b*c)
```

3.680.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(128) = 256.

Time = 0.52 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.99

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx=\frac{2\sqrt{3}\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)-\left(\frac{b^2c^2-2}{}\right.}{}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2*
sqrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) -
sqrt(3)*(b*c - a*d))/(b*c - a*d)) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^
(1/3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3)
- (b*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/d^2)^(1/3)) + 2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*((
b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))
 + 3*(b*x^3 + a)^(2/3))/d
```

3.680.6 Sympy [F]

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{x^2(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**2*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**2*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.680.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.680.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(128) = 256$.

Time = 0.31 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.60

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\left(bcd\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ad^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bcd^2-ad^3)} + \frac{(bx^3+a)^{\frac{2}{3}}}{2\,d} + \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^3} - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^3}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*(b*c*d*(-(b*c - a*d)/d)^(1/3) - a*d^2*(-(b*c - a*d)/d)^(1/3))*(-(b*c -
 a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d
^2 - a*d^3) + 1/2*(b*x^3 + a)^(2/3)/d + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/
3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*
c - a*d)/d)^(1/3))/d^3 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3
) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3
```

3.680.9 Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.47

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\left(b\,x^3+a\right)^{2/3}}{2\,d} + \frac{\ln\left(\frac{\left(b\,x^3+a\right)^{1/3}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{d}-\frac{\left(a\,d-b\,c\right)^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{9\,d^{10/3}}\right)\,\left(a\,d-b\,c\right)^{2/3}}{3\,d^{5/3}} - \frac{\ln\left(\frac{\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)^2}{d}-\frac{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{7/3}}{d^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{2/3}}{3\,d^{5/3}} + \frac{\ln\left(\frac{\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)^2}{d}-\frac{{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(a\,d-b\,c\right)^{7/3}}{4\,d^{4/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,\left(a\,d-b\,c\right)^{2/3}}{d^{5/3}}$$

input
```
int((x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
(a + b*x^3)^(2/3)/(2*d) + (log(((a + b*x^3)^(1/3)*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d))/d - ((a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(10/3)))*(a*d
- b*c)^(2/3))/(3*d^(5/3)) - (log(((a + b*x^3)^(1/3)*(a*d - b*c)^2)/d - (((
3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(a
*d - b*c)^(2/3))/(3*d^(5/3)) + (log(((a + b*x^3)^(1/3)*(a*d - b*c)^2)/d -
((3^(1/2)*1i - 1)^2*(a*d - b*c)^(7/3))/(4*d^(4/3)))*((3^(1/2)*1i)/6 - 1/6)
*(a*d - b*c)^(2/3))/d^(5/3)
```

3.680. $\int \frac{x^2 (a+bx^3)^{2/3}}{c+dx^3} \, dx$

3.681 $\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx$

3.681.1 Optimal result

Integrand size = 24, antiderivative size = 245

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx = \frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c} - \frac{(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{2/3}} - \frac{a^{2/3}\log(x)}{2c} + \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}} + \frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{2/3}}$$

output
```
-1/2*a^(2/3)*ln(x)/c+1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c/d^(2/3)+1/2*a^(2/3
)*ln(a^(1/3)-(b*x^3+a)^(1/3))/c-1/2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d
^(1/3)*(b*x^3+a)^(1/3))/c/d^(2/3)+1/3*a^(2/3)*arctan(1/3*(a^(1/3)+2*(b*x^3
+a)^(1/3))/a^(1/3)*3^(1/2))/c*3^(1/2)-1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2
*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/d^(2/3)*3^(1/2)
```

3.681.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx = \frac{2\sqrt{3}a^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + \frac{-2\sqrt{3}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)+2a^{2/3}d^{2/3}\log\Big(}{}}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x*(c + d*x^3)),x]
```

output
```
(2*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + (
-2*Sqrt[3]*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*
c - a*d)^(1/3))/Sqrt[3]] + 2*a^(2/3)*d^(2/3)*Log[-a^(1/3) + (a + b*x^3)^(1
/3)] - 2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/
3)] - a^(2/3)*d^(2/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3
)^(2/3)] + (b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(
1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/d^(2/3))/(6*c)
```

3.681.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 94, 67, 16, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{2/3}}{x^3(dx^3+c)}dx^3$$

$$\Big\downarrow \text{94}$$

$$\frac{1}{3}\left(\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{c}+\frac{a\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{c}\right)$$

$$\Big\downarrow \ 67$$

$$\frac{1}{3}\left(\frac{a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{c}+\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}}}{c}\right.$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\left(\frac{a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{c}+\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}d}{c}\right.$$

$$\Big\downarrow \ 68$$

$$\frac{1}{3}\left(\frac{a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{c}+\frac{(bc-ad)\left(-\frac{3\int\frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+}}{2d^{2/3}}\right.}{}\right.$$

$$\Big\downarrow \ 16$$

3.681. $\int \frac{\left(a+bx^3\right)^{2/3}}{x\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(\frac{a\left(\frac{3}{2}\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}+\frac{(bc-ad)\left(\frac{3\int\frac{}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\sqrt[3]{}}}{}\right.}{}\right.$$

$$\Big\downarrow \textcolor{blue}{1082}$$

$$\frac{1}{3}\left(\frac{(bc-ad)\left(\frac{3\int\frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{c}+\frac{a\left(-\frac{3\int\frac{}{-x}}{}\right.}{}\right.$$

$$\Big\downarrow \textcolor{blue}{217}$$

$$\frac{1}{3}\left(\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{c}+\frac{a\left(\frac{\sqrt{3}\,\mathrm{arcta}}{}\right.}{}\right.$$

input
```
Int[(a + b*x^3)^(2/3)/(x*(c + d*x^3)),x]
```

3.681. $\int\frac{\left(a+bx^3\right)^{2/3}}{x\left(c+dx^3\right)}\,dx$

output
```
((a*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3)
 - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3))
))/c + ((b*c - a*d)*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/
(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]
/(2*d^(2/3)*(b*c - a*d)^(1/3)) - (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b
*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(1/3))))/c)/3
```

3.681.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.681. $\int \frac{\left(a+bx^3\right)^{2/3}}{x\left(c+dx^3\right)}\,dx$

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.681.4 Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{-a^{\frac{2}{3}} \left(-2 \arctan \left(\frac{\left(a^{\frac{1}{3}}+2 \left(b \,x^{3}+a \right)^{\frac{1}{3}}\right) \sqrt{3}}{3 a^{\frac{1}{3}}}\right) \sqrt{3}+\ln \left(\left(b \,x^{3}+a \right)^{\frac{2}{3}}+a^{\frac{1}{3}} \left(b \,x^{3}+a \right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2 \ln \left(\left(b \,x^{3}+a \right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)\right) d \left(\frac{a d -b c}{d}\right)}{}$

input
```
int((b*x^3+a)^(2/3)/x/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/(1/d*(a*d-b*c))^(1/3)*(-a^(2/3)*(-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1
/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a
^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*d*(1/d*(a*d-b*c))^(1/3)+(a*d-b*c)*(
-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-
b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1
/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/c
/d
```

3.681.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(192) = 384$.

Time = 0.54 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx^3)^{2/3}}{x\,(c+dx^3)}\,dx =$$

$$-\frac{2\sqrt{3}\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)-2\sqrt{3}(a^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}a+2}{}\right.}{}$$

input
```
integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(
2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)
 + sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 2*sqrt(3)*(a^2)^(1/3)*arctan(1/3*(s
qrt(3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(1/3))/a) + (-(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + (a^2)^(1/3)*log((b*x^3 + a)^(2/
3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 2*(-(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)
^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) - 2*(a^2)^(1/3)*log((b*x^3 + a)^(1
/3)*a - (a^2)^(2/3)))/c
```

3.681.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x\,(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x\,(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x*(c + d*x**3)), x)
```

3.681.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x), x)
```

3.681.8 Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.39

$$\begin{aligned}\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx =& -\frac{\left(bc\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ad\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3(bc^2-acd)} \\ &+ \frac{\sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,c} \\ &- \frac{a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,c} + \frac{a^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,c} \\ &- \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd^2} \\ &+ \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,cd^2}\end{aligned}$$

input
```
integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b*c*(-(b*c - a*d)/d)^(1/3) - a*d*(-(b*c - a*d)/d)^(1/3))*(-(b*c - a*
d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^2 -
a*c*d) + 1/3*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(
1/3))/a^(1/3))/c - 1/6*a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a
^(1/3) + a^(2/3))/c + 1/3*a^(2/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/c
- 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(
1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c*d^2) + 1/6*(-b*c
*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d
)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c*d^2)
```

3.681.9 Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 1963, normalized size of antiderivative = 8.01

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^3)^(2/3)/(x*(c + d*x^3)),x)
```

output
```
log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4
*a^3*b^7*c^2*d^2) - (a^2/(27*c^3))^(2/3)*(((a + b*x^3)^(1/3)*(54*a^2*b^6*c
^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - (243*a*b^6*c^6*d^3 -
729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^(2/3))*(a^2/(27*
c^3))^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4
 - 9*a*b^8*c^5*d))*(a^2/(27*c^3))^(1/3) + log((a + b*x^3)^(1/3)*(2*a^5*b^5
*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - (-(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3)*(((a + b*x^3)^(1/3)*(54*a^2*b^6*
c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - (243*a*b^6*c^6*d^3 -
 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d)/(27*c^3*d^2))^(2/3))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(
1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*
b^8*c^5*d))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) - log((a
 + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b
^7*c^2*d^2) + ((3^(1/2)*1i)/2 + 1/2)^2*(a^2/(27*c^3))^(2/3)*(((3^(1/2)*1i)
/2 + 1/2)*((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 5
4*a^4*b^4*c^2*d^5) - ((3^(1/2)*1i)/2 + 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2
*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^(2/3))*(a^2/(27*c^3))^(
1/3) - 36*a^2*b^7*c^4*d^2 + 54*a^3*b^6*c^3*d^3 - 27*a^4*b^5*c^2*d^4 + 9*a*
b^8*c^5*d))*((3^(1/2)*1i)/2 + 1/2)*(a^2/(27*c^3))^(1/3) + log((a + b*x^...
```

3.682 $\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx$

3.682.1 Optimal result

Integrand size = 24, antiderivative size = 347

$$
\begin{aligned}
\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx &= \frac{d(a+bx^3)^{2/3}}{2c^2} + \frac{(2bc-3ad)(a+bx^3)^{2/3}}{6ac^2} \\
&\quad - \frac{(a+bx^3)^{5/3}}{3acx^3} + \frac{(2bc-3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}c^2} \\
&\quad + \frac{\sqrt[3]{d}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2} - \frac{(2bc-3ad)\log(x)}{6\sqrt[3]{a}c^2} \\
&\quad - \frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^2} + \frac{(2bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}c^2} \\
&\quad + \frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2}
\end{aligned}
$$

output
```
1/2*d*(b*x^3+a)^(2/3)/c^2+1/6*(-3*a*d+2*b*c)*(b*x^3+a)^(2/3)/a/c^2-1/3*(b*
x^3+a)^(5/3)/a/c/x^3-1/6*(-3*a*d+2*b*c)*ln(x)/a^(1/3)/c^2-1/6*d^(1/3)*(-a*
d+b*c)^(2/3)*ln(d*x^3+c)/c^2+1/6*(-3*a*d+2*b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3)
)/a^(1/3)/c^2+1/2*d^(1/3)*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*
x^3+a)^(1/3))/c^2+1/9*(-3*a*d+2*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3)
)/a^(1/3)*3^(1/2))/a^(1/3)/c^2*3^(1/2)+1/3*d^(1/3)*(-a*d+b*c)^(2/3)*arctan
(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2*3^(1/2)
```

3.682.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(c+dx^3\right)}\,dx = \frac{-\frac{6c\left(a+bx^3\right)^{2/3}}{x^3}+\frac{2\sqrt{3}(2bc-3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+6\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}\arctan\Bigg(}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x]
```

output
```
((-6*c*(a + b*x^3)^(2/3))/x^3 + (2*Sqrt[3]*(2*b*c - 3*a*d)*ArcTan[(1 + (2*
(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) + 6*Sqrt[3]*d^(1/3)*(b*c - a
*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqr
t[3]] + (2*(2*b*c - 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(1/3) + 6*
d^(1/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3
)] + ((-2*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^
3)^(2/3)])/a^(1/3) - 3*d^(1/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d
^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(
18*c^2)
```

3.682.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {948, 114, 27, 174, 60, 67, 16, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(c+dx^3\right)}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)^{2/3}}{x^6\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \; 114$$

$$\frac{1}{3}\left(-\frac{\int -\frac{\left(bx^3+a\right)^{2/3}\left(2bdx^3+2bc-3ad\right)}{3x^3\left(dx^3+c\right)}dx^3}{ac}-\frac{\left(a+bx^3\right)^{5/3}}{acx^3}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{\int \frac{\left(bx^3+a\right)^{2/3}\left(2bdx^3+2bc-3ad\right)}{x^3\left(dx^3+c\right)}dx^3}{3ac}-\frac{\left(a+bx^3\right)^{5/3}}{acx^3}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(\frac{\frac{3ad^2\int \frac{\left(bx^3+a\right)^{2/3}}{dx^3+c}dx^3}{c}+\frac{(2bc-3ad)\int \frac{\left(bx^3+a\right)^{2/3}}{x^3}dx^3}{c}}{3ac}-\frac{\left(a+bx^3\right)^{5/3}}{acx^3}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3}\left(\frac{\frac{3ad^2\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{d}\right)}{c}+\frac{(2bc-3ad)\left(a\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)}{c}}{3ac}-\frac{\left(a+bx^3\right)^{5/3}}{acx^3}\right)$$

$$\Big\downarrow \; 67$$

$$\frac{1}{3}\left(\frac{\frac{(2bc-3ad)\left(a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)}{c}+3ad^2}{3ac}\right.$$

$$\Big\downarrow \; 16$$

3.682. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(\frac{\frac{(2bc-3ad)\left(a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)}{c}+\frac{3ad^2\left(\frac{3\left(a+bx^3\right)}{2d}\right.}{}}{3ac}\right.$$

$$\Big\downarrow \; 68$$

$$\frac{1}{3}\left(\frac{(2bc-3ad)\left(a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)}{c}+\frac{3ad^2\left(\frac{3\left(a+bx^3\right)}{2d}\right.}{}\right.$$

$$\Big\downarrow \; 16$$

3.682. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(\frac{\frac{(2bc-3ad)\left(a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)}{c}+\frac{3ad^2\left(\frac{3\left(a+bx^3\right)}{2d}\right.}{}}{3a}\right.$$

$$\Big\downarrow 1082$$

$$\frac{1}{3}\left(\frac{\frac{3ad^2\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{d}\right)}{c}+\frac{(2b}{}}{3ac}\right.$$

$$\Big\downarrow 217$$

3.682. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(\frac{\frac{3ad^2\left(\frac{3\left(a+bx^3\right)^{2/3}}{2d}-\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{d}\right)}{c}+\frac{(2b}{}}{3ac}\right.$$

input
```
Int[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x]
```

output
```
(-((a + b*x^3)^(5/3)/(a*c*x^3)) + (((2*b*c - 3*a*d)*((3*(a + b*x^3)^(2/3))
/2 + a*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1
/3) - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/
3)))))/c + (3*a*d^2*((3*(a + b*x^3)^(2/3))/(2*d) - ((b*c - a*d)*(-((Sqrt[3
]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(
d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/3))
 - (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c
- a*d)^(1/3))))/d))/c)/(3*a*c))/3
```

3.682. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(c+dx^3\right)}\,dx$

3.682.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

3.682. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(c+dx^3\right)}\,dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.682.4 Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{\left(-2\left(b\,x^{3}+a\right)^{\frac{2}{3}} a^{\frac{1}{3}} c+x^{3} \left(-2 \arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3 a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right.\right.\right.}{}$

input
```
int((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/a^(1/3)*((-2*(b*x^3+a)^(2/3)*a^(1/3)*c+x^3*(-2*arctan(1/3*(a^(1/3)+2*(
b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3
+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*(a*d-2/3*b*c))*(1/d*(a*d
-b*c))^(1/3)+2*x^3*(arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^
(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(
1/3))-1/2*ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a
*d-b*c))^(2/3)))*(d*a^(4/3)-a^(1/3)*b*c))/(1/d*(a*d-b*c))^(1/3)/c^2/x^3
```

3.682.5 Fricas [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 1030, normalized size of antiderivative = 2.97

$$\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/18*(3*sqrt(1/3)*(2*a*b*c - 3*a^2*d)*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x^
3 - 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-
a)^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3
) - 6*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*arctan(1/3*(
sqrt(3)*(b*c - a*d) - 2*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*
(b*x^3 + a)^(1/3))/(b*c - a*d)) + (2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^
3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(2*b*c - 3*a
*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) + 3*(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) -
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b
*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 6*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)^(1/3)*a*x^3*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)^(2/3)) + 6*(b*x^3 + a)^(2/3)*a*c)/(a*c^2*x^3), 1/1
8*(6*sqrt(1/3)*(2*a*b*c - 3*a^2*d)*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3
)*(2*(b*x^3 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 6*sqrt(3)*(b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*arctan(1/3*(sqrt(3)*(b*c - a*d
) - 2*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3))
/(b*c - a*d)) - (2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*
x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(2*b*c - 3*a*d)*(-a)^(2/3)*x^3
*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2...
```

3.682.6 Sympy [F]

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(c+dx^3\right)}\,dx=\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^4\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**4/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**4*(c + d*x**3)), x)
```

3.682.7 Maxima [F]

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{(dx^3+c)x^4}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^4), x)
```

3.682.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx = \frac{\left(bcd\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ad^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3-ac^2d)} + \frac{\sqrt{3}(2\,bc-3\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{1}{3}}c^2} - \frac{(2\,bc-3\,ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{1}{3}}c^2} + \frac{\left(2\,a^{\frac{1}{3}}bc-3\,a^{\frac{4}{3}}d\right)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{2}{3}}c^2} + \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^2d} - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^2d} - \frac{(bx^3+a)^{\frac{2}{3}}}{3\,cx^3}$$

input
```
integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*(b*c*d*(-(b*c - a*d)/d)^(1/3) - a*d^2*(-(b*c - a*d)/d)^(1/3))*(-(b*c -
 a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^3
 - a*c^2*d) + 1/9*sqrt(3)*(2*b*c - 3*a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a
)^(1/3) + a^(1/3))/a^(1/3))/(a^(1/3)*c^2) - 1/18*(2*b*c - 3*a*d)*log((b*x^
3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(1/3)*c^2) + 1/9*(2
*a^(1/3)*b*c - 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)
*c^2) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3
+ a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c^2*d) - 1/6
*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
 - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c^2*d) - 1/3*(b*x^3 + a)^(2/3)
/(c*x^3)
```

3.682.9 Mupad [B] (verification not implemented)

Time = 14.44 (sec) , antiderivative size = 1908, normalized size of antiderivative = 5.50

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(c+dx^3\right)}\,dx=\text{Too large to display}$$

input

```
int((a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x)
```

output

```
log(- ((((6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2
 - 6*a*b*c*d) - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d*(a*
d - b*c)^2)/c^6)^(2/3))*((d*(a*d - b*c)^2)/c^6)^(1/3))/3 - (a*b^5*d^4*(27*
a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*((d*(a*d -
 b*c)^2)/c^6)^(2/3))/9 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(3*a^2
*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((a^2*d^3 + b^2*c^2*d - 2*a*b*c
*d^2)/(27*c^6))^(1/3) + log(- ((((6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^
2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) - 3*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c
^2 - 3*a*b*c*d)*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a
*c^6))^(1/3))/9 - (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 7
2*a^2*b*c*d^2))/(3*c))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(
a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(2
7*c^5))*(-(27*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*
a*c^6))^(1/3) - log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(6*b^
4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d)
- 3*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)
*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1/3))/9
 + (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))
/(3*c))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^(1/3
)*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^...
```

3.683 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(c+dx^3\right)}\,dx$

3.683.1 Optimal result

Integrand size = 24, antiderivative size = 370

$$\begin{aligned}\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx &= \frac{(bc+6ad)\,(a+bx^3)^{2/3}}{18ac^2x^3}-\frac{(a+bx^3)^{5/3}}{6acx^6}\\ &\quad-\frac{(b^2c^2+6abcd-9a^2d^2)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}c^3}\\ &\quad-\frac{d^{4/3}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3}+\frac{(b^2c^2+6abcd-9a^2d^2)\log(x)}{18a^{4/3}c^3}\\ &\quad+\frac{d^{4/3}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^3}-\frac{(b^2c^2+6abcd-9a^2d^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3}\\ &\quad-\frac{d^{4/3}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}\end{aligned}$$

output
```
1/18*(6*a*d+b*c)*(b*x^3+a)^(2/3)/a/c^2/x^3-1/6*(b*x^3+a)^(5/3)/a/c/x^6+1/1
8*(-9*a^2*d^2+6*a*b*c*d+b^2*c^2)*ln(x)/a^(4/3)/c^3+1/6*d^(4/3)*(-a*d+b*c)^
(2/3)*ln(d*x^3+c)/c^3-1/18*(-9*a^2*d^2+6*a*b*c*d+b^2*c^2)*ln(a^(1/3)-(b*x^
3+a)^(1/3))/a^(4/3)/c^3-1/2*d^(4/3)*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d
^(1/3)*(b*x^3+a)^(1/3))/c^3-1/27*(-9*a^2*d^2+6*a*b*c*d+b^2*c^2)*arctan(1/3
*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)/c^3*3^(1/2)-1/3*d^(4
/3)*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1
/3))*3^(1/2))/c^3*3^(1/2)
```

3.683.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx = \frac{\frac{3c(a+bx^3)^{2/3}(-3ac-2bcx^3+6adx^3)}{ax^6} - \frac{2\sqrt{3}(b^2c^2+6abcd-9a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} - 18\sqrt{3}d^{4/3}(bc}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x]
```

output
```
((3*c*(a + b*x^3)^(2/3)*(-3*a*c - 2*b*c*x^3 + 6*a*d*x^3))/(a*x^6) - (2*Sqr
t[3]*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a
^(1/3))/Sqrt[3]])/a^(4/3) - 18*Sqrt[3]*d^(4/3)*(b*c - a*d)^(2/3)*ArcTan[(1
 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - (2*(b^2*c^2
 + 6*a*b*c*d - 9*a^2*d^2)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(4/3) - 18*
d^(4/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3
)] + ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^
(1/3) + (a + b*x^3)^(2/3)])/a^(4/3) + 9*d^(4/3)*(b*c - a*d)^(2/3)*Log[(b*c
 - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a +
 b*x^3)^(2/3)])/(54*c^3)
```

3.683.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {948, 114, 27, 166, 27, 174, 67, 16, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^7 \left(c+dx^3\right)} \, dx$$

$$\Big\downarrow \textbf{948}$$

$$\frac{1}{3} \int \frac{\left(bx^3+a\right)^{2/3}}{x^9 \left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \textbf{114}$$

$$\frac{1}{3} \left(-\frac{\int \frac{\left(bx^3+a\right)^{2/3} \left(bdx^3+bc+6ad\right)}{3x^6 \left(dx^3+c\right)}dx^3}{2ac}-\frac{\left(a+bx^3\right)^{5/3}}{2acx^6}\right)$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{3} \left(-\frac{\int \frac{\left(bx^3+a\right)^{2/3} \left(bdx^3+bc+6ad\right)}{x^6 \left(dx^3+c\right)}dx^3}{6ac}-\frac{\left(a+bx^3\right)^{5/3}}{2acx^6}\right)$$

$$\Big\downarrow \textbf{166}$$

$$\frac{1}{3} \left(-\frac{\frac{\int \frac{2 \left(bd (bc-3ad) x^3+b^2 c^2-9 a^2 d^2+6abcd\right)}{3x^3 \sqrt[3]{bx^3+a} \left(dx^3+c\right)}dx^3}{c}-\frac{\left(a+bx^3\right)^{2/3} (6ad+bc)}{cx^3}}{6ac}-\frac{\left(a+bx^3\right)^{5/3}}{2acx^6}\right)$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{3} \left(-\frac{\frac{2 \int \frac{bd (bc-3ad) x^3+b^2 c^2-9 a^2 d^2+6abcd}{x^3 \sqrt[3]{bx^3+a} \left(dx^3+c\right)}dx^3}{3c}-\frac{\left(a+bx^3\right)^{2/3} (6ad+bc)}{cx^3}}{6ac}-\frac{\left(a+bx^3\right)^{5/3}}{2acx^6}\right)$$

$$\Big\downarrow \textbf{174}$$

$$\frac{1}{3}\left(-\frac{\frac{2\left(\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{c}-\frac{9ad^2(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{c}\right)}{3c}-\frac{\left(a+bx^3\right)^{2/3}(6ad+bc)}{cx^3}}{6ac}-\frac{\left(a+bx^3\right)^{5/3}}{2acx^6}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(-\frac{\frac{2\left(\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{9ad^2(bc-ad)\int}{}\right.}{3c}}{6ac}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(-\frac{\frac{2\left(\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{9ad^2(bc-ad)\int \frac{}{\sqrt[3]{bx^3}}}{c}\right.}{3c}}{6ac}\right.$$

$$\Big\downarrow \text{68}$$

3.683. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(-\frac{\frac{2\left(\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{9ad^2(bc-ad)\left(-\frac{3\int \frac{}{\sqrt[3]{b}}}{}\right.}{}\right.}{3c}}{}\right.$$

$$\Big\downarrow \color{blue}{16}$$

3.683. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(-\frac{\frac{2\left(\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{9ad^2(bc-ad)\left(\frac{3\int \frac{}{x^6+\frac{(bc}{}}}{}\right.}{}\right.}{3c}}{(}\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(-\frac{\frac{2\left(\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{9ad^2(bc-ad)\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}}{\sqrt[3]{}}\right.}{d^{2/3}\sqrt[3]{bc-}}\right.}{}\right.}{3c}}{6ac}\right.$$

$$\Big\downarrow \text{217}$$

3.683. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(-\frac{\frac{2\left(\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{9ad^2(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a}}{\sqrt[3]{bc}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-a}}\right.}{}\right)}{3c}}{6ac}\right.$$

input
```
Int[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x]
```

output
```
(-1/2*(a + b*x^3)^(5/3)/(a*c*x^6) - (-(((b*c + 6*a*d)*(a + b*x^3)^(2/3))/(
c*x^3)) + (2*(((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*((Sqrt[3]*ArcTan[(1 + (2*
(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^3]/(2*a^(1/3)) + (3*
Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3))))/c - (9*a*d^2*(b*c - a*d)*(
-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sq
rt[3]])/(d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*
d)^(1/3)) - (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2
/3)*(b*c - a*d)^(1/3))))/c))/(3*c))/(6*a*c))/3
```

3.683. $\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx$

3.683.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

3.683. $\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.683.4 Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{\frac{\left(-\left(b x^{3}+a\right)^{\frac{2}{3}} a^{\frac{4}{3}} c \left(-6 a d x^{3}+2 b c x^{3}+3 a c\right)+\frac{x^{6} \left(9 a^{2} d^{2}-6 a b c d-b^{2} c^{2}\right) \left(2 \arctan \left(\frac{\left(a^{\frac{1}{3}}+2 \left(b x^{3}+a\right)^{\frac{1}{3}}\right) \sqrt{3}}{3 a^{\frac{1}{3}}}\right) \sqrt{3}+2 \ln \left(\left(b x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)\right.}{3}\right.}{3}}{}$

input
```
int((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/(1/d*(a*d-b*c))^(1/3)*(1/3*(-(b*x^3+a)^(2/3)*a^(4/3)*c*(-6*a*d*x^3+2*b
*c*x^3+3*a*c)+1/3*x^6*(9*a^2*d^2-6*a*b*c*d-b^2*c^2)*(2*arctan(1/3*(a^(1/3)
+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-a^(1/3))
-ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3)))*a)*(1/d*(a*d-b*c))^(
1/3)-x^6*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(
1/d*(a*d-b*c))^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))-
ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(
2/3)))*a^(7/3)*(a*d-b*c)*d)/c^3/x^6/a^(7/3)
```

3.683.5 Fricas [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 1151, normalized size of antiderivative = 3.11

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(c+dx^3\right)}\,dx=\text{Too large to display}$$

input
```
integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/54*(18*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*ar
ctan(-1/3*(sqrt(3)*(b*c - a*d) + 2*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2
*d^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + 9*(-b^2*c^2*d + 2*a*b*c*d^2
- a^2*d^3)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*
c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d
^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 18*(-b^2*c^2*d + 2*a*b*c*d^2 - a^
2*d^3)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*
d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) + 3*sqrt(1/3)*(a*b^2*c^2 + 6*a^2*b*c*d -
 9*a^3*d^2)*x^6*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)
^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^
3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - (b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*a^(2/
3)*x^6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b
^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b*x^3 + a)^(1/3) - a^(1/3
)) + 3*(3*a^2*c^2 + 2*(a*b*c^2 - 3*a^2*c*d)*x^3)*(b*x^3 + a)^(2/3))/(a^2*c
^3*x^6), -1/54*(18*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*
d*x^6*arctan(-1/3*(sqrt(3)*(b*c - a*d) + 2*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d
^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + 9*(-b^2*c^2*d + 2*a*
b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2)
+ (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2
*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 18*(-b^2*c^2*d + 2*a*b...
```

3.683.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^7\left(c+dx^3\right)}\,dx=\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^7\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**7/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**7*(c + d*x**3)), x)
```

3.683.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^7\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{(dx^3+c)x^7}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^7), x)
```

3.683.8 Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.32

$$
\begin{aligned}
\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx ={}& -\frac{\left(bcd^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ad^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^4-ac^3d)} \\
& - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^3} \\
& + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^3} \\
& - \frac{\sqrt{3}(b^2c^2+6\,abcd-9\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{4}{3}}c^3} \\
& + \frac{(b^2c^2+6\,abcd-9\,a^2d^2)\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{54\,a^{\frac{4}{3}}c^3} \\
& - \frac{\left(a^{\frac{1}{3}}b^2c^2 + 6\,a^{\frac{4}{3}}bcd - 9\,a^{\frac{7}{3}}d^2\right)\log\left(\left|(bx^3+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{5}{3}}c^3} \\
& - \frac{2\,(bx^3+a)^{\frac{5}{3}}b^2c + (bx^3+a)^{\frac{2}{3}}ab^2c - 6\,(bx^3+a)^{\frac{5}{3}}abd + 6\,(bx^3+a)^{\frac{2}{3}}a^2bd}{18\,ab^2c^2x^6}
\end{aligned}
$$

input

```
integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

output

```
-1/3*(b*c*d^2*(-(b*c - a*d)/d)^(1/3) - a*d^3*(-(b*c - a*d)/d)^(1/3))*(-(b*
c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*
c^4 - a*c^3*d) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(
2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^3
+ 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(
-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^3 - 1/27*sqrt(3)*(b^2*c^
2 + 6*a*b*c*d - 9*a^2*d^2)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/
3))/a^(1/3))/(a^(4/3)*c^3) + 1/54*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*c^3) - 1/2
7*(a^(1/3)*b^2*c^2 + 6*a^(4/3)*b*c*d - 9*a^(7/3)*d^2)*log(abs((b*x^3 + a)^
(1/3) - a^(1/3)))/(a^(5/3)*c^3) - 1/18*(2*(b*x^3 + a)^(5/3)*b^2*c + (b*x^3
 + a)^(2/3)*a*b^2*c - 6*(b*x^3 + a)^(5/3)*a*b*d + 6*(b*x^3 + a)^(2/3)*a^2*
b*d)/(a*b^2*c^2*x^6)
```

3.683.9 Mupad [B] (verification not implemented)

Time = 19.14 (sec) , antiderivative size = 2788, normalized size of antiderivative = 7.54

$$\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx = \text{Too large to display}$$

input

```
int((a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x)
```

output

```
log(((((27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^4*(a*d - b
*c)^2)/c^9)^(2/3) - (b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4
+ b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2
*c^2))*(-(d^4*(a*d - b*c)^2)/c^9)^(1/3))/3 - (b^5*d^4*(729*a^6*d^6 + b^6*c
^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*
a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(d^4*(a*d - b*c)^2)/c^9)^
(2/3))/9 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c
^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*(-(a^2*d^6 + b^2*c
^2*d^4 - 2*a*b*c*d^5)/(27*c^9))^(1/3) + log((((((a*b^4*c^4*d^3*(2*a^2*d^2
+ b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2
/3))/3 - (b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 +
 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2))*(-(b
^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(1/3))/27 - (b^5*d^4*(729*a^6
*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c
^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(b^2*c^2 - 9*
a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/729 + (2*b^5*d^7*(a + b*x^3)^(1/3
)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2
)/(729*a^2*c^10))*(-(b^6*c^6 - 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*
b^3*c^3*d^3 - 729*a^4*b^2*c^2*d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19
683*a^4*c^9))^(1/3) - (((a + b*x^3)^(2/3)*(b^2*c + 6*a*b*d))/(18*c^2) -...
```

3.683. $\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx$

3.684 $\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx$

3.684.1 Optimal result

Integrand size = 24, antiderivative size = 334

$$\begin{aligned}\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx &= -\frac{(3bc-ad)x(a+bx^3)^{2/3}}{9bd^2} \\ &\quad + \frac{x^4(a+bx^3)^{2/3}}{6d} + \frac{(9b^2c^2-6abcd-a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}d^3} \\ &\quad - \frac{c^{4/3}(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3} - \frac{c^{4/3}(bc-ad)^{2/3}\log(c+dx^3)}{6d^3} \\ &\quad + \frac{c^{4/3}(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3} \\ &\quad - \frac{(9b^2c^2-6abcd-a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{4/3}d^3}\end{aligned}$$

output
```
-1/9*(-a*d+3*b*c)*x*(b*x^3+a)^(2/3)/b/d^2+1/6*x^4*(b*x^3+a)^(2/3)/d-1/6*c^
(4/3)*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^3+1/2*c^(4/3)*(-a*d+b*c)^(2/3)*ln((-a
*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^3-1/18*(-a^2*d^2-6*a*b*c*d+9*b^
2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)/d^3+1/27*(-a^2*d^2-6*a*b*c*d
+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d^
3*3^(1/2)-1/3*c^(4/3)*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/
c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)
```

3.684.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.29 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.58

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\frac{6d(a+bx^3)^{2/3}(-6bcx+2adx+3bdx^4)}{b} + \frac{4\sqrt{3}(9b^2c^2-6abcd-a^2d^2)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}_x}{\sqrt[3]{b}_{x+2}\sqrt[3]{a+bx^3}}\right)}{b^{4/3}} + 18\sqrt{-}}{}$$

input
```
Integrate[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
((6*d*(a + b*x^3)^(2/3)*(-6*b*c*x + 2*a*d*x + 3*b*d*x^4))/b + (4*Sqrt[3]*(
9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2
*(a + b*x^3)^(1/3))])/b^(4/3) + 18*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(4/3)*(b*c -
 a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x -
(3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(-9*b^2*c^2 + 6*a*b*c*d +
 a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(4/3) - (18*I)*(-I + Sq
rt[3])*c^(4/3)*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3
])*c^(1/3)*(a + b*x^3)^(1/3)] + (2*(9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*Log[b
^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(4/3) + 9
*(1 + I*Sqrt[3])*c^(4/3)*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (
-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqr
t[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(108*d^3)
```

3.684.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {978, 27, 1052, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx$$

$$\Big\downarrow \text{978}$$

$$\frac{x^4 \left(a+b x^3\right)^{2/3}}{6 d}-\frac{\int \frac{2 x^3 \left((3 b c-a d) x^3+2 a c\right)}{\sqrt[3]{b x^3+a} \left(d x^3+c\right)}dx}{6 d}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4 \left(a+b x^3\right)^{2/3}}{6 d}-\frac{\int \frac{x^3 \left((3 b c-a d) x^3+2 a c\right)}{\sqrt[3]{b x^3+a} \left(d x^3+c\right)}dx}{3 d}$$

$$\Big\downarrow \text{1052}$$

$$\frac{x^4 \left(a+b x^3\right)^{2/3}}{6 d}-\frac{\frac{x \left(a+b x^3\right)^{2/3} (3 b c-a d)}{3 b d}-\frac{\int \frac{\left(9 b^2 c^2-6 a b d c-a^2 d^2\right) x^3+a c (3 b c-a d)}{\sqrt[3]{b x^3+a} \left(d x^3+c\right)}dx}{3 b d}}{3 d}$$

$$\Big\downarrow \text{1026}$$

$$\frac{x^4 \left(a+b x^3\right)^{2/3}}{6 d}-\frac{\frac{x \left(a+b x^3\right)^{2/3} (3 b c-a d)}{3 b d}-\frac{\frac{\left(-a^2 d^2-6 a b c d+9 b^2 c^2\right) \int \frac{1}{\sqrt[3]{b x^3+a}}dx}{d}-\frac{9 b c^2 (b c-a d) \int \frac{1}{\sqrt[3]{b x^3+a} \left(d x^3+c\right)}dx}{d}}{3 b d}}{3 d}$$

$$\Big\downarrow \text{769}$$

$$\frac{x^4\left(a+bx^3\right)^{2/3}}{6d}-\frac{\frac{x\left(a+bx^3\right)^{2/3}(3bc-ad)}{3bd}-\frac{\frac{\left(-a^2d^2-6abcd+9b^2c^2\right)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{9bc^2(bc-ad)\int\frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}d}{d}}{3bd}}{3d}$$

$$\Big\downarrow \; 901$$

$$\frac{x^4\left(a+bx^3\right)^{2/3}}{6d}-\frac{\frac{x\left(a+bx^3\right)^{2/3}(3bc-ad)}{3bd}-\frac{\frac{\left(-a^2d^2-6abcd+9b^2c^2\right)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-9bc^2(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-a}}{\sqrt[3]{c}\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-a}}\right.}{3bd}}{3d}$$

input
```
Int[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(x^4*(a + b*x^3)^(2/3))/(6*d) - ((((3*b*c - a*d)*x*(a + b*x^3)^(2/3))/(3*b*
d) - ((-9*b*c^2*(b*c - a*d)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*
(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c +
 d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3)
 - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d + ((9*b^2*c^2 - 6*a
*b*c*d - a^2*d^2)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(
Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/d)/
(3*b*d))/(3*d)
```

3.684. $\int \frac{x^6\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

3.684.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

3.684. $\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx$

3.684.4 Maple [A] (verified)

Time = 5.32 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{\Bigg(-6x\left(\left(\frac{3d\,x^{3}}{2}-3c\right)b^{\frac{7}{3}}+b^{\frac{4}{3}}ad\right)d\left(b\,x^{3}+a\right)^{\frac{2}{3}}+b\Bigg(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\Bigg(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+(b}{x^{2}}}{}$

input
```
int(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/54/((a*d-b*c)/c)^(1/3)/b^(7/3)*((-6*x*((3/2*d*x^3-3*c)*b^(7/3)+b^(4/3)*
a*d)*d*(b*x^3+a)^(2/3)+b*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^
3+a)^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)
^(2/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*(a^2*d^2+6*a*b*c*d-9*b^
2*c^2))*((a*d-b*c)/c)^(1/3)+18*(b^(10/3)*c-b^(7/3)*a*d)*(arctan(1/3*3^(1/2
)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)
+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((((a*d-b*c)/c)^(2/3)
*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*c)/d^3
```

3.684.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(276) = 552.

Time = 1.59 (sec) , antiderivative size = 1164, normalized size of antiderivative = 3.49

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output

```
[-1/54*(18*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2*c*arctan(
-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2
)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) - 18*(b^2*c^3 - 2*a*b*c^2*d +
a^2*c*d^2)^(1/3)*b^2*c*log(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*x -
(b*x^3 + a)^(1/3)*(b*c^2 - a*c*d))/x) + 9*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d
^2)^(1/3)*b^2*c*log(-((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*(b*c - a*d
)*x^2 + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x + (b
*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2) + 3*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c
*d - a^2*b*d^2)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)
*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^
(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(9*b^2*c^2 - 6*a*b*c*d - a^2*
d^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*
c*d - a^2*d^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b
*x^3 + a)^(2/3))/x^2) - 3*(3*b^2*d^2*x^4 - 2*(3*b^2*c*d - a*b*d^2)*x)*(b*x
^3 + a)^(2/3))/(b^2*d^3), -1/54*(18*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c
*d^2)^(1/3)*b^2*c*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b^2*c^3
- 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) - 18*
(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2*c*log(((b^2*c^3 - 2*a*b*c^2*
d + a^2*c*d^2)^(2/3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d))/x) + 9*(b^2*c^
3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2*c*log(-((b^2*c^3 - 2*a*b*c^2*d +...
```

3.684.6 Sympy [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^6(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input

```
integrate(x**6*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output

```
Integral(x**6*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.684.7 Maxima [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}x^6}{dx^3+c}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)
```

3.684.8 Giac [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}x^6}{dx^3+c}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)
```

3.684.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{x^6\,(b\,x^3+a)^{2/3}}{d\,x^3+c}\,dx$$

input
```
int((x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
int((x^6*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

3.685 $\int \frac{x^3\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

3.685.1 Optimal result

Integrand size = 24, antiderivative size = 272

$$\begin{aligned}\int \frac{x^3\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx &= \frac{x\left(a+bx^3\right)^{2/3}}{3d} - \frac{(3bc-2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}d^2} \\ &+ \frac{\sqrt[3]{c}(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt[3]{c}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^2} \\ &- \frac{\sqrt[3]{c}(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2} \\ &+ \frac{(3bc-2ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{b}d^2}\end{aligned}$$

output
```
1/3*x*(b*x^3+a)^(2/3)/d+1/6*c^(1/3)*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^2-1/2*c
^(1/3)*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2
+1/6*(-2*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)/d^2-1/9*(-2*a*d
+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)/d^2*3^
(1/2)+1/3*c^(1/3)*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1
/3)/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)
```

3.685.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.05 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.71

$$\int \frac{x^3\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{12dx\left(a+bx^3\right)^{2/3}-\frac{4\sqrt{3}(3bc-2ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b_x}}{\sqrt[3]{b_{x+2}}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}-6\sqrt{-6+6i\sqrt{3}}\sqrt[3]{c}(bc-a}{}$$

input
```
Integrate[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(12*d*x*(a + b*x^3)^(2/3) - (4*Sqrt[3]*(3*b*c - 2*a*d)*ArcTan[(Sqrt[3]*b^(
1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(1/3) - 6*Sqrt[-6 + (6*I)*Sq
rt[3]]*c^(1/3)*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(
b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(3*b
*c - 2*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(1/3) + 6*(1 + I*Sqrt
[3])*c^(1/3)*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])
*c^(1/3)*(a + b*x^3)^(1/3)] + (2*(-3*b*c + 2*a*d)*Log[b^(2/3)*x^2 + b^(1/3
)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(1/3) - (3*I)*(-I + Sqrt[3])
*c^(1/3)*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*
c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a
 + b*x^3)^(2/3)])/(36*d^2)
```

3.685.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {978, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+bx^3\right)^{2/3}}{c+dx^3}dx$$
$$\Big\downarrow \text{978}$$
$$\frac{x\left(a+bx^3\right)^{2/3}}{3d}-\frac{\int \frac{(3bc-2ad)x^3+ac}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{3d}$$

$$\Big\downarrow \text{1026}$$

$$\frac{x\left(a+bx^3\right)^{2/3}}{3d}-\frac{\frac{(3bc-2ad)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{3c(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{d}}{3d}$$

$$\Big\downarrow \text{769}$$

$$\frac{x\left(a+bx^3\right)^{2/3}}{3d}-\frac{\frac{(3bc-2ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{3c(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{d}}{3d}$$

$$\Big\downarrow \text{901}$$

$$\frac{x\left(a+bx^3\right)^{2/3}}{3d}-\frac{\frac{(3bc-2ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{3c(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\log\left(\frac{x}{}\right.\right.}{d}}{3d}$$

input
```
Int[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^(2/3))/(3*d) - ((-3*c*(b*c - a*d)*(ArcTan[(1 + (2*(b*c - a*
d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c -
a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a
*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/
d + ((3*b*c - 2*a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]
]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/
d)/(3*d)
```

3.685. $\int \frac{x^3\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

3.685.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.685.4 Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.44

method	result
pseudoelliptic	$-\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(ad-\frac{3bc}{2}\right)\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{3}+\left(ad\,b^{\frac{1}{3}}-b^{\frac{4}{3}}c\right)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)+ 2\sqrt{3}\left(\frac{ad-b}{c}\right.$ $-$

3.685. $\int \frac{x^3\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

input

```
int(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output

```
-1/3*(-1/3*((a*d-b*c)/c)^(1/3)*(a*d-3/2*b*c)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^
3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(a*d*b^(1/3)-b^(4/3)*c)*ln((((a*d-b*c)/
c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+2/3*3^(1/2)*((a*d-b*c)/c)^(1/3)*(a*d-3/2*b*
c)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+2/3*((a*d-b
*c)/c)^(1/3)*(a*d-3/2*b*c)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-(b*x^3+a)^(2
/3)*x*((a*d-b*c)/c)^(1/3)*d*b^(1/3)+(a*d*b^(1/3)-b^(4/3)*c)*(arctan(1/3*3^
(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(
1/2)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x
+(b*x^3+a)^(2/3))/x^2)))/b^(1/3)/((a*d-b*c)/c)^(1/3)/d^2
```

3.685.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(219) = 438$.

Time = 0.60 (sec) , antiderivative size = 1091, normalized size of antiderivative = 4.01

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Too large to display}$$

input

```
integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c - 2*a*b*d)*sqrt((-
b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)
*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2
/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 6*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2*d - a^2
*c*d^2)^(1/3)*b*arctan(1/3*(sqrt(3)*(b*c - a*d)*x - 2*sqrt(3)*(-b^2*c^3 +
2*a*b*c^2*d - a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) + 2*(3*
b*c - 2*a*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (3*b*c
 - 2*a*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x
+ (b*x^3 + a)^(2/3))/x^2) + 6*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b
*log(((-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(2/3)*x - (b*x^3 + a)^(1/3)*(b*
c^2 - a*c*d))/x) - 3*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b
^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2 - (-b^2*c^3 + 2*a*
b*c^2*d - a^2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x - (b*x^3 + a)^(2/3)*(b*c^2
- a*c*d))/x^2))/(b*d^2), 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x + 6*sqrt(1/3)*(3*
b^2*c - 2*a*b*d)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(
b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 6*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2
*d - a^2*c*d^2)^(1/3)*b*arctan(1/3*(sqrt(3)*(b*c - a*d)*x - 2*sqrt(3)*(-b^
2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x))
 + 2*(3*b*c - 2*a*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x)
- (3*b*c - 2*a*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-...
```

3.685.6 Sympy [F]

$$\int \frac{x^3 (a+b x^3)^{2/3}}{c+d x^3} \, dx = \int \frac{x^3 (a+b x^3)^{\frac{2}{3}}}{c+d x^3} \, dx$$

input
```
integrate(x**3*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**3*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.685.7 Maxima [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}x^3}{dx^3+c}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)
```

3.685.8 Giac [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}x^3}{dx^3+c}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)
```

3.685.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^3\,(b\,x^3+a)^{2/3}}{d\,x^3+c}\,dx$$

input
```
int((x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
int((x^3*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

3.686 $\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx$

3.686.1 Optimal result

Integrand size = 21, antiderivative size = 233

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d} - \frac{(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d} - \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6c^{2/3}d} + \frac{(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}d} - \frac{b^{2/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d}$$

output

```
-1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(2/3)/d+1/2*(-a*d+b*c)^(2/3)*ln((-a*d+
b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d-1/2*b^(2/3)*ln(-b^(1/3)*x+
(b*x^3+a)^(1/3))/d+1/3*b^(2/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*
3^(1/2))/d*3^(1/2)-1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x
/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/d*3^(1/2)
```

3.686.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{4\sqrt{3}b^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+\frac{2\sqrt{-6+6i\sqrt{3}}(bc-ad)^{2/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}}\right.}{c^{2/3}}}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

output
```
(4*Sqrt[3]*b^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(
1/3))] + (2*Sqrt[-6 + (6*I)*Sqrt[3]]*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*
d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*
x^3)^(1/3))]])/c^(2/3) - 4*b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] -
((2*I)*(-I + Sqrt[3])*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I
*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/c^(2/3) + 2*b^(2/3)*Log[b^(2/3)*x^2
+ b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((1 + I*Sqrt[3])*(b*c
 - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c
- a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/
3)])/c^(2/3))/(12*d)
```

3.686.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {916, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}dx$$

$$\Big\downarrow \; 916$$

$$\frac{b\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \; 769$$

$$\frac{b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \text{901}$$

$$\frac{b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{d}$$

input
```
Int[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

output
```
-(((b*c - a*d)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(
1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^
(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)
^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d) + (b*(ArcTan[(1 + (2*b^(1/3)*x)
/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b
*x^3)^(1/3)]/(2*b^(1/3))))/d
```

3.686. $\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx$

3.686.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 916
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[b/d   Int[(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*
x^n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c -
a*d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

3.686.4 Maple [A] (verified)

Time = 4.78 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.45

method	result
pseudoelliptic	$\frac{\frac{b^{\frac{2}{3}}\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)c\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{2}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)(ad-bc)-\sqrt{3}\,b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\right.}{3b^{\frac{1}{3}}}\right.}{}$

input
```
int((b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(1/3)*(1/2*b^(2/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/
3)*x+(b*x^3+a)^(2/3))/x^2)*c*((a*d-b*c)/c)^(1/3)+ln((((a*d-b*c)/c)^(1/3)*x
+(b*x^3+a)^(1/3))/x)*(a*d-b*c)-3^(1/2)*b^(2/3)*arctan(1/3*3^(1/2)*(b^(1/3)
*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*c*((a*d-b*c)/c)^(1/3)-b^(2/3)*ln((-b^(1/3
)*x+(b*x^3+a)^(1/3))/x)*c*((a*d-b*c)/c)^(1/3)+(arctan(1/3*3^(1/2)*(((a*d-b
*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)-1/2*ln(((
(a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/
3))/x^2))*(a*d-b*c))/c/d
```

3.686.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(186) = 372$.

Time = 0.50 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx =$$

$$-\frac{2\sqrt{3}\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}(-b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}bx}{}\right.}{}$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(s
qrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*arctan(-
1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((
b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*
log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*log(-((-b^2)
^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2
) + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2
- 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d
```

3.686.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.686.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

3.686.8 Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

3.686.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(b\,x^3+a)^{2/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(2/3)/(c + d*x^3), x)
```

3.687 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^3\left(c+dx^3\right)}\,dx$

3.687.1 Optimal result

Integrand size = 24, antiderivative size = 169

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^3\left(c+dx^3\right)}\,dx=-\frac{\left(a+bx^3\right)^{2/3}}{2cx^2}+\frac{(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}}+\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{5/3}}-\frac{(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}}$$

output
```
-1/2*(b*x^3+a)^(2/3)/c/x^2+1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(5/3)-1/2*(-
a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)+1/3*
(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3
))*3^(1/2))/c^(5/3)*3^(1/2)
```

3.687.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.59 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \frac{-\frac{6c^{2/3}(a+bx^3)^{2/3}}{x^2} - 2\sqrt{-6+6i\sqrt{3}}(bc-ad)^{2/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+b}}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]
```

output
```
((-6*c^(2/3)*(a + b*x^3)^(2/3))/x^2 - 2*Sqrt[-6 + (6*I)*Sqrt[3]]*(b*c - a*
d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*
I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 2*(1 + I*Sqrt[3])*(b*c - a*d)^(
2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)
] - I*(-I + Sqrt[3])*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 -
 I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3]
)*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(5/3))
```

3.687.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {975, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}dx$$

$$\Big\downarrow \; 975$$

$$\frac{\int \frac{2(bc-ad)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{2c}-\frac{(a+bx^3)^{2/3}}{2cx^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{c}-\frac{(a+bx^3)^{2/3}}{2cx^2}$$

$$\Big\downarrow 901$$

$$\frac{(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{\left(a+bx^3\right)^{2/3}}{2cx^2}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]
```

output
```
-1/2*(a + b*x^3)^(2/3)/(c*x^2) + ((b*c - a*d)*(ArcTan[(1 + (2*(b*c - a*d)^
(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d
)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)
^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c
```

3.687.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

3.687.4 Maple [A] (verified)

Time = 5.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.31

method	result
pseudoelliptic	$\frac{-2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)(ad-bc)x^{2}-3\left(b\,x^{3}+a\right)^{\frac{2}{3}}c\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+x^{2}\left(-2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}c^{2}x^{2}}$

input
```
int((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/c)^(1/3)*(-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*
(a*d-b*c)*x^2-3*(b*x^3+a)^(2/3)*c*((a*d-b*c)/c)^(1/3)+x^2*(-2*arctan(1/3*3
^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^
(1/2)+ln(((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b
*x^3+a)^(2/3))/x^2))*(a*d-b*c))/c^2/x^2
```

3.687.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^3\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.687.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3\,(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^3\,(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**3/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**3*(c + d*x**3)), x)
```

3.687.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)
```

3.687.8 Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)
```

3.687.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^3\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^3*(c + d*x^3)), x)
```

3.688 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(c+dx^3\right)}\,dx$

3.688.1 Optimal result

Integrand size = 24, antiderivative size = 206

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(c+dx^3\right)}\,dx=-\frac{\left(a+bx^3\right)^{2/3}}{5cx^5}-\frac{(2bc-5ad)\left(a+bx^3\right)^{2/3}}{10ac^2x^2}-\frac{d(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}}-\frac{d(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{8/3}}+\frac{d(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}}$$

output
```
-1/5*(b*x^3+a)^(2/3)/c/x^5-1/10*(-5*a*d+2*b*c)*(b*x^3+a)^(2/3)/a/c^2/x^2-1
/6*d*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(8/3)+1/2*d*(-a*d+b*c)^(2/3)*ln((-a*d+
b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)-1/3*d*(-a*d+b*c)^(2/3)*arcta
n(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)*3^
(1/2)
```

3.688.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.86 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(c+dx^3\right)}\,dx = \frac{\frac{6c^{2/3}\left(a+bx^3\right)^{2/3}\left(-2ac-2bcx^3+5adx^3\right)}{ax^5}+10\sqrt{-6+6i\sqrt{3}}d(bc-ad)^{2/3}\arctan\left(\frac{3\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{bc-ad}x-(}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x]
```

output
```
((6*c^(2/3)*(a + b*x^3)^(2/3)*(-2*a*c - 2*b*c*x^3 + 5*a*d*x^3))/(a*x^5) +
10*Sqrt[-6 + (6*I)*Sqrt[3]]*d*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3
)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1
/3))] - (10*I)*(-I + Sqrt[3])*d*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*
x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + 5*(1 + I*Sqrt[3])*d*(b*c
- a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c -
 a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3
)])/(60*c^(8/3))
```

3.688.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {975, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{-3bdx^3+2bc-5ad}{x^3\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{5c}-\frac{\left(a+bx^3\right)^{2/3}}{5cx^5}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int \frac{10ad(bc-ad)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{2ac}-\frac{\left(a+bx^3\right)^{2/3}(2bc-5ad)}{2acx^2}}{5c}-\frac{\left(a+bx^3\right)^{2/3}}{5cx^5}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{5d(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{c}-\frac{\left(a+bx^3\right)^{2/3}(2bc-5ad)}{2acx^2}}{5c}-\frac{\left(a+bx^3\right)^{2/3}}{5cx^5}$$

$$\Big\downarrow \ 901$$

$$\frac{-\frac{5d(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{\left(a+bx^3\right)^{2/3}(2bc-5ad)}{2acx^2}}{5c}-\frac{\left(a+bx^3\right)^{2/3}}{5cx^5}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x]
```

output
```
-1/5*(a + b*x^3)^(2/3)/(c*x^5) + (-1/2*((2*b*c - 5*a*d)*(a + b*x^3)^(2/3))
/(a*c*x^2) - (5*d*(b*c - a*d)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3
)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c
 + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3
) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c)/(5*c)
```

3.688. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(c+dx^3\right)}\,dx$

3.688.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.688.4 Maple [A] (verified)

Time = 4.85 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.19

method	result
pseudoelliptic	$-\frac{-2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a(ad-bc)d\,x^{5}+\frac{6\left(\left(-\frac{5ad}{2}+bc\right)x^{3}+ac\right)c\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{5}+x^{5}da\left(-2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad}{\ldots}\right.\right.}{\ldots}\right.\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x^{5}c^{3}a}$

3.688. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(c+dx^3\right)}\,dx$

input
```
int((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/((a*d-b*c)/c)^(1/3)*(-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)
*a*(a*d-b*c)*d*x^5+6/5*((-5/2*a*d+b*c)*x^3+a*c)*c*(b*x^3+a)^(2/3)*((a*d-b*
c)/c)^(1/3)+x^5*d*a*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3
+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*
d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a*d-b*c))/x^5/c^
3/a
```

3.688.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.688.6 Sympy [F]

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(c+dx^3\right)}\,dx=\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^6\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**6/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**6*(c + d*x**3)), x)
```

3.688.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)
```

3.688.8 Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)
```

3.688.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^6\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^6*(c + d*x^3)), x)
```

3.689 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(c+dx^3\right)}\,dx$

3.689.1 Optimal result

Integrand size $=24$, antiderivative size $=257$

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(c+dx^3\right)}\,dx =& -\frac{\left(a+bx^3\right)^{2/3}}{8cx^8}-\frac{(bc-4ad)\left(a+bx^3\right)^{2/3}}{20ac^2x^5}\\ &+\frac{\left(3b^2c^2+8abcd-20a^2d^2\right)\left(a+bx^3\right)^{2/3}}{40a^2c^3x^2}+\frac{d^2(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{11/3}}\\ &+\frac{d^2(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{11/3}}-\frac{d^2(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}}\end{aligned}$$

output
```
-1/8*(b*x^3+a)^(2/3)/c/x^8-1/20*(-4*a*d+b*c)*(b*x^3+a)^(2/3)/a/c^2/x^5+1/4
0*(-20*a^2*d^2+8*a*b*c*d+3*b^2*c^2)*(b*x^3+a)^(2/3)/a^2/c^3/x^2+1/6*d^2*(-
a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(11/3)-1/2*d^2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)
^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(11/3)+1/3*d^2*(-a*d+b*c)^(2/3)*arctan
(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(11/3)*3^
(1/2)
```

3.689.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.25 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(c+dx^3\right)}\,dx = \frac{-\frac{3c^{2/3}\left(a+bx^3\right)^{2/3}\left(-3b^2c^2x^6+2abcx^3\left(c-4dx^3\right)+a^2\left(5c^2-8cdx^3+20d^2x^6\right)\right)}{a^2x^8}-20\sqrt{-6+6i\sqrt{3}}d^2(bc-ad}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
```

output
```
((-3*c^(2/3)*(a + b*x^3)^(2/3)*(-3*b^2*c^2*x^6 + 2*a*b*c*x^3*(c - 4*d*x^3)
 + a^2*(5*c^2 - 8*c*d*x^3 + 20*d^2*x^6)))/(a^2*x^8) - 20*Sqrt[-6 + (6*I)*S
qrt[3]]*d^2*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c
 - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 20*(1 + I*
Sqrt[3])*d^2*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])
*c^(1/3)*(a + b*x^3)^(1/3)] - (10*I)*(-I + Sqrt[3])*d^2*(b*c - a*d)^(2/3)*
Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x
*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(120*c^(1
1/3))
```

3.689.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {975, 27, 1053, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{2\left(-3bdx^3+bc-4ad\right)}{x^6\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{8c}-\frac{\left(a+bx^3\right)^{2/3}}{8cx^8}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-3bdx^3+bc-4ad}{x^6\sqrt[3]{bx^3+a(dx^3+c)}}dx}{4c}-\frac{\left(a+bx^3\right)^{2/3}}{8cx^8}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{-\frac{\int \frac{3bd(bc-4ad)x^3+3b^2c^2-20a^2d^2+8abcd}{x^3\sqrt[3]{bx^3+a\left(dx^3+c\right)}}dx}{5ac}-\frac{\left(a+bx^3\right)^{2/3}(bc-4ad)}{5acx^5}}{4c}-\frac{\left(a+bx^3\right)^{2/3}}{8cx^8}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{-\frac{-\frac{\int \frac{40a^2d^2(bc-ad)}{\sqrt[3]{bx^3+a\left(dx^3+c\right)}}dx}{2ac}-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{3b^2c}{a}-\frac{20ad^2}{c}+8bd\right)}{2x^2}}{5ac}-\frac{\left(a+bx^3\right)^{2/3}(bc-4ad)}{5acx^5}}{4c}-\frac{\left(a+bx^3\right)^{2/3}}{8cx^8}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{-\frac{20ad^2(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a\left(dx^3+c\right)}}dx}{c}-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{3b^2c}{a}-\frac{20ad^2}{c}+8bd\right)}{2x^2}}{5ac}-\frac{\left(a+bx^3\right)^{2/3}(bc-4ad)}{5acx^5}}{4c}-\frac{\left(a+bx^3\right)^{2/3}}{8cx^8}$$

$$\Big\downarrow \text{ 901}$$

$$\frac{-\frac{-\frac{20ad^2(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{3b^2c}{a}-\frac{20ad^2}{c}+8bd\right)}{2x^2}}{5ac}-}{4c}$$

$$\frac{\left(a+bx^3\right)^{2/3}}{8cx^8}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
```

output
```
-1/8*(a + b*x^3)^(2/3)/(c*x^8) + (-1/5*((b*c - 4*a*d)*(a + b*x^3)^(2/3))/(
a*c*x^5) - (-1/2*(((3*b^2*c)/a + 8*b*d - (20*a*d^2)/c)*(a + b*x^3)^(2/3))/
x^2 - (20*a*d^2*(b*c - a*d)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*
(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c +
 d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3)
- (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c)/(5*a*c))/(4*c)
```

3.689. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(c+dx^3\right)}\,dx$

3.689.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.689.4 Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{-3c\left(\left(4a^2d^2-\frac{8}{5}abcd-\frac{3}{5}b^2c^2\right)x^6+\frac{2\left(-4a^2cd+b\,c^2a\right)x^3}{5}+a^2c^2\right)\left(b\,x^3+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+4a^2d^2x^8(ad-bc)\left(-2\arctan\left(\frac{\sqrt{3}\left(\left(\right.\right.}{}\right.\right.}{24\left(\frac{ad-bc}{c}\right)}$

3.689. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(c+dx^3\right)}\,dx$

input
```
int((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/24/((a*d-b*c)/c)^(1/3)*(-3*c*((4*a^2*d^2-8/5*a*b*c*d-3/5*b^2*c^2)*x^6+2/
5*(-4*a^2*c*d+a*b*c^2)*x^3+a^2*c^2)*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)+4*
a^2*d^2*x^8*(a*d-b*c)*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x
^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((
a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)
/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))/x^8/c^4/a^2
```

3.689.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.689.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^9(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**9/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**9*(c + d*x**3)), x)
```

3.689.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9\,(c+dx^3)}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^9}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)
```

3.689.8 Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9\,(c+dx^3)}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^9}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)
```

3.689.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^9\,(c+dx^3)}\,dx=\int \frac{(b\,x^3+a)^{2/3}}{x^9\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^9*(c + d*x^3)), x)
```

3.690 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(c+dx^3\right)}\,dx$

3.690.1 Optimal result

Integrand size = 24, antiderivative size = 320

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(c+dx^3\right)}\,dx =& -\frac{\left(a+bx^3\right)^{2/3}}{11cx^{11}}-\frac{(2bc-11ad)\left(a+bx^3\right)^{2/3}}{88ac^2x^8}\\&+\frac{\left(6b^2c^2+11abcd-44a^2d^2\right)\left(a+bx^3\right)^{2/3}}{220a^2c^3x^5}\\&-\frac{\left(18b^3c^3+33ab^2c^2d+88a^2bcd^2-220a^3d^3\right)\left(a+bx^3\right)^{2/3}}{440a^3c^4x^2}\\&-\frac{d^3(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{14/3}}-\frac{d^3(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{14/3}}\\&+\frac{d^3(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{14/3}}\end{aligned}$$

output

```
-1/11*(b*x^3+a)^(2/3)/c/x^11-1/88*(-11*a*d+2*b*c)*(b*x^3+a)^(2/3)/a/c^2/x^
8+1/220*(-44*a^2*d^2+11*a*b*c*d+6*b^2*c^2)*(b*x^3+a)^(2/3)/a^2/c^3/x^5-1/4
40*(-220*a^3*d^3+88*a^2*b*c*d^2+33*a*b^2*c^2*d+18*b^3*c^3)*(b*x^3+a)^(2/3)
/a^3/c^4/x^2-1/6*d^3*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(14/3)+1/2*d^3*(-a*d+b
*c)^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(14/3)-1/3*d^3*
(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3
))*3^(1/2))/c^(14/3)*3^(1/2)
```

3.690.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.92 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.32

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(c+dx^3\right)}\,dx = \frac{\frac{3c^{2/3}\left(a+bx^3\right)^{2/3}\left(-18b^3c^3x^9+3ab^2c^2x^6\left(4c-11dx^3\right)-2a^2bcx^3\left(5c^2-11cdx^3+44d^2x^6\right)+a^3\left(-40c^3+55c^2dx^3-88cd^2x^6\right.\right.}{a^3x^{11}}}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

output
```
((3*c^(2/3)*(a + b*x^3)^(2/3)*(-18*b^3*c^3*x^9 + 3*a*b^2*c^2*x^6*(4*c - 11
*d*x^3) - 2*a^2*b*c*x^3*(5*c^2 - 11*c*d*x^3 + 44*d^2*x^6) + a^3*(-40*c^3 +
 55*c^2*d*x^3 - 88*c*d^2*x^6 + 220*d^3*x^9)))/(a^3*x^11) + 220*Sqrt[-6 + (
6*I)*Sqrt[3]]*d^3*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3
]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - (220
*I)*(-I + Sqrt[3])*d^3*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 +
I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + 110*(1 + I*Sqrt[3])*d^3*(b*c - a*d
)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)
^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(
1320*c^(14/3))
```

3.690.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {975, 1053, 27, 1053, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int \frac{-9bdx^3+2bc-11ad}{x^9\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{11c}-\frac{\left(a+bx^3\right)^{2/3}}{11cx^{11}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int \frac{2\left(3bd(2bc-11ad)x^3+6b^2c^2-44a^2d^2+11abcd\right)}{x^6\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{8ac}-\frac{\left(a+bx^3\right)^{2/3}(2bc-11ad)}{8acx^8}}{11c}-\frac{\left(a+bx^3\right)^{2/3}}{11cx^{11}}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\int \frac{3bd(2bc-11ad)x^3+6b^2c^2-44a^2d^2+11abcd}{x^6\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{4ac}-\frac{\left(a+bx^3\right)^{2/3}(2bc-11ad)}{8acx^8}}{11c}-\frac{\left(a+bx^3\right)^{2/3}}{11cx^{11}}$$

$$\Big\downarrow \ 1053$$

$$\frac{-\frac{-\frac{\int \frac{18b^3c^3+33ab^2dc^2+88a^2bd^2c-220a^3d^3+3bd\left(6b^2c^2+11abdc-44a^2d^2\right)x^3}{x^3\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{5ac}-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{6b^2c}{a}-\frac{44ad^2}{c}+11bd\right)}{5x^5}}{4ac}-\frac{\left(a+bx^3\right)^{2/3}(2bc-11ad)}{8acx^8}}{11c}-\frac{\left(a+bx^3\right)^{2/3}}{11cx^{11}}$$

$$\Big\downarrow \ 1053$$

$$\frac{-\frac{-\frac{-\frac{\int \frac{440a^3d^3(bc-ad)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{2ac}-\frac{\left(a+bx^3\right)^{2/3}\left(-220a^3d^3+88a^2bcd^2+33ab^2c^2d+18b^3c^3\right)}{2acx^2}}{5ac}-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{6b^2c}{a}-\frac{44ad^2}{c}+11bd\right)}{5x^5}}{4ac}-\frac{\left(a+bx^3\right)^{2/3}(2bc-11a}{8acx^8}}{11c}$$

$$\frac{\left(a+bx^3\right)^{2/3}}{11cx^{11}}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{-\frac{-\frac{220a^2d^3(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{c}-\frac{\left(a+bx^3\right)^{2/3}\left(-220a^3d^3+88a^2bcd^2+33ab^2c^2d+18b^3c^3\right)}{2acx^2}}{5ac}-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{6b^2c}{a}-\frac{44ad^2}{c}+11bd\right)}{5x^5}}{4ac}-(a+bx}{11c}$$

$$\frac{\left(a+bx^3\right)^{2/3}}{11cx^{11}}$$

$$\Big\downarrow \ 901$$

3.690. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(c+dx^3\right)}\,dx$

$$-\frac{-\frac{-\frac{\frac{220a^2d^3(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{\left(a+bx^3\right)^{2/3}\left(-220a^3d^3+88a^2bcd^2}{2acx^2}}{5ac}}{4ac}}{11c}$$

$$\frac{\left(a+bx^3\right)^{2/3}}{11cx^{11}}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

output
```
-1/11*(a + b*x^3)^(2/3)/(c*x^11) + (-1/8*((2*b*c - 11*a*d)*(a + b*x^3)^(2/
3))/(a*c*x^8) - (-1/5*(((6*b^2*c)/a + 11*b*d - (44*a*d^2)/c)*(a + b*x^3)^(
2/3))/x^5 - (-1/2*((18*b^3*c^3 + 33*a*b^2*c^2*d + 88*a^2*b*c*d^2 - 220*a^3
*d^3)*(a + b*x^3)^(2/3))/(a*c*x^2) - (220*a^2*d^3*(b*c - a*d)*(ArcTan[(1 +
 (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^
(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) -
Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a
*d)^(1/3))))/c)/(5*a*c))/(4*a*c))/(11*c)
```

3.690.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

3.690. $\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(c+dx^3\right)}\,dx$

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.690.4 Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{6\left(\left(b\,x^{3}+a\right)\left(\frac{9}{20}b^{2}x^{6}-\frac{3}{4}ab\,x^{3}+a^{2}\right)c^{3}-\frac{11x^{3}\left(-\frac{3b\,x^{3}}{5}+a\right)d\left(b\,x^{3}+a\right)a\,c^{2}}{8}+\frac{11\left(b\,x^{3}+a\right)a^{2}c\,d^{2}x^{6}}{5}-\frac{11a^{3}d^{3}x^{9}}{2}\right)c\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(\frac{ad-}{c}\right.}{}$

input
```
int((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/66/((a*d-b*c)/c)^(1/3)*(6*((b*x^3+a)*(9/20*b^2*x^6-3/4*a*b*x^3+a^2)*c^3
-11/8*x^3*(-3/5*b*x^3+a)*d*(b*x^3+a)*a*c^2+11/5*(b*x^3+a)*a^2*c*d^2*x^6-11
/2*a^3*d^3*x^9)*c*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)+11*a^3*d^3*x^11*(a*d
-b*c)*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a
*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3
)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln(((((a*d-b*c)/c)^(1/3)*x+(b*x
^3+a)^(1/3))/x)))/x^11/c^5/a^3
```

3.690.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.690.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx=\int \frac{(a+bx^3)^{\frac{2}{3}}}{x^{12}(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**12/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**12*(c + d*x**3)), x)
```

3.690.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^{12}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)
```

3.690.8 Giac [F]

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^{12}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)
```

3.690.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(c+dx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^{12}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^12*(c + d*x^3)), x)
```

3.691 $\int \frac{x^7\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

3.691.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx=\frac{x^8(a+bx^3)^{2/3}\operatorname{AppellF1}\left(\frac{8}{3},-\frac{2}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
1/8*x^8*(b*x^3+a)^(2/3)*AppellF1(8/3,-2/3,1,11/3,-b*x^3/a,-d*x^3/c)/c/(1+b
*x^3/a)^(2/3)
```

3.691.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(64) = 128.

Time = 8.42 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.83

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx=\frac{x^2\left(5c(a+bx^3)\left(-7bc+2ad+4bdx^3\right)+5ac(7bc-2ad)\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},1\right.\right.}{14}$$

input
```
Integrate[(x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(x^2*(5*c*(a + b*x^3)*(-7*b*c + 2*a*d + 4*b*d*x^3) + 5*a*c*(7*b*c - 2*a*d)
*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)] - 2*(-14*b^2*c^2 + 7*a*b*c*d + 2*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*Ap
pellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(140*b*c*d^2*(a + b
*x^3)^(1/3))
```

3.691.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \left(a+bx^3\right)^{2/3}}{c+dx^3} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\left(a+bx^3\right)^{2/3} \int \frac{x^7 \left(\frac{bx^3}{a}+1\right)^{2/3}}{dx^3+c}dx}{\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^8 \left(a+bx^3\right)^{2/3} \text{AppellF1}\left(\frac{8}{3},-\frac{2}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c \left(\frac{bx^3}{a}+1\right)^{2/3}}$$

input
```
Int[(x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(x^8*(a + b*x^3)^(2/3)*AppellF1[8/3, -2/3, 1, 11/3, -((b*x^3)/a), -((d*x^3
)/c)])/(8*c*(1 + (b*x^3)/a)^(2/3))
```

3.691.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.691.4 Maple [F]

$$\int \frac{x^{7} \left(b \,x^{3}+a \right)^{\frac{2}{3}}}{d \,x^{3}+c}d x$$

input
```
int(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

3.691.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^7 (a+bx^3)^{2/3}}{c+dx^3} \, dx = \text{Timed out}$$

input
```
integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.691.6 Sympy [F]

$$\int \frac{x^7 (a+b x^3)^{2/3}}{c+d x^3} \, dx = \int \frac{x^7 (a+b x^3)^{\frac{2}{3}}}{c+d x^3} \, dx$$

input
```
integrate(x**7*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**7*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.691.7 Maxima [F]

$$\int \frac{x^7 (a+b x^3)^{2/3}}{c+d x^3} \, dx = \int \frac{(b x^3+a)^{\frac{2}{3}} x^7}{d x^3+c} \, dx$$

input
```
integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^7/(d*x^3 + c), x)
```

3.691.8 Giac [F]

$$\int \frac{x^7 (a+b x^3)^{2/3}}{c+d x^3} \, dx = \int \frac{(b x^3+a)^{\frac{2}{3}} x^7}{d x^3+c} \, dx$$

input
```
integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^7/(d*x^3 + c), x)
```

3.691.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^7\,\left(b\,x^3+a\right)^{2/3}}{d\,x^3+c}\,dx$$

input
```
int((x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
int((x^7*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

3.692 $\int \frac{x^4\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

3.692.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx=\frac{x^5(a+bx^3)^{2/3}\operatorname{AppellF1}\left(\frac{5}{3},-\frac{2}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
1/5*x^5*(b*x^3+a)^(2/3)*AppellF1(5/3,-2/3,1,8/3,-b*x^3/a,-d*x^3/c)/c/(1+b*
x^3/a)^(2/3)
```

3.692.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(64) = 128.

Time = 8.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx=\frac{5cx^2(a+bx^3)-5acx^2\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2(-2bc+ad)x}{20cd\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(5*c*x^2*(a + b*x^3) - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3,
1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*(-2*b*c + a*d)*x^5*(1 + (b*x^3)/a)
^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*c*d*(a
+ b*x^3)^(1/3))
```

3.692.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\left(a+b x^3\right)^{2/3} \int \frac{x^4 \left(\frac{b x^3}{a}+1\right)^{2/3}}{d x^3+c}dx}{\left(\frac{b x^3}{a}+1\right)^{2/3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^5 \left(a+b x^3\right)^{2/3} \text{AppellF1}\left(\frac{5}{3},-\frac{2}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right)}{5 c \left(\frac{b x^3}{a}+1\right)^{2/3}}$$

input
```
Int[(x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(x^5*(a + b*x^3)^(2/3)*AppellF1[5/3, -2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)
/c)])/(5*c*(1 + (b*x^3)/a)^(2/3))
```

3.692.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.692.4 Maple [F]

$$\int \frac{x^{4} \left(b \,x^{3}+a \right)^{\frac{2}{3}}}{d \,x^{3}+c}d x$$

input
```
int(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

3.692.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^4 \left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx=\text{Timed out}$$

input
```
integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.692.6 Sympy [F]

$$\int \frac{x^4 \left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx=\int \frac{x^{4} \left(a + b x^{3}\right)^{\frac{2}{3}}}{c + d x^{3}}\, dx$$

input
```
integrate(x**4*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**4*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.692.7 Maxima [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}x^4}{dx^3+c}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^4/(d*x^3 + c), x)
```

3.692.8 Giac [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}x^4}{dx^3+c}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^4/(d*x^3 + c), x)
```

3.692.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^4\,(b\,x^3+a)^{2/3}}{d\,x^3+c}\,dx$$

input
```
int((x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
int((x^4*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

3.693 $\int \frac{x\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

3.693.1 Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx=\frac{x^2(a+bx^3)^{2/3}\operatorname{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
1/2*x^2*(b*x^3+a)^(2/3)*AppellF1(2/3,-2/3,1,5/3,-b*x^3/a,-d*x^3/c)/c/(1+b*
x^3/a)^(2/3)
```

3.693.2 Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx=\frac{x^2(a+bx^3)^{2/3}\operatorname{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(\frac{a+bx^3}{a}\right)^{2/3}}$$

input
```
Integrate[(x*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(x^2*(a + b*x^3)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*c*((a + b*x^3)/a)^(2/3))
```

3.693.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}dx$$

$$\Big\downarrow \textcolor{blue}{1013}$$

$$\frac{(a+bx^3)^{2/3}\int \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}}{dx^3+c}dx}{\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

$$\Big\downarrow \textcolor{blue}{1012}$$

$$\frac{x^2(a+bx^3)^{2/3}\operatorname{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

input
```
Int[(x*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

output
```
(x^2*(a + b*x^3)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*c*(1 + (b*x^3)/a)^(2/3))
```

3.693.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.693.4 Maple [F]

$$\int \frac{x(b\,x^3+a)^{\frac{2}{3}}}{d\,x^3+c}dx$$

input
```
int(x*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

3.693.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.693.6 Sympy [F]

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.693.7 Maxima [F]

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}x}{dx^3+c}\,dx$$

input
```
integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)*x/(d*x^3 + c), x)
```

3.693.8 Giac [F]

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}x}{dx^3+c}\,dx$$

input
```
integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*x/(d*x^3 + c), x)
```

3.693.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{x\,(b\,x^3+a)^{2/3}}{d\,x^3+c}\,dx$$

input
```
int((x*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

output
```
int((x*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

3.694 $\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx$

3.694.1 Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}\operatorname{AppellF1}\left(-\frac{1}{3},-\frac{2}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
-(b*x^3+a)^(2/3)*AppellF1(-1/3,-2/3,1,2/3,-b*x^3/a,-d*x^3/c)/c/x/(1+b*x^3/
a)^(2/3)
```

3.694.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 138 vs. 2(62) = 124.

Time = 10.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.23

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = \frac{-10c(a+bx^3)-5(-2bc+ad)x^3\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx^6}{10c^2x\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x]
```

output
```
(-10*c*(a + b*x^3) - 5*(-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2
/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^6*(1 + (b*x^3)/a)^(
1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*c^2*x*(a
+ b*x^3)^(1/3))
```

3.694.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{(a+bx^3)^{2/3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{2/3}}{x^2(dx^3+c)}dx}{\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{(a+bx^3)^{2/3}\operatorname{AppellF1}\left(-\frac{1}{3},-\frac{2}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x]
```

output
```
-(((a + b*x^3)^(2/3)*AppellF1[-1/3, -2/3, 1, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*x*(1 + (b*x^3)/a)^(2/3)))
```

3.694.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.694.4 Maple [F]

$$\int \frac{\left(b \,x^{3}+a \right)^{\frac{2}{3}}}{x^{2} \left(d \,x^{3}+c \right)}d x$$

input
```
int((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x)
```

3.694.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^2\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.694.6 Sympy [F]

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^2\left(c+dx^3\right)}\,dx=\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^2\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**2/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**2*(c + d*x**3)), x)
```

3.694.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2\left(c+dx^3\right)}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^2), x)
```

3.694.8 Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2\left(c+dx^3\right)}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^2), x)
```

3.694.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^2\left(c+dx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^2\,\left(d\,x^3+c\right)}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(2/3)/(x^2*(c + d*x^3)), x)
```

3.695 $\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx$

3.695.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}\operatorname{AppellF1}\left(-\frac{4}{3},-\frac{2}{3},1,-\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
-1/4*(b*x^3+a)^(2/3)*AppellF1(-4/3,-2/3,1,-1/3,-b*x^3/a,-d*x^3/c)/c/x^4/(1
+b*x^3/a)^(2/3)
```

3.695.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(64) = 128.

Time = 10.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.83

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx = \frac{-5c(a+bx^3)\left(2bcx^3+a(c-4dx^3)\right)+5(b^2c^2-4abcd+2a^2d^2)\,x^6\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{AppellF1}}{20ac^3x^4}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x]
```

output
```
(-5*c*(a + b*x^3)*(2*b*c*x^3 + a*(c - 4*d*x^3)) + 5*(b^2*c^2 - 4*a*b*c*d +
 2*a^2*d^2)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)
/a), -((d*x^3)/c)] + 2*b*d*(b*c - 2*a*d)*x^9*(1 + (b*x^3)/a)^(1/3)*AppellF
1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*a*c^3*x^4*(a + b*x^3)
^(1/3))
```

3.695.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}dx$$
$$\Big\downarrow \text{1013}$$
$$\frac{(a+bx^3)^{2/3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{2/3}}{x^5(dx^3+c)}dx}{\left(\frac{bx^3}{a}+1\right)^{2/3}}$$
$$\Big\downarrow \text{1012}$$
$$-\frac{(a+bx^3)^{2/3}\operatorname{AppellF1}\left(-\frac{4}{3},-\frac{2}{3},1,-\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

input
```
Int[(a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x]
```

output
```
-1/4*((a + b*x^3)^(2/3)*AppellF1[-4/3, -2/3, 1, -1/3, -((b*x^3)/a), -((d*x
^3)/c)])/(c*x^4*(1 + (b*x^3)/a)^(2/3))
```

3.695.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.695.4 Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{2}{3}}}{x^{5}\,(d\,x^{3}+c)}dx$$

input
```
int((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x)
```

3.695.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.695.6 Sympy [F]

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^5\left(c+dx^3\right)}\,dx=\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^5\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/x**5/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(x**5*(c + d*x**3)), x)
```

3.695.7 Maxima [F]

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^5\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{(dx^3+c)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^5), x)
```

3.695.8 Giac [F]

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^5\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{(dx^3+c)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^5), x)
```

3.695.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^5\,(d\,x^3+c)}\,dx$$

input

```
int((a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x)
```

output

```
int((a + b*x^3)^(2/3)/(x^5*(c + d*x^3)), x)
```

3.696 $\int \frac{x^8 \left(a+bx^3\right)^{4/3}}{c+dx^3} \, dx$

3.696.1 Optimal result

Integrand size = 24, antiderivative size = 251

$$\begin{aligned}\int \frac{x^8 (a+bx^3)^{4/3}}{c+dx^3} \, dx = & -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4}+\frac{c^2(a+bx^3)^{4/3}}{4d^3} \\ & -\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{c^2(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{13/3}} \\ & -\frac{c^2(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{13/3}}+\frac{c^2(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}}\end{aligned}$$

output
```
-c^2*(-a*d+b*c)*(b*x^3+a)^(1/3)/d^4+1/4*c^2*(b*x^3+a)^(4/3)/d^3-1/7*(a*d+b
*c)*(b*x^3+a)^(7/3)/b^2/d^2+1/10*(b*x^3+a)^(10/3)/b^2/d-1/6*c^2*(-a*d+b*c)
^(4/3)*ln(d*x^3+c)/d^(13/3)+1/2*c^2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d
^(1/3)*(b*x^3+a)^(1/3))/d^(13/3)-1/3*c^2*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*
d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(13/3)*3^(1/2)
```

3.696.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.23

$$\int \frac{x^8 (a+bx^3)^{4/3}}{c+dx^3} \, dx = \frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(-6a^3d^3+2a^2bd^2(-10c+dx^3)+ab^2d(175c^2-40cdx^3+22d^2x^6)+b^3(-140c^3+35c^2dx^3-20cd^2x^6-}{b^2}}{}$$

input
```
Integrate[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
((3*d^(1/3)*(a + b*x^3)^(1/3)*(-6*a^3*d^3 + 2*a^2*b*d^2*(-10*c + d*x^3) +
a*b^2*d*(175*c^2 - 40*c*d*x^3 + 22*d^2*x^6) + b^3*(-140*c^3 + 35*c^2*d*x^3
 - 20*c*d^2*x^6 + 14*d^3*x^9)))/b^2 - 140*Sqrt[3]*c^2*(b*c - a*d)^(4/3)*Ar
cTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 140*
c^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] -
 70*c^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3
)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(420*d^(13/3))
```

3.696.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 \left(a+bx^3\right)^{4/3}}{c+dx^3} dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{x^6 \left(bx^3+a\right)^{4/3}}{dx^3+c} dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3} \int \left(\frac{\left(bx^3+a\right)^{7/3}}{bd} + \frac{(-bc-ad)\left(bx^3+a\right)^{4/3}}{bd^2} + \frac{c^2 \left(bx^3+a\right)^{4/3}}{d^2 \left(dx^3+c\right)}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}c^2(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{13/3}}-\frac{3\left(a+bx^3\right)^{7/3}(ad+bc)}{7b^2d^2}+\frac{3\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{c^2(bc-ad)^{4/}}{2d}\right.$$

input

```
Int[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output

```
((-3*c^2*(b*c - a*d)*(a + b*x^3)^(1/3))/d^4 + (3*c^2*(a + b*x^3)^(4/3))/(4
*d^3) - (3*(b*c + a*d)*(a + b*x^3)^(7/3))/(7*b^2*d^2) + (3*(a + b*x^3)^(10
/3))/(10*b^2*d) - (Sqrt[3]*c^2*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a
 + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/d^(13/3) - (c^2*(b*c - a*d)^
(4/3)*Log[c + d*x^3])/(2*d^(13/3)) + (3*c^2*(b*c - a*d)^(4/3)*Log[(b*c - a
*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3)))/3
```

3.696.3.1 Defintions of rubi rules used

rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.696.4 Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{\frac{9\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} d\left(\left(-\frac{7b\,x^{3}}{3}+a\right)\left(b\,x^{3}+a\right)^{2} d^{3}+\frac{10bc\left(b\,x^{3}+a\right)^{2} d^{2}}{3}-\frac{175b^{2}\left(\frac{b\,x^{3}}{5}+a\right)c^{2}d}{6}+\frac{70b^{3}c^{3}}{3}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{35}+b^{2}c^{2}(ad-bc)^{2}\left(2\,\mathrm{arct}\right.}{6\left(\right.}$

input
```
int(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(1/d*(a*d-b*c))^(2/3)*(9/35*(1/d*(a*d-b*c))^(2/3)*d*((-7/3*b*x^3+a)*(
b*x^3+a)^2*d^3+10/3*b*c*(b*x^3+a)^2*d^2-175/6*b^2*(1/5*b*x^3+a)*c^2*d+70/3
*b^3*c^3)*(b*x^3+a)^(1/3)+b^2*c^2*(a*d-b*c)^2*(2*arctan(1/3*3^(1/2)*(2*(b*
x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x
^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2
*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/b^2/d^5
```

3.696.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.47

$$\int \frac{x^8\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx=\frac{140\sqrt{3}(b^3c^3-ab^2c^2d)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+70\,(}{}$$

input
```
integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/420*(140*sqrt(3)*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-
1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c -
 a*d))/(b*c - a*d)) + 70*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*lo
g((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c -
a*d)/d)^(2/3)) - 140*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*log((b
*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) + 3*(14*b^3*d^3*x^9 - 2*(10*b^3*
c*d^2 - 11*a*b^2*d^3)*x^6 - 140*b^3*c^3 + 175*a*b^2*c^2*d - 20*a^2*b*c*d^2
 - 6*a^3*d^3 + (35*b^3*c^2*d - 40*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^3)*(b*x^3 +
 a)^(1/3))/(b^2*d^4)
```

3.696.6 Sympy [F]

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{x^8(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**8*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**8*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.696.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.696.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.57

$$\int \frac{x^8 (a+bx^3)^{4/3}}{c+dx^3}\,dx =$$

$$-\frac{(b^{24}c^4d^6 - 2\,ab^{23}c^3d^7 + a^2b^{22}c^2d^8)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{23}cd^{10} - ab^{22}d^{11})}$$

$$+\frac{\sqrt{3}(bc^3 - ac^2d)(-bcd^2 + ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^5}$$

$$+\frac{(bc^3 - ac^2d)(-bcd^2 + ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^5}$$

$$-\frac{140\,(bx^3+a)^{\frac{1}{3}}b^{21}c^3d^6 - 35\,(bx^3+a)^{\frac{4}{3}}b^{20}c^2d^7 - 140\,(bx^3+a)^{\frac{1}{3}}ab^{20}c^2d^7 + 20\,(bx^3+a)^{\frac{7}{3}}b^{19}cd^8 - 14\,(bx^3+}{140\,b^{20}d^{10}}$$

input
```
integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b^24*c^4*d^6 - 2*a*b^23*c^3*d^7 + a^2*b^22*c^2*d^8)*(-(b*c - a*d)/d)
^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^23*c*d^10 -
 a*b^22*d^11) + 1/3*sqrt(3)*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*arc
tan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*
d)/d)^(1/3))/d^5 + 1/6*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x
^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d
)^(2/3))/d^5 - 1/140*(140*(b*x^3 + a)^(1/3)*b^21*c^3*d^6 - 35*(b*x^3 + a)^
(4/3)*b^20*c^2*d^7 - 140*(b*x^3 + a)^(1/3)*a*b^20*c^2*d^7 + 20*(b*x^3 + a)
^(7/3)*b^19*c*d^8 - 14*(b*x^3 + a)^(10/3)*b^18*d^9 + 20*(b*x^3 + a)^(7/3)*
a*b^18*d^9)/(b^20*d^10)
```

3.696.9 Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.90

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx = \left(\frac{a^2}{4\,b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{4\,b^2\,d}\right)\,{\left(b\,x^3+a\right)}^{4/3}$$

$$-\left(\frac{2\,a}{7\,b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{7\,b^4\,d^2}\right)\,{\left(b\,x^3+a\right)}^{7/3}+\frac{{\left(b\,x^3+a\right)}^{10/3}}{10\,b^2\,d}$$

$$+\frac{c^2\,\ln\left(\frac{3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4\right)}{d^2}-\frac{c^2\,{\left(a\,d-b\,c\right)}^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{13/3}}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{3\,d^{13/3}}$$

$$-\frac{\left(\frac{a^2}{b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{b^2\,d}\right)\,{\left(b\,x^3+a\right)}^{1/3}\,\left(b^3\,c-a\,b^2\,d\right)}{b^2\,d}$$

$$-\frac{c^2\,\ln\left(\frac{3\,c^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{7/3}}{d^{7/3}}+\frac{3\,c^2\,{\left(b\,x^3+a\right)}^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{3\,d^{13/3}}$$

$$+\frac{c^2\,\ln\left(\frac{3\,c^2\,{\left(b\,x^3+a\right)}^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d^2}-\frac{9\,c^2\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{7/3}}{d^{7/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{d^{13/3}}$$

input
```
int((x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

output
```
(a^2/(4*b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a
*b^2*d))/(4*b^2*d))*(a + b*x^3)^(4/3) - ((2*a)/(7*b^2*d) + (b^3*c - a*b^2*
d)/(7*b^4*d^2))*(a + b*x^3)^(7/3) + (a + b*x^3)^(10/3)/(10*b^2*d) + (c^2*l
og((3*(a + b*x^3)^(1/3)*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))/d^2 - (c^2*
(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(13/3)))*(a*d - b*c)^(4/3))/
(3*d^(13/3)) - ((a^2/(b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^
2))*(b^3*c - a*b^2*d))/(b^2*d))*(a + b*x^3)^(1/3)*(b^3*c - a*b^2*d))/(b^2*
d) - (c^2*log((3*c^2*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7/3))/d^(7/3) + (
3*c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^2)*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c)^(4/3))/(3*d^(13/3)) + (c^2*log((3*c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2
)/d^2 - (9*c^2*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(7/3))/d^(7/3))*((3^(1/2
)*1i)/6 - 1/6)*(a*d - b*c)^(4/3))/d^(13/3)
```

3.697 $\int \frac{x^5\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

3.697.1 Optimal result

Integrand size = 24, antiderivative size = 211

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3}-\frac{c(a+bx^3)^{4/3}}{4d^2}+\frac{(a+bx^3)^{7/3}}{7bd}+\frac{c(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}}+\frac{c(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}$$

output

```
c*(-a*d+b*c)*(b*x^3+a)^(1/3)/d^3-1/4*c*(b*x^3+a)^(4/3)/d^2+1/7*(b*x^3+a)^(
7/3)/b/d+1/6*c*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/d^(10/3)-1/2*c*(-a*d+b*c)^(4/3
)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(10/3)+1/3*c*(-a*d+b*c)^(
4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^
(10/3)*3^(1/2)
```

3.697.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.23

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(4a^2d^2+abd(-35c+8dx^3)+b^2(28c^2-7cdx^3+4d^2x^6))}{b}+28\sqrt{3}c(bc-ad)^{4/3}\arctan\Bigg(}{}$$

input

```
Integrate[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output

```
((3*d^(1/3)*(a + b*x^3)^(1/3)*(4*a^2*d^2 + a*b*d*(-35*c + 8*d*x^3) + b^2*(
28*c^2 - 7*c*d*x^3 + 4*d^2*x^6)))/b + 28*Sqrt[3]*c*(b*c - a*d)^(4/3)*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 28*c*(b
*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 14*c*
(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b
*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(84*d^(10/3))
```

3.697.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {948, 90, 60, 60, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+bx^3\right)^{4/3}}{c+dx^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3\left(bx^3+a\right)^{4/3}}{dx^3+c}dx^3$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{7/3}}{7bd}-\frac{c\int \frac{\left(bx^3+a\right)^{4/3}}{dx^3+c}dx^3}{d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{7/3}}{7bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\int\frac{\sqrt[3]{bx^3+a}}{dx^3+c}dx^3}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{7/3}}{7bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\int\frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{d}\right)}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{70}$$

3.697. $\int \frac{x^5\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{7/3}}{7bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int\frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}\right.}{d}\right.}{d}\right.}{d}\right.$$

↓ 16

3.697. $\int \frac{x^5\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{7/3}}{7bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int\frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}\right.}{d}\right.}{d}\right.}{d}\right.$$

$$\downarrow 1082$$

3.697. $\int\frac{x^5\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{7/3}}{7bd}-\frac{c\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int\frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log}{}\right.}{d}\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \; 217$$

3.697. $\int \frac{x^5\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

$$\frac{1}{3}\Bigg(\frac{3\left(a+bx^3\right)^{7/3}}{7bd}-\frac{c\Bigg(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\Bigg(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\Bigg(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log}{}}{d}}{d}}{d}$$

3.697. $\int \frac{x^5\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

input
```
Int[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
((3*(a + b*x^3)^(7/3))/(7*b*d) - (c*((3*(a + b*x^3)^(4/3))/(4*d) - ((b*c -
 a*d)*((3*(a + b*x^3)^(1/3))/d - ((b*c - a*d)*(-((Sqrt[3]*ArcTan[(1 - (2*d
^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(1/3)*(b*c - a*d
)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)^(2/3)) + (3*Log[(b*c - a
*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3)*(b*c - a*d)^(2/3))))/d)
)/d))/d)/3
```

3.697.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.697.4 Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{\frac{6\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} d\left(\left(b\,x^{3}+a\right)^{2} d^{2}-\frac{35 b\left(\frac{b\,x^{3}}{5}+a\right) c d}{4}+7 b^{2} c^{2}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{7}+bc(ad-bc)^{2}\Bigg(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{6\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} d^{4} b}$

input
```
int(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/(1/d*(a*d-b*c))^(2/3)*(6/7*(1/d*(a*d-b*c))^(2/3)*d*((b*x^3+a)^2*d^2-35
/4*b*(1/5*b*x^3+a)*c*d+7*b^2*c^2)*(b*x^3+a)^(1/3)+b*c*(a*d-b*c)^2*(2*arcta
n(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1
/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d
*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/d^4/b
```

3.697.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.41

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx=\frac{28\sqrt{3}(b^2c^2-abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+14\,(b^2c^2-}{}$$

input
```
integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/84*(28*sqrt(3)*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*
sqrt(3)*(b*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(
b*c - a*d)) + 14*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)
^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3))
- 28*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b
*c - a*d)/d)^(1/3)) + 3*(4*b^2*d^2*x^6 + 28*b^2*c^2 - 35*a*b*c*d + 4*a^2*d
^2 - (7*b^2*c*d - 8*a*b*d^2)*x^3)*(b*x^3 + a)^(1/3))/(b*d^3)
```

3.697.6 Sympy [F]

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{x^5(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**5*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**5*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.697.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.697.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(171) = 342$.

Time = 0.30 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.65

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{(b^{10}c^3d^4 - 2\,ab^9c^2d^5 + a^2b^8cd^6)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^9cd^7 - ab^8d^8)}$$
$$- \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc^2-acd)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^4}$$
$$- \frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc^2-acd)\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^4}$$
$$+ \frac{28\,(bx^3+a)^{\frac{1}{3}}b^8c^2d^4 - 7\,(bx^3+a)^{\frac{4}{3}}b^7cd^5 - 28\,(bx^3+a)^{\frac{1}{3}}ab^7cd^5 + 4\,(bx^3+a)^{\frac{7}{3}}b^6d^6}{28\,b^7d^7}$$

input
```
integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*(b^10*c^3*d^4 - 2*a*b^9*c^2*d^5 + a^2*b^8*c*d^6)*(-(b*c - a*d)/d)^(1/3
)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^9*c*d^7 - a*b^8*
d^8) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*arctan(1/3*sqr
t(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3)
)/d^4 - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*log((b*x^3 + a)^(2/3)
 + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^4
+ 1/28*(28*(b*x^3 + a)^(1/3)*b^8*c^2*d^4 - 7*(b*x^3 + a)^(4/3)*b^7*c*d^5 -
 28*(b*x^3 + a)^(1/3)*a*b^7*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^6*d^6)/(b^7*d^7)
```

3.697.9 Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.65

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{{\left(b\,x^3+a\right)}^{7/3}}{7\,b\,d}$$
$$-{\left(b\,x^3+a\right)}^{4/3}\,\left(\frac{a}{4\,b\,d}+\frac{b^2\,c-a\,b\,d}{4\,b^2\,d^2}\right)-\frac{c\,\ln\left(\frac{3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(a^2\,c\,d^2-2\,a\,b\,c^2\,d+b^2\,c^3\right)}{d}-\frac{c\,{\left(a\,d-b\,c\right)}^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{10/3}}\right)\,(a\,d -}{3\,d^{10/3}}$$

input

```
int((x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

output

```
(a + b*x^3)^(7/3)/(7*b*d) - (a + b*x^3)^(4/3)*(a/(4*b*d) + (b^2*c - a*b*d)
/(4*b^2*d^2)) - (c*log((3*(a + b*x^3)^(1/3)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c
^2*d))/d - (c*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(10/3)))*(a*d
- b*c)^(4/3))/(3*d^(10/3)) - (c*log((3*c*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/
d - (3*c*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/2)*1i)/
2 - 1/2)*(a*d - b*c)^(4/3))/(3*d^(10/3)) + (c*log((3*c*(a + b*x^3)^(1/3)*(
a*d - b*c)^2)/d + (3*c*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*
((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(4/3))/(3*d^(10/3)) + ((a + b*x^3)^(1/3
)*(b^2*c - a*b*d)*(a/(b*d) + (b^2*c - a*b*d)/(b^2*d^2)))/(b*d)
```

3.697. $\int \frac{x^5\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

3.698 $\int \frac{x^2\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

3.698.1 Optimal result

Integrand size = 24, antiderivative size = 187

$$\begin{aligned}\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx &= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2}+\frac{(a+bx^3)^{4/3}}{4d}\\ &\quad-\frac{(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{7/3}}\\ &\quad+\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}\end{aligned}$$

output
```
-(-a*d+b*c)*(b*x^3+a)^(1/3)/d^2+1/4*(b*x^3+a)^(4/3)/d-1/6*(-a*d+b*c)^(4/3)
*ln(d*x^3+c)/d^(7/3)+1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x
^3+a)^(1/3))/d^(7/3)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a
)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(7/3)*3^(1/2)
```

3.698.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.18

$$\int \frac{x^2 (a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(-4bc+5ad+bdx^3)-4\sqrt{3}(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) -}{}$$

input
```
Integrate[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(3*d^(1/3)*(a + b*x^3)^(1/3)*(-4*b*c + 5*a*d + b*d*x^3) - 4*Sqrt[3]*(b*c -
 a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/S
qrt[3]] + 4*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^
(1/3)] - 2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(
1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(12*d^(7/3))
```

3.698.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {946, 60, 60, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (a+bx^3)^{4/3}}{c+dx^3}dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{4/3}}{dx^3+c}dx^3$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3}\left(\frac{3(a+bx^3)^{4/3}}{4d}-\frac{(bc-ad)\int \frac{\sqrt[3]{bx^3+a}}{dx^3+c}dx^3}{d}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\int\frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{d}\right)}{d}\right)$$

$$\big\downarrow \ 70$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int\frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int\frac{}{\sqrt[3]{bc-}}}{} \right.}{d}\right.}{d}\right.$$

$$\big\downarrow \ 16$$

3.698. $\int \frac{x^2\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(c+dx^3}{2\sqrt[3]{d}(bc-ad}\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \textcolor{blue}{1082}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}\right.}{2\sqrt[3]{d}}\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \textcolor{blue}{217}$$

3.698. $\int \frac{x^2\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-a}\right.}{2\sqrt[3]{d}}\right.}{d}\right.}{d}\right.$$

input
```
Int[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
((3*(a + b*x^3)^(4/3))/(4*d) - ((b*c - a*d)*((3*(a + b*x^3)^(1/3))/d - ((b
*c - a*d)*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d
)^(1/3))/Sqrt[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3
)*(b*c - a*d)^(2/3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3
)])/(2*d^(1/3)*(b*c - a*d)^(2/3))))/d))/d)/3
```

3.698. $\int \frac{x^2\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

3.698.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.698. $\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx$

3.698.4 Maple [A] (verified)

Time = 5.01 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.09

method	result
pseudoelliptic	$\frac{\frac{5\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}d\left(\left(\frac{b\,x^{3}}{5}+a\right)d-\frac{4bc}{5}\right)}{4}-\frac{(ad-bc)^{2}\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)\right.\right.}{6}}{d^{3}\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}}$

input
```
int(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
5/4/(1/d*(a*d-b*c))^(2/3)*((1/d*(a*d-b*c))^(2/3)*(b*x^3+a)^(1/3)*d*((1/5*b
*x^3+a)*d-4/5*b*c)-2/15*(a*d-b*c)^2*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/
3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3
)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3
+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/d^3
```

3.698.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.32

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx=\frac{4\sqrt{3}(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+2\,(bc-ad)\left(\right.}{}$$

input
```
integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/12*(4*sqrt(3)*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*
(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a
*d)) + 2*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3
 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 4*(b*c - a*
d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3))
+ 3*(b*d*x^3 - 4*b*c + 5*a*d)*(b*x^3 + a)^(1/3))/d^2
```

3.698.6 Sympy [F]

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^2(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input

```
integrate(x**2*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output

```
Integral(x**2*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.698.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.698.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.59

$$\int \frac{x^2 (a+bx^3)^{4/3}}{c+dx^3}\,dx =$$
$$-\frac{(b^2c^2d^2-2\,abcd^3+a^2d^4)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bcd^4-ad^5)}$$
$$+\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^3}$$
$$+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^3}$$
$$-\frac{4\,(bx^3+a)^{\frac{1}{3}}bcd^2-(bx^3+a)^{\frac{4}{3}}d^3-4\,(bx^3+a)^{\frac{1}{3}}ad^3}{4\,d^4}$$

input
```
integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-(b*c - a*d)/d)^(1/3)*log(abs(
(b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^4 - a*d^5) + 1/3*sqrt(
3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^
(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^3 + 1/6*(-b*c*d^
2 + a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(
b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3 - 1/4*(4*(b*x^3 + a)^(1/
3)*b*c*d^2 - (b*x^3 + a)^(4/3)*d^3 - 4*(b*x^3 + a)^(1/3)*a*d^3)/d^4
```

3.698.9 Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.63

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{\left(b\,x^3+a\right)^{4/3}}{4\,d}$$

$$+\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\left(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2\right)-\frac{\left(a\,d-b\,c\right)^{4/3}\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\left(a\,d-b\,c\right)^{4/3}}{3\,d^{7/3}}$$

$$+\frac{\left(b\,x^3+a\right)^{1/3}\left(a\,d-b\,c\right)}{d^2}$$

$$-\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\left(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2\right)+\frac{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(a\,d-b\,c\right)^{4/3}\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(a\,d-b\,c\right)^{4/3}}{3\,d^{7/3}}$$

$$+\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\left(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2\right)-\frac{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(a\,d-b\,c\right)^{4/3}\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{d^{7/3}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(a\,d-b\,c\right)}{d^{7/3}}$$

input
```
int((x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

output
```
(a + b*x^3)^(4/3)/(4*d) + (log((a + b*x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*c^2 -
6*a*b*c*d) - ((a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*(a*d -
 b*c)^(4/3))/(3*d^(7/3)) + ((a + b*x^3)^(1/3)*(a*d - b*c))/d^2 - (log((a +
 b*x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*c^2 - 6*a*b*c*d) + (((3^(1/2)*1i)/2 + 1/2
)*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*((3^(1/2)*1i)/2 +
1/2)*(a*d - b*c)^(4/3))/(3*d^(7/3)) + (log((a + b*x^3)^(1/3)*(3*a^2*d^2 +
3*b^2*c^2 - 6*a*b*c*d) - (((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3)*(9*a*d^
3 - 9*b*c*d^2))/d^(7/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3))/d^(7/3)
```

3.699 $\int \frac{\left(a+bx^3\right)^{4/3}}{x\left(c+dx^3\right)}\,dx$

3.699.1 Optimal result

Integrand size = 24, antiderivative size = 261

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{4/3}}{x\left(c+dx^3\right)}\,dx &= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c} \\ &+ \frac{(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{4/3}} - \frac{a^{4/3}\log(x)}{2c} + \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}} \\ &+ \frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{4/3}}\end{aligned}$$

output

```
b*(b*x^3+a)^(1/3)/d-1/2*a^(4/3)*ln(x)/c+1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c
/d^(4/3)+1/2*a^(4/3)*ln(a^(1/3)-(b*x^3+a)^(1/3))/c-1/2*(-a*d+b*c)^(4/3)*ln
((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c/d^(4/3)-1/3*a^(4/3)*arctan(1/
3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/c*3^(1/2)+1/3*(-a*d+b*c)^(4
/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/d
^(4/3)*3^(1/2)
```

3.699.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx = \frac{6bc\sqrt[3]{d}\sqrt[3]{a+bx^3}-2\sqrt{3}a^{4/3}d^{4/3}\arctan\Bigg(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\Bigg)+2\sqrt{3}(bc-ad)^{4/3}\arctan\Bigg(}{}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]
```

output
```
(6*b*c*d^(1/3)*(a + b*x^3)^(1/3) - 2*Sqrt[3]*a^(4/3)*d^(4/3)*ArcTan[(1 + (
2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Sqrt[3]*(b*c - a*d)^(4/3)*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*a^(4/
3)*d^(4/3)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - 2*(b*c - a*d)^(4/3)*Log[(b*
c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - a^(4/3)*d^(4/3)*Log[a^(2/3)
+ a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + (b*c - a*d)^(4/3)*Log[(
b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(
a + b*x^3)^(2/3)])/(6*c*d^(4/3))
```

3.699.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {948, 95, 174, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{4/3}}{x^3(dx^3+c)}dx^3$$

$$\Big\downarrow \; 95$$

$$\frac{1}{3}\left(\frac{\int \frac{a^2d-b(bc-2ad)x^3}{x^3(bx^3+a)^{2/3}(dx^3+c)}dx^3}{d}+\frac{3b\sqrt[3]{a+bx^3}}{d}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(\frac{\frac{a^2 d\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3}{c}-\frac{(bc-ad)^2\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{c}}{d}+\frac{3b\sqrt[3]{a+bx^3}}{d}\right)$$

$$\Big\downarrow \; 69$$

$$\frac{1}{3}\left(\frac{\frac{a^2d\left(-\frac{3\int\frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{(bc-ad)^2\int\frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}}{c}}{d}\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(\frac{\frac{a^2d\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{(bc-ad)^2\int\frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{c}}{d}+3\right.$$

$$\Big\downarrow \; 70$$

$$\frac{1}{3}\left(\frac{\frac{a^2d\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-(bc-ad)^2\left(\frac{3\int\frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\sqrt[3]{bc-a}}}{2d^{2/3}\sqrt[3]{b}}\right.}{d}\right.$$

$$\Big\downarrow \; 16$$

3.699. $\int \frac{\left(a+bx^3\right)^{4/3}}{x\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(\frac{\frac{a^2d\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{(bc-ad)^2\left(\frac{3\int\frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-a}}{}}}{2d^{2/3}\sqrt[3]{b}}\right.}{}}{d}\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}\left(\frac{\frac{a^2d\left(\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{(bc-ad)^2\left(\frac{3\int\frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log(}{2\sqrt[3]{d}(}\right.}{c}}{d}\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(\frac{\frac{a^2d\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{(bc-ad)^2\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log(}{2\sqrt[3]{d}(}\right.}{c}}{d}\right.$$

3.699. $\int \frac{\left(a+bx^3\right)^{4/3}}{x\left(c+dx^3\right)}\,dx$

input
```
Int[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]
```

output
```
((3*b*(a + b*x^3)^(1/3))/d + ((a^2*d*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3
)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/
3) - (a + b*x^3)^(1/3)])/(2*a^(2/3))))/c - ((b*c - a*d)^2*(-((Sqrt[3]*ArcT
an[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(1/3
)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)^(2/3)) + (3*
Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3)*(b*c - a*d)
^(2/3))))/c)/d)/3
```

3.699.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

3.699. $\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.699.4 Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{-3\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}bcd+\frac{d^{2}\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln}{2}}{}$

input
```
int((b*x^3+a)^(4/3)/x/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(1/d*(a*d-b*c))^(2/3)*(-3*(b*x^3+a)^(1/3)*(1/d*(a*d-b*c))^(2/3)*b*c*d
+1/2*d^2*(1/d*(a*d-b*c))^(2/3)*(2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a
^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3)
)-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*a^(4/3)-1/2*(a*d-b*c)^2*(2*arctan(1/3*3^(
1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1
/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c
))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/c/d^2
```

3.699.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx =$$

$$-\frac{2\sqrt{3}a^{\frac{4}{3}}d\arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+a^{\frac{4}{3}}d\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2a^{\frac{4}{3}}d\log\bigl((bx^3+a}{}$$

input
```
integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*a^(4/3)*d*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3)
+ sqrt(3)*a)/a) + a^(4/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1
/3) + a^(2/3)) - 2*a^(4/3)*d*log((b*x^3 + a)^(1/3) - a^(1/3)) - 2*sqrt(3)*
(b*c - a*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)
*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 6*(b*x^3 +
a)^(1/3)*b*c - (b*c - a*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (
b*x^3 + a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) + 2*(b*c -
 a*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3))
)/(c*d)
```

3.699.6 Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x*(c + d*x**3)), x)
```

3.699.7 Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x), x)
```

3.699.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.37

$$\begin{aligned}\int \frac{(a+bx^3)^{4/3}}{x\,(c+dx^3)}\,dx = & -\frac{\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,c} \\ & - \frac{a^{\frac{4}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,c} + \frac{a^{\frac{4}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,c} \\ & + \frac{(b^2c^2-2\,abcd+a^2d^2)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2d-acd^2)} \\ & + \frac{(bx^3+a)^{\frac{1}{3}}b}{d} - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd^2} \\ & - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,cd^2}\end{aligned}$$

input
```
integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^
(1/3))/c - 1/6*a^(4/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) +
 a^(2/3))/c + 1/3*a^(4/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/c + 1/3*(b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(
1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^2*d - a*c*d^2) + (b*x^3 + a)^(1/3)*b/
d - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2
*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c*d^
2) - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x
^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c*d^2)
```

3.699.9 Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 796, normalized size of antiderivative = 3.05

$$\int \frac{(a+bx^3)^{4/3}}{x\,(c+dx^3)}\,dx = \ln\left(c\,d\left(-\frac{(a\,d-b\,c)^4}{c^3\,d^4}\right)^{1/3} + a\,d\,\left(b\,x^3+a\right)^{1/3}\right.$$

$$\left.-b\,c\,\left(b\,x^3+a\right)^{1/3}\right)\left(-\frac{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}{27\,c^3\,d^4}\right)^{1/3}+\ln\left(c\left(\frac{a^4}{c^3}\right)^{1/3}-a\,\left(b\,x^3+\right.\right.$$

input
```
int((a + b*x^3)^(4/3)/(x*(c + d*x^3)),x)
```

output

```
log(c*d*(-(a*d - b*c)^4/(c^3*d^4))^(1/3) + a*d*(a + b*x^3)^(1/3) - b*c*(a
+ b*x^3)^(1/3))*(-(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d -
 4*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3) + log(c*(a^4/c^3)^(1/3) - a*(a + b*x^3
)^(1/3))*(a^4/(27*c^3))^(1/3) + (b*(a + b*x^3)^(1/3))/d - log(c*(a^4/c^3)^
(1/3) + 2*a*(a + b*x^3)^(1/3) + 3^(1/2)*c*(a^4/c^3)^(1/3)*1i)*((3^(1/2)*1i
)/2 + 1/2)*(a^4/(27*c^3))^(1/3) + log(c*(a^4/c^3)^(1/3)*1i + a*(a + b*x^3)
^(1/3)*2i + 3^(1/2)*c*(a^4/c^3)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a^4/(27*c^3
))^(1/3) + log((3*a^2*b^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(2*a^4*d^4 + b^4
*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + 3*a^2*b^4*c
*((3^(1/2)*1i)/2 - 1/2)*(-(a*d - b*c)^4/(c^3*d^4))^(1/3)*(2*a^5*d^5 - b^5*
c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 6*a^4*b*c*
d^4))*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*
a*b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3) - log((3*a^2*b^4*(a + b*x
^3)^(1/3)*(a*d - b*c)^2*(2*a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3
*c^3*d - 4*a^3*b*c*d^3))/d - 3*a^2*b^4*c*((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b
*c)^4/(c^3*d^4))^(1/3)*(2*a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*
b^2*c^2*d^3 + 5*a*b^4*c^4*d - 6*a^4*b*c*d^4))*((3^(1/2)*1i)/2 + 1/2)*(-(a^
4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c
^3*d^4))^(1/3)
```

3.699. $\int \frac{\left(a+bx^3\right)^{4/3}}{x(c+dx^3)}\,dx$

3.700 $\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx$

3.700.1 Optimal result

Integrand size = 24, antiderivative size = 399

$$\begin{aligned}
\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx &= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d(a+bx^3)^{4/3}}{4c^2} \\
&+ \frac{(4bc-3ad)\,(a+bx^3)^{4/3}}{12ac^2} - \frac{(a+bx^3)^{7/3}}{3acx^3} - \frac{\sqrt[3]{a}(4bc-3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}c^2} \\
&- \frac{(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2\sqrt[3]{d}} - \frac{\sqrt[3]{a}(4bc-3ad)\log(x)}{6c^2} \\
&- \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6c^2\sqrt[3]{d}} + \frac{\sqrt[3]{a}(4bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6c^2} \\
&+ \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{d}}
\end{aligned}$$

output
```
1/3*(-3*a*d+4*b*c)*(b*x^3+a)^(1/3)/c^2-(-a*d+b*c)*(b*x^3+a)^(1/3)/c^2+1/4*
d*(b*x^3+a)^(4/3)/c^2+1/12*(-3*a*d+4*b*c)*(b*x^3+a)^(4/3)/a/c^2-1/3*(b*x^3
+a)^(7/3)/a/c/x^3-1/6*a^(1/3)*(-3*a*d+4*b*c)*ln(x)/c^2-1/6*(-a*d+b*c)^(4/3
)*ln(d*x^3+c)/c^2/d^(1/3)+1/6*a^(1/3)*(-3*a*d+4*b*c)*ln(a^(1/3)-(b*x^3+a)^
(1/3))/c^2+1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3
))/c^2/d^(1/3)-1/9*a^(1/3)*(-3*a*d+4*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^
(1/3))/a^(1/3)*3^(1/2))/c^2*3^(1/2)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d
^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/d^(1/3)*3^(1/2)
```

3.700.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx = \frac{-\frac{6ac\sqrt[3]{a+bx^3}}{x^3}+2\sqrt{3}\sqrt[3]{a}(-4bc+3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{6\sqrt{3}(bc-ad)^{4/3}\arctan\left(\frac{1-}{}\right)}{\sqrt[3]{}}}{}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]
```

output
```
((-6*a*c*(a + b*x^3)^(1/3))/x^3 + 2*Sqrt[3]*a^(1/3)*(-4*b*c + 3*a*d)*ArcTa
n[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - (6*Sqrt[3]*(b*c - a*d)^(4
/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/d^(1/3) - 2*a^(1/3)*(-4*b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] +
(6*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/d
^(1/3) + a^(1/3)*(-4*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3)
+ (a + b*x^3)^(2/3)] - (3*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3
)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/d^(1/3
))/(18*c^2)
```

3.700.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {948, 114, 27, 174, 60, 60, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^4\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)^{4/3}}{x^6\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \text{ 114}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{\left(bx^3+a\right)^{4/3}\left(4bdx^3+4bc-3ad\right)}{3x^3\left(dx^3+c\right)}dx^3}{ac}-\frac{\left(a+bx^3\right)^{7/3}}{acx^3}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{\int \frac{\left(bx^3+a\right)^{4/3}\left(4bdx^3+4bc-3ad\right)}{x^3\left(dx^3+c\right)}dx^3}{3ac}-\frac{\left(a+bx^3\right)^{7/3}}{acx^3}\right)$$

$$\Big\downarrow \text{ 174}$$

$$\frac{1}{3}\left(\frac{\frac{3ad^2\int \frac{\left(bx^3+a\right)^{4/3}}{dx^3+c}dx^3}{c}+\frac{(4bc-3ad)\int \frac{\left(bx^3+a\right)^{4/3}}{x^3}dx^3}{c}}{3ac}-\frac{\left(a+bx^3\right)^{7/3}}{acx^3}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}\left(\frac{\frac{3ad^2\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\int \frac{\sqrt[3]{bx^3+a}}{dx^3+c}dx^3}{d}\right)}{c}+\frac{(4bc-3ad)\left(a\int \frac{\sqrt[3]{bx^3+a}}{x^3}dx^3+\frac{3}{4}\left(a+bx^3\right)^{4/3}\right)}{c}}{3ac}-\frac{\left(a+bx^3\right)^{7/3}}{acx^3}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}\left(\frac{\frac{3ad^2\left(\frac{3\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\int\frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{d}\right)}{d}\right)}{c}+\frac{(4bc-3ad)\left(a\left(a\int\frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3+3\sqrt[3]{a+bx}\right.\right.}{c}}{3ac}\right.$$

$$\Big\downarrow \ 69$$

$$\frac{1}{3}\left(\frac{\frac{(4bc-3ad)\left(a\left(a\left(-\frac{3\int\frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+\frac{3}{4}(\right.}{c}}{3ac}\right.$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\left(\frac{\frac{(4bc-3ad)\left(a\left(a\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+\frac{3}{4}\left(a+bx^3\right)^{4/3}\right.}{c}}{3ac}\right.$$

$$\Big\downarrow \ 70$$

3.700. $\int \frac{\left(a+bx^3\right)^{4/3}}{x^4\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(\frac{\frac{(4bc-3ad)\left(a\left(a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+\frac{3}{4}\left(a+bx^3\right)^{4/3}}{}}{c}\right.$$

$$\downarrow 16$$

3.700. $\int \frac{\left(a+bx^3\right)^{4/3}}{x^4\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\Bigg(\frac{(4bc-3ad)\Bigg(a\Bigg(a\Bigg(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\Bigg)+3\sqrt[3]{a+bx^3}\Bigg)+\frac{3}{4}\left(a+bx^3\right)^{4/3}}{c}$$

↓ 1082

3.700. $\int\frac{\left(a+bx^3\right)^{4/3}}{x^4\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(\frac{(4bc-3ad)\left(a\left(a\left(\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+\frac{3}{4}\left(a+bx^3\right)^{4/3}\right)}{c}+3ad^2\left(\right.\right.$$

$$\Bigg\downarrow \text{217}$$

3.700. $\int\frac{\left(a+bx^3\right)^{4/3}}{x^4\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(\frac{(4bc-3ad)\left(a\left(a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)+\frac{3}{4}\left(a+bx^3\right)^{4/3}\right)}{c}+\frac{3ad^2\left(\frac{3}{}\right.}{}\right.$$

3.700. $\int \frac{\left(a+bx^3\right)^{4/3}}{x^4\left(c+dx^3\right)}\,dx$

input

```
Int[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]
```

output

```
(-((a + b*x^3)^(7/3)/(a*c*x^3)) + (((4*b*c - 3*a*d)*((3*(a + b*x^3)^(4/3))
/4 + a*(3*(a + b*x^3)^(1/3) + a*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/
3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) -
(a + b*x^3)^(1/3)])/(2*a^(2/3))))))/c + (3*a*d^2*((3*(a + b*x^3)^(4/3))/(4
*d) - ((b*c - a*d)*((3*(a + b*x^3)^(1/3))/d - ((b*c - a*d)*(-((Sqrt[3]*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(1/
3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)^(2/3)) + (3
*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3)*(b*c - a*d
)^(2/3))))/d))/d))/c)/(3*a*c))/3
```

3.700.3.1 Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.700. $\int \frac{\left(a+bx^3\right)^{4/3}}{x^4\left(c+dx^3\right)}\,dx$

3.700.4 Maple [A] (verified)

Time = 5.04 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$\frac{-\frac{x^{3}(ad-bc)^{2}\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)}{2}-x^{3}\sqrt{3}\,(ad-bc)^{2}\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)+}{}$

input
```
int((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-1/2*x^3*(a*d-b*c)^2*ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+
a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-x^3*3^(1/2)*(a*d-b*c)^2*arctan(1/3*3^(1/2)
*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))+1/2*x^3*
(-4/3*a^(1/3)*b*c+d*a^(4/3))*d*(1/d*(a*d-b*c))^(2/3)*ln((b*x^3+a)^(2/3)+a^
(1/3)*(b*x^3+a)^(1/3)+a^(2/3))+x^3*(a*d-b*c)^2*ln((b*x^3+a)^(1/3)-(1/d*(a*
d-b*c))^(1/3))-d*(1/d*(a*d-b*c))^(2/3)*(-x^3*(-4/3*a^(1/3)*b*c+d*a^(4/3))*
3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))+x^3*(-4/3*
a^(1/3)*b*c+d*a^(4/3))*ln((b*x^3+a)^(1/3)-a^(1/3))+(b*x^3+a)^(1/3)*a*c))/(
1/d*(a*d-b*c))^(2/3)/c^2/d/x^3
```

3.700.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+bx^{3}\right)^{4/3}}{x^{4}\left(c+dx^{3}\right)}\,dx=\frac{6\sqrt{3}(bc-ad)x^{3}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^{3}+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+2\sqrt{3}(4\,bc-}{}$$

input
```
integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/18*(6*sqrt(3)*(b*c - a*d)*x^3*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt
(3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c
 - a*d)) + 2*sqrt(3)*(4*b*c - 3*a*d)*(-a)^(1/3)*x^3*arctan(1/3*(2*sqrt(3)*
(b*x^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + (4*b*c - 3*a*d)*(-a)^(1/3)*
x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 3
*(b*c - a*d)*x^3*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a
)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 2*(4*b*c - 3*a*
d)*(-a)^(1/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*(b*c - a*d)*x^3*
(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 6
*(b*x^3 + a)^(1/3)*a*c)/(c^2*x^3)
```

3.700.6 Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^4\,(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^4\,(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x**4/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x**4*(c + d*x**3)), x)
```

3.700.7 Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^4\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^4}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^4), x)
```

3.700.8 Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.99

$$
\begin{aligned}
\int \frac{(a+bx^3)^{4/3}}{x^4\,(c+dx^3)}\,dx =
&-\frac{(b^2c^2-2\,abcd+a^2d^2)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3-ac^2d)} \\
&-\frac{\sqrt{3}\left(4\,a^{\frac{1}{3}}bc-3\,a^{\frac{4}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,c^2} \\
&-\frac{\left(4\,a^{\frac{1}{3}}bc-3\,a^{\frac{4}{3}}d\right)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,c^2} \\
&+\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^2d} \\
&+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^2d} \\
&+\frac{(4\,abc-3\,a^2d)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{2}{3}}c^2}-\frac{(bx^3+a)^{\frac{1}{3}}a}{3\,cx^3}
\end{aligned}
$$

input

```
integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

output

```
-1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3
 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^3 - a*c^2*d) - 1/9*sqrt(3)*(4*
a^(1/3)*b*c - 3*a^(4/3)*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/
3))/a^(1/3))/c^2 - 1/18*(4*a^(1/3)*b*c - 3*a^(4/3)*d)*log((b*x^3 + a)^(2/3
) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/c^2 + 1/3*sqrt(3)*(-b*c*d^2 + a*d
^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a
*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/
3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^
(1/3) + (-(b*c - a*d)/d)^(2/3))/(c^2*d) + 1/9*(4*a*b*c - 3*a^2*d)*log(abs(
(b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*c^2) - 1/3*(b*x^3 + a)^(1/3)*a/(c*x
^3)
```

3.700.9 Mupad [B] (verification not implemented)

Time = 14.72 (sec) , antiderivative size = 2047, normalized size of antiderivative = 5.13

$$\int \frac{(a+bx^3)^{4/3}}{x^4\,(c+dx^3)}\,dx = \text{Too large to display}$$

input

```
int((a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x)
```

output

```
log(c^2*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3) + 3*a*d*(a + b*x^3)^(1/3) - 4*b
*c*(a + b*x^3)^(1/3))*(-(27*a^4*d^3 - 64*a*b^3*c^3 + 144*a^2*b^2*c^2*d - 1
08*a^3*b*c*d^2)/(729*c^6))^(1/3) + log((((81*a*b^4*c^4*d^3*(2*a^2*d^2 + b
^2*c^2 - 3*a*b*c*d)*((a*d - b*c)^4/(c^6*d))^(1/3) - 108*a*b^5*c^3*d^3*(a +
 b*x^3)^(1/3)*(a*d - b*c)^2)*((a*d - b*c)^4/(c^6*d))^(2/3))/9 + (a*b^5*d^2
*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 16
2*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3
- (a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 38
8*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^
4))/(9*c^4))*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a
^3*b*c*d^3)/(27*c^6*d))^(1/3) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i
)/2 + 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 - 81*a*b^4*c
^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((a*d - b*
c)^4/(c^6*d))^(1/3))*((a*d - b*c)^4/(c^6*d))^(2/3))/9 + (a*b^5*d^2*(27*a^5
*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^4*
c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3 - (a*b^4
*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^
3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c
^4))*((3^(1/2)*1i)/2 - 1/2)*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*
b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c^6*d))^(1/3) - log((a*b^4*d^2*(a + b*x^...
```

3.701 $\int \frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}\,dx$

3.701.1 Optimal result

Integrand size = 24, antiderivative size = 440

$$\begin{aligned}
\int \frac{(a+bx^3)^{4/3}}{x^7\left(c+dx^3\right)}\,dx &= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} \\
&+ \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} \\
&- \frac{\left(a+bx^3\right)^{7/3}}{6acx^6} - \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}c^3} \\
&+ \frac{d^{2/3}(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3} - \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\log(x)}{18a^{2/3}c^3} \\
&+ \frac{d^{2/3}(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{2/3}c^3} \\
&- \frac{d^{2/3}(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}
\end{aligned}$$

output
```
d*(-a*d+b*c)*(b*x^3+a)^(1/3)/c^3+1/9*(9*a^2*d^2-12*a*b*c*d+2*b^2*c^2)*(b*x
^3+a)^(1/3)/a/c^3-1/18*(-6*a*d+b*c)*(b*x^3+a)^(4/3)/a/c^2/x^3-1/6*(b*x^3+a
)^(7/3)/a/c/x^6-1/18*(9*a^2*d^2-12*a*b*c*d+2*b^2*c^2)*ln(x)/a^(2/3)/c^3+1/
6*d^(2/3)*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^3+1/18*(9*a^2*d^2-12*a*b*c*d+2*b^
2*c^2)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(2/3)/c^3-1/2*d^(2/3)*(-a*d+b*c)^(4/3
)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c^3-1/27*(9*a^2*d^2-12*a*b*
c*d+2*b^2*c^2)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(
2/3)/c^3*3^(1/2)+1/3*d^(2/3)*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x
^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^3*3^(1/2)
```

3.701.2 Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx = \frac{\frac{3c\sqrt[3]{a+bx^3}(-3ac-7bcx^3+6adx^3)}{x^6} - \frac{2\sqrt{3}(2b^2c^2-12abcd+9a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + 18\sqrt{3}d^{2/3}}{}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]
```

output
```
((3*c*(a + b*x^3)^(1/3)*(-3*a*c - 7*b*c*x^3 + 6*a*d*x^3))/x^6 - (2*Sqrt[3]
*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^
(1/3))/Sqrt[3]])/a^(2/3) + 18*Sqrt[3]*d^(2/3)*(b*c - a*d)^(4/3)*ArcTan[(1
- (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + (2*(2*b^2*c^
2 - 12*a*b*c*d + 9*a^2*d^2)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(2/3) - 1
8*d^(2/3)*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1
/3)] + ((-2*b^2*c^2 + 12*a*b*c*d - 9*a^2*d^2)*Log[a^(2/3) + a^(1/3)*(a + b
*x^3)^(1/3) + (a + b*x^3)^(2/3)])/a^(2/3) + 9*d^(2/3)*(b*c - a*d)^(4/3)*Lo
g[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3
)*(a + b*x^3)^(2/3)])/(54*c^3)
```

3.701.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {948, 114, 27, 166, 27, 174, 60, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)^{4/3}}{x^9\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \ 114$$

$$\frac{1}{3}\left(-\frac{\int -\frac{\left(bx^3+a\right)^{4/3}\left(bdx^3+bc-6ad\right)}{3x^6\left(dx^3+c\right)}dx^3}{2ac}-\frac{\left(a+bx^3\right)^{7/3}}{2acx^6}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\frac{\int \frac{\left(bx^3+a\right)^{4/3}\left(bdx^3+bc-6ad\right)}{x^6\left(dx^3+c\right)}dx^3}{6ac}-\frac{\left(a+bx^3\right)^{7/3}}{2acx^6}\right)$$

$$\Big\downarrow \ 166$$

$$\frac{1}{3}\left(\frac{\frac{\int \frac{2\sqrt[3]{bx^3+a}\left(bd(2bc-3ad)x^3+2b^2c^2+9a^2d^2-12abcd\right)}{3x^3\left(dx^3+c\right)}dx^3}{c}-\frac{\left(a+bx^3\right)^{4/3}(bc-6ad)}{cx^3}}{6ac}-\frac{\left(a+bx^3\right)^{7/3}}{2acx^6}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\frac{\frac{2\int \frac{\sqrt[3]{bx^3+a}\left(bd(2bc-3ad)x^3+2b^2c^2+9a^2d^2-12abcd\right)}{x^3\left(dx^3+c\right)}dx^3}{3c}-\frac{\left(a+bx^3\right)^{4/3}(bc-6ad)}{cx^3}}{6ac}-\frac{\left(a+bx^3\right)^{7/3}}{2acx^6}\right)$$

$$\Big\downarrow \ 174$$

$$\frac{1}{3}\left(\frac{\frac{2\left(\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\int \frac{\sqrt[3]{bx^3+a}}{x^3}dx^3}{c}+\frac{9ad^2(bc-ad)\int \frac{\sqrt[3]{bx^3+a}}{dx^3+c}dx^3}{c}\right)}{3c}-\frac{\left(a+bx^3\right)^{4/3}(bc-6ad)}{cx^3}}{6ac}-\frac{\left(a+bx^3\right)^{7/3}}{2acx^6}\right)$$

$$\Big\downarrow 60$$

$$\frac{1}{3}\left(\frac{\frac{2\left(\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\left(a\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{9ad^2(bc-ad)\left(\frac{3\sqrt[3]{a+bx^3}}{d}-\frac{(bc-ad)\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{d}\right)}{c}\right)}{3c}-}{6ac}\right.$$

$$\Big\downarrow 69$$

$$\frac{1}{3}\left(\frac{\frac{2\left(\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\left(a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+b}\right.}{c}\right.}{3c}}{6ac}\right.$$

$$\Big\downarrow 16$$

3.701. $\int \frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(\frac{\frac{2\left(\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\left(a\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+9ad\right.}{3c}}{6ac}\right.$$

$$\Big\downarrow 70$$

3.701. $\int \frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(\frac{2\left(\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\left(a\left(-\frac{3\int\frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{9ad}{\,}\right.}{\,}\right.$$

↓ 16

3.701. $\int \frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(\frac{2\left(\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\left(a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{9ad}{}\right.}{}\right.$$

↓ 1082

3.701. $\int \frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(\frac{2\left(\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\left(a\left(\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{9ad^2(bc-ad)\Big(\frac{3\sqrt[3]{}}{}}{}}{3c}}{}\right.$$

↓ 217

3.701. $\int\frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(\frac{2\left(\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\left(a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)}{c}+\frac{9ad^2(bc-ad)\left(\frac{3\sqrt[3]{}}{}\right.}{}\right.}{3c}\right.$$

3.701. $\int \frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}\,dx$

input
```
Int[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]
```

output
```
(-1/2*(a + b*x^3)^(7/3)/(a*c*x^6) + (-(((b*c - 6*a*d)*(a + b*x^3)^(4/3))/(
c*x^3)) + (2*(((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(3*(a + b*x^3)^(1/3) +
 a*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3
)) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3
)))))/c + (9*a*d^2*(b*c - a*d)*((3*(a + b*x^3)^(1/3))/d - ((b*c - a*d)*(-(
(Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt
[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)
^(2/3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3
)*(b*c - a*d)^(2/3))))/d))/c))/(3*c))/(6*a*c))/3
```

3.701.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.701. $\int \frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.701.4 Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{-\frac{x^{6}\left(b^{2} c^{2} a^{\frac{2}{3}}+a^{\frac{5}{3}}\left(a d-2 b c\right) d\right) \ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{a d-b c}{d}\right)^{\frac{2}{3}}\right)}{2}-x^{6} \sqrt{3}\,\left(b^{2} c^{2} a^{\frac{2}{3}}+a^{\frac{5}{3}}\left(a d-2 b c\right) d\right) \arctan\Bigg(}{}$

input
```
int((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-1/2*x^6*(b^2*c^2*a^(2/3)+a^(5/3)*(a*d-2*b*c)*d)*ln((b*x^3+a)^(2/3)+
(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-x^6*3^(1/2)*(
b^2*c^2*a^(2/3)+a^(5/3)*(a*d-2*b*c)*d)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/
3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))+1/2*(1/d*(a*d-b*c))^(2/3)
*x^6*(a^2*d^2-4/3*a*b*c*d+2/9*b^2*c^2)*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a
)^(1/3)+a^(2/3))+x^6*(b^2*c^2*a^(2/3)+a^(5/3)*(a*d-2*b*c)*d)*ln((b*x^3+a)^
(1/3)-(1/d*(a*d-b*c))^(1/3))+1/2*(1/d*(a*d-b*c))^(2/3)*(2*x^6*(a^2*d^2-4/3
*a*b*c*d+2/9*b^2*c^2)*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/
3)*3^(1/2))-2*x^6*(a^2*d^2-4/3*a*b*c*d+2/9*b^2*c^2)*ln((b*x^3+a)^(1/3)-a^(
1/3))+(b*x^3+a)^(1/3)*(7/3*b*c*x^3*a^(2/3)+a^(5/3)*(-2*d*x^3+c))*c))/a^(2/
3)/(1/d*(a*d-b*c))^(2/3)/c^3/x^6
```

3.701.5 Fricas [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}\,dx=\frac{18\sqrt{3}(a^2bc-a^3d)(bcd^2-ad^3)^{\frac{1}{3}}x^6\arctan\left(-\frac{2\sqrt{3}(bcd^2-ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}-\sqrt{3}(bcd-ad^2)}{3\,(bcd-ad^2)}\right)-2\sqrt{}}{}$$

input
```
integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/54*(18*sqrt(3)*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(1/3)*x^6*arctan(-1/3
*(2*sqrt(3)*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a
*d^2))/(b*c*d - a*d^2)) - 2*sqrt(3)*(2*a*b^2*c^2 - 12*a^2*b*c*d + 9*a^3*d^
2)*(a^2)^(1/6)*x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(
3)*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) - (2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d
^2)*(a^2)^(2/3)*x^6*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^
(1/3)*(a^2)^(2/3)) + 2*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(a^2)^(2/3)*x^
6*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 9*(a^2*b*c - a^3*d)*(b*c*d^2 -
a*d^3)^(1/3)*x^6*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^
3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 18*(a^2*b*c - a^3*d)*(b*c*d^2
- a*d^3)^(1/3)*x^6*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*
(3*a^3*c^2 + (7*a^2*b*c^2 - 6*a^3*c*d)*x^3)*(b*x^3 + a)^(1/3))/(a^2*c^3*x^
6)
```

3.701.6 Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^7\,(c+dx^3)}\,dx=\int \frac{(a+bx^3)^{\frac{4}{3}}}{x^7\,(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x**7/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x**7*(c + d*x**3)), x)
```

3.701.7 Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^7\,(c+dx^3)}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^7}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^7), x)
```

3.701.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx = \frac{(b^2c^2d-2\,abcd^2+a^2d^3)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^4-ac^3d)}$$
$$-\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^3}$$
$$-\frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^3}$$
$$-\frac{\sqrt{3}(2\,b^2c^2-12\,abcd+9\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{2}{3}}c^3}$$
$$-\frac{(2\,b^2c^2-12\,abcd+9\,a^2d^2)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54\,a^{\frac{2}{3}}c^3}$$
$$+\frac{(2\,b^2c^2-12\,abcd+9\,a^2d^2)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{2}{3}}c^3}$$
$$-\frac{7\,(bx^3+a)^{\frac{4}{3}}b^2c-4\,(bx^3+a)^{\frac{1}{3}}ab^2c-6\,(bx^3+a)^{\frac{4}{3}}abd+6\,(bx^3+a)^{\frac{1}{3}}a^2bd}{18\,b^2c^2x^6}$$

input
```
integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*
x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^4 - a*c^3*d) - 1/3*sqrt(3)*
(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/
3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^3 - 1/6*(-b*c*d^2 +
 a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
 - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^3 - 1/27*sqrt(3)*(2*b^2*c^2 -
 12*a*b*c*d + 9*a^2*d^2)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3)
)/a^(1/3))/(a^(2/3)*c^3) - 1/54*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3)*c^3) + 1/
27*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*log(abs((b*x^3 + a)^(1/3) - a^(1/3
)))/(a^(2/3)*c^3) - 1/18*(7*(b*x^3 + a)^(4/3)*b^2*c - 4*(b*x^3 + a)^(1/3)*
a*b^2*c - 6*(b*x^3 + a)^(4/3)*a*b*d + 6*(b*x^3 + a)^(1/3)*a^2*b*d)/(b^2*c^
2*x^6)
```

3.701.9 Mupad [B] (verification not implemented)

Time = 16.70 (sec) , antiderivative size = 2841, normalized size of antiderivative = 6.46

$$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx = \text{Too large to display}$$

input

```
int((a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x)
```

output

```
log(((((18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 9*a
*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((9*a^2*d^2 + 2*b^2*c^2 - 1
2*a*b*c*d)^3/(a^2*c^9))^(1/3))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^
2*c^9))^(2/3))/729 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*
d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645
*a^5*b*c*d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9)
)^(1/3))/27 - (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170
*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^
2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((729*a^6*d^6 + 8
*b^6*c^6 + 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^4*b^2*c^2*d
^4 - 144*a*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^(1/3) + log((((
18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 81*a*b^4*c^
4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))*
(-(d^2*(a*d - b*c)^4)/c^9)^(2/3))/9 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 -
3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a
*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4))*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3)
)/3 - (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6
 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 -
1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*(-(a^4*d^6 + b^4*c^4*d^2
- 4*a*b^3*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5)/(27*c^9))^(1/3) ...
```

3.702 $\int \frac{x^4 \left(a+bx^3\right)^{4/3}}{c+dx^3} \, dx$

3.702.1 Optimal result

Integrand size = 24, antiderivative size = 334

$$\int \frac{x^4 (a+bx^3)^{4/3}}{c+dx^3} \, dx = -\frac{(6bc-7ad)x^2\sqrt[3]{a+bx^3}}{18d^2} + \frac{bx^5\sqrt[3]{a+bx^3}}{6d} - \frac{(9b^2c^2-12abcd+2a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{2/3}d^3} + \frac{c^{2/3}(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3} - \frac{c^{2/3}(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^3} - \frac{(9b^2c^2-12abcd+2a^2d^2)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{2/3}d^3} + \frac{c^{2/3}(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3}$$

output
```
-1/18*(-7*a*d+6*b*c)*x^2*(b*x^3+a)^(1/3)/d^2+1/6*b*x^5*(b*x^3+a)^(1/3)/d-1
/6*c^(2/3)*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/d^3-1/18*(2*a^2*d^2-12*a*b*c*d+9*b
^2*c^2)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)/d^3+1/2*c^(2/3)*(-a*d+b*c)^(
4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^3-1/27*(2*a^2*d^2-12
*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^
(2/3)/d^3*3^(1/2)+1/3*c^(2/3)*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^
(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)
```

3.702.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.38 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.57

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{6dx^2\sqrt[3]{a+bx^3}(-6bc+7ad+3bdx^3) - \frac{4\sqrt{3}(9b^2c^2-12abcd+2a^2d^2)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}_x}{\sqrt[3]{b}_{x+2}\sqrt[3]{a+bx^3}}\right)}{b^{2/3}}}{}$$

input
```
Integrate[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(6*d*x^2*(a + b*x^3)^(1/3)*(-6*b*c + 7*a*d + 3*b*d*x^3) - (4*Sqrt[3]*(9*b^
2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*
(a + b*x^3)^(1/3))])/b^(2/3) - 18*Sqrt[-6 - (6*I)*Sqrt[3]]*c^(2/3)*(b*c -
a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (
3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - (4*(9*b^2*c^2 - 12*a*b*c*d +
2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(2/3) + (18*I)*(I + Sq
rt[3])*c^(2/3)*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3
])*c^(1/3)*(a + b*x^3)^(1/3)] + (2*(9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*Lo
g[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(2/3)
+ 9*(1 - I*Sqrt[3])*c^(2/3)*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(2/3)*x^2
+ (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I +
Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(108*d^3)
```

3.702.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {977, 25, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(a+b x^3\right)^{4/3}}{c+d x^3} \, dx$$

$$\Big\downarrow \text{977}$$

$$\frac{\int -\frac{x^4 \left(b (6 b c-7 a d) x^3+a (5 b c-6 a d)\right)}{\left(b x^3+a\right)^{2/3} \left(d x^3+c\right)}dx}{6 d}+\frac{b x^5 \sqrt[3]{a+b x^3}}{6 d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b x^5 \sqrt[3]{a+b x^3}}{6 d}-\frac{\int \frac{x^4 \left(b (6 b c-7 a d) x^3+a (5 b c-6 a d)\right)}{\left(b x^3+a\right)^{2/3} \left(d x^3+c\right)}dx}{6 d}$$

$$\Big\downarrow \text{1052}$$

$$\frac{b x^5 \sqrt[3]{a+b x^3}}{6 d}-\frac{\frac{x^2 \sqrt[3]{a+b x^3} (6 b c-7 a d)}{3 d}-\frac{\int \frac{2 b x \left(\left(9 b^2 c^2-12 a b d c+2 a^2 d^2\right) x^3+a c (6 b c-7 a d)\right)}{\left(b x^3+a\right)^{2/3} \left(d x^3+c\right)}dx}{3 b d}}{6 d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b x^5 \sqrt[3]{a+b x^3}}{6 d}-\frac{\frac{x^2 \sqrt[3]{a+b x^3} (6 b c-7 a d)}{3 d}-\frac{2 \int \frac{x \left(\left(9 b^2 c^2-12 a b d c+2 a^2 d^2\right) x^3+a c (6 b c-7 a d)\right)}{\left(b x^3+a\right)^{2/3} \left(d x^3+c\right)}dx}{3 d}}{6 d}$$

$$\Big\downarrow \text{1054}$$

$$\frac{b x^5 \sqrt[3]{a+b x^3}}{6 d}-\frac{\frac{x^2 \sqrt[3]{a+b x^3} (6 b c-7 a d)}{3 d}-\frac{2 \int \left(\frac{\left(9 b^2 c^2-12 a b d c+2 a^2 d^2\right) x}{d \left(b x^3+a\right)^{2/3}}-\frac{9 \left(b^2 c^3-2 a b d c^2+a^2 d^2 c\right) x}{d \left(b x^3+a\right)^{2/3} \left(d x^3+c\right)}\right)dx}{3 d}}{6 d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx^5\sqrt[3]{a+bx^3}}{6d}-$$

$$\frac{\frac{x^2\sqrt[3]{a+bx^3}(6bc-7ad)}{3d}-\frac{2\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\left(2a^2d^2-12abcd+9b^2c^2\right)}{\sqrt{3}b^{2/3}d}-\frac{\left(2a^2d^2-12abcd+9b^2c^2\right)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}+\frac{3\sqrt{3}c^{2/3}}{}\right.}{}}{6d}$$

input
```
Int[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(b*x^5*(a + b*x^3)^(1/3))/(6*d) - (((6*b*c - 7*a*d)*x^2*(a + b*x^3)^(1/3))
/(3*d) - (2*(-(((9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(1/3
)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)*d)) + (3*Sqrt[3]*c^(2/3
)*(b*c - a*d)^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^
3)^(1/3)))/Sqrt[3]])/d - (3*c^(2/3)*(b*c - a*d)^(4/3)*Log[c + d*x^3])/(2*d
) - ((9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*Log[b^(1/3)*x - (a + b*x^3)^(1/3
)])/(2*b^(2/3)*d) + (9*c^(2/3)*(b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)
/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d)))/(3*d))/(6*d)
```

3.702.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 977
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c -
a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p,
q, x]
```

---

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.702.4 Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.41

method	result
pseudoelliptic	$\frac{\frac{\left(-a^{2} d^{2} b^{\frac{2}{3}}-\left(-2 a d+b c\right) c\, b^{\frac{5}{3}}\right) \ln \left(\frac{\left(\frac{a d-b c}{c}\right)^{\frac{2}{3}} x^{2}-\left(\frac{a d-b c}{c}\right)^{\frac{1}{3}} \left(b\, x^{3}+a\right)^{\frac{1}{3}} x+\left(b\, x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}-\left(a^{2} d^{2} b^{\frac{2}{3}}+\left(-2 a d+b c\right) c\, b^{\frac{5}{3}}\right) \sqrt{3}\, \arctan \left(\right.}{}$

input
```
int(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*(1/2*(-a^2*d^2*b^(2/3)-(-2*a*d+b*c)*c*b^(5/3))*ln((((a*d-b*c)/c)^(2/3)
*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-(a^2*d^2*
b^(2/3)+(-2*a*d+b*c)*c*b^(5/3))*3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^
(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)+1/9*((a*d-b*c)/c)^(2/3)*
(a^2*d^2-6*a*b*c*d+9/2*b^2*c^2)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+
(b*x^3+a)^(2/3))/x^2)+(a^2*d^2*b^(2/3)+(-2*a*d+b*c)*c*b^(5/3))*ln((((a*d-b
*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-((a*d-b*c)/c)^(2/3)*(-2/9*3^(1/2)*(a^2*
d^2-6*a*b*c*d+9/2*b^2*c^2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3)
)/b^(1/3)/x)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*(b^2*c^2-4/3*a*b*c*d+2/9*a
^2*d^2)+x^2*(b*x^3+a)^(1/3)*(-7/6*a*d*b^(2/3)+(-1/2*d*x^3+c)*b^(5/3))*d))/
b^(2/3)/((a*d-b*c)/c)^(2/3)/d^3
```

3.702.5 Fricas [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.65

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{2\sqrt{3}(9\,b^3c^2-12\,ab^2cd+2\,a^2bd^2)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)}{3\,b^2x}\right.}{}$$

input
```
integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/54*(2*sqrt(3)*(9*b^3*c^2 - 12*a*b^2*c*d + 2*a^2*b*d^2)*sqrt(-(-b^2)^(1/3
))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b
^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) - 18*sqrt(3)*(b^3*c - a*b^2*d)*(-b
*c^3 + a*c^2*d)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c^2 - a*c*d)*x + 2*sqrt(3)*(
-b*c^3 + a*c^2*d)^(2/3)*(b*x^3 + a)^(1/3))/((b*c^2 - a*c*d)*x)) - 2*(9*b^2
*c^2 - 12*a*b*c*d + 2*a^2*d^2)*(-b^2)^(2/3)*log(-((-b^2)^(2/3)*x - (b*x^3
+ a)^(1/3)*b)/x) + (9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*(-b^2)^(2/3)*log(-
((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)
*b)/x^2) - 18*(b^3*c - a*b^2*d)*(-b*c^3 + a*c^2*d)^(1/3)*log(((b*x^3 + a)^
(1/3)*c + (-b*c^3 + a*c^2*d)^(1/3)*x)/x) + 9*(b^3*c - a*b^2*d)*(-b*c^3 + a
*c^2*d)^(1/3)*log(((b*x^3 + a)^(2/3)*c^2 - (-b*c^3 + a*c^2*d)^(1/3)*(b*x^3
 + a)^(1/3)*c*x + (-b*c^3 + a*c^2*d)^(2/3)*x^2)/x^2) + 3*(3*b^3*d^2*x^5 -
(6*b^3*c*d - 7*a*b^2*d^2)*x^2)*(b*x^3 + a)^(1/3))/(b^2*d^3)
```

3.702.6 Sympy [F]

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{x^4(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**4*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**4*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.702.7 Maxima [F]

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}x^4}{dx^3+c}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)
```

3.702.8 Giac [F]

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}x^4}{dx^3+c}\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)
```

3.702.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{x^4\,{\left(b\,x^3+a\right)}^{4/3}}{d\,x^3+c}\,dx$$

input
```
int((x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

output
```
int((x^4*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

3.703 $\int \frac{x\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

3.703.1 Optimal result

Integrand size = 22, antiderivative size = 277

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{bx^2\sqrt[3]{a+bx^3}}{3d}+\frac{\sqrt[3]{b}(3bc-4ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^2}$$

$$-\frac{(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}d^2}+\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6\sqrt[3]{c}d^2}$$

$$+\frac{\sqrt[3]{b}(3bc-4ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6d^2}-\frac{(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}d^2}$$

output
```
1/3*b*x^2*(b*x^3+a)^(1/3)/d+1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(1/3)/d^2+1
/6*b^(1/3)*(-4*a*d+3*b*c)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d^2-1/2*(-a*d+b*c)
^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(1/3)/d^2+1/9*b^(1
/3)*(-4*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2
*3^(1/2)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(
b*x^3+a)^(1/3))*3^(1/2))/c^(1/3)/d^2*3^(1/2)
```

3.703.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.54 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.69

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{12bdx^2\sqrt[3]{a+bx^3}+4\sqrt{3}\sqrt[3]{b}(3bc-4ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+\frac{6\sqrt{-6-6i\sqrt{3}}(bc-ad)}{}}{}$$

input

```
Integrate[(x*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output

```
(12*b*d*x^2*(a + b*x^3)^(1/3) + 4*Sqrt[3]*b^(1/3)*(3*b*c - 4*a*d)*ArcTan[(
Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + (6*Sqrt[-6 - (6*I)
*Sqrt[3]]*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
 a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/c^(1/3) + 4*b
^(1/3)*(3*b*c - 4*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + (6*(1 - I*S
qrt[3])*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1
/3)*(a + b*x^3)^(1/3)])/c^(1/3) - 2*b^(1/3)*(3*b*c - 4*a*d)*Log[b^(2/3)*x^
2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((3*I)*(I + Sqrt[3]
)*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)
*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^
3)^(2/3)])/c^(1/3))/(36*d^2)
```

3.703.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {977, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx^3\right)^{4/3}}{c+dx^3}dx$$

$$\Big\downarrow \text{977}$$

$$\frac{\int -\frac{x\left(b(3bc-4ad)x^3+a(2bc-3ad)\right)}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{3d}+\frac{bx^2\sqrt[3]{a+bx^3}}{3d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx^2\sqrt[3]{a+bx^3}}{3d}-\frac{\int \frac{x\left(b(3bc-4ad)x^3+a(2bc-3ad)\right)}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{3d}$$

$$\Big\downarrow \; 1054$$

$$\frac{bx^2\sqrt[3]{a+bx^3}}{3d}-\frac{\int \left(\frac{b(3bc-4ad)x}{d\left(bx^3+a\right)^{2/3}}-\frac{3\left(b^2c^2-2abdc+a^2d^2\right)x}{d\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}\right)dx}{3d}$$

$$\Big\downarrow \; 2009$$

$$\frac{bx^2\sqrt[3]{a+bx^3}}{3d}-\frac{\frac{\sqrt{3}(bc-ad)^{4/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{c}d}-\frac{\sqrt[3]{b}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-4ad)}{\sqrt{3}d}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{2\sqrt[3]{c}d}+\frac{3(bc-ad)^{4/3}\log\left(x\right.}{}}{3d}$$

input
```
Int[(x*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(b*x^2*(a + b*x^3)^(1/3))/(3*d) - (-((b^(1/3)*(3*b*c - 4*a*d)*ArcTan[(1 +
(2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d)) + (Sqrt[3]*(b*c -
a*d)^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3))
)/Sqrt[3]])/(c^(1/3)*d) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(2*c^(1/3)*d)
 - (b^(1/3)*(3*b*c - 4*a*d)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*d) + (3
*(b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*c^(1/3)*d))/(3*d)
```

3.703.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 977
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c -
a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p,
q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.703.4 Maple [A] (verified)

Time = 5.19 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$-\frac{-\frac{(ad-bc)^{2}\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}-\sqrt{3}\,(ad-bc)^{2}\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right.}{}$

input
```
int(x*(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a*d-b*c)/c)^(2/3)*(-1/2*(a*d-b*c)^2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a
*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-3^(1/2)*(a*d-b*c)
^2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)
/c)^(1/3)/x)-2/3*((a*d-b*c)/c)^(2/3)*c*(a*d*b^(1/3)-3/4*b^(4/3)*c)*ln((b^(
2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(a*d-b*c)^2*ln(((
(a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+4/3*(-3^(1/2)*(a*d*b^(1/3)-3/4*b^
(4/3)*c)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+(a*d*
b^(1/3)-3/4*b^(4/3)*c)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-3/4*b*x^2*(b*x^3
+a)^(1/3)*d)*((a*d-b*c)/c)^(2/3)*c)/c/d^2
```

3.703.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.43

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{6\left(bx^3+a\right)^{\frac{1}{3}}bdx^2 - 6\sqrt{3}(bc-ad)\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}}{3\,(bc-ad)x}\right)}{}$$

input
```
integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/18*(6*(b*x^3 + a)^(1/3)*b*d*x^2 - 6*sqrt(3)*(b*c - a*d)*((b*c - a*d)/c)^
(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*(
(b*c - a*d)/c)^(2/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(3*b*c - 4*a*d)*(-b)^(1
/3)*arctan(1/3*(sqrt(3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b)^(2/3))/(b*x
)) - 2*(3*b*c - 4*a*d)*(-b)^(1/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x
) - 6*(b*c - a*d)*((b*c - a*d)/c)^(1/3)*log(-(x*((b*c - a*d)/c)^(1/3) - (b
*x^3 + a)^(1/3))/x) + (3*b*c - 4*a*d)*(-b)^(1/3)*log(((-b)^(2/3)*x^2 - (b*
x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(b*c - a*d)*((b*
c - a*d)/c)^(1/3)*log((x^2*((b*c - a*d)/c)^(2/3) + (b*x^3 + a)^(1/3)*x*((b
*c - a*d)/c)^(1/3) + (b*x^3 + a)^(2/3))/x^2))/d^2
```

3.703.6 Sympy [F]

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.703.7 Maxima [F]

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}x}{dx^3+c}\,dx$$

input
```
integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)
```

3.703.8 Giac [F]

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}x}{dx^3+c}\,dx$$

input
```
integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)
```

3.703.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{x\,{\left(b\,x^3+a\right)}^{4/3}}{d\,x^3+c}\,dx$$

input

```
int((x*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

output

```
int((x*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

3.704 $\int \frac{\left(a+bx^3\right)^{4/3}}{x^2\left(c+dx^3\right)}\,dx$

3.704.1 Optimal result

Integrand size = 24, antiderivative size = 254

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{4/3}}{x^2\left(c+dx^3\right)}\,dx ={}& -\frac{a\sqrt[3]{a+bx^3}}{cx}-\frac{b^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d}\\
&+\frac{(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}d}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{4/3}d}\\
&-\frac{b^{4/3}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d}+\frac{(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}d}\end{aligned}$$

output

```
-a*(b*x^3+a)^(1/3)/c/x-1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(4/3)/d-1/2*b^(4
/3)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d+1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/
3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(4/3)/d-1/3*b^(4/3)*arctan(1/3*(1+2*b^(1/3
)*x/(b*x^3+a)^(1/3))*3^(1/2))/d*3^(1/2)+1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1
+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(4/3)/d*3^(1/2)
```

3.704.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.25 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.80

$$\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx = \frac{-12a\sqrt[3]{c}d\sqrt[3]{a+bx^3} - 4\sqrt{3}b^{4/3}c^{4/3}x\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 2\sqrt{-6-6i\sqrt{3}}(bc-}{}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x]
```

output
```
(-12*a*c^(1/3)*d*(a + b*x^3)^(1/3) - 4*Sqrt[3]*b^(4/3)*c^(4/3)*x*ArcTan[(S
qrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*Sqrt[-6 - (6*I)*S
qrt[3]]*(b*c - a*d)^(4/3)*x*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
 a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - 4*b^(4/3)*c^
(4/3)*x*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + (2*I)*(I + Sqrt[3])*(b*c -
 a*d)^(4/3)*x*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x
^3)^(1/3)] + 2*b^(4/3)*c^(4/3)*x*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(
1/3) + (a + b*x^3)^(2/3)] + (1 - I*Sqrt[3])*(b*c - a*d)^(4/3)*x*Log[2*(b*c
 - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^
3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(4/3)*d*x)
```

3.704.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {974, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx$$

$$\Big\downarrow \; 974$$

$$\frac{\int \frac{x(b^2cx^3+a(2bc-ad))}{(bx^3+a)^{2/3}(dx^3+c)}dx}{c} - \frac{a\sqrt[3]{a+bx^3}}{cx}$$

$$\Big\downarrow \; 1054$$

$$\frac{\int \left(\frac{b^2 c x}{d(bx^3+a)^{2/3}}-\frac{(b^2c^2-2abdc+a^2d^2)x}{d(bx^3+a)^{2/3}(dx^3+c)}\right)dx}{c}-\frac{a\sqrt[3]{a+bx^3}}{cx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^{4/3}c\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}+\frac{(bc-ad)^{4/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}d}-\frac{b^{4/3}c\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d}-\frac{(bc-ad)^{4/3}\log(c+dx}{6\sqrt[3]{c}d}}{c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{cx}$$

input
```
Int[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x]
```

output
```
-((a*(a + b*x^3)^(1/3))/(c*x)) + (-((b^(4/3)*c*ArcTan[(1 + (2*b^(1/3)*x)/(
a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d)) + ((b*c - a*d)^(4/3)*ArcTan[(1 +
(2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^
(1/3)*d) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^(1/3)*d) - (b^(4/3)*c*L
og[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*d) + ((b*c - a*d)^(4/3)*Log[((b*c -
a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(1/3)*d))/c
```

3.704.3.1 Defintions of rubi rules used

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

3.704. $\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.704.4 Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.31

method	result
pseudoelliptic	$-\frac{6\left(b x^{3}+a\right)^{\frac{1}{3}} c \left(\frac{a d-b c}{c}\right)^{\frac{2}{3}} a d+x \Bigg(-\left(\frac{a d-b c}{c}\right)^{\frac{2}{3}} c^{2} \Bigg(2\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\, \left(b^{\frac{1}{3}} x+2 \left(b x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right)+\ln\Bigg(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}} \left(b x^{3}+a\right)^{\frac{1}{3}} x+\big(b x}{x^{2}}}{}$

input
```
int((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(6*(b*x^3+a)^(1/3)*c*((a*d-b*c)/c)^(2/3)*a*d+x*(-((a*d-b*c)/c)^(2/3)*
c^2*(2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)
+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^
(1/3)*x+(b*x^3+a)^(1/3))/x))*b^(4/3)+(a*d-b*c)^2*(2*arctan(1/3*3^(1/2)*(((
a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((
((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2
/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x))))/((a*d-b*c)/c)
^(2/3)/c^2/x/d
```

3.704.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^2\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.704.6 Sympy [F]

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^2\left(c+dx^3\right)}\,dx=\int \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{x^2\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x**2/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x**2*(c + d*x**3)), x)
```

3.704.7 Maxima [F]

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^2\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)
```

3.704.8 Giac [F]

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^2\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)
```

3.704.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^2\,(c+dx^3)}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{4/3}}{x^2\,\left(d\,x^3+c\right)}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(4/3)/(x^2*(c + d*x^3)), x)
```

3.705 $\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx$

3.705.1 Optimal result

Integrand size = 24, antiderivative size = 201

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx = -\frac{a\sqrt[3]{a+bx^3}}{4cx^4}-\frac{(5bc-4ad)\sqrt[3]{a+bx^3}}{4c^2x}-\frac{(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}}+\frac{(bc-ad)^{4/3}\log(c+dx^3)}{6c^{7/3}}-\frac{(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}}$$

output
```
-1/4*a*(b*x^3+a)^(1/3)/c/x^4-1/4*(-4*a*d+5*b*c)*(b*x^3+a)^(1/3)/c^2/x+1/6*
(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(7/3)-1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1
/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(7/3)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+
2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(7/3)*3^(1/2)
```

3.705.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.06 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.63

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^5\left(c+dx^3\right)}\,dx = \frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}\left(-ac-5bcx^3+4adx^3\right)}{x^4}+2\sqrt{-6-6i\sqrt{3}}(bc-ad)^{4/3}\arctan\left(\frac{3\sqrt[3]{bc}}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x]
```

output
```
((3*c^(1/3)*(a + b*x^3)^(1/3)*(-(a*c) - 5*b*c*x^3 + 4*a*d*x^3))/x^4 + 2*Sq
rt[-6 - (6*I)*Sqrt[3]]*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(S
qrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] +
 2*(1 - I*Sqrt[3])*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sq
rt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + I*(I + Sqrt[3])*(b*c - a*d)^(4/3)*Log[
2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a
+ b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(7/3))
```

3.705.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {974, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^5\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{974}$$

$$\frac{\int \frac{b(4bc-3ad)x^3+a(5bc-4ad)}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4c}-\frac{a\sqrt[3]{a+bx^3}}{4cx^4}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int -\frac{4a(bc-ad)^2x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}(5bc-4ad)}{cx}}{4c}-\frac{a\sqrt[3]{a+bx^3}}{4cx^4}$$

$$\Big\downarrow 27$$

$$\frac{\frac{4(bc-ad)^2\int \frac{x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{c}-\frac{\sqrt[3]{a+bx^3}(5bc-4ad)}{cx}}{4c}-\frac{a\sqrt[3]{a+bx^3}}{4cx^4}$$

$$\Big\downarrow 992$$

$$\frac{\frac{4(bc-ad)^2\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c}-\frac{\sqrt[3]{a+bx^3}(5bc-4ad)}{cx}}{4c}-\frac{a\sqrt[3]{a+bx^3}}{4cx^4}$$

input

```
Int[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x]
```

output

```
-1/4*(a*(a + b*x^3)^(1/3))/(c*x^4) + (-(((5*b*c - 4*a*d)*(a + b*x^3)^(1/3)
)/(c*x)) + (4*(b*c - a*d)^2*(-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3
)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[
c + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/
3) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3))))/c)/(4*c)
```

3.705.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 974

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

3.705. $\int \frac{\left(a+bx^3\right)^{4/3}}{x^5\left(c+dx^3\right)}dx$

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.705.4 Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.21

method	result
pseudoelliptic	$\frac{-2x^4(ad-bc)^2 \ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)-\frac{3\left(b\,x^3+a\right)^{\frac{1}{3}}\left((-4ad+5bc)x^3+ac\right)c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}}{2}+x^4\left(2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x}{3\left(\frac{ad-bc}{c}\right.}\right.\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^4 c^3}$

input
```
int((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/c)^(2/3)*(-2*x^4*(a*d-b*c)^2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x
^3+a)^(1/3))/x)-3/2*(b*x^3+a)^(1/3)*((-4*a*d+5*b*c)*x^3+a*c)*c*((a*d-b*c)/
c)^(2/3)+x^4*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3
))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c
)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a*d-b*c)^2)/x^4/c^3
```

3.705.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^5\left(c+dx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.705.6 Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^5\left(c+dx^3\right)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^5\left(c+dx^3\right)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x**5/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x**5*(c + d*x**3)), x)
```

3.705.7 Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^5\left(c+dx^3\right)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)
```

3.705.8 Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^5}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)
```

3.705.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx=\int \frac{(b\,x^3+a)^{4/3}}{x^5\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(4/3)/(x^5*(c + d*x^3)), x)
```

3.706 $\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx$

3.706.1 Optimal result

Integrand size $=24$, antiderivative size $=250$

$$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx = -\frac{a\sqrt[3]{a+bx^3}}{7cx^7}-\frac{(8bc-7ad)\sqrt[3]{a+bx^3}}{28c^2x^4} - \frac{(4b^2c^2-35abcd+28a^2d^2)\sqrt[3]{a+bx^3}}{28ac^3x} + \frac{d(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{10/3}} - \frac{d(bc-ad)^{4/3}\log(c+dx^3)}{6c^{10/3}} + \frac{d(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{10/3}}$$

output
```
-1/7*a*(b*x^3+a)^(1/3)/c/x^7-1/28*(-7*a*d+8*b*c)*(b*x^3+a)^(1/3)/c^2/x^4-1
/28*(28*a^2*d^2-35*a*b*c*d+4*b^2*c^2)*(b*x^3+a)^(1/3)/a/c^3/x-1/6*d*(-a*d+
b*c)^(4/3)*ln(d*x^3+c)/c^(10/3)+1/2*d*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)
*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(10/3)+1/3*d*(-a*d+b*c)^(4/3)*arctan(1/3*(1+
2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(10/3)*3^(1/2)
```

3.706.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.41 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx = \frac{-\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}\left(4b^2c^2x^6+abcx^3\left(8c-35dx^3\right)+a^2\left(4c^2-7cdx^3+28d^2x^6\right)\right)}{ax^7}-14\sqrt{-6-6i\sqrt{3}}d(bc-ad)}{}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x]
```

output
```
((-3*c^(1/3)*(a + b*x^3)^(1/3)*(4*b^2*c^2*x^6 + a*b*c*x^3*(8*c - 35*d*x^3)
 + a^2*(4*c^2 - 7*c*d*x^3 + 28*d^2*x^6)))/(a*x^7) - 14*Sqrt[-6 - (6*I)*Sqr
t[3]]*d*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a
*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (14*I)*(I + Sq
rt[3])*d*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(
1/3)*(a + b*x^3)^(1/3)] + 7*(1 - I*Sqrt[3])*d*(b*c - a*d)^(4/3)*Log[2*(b*c
 - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^
3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(84*c^(10/3))
```

3.706.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {974, 1053, 25, 27, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}dx$$

$$\Big\downarrow \text{974}$$

$$\frac{\int \frac{b(7bc-6ad)x^3+a(8bc-7ad)}{x^5(bx^3+a)^{2/3}(dx^3+c)}dx}{7c}-\frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int -\frac{a\left(-3bd(8bc-7ad)x^3+4b^2c^2+28a^2d^2-35abcd\right)}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4ac}-\frac{\sqrt[3]{a+bx^3}(8bc-7ad)}{4cx^4}}{7c}-\frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{a\left(-3bd(8bc-7ad)x^3+4b^2c^2+28a^2d^2-35abcd\right)}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4ac}-\frac{\sqrt[3]{a+bx^3}(8bc-7ad)}{4cx^4}}{7c}-\frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{-3bd(8bc-7ad)x^3+4b^2c^2+28a^2d^2-35abcd}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4c}-\frac{\sqrt[3]{a+bx^3}(8bc-7ad)}{4cx^4}}{7c}-\frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

$$\Big\downarrow \ 1053$$

$$\frac{\frac{-\frac{\int \frac{28ad(bc-ad)^2x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{4b^2c}{a}+\frac{28ad^2}{c}-35bd\right)}{x}}{4c}-\frac{\sqrt[3]{a+bx^3}(8bc-7ad)}{4cx^4}}{7c}-\frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{-\frac{28d(bc-ad)^2\int \frac{x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{c}-\frac{\sqrt[3]{a+bx^3}\left(\frac{4b^2c}{a}+\frac{28ad^2}{c}-35bd\right)}{x}}{4c}-\frac{\sqrt[3]{a+bx^3}(8bc-7ad)}{4cx^4}}{7c}-\frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

$$\Big\downarrow \ 992$$

$$\frac{\frac{-\frac{28d(bc-ad)^2\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c}-\frac{\sqrt[3]{a+bx^3}\left(\frac{4b^2c}{a}+\frac{28ad^2}{c}-35bd\right)}{x}}{4c}}{7c}-$$

$$\frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

input
```
Int[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x]
```

3.706. $\int \frac{\left(a+bx^3\right)^{4/3}}{x^8\left(c+dx^3\right)}\,dx$

output
```
-1/7*(a*(a + b*x^3)^(1/3))/(c*x^7) + (-1/4*((8*b*c - 7*a*d)*(a + b*x^3)^(1
/3))/(c*x^4) + (-((((4*b^2*c)/a - 35*b*d + (28*a*d^2)/c)*(a + b*x^3)^(1/3)
)/x) - (28*d*(b*c - a*d)^2*(-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)
*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c
 + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3
) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3))))/c)/(4*c))/(7*c)
```

3.706.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.706. $\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx$

3.706.4 Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{\frac{6\left(\left(b x^{3}+a\right)^{2} c^{2}-\frac{7 a d x^{3}\left(5 b x^{3}+a\right) c}{4}+7 a^{2} d^{2} x^{6}\right) c \left(\frac{a d -b c}{c}\right)^{\frac{2}{3}} \left(b x^{3}+a\right)^{\frac{1}{3}}}{7}+a d x^{7} \left(a d -b c \right)^{2} \Bigg(2 \arctan \Bigg(\frac{\sqrt{3}\,\Big(\Big(\frac{a d -b c}{c}\Big)^{\frac{1}{3}} x -2 \big(b x^{3}+}{3 \left(\frac{a d -b c}{c}\right)^{\frac{1}{3}} x}}{6 \left(\frac{a d -b c}{c}\right)^{\frac{2}{3}} x^{7} c^{4}}$

input
```
int((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/((a*d-b*c)/c)^(2/3)*(6/7*((b*x^3+a)^2*c^2-7/4*a*d*x^3*(5*b*x^3+a)*c+7
*a^2*d^2*x^6)*c*((a*d-b*c)/c)^(2/3)*(b*x^3+a)^(1/3)+a*d*x^7*(a*d-b*c)^2*(2
*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c
)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+
a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/
3))/x)))/x^7/c^4/a
```

3.706.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^8\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.706.6 Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^8\,(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^8\,(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x**8/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x**8*(c + d*x**3)), x)
```

3.706.7 Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^8}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)
```

3.706.8 Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^8}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)
```

3.706.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{x^8\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(4/3)/(x^8*(c + d*x^3)), x)
```

3.707 $\int \frac{\left(a+bx^3\right)^{4/3}}{x^{11}\left(c+dx^3\right)}\,dx$

3.707.1 Optimal result

Integrand size = 24, antiderivative size = 318

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{4/3}}{x^{11}\left(c+dx^3\right)}\,dx &= -\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}-\frac{(11bc-10ad)\sqrt[3]{a+bx^3}}{70c^2x^7}\\ &\quad-\frac{\left(2b^2c^2-40abcd+35a^2d^2\right)\sqrt[3]{a+bx^3}}{140ac^3x^4}\\ &\quad+\frac{\left(6b^3c^3+20ab^2c^2d-175a^2bcd^2+140a^3d^3\right)\sqrt[3]{a+bx^3}}{140a^2c^4x}\\ &\quad-\frac{d^2(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{13/3}}+\frac{d^2(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{13/3}}\\ &\quad-\frac{d^2(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{13/3}}\end{aligned}$$

output
```
-1/10*a*(b*x^3+a)^(1/3)/c/x^10-1/70*(-10*a*d+11*b*c)*(b*x^3+a)^(1/3)/c^2/x
^7-1/140*(35*a^2*d^2-40*a*b*c*d+2*b^2*c^2)*(b*x^3+a)^(1/3)/a/c^3/x^4+1/140
*(140*a^3*d^3-175*a^2*b*c*d^2+20*a*b^2*c^2*d+6*b^3*c^3)*(b*x^3+a)^(1/3)/a^
2/c^4/x+1/6*d^2*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(13/3)-1/2*d^2*(-a*d+b*c)^(
4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(13/3)-1/3*d^2*(-a*d
+b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^
(1/2))/c^(13/3)*3^(1/2)
```

3.707.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.73 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.32

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^{11}\left(c+dx^3\right)}\,dx = \frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(6b^3c^3x^9-2ab^2c^2x^6(c-10dx^3)+a^2bcx^3(-22c^2+40cdx^3-175d^2x^6)+a^3(-14c^3+20c^2dx^3-35cd^2x}{a^2x^{10}}}{}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
```

output
```
((3*c^(1/3)*(a + b*x^3)^(1/3)*(6*b^3*c^3*x^9 - 2*a*b^2*c^2*x^6*(c - 10*d*x
^3) + a^2*b*c*x^3*(-22*c^2 + 40*c*d*x^3 - 175*d^2*x^6) + a^3*(-14*c^3 + 20
*c^2*d*x^3 - 35*c*d^2*x^6 + 140*d^3*x^9)))/(a^2*x^10) + 70*Sqrt[-6 - (6*I)
*Sqrt[3]]*d^2*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b
*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 70*(1 -
I*Sqrt[3])*d^2*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3
])*c^(1/3)*(a + b*x^3)^(1/3)] + (35*I)*(I + Sqrt[3])*d^2*(b*c - a*d)^(4/3)
*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*
x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(420*c^(
13/3))
```

3.707.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {974, 1053, 27, 1053, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^{11}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{974}$$

$$\frac{\int \frac{b(10bc-9ad)x^3+a(11bc-10ad)}{x^8\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{10c}-\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int -\frac{2a\left(-3bd(11bc-10ad)x^3+2b^2c^2+35a^2d^2-40abcd\right)}{x^5\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{7ac}-\frac{\sqrt[3]{a+bx^3}(11bc-10ad)}{7cx^7}}{10c}-\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2\int \frac{-3bd(11bc-10ad)x^3+2b^2c^2+35a^2d^2-40abcd}{x^5\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{7c}-\frac{\sqrt[3]{a+bx^3}(11bc-10ad)}{7cx^7}}{10c}-\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \ 1053$$

$$\frac{\frac{2\left(-\frac{\int \frac{6b^3c^3+20ab^2dc^2-175a^2bd^2c+140a^3d^3+3bd\left(2b^2c^2-40abdc+35a^2d^2\right)x^3}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{2b^2c}{a}+\frac{35ad^2}{c}-40bd\right)}{4x^4}\right)}{7c}-\frac{\sqrt[3]{a+bx^3}(11bc-10ad)}{7cx^7}}{10c}-$$

$$\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \ 1053$$

$$\frac{\frac{2\left(-\frac{-\frac{\int \frac{140a^2d^2(bc-ad)^2x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}\left(140a^3d^3-175a^2bcd^2+20ab^2c^2d+6b^3c^3\right)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{2b^2c}{a}+\frac{35ad^2}{c}-40bd\right)}{4x^4}\right)}{7c}-\frac{\sqrt[3]{a+bx^3}(11bc}{7cx^7}}{10c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2\left(-\frac{-\frac{140ad^2(bc-ad)^2\int \frac{x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{c}-\frac{\sqrt[3]{a+bx^3}\left(140a^3d^3-175a^2bcd^2+20ab^2c^2d+6b^3c^3\right)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{2b^2c}{a}+\frac{35ad^2}{c}-40bd\right)}{4x^4}\right)}{7c}-\sqrt[3]{}}{10c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

$$\Big\downarrow \ 992$$

3.707. $\int \frac{\left(a+bx^3\right)^{4/3}}{x^{11}\left(c+dx^3\right)}\,dx$

$$\frac{\frac{2\left(-\frac{-\frac{\sqrt[3]{a+bx^3}\left(140a^3d^3-175a^2bcd^2+20ab^2c^2d+6b^3c^3\right)}{acx}-\frac{140ad^2(bc-ad)^2\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-a}}{\sqrt[3]{c}}\right.}{2\sqrt[3]{c}(b}\right.}{c}}{4ac}\right.}{7c}}{10c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

input

```
Int[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
```

output

```
-1/10*(a*(a + b*x^3)^(1/3))/(c*x^10) + (-1/7*((11*b*c - 10*a*d)*(a + b*x^3
)^(1/3))/(c*x^7) + (2*(-1/4*(((2*b^2*c)/a - 40*b*d + (35*a*d^2)/c)*(a + b*
x^3)^(1/3))/x^4 - (-(((6*b^3*c^3 + 20*a*b^2*c^2*d - 175*a^2*b*c*d^2 + 140*
a^3*d^3)*(a + b*x^3)^(1/3))/(a*c*x)) - (140*a*d^2*(b*c - a*d)^2*(-(ArcTan[
(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3
]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3
)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*
c - a*d)^(2/3))))/c)/(4*a*c)))/(7*c))/(10*c)
```

3.707. $\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx$

3.707.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.707.4 Maple [A] (verified)

Time = 5.05 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$\frac{-\frac{3\left(\left(-\frac{3b\,x^{3}}{7}+a\right)\left(b\,x^{3}+a\right)^{2}c^{3}-\frac{10a d\,x^{3}\left(b\,x^{3}+a\right)^{2}c^{2}}{7}+\frac{5a^{2}d^{2}x^{6}\left(5b\,x^{3}+a\right)c}{2}-10a^{3}d^{3}x^{9}\right)c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{5}+a^{2}d^{2}x^{10}(ad-bc)}{6}$

input
```
int((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/c)^(2/3)*(-3/5*((-3/7*b*x^3+a)*(b*x^3+a)^2*c^3-10/7*a*d*x^3
*(b*x^3+a)^2*c^2+5/2*a^2*d^2*x^6*(5*b*x^3+a)*c-10*a^3*d^3*x^9)*c*((a*d-b*c
)/c)^(2/3)*(b*x^3+a)^(1/3)+a^2*d^2*x^10*(a*d-b*c)^2*(2*arctan(1/3*3^(1/2)*
(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+l
n((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)
^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))/x^10/c^5/a^
2
```

3.707.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.707.6 Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^{11}(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x**11/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x**11*(c + d*x**3)), x)
```

3.707.7 Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)
```

3.707.8 Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)
```

3.707.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{x^{11}\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(4/3)/(x^11*(c + d*x^3)), x)
```

3.708 $\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx$

3.708.1 Optimal result

Integrand size = 24, antiderivative size = 392

$$\begin{aligned}\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx = {} & -\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}-\frac{(14bc-13ad)\sqrt[3]{a+bx^3}}{130c^2x^{10}} \\ & -\frac{(4b^2c^2-143abcd+130a^2d^2)\sqrt[3]{a+bx^3}}{910ac^3x^7} \\ & +\frac{(12b^3c^3+26ab^2c^2d-520a^2bcd^2+455a^3d^3)\sqrt[3]{a+bx^3}}{1820a^2c^4x^4} \\ & -\frac{(36b^4c^4+78ab^3c^3d+260a^2b^2c^2d^2-2275a^3bcd^3+1820a^4d^4)\sqrt[3]{a+bx^3}}{1820a^3c^5x} \\ & +\frac{d^3(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{16/3}}-\frac{d^3(bc-ad)^{4/3}\log(c+dx^3)}{6c^{16/3}} \\ & +\frac{d^3(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{16/3}}\end{aligned}$$

output

```
-1/13*a*(b*x^3+a)^(1/3)/c/x^13-1/130*(-13*a*d+14*b*c)*(b*x^3+a)^(1/3)/c^2/
x^10-1/910*(130*a^2*d^2-143*a*b*c*d+4*b^2*c^2)*(b*x^3+a)^(1/3)/a/c^3/x^7+1
/1820*(455*a^3*d^3-520*a^2*b*c*d^2+26*a*b^2*c^2*d+12*b^3*c^3)*(b*x^3+a)^(1
/3)/a^2/c^4/x^4-1/1820*(1820*a^4*d^4-2275*a^3*b*c*d^3+260*a^2*b^2*c^2*d^2+
78*a*b^3*c^3*d+36*b^4*c^4)*(b*x^3+a)^(1/3)/a^3/c^5/x-1/6*d^3*(-a*d+b*c)^(4
/3)*ln(d*x^3+c)/c^(16/3)+1/2*d^3*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^
(1/3)-(b*x^3+a)^(1/3))/c^(16/3)+1/3*d^3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(
-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(16/3)*3^(1/2)
```

3.708.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.75 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.22

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx = \frac{-\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}\left(36b^4c^4x^{12}+6ab^3c^3x^9\left(-2c+13dx^3\right)+2a^2b^2c^2x^6\left(4c^2-13cdx^3+130d^2x^6\right)+a^3bcx^3\left(196c^3-286c^2\right.}{a^3x^{13}}}{}$$

input

```
Integrate[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]
```

output

```
((-3*c^(1/3)*(a + b*x^3)^(1/3)*(36*b^4*c^4*x^12 + 6*a*b^3*c^3*x^9*(-2*c +
13*d*x^3) + 2*a^2*b^2*c^2*x^6*(4*c^2 - 13*c*d*x^3 + 130*d^2*x^6) + a^3*b*c
*x^3*(196*c^3 - 286*c^2*d*x^3 + 520*c*d^2*x^6 - 2275*d^3*x^9) + a^4*(140*c
^4 - 182*c^3*d*x^3 + 260*c^2*d^2*x^6 - 455*c*d^3*x^9 + 1820*d^4*x^12)))/(a
^3*x^13) - 910*Sqrt[-6 - (6*I)*Sqrt[3]]*d^3*(b*c - a*d)^(4/3)*ArcTan[(3*(b
*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*
(a + b*x^3)^(1/3))] + (910*I)*(I + Sqrt[3])*d^3*(b*c - a*d)^(4/3)*Log[2*(b
*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + 455*(1 -
I*Sqrt[3])*d^3*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqr
t[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2
/3)*(a + b*x^3)^(2/3)])/(5460*c^(16/3))
```

3.708.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {974, 1053, 25, 27, 1053, 27, 1053, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^{14}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{974}$$

$$\frac{\int \frac{b(13bc-12ad)x^3+a(14bc-13ad)}{x^{11}(bx^3+a)^{2/3}(dx^3+c)}dx}{13c}-\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int -\frac{a\left(-9bd(14bc-13ad)x^3+4b^2c^2+130a^2d^2-143abcd\right)}{x^8\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{10ac}-\frac{\sqrt[3]{a+bx^3}(14bc-13ad)}{10cx^{10}}}{13c}-\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a\left(-9bd(14bc-13ad)x^3+4b^2c^2+130a^2d^2-143abcd\right)}{x^8\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{10ac}-\frac{\sqrt[3]{a+bx^3}(14bc-13ad)}{10cx^{10}}}{13c}-\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{-9bd(14bc-13ad)x^3+4b^2c^2+130a^2d^2-143abcd}{x^8\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{10c}-\frac{\sqrt[3]{a+bx^3}(14bc-13ad)}{10cx^{10}}}{13c}-\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{\frac{-\frac{\int \frac{2\left(12b^3c^3+26ab^2dc^2-520a^2bd^2c+455a^3d^3+3bd\left(4b^2c^2-143abdc+130a^2d^2\right)x^3\right)}{x^5\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{7ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{4b^2c}{a}+\frac{130ad^2}{c}-143bd\right)}{7x^7}}{10c}-\frac{\sqrt[3]{a+bx^3}(14bc-13a}{10cx^{10}}}{13c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{-\frac{2\int \frac{12b^3c^3+26ab^2dc^2-520a^2bd^2c+455a^3d^3+3bd\left(4b^2c^2-143abdc+130a^2d^2\right)x^3}{x^5\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{7ac}-\frac{\sqrt[3]{a+bx^3}\left(\frac{4b^2c}{a}+\frac{130ad^2}{c}-143bd\right)}{7x^7}}{10c}-\frac{\sqrt[3]{a+bx^3}(14bc-13ad)}{10cx^{10}}}{13c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{\frac{-\frac{2\left(-\frac{\int \frac{36b^4c^4+78ab^3dc^3+260a^2b^2d^2c^2-2275a^3bd^3c+1820a^4d^4+3bd\left(12b^3c^3+26ab^2dc^2-520a^2bd^2c+455a^3d^3\right)x^3}{x^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(455a^3d^3-520a^2bcd^2+2\right.}{4acx^4}\right.}{7ac}}{10c}}{13c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$$\Big\downarrow \text{1053}$$

$$\frac{\frac{-\frac{2\left(-\frac{-\frac{\int \frac{1820a^3d^3(bc-ad)^2x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}\left(1820a^4d^4-2275a^3bcd^3+260a^2b^2c^2d^2+78ab^3c^3d+36b^4c^4\right)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(455a^3d^3-520a^2bcd^2+26ab^2c^2\right.}{4acx^4}\right.}{7ac}}{10c}}{13c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{-\frac{2\left(-\frac{-\frac{1820a^2d^3(bc-ad)^2\int \frac{x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{c}-\frac{\sqrt[3]{a+bx^3}\left(1820a^4d^4-2275a^3bcd^3+260a^2b^2c^2d^2+78ab^3c^3d+36b^4c^4\right)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}\left(455a^3d^3-5\right.}{4}\right.}{7ac}}{10c}}{13c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

$$\Big\downarrow \text{992}$$

3.708. $\int \frac{\left(a+bx^3\right)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx$

$$-\frac{\frac{2\left(-\frac{\sqrt[3]{a+bx^3}\left(455a^3d^3-520a^2bcd^2+26ab^2c^2d+12b^3c^3\right)}{4acx^4}-\frac{-\frac{1820a^2d^3(bc-ad)^2\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc}}{\sqrt[3]{}}\right.}{2\sqrt[3]{}}\right.}{c}}{}\right.}{7ac}}{10c}$$

$$\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

input
```
Int[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]
```

output
```
-1/13*(a*(a + b*x^3)^(1/3))/(c*x^13) + (-1/10*((14*b*c - 13*a*d)*(a + b*x^
3)^(1/3))/(c*x^10) + (-1/7*(((4*b^2*c)/a - 143*b*d + (130*a*d^2)/c)*(a + b
*x^3)^(1/3))/x^7 - (2*(-1/4*((12*b^3*c^3 + 26*a*b^2*c^2*d - 520*a^2*b*c*d^
2 + 455*a^3*d^3)*(a + b*x^3)^(1/3))/(a*c*x^4) - (-(((36*b^4*c^4 + 78*a*b^3
*c^3*d + 260*a^2*b^2*c^2*d^2 - 2275*a^3*b*c*d^3 + 1820*a^4*d^4)*(a + b*x^3
)^(1/3))/(a*c*x)) - (1820*a^2*d^3*(b*c - a*d)^2*(-(ArcTan[(1 + (2*(b*c - a
*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c -
 a*d)^(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c -
 a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3)))
)/c)/(4*a*c))/(7*a*c))/(10*c))/(13*c)
```

3.708. $\int \frac{\left(a+bx^3\right)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx$

3.708.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.708. $\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx$

3.708.4 Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$-\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\left(\left(\frac{9}{35}b^2x^6-\frac{3}{5}ab\,x^3+a^2\right)\left(b\,x^3+a\right)^2c^4-\frac{13x^3\left(-\frac{3b\,x^3}{7}+a\right)d\left(b\,x^3+a\right)^2a\,c^3}{10}+\frac{13\left(b\,x^3+a\right)^2a^2c^2d^2x^6}{7}-\frac{13a^3d^3x^9(5b\,x^3}{4}\right.}{}$

input
```
int((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/13/((a*d-b*c)/c)^(2/3)*(((a*d-b*c)/c)^(2/3)*((9/35*b^2*x^6-3/5*a*b*x^3+
a^2)*(b*x^3+a)^2*c^4-13/10*x^3*(-3/7*b*x^3+a)*d*(b*x^3+a)^2*a*c^3+13/7*(b*
x^3+a)^2*a^2*c^2*d^2*x^6-13/4*a^3*d^3*x^9*(5*b*x^3+a)*c+13*a^4*d^4*x^12)*c
*(b*x^3+a)^(1/3)+13/6*a^3*d^3*x^13*(a*d-b*c)^2*(2*arctan(1/3*3^(1/2)*(((a*
d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((
a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3
))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))/x^13/c^6/a^3
```

3.708.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.708.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(4/3)/x**14/(d*x**3+c),x)
```

output
```
Timed out
```

3.708.7 Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^{14}}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)
```

3.708.8 Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^{14}}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)
```

3.708.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{x^{14}\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(4/3)/(x^14*(c + d*x^3)), x)
```

3.709 $\int \frac{x^6\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

3.709.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx=\frac{ax^7\sqrt[3]{a+bx^3}\,\mathrm{AppellF1}\left(\frac{7}{3},-\frac{4}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/7*a*x^7*(b*x^3+a)^(1/3)*AppellF1(7/3,-4/3,1,10/3,-b*x^3/a,-d*x^3/c)/c/(1
+b*x^3/a)^(1/3)
```

3.709.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 343 vs. 2(65) = 130.

Time = 9.84 (sec) , antiderivative size = 343, normalized size of antiderivative = 5.28

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx=\frac{x\left(2(a+bx^3)\left(2a^2d^2+3abd(-8c+3dx^3)+b^2(20c^2-8cdx^3+5d^2x^6)\right)-\frac{(20b^3c^3-30a}{}\right.}{}$$

input
```
Integrate[(x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(x*(2*(a + b*x^3)*(2*a^2*d^2 + 3*a*b*d*(-8*c + 3*d*x^3) + b^2*(20*c^2 - 8*
c*d*x^3 + 5*d^2*x^6)) - ((20*b^3*c^3 - 30*a*b^2*c^2*d + 8*a^2*b*c*d^2 + a^
3*d^3)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
-((d*x^3)/c)])/c + (16*a^2*c^2*(10*b^2*c^2 - 12*a*b*c*d + a^2*d^2)*AppellF
1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4
/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(80*b*d^3*(a + b*x^3)^(2/3))
```

3.709.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \left(a+b x^3\right)^{4/3}}{c+d x^3} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{a \sqrt[3]{a+b x^3} \int \frac{x^6 \left(\frac{b x^3}{a}+1\right)^{4/3}}{d x^3+c}dx}{\sqrt[3]{\frac{b x^3}{a}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{a x^7 \sqrt[3]{a+b x^3} \operatorname{AppellF1}\left(\frac{7}{3},-\frac{4}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right)}{7 c \sqrt[3]{\frac{b x^3}{a}+1}}$$

input
```
Int[(x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(a*x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -4/3, 1, 10/3, -((b*x^3)/a), -((d*x
^3)/c)])/(7*c*(1 + (b*x^3)/a)^(1/3))
```

3.709.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.709.4 Maple [F]

$$\int \frac{x^{6} \left(b\,x^{3}+a \right)^{\frac{4}{3}}}{d\,x^{3}+c}d x$$

input
```
int(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

3.709.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx=\text{Timed out}$$

input
```
integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.709.6 Sympy [F]

$$\int \frac{x^6 (a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^6 (a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**6*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**6*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.709.7 Maxima [F]

$$\int \frac{x^6 (a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}} x^6}{dx^3+c}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*x^6/(d*x^3 + c), x)
```

3.709.8 Giac [F]

$$\int \frac{x^6 (a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}} x^6}{dx^3+c}\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*x^6/(d*x^3 + c), x)
```

3.709.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{x^6\,{\left(b\,x^3+a\right)}^{4/3}}{d\,x^3+c}\,dx$$

input
```
int((x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

output
```
int((x^6*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

3.710 $\int \frac{x^3\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

3.710.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx=\frac{ax^4\sqrt[3]{a+bx^3}\operatorname{AppellF1}\left(\frac{4}{3},-\frac{4}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/4*a*x^4*(b*x^3+a)^(1/3)*AppellF1(4/3,-4/3,1,7/3,-b*x^3/a,-d*x^3/c)/c/(1+
b*x^3/a)^(1/3)
```

3.710.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(65) = 130.

Time = 9.29 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.31

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx=\frac{x\Biggl(4(a+bx^3)\left(-5bc+6ad+2bdx^3\right)+\frac{(10b^2c^2-15abcd+4a^2d^2)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\bigl(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-}{c}}{}$$

input
```
Integrate[(x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(x*(4*(a + b*x^3)*(-5*b*c + 6*a*d + 2*b*d*x^3) + ((10*b^2*c^2 - 15*a*b*c*d
 + 4*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^
3)/a), -((d*x^3)/c)])/c + (16*a^2*c^2*(-5*b*c + 6*a*d)*AppellF1[1/3, 2/3,
1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2,
7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x
^3)/a), -((d*x^3)/c)]))))/(40*d^2*(a + b*x^3)^(2/3))
```

3.710.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+bx^3\right)^{4/3}}{c+dx^3} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{a \sqrt[3]{a+bx^3} \int \frac{x^3 \left(\frac{bx^3}{a}+1\right)^{4/3}}{dx^3+c}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{ax^4 \sqrt[3]{a+bx^3} \operatorname{AppellF1}\left(\frac{4}{3},-\frac{4}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

output
```
(a*x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -4/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(4*c*(1 + (b*x^3)/a)^(1/3))
```

3.710.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.710.4 Maple [F]

$$\int \frac{x^3 (b\,x^3+a)^{\frac{4}{3}}}{d\,x^3+c}dx$$

input
```
int(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

3.710.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.710.6 Sympy [F]

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{x^3(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate(x**3*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**3*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.710.7 Maxima [F]

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}x^3}{dx^3+c}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*x^3/(d*x^3 + c), x)
```

3.710.8 Giac [F]

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}x^3}{dx^3+c}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*x^3/(d*x^3 + c), x)
```

3.710.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^3\,(b\,x^3+a)^{4/3}}{d\,x^3+c}\,dx$$

input
```
int((x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

output
```
int((x^3*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

3.711 $\int \frac{\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

3.711.1 Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx=\frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/
a)^(1/3)
```

3.711.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 346 vs. $2(60)=120$.

Time = 0.30 (sec) , antiderivative size = 346, normalized size of antiderivative = 5.77

$$\int \frac{\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx=\frac{x\Bigg(\frac{b(-2bc+3ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{4\Big(-4ac\left(2a^2d+abdx^3+b^2x^3\left(c+dx^3\right)\right)\text{AppellF1}}{\left(c+dx^3\right)\Big(-4ac\,\text{AppellF1}\Big(}}{8d}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

output
```
(x*((b*(-2*b*c + 3*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(-4*a*c*(2*a^2*d + a*b*d*x^3 + b^2*
x^3*(c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] +
b*x^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/
a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])))/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/
(8*d*(a + b*x^3)^(2/3))
```

3.711.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{c+dx^3}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{a\sqrt[3]{a+bx^3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{4/3}}{dx^3+c}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

output
```
(a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*(1 + (b*x^3)/a)^(1/3))
```

3.711.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.711.4 Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{4}{3}}}{d\,x^3+c}dx$$

input
```
int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

3.711.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.711.6 Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.711.7 Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

3.711.8 Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

3.711.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{4/3}}{d\,x^3+c} \,d x$$

input

```
int((a + b*x^3)^(4/3)/(c + d*x^3),x)
```

output

```
int((a + b*x^3)^(4/3)/(c + d*x^3), x)
```

3.712 $\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx$

3.712.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx=-\frac{a\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{4}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/2*a*(b*x^3+a)^(1/3)*AppellF1(-2/3,-4/3,1,1/3,-b*x^3/a,-d*x^3/c)/c/x^2/(
1+b*x^3/a)^(1/3)
```

3.712.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 341 vs. 2(65) = 130.

Time = 10.36 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.25

$$\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx=$$
$$-\frac{b(-2bc+ad)x^6\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4ac\left(-4ac(ac-2bcx^3+3adx^3+bdx^6)\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3}\right.\right.}{(c+dx^3)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\right.\right.}}{8c^2x^2(a+bx^3)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x]
```

output
```
-1/8*(b*(-2*b*c + a*d)*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), -((d*x^3)/c)] + (4*a*c*(-4*a*c*(a*c - 2*b*c*x^3 + 3*a*d*x^
3 + b*d*x^6)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*
(a + b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -(
(d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
)))/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
+ 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(c^2*x^
2*(a + b*x^3)^(2/3))
```

3.712.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^3\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{a\sqrt[3]{a+bx^3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{4/3}}{x^3\left(dx^3+c\right)}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{a\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{4}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x]
```

output
```
-1/2*(a*(a + b*x^3)^(1/3)*AppellF1[-2/3, -4/3, 1, 1/3, -((b*x^3)/a), -((d*
x^3)/c)])/(c*x^2*(1 + (b*x^3)/a)^(1/3))
```

3.712.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.712.4 Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{x^{3}\left(d\,x^{3}+c\right)}d x$$

input
```
int((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x)
```

3.712.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.712.6 Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^3\,(c+dx^3)}\,dx=\int \frac{(a+bx^3)^{\frac{4}{3}}}{x^3\,(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x**3/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x**3*(c + d*x**3)), x)
```

3.712.7 Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^3\,(c+dx^3)}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^3), x)
```

3.712.8 Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^3\,(c+dx^3)}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^3}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^3), x)
```

3.712.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx=\int \frac{(b\,x^3+a)^{4/3}}{x^3\,(d\,x^3+c)}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(4/3)/(x^3*(c + d*x^3)), x)
```

3.713 $\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx$

3.713.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx = -\frac{a\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{5}{3},-\frac{4}{3},1,-\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/5*a*(b*x^3+a)^(1/3)*AppellF1(-5/3,-4/3,1,-2/3,-b*x^3/a,-d*x^3/c)/c/x^5/
(1+b*x^3/a)^(1/3)
```

3.713.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 286 vs. 2(65) = 130.

Time = 10.38 (sec) , antiderivative size = 286, normalized size of antiderivative = 4.40

$$\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx = \frac{-\frac{4(a+bx^3)(2ac+6bcx^3-5adx^3)}{c^2x^5}+\frac{bd(-6bc+5ad)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}-\frac{}{c(c+dx^3)\big(-}}{40\,(a}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x]
```

output
```
((-4*(a + b*x^3)*(2*a*c + 6*b*c*x^3 - 5*a*d*x^3))/(c^2*x^5) + (b*d*(-6*b*c
 + 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a
), -((d*x^3)/c)])/c^3 - (16*a*(4*b^2*c^2 - 15*a*b*c*d + 10*a^2*d^2)*x*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(c + d*x^3)*(-4*a*c
*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*Appel
lF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/
3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(40*(a + b*x^3)^(2/3))
```

3.713.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^6\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{a\sqrt[3]{a+bx^3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{4/3}}{x^6\left(dx^3+c\right)}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{a\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{5}{3},-\frac{4}{3},1,-\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x]
```

output
```
-1/5*(a*(a + b*x^3)^(1/3)*AppellF1[-5/3, -4/3, 1, -2/3, -((b*x^3)/a), -((d
*x^3)/c)])/(c*x^5*(1 + (b*x^3)/a)^(1/3))
```

3.713.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.713.4 Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{x^{6}\left(d\,x^{3}+c\right)}dx$$

input
```
int((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x)
```

3.713.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^6\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.713.6 Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^6\,(c+dx^3)}\,dx=\int \frac{(a+bx^3)^{\frac{4}{3}}}{x^6\,(c+dx^3)}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/x**6/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(x**6*(c + d*x**3)), x)
```

3.713.7 Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^6\,(c+dx^3)}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^6), x)
```

3.713.8 Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^6\,(c+dx^3)}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^6}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^6), x)
```

3.713.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^6 (c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{x^6\,\left(d\,x^3+c\right)}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x)
```

output
```
int((a + b*x^3)^(4/3)/(x^6*(c + d*x^3)), x)
```

3.714 $\int \frac{x^{14}}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

3.714.1 Optimal result

Integrand size = 24, antiderivative size = 290

$$\begin{aligned}\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = &-\frac{(bc+ad)\,(b^2c^2+a^2d^2)\,(a+bx^3)^{2/3}}{2b^4d^4}\\ &+\frac{(b^2c^2+2abcd+3a^2d^2)\,(a+bx^3)^{5/3}}{5b^4d^3}-\frac{(bc+3ad)\,(a+bx^3)^{8/3}}{8b^4d^2}\\ &+\frac{(a+bx^3)^{11/3}}{11b^4d}-\frac{c^4\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{14/3}\sqrt[3]{bc-ad}}\\ &+\frac{c^4\log(c+dx^3)}{6d^{14/3}\sqrt[3]{bc-ad}}-\frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}\sqrt[3]{bc-ad}}\end{aligned}$$

output

```
-1/2*(a*d+b*c)*(a^2*d^2+b^2*c^2)*(b*x^3+a)^(2/3)/b^4/d^4+1/5*(3*a^2*d^2+2*
a*b*c*d+b^2*c^2)*(b*x^3+a)^(5/3)/b^4/d^3-1/8*(3*a*d+b*c)*(b*x^3+a)^(8/3)/b
^4/d^2+1/11*(b*x^3+a)^(11/3)/b^4/d+1/6*c^4*ln(d*x^3+c)/d^(14/3)/(-a*d+b*c)
^(1/3)-1/2*c^4*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(14/3)/(-a*d
+b*c)^(1/3)-1/3*c^4*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/
3))*3^(1/2))/d^(14/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

3.714.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.06

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$=\frac{-3d^{2/3}\sqrt[3]{bc-ad}(a+bx^3)^{2/3}\left(81a^3d^3+9a^2bd^2(11c-6dx^3)+3ab^2d(44c^2-22cdx^3+15d^2x^6)+b^3(220c^3\right.}{}$$

input
```
Integrate[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3)*(81*a^3*d^3 + 9*a^2*b*d^2*
(11*c - 6*d*x^3) + 3*a*b^2*d*(44*c^2 - 22*c*d*x^3 + 15*d^2*x^6) + b^3*(220
*c^3 - 88*c^2*d*x^3 + 55*c*d^2*x^6 - 40*d^3*x^9)) - 440*Sqrt[3]*b^4*c^4*Ar
cTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 440*
b^4*c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 220*b^4*c^4*L
og[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/
3)*(a + b*x^3)^(2/3)])/(1320*b^4*d^(14/3)*(b*c - a*d)^(1/3))
```

3.714.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^{12}}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{c^4}{d^4\sqrt[3]{bx^3+a}\,(dx^3+c)}+\frac{(bx^3+a)^{8/3}}{b^3d}+\frac{(-bc-3ad)\,(bx^3+a)^{5/3}}{b^3d^2}+\frac{(b^2c^2+2abdc+3a^2d^2)\,(bx^3+a)^{2/3}}{b^3d^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{3(a+bx^3)^{2/3}\,(ad+bc)\,(a^2d^2+b^2c^2)}{2b^4d^4}+\frac{3(a+bx^3)^{5/3}\,(3a^2d^2+2abcd+b^2c^2)}{5b^4d^3}-\frac{\sqrt{3}c^4\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a}}{\sqrt[3]{bc-}}}{\sqrt{3}}\right.}{d^{14/3}\sqrt[3]{bc-ad}}\right.$$

input
```
Int[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((-3*(b*c + a*d)*(b^2*c^2 + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^4*d^4) + (3*(
b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*(a + b*x^3)^(5/3))/(5*b^4*d^3) - (3*(b*c
+ 3*a*d)*(a + b*x^3)^(8/3))/(8*b^4*d^2) + (3*(a + b*x^3)^(11/3))/(11*b^4*d
) - (Sqrt[3]*c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/
3))/Sqrt[3]])/(d^(14/3)*(b*c - a*d)^(1/3)) + (c^4*Log[c + d*x^3])/(2*d^(14
/3)*(b*c - a*d)^(1/3)) - (3*c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3
)^(1/3)])/(2*d^(14/3)*(b*c - a*d)^(1/3)))/3
```

3.714.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.714. $\int \frac{x^{14}}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

3.714.4 Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{\frac{243\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(\frac{\left(-40d^{3}x^{9}+55c\,d^{2}x^{6}-88c^{2}d\,x^{3}+220c^{3}\right)b^{3}}{81}+\frac{44da\left(\frac{15}{44}d^{2}x^{6}-\frac{1}{2}cd\,x^{3}+c^{2}\right)b^{2}}{27}+\frac{11\left(-\frac{6d\,x^{3}}{11}+c\right)d^{2}a^{2}b}{9}+a^{3}d^{3}\right)d\left(b\,x^{3}+\right.}{220}}{}$

input
```
int(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(1/d*(a*d-b*c))^(1/3)*(243/220*(1/d*(a*d-b*c))^(1/3)*(1/81*(-40*d^3*x
^9+55*c*d^2*x^6-88*c^2*d*x^3+220*c^3)*b^3+44/27*d*a*(15/44*d^2*x^6-1/2*c*d
*x^3+c^2)*b^2+11/9*(-6/11*d*x^3+c)*d^2*a^2*b+a^3*d^3)*d*(b*x^3+a)^(2/3)+b^
4*c^4*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/
d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^
3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/
3))))/d^5/b^4
```

3.714.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1004, normalized size of antiderivative = 3.46

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/1320*(220*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(2/3)*d^2 +
(-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) -
 440*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2
+ a*d^3)^(1/3)) + 660*sqrt(1/3)*(b^5*c^5*d - a*b^4*c^4*d^2)*sqrt((-b*c*d^2
 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1
/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c
*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3
)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^
3 + c)) - 3*(220*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 - 18*
a^3*b*c*d^5 - 81*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b^4*c^2*
d^4 - 2*a*b^3*c*d^5 - 9*a^2*b^2*d^6)*x^6 - 2*(44*b^4*c^3*d^3 - 11*a*b^3*c^
2*d^4 - 6*a^2*b^2*c*d^5 - 27*a^3*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(b^5*c*d^6
 - a*b^4*d^7), 1/1320*(220*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a
)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a
*d^3)^(2/3)) - 440*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(1/3)*
d - (-b*c*d^2 + a*d^3)^(1/3)) + 1320*sqrt(1/3)*(b^5*c^5*d - a*b^4*c^4*d^2)
*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 +
a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c
 - a*d))/d) - 3*(220*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 -
 18*a^3*b*c*d^5 - 81*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b...
```

3.714.6 Sympy [F]

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**14/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**14/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.714.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.714.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.57

$$\begin{aligned}\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = {} & -\frac{b^{48}c^4d^7\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{49}cd^{11}-ab^{48}d^{12})} \\ & -\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^4\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^6-\sqrt{3}ad^7} \\ & +\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^4\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^6-ad^7)} \\ & -\frac{220\,(bx^3+a)^{\frac{2}{3}}b^{43}c^3d^7-88\,(bx^3+a)^{\frac{5}{3}}b^{42}c^2d^8+220\,(bx^3+a)^{\frac{2}{3}}ab^{42}c^2d^8+55\,(bx^3+a)^{\frac{8}{3}}b^{41}cd^9-176\,(bx}{}\end{aligned}$$

input
```
integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*b^48*c^4*d^7*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*
c - a*d)/d)^(1/3)))/(b^49*c*d^11 - a*b^48*d^12) - (-b*c*d^2 + a*d^3)^(2/3)
*c^4*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(
b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^6 - sqrt(3)*a*d^7) + 1/6*(-b*c*d^2 + a
*d^3)^(2/3)*c^4*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)
^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^6 - a*d^7) - 1/440*(220*(b*x^3 + a
)^(2/3)*b^43*c^3*d^7 - 88*(b*x^3 + a)^(5/3)*b^42*c^2*d^8 + 220*(b*x^3 + a)
^(2/3)*a*b^42*c^2*d^8 + 55*(b*x^3 + a)^(8/3)*b^41*c*d^9 - 176*(b*x^3 + a)^
(5/3)*a*b^41*c*d^9 + 220*(b*x^3 + a)^(2/3)*a^2*b^41*c*d^9 - 40*(b*x^3 + a)
^(11/3)*b^40*d^10 + 165*(b*x^3 + a)^(8/3)*a*b^40*d^10 - 264*(b*x^3 + a)^(5
/3)*a^2*b^40*d^10 + 220*(b*x^3 + a)^(2/3)*a^3*b^40*d^10)/(b^44*d^11)
```

3.714.9 Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.51

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \left(\frac{6\,a^2}{5\,b^4\,d}+\frac{\left(\frac{4\,a}{b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{b^8\,d^2}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{5\,b^4\,d}\right)\,{\left(b\,x^3+a\right)}^{5/3}$$
$$-\left(\frac{a}{2\,b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{8\,b^8\,d^2}\right)\,{\left(b\,x^3+a\right)}^{8/3}$$
$$-{\left(b\,x^3+a\right)}^{2/3}\,\left(\frac{2\,a^3}{b^4\,d}+\frac{\left(\frac{6\,a^2}{b^4\,d}+\frac{\left(\frac{4\,a}{b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{b^8\,d^2}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{b^4\,d}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{2\,b^4\,d}\right)+\frac{{\left(b\,x^3+a\right)}^{11/3}}{11\,b^4\,d}+\frac{c^4\,\ln\left(\frac{c^8\,(b\,x^3}{d}\right.}{3\,d^{14/}}$$

input
```
int(x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
((6*a^2)/(5*b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c
 - a*b^4*d))/(5*b^4*d))*(a + b*x^3)^(5/3) - (a/(2*b^4*d) + (b^5*c - a*b^4*
d)/(8*b^8*d^2))*(a + b*x^3)^(8/3) - (a + b*x^3)^(2/3)*((2*a^3)/(b^4*d) + (
((6*a^2)/(b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c -
 a*b^4*d))/(b^4*d))*(b^5*c - a*b^4*d))/(2*b^4*d)) + (a + b*x^3)^(11/3)/(11
*b^4*d) + (c^4*log((c^8*(a + b*x^3)^(1/3))/d^7 - (c^8*(a*d - b*c)^(1/3))/d
^(22/3)))/(3*d^(14/3)*(a*d - b*c)^(1/3)) - (log((c^8*(a + b*x^3)^(1/3))/d^
7 - (c^8*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(22/3)))*(3^(1/2)*c^4*
1i + c^4))/(6*d^(14/3)*(a*d - b*c)^(1/3)) + (c^4*log((c^8*(a + b*x^3)^(1/3
))/d^7 - (c^8*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(22/3)))*((3^(1/2
)*1i)/6 - 1/6))/(d^(14/3)*(a*d - b*c)^(1/3))
```

3.715 $\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

3.715.1 Optimal result

Integrand size = 24, antiderivative size = 244

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\,(a+bx^3)^{5/3}}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} + \frac{c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}\sqrt[3]{bc-ad}} - \frac{c^3\log\left(c+dx^3\right)}{6d^{11/3}\sqrt[3]{bc-ad}} + \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}\sqrt[3]{bc-ad}}$$

output
```
1/2*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(2/3)/b^3/d^3-1/5*(2*a*d+b*c)*(b*x
^3+a)^(5/3)/b^3/d^2+1/8*(b*x^3+a)^(8/3)/b^3/d-1/6*c^3*ln(d*x^3+c)/d^(11/3)
/(-a*d+b*c)^(1/3)+1/2*c^3*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(
11/3)/(-a*d+b*c)^(1/3)+1/3*c^3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a
*d+b*c)^(1/3))*3^(1/2))/d^(11/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

3.715.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.08

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{3d^{2/3}\sqrt[3]{bc-ad}(a+bx^3)^{2/3}\left(9a^2d^2-6abd(-2c+dx^3)+b^2(20c^2-8cdx^3+5d^2x^6)\right)+40\sqrt{3}b^3c^3\arctan\left(\right.}{}$$

input
```
Integrate[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3)*(9*a^2*d^2 - 6*a*b*d*(-2*c
+ d*x^3) + b^2*(20*c^2 - 8*c*d*x^3 + 5*d^2*x^6)) + 40*Sqrt[3]*b^3*c^3*ArcT
an[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 40*b^3
*c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - 20*b^3*c^3*Log[(
b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(
a + b*x^3)^(2/3)])/(120*b^3*d^(11/3)*(b*c - a*d)^(1/3))
```

3.715.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \color{blue}{948}$$

$$\frac{1}{3}\int \frac{x^9}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx^3$$

$$\Big\downarrow \color{blue}{99}$$

$$\frac{1}{3}\int \left(-\frac{c^3}{d^3\sqrt[3]{bx^3+a}\,(dx^3+c)}+\frac{(bx^3+a)^{5/3}}{b^2d}+\frac{(-bc-2ad)\,(bx^3+a)^{2/3}}{b^2d^2}+\frac{b^2c^2+abdc+a^2d^2}{b^2d^3\sqrt[3]{bx^3+a}}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^3d^3}+\frac{\sqrt{3}c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{11/3}\sqrt[3]{bc-ad}}-\frac{3\left(a+bx^3\right)^{5/3}(2ad+bc)}{5b^3d^2}+\frac{3(a+}{8}\right.$$

input
```
Int[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((3*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^3*d^3) - (3*(b*c
 + 2*a*d)*(a + b*x^3)^(5/3))/(5*b^3*d^2) + (3*(a + b*x^3)^(8/3))/(8*b^3*d)
 + (Sqrt[3]*c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]])/(d^(11/3)*(b*c - a*d)^(1/3)) - (c^3*Log[c + d*x^3])/(2*d^(11/
3)*(b*c - a*d)^(1/3)) + (3*c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)
^(1/3)])/(2*d^(11/3)*(b*c - a*d)^(1/3)))/3
```

3.715.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.715.4 Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{\frac{27\left(\frac{\left(5d^{2}x^{6}-8cdx^{3}+20c^{2}\right)b^{2}}{9}+\frac{4\left(-\frac{dx^{3}}{2}+c\right)dab}{3}+a^{2}d^{2}\right)\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}d\left(bx^{3}+a\right)^{\frac{2}{3}}}{20}+b^{3}c^{3}\left(-2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right.}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}b^{3}d^{4}}$

input
```
int(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/(1/d*(a*d-b*c))^(1/3)*(27/20*(1/9*(5*d^2*x^6-8*c*d*x^3+20*c^2)*b^2+4/3
*(-1/2*d*x^3+c)*d*a*b+a^2*d^2)*(1/d*(a*d-b*c))^(1/3)*d*(b*x^3+a)^(2/3)+b^3
*c^3*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d
*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3
+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3
))))/b^3/d^4
```

3.715.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 873, normalized size of antiderivative = 3.58

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$=\left[-\frac{20\left(bcd^{2}-ad^{3}\right)^{\frac{2}{3}}b^{3}c^{3}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}d^{2}-\left(bcd^{2}-ad^{3}\right)^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}d+\left(bcd^{2}-ad^{3}\right)^{\frac{2}{3}}\right)-40\left(bcd^{2}-ad\right.}{}\right.$$

$$-\frac{20\left(bcd^{2}-ad^{3}\right)^{\frac{2}{3}}b^{3}c^{3}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}d^{2}-\left(bcd^{2}-ad^{3}\right)^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}d+\left(bcd^{2}-ad^{3}\right)^{\frac{2}{3}}\right)-40\left(bcd^{2}-ad^{3}\right.}{}$$

input
```
integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/120*(20*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 + a)^(2/3)*d^2 - (b
*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 40*
(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3
)^(1/3)) - 60*sqrt(1/3)*(b^4*c^4*d - a*b^3*c^3*d^2)*sqrt(-(b*c*d^2 - a*d^3
)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(
b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^
2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b
*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3
*(20*b^3*c^3*d^2 - 8*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 - 9*a^3*d^5 + 5*(b^3*c*
d^4 - a*b^2*d^5)*x^6 - 2*(4*b^3*c^2*d^3 - a*b^2*c*d^4 - 3*a^2*b*d^5)*x^3)*
(b*x^3 + a)^(2/3))/(b^4*c*d^5 - a*b^3*d^6), -1/120*(20*(b*c*d^2 - a*d^3)^(
2/3)*b^3*c^3*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 +
a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 40*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3
*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 120*sqrt(1/3)*(b^4*c
^4*d - a*b^3*c^3*d^2)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqr
t(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a
*d^3)^(1/3)/(b*c - a*d))/d) - 3*(20*b^3*c^3*d^2 - 8*a*b^2*c^2*d^3 - 3*a^2*
b*c*d^4 - 9*a^3*d^5 + 5*(b^3*c*d^4 - a*b^2*d^5)*x^6 - 2*(4*b^3*c^2*d^3 - a
*b^2*c*d^4 - 3*a^2*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(b^4*c*d^5 - a*b^3*d^6)]
```

3.715.6 Sympy [F]

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**11/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**11/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.715.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.715.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.52

$$\begin{aligned}\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx &= \frac{b^{27}c^3d^5\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{28}cd^8-ab^{27}d^9)} \\ &\quad+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^5-\sqrt{3}ad^6} \\ &\quad-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^5-ad^6)} \\ &\quad+\frac{20\,(bx^3+a)^{\frac{2}{3}}b^{23}c^2d^5-8\,(bx^3+a)^{\frac{5}{3}}b^{22}cd^6+20\,(bx^3+a)^{\frac{2}{3}}ab^{22}cd^6+5\,(bx^3+a)^{\frac{8}{3}}b^{21}d^7-16\,(bx^3+a)^{\frac{5}{3}}a}{40\,b^{24}d^8}\end{aligned}$$

input

```
integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b^27*c^3*d^5*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^28*c*d^8 - a*b^27*d^9) + (-b*c*d^2 + a*d^3)^(2/3)*c^
3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c
 - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^5 - sqrt(3)*a*d^6) - 1/6*(-b*c*d^2 + a*d^
3)^(2/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1
/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^5 - a*d^6) + 1/40*(20*(b*x^3 + a)^(2/
3)*b^23*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^22*c*d^6 + 20*(b*x^3 + a)^(2/3)*a*
b^22*c*d^6 + 5*(b*x^3 + a)^(8/3)*b^21*d^7 - 16*(b*x^3 + a)^(5/3)*a*b^21*d^
7 + 20*(b*x^3 + a)^(2/3)*a^2*b^21*d^7)/(b^24*d^8)
```

3.715.9 Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.39

$$\begin{aligned}\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx &= \left(\frac{3\,a^2}{2\,b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{2\,b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{2/3}\\ &\quad-\left(\frac{3\,a}{5\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{5\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{5/3}\\ &\quad+\frac{{\left(b\,x^3+a\right)}^{8/3}}{8\,b^3\,d}-\frac{c^3\,\ln\left(\frac{c^6\,{\left(b\,x^3+a\right)}^{1/3}}{d^5}+\frac{b\,c^7-a\,c^6\,d}{d^{16/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{3\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{1/3}}\\ &\quad+\frac{\ln\left(\frac{c^6\,{\left(b\,x^3+a\right)}^{1/3}}{d^5}-\frac{c^6\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(a\,d-b\,c\right)}^{1/3}}{4\,d^{16/3}}\right)\,\left(c^3+\sqrt{3}\,c^3\,1{}\mathrm{i}\right)}{6\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{1/3}}\\ &\quad-\frac{c^3\,\ln\left(\frac{c^6\,{\left(b\,x^3+a\right)}^{1/3}}{d^5}+\frac{c^6\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{16/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{1/3}}\end{aligned}$$

input
```
int(x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output

```
((3*a^2)/(2*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
 - a*b^3*d))/(2*b^3*d))*(a + b*x^3)^(2/3) - ((3*a)/(5*b^3*d) + (b^4*c - a*
b^3*d)/(5*b^6*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^3*d) - (c^3
*log((c^6*(a + b*x^3)^(1/3))/d^5 + (b*c^7 - a*c^6*d)/(d^(16/3)*(a*d - b*c)
^(2/3))))/(3*d^(11/3)*(a*d - b*c)^(1/3)) + (log((c^6*(a + b*x^3)^(1/3))/d^
5 - (c^6*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(16/3)))*(3^(1/2)*c^3*
1i + c^3))/(6*d^(11/3)*(a*d - b*c)^(1/3)) - (c^3*log((c^6*(a + b*x^3)^(1/3
))/d^5 + (c^6*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/d^(16/3))*((3^(1/2
)*1i)/2 - 1/2))/(3*d^(11/3)*(a*d - b*c)^(1/3))
```

3.715. $\int \frac{x^{11}}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

3.716 $\int \frac{x^8}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

3.716.1 Optimal result

Integrand size = 24, antiderivative size = 203

$$\begin{aligned}\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = &-\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2}+\frac{\left(a+bx^3\right)^{5/3}}{5b^2d} \\ &-\frac{c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}\sqrt[3]{bc-ad}}+\frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}} \\ &-\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}}\end{aligned}$$

output
```
-1/2*(a*d+b*c)*(b*x^3+a)^(2/3)/b^2/d^2+1/5*(b*x^3+a)^(5/3)/b^2/d+1/6*c^2*l
n(d*x^3+c)/d^(8/3)/(-a*d+b*c)^(1/3)-1/2*c^2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b
*x^3+a)^(1/3))/d^(8/3)/(-a*d+b*c)^(1/3)-1/3*c^2*arctan(1/3*(1-2*d^(1/3)*(b
*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(8/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

3.716.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.14

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{-3d^{2/3}\sqrt[3]{bc-ad}(a+bx^3)^{2/3}\left(5bc+3ad-2bdx^3\right)-10\sqrt{3}b^2c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-10b^2c^2\log\Bigl(}{30b^2d^{8/3}\sqrt[3]{bc}}$$

input
```
Integrate[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3)*(5*b*c + 3*a*d - 2*b*d*x^3
) - 10*Sqrt[3]*b^2*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*
d)^(1/3))/Sqrt[3]] - 10*b^2*c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3
)^(1/3)] + 5*b^2*c^2*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a
+ b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(30*b^2*d^(8/3)*(b*c - a*d)^(
1/3))
```

3.716.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{c^2}{d^2\sqrt[3]{bx^3+a}\,(dx^3+c)}+\frac{(bx^3+a)^{2/3}}{bd}+\frac{-bc-ad}{bd^2\sqrt[3]{bx^3+a}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{8/3}\sqrt[3]{bc-ad}}-\frac{3\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^2d^2}+\frac{3\left(a+bx^3\right)^{5/3}}{5b^2d}+\frac{c^2\log\left(c+dx^3\right)}{2d^{8/3}\sqrt[3]{bc-ad}}-\frac{3c^2\log}{}\right.$$

input
```
Int[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((-3*(b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^2*d^2) + (3*(a + b*x^3)^(5/3))/(5
*b^2*d) - (Sqrt[3]*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*
d)^(1/3))/Sqrt[3]])/(d^(8/3)*(b*c - a*d)^(1/3)) + (c^2*Log[c + d*x^3])/(2*
d^(8/3)*(b*c - a*d)^(1/3)) - (3*c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b
*x^3)^(1/3)])/(2*d^(8/3)*(b*c - a*d)^(1/3)))/3
```

3.716.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.716.4 Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{\frac{9\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}} d\left(\frac{\left(-2d\,x^{3}+5c\right)b}{3}+ad\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{5}+b^{2}c^{2}\left(-2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\right.\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}} b^{2}d^{3}}$

input
```
int(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(9/5*(1/d*(a*d-b*c))^(1/3)*d*(1/3*(-2*d*x^3+5*c)*b+a*d)*(b*x^3+a)^(2/
3)+b^2*c^2*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3)
)/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*
(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c)
)^(1/3))))/(1/d*(a*d-b*c))^(1/3)/b^2/d^3
```

3.716.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(164) = 328$.

Time = 0.32 (sec) , antiderivative size = 768, normalized size of antiderivative = 3.78

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \left[\frac{5\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}b^{2}c^{2}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}d^{2}+\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}d+\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}\right)-10\left(-bcd^{2}\right.}{}\right.$$

input
```
integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/30*(5*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*
c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*
(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d
^3)^(1/3)) + 15*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((-b*c*d^2 + a*d
^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2
*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a
*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3
)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)
) - 3*(5*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3*a^2*d^4 - 2*(b^2*c*d^3 - a*b*d^4)*x
^3)*(b*x^3 + a)^(2/3))/(b^3*c*d^4 - a*b^2*d^5), 1/30*(5*(-b*c*d^2 + a*d^3)
^(2/3)*b^2*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3
 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*(-b*c*d^2 + a*d^3)^(2/3)*b^
2*c^2*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) + 30*sqrt(1/3)*(
b^3*c^3*d - a*b^2*c^2*d^2)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arc
tan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b
*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d) - 3*(5*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3
*a^2*d^4 - 2*(b^2*c*d^3 - a*b*d^4)*x^3)*(b*x^3 + a)^(2/3))/(b^3*c*d^4 - a*
b^2*d^5)]
```

3.716.6 Sympy [F]

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**8/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**8/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.716.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.716.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.54

$$\begin{aligned}
\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx
&= -\frac{b^{12}c^2d^3\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{13}cd^5-ab^{12}d^6)} \\
&\quad - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^4-\sqrt{3}ad^5} \\
&\quad + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^4-ad^5)} \\
&\quad - \frac{5\,(bx^3+a)^{\frac{2}{3}}b^9cd^3-2\,(bx^3+a)^{\frac{5}{3}}b^8d^4+5\,(bx^3+a)^{\frac{2}{3}}ab^8d^4}{10\,b^{10}d^5}
\end{aligned}$$

input
```
integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*b^12*c^2*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*
c - a*d)/d)^(1/3)))/(b^13*c*d^5 - a*b^12*d^6) - (-b*c*d^2 + a*d^3)^(2/3)*c
^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*
c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^4 - sqrt(3)*a*d^5) + 1/6*(-b*c*d^2 + a*d
^3)^(2/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(
1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^4 - a*d^5) - 1/10*(5*(b*x^3 + a)^(2/
3)*b^9*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^8*d^4 + 5*(b*x^3 + a)^(2/3)*a*b^8*d^4
)/(b^10*d^5)
```

3.716.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.32

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\left(b\,x^3+a\right)^{5/3}}{5\,b^2\,d}-\left(\frac{a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{2\,b^4\,d^2}\right)\,\left(b\,x^3+a\right)^{2/3}$$
$$+\frac{c^2\,\ln\left(\frac{c^4\,\left(b\,x^3+a\right)^{1/3}}{d^3}+\frac{b\,c^5-a\,c^4\,d}{d^{10/3}\,\left(a\,d-b\,c\right)^{2/3}}\right)}{3\,d^{8/3}\,\left(a\,d-b\,c\right)^{1/3}}$$
$$-\frac{\ln\left(\frac{c^4\,\left(b\,x^3+a\right)^{1/3}}{d^3}-\frac{c^4\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,\left(a\,d-b\,c\right)^{1/3}}{4\,d^{10/3}}\right)\,\left(c^2+\sqrt{3}\,c^2\,1\mathrm{i}\right)}{6\,d^{8/3}\,\left(a\,d-b\,c\right)^{1/3}}$$
$$+\frac{c^2\,\ln\left(\frac{c^4\,\left(b\,x^3+a\right)^{1/3}}{d^3}-\frac{c^4\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,\left(a\,d-b\,c\right)^{1/3}}{4\,d^{10/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{d^{8/3}\,\left(a\,d-b\,c\right)^{1/3}}$$

input
```
int(x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
(a + b*x^3)^(5/3)/(5*b^2*d) - (a/(b^2*d) + (b^3*c - a*b^2*d)/(2*b^4*d^2))*
(a + b*x^3)^(2/3) + (c^2*log((c^4*(a + b*x^3)^(1/3))/d^3 + (b*c^5 - a*c^4*
d)/(d^(10/3)*(a*d - b*c)^(2/3))))/(3*d^(8/3)*(a*d - b*c)^(1/3)) - (log((c^
4*(a + b*x^3)^(1/3))/d^3 - (c^4*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d
^(10/3)))*(3^(1/2)*c^2*1i + c^2))/(6*d^(8/3)*(a*d - b*c)^(1/3)) + (c^2*log
((c^4*(a + b*x^3)^(1/3))/d^3 - (c^4*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(1/3))/
(4*d^(10/3)))*((3^(1/2)*1i)/6 - 1/6))/(d^(8/3)*(a*d - b*c)^(1/3))
```

3.717 $\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

3.717.1 Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{(a+bx^3)^{2/3}}{2bd} + \frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\log(c+dx^3)}{6d^{5/3}\sqrt[3]{bc-ad}} + \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}$$

output
```
1/2*(b*x^3+a)^(2/3)/b/d-1/6*c*ln(d*x^3+c)/d^(5/3)/(-a*d+b*c)^(1/3)+1/2*c*l
n((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(5/3)/(-a*d+b*c)^(1/3)+1/3*c
*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(5/3
)/(-a*d+b*c)^(1/3)*3^(1/2)
```

3.717.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.21

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{3d^{2/3}\sqrt[3]{bc-ad}(a+bx^3)^{2/3} + 2\sqrt{3}bc\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + 2bc\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) -}{6bd^{5/3}\sqrt[3]{bc-ad}}$$

input
```
Integrate[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3) + 2*Sqrt[3]*b*c*ArcTan[(1 -
 (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*b*c*Log[(b*
c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - b*c*Log[(b*c - a*d)^(2/3) -
d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/
(6*b*d^(5/3)*(b*c - a*d)^(1/3))
```

3.717.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 90, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}dx$$

$$\Big\downarrow \color{blue}{948}$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx^3$$

$$\Big\downarrow \color{blue}{90}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2bd}-\frac{c\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{d}\right)$$

$$\Big\downarrow \color{blue}{68}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2bd}-\frac{c\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}}{d}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2bd}-\frac{c\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{}\right.}{2d^{2/3}\sqrt[3]{bc-a}}\right.}{d}\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2bd}-\frac{c\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{d}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{2/3}}{2bd}-\frac{c\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{d}\right)$$

3.717. $\int \frac{x^5}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

input
```
Int[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((3*(a + b*x^3)^(2/3))/(2*b*d) - (c*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a
+ b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d)^(1/3)))
+ Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/3)) - (3*Log[(b*c - a*d)^(1/3)
+ d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(1/3))))/d)/3
```

3.717.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.717.4 Maple [A] (verified)

Time = 4.85 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$\frac{-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)bc+3\left(b\,x^{3}+a\right)^{\frac{2}{3}}d\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}-2c\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)b+c\ln\left(\left(b\,x^{3}\right.\right.}{6b\,d^{2}\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}$

input
```
int(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3
))/(1/d*(a*d-b*c))^(1/3))*b*c+3*(b*x^3+a)^(2/3)*d*(1/d*(a*d-b*c))^(1/3)-2*
c*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))*b+c*ln((b*x^3+a)^(2/3)+(1/d*(a
*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))*b)/b/d^2/(1/d*(a*d-b
*c))^(1/3)
```

3.717.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(134) = 268.

Time = 0.27 (sec) , antiderivative size = 667, normalized size of antiderivative = 3.97

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \left[-\frac{\left(bcd^2-ad^3\right)^{\frac{2}{3}} bc \log \left(\left(bx^3+a\right)^{\frac{2}{3}} d^2-\left(bcd^2-ad^3\right)^{\frac{1}{3}} \left(bx^3+a\right)^{\frac{1}{3}} d+\left(bcd^2-ad^3\right)^{\frac{2}{3}}\right)-2 \left(bcd^2-ad^3\right)^{\frac{2}{3}} bc}{}\right.$$

$$-\frac{\left(bcd^2-ad^3\right)^{\frac{2}{3}} bc \log \left(\left(bx^3+a\right)^{\frac{2}{3}} d^2-\left(bcd^2-ad^3\right)^{\frac{1}{3}} \left(bx^3+a\right)^{\frac{1}{3}} d+\left(bcd^2-ad^3\right)^{\frac{2}{3}}\right)-2 \left(bcd^2-ad^3\right)^{\frac{2}{3}} bc}{}$$

input
```
integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/6*((b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 -
a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b*c*d^2 -
 a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*s
qrt(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)
)*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2
/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d
^3)^(1/3)*(b*c - a*d))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c
*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(b*c*d^2 - a*d^3)*
(b*x^3 + a)^(2/3))/(b^2*c*d^3 - a*b*d^4), -1/6*((b*c*d^2 - a*d^3)^(2/3)*b*
c*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d
+ (b*c*d^2 - a*d^3)^(2/3)) - 2*(b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)
^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 6*sqrt(1/3)*(b^2*c^2*d - a*b*c*d^2)*
sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^
(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d
))/d) - 3*(b*c*d^2 - a*d^3)*(b*x^3 + a)^(2/3))/(b^2*c*d^3 - a*b*d^4)]
```

3.717. $\int \frac{x^5}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

3.717.6 Sympy [F]

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**5/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**5/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.717.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.717.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.53

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{\frac{2\,bcd\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{bcd^2-ad^3}+\frac{6\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^3-\sqrt{3}ad^4}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\log\Bigg(}{}}{6\,b}$$

input
```
integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/6*(2*b*c*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a
*d)/d)^(1/3)))/(b*c*d^2 - a*d^3) + 6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1
/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)
^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) - (-b*c*d^2 + a*d^3)^(2/3)*b*c*l
og((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c -
 a*d)/d)^(2/3))/(b*c*d^3 - a*d^4) + 3*(b*x^3 + a)^(2/3)/d)/b
```

3.717.9 Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.30

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\left(b\,x^3+a\right)^{2/3}}{2\,b\,d} + \frac{\ln\left(\frac{c^2\,\left(b\,x^3+a\right)^{1/3}}{d}-\frac{c^2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,\left(a\,d-b\,c\right)^{1/3}}{4\,d^{4/3}}\right)\,\left(c-\sqrt{3}\,c\,1\mathrm{i}\right)}{6\,d^{5/3}\,\left(a\,d-b\,c\right)^{1/3}} + \frac{\ln\left(\frac{c^2\,\left(b\,x^3+a\right)^{1/3}}{d}-\frac{c^2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,\left(a\,d-b\,c\right)^{1/3}}{4\,d^{4/3}}\right)\,\left(c+\sqrt{3}\,c\,1\mathrm{i}\right)}{6\,d^{5/3}\,\left(a\,d-b\,c\right)^{1/3}} - \frac{c\,\ln\left(\frac{c^2\,\left(b\,x^3+a\right)^{1/3}}{d}+\frac{b\,c^3-a\,c^2\,d}{d^{4/3}\,\left(a\,d-b\,c\right)^{2/3}}\right)}{3\,d^{5/3}\,\left(a\,d-b\,c\right)^{1/3}}$$

input
```
int(x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
(a + b*x^3)^(2/3)/(2*b*d) + (log((c^2*(a + b*x^3)^(1/3))/d - (c^2*(3^(1/2)
*1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(4/3)))*(c - 3^(1/2)*c*1i))/(6*d^(5/3)*
(a*d - b*c)^(1/3)) + (log((c^2*(a + b*x^3)^(1/3))/d - (c^2*(3^(1/2)*1i + 1
)^2*(a*d - b*c)^(1/3))/(4*d^(4/3)))*(c + 3^(1/2)*c*1i))/(6*d^(5/3)*(a*d -
b*c)^(1/3)) - (c*log((c^2*(a + b*x^3)^(1/3))/d + (b*c^3 - a*c^2*d)/(d^(4/3
)*(a*d - b*c)^(2/3))))/(3*d^(5/3)*(a*d - b*c)^(1/3))
```

3.718 $\int \frac{x^2}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

3.718.1 Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}$$

output
```
1/6*ln(d*x^3+c)/d^(2/3)/(-a*d+b*c)^(1/3)-1/2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(
b*x^3+a)^(1/3))/d^(2/3)/(-a*d+b*c)^(1/3)-1/3*arctan(1/3*(1-2*d^(1/3)*(b*x^
3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(2/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

3.718.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$=\frac{-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-}\right.}{6d^{2/3}\sqrt[3]{bc-ad}}$$

input
```
Integrate[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/S
qrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c
- a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a +
b*x^3)^(2/3)])/(6*d^(2/3)*(b*c - a*d)^(1/3))
```

3.718.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {946, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}dx$$

$$\Big\downarrow \, 946$$

$$\frac{1}{3}\int \frac{1}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx^3$$

$$\Big\downarrow \, 68$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right.$$

$$\Big\downarrow \, 16$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)$$

$$\Big\downarrow \, 1082$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)$$

input
```
Int[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/S
qrt[3]])/(d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a
*d)^(1/3)) - (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(
2/3)*(b*c - a*d)^(1/3)))/3
```

3.718.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.718.4 Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)-\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\right.\right.}{6d\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}$

input
```
int(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(
a*d-b*c))^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))-ln((b
*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))
)/d/(1/d*(a*d-b*c))^(1/3)
```

3.718.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(114) = 228$.

Time = 0.26 (sec) , antiderivative size = 592, normalized size of antiderivative = 4.08

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}(bcd-ad^2)\sqrt{\frac{(-bcd^2+ad^3)^{\frac{1}{3}}}{bc-ad}}\log\left(\frac{2\,bd^2x^3-bcd+3\,ad^2+3\,\sqrt{\frac{1}{3}}\left(2\,(-bcd^2+ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}(bcd-ad^2)+(-bcd^2+a\right.}{dx^3+c}\right.}{}\right.$$

input
```
integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d
))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^
(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 +
a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(
-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) + (-b*c*d^2 + a*d^
3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^
(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^
3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)))/(b*c*d^2 - a*d^3), 1/6*(6*sqrt
(1/3)*(b*c*d - a*d^2)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(s
qrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^
2 + a*d^3)^(1/3)/(b*c - a*d))/d) + (-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a
)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a
*d^3)^(2/3)) - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*
d^2 + a*d^3)^(1/3)))/(b*c*d^2 - a*d^3)]
```

3.718.6 Sympy [F]

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**2/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**2/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.718.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.718.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.56

$$\begin{aligned}
&\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx \\
&= -\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3} \\
&\quad+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,\left(bcd^2-ad^3\right)} \\
&\quad-\frac{\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,\left(bc-ad\right)}
\end{aligned}$$

input
```
integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c
 - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3
) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)
*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2 - a*d^3) - 1/3*
(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3))
)/(b*c - a*d)
```

3.718.9 Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.43

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\ln\left(d\,(b\,x^3+a)^{1/3}-\frac{9\,a\,d^3-9\,b\,c\,d^2}{9\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{2/3}\,(a\,d-b\,c)^{1/3}} + \frac{\ln\left(d\,(b\,x^3+a)^{1/3}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,(9\,a\,d^3-9\,b\,c\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,d^{2/3}\,(a\,d-b\,c)^{1/3}} - \frac{\ln\left(d\,(b\,x^3+a)^{1/3}-\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,(9\,a\,d^3-9\,b\,c\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,d^{2/3}\,(a\,d-b\,c)^{1/3}}$$

input
```
int(x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
log(d*(a + b*x^3)^(1/3) - (9*a*d^3 - 9*b*c*d^2)/(9*d^(4/3)*(a*d - b*c)^(2/
3)))/(3*d^(2/3)*(a*d - b*c)^(1/3)) + (log(d*(a + b*x^3)^(1/3) - ((3^(1/2)*
1i - 1)^2*(9*a*d^3 - 9*b*c*d^2))/(36*d^(4/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*
1i - 1))/(6*d^(2/3)*(a*d - b*c)^(1/3)) - (log(d*(a + b*x^3)^(1/3) - ((3^(1
/2)*1i + 1)^2*(9*a*d^3 - 9*b*c*d^2))/(36*d^(4/3)*(a*d - b*c)^(2/3)))*(3^(1
/2)*1i + 1))/(6*d^(2/3)*(a*d - b*c)^(1/3))
```

3.719 $\int \frac{1}{x\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

3.719.1 Optimal result

Integrand size = 24, antiderivative size = 244

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}c} + \frac{\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{bc-ad}} - \frac{\log(x)}{2\sqrt[3]{a}c} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}c} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}}$$

output
```
-1/2*ln(x)/a^(1/3)/c-1/6*d^(1/3)*ln(d*x^3+c)/c/(-a*d+b*c)^(1/3)+1/2*ln(a^(
1/3)-(b*x^3+a)^(1/3))/a^(1/3)/c+1/2*d^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b
*x^3+a)^(1/3))/c/(-a*d+b*c)^(1/3)+1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3
))/a^(1/3)*3^(1/2))/a^(1/3)/c*3^(1/2)+1/3*d^(1/3)*arctan(1/3*(1-2*d^(1/3)*
(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/(-a*d+b*c)^(1/3)*3^(1/2)
```

3.719.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.27

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$=\frac{\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{2\sqrt{3}\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{bc-ad}}+\frac{2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{a}}+\frac{2\sqrt[3]{d}\log\left(\sqrt[3]{bc-a}\right.}{\sqrt[3]{bc}}}{}$$

input
```
Integrate[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) +
 (2*Sqrt[3]*d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^
(1/3))/Sqrt[3]])/(b*c - a*d)^(1/3) + (2*Log[-a^(1/3) + (a + b*x^3)^(1/3)])
/a^(1/3) + (2*d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/
(b*c - a*d)^(1/3) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^
(2/3)]/a^(1/3) - (d^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3
)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(1/3))/(6*c)
```

3.719.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 97, 67, 16, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$\Big\downarrow \text{948}$$
$$\frac{1}{3}\int \frac{1}{x^3\sqrt[3]{bx^3+a}\,(dx^3+c)}dx^3$$
$$\Big\downarrow \text{97}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{c}-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{c}\right)$$

$$\Big\downarrow \ 67$$

$$\frac{1}{3}\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}}{c}-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{c}\right)$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}}{c}-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{c}\right)$$

$$\Big\downarrow \ 68$$

$$\frac{1}{3}\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}}{c}-\frac{d\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{b}}{2d^{2/3}\sqrt[3]{bc-ad}}\right.}{}\right.$$

$$\Big\downarrow \ 16$$

3.719. $\int \frac{1}{x\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

$$\frac{1}{3}\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}}{c}-\frac{d\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3}}{\sqrt[3]{d}}}}{2d}\right.}{}\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}\left(\frac{-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}}{c}-\frac{d\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c}{2d^{2/3}\sqrt[3]{}}\right.}{c}\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}}{c}-\frac{d\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx}{2d^{2/3}\sqrt[3]{bc}}\right.}{c}\right.$$

input
```
Int[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

3.719. $\int \frac{1}{x\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

output
```
(((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) -
Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3)))/c
 - (d*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1
/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b
*c - a*d)^(1/3)) - (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/
(2*d^(2/3)*(b*c - a*d)^(1/3))))/c)/3
```

3.719.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.719. $\int \frac{1}{x\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.719.4 Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$-\frac{\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)\right)\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}+2}{6}$

input
```
int(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(1/d*(a*d-b*c))^(1/3)/a^(1/3)*((-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1
/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a
^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*(1/d*(a*d-b*c))^(1/3)+2*(arctan(1/3
*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*
3^(1/2)+ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))-1/2*ln((b*x^3+a)^(2/3)+(
1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3)))*a^(1/3))/c
```

3.719.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.57

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \left[\frac{3\,\sqrt{\frac{1}{3}}a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3+3\,\sqrt{\frac{1}{3}}\left(2\,(bx^3+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^3}\right)-2\,\sqrt{3}a\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}\operatorname{arctan}}{}\right.$$

$$-\frac{2\,\sqrt{3}a\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+a\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left(-(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(\frac{d}{bc-ad}\right.\right.}{}$$

input
```
integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3
+ a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(
b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - 2*sqrt(3)*a*(d/(b*c - a*d))^(1/3)*a
rctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3*sqrt(3)) -
 a*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d)
)^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) + 2*a*(
d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(
1/3)*d) - a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2
/3)) + 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*c), -1/6*(2*sqrt(3)*
a*(d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d
))^(1/3) - 1/3*sqrt(3)) + a*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(
b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b
*c - a*d))^(1/3)) - 2*a*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*
d))^(2/3) + (b*x^3 + a)^(1/3)*d) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2
*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) + (
b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^
(1/3)))/(a*c)]
```

3.719.6 Sympy [F]

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/x/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.719.7 Maxima [F]

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x}\,dx$$

input
```
integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x), x)
```

3.719.8 Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.34

$$
\begin{aligned}
&\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx \\
&= \frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} \\
&\quad+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2d-\sqrt{3}acd^2} \\
&\quad-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2d-acd^2)} \\
&\quad+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}c} \\
&\quad-\frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}c}+\frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{1}{3}}c}
\end{aligned}
$$

input

```
integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output

```
1/3*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^
(1/3)))/(b*c^2 - a*c*d) + (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(
b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3
)*b*c^2*d - sqrt(3)*a*c*d^2) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a
)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3
))/(b*c^2*d - a*c*d^2) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/
3) + a^(1/3))/a^(1/3))/(a^(1/3)*c) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*a^(1/3) + a^(2/3))/(a^(1/3)*c) + 1/3*log(abs((b*x^3 + a)^(1/3) -
a^(1/3)))/(a^(1/3)*c)
```

3.719.9 Mupad [B] (verification not implemented)

Time = 10.23 (sec) , antiderivative size = 702, normalized size of antiderivative = 2.88

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \ln\left(b^5\,d^4\,\left(b\,x^3+a\right)^{1/3}\right.$$

$$-\frac{d\left(27\,b^4\,c^2\,d^3\,\left(b\,x^3+a\right)^{1/3}\left(2\,a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)-243\,a\,b^4\,c^4\,d^3\,\left(\frac{d}{27\,b\,c^4-27\,a\,c^3\,d}\right)^{2/3}\left(2\,a^2\,d^2-3\,a\,b\,c\,d\right.\right.}{27\,b\,c^4-27\,a\,c^3\,d}$$

input
```
int(1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
log(b^5*d^4*(a + b*x^3)^(1/3) - (d*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^
2*d^2 + b^2*c^2 - 2*a*b*c*d) - 243*a*b^4*c^4*d^3*(d/(27*b*c^4 - 27*a*c^3*d
))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)))/(27*b*c^4 - 27*a*c^3*d))*(d/(
27*b*c^4 - 27*a*c^3*d))^(1/3) + log((a + b*x^3)^(1/3) - a*c^2*(1/(a*c^3))^
(2/3))*(1/(27*a*c^3))^(1/3) + (log(b^5*d^4*(a + b*x^3)^(1/3) - (d*(3^(1/2)
*1i - 1)^3*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^2*d^2 + b^2*c^2 - 2*a*b*
c*d) - (243*a*b^4*c^4*d^3*(3^(1/2)*1i - 1)^2*(d/(27*b*c^4 - 27*a*c^3*d))^(
2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/(8*(27*b*c^4 - 27*a*c^3*d)))*(
3^(1/2)*1i - 1)*(d/(27*b*c^4 - 27*a*c^3*d))^(1/3))/2 - (log(b^5*d^4*(a + b
*x^3)^(1/3) + (d*(3^(1/2)*1i + 1)^3*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a
^2*d^2 + b^2*c^2 - 2*a*b*c*d) - (243*a*b^4*c^4*d^3*(3^(1/2)*1i + 1)^2*(d/(
27*b*c^4 - 27*a*c^3*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/(8*(2
7*b*c^4 - 27*a*c^3*d)))*(3^(1/2)*1i + 1)*(d/(27*b*c^4 - 27*a*c^3*d))^(1/3)
)/2 - log((a + b*x^3)^(1/3)*2i + a*c^2*(1/(a*c^3))^(2/3)*1i + 3^(1/2)*a*c^
2*(1/(a*c^3))^(2/3))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a*c^3))^(1/3) + log((a
+ b*x^3)^(1/3)*2i + a*c^2*(1/(a*c^3))^(2/3)*1i - 3^(1/2)*a*c^2*(1/(a*c^3))
^(2/3))*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a*c^3))^(1/3)
```

3.720 $\int \frac{1}{x^4 \sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

3.720.1 Optimal result

Integrand size = 24, antiderivative size = 296

$$\begin{aligned}\int \frac{1}{x^4 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = & -\frac{(a+bx^3)^{2/3}}{3acx^3} - \frac{(bc+3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}c^2} \\ & - \frac{d^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2\sqrt[3]{bc-ad}} + \frac{(bc+3ad)\log(x)}{6a^{4/3}c^2} \\ & + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}} - \frac{(bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2} \\ & - \frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{bc-ad}}\end{aligned}$$

output
```
-1/3*(b*x^3+a)^(2/3)/a/c/x^3+1/6*(3*a*d+b*c)*ln(x)/a^(4/3)/c^2+1/6*d^(4/3)
*ln(d*x^3+c)/c^2/(-a*d+b*c)^(1/3)-1/6*(3*a*d+b*c)*ln(a^(1/3)-(b*x^3+a)^(1/
3))/a^(4/3)/c^2-1/2*d^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c
^2/(-a*d+b*c)^(1/3)-1/9*(3*a*d+b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))
/a^(1/3)*3^(1/2))/a^(4/3)/c^2*3^(1/2)-1/3*d^(4/3)*arctan(1/3*(1-2*d^(1/3)*
(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/(-a*d+b*c)^(1/3)*3^(1/2)
```

3.720.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx =$$

$$-\frac{\frac{6c(a+bx^3)^{2/3}}{ax^3} + \frac{2\sqrt{3}(bc+3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} + \frac{6\sqrt{3}d^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{bc-ad}} + \frac{2(bc+3ad)\log\left(-\sqrt[3]{a}+\right.}{a^{4/3}}}{}$$

input
```
Integrate[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
-1/18*((6*c*(a + b*x^3)^(2/3))/(a*x^3) + (2*Sqrt[3]*(b*c + 3*a*d)*ArcTan[(
1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(4/3) + (6*Sqrt[3]*d^(4/3)*
ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*
c - a*d)^(1/3) + (2*(b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(4/
3) + (6*d^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c -
 a*d)^(1/3) - ((b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a
+ b*x^3)^(2/3)])/a^(4/3) - (3*d^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c
 - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^
(1/3))/c^2
```

3.720.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {948, 114, 27, 174, 67, 16, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{1}{x^6\sqrt[3]{bx^3+a}\,(dx^3+c)}dx^3$$

$$\Big\downarrow \ 114$$

$$\frac{1}{3}\left(-\frac{\int \frac{bdx^3+bc+3ad}{3x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{ac}-\frac{\left(a+bx^3\right)^{2/3}}{acx^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{bdx^3+bc+3ad}{x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{3ac}-\frac{\left(a+bx^3\right)^{2/3}}{acx^3}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+bc)\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{c}-\frac{3ad^2\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{c}}{3ac}-\frac{\left(a+bx^3\right)^{2/3}}{acx^3}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\Bigg(-\frac{\frac{(3ad+bc)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{3ad^2\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3\right.}}{c}}{3ac}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\Bigg(-\frac{\frac{(3ad+bc)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{3ad^2\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{c}}{3ac}-$$

$$\Big\downarrow \text{68}$$

3.720. $\int \frac{1}{x^4\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+bc)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{3ad^2\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3}}}{2d^{2/3}\sqrt[3]{bc-}}\right.}{3ac}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+bc)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{3ad^2\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-}}{}}}{}\right.}{3ac}\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+bc)\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{3ad^2\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{}{2}\right.}{3ac}\right.$$

$$\Big\downarrow \text{217}$$

3.720. $\int \frac{1}{x^4\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

$$\frac{1}{3}\left(-\frac{(3ad+bc)\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}-\frac{3ad^2\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\mathrm{lo}}{2d^{2/3}}\right.}{3ac}\right.$$

input
```
Int[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-((a + b*x^3)^(2/3)/(a*c*x^3)) - (((b*c + 3*a*d)*((Sqrt[3]*ArcTan[(1 + (2
*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^3]/(2*a^(1/3)) + (3
*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3))))/c - (3*a*d^2*(-((Sqrt[3]*
ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^
(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/3)) -
 (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c -
a*d)^(1/3))))/c)/(3*a*c))/3
```

3.720.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.720. $\int \frac{1}{x^4 \sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.720.4 Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{\left(2\left(b x^3+a\right)^{\frac{2}{3}} a^{\frac{1}{3}} c-x^3\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b x^3+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b x^3+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b x^3+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b x^3+a\right)^{\frac{1}{3}}\right.\right.\right.}{}$

input

```
int(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output

```
-1/6/a^(4/3)*((2*(b*x^3+a)^(2/3)*a^(1/3)*c-x^3*(-2*arctan(1/3*(a^(1/3)+2*(
b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3
+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*(1/3*b*c+a*d))*(1/d*(a*d
-b*c))^(1/3)+(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/
3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3
)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*
c))^(1/3)))*x^3*a^(4/3)*d)/(1/d*(a*d-b*c))^(1/3)/c^2/x^3
```

3.720.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 837, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input

```
integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output

```
[1/18*(6*sqrt(3)*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^
3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - 3*a^2*d*x^3*(-d/(b*c
- a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) +
(b*x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*a^2*d*x^3*(-
d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^
(1/3)*d) + 3*sqrt(1/3)*(a*b*c + 3*a^2*d)*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x
^3 - 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (
-a)^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^
3) + (b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3
)*(-a)^(1/3) + (-a)^(2/3)) - 2*(b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a
)^(1/3) + (-a)^(1/3)) - 6*(b*x^3 + a)^(2/3)*a*c)/(a^2*c^2*x^3), 1/18*(6*sq
rt(3)*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3
)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - 3*a^2*d*x^3*(-d/(b*c - a*d))^(1/
3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)
^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*a^2*d*x^3*(-d/(b*c - a*
d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) -
6*sqrt(1/3)*(a*b*c + 3*a^2*d)*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*
(b*x^3 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + (b*c + 3*a*d)*(-a)^
(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3
)) - 2*(b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3))...
```

3.720.6 Sympy [F]

$$\int \frac{1}{x^4 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^4 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input

```
integrate(1/x**4/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output

```
Integral(1/(x**4*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.720.7 Maxima [F]

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^4), x)
```

3.720.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.28

$$\begin{aligned}
&\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx \\
&= -\frac{d^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3-ac^2d)} \\
&\quad-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^3-\sqrt{3}ac^2d} \\
&\quad+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^3-ac^2d)} \\
&\quad-\frac{\sqrt{3}(bc+3\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{4}{3}}c^2} \\
&\quad+\frac{(bc+3\,ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{4}{3}}c^2} \\
&\quad-\frac{\left(a^{\frac{1}{3}}bc+3\,a^{\frac{4}{3}}d\right)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{5}{3}}c^2}-\frac{(bx^3+a)^{\frac{2}{3}}}{3\,acx^3}
\end{aligned}$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*d^2*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/
d)^(1/3)))/(b*c^3 - a*c^2*d) - (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)
*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(s
qrt(3)*b*c^3 - sqrt(3)*a*c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3
+ a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(
2/3))/(b*c^3 - a*c^2*d) - 1/9*sqrt(3)*(b*c + 3*a*d)*arctan(1/3*sqrt(3)*(2*
(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(4/3)*c^2) + 1/18*(b*c + 3*a*d)*l
og((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*c^2)
- 1/9*(a^(1/3)*b*c + 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a
^(5/3)*c^2) - 1/3*(b*x^3 + a)^(2/3)/(a*c*x^3)
```

3.720.9 Mupad [B] (verification not implemented)

Time = 15.39 (sec) , antiderivative size = 1929, normalized size of antiderivative = 6.52

$$\int \frac{1}{x^4 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
log(- (((((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c
^2*d^2 + 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 3*a*b^4*c^4*d^3*(2*a^2*d^2
 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))*(-(3*a*d + b*c
)^3/(a^4*c^6))^(1/3))/9 + (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d +
 18*a^2*b*c*d^2))/(3*a^2*c))*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4*b
^5*d^7*(a + b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(-(27*a^3*d^3 + b^
3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2)/(729*a^4*c^6))^(1/3) + log(- (-d^4
/(27*b*c^7 - 27*a*c^6*d))^(2/3)*((-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3)*((3*
b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4*a*
b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 243*a*b^4*c^4*d^3*(-d^4/(27*b*c^7 - 27*
a*c^6*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)) + (b^5*d^4*(b^3*c^3 - 2
7*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^2*c)) - (4*b^5*d^7*(a +
b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(-d^4/(27*b*c^7 - 27*a*c^6*d))
^(1/3) - log((((((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^
2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 3*a*b^4*c^4*d^3*((3
^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a
^4*c^6))^(2/3))*((3^(1/2)*1i)/2 + 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(1/3))
/9 - (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*
a^2*c))*((3^(1/2)*1i)/2 - 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4
*b^5*d^7*(a + b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*((3^(1/2)*1i)...
```

3.721 $\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

3.721.1 Optimal result

Integrand size = 24, antiderivative size = 273

$$\begin{aligned}\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx &= \frac{x(a+bx^3)^{2/3}}{3bd} - \frac{(3bc+ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d^2} \\ &\quad + \frac{c^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2\sqrt[3]{bc-ad}} + \frac{c^{4/3}\log\left(c+dx^3\right)}{6d^2\sqrt[3]{bc-ad}} \\ &\quad - \frac{c^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d^2\sqrt[3]{bc-ad}} \\ &\quad + \frac{(3bc+ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}d^2}\end{aligned}$$

output
```
1/3*x*(b*x^3+a)^(2/3)/b/d+1/6*c^(4/3)*ln(d*x^3+c)/d^2/(-a*d+b*c)^(1/3)-1/2
*c^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2/(-a*d+b*c)^(1/
3)+1/6*(a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)/d^2-1/9*(a*d+3*b
*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d^2*3^(1/2
)+1/3*c^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*
3^(1/2))/d^2/(-a*d+b*c)^(1/3)*3^(1/2)
```

3.721.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.82 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.71

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{\frac{12dx(a+bx^3)^{2/3}}{b} - \frac{4\sqrt{3}(3bc+ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}_x}{\sqrt[3]{b}_{x+2}\sqrt[3]{a+bx^3}}\right)}{b^{4/3}} - \frac{6\sqrt{-6+6i\sqrt{3}}c^{4/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}_x}{\sqrt{3}\sqrt[3]{bc-ad}_{x-(3i+\sqrt{3})}\sqrt[3]{c}\sqrt[3]{a+bx}}\right.}{\sqrt[3]{bc-ad}}}{}$$

input
```
Integrate[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((12*d*x*(a + b*x^3)^(2/3))/b - (4*Sqrt[3]*(3*b*c + a*d)*ArcTan[(Sqrt[3]*b
^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]])/b^(4/3) - (6*Sqrt[-6 + (6*I)
*Sqrt[3]]*c^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3
)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c - a*d)^(1/3) + (4*
(3*b*c + a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]])/b^(4/3) + (6*(1 + I*S
qrt[3])*c^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b
*x^3)^(1/3)]])/(b*c - a*d)^(1/3) - (2*(3*b*c + a*d)*Log[b^(2/3)*x^2 + b^(1/
3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]])/b^(4/3) - ((3*I)*(-I + Sqrt[3
])*c^(4/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a
*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]
)/(b*c - a*d)^(1/3))/(36*d^2)
```

3.721.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {979, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \; 979$$

$$\frac{x\left(a+bx^3\right)^{2/3}}{3bd}-\frac{\int \frac{(3bc+ad)x^3+ac}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3bd}$$

$$\Big\downarrow \text{1026}$$

$$\frac{x\left(a+bx^3\right)^{2/3}}{3bd}-\frac{\frac{(ad+3bc)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{3bc^2\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{d}}{3bd}$$

$$\Big\downarrow \text{769}$$

$$\frac{x\left(a+bx^3\right)^{2/3}}{3bd}-\frac{\frac{(ad+3bc)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{3bc^2\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{d}}{3bd}$$

$$\Big\downarrow \text{901}$$

$$\frac{x\left(a+bx^3\right)^{2/3}}{3bd}-\frac{\frac{(ad+3bc)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{3bc^2\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-}}{\sqrt[3]{c}}\right.}{2c^{2/3}}\right.}{d}}{3bd}$$

input
```
Int[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(x*(a + b*x^3)^(2/3))/(3*b*d) - ((-3*b*c^2*(ArcTan[(1 + (2*(b*c - a*d)^(1/
3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(
1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1
/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d + ((
3*b*c + a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[
3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/d)/(3*b*
d)
```

3.721. $\int \frac{x^6}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

3.721.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.721.4 Maple [A] (verified)

Time = 4.97 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$\left((bx^3+a)^{\frac{2}{3}}dx\,b^{\frac{1}{3}}+\frac{\left(\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{2}}{3}$

input
```
int(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/b^(4/3)/((a*d-b*c)/c)^(1/3)*(((b*x^3+a)^(2/3)*d*x*b^(1/3)+1/3*(3^(1/2)
*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((-b^(1/3)*
x+(b*x^3+a)^(1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3
+a)^(2/3))/x^2))*(a*d+3*b*c))*((a*d-b*c)/c)^(1/3)+b^(4/3)*(arctan(1/3*3^(1
/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/
2)+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((((a*d-b*c)/c)^(2/
3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*c)/d^2
```

3.721.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 826, normalized size of antiderivative = 3.03

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/18*(6*sqrt(3)*b^2*c*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*
sqrt(3)*(b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 6*b^2*c*(-c/(b*c -
a*d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)
*c)/x) + 3*b^2*c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x^2*(-c/(b*c - a
*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x
^3 + a)^(2/3)*c)/x^2) - 6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c +
 a*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3
*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^
(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) - 2*(3*b*c + a*d)*b^(2/3)*log(-(b^(1/3)*x
 - (b*x^3 + a)^(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3
 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d^2), -1/18*(6*sqrt(
3)*b^2*c*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(b*x^3
+ a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 6*b^2*c*(-c/(b*c - a*d))^(1/3)*log
(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) + 3*b^2*
c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x^2*(-c/(b*c - a*d))^(1/3) - (b
*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(2/3)*c
)/x^2) - 6*(b*x^3 + a)^(2/3)*b*d*x - 2*(3*b*c + a*d)*b^(2/3)*log(-(b^(1/3)
*x - (b*x^3 + a)^(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x
^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*sqrt(1/3)*(3*b^2*c +
 a*b*d)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))...
```

3.721.6 Sympy [F]

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**6/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**6/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.721.7 Maxima [F]

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.721.8 Giac [F]

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.721.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^6}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(x^6/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.722 $\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

3.722.1 Optimal result

Integrand size = 24, antiderivative size = 233

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d} - \frac{\sqrt[3]{c}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d\sqrt[3]{bc-ad}} - \frac{\sqrt[3]{c}\log\left(c+dx^3\right)}{6d\sqrt[3]{bc-ad}} + \frac{\sqrt[3]{c}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d\sqrt[3]{bc-ad}} - \frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}d}$$

output
```
-1/6*c^(1/3)*ln(d*x^3+c)/d/(-a*d+b*c)^(1/3)+1/2*c^(1/3)*ln((-a*d+b*c)^(1/3
)*x/c^(1/3)-(b*x^3+a)^(1/3))/d/(-a*d+b*c)^(1/3)-1/2*ln(-b^(1/3)*x+(b*x^3+a
)^(1/3))/b^(1/3)/d+1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))
/b^(1/3)/d*3^(1/2)-1/3*c^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/
(b*x^3+a)^(1/3))*3^(1/2))/d/(-a*d+b*c)^(1/3)*3^(1/2)
```

3.722.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.10 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.82

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}_x}{\sqrt[3]{b}_{x+2}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}} + \frac{2\sqrt{-6+6i\sqrt{3}}\sqrt[3]{c}\arctan\left(\frac{3\sqrt[3]{bc-ad}_x}{\sqrt{3}\sqrt[3]{bc-ad}_{x-\left(3i+\sqrt{3}\right)}\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}} - \frac{4\log\left(-\sqrt[3]{b}_{x+}\sqrt[3]{a}\right.}{\sqrt[3]{b}}}{}$$

input
```
Integrate[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((4*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])
/b^(1/3) + (2*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)
*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/
3))]])/(b*c - a*d)^(1/3) - (4*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(1/3
) - ((2*I)*(-I + Sqrt[3])*c^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[
3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(1/3) + (2*Log[b^(2/3)*x^2 + b
^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(1/3) + ((1 + I*Sqrt[3]
)*c^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*
d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])
/(b*c - a*d)^(1/3))/(12*d)
```

3.722.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {983, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \text{983}$$

$$\frac{\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d} - \frac{c\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \text{769}$$

$$\frac{\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}}{d}-\frac{c\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \text{901}$$

$$\frac{\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}}{d}-\frac{c\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{d}$$

input
```
Int[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
-((c*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqr
t[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c
 - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2
*c^(2/3)*(b*c - a*d)^(1/3))))/d) + (ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^
(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/
(2*b^(1/3)))/d
```

3.722. $\int \frac{x^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

3.722.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

3.722.4 Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.23

method	result
pseudoelliptic	$\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)\right)\left(\frac{ad-bc}{c}\right)$

input
```
int(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/c)^(1/3)*((-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^
3+a)^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)
^(2/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*((a*d-b*c)/c)^(1/3)+(-2
*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c
)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+
a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/
3))/x))*b^(1/3))/b^(1/3)/d
```

3.722.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 761, normalized size of antiderivative = 3.27

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3 - 3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2 - 3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3 - (bx^3+a)^{\frac{1}{3}}bx^2 + 2\,(bx^3+a)^{\frac{2}{3}}(-b\right.\right.}{}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right) - 2\sqrt{3}b\left(\frac{c}{bc-ad}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(\right.}{3\,x}\right.}{}$$

input
```
integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(
-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 +
2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*sqrt(3)*b*
(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*
(c/(b*c - a*d))^(1/3))/x) + 2*b*(c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*
(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) - b*(c/(b*c - a*d))^(1/3)*
log(((b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)
*x*(c/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) - 2*(-b)^(2/3)*log(((
-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*
x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d), -1/6*(6*sqrt
(1/3)*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a
)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) - 2*sqrt(3)*b*(c/(b*c - a*d))^(1/3)*arctan
(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) -
2*b*(c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x
^3 + a)^(1/3)*c)/x) + b*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c
 - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (
b*x^3 + a)^(2/3)*c)/x^2) + 2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1
/3))/x) - (-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x
+ (b*x^3 + a)^(2/3))/x^2))/(b*d)]
```

3.722.6 Sympy [F]

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**3/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.722.7 Maxima [F]

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^3}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.722.8 Giac [F]

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^3}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.722.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^3}{\left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

input

```
int(x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output

```
int(x^3/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.723 $\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

3.723.1 Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx=\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

output
```
1/6*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(1/3)-1/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)
-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(1/3)+1/3*arctan(1/3*(1+2*(-a*d+b*c)^
(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

3.723.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

$$=\frac{-2\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+\left(1+i\sqrt{3}\right)\Big(2\log\Big(2\sqrt[3]{bc-ad}x+(1+i}{12c^{2}}$$

input
```
Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c
- a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (1 + I*Sqrt
[3])*(2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1
/3)] - Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^
(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]))/(
12*c^(2/3)*(b*c - a*d)^(1/3))
```

3.723.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \textcolor{blue}{901}$$

$$\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

input
```
Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/
(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d
)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/
3)*(b*c - a*d)^(1/3))
```

3.723.3.1 Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

3.723.4 Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-\ln\Bigg(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x}{x^2}}{6\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}c}$

input
```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-
b*c)/c)^(1/3)/x)*3^(1/2)+2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-l
n((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)
^(2/3))/x^2))/((a*d-b*c)/c)^(1/3)/c
```

3.723.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.723.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.723.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.723.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.723.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.724 $\int \frac{1}{x^3 \sqrt[3]{a+bx^3}(c+dx^3)} \, dx$

3.724.1 Optimal result

Integrand size = 24, antiderivative size = 176

$$\int \frac{1}{x^3 \sqrt[3]{a+bx^3}\,(c+dx^3)} \, dx = -\frac{(a+bx^3)^{2/3}}{2acx^2} - \frac{d \arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}} - \frac{d \log(c+dx^3)}{6c^{5/3}\sqrt[3]{bc-ad}} + \frac{d \log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{5/3}\sqrt[3]{bc-ad}}$$

output
```
-1/2*(b*x^3+a)^(2/3)/a/c/x^2-1/6*d*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(1/3)+1/
2*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(1/3
)-1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2
))/c^(5/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

3.724.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{-6c^{2/3}\sqrt[3]{bc-ad}(a+bx^3)^{2/3}+2\sqrt{-6+6i\sqrt{3}}adx^2\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)-2i(-}{}$$

input
```
Integrate[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-6*c^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3) + 2*Sqrt[-6 + (6*I)*Sqrt[3
]]*a*d*x^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (
3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - (2*I)*(-I + Sqrt[3])*a*d*x^2*
Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + a
*(d + I*Sqrt[3]*d)*x^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1
/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b
*x^3)^(2/3)])/(12*a*c^(5/3)*(b*c - a*d)^(1/3)*x^2)
```

3.724.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {980, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}dx$$

$$\Big\downarrow \ 980$$

$$\frac{\int -\frac{2ad}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{2ac}-\frac{\left(a+bx^3\right)^{2/3}}{2acx^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{c}-\frac{\left(a+bx^3\right)^{2/3}}{2acx^2}$$

$$\Big\downarrow \ 901$$

$$-\frac{d\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{\left(a+bx^3\right)^{2/3}}{2acx^2}$$

input
```
Int[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
-1/2*(a + b*x^3)^(2/3)/(a*c*x^2) - (d*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)
/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3))
 + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x
)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c
```

3.724.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

3.724.4 Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.23

method	result
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}\,a d\,x^{2}+a\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)d\,x^{2}-\frac{a\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{x^{2}}\right.}{2}}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}a\,c^{2}x^{2}}$

input
```
int(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a*d-b*c)/c)^(1/3)*(arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x
^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)*a*d*x^2+a*ln((((a*d-b*c)/c)^(1
/3)*x+(b*x^3+a)^(1/3))/x)*d*x^2-1/2*a*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*
c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*d*x^2+3/2*(b*x^3+a)^(2
/3)*c*((a*d-b*c)/c)^(1/3))/a/c^2/x^2
```

3.724.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.724.6 Sympy [F]

$$\int \frac{1}{x^3 \sqrt[3]{a+bx^3} (c+dx^3)} \, dx = \int \frac{1}{x^3 \sqrt[3]{a+bx^3} (c+dx^3)} \, dx$$

input
```
integrate(1/x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**3*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.724.7 Maxima [F]

$$\int \frac{1}{x^3 \sqrt[3]{a+bx^3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)
```

3.724.8 Giac [F]

$$\int \frac{1}{x^3 \sqrt[3]{a+bx^3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)
```

3.724.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{x^3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.725 $\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

3.725.1 Optimal result

Integrand size = 24, antiderivative size = 214

$$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{5acx^5} + \frac{(3bc+5ad)(a+bx^3)^{2/3}}{10a^2c^2x^2} + \frac{d^2 \arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}} + \frac{d^2 \log(c+dx^3)}{6c^{8/3}\sqrt[3]{bc-ad}} - \frac{d^2 \log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{8/3}\sqrt[3]{bc-ad}}$$

output
```
-1/5*(b*x^3+a)^(2/3)/a/c/x^5+1/10*(5*a*d+3*b*c)*(b*x^3+a)^(2/3)/a^2/c^2/x^
2+1/6*d^2*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(1/3)-1/2*d^2*ln((-a*d+b*c)^(1/3)
*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(1/3)+1/3*d^2*arctan(1/3*(1
+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)
^(1/3)*3^(1/2)
```

3.725.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.62 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$= \frac{\frac{6c^{2/3}(a+bx^3)^{2/3}(-2ac+3bcx^3+5adx^3)}{a^2x^5} - \frac{10\sqrt{-6+6i\sqrt{3}}d^2\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}} + \frac{10\left(1+i\sqrt{3}\right)d^2\log\left(2\right.}{}}{}$$

input
```
Integrate[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((6*c^(2/3)*(a + b*x^3)^(2/3)*(-2*a*c + 3*b*c*x^3 + 5*a*d*x^3))/(a^2*x^5)
- (10*Sqrt[-6 + (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]
*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c -
 a*d)^(1/3) + (10*(1 + I*Sqrt[3])*d^2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*S
qrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(1/3) - ((5*I)*(-I + Sqrt[
3])*d^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)
^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(
b*c - a*d)^(1/3))/(60*c^(8/3))
```

3.725.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {980, 25, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$\Big\downarrow \textbf{980}$$
$$\frac{\int -\frac{3bdx^3+3bc+5ad}{x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx}{5ac} - \frac{(a+bx^3)^{2/3}}{5acx^5}$$
$$\Big\downarrow \textbf{25}$$

$$-\frac{\int \frac{3bdx^3+3bc+5ad}{x^3\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{5ac}-\frac{\left(a+bx^3\right)^{2/3}}{5acx^5}$$

$$\Big\downarrow \text{ 1053}$$

$$-\frac{-\frac{\int \frac{10a^2d^2}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{2ac}-\frac{\left(a+bx^3\right)^{2/3}(5ad+3bc)}{2acx^2}}{5ac}-\frac{\left(a+bx^3\right)^{2/3}}{5acx^5}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{-\frac{5ad^2\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{c}-\frac{\left(a+bx^3\right)^{2/3}(5ad+3bc)}{2acx^2}}{5ac}-\frac{\left(a+bx^3\right)^{2/3}}{5acx^5}$$

$$\Big\downarrow \text{ 901}$$

$$-\frac{-\frac{5ad^2\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{\left(a+bx^3\right)^{2/3}(5ad+3bc)}{2acx^2}}{5ac}-\frac{\left(a+bx^3\right)^{2/3}}{5acx^5}$$

input
```
Int[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
-1/5*(a + b*x^3)^(2/3)/(a*c*x^5) - (-1/2*((3*b*c + 5*a*d)*(a + b*x^3)^(2/3
))/(a*c*x^2) - (5*a*d^2*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a +
 b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x
^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a
 + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c)/(5*a*c)
```

3.725. $\int \frac{1}{x^6\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

3.725.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.725. $\int \frac{1}{x^6 \sqrt[3]{a+bx^3}(c+dx^3)} \, dx$

3.725.4 Maple [A] (verified)

Time = 4.99 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$-\frac{6c\left(\frac{(-5ad-3bc)x^3}{2}+ac\right)\left(b\,x^3+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+5a^2d^2x^5\Bigg(-2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\Bigg(\Big(\frac{ad-bc}{c}}{30\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}a^2c^3x^5}$

input
```
int(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/30*(6*c*(1/2*(-5*a*d-3*b*c)*x^3+a*c)*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3
)+5*a^2*d^2*x^5*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^
(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln(((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*
c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln(((((a*d-b*c)/c)^(1
/3)*x+(b*x^3+a)^(1/3))/x)))/((a*d-b*c)/c)^(1/3)/a^2/c^3/x^5
```

3.725.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.725.6 Sympy [F]

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/x**6/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**6*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.725.7 Maxima [F]

$$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)
```

3.725.8 Giac [F]

$$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)
```

3.725.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^6\,(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.726 $\int \frac{1}{x^9 \sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

3.726.1 Optimal result

Integrand size = 24, antiderivative size = 262

$$\begin{aligned}\int \frac{1}{x^9 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx ={}& -\frac{(a+bx^3)^{2/3}}{8acx^8}+\frac{(3bc+4ad)\,(a+bx^3)^{2/3}}{20a^2c^2x^5} \\ &-\frac{(9b^2c^2+12abcd+20a^2d^2)\,(a+bx^3)^{2/3}}{40a^3c^3x^2} \\ &-\frac{d^3\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{11/3}\sqrt[3]{bc-ad}}-\frac{d^3\log(c+dx^3)}{6c^{11/3}\sqrt[3]{bc-ad}} \\ &+\frac{d^3\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}\sqrt[3]{bc-ad}}\end{aligned}$$

output
```
-1/8*(b*x^3+a)^(2/3)/a/c/x^8+1/20*(4*a*d+3*b*c)*(b*x^3+a)^(2/3)/a^2/c^2/x^
5-1/40*(20*a^2*d^2+12*a*b*c*d+9*b^2*c^2)*(b*x^3+a)^(2/3)/a^3/c^3/x^2-1/6*d
^3*ln(d*x^3+c)/c^(11/3)/(-a*d+b*c)^(1/3)+1/2*d^3*ln((-a*d+b*c)^(1/3)*x/c^(
1/3)-(b*x^3+a)^(1/3))/c^(11/3)/(-a*d+b*c)^(1/3)-1/3*d^3*arctan(1/3*(1+2*(-
a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(11/3)/(-a*d+b*c)^(1/
3)*3^(1/2)
```

3.726.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.59 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{-\frac{3c^{2/3}(a+bx^3)^{2/3}(9b^2c^2x^6-6abcx^3(c-2dx^3)+a^2(5c^2-8cdx^3+20d^2x^6))}{a^3x^8}+\frac{20\sqrt{-6+6i\sqrt{3}}d^3\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a}}\right.}{\sqrt[3]{bc-ad}}}{}$$

input
```
Integrate[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
((-3*c^(2/3)*(a + b*x^3)^(2/3)*(9*b^2*c^2*x^6 - 6*a*b*c*x^3*(c - 2*d*x^3)
+ a^2*(5*c^2 - 8*c*d*x^3 + 20*d^2*x^6)))/(a^3*x^8) + (20*Sqrt[-6 + (6*I)*S
qrt[3]]*d^3*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x -
(3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c - a*d)^(1/3) - ((20*I)*(
-I + Sqrt[3])*d^3*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a +
 b*x^3)^(1/3)])/(b*c - a*d)^(1/3) + (10*(1 + I*Sqrt[3])*d^3*Log[2*(b*c - a
*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(
1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(1/3))/(120
*c^(11/3))
```

3.726.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {980, 27, 1053, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \text{980}$$

$$\frac{\int -\frac{2(3bdx^3+3bc+4ad)}{x^6\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{8ac}-\frac{(a+bx^3)^{2/3}}{8acx^8}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{3bdx^3+3bc+4ad}{x^6\sqrt[3]{bx^3+a}(dx^3+c)}dx}{4ac}-\frac{\left(a+bx^3\right)^{2/3}}{8acx^8}$$

$$\Big\downarrow \text{1053}$$

$$-\frac{-\frac{\int \frac{3bd(3bc+4ad)x^3+9b^2c^2+20a^2d^2+12abcd}{x^3\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{5ac}-\frac{\left(a+bx^3\right)^{2/3}(4ad+3bc)}{5acx^5}}{4ac}-\frac{\left(a+bx^3\right)^{2/3}}{8acx^8}$$

$$\Big\downarrow \text{1053}$$

$$-\frac{-\frac{-\frac{\int \frac{40a^3d^3}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{2ac}-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{9b^2c}{a}+\frac{20ad^2}{c}+12bd\right)}{2x^2}}{5ac}-\frac{\left(a+bx^3\right)^{2/3}(4ad+3bc)}{5acx^5}}{4ac}-\frac{\left(a+bx^3\right)^{2/3}}{8acx^8}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{-\frac{20a^2d^3\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{c}-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{9b^2c}{a}+\frac{20ad^2}{c}+12bd\right)}{2x^2}}{5ac}-\frac{\left(a+bx^3\right)^{2/3}(4ad+3bc)}{5acx^5}}{4ac}-\frac{\left(a+bx^3\right)^{2/3}}{8acx^8}$$

$$\Big\downarrow \text{901}$$

$$-\frac{-\frac{-\frac{20a^2d^3\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{9b^2c}{a}+\frac{20ad^2}{c}+12bd\right)}{2x^2}}{5ac}-\ (}{4ac}$$

$$\frac{\left(a+bx^3\right)^{2/3}}{8acx^8}$$

input
```
Int[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

3.726. $\int \frac{1}{x^9\sqrt[3]{a+bx^3}\left(c+dx^3\right)}dx$

output

```
-1/8*(a + b*x^3)^(2/3)/(a*c*x^8) - (-1/5*((3*b*c + 4*a*d)*(a + b*x^3)^(2/3
))/(a*c*x^5) - (-1/2*(((9*b^2*c)/a + 12*b*d + (20*a*d^2)/c)*(a + b*x^3)^(2
/3))/x^2 - (20*a^2*d^3*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a +
b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^
3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a
+ b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c)/(5*a*c))/(4*a*c)
```

3.726.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 980

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.726. $\int \frac{1}{x^9 \sqrt[3]{a+bx^3}(c+dx^3)} \, dx$

3.726.4 Maple [A] (verified)

Time = 4.98 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{3\left(\left(4a^2d^2+\frac{12}{5}abcd+\frac{9}{5}b^2c^2\right)x^6+\frac{2\left(-4a^2cd-3b\,c^2a\right)x^3}{5}+a^2c^2\right)c\left(b\,x^3+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+4a^3d^3x^8\left(2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\right.\right.}{3\left(\frac{ad}{}\right.}\right.\right.}{24\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}$

input
```
int(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/24/((a*d-b*c)/c)^(1/3)*(3*((4*a^2*d^2+12/5*a*b*c*d+9/5*b^2*c^2)*x^6+2/5
*(-4*a^2*c*d-3*a*b*c^2)*x^3+a^2*c^2)*c*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)
+4*a^3*d^3*x^8*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1
/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^
(1/3))/x)-ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*
x+(b*x^3+a)^(2/3))/x^2)))/x^8/c^4/a^3
```

3.726.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.726.6 Sympy [F]

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/x**9/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**9*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.726.7 Maxima [F]

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^9}\,dx$$

input
```
integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)
```

3.726.8 Giac [F]

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^9}\,dx$$

input
```
integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)
```

3.726.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{x^9\,(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.727 $\int \frac{x^7}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

3.727.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{8}{3},\frac{1}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}$$

output
```
1/8*x^8*(1+b*x^3/a)^(1/3)*AppellF1(8/3,1/3,1,11/3,-b*x^3/a,-d*x^3/c)/c/(b*
x^3+a)^(1/3)
```

3.727.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 144 vs. 2(64) = 128.

Time = 8.46 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.25

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$=\frac{5cx^2(a+bx^3)-5acx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2(2bc+ad)x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\,(}{20bcd\sqrt[3]{a+bx^3}}$$

input
```
Integrate[x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(5*c*x^2*(a + b*x^3) - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3,
1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] - 2*(2*b*c + a*d)*x^5*(1 + (b*x^3)/a)^
(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*b*c*d*(a
 + b*x^3)^(1/3))
```

3.727.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x^7}{\sqrt[3]{\frac{bx^3}{a}+1}(dx^3+c)}dx}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{8}{3},\frac{1}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}$$

input
```
Int[x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(x^8*(1 + (b*x^3)/a)^(1/3)*AppellF1[8/3, 1/3, 1, 11/3, -((b*x^3)/a), -((d*
x^3)/c)])/(8*c*(a + b*x^3)^(1/3))
```

3.727.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.727.4 Maple [F]

$$\int \frac{x^7}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}dx$$

input
```
int(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

3.727.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.727.6 Sympy [F]

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x**7/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**7/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.727.7 Maxima [F]

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^7}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^7/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.727.8 Giac [F]

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^7}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^7/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.727.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^7}{\left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(x^7/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.728 $\int \frac{x^4}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

3.728.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx=\frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

output
```
1/5*x^5*(1+b*x^3/a)^(1/3)*AppellF1(5/3,1/3,1,8/3,-b*x^3/a,-d*x^3/c)/c/(b*x
^3+a)^(1/3)
```

3.728.2 Mathematica [A] (verified)

Time = 7.95 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx=\frac{x^5\sqrt[3]{\frac{a+bx^3}{a}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

input
```
Integrate[x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(x^5*((a + b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x
^3)/c)])/(5*c*(a + b*x^3)^(1/3))
```

3.728.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}dx$$

$$\Big\downarrow \textcolor{blue}{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x^4}{\sqrt[3]{\frac{bx^3}{a}+1}(dx^3+c)}dx}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \textcolor{blue}{1012}$$

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

input
```
Int[x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x
^3)/c)])/(5*c*(a + b*x^3)^(1/3))
```

3.728.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.728.4 Maple [F]

$$\int \frac{x^{4}}{\left(b \,x^{3}+a \right)^{\frac{1}{3}} \left(d \,x^{3}+c \right)}d x$$

input
```
int(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

3.728.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[3]{a+bx^3} (c+dx^3)} \, dx=\text{Timed out}$$

input
```
integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.728.6 Sympy [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^3} (c+dx^3)} \, dx=\int \frac{x^{4}}{\sqrt[3]{a + b x^{3}} \left(c + d x^{3}\right)}\, dx$$

input
```
integrate(x**4/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x**4/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.728.7 Maxima [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^4}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.728.8 Giac [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^4}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.728.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^4}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(x^4/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.729 $\int \frac{x}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

3.729.1 Optimal result

Integrand size $= 22$, antiderivative size $= 64$

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

output

```
1/2*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1/3,1,5/3,-b*x^3/a,-d*x^3/c)/c/(b*x
^3+a)^(1/3)
```

3.729.2 Mathematica [A] (verified)

Time $= 10.03$ (sec) , antiderivative size $= 65$, normalized size of antiderivative $= 1.02$

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\frac{x^2\sqrt[3]{\frac{a+bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

input

```
Integrate[x/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output

```
(x^2*((a + b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x
^3)/c)])/(2*c*(a + b*x^3)^(1/3))
```

3.729.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \textcolor{blue}{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x}{\sqrt[3]{\frac{bx^3}{a}+1}(dx^3+c)}dx}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \textcolor{blue}{1012}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

input
```
Int[x/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x
^3)/c)])/(2*c*(a + b*x^3)^(1/3))
```

3.729.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.729.4 Maple [F]

$$\int \frac{x}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(d\,x^{3}+c\right)}d x$$

input
```
int(x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

3.729.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.729.6 Sympy [F]

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(x/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(x/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.729.7 Maxima [F]

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.729.8 Giac [F]

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.729.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(x/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.730 $\int \frac{1}{x^2 \sqrt[3]{a+bx^3}(c+dx^3)} \, dx$

3.730.1 Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3} \left(c+dx^3\right)} \, dx = -\frac{\sqrt[3]{1+\frac{bx^3}{a}} \text{AppellF1}\left(-\frac{1}{3},\frac{1}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{a+bx^3}}$$

output
```
-(1+b*x^3/a)^(1/3)*AppellF1(-1/3,1/3,1,2/3,-b*x^3/a,-d*x^3/c)/c/x/(b*x^3+a
)^(1/3)
```

3.730.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(62) = 124.

Time = 10.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3} \left(c+dx^3\right)} \, dx$$
$$= \frac{-10c(a+bx^3)+5(bc-ad)x^3\sqrt[3]{1+\frac{bx^3}{a}}\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx^6\sqrt[3]{1+\frac{bx^3}{a}}\text{AppellF1}(}{10ac^2x\sqrt[3]{a+bx^3}}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-10*c*(a + b*x^3) + 5*(b*c - a*d)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3,
 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^6*(1 + (b*x^3)/a)^(1/3
)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*a*c^2*x*(a +
 b*x^3)^(1/3))
```

3.730.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{1}{x^2\sqrt[3]{\frac{bx^3}{a}+1}(dx^3+c)}dx}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{a+bx^3}}$$

input
```
Int[1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
-(((1 + (b*x^3)/a)^(1/3)*AppellF1[-1/3, 1/3, 1, 2/3, -((b*x^3)/a), -((d*x^
3)/c)])/(c*x*(a + b*x^3)^(1/3)))
```

3.730.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.730.4 Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{3}+a \right)^{\frac{1}{3}} \left(d\,x^{3}+c \right)}d x$$

input
```
int(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

3.730.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3} \left(c+dx^3\right)} \, dx=\text{Timed out}$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.730.6 Sympy [F]

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^2 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**2*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.730.7 Maxima [F]

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^2), x)
```

3.730.8 Giac [F]

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^2), x)
```

3.730.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^2\,{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

input

```
int(1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output

```
int(1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.731 $\int \frac{1}{x^5 \sqrt[3]{a+bx^3}(c+dx^3)} \, dx$

3.731.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}\,(c+dx^3)} \, dx = -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\ \mathrm{AppellF1}\left(-\frac{4}{3},\frac{1}{3},1,-\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\sqrt[3]{a+bx^3}}$$

output
```
-1/4*(1+b*x^3/a)^(1/3)*AppellF1(-4/3,1/3,1,-1/3,-b*x^3/a,-d*x^3/c)/c/x^4/(
b*x^3+a)^(1/3)
```

3.731.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 183 vs. 2(64) = 128.

Time = 10.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.86

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}\,(c+dx^3)} \, dx$$
$$= \frac{5c(a+bx^3)\left(-ac+2bcx^3+4adx^3\right)+5\left(-b^2c^2-2abcd+2a^2d^2\right)x^6\sqrt[3]{1+\frac{bx^3}{a}}\ \mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\right.}{20a^2c^3x^4\sqrt[3]{a+bx^3}}$$

input
```
Integrate[1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(5*c*(a + b*x^3)*(-(a*c) + 2*b*c*x^3 + 4*a*d*x^3) + 5*(-(b^2*c^2) - 2*a*b*
c*d + 2*a^2*d^2)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b
*x^3)/a), -((d*x^3)/c)] - 2*b*d*(b*c + 2*a*d)*x^9*(1 + (b*x^3)/a)^(1/3)*Ap
pellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*a^2*c^3*x^4*(a +
 b*x^3)^(1/3))
```

3.731.3 Rubi [A] (verified)

Time $= 0.21$ (sec) , antiderivative size $= 64$, normalized size of antiderivative $= 1.00$, number of steps used $= 2$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used $= \{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3} \left(c+dx^3\right)} \, dx$$

$$\Big\downarrow \; 1013$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1} \int \frac{1}{x^5 \sqrt[3]{\frac{bx^3}{a}+1} (dx^3+c)}dx}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \; 1012$$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1} \operatorname{AppellF1}\left(-\frac{4}{3},\frac{1}{3},1,-\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4 \sqrt[3]{a+bx^3}}$$

input
```
Int[1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
-1/4*((1 + (b*x^3)/a)^(1/3)*AppellF1[-4/3, 1/3, 1, -1/3, -((b*x^3)/a), -((
d*x^3)/c)])/(c*x^4*(a + b*x^3)^(1/3))
```

3.731.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.731.4 Maple [F]

$$\int \frac{1}{x^{5} \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

3.731.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3} \left(c+dx^3\right)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.731.6 Sympy [F]

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^5 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/x**5/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**5*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.731.7 Maxima [F]

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^5), x)
```

3.731.8 Giac [F]

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^5), x)
```

3.731.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{x^5\,{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.732 $\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

3.732.1 Optimal result

Integrand size = 24, antiderivative size = 241

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{4/3}}{4b^3d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} + \frac{c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}(bc-ad)^{2/3}} + \frac{c^3\log\left(c+dx^3\right)}{6d^{10/3}(bc-ad)^{2/3}} - \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}(bc-ad)^{2/3}}$$

output
```
(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(1/3)/b^3/d^3-1/4*(2*a*d+b*c)*(b*x^3+a
)^(4/3)/b^3/d^2+1/7*(b*x^3+a)^(7/3)/b^3/d+1/6*c^3*ln(d*x^3+c)/d^(10/3)/(-a
*d+b*c)^(2/3)-1/2*c^3*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(10/3
)/(-a*d+b*c)^(2/3)+1/3*c^3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b
*c)^(1/3))*3^(1/2))/d^(10/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

3.732.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.10

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{3\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{a+bx^3}(18a^2d^2+3abd(7c-2dx^3)+b^2(28c^2-7cdx^3+4d}{}$$

input

```
Integrate[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output

```
(3*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3)*(18*a^2*d^2 + 3*a*b*d*(7*c
- 2*d*x^3) + b^2*(28*c^2 - 7*c*d*x^3 + 4*d^2*x^6)) + 28*Sqrt[3]*b^3*c^3*Ar
cTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 28*b
^3*c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 14*b^3*c^3*Log
[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)
*(a + b*x^3)^(2/3)])/(84*b^3*d^(10/3)*(b*c - a*d)^(2/3))
```

3.732.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^9}{(bx^3+a)^{2/3}(dx^3+c)}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(-\frac{c^3}{d^3(bx^3+a)^{2/3}(dx^3+c)}+\frac{(bx^3+a)^{4/3}}{b^2d}+\frac{(-bc-2ad)\sqrt[3]{bx^3+a}}{b^2d^2}+\frac{b^2c^2+abdc+a^2d^2}{b^2d^3(bx^3+a)^{2/3}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\Bigg(\frac{3\sqrt[3]{a+bx^3}\left(a^2d^2+abcd+b^2c^2\right)}{b^3d^3}+\frac{\sqrt{3}c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{10/3}(bc-ad)^{2/3}}-\frac{3\left(a+bx^3\right)^{4/3}(2ad+bc)}{4b^3d^2}+\frac{3(a+bx}{7b^3}$$

input
```
Int[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((3*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(1/3))/(b^3*d^3) - (3*(b*c +
 2*a*d)*(a + b*x^3)^(4/3))/(4*b^3*d^2) + (3*(a + b*x^3)^(7/3))/(7*b^3*d) +
 (Sqrt[3]*c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))
/Sqrt[3]])/(d^(10/3)*(b*c - a*d)^(2/3)) + (c^3*Log[c + d*x^3])/(2*d^(10/3)
*(b*c - a*d)^(2/3)) - (3*c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(
1/3)])/(2*d^(10/3)*(b*c - a*d)^(2/3)))/3
```

3.732.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.732.4 Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{\frac{9\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}d\left(\frac{\left(2d^{2}x^{6}-\frac{7}{2}cd\,x^{3}+14c^{2}\right)b^{2}}{9}+\frac{7\left(-\frac{2d\,x^{3}}{7}+c\right)dab}{6}+a^{2}d^{2}\right)}{14}+\frac{b^{3}c^{3}\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right.\right.}{}}{b^{3}d^{4}\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}}$

input
```
int(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
9/14*((1/d*(a*d-b*c))^(2/3)*(b*x^3+a)^(1/3)*d*(1/9*(2*d^2*x^6-7/2*c*d*x^3+
14*c^2)*b^2+7/6*(-2/7*d*x^3+c)*d*a*b+a^2*d^2)+7/27*b^3*c^3*(2*arctan(1/3*3
^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^
(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b
*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/(1/d*(a*d-b*c))^
(2/3)/b^3/d^4
```

3.732.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(201) = 402.

Time = 0.35 (sec) , antiderivative size = 1322, normalized size of antiderivative = 5.49

$$\int \frac{x^{11}}{\left(a+bx^{3}\right)^{2/3}\left(c+dx^{3}\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output

```
[1/84*(14*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 +
 a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*
c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) -
 28*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(1
/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 42*sqr
t(1/3)*(b^4*c^4*d - a*b^3*c^3*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^
3)^(1/3)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x
^3 - 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*
b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3
)^(2/3)*(b*x^3 + a)^(1/3))*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)
/d) - 3*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c
- a*d))/(d*x^3 + c)) + 3*(28*b^4*c^4*d - 35*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*
d^3 - 15*a^3*b*c*d^4 + 18*a^4*d^5 + 4*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b
^2*d^5)*x^6 - (7*b^4*c^3*d^2 - 8*a*b^3*c^2*d^3 - 5*a^2*b^2*c*d^4 + 6*a^3*b
*d^5)*x^3)*(b*x^3 + a)^(1/3))/(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6),
 1/84*(14*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 +
 a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*
c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) -
 28*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(1
/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 84*...
```

3.732.6 Sympy [F]

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x^{11}}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

input

```
integrate(x**11/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output

```
Integral(x**11/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.732.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.732.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.54

$$\begin{aligned}\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx &= \frac{b^{24}c^3d^4\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{25}cd^7-ab^{24}d^8\right)}\\
&-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^4-\sqrt{3}ad^5}\\
&-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^4-ad^5\right)}\\
&+\frac{28\left(bx^3+a\right)^{\frac{1}{3}}b^{20}c^2d^4-7\left(bx^3+a\right)^{\frac{4}{3}}b^{19}cd^5+28\left(bx^3+a\right)^{\frac{1}{3}}ab^{19}cd^5+4\left(bx^3+a\right)^{\frac{7}{3}}b^{18}d^6-14\left(bx^3+a\right)^{\frac{4}{3}}ab^{18}}{28\,b^{21}d^7}\end{aligned}$$

input
```
integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b^24*c^3*d^4*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^25*c*d^7 - a*b^24*d^8) - (-b*c*d^2 + a*d^3)^(1/3)*c^
3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c
 - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^4 - sqrt(3)*a*d^5) - 1/6*(-b*c*d^2 + a*d^
3)^(1/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1
/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^4 - a*d^5) + 1/28*(28*(b*x^3 + a)^(1/
3)*b^20*c^2*d^4 - 7*(b*x^3 + a)^(4/3)*b^19*c*d^5 + 28*(b*x^3 + a)^(1/3)*a*
b^19*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^18*d^6 - 14*(b*x^3 + a)^(4/3)*a*b^18*d^
6 + 28*(b*x^3 + a)^(1/3)*a^2*b^18*d^6)/(b^21*d^7)
```

3.732.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.37

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \left(\frac{3\,a^2}{b^3\,d} + \frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{1/3} - \left(\frac{3\,a}{4\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{4\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{4/3} + \frac{{\left(b\,x^3+a\right)}^{7/3}}{7\,b^3\,d} + \frac{\ln\left(\frac{3\,c^3\,{\left(b\,x^3+a\right)}^{1/3}}{d}+\frac{3\,c^3\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{2\,d^{4/3}}\right)\,\left(c^3+\sqrt{3}\,c^3\,1{}\mathrm{i}\right)}{6\,d^{10/3}\,{\left(a\,d-b\,c\right)}^{2/3}} - \frac{c^3\,\ln\left(\frac{3\,c^3\,{\left(b\,x^3+a\right)}^{1/3}}{d}-\frac{3\,c^3\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{4/3}}\right)}{3\,d^{10/3}\,{\left(a\,d-b\,c\right)}^{2/3}} - \frac{c^3\,\ln\left(\frac{3\,c^3\,{\left(b\,x^3+a\right)}^{1/3}}{d}-\frac{3\,c^3\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{4/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,d^{10/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$

input
```
int(x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
((3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c -
 a*b^3*d))/(b^3*d))*(a + b*x^3)^(1/3) - ((3*a)/(4*b^3*d) + (b^4*c - a*b^3*
d)/(4*b^6*d^2))*(a + b*x^3)^(4/3) + (a + b*x^3)^(7/3)/(7*b^3*d) + (log((3*
c^3*(a + b*x^3)^(1/3))/d + (3*c^3*(3^(1/2)*1i + 1)*(a*d - b*c)^(1/3))/(2*d
^(4/3)))*(3^(1/2)*c^3*1i + c^3))/(6*d^(10/3)*(a*d - b*c)^(2/3)) - (c^3*log
((3*c^3*(a + b*x^3)^(1/3))/d - (3*c^3*(a*d - b*c)^(1/3))/d^(4/3)))/(3*d^(1
0/3)*(a*d - b*c)^(2/3)) - (c^3*log((3*c^3*(a + b*x^3)^(1/3))/d - (3*c^3*((
3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(1/3))/d^(4/3))*((3^(1/2)*1i)/2 - 1/2))/(
3*d^(10/3)*(a*d - b*c)^(2/3))
```

3.732. $\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

3.733 $\int \frac{x^8}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

3.733.1 Optimal result

Integrand size = 24, antiderivative size = 201

$$\int \frac{x^8}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} - \frac{c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}(bc-ad)^{2/3}} - \frac{c^2\log\left(c+dx^3\right)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}}$$

output
```
-(a*d+b*c)*(b*x^3+a)^(1/3)/b^2/d^2+1/4*(b*x^3+a)^(4/3)/b^2/d-1/6*c^2*ln(d*
x^3+c)/d^(7/3)/(-a*d+b*c)^(2/3)+1/2*c^2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3
+a)^(1/3))/d^(7/3)/(-a*d+b*c)^(2/3)-1/3*c^2*arctan(1/3*(1-2*d^(1/3)*(b*x^3
+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(7/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

3.733.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.15

$$\int \frac{x^8}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{-3\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{a+bx^3}\left(4bc+3ad-bdx^3\right)-4\sqrt{3}b^2c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{}}{\sqrt[3]{bc}}}{\sqrt{}}\right.}{}$$

input
```
Integrate[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(-3*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3)*(4*b*c + 3*a*d - b*d*x^3)
- 4*Sqrt[3]*b^2*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^
(1/3))/Sqrt[3]] + 4*b^2*c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1
/3)] - 2*b^2*c^2*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*
x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(12*b^2*d^(7/3)*(b*c - a*d)^(2/3)
)
```

3.733.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}\int \left(\frac{c^2}{d^2\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}+\frac{\sqrt[3]{bx^3+a}}{bd}+\frac{-bc-ad}{bd^2\left(bx^3+a\right)^{2/3}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{7/3}(bc-ad)^{2/3}}-\frac{3\sqrt[3]{a+bx^3}(ad+bc)}{b^2d^2}+\frac{3\left(a+bx^3\right)^{4/3}}{4b^2d}-\frac{c^2\log\left(c+dx^3\right)}{2d^{7/3}(bc-ad)^{2/3}}+\frac{3c^2\log\left(\right.}{}\right.$$

input
```
Int[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((-3*(b*c + a*d)*(a + b*x^3)^(1/3))/(b^2*d^2) + (3*(a + b*x^3)^(4/3))/(4*b
^2*d) - (Sqrt[3]*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)
^(1/3))/Sqrt[3]])/(d^(7/3)*(b*c - a*d)^(2/3)) - (c^2*Log[c + d*x^3])/(2*d^
(7/3)*(b*c - a*d)^(2/3)) + (3*c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x
^3)^(1/3)])/(2*d^(7/3)*(b*c - a*d)^(2/3)))/3
```

3.733.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.733.4 Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{\frac{9d\left(\left(-\frac{b\,x^{3}}{3}+a\right)d+\frac{4bc}{3}\right)\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{2}+b^{2}c^{2}\left(2\arctan\left(\frac{\sqrt{3}\,\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-}{d}\right.\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}b^{2}d^{3}}$

input
```
int(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(1/d*(a*d-b*c))^(2/3)*(9/2*d*((-1/3*b*x^3+a)*d+4/3*b*c)*(1/d*(a*d-b*c
))^(2/3)*(b*x^3+a)^(1/3)+b^2*c^2*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+
(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(
1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)
^(1/3)-(1/d*(a*d-b*c))^(1/3))))/b^2/d^3
```

3.733.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(164) = 328.

Time = 0.32 (sec) , antiderivative size = 1156, normalized size of antiderivative = 5.75

$$\int \frac{x^8}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output

```
[-1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 +
a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c
- a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 4*
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a)^(1/3)*(
b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)) + 6*sqrt(1/3)*
(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3
)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 + 3*
sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2
+ a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(
b*x^3 + a)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d) + 3*(
b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d
*x^3 + c)) + 3*(4*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + 3*a^3*d^4
- (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(1/3))/(b^4*c
^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5), -1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 +
 a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 4*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^
(2/3)*b^2*c^2*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)^(2/3)) - 12*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)*arctan(-sqrt(1/3)*((b^2*c^2*...
```

3.733.6 Sympy [F]

$$\int \frac{x^8}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{x^8}{(a+bx^3)^{\frac{2}{3}}\,(c+dx^3)}\,dx$$

input

```
integrate(x**8/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output

```
Integral(x**8/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.733.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.733.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.55

$$\begin{aligned}\int \frac{x^8}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx &= -\frac{b^{10}c^2d^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{11}cd^4-ab^{10}d^5\right)}\\
&\quad+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^3-\sqrt{3}ad^4}\\
&\quad+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^2\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^3-ad^4\right)}\\
&\quad-\frac{4\left(bx^3+a\right)^{\frac{1}{3}}b^7cd^2-\left(bx^3+a\right)^{\frac{4}{3}}b^6d^3+4\left(bx^3+a\right)^{\frac{1}{3}}ab^6d^3}{4\,b^8d^4}\end{aligned}$$

input

```
integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*b^10*c^2*d^2*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*
c - a*d)/d)^(1/3)))/(b^11*c*d^4 - a*b^10*d^5) + (-b*c*d^2 + a*d^3)^(1/3)*c
^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*
c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) + 1/6*(-b*c*d^2 + a*d
^3)^(1/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(
1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^3 - a*d^4) - 1/4*(4*(b*x^3 + a)^(1/3
)*b^7*c*d^2 - (b*x^3 + a)^(4/3)*b^6*d^3 + 4*(b*x^3 + a)^(1/3)*a*b^6*d^3)/(
b^8*d^4)
```

3.733.9 Mupad [B] (verification not implemented)

Time = 8.67 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.45

$$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{(b\,x^3+a)^{4/3}}{4\,b^2\,d} - \left(\frac{2\,a}{b^2\,d} + \frac{b^3\,c - a\,b^2\,d}{b^4\,d^2}\right)(b\,x^3+a)^{1/3} - \frac{\ln\left(3\,c^2\,(b\,x^3+a)^{1/3} + \frac{\left(c^2+\sqrt{3}\,c^2\,1\mathrm{i}\right)(9\,a\,d^3-9\,b\,c\,d^2)}{6\,d^{7/3}\,(a\,d-b\,c)^{2/3}}\right)\left(c^2+\sqrt{3}\,c^2\,1\mathrm{i}\right)}{6\,d^{7/3}\,(a\,d-b\,c)^{2/3}} + \frac{c^2\,\ln\left(3\,c^2\,(b\,x^3+a)^{1/3} - \frac{c^2\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{7/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{7/3}\,(a\,d-b\,c)^{2/3}} + \frac{c^2\,\ln\left(3\,c^2\,(b\,x^3+a)^{1/3} - \frac{c^2\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)(9\,a\,d^3-9\,b\,c\,d^2)}{d^{7/3}\,(a\,d-b\,c)^{2/3}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{d^{7/3}\,(a\,d-b\,c)^{2/3}}$$

input
```
int(x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
(a + b*x^3)^(4/3)/(4*b^2*d) - ((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2)
)*(a + b*x^3)^(1/3) - (log(3*c^2*(a + b*x^3)^(1/3) + ((3^(1/2)*c^2*1i + c^
2)*(9*a*d^3 - 9*b*c*d^2))/(6*d^(7/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*c^2*1i +
 c^2))/(6*d^(7/3)*(a*d - b*c)^(2/3)) + (c^2*log(3*c^2*(a + b*x^3)^(1/3) -
(c^2*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)*(a*d - b*c)^(2/3))))/(3*d^(7/3)*(a*
d - b*c)^(2/3)) + (c^2*log(3*c^2*(a + b*x^3)^(1/3) - (c^2*((3^(1/2)*1i)/6
- 1/6)*(9*a*d^3 - 9*b*c*d^2))/(d^(7/3)*(a*d - b*c)^(2/3)))*((3^(1/2)*1i)/6
 - 1/6))/(d^(7/3)*(a*d - b*c)^(2/3))
```

3.734 $\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

3.734.1 Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}(bc-ad)^{2/3}} + \frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}$$

output
```
(b*x^3+a)^(1/3)/b/d+1/6*c*ln(d*x^3+c)/d^(4/3)/(-a*d+b*c)^(2/3)-1/2*c*ln((-
a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(4/3)/(-a*d+b*c)^(2/3)+1/3*c*arc
tan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(4/3)/(-
a*d+b*c)^(2/3)*3^(1/2)
```

3.734.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.22

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{6\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{a+bx^3}+2\sqrt{3}bc\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-2bc\log\left(\right.}{6}$$

input
```
Integrate[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(6*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3) + 2*Sqrt[3]*b*c*ArcTan[(1 -
 (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*b*c*Log[(b*
c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + b*c*Log[(b*c - a*d)^(2/3) -
d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/
(6*b*d^(4/3)*(b*c - a*d)^(2/3))
```

3.734.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 90, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{x^3}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \ 90$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{bd}-\frac{c\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{d}\right)$$

$$\Big\downarrow \ 70$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{bd}-\frac{c\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{d}(bc-ad)^{2/3}}-\right.}{d}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{bd}-\frac{c\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a\right.}}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right.}{d}\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{bd}-\frac{c\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right)}{d}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{3\sqrt[3]{a+bx^3}}{bd}-\frac{c\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log\left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right)}{d}\right)$$

3.734. $\int \frac{x^5}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

input
```
Int[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((3*(a + b*x^3)^(1/3))/(b*d) - (c*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a +
b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) -
Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)^(2/3)) + (3*Log[(b*c - a*d)^(1/3) +
d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3)*(b*c - a*d)^(2/3))))/d)/3
```

3.734.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.734.4 Maple [A] (verified)

Time = 5.04 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{2\sqrt{3}\,\arctan\Bigg(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}}\Bigg) b c+6\left(b\,x^{3}+a\right)^{\frac{1}{3}} d\left(\frac{a d-b c}{d}\right)^{\frac{2}{3}}-2 c \ln\Bigg(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}\Bigg) b+c \ln\Bigg((b\,x^{3}+}{6 b\,d^{2}\left(\frac{a d-b c}{d}\right)^{\frac{2}{3}}}$

input
```
int(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3)
)/(1/d*(a*d-b*c))^(1/3))*b*c+6*(b*x^3+a)^(1/3)*d*(1/d*(a*d-b*c))^(2/3)-2*c
*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))*b+c*ln((b*x^3+a)^(2/3)+(1/d*(a*
d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))*b)/b/d^2/(1/d*(a*d-b*
c))^(2/3)
```

3.734.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(133) = 266$.

Time = 0.32 (sec) , antiderivative size = 1060, normalized size of antiderivative = 6.42

$$\int \frac{x^5}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(2/3
)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d)
 + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 2*(-b^2
*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(1/3)*(b*c*d -
a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 3*sqrt(1/3)*(b^2*c^
2*d - a*b*c*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)*log((b
^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 - 3*sqrt(1/3)*(
2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)
^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a
)^(1/3))*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d) - 3*(-b^2*c^2*
d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*x^3 + c
)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x^3 + a)^(1/3))/(b^3*c^2*d^2
 - 2*a*b^2*c*d^3 + a^2*b*d^4), 1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(
2/3)*b*c*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^
2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3
)*(b*x^3 + a)^(1/3)) - 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*lo
g(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3
)^(2/3)) - 6*sqrt(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt(-(-b^2*c^2*d + 2*a*b*c
*d^2 - a^2*d^3)^(1/3)/d)*arctan(sqrt(1/3)*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2
*d^3)^(1/3)*(b*c - a*d) + 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*...
```

3.734.6 Sympy [F]

$$\int \frac{x^5}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x^5}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

input
```
integrate(x**5/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**5/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.734.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.734.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.53

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\frac{6\left(-bcd^2+ad^3\right)^{\frac{1}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}bc\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{bcd^2-ad^3}-}{6\,b}$$

input
```
integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/6*(6*(-b*c*d^2 + a*d^3)^(1/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/
3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^2 - sq
rt(3)*a*d^3) + (-b*c*d^2 + a*d^3)^(1/3)*b*c*log((b*x^3 + a)^(2/3) + (b*x^3
 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2 - a*
d^3) - 2*b*c*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a
*d)/d)^(1/3))))/(b*c*d - a*d^2) - 6*(b*x^3 + a)^(1/3)/d)/b
```

3.734.9 Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.41

$$
\begin{aligned}
\int \frac{x^5}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx &= \frac{\left(b\,x^3+a\right)^{1/3}}{b\,d} \\
&\quad - \frac{c\,\ln\left(3\,c\,d\,\left(b\,x^3+a\right)^{1/3}-\frac{c\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{3\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}} \\
&\quad + \frac{\ln\left(3\,c\,d\,\left(b\,x^3+a\right)^{1/3}+\frac{\left(9\,a\,d^3-9\,b\,c\,d^2\right)\,\left(c-\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(c-\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}} \\
&\quad + \frac{\ln\left(3\,c\,d\,\left(b\,x^3+a\right)^{1/3}+\frac{\left(9\,a\,d^3-9\,b\,c\,d^2\right)\,\left(c+\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(c+\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}
\end{aligned}
$$

input
```
int(x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
(a + b*x^3)^(1/3)/(b*d) - (c*log(3*c*d*(a + b*x^3)^(1/3) - (c*(9*a*d^3 - 9
*b*c*d^2))/(3*d^(4/3)*(a*d - b*c)^(2/3))))/(3*d^(4/3)*(a*d - b*c)^(2/3)) +
 (log(3*c*d*(a + b*x^3)^(1/3) + ((9*a*d^3 - 9*b*c*d^2)*(c - 3^(1/2)*c*1i))
/(6*d^(4/3)*(a*d - b*c)^(2/3)))*(c - 3^(1/2)*c*1i))/(6*d^(4/3)*(a*d - b*c)
^(2/3)) + (log(3*c*d*(a + b*x^3)^(1/3) + ((9*a*d^3 - 9*b*c*d^2)*(c + 3^(1/
2)*c*1i))/(6*d^(4/3)*(a*d - b*c)^(2/3)))*(c + 3^(1/2)*c*1i))/(6*d^(4/3)*(a
*d - b*c)^(2/3))
```

3.735 $\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

3.735.1 Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}} - \frac{\log(c+dx^3)}{6\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}$$

output
```
-1/6*ln(d*x^3+c)/d^(1/3)/(-a*d+b*c)^(2/3)+1/2*ln((-a*d+b*c)^(1/3)+d^(1/3)*
(b*x^3+a)^(1/3))/d^(1/3)/(-a*d+b*c)^(2/3)-1/3*arctan(1/3*(1-2*d^(1/3)*(b*x
^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(1/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

3.735.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx =$$

$$-\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) + \log\bigl((bc-ad)^{2/3} - \sqrt[3]{d}\sqrt[3]{bc-ad}}{6\sqrt[3]{d}(bc-ad)^{2/3}}$$

input
```
Integrate[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(
b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(
a + b*x^3)^(2/3)])/(d^(1/3)*(b*c - a*d)^(2/3))
```

3.735.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {946, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \text{70}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log \left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log \left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log \left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log \left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log \left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right)$$

$$\Big\downarrow \textcolor{blue}{217}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan \left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log \left(c+dx^3\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{3\log \left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}\right)$$

input
```
Int[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/S
qrt[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a
*d)^(2/3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(
1/3)*(b*c - a*d)^(2/3)))/3
```

3.735.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.735.4 Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{-2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)-\ln((b\,x^{3}+a)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}(b\,x^{3}+a)^{\frac{1}{3}}+(}{6d\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}}$

input
```
int(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*
(a*d-b*c))^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))-ln((
b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3)
))/d/(1/d*(a*d-b*c))^(2/3)
```

3.735.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(114) = 228$.

Time = 0.33 (sec) , antiderivative size = 927, normalized size of antiderivative = 6.39

$$\int \frac{x^2}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \left[-\frac{3\sqrt{\frac{1}{3}}(bcd-ad^2)\sqrt{-\frac{(b^2c^2d-2\,abcd^2+a^2d^3)^{\frac{1}{3}}}{d}}\,\log\left(\frac{b^2c^2-4\,abcd+3\,a^2d^2-2\,(b^2cd-abd^2)x}{}\right.}{}\right.$$

input
```
integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)^(1/3)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x
^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b
*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^
(2/3)*(b*x^3 + a)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d
) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a
*d))/(d*x^3 + c)) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3
+ a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*
c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) -
2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(1/3)*(b*c*d
- a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)))/(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3), 1/6*(6*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((b^2*c^2*d - 2*a*b
*c*d^2 + a^2*d^3)^(1/3)/d)*arctan(-sqrt(1/3)*((b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)^(1/3)*(b*c - a*d) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b
*x^3 + a)^(1/3))*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)/(b^2*c^
2 - 2*a*b*c*d + a^2*d^2)) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(
-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(
1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(
1/3)) + 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(1/3)
*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)))/(b^2*c^2...
```

3.735.6 Sympy [F]

$$\int \frac{x^2}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x^2}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

input
```
integrate(x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**2/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.735.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.735.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.52

$$\int \frac{x^2}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd-\sqrt{3}ad^2}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd-ad^2\right)}-\frac{\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc-ad\right)}$$

---

input
```
integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
(-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c
- a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d - sqrt(3)*a*d^2) +
 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-
(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d - a*d^2) - 1/3*(-(b*
c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*
c - a*d)
```

3.735.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.47

$$\int \frac{x^2}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \frac{\ln\left(3\,d^2\,{\left(b\,x^3+a\right)}^{1/3}-\frac{9\,a\,d^3-9\,b\,c\,d^2}{3\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{3\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$
$$+\frac{\ln\left(3\,d^2\,{\left(b\,x^3+a\right)}^{1/3}-\frac{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{6\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$
$$-\frac{\ln\left(3\,d^2\,{\left(b\,x^3+a\right)}^{1/3}+\frac{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{6\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$

input
```
int(x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
log(3*d^2*(a + b*x^3)^(1/3) - (9*a*d^3 - 9*b*c*d^2)/(3*d^(1/3)*(a*d - b*c)
^(2/3)))/(3*d^(1/3)*(a*d - b*c)^(2/3)) + (log(3*d^2*(a + b*x^3)^(1/3) - ((
3^(1/2)*1i - 1)*(9*a*d^3 - 9*b*c*d^2))/(6*d^(1/3)*(a*d - b*c)^(2/3)))*(3^(
1/2)*1i - 1))/(6*d^(1/3)*(a*d - b*c)^(2/3)) - (log(3*d^2*(a + b*x^3)^(1/3)
 + ((3^(1/2)*1i + 1)*(9*a*d^3 - 9*b*c*d^2))/(6*d^(1/3)*(a*d - b*c)^(2/3)))
*(3^(1/2)*1i + 1))/(6*d^(1/3)*(a*d - b*c)^(2/3))
```

3.736 $\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

3.736.1 Optimal result

Integrand size = 24, antiderivative size = 245

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}c} + \frac{d^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{2/3}} - \frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}}$$

output
```
-1/2*ln(x)/a^(2/3)/c+1/6*d^(2/3)*ln(d*x^3+c)/c/(-a*d+b*c)^(2/3)+1/2*ln(a^(
1/3)-(b*x^3+a)^(1/3))/a^(2/3)/c-1/2*d^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b
*x^3+a)^(1/3))/c/(-a*d+b*c)^(2/3)-1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3
))/a^(1/3)*3^(1/2))/a^(2/3)/c*3^(1/2)+1/3*d^(2/3)*arctan(1/3*(1-2*d^(1/3)*
(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/(-a*d+b*c)^(2/3)*3^(1/2)
```

3.736.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.26

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{-\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2\sqrt{3}d^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{(bc-ad)^{2/3}} + \frac{2\log\left(-\sqrt[3]{a}+\right.}{a}}{}$$

input
```
Integrate[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((-2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)
+ (2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)
^(1/3))/Sqrt[3]])/(b*c - a*d)^(2/3) + (2*Log[-a^(1/3) + (a + b*x^3)^(1/3)]
)/a^(2/3) - (2*d^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(b*c - a*d)^(2/3) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)
^(2/3)]/a^(2/3) + (d^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/
3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(2/3))/(6*c
)
```

3.736.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {948, 97, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3}{c}-\frac{d\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{c}\right)$$

$$\Big\downarrow 69$$

$$\frac{1}{3}\left(\frac{-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2a^{2/3}}}{c}-\frac{d\int \frac{1}{(bx^3+a)^{2/3}(dx^3+c)}dx^3}{c}\right)$$

$$\Big\downarrow 16$$

$$\frac{1}{3}\left(\frac{-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}}{c}-\frac{d\int \frac{1}{(bx^3+a)^{2/3}(dx^3+c)}dx^3}{c}\right)$$

$$\Big\downarrow 70$$

$$\frac{1}{3}\left(\frac{-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}}{c}-\frac{d\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx}}{\sqrt[3]{d}}}}{2d^{2/3}\sqrt[3]{bc-ad}}\right.}{}\right.$$

$$\Big\downarrow 16$$

$$\frac{1}{3}\left(\frac{-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}}{c}-\frac{d\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx}}{\sqrt[3]{d}}}}{2d^{2/3}\sqrt[3]{bc-ad}}\right.}{}\right.$$

$$\Big\downarrow 1082$$

3.736. $\int \frac{1}{x(a+bx^3)^{2/3}(c+dx^3)}\,dx$

$$\frac{1}{3}\left(\frac{\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}}{c}-\frac{d\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log(c+d}{2\sqrt[3]{d}(bc-a}\right.}{c}\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}}{c}-\frac{d\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log(c+}{2\sqrt[3]{d}(bc-}\right.}{c}\right.$$

input
```
Int[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3))
 - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3))
)/c - (d*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)
^(1/3))/Sqrt[3]])/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)
*(b*c - a*d)^(2/3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)
])/(2*d^(1/3)*(b*c - a*d)^(2/3))))/c)/3
```

3.736. $\int \frac{1}{x(a+bx^3)^{2/3}(c+dx^3)}\,dx$

3.736.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.736. $\int \frac{1}{x(a+bx^3)^{2/3}(c+dx^3)}\,dx$

3.736.4 Maple [A] (verified)

Time = 4.65 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\right.\right.}{6a}$

input
```
int(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*((2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*
(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+
a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3)
))*a^(2/3)-(2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1
/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/
3)-a^(1/3)))*(1/d*(a*d-b*c))^(2/3))/a^(2/3)/(1/d*(a*d-b*c))^(2/3)/c
```

3.736.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(192) = 384.

Time = 0.32 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.93

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx =$$

$$-\frac{2\sqrt{3}a^{2}\left(-\frac{d^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^{3}+a\right)^{\frac{1}{3}}\left(bc-ad\right)\left(-\frac{d^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{2}{3}}-\sqrt{3}d}{3\,d}\right)+a^{2}\left(-\frac{d^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}}{}$$

input
```
integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*a^2*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-1
/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d^2/(b^2*c^2 - 2*a*b*c*d + a
^2*d^2))^(2/3) - sqrt(3)*d)/d) + a^2*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)
)^(1/3)*log((b*x^3 + a)^(2/3)*d^2 + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2)*(-d^
2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)) - 2*a^2*(-d^2/(b^2*c^2 - 2*
a*b*c*d + a^2*d^2))^(1/3)*log((b*x^3 + a)^(1/3)*d - (b*c - a*d)*(-d^2/(b^2
*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)) + 2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*(
a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(2/3
))/a^2) + (a^2)^(2/3)*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a
)^(1/3)*(a^2)^(2/3)) - 2*(a^2)^(2/3)*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)
))/(a^2*c)
```

3.736.6 Sympy [F]

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{x\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

input
```
integrate(1/x/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.736.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)x}\,dx$$

input
```
integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x), x)
```

3.736.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.31

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = {} & \frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)} \\
& -\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2-\sqrt{3}acd} \\
& -\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^2-acd\right)} \\
& -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}c} \\
& -\frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}c}+\frac{\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{2}{3}}c}
\end{aligned}
$$

input
```
integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*d*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^
(1/3)))/(b*c^2 - a*c*d) - (-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(
b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3
)*b*c^2 - sqrt(3)*a*c*d) - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2
/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(
b*c^2 - a*c*d) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(
1/3))/a^(1/3))/(a^(2/3)*c) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)
*a^(1/3) + a^(2/3))/(a^(2/3)*c) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3))
)/(a^(2/3)*c)
```

3.736.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 1413, normalized size of antiderivative = 5.77

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
log((((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - (243*a*b^6*
c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(1/(27*a^2*c^3))^(1/3
))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c^3))^(1/3) + 6*b^4*
d^5*(a + b*x^3)^(1/3))*(1/(27*a^2*c^3))^(1/3) + log(- (((81*b^6*c^5*d^3 -
162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - (-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2
- 54*a*b*c^4*d))^(1/3)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*
b^4*c^4*d^5))*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(2/3) -
9*b^5*c^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3) -
 6*b^4*d^5*(a + b*x^3)^(1/3))*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*
c^4*d))^(1/3) + log(((3^(1/2)*1i)/2 - 1/2)*(((3^(1/2)*1i)/2 - 1/2)^2*((81*
b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - ((3^(1/2)*1i)/2 - 1/2
)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(1/(27*a
^2*c^3))^(1/3))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c^3))^(
1/3) + 6*b^4*d^5*(a + b*x^3)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^2*c^3)
)^(1/3) - log(6*b^4*d^5*(a + b*x^3)^(1/3) - ((3^(1/2)*1i)/2 + 1/2)*(((3^(1
/2)*1i)/2 + 1/2)^2*((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3)
 + ((3^(1/2)*1i)/2 + 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a
^3*b^4*c^4*d^5)*(1/(27*a^2*c^3))^(1/3))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2
*d^4)*(1/(27*a^2*c^3))^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^2*c^3))^(1/3
) + (log(6*b^4*d^5*(a + b*x^3)^(1/3) + ((3^(1/2)*1i - 1)*(((3^(1/2)*1i ...
```

3.737 $\int \frac{1}{x^4\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

3.737.1 Optimal result

Integrand size = 24, antiderivative size = 299

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx &= -\frac{\sqrt[3]{a+bx^3}}{3acx^3}+\frac{(2bc+3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}c^2} \\
&\quad -\frac{d^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2(bc-ad)^{2/3}}+\frac{(2bc+3ad)\log(x)}{6a^{5/3}c^2}-\frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}} \\
&\quad -\frac{(2bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{5/3}c^2}+\frac{d^{5/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{2/3}}
\end{aligned}$$

output
```
-1/3*(b*x^3+a)^(1/3)/a/c/x^3+1/6*(3*a*d+2*b*c)*ln(x)/a^(5/3)/c^2-1/6*d^(5/
3)*ln(d*x^3+c)/c^2/(-a*d+b*c)^(2/3)-1/6*(3*a*d+2*b*c)*ln(a^(1/3)-(b*x^3+a)
^(1/3))/a^(5/3)/c^2+1/2*d^(5/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3
))/c^2/(-a*d+b*c)^(2/3)+1/9*(3*a*d+2*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^
(1/3))/a^(1/3)*3^(1/2))/a^(5/3)/c^2*3^(1/2)-1/3*d^(5/3)*arctan(1/3*(1-2*d^
(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/(-a*d+b*c)^(2/3)*3^(1
/2)
```

3.737.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^4 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)} \, dx = \frac{-\frac{6c\sqrt[3]{a+bx^3}}{ax^3} + \frac{2\sqrt{3}(2bc+3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} - \frac{6\sqrt{3}d^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{}}{\sqrt[3]{bc}}}{}\right)}{(bc-ad)^{2/3}}}{}$$

input
```
Integrate[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((-6*c*(a + b*x^3)^(1/3))/(a*x^3) + (2*Sqrt[3]*(2*b*c + 3*a*d)*ArcTan[(1 +
 (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(5/3) - (6*Sqrt[3]*d^(5/3)*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*c -
 a*d)^(2/3) - (2*(2*b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(5/3
) + (6*d^(5/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c -
a*d)^(2/3) + ((2*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a
 + b*x^3)^(2/3)])/a^(5/3) - (3*d^(5/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*
c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)
^(2/3))/(18*c^2)
```

3.737.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {948, 114, 27, 174, 69, 16, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3} \int \frac{1}{x^6 \left(bx^3+a\right)^{2/3} \left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{2bdx^3+2bc+3ad}{3x^3\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{ac}-\frac{\sqrt[3]{a+bx^3}}{acx^3}\right)$$

$$\Big\downarrow 27$$

$$\frac{1}{3}\left(-\frac{\int \frac{2bdx^3+2bc+3ad}{x^3\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{3ac}-\frac{\sqrt[3]{a+bx^3}}{acx^3}\right)$$

$$\Big\downarrow 174$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+2bc)\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3}{c}-\frac{3ad^2\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{c}}{3ac}-\frac{\sqrt[3]{a+bx^3}}{acx^3}\right)$$

$$\Big\downarrow 69$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+2bc)\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{3ad^2\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx\right.}}{c}}{3ac}\right.$$

$$\Big\downarrow 16$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+2bc)\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{3ad^2\int \frac{1}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx^3}{c}}{3ac}-\right.$$

$$\Big\downarrow 70$$

3.737. $\int \frac{1}{x^4\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(-\frac{(3ad+2bc)\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{3ad^2\left(\frac{3\int\frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\sqrt[3]{bc}}}{2d^{2/}}\right.}{3ac}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(-\frac{(3ad+2bc)\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{3ad^2\left(\frac{3\int\frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\sqrt[3]{bc}}}{2d^{2/}}\right.}{3ac}\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(-\frac{(3ad+2bc)\left(\frac{3\int\frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{3ad^2\left(\frac{3\int\frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{}{2}\right.}{3ac}\right.$$

$$\Big\downarrow \text{217}$$

3.737. $\int \frac{1}{x^4(a+bx^3)^{2/3}(c+dx^3)}\,dx$

$$\frac{1}{3}\left(-\frac{(3ad+2bc)\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)}{c}-\frac{3ad^2\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}-\right.}{3ac}\right.$$

input
```
Int[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(-((a + b*x^3)^(1/3)/(a*c*x^3)) - (((2*b*c + 3*a*d)*(-((Sqrt[3]*ArcTan[(1
+ (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3]/(2*a^(2/3))
 + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3))))/c - (3*a*d^2*(-((Sqr
t[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]
)/(d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(2*d^(1/3)*(b*c - a*d)^(2/
3)) + (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1/3)*(b
*c - a*d)^(2/3))))/c)/(3*a*c))/3
```

3.737.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.737. $\int \frac{1}{x^4(a+bx^3)^{2/3}(c+dx^3)}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.737.4 Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$-2(b\,x^3+a)^{\frac{1}{3}}\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}c\,a^{\frac{2}{3}}+x^3\,\Big(-d\,\Big(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^3+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\Big(\left(b\,x^3+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}(b\,x^3+a$

input
```
int(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*(b*x^3+a)^(1/3)*(1/d*(a*d-b*c))^(2/3)*c*a^(2/3)+x^3*(-d*(2*arctan(
1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3
))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(
a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3)))*a^(5/3)+1/3*
(1/d*(a*d-b*c))^(2/3)*(3*a*d+2*b*c)*(2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/
3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^
(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))))/a^(5/3)/(1/d*(a*d-b*c))^(2/3)/c^2/
x^3
```

3.737.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(239) = 478$.

Time = 0.79 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.88

$$\int \frac{1}{x^4\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx =$$

$$-\,6\sqrt{3}a^3d\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}x^3\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3\,d}\right)+3\,a^3d\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)$$

input
```
integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/18*(6*sqrt(3)*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x^3*arc
tan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(d^2/(b^2*c^2 - 2*a*b*c*
d + a^2*d^2))^(2/3) - sqrt(3)*d)/d) + 3*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d +
a^2*d^2))^(1/3)*x^3*log((b*x^3 + a)^(2/3)*d^2 - (b*x^3 + a)^(1/3)*(b*c*d -
 a*d^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3) + (b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)) - 6*a^3*d*(d^2/(b
^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x^3*log((b*x^3 + a)^(1/3)*d + (b*c -
a*d)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)) - 2*sqrt(3)*(2*a*b*c + 3
*a^2*d)*x^3*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3)*a - 2*sq
rt(3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2) - (-a^2)^(2
/3)*(2*b*c + 3*a*d)*x^3*log((b*x^3 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^3
+ a)^(1/3)*(-a^2)^(2/3)) + 2*(-a^2)^(2/3)*(2*b*c + 3*a*d)*x^3*log((b*x^3 +
 a)^(1/3)*a - (-a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)*a^2*c)/(a^3*c^2*x^3)
```

3.737.6 Sympy [F]

$$\int \frac{1}{x^4 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)}\,dx = \int \frac{1}{x^4 \left(a+bx^3\right)^{\frac{2}{3}} \left(c+dx^3\right)}\,dx$$

input
```
integrate(1/x**4/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**4*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.737.7 Maxima [F]

$$\int \frac{1}{x^4 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^4), x)
```

3.737.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)} \, dx = -\frac{d^2 \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log \left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3-ac^2d)} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}} d \arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^3-\sqrt{3}ac^2d} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}} d \log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^3-ac^2d)} + \frac{\sqrt{3}(2\,bc+3\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{5}{3}}c^2} + \frac{(2\,bc+3\,ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{5}{3}}c^2} - \frac{(2\,bc+3\,ad)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{5}{3}}c^2} - \frac{(bx^3+a)^{\frac{1}{3}}}{3\,acx^3}$$

input
```
integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*d^2*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/
d)^(1/3)))/(b*c^3 - a*c^2*d) + (-b*c*d^2 + a*d^3)^(1/3)*d*arctan(1/3*sqrt(
3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/
(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*d*log((b*
x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/
d)^(2/3))/(b*c^3 - a*c^2*d) + 1/9*sqrt(3)*(2*b*c + 3*a*d)*arctan(1/3*sqrt(
3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(5/3)*c^2) + 1/18*(2*b*c +
3*a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(5/
3)*c^2) - 1/9*(2*b*c + 3*a*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/
3)*c^2) - 1/3*(b*x^3 + a)^(1/3)/(a*c*x^3)
```

3.737.9 Mupad [B] (verification not implemented)

Time = 15.75 (sec) , antiderivative size = 1959, normalized size of antiderivative = 6.55

$$\int \frac{1}{x^4 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)}\, dx = \text{Too large to display}$$

input

```
int(1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output

```
log(- (((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c
*d))/a - 81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(d^5/(c^6*(a*d
 - b*c)^2))^(1/3))*(d^5/(c^6*(a*d - b*c)^2))^(2/3))/9 + (b^5*d^4*(8*b^3*c^
3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*(d^5/(c^6*(a
*d - b*c)^2))^(1/3))/3 - (2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*
c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4))*(d^5/(27*b^2*c^8 + 27
*a^2*c^6*d^2 - 54*a*b*c^7*d))^(1/3) + log(- (((((27*b^5*c^3*d^3*(a + b*x^3
)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - 27*a*b^4*c^4*d^3*(2*a^2*d^2
 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))*(-(3*a*d + 2
*b*c)^3/(a^5*c^6))^(2/3))/81 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2
*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))
/9 - (2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d
 + 36*a^2*b*c*d^2))/(9*a^3*c^4))*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^2*c^2*
d + 54*a^2*b*c*d^2)/(729*a^5*c^6))^(1/3) + (log(((3^(1/2)*1i - 1)*((((27*b
^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - (81*a*
b^4*c^4*d^3*(3^(1/2)*1i - 1)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(d^5/(c^6*(
a*d - b*c)^2))^(1/3))/2)*(3^(1/2)*1i - 1)^2*(d^5/(c^6*(a*d - b*c)^2))^(2/3
))/36 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2
))/(3*a^3*c))*(d^5/(c^6*(a*d - b*c)^2))^(1/3))/6 + (2*b^4*d^6*(a + b*x^3)^
(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^...
```

3.738 $\int \frac{x^7}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

3.738.1 Optimal result

Integrand size = 24, antiderivative size = 279

$$\int \frac{x^7}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\frac{x^2\sqrt[3]{a+bx^3}}{3bd}+\frac{(3bc+2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}d^2}-\frac{c^{5/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2(bc-ad)^{2/3}}+\frac{c^{5/3}\log\left(c+dx^3\right)}{6d^2(bc-ad)^{2/3}}+\frac{(3bc+2ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{5/3}d^2}-\frac{c^{5/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2(bc-ad)^{2/3}}$$

output

```
1/3*x^2*(b*x^3+a)^(1/3)/b/d+1/6*c^(5/3)*ln(d*x^3+c)/d^2/(-a*d+b*c)^(2/3)+1
/6*(2*a*d+3*b*c)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/d^2-1/2*c^(5/3)*ln(
(-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2/(-a*d+b*c)^(2/3)+1/9*(2*a*
d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)/d^2*3
^(1/2)-1/3*c^(5/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1
/3))*3^(1/2))/d^2/(-a*d+b*c)^(2/3)*3^(1/2)
```

3.738.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.26 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.69

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{\frac{12dx^2\sqrt[3]{a+bx^3}}{b} + \frac{4\sqrt{3}(3bc+2ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{5/3}} + \frac{6\sqrt{-6-6i\sqrt{3}}c^{5/3}\arctan\left(-\right.}{}}{}$$

input
```
Integrate[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((12*d*x^2*(a + b*x^3)^(1/3))/b + (4*Sqrt[3]*(3*b*c + 2*a*d)*ArcTan[(Sqrt[
3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(5/3) + (6*Sqrt[-6 - (
6*I)*Sqrt[3]]*c^(5/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^
(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c - a*d)^(2/3) +
 (4*(3*b*c + 2*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(5/3) + (6*(1
 - I*Sqrt[3])*c^(5/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*
(a + b*x^3)^(1/3)])/(b*c - a*d)^(2/3) - (2*(3*b*c + 2*a*d)*Log[b^(2/3)*x^2
 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(5/3) + ((3*I)*(I +
 Sqrt[3])*c^(5/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(
b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)
^(2/3)])/(b*c - a*d)^(2/3))/(36*d^2)
```

3.738.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {979, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}dx$$

$$\Big\downarrow \; 979$$

$$\frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{\int \frac{x((3bc+2ad)x^3+2ac)}{(bx^3+a)^{2/3}(dx^3+c)}dx}{3bd}$$

$$\Big\downarrow \text{1054}$$

$$\frac{x^2\sqrt[3]{a+bx^3}}{3bd}-\frac{\int \left(\frac{(3bc+2ad)x}{d(bx^3+a)^{2/3}}-\frac{3bc^2x}{d(bx^3+a)^{2/3}(dx^3+c)}\right)dx}{3bd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2\sqrt[3]{a+bx^3}}{3bd}-\frac{-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(2ad+3bc)}{\sqrt{3}b^{2/3}d}+\frac{\sqrt{3}bc^{5/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{d(bc-ad)^{2/3}}-\frac{(2ad+3bc)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{bc^{5/3}\log(c+d}{2d(bc-ad)^{2}}}{3bd}$$

input

```
Int[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output

```
(x^2*(a + b*x^3)^(1/3))/(3*b*d) - (-(((3*b*c + 2*a*d)*ArcTan[(1 + (2*b^(1/
3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)*d)) + (Sqrt[3]*b*c^(5/
3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3
]])/(d*(b*c - a*d)^(2/3)) - (b*c^(5/3)*Log[c + d*x^3])/(2*d*(b*c - a*d)^(2
/3)) - ((3*b*c + 2*a*d)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*b^(2/3)*d)
+ (3*b*c^(5/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*
d*(b*c - a*d)^(2/3)))/(3*b*d)
```

3.738.3.1 Defintions of rubi rules used

rule 979

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

3.738. $\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.738.4 Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$\frac{(b x^3+a)^{\frac{1}{3}} x^2 \left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} d b^{\frac{2}{3}}+\frac{c\left(2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x-2\left(b x^3+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x}\right)\sqrt{3}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b x^3+a\right)^{\frac{1}{3}} x+(b}{x^2}\right.\right.}{2}}{}$

input
```
int(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*((b*x^3+a)^(1/3)*x^2*((a*d-b*c)/c)^(2/3)*d*b^(2/3)+1/2*c*(2*arctan(1/3
*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*
3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+
(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x))*b^(
5/3)-1/6*((a*d-b*c)/c)^(2/3)*(2*a*d+3*b*c)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(
b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^
(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)))/b^(5/
3)/((a*d-b*c)/c)^(2/3)/d^2
```

3.738.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(225) = 450$.

Time = 0.65 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.00

$$\int \frac{x^7}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\frac{6\sqrt{3}b^3c\left(-\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(-\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}+\sqrt{}}{3\,cx}\right.}{}$$

input
```
integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/18*(6*sqrt(3)*b^3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(
-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2))^(2/3) + sqrt(3)*c*x)/(c*x)) + 6*(b*x^3 + a)^(1/3)*b^2*d*x^2 + 6
*b^3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(((b*c - a*d)*(-c^2
/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x + (b*x^3 + a)^(1/3)*c)/x) - 3*b^
3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(((b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)*x^2 + (b*x^3 + a)
^(2/3)*c^2 - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2))^(1/3)*x)/x^2) - 2*sqrt(3)*(3*b^2*c + 2*a*b*d)*(b^2)^(1/6)*arct
an(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3))
*(b^2)^(1/6)/(b^2*x)) + 2*(b^2)^(2/3)*(3*b*c + 2*a*d)*log(-((b^2)^(2/3)*x
- (b*x^3 + a)^(1/3)*b)/x) - (b^2)^(2/3)*(3*b*c + 2*a*d)*log(((b^2)^(1/3)*b
*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2))/(b^3*d
^2)
```

3.738.6 Sympy [F]

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\int \frac{x^7}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**7/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**7/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.738.7 Maxima [F]

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\int \frac{x^7}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.738.8 Giac [F]

$$\int \frac{x^7}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x^7}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.738.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x^7}{\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(x^7/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.739 $\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

3.739.1 Optimal result

Integrand size = 24, antiderivative size = 234

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d} + \frac{c^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d(bc-ad)^{2/3}} - \frac{c^{2/3}\log\left(c+dx^3\right)}{6d(bc-ad)^{2/3}} - \frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d} + \frac{c^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{2/3}}$$

output

```
-1/6*c^(2/3)*ln(d*x^3+c)/d/(-a*d+b*c)^(2/3)-1/2*ln(b^(1/3)*x-(b*x^3+a)^(1/
3))/b^(2/3)/d+1/2*c^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d
/(-a*d+b*c)^(2/3)-1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/
b^(2/3)/d*3^(1/2)+1/3*c^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(
b*x^3+a)^(1/3))*3^(1/2))/d/(-a*d+b*c)^(2/3)*3^(1/2)
```

3.739.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.47 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.81

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{-\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{2/3}} - \frac{2\sqrt{-6-6i\sqrt{3}}c^{2/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}}\right.}{(bc-ad)^{2/3}}}{}$$

input
```
Integrate[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((-4*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]
)/b^(2/3) - (2*Sqrt[-6 - (6*I)*Sqrt[3]]*c^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3
)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1
/3))]])/(b*c - a*d)^(2/3) - (4*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(2/
3) + ((2*I)*(I + Sqrt[3])*c^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[
3])*c^(1/3)*(a + b*x^3)^(1/3)]])/(b*c - a*d)^(2/3) + (2*Log[b^(2/3)*x^2 + b
^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(2/3) + ((1 - I*Sqrt[3]
)*c^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*
d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])
/(b*c - a*d)^(2/3))/(12*d)
```

3.739.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {983, 853, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{983}$$

$$\frac{\int \frac{x}{(bx^3+a)^{2/3}}dx}{d} - \frac{c\int \frac{x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \text{853}$$

$$\frac{-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}}{d}-\frac{c\int\frac{x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \; 992$$

$$\frac{-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}}{d}-\frac{c\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{d}$$

input
```
Int[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(-(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3))
) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3)))/d - (c*(-(ArcTan[(1 +
(2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(
1/3)*(b*c - a*d)^(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) -
Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a
*d)^(2/3))))/d
```

3.739. $\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

3.739.3.1 Defintions of rubi rules used

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

3.739.4 Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.23

method	result
pseudoelliptic	$\left(-\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}-\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x}\right)\sqrt{3}+\ln\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x\right.\right.$

input
```
int(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/b^(2/3)*((-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+
a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x
-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(1/3)
*x+(b*x^3+a)^(1/3))/x))*b^(2/3)+1/2*(2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)
*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x
+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*((a*d-b*c)/c)
^(2/3))/((a*d-b*c)/c)^(2/3)/d
```

3.739.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(187) = 374$.

Time = 0.28 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.26

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{2\sqrt{3}b^2\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}+\sqrt{3}cx}{3\,cx}\right)}{}$$

input
```
integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-1/3
*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*
d^2))^(2/3) + sqrt(3)*c*x)/(c*x)) + 2*b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*
d^2))^(1/3)*log(-((b*c - a*d)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*
x - (b*x^3 + a)^(1/3)*c)/x) - b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1
/3)*log(((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d
^2))^(2/3)*x^2 + (b*x^3 + a)^(2/3)*c^2 + (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d)
*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x)/x^2) + 2*sqrt(3)*b*sqrt(-(
-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqrt(3)*(b*x^3 + a)
^(1/3)*(-b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) - 2*(-b^2)^(2/3)*log(-((
-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(2/3)*log(-((-b^2)^(1/3)*
b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/(b^2
*d)
```

3.739.6 Sympy [F]

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\int \frac{x^4}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**4/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**4/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.739.7 Maxima [F]

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\int \frac{x^4}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.739.8 Giac [F]

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\int \frac{x^4}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.739.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\int \frac{x^4}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)}\,dx$$

input

```
int(x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output

```
int(x^4/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.740 $\int \frac{x}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

3.740.1 Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{x}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=-\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}$$

output
```
1/6*ln(d*x^3+c)/c^(1/3)/(-a*d+b*c)^(2/3)-1/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)
-(b*x^3+a)^(1/3))/c^(1/3)/(-a*d+b*c)^(2/3)-1/3*arctan(1/3*(1+2*(-a*d+b*c)^
(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(1/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

3.740.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.71

$$\int \frac{x}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\frac{2\sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+\left(1-i\sqrt{3}\right)\left(2\right.}{}$$

input
```
Integrate[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(2*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
 a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (1 - I*Sqrt[
3])*(2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/
3)] - Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(
1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]))/(1
2*c^(1/3)*(b*c - a*d)^(2/3))
```

3.740.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{992}$$

$$-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}$$

input
```
Int[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]
]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c -
a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^
(1/3)*(b*c - a*d)^(2/3))
```

3.740.3.1 Defintions of rubi rules used

rule 992

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

3.740.4 Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}{x}\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}c}$

input

```
int(x/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output

```
1/6*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-
b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(
b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln(((((a*d-b*c)/c)^(1/3)*x+(b*x^3+
a)^(1/3))/x))/((a*d-b*c)/c)^(2/3)/c
```

3.740.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \text{Timed out}$$

input

```
integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output

```
Timed out
```

3.740.6 Sympy [F]

$$\int \frac{x}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

input
```
integrate(x/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.740.7 Maxima [F]

$$\int \frac{x}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.740.8 Giac [F]

$$\int \frac{x}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.740.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x}{\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(x/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(x/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.741 $\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

3.741.1 Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{acx}+\frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}(bc-ad)^{2/3}} -\frac{d\log\left(c+dx^3\right)}{6c^{4/3}(bc-ad)^{2/3}}+\frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}(bc-ad)^{2/3}}$$

output
```
-(b*x^3+a)^(1/3)/a/c/x-1/6*d*ln(d*x^3+c)/c^(4/3)/(-a*d+b*c)^(2/3)+1/2*d*ln
((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(4/3)/(-a*d+b*c)^(2/3)+1/3*
d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(
4/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

3.741.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^2 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)}\,dx = \frac{-12\sqrt[3]{c}(bc-ad)^{2/3}\sqrt[3]{a+bx^3}-2\sqrt{-6-6i\sqrt{3}}adx\arctan\left(\frac{3\sqrt[3]{}}{\sqrt{3}\sqrt[3]{bc-ad}x-}\right.}{}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(-12*c^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3) - 2*Sqrt[-6 - (6*I)*Sqrt[
3]]*a*d*x*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3
*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (2*I)*(I + Sqrt[3])*a*d*x*Log[
2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + a*(d
- I*Sqrt[3]*d)*x*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b
*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^
(2/3)])/(12*a*c^(4/3)*(b*c - a*d)^(2/3)*x)
```

3.741.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {980, 25, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{980}$$

$$\frac{\int -\frac{adx}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{adx}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{x}{(bx^3+a)^{2/3}(dx^3+c)}dx}{c}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

$$\Big\downarrow 992$$

$$-\frac{d\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

input
```
Int[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
-((a + b*x^3)^(1/3)/(a*c*x)) - (d*(-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(
c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3)))
+ Log[c + d*x^3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)
/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3))))/c
```

3.741.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

3.741.4 Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$\frac{-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x}\right) adx+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) adx-\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} \left(b\,x^{3}}{x^{2}}\right.}{2}}{3\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} a\,c^{2} x}$

input
```
int(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(2/3)*(-3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*
x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*a*d*x+ln((((a*d-b*c)/c)^(1/3)*
x+(b*x^3+a)^(1/3))/x)*a*d*x-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^
(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a*d*x-3*(b*x^3+a)^(1/3)*c*((
a*d-b*c)/c)^(2/3))/a/c^2/x
```

3.741.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.741.6 Sympy [F]

$$\int \frac{1}{x^2 \left(a+b x^3\right)^{2/3} \left(c+d x^3\right)}\,dx = \int \frac{1}{x^2 \left(a+b x^3\right)^{\frac{2}{3}} \left(c+d x^3\right)}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**2*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.741.7 Maxima [F]

$$\int \frac{1}{x^2 \left(a+b x^3\right)^{2/3} \left(c+d x^3\right)}\,dx = \int \frac{1}{(b x^3+a)^{\frac{2}{3}} (d x^3+c) x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)
```

3.741.8 Giac [F]

$$\int \frac{1}{x^2 \left(a+b x^3\right)^{2/3} \left(c+d x^3\right)}\,dx = \int \frac{1}{(b x^3+a)^{\frac{2}{3}} (d x^3+c) x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)
```

3.741.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)} \, dx = \int \frac{1}{x^2 \left(b\,x^3+a\right)^{2/3} \left(d\,x^3+c\right)} \, dx$$

input
```
int(1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.742 $\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

3.742.1 Optimal result

Integrand size = 24, antiderivative size = 215

$$\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=-\frac{\sqrt[3]{a+bx^3}}{4acx^4}+\frac{(3bc+4ad)\sqrt[3]{a+bx^3}}{4a^2c^2x}-\frac{d^2\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}(bc-ad)^{2/3}}+\frac{d^2\log\left(c+dx^3\right)}{6c^{7/3}(bc-ad)^{2/3}}-\frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}(bc-ad)^{2/3}}$$

output
```
-1/4*(b*x^3+a)^(1/3)/a/c/x^4+1/4*(4*a*d+3*b*c)*(b*x^3+a)^(1/3)/a^2/c^2/x+1
/6*d^2*ln(d*x^3+c)/c^(7/3)/(-a*d+b*c)^(2/3)-1/2*d^2*ln((-a*d+b*c)^(1/3)*x/
c^(1/3)-(b*x^3+a)^(1/3))/c^(7/3)/(-a*d+b*c)^(2/3)-1/3*d^2*arctan(1/3*(1+2*
(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(7/3)/(-a*d+b*c)^(2
/3)*3^(1/2)
```

3.742.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)}\,dx = \frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(-ac+3bcx^3+4adx^3)}{a^2x^4} + \frac{2\sqrt{-6-6i\sqrt{3}}d^2 \arctan\Bigg(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}}}{(bc-ad)^{2/3}}}{}$$

input
```
Integrate[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
((3*c^(1/3)*(a + b*x^3)^(1/3)*(-(a*c) + 3*b*c*x^3 + 4*a*d*x^3))/(a^2*x^4)
+ (2*Sqrt[-6 - (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*
(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c -
a*d)^(2/3) + (2*(1 - I*Sqrt[3])*d^2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqr
t[3])*c^(1/3)*(a + b*x^3)^(1/3)]])/(b*c - a*d)^(2/3) + (I*(I + Sqrt[3])*d^2
*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*
x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(b*c - a
*d)^(2/3))/(12*c^(7/3))
```

3.742.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {980, 25, 1053, 27, 992}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)}dx$$

$$\Big\downarrow \text{980}$$

$$\frac{\int -\frac{3bdx^3+3bc+4ad}{x^2(bx^3+a)^{2/3}(dx^3+c)}dx}{4ac} - \frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{3bdx^3+3bc+4ad}{x^2(bx^3+a)^{2/3}(dx^3+c)}dx}{4ac} - \frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

$$\Big\downarrow 1053$$

$$-\frac{-\frac{\int \frac{4a^2d^2x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{ac}-\frac{\sqrt[3]{a+bx^3}(4ad+3bc)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

$$\Big\downarrow 27$$

$$-\frac{-\frac{4ad^2\int \frac{x}{\left(bx^3+a\right)^{2/3}\left(dx^3+c\right)}dx}{c}-\frac{\sqrt[3]{a+bx^3}(4ad+3bc)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

$$\Big\downarrow 992$$

$$-\frac{-\frac{4ad^2\left(-\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}\right)}{c}-\frac{\sqrt[3]{a+bx^3}(4ad+3bc)}{acx}}{4ac}-\frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

input
```
Int[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
-1/4*(a + b*x^3)^(1/3)/(a*c*x^4) - (-(((3*b*c + 4*a*d)*(a + b*x^3)^(1/3))/
(a*c*x)) - (4*a*d^2*(-(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b
*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c + d*x^
3]/(6*c^(1/3)*(b*c - a*d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a
+ b*x^3)^(1/3)]/(2*c^(1/3)*(b*c - a*d)^(2/3))))/c)/(4*a*c)
```

3.742.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 992
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.742.4 Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\left(\left(-4ad-3bc\right)x^{3}+ac\right)c\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\frac{2a^{2}d^{2}x^{4}\left(-2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+}{x}\right.\right.}{3}}{4\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}a^{2}c^{3}x^{4}}$

input
```
int(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4/((a*d-b*c)/c)^(2/3)*(((a*d-b*c)/c)^(2/3)*((-4*a*d-3*b*c)*x^3+a*c)*c*(
b*x^3+a)^(1/3)+2/3*a^2*d^2*x^4*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)
*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+2*ln((((a*d-b*c)/c)^(
1/3)*x+(b*x^3+a)^(1/3))/x)-ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)
*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/a^2/c^3/x^4
```

3.742.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.742.6 Sympy [F]

$$\int \frac{1}{x^5 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)}\,dx = \int \frac{1}{x^5 \left(a+bx^3\right)^{\frac{2}{3}} \left(c+dx^3\right)}\,dx$$

input
```
integrate(1/x**5/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**5*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.742.7 Maxima [F]

$$\int \frac{1}{x^5 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)
```

3.742.8 Giac [F]

$$\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)
```

3.742.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{x^5\,\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.743 $\int \frac{x^6}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

3.743.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^6}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{7}{3},\frac{2}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\left(a+bx^3\right)^{2/3}}$$

output
```
1/7*x^7*(1+b*x^3/a)^(2/3)*AppellF1(7/3,2/3,1,10/3,-b*x^3/a,-d*x^3/c)/c/(b*
x^3+a)^(2/3)
```

3.743.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 249 vs. 2(64) = 128.

Time = 9.49 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.89

$$\int \frac{x^6}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\frac{x\left(-\frac{(2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{bc}+4\left(\frac{a}{b}+x^3+\frac{}{b(c+dx^3)\left(-4ac\,A\right.}\right.\right.}{8}$$

input
```
Integrate[x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(x*(-(((2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
-((b*x^3)/a), -((d*x^3)/c)])/(b*c)) + 4*(a/b + x^3 + (4*a^2*c^2*AppellF1[1
/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(b*(c + d*x^3)*(-4*a*c*Appel
lF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/
3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
7/3, -((b*x^3)/a), -((d*x^3)/c)]))))))/(8*d*(a + b*x^3)^(2/3))
```

3.743.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{x^6}{\left(\frac{bx^3}{a}+1\right)^{2/3}(dx^3+c)}dx}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^7\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{7}{3},\frac{2}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c(a+bx^3)^{2/3}}$$

input
```
Int[x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(x^7*(1 + (b*x^3)/a)^(2/3)*AppellF1[7/3, 2/3, 1, 10/3, -((b*x^3)/a), -((d*
x^3)/c)])/(7*c*(a + b*x^3)^(2/3))
```

3.743.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.743.4 Maple [F]

$$\int \frac{x^{6}}{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

3.743.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.743.6 Sympy [F]

$$\int \frac{x^6}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x^6}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

input
```
integrate(x**6/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**6/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.743.7 Maxima [F]

$$\int \frac{x^6}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x^6}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^6/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.743.8 Giac [F]

$$\int \frac{x^6}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x^6}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^6/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.743.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x^6}{{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)} \,d x$$

input

```
int(x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output

```
int(x^6/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.744 $\int \frac{x^3}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

3.744.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(a+bx^3\right)^{2/3}}$$

output
```
1/4*x^4*(1+b*x^3/a)^(2/3)*AppellF1(4/3,2/3,1,7/3,-b*x^3/a,-d*x^3/c)/c/(b*x
^3+a)^(2/3)
```

3.744.2 Mathematica [A] (verified)

Time = 8.96 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\frac{x^4\left(\frac{a+bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(x^4*((a + b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x
^3)/c)])/(4*c*(a + b*x^3)^(2/3))
```

3.744.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}dx$$

$$\Big\downarrow \ 1013$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{x^3}{\left(\frac{bx^3}{a}+1\right)^{2/3}(dx^3+c)}dx}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \ 1012$$

$$\frac{x^4\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c(a+bx^3)^{2/3}}$$

input
```
Int[x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x
^3)/c)])/(4*c*(a + b*x^3)^(2/3))
```

3.744.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

3.744.4 Maple [F]

$$\int \frac{x^{3}}{\left(b\,x^{3}+a \right)^{\frac{2}{3}} \left(d\,x^{3}+c \right)}d x$$

input
```
int(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

3.744.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.744.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\int \frac{x^{3}}{\left(a+bx^{3}\right)^{\frac{2}{3}}\left(c+dx^{3}\right)}\,dx$$

input
```
integrate(x**3/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(x**3/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.744.7 Maxima [F]

$$\int \frac{x^3}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x^3}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.744.8 Giac [F]

$$\int \frac{x^3}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x^3}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.744.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x^3}{\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(x^3/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.745 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

3.745.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\,(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(b*x^3+a)^
(2/3)
```

3.745.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 0.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx =$$

$$-\frac{4acx\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(a+bx^3)^{2/3}(c+dx^3)\,\big(-4ac\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\,\big(3ad\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}$$

input
```
Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(-4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((a + b*
x^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

3.745.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}(dx^3+c)}dx}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c*(a + b*x^3)^(2/3))
```

3.745.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.745.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

3.745.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.745.6 Sympy [F]

$$\int \frac{1}{\left(a+bx^3\right)^{2/3} \left(c+dx^3\right)} \, dx=\int \frac{1}{\left(a+bx^{3}\right)^{\frac{2}{3}} \left(c+dx^{3}\right)}\, dx$$

input
```
integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.745.7 Maxima [F]

$$\int \frac{1}{\left(a+bx^3\right)^{2/3} \left(c+dx^3\right)} \, dx=\int \frac{1}{\left(bx^{3}+a\right)^{\frac{2}{3}} \left(dx^{3}+c\right)}\, dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.745.8 Giac [F]

$$\int \frac{1}{\left(a+bx^3\right)^{2/3} \left(c+dx^3\right)} \, dx=\int \frac{1}{\left(bx^{3}+a\right)^{\frac{2}{3}} \left(dx^{3}+c\right)}\, dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.745.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)} \,dx$$

input

```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output

```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.746 $\int \frac{1}{x^3\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

3.746.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^3\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=-\frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(-\frac{2}{3},\frac{2}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\left(a+bx^3\right)^{2/3}}$$

output
```
-1/2*(1+b*x^3/a)^(2/3)*AppellF1(-2/3,2/3,1,1/3,-b*x^3/a,-d*x^3/c)/c/x^2/(b
*x^3+a)^(2/3)
```

3.746.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 338 vs. 2(64) = 128.

Time = 10.28 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.28

$$\int \frac{1}{x^3\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\frac{-bdx^6\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left(-4ac(ac+2bcx^3+3ad}{(c+d}}{}$$

input
```
Integrate[1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(-(b*d*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
-((d*x^3)/c)]) + (4*c*(-4*a*c*(a*c + 2*b*c*x^3 + 3*a*d*x^3 + b*d*x^6)*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(a + b*x^3)*(c +
d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b
*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*
(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d
*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4
/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(8*a*c^2*x^2*(a + b*x^3)^
(2/3))
```

3.746.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\left(\frac{b x^3}{a}+1\right)^{2/3} \int \frac{1}{x^3 \left(\frac{b x^3}{a}+1\right)^{2/3} \left(d x^3+c\right)}dx}{\left(a+b x^3\right)^{2/3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\left(\frac{b x^3}{a}+1\right)^{2/3} \text{AppellF1}\left(-\frac{2}{3},\frac{2}{3},1,\frac{1}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right)}{2 c x^2 \left(a+b x^3\right)^{2/3}}$$

input
```
Int[1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
-1/2*((1 + (b*x^3)/a)^(2/3)*AppellF1[-2/3, 2/3, 1, 1/3, -((b*x^3)/a), -((d
*x^3)/c)])/(c*x^2*(a + b*x^3)^(2/3))
```

3.746.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.746.4 Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

3.746.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.746.6 Sympy [F]

$$\int \frac{1}{x^3 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)}\,dx = \int \frac{1}{x^3 \left(a+bx^3\right)^{\frac{2}{3}} \left(c+dx^3\right)}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**3*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.746.7 Maxima [F]

$$\int \frac{1}{x^3 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^3), x)
```

3.746.8 Giac [F]

$$\int \frac{1}{x^3 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^3), x)
```

3.746.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)}\,dx=\int \frac{1}{x^3 \left(b\,x^3+a\right)^{2/3} \left(d\,x^3+c\right)}\,dx$$

input
```
int(1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.747 $\int \frac{x^{14}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

3.747.1 Optimal result

Integrand size = 24, antiderivative size = 347

$$\begin{aligned}\int \frac{x^{14}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}}+\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^4d} \\ &+\frac{a(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^4d^2}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^3}-\frac{2a\left(a+bx^3\right)^{5/3}}{5b^4d} \\ &-\frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^4d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^4d}+\frac{c^4\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}(bc-ad)^{4/3}} \\ &-\frac{c^4\log\left(c+dx^3\right)}{6d^{11/3}(bc-ad)^{4/3}}+\frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}(bc-ad)^{4/3}}\end{aligned}$$

output
```
-a^4/b^4/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/2*a^2*(b*x^3+a)^(2/3)/b^4/d+1/2*a*(a
*d+b*c)*(b*x^3+a)^(2/3)/b^4/d^2+1/2*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(2
/3)/b^4/d^3-2/5*a*(b*x^3+a)^(5/3)/b^4/d-1/5*(a*d+b*c)*(b*x^3+a)^(5/3)/b^4/
d^2+1/8*(b*x^3+a)^(8/3)/b^4/d-1/6*c^4*ln(d*x^3+c)/d^(11/3)/(-a*d+b*c)^(4/3
)+1/2*c^4*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(11/3)/(-a*d+b*c)
^(4/3)+1/3*c^4*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3
^(1/2))/d^(11/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

3.747.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.01

$$\int \frac{x^{14}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\frac{3d^{2/3}(-81a^4d^3+9a^3bd^2(c-3dx^3)+3a^2b^2d(4c^2+cdx^3+3d^2x^6)+b^4cx^3(20c^2-8cdx^3+5d^2x^6)+ab^3(20c^3+}{b^4(bc-ad)\sqrt[3]{a+bx^3}}}{}$$

input
```
Integrate[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((3*d^(2/3)*(-81*a^4*d^3 + 9*a^3*b*d^2*(c - 3*d*x^3) + 3*a^2*b^2*d*(4*c^2
+ c*d*x^3 + 3*d^2*x^6) + b^4*c*x^3*(20*c^2 - 8*c*d*x^3 + 5*d^2*x^6) + a*b^
3*(20*c^3 + 4*c^2*d*x^3 - c*d^2*x^6 - 5*d^3*x^9)))/(b^4*(b*c - a*d)*(a + b
*x^3)^(1/3)) + (40*Sqrt[3]*c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(
b*c - a*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) + (40*c^4*Log[(b*c - a*d)^(1
/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) - (20*c^4*Log[(b*c - a
*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x
^3)^(2/3)])/(b*c - a*d)^(4/3))/(120*d^(11/3))
```

3.747.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{x^{12}}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \; 98$$

$$\frac{1}{3}\int \left(\frac{x^6}{bd\sqrt[3]{bx^3+a}}-\frac{(bc+ad)x^3}{b^2d^2\sqrt[3]{bx^3+a}}+\frac{b^2c^2+abdc+a^2d^2}{b^3d^3\sqrt[3]{bx^3+a}}+\frac{c^4}{d^3(ad-bc)\sqrt[3]{bx^3+a}\,(dx^3+c)}+\frac{a^4}{b^3(bc-ad)\,(bx^3\right.$$

$$\Big\downarrow \color{blue}{2009}$$

$$\frac{1}{3}\left(-\frac{3a^4}{b^4\sqrt[3]{a+bx^3}(bc-ad)}+\frac{3a^2\left(a+bx^3\right)^{2/3}}{2b^4d}+\frac{3\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^4d^3}+\frac{\sqrt{3}c^4\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a}}{\sqrt[3]{bc-}}}{\sqrt{3}}\right.}{d^{11/3}(bc-ad)^{4/3}}\right.$$

input

```
Int[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output

```
((-3*a^4)/(b^4*(b*c - a*d)*(a + b*x^3)^(1/3)) + (3*a^2*(a + b*x^3)^(2/3))/
(2*b^4*d) + (3*a*(b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^4*d^2) + (3*(b^2*c^2
+ a*b*c*d + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^4*d^3) - (6*a*(a + b*x^3)^(5/
3))/(5*b^4*d) - (3*(b*c + a*d)*(a + b*x^3)^(5/3))/(5*b^4*d^2) + (3*(a + b*
x^3)^(8/3))/(8*b^4*d) + (Sqrt[3]*c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1
/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(11/3)*(b*c - a*d)^(4/3)) - (c^4*Log[
c + d*x^3])/(2*d^(11/3)*(b*c - a*d)^(4/3)) + (3*c^4*Log[(b*c - a*d)^(1/3)
+ d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(11/3)*(b*c - a*d)^(4/3)))/3
```

3.747.3.1 Defintions of rubi rules used

rule 98

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.747.4 Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{-\frac{243\Bigg(a\left(\frac{5}{81}b^{3}x^{9}-\frac{1}{9}a\,b^{2}x^{6}+\frac{1}{3}a^{2}b\,x^{3}+a^{3}\right)d^{3}-\frac{b\left(\frac{5}{9}b^{2}x^{6}-\frac{2}{3}ab\,x^{3}+a^{2}\right)\left(b\,x^{3}+a\right)c\,d^{2}}{9}-\frac{4b^{2}\left(-\frac{2b\,x^{3}}{3}+a\right)\left(b\,x^{3}+a\right)c^{2}d}{27}-\frac{20b^{3}c^{3}\big(b\,x^{3}+}{81}}{20}}{}$

input
```
int(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(1/d*(a*d-b*c))^(1/3)*(-243/20*(a*(5/81*b^3*x^9-1/9*a*b^2*x^6+1/3*a^2
*b*x^3+a^3)*d^3-1/9*b*(5/9*b^2*x^6-2/3*a*b*x^3+a^2)*(b*x^3+a)*c*d^2-4/27*b
^2*(-2/3*b*x^3+a)*(b*x^3+a)*c^2*d-20/81*b^3*c^3*(b*x^3+a))*d*(1/d*(a*d-b*c
))^(1/3)+b^4*c^4*(b*x^3+a)^(1/3)*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)
+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+
(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a
)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/(b*x^3+a)^(1/3)/d^4/(a*d-b*c)/b^4
```

3.747.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(291) = 582$.

Time = 0.36 (sec) , antiderivative size = 1300, normalized size of antiderivative = 3.75

$$\int \frac{x^{14}}{\left(a+bx^{3}\right)^{4/3}\left(c+dx^{3}\right)}\,dx=\text{Too large to display}$$

input
```
integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/120*(60*sqrt(1/3)*(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*
c^4*d^2)*x^3)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3
- b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2
/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c -
a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2
/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) + 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d
^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*
x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 40*(b^5*c^4*x^3 + a*b^4*c^4
)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1
/3)) - 3*(20*a*b^4*c^4*d^2 - 8*a^2*b^3*c^3*d^3 - 3*a^3*b^2*c^2*d^4 - 90*a^
4*b*c*d^5 + 81*a^5*d^6 + 5*(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x^9
 - (8*b^5*c^3*d^3 - 7*a*b^4*c^2*d^4 - 10*a^2*b^3*c*d^5 + 9*a^3*b^2*d^6)*x^
6 + (20*b^5*c^4*d^2 - 16*a*b^4*c^3*d^3 - a^2*b^3*c^2*d^4 - 30*a^3*b^2*c*d^
5 + 27*a^4*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(a*b^6*c^2*d^5 - 2*a^2*b^5*c*d^6
 + a^3*b^4*d^7 + (b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^3), -1/120*
(120*sqrt(1/3)*(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*c^4*d^2
)*x^3)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*
x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3
)/(b*c - a*d))/d) + 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^(2/3)*
log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*...
```

3.747.6 Sympy [F]

$$\int \frac{x^{14}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \int \frac{x^{14}}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

input
```
integrate(x**14/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**14/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.747.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{14}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.747.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.24

$$\int \frac{x^{14}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^4\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^5-2\sqrt{3}abcd^6+\sqrt{3}a^2d^7}$$

$$-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^4\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^2d^5-2\,abcd^6+a^2d^7\right)}$$

$$+\frac{c^4\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^2d^3-2\,abcd^4+a^2d^5\right)}-\frac{a^4}{\left(b^5c-ab^4d\right)\left(bx^3+a\right)^{\frac{1}{3}}}$$

$$+\frac{20\left(bx^3+a\right)^{\frac{2}{3}}b^{30}c^2d^5-8\left(bx^3+a\right)^{\frac{5}{3}}b^{29}cd^6+40\left(bx^3+a\right)^{\frac{2}{3}}ab^{29}cd^6+5\left(bx^3+a\right)^{\frac{8}{3}}b^{28}d^7-24\left(bx^3+a\right)^{\frac{5}{3}}ab^{28}}{40\,b^{32}d^8}$$

input
```
integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
(-b*c*d^2 + a*d^3)^(2/3)*c^4*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(
b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^5 - 2*sqrt
(3)*a*b*c*d^6 + sqrt(3)*a^2*d^7) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^4*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3))/(b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7) + 1/3*c^4*(-(b*c - a*d)/d
)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d^3
- 2*a*b*c*d^4 + a^2*d^5) - a^4/((b^5*c - a*b^4*d)*(b*x^3 + a)^(1/3)) + 1/4
0*(20*(b*x^3 + a)^(2/3)*b^30*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^29*c*d^6 + 40
*(b*x^3 + a)^(2/3)*a*b^29*c*d^6 + 5*(b*x^3 + a)^(8/3)*b^28*d^7 - 24*(b*x^3
 + a)^(5/3)*a*b^28*d^7 + 60*(b*x^3 + a)^(2/3)*a^2*b^28*d^7)/(b^32*d^8)
```

3.747.9 Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.63

$$\int \frac{x^{14}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \left(\frac{3\,a^2}{b^4\,d}+\frac{\left(\frac{4\,a}{b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{b^8\,d^2}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{2\,b^4\,d}\right)\,{\left(b\,x^3+a\right)}^{2/3}$$

$$-\left(\frac{4\,a}{5\,b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{5\,b^8\,d^2}\right)\,{\left(b\,x^3+a\right)}^{5/3}+\frac{{\left(b\,x^3+a\right)}^{8/3}}{8\,b^4\,d}+\frac{a^4}{b^4\,{\left(b\,x^3+a\right)}^{1/3}\,\left(a\,d-b\,c\right)}$$

$$+\frac{c^4\,\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^8\,d^5-b\,c^9\,d^4\right)-\frac{c^8\,\left(9\,a^4\,d^{15}-36\,a^3\,b\,c\,d^{14}+54\,a^2\,b^2\,c^2\,d^{13}-36\,a\,b^3\,c^3\,d^{12}+9\,b^4\,c^4\,d^{11}\right)}{9\,d^{22/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)}{3\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{4/3}}$$

$$-\frac{\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^8\,d^5-b\,c^9\,d^4\right)-\frac{{\left(c^4+\sqrt{3}\,c^4\,1{}\mathrm{i}\right)}^2\,\left(9\,a^4\,d^{15}-36\,a^3\,b\,c\,d^{14}+54\,a^2\,b^2\,c^2\,d^{13}-36\,a\,b^3\,c^3\,d^{12}+9\,b^4\,c^4\,d^{11}\right)}{36\,d^{22/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(c^4+\right.}{6\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{4/3}}$$

$$+\frac{c^4\,\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^8\,d^5-b\,c^9\,d^4\right)-\frac{c^8\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2\,\left(9\,a^4\,d^{15}-36\,a^3\,b\,c\,d^{14}+54\,a^2\,b^2\,c^2\,d^{13}-36\,a\,b^3\,c^3\,d^{12}+9\,b^4\,c^4\,d^{11}\right)}{d^{22/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(-\right.}{d^{11/3}\,{\left(a\,d-b\,c\right)}^{4/3}}$$

input
```
int(x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output

```
((3*a^2)/(b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c -
 a*b^4*d))/(2*b^4*d))*(a + b*x^3)^(2/3) - ((4*a)/(5*b^4*d) + (b^5*c - a*b^
4*d)/(5*b^8*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^4*d) + a^4/(b
^4*(a + b*x^3)^(1/3)*(a*d - b*c)) + (c^4*log((a + b*x^3)^(1/3)*(a*c^8*d^5
- b*c^9*d^4) - (c^8*(9*a^4*d^15 + 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 54*
a^2*b^2*c^2*d^13 - 36*a^3*b*c*d^14))/(9*d^(22/3)*(a*d - b*c)^(8/3))))/(3*d
^(11/3)*(a*d - b*c)^(4/3)) - (log((a + b*x^3)^(1/3)*(a*c^8*d^5 - b*c^9*d^4
) - ((3^(1/2)*c^4*1i + c^4)^2*(9*a^4*d^15 + 9*b^4*c^4*d^11 - 36*a*b^3*c^3*
d^12 + 54*a^2*b^2*c^2*d^13 - 36*a^3*b*c*d^14))/(36*d^(22/3)*(a*d - b*c)^(8
/3)))*(3^(1/2)*c^4*1i + c^4))/(6*d^(11/3)*(a*d - b*c)^(4/3)) + (c^4*log((a
 + b*x^3)^(1/3)*(a*c^8*d^5 - b*c^9*d^4) - (c^8*((3^(1/2)*1i)/6 - 1/6)^2*(9
*a^4*d^15 + 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 54*a^2*b^2*c^2*d^13 - 36*
a^3*b*c*d^14))/(d^(22/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/6 - 1/6))/(d^(1
1/3)*(a*d - b*c)^(4/3))
```

3.747. $\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

3.748 $\int \frac{x^{11}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

3.748.1 Optimal result

Integrand size = 24, antiderivative size = 253

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{a\left(a+bx^3\right)^{2/3}}{2b^3d} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^3d} - \frac{c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}(bc-ad)^{4/3}} + \frac{c^3\log\left(c+dx^3\right)}{6d^{8/3}(bc-ad)^{4/3}} - \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}(bc-ad)^{4/3}}$$

output
```
a^3/b^3/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/2*a*(b*x^3+a)^(2/3)/b^3/d-1/2*(a*d+b*
c)*(b*x^3+a)^(2/3)/b^3/d^2+1/5*(b*x^3+a)^(5/3)/b^3/d+1/6*c^3*ln(d*x^3+c)/d
^(8/3)/(-a*d+b*c)^(4/3)-1/2*c^3*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3
))/d^(8/3)/(-a*d+b*c)^(4/3)-1/3*c^3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3
)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(8/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

3.748.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.18

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\frac{3d^{2/3}(18a^3d^2+b^3cx^3(-5c+2dx^3)+3a^2bd(-c+2dx^3)-ab^2(5c^2+cdx^3+2d^2x^6))}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{10\sqrt{3}c^3\arctan\left(\frac{1-2}{}\right.}{(bc-a}}{}$$

input
```
Integrate[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((3*d^(2/3)*(18*a^3*d^2 + b^3*c*x^3*(-5*c + 2*d*x^3) + 3*a^2*b*d*(-c + 2*d
*x^3) - a*b^2*(5*c^2 + c*d*x^3 + 2*d^2*x^6)))/(b^3*(b*c - a*d)*(a + b*x^3)
^(1/3)) - (10*Sqrt[3]*c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
 a*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) - (10*c^3*Log[(b*c - a*d)^(1/3) +
 d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) + (5*c^3*Log[(b*c - a*d)^(2
/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2
/3)])/(b*c - a*d)^(4/3))/(30*d^(8/3))
```

3.748.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow 948$$

$$\frac{1}{3}\int \frac{x^9}{(bx^3+a)^{4/3}(dx^3+c)}dx^3$$

$$\Big\downarrow 98$$

$$\frac{1}{3}\int \left(-\frac{a^3}{b^2(bc-ad)(bx^3+a)^{4/3}}+\frac{x^3}{bd\sqrt[3]{bx^3+a}}+\frac{-bc-ad}{b^2d^2\sqrt[3]{bx^3+a}}-\frac{c^3}{d^2(ad-bc)\sqrt[3]{bx^3+a}(dx^3+c)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3a^3}{b^3\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\sqrt{3}c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{8/3}(bc-ad)^{4/3}}-\frac{3(a+bx^3)^{2/3}(ad+bc)}{2b^3d^2}-\frac{3a(a+bx^3)^{2/3}}{2b^3d}+3(\right.$$

input
```
Int[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((3*a^3)/(b^3*(b*c - a*d)*(a + b*x^3)^(1/3)) - (3*a*(a + b*x^3)^(2/3))/(2*
b^3*d) - (3*(b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^3*d^2) + (3*(a + b*x^3)^(5
/3))/(5*b^3*d) - (Sqrt[3]*c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b
*c - a*d)^(1/3))/Sqrt[3]])/(d^(8/3)*(b*c - a*d)^(4/3)) + (c^3*Log[c + d*x^
3])/(2*d^(8/3)*(b*c - a*d)^(4/3)) - (3*c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)])/(2*d^(8/3)*(b*c - a*d)^(4/3)))/3
```

3.748.3.1 Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.748. $\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

3.748.4 Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{-\frac{54d\left(a\left(-\frac{1}{9}b^{2}x^{6}+\frac{1}{3}abx^{3}+a^{2}\right)d^{2}-\frac{b\left(-\frac{2bx^{3}}{3}+a\right)\left(bx^{3}+a\right)cd}{6}-\frac{5b^{2}c^{2}\left(bx^{3}+a\right)}{18}\right)\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}{5}+b^{3}c^{3}\left(bx^{3}+a\right)^{\frac{1}{3}}\Bigg(-2\arctan\Bigg(\frac{\sqrt{3}\,\big(2}{}}{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(bx^{3}+a\right.}$

input
```
int(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/(1/d*(a*d-b*c))^(1/3)/(b*x^3+a)^(1/3)*(-54/5*d*(a*(-1/9*b^2*x^6+1/3*a*
b*x^3+a^2)*d^2-1/6*b*(-2/3*b*x^3+a)*(b*x^3+a)*c*d-5/18*b^2*c^2*(b*x^3+a))*
(1/d*(a*d-b*c))^(1/3)+b^3*c^3*(b*x^3+a)^(1/3)*(-2*arctan(1/3*3^(1/2)*(2*(b
*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*
x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-
2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/d^3/(a*d-b*c)/b^3
```

3.748.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(208) = 416.

Time = 0.36 (sec) , antiderivative size = 1141, normalized size of antiderivative = 4.51

$$\int \frac{x^{11}}{\left(a+bx^{3}\right)^{4/3}\left(c+dx^{3}\right)}\,dx=\text{Too large to display}$$

input
```
integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/30*(15*sqrt(1/3)*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*d - a*b^4*c
^3*d^2)*x^3)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 -
 b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3
) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d
))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*
(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a
*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a
)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) + 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d
^2 - a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 3*(
5*a*b^3*c^3*d^2 - 2*a^2*b^2*c^2*d^3 - 21*a^3*b*c*d^4 + 18*a^4*d^5 - 2*(b^4
*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 + (5*b^4*c^3*d^2 - 4*a*b^3*c^2
*d^3 - 7*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(a*b^5*c^2*d
^4 - 2*a^2*b^4*c*d^5 + a^3*b^3*d^6 + (b^6*c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^
4*d^6)*x^3), 1/30*(30*sqrt(1/3)*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*
d - a*b^4*c^3*d^2)*x^3)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(s
qrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 -
 a*d^3)^(1/3)/(b*c - a*d))/d) + 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d
^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^
(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2
 - a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*...
```

3.748.6 Sympy [F]

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx=\int \frac{x^{11}}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

input
```
integrate(x**11/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**11/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.748.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.748.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.47

$$\begin{aligned}\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx =& -\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^4-2\sqrt{3}abcd^5+\sqrt{3}a^2d^6} \\ &+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^2d^4-2\,abcd^5+a^2d^6)} \\ &-\frac{c^3\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^2c^2d^2-2\,abcd^3+a^2d^4)}+\frac{a^3}{(b^4c-ab^3d)(bx^3+a)^{\frac{1}{3}}} \\ &-\frac{5\,(bx^3+a)^{\frac{2}{3}}b^{13}cd^3-2\,(bx^3+a)^{\frac{5}{3}}b^{12}d^4+10\,(bx^3+a)^{\frac{2}{3}}ab^{12}d^4}{10\,b^{15}d^5}\end{aligned}$$

input
```
integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-(-b*c*d^2 + a*d^3)^(2/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-
(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^4 - 2*sqr
t(3)*a*b*c*d^5 + sqrt(3)*a^2*d^6) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d
)/d)^(2/3))/(b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6) - 1/3*c^3*(-(b*c - a*d)/
d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d^2
 - 2*a*b*c*d^3 + a^2*d^4) + a^3/((b^4*c - a*b^3*d)*(b*x^3 + a)^(1/3)) - 1/
10*(5*(b*x^3 + a)^(2/3)*b^13*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^12*d^4 + 10*(b*
x^3 + a)^(2/3)*a*b^12*d^4)/(b^15*d^5)
```

3.748.9 Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.95

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{{\left(b\,x^3+a\right)}^{5/3}}{5\,b^3\,d}$$

$$-\left(\frac{3\,a}{2\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{2\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{2/3}-\frac{a^3}{b^3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(a\,d-b\,c\right)}$$

$$-\frac{c^3\,\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^6\,d^4-b\,c^7\,d^3\right)-\frac{c^6\,\left(9\,a^4\,d^{12}-36\,a^3\,b\,c\,d^{11}+54\,a^2\,b^2\,c^2\,d^{10}-36\,a\,b^3\,c^3\,d^9+9\,b^4\,c^4\,d^8\right)}{9\,d^{16/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)}{3\,d^{8/3}\,{\left(a\,d-b\,c\right)}^{4/3}}$$

$$+\frac{\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^6\,d^4-b\,c^7\,d^3\right)-\frac{{\left(c^3+\sqrt{3}\,c^3\,1{}\mathrm{i}\right)}^2\,\left(9\,a^4\,d^{12}-36\,a^3\,b\,c\,d^{11}+54\,a^2\,b^2\,c^2\,d^{10}-36\,a\,b^3\,c^3\,d^9+9\,b^4\,c^4\,d^8\right)}{36\,d^{16/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(c^3+\right.}{6\,d^{8/3}\,{\left(a\,d-b\,c\right)}^{4/3}}$$

$$-\frac{c^3\,\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^6\,d^4-b\,c^7\,d^3\right)-\frac{c^6\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2\,\left(9\,a^4\,d^{12}-36\,a^3\,b\,c\,d^{11}+54\,a^2\,b^2\,c^2\,d^{10}-36\,a\,b^3\,c^3\,d^9+9\,b^4\,c^4\,d^8\right)}{9\,d^{16/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(-\right.}{3\,d^{8/3}\,{\left(a\,d-b\,c\right)}^{4/3}}$$

input
```
int(x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
(a + b*x^3)^(5/3)/(5*b^3*d) - ((3*a)/(2*b^3*d) + (b^4*c - a*b^3*d)/(2*b^6*
d^2))*(a + b*x^3)^(2/3) - a^3/(b^3*(a + b*x^3)^(1/3)*(a*d - b*c)) - (c^3*l
og((a + b*x^3)^(1/3)*(a*c^6*d^4 - b*c^7*d^3) - (c^6*(9*a^4*d^12 + 9*b^4*c^
4*d^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/(9*d^(1
6/3)*(a*d - b*c)^(8/3))))/(3*d^(8/3)*(a*d - b*c)^(4/3)) + (log((a + b*x^3)
^(1/3)*(a*c^6*d^4 - b*c^7*d^3) - ((3^(1/2)*c^3*1i + c^3)^2*(9*a^4*d^12 + 9
*b^4*c^4*d^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/
(36*d^(16/3)*(a*d - b*c)^(8/3)))*(3^(1/2)*c^3*1i + c^3))/(6*d^(8/3)*(a*d -
 b*c)^(4/3)) - (c^3*log((a + b*x^3)^(1/3)*(a*c^6*d^4 - b*c^7*d^3) - (c^6*(
(3^(1/2)*1i)/2 - 1/2)^2*(9*a^4*d^12 + 9*b^4*c^4*d^8 - 36*a*b^3*c^3*d^9 + 5
4*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/(9*d^(16/3)*(a*d - b*c)^(8/3)))*((3
^(1/2)*1i)/2 - 1/2))/(3*d^(8/3)*(a*d - b*c)^(4/3))
```

3.748. $\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

3.749 $\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

3.749.1 Optimal result

Integrand size = 24, antiderivative size = 203

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}}+\frac{(a+bx^3)^{2/3}}{2b^2d}+\frac{c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}(bc-ad)^{4/3}}-\frac{c^2\log(c+dx^3)}{6d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}}$$

output
```
-a^2/b^2/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/2*(b*x^3+a)^(2/3)/b^2/d-1/6*c^2*ln(d
*x^3+c)/d^(5/3)/(-a*d+b*c)^(4/3)+1/2*c^2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^
3+a)^(1/3))/d^(5/3)/(-a*d+b*c)^(4/3)+1/3*c^2*arctan(1/3*(1-2*d^(1/3)*(b*x^
3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(5/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

3.749.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.25

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\frac{3d^{2/3}\left(-3a^2d+b^2cx^3+ab(c-dx^3)\right)}{b^2(bc-ad)\sqrt[3]{a+bx^3}}+\frac{2\sqrt{3}c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{(bc-ad)^{4/3}}+\frac{2c^2\log\left(\sqrt[3]{bc-a\right.}{(bc-}}{6d^{5/}}$$

input
```
Integrate[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((3*d^(2/3)*(-3*a^2*d + b^2*c*x^3 + a*b*(c - d*x^3)))/(b^2*(b*c - a*d)*(a
+ b*x^3)^(1/3)) + (2*Sqrt[3]*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))
/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) + (2*c^2*Log[(b*c - a*d)^(
1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) - (c^2*Log[(b*c - a*d
)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3
)^(2/3)])/(b*c - a*d)^(4/3))/(6*d^(5/3))
```

3.749.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^3+a)^{4/3}(dx^3+c)}dx^3$$

$$\Big\downarrow \text{98}$$

$$\frac{1}{3}\int \left(\frac{a^2}{b(bc-ad)(bx^3+a)^{4/3}}+\frac{1}{bd\sqrt[3]{bx^3+a}}+\frac{c^2}{d(ad-bc)\sqrt[3]{bx^3+a}(dx^3+c)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{3a^2}{b^2\sqrt[3]{a+bx^3}(bc-ad)}+\frac{\sqrt{3}c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{5/3}(bc-ad)^{4/3}}+\frac{3(a+bx^3)^{2/3}}{2b^2d}-\frac{c^2\log(c+dx^3)}{2d^{5/3}(bc-ad)^{4/3}}+3c^2\log\right.$$

input
```
Int[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((-3*a^2)/(b^2*(b*c - a*d)*(a + b*x^3)^(1/3)) + (3*(a + b*x^3)^(2/3))/(2*b
^2*d) + (Sqrt[3]*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)
^(1/3))/Sqrt[3]])/(d^(5/3)*(b*c - a*d)^(4/3)) - (c^2*Log[c + d*x^3])/(2*d^
(5/3)*(b*c - a*d)^(4/3)) + (3*c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x
^3)^(1/3)])/(2*d^(5/3)*(b*c - a*d)^(4/3)))/3
```

3.749.3.1 Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.749.4 Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{-9\left(-\frac{b^{2} c\,x^{3}}{3}-\frac{a\left(-d\,x^{3}+c\right)b}{3}+a^{2} d\right)\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}} d+b^{2} c^{2}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\Bigg(\Bigg(}{6\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}} d^{2}\left(a d-b c\right) b^{2}}$

input
```
int(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output

```
-1/6*(-9*(-1/3*b^2*c*x^3-1/3*a*(-d*x^3+c)*b+a^2*d)*(1/d*(a*d-b*c))^(1/3)*d
+b^2*c^2*(b*x^3+a)^(1/3)*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a
*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*
d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-
(1/d*(a*d-b*c))^(1/3))))/(1/d*(a*d-b*c))^(1/3)/(b*x^3+a)^(1/3)/d^2/(a*d-b*
c)/b^2
```

3.749.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(167) = 334$.

Time = 0.34 (sec) , antiderivative size = 1004, normalized size of antiderivative = 4.95

$$\int \frac{x^8}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

input

```
integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output

```
[-1/6*(3*sqrt(1/3)*(a*b^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2
*d^2)*x^3)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b
*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3)
 + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d
))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)
*(b*x^3 + a)^(1/3))/(d*x^3 + c)) + (b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a
*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 +
a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c
*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) -
3*(a*b^2*c^2*d^2 - 4*a^2*b*c*d^3 + 3*a^3*d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^
3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(2/3))/(a*b^4*c^2*d^3 - 2*a^2*b^3*c*d^4 +
a^3*b^2*d^5 + (b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*x^3), -1/6*(6*sq
rt(1/3)*(a*b^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2*d^2)*x^3)*
sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a
)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c
- a*d))/d) + (b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3
 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2
 + a*d^3)^(2/3)) - 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^(2/3)*lo
g((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(a*b^2*c^2*d^2 - 4*a
^2*b*c*d^3 + 3*a^3*d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)...
```

3.749.6 Sympy [F]

$$\int \frac{x^8}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{x^8}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

input
```
integrate(x**8/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**8/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.749.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.749.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.60

$$\begin{aligned}\int \frac{x^8}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^3-2\sqrt{3}abcd^4+\sqrt{3}a^2d^5} \\ &\quad-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^2\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^2d^3-2\,abcd^4+a^2d^5\right)} \\ &\quad+\frac{c^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^2d-2\,abcd^2+a^2d^3\right)}-\frac{a^2}{\left(b^3c-ab^2d\right)\left(bx^3+a\right)^{\frac{1}{3}}}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{2\,b^2d}\end{aligned}$$

input
```
integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
(-b*c*d^2 + a*d^3)^(2/3)*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(
b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^3 - 2*sqrt
(3)*a*b*c*d^4 + sqrt(3)*a^2*d^5) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^2*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3))/(b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5) + 1/3*c^2*(-(b*c - a*d)/d
)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d -
2*a*b*c*d^2 + a^2*d^3) - a^2/((b^3*c - a*b^2*d)*(b*x^3 + a)^(1/3)) + 1/2*(
b*x^3 + a)^(2/3)/(b^2*d)
```

3.749.9 Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.21

$$
\begin{aligned}
\int \frac{x^8}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx &= \frac{(b\,x^3+a)^{2/3}}{2\,b^2\,d} + \frac{a^2}{b^2\,(b\,x^3+a)^{1/3}\,(a\,d-b\,c)} \\
&+ \frac{c^2\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^4\,d^3-b\,c^5\,d^2)-\frac{c^4\,(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\,a\,b^3\,c^3\,d^6+9\,b^4\,c^4\,d^5)}{9\,d^{10/3}\,(a\,d-b\,c)^{8/3}}\right)}{3\,d^{5/3}\,(a\,d-b\,c)^{4/3}} \\
&- \frac{\ln\left((b\,x^3+a)^{1/3}\,(a\,c^4\,d^3-b\,c^5\,d^2)-\frac{\left(c^2+\sqrt{3}\,c^2\,1\mathrm{i}\right)^2\,(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\,a\,b^3\,c^3\,d^6+9\,b^4\,c^4\,d^5)}{36\,d^{10/3}\,(a\,d-b\,c)^{8/3}}\right)\,(c^2+\sqrt{3}}{6\,d^{5/3}\,(a\,d-b\,c)^{4/3}} \\
&+ \frac{c^2\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^4\,d^3-b\,c^5\,d^2)-\frac{c^4\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\,(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\,a\,b^3\,c^3\,d^6+9\,b^4\,c^4\,d^5)}{d^{10/3}\,(a\,d-b\,c)^{8/3}}\right)\,\left(-\frac{1}{6}\right.}{d^{5/3}\,(a\,d-b\,c)^{4/3}}
\end{aligned}
$$

input
```
int(x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output

```
(a + b*x^3)^(2/3)/(2*b^2*d) + a^2/(b^2*(a + b*x^3)^(1/3)*(a*d - b*c)) + (c
^2*log((a + b*x^3)^(1/3)*(a*c^4*d^3 - b*c^5*d^2) - (c^4*(9*a^4*d^9 + 9*b^4
*c^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8))/(9*d^(
10/3)*(a*d - b*c)^(8/3))))/(3*d^(5/3)*(a*d - b*c)^(4/3)) - (log((a + b*x^3
)^(1/3)*(a*c^4*d^3 - b*c^5*d^2) - ((3^(1/2)*c^2*1i + c^2)^2*(9*a^4*d^9 + 9
*b^4*c^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8))/(3
6*d^(10/3)*(a*d - b*c)^(8/3)))*(3^(1/2)*c^2*1i + c^2))/(6*d^(5/3)*(a*d - b
*c)^(4/3)) + (c^2*log((a + b*x^3)^(1/3)*(a*c^4*d^3 - b*c^5*d^2) - (c^4*((3
^(1/2)*1i)/6 - 1/6)^2*(9*a^4*d^9 + 9*b^4*c^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a
^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8))/(d^(10/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*
1i)/6 - 1/6))/(d^(5/3)*(a*d - b*c)^(4/3))
```

3.749. $\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

3.750 $\int \frac{x^5}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

3.750.1 Optimal result

Integrand size = 24, antiderivative size = 174

$$\int \frac{x^5}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} - \frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}(bc-ad)^{4/3}} + \frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}}$$

output
```
a/b/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/6*c*ln(d*x^3+c)/d^(2/3)/(-a*d+b*c)^(4/3)-
1/2*c*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(2/3)/(-a*d+b*c)^(4/3
)-1/3*c*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))
/d^(2/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

3.750.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.30

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{6}\left(\frac{6a}{(b^2c-abd)\sqrt[3]{a+bx^3}} - \frac{2\sqrt{3}c\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}(bc-ad)^{4/3}} - \frac{2c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{d^{2/3}(bc-ad)^{4/3}} + \frac{c\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{d^{2/3}(bc-ad)^{4/3}}\right)$$

input
```
Integrate[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((6*a)/((b^2*c - a*b*d)*(a + b*x^3)^(1/3)) - (2*Sqrt[3]*c*ArcTan[(1 - (2*d
^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d
)^(4/3)) - (2*c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(d^(2/
3)*(b*c - a*d)^(4/3)) + (c*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/
3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(d^(2/3)*(b*c - a*d)^(4
/3)))/6
```

3.750.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 87, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{x^3}{(bx^3+a)^{4/3}(dx^3+c)}dx^3$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{3}\left(\frac{c\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{bc-ad}+\frac{3a}{b\sqrt[3]{a+bx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{68}$$

$$\frac{1}{3}\left(\frac{c\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}\right.}{bc-ad}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{c\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}+\frac{}{b}\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{c\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}+\frac{3a}{b\sqrt[3]{a+bx^3}(bc-ad}\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{c\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}+\frac{3a}{b\sqrt[3]{a+bx^3}(bc-a}\right.$$

input
```
Int[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

3.750. $\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

output
```
((3*a)/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) + (c*(-((Sqrt[3]*ArcTan[(1 - (2*d
^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d
)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/3)) - (3*Log[(b*c - a
*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(1/3))))/(b
*c - a*d))/3
```

3.750.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.750.4 Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$-\frac{\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)bc\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3}+\frac{\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)bc\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{3}-\frac{\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-b}{d}\right.\right.}{}}{\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}(ad-bc)db}$

input
```
int(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/(1/d*(a*d-b*c))^(1/3)*(1/3*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3
)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*b*c*(b*x^3+a)^(1/3)+1/3*ln
((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))*b*c*(b*x^3+a)^(1/3)-1/6*ln((b*x^3+
a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))*b*c*
(b*x^3+a)^(1/3)+a*d*(1/d*(a*d-b*c))^(1/3))/(b*x^3+a)^(1/3)/(a*d-b*c)/d/b
```

3.750.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(141) = 282.

Time = 0.34 (sec) , antiderivative size = 872, normalized size of antiderivative = 5.01

$$\int \frac{x^5}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\left[-\frac{3\sqrt{\frac{1}{3}}(ab^2c^2d-a^2bcd^2+(b^3c^2d-ab^2cd^2)x^3)\sqrt{-\frac{(bcd^2-ad^3)^{\frac{1}{3}}}{bc-ad}}\log\left(\frac{2\,bd^2x^3-bc}{}\right.}{}\right.$$

input
```
integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/6*(3*sqrt(1/3)*(a*b^2*c^2*d - a^2*b*c*d^2 + (b^3*c^2*d - a*b^2*c*d^2)*
x^3)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d +
 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x
^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(
-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 +
 a)^(1/3))/(d*x^3 + c)) - (b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log(
(b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c
*d^2 - a*d^3)^(2/3)) + 2*(b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log((
b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 6*(a*b*c*d^2 - a^2*d^3)*(b
*x^3 + a)^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + a^3*b*d^4 + (b^4*c^2*d
^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^3), 1/6*(6*sqrt(1/3)*(a*b^2*c^2*d - a^
2*b*c*d^2 + (b^3*c^2*d - a*b^2*c*d^2)*x^3)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b
*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/
3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))/d) + (b^2*c*x^3 + a*b*c)*(b*
c*d^2 - a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(
b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b^2*c*x^3 + a*b*c)*(b*c
*d^2 - a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 6
*(a*b*c*d^2 - a^2*d^3)*(b*x^3 + a)^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3
 + a^3*b*d^4 + (b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^3)]
```

3.750.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^5}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**5/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**5/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.750.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.750.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(141) = 282$.

Time = 0.30 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.73

$$\int \frac{x^5}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx =$$
$$-\frac{\frac{6\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^2-2\sqrt{3}abcd^3+\sqrt{3}a^2d^4}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{b^2c^2d^2-2\,abcd^3+a^2d^4}+}{6\,b}$$

input
```
integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/6*(6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/
3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^2
- 2*sqrt(3)*a*b*c*d^3 + sqrt(3)*a^2*d^4) - (-b*c*d^2 + a*d^3)^(2/3)*b*c*lo
g((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c -
a*d)/d)^(2/3))/(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4) + 2*b*c*(-(b*c - a*d)
/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2 -
2*a*b*c*d + a^2*d^2) - 6*a/((b*x^3 + a)^(1/3)*(b*c - a*d)))/b
```

3.750.9 Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.37

$$
\begin{aligned}
\int \frac{x^5}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx &= -\frac{a}{b\,(b\,x^3+a)^{1/3}\,(a\,d-b\,c)} \\
&- \frac{c\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^2\,d^2-b\,c^3\,d)-\frac{c^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{9\,d^{4/3}\,(a\,d-b\,c)^{8/3}}\right)}{3\,d^{2/3}\,(a\,d-b\,c)^{4/3}} \\
&+ \frac{\ln\left((b\,x^3+a)^{1/3}\,(a\,c^2\,d^2-b\,c^3\,d)-\frac{\left(c-\sqrt{3}\,c\,1\mathrm{i}\right)^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{8/3}}\right)\,(c-\sqrt{3}\,c\,1\mathrm{i}}{6\,d^{2/3}\,(a\,d-b\,c)^{4/3}} \\
&+ \frac{\ln\left((b\,x^3+a)^{1/3}\,(a\,c^2\,d^2-b\,c^3\,d)-\frac{\left(c+\sqrt{3}\,c\,1\mathrm{i}\right)^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{8/3}}\right)\,(c+\sqrt{3}\,c\,1\mathrm{i}}{6\,d^{2/3}\,(a\,d-b\,c)^{4/3}}
\end{aligned}
$$

input
```
int(x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
(log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d) - ((c - 3^(1/2)*c*1i)^2*(9*a^
4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c
*d^5))/(36*d^(4/3)*(a*d - b*c)^(8/3)))*(c - 3^(1/2)*c*1i))/(6*d^(2/3)*(a*d
 - b*c)^(4/3)) - (c*log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d) - (c^2*(9*
a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b
*c*d^5))/(9*d^(4/3)*(a*d - b*c)^(8/3))))/(3*d^(2/3)*(a*d - b*c)^(4/3)) - a
/(b*(a + b*x^3)^(1/3)*(a*d - b*c)) + (log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b
*c^3*d) - ((c + 3^(1/2)*c*1i)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*
d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(36*d^(4/3)*(a*d - b*c)^(8/3))
)*(c + 3^(1/2)*c*1i))/(6*d^(2/3)*(a*d - b*c)^(4/3))
```

3.751 $\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

3.751.1 Optimal result

Integrand size = 24, antiderivative size = 167

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}$$

output
```
-1/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*d^(1/3)*ln(d*x^3+c)/(-a*d+b*c)^(4/3)+1/2
*d^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/(-a*d+b*c)^(4/3)+1/3
*d^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2)
)/(-a*d+b*c)^(4/3)*3^(1/2)
```

3.751.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.32

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3(bc-ad)^{4/3}} - \frac{\sqrt[3]{d}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{6(bc-ad)^{4/3}}$$

input
```
Integrate[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
-(1/((b*c - a*d)*(a + b*x^3)^(1/3))) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a
+ b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c - a*d)^(4/3)) +
 (d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*(b*c - a*
d)^(4/3)) - (d^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a
+ b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*(b*c - a*d)^(4/3))
```

3.751.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {946, 61, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{3}\int \frac{1}{(bx^3+a)^{4/3}(dx^3+c)}dx^3$$

$$\Big\downarrow \; 61$$

$$\frac{1}{3}\left(-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx^3}{bc-ad}-\frac{3}{\sqrt[3]{a+bx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{ 68}$$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2d^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-a}}\right.}{bc-ad}\right.$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\sqrt[3]{bx^3+a}}{\sqrt[3]{d}}}d\sqrt[3]{bx^3+a}}{2d}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}-\right.$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{3}\left(-\frac{d\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}-\frac{3}{\sqrt[3]{a+bx^3}(bc-a}\right.$$

$$\Big\downarrow \text{ 217}$$

3.751. $\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

$$\frac{1}{3}\left(-\frac{d\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}-\frac{3}{\sqrt[3]{a+bx^3}(bc-a}\right.$$

input
```
Int[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(-3/((b*c - a*d)*(a + b*x^3)^(1/3)) - (d*(-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3
)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d)^(1/
3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/3)) - (3*Log[(b*c - a*d)^(
1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(1/3))))/(b*c -
a*d))/3
```

3.751.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

3.751. $\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.751.4 Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$\frac{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}+\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)-\ln (\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}{\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(6ad-6bc\right)}$

input
```
int(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
2*(3*(1/d*(a*d-b*c))^(1/3)+1/2*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1
/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-(
1/d*(a*d-b*c))^(1/3))-ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(
1/3)+(1/d*(a*d-b*c))^(2/3)))*(b*x^3+a)^(1/3))/(1/d*(a*d-b*c))^(1/3)/(b*x^3
+a)^(1/3)/(6*a*d-6*b*c)
```

3.751.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.57

$$\int \frac{x^2}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx =$$

$$-\frac{2\sqrt{3}(bx^3+a)\Big(-\frac{d}{bc-ad}\Big)^{\frac{1}{3}}\arctan\Big(\frac{2}{3}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\Big(-\frac{d}{bc-ad}\Big)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\Big)-(bx^3+a)\Big(-\frac{d}{bc-ad}\Big)^{\frac{1}{3}}\log\Big(-(bx^3+}{}$$

input
```
integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(b*x^3 + a)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x
^3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - (b*x^3 + a)*(-d/(b*c
 - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) +
 (b*x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 2*(b*x^3 + a)
*(-d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 +
a)^(1/3)*d) + 6*(b*x^3 + a)^(2/3))/((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)
```

3.751.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \int \frac{x^2}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

input
```
integrate(x**2/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**2/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.751.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.751.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(135) = 270.

Time = 0.30 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.71

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^2-2\,abcd+a^2d^2\right)} \\ &+ \frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d-2\sqrt{3}abcd^2+\sqrt{3}a^2d^3} \\ &- \frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^2d-2\,abcd^2+a^2d^3\right)} \\ &- \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(bc-ad\right)}\end{aligned}$$

input
```
integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^
(1/3)))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + (-b*c*d^2 + a*d^3)^(2/3)*arctan(
1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d
)^(1/3))/(sqrt(3)*b^2*c^2*d - 2*sqrt(3)*a*b*c*d^2 + sqrt(3)*a^2*d^3) - 1/6
*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
 - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3) - 1/((b*x^3 + a)^(1/3)*(b*c - a*d))
```

3.751.9 Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.33

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)}$$
$$+\frac{d^{1/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,d^4-b\,c\,d^3\right)-\frac{d^{2/3}\,\left(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2\right)}{9\,\left(a\,d-b\,c\right)^{8/3}}\right)}{3\,\left(a\,d-b\,c\right)^{4/3}}$$
$$-\frac{d^{1/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,d^4-b\,c\,d^3\right)-\frac{d^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)^2\,\left(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2\right)}{9\,\left(a\,d-b\,c\right)^{8/3}}\right)\,\left(\frac{1}{2}+\right.}{3\,\left(a\,d-b\,c\right)^{4/3}}$$
$$+\frac{d^{1/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,d^4-b\,c\,d^3\right)-\frac{d^{2/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)^2\,\left(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2\right)}{\left(a\,d-b\,c\right)^{8/3}}\right)\,\left(-\frac{1}{6}\right.}{\left(a\,d-b\,c\right)^{4/3}}$$

input
```
int(x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
1/((a + b*x^3)^(1/3)*(a*d - b*c)) + (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4
- b*c*d^3) - (d^(2/3)*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a
^2*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(9*(a*d - b*c)^(8/3))))/(3*(a*d - b*c)^(
4/3)) - (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 - b*c*d^3) - (d^(2/3)*((3^(1
/2)*1i)/2 + 1/2)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*
b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(9*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/2 + 1/
2))/(3*(a*d - b*c)^(4/3)) + (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 - b*c*d^
3) - (d^(2/3)*((3^(1/2)*1i)/6 - 1/6)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b
^3*c^3*d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(a*d - b*c)^(8/3))*((3^
(1/2)*1i)/6 - 1/6))/(a*d - b*c)^(4/3)
```

3.752 $\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

3.752.1 Optimal result

Integrand size = 24, antiderivative size = 271

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}c} - \frac{d^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{4/3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c} - \frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}}$$

output

```
b/a/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/2*ln(x)/a^(4/3)/c+1/6*d^(4/3)*ln(d*x^3+c)
/c/(-a*d+b*c)^(4/3)+1/2*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(4/3)/c-1/2*d^(4/3)*
ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c/(-a*d+b*c)^(4/3)+1/3*arctan
(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)/c*3^(1/2)-1/3*d^
(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c
/(-a*d+b*c)^(4/3)*3^(1/2)
```

3.752.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.29

$$\int \frac{1}{x(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{6}\left(\frac{6b}{(abc-a^2d)\sqrt[3]{a+bx^3}} + \frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}c} - \frac{2\sqrt{3}d^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{c(bc-ad)^{4/3}} + \frac{2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{a^{4/3}c} - \frac{2d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{c(bc-ad)^{4/3}} - \frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{a^{4/3}c} + \frac{d^{4/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{c(bc-ad)^{4/3}}\right)$$

input
```
Integrate[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((6*b)/((a*b*c - a^2*d)*(a + b*x^3)^(1/3)) + (2*Sqrt[3]*ArcTan[(1 + (2*(a
 + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/(a^(4/3)*c) - (2*Sqrt[3]*d^(4/3)*ArcTan
[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(c*(b*c -
 a*d)^(4/3)) + (2*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/(a^(4/3)*c) - (2*d^(4
/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(c*(b*c - a*d)^(4/
3)) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(a^(4/3
)*c) + (d^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x
^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(c*(b*c - a*d)^(4/3)))/6
```

3.752.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {948, 96, 25, 174, 67, 16, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx^3$$

$$\Big\downarrow \text{ 96}$$

$$\frac{1}{3}\left(\frac{3b}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int -\frac{bdx^3+bc-ad}{x^3\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{a(bc-ad)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}\left(\frac{\int \frac{bdx^3+bc-ad}{x^3\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{a(bc-ad)}+\frac{3b}{a\sqrt[3]{a+bx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{ 174}$$

$$\frac{1}{3}\left(\frac{\frac{ad^2\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{c}+\frac{(bc-ad)\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{c}}{a(bc-ad)}+\frac{3b}{a\sqrt[3]{a+bx^3}(bc-ad)}\right)$$

$$\Big\downarrow \text{ 67}$$

$$\frac{1}{3}\left(\frac{\frac{(bc-ad)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}+\frac{ad^2\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}}{c}}{a(bc-ad)}\right.$$

---

3. 752. $\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}dx$

$$\Big\downarrow 16$$

$$\frac{1}{3}\left(\frac{\frac{(bc-ad)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}+\frac{ad^2\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{c}}{a(bc-ad)}+\frac{}{a\sqrt[3]{}}\right.$$

$$\Big\downarrow 68$$

$$\frac{1}{3}\left(\frac{\frac{(bc-ad)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}+\frac{ad^2\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\sqrt[3]{bx^3+a}}}{2d^{2/3}\sqrt[3]{bc-ad}}\right.}{}}{a(bc-ad)}\right.$$

$$\Big\downarrow 16$$

$$\frac{1}{3}\left(\frac{\frac{(bc-ad)\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{c}+\frac{ad^2\left(\frac{3\int \frac{1}{x^6+\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{}}}}{2d}\right.}{}}{a(bc-ad)}\right.$$

$$\Big\downarrow 1082$$

3.752. $\int \frac{1}{x(a+bx^3)^{4/3}(c+dx^3)}\,dx$

$$\frac{1}{3}\left(\frac{\frac{ad^2\left(\frac{3\int\frac{1}{-x^6-3}d\left(1-\frac{2\sqrt[3]{d}\sqrt[3]{bx^3+a}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{c}+\frac{(bc-ad)\left(-\frac{3\int\frac{1}{-x^6-3}d\left(2\sqrt[3]{\right.}{\sqrt[3]{a}}\right.}{}}{a(bc-ad)}\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{\frac{ad^2\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}\right)}{c}+\frac{(bc-ad)\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+}}{\sqrt[3]{}}}{\sqrt{}}\right.}{\sqrt[3]{a}}\right.}{a(bc-ad)}\right.$$

input
```
Int[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((3*b)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) + (((b*c - a*d)*((Sqrt[3]*ArcTan[
(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^3]/(2*a^(1/3
)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3))))/c + (a*d^2*(-((Sqr
t[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]
)/(d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/
3)) - (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b
*c - a*d)^(1/3))))/c)/(a*(b*c - a*d)))/3
```

3.752. $\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

3.752.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.752. $\int \frac{1}{x(a+bx^3)^{4/3}(c+dx^3)}\,dx$

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.752.4 Maple [A] (verified)

Time = 4.74 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{\Bigg(-\Bigg(-2\arctan\Bigg(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\Bigg)\sqrt{3}+\ln\Big(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\Big)-2\ln\Big(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\Big)\Bigg)(ad-bc)(b\,x}{}$

input
```
int(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/(b*x^3+a)^(1/3)/(1/d*(a*d-b*c))^(1/3)*((-(-2*arctan(1/3*(a^(1/3)+2*(b*
x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a
)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*(a*d-b*c)*(b*x^3+a)^(1/3)-
6*a^(1/3)*b*c)*(1/d*(a*d-b*c))^(1/3)+(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(
1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2
/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x
^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3)))*a^(4/3)*d*(b*x^3+a)^(1/3))/a^(4/3)/(a*
d-b*c)/c
```

3.752.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(216) = 432$.

Time = 0.34 (sec) , antiderivative size = 975, normalized size of antiderivative = 3.60

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 3*sqrt(1/3)*(a^2*b*c - a^3*d + (a*b^2*c
- a^2*b*d)*x^3)*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)
^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^
3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) + 2*sqrt(3)*(a^2*b*d*x^3 + a^3*d)*(d/(b*c
 - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3)
- 1/3*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3
+ a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*c - a*b*d)*x^3
 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (a^2*b*d*x^3
+ a^3*d)*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c
- a*d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) -
 2*(a^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a
*d))^(2/3) + (b*x^3 + a)^(1/3)*d))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a
^3*b*c*d)*x^3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 2*sqrt(3)*(a^2*b*d*x^3 +
a^3*d)*(d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c
- a*d))^(1/3) - 1/3*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/
3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*
c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) +
 (a^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c -
 a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c -
a*d))^(1/3)) - 2*(a^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c -...
```

3.752.6 Sympy [F]

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \int \frac{1}{x\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

input
```
integrate(1/x/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.752.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x}\,dx$$

input
```
integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x), x)
```

3.752.8 Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.44

$$\begin{aligned}\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= -\frac{d^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^3-2\,abc^2d+a^2cd^2\right)} \\ &\quad - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^3-2\sqrt{3}abc^2d+\sqrt{3}a^2cd^2} \\ &\quad + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^3-2\,abc^2d+a^2cd^2\right)} \\ &\quad + \frac{b}{(bx^3+a)^{\frac{1}{3}}(abc-a^2d)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}c} \\ &\quad - \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}c} + \frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{4}{3}}c}\end{aligned}$$

input
```
integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*d^2*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/
d)^(1/3)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2) - (-b*c*d^2 + a*d^3)^(2/3)*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b^2*c^3 - 2*sqrt(3)*a*b*c^2*d + sqrt(3)*a^2*c*d^2
) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)
*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^3 - 2*a*b*c^2*d +
 a^2*c*d^2) + b/((b*x^3 + a)^(1/3)*(a*b*c - a^2*d)) + 1/3*sqrt(3)*arctan(1
/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(4/3)*c) - 1/6*log(
(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*c) + 1/3
*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(4/3)*c)
```

3.752.9 Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 3804, normalized size of antiderivative = 14.04

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
log(9*a^7*b^14*c^11*d^4 - ((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a
^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937
*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 98
55*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 4
86*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) - (-d^4/(27*b^4*c^7 + 27*a^4*
c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(2/3
)*(243*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13
*d^5 - 51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*
b^10*c^10*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255
*a^18*b^7*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 48
6*a^21*b^4*c^4*d^14))*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d
^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3) - 90*a^8*b^13*c^10*d^5
+ 405*a^9*b^12*c^9*d^6 - 1071*a^10*b^11*c^8*d^7 + 1827*a^11*b^10*c^7*d^8 -
 2079*a^12*b^9*c^6*d^9 + 1575*a^13*b^8*c^5*d^10 - 765*a^14*b^7*c^4*d^11 +
216*a^15*b^6*c^3*d^12 - 27*a^16*b^5*c^2*d^13)*(-d^4/(27*b^4*c^7 + 27*a^4*c
^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3)
 + log(9*a^7*b^14*c^11*d^4 - ((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 29
7*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8
937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 -
 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^...
```

3.753 $\int \frac{1}{x^4\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

3.753.1 Optimal result

Integrand size = 24, antiderivative size = 357

$$\int \frac{1}{x^4\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}}-\frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}}-\frac{1}{3acx^3\sqrt[3]{a+bx^3}}-\frac{(4bc+3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}c^2}+\frac{d^{7/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2(bc-ad)^{4/3}}+\frac{(4bc+3ad)\log(x)}{6a^{7/3}c^2}-\frac{d^{7/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{4/3}}-\frac{(4bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{7/3}c^2}+\frac{d^{7/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{4/3}}$$

output
```
-d^2/c^2/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/3*(-3*a*d-4*b*c)/a^2/c^2/(b*x^3+a)^(
1/3)-1/3/a/c/x^3/(b*x^3+a)^(1/3)+1/6*(3*a*d+4*b*c)*ln(x)/a^(7/3)/c^2-1/6*d
^(7/3)*ln(d*x^3+c)/c^2/(-a*d+b*c)^(4/3)-1/6*(3*a*d+4*b*c)*ln(a^(1/3)-(b*x^
3+a)^(1/3))/a^(7/3)/c^2+1/2*d^(7/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^
(1/3))/c^2/(-a*d+b*c)^(4/3)-1/9*(3*a*d+4*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3
+a)^(1/3))/a^(1/3)*3^(1/2))/a^(7/3)/c^2*3^(1/2)+1/3*d^(7/3)*arctan(1/3*(1-
2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/(-a*d+b*c)^(4/3)*
3^(1/2)
```

3.753.2 Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^4 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \frac{\frac{6c(-a^2d+4b^2cx^3+ab(c-dx^3))}{a^2(-bc+ad)x^3\sqrt[3]{a+bx^3}} - \frac{2\sqrt{3}(4bc+3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}} + \frac{6\sqrt{3}d^{7/3}\arctan}{(}}{}$$

input
```
Integrate[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((6*c*(-(a^2*d) + 4*b^2*c*x^3 + a*b*(c - d*x^3)))/(a^2*(-(b*c) + a*d)*x^3*
(a + b*x^3)^(1/3)) - (2*Sqrt[3]*(4*b*c + 3*a*d)*ArcTan[(1 + (2*(a + b*x^3)
^(1/3))/a^(1/3))/Sqrt[3]])/a^(7/3) + (6*Sqrt[3]*d^(7/3)*ArcTan[(1 - (2*d^(
1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) - (
2*(4*b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(7/3) + (6*d^(7/3)*
Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) + ((
4*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
])/a^(7/3) - (3*d^(7/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*
(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/(18*c^2
)
```

3.753.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {948, 114, 27, 174, 61, 67, 16, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3)^{4/3} (c+dx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{1}{x^6 (bx^3+a)^{4/3} (dx^3+c)}dx^3$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(-\frac{\int \frac{4bdx^3+4bc+3ad}{3x^3(bx^3+a)^{4/3}(dx^3+c)}dx^3}{ac}-\frac{1}{acx^3\sqrt[3]{a+bx^3}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(-\frac{\int \frac{4bdx^3+4bc+3ad}{x^3(bx^3+a)^{4/3}(dx^3+c)}dx^3}{3ac}-\frac{1}{acx^3\sqrt[3]{a+bx^3}}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+4bc)\int \frac{1}{x^3\left(bx^3+a\right)^{4/3}}dx^3}{c}-\frac{3ad^2\int \frac{1}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx^3}{c}}{3ac}-\frac{1}{acx^3\sqrt[3]{a+bx^3}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+4bc)\left(\frac{\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{c}-\frac{3ad^2\left(-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx^3}{bc-ad}-\frac{3}{\sqrt[3]{a+bx^3}(bc-ad)}\right)}{c}}{3ac}-\frac{1}{acx^3\sqrt[3]{a+}}\right.$$

$$\Big\downarrow \; 67$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+4bc)\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{c}-\frac{3ad^2\left(\right.}{}}{3ac}\right.$$

$$\Big\downarrow \; 16$$

3.753. $\int \frac{1}{x^4(a+bx^3)^{4/3}(c+dx^3)}\,dx$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+4bc)\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{c}-\frac{3ad^2\left(-\frac{d\int \frac{}{\sqrt[3]{bx}}}{}\right.}{}}{3ac}\right.$$

$$\Big\downarrow \text{ 68}$$

$$\frac{1}{3}\left(-\frac{(3ad+4bc)\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{c}-\frac{3ad^2\left(-\frac{d\left(-\frac{3\int}{}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{ 16}$$

3.753. $\int \frac{1}{x^4(a+bx^3)^{4/3}(c+dx^3)}\, dx$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+4bc)\left(\frac{\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{c}-\frac{3ad^2\left(-\frac{d\left(\frac{3\int}{x^6}\right.}{}\right.}{}}{}\right.$$

$$\downarrow 1082$$

$$\frac{1}{3}\left(-\frac{\frac{(3ad+4bc)\left(\frac{-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{c}-\frac{3ad^2\left(-\frac{d\left(\frac{3\int \frac{1}{-x^6-3}d\left(1-\frac{2}{}\right.}{d^{2/3}\sqrt[3]{}}\right.}{}\right.}{3ac}}{}\right.$$

$$\downarrow 217$$

3.753. $\int \frac{1}{x^4\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

$$\frac{1}{3}\left(-\frac{(3ad+4bc)\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^3}}\right)}{c}-\frac{3ad^2\left(-\frac{d\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}}{\sqrt[3]{}}}{}\right)}{d^{2/3}\sqrt[3]{bc}}\right)}{}\right)}{3ac}\right)$$

input
```
Int[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(-(1/(a*c*x^3*(a + b*x^3)^(1/3))) - (((4*b*c + 3*a*d)*(3/(a*(a + b*x^3)^(1
/3)) + ((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1
/3) - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/
3)))/a))/c - (3*a*d^2*(-3/((b*c - a*d)*(a + b*x^3)^(1/3)) - (d*(-((Sqrt[3]
*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d
^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(2*d^(2/3)*(b*c - a*d)^(1/3))
- (3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c -
 a*d)^(1/3))))/(b*c - a*d)))/c)/(3*a*c))/3
```

3.753. $\int \frac{1}{x^4(a+bx^3)^{4/3}(c+dx^3)}\,dx$

3.753.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

3.753. $\int \frac{1}{x^4\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.753.4 Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$-\frac{\left(x^{3} \left(\arctan\left(\frac{\left(a^{\frac{1}{3}}+2 \left(b \,x^{3}+a \right)^{\frac{1}{3}}\right) \sqrt{3}}{3 a^{\frac{1}{3}}}\right) \sqrt{3}+\ln\left(\left(b \,x^{3}+a \right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{\ln\left(\left(b \,x^{3}+a \right)^{\frac{2}{3}}+a^{\frac{1}{3}} \left(b \,x^{3}+a \right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2}\right)\right) \left(a d +\frac{4 b c}{3}\right) (a d -}{}$

input
```
int(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((x^3*(arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2
)+ln((b*x^3+a)^(1/3)-a^(1/3))-1/2*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/
3)+a^(2/3)))*(a*d+4/3*b*c)*(a*d-b*c)*(b*x^3+a)^(1/3)-(-a^(7/3)*d+b*((-d*x^
3+c)*a^(4/3)+4*b*c*x^3*a^(1/3)))*c)*(1/d*(a*d-b*c))^(1/3)-a^(7/3)*x^3*(arc
tan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^
(1/3))*3^(1/2)+ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))-1/2*ln((b*x^3+a)^
(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3)))*d^2*(b
*x^3+a)^(1/3))/a^(7/3)/(1/d*(a*d-b*c))^(1/3)/(b*x^3+a)^(1/3)/(a*d-b*c)/c^2
/x^3
```

3.753.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(292) = 584.

Time = 0.82 (sec) , antiderivative size = 1386, normalized size of antiderivative = 3.88

$$\int \frac{1}{x^4 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\, dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*((4*a*b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2
*b^2*c^2 - a^3*b*c*d - 3*a^4*d^2)*x^3)*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3
*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1
/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) - 6
*sqrt(3)*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*s
qrt(3)*(b*x^3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) + ((4*b^3*c
^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*
x^3)*(-a)^(2/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a
)^(2/3)) - 2*((4*b^3*c^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a
^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(1/3) + (-a)^(1/3))
+ 3*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^
(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d - (b*c - a*
d)*(-d/(b*c - a*d))^(1/3)) - 6*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*
d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) -
6*(a^2*b*c^2 - a^3*c*d + (4*a*b^2*c^2 - a^2*b*c*d)*x^3)*(b*x^3 + a)^(2/3))
/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/18*(6
*sqrt(1/3)*((4*a*b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2*b^2*c^2
 - a^3*b*c*d - 3*a^4*d^2)*x^3)*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*
x^3 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 6*sqrt(3)*(a^3*b*d^2*x
^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)...
```

3.753.6 Sympy [F]

$$\int \frac{1}{x^4 \left(a + bx^3\right)^{4/3} \left(c + dx^3\right)} \, dx = \int \frac{1}{x^4 \left(a + bx^3\right)^{\frac{4}{3}} \left(c + dx^3\right)} \, dx$$

input
```
integrate(1/x**4/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**4*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.753.7 Maxima [F]

$$\int \frac{1}{x^4 \left(a + bx^3\right)^{4/3} \left(c + dx^3\right)} \, dx = \int \frac{1}{\left(bx^3 + a\right)^{\frac{4}{3}} \left(dx^3 + c\right) x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^4), x)
```

3.753.8 Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.35

$$\begin{aligned}\int \frac{1}{x^4 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\,dx &= \frac{d^3 \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}} \log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^4 - 2\,abc^3d + a^2c^2d^2\right)} \\ &+ \frac{(-bcd^2+ad^3)^{\frac{2}{3}} d \arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^4 - 2\sqrt{3}abc^3d + \sqrt{3}a^2c^2d^2} \\ &- \frac{(-bcd^2+ad^3)^{\frac{2}{3}} d \log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^4 - 2\,abc^3d + a^2c^2d^2\right)} \\ &- \frac{4\,(bx^3+a)b^2c - 3\,ab^2c - (bx^3+a)abd}{3\left(a^2bc^2 - a^3cd\right)\left((bx^3+a)^{\frac{4}{3}} - (bx^3+a)^{\frac{1}{3}}a\right)} \\ &- \frac{\sqrt{3}(4\,bc+3\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}c^2} \\ &+ \frac{(4\,bc+3\,ad)\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{18\,a^{\frac{7}{3}}c^2} \\ &- \frac{\left(4\,a^{\frac{1}{3}}bc + 3\,a^{\frac{4}{3}}d\right)\log\left(\left|(bx^3+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{8}{3}}c^2}\end{aligned}$$

input
```
integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) + (-b*c*d^2 + a*d^3)^(2/3)
*d*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*
c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^4 - 2*sqrt(3)*a*b*c^3*d + sqrt(3)*a^2*c^
2*d^2) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a
)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^4 - 2*a*b*
c^3*d + a^2*c^2*d^2) - 1/3*(4*(b*x^3 + a)*b^2*c - 3*a*b^2*c - (b*x^3 + a)*
a*b*d)/((a^2*b*c^2 - a^3*c*d)*((b*x^3 + a)^(4/3) - (b*x^3 + a)^(1/3)*a)) -
 1/9*sqrt(3)*(4*b*c + 3*a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(
1/3))/a^(1/3))/(a^(7/3)*c^2) + 1/18*(4*b*c + 3*a*d)*log((b*x^3 + a)^(2/3)
+ (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(7/3)*c^2) - 1/9*(4*a^(1/3)*b*c
+ 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(8/3)*c^2)
```

3.753.9 Mupad [B] (verification not implemented)

Time = 11.00 (sec) , antiderivative size = 5875, normalized size of antiderivative = 16.46

$$\int \frac{1}{x^4 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
log((d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c
^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((a + b*x^3)
^(1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*
a^16*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^1
8*d^7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 937107
63*a^21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c
^13*d^12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 814
8762*a^26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + (d^7/(27*b^4*c^10 +
 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*
d))^(2/3)*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 3108
92985*a^21*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23
*b^11*c^20*d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*
d^9 - 2582803260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 4782
96900*a^28*b^6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4
*c^13*d^14))*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*
a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3) - 3254256*a^14*b^16*c^19*d^5 + 1
0156428*a^15*b^15*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^
13*c^16*d^8 + 15529887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 +
 5412825*a^20*b^10*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22
*b^8*c^11*d^13 + 7801029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15...
```

3.754 $\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

3.754.1 Optimal result

Integrand size = 24, antiderivative size = 322

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{ax^4}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(bc-4ad)x(a+bx^3)^{2/3}}{3b^2d(bc-ad)} - \frac{(3bc+4ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}d^2} + \frac{c^{7/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2(bc-ad)^{4/3}} + \frac{c^{7/3}\log(c+dx^3)}{6d^2(bc-ad)^{4/3}} - \frac{c^{7/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2(bc-ad)^{4/3}} + \frac{(3bc+4ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{7/3}d^2}$$

output
```
a*x^4/b/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/3*(-4*a*d+b*c)*x*(b*x^3+a)^(2/3)/b^2/
d/(-a*d+b*c)+1/6*c^(7/3)*ln(d*x^3+c)/d^2/(-a*d+b*c)^(4/3)-1/2*c^(7/3)*ln((
-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2/(-a*d+b*c)^(4/3)+1/6*(4*a*d
+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)/d^2-1/9*(4*a*d+3*b*c)*arcta
n(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)/d^2*3^(1/2)+1/3*c^(
7/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/
d^2/(-a*d+b*c)^(4/3)*3^(1/2)
```

3.754.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.81 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.57

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\frac{12d(-4a^2dx+b^2cx^4+abx(c-dx^3))}{b^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4\sqrt{3}(3bc+4ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{7/3}} - \frac{6\sqrt{-6+6i\sqrt{}}}{}}{}$$

input
```
Integrate[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((12*d*(-4*a^2*d*x + b^2*c*x^4 + a*b*x*(c - d*x^3)))/(b^2*(b*c - a*d)*(a +
 b*x^3)^(1/3)) - (4*Sqrt[3]*(3*b*c + 4*a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^
(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(7/3) - (6*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(
7/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I +
Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c - a*d)^(4/3) + (4*(3*b*c + 4*a*
d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(7/3) + (6*(1 + I*Sqrt[3])*c^(
7/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)
])/(b*c - a*d)^(4/3) - (2*(3*b*c + 4*a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a +
 b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(7/3) - ((3*I)*(-I + Sqrt[3])*c^(7/3
)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)
*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c -
a*d)^(4/3))/(36*d^2)
```

3.754.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {970, 1052, 25, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{970}$$

$$\frac{ax^4}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int \frac{x^3\left(4ac-(bc-4ad)x^3\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{b(bc-ad)}$$

$$\Big\downarrow 1052$$

$$\frac{ax^4}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{-\frac{\int -\frac{(bc-ad)(3bc+4ad)x^3+ac(bc-4ad)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3bd}-\frac{x\left(a+bx^3\right)^{2/3}(bc-4ad)}{3bd}}{b(bc-ad)}$$

$$\Big\downarrow 25$$

$$\frac{ax^4}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\frac{\int \frac{(bc-ad)(3bc+4ad)x^3+ac(bc-4ad)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3bd}-\frac{x\left(a+bx^3\right)^{2/3}(bc-4ad)}{3bd}}{b(bc-ad)}$$

$$\Big\downarrow 1026$$

$$\frac{ax^4}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\frac{\frac{(bc-ad)(4ad+3bc)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{3b^2c^3\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{d}}{3bd}-\frac{x\left(a+bx^3\right)^{2/3}(bc-4ad)}{3bd}}{b(bc-ad)}$$

$$\Big\downarrow 769$$

$$\frac{ax^4}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\frac{\frac{(bc-ad)(4ad+3bc)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{3b^2c^3\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{d}}{3bd}-\frac{x\left(a+bx^3\right)^{2/3}(bc-4ad)}{3bd}}{b(bc-ad)}$$

$$\Big\downarrow 901$$

3.754. $\int \frac{x^9}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}dx$

$$\frac{\frac{ax^4}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\frac{(bc-ad)(4ad+3bc)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{3b^2c^3\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(x\sqrt[3]{\right.}}{}\right.}{d}}{3bd}}{b(bc-ad)}$$

input
```
Int[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(a*x^4)/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) - (-1/3*((b*c - 4*a*d)*x*(a + b*
x^3)^(2/3))/(b*d) + ((-3*b^2*c^3*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(
1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Lo
g[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(
1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d + ((b*c - a*d)
*(3*b*c + 4*a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(S
qrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/d)/(
3*b*d))/(b*(b*c - a*d))
```

3.754.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

3.754. $\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

3.754.4 Maple [A] (verified)

Time = 5.41 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.53

method	result
pseudoelliptic	$\frac{-\frac{4\left(ad+\frac{3bc}{4}\right)b^{2}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}(ad-bc)\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{3}-2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)c^{2}b^{\frac{13}{3}}(}{}$

input
```
int(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output

```
1/6*(-4/3*(a*d+3/4*b*c)*b^2*((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*(a*d-b*c)*
ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*
d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*c^2*b^(13/3)*(b*x^3+a)^(1/3)+8/3*(a*
d+3/4*b*c)*b^2*3^(1/2)*((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*(a*d-b*c)*arcta
n(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+8/3*(a*d+3/4*b*c)*b
^2*((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*(a*d-b*c)*ln((-b^(1/3)*x+(b*x^3+a)^
(1/3))/x)+(8*x*d*(-1/4*b^2*c*x^3-1/4*a*(-d*x^3+c)*b+a^2*d)*((a*d-b*c)/c)^(
1/3)+(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*
d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)
*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*b^2*(b*x^3+a)^(1/3)*c^2)*b^(7/3)
)/(b*x^3+a)^(1/3)/((a*d-b*c)/c)^(1/3)/(a*d-b*c)/d^2/b^(13/3)
```

3.754.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(267) = 534$.

Time = 0.80 (sec) , antiderivative size = 1329, normalized size of antiderivative = 4.13

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Too large to display}$$

input

```
integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*(3*a*b^3*c^2 + a^2*b^2*c*d - 4*a^3*b*d^2 + (3*b^4*c^2 +
 a*b^3*c*d - 4*a^2*b^2*d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 +
 a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2
 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) - 6*sqrt(3)*(b^4
*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt
(3)*(b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) + 2*(3*a*b^2*c^2 + a^2*b*c
*d - 4*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(
b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 +
 (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*
x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*(b^4*c^2*x^3 + a*b^
3*c^2)*(c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (
b*x^3 + a)^(1/3)*c)/x) + 3*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)
*log(((b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d
)*x*(c/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) + 6*((b^3*c*d - a*b^
2*d^2)*x^4 + (a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*c*d^2
- a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^3), -1/18*(6*sqrt(3)*(b^4*c^2*x^
3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*
x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) - 2*(3*a*b^2*c^2 + a^2*b*c*d - 4*
a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)
*x - (b*x^3 + a)^(1/3))/x) + (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*...
```

3.754.6 Sympy [F]

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^9}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**9/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**9/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.754.7 Maxima [F]

$$\int \frac{x^9}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{x^9}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.754.8 Giac [F]

$$\int \frac{x^9}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{x^9}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.754.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{x^9}{\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(x^9/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.755 $\int \frac{x^6}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

3.755.1 Optimal result

Integrand size = 24, antiderivative size = 260

$$\begin{aligned}\int \frac{x^6}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{ax}{b(bc-ad)\sqrt[3]{a+bx^3}}+\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d} \\ &\quad-\frac{c^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d(bc-ad)^{4/3}}-\frac{c^{4/3}\log\left(c+dx^3\right)}{6d(bc-ad)^{4/3}} \\ &\quad+\frac{c^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{4/3}}-\frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}d}\end{aligned}$$

output
```
a*x/b/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*c^(4/3)*ln(d*x^3+c)/d/(-a*d+b*c)^(4/3
)+1/2*c^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d/(-a*d+b*c)^
(4/3)-1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)/d+1/3*arctan(1/3*(1+2*b^(
1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d*3^(1/2)-1/3*c^(4/3)*arctan(1/3*
(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/d/(-a*d+b*c)^(4/
3)*3^(1/2)
```

3.755.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.68 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.79

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{12}\left(\frac{12ax}{(b^2c-abd)\sqrt[3]{a+bx^3}}+\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{4/3}d}\right.$$
$$+\frac{2\sqrt{-6+6i\sqrt{3}}c^{4/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{d(bc-ad)^{4/3}}$$
$$-\frac{4\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{b^{4/3}d}$$
$$-\frac{2i(-i+\sqrt{3})\,c^{4/3}\log\left(2\sqrt[3]{bc-ad}x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{d(bc-ad)^{4/3}}$$
$$+\frac{2\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{b^{4/3}d}$$
$$\left.+\frac{\left(1+i\sqrt{3}\right)c^{4/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{d(bc-ad)^{4/3}}\right)$$

input
```
Integrate[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((12*a*x)/((b^2*c - a*b*d)*(a + b*x^3)^(1/3)) + (4*Sqrt[3]*ArcTan[(Sqrt[3]
*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(b^(4/3)*d) + (2*Sqrt[-6 +
 (6*I)*Sqrt[3]]*c^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d
)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(d*(b*c - a*d)^(4
/3)) - (4*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(b^(4/3)*d) - ((2*I)*(-I
+ Sqrt[3])*c^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a
+ b*x^3)^(1/3)])/(d*(b*c - a*d)^(4/3)) + (2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a
 + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(b^(4/3)*d) + ((1 + I*Sqrt[3])*c^(4/
3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3
)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(d*(b*
c - a*d)^(4/3)))/12
```

3.755.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {970, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \; 970$$

$$\frac{ax}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int \frac{ac-(bc-ad)x^3}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{b(bc-ad)}$$

$$\Big\downarrow \; 1026$$

$$\frac{ax}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\frac{bc^2\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}}{b(bc-ad)}$$

$$\Big\downarrow \; 769$$

$$\frac{ax}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\frac{bc^2\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}-\frac{(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}}{b(bc-ad)}$$

$$\Big\downarrow \; 901$$

$$\frac{ax}{b\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\frac{bc^2\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{d}-\frac{(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\log\left(\sqrt[3]{}\right.\right.}{d}}{b(bc-ad)}$$

input
```
Int[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(a*x)/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) - ((b*c^2*(ArcTan[(1 + (2*(b*c - a
*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c -
 a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c -
a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))
/d - ((b*c - a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(
Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/d)/
(b*(b*c - a*d))
```

3.755.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

3.755. $\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.755.4 Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.32

method	result
pseudoelliptic	$-\frac{\Bigg(-\Bigg(-2\sqrt{3}\,\arctan\Bigg(\frac{\sqrt{3}\,\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\Bigg)+\ln\Bigg(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\Bigg)-2\ln\Bigg(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\Bigg)\Bigg)(a}{}$

input
```
int(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(b*x^3+a)^(1/3)*((-(-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3
+a)^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^
(2/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*(a*d-b*c)*(b*x^3+a)^(1/3
)+6*a*d*x*b^(1/3))*((a*d-b*c)/c)^(1/3)+(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/
c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c
)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)
-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x))*b^(4/3)*c*(b*x^3+a)^(1/3
))/((a*d-b*c)/c)^(1/3)/b^(4/3)/(a*d-b*c)/d
```

3.755.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(211) = 422$.

Time = 0.31 (sec) , antiderivative size = 1127, normalized size of antiderivative = 4.33

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x + 3*sqrt(1/3)*(a*b^2*c - a^2*b*d + (b^3*
c - a*b^2*d)*x^3)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b
)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*
(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*sqrt(3)*(b^3
*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3
)*(b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c - a*b*d)*x^3 +
a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + ((b^
2*c - a*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3
+ a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 2*(b^3*c*x^3 + a*b^2*c
)*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x
^3 + a)^(1/3)*c)/x) + (b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-((
b*c - a*d)*x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-
c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2))/(a*b^3*c*d - a^2*b^2*d^2
 + (b^4*c*d - a*b^3*d^2)*x^3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x - 6*sqrt(1
/3)*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-(-b)^(1/3)/b)*arctan
(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) +
2*sqrt(3)*(b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3
)*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c -
 a*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1
/3))/x) + ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(2...
```

3.755.6 Sympy [F]

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^6}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**6/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**6/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.755.7 Maxima [F]

$$\int \frac{x^6}{(a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.755.8 Giac [F]

$$\int \frac{x^6}{(a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.755.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{x^6}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(x^6/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.756 $\int \frac{x^3}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

3.756.1 Optimal result

Integrand size = 24, antiderivative size = 172

$$\int \frac{x^3}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = -\frac{x}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{c}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} + \frac{\sqrt[3]{c}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} - \frac{\sqrt[3]{c}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}$$

output
```
-x/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/6*c^(1/3)*ln(d*x^3+c)/(-a*d+b*c)^(4/3)-1/2
*c^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/(-a*d+b*c)^(4/3)+1
/3*c^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(
1/2))/(-a*d+b*c)^(4/3)*3^(1/2)
```

3.756.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.84 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.87

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{12}\left(-\frac{12x}{(bc-ad)\sqrt[3]{a+bx^3}}\right.$$

$$-\frac{2\sqrt{-6+6i\sqrt{3}}\sqrt[3]{c}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{(bc-ad)^{4/3}}$$

$$+\frac{2\left(1+i\sqrt{3}\right)\sqrt[3]{c}\log\left(2\sqrt[3]{bc-ad}x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{4/3}}$$

$$-\frac{i\left(-i+\sqrt{3}\right)\sqrt[3]{c}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{4/3}}$$

input
```
Integrate[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((-12*x)/((b*c - a*d)*(a + b*x^3)^(1/3)) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(
1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I +
Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c - a*d)^(4/3) + (2*(1 + I*Sqrt[3
])*c^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)
^(1/3)]])/(b*c - a*d)^(4/3) - (I*(-I + Sqrt[3])*c^(1/3)*Log[2*(b*c - a*d)^(
2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3)
+ I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(b*c - a*d)^(4/3))/12
```

3.756.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {971, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{971}$$

$$\frac{\int \frac{c}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{bc-ad}-\frac{x}{\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{bc-ad}-\frac{x}{\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \text{901}$$

$$\frac{c\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}-\frac{x}{\sqrt[3]{a+bx^3}(bc-ad)}$$

input
```
Int[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
-(x/((b*c - a*d)*(a + b*x^3)^(1/3))) + (c*(ArcTan[(1 + (2*(b*c - a*d)^(1/3
)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1
/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/
3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(b*c -
a*d)
```

3.756.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

3.756.4 Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.26

method	result
pseudoelliptic	$-\frac{2\left(-3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\frac{\left(2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\right.}{}\right.\right.}{2}\right.}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(6ad-6bc\right)}$

input
```
int(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-2*(-3*((a*d-b*c)/c)^(1/3)*x+1/2*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3
)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+2*ln((((a*d-b*c)/c)^
(1/3)*x+(b*x^3+a)^(1/3))/x)-ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3
)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^
(1/3)/(b*x^3+a)^(1/3)/(6*a*d-6*b*c)
```

3.756.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.756.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^3}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x**3/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**3/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.756.7 Maxima [F]

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.756.8 Giac [F]

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\int \frac{x^3}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.756.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\int \frac{x^3}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

input
```
int(x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(x^3/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.757 $\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

3.757.1 Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}}-\frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}}-\frac{d\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{4/3}}+\frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}$$

output

```
b*x/a/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*d*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(4/3
)+1/2*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^
(4/3)-1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^
(1/2))/c^(2/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

3.757.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.83

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{12}\left(\frac{12bx}{(abc-a^2d)\sqrt[3]{a+bx^3}} + \frac{2\sqrt{-6+6i\sqrt{3}}d\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{4/3}} - \frac{2i\left(-i+\sqrt{3}\right)d\log\left(2\sqrt[3]{bc-ad}x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{4/3}} + \frac{\left(d+i\sqrt{3}d\right)\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{4/3}}\right)$$

input

```
Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output

```
((12*b*x)/((a*b*c - a^2*d)*(a + b*x^3)^(1/3)) + (2*Sqrt[-6 + (6*I)*Sqrt[3]
]*d*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + S
qrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(c^(2/3)*(b*c - a*d)^(4/3)) - ((2*I)*
(-I + Sqrt[3])*d*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a +
b*x^3)^(1/3)]])/(c^(2/3)*(b*c - a*d)^(4/3)) + ((d + I*Sqrt[3]*d)*Log[2*(b*c
 - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^
3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(c^(2/3)*(b*c - a*d
)^(4/3)))/12
```

3.757.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {907, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \textcolor{blue}{907}$$

$$\frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{bc-ad}$$

$$\Big\downarrow \textcolor{blue}{901}$$

$$\frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{d\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*(ArcTan[(1 + (2*(b*c - a*d)^(
1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)
^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^
(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(b*c
 - a*d)
```

3.757.3.1 Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

3.757.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x}\right) a d\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) a d\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^{2}}{}\right.}{}}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(ad-bc\right) c a}$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(((a*d
-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*a*d*(b*x^3+a)^(
1/3)+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*a*d*(b*x^3+a)^(1/3)-1/2
*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+
a)^(2/3))/x^2)*a*d*(b*x^3+a)^(1/3)-3*b*x*c*((a*d-b*c)/c)^(1/3))/(a*d-b*c)/
c/a
```

3.757.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.757.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.757.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.757.8 Giac [F]

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.757.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.758 $\int \frac{1}{x^3\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

3.758.1 Optimal result

Integrand size = 24, antiderivative size = 229

$$\int \frac{1}{x^3\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{b}{a(bc-ad)x^2\sqrt[3]{a+bx^3}} - \frac{(3bc-ad)\left(a+bx^3\right)^{2/3}}{2a^2c(bc-ad)x^2} + \frac{d^2\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}(bc-ad)^{4/3}} + \frac{d^2\log\left(c+dx^3\right)}{6c^{5/3}(bc-ad)^{4/3}} - \frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}(bc-ad)^{4/3}}$$

output
```
b/a/(-a*d+b*c)/x^2/(b*x^3+a)^(1/3)-1/2*(-a*d+3*b*c)*(b*x^3+a)^(2/3)/a^2/c/
(-a*d+b*c)/x^2+1/6*d^2*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(4/3)-1/2*d^2*ln((-a
*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(4/3)+1/3*d^2*
arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/
3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

3.758.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.74 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^3\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\frac{6c^{2/3}(-a^2d+3b^2cx^3+ab(c-dx^3))}{a^2(-bc+ad)x^2\sqrt[3]{a+bx^3}} - \frac{2\sqrt{-6+6i\sqrt{3}}d^2\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a}}\right.}{(bc-ad)^{4/3}}}{}$$

input
```
Integrate[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((6*c^(2/3)*(-(a^2*d) + 3*b^2*c*x^3 + a*b*(c - d*x^3)))/(a^2*(-(b*c) + a*d
)*x^2*(a + b*x^3)^(1/3)) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c
- a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a
+ b*x^3)^(1/3))])/(b*c - a*d)^(4/3) + (2*(1 + I*Sqrt[3])*d^2*Log[2*(b*c -
a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/
3) - (I*(-I + Sqrt[3])*d^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*
c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a
 + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/(12*c^(5/3))
```

3.758.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {972, 25, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{b}{ax^2\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int -\frac{3bdx^3+3bc-ad}{x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx}{a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3bdx^3+3bc-ad}{x^3\sqrt[3]{bx^3+a(dx^3+c)}}dx}{a(bc-ad)}+\frac{b}{ax^2\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow 1053$$

$$\frac{-\frac{\int -\frac{2a^2d^2}{\sqrt[3]{bx^3+a(dx^3+c)}}dx}{2ac}-\frac{(a+bx^3)^{2/3}(3bc-ad)}{2acx^2}}{a(bc-ad)}+\frac{b}{ax^2\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow 27$$

$$\frac{\frac{ad^2\int \frac{1}{\sqrt[3]{bx^3+a(dx^3+c)}}dx}{c}-\frac{(a+bx^3)^{2/3}(3bc-ad)}{2acx^2}}{a(bc-ad)}+\frac{b}{ax^2\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow 901$$

$$\frac{\frac{ad^2\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{(a+bx^3)^{2/3}(3bc-ad)}{2acx^2}}{a(bc-ad)}+\frac{b}{ax^2\sqrt[3]{a+bx^3}(bc-ad)}$$

input
```
Int[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
b/(a*(b*c - a*d)*x^2*(a + b*x^3)^(1/3)) + (-1/2*((3*b*c - a*d)*(a + b*x^3)
^(2/3))/(a*c*x^2) + (a*d^2*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(
a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c +
d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
 (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c)/(a*(b*c - a*d))
```

3.758. $\int \frac{1}{x^3(a+bx^3)^{4/3}(c+dx^3)}\,dx$

3.758.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.758. $\int \frac{1}{x^3(a+bx^3)^{4/3}(c+dx^3)}\,dx$

3.758.4 Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{-3\left(\left(abd-3b^{2}c\right)x^{3}+a^{2}d-abc\right)c\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+a^{2}d^{2}x^{2}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\right.\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}c^{2}x^{2}(ad-bc)a^{2}}$

input
```
int(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-3*((a*b*d-3*b^2*c)*x^3+a^2*d-a*b*c)*c*((a*d-b*c)/c)^(1/3)+a^2*d^2*x^
2*(b*x^3+a)^(1/3)*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a
)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-
b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln(((((a*d-b*c)/c)^
(1/3)*x+(b*x^3+a)^(1/3))/x)))/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)/c^2/x^2/
(a*d-b*c)/a^2
```

3.758.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^{3}\left(a+bx^{3}\right)^{4/3}\left(c+dx^{3}\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.758.6 Sympy [F]

$$\int \frac{1}{x^{3}\left(a+bx^{3}\right)^{4/3}\left(c+dx^{3}\right)}\,dx=\int \frac{1}{x^{3}\left(a+bx^{3}\right)^{\frac{4}{3}}\left(c+dx^{3}\right)}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**3*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.758.7 Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)
```

3.758.8 Giac [F]

$$\int \frac{1}{x^3 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)
```

3.758.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{x^3 \, (b\, x^3+a)^{4/3} \, (d\, x^3+c)} \, dx$$

input
```
int(1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.759 $\int \frac{1}{x^6\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

3.759.1 Optimal result

Integrand size = 24, antiderivative size = 287

$$\int \frac{1}{x^6\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{b}{a(bc-ad)x^5\sqrt[3]{a+bx^3}} - \frac{(6bc-ad)\left(a+bx^3\right)^{2/3}}{5a^2c(bc-ad)x^5} + \frac{\left(18b^2c^2-3abcd-5a^2d^2\right)\left(a+bx^3\right)^{2/3}}{10a^3c^2(bc-ad)x^2} - \frac{d^3\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}(bc-ad)^{4/3}} - \frac{d^3\log\left(c+dx^3\right)}{6c^{8/3}(bc-ad)^{4/3}} + \frac{d^3\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}(bc-ad)^{4/3}}$$

output
```
b/a/(-a*d+b*c)/x^5/(b*x^3+a)^(1/3)-1/5*(-a*d+6*b*c)*(b*x^3+a)^(2/3)/a^2/c/
(-a*d+b*c)/x^5+1/10*(-5*a^2*d^2-3*a*b*c*d+18*b^2*c^2)*(b*x^3+a)^(2/3)/a^3/
c^2/(-a*d+b*c)/x^2-1/6*d^3*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(4/3)+1/2*d^3*ln
((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(4/3)-1/3*
d^3*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c
^(8/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

3.759.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.70 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^6 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\, dx = \frac{\frac{6c^{2/3}\left(-18b^3c^2x^6+3ab^2cx^3\left(-2c+dx^3\right)+a^3d\left(-2c+5dx^3\right)+a^2b\left(2c^2+cdx^3+5d^2x^6\right)\right)}{a^3(-bc+ad)x^5\sqrt[3]{a+bx^3}} + \frac{10\sqrt{-6+6i\sqrt{}}}{}}{}$$

input
```
Integrate[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((6*c^(2/3)*(-18*b^3*c^2*x^6 + 3*a*b^2*c*x^3*(-2*c + d*x^3) + a^3*d*(-2*c
+ 5*d*x^3) + a^2*b*(2*c^2 + c*d*x^3 + 5*d^2*x^6)))/(a^3*(-(b*c) + a*d)*x^5
*(a + b*x^3)^(1/3)) + (10*Sqrt[-6 + (6*I)*Sqrt[3]]*d^3*ArcTan[(3*(b*c - a*
d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*
x^3)^(1/3))]])/(b*c - a*d)^(4/3) - ((10*I)*(-I + Sqrt[3])*d^3*Log[2*(b*c -
a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]])/(b*c - a*d)^(4/
3) + (5*(1 + I*Sqrt[3])*d^3*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])
*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(
a + b*x^3)^(2/3)]])/(b*c - a*d)^(4/3))/(60*c^(8/3))
```

3.759.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {972, 25, 1053, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{b}{ax^5\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int -\frac{6bdx^3+6bc-ad}{x^6\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{6bdx^3+6bc-ad}{x^6\sqrt[3]{bx^3+a}(dx^3+c)}dx}{a(bc-ad)}+\frac{b}{ax^5\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int \frac{3bd(6bc-ad)x^3+18b^2c^2-5a^2d^2-3abcd}{x^3\sqrt[3]{bx^3+a}(dx^3+c)}dx}{5ac}-\frac{(a+bx^3)^{2/3}(6bc-ad)}{5acx^5}}{a(bc-ad)}+\frac{b}{ax^5\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{-\frac{\int -\frac{10a^3d^3}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{2ac}-\frac{(a+bx^3)^{2/3}\left(\frac{18b^2c}{a}-\frac{5ad^2}{c}-3bd\right)}{2x^2}}{5ac}-\frac{(a+bx^3)^{2/3}(6bc-ad)}{5acx^5}}{a(bc-ad)}+\frac{b}{ax^5\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\frac{5a^2d^3\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{c}-\frac{(a+bx^3)^{2/3}\left(\frac{18b^2c}{a}-\frac{5ad^2}{c}-3bd\right)}{2x^2}}{5ac}-\frac{(a+bx^3)^{2/3}(6bc-ad)}{5acx^5}}{a(bc-ad)}+\frac{b}{ax^5\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \text{901}$$

$$\frac{-\frac{\frac{5a^2d^3\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{(a+bx^3)^{2/3}\left(\frac{18b^2c}{a}-\frac{5ad^2}{c}-3bd\right)}{2x^2}}{5ac}-(a+bx^3}{a(bc-ad)}$$

$$\frac{b}{ax^5\sqrt[3]{a+bx^3}(bc-ad)}$$

input
```
Int[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

3.759. $\int \frac{1}{x^6(a+bx^3)^{4/3}(c+dx^3)}\,dx$

output
```
b/(a*(b*c - a*d)*x^5*(a + b*x^3)^(1/3)) + (-1/5*((6*b*c - a*d)*(a + b*x^3)
^(2/3))/(a*c*x^5) - (-1/2*(((18*b^2*c)/a - 3*b*d - (5*a*d^2)/c)*(a + b*x^3
)^(2/3))/x^2 + (5*a^2*d^3*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a
 + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d
*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
(a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/c)/(5*a*c))/(a*(b*c - a
*d))
```

3.759.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.759.4 Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{-\frac{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} c\left(d\left(-\frac{5d\,x^{3}}{2}+c\right)a^{3}-\left(\frac{5}{2}d^{2}x^{6}+\frac{1}{2}cd\,x^{3}+c^{2}\right)b\,a^{2}+3x^{3}\left(-\frac{d\,x^{3}}{2}+c\right)b^{2}ca+9b^{3}c^{2}x^{6}\right)}{5}+\frac{a^{3}d^{3}x^{5}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(2\arctan\left(\frac{\sqrt{3}\left(\left(\right.\right.}{}\right.\right.}{}}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)}$

input
```
int(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-3/5*((a*d-b*c)/c)^(1/3)*c*(d*(-5/2*d*x^3+c)*a^3-(5/2*d^2*x^6+1/2*c*d
*x^3+c^2)*b*a^2+3*x^3*(-1/2*d*x^3+c)*b^2*c*a+9*b^3*c^2*x^6)+1/2*a^3*d^3*x^
5*(b*x^3+a)^(1/3)*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)
^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+2*ln(((((a*d-b*c)/c)^(1/3)*x+(b*x^3+
a)^(1/3))/x)-ln(((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/
3)*x+(b*x^3+a)^(2/3))/x^2)))/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)/x^5/c^3/(
a*d-b*c)/a^3
```

3.759.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^6\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.759.6 Sympy [F]

$$\int \frac{1}{x^6 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\,dx=\int \frac{1}{x^6 \left(a+bx^3\right)^{\frac{4}{3}} \left(c+dx^3\right)}\,dx$$

input
```
integrate(1/x**6/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**6*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.759.7 Maxima [F]

$$\int \frac{1}{x^6 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\,dx=\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)
```

3.759.8 Giac [F]

$$\int \frac{1}{x^6 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\,dx=\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)
```

3.759.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\,dx=\int \frac{1}{x^6\,{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)} \,d x$$

input
```
int(1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.760 $\int \frac{1}{x^9\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

3.760.1 Optimal result

Integrand size = 24, antiderivative size = 351

$$\begin{aligned}\int \frac{1}{x^9\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{b}{a(bc-ad)x^8\sqrt[3]{a+bx^3}}\\
&-\frac{(9bc-ad)\left(a+bx^3\right)^{2/3}}{8a^2c(bc-ad)x^8}+\frac{(9bc-4ad)(3bc+ad)\left(a+bx^3\right)^{2/3}}{20a^3c^2(bc-ad)x^5}\\
&-\frac{\left(81b^3c^3-9ab^2c^2d-12a^2bcd^2-20a^3d^3\right)\left(a+bx^3\right)^{2/3}}{40a^4c^3(bc-ad)x^2}\\
&+\frac{d^4\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{11/3}(bc-ad)^{4/3}}+\frac{d^4\log\left(c+dx^3\right)}{6c^{11/3}(bc-ad)^{4/3}}\\
&-\frac{d^4\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}(bc-ad)^{4/3}}\end{aligned}$$

output
```
b/a/(-a*d+b*c)/x^8/(b*x^3+a)^(1/3)-1/8*(-a*d+9*b*c)*(b*x^3+a)^(2/3)/a^2/c/
(-a*d+b*c)/x^8+1/20*(-4*a*d+9*b*c)*(a*d+3*b*c)*(b*x^3+a)^(2/3)/a^3/c^2/(-a
*d+b*c)/x^5-1/40*(-20*a^3*d^3-12*a^2*b*c*d^2-9*a*b^2*c^2*d+81*b^3*c^3)*(b*
x^3+a)^(2/3)/a^4/c^3/(-a*d+b*c)/x^2+1/6*d^4*ln(d*x^3+c)/c^(11/3)/(-a*d+b*c
)^(4/3)-1/2*d^4*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(11/3)/(-
a*d+b*c)^(4/3)+1/3*d^4*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a
)^(1/3))*3^(1/2))/c^(11/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

3.760.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.48 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^9 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\, dx = \frac{-\frac{3c^{2/3}\left(-81b^4c^3x^9+9ab^3c^2x^6\left(-3c+dx^3\right)+3a^2b^2cx^3\left(3c^2+cdx^3+4d^2x^6\right)+a^4d\left(5c^2-8cdx^3+20d^2x^6\right)\right.}{a^4(-bc+ad)x^8\sqrt[3]{a+bx^3}}}{}$$

input
```
Integrate[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((-3*c^(2/3)*(-81*b^4*c^3*x^9 + 9*a*b^3*c^2*x^6*(-3*c + d*x^3) + 3*a^2*b^2
*c*x^3*(3*c^2 + c*d*x^3 + 4*d^2*x^6) + a^4*d*(5*c^2 - 8*c*d*x^3 + 20*d^2*x
^6) + a^3*b*(-5*c^3 - c^2*d*x^3 + 4*c*d^2*x^6 + 20*d^3*x^9)))/(a^4*(-(b*c)
 + a*d)*x^8*(a + b*x^3)^(1/3)) - (20*Sqrt[-6 + (6*I)*Sqrt[3]]*d^4*ArcTan[(
3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1
/3)*(a + b*x^3)^(1/3))])/(b*c - a*d)^(4/3) + (20*(1 + I*Sqrt[3])*d^4*Log[2
*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c -
a*d)^(4/3) - ((10*I)*(-I + Sqrt[3])*d^4*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1
- I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3
])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/(120*c^(11/3))
```

3.760.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {972, 25, 1053, 27, 1053, 1053, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{b}{ax^8\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int -\frac{9bdx^3+9bc-ad}{x^9\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{a(bc-ad)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{9bdx^3+9bc-ad}{x^9\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{a(bc-ad)}+\frac{b}{ax^8\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \ 1053$$

$$\frac{-\frac{\int \frac{2\left(3bd(9bc-ad)x^3+(9bc-4ad)(3bc+ad)\right)}{x^6\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{8ac}-\frac{\left(a+bx^3\right)^{2/3}(9bc-ad)}{8acx^8}}{a(bc-ad)}+\frac{b}{ax^8\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\int \frac{3bd(9bc-ad)x^3+(9bc-4ad)(3bc+ad)}{x^6\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{4ac}-\frac{\left(a+bx^3\right)^{2/3}(9bc-ad)}{8acx^8}}{a(bc-ad)}+\frac{b}{ax^8\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \ 1053$$

$$\frac{-\frac{-\frac{\int \frac{81b^3c^3-9ab^2dc^2-12a^2bd^2c-20a^3d^3+3bd(9bc-4ad)(3bc+ad)x^3}{x^3\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{5ac}-\frac{\left(a+bx^3\right)^{2/3}(9bc-4ad)(ad+3bc)}{5acx^5}}{4ac}-\frac{\left(a+bx^3\right)^{2/3}(9bc-ad)}{8acx^8}}{a(bc-ad)}+$$
$$\frac{b}{ax^8\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \ 1053$$

$$\frac{-\frac{-\frac{-\frac{\int -\frac{40a^4d^4}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{2ac}-\frac{\left(a+bx^3\right)^{2/3}\left(-20a^3d^3-12a^2bcd^2-9ab^2c^2d+81b^3c^3\right)}{2acx^2}}{5ac}-\frac{\left(a+bx^3\right)^{2/3}(9bc-4ad)(ad+3bc)}{5acx^5}}{4ac}-\frac{\left(a+bx^3\right)^{2/3}(9bc-ad)}{8acx^8}}{a(bc-ad)}+$$
$$\frac{b}{ax^8\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{-\frac{\frac{20a^3d^4\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{c}-\frac{\left(a+bx^3\right)^{2/3}\left(-20a^3d^3-12a^2bcd^2-9ab^2c^2d+81b^3c^3\right)}{2acx^2}}{5ac}-\frac{\left(a+bx^3\right)^{2/3}(9bc-4ad)(ad+3bc)}{5acx^5}}{4ac}-\frac{\left(a+bx^3\right)^{2/3}(9bc-ad)}{8acx^8}}{a(bc-ad)}$$
$$\frac{b}{ax^8\sqrt[3]{a+bx^3}(bc-ad)}$$

$$\Big\downarrow \ 901$$

3.760. $\int \frac{1}{x^9\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

$$-\frac{-\frac{-\frac{20a^3d^4\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{c}-\frac{\left(a+bx^3\right)^{2/3}\left(-20a^3d^3-12a^2bcd^2-9ab^2c^2d+\right.}{2acx^2}}{5ac}}{4ac}}{a(bc-ad)}$$

$$\frac{b}{ax^8\sqrt[3]{a+bx^3}(bc-ad)}$$

input
```
Int[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
b/(a*(b*c - a*d)*x^8*(a + b*x^3)^(1/3)) + (-1/8*((9*b*c - a*d)*(a + b*x^3)
^(2/3))/(a*c*x^8) - (-1/5*((9*b*c - 4*a*d)*(3*b*c + a*d)*(a + b*x^3)^(2/3)
)/(a*c*x^5) - (-1/2*((81*b^3*c^3 - 9*a*b^2*c^2*d - 12*a^2*b*c*d^2 - 20*a^3
*d^3)*(a + b*x^3)^(2/3))/(a*c*x^2) + (20*a^3*d^4*(ArcTan[(1 + (2*(b*c - a*
d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c -
a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a
*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/
c)/(5*a*c))/(4*a*c))/(a*(b*c - a*d))
```

3.760.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.760.4 Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{\frac{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(d\left(4d^2x^6-\frac{8}{5}cd\,x^3+c^2\right)a^4-b\left(-4d^3x^9-\frac{4}{5}c\,d^2x^6+\frac{1}{5}c^2d\,x^3+c^3\right)a^3+\frac{9x^3\left(\frac{4}{3}d^2x^6+\frac{1}{3}cd\,x^3+c^2\right)b^2c\,a^2}{5}-\frac{27x^6\left(-\frac{d\,x^3}{3}+c\right)}{5}\right.}{4}}{}$

input
```
int(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)*(3/4*((a*d-b*c)/c)^(1/3)*(d*(4*d^
2*x^6-8/5*c*d*x^3+c^2)*a^4-b*(-4*d^3*x^9-4/5*c*d^2*x^6+1/5*c^2*d*x^3+c^3)*
a^3+9/5*x^3*(4/3*d^2*x^6+1/3*c*d*x^3+c^2)*b^2*c*a^2-27/5*x^6*(-1/3*d*x^3+c
)*b^3*c^2*a-81/5*b^4*c^3*x^9)*c+a^4*d^4*x^8*(b*x^3+a)^(1/3)*(2*arctan(1/3*
3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3
^(1/2)+2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln((((a*d-b*c)/c)^(
2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/x^8
/c^4/(a*d-b*c)/a^4
```

3.760.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^9 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.760.6 Sympy [F]

$$\int \frac{1}{x^9 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)} \, dx = \int \frac{1}{x^9 \left(a+bx^3\right)^{\frac{4}{3}} \left(c+dx^3\right)} \, dx$$

input
```
integrate(1/x**9/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**9*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.760.7 Maxima [F]

$$\int \frac{1}{x^9 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^9} \, dx$$

input
```
integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)
```

3.760.8 Giac [F]

$$\int \frac{1}{x^9\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^9}\,dx$$

input
```
integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)
```

3.760.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^9\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{x^9\,\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.761 $\int \frac{x^{10}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

3.761.1 Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^{10}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\frac{x^{11}\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{11}{3},\frac{4}{3},1,\frac{14}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}$$

output
```
1/11*x^11*(1+b*x^3/a)^(1/3)*AppellF1(11/3,4/3,1,14/3,-b*x^3/a,-d*x^3/c)/a/
c/(b*x^3+a)^(1/3)
```

3.761.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 194 vs. 2(67) = 134.

Time = 10.23 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.90

$$\int \frac{x^{10}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\frac{x^2\Bigg(5c\left(-5a^2d+b^2cx^3+ab\left(c-dx^3\right)\right)+5ac(-bc+5ad)\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}}{2}$$

input
```
Integrate[x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(x^2*(5*c*(-5*a^2*d + b^2*c*x^3 + a*b*(c - d*x^3)) + 5*a*c*(-(b*c) + 5*a*d
)*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)
/c)] + 2*(-2*b^2*c^2 - a*b*c*d + 5*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*Appe
llF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(20*b^2*c*d*(b*c - a*
d)*(a + b*x^3)^(1/3))
```

3.761.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x^{10}}{\left(\frac{bx^3}{a}+1\right)^{4/3}\left(dx^3+c\right)}dx}{a\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^{11}\sqrt[3]{\frac{bx^3}{a}+1}\text{AppellF1}\left(\frac{11}{3},\frac{4}{3},1,\frac{14}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}$$

input
```
Int[x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(x^11*(1 + (b*x^3)/a)^(1/3)*AppellF1[11/3, 4/3, 1, 14/3, -((b*x^3)/a), -((
d*x^3)/c)])/(11*a*c*(a + b*x^3)^(1/3))
```

3.761.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.761.4 Maple [F]

$$\int \frac{x^{10}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

3.761.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.761.6 Sympy [F]

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx=\int \frac{x^{10}}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

input
```
integrate(x**10/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**10/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.761.7 Maxima [F]

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx=\int \frac{x^{10}}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^10/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.761.8 Giac [F]

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx=\int \frac{x^{10}}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^10/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.761.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\int \frac{x^{10}}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)} \,d x$$

input
```
int(x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(x^10/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.762 $\int \frac{x^7}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

3.762.1 Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^7}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}$$

output

```
1/8*x^8*(1+b*x^3/a)^(1/3)*AppellF1(8/3,4/3,1,11/3,-b*x^3/a,-d*x^3/c)/a/c/(
b*x^3+a)^(1/3)
```

3.762.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 144 vs. $2(67) = 134$.

Time = 10.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.15

$$\int \frac{x^7}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\frac{5acx^2-5acx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+(bc-2ad)x^5\sqrt[3]{}}{5bc(bc-ad)\sqrt[3]{a+bx^3}}$$

input

```
Integrate[x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output

```
(5*a*c*x^2 - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -(
(b*x^3)/a), -((d*x^3)/c)] + (b*c - 2*a*d)*x^5*(1 + (b*x^3)/a)^(1/3)*Appell
F1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(5*b*c*(b*c - a*d)*(a +
b*x^3)^(1/3))
```

3.762.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x^7}{\left(\frac{bx^3}{a}+1\right)^{4/3}(dx^3+c)}dx}{a\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}$$

input
```
Int[x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(x^8*(1 + (b*x^3)/a)^(1/3)*AppellF1[8/3, 4/3, 1, 11/3, -((b*x^3)/a), -((d*
x^3)/c)])/(8*a*c*(a + b*x^3)^(1/3))
```

3.762.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.762.4 Maple [F]

$$\int \frac{x^{7}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

3.762.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.762.6 Sympy [F]

$$\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\int \frac{x^{7}}{\left(a+bx^{3}\right)^{\frac{4}{3}}\left(c+dx^{3}\right)}\,dx$$

input
```
integrate(x**7/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**7/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.762.7 Maxima [F]

$$\int \frac{x^7}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{x^7}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^7/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.762.8 Giac [F]

$$\int \frac{x^7}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{x^7}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^7/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.762.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{x^7}{\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(x^7/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.763 $\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

3.763.1 Optimal result

Integrand size $=24$, antiderivative size $=67$

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}$$

output

```
1/5*x^5*(1+b*x^3/a)^(1/3)*AppellF1(5/3,4/3,1,8/3,-b*x^3/a,-d*x^3/c)/a/c/(b
*x^3+a)^(1/3)
```

3.763.2 Mathematica [A] (verified)

Time $=10.10$ (sec) , antiderivative size $=129$, normalized size of antiderivative $=1.93$

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\frac{x^2\left(-5c+5c\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+dx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{A}\right.}{5c(bc-ad)\sqrt[3]{a+bx^3}}$$

input

```
Integrate[x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output

```
(x^2*(-5*c + 5*c*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3
)/a), -((d*x^3)/c)] + d*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/
3, -((b*x^3)/a), -((d*x^3)/c)]))/(5*c*(b*c - a*d)*(a + b*x^3)^(1/3))
```

3.763.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x^4}{\left(\frac{bx^3}{a}+1\right)^{4/3}(dx^3+c)}dx}{a\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}$$

input
```
Int[x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 4/3, 1, 8/3, -((b*x^3)/a), -((d*x
^3)/c)])/(5*a*c*(a + b*x^3)^(1/3))
```

3.763.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.763.4 Maple [F]

$$\int \frac{x^{4}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

3.763.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.763.6 Sympy [F]

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\int \frac{x^{4}}{\left(a+bx^{3}\right)^{\frac{4}{3}}\left(c+dx^{3}\right)}\,dx$$

input
```
integrate(x**4/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x**4/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.763.7 Maxima [F]

$$\int \frac{x^4}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{x^4}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.763.8 Giac [F]

$$\int \frac{x^4}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{x^4}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.763.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{x^4}{\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(x^4/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.764 $\int \frac{x}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

3.764.1 Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{4}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}$$

output
```
1/2*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,4/3,1,5/3,-b*x^3/a,-d*x^3/c)/a/c/(b
*x^3+a)^(1/3)
```

3.764.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(67) = 134.

Time = 10.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.10

$$\int \frac{x}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\frac{x^2\Bigg(-10bc+5(bc+ad)\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx^3}{10ac(-bc+ad)\sqrt[3]{a+bx^3}}$$

input
```
Integrate[x/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(x^2*(-10*b*c + 5*(b*c + a*d)*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1,
5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF
1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(10*a*c*(-(b*c) + a*d)*(
a + b*x^3)^(1/3))
```

3.764.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x}{\left(\frac{bx^3}{a}+1\right)^{4/3}(dx^3+c)}dx}{a\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{4}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}$$

input
```
Int[x/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 4/3, 1, 5/3, -((b*x^3)/a), -((d*x
^3)/c)])/(2*a*c*(a + b*x^3)^(1/3))
```

3.764.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.764.4 Maple [F]

$$\int \frac{x}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

3.764.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.764.6 Sympy [F]

$$\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(x/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(x/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.764.7 Maxima [F]

$$\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.764.8 Giac [F]

$$\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(x/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.764.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx=\int \frac{x}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

input

```
int(x/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output

```
int(x/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.765 $\int \frac{1}{x^2(a+bx^3)^{4/3}(c+dx^3)}\,dx$

3.765.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{1}{x^2\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=-\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(-\frac{1}{3},\frac{4}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt[3]{a+bx^3}}$$

output
```
-(1+b*x^3/a)^(1/3)*AppellF1(-1/3,4/3,1,2/3,-b*x^3/a,-d*x^3/c)/a/c/x/(b*x^3
+a)^(1/3)
```

3.765.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(65) = 130.

Time = 10.21 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.97

$$\int \frac{1}{x^2\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\frac{10c(-a^2d+2b^2cx^3+ab(c-dx^3))-5(2b^2c^2-abcd+a^2d^2)\,x^3\sqrt[3]{1+\frac{bx^3}{a}}}{10}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(10*c*(-(a^2*d) + 2*b^2*c*x^3 + a*b*(c - d*x^3)) - 5*(2*b^2*c^2 - a*b*c*d
+ a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/
a), -((d*x^3)/c)] + 2*b*d*(-2*b*c + a*d)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF
1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*a^2*c^2*(-(b*c) + a*d
)*x*(a + b*x^3)^(1/3))
```

3.765.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)} \, dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1} \int \frac{1}{x^2 \left(\frac{bx^3}{a}+1\right)^{4/3} \left(dx^3+c\right)}dx}{a \sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1} \operatorname{AppellF1}\left(-\frac{1}{3},\frac{4}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx \sqrt[3]{a+bx^3}}$$

input
```
Int[1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
-(((1 + (b*x^3)/a)^(1/3)*AppellF1[-1/3, 4/3, 1, 2/3, -((b*x^3)/a), -((d*x^
3)/c)])/(a*c*x*(a + b*x^3)^(1/3)))
```

3.765.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.765.4 Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

3.765.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.765.6 Sympy [F]

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{x^2 (a+bx^3)^{\frac{4}{3}} (c+dx^3)}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**2*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.765.7 Maxima [F]

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^2), x)
```

3.765.8 Giac [F]

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^2), x)
```

3.765.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{x^2 (b\,x^3+a)^{4/3} (d\,x^3+c)} \, dx$$

input
```
int(1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.766 $\int \frac{1}{x^5(a+bx^3)^{4/3}(c+dx^3)}\,dx$

3.766.1 Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{1}{x^5 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\,dx=-\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(-\frac{4}{3},\frac{4}{3},1,-\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acx^4\sqrt[3]{a+bx^3}}$$

output
```
-1/4*(1+b*x^3/a)^(1/3)*AppellF1(-4/3,4/3,1,-1/3,-b*x^3/a,-d*x^3/c)/a/c/x^4
/(b*x^3+a)^(1/3)
```

3.766.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 264 vs. $2(67) = 134$.

Time = 10.32 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.94

$$\int \frac{1}{x^5 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\,dx=\frac{5c(-10b^3c^2x^6+ab^2cx^3(-5c+2dx^3)+a^3d(-c+4dx^3)+a^2b(c^2+cdx^3+}{}$$

input
```
Integrate[1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(5*c*(-10*b^3*c^2*x^6 + a*b^2*c*x^3*(-5*c + 2*d*x^3) + a^3*d*(-c + 4*d*x^3
) + a^2*b*(c^2 + c*d*x^3 + 4*d^2*x^6)) + 5*(5*b^3*c^3 - a*b^2*c^2*d - 2*a^
2*b*c*d^2 + 2*a^3*d^3)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3
, -((b*x^3)/a), -((d*x^3)/c)] - 2*b*d*(-5*b^2*c^2 + a*b*c*d + 2*a^2*d^2)*x
^9*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3
)/c)])/(20*a^3*c^3*(-(b*c) + a*d)*x^4*(a + b*x^3)^(1/3))
```

3.766.3 Rubi [A] (verified)

Time $= 0.22$ (sec) , antiderivative size $= 67$, normalized size of antiderivative $= 1.00$, number of steps used $= 2$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used $= \{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[3]{\frac{b x^3}{a}+1} \int \frac{1}{x^5 \left(\frac{b x^3}{a}+1\right)^{4/3} \left(d x^3+c\right)}dx}{a \sqrt[3]{a+b x^3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt[3]{\frac{b x^3}{a}+1} \text{AppellF1}\left(-\frac{4}{3},\frac{4}{3},1,-\frac{1}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right)}{4 a c x^4 \sqrt[3]{a+b x^3}}$$

input
```
Int[1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
-1/4*((1 + (b*x^3)/a)^(1/3)*AppellF1[-4/3, 4/3, 1, -1/3, -((b*x^3)/a), -((
d*x^3)/c)])/(a*c*x^4*(a + b*x^3)^(1/3))
```

3.766.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.766.4 Maple [F]

$$\int \frac{1}{x^{5} \left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

3.766.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.766.6 Sympy [F]

$$\int \frac{1}{x^5 \left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx=\int \frac{1}{x^{5} \left(a + b x^{3}\right)^{\frac{4}{3}} \left(c + d x^{3}\right)}\, dx$$

input
```
integrate(1/x**5/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/(x**5*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.766.7 Maxima [F]

$$\int \frac{1}{x^5 \left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx=\int \frac{1}{{\left(b x^{3} + a\right)}^{\frac{4}{3}} {\left(d x^{3} + c\right)} x^{5}} \,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^5), x)
```

3.766.8 Giac [F]

$$\int \frac{1}{x^5 \left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx=\int \frac{1}{{\left(b x^{3} + a\right)}^{\frac{4}{3}} {\left(d x^{3} + c\right)} x^{5}} \,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^5), x)
```

3.766.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{x^5 \left(b\,x^3+a\right)^{4/3} \left(d\,x^3+c\right)}\,dx$$

input
```
int(1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.767 $\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx$

3.767.1 Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx=-\frac{(bc+ad)x^4}{4b^2d^2}+\frac{x^8}{8bd}-\frac{a^3\log(a+bx^4)}{4b^3(bc-ad)}+\frac{c^3\log(c+dx^4)}{4d^3(bc-ad)}$$

output
```
-1/4*(a*d+b*c)*x^4/b^2/d^2+1/8*x^8/b/d-1/4*a^3*ln(b*x^4+a)/b^3/(-a*d+b*c)+
1/4*c^3*ln(d*x^4+c)/d^3/(-a*d+b*c)
```

3.767.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx =\frac{bd(bc-ad)x^4(-2bc-2ad+bdx^4)-2a^3d^3\log(a+bx^4)+2b^3c^3\log(c+dx^4)}{8b^3d^3(bc-ad)}$$

input
```
Integrate[x^15/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(b*d*(b*c - a*d)*x^4*(-2*b*c - 2*a*d + b*d*x^4) - 2*a^3*d^3*Log[a + b*x^4]
 + 2*b^3*c^3*Log[c + d*x^4])/(8*b^3*d^3*(b*c - a*d))
```

3.767.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{4}\int \frac{x^{12}}{(bx^4+a)(dx^4+c)}dx^4$$

$$\Big\downarrow \ 93$$

$$\frac{1}{4}\int \left(\frac{x^4}{bd}+\frac{-bc-ad}{b^2d^2}-\frac{a^3}{b^2(bc-ad)(bx^4+a)}-\frac{c^3}{d^2(ad-bc)(dx^4+c)}\right)dx^4$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(-\frac{a^3\log(a+bx^4)}{b^3(bc-ad)}-\frac{x^4(ad+bc)}{b^2d^2}+\frac{c^3\log(c+dx^4)}{d^3(bc-ad)}+\frac{x^8}{2bd}\right)$$

input
```
Int[x^15/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-(((b*c + a*d)*x^4)/(b^2*d^2)) + x^8/(2*b*d) - (a^3*Log[a + b*x^4])/(b^3*
(b*c - a*d)) + (c^3*Log[c + d*x^4])/(d^3*(b*c - a*d)))/4
```

3.767.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.767.4 Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\left(-bd\,x^{4}+ad+bc\right)^{2}}{8b^{3}d^{3}}+\frac{a^{3}\ln\left(b\,x^{4}+a\right)}{4b^{3}\left(ad-bc\right)}-\frac{c^{3}\ln\left(d\,x^{4}+c\right)}{4d^{3}\left(ad-bc\right)}$	78
norman	$\frac{x^{8}}{8bd}-\frac{\left(ad+bc\right)x^{4}}{4b^{2}d^{2}}+\frac{a^{3}\ln\left(b\,x^{4}+a\right)}{4b^{3}\left(ad-bc\right)}-\frac{c^{3}\ln\left(d\,x^{4}+c\right)}{4d^{3}\left(ad-bc\right)}$	83
parallelrisch	$\frac{a\,b^{2}d^{3}x^{8}-x^{8}b^{3}c\,d^{2}-2a^{2}b\,d^{3}x^{4}+2b^{3}c^{2}d\,x^{4}+2a^{3}\ln\left(b\,x^{4}+a\right)d^{3}-2c^{3}\ln\left(d\,x^{4}+c\right)b^{3}}{8b^{3}d^{3}\left(ad-bc\right)}$	99
risch	$\frac{x^{8}}{8bd}-\frac{a\,x^{4}}{4b^{2}d}-\frac{c\,x^{4}}{4b\,d^{2}}+\frac{a^{2}}{8b^{3}d}+\frac{ac}{4b^{2}d^{2}}+\frac{c^{2}}{8b\,d^{3}}-\frac{c^{3}\ln\left(d\,x^{4}+c\right)}{4d^{3}\left(ad-bc\right)}+\frac{a^{3}\ln\left(-b\,x^{4}-a\right)}{4b^{3}\left(ad-bc\right)}$	124

input
```
int(x^15/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-b*d*x^4+a*d+b*c)^2/b^3/d^3+1/4*a^3/b^3/(a*d-b*c)*ln(b*x^4+a)-1/4*c^3
/d^3/(a*d-b*c)*ln(d*x^4+c)
```

3.767.5 Fricas [A] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx = \frac{(b^3cd^2-ab^2d^3)x^8-2\,a^3d^3\log\left(bx^4+a\right)+2\,b^3c^3\log\left(dx^4+c\right)-2\,(b^3c^2d-a^2bd^3)x^4}{8\,(b^4cd^3-ab^3d^4)}$$

input
```
integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/8*((b^3*c*d^2 - a*b^2*d^3)*x^8 - 2*a^3*d^3*log(b*x^4 + a) + 2*b^3*c^3*lo
g(d*x^4 + c) - 2*(b^3*c^2*d - a^2*b*d^3)*x^4)/(b^4*c*d^3 - a*b^3*d^4)
```

3.767.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx=\text{Timed out}$$

input
```
integrate(x**15/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.767.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx=-\frac{a^3\log(bx^4+a)}{4(b^4c-ab^3d)}+\frac{c^3\log(dx^4+c)}{4(bcd^3-ad^4)}+\frac{bdx^8-2(bc+ad)x^4}{8\,b^2d^2}$$

input
```
integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/4*a^3*log(b*x^4 + a)/(b^4*c - a*b^3*d) + 1/4*c^3*log(d*x^4 + c)/(b*c*d^
3 - a*d^4) + 1/8*(b*d*x^8 - 2*(b*c + a*d)*x^4)/(b^2*d^2)
```

3.767.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx=-\frac{a^3\log(|bx^4+a|)}{4(b^4c-ab^3d)}+\frac{c^3\log(|dx^4+c|)}{4(bcd^3-ad^4)}+\frac{bdx^8-2\,bcx^4-2\,adx^4}{8\,b^2d^2}$$

input
```
integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/4*a^3*log(abs(b*x^4 + a))/(b^4*c - a*b^3*d) + 1/4*c^3*log(abs(d*x^4 + c
))/(b*c*d^3 - a*d^4) + 1/8*(b*d*x^8 - 2*b*c*x^4 - 2*a*d*x^4)/(b^2*d^2)
```

3.767.9 Mupad [B] (verification not implemented)

Time = 10.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx = \frac{x^8}{8\,b\,d}-\frac{c^3\,\ln\left(d\,x^4+c\right)}{4\,\left(a\,d^4-b\,c\,d^3\right)}-\frac{a^3\,\ln\left(b\,x^4+a\right)}{4\,\left(b^4\,c-a\,b^3\,d\right)}-\frac{x^4\,\left(a\,d+b\,c\right)}{4\,b^2\,d^2}$$

input

```
int(x^15/((a + b*x^4)*(c + d*x^4)),x)
```

output

```
x^8/(8*b*d) - (c^3*log(c + d*x^4))/(4*(a*d^4 - b*c*d^3)) - (a^3*log(a + b*
x^4))/(4*(b^4*c - a*b^3*d)) - (x^4*(a*d + b*c))/(4*b^2*d^2)
```

3.768 $\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx$

3.768.1 Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \frac{x^4}{4bd} + \frac{a^2\log(a+bx^4)}{4b^2(bc-ad)} - \frac{c^2\log(c+dx^4)}{4d^2(bc-ad)}$$

output
```
1/4*x^4/b/d+1/4*a^2*ln(b*x^4+a)/b^2/(-a*d+b*c)-1/4*c^2*ln(d*x^4+c)/d^2/(-a
*d+b*c)
```

3.768.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \frac{a^2d^2\log(a+bx^4) - b(d(-bc+ad)x^4 + bc^2\log(c+dx^4))}{4b^2d^2(bc-ad)}$$

input
```
Integrate[x^11/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(a^2*d^2*Log[a + b*x^4] - b*(d*(-(b*c) + a*d)*x^4 + b*c^2*Log[c + d*x^4]))
/(4*b^2*d^2*(b*c - a*d))
```

3.768.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \textcolor{blue}{948}$$

$$\frac{1}{4}\int \frac{x^8}{(bx^4+a)(dx^4+c)}dx^4$$

$$\Big\downarrow \textcolor{blue}{93}$$

$$\frac{1}{4}\int \left(\frac{a^2}{b(bc-ad)(bx^4+a)}+\frac{1}{bd}+\frac{c^2}{d(ad-bc)(dx^4+c)}\right)dx^4$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{1}{4}\left(\frac{a^2\log(a+bx^4)}{b^2(bc-ad)}-\frac{c^2\log(c+dx^4)}{d^2(bc-ad)}+\frac{x^4}{bd}\right)$$

input
```
Int[x^11/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(x^4/(b*d) + (a^2*Log[a + b*x^4])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x^4])
/(d^2*(b*c - a*d)))/4
```

3.768.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.768.4 Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{x^{4}}{4bd}-\frac{a^{2}\ln\left(b\,x^{4}+a\right)}{4\left(ad-bc\right)b^{2}}+\frac{c^{2}\ln\left(d\,x^{4}+c\right)}{4\left(ad-bc\right)d^{2}}$	65
norman	$\frac{x^{4}}{4bd}-\frac{a^{2}\ln\left(b\,x^{4}+a\right)}{4\left(ad-bc\right)b^{2}}+\frac{c^{2}\ln\left(d\,x^{4}+c\right)}{4\left(ad-bc\right)d^{2}}$	65
risch	$\frac{x^{4}}{4bd}+\frac{c^{2}\ln\left(d\,x^{4}+c\right)}{4\left(ad-bc\right)d^{2}}-\frac{a^{2}\ln\left(-b\,x^{4}-a\right)}{4b^{2}\left(ad-bc\right)}$	68
parallelrisch	$-\frac{-ab\,d^{2}x^{4}+b^{2}cd\,x^{4}+a^{2}\ln\left(b\,x^{4}+a\right)d^{2}-c^{2}\ln\left(d\,x^{4}+c\right)b^{2}}{4b^{2}d^{2}\left(ad-bc\right)}$	70

input
```
int(x^11/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4/b/d-1/4*a^2/(a*d-b*c)/b^2*ln(b*x^4+a)+1/4*c^2/(a*d-b*c)/d^2*ln(d*x
^4+c)
```

3.768.5 Fricas [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=\frac{(b^2cd-abd^2)x^4+a^2d^2\log\left(bx^4+a\right)-b^2c^2\log\left(dx^4+c\right)}{4\left(b^3cd^2-ab^2d^3\right)}$$

input
```
integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*((b^2*c*d - a*b*d^2)*x^4 + a^2*d^2*log(b*x^4 + a) - b^2*c^2*log(d*x^4
+ c))/(b^3*c*d^2 - a*b^2*d^3)
```

3.768.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**11/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.768.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \frac{x^4}{4\,bd} + \frac{a^2\log(bx^4+a)}{4\,(b^3c-ab^2d)} - \frac{c^2\log(dx^4+c)}{4\,(bcd^2-ad^3)}$$

input
```
integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/4*x^4/(b*d) + 1/4*a^2*log(b*x^4 + a)/(b^3*c - a*b^2*d) - 1/4*c^2*log(d*x
^4 + c)/(b*c*d^2 - a*d^3)
```

3.768.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \frac{x^4}{4\,bd} + \frac{a^2\log(|bx^4+a|)}{4\,(b^3c-ab^2d)} - \frac{c^2\log(|dx^4+c|)}{4\,(bcd^2-ad^3)}$$

input
```
integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/4*x^4/(b*d) + 1/4*a^2*log(abs(b*x^4 + a))/(b^3*c - a*b^2*d) - 1/4*c^2*lo
g(abs(d*x^4 + c))/(b*c*d^2 - a*d^3)
```

3.768.9 Mupad [B] (verification not implemented)

Time = 9.91 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \frac{a^2\,\ln\left(b\,x^4+a\right)}{4\,b^3\,c-4\,a\,b^2\,d}+\frac{c^2\,\ln\left(d\,x^4+c\right)}{4\,a\,d^3-4\,b\,c\,d^2}+\frac{x^4}{4\,b\,d}$$

input
```
int(x^11/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
(a^2*log(a + b*x^4))/(4*b^3*c - 4*a*b^2*d) + (c^2*log(c + d*x^4))/(4*a*d^3
 - 4*b*c*d^2) + x^4/(4*b*d)
```

3.769 $\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx$

3.769.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx=-\frac{a\log(a+bx^4)}{4b(bc-ad)}+\frac{c\log(c+dx^4)}{4d(bc-ad)}$$

output
```
-1/4*a*ln(b*x^4+a)/b/(-a*d+b*c)+1/4*c*ln(d*x^4+c)/d/(-a*d+b*c)
```

3.769.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx=-\frac{ad\log(a+bx^4)-bc\log(c+dx^4)}{4b^2cd-4abd^2}$$

input
```
Integrate[x^7/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
-((a*d*Log[a + b*x^4] - b*c*Log[c + d*x^4])/(4*b^2*c*d - 4*a*b*d^2))
```

3.769.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow 948$$

$$\frac{1}{4}\int \frac{x^4}{(bx^4+a)(dx^4+c)}dx^4$$

$$\Big\downarrow 86$$

$$\frac{1}{4}\int \left(\frac{c}{(bc-ad)(dx^4+c)}-\frac{a}{(bc-ad)(bx^4+a)}\right)dx^4$$

$$\Big\downarrow 2009$$

$$\frac{1}{4}\left(\frac{c\log(c+dx^4)}{d(bc-ad)}-\frac{a\log(a+bx^4)}{b(bc-ad)}\right)$$

input
```
Int[x^7/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-((a*Log[a + b*x^4])/(b*(b*c - a*d))) + (c*Log[c + d*x^4])/(d*(b*c - a*d)
))/4
```

3.769.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.769.4 Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$\frac{a\ln(b\,x^4+a)d-c\ln(d\,x^4+c)b}{4(ad-bc)bd}$	43
default	$\frac{a\ln(b\,x^4+a)}{4(ad-bc)b}-\frac{c\ln(d\,x^4+c)}{4(ad-bc)d}$	50
norman	$\frac{a\ln(b\,x^4+a)}{4(ad-bc)b}-\frac{c\ln(d\,x^4+c)}{4(ad-bc)d}$	50
risch	$-\frac{c\ln(-d\,x^4-c)}{4(ad-bc)d}+\frac{a\ln(b\,x^4+a)}{4(ad-bc)b}$	53

input
```
int(x^7/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*(a*ln(b*x^4+a)*d-c*ln(d*x^4+c)*b)/(a*d-b*c)/b/d
```

3.769.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{x^7}{(a+bx^4)\,(c+dx^4)}\,dx=-\frac{ad\log\left(bx^4+a\right)-bc\log\left(dx^4+c\right)}{4\,(b^2cd-abd^2)}$$

input
```
integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
-1/4*(a*d*log(b*x^4 + a) - b*c*log(d*x^4 + c))/(b^2*c*d - a*b*d^2)
```

3.769.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**7/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.769.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\log(bx^4+a)}{4(b^2c-abd)} + \frac{c\log(dx^4+c)}{4(bcd-ad^2)}$$

input
```
integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/4*a*log(b*x^4 + a)/(b^2*c - a*b*d) + 1/4*c*log(d*x^4 + c)/(b*c*d - a*d^
2)
```

3.769.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\log(|bx^4+a|)}{4(b^2c-abd)} + \frac{c\log(|dx^4+c|)}{4(bcd-ad^2)}$$

input
```
integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/4*a*log(abs(b*x^4 + a))/(b^2*c - a*b*d) + 1/4*c*log(abs(d*x^4 + c))/(b*
c*d - a*d^2)
```

3.769.9 Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\,\ln\left(b\,x^4+a\right)}{4\,b^2\,c-4\,a\,b\,d}-\frac{c\,\ln\left(d\,x^4+c\right)}{4\,a\,d^2-4\,b\,c\,d}$$

input

```
int(x^7/((a + b*x^4)*(c + d*x^4)),x)
```

output

```
- (a*log(a + b*x^4))/(4*b^2*c - 4*a*b*d) - (c*log(c + d*x^4))/(4*a*d^2 - 4
*b*c*d)
```

3.770 $\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx$

3.770.1 Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{\log(a+bx^4)}{4(bc-ad)} - \frac{\log(c+dx^4)}{4(bc-ad)}$$

output
```
1/4*ln(b*x^4+a)/(-a*d+b*c)-1/4*ln(d*x^4+c)/(-a*d+b*c)
```

3.770.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{\log(a+bx^4)-\log(c+dx^4)}{4bc-4ad}$$

input
```
Integrate[x^3/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(Log[a + b*x^4] - Log[c + d*x^4])/(4*b*c - 4*a*d)
```

3.770.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {946, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{4}\int \frac{1}{(bx^4+a)(dx^4+c)}dx^4$$

$$\Big\downarrow \; 47$$

$$\frac{1}{4}\left(\frac{b\int \frac{1}{bx^4+a}dx^4}{bc-ad}-\frac{d\int \frac{1}{dx^4+c}dx^4}{bc-ad}\right)$$

$$\Big\downarrow \; 16$$

$$\frac{1}{4}\left(\frac{\log\left(a+bx^4\right)}{bc-ad}-\frac{\log\left(c+dx^4\right)}{bc-ad}\right)$$

input
```
Int[x^3/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(Log[a + b*x^4]/(b*c - a*d) - Log[c + d*x^4]/(b*c - a*d))/4
```

3.770.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.770.4 Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

method	result	size
parallelrisch	$-\frac{\ln\left(b\,x^{4}+a\right)-\ln\left(d\,x^{4}+c\right)}{4\left(a d -b c \right)}$	32
default	$-\frac{\ln\left(b\,x^{4}+a\right)}{4\left(a d -b c \right)}+\frac{\ln\left(d\,x^{4}+c\right)}{4 a d -4 b c}$	42
norman	$-\frac{\ln\left(b\,x^{4}+a\right)}{4\left(a d -b c \right)}+\frac{\ln\left(d\,x^{4}+c\right)}{4 a d -4 b c}$	42
risch	$-\frac{\ln\left(-b\,x^{4}-a\right)}{4\left(a d -b c \right)}+\frac{\ln\left(d\,x^{4}+c\right)}{4 a d -4 b c}$	45

input
```
int(x^3/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(ln(b*x^4+a)-ln(d*x^4+c))/(a*d-b*c)
```

3.770.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{\log\left(bx^4+a\right)-\log\left(dx^4+c\right)}{4\,(bc-ad)}$$

input
```
integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*(log(b*x^4 + a) - log(d*x^4 + c))/(b*c - a*d)
```

3.770.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(36) = 72.

Time = 0.91 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.07

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{\log\left(x^4+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{4(ad-bc)} - \frac{\log\left(x^4+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{4(ad-bc)}$$

input
```
integrate(x**3/(b*x**4+a)/(d*x**4+c),x)
```

output
```
log(x**4 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c*
*2/(a*d - b*c) + b*c)/(2*b*d))/(4*(a*d - b*c)) - log(x**4 + (a**2*d**2/(a*
d - b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b
*d))/(4*(a*d - b*c))
```

3.770.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{\log(bx^4+a)}{4(bc-ad)} - \frac{\log(dx^4+c)}{4(bc-ad)}$$

input
```
integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/4*log(b*x^4 + a)/(b*c - a*d) - 1/4*log(d*x^4 + c)/(b*c - a*d)
```

3.770.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{b\log\left(|bx^4+a|\right)}{4\left(b^2c-abd\right)} - \frac{d\log\left(|dx^4+c|\right)}{4\left(bcd-ad^2\right)}$$

input
```
integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/4*b*log(abs(b*x^4 + a))/(b^2*c - a*b*d) - 1/4*d*log(abs(d*x^4 + c))/(b*c
*d - a*d^2)
```

3.770.9 Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 1012, normalized size of antiderivative = 22.49

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx =$$

$$-\,\operatorname{atan}\left(\frac{\frac{\left(\frac{x^4\left(96\,c\,b^5\,d^4+96\,a\,b^4\,d^5\right)+\frac{\frac{x^4\left(512\,a^3\,b^4\,d^7+1536\,a^2\,b^5\,c\,d^6+1536\,a\,b^6\,c^2\,d^5+512\,b^7\,c^3\,d^4\right)+1024\,a\,b^6\,c^3\,d^4+1024\,a^3\,b^4\,c\,d^6+2048\,a^2\,b^5\,c^2\,d^5}{4\,a\,d-4\,b\,c}+}{4\,a\,d-4\,b\,c}}{4\,a\,d-4\,b\,c}\right.}{4\,a\,d-4\,b\,c}}{\frac{\frac{x^4\left(96\,c\,b^5\,d^4+96\,a\,b^4\,d^5\right)+\frac{\frac{x^4\left(512\,a^3\,b^4\,d^7+1536\,a^2\,b^5\,c\,d^6+1536\,a\,b^6\,c^2\,d^5+512\,b^7\,c^3\,d^4\right)+1024\,a\,b^6\,c^3\,d^4+1024\,a^3\,b^4\,c\,d^6+2048\,a^2\,b^5\,c^2\,d}{4\,a\,d-4\,b\,c}}{4\,a\,d-4\,b\,c}}{4\,a\,d-4\,b\,c}}{4\,a\,d-4\,b\,c}}\right.$$

input
```
int(x^3/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
-(atan(((((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) + ((x^4*(512*a^3*b^4*d^7 + 51
2*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*
d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + x^4*(38
4*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) + 512*a*b^5*c^2*d^4 + 5
12*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) + 8*b^
4*d^4*x^4)*1i)/(4*a*d - 4*b*c) - (((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) - (x
^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) - (x^4*(512*a^3*b
^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024
*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c
) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^
4)/(4*a*d - 4*b*c) - 8*b^4*d^4*x^4)*1i)/(4*a*d - 4*b*c))/(((x^4*(96*a*b^4*
d^5 + 96*b^5*c*d^4) + ((x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^
6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6
+ 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + x^4*(384*a^2*b^4*d^6 + 384*b^6*c
^2*d^4 + 768*a*b^5*c*d^5) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d
- 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) + 8*b^4*d^4*x^4)/(4*a*d - 4*b*c
) + ((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) - (x^4*(384*a^2*b^4*d^6 + 384*b^6*
c^2*d^4 + 768*a*b^5*c*d^5) - (x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 153
6*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*
c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + 512*a*b^5*c^2*d^4 + 512...
```

3.770. $\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx$

3.771 $\int \frac{1}{x(a+bx^4)(c+dx^4)}\,dx$

3.771.1 Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{1}{x(a+bx^4)(c+dx^4)}\,dx = \frac{\log(x)}{ac} - \frac{b\log(a+bx^4)}{4a(bc-ad)} + \frac{d\log(c+dx^4)}{4c(bc-ad)}$$

output
```
ln(x)/a/c-1/4*b*ln(b*x^4+a)/a/(-a*d+b*c)+1/4*d*ln(d*x^4+c)/c/(-a*d+b*c)
```

3.771.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(a+bx^4)(c+dx^4)}\,dx = \frac{4bc\log(x) - 4ad\log(x) - bc\log(a+bx^4) + ad\log(c+dx^4)}{4abc^2 - 4a^2cd}$$

input
```
Integrate[1/(x*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(4*b*c*Log[x] - 4*a*d*Log[x] - b*c*Log[a + b*x^4] + a*d*Log[c + d*x^4])/(4
*a*b*c^2 - 4*a^2*c*d)
```

3.771.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(bx^4+a\right)\left(dx^4+c\right)}dx^4$$

$$\Big\downarrow \text{93}$$

$$\frac{1}{4}\int \left(\frac{b^2}{a(ad-bc)\left(bx^4+a\right)}+\frac{d^2}{c(bc-ad)\left(dx^4+c\right)}+\frac{1}{acx^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{b\log\left(a+bx^4\right)}{a(bc-ad)}+\frac{d\log\left(c+dx^4\right)}{c(bc-ad)}+\frac{\log\left(x^4\right)}{ac}\right)$$

input
```
Int[1/(x*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(Log[x^4]/(a*c) - (b*Log[a + b*x^4])/(a*(b*c - a*d)) + (d*Log[c + d*x^4])/
(c*(b*c - a*d)))/4
```

3.771.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.771.4 Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{4\ln(x) a d-4\ln(x) b c+b\ln(b\,x^{4}+a) c-d\ln(d\,x^{4}+c) a}{4 a c (a d-b c)}$	55
default	$\frac{\ln(x)}{a c}+\frac{b\ln(b\,x^{4}+a)}{4(a d-b c) a}-\frac{d\ln(d\,x^{4}+c)}{4(a d-b c) c}$	59
norman	$\frac{\ln(x)}{a c}+\frac{b\ln(b\,x^{4}+a)}{4(a d-b c) a}-\frac{d\ln(d\,x^{4}+c)}{4(a d-b c) c}$	59
risch	$\frac{\ln(x)}{a c}+\frac{b\ln(b\,x^{4}+a)}{4(a d-b c) a}-\frac{d\ln(d\,x^{4}+c)}{4(a d-b c) c}$	59

input
```
int(1/x/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*ln(x)*a*d-4*ln(x)*b*c+b*ln(b*x^4+a)*c-d*ln(d*x^4+c)*a)/a/c/(a*d-b*c
)
```

3.771.5 Fricas [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(a+bx^4)(c+dx^4)}\,dx = -\frac{bc\log(bx^4+a)-ad\log(dx^4+c)-4(bc-ad)\log(x)}{4(abc^2-a^2cd)}$$

input
```
integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
-1/4*(b*c*log(b*x^4 + a) - a*d*log(d*x^4 + c) - 4*(b*c - a*d)*log(x))/(a*b
*c^2 - a^2*c*d)
```

3.771.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.771.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=-\frac{b\log\left(bx^4+a\right)}{4\left(abc-a^2d\right)}+\frac{d\log\left(dx^4+c\right)}{4\left(bc^2-acd\right)}+\frac{\log\left(x^4\right)}{4\,ac}$$

input
```
integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/4*b*log(b*x^4 + a)/(a*b*c - a^2*d) + 1/4*d*log(d*x^4 + c)/(b*c^2 - a*c*
d) + 1/4*log(x^4)/(a*c)
```

3.771.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=-\frac{b^2\log\left(\left|bx^4+a\right|\right)}{4\left(ab^2c-a^2bd\right)}+\frac{d^2\log\left(\left|dx^4+c\right|\right)}{4\left(bc^2d-acd^2\right)}+\frac{\log\left(x^4\right)}{4\,ac}$$

input
```
integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/4*b^2*log(abs(b*x^4 + a))/(a*b^2*c - a^2*b*d) + 1/4*d^2*log(abs(d*x^4 +
 c))/(b*c^2*d - a*c*d^2) + 1/4*log(x^4)/(a*c)
```

3.771.9 Mupad [B] (verification not implemented)

Time = 10.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=\frac{b\,\ln\left(b\,x^4+a\right)}{4\,a^2\,d-4\,a\,b\,c}+\frac{d\,\ln\left(d\,x^4+c\right)}{4\,b\,c^2-4\,a\,c\,d}+\frac{\ln\left(x\right)}{a\,c}$$

input
```
int(1/(x*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
(b*log(a + b*x^4))/(4*a^2*d - 4*a*b*c) + (d*log(c + d*x^4))/(4*b*c^2 - 4*a
*c*d) + log(x)/(a*c)
```

3.772 $\int \frac{1}{x^5\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$

3.772.1 Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{1}{x^5\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=-\frac{1}{4acx^4}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^4\right)}{4a^2(bc-ad)}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}$$

output
```
-1/4/a/c/x^4-(a*d+b*c)*ln(x)/a^2/c^2+1/4*b^2*ln(b*x^4+a)/a^2/(-a*d+b*c)-1/
4*d^2*ln(d*x^4+c)/c^2/(-a*d+b*c)
```

3.772.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^5\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=-\frac{1}{4acx^4}+\frac{(-bc-ad)\log(x)}{a^2c^2}-\frac{b^2\log\left(a+bx^4\right)}{4a^2(-bc+ad)}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}$$

input
```
Integrate[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
-1/4*1/(a*c*x^4) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^4]
)/(4*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^4])/(4*c^2*(b*c - a*d))
```

3.772.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^4) (c+dx^4)} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{4} \int \frac{1}{x^8 (bx^4+a) (dx^4+c)} dx^4$$

$$\Big\downarrow \; 93$$

$$\frac{1}{4} \int \left(-\frac{b^3}{a^2(ad-bc) (bx^4+a)} - \frac{d^3}{c^2(bc-ad) (dx^4+c)} + \frac{-bc-ad}{a^2c^2x^4} + \frac{1}{acx^8}\right) dx^4$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\left(\frac{b^2 \log(a+bx^4)}{a^2(bc-ad)} - \frac{\log(x^4)(ad+bc)}{a^2c^2} - \frac{d^2\log(c+dx^4)}{c^2(bc-ad)} - \frac{1}{acx^4}\right)$$

input

```
Int[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

output

```
(-(1/(a*c*x^4)) - ((b*c + a*d)*Log[x^4])/(a^2*c^2) + (b^2*Log[a + b*x^4])/
(a^2*(b*c - a*d)) - (d^2*Log[c + d*x^4])/(c^2*(b*c - a*d)))/4
```

3.772.3.1 Defintions of rubi rules used

rule 93

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.772.4 Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

method	result	size
norman	$-\frac{1}{4ac\,x^{4}}-\frac{b^{2}\ln\left(b\,x^{4}+a\right)}{4a^{2}\left(ad-bc\right)}+\frac{d^{2}\ln\left(d\,x^{4}+c\right)}{4c^{2}\left(ad-bc\right)}-\frac{\left(ad+bc\right)\ln\left(x\right)}{a^{2}c^{2}}$	82
default	$-\frac{1}{4ac\,x^{4}}+\frac{\left(-ad-bc\right)\ln\left(x\right)}{a^{2}c^{2}}-\frac{b^{2}\ln\left(b\,x^{4}+a\right)}{4a^{2}\left(ad-bc\right)}+\frac{d^{2}\ln\left(d\,x^{4}+c\right)}{4c^{2}\left(ad-bc\right)}$	83
risch	$-\frac{1}{4ac\,x^{4}}-\frac{\ln\left(x\right)d}{a\,c^{2}}-\frac{\ln\left(x\right)b}{a^{2}c}+\frac{d^{2}\ln\left(-d\,x^{4}-c\right)}{4c^{2}\left(ad-bc\right)}-\frac{b^{2}\ln\left(b\,x^{4}+a\right)}{4a^{2}\left(ad-bc\right)}$	90
parallelrisch	$-\frac{4\ln\left(x\right)x^{4}a^{2}d^{2}-4\ln\left(x\right)x^{4}b^{2}c^{2}+b^{2}\ln\left(b\,x^{4}+a\right)c^{2}x^{4}-d^{2}\ln\left(d\,x^{4}+c\right)a^{2}x^{4}+a^{2}cd-b\,c^{2}a}{4a^{2}c^{2}x^{4}\left(ad-bc\right)}$	99

input
```
int(1/x^5/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/c/x^4-1/4*b^2/a^2/(a*d-b*c)*ln(b*x^4+a)+1/4*d^2/c^2/(a*d-b*c)*ln(d*
x^4+c)-(a*d+b*c)*ln(x)/a^2/c^2
```

3.772.5 Fricas [A] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^5\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{b^2c^2x^4\log\left(bx^4+a\right)-a^2d^2x^4\log\left(dx^4+c\right)-4\left(b^2c^2-a^2d^2\right)x^4\log\left(x\right)-abc^2+a^2cd}{4\left(a^2bc^3-a^3c^2d\right)x^4}$$

input
```
integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*(b^2*c^2*x^4*log(b*x^4 + a) - a^2*d^2*x^4*log(d*x^4 + c) - 4*(b^2*c^2
- a^2*d^2)*x^4*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^4)
```

3.772.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^4) (c+dx^4)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**5/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.772.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (a+bx^4) (c+dx^4)} \, dx = \frac{b^2 \log (bx^4+a)}{4 (a^2bc-a^3d)} - \frac{d^2 \log (dx^4+c)}{4 (bc^3-ac^2d)} - \frac{(bc+ad) \log (x^4)}{4 a^2c^2} - \frac{1}{4 acx^4}$$

input
```
integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/4*b^2*log(b*x^4 + a)/(a^2*b*c - a^3*d) - 1/4*d^2*log(d*x^4 + c)/(b*c^3 -
 a*c^2*d) - 1/4*(b*c + a*d)*log(x^4)/(a^2*c^2) - 1/4/(a*c*x^4)
```

3.772.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.29

$$\begin{aligned}\int \frac{1}{x^5 (a+bx^4) (c+dx^4)} \, dx = {} & \frac{b^3 \log (|bx^4+a|)}{4 (a^2b^2c-a^3bd)} - \frac{d^3 \log (|dx^4+c|)}{4 (bc^3d-ac^2d^2)} \\ & - \frac{(bc+ad) \log (x^4)}{4 a^2c^2} + \frac{bcx^4+adx^4-ac}{4 a^2c^2x^4}\end{aligned}$$

input
```
integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/4*b^3*log(abs(b*x^4 + a))/(a^2*b^2*c - a^3*b*d) - 1/4*d^3*log(abs(d*x^4
+ c))/(b*c^3*d - a*c^2*d^2) - 1/4*(b*c + a*d)*log(x^4)/(a^2*c^2) + 1/4*(b*
c*x^4 + a*d*x^4 - a*c)/(a^2*c^2*x^4)
```

3.772.9 Mupad [B] (verification not implemented)

Time = 11.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (a+bx^4) (c+dx^4)}\,dx = -\frac{b^2\,\ln\left(b\,x^4+a\right)}{4\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{d^2\,\ln\left(d\,x^4+c\right)}{4\,\left(b\,c^3-a\,c^2\,d\right)} -\frac{1}{4\,a\,c\,x^4}-\frac{\ln\left(x\right)\,\left(a\,d+b\,c\right)}{a^2\,c^2}$$

input
```
int(1/(x^5*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
- (b^2*log(a + b*x^4))/(4*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^4))/(4*(b*
c^3 - a*c^2*d)) - 1/(4*a*c*x^4) - (log(x)*(a*d + b*c))/(a^2*c^2)
```

3.773 $\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx$

3.773.1 Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx=-\frac{(bc+ad)x^2}{2b^2d^2}+\frac{x^6}{6bd}-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)}+\frac{c^{5/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}$$

output
```
-1/2*(a*d+b*c)*x^2/b^2/d^2+1/6*x^6/b/d-1/2*a^(5/2)*arctan(x^2*b^(1/2)/a^(1
/2))/b^(5/2)/(-a*d+b*c)+1/2*c^(5/2)*arctan(x^2*d^(1/2)/c^(1/2))/d^(5/2)/(-
a*d+b*c)
```

3.773.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx=\frac{1}{6}\left(\frac{x^2(-3bc-3ad+bdx^4)}{b^2d^2}+\frac{3a^{5/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{b^{5/2}(-bc+ad)}+\frac{3c^{5/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{d^{5/2}(bc-ad)}\right)$$

input
```
Integrate[x^13/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((x^2*(-3*b*c - 3*a*d + b*d*x^4))/(b^2*d^2) + (3*a^(5/2)*ArcTan[(Sqrt[b]*x
^2)/Sqrt[a]])/(b^(5/2)*(-(b*c) + a*d)) + (3*c^(5/2)*ArcTan[(Sqrt[d]*x^2)/S
qrt[c]])/(d^(5/2)*(b*c - a*d)))/6
```

3.773.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {965, 381, 27, 444, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^{12}}{(bx^4+a)(dx^4+c)}dx^2$$

$$\Big\downarrow \text{381}$$

$$\frac{1}{2}\left(\frac{x^6}{3bd}-\frac{\int \frac{3x^4((bc+ad)x^4+ac)}{(bx^4+a)(dx^4+c)}dx^2}{3bd}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{x^6}{3bd}-\frac{\int \frac{x^4((bc+ad)x^4+ac)}{(bx^4+a)(dx^4+c)}dx^2}{bd}\right)$$

$$\Big\downarrow \text{444}$$

$$\frac{1}{2}\left(\frac{x^6}{3bd}-\frac{\frac{x^2(ad+bc)}{bd}-\frac{\int \frac{\left(b^2c^2+abdc+a^2d^2\right)x^4+ac(bc+ad)}{\left(bx^4+a\right)\left(dx^4+c\right)}dx^2}{bd}}{bd}\right)$$

$$\Big\downarrow \text{397}$$

$$\frac{1}{2}\left(\frac{x^6}{3bd}-\frac{\frac{x^2(ad+bc)}{bd}-\frac{\frac{b^2c^3\int \frac{1}{dx^4+c}dx^2}{bc-ad}-\frac{a^3d^2\int \frac{1}{bx^4+a}dx^2}{bc-ad}}{bd}}{bd}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{x^6}{3bd}-\frac{\frac{x^2(ad+bc)}{bd}-\frac{\frac{b^2c^{5/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}-\frac{a^{5/2}d^2\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}}{bd}}{bd}\right)$$

input

```
Int[x^13/((a + b*x^4)*(c + d*x^4)),x]
```

output

```
(x^6/(3*b*d) - (((b*c + a*d)*x^2)/(b*d) - (-((a^(5/2)*d^2*ArcTan[(Sqrt[b]*
x^2)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d))) + (b^2*c^(5/2)*ArcTan[(Sqrt[d]*x^2)/
Sqrt[c]])/(Sqrt[d]*(b*c - a*d)))/(b*d))/(b*d))/2
```

3.773.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 381

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397

```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.773.4 Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

method	result
default	$-\frac{-\frac{bd\,x^{6}}{6}+\frac{(ad+bc)x^{2}}{2}}{b^{2}d^{2}}+\frac{a^{3}\arctan\left(\frac{b\,x^{2}}{\sqrt{ab}}\right)}{2b^{2}(ad-bc)\sqrt{ab}}-\frac{c^{3}\arctan\left(\frac{d\,x^{2}}{\sqrt{cd}}\right)}{2d^{2}(ad-bc)\sqrt{cd}}$
risch	$\frac{x^{6}}{6bd}-\frac{a\,x^{2}}{2b^{2}d}-\frac{c\,x^{2}}{2b\,d^{2}}+\frac{\sqrt{-cd}\,c^{2}\ln\left(\left(-a^{6}d^{8}+a\,d^{3}c^{5}b^{5}\right)x^{2}+(-cd)^{\frac{3}{2}}a\,b^{5}c^{4}d+(-cd)^{\frac{3}{2}}b^{6}c^{5}+a^{6}d^{7}\sqrt{-cd}+b^{6}c^{6}\sqrt{-cd}\,d\right)}{4d^{3}(ad-bc)}-\sqrt{-cd}\,c$

input
```
int(x^13/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2/d^2*(-1/6*b*d*x^6+1/2*(a*d+b*c)*x^2)+1/2*a^3/b^2/(a*d-b*c)/(a*b)^(1
/2)*arctan(b*x^2/(a*b)^(1/2))-1/2*c^3/d^2/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x
^2/(c*d)^(1/2))
```

3.773.5 Fricas [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 576, normalized size of antiderivative = 5.14

$$\int \frac{x^{13}}{(a+bx^{4})\,(c+dx^{4})}\,dx$$

$$= \left[\frac{2\,(b^{2}cd-abd^{2})x^{6}-3\,a^{2}d^{2}\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{4}+2\,bx^{2}\sqrt{-\frac{a}{b}}-a}{bx^{4}+a}\right)-3\,b^{2}c^{2}\sqrt{-\frac{c}{d}}\log\left(\frac{dx^{4}-2\,dx^{2}\sqrt{-\frac{c}{d}}-c}{dx^{4}+c}\right)-6\,(b^{2}c^{2}-a}{12\,(b^{3}cd^{2}-ab^{2}d^{3})}\right.$$

input
```
integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 3*a^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x
^2*sqrt(-a/b) - a)/(b*x^4 + a)) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^
2*sqrt(-c/d) - c)/(d*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a
*b^2*d^3), 1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 6*a^2*d^2*sqrt(a/b)*arctan(b*
x^2*sqrt(a/b)/a) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) -
c)/(d*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/12
*(2*(b^2*c*d - a*b*d^2)*x^6 + 6*b^2*c^2*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c
) - 3*a^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a))
 - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/6*((b^2*c*d - a*b
*d^2)*x^6 - 3*a^2*d^2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + 3*b^2*c^2*sqrt
(c/d)*arctan(d*x^2*sqrt(c/d)/c) - 3*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 -
a*b^2*d^3)]
```

3.773.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**13/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.773.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a^3\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(b^3c-ab^2d)\sqrt{ab}} + \frac{c^3\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bcd^2-ad^3)\sqrt{cd}} + \frac{bdx^6-3\,(bc+ad)x^2}{6\,b^2d^2}$$

input
```
integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/2*a^3*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b)) + 1/2*c^3*a
rctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt(c*d)) + 1/6*(b*d*x^6 - 3*(b
*c + a*d)*x^2)/(b^2*d^2)
```

3.773.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(b^3c-ab^2d)\sqrt{ab}} + \frac{c^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bcd^2-ad^3)\sqrt{cd}} + \frac{b^2d^2x^6-3\,b^2cdx^2-3\,abd^2x^2}{6\,b^3d^3}$$

input
```
integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*a^3*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b)) + 1/2*c^3*a
rctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt(c*d)) + 1/6*(b^2*d^2*x^6 -
3*b^2*c*d*x^2 - 3*a*b*d^2*x^2)/(b^3*d^3)
```

3.773.9 Mupad [B] (verification not implemented)

Time = 10.77 (sec) , antiderivative size = 532, normalized size of antiderivative = 4.75

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx$$

$$= \frac{\ln\left(d^{10}\,{\left(-a^5\,b^5\right)}^{5/2}+b^{20}\,c^{10}\,\sqrt{-a^5\,b^5}-a^2\,b^{23}\,c^{10}\,x^2-a^{12}\,b^{13}\,d^{10}\,x^2+2\,b^{10}\,c^5\,d^5\,{\left(-a^5\,b^5\right)}^{3/2}+2\,a^7\,b^{18}\,c^5\,d^5\right.}{4\,b^6\,c-4\,a\,b^5\,d}$$

$$-\frac{\ln\left(d^{10}\,{\left(-a^5\,b^5\right)}^{5/2}+b^{20}\,c^{10}\,\sqrt{-a^5\,b^5}+a^2\,b^{23}\,c^{10}\,x^2+a^{12}\,b^{13}\,d^{10}\,x^2+2\,b^{10}\,c^5\,d^5\,{\left(-a^5\,b^5\right)}^{3/2}-2\,a^7\,b^{18}\,c^5\right.}{4\,\left(b^6\,c-a\,b^5\,d\right)}$$

$$-\frac{\ln\left(b^{10}\,{\left(-c^5\,d^5\right)}^{5/2}+a^{10}\,d^{20}\,\sqrt{-c^5\,d^5}+a^{10}\,c^2\,d^{23}\,x^2+b^{10}\,c^{12}\,d^{13}\,x^2+2\,a^5\,b^5\,d^{10}\,{\left(-c^5\,d^5\right)}^{3/2}-2\,a^5\,b^5\,c^7\,d\right.}{4\,\left(a\,d^6-b\,c\,d^5\right)}$$

$$+\frac{\ln\left(b^{10}\,{\left(-c^5\,d^5\right)}^{5/2}+a^{10}\,d^{20}\,\sqrt{-c^5\,d^5}-a^{10}\,c^2\,d^{23}\,x^2-b^{10}\,c^{12}\,d^{13}\,x^2+2\,a^5\,b^5\,d^{10}\,{\left(-c^5\,d^5\right)}^{3/2}+2\,a^5\,b^5\,c^7\,d\right.}{4\,a\,d^6-4\,b\,c\,d^5}$$

$$+\frac{x^6}{6\,b\,d}-\frac{x^2\,\left(a\,d+b\,c\right)}{2\,b^2\,d^2}$$

input
```
int(x^13/((a + b*x^4)*(c + d*x^4)),x)
```

output

```
(log(d^10*(-a^5*b^5)^(5/2) + b^20*c^10*(-a^5*b^5)^(1/2) - a^2*b^23*c^10*x^
2 - a^12*b^13*d^10*x^2 + 2*b^10*c^5*d^5*(-a^5*b^5)^(3/2) + 2*a^7*b^18*c^5*
d^5*x^2)*(-a^5*b^5)^(1/2))/(4*b^6*c - 4*a*b^5*d) - (log(d^10*(-a^5*b^5)^(5
/2) + b^20*c^10*(-a^5*b^5)^(1/2) + a^2*b^23*c^10*x^2 + a^12*b^13*d^10*x^2
+ 2*b^10*c^5*d^5*(-a^5*b^5)^(3/2) - 2*a^7*b^18*c^5*d^5*x^2)*(-a^5*b^5)^(1/
2))/(4*(b^6*c - a*b^5*d)) - (log(b^10*(-c^5*d^5)^(5/2) + a^10*d^20*(-c^5*d
^5)^(1/2) + a^10*c^2*d^23*x^2 + b^10*c^12*d^13*x^2 + 2*a^5*b^5*d^10*(-c^5*
d^5)^(3/2) - 2*a^5*b^5*c^7*d^18*x^2)*(-c^5*d^5)^(1/2))/(4*(a*d^6 - b*c*d^5
)) + (log(b^10*(-c^5*d^5)^(5/2) + a^10*d^20*(-c^5*d^5)^(1/2) - a^10*c^2*d^
23*x^2 - b^10*c^12*d^13*x^2 + 2*a^5*b^5*d^10*(-c^5*d^5)^(3/2) + 2*a^5*b^5*
c^7*d^18*x^2)*(-c^5*d^5)^(1/2))/(4*a*d^6 - 4*b*c*d^5) + x^6/(6*b*d) - (x^2
*(a*d + b*c))/(2*b^2*d^2)
```

3.773. $\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx$

3.774 $\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx$

3.774.1 Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \frac{x^2}{2bd} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)} - \frac{c^{3/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}$$

output
```
1/2*x^2/b/d+1/2*a^(3/2)*arctan(x^2*b^(1/2)/a^(1/2))/b^(3/2)/(-a*d+b*c)-1/2
*c^(3/2)*arctan(x^2*d^(1/2)/c^(1/2))/d^(3/2)/(-a*d+b*c)
```

3.774.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \frac{\left(-\frac{a}{b}+\frac{c}{d}\right)x^2 + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{b^{3/2}} - \frac{c^{3/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{d^{3/2}}}{2bc-2ad}$$

input
```
Integrate[x^9/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((-(a/b) + c/d)*x^2 + (a^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/b^(3/2) - (c
^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/d^(3/2))/(2*b*c - 2*a*d)
```

3.774.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {965, 381, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx$$

$$\Big\downarrow \; 965$$

$$\frac{1}{2}\int \frac{x^8}{(bx^4+a)(dx^4+c)}dx^2$$

$$\Big\downarrow \; 381$$

$$\frac{1}{2}\left(\frac{x^2}{bd}-\frac{\int \frac{(bc+ad)x^4+ac}{(bx^4+a)(dx^4+c)}dx^2}{bd}\right)$$

$$\Big\downarrow \; 397$$

$$\frac{1}{2}\left(\frac{x^2}{bd}-\frac{\frac{bc^2\int \frac{1}{dx^4+c}dx^2}{bc-ad}-\frac{a^2d\int \frac{1}{bx^4+a}dx^2}{bc-ad}}{bd}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}\left(\frac{x^2}{bd}-\frac{\frac{bc^{3/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}-\frac{a^{3/2}d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}}{bd}\right)$$

input
```
Int[x^9/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(x^2/(b*d) - (-((a^(3/2)*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*(b*c -
a*d))) + (b*c^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d)))/
(b*d))/2
```

3.774.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.774.4 Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

method	result
default	$\frac{x^{2}}{2 b d}-\frac{a^{2} \arctan \left(\frac{b \,x^{2}}{\sqrt{a b}}\right)}{2 \left(a d -b c \right) b \sqrt{a b}}+\frac{c^{2} \arctan \left(\frac{d \,x^{2}}{\sqrt{c d}}\right)}{2 \left(a d -b c \right) d \sqrt{c d}}$
risch	$\frac{x^{2}}{2 b d}+\frac{\sqrt{-a b}\, a \ln \left(\left(b^{3} c \,d^{3} a^{3}-b^{6} c^{4}\right) x^{2}+\left(-a b \right)^{\frac{3}{2}} a^{3} d^{4}+\left(-a b \right)^{\frac{3}{2}} a^{2} b c \,d^{3}+a^{4} \sqrt{-a b}\, d^{4} b +b^{5} c^{4} \sqrt{-a b}\right)}{4 b^{2} \left(a d -b c \right)}-\sqrt{-a b}\, a \ln \Big(\left(b^{3} c \,d^{3} a^{3}-b^{6} c^{4}\right) x$

input
```
int(x^9/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/b/d-1/2*a^2/(a*d-b*c)/b/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))+1/2*
c^2/(a*d-b*c)/d/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

3.774.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 416, normalized size of antiderivative = 4.52

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \left[-\frac{ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4-2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)+bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4+2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)-2\,(bc-ad)x^2}{4\,(b^2cd-abd^2)}, \frac{2\,ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{}}{a}\right.}{} \right.$$
$$-\frac{2\,bc\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right)+ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4-2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)-2\,(bc-ad)x^2}{4\,(b^2cd-abd^2)}, \frac{ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)-b}{2\,(}$$

input
```
integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[-1/4*(a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) +
b*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) - 2*(b*c
- a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/4*(2*a*d*sqrt(a/b)*arctan(b*x^2*sqrt(a/
b)/a) - b*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) +
 2*(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), -1/4*(2*b*c*sqrt(c/d)*arctan(d*x^
2*sqrt(c/d)/c) + a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^
4 + a)) - 2*(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/2*(a*d*sqrt(a/b)*arcta
n(b*x^2*sqrt(a/b)/a) - b*c*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) + (b*c - a*
d)*x^2)/(b^2*c*d - a*b*d^2)]
```

3.774.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**9/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.774.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \frac{a^2\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(b^2c-abd)\sqrt{ab}} - \frac{c^2\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bcd-ad^2)\sqrt{cd}} + \frac{x^2}{2\,bd}$$

input
```
integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/2*a^2*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - 1/2*c^2*arct
an(d*x^2/sqrt(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + 1/2*x^2/(b*d)
```

3.774.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \frac{a^2\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(b^2c-abd)\sqrt{ab}} - \frac{c^2\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bcd-ad^2)\sqrt{cd}} + \frac{x^2}{2\,bd}$$

input
```
integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*a^2*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - 1/2*c^2*arct
an(d*x^2/sqrt(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + 1/2*x^2/(b*d)
```

3.774.9 Mupad [B] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 518, normalized size of antiderivative = 5.63

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx$$

$$= \frac{\ln\left(b^9\,c^6\,\sqrt{-a^3\,b^3} - a^3\,d^6\,{\left(-a^3\,b^3\right)}^{3/2} + a\,b^{11}\,c^6\,x^2 + a^7\,b^5\,d^6\,x^2 + 2\,b^3\,c^3\,d^3\,{\left(-a^3\,b^3\right)}^{3/2} - 2\,a^4\,b^8\,c^3\,d^3\,x^2\right)\,\sqrt{}}{4\,b^4\,c - 4\,a\,b^3\,d}$$

$$- \frac{\ln\left(a^3\,d^6\,{\left(-a^3\,b^3\right)}^{3/2} - b^9\,c^6\,\sqrt{-a^3\,b^3} + a\,b^{11}\,c^6\,x^2 + a^7\,b^5\,d^6\,x^2 - 2\,b^3\,c^3\,d^3\,{\left(-a^3\,b^3\right)}^{3/2} - 2\,a^4\,b^8\,c^3\,d^3\,x^2\right)}{4\,\left(b^4\,c - a\,b^3\,d\right)}$$

$$- \frac{\ln\left(b^6\,c^3\,{\left(-c^3\,d^3\right)}^{3/2} - a^6\,d^9\,\sqrt{-c^3\,d^3} + a^6\,c\,d^{11}\,x^2 + b^6\,c^7\,d^5\,x^2 - 2\,a^3\,b^3\,d^3\,{\left(-c^3\,d^3\right)}^{3/2} - 2\,a^3\,b^3\,c^4\,d^8\,x^2\right)}{4\,\left(a\,d^4 - b\,c\,d^3\right)}$$

$$+ \frac{\ln\left(a^6\,d^9\,\sqrt{-c^3\,d^3} - b^6\,c^3\,{\left(-c^3\,d^3\right)}^{3/2} + a^6\,c\,d^{11}\,x^2 + b^6\,c^7\,d^5\,x^2 + 2\,a^3\,b^3\,d^3\,{\left(-c^3\,d^3\right)}^{3/2} - 2\,a^3\,b^3\,c^4\,d^8\,x^2\right)}{4\,a\,d^4 - 4\,b\,c\,d^3}$$

$$+ \frac{x^2}{2\,b\,d}$$

input
```
int(x^9/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
(log(b^9*c^6*(-a^3*b^3)^(1/2) - a^3*d^6*(-a^3*b^3)^(3/2) + a*b^11*c^6*x^2
+ a^7*b^5*d^6*x^2 + 2*b^3*c^3*d^3*(-a^3*b^3)^(3/2) - 2*a^4*b^8*c^3*d^3*x^2
)*(-a^3*b^3)^(1/2))/(4*b^4*c - 4*a*b^3*d) - (log(a^3*d^6*(-a^3*b^3)^(3/2)
- b^9*c^6*(-a^3*b^3)^(1/2) + a*b^11*c^6*x^2 + a^7*b^5*d^6*x^2 - 2*b^3*c^3*
d^3*(-a^3*b^3)^(3/2) - 2*a^4*b^8*c^3*d^3*x^2)*(-a^3*b^3)^(1/2))/(4*(b^4*c
- a*b^3*d)) - (log(b^6*c^3*(-c^3*d^3)^(3/2) - a^6*d^9*(-c^3*d^3)^(1/2) + a
^6*c*d^11*x^2 + b^6*c^7*d^5*x^2 - 2*a^3*b^3*d^3*(-c^3*d^3)^(3/2) - 2*a^3*b
^3*c^4*d^8*x^2)*(-c^3*d^3)^(1/2))/(4*(a*d^4 - b*c*d^3)) + (log(a^6*d^9*(-c
^3*d^3)^(1/2) - b^6*c^3*(-c^3*d^3)^(3/2) + a^6*c*d^11*x^2 + b^6*c^7*d^5*x^
2 + 2*a^3*b^3*d^3*(-c^3*d^3)^(3/2) - 2*a^3*b^3*c^4*d^8*x^2)*(-c^3*d^3)^(1/
2))/(4*a*d^4 - 4*b*c*d^3) + x^2/(2*b*d)
```

3.775 $\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx$

3.775.1 Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx=-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)}$$

output
```
-1/2*arctan(x^2*b^(1/2)/a^(1/2))*a^(1/2)/(-a*d+b*c)/b^(1/2)+1/2*arctan(x^2
*d^(1/2)/c^(1/2))*c^(1/2)/(-a*d+b*c)/d^(1/2)
```

3.775.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx=\frac{-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{d}}}{2bc-2ad}$$

input
```
Integrate[x^5/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-((Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[b]) + (Sqrt[c]*ArcTan[(Sqr
t[d]*x^2)/Sqrt[c]])/Sqrt[d])/(2*b*c - 2*a*d)
```

3.775.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {965, 383, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^4+a)(dx^4+c)}dx^2$$

$$\Big\downarrow \text{383}$$

$$\frac{1}{2}\left(\frac{c\int \frac{1}{dx^4+c}dx^2}{bc-ad}-\frac{a\int \frac{1}{bx^4+a}dx^2}{bc-ad}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}\right)$$

input
```
Int[x^5/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-((Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d))) + (Sqrt[
c]*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d)))/2
```

3.775.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.775.4 Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

method	result
default	$\frac{a\arctan\left(\frac{b\,x^{2}}{\sqrt{ab}}\right)}{2(ad-bc)\sqrt{ab}}-\frac{c\arctan\left(\frac{d\,x^{2}}{\sqrt{cd}}\right)}{2(ad-bc)\sqrt{cd}}$
risch	$\frac{\sqrt{-ab}\,\ln\left(\left(-a\,b^{3}cd+b^{4}c^{2}\right)x^{2}+\left(-ab\right)^{\frac{3}{2}}a\,d^{2}+\left(-ab\right)^{\frac{3}{2}}bcd+a^{2}\sqrt{-ab}\,d^{2}b+b^{3}c^{2}\sqrt{-ab}\right)}{4b(ad-bc)}-\frac{\sqrt{-ab}\,\ln\left(\left(-a\,b^{3}cd+b^{4}c^{2}\right)x^{2}-\left(-ab\right)^{\frac{3}{2}}a\,d^{2}-\right.}{4b(ad-}$

input
```
int(x^5/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*a/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))-1/2*c/(a*d-b*c)/(c*d
)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

3.775.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 4.11

$$\int\frac{x^5}{(a+bx^4)(c+dx^4)}\,dx=\left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)}{4\,(bc-ad)},\right.$$
$$-\frac{2\,\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)}{4\,(bc-ad)},\frac{2\,\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right)-\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2\,bx^2\sqrt{-}}{bx^4+a}\right.}{4\,(bc-ad)}$$
$$\left.-\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right)}{2\,(bc-ad)}\right]$$

input
```
integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) + sqrt
(-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)))/(b*c - a*d), -1/
4*(2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + sqrt(-c/d)*log((d*x^4 - 2*d*x^2
*sqrt(-c/d) - c)/(d*x^4 + c)))/(b*c - a*d), 1/4*(2*sqrt(c/d)*arctan(d*x^2*
sqrt(c/d)/c) - sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)
))/(b*c - a*d), -1/2*(sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) - sqrt(c/d)*arct
an(d*x^2*sqrt(c/d)/c))/(b*c - a*d)]
```

3.775.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**5/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.775.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)} + \frac{c\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc-ad)\sqrt{cd}}$$

input
```
integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/2*a*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + 1/2*c*arctan(d*x^
2/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

3.775.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)}+\frac{c\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc-ad)\sqrt{cd}}$$

input
```
integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*a*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + 1/2*c*arctan(d*x^
2/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

3.775.9 Mupad [B] (verification not implemented)

Time = 10.43 (sec) , antiderivative size = 379, normalized size of antiderivative = 4.80

$$\begin{aligned}\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx &= \frac{\ln\left(d^2\,(-a\,b)^{5/2}+b^4\,c^2\,\sqrt{-a\,b}-b^5\,c^2\,x^2+2\,b^2\,c\,d\,(-a\,b)^{3/2}-a^2\,b^3\,d^2\,x^2+2\,a\,b^4\,c\,d\,x^2\right)\sqrt{-a\,b}}{4\,b^2\,c-4\,a\,b\,d}\\ &\quad-\frac{\ln\left(d^2\,(-a\,b)^{5/2}+b^4\,c^2\,\sqrt{-a\,b}+b^5\,c^2\,x^2+2\,b^2\,c\,d\,(-a\,b)^{3/2}+a^2\,b^3\,d^2\,x^2-2\,a\,b^4\,c\,d\,x^2\right)\sqrt{-a\,b}}{4\,\left(b^2\,c-a\,b\,d\right)}\\ &\quad-\frac{\ln\left(b^2\,(-c\,d)^{5/2}+a^2\,d^4\,\sqrt{-c\,d}+a^2\,d^5\,x^2+2\,a\,b\,d^2\,(-c\,d)^{3/2}+b^2\,c^2\,d^3\,x^2-2\,a\,b\,c\,d^4\,x^2\right)\sqrt{-c\,d}}{4\,\left(a\,d^2-b\,c\,d\right)}\\ &\quad+\frac{\ln\left(b^2\,(-c\,d)^{5/2}+a^2\,d^4\,\sqrt{-c\,d}-a^2\,d^5\,x^2+2\,a\,b\,d^2\,(-c\,d)^{3/2}-b^2\,c^2\,d^3\,x^2+2\,a\,b\,c\,d^4\,x^2\right)\sqrt{-c\,d}}{4\,a\,d^2-4\,b\,c\,d}\end{aligned}$$

input
```
int(x^5/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
(log(d^2*(-a*b)^(5/2) + b^4*c^2*(-a*b)^(1/2) - b^5*c^2*x^2 + 2*b^2*c*d*(-a
*b)^(3/2) - a^2*b^3*d^2*x^2 + 2*a*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*b^2*c - 4*
a*b*d) - (log(d^2*(-a*b)^(5/2) + b^4*c^2*(-a*b)^(1/2) + b^5*c^2*x^2 + 2*b^
2*c*d*(-a*b)^(3/2) + a^2*b^3*d^2*x^2 - 2*a*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*(
b^2*c - a*b*d)) - (log(b^2*(-c*d)^(5/2) + a^2*d^4*(-c*d)^(1/2) + a^2*d^5*x
^2 + 2*a*b*d^2*(-c*d)^(3/2) + b^2*c^2*d^3*x^2 - 2*a*b*c*d^4*x^2)*(-c*d)^(1
/2))/(4*(a*d^2 - b*c*d)) + (log(b^2*(-c*d)^(5/2) + a^2*d^4*(-c*d)^(1/2) -
a^2*d^5*x^2 + 2*a*b*d^2*(-c*d)^(3/2) - b^2*c^2*d^3*x^2 + 2*a*b*c*d^4*x^2)*
(-c*d)^(1/2))/(4*a*d^2 - 4*b*c*d)
```

3.776 $\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx$

3.776.1 Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{c}(bc-ad)}$$

output

```
1/2*arctan(x^2*b^(1/2)/a^(1/2))*b^(1/2)/(-a*d+b*c)/a^(1/2)-1/2*arctan(x^2*
d^(1/2)/c^(1/2))*d^(1/2)/(-a*d+b*c)/c^(1/2)
```

3.776.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx = \frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{c}}}{2bc-2ad}$$

input

```
Integrate[x/((a + b*x^4)*(c + d*x^4)),x]
```

output

```
((Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d
]*x^2)/Sqrt[c]])/Sqrt[c])/(2*b*c - 2*a*d)
```

3.776.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {965, 303, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx$$

$$\Big\downarrow \; 965$$

$$\frac{1}{2}\int \frac{1}{(bx^4+a)(dx^4+c)}dx^2$$

$$\Big\downarrow \; 303$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{bx^4+a}dx^2}{bc-ad}-\frac{d\int \frac{1}{dx^4+c}dx^2}{bc-ad}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}\left(\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}\right)$$

input
```
Int[x/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*
ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/2
```

3.776.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.776.4 Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

method	result
default	$-\frac{b\arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2(ad-bc)\sqrt{ab}}+\frac{d\arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2(ad-bc)\sqrt{cd}}$
risch	$\frac{\sqrt{-cd}\,\ln\left(\left(-ac\,d^3+b\,c^2d^2\right)x^2+\left(-cd\right)^{\frac{3}{2}}ad+\left(-cd\right)^{\frac{3}{2}}bc+2\sqrt{-cd}\,b\,c^2d\right)}{4c(ad-bc)}-\frac{\sqrt{-cd}\,\ln\left(\left(-ac\,d^3+b\,c^2d^2\right)x^2-\left(-cd\right)^{\frac{3}{2}}ad-\left(-cd\right)^{\frac{3}{2}}bc-2\sqrt{-}\right)}{4c(ad-bc)}$

input
```
int(x/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))+1/2*d/(a*d-b*c)/(c*
d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

3.776.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 4.11

$$\int \frac{x}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

$$=\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)}{4\,(bc-ad)},\frac{2\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)-\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax}{bx}\right.}{4\,(bc-ad)}\right.$$

$$-\frac{2\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)}{4\,(bc-ad)},$$

$$\left.-\frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right)-\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)}{2\,(bc-ad)}\right]$$

input
```
integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a)) + sqrt
(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)))/(b*c - a*d), 1/4
*(2*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)) - sqrt(-b/a)*log((b*x^4 - 2*a*x^
2*sqrt(-b/a) - a)/(b*x^4 + a)))/(b*c - a*d), -1/4*(2*sqrt(b/a)*arctan(a*sq
rt(b/a)/(b*x^2)) + sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4
+ c)))/(b*c - a*d), -1/2*(sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - sqrt(d/c
)*arctan(c*sqrt(d/c)/(d*x^2)))/(b*c - a*d)]
```

3.776.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.776.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx = \frac{b\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)} - \frac{d\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc-ad)\sqrt{cd}}$$

input
```
integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/2*b*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - 1/2*d*arctan(d*x^2
/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

3.776.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{x}{(a+bx^4)\,(c+dx^4)}\,dx = \frac{b\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)} - \frac{d\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc-ad)\sqrt{cd}}$$

input

```
integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output

```
1/2*b*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - 1/2*d*arctan(d*x^2
/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

3.776.9 Mupad [B] (verification not implemented)

Time = 10.41 (sec) , antiderivative size = 399, normalized size of antiderivative = 5.05

$$\begin{aligned}\int \frac{x}{(a+bx^4)\,(c+dx^4)}\,dx
&= \frac{\ln\left(a^2\,d^2\,(-a\,b)^{5/2}+b^2\,c^2\,(-a\,b)^{5/2}+2\,c\,d\,(-a\,b)^{7/2}-a^2\,b^5\,c^2\,x^2-a^4\,b^3\,d^2\,x^2+2\,a^3\,b^4\,c\,d\,x^2\right)\,\sqrt{-a\,b}}{4\,a^2\,d-4\,a\,b\,c}\\
&\quad-\frac{\ln\left(a^2\,d^2\,(-a\,b)^{5/2}+b^2\,c^2\,(-a\,b)^{5/2}+2\,c\,d\,(-a\,b)^{7/2}+a^2\,b^5\,c^2\,x^2+a^4\,b^3\,d^2\,x^2-2\,a^3\,b^4\,c\,d\,x^2\right)\,\sqrt{-a\,b}}{4\,\left(a^2\,d-a\,b\,c\right)}\\
&\quad-\frac{\ln\left(a^2\,d^2\,(-c\,d)^{5/2}+b^2\,c^2\,(-c\,d)^{5/2}+2\,a\,b\,(-c\,d)^{7/2}+a^2\,c^2\,d^5\,x^2+b^2\,c^4\,d^3\,x^2-2\,a\,b\,c^3\,d^4\,x^2\right)\,\sqrt{-c\,d}}{4\,\left(b\,c^2-a\,c\,d\right)}\\
&\quad+\frac{\ln\left(a^2\,d^2\,(-c\,d)^{5/2}+b^2\,c^2\,(-c\,d)^{5/2}+2\,a\,b\,(-c\,d)^{7/2}-a^2\,c^2\,d^5\,x^2-b^2\,c^4\,d^3\,x^2+2\,a\,b\,c^3\,d^4\,x^2\right)\,\sqrt{-c\,d}}{4\,b\,c^2-4\,a\,c\,d}\end{aligned}$$

input

```
int(x/((a + b*x^4)*(c + d*x^4)),x)
```

output

```
(log(a^2*d^2*(-a*b)^(5/2) + b^2*c^2*(-a*b)^(5/2) + 2*c*d*(-a*b)^(7/2) - a^
2*b^5*c^2*x^2 - a^4*b^3*d^2*x^2 + 2*a^3*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*a^2*
d - 4*a*b*c) - (log(a^2*d^2*(-a*b)^(5/2) + b^2*c^2*(-a*b)^(5/2) + 2*c*d*(-
a*b)^(7/2) + a^2*b^5*c^2*x^2 + a^4*b^3*d^2*x^2 - 2*a^3*b^4*c*d*x^2)*(-a*b)
^(1/2))/(4*(a^2*d - a*b*c)) - (log(a^2*d^2*(-c*d)^(5/2) + b^2*c^2*(-c*d)^(
5/2) + 2*a*b*(-c*d)^(7/2) + a^2*c^2*d^5*x^2 + b^2*c^4*d^3*x^2 - 2*a*b*c^3*
d^4*x^2)*(-c*d)^(1/2))/(4*(b*c^2 - a*c*d)) + (log(a^2*d^2*(-c*d)^(5/2) + b
^2*c^2*(-c*d)^(5/2) + 2*a*b*(-c*d)^(7/2) - a^2*c^2*d^5*x^2 - b^2*c^4*d^3*x
^2 + 2*a*b*c^3*d^4*x^2)*(-c*d)^(1/2))/(4*b*c^2 - 4*a*c*d)
```

3.777 $\int \frac{1}{x^3(a+bx^4)(c+dx^4)}\,dx$

3.777.1 Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{1}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=-\frac{1}{2acx^2}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)}+\frac{d^{3/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}$$

output
```
-1/2/a/c/x^2-1/2*b^(3/2)*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d+b*c)+1/
2*d^(3/2)*arctan(x^2*d^(1/2)/c^(1/2))/c^(3/2)/(-a*d+b*c)
```

3.777.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.84

$$\int \frac{1}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$
$$=\frac{\frac{b}{a}-\frac{d}{c}-\frac{b^{3/2}x^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/2}}-\frac{b^{3/2}x^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/2}}+\frac{d^{3/2}x^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{3/2}}+\frac{d^{3/2}x^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{3/2}}}{2(-bc+ad)x^2}$$

input
```
Integrate[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(b/a - d/c - (b^(3/2)*x^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2)
 - (b^(3/2)*x^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2) + (d^(3/2
)*x^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2) + (d^(3/2)*x^2*ArcT
an[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2))/(2*(-(b*c) + a*d)*x^2)
```

3.777.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {965, 382, 25, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^4) (c+dx^4)} \, dx$$

$$\Big\downarrow 965$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^4+a) (dx^4+c)}dx^2$$

$$\Big\downarrow 382$$

$$\frac{1}{2} \left(\frac{\int -\frac{bdx^4+bc+ad}{(bx^4+a)(dx^4+c)}dx^2}{ac}-\frac{1}{acx^2}\right)$$

$$\Big\downarrow 25$$

$$\frac{1}{2} \left(-\frac{\int \frac{bdx^4+bc+ad}{(bx^4+a)(dx^4+c)}dx^2}{ac}-\frac{1}{acx^2}\right)$$

$$\Big\downarrow 397$$

$$\frac{1}{2} \left(-\frac{\frac{b^2c\int \frac{1}{bx^4+a}dx^2}{bc-ad}-\frac{ad^2\int \frac{1}{dx^4+c}dx^2}{bc-ad}}{ac}-\frac{1}{acx^2}\right)$$

$$\Big\downarrow 218$$

$$\frac{1}{2} \left(-\frac{\frac{b^{3/2}c\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{ad^{3/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{ac}-\frac{1}{acx^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-(1/(a*c*x^2)) - ((b^(3/2)*c*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*(b*c
 - a*d)) - (a*d^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
)/(a*c))/2
```

3.777. $\int \frac{1}{x^3(a+bx^4)(c+dx^4)} \, dx$

3.777.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.777.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

method	result
default	$\frac{b^{2} \arctan\left(\frac{b\,x^{2}}{\sqrt{a b}}\right)}{2 \left(a d -b c \right) a \sqrt{a b}}-\frac{1}{2 a c\,x^{2}}-\frac{d^{2} \arctan\left(\frac{d\,x^{2}}{\sqrt{c d}}\right)}{2 \left(a d -b c \right) c \sqrt{c d}}$
risch	$-\frac{1}{2 a c\,x^{2}}+\frac{\left(\sum_{\_R =\operatorname{RootOf}\left(\left(d^{2} a^{5}-2 a^{4} c b d +a^{3} b^{2} c^{2}\right) \_Z^{2}+b^{3}\right)}\_R \ln\left(\left(\left(-5 c^{3} a^{7} d^{4}+18 c^{4} a^{6} b\,d^{3}-26 a^{5} c^{5} b^{2} d^{2}+18 c^{6} a^{4} b^{3} d -5 c^{7} a^{3} b^{4}\right) \_\right.\right.\right.}{}$

input
```
int(1/x^3/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2/(a*d-b*c)/a/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))-1/2/a/c/x^2-1/2*
d^2/(a*d-b*c)/c/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

3.777.5 Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.70

$$\int \frac{1}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \left[-\frac{bcx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4+2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+adx^2\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4-2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)+2\,bc-2\,ad}{4\,(abc^2-a^2cd)x^2},\right.$$

$$-\frac{2\,adx^2\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)+bcx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4+2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+2\,bc-2\,ad}{4\,(abc^2-a^2cd)x^2},\ 2\,bcx^2\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right)-$$

input
```
integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[-1/4*(b*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a)
) + a*d*x^2*sqrt(-d/c)*log((d*x^4 - 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) +
 2*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), -1/4*(2*a*d*x^2*sqrt(d/c)*arcta
n(c*sqrt(d/c)/(d*x^2)) + b*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a
) - a)/(b*x^4 + a)) + 2*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), 1/4*(2*b*c
*x^2*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - a*d*x^2*sqrt(-d/c)*log((d*x^4
 - 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) - 2*b*c + 2*a*d)/((a*b*c^2 - a^2*c
*d)*x^2), 1/2*(b*c*x^2*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - a*d*x^2*sqr
t(d/c)*arctan(c*sqrt(d/c)/(d*x^2)) - b*c + a*d)/((a*b*c^2 - a^2*c*d)*x^2)]
```

3.777.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^4) (c+dx^4)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.777.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 (a+bx^4) (c+dx^4)} \, dx = -\frac{b^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(abc-a^2d)\sqrt{ab}} + \frac{d^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc^2-acd)\sqrt{cd}} - \frac{1}{2\,acx^2}$$

input
```
integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/2*b^2*arctan(b*x^2/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + 1/2*d^2*arc
tan(d*x^2/sqrt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/2/(a*c*x^2)
```

3.777.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 (a+bx^4) (c+dx^4)} \, dx = -\frac{b^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(abc-a^2d)\sqrt{ab}} + \frac{d^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc^2-acd)\sqrt{cd}} - \frac{1}{2\,acx^2}$$

input
```
integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*b^2*arctan(b*x^2/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + 1/2*d^2*arc
tan(d*x^2/sqrt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/2/(a*c*x^2)
```

3.777.9 Mupad [B] (verification not implemented)

Time = 10.19 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.85

$$\int \frac{1}{x^3 (a+bx^4) (c+dx^4)}\,dx = \frac{\ln\left(c^3\,x^2\,{\left(-a^3\,b^3\right)}^{3/2}-a^8\,b\,d^3+a^5\,b^4\,c^3+a^6\,d^3\,x^2\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{4\,a^4\,d-4\,a^3\,b\,c} - \frac{\ln\left(c^3\,x^2\,{\left(-a^3\,b^3\right)}^{3/2}+a^8\,b\,d^3-a^5\,b^4\,c^3+a^6\,d^3\,x^2\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{4\,\left(a^4\,d-a^3\,b\,c\right)} - \frac{1}{2\,a\,c\,x^2} - \frac{\ln\left(a^3\,x^2\,{\left(-c^3\,d^3\right)}^{3/2}+b^3\,c^8\,d-a^3\,c^5\,d^4+b^3\,c^6\,x^2\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{4\,\left(b\,c^4-a\,c^3\,d\right)} + \frac{\ln\left(a^3\,x^2\,{\left(-c^3\,d^3\right)}^{3/2}-b^3\,c^8\,d+a^3\,c^5\,d^4+b^3\,c^6\,x^2\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{4\,b\,c^4-4\,a\,c^3\,d}$$

input
```
int(1/(x^3*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
(log(c^3*x^2*(-a^3*b^3)^(3/2) - a^8*b*d^3 + a^5*b^4*c^3 + a^6*d^3*x^2*(-a^
3*b^3)^(1/2))*(-a^3*b^3)^(1/2))/(4*a^4*d - 4*a^3*b*c) - (log(c^3*x^2*(-a^3
*b^3)^(3/2) + a^8*b*d^3 - a^5*b^4*c^3 + a^6*d^3*x^2*(-a^3*b^3)^(1/2))*(-a^
3*b^3)^(1/2))/(4*(a^4*d - a^3*b*c)) - 1/(2*a*c*x^2) - (log(a^3*x^2*(-c^3*d
^3)^(3/2) + b^3*c^8*d - a^3*c^5*d^4 + b^3*c^6*x^2*(-c^3*d^3)^(1/2))*(-c^3*
d^3)^(1/2))/(4*(b*c^4 - a*c^3*d)) + (log(a^3*x^2*(-c^3*d^3)^(3/2) - b^3*c^
8*d + a^3*c^5*d^4 + b^3*c^6*x^2*(-c^3*d^3)^(1/2))*(-c^3*d^3)^(1/2))/(4*b*c
^4 - 4*a*c^3*d)
```

3.778 $\int \frac{1}{x^7(a+bx^4)(c+dx^4)}\,dx$

3.778.1 Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{1}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=-\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}-\frac{d^{5/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}$$

output
```
-1/6/a/c/x^6+1/2*(a*d+b*c)/a^2/c^2/x^2+1/2*b^(5/2)*arctan(x^2*b^(1/2)/a^(1
/2))/a^(5/2)/(-a*d+b*c)-1/2*d^(5/2)*arctan(x^2*d^(1/2)/c^(1/2))/c^(5/2)/(-
a*d+b*c)
```

3.778.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.72

$$\int \frac{1}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$
$$=\frac{\frac{b}{a}-\frac{d}{c}-\frac{3b^2x^4}{a^2}+\frac{3d^2x^4}{c^2}+\frac{3b^{5/2}x^6\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/2}}+\frac{3b^{5/2}x^6\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/2}}-\frac{3d^{5/2}x^6\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{5/2}}-}{6(-bc+ad)x^6}$$

input
```
Integrate[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(b/a - d/c - (3*b^2*x^4)/a^2 + (3*d^2*x^4)/c^2 + (3*b^(5/2)*x^6*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(5/2) + (3*b^(5/2)*x^6*ArcTan[1 + (Sqrt[2
]*b^(1/4)*x)/a^(1/4)])/a^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 - (Sqrt[2]*d^(1/4
)*x)/c^(1/4)])/c^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(
1/4)])/c^(5/2))/(6*(-(b*c) + a*d)*x^6)
```

3.778.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {965, 382, 27, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+b x^4\right) \left(c+d x^4\right)} \, dx$$

$$\Big\downarrow \color{blue}{965}$$

$$\frac{1}{2} \int \frac{1}{x^8 \left(b x^4+a\right) \left(d x^4+c\right)}dx^2$$

$$\Big\downarrow \color{blue}{382}$$

$$\frac{1}{2} \left(\frac{\int -\frac{3 \left(b d x^4+b c+a d\right)}{x^4 \left(b x^4+a\right) \left(d x^4+c\right)}dx^2}{3 a c}-\frac{1}{3 a c x^6}\right)$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{1}{2} \left(-\frac{\int \frac{b d x^4+b c+a d}{x^4 \left(b x^4+a\right) \left(d x^4+c\right)}dx^2}{a c}-\frac{1}{3 a c x^6}\right)$$

$$\Big\downarrow \color{blue}{445}$$

$$\frac{1}{2} \left(-\frac{-\frac{\int \frac{b d (b c+a d) x^4+b^2 c^2+a^2 d^2+a b c d}{\left(b x^4+a\right) \left(d x^4+c\right)}dx^2}{a c}-\frac{a d+b c}{a c x^2}}{a c}-\frac{1}{3 a c x^6}\right)$$

$$\Big\downarrow \color{blue}{397}$$

$$\frac{1}{2} \left(-\frac{-\frac{\frac{b^3 c^2 \int \frac{1}{b x^4+a}dx^2}{b c-a d}-\frac{a^2 d^3 \int \frac{1}{d x^4+c}dx^2}{b c-a d}}{a c}-\frac{a d+b c}{a c x^2}}{a c}-\frac{1}{3 a c x^6}\right)$$

$$\Big\downarrow 218$$

$$\frac{1}{2}\left(-\frac{-\frac{\frac{b^{5/2}c^2\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{a^2d^{5/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{ac}-\frac{ad+bc}{acx^2}}{ac}-\frac{1}{3acx^6}\right)$$

input
```
Int[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-1/3*1/(a*c*x^6) - (-((b*c + a*d)/(a*c*x^2)) - ((b^(5/2)*c^2*ArcTan[(Sqrt
[b]*x^2)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (a^2*d^(5/2)*ArcTan[(Sqrt[d]*x^
2)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(a*c))/(a*c))/2
```

3.778.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.778.4 Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

method	result
default	$-\frac{1}{6ac\,x^6}-\frac{-ad-bc}{2x^2a^2c^2}-\frac{b^3\arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2a^2(ad-bc)\sqrt{ab}}+\frac{d^3\arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2c^2(ad-bc)\sqrt{cd}}$
risch	$\frac{\frac{(ad+bc)x^4}{2a^2c^2}-\frac{1}{6ac}}{x^6}+\left(\sum_{\_R=\operatorname{RootOf}\left(\left(d^2c^5a^2-2ab\,c^6d+b^2c^7\right)\_Z^2+d^5\right)}\_R\ln\left(\left(\left(5c^5a^9d^4-18c^6a^8b\,d^3+26c^7a^7b^2d^2-18c^8a^6b^3d+5c^9a$

input
```
int(1/x^7/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/a/c/x^6-1/2*(-a*d-b*c)/x^2/a^2/c^2-1/2*b^3/a^2/(a*d-b*c)/(a*b)^(1/2)*
arctan(b*x^2/(a*b)^(1/2))+1/2*d^3/c^2/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(
c*d)^(1/2))
```

3.778.5 Fricas [A] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 592, normalized size of antiderivative = 5.29

$$\int \frac{1}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

$$= \left[-\frac{3\,b^2c^2x^6\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right) + 3\,a^2d^2x^6\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right) - 6\,(b^2c^2-a^2d^2)x^4 + 2\,abc}{12\,(a^2bc^3-a^3c^2d)x^6} \right.$$

$$-\frac{6\,b^2c^2x^6\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right) + 3\,a^2d^2x^6\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right) - 6\,(b^2c^2-a^2d^2)x^4 + 2\,abc^2 - 2\,a^2c}{12\,(a^2bc^3-a^3c^2d)x^6}$$

$$\left. -\frac{3\,b^2c^2x^6\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right) - 3\,a^2d^2x^6\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right) - 3\,(b^2c^2-a^2d^2)x^4 + abc^2 - a^2cd}{6\,(a^2bc^3-a^3c^2d)x^6} \right]$$

input

```
integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output

```
[-1/12*(3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x
^4 + a)) + 3*a^2*d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(
d*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c
^3 - a^3*c^2*d)*x^6), 1/12*(6*a^2*d^2*x^6*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*
x^2)) - 3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x
^4 + a)) + 6*(b^2*c^2 - a^2*d^2)*x^4 - 2*a*b*c^2 + 2*a^2*c*d)/((a^2*b*c^3
- a^3*c^2*d)*x^6), -1/12*(6*b^2*c^2*x^6*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^
2)) + 3*a^2*d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4
 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c^3 -
a^3*c^2*d)*x^6), -1/6*(3*b^2*c^2*x^6*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2))
 - 3*a^2*d^2*x^6*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)) - 3*(b^2*c^2 - a^2*
d^2)*x^4 + a*b*c^2 - a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^6)]
```

3.778.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x**7/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.778.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=\frac{b^3\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(a^2bc-a^3d\right)\sqrt{ab}}-\frac{d^3\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bc^3-ac^2d\right)\sqrt{cd}}+\frac{3\left(bc+ad\right)x^4-ac}{6\,a^2c^2x^6}$$

input
```
integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/2*b^3*arctan(b*x^2/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - 1/2*d^3*ar
ctan(d*x^2/sqrt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/6*(3*(b*c + a*d)*x
^4 - a*c)/(a^2*c^2*x^6)
```

3.778.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=\frac{b^3\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(a^2bc-a^3d\right)\sqrt{ab}}-\frac{d^3\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bc^3-ac^2d\right)\sqrt{cd}}+\frac{3\,bcx^4+3\,adx^4-ac}{6\,a^2c^2x^6}$$

input
```
integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*b^3*arctan(b*x^2/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - 1/2*d^3*ar
ctan(d*x^2/sqrt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/6*(3*b*c*x^4 + 3*a
*d*x^4 - a*c)/(a^2*c^2*x^6)
```

3.778.9 Mupad [B] (verification not implemented)

Time = 9.93 (sec) , antiderivative size = 535, normalized size of antiderivative = 4.78

$$\int \frac{1}{x^7 (a+bx^4)(c+dx^4)}\,dx$$

$$= \frac{\ln \Big(c^{10} \left(-a^5 b^5\right)^{5/2} + a^{20} d^{10} \sqrt{-a^5 b^5} - a^{12} b^{13} c^{10} x^2 - a^{22} b^3 d^{10} x^2 + 2 a^{10} c^5 d^5 \left(-a^5 b^5\right)^{3/2} + 2 a^{17} b^8 c^5 d^5}{4 a^6 d - 4 a^5 b c}$$

$$- \frac{\ln \Big(c^{10} \left(-a^5 b^5\right)^{5/2} + a^{20} d^{10} \sqrt{-a^5 b^5} + a^{12} b^{13} c^{10} x^2 + a^{22} b^3 d^{10} x^2 + 2 a^{10} c^5 d^5 \left(-a^5 b^5\right)^{3/2} - 2 a^{17} b^8 c^5}{4 \left(a^6 d - a^5 b c\right)}$$

$$- \frac{\frac{1}{6 a c} - \frac{x^4 (a d + b c)}{2 a^2 c^2}}{x^6}$$

$$- \frac{\ln \Big(a^{10} \left(-c^5 d^5\right)^{5/2} + b^{10} c^{20} \sqrt{-c^5 d^5} + a^{10} c^{12} d^{13} x^2 + b^{10} c^{22} d^3 x^2 + 2 a^5 b^5 c^{10} \left(-c^5 d^5\right)^{3/2} - 2 a^5 b^5 c^{17}}{4 \left(b c^6 - a c^5 d\right)}$$

$$+ \frac{\ln \Big(a^{10} \left(-c^5 d^5\right)^{5/2} + b^{10} c^{20} \sqrt{-c^5 d^5} - a^{10} c^{12} d^{13} x^2 - b^{10} c^{22} d^3 x^2 + 2 a^5 b^5 c^{10} \left(-c^5 d^5\right)^{3/2} + 2 a^5 b^5 c^{17}}{4 b c^6 - 4 a c^5 d}$$

input
```
int(1/(x^7*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
(log(c^10*(-a^5*b^5)^(5/2) + a^20*d^10*(-a^5*b^5)^(1/2) - a^12*b^13*c^10*x
^2 - a^22*b^3*d^10*x^2 + 2*a^10*c^5*d^5*(-a^5*b^5)^(3/2) + 2*a^17*b^8*c^5*
d^5*x^2)*(-a^5*b^5)^(1/2))/(4*a^6*d - 4*a^5*b*c) - (log(c^10*(-a^5*b^5)^(5
/2) + a^20*d^10*(-a^5*b^5)^(1/2) + a^12*b^13*c^10*x^2 + a^22*b^3*d^10*x^2
+ 2*a^10*c^5*d^5*(-a^5*b^5)^(3/2) - 2*a^17*b^8*c^5*d^5*x^2)*(-a^5*b^5)^(1/
2))/(4*(a^6*d - a^5*b*c)) - (1/(6*a*c) - (x^4*(a*d + b*c))/(2*a^2*c^2))/x^
6 - (log(a^10*(-c^5*d^5)^(5/2) + b^10*c^20*(-c^5*d^5)^(1/2) + a^10*c^12*d^
13*x^2 + b^10*c^22*d^3*x^2 + 2*a^5*b^5*c^10*(-c^5*d^5)^(3/2) - 2*a^5*b^5*c
^17*d^8*x^2)*(-c^5*d^5)^(1/2))/(4*(b*c^6 - a*c^5*d)) + (log(a^10*(-c^5*d^5
)^(5/2) + b^10*c^20*(-c^5*d^5)^(1/2) - a^10*c^12*d^13*x^2 - b^10*c^22*d^3*
x^2 + 2*a^5*b^5*c^10*(-c^5*d^5)^(3/2) + 2*a^5*b^5*c^17*d^8*x^2)*(-c^5*d^5)
^(1/2))/(4*b*c^6 - 4*a*c^5*d)
```

3.779 $\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx$

3.779.1 Optimal result

Integrand size = 22, antiderivative size = 457

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \frac{x}{bd} - \frac{a^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)} + \frac{a^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)} + \frac{c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{5/4}(bc-ad)} - \frac{c^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{5/4}(bc-ad)} - \frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)} + \frac{a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)} + \frac{c^{5/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}d^{5/4}(bc-ad)} - \frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}d^{5/4}(bc-ad)}$$

output
```
x/b/d+1/4*a^(5/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(5/4)/(-a*d+b*c)*
2^(1/2)+1/4*a^(5/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(5/4)/(-a*d+b*c)
*2^(1/2)-1/4*c^(5/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(5/4)/(-a*d+b*
c)*2^(1/2)-1/4*c^(5/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(5/4)/(-a*d+b
*c)*2^(1/2)-1/8*a^(5/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))
/b^(5/4)/(-a*d+b*c)*2^(1/2)+1/8*a^(5/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/
2)+x^2*b^(1/2))/b^(5/4)/(-a*d+b*c)*2^(1/2)+1/8*c^(5/4)*ln(-c^(1/4)*d^(1/4)
*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/d^(5/4)/(-a*d+b*c)*2^(1/2)-1/8*c^(5/4)*ln(
c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/d^(5/4)/(-a*d+b*c)*2^(1/2)
```

3.779.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx$$

$$= -\frac{8ax}{b} + \frac{8cx}{d} - \frac{2\sqrt{2}a^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{5/4}} + \frac{2\sqrt{2}a^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{5/4}} + \frac{2\sqrt{2}c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{d^{5/4}} - \frac{2\sqrt{2}c^{5/4}\arctan\left(\right.}{d^{5/}}$$

input
```
Integrate[x^8/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((-8*a*x)/b + (8*c*x)/d - (2*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/b^(5/4) + (2*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^
(1/4)])/b^(5/4) + (2*Sqrt[2]*c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/d^(5/4) - (2*Sqrt[2]*c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/
d^(5/4) - (Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/b^(5/4) + (Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/b^(5/4) + (Sqrt[2]*c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)
*d^(1/4)*x + Sqrt[d]*x^2])/d^(5/4) - (Sqrt[2]*c^(5/4)*Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/d^(5/4))/(8*b*c - 8*a*d)
```

3.779.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 449, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {979, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx$$

$$\Big\downarrow \text{ 979}$$

$$\frac{x}{bd}-\frac{\int \frac{(bc+ad)x^4+ac}{(bx^4+a)(dx^4+c)}dx}{bd}$$

$$\Big\downarrow \text{ 1020}$$

$$\frac{x}{bd}-\frac{\frac{bc^2\int \frac{1}{dx^4+c}dx}{bc-ad}-\frac{a^2d\int \frac{1}{bx^4+a}dx}{bc-ad}}{bd}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}}{bd}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right.}{bc-ad}}{bd}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-a^2d\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\right.}{bd}$$

$$\Big\downarrow \text{217}$$

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}}{bd}$$

$$\Big\downarrow \text{1479}$$

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-a^2d\left(\right.}{bd}$$

$$\Big\downarrow \text{25}$$

3.779. $\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx$

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-a^2d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{}}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{}}{}\right)}}{2\sqrt{2}\sqrt[4]{a}}\right.}{bd}$$

$$\Big\downarrow \ 27$$

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-a^2d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{}}{x^2+\sqrt{2}}}{2}}{2\sqrt{a}}\right.}{bd}$$

$$\Big\downarrow \ 1103$$

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-a^2d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right.}{bd}$$

input
```
Int[x^8/((a + b*x^4)*(c + d*x^4)),x]
```

3.779. $\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx$

output

```
x/(b*d) - (-((a^2*d*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a
^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4
)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*
d)) + (b*c^2*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*
d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/
4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(b*
d)
```

3.779.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 979

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.779.4 Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.51

method	result
default	$\frac{x}{bd}-\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b(ad-bc)}+\frac{c\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,(\ln(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+}}{}$
risch	$\frac{x}{bd}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\left(a^{4}d^{5}-4a^{3}bc\,d^{4}+6a^{2}b^{2}c^{2}d^{3}-4a\,b^{3}c^{3}d^{2}+b^{4}c^{4}d\right)\_Z^{4}+b^{4}c^{5}\right)}\_R\ln(((-a^{5}b\,d^{6}+3a^{4}b^{2}c\,d^{5}-2a^{3}b^{3}c^{2}d^{4}-2a^{2}b^{4}c^{3}d^{3}+}{4bd}$

input
```
int(x^8/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output

```
x/b/d-1/8/b*a/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)
+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a
/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8/d*c/(a*d-b*c)*(c/d)^
(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)
*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2
)/(c/d)^(1/4)*x-1))
```

3.779.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 1196, normalized size of antiderivative = 2.62

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input

```
integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output

```
1/4*((-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3
+ a^4*b^5*d^4))^(1/4)*b*d*log(a*x + (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2
*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)) - (-
a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b
^5*d^4))^(1/4)*b*d*log(a*x - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^
2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)) - I*(-a^5/(
b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^
4))^(1/4)*b*d*log(a*x - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2
 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(I*b^2*c - I*a*b*d)) + I*(-a^5/(b
^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4
))^(1/4)*b*d*log(a*x - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2
- 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(-I*b^2*c + I*a*b*d)) - (-c^5/(b^4
*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))
^(1/4)*b*d*log(c*x + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*
d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) + (-c^5/(b^4*c^4*d^
5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*
b*d*log(c*x - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4
*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) + I*(-c^5/(b^4*c^4*d^5 - 4
*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*l
og(c*x - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a...
```

3.779.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**8/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.779.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}a^{\frac{5}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}} - \sqrt{2}a^{\frac{5}{4}}\log\left(\sqrt{b}x\right.}{8\left(b^2c-abd\right)}$$
$$- \frac{\frac{2\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}c^{\frac{5}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{d^{\frac{1}{4}}} - \sqrt{2}c^{\frac{5}{4}}\log\left(\right.}{8\left(bcd-ad^2\right)}$$
$$+ \frac{x}{bd}$$

input
```
integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output

```
1/8*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b
^(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqr
t(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(5/4)*log(sqrt(b)*x^2 + sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(5/4)*log(sqrt(b)*x^2 - sq
rt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4))/(b^2*c - a*b*d) - 1/8*(2*sqrt(
2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt
(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*c^(3/2)*arctan(1/2*sq
rt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(
sqrt(c)*sqrt(d)) + sqrt(2)*c^(5/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/
4)*x + sqrt(c))/d^(1/4) - sqrt(2)*c^(5/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4
)*d^(1/4)*x + sqrt(c))/d^(1/4))/(b*c*d - a*d^2) + x/(b*d)
```

3.779. $\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx$

3.779.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.03

$$
\begin{aligned}
\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = {} & \frac{(ab^3)^{\frac{1}{4}}\, a \arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^3c-\sqrt{2}ab^2d\right)} \\
& + \frac{(ab^3)^{\frac{1}{4}}\, a \arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^3c-\sqrt{2}ab^2d\right)} \\
& - \frac{(cd^3)^{\frac{1}{4}}\, c \arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^2-\sqrt{2}ad^3\right)} \\
& - \frac{(cd^3)^{\frac{1}{4}}\, c \arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^2-\sqrt{2}ad^3\right)} \\
& + \frac{(ab^3)^{\frac{1}{4}}\, a \log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^3c-\sqrt{2}ab^2d\right)} \\
& - \frac{(ab^3)^{\frac{1}{4}}\, a \log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^3c-\sqrt{2}ab^2d\right)} \\
& - \frac{(cd^3)^{\frac{1}{4}}\, c \log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^2-\sqrt{2}ad^3\right)} \\
& + \frac{(cd^3)^{\frac{1}{4}}\, c \log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^2-\sqrt{2}ad^3\right)} + \frac{x}{bd}
\end{aligned}
$$

input
```
integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output

```
1/2*(a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) + 1/2*(a*b^3)^(1/4)*a*arctan(1/2*s
qrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^3*c - sqrt(2)*a
*b^2*d) - 1/2*(c*d^3)^(1/4)*c*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4
))/(c/d)^(1/4))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) - 1/2*(c*d^3)^(1/4)*c*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^2
 - sqrt(2)*a*d^3) + 1/4*(a*b^3)^(1/4)*a*log(x^2 + sqrt(2)*x*(a/b)^(1/4) +
sqrt(a/b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) - 1/4*(a*b^3)^(1/4)*a*log(x^2
 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) -
1/4*(c*d^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*
b*c*d^2 - sqrt(2)*a*d^3) + 1/4*(c*d^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(c/d)^(
1/4) + sqrt(c/d))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) + x/(b*d)
```

3.779.9 Mupad [B] (verification not implemented)

Time = 9.94 (sec) , antiderivative size = 6361, normalized size of antiderivative = 13.92

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input

```
int(x^8/((a + b*x^4)*(c + d*x^4)),x)
```

output

```
atan(((-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2
*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 -
 a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (4*x*(-a^5/(256*b^9*c^4 + 256*a^4
*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^
(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 5
12*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-
a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2
*d^2 - 1024*a*b^8*c^3*d))^(1/4) - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))
*1i - (-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2
*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 -
 a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (4*x*(-a^5/(256*b^9*c^4 + 256*a^4
*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^
(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 5
12*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-
a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2
*d^2 - 1024*a*b^8*c^3*d))^(1/4) + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))
*1i)/((-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2
*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 -
 a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (4*x*(-a^5/(256*b^9*c^4 + 256*a^4
*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d...
```

3.779. $\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx$

3.780 $\int \frac{x^6}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$

3.780.1 Optimal result

Integrand size = 22, antiderivative size = 449

$$
\begin{aligned}
\int \frac{x^6}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{a^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)} - \frac{a^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)} \\
&\quad - \frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{3/4}(bc-ad)} + \frac{c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{3/4}(bc-ad)} \\
&\quad - \frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)} \\
&\quad + \frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)} \\
&\quad + \frac{c^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}d^{3/4}(bc-ad)} \\
&\quad - \frac{c^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}d^{3/4}(bc-ad)}
\end{aligned}
$$

output
```
-1/4*a^(3/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(3/4)/(-a*d+b*c)*2^(1/
2)-1/4*a^(3/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(3/4)/(-a*d+b*c)*2^(1
/2)+1/4*c^(3/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(3/4)/(-a*d+b*c)*2^
(1/2)+1/4*c^(3/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(3/4)/(-a*d+b*c)*2
^(1/2)-1/8*a^(3/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/b^(3
/4)/(-a*d+b*c)*2^(1/2)+1/8*a^(3/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*b^(1/2))/b^(3/4)/(-a*d+b*c)*2^(1/2)+1/8*c^(3/4)*ln(-c^(1/4)*d^(1/4)*x*2^
(1/2)+c^(1/2)+x^2*d^(1/2))/d^(3/4)/(-a*d+b*c)*2^(1/2)-1/8*c^(3/4)*ln(c^(1/
4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/d^(3/4)/(-a*d+b*c)*2^(1/2)
```

3.780.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = \frac{2a^{3/4}d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2a^{3/4}d^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2b^{3/4}c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+2b}{}$$

input
```
Integrate[x^6/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(2*a^(3/4)*d^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(3/4)*d^(
3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(3/4)*c^(3/4)*ArcTan[1
- (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(3/4)*c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(
1/4)*x)/c^(1/4)] - a^(3/4)*d^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2] - b^(3/4)*c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x +
Sqrt[d]*x^2])/(4*Sqrt[2]*b^(3/4)*d^(3/4)*(b*c - a*d))
```

3.780.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {981, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{981}$$

$$\frac{c\int \frac{x^2}{dx^4+c}dx}{bc-ad}-\frac{a\int \frac{x^2}{bx^4+a}dx}{bc-ad}$$

$$\Big\downarrow \text{826}$$

$$\frac{c\left(\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1476}$$

$$\frac{c\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1082}$$

$$\frac{c\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}$$

$$\Big\downarrow \ 217$$

$$\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}$$

$$\Big\downarrow \ 1479$$

3.780. $\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$

$$\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{bc-ad}-$$

$$\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{bc-ad}$$

$$\Big\downarrow \ 25$$

$$\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{bc-ad}-$$

$$\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{bc-ad}$$

$$\Big\downarrow \ 27$$

3.780. $\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$

$$\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{bc-ad}$$

$$\Big\downarrow \ 1103$$

$$\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{bc-ad}$$

input

```
Int[x^6/((a + b*x^4)*(c + d*x^4)),x]
```

3.780. $\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$

output
```
-((a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))
) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*
Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(S
qrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/(b*c - a*d)) + (c*((-(Ar
cTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[
1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d]) -
(-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1
/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*S
qrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d])))/(b*c - a*d)
```

3.780.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 981
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[(-a)*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(a + b*x^n),
x], x] + Simp[c*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(c + d*x^n), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
 m, 2*n - 1]
```

3.780. $\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.780.4 Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.50

method	result
default	$\frac{a\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{c\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\right.}{8(ad-bc)d(}$
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^{4}d^{7}-4a^{3}bc\,d^{6}+6a^{2}b^{2}c^{2}d^{5}-4a\,b^{3}c^{3}d^{4}+b^{4}c^{4}d^{3}\right)\_Z^{4}+c^{3}\right)}\_R\ln\left(\left(\left(2a^{4}b^{3}d^{7}-8a^{3}b^{4}c\,d^{6}+12a^{2}b^{5}c^{2}d^{5}-8a\,b^{6}c^{3}d^{4}+2b^{7}c\right.\right.\right.\right.}{4}$

input
```
int(x^6/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)
^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/8*c/(a*d-b*c)/d/(c/d)^(1/4)*
2^(1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2+(c/d)^(1/4)*x*2^(
1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d
)^(1/4)*x-1))
```

3.780.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 1427, normalized size of antiderivative = 3.18

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
-1/4*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3
+ a^4*b^3*d^4))^(1/4)*log(a^2*x + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d
^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a
^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/4*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d +
 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*log(a^2*x - (b^
5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*
a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) -
 1/4*I*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^
3 + a^4*b^3*d^4))^(1/4)*log(a^2*x - (I*b^5*c^3 - 3*I*a*b^4*c^2*d + 3*I*a^2
*b^3*c*d^2 - I*a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2
*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/4*I*(-a^3/(b^7*c^4 - 4*a
*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*log
(a^2*x - (-I*b^5*c^3 + 3*I*a*b^4*c^2*d - 3*I*a^2*b^3*c*d^2 + I*a^3*b^2*d^3
)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a
^4*b^3*d^4))^(3/4)) + 1/4*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2
*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*log(c^2*x + (b^3*c^3*d^2 - 3*a*
b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^
4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/4)) - 1/4*(-c^3/(b^4*
c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^
(1/4)*log(c^2*x - (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*...
```

3.780.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**6/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.780.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.81

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx =$$

$$-\frac{a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^2-\right.}{a^{\frac{1}{4}}}\right.}{8(bc-ad)}$$

$$+\frac{c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}x^2-\right.}{c^{\frac{1}{4}}}\right.}{8(bc-ad)}$$

input
```
integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/8*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^
(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*c - a*d) + 1/8*c
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqr
t(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(s
qrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*
d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)
*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c - a*d)
```

3.780. $\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$

3.780.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.01

$$
\begin{aligned}
\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = &-\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)} \\
&-\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)} \\
&+\frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)} \\
&+\frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)} \\
&+\frac{(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)} \\
&-\frac{(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)} \\
&-\frac{(cd^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)} \\
&+\frac{(cd^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)}
\end{aligned}
$$

input
```
integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^
3*d) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c
/d)^(1/4))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4) + 1/2*(c*d^3)^(3/4)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^3 - sqrt
(2)*a*d^4) + 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b)
)/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/4*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/4*(c*d^3)
^(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d^3 - sqr
t(2)*a*d^4) + 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d
))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4)
```

3.780.9 Mupad [B] (verification not implemented)

Time = 10.09 (sec) , antiderivative size = 2553, normalized size of antiderivative = 5.69

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^6/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
- 2*atan((4*b^4*c^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*
c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4) + 4*a^3*b*d^3*x*(-
a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2
*d^2 - 1024*a*b^6*c^3*d))^(1/4) + 2048*a^4*b^4*d^7*x*(-a^3/(256*b^7*c^4 +
256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c
^3*d))^(5/4) + 2048*b^8*c^4*d^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1
024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4) - 8192
*a*b^7*c^3*d^4*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3
 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4) - 8192*a^3*b^5*c*d^6*x*
(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c
^2*d^2 - 1024*a*b^6*c^3*d))^(5/4) + 12288*a^2*b^6*c^2*d^5*x*(-a^3/(256*b^7
*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*
a*b^6*c^3*d))^(5/4))/(a^3*d^2 + a*b^2*c^2 + a^2*b*c*d))*(-a^3/(256*b^7*c^4
 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^
6*c^3*d))^(1/4) - atan((b^4*c^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1
024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4)*4i + a
^3*b*d^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 153
6*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4)*4i + a^4*b^4*d^7*x*(-a^3/(256
*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1
024*a*b^6*c^3*d))^(5/4)*2048i + b^8*c^4*d^3*x*(-a^3/(256*b^7*c^4 + 256*...
```

3.780. $\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$

3.781 $\int \frac{x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$

3.781.1 Optimal result

Integrand size = 22, antiderivative size = 449

$$
\begin{aligned}
\int \frac{x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = {} & \frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)} \\
& -\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)} \\
& +\frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)} \\
& -\frac{\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)} \\
& -\frac{\sqrt[4]{c}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)} \\
& +\frac{\sqrt[4]{c}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)}
\end{aligned}
$$

output
```
-1/4*a^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(1/4)/(-a*d+b*c)*2^(1/
2)-1/4*a^(1/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(1/4)/(-a*d+b*c)*2^(1
/2)+1/4*c^(1/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(1/4)/(-a*d+b*c)*2^
(1/2)+1/4*c^(1/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(1/4)/(-a*d+b*c)*2
^(1/2)+1/8*a^(1/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/b^(1
/4)/(-a*d+b*c)*2^(1/2)-1/8*a^(1/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*b^(1/2))/b^(1/4)/(-a*d+b*c)*2^(1/2)-1/8*c^(1/4)*ln(-c^(1/4)*d^(1/4)*x*2^
(1/2)+c^(1/2)+x^2*d^(1/2))/d^(1/4)/(-a*d+b*c)*2^(1/2)+1/8*c^(1/4)*ln(c^(1/
4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/d^(1/4)/(-a*d+b*c)*2^(1/2)
```

3.781.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = \frac{2\sqrt[4]{a}\sqrt[4]{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2\sqrt[4]{a}\sqrt[4]{d}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2\sqrt[4]{b}\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+2\sqrt[4]{b}\sqrt[4]{c}}{}$$

input
```
Integrate[x^4/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(2*a^(1/4)*d^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(1/4)*d^(
1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(1/4)*c^(1/4)*ArcTan[1
- (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(1/4)*c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(
1/4)*x)/c^(1/4)] + a^(1/4)*d^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2] - a^(1/4)*d^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2] - b^(1/4)*c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2] + b^(1/4)*c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x +
Sqrt[d]*x^2])/(4*Sqrt[2]*b^(1/4)*d^(1/4)*(b*c - a*d))
```

3.781.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {981, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{ 981}$$

$$\frac{c\int \frac{1}{dx^4+c}dx}{bc-ad}-\frac{a\int \frac{1}{bx^4+a}dx}{bc-ad}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{c\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}$$

$$\Big\downarrow \text{217}$$

$$\frac{c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1479}$$

3.781. $\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx$

$$\frac{c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}$$

$$\Big\downarrow \, 25$$

$$\frac{c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}$$

$$\Big\downarrow \, 27$$

3.781. $\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx$

$$\frac{c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1103$$

$$\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}$$

input
```
Int[x^4/((a + b*x^4)*(c + d*x^4)),x]
```

3.781. $\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx$

output
```
-((a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))
) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*
Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(S
qrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d)) + (c*((-(Ar
cTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[
1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) +
(-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1
/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*S
qrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d)
```

3.781.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 981
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[(-a)*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(a + b*x^n),
x], x] + Simp[c*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(c + d*x^n), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
 m, 2*n - 1]
```

3.781. $\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.781.4 Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.49

method	result
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8 a d -8 b c}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2}{8(}$
risch	$\frac{\left(\sum_{\_R =\operatorname{RootOf}\left(\left(a^{4} b\,d^{4}-4 a^{3} b^{2} c\,d^{3}+6 a^{2} b^{3} c^{2} d^{2}-4 a\,b^{4} c^{3} d +b^{5} c^{4}\right) \_Z^{4}+a \right)}\_R \ln\left(\left(\left(-a^{5} b\,d^{6}+3 a^{4} b^{2} c\,d^{5}-2 a^{3} b^{3} c^{2} d^{4}-2 a^{2} b^{4} c^{3} d^{3}+3 a\,b^{5} c^{4}\right.\right.\right.}{}$

input
```
int(x^4/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output

```
1/8/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/
2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*
x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/8/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*
(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/
d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*
x-1))
```

3.781.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1067, normalized size of antiderivative = 2.38

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx =$$

$$-\frac{1}{4}\left(-\frac{a}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}\right)^{\frac{1}{4}}\log\Bigg((bc-ad)\Bigg(-\frac{a}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d}$$

$$+x\Bigg)$$

$$+\frac{1}{4}\left(-\frac{a}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}\right)^{\frac{1}{4}}\log\Bigg(-(bc-ad)\Bigg(-\frac{a}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2}$$

$$+x\Bigg)$$

$$+\frac{1}{4}i\left(-\frac{a}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}\right)^{\frac{1}{4}}\log\Bigg(-(i\,bc-i\,ad)\Bigg(-\frac{}{b^5c^4-4\,ab^4c^3d+6\,a^2}$$

$$+x\Bigg)$$

$$-\frac{1}{4}i\left(-\frac{a}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}\right)^{\frac{1}{4}}\log\Bigg(-(-i\,bc+i\,ad)\Bigg(-\frac{}{b^5c^4-4\,ab^4c^3d+6}$$

$$+x\Bigg)$$

$$+\frac{1}{4}\left(-\frac{c}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}\right)^{\frac{1}{4}}\log\Bigg((bc-ad)\Bigg(-\frac{c}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c}$$

$$+x\Bigg)$$

$$-\frac{1}{4}\left(-\frac{c}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}\right)^{\frac{1}{4}}\log\Bigg(-(bc-ad)\Bigg(-\frac{c}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b}$$

$$+x\Bigg)$$

$$-\frac{1}{4}i\left(-\frac{c}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}\right)^{\frac{1}{4}}\log\Bigg(-(i\,bc-i\,ad)\Bigg(-\frac{}{b^4c^4d-4\,ab^3c^3d^2+6}$$

$$+x\Bigg)$$

$$+\frac{1}{4}i\left(-\frac{c}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}\right)^{\frac{1}{4}}\log\Bigg(-(-i\,bc+i\,ad)\Bigg(-\frac{}{b^4c^4d-4\,ab^3c^3d^2+}$$

$$+x\Bigg)$$

input
```
integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
-1/4*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4))^(1/4)*log((b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3
*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4) + x) + 1/4*(-a/(b^5*c^4 - 4
*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*log
(-(b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2
*c*d^3 + a^4*b*d^4))^(1/4) + x) + 1/4*I*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a
^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*log(-(I*b*c - I*a*d)*
(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b
*d^4))^(1/4) + x) - 1/4*I*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*log(-(-I*b*c + I*a*d)*(-a/(b^5*c^4
- 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)
+ x) + 1/4*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*
c*d^4 + a^4*d^5))^(1/4)*log((b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 +
 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + x) - 1/4*(-c/(b^4*c
^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/
4)*log(-(b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 -
 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + x) - 1/4*I*(-c/(b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log(-(I*b*c - I
*a*d)*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4
 + a^4*d^5))^(1/4) + x) + 1/4*I*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^...
```

3.781.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.781.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.80

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx =$$

$$-\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\right.}{}}{8\,(bc-ad)}$$

$$+\frac{\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}c^{\frac{1}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}-\frac{\sqrt{2}c^{\frac{1}{4}}\log\left(\sqrt{\right.}}{}}{8\,(bc-ad)}$$

input
```
integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/8*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*
b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*
arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sq
rt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(b)*x^2 + sqrt(2)*
a^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(sqrt(b)*x^2 - s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4))/(b*c - a*d) + 1/8*(2*sqrt(2)*
sqrt(c)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sq
rt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(
2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqr
t(c)*sqrt(d)) + sqrt(2)*c^(1/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*
x + sqrt(c))/d^(1/4) - sqrt(2)*c^(1/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d
^(1/4)*x + sqrt(c))/d^(1/4))/(b*c - a*d)
```

3.781.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.97

$$
\begin{aligned}
\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = & -\frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^2c-\sqrt{2}abd\right)} \\
& -\frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^2c-\sqrt{2}abd\right)} \\
& +\frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)} \\
& +\frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)} \\
& -\frac{(ab^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^2c-\sqrt{2}abd\right)} \\
& +\frac{(ab^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^2c-\sqrt{2}abd\right)} \\
& +\frac{(cd^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)} \\
& -\frac{(cd^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)}
\end{aligned}
$$

input

```
integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output

```
-1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) - 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2
)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d)
 + 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^
(1/4))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) + 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d - sqrt(2)*a*d^
2) - 1/4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(
2)*b^2*c - sqrt(2)*a*b*d) + 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1
/4) + sqrt(a/b))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) + 1/4*(c*d^3)^(1/4)*log(x
^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) -
1/4*(c*d^3)^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*
c*d - sqrt(2)*a*d^2)
```

3.781.9 Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 5889, normalized size of antiderivative = 13.12

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input

```
int(x^4/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
- atan((a^2*d^2*x*1i + b^2*c^2*x*1i - (a^6*b*d^6*x*256i)/(256*b^5*c^4 + 25
6*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d
) - (a*b^6*c^5*d*x*256i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3
 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) + (a^5*b^2*c*d^5*x*768i)/(256*
b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024
*a*b^4*c^3*d) + (a^2*b^5*c^4*d^2*x*768i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 10
24*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - (a^3*b^4*c^3
*d^3*x*512i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*
b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - (a^4*b^3*c^2*d^4*x*512i)/(256*b^5*c^4 +
256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3
*d))/((-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3
*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*((a*(1024*a^6*b*d^7 + 1024*b^7*c^6*d -
 6144*a*b^6*c^5*d^2 - 6144*a^5*b^2*c*d^6 + 15360*a^2*b^5*c^4*d^3 - 20480*a
^3*b^4*c^3*d^4 + 15360*a^4*b^3*c^2*d^5))/(256*b^5*c^4 + 256*a^4*b*d^4 - 10
24*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - 4*b^3*c^3 -
4*a^3*d^3 + 4*a*b^2*c^2*d + 4*a^2*b*c*d^2)))*(-a/(256*b^5*c^4 + 256*a^4*b*
d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)
*2i - atan((a^2*d^2*x*1i + b^2*c^2*x*1i - (b^6*c^6*d*x*256i)/(256*a^4*d^5
+ 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c
*d^4) - (a^5*b*c*d^6*x*256i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*...
```

3.781. $\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx$

3.782 $\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$

3.782.1 Optimal result

Integrand size = 22, antiderivative size = 449

$$\begin{aligned}\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = & -\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)} \\ & +\frac{\sqrt[4]{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)} \\ & +\frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)} \\ & -\frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)} \\ & -\frac{\sqrt[4]{d}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)} \\ & +\frac{\sqrt[4]{d}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}\end{aligned}$$

output
```
1/4*b^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)*2^(1/2
)+1/4*b^(1/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)*2^(1/
2)-1/4*d^(1/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(1/4)/(-a*d+b*c)*2^(
1/2)-1/4*d^(1/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(1/4)/(-a*d+b*c)*2^
(1/2)+1/8*b^(1/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(1/
4)/(-a*d+b*c)*2^(1/2)-1/8*b^(1/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*b^(1/2))/a^(1/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(1/4)*ln(-c^(1/4)*d^(1/4)*x*2^(
1/2)+c^(1/2)+x^2*d^(1/2))/c^(1/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(1/4)*ln(c^(1/4
)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(1/4)/(-a*d+b*c)*2^(1/2)
```

3.782.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{-2\sqrt[4]{b}\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{b}\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\sqrt[4]{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-2\sqrt[4]{a}}{}$$

input
```
Integrate[x^2/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-2*b^(1/4)*c^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(1/4)*c^
(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*d^(1/4)*ArcTan[1
 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(1/4)*d^(1/4)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*x)/c^(1/4)] + b^(1/4)*c^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2] - b^(1/4)*c^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2] - a^(1/4)*d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2] + a^(1/4)*d^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(b*c - a*d))
```

3.782.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {982, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{982}$$

$$\frac{b\int \frac{x^2}{bx^4+a}dx}{bc-ad}-\frac{d\int \frac{x^2}{dx^4+c}dx}{bc-ad}$$

$$\Big\downarrow \text{826}$$

$$\frac{b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1476}$$

$$\frac{b\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1082}$$

$$\frac{b\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{217}$$

$$\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1479}$$

3.782. $\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$

$$\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{bc-ad}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{bc-ad}$$

$$\Big\downarrow \; 27$$

3.782. $\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$

$$\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1103}$$

$$\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{bc-ad}$$

input
```
Int[x^2/((a + b*x^4)*(c + d*x^4)),x]
```

3.782. $\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$

output
```
(b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sq
rt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqr
t[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/(b*c - a*d) - (d*((-(ArcTa
n[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d]) - (-1
/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)
*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt
[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d])))/(b*c - a*d)
```

3.782.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.782.4 Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.49

method	result
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\right.\right.}{8(ad-bc)(}$
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^4d^4c-4a^3b\,d^3c^2+6a^2b^2d^2c^3-4a\,b^3d\,c^4+b^4c^5\right)\_Z^4+d\right)}\_R\ln\left(\left(\left(a^6d^6-4a^5bc\,d^5+7a^4b^2c^2d^4-8a^3b^3c^3d^3+7a^2b^4c^4d^2-\right.\right.\right.\right.}{4}$

input
```
int(x^2/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1
/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)
*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)
*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2+(c/d)^(1/4)*x*2^(1/2)+(c
/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)
*x-1))
```

3.782.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1331, normalized size of antiderivative = 2.96

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 +
 a^5*d^4))^(1/4)*log(b*x + (a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 -
a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*
d^3 + a^5*d^4))^(3/4)) - 1/4*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*
c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(b*x - (a*b^3*c^3 - 3*a^2*b^2
*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3
*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4)) + 1/4*I*(-b/(a*b^4*c^4 - 4
*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(b
*x - (I*a*b^3*c^3 - 3*I*a^2*b^2*c^2*d + 3*I*a^3*b*c*d^2 - I*a^4*d^3)*(-b/(
a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)
)^(3/4)) - 1/4*I*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*
a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(b*x - (-I*a*b^3*c^3 + 3*I*a^2*b^2*c^2*d
- 3*I*a^3*b*c*d^2 + I*a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^
2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4)) - 1/4*(-d/(b^4*c^5 - 4*a*b^3*
c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log(d*x +
(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a
*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4)) + 1/
4*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4
*c*d^4))^(1/4)*log(d*x - (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*
c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d...
```

3.782.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.782.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$$

$$= \frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log(\sqrt{b}x^2-\sqrt{2}a}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{8\,(bc-ad)}$$

$$- \frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}} + \frac{\sqrt{2}\log(\sqrt{d}x^2-}{c^{\frac{1}{4}}}\right.}{8\,(bc-ad)}$$

input
```
integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output

```
1/8*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(
1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sq
rt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*c - a*d) - 1/8*d*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt
(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2*
sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sq
rt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d
^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*
c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c - a*d)
```

3.782. $\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$

3.782.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^3c-\sqrt{2}a^2b^2d\right)} + \frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^3c-\sqrt{2}a^2b^2d\right)} - \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2d^2-\sqrt{2}acd^3\right)} - \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2d^2-\sqrt{2}acd^3\right)} - \frac{(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c-\sqrt{2}a^2b^2d\right)} + \frac{(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c-\sqrt{2}a^2b^2d\right)} + \frac{(cd^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2d^2-\sqrt{2}acd^3\right)} - \frac{(cd^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2d^2-\sqrt{2}acd^3\right)}$$

input

```
integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output

```
1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/2*(a*b^3)^(3/4)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c - sqrt(2)
*a^2*b^2*d) - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1
/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/2*(c*d^3)^(3/4
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c
^2*d^2 - sqrt(2)*a*c*d^3) - 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1
/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/4*(a*b^3)^(3/4)
*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c - sqrt(2)*a
^2*b^2*d) + 1/4*(c*d^3)^(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))
/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(
2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3)
```

3.782.9 Mupad [B] (verification not implemented)

Time = 9.98 (sec) , antiderivative size = 6633, normalized size of antiderivative = 14.77

$$\int \frac{x^2}{(a+bx^4)\,(c+dx^4)}\,dx = \text{Too large to display}$$

input

```
int(x^2/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
atan(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-b/(256*a^5*d^4 + 256*a*b^
4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/
4)*(x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2
*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*
d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7
+ 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*b^9*c^6*d^4 + 256*a
^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3
*d^7 - 768*a^5*b^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b
^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*1i + (x*(4*a*b^
6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2
*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(256*a*b^9*c^
6*d^4 - x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3
*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^
4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*
d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*d^9 - 7
68*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b
^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*
a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*1i)/((x*(4*a*b^6*c^2*d^5 + 4*a^
2*b^5*c*d^6) + (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 153
6*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(x*(-b/(256*a^5*d^4 + 256*...
```

3.782. $\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$

3.783 $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

3.783.1 Optimal result

Integrand size = 19, antiderivative size = 449

$$\begin{aligned}\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = & -\frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}\\ & +\frac{d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}\\ & -\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}\\ & +\frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}\\ & +\frac{d^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}\\ & -\frac{d^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}\end{aligned}$$

output

```
1/4*b^(3/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/2
)+1/4*b^(3/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/
2)-1/4*d^(3/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^(
1/2)-1/4*d^(3/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^
(1/2)-1/8*b^(3/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/
4)/(-a*d+b*c)*2^(1/2)+1/8*b^(3/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*b^(1/2))/a^(3/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(3/4)*ln(-c^(1/4)*d^(1/4)*x*2^(
1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(3/4)*ln(c^(1/4
)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)
```

3.783.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$$

$$= \frac{-2b^{3/4}c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2b^{3/4}c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2a^{3/4}d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-2a}{}$$

input

```
Integrate[1/((a + b*x^4)*(c + d*x^4)),x]
```

output

```
(-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(3/4)*c^
(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4)*d^(3/4)*ArcTan[1
 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

3.783.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {917, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{917}$$

$$\frac{b\int \frac{1}{bx^4+a}dx}{bc-ad}-\frac{d\int \frac{1}{dx^4+c}dx}{bc-ad}$$

$$\Big\downarrow \text{755}$$

$$\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1476}$$

$$\frac{b\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1082}$$

$$\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \ 217$$

$$\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \ 1479$$

3.783. $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

$$\frac{b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow 25$$

$$\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow 27$$

3.783. $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

$$\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1103$$

$$\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^4)*(c + d*x^4)),x]
```

3.783. $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

output

```
(b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sq
rt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqr
t[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - (d*((-(ArcTa
n[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1
/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)
*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt
[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d)
```

3.783.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 917

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^n), x], x] - Simp[d/(b*c - a*d)   Int[1/(c
 + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.783.4 Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.50

method	result
default	$-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)a}+\frac{d\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right.\right.}{}$
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^{4}c^{3}d^{4}-4a^{3}b\,c^{4}d^{3}+6a^{2}b^{2}c^{5}d^{2}-4a\,b^{3}c^{6}d+b^{4}c^{7}\right)\_Z^{4}+d^{3}\right)}\_R\ln\left(\left(\left(-a^{7}d^{7}+4c\,a^{6}b\,d^{6}-6c^{2}a^{5}b^{2}d^{5}+3a^{4}b^{3}c^{3}d^{4}+3a^{3}b^{4}c^{4}\right.\right.\right.\right.}{}$

input
```
int(1/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(
1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8*d/(a*d-b*c)*(c/d)^(1/4)/c
*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^
(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/
d)^(1/4)*x-1))
```

3.783.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 1171, normalized size of antiderivative = 2.61

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d
^3 + a^7*d^4))^(1/4)*log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*
b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-b
^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^
7*d^4))^(1/4)*log(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3
*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*I*(-b^3/(a
^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4
))^(1/4)*log(b*x - (I*a*b*c - I*a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*
d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) + 1/4*I*(-b^3/(a^
3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)
)^(1/4)*log(b*x - (-I*a*b*c + I*a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*
d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-d^3/(b^4*
c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^
(1/4)*log(d*x + (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2
*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*(-d^3/(b^4*c^7 - 4
*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*l
og(d*x - (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^
2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*I*(-d^3/(b^4*c^7 - 4*a*b^
3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*
x - (I*b*c^2 - I*a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*...
```

3.783.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.783.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+bx^4)\,(c+dx^4)}\,dx$$

$$= \frac{\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^2-\right.}{a^{\frac{3}{4}}}}{8\,(bc-ad)}$$

$$-\frac{\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{d}x\right.}{}}{8\,(bc-ad)}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/8*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 + s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*
x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(b*c - a*d) - 1/8*(2*s
qrt(2)*d*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(s
qrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d*arctan(1/2*
sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sq
rt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 + sqrt(2)*c
^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 - sq
rt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c - a*d)
```

3.783.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.97

$$
\begin{aligned}
\int \frac{1}{(a+bx^4)\,(c+dx^4)}\,dx = {} & \frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} \\
& + \frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} \\
& - \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} \\
& - \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} \\
& + \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\,\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} \\
& - \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\,\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} \\
& - \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\,\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} \\
& + \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\,\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}
\end{aligned}
$$

input

```
integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) + 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d)
- 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(
1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) - 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d
) + 1/4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2
)*a*b*c - sqrt(2)*a^2*d) - 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/
4) + sqrt(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/4*(c*d^3)^(1/4)*log(x^
2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1
/4*(c*d^3)^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c
^2 - sqrt(2)*a*c*d)
```

3.783.9 Mupad [B] (verification not implemented)

Time = 10.39 (sec) , antiderivative size = 6153, normalized size of antiderivative = 13.70

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^4)*(c + d*x^4)),x)
```

output

```
- atan((((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*
a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c
^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3
/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*
b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4
*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c
^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2
*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 -
4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a
^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6
+ 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 10
24*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i - ((
-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^
5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1
024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3
/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^
2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 -
20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 2
0480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x
*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^...
```

3.783. $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

3.784 $\int \frac{1}{x^2(a+bx^4)(c+dx^4)}\,dx$

3.784.1 Optimal result

Integrand size = 22, antiderivative size = 460

$$\begin{aligned}\int \frac{1}{x^2(a+bx^4)(c+dx^4)}\,dx = &-\frac{1}{acx}+\frac{b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)}\\
&-\frac{b^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)}\\
&-\frac{d^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{5/4}(bc-ad)}+\frac{d^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{5/4}(bc-ad)}\\
&-\frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)}\\
&+\frac{b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)}\\
&+\frac{d^{5/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{5/4}(bc-ad)}\\
&-\frac{d^{5/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{5/4}(bc-ad)}\end{aligned}$$

output
```
-1/a/c/x-1/4*b^(5/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*
c)*2^(1/2)-1/4*b^(5/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b
*c)*2^(1/2)+1/4*d^(5/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(5/4)/(-a*d
+b*c)*2^(1/2)+1/4*d^(5/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(5/4)/(-a*
d+b*c)*2^(1/2)-1/8*b^(5/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/
2))/a^(5/4)/(-a*d+b*c)*2^(1/2)+1/8*b^(5/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^
(1/2)+x^2*b^(1/2))/a^(5/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(5/4)*ln(-c^(1/4)*d^(1
/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(5/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(5/4)*
ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(5/4)/(-a*d+b*c)*2^(1/
2)
```

3.784.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

$$= \frac{8b}{a} - \frac{8d}{c} - \frac{2\sqrt{2}b^{5/4}x\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/4}} + \frac{2\sqrt{2}b^{5/4}x\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/4}} + \frac{2\sqrt{2}d^{5/4}x\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{5/4}} - \frac{2\sqrt{2}d^{5/4}x\arctan}{c^{5/}}$$

input
```
Integrate[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
((8*b)/a - (8*d)/c - (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/a^(5/4) + (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/a^(5/4) + (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/
4)])/c^(5/4) - (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/c^(5/4) + (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/a^(5/4) - (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/a^(5/4) - (Sqrt[2]*d^(5/4)*x*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4) + (Sqrt[2]*d^(5/4)*x*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4))/(-8*b*c*x + 8*a*d*
x)
```

3.784.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {980, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^4) (c+dx^4)} \, dx$$

$$\Big\downarrow \text{ 980}$$

$$\frac{\int -\frac{x^2 (bdx^4+bc+ad)}{(bx^4+a)(dx^4+c)}dx}{ac}-\frac{1}{acx}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{x^2 (bdx^4+bc+ad)}{(bx^4+a)(dx^4+c)}dx}{ac}-\frac{1}{acx}$$

$$\Big\downarrow \text{ 1054}$$

$$-\frac{\int \left(\frac{b^2cx^2}{(bc-ad)(bx^4+a)}+\frac{ad^2x^2}{(ad-bc)(dx^4+c)}\right)dx}{ac}-\frac{1}{acx}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\frac{b^{5/4}c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{b^{5/4}c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{ad^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{ad^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{b^{5/4}c\log\Big(-}{}}{}$$

$$\frac{1}{acx}$$

input
```
Int[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

output

```
-(1/(a*c*x)) - (-1/2*(b^(5/4)*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
Sqrt[2]*a^(1/4)*(b*c - a*d)) + (b^(5/4)*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)) + (a*d^(5/4)*ArcTan[1 - (Sqrt[2]*
d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) - (a*d^(5/4)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(5/4
)*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(
1/4)*(b*c - a*d)) - (b^(5/4)*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (a*d^(5/4)*Log[Sqrt[c] - Sq
rt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) +
(a*d^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt
[2]*c^(1/4)*(b*c - a*d)))/(a*c)
```

3.784.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 980

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.784.4 Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.52

method	result
default	$\frac{b\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)a\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{1}{acx}-\frac{d\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\,\mathrm{ar}\right.}{8(ad}$
risch	Expression too large to display

input
```
int(1/x^2/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8*b/(a*d-b*c)/a/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)
^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/a/c/x-1/8*d/(a*d-b*c)/c/(c/d
)^(1/4)*2^(1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2+(c/d)^(1/
4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1
/2)/(c/d)^(1/4)*x-1))
```

3.784.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 1461, normalized size of antiderivative = 3.18

$$\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output

```
-1/4*((-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c
*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*x + (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d +
3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*
c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - (-b^5/(a^5*b^4*c^4 - 4*a^6*b^
3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^
4*x - (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5
*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))
^(3/4)) + I*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a
^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*x - (I*a^4*b^3*c^3 - 3*I*a^5*b^
2*c^2*d + 3*I*a^6*b*c*d^2 - I*a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*
d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - I*(-b^5/(a^5*b^
4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1
/4)*a*c*x*log(b^4*x - (-I*a^4*b^3*c^3 + 3*I*a^5*b^2*c^2*d - 3*I*a^6*b*c*d^
2 + I*a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 -
4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*
b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*log(d^4*x + (b^3
*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a
*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) +
(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4
*c^5*d^4))^(1/4)*a*c*x*log(d^4*x - (b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c...
```

3.784.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^4) (c+dx^4)}\, dx = \text{Timed out}$$

input

```
integrate(1/x**2/(b*x**4+a)/(d*x**4+c),x)
```

output

```
Timed out
```

3.784.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx =$$

$$-\frac{b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^2-\right.}{a}\right.}{8\left(abc-a^2d\right)}$$

$$+\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}x^2-\right.}{c}\right.}{8\left(bc^2-acd\right)}$$

$$-\frac{1}{acx}$$

input
```
integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/8*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*
a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 -
 sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b*c - a^2*d) +
 1/8*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1
/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*ar
ctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt
(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*
c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 -
 sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c^2 - a*c*d) -
 1/(a*c*x)
```

3.784.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 (a+bx^4) (c+dx^4)}\,dx = -\frac{(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)} - \frac{(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)} + \frac{(cd^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^3d-\sqrt{2}ac^2d^2\right)} + \frac{(cd^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^3d-\sqrt{2}ac^2d^2\right)} + \frac{(ab^3)^{\frac{3}{4}} \log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)} - \frac{(ab^3)^{\frac{3}{4}} \log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)} - \frac{(cd^3)^{\frac{3}{4}} \log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^3d-\sqrt{2}ac^2d^2\right)} + \frac{(cd^3)^{\frac{3}{4}} \log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^3d-\sqrt{2}ac^2d^2\right)} - \frac{1}{acx}$$

input

```
integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/2*(a*b^3)^(3/4)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c - sqrt
(2)*a^3*b*d) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(
1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/2*(c*d^3)^(3/
4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*
c^3*d - sqrt(2)*a*c^2*d^2) + 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/4*(a*b^3)^(3/4
)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^2*c - sqrt(2
)*a^3*b*d) - 1/4*(c*d^3)^(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d)
)/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt
(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) - 1/(
a*c*x)
```

3.784.9 Mupad [B] (verification not implemented)

Time = 10.36 (sec) , antiderivative size = 5962, normalized size of antiderivative = 12.96

$$\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
2*atan(((-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a
^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12
*b^8*c^11*d^9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3
 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 +
 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*
c^8*d))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^
14*b^10*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*
a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 10
24*a^20*b^4*c^12*d^12)*1i - 256*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^
5 - 768*a^13*b^10*c^17*d^6 + 256*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9
 - 768*a^17*b^6*c^13*d^10 + 768*a^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^1
2)*1i) + (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*
a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^1
2*b^8*c^11*d^9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^
3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9
+ 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3
*c^8*d))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a
^14*b^10*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096
*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1
024*a^20*b^4*c^12*d^12)*1i + 256*a^11*b^12*c^19*d^4 - 768*a^12*b^11*c^1...
```

3.784. $\int \frac{1}{x^2(a+bx^4)(c+dx^4)}\,dx$

3.785 $\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$

3.785.1 Optimal result

Integrand size = 22, antiderivative size = 462

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= -\frac{1}{3acx^3}+\frac{b^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)} \\ &\quad-\frac{b^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)}-\frac{d^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)} \\ &\quad+\frac{d^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)} \\ &\quad+\frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)} \\ &\quad-\frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)} \\ &\quad-\frac{d^{7/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{7/4}(bc-ad)} \\ &\quad+\frac{d^{7/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{7/4}(bc-ad)}\end{aligned}$$

output
```
-1/3/a/c/x^3-1/4*b^(7/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*
d+b*c)*2^(1/2)-1/4*b^(7/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a
*d+b*c)*2^(1/2)+1/4*d^(7/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(
-a*d+b*c)*2^(1/2)+1/4*d^(7/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/
(-a*d+b*c)*2^(1/2)+1/8*b^(7/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b
^(1/2))/a^(7/4)/(-a*d+b*c)*2^(1/2)-1/8*b^(7/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2
)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(7/4)*ln(-c^(1/4)*
d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(7
/4)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)*2
^(1/2)
```

3.785.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$
$$=\frac{\frac{8b}{a}-\frac{8d}{c}-\frac{6\sqrt{2}b^{7/4}x^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{6\sqrt{2}b^{7/4}x^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{6\sqrt{2}d^{7/4}x^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}}-\frac{6\sqrt{2}d^{7/4}x^3\,\mathrm{arc}}{}}{}$$

input
```
Integrate[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
((8*b)/a - (8*d)/c - (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/a^(7/4) + (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/a^(7/4) + (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)
/c^(1/4)])/c^(7/4) - (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)
/c^(1/4)])/c^(7/4) - (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*d^(7/4)*x^3*
Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4) - (3*Sqrt[
2]*d^(7/4)*x^3*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(
7/4))/(24*(-(b*c) + a*d)*x^3)
```

3.785.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {980, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+b x^4\right) \left(c+d x^4\right)} \, dx$$

$$\Big\downarrow \text{980}$$

$$\frac{\int -\frac{3\left(bdx^4+bc+ad\right)}{\left(bx^4+a\right)\left(dx^4+c\right)}dx}{3ac}-\frac{1}{3acx^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{bdx^4+bc+ad}{\left(bx^4+a\right)\left(dx^4+c\right)}dx}{ac}-\frac{1}{3acx^3}$$

$$\Big\downarrow \text{1020}$$

$$-\frac{\frac{b^2c\int \frac{1}{bx^4+a}dx}{bc-ad}-\frac{ad^2\int \frac{1}{dx^4+c}dx}{bc-ad}}{ac}-\frac{1}{3acx^3}$$

$$\Big\downarrow \text{755}$$

$$-\frac{\frac{b^2c\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}}{ac}-\frac{1}{3acx^3}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{\frac{b^2c\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}}{2\sqrt{c}}\right.}{bc-ad}}{ac}$$

$$\frac{1}{3acx^3}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{\frac{b^2c\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int\frac{1}{-\left(1-\sqrt{2}\sqrt[4]{}\right.}}{}\right.}{}}{ac}$$

$$\frac{1}{3acx^3}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{\frac{b^2c\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}}{ac}-$$

$$\frac{1}{3acx^3}$$

$$\Big\downarrow \text{ 1479}$$

$$-\frac{\frac{b^2c\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(-\frac{\int-\frac{}{\sqrt[4]{d}\left(\right.}}{}\right.}{}}{ac}$$

$$\frac{1}{3acx^3}$$

$$\Big\downarrow \text{ 25}$$

3.785. $\int \frac{1}{x^4(a+bx^4)(c+dx^4)}\,dx$

$$-\frac{\frac{b^2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}}{}\right.}}{2\sqrt{2}\sqrt[4]{}}\right.}{}}{ac}-\frac{1}{3acx^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{b^2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\int \frac{\sqrt{2}}{x^2+}}{2\sqrt{c}}\right.}{}}{ac}-\frac{1}{3acx^3}$$

$$\Big\downarrow \; 1103$$

$$-\frac{\frac{b^2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right.}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\right.}{}}{ac}-\frac{1}{3acx^3}$$

input

```
Int[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

3.785. $\int \frac{1}{x^4(a+bx^4)(c+dx^4)}\,dx$

output

```
-1/3*1/(a*c*x^3) - ((b^2*c*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sq
rt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]
*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b
*c - a*d) - (a*d^2*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^
(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)
*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + S
qrt[d]*x^2]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1
/4)*x + Sqrt[d]*x^2]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d
))/(a*c)
```

3.785.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 980

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.785.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.52

method	result
default	$\frac{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^{2}\left(ad-bc\right)}-\frac{1}{3ac\,x^{3}}-\frac{d^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.\right.}{}$
risch	$-\frac{1}{3ac\,x^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^{4}a^{11}-4a^{10}bc\,d^{3}+6a^{9}b^{2}c^{2}d^{2}-4a^{8}b^{3}c^{3}d+a^{7}b^{4}c^{4}\right)\_Z^{4}+b^{7}\right)}\_R\ln\left(\left(\left(-5a^{15}c^{7}d^{8}+38a^{14}b\,c^{8}d^{7}-128a^{13}b^{2}\right.\right.\right.\right.}{}$

input

```
int(1/x^4/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output

```
1/8/a^2*b^2/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(
a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b
)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/3/a/c/x^3-1/8/c^2*d^2/(a
*d-b*c)*(c/d)^(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x
^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2
*arctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

3.785.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.94 (sec) , antiderivative size = 1255, normalized size of antiderivative = 2.72

$$\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Too large to display}$$

input

```
integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output

```
-1/12*(3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10
*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x + (-b^7/(a^7*b^4*c^4 - 4*a^8
*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*(a^2*b*
c - a^3*d)) - 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 -
 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x - (-b^7/(a^7*b^4*c^4
- 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*
(a^2*b*c - a^3*d)) - 3*I*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*
c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x - (-b^7/(a^7
*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4
))^(1/4)*(I*a^2*b*c - I*a^3*d)) + 3*I*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d
 + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x
 - (-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d
^3 + a^11*d^4))^(1/4)*(-I*a^2*b*c + I*a^3*d)) - 3*(-d^7/(b^4*c^11 - 4*a*b^
3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x
^3*log(d^2*x + (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^
3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) + 3*(-d^7/(b^4*c^11 -
 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4
)*a*c*x^3*log(d^2*x - (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2
 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) + 3*I*(-d^7/(b
^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^...
```

3.785.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**4/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.785.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{\frac{2\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{b}\right.}{8\left(abc-a^2d\right)}+\frac{\frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{}\right.}{8\left(bc^2-acd\right)}-\frac{1}{3\,acx^3}$$

input
```
integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output

```
-1/8*(2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^
2*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(sqrt(b)*x^
2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(7/4)*log(sqr
t(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(a*b*c - a^2*d) +
 1/8*(2*sqrt(2)*d^2*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1
/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d^
2*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*d^(7/4)*log(sqrt(d)*x^
2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(7/4)*log(sqr
t(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c^2 - a*c*d) -
 1/3/(a*c*x^3)
```

3.785. $\int \frac{1}{x^4(a+bx^4)(c+dx^4)}\,dx$

3.785.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = &-\frac{\left(ab^3\right)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2bc-\sqrt{2}a^3d\right)} \\ &- \frac{\left(ab^3\right)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2bc-\sqrt{2}a^3d\right)} \\ &+ \frac{\left(cd^3\right)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^3-\sqrt{2}ac^2d\right)} \\ &+ \frac{\left(cd^3\right)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^3-\sqrt{2}ac^2d\right)} \\ &- \frac{\left(ab^3\right)^{\frac{1}{4}} b \log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^2bc-\sqrt{2}a^3d\right)} \\ &+ \frac{\left(ab^3\right)^{\frac{1}{4}} b \log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^2bc-\sqrt{2}a^3d\right)} \\ &+ \frac{\left(cd^3\right)^{\frac{1}{4}} d \log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^3-\sqrt{2}ac^2d\right)} \\ &- \frac{\left(cd^3\right)^{\frac{1}{4}} d \log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^3-\sqrt{2}ac^2d\right)} - \frac{1}{3\,acx^3}\end{aligned}$$

input

```
integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output

```
-1/2*(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^
(1/4))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) - 1/2*(a*b^3)^(1/4)*b*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b*c - sqrt(2
)*a^3*d) + 1/2*(c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/
4))/(c/d)^(1/4))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) + 1/2*(c*d^3)^(1/4)*d*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^3
- sqrt(2)*a*c^2*d) - 1/4*(a*b^3)^(1/4)*b*log(x^2 + sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) + 1/4*(a*b^3)^(1/4)*b*log(x^
2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) +
 1/4*(c*d^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)
*b*c^3 - sqrt(2)*a*c^2*d) - 1/4*(c*d^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(c/d)^
(1/4) + sqrt(c/d))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) - 1/3/(a*c*x^3)
```

3.785.9 Mupad [B] (verification not implemented)

Time = 10.65 (sec) , antiderivative size = 7459, normalized size of antiderivative = 16.15

$$\int \frac{1}{x^4 (a+bx^4) (c+dx^4)} \, dx = \text{Too large to display}$$

input

```
int(1/(x^4*(a + b*x^4)*(c + d*x^4)),x)
```

output

```
- atan((a^2*b^5*d^7*x*1i + b^7*c^2*d^5*x*1i - (a^2*b^16*c^11*x*256i)/(256*
a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1
024*a^10*b*c*d^3) - (a^4*b^14*c^9*d^2*x*1536i)/(256*a^11*d^4 + 256*a^7*b^4
*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a
^5*b^13*c^8*d^3*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^
3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^6*b^12*c^7*d^4*x*256i
)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*
d^2 - 1024*a^10*b*c*d^3) - (a^7*b^11*c^6*d^5*x*256i)/(256*a^11*d^4 + 256*a
^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3
) + (a^8*b^10*c^5*d^6*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*
b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^9*b^9*c^4*d^7*x
*1536i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^
2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^10*b^8*c^3*d^8*x*1024i)/(256*a^11*d^4
+ 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*
b*c*d^3) - (a^11*b^7*c^2*d^9*x*256i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 102
4*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^3*b^15*c^
10*d*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*
a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))/((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c
^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)
*((b^7*(1024*a^4*b^8*c^12 + 1024*a^12*c^4*d^8 - 5120*a^5*b^7*c^11*d - 5...
```

3.785. $\int \frac{1}{x^4(a+bx^4)(c+dx^4)}\,dx$

3.786 $\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$

3.786.1 Optimal result

Integrand size = 22, antiderivative size = 479

$$
\begin{aligned}
\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = {} & -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{b^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)} \\
& +\frac{b^{9/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)}+\frac{d^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{9/4}(bc-ad)} \\
& -\frac{d^{9/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{9/4}(bc-ad)} \\
& +\frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)} \\
& -\frac{b^{9/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)} \\
& -\frac{d^{9/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{9/4}(bc-ad)} \\
& +\frac{d^{9/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{9/4}(bc-ad)}
\end{aligned}
$$

output
```
-1/5/a/c/x^5+(a*d+b*c)/a^2/c^2/x+1/4*b^(9/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a
^(1/4))/a^(9/4)/(-a*d+b*c)*2^(1/2)+1/4*b^(9/4)*arctan(1+b^(1/4)*x*2^(1/2)/
a^(1/4))/a^(9/4)/(-a*d+b*c)*2^(1/2)-1/4*d^(9/4)*arctan(-1+d^(1/4)*x*2^(1/2
)/c^(1/4))/c^(9/4)/(-a*d+b*c)*2^(1/2)-1/4*d^(9/4)*arctan(1+d^(1/4)*x*2^(1/
2)/c^(1/4))/c^(9/4)/(-a*d+b*c)*2^(1/2)+1/8*b^(9/4)*ln(-a^(1/4)*b^(1/4)*x*2
^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(9/4)/(-a*d+b*c)*2^(1/2)-1/8*b^(9/4)*ln(a^(1
/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(9/4)/(-a*d+b*c)*2^(1/2)-1/8*
d^(9/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(9/4)/(-a*d+b
*c)*2^(1/2)+1/8*d^(9/4)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/
c^(9/4)/(-a*d+b*c)*2^(1/2)
```

3.786.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^6 (a+bx^4)(c+dx^4)}\,dx$$

$$= \frac{8b}{a} - \frac{8d}{c} - \frac{40b^2x^4}{a^2} + \frac{40d^2x^4}{c^2} + \frac{10\sqrt{2}b^{9/4}x^5\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{9/4}} - \frac{10\sqrt{2}b^{9/4}x^5\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{9/4}} - \frac{10\sqrt{2}d^{9/4}x^5\arctan\left(1-\right.}{c^{9/4}}$$

input
```
Integrate[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
((8*b)/a - (8*d)/c - (40*b^2*x^4)/a^2 + (40*d^2*x^4)/c^2 + (10*Sqrt[2]*b^(
9/4)*x^5*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*b^
(9/4)*x^5*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*d
^(9/4)*x^5*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(9/4) + (10*Sqrt[2]*
d^(9/4)*x^5*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(9/4) - (5*Sqrt[2]*
b^(9/4)*x^5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(9/4
) + (5*Sqrt[2]*b^(9/4)*x^5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/a^(9/4) + (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/c^(9/4) - (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(9/4))/(40*(-(b*c) + a*d)*x^5)
```

3.786.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {980, 27, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{980}$$

$$\frac{\int -\frac{5(bdx^4+bc+ad)}{x^2(bx^4+a)(dx^4+c)}dx}{5ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{bdx^4+bc+ad}{x^2(bx^4+a)(dx^4+c)}dx}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{1053}$$

$$-\frac{-\frac{\int \frac{x^2\left(bd(bc+ad)x^4+b^2c^2+a^2d^2+abcd\right)}{\left(bx^4+a\right)\left(dx^4+c\right)}dx}{ac}-\frac{ad+bc}{acx}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{1054}$$

$$-\frac{-\frac{\int \left(\frac{c^2x^2b^3}{(bc-ad)\left(bx^4+a\right)}+\frac{a^2d^3x^2}{(ad-bc)\left(dx^4+c\right)}\right)dx}{ac}-\frac{ad+bc}{acx}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{\frac{a^2d^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{a^2d^{9/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{a^2d^{9/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{a^2d^{9/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}-}{ac}}{a}$$

$$\frac{1}{5acx^5}$$

input
```
Int[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

output

```
-1/5*1/(a*c*x^5) - (-((b*c + a*d)/(a*c*x)) - (-1/2*(b^(9/4)*c^2*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)) + (b^(9/4)*c^
2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d))
 + (a^2*d^(9/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(1/4
)*(b*c - a*d)) - (a^2*d^(9/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*
Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(9/4)*c^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (b^(9/4)*c^2*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*
(b*c - a*d)) - (a^2*d^(9/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (a^2*d^(9/4)*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)))/(a
*c))/(a*c)
```

3.786.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.786.4 Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.54

method	result
default	$-\frac{1}{5ac\,x^{5}}-\frac{-ad-bc}{a^{2}c^{2}x}-\frac{b^{2}\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^{2}(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+$
risch	Expression too large to display

input
```
int(1/x^6/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/5/a/c/x^5-1/a^2/c^2*(-a*d-b*c)/x-1/8*b^2/a^2/(a*d-b*c)/(a/b)^(1/4)*2^(1
/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)
+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1
/4)*x-1))+1/8*d^2/c^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*(ln((x^2-(c/d)^(1/4)*x
*2^(1/2)+(c/d)^(1/2))/(x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^
(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

3.786.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.97 (sec) , antiderivative size = 1526, normalized size of antiderivative = 3.19

$$\int \frac{1}{x^{6}\left(a+bx^{4}\right)\left(c+dx^{4}\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output

```
1/20*(5*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^1
2*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^5*log(b^7*x + (a^7*b^3*c^3 - 3*a^8*
b^2*c^2*d + 3*a^9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*
d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) - 5*(-b^9/(a^9
*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d
^4))^(1/4)*a^2*c^2*x^5*log(b^7*x - (a^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^9*
b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2
*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) + 5*I*(-b^9/(a^9*b^4*c^4 - 4*a^1
0*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c
^2*x^5*log(b^7*x - (I*a^7*b^3*c^3 - 3*I*a^8*b^2*c^2*d + 3*I*a^9*b*c*d^2 -
I*a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4
*a^12*b*c*d^3 + a^13*d^4))^(3/4)) - 5*I*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^
3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^5*l
og(b^7*x - (-I*a^7*b^3*c^3 + 3*I*a^8*b^2*c^2*d - 3*I*a^9*b*c*d^2 + I*a^10*
d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b
*c*d^3 + a^13*d^4))^(3/4)) - 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^
2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^5*log(d^7*x
+ (b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4*c^
13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4)
)^(3/4)) + 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*...
```

3.786.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^4) (c+dx^4)} \, dx = \text{Timed out}$$

input

```
integrate(1/x**6/(b*x**4+a)/(d*x**4+c),x)
```

output

```
Timed out
```

3.786.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

$$= \frac{b^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^2-\sqrt{2}\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}}{8\left(a^2bc-a^3d\right)}$$

$$-\frac{d^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}x^2-\right.}{c}}{8\left(bc^3-ac^2d\right)}$$

$$+\frac{5\left(bc+ad\right)x^4-ac}{5\,a^2c^2x^5}$$

input
```
integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/8*b^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 -
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^2*b*c - a^3*d)
- 1/8*d^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(
1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*a
rctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqr
t(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)
*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2
- sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c^3 - a*c^2*d
) + 1/5*(5*(b*c + a*d)*x^4 - a*c)/(a^2*c^2*x^5)
```

3.786.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^3bc-\sqrt{2}a^4d\right)} + \frac{\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^3bc-\sqrt{2}a^4d\right)} - \frac{\left(cd^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^4-\sqrt{2}ac^3d\right)} - \frac{\left(cd^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^4-\sqrt{2}ac^3d\right)} - \frac{\left(ab^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^3bc-\sqrt{2}a^4d\right)} + \frac{\left(ab^3\right)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^3bc-\sqrt{2}a^4d\right)} + \frac{\left(cd^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^4-\sqrt{2}ac^3d\right)} - \frac{\left(cd^3\right)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^4-\sqrt{2}ac^3d\right)} + \frac{5\,bcx^4+5\,adx^4-ac}{5\,a^2c^2x^5}$$

input
```
integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4
*d) - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/
d)^(1/4))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d) - 1/2*(c*d^3)^(3/4)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^4 - sqrt(2)
*a*c^3*d) - 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/4*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/4*(c*d^3)^
(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^4 - sqrt(2
)*a*c^3*d) - 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d)
)/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d) + 1/5*(5*b*c*x^4 + 5*a*d*x^4 - a*c)/(a
^2*c^2*x^5)
```

3.786.9 Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 4547, normalized size of antiderivative = 9.49

$$\int \frac{1}{x^6\,(a+bx^4)\,(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^6*(a + b*x^4)*(c + d*x^4)),x)
```

output

```
- 2*atan((1024*a^11*b^10*c^13*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 10
24*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 4*
a^11*b^6*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d
 + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4) + 1024*a^21*c^3*d^10*
x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*
b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^12*b^9*c^12*d*x*(-b^9/(25
6*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2
 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^20*b*c^4*d^9*x*(-b^9/(256*a^13*d^4 +
 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12
*b*c*d^3))^(5/4) + 4*a^8*b^9*c^3*d^6*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c
^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/
4) + 6144*a^13*b^8*c^11*d^2*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024
*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096
*a^14*b^7*c^10*d^3*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3
*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 1024*a^15*b^6
*c^9*d^4*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1
536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 1024*a^17*b^4*c^7*d^6*x
*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b
^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^18*b^3*c^6*d^7*x*(-b^9/(25
6*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*...
```

3.786. $\int \frac{1}{x^6(a+bx^4)(c+dx^4)}\,dx$

3.787 $\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.787.1 Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = -\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd}+\frac{a\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}$$

output
```
1/6*(d*x^4+c)^(3/2)/b/d+1/2*a*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(
1/2))*(-a*d+b*c)^(1/2)/b^(5/2)-1/2*a*(d*x^4+c)^(1/2)/b^2
```

3.787.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{\sqrt{c+dx^4}\left(-3ad+b\left(c+dx^4\right)\right)}{6b^2d}+\frac{a\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{2b^{5/2}}$$

input
```
Integrate[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(Sqrt[c + d*x^4]*(-3*a*d + b*(c + d*x^4)))/(6*b^2*d) + (a*Sqrt[-(b*c) + a*
d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(2*b^(5/2))
```

3.787.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {948, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{4}\int \frac{x^4\sqrt{dx^4+c}}{bx^4+a}dx^4$$

$$\Big\downarrow \; 90$$

$$\frac{1}{4}\left(\frac{2\left(c+dx^4\right)^{3/2}}{3bd}-\frac{a\int \frac{\sqrt{dx^4+c}}{bx^4+a}dx^4}{b}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{4}\left(\frac{2\left(c+dx^4\right)^{3/2}}{3bd}-\frac{a\left(\frac{(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{b}+\frac{2\sqrt{c+dx^4}}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{4}\left(\frac{2\left(c+dx^4\right)^{3/2}}{3bd}-\frac{a\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{bd}+\frac{2\sqrt{c+dx^4}}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{4}\left(\frac{2\left(c+dx^4\right)^{3/2}}{3bd}-\frac{a\left(\frac{2\sqrt{c+dx^4}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}\right)$$

input
```
Int[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output

```
((2*(c + d*x^4)^(3/2))/(3*b*d) - (a*((2*Sqrt[c + d*x^4])/b - (2*Sqrt[b*c -
 a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/b^(3/2)))/b)/4
```

3.787.3.1 Defintions of rubi rules used

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.787. $\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.787.4 Maple [A] (verified)

Time = 6.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{dx^4+c}\,\left(-bdx^4+3ad-bc\right)}{3}+\frac{ad(ad-bc)\arctan\left(\frac{b\sqrt{dx^4+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{2b^2d}$
risch	$-\frac{\left(-bdx^4+3ad-bc\right)\sqrt{dx^4+c}}{6db^2}+\frac{(ad-bc)a\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-a}}{}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}}\right)}{}$
elliptic	$\frac{\left(dx^4+c\right)^{\frac{3}{2}}}{6bd}-\frac{a\left(\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{-\frac{d\sqrt{-ab}}{b}+d\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)}\right)}{b}\right)}{}$
default	$\frac{\left(dx^4+c\right)^{\frac{3}{2}}}{6bd}-\frac{a\left(\frac{\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{-\frac{d\sqrt{-ab}}{b}+d\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2}\right)}{b}}{}\right)}{}$

input
```
int(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*(-1/3*(d*x^4+c)^(1/2)*(-b*d*x^4+3*a*d-b*c)+a*d*(a*d-b*c)/((a*d-b*c
)*b)^(1/2)*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2)))/d
```

3.787.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.10

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+2\,(bdx^4+bc-3\,ad)\sqrt{dx^4+c}}{12\,b^2d}, \quad 3\,ad\sqrt{-\frac{bc-ad}{b}}\arctan\Bigl(-\right.$$

input
```
integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/12*(3*a*d*sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4
 + c)*b*sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*(b*d*x^4 + b*c - 3*a*d)*sqrt
(d*x^4 + c))/(b^2*d), 1/6*(3*a*d*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 +
 c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b*d*x^4 + b*c - 3*a*d)*sqrt(d*x
^4 + c))/(b^2*d)]
```

3.787.6 Sympy [A] (verification not implemented)

Time = 5.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.31

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \begin{cases} \frac{2\left(-\frac{ad^2\sqrt{c+dx^4}}{4b^2}+\frac{ad^2(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^4\right)^{\frac{3}{2}}}{12b}\right)}{d^2} & \text{for } d\neq 0 \\ \sqrt{c}\left(-\frac{a\left(\begin{cases}\frac{x^4}{a} & \text{for } b=0 \\ \frac{\log\left(a+bx^4\right)}{b} & \text{otherwise}\end{cases}\right)}{4b}+\frac{x^4}{4b}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x**7*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Piecewise((2*(-a*d**2*sqrt(c + d*x**4)/(4*b**2) + a*d**2*(a*d - b*c)*atan(
sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*b**3*sqrt((a*d - b*c)/b)) + d*(c
+ d*x**4)**(3/2)/(12*b))/d**2, Ne(d, 0)), (sqrt(c)*(-a*Piecewise((x**4/a,
Eq(b, 0)), (log(a + b*x**4)/b, True))/(4*b) + x**4/(4*b)), True))
```

3.787.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.787.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = -\frac{(abc-a^2d)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}b^2} + \frac{(dx^4+c)^{\frac{3}{2}}b^2d^2 - 3\sqrt{dx^4+c}abd^3}{6\,b^3d^3}$$

input
```
integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
-1/2*(a*b*c - a^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(
-b^2*c + a*b*d)*b^2) + 1/6*((d*x^4 + c)^(3/2)*b^2*d^2 - 3*sqrt(d*x^4 + c)*
a*b*d^3)/(b^3*d^3)
```

3.787.9 Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{\left(d\,x^4+c\right)^{3/2}}{6\,b\,d} - \frac{a\,\sqrt{d\,x^4+c}}{2\,b^2} + \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^4+c}\,\sqrt{a\,d-b\,c}}{a^2\,d-a\,b\,c}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{5/2}}$$

input
```
int((x^7*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
(c + d*x^4)^(3/2)/(6*b*d) - (a*(c + d*x^4)^(1/2))/(2*b^2) + (a*atan((a*b^(
1/2)*(c + d*x^4)^(1/2)*(a*d - b*c)^(1/2))/(a^2*d - a*b*c))*(a*d - b*c)^(1/
2))/(2*b^(5/2))
```

3.788 $\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.788.1 Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{x^2\sqrt{c+dx^4}}{4b} - \frac{\sqrt{a}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2} + \frac{(bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}}$$

output
```
1/4*(-2*a*d+b*c)*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b^2/d^(1/2)-1/2*arct
an(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*a^(1/2)*(-a*d+b*c)^(1/2)/
b^2+1/4*x^2*(d*x^4+c)^(1/2)/b
```

3.788.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.18

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$$
$$= \frac{b\sqrt{d}x^2\sqrt{c+dx^4} - 2\sqrt{a}\sqrt{d}\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right) + (bc-2ad)\log\Bigg(\sqrt{d}x^2+\sqrt{c+dx^4}\Bigg)}{4b^2\sqrt{d}}$$

input
```
Integrate[(x^5*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(b*Sqrt[d]*x^2*Sqrt[c + d*x^4] - 2*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[
(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*
d])] + (b*c - 2*a*d)*Log[Sqrt[d]*x^2 + Sqrt[c + d*x^4]])/(4*b^2*Sqrt[d])
```

3.788.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {965, 380, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{x^4\sqrt{dx^4+c}}{bx^4+a}dx^2$$

$$\Big\downarrow \text{ 380}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2b}-\frac{\int \frac{ac-(bc-2ad)x^4}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{2b}\right)$$

$$\Big\downarrow \text{ 398}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2b}-\frac{\frac{2a(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}-\frac{(bc-2ad)\int \frac{1}{\sqrt{dx^4+c}}dx^2}{b}}{2b}\right)$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2b}-\frac{\frac{2a(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}-\frac{(bc-2ad)\int \frac{1}{1-dx^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}}{2b}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2b}-\frac{\frac{2a(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}-\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}}{2b}\right)$$

$$\Big\downarrow \; 291$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2b}-\frac{\frac{2a(bc-ad)\int\frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}-\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}}{2b}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2b}-\frac{\frac{2\sqrt{a}\sqrt{bc-ad}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{b}-\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}}{2b}\right)$$

input
```
Int[(x^5*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
((x^2*Sqrt[c + d*x^4])/(2*b) - ((2*Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*
c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/b - ((b*c - 2*a*d)*ArcTanh[(Sqrt
[d]*x^2)/Sqrt[c + d*x^4]])/(b*Sqrt[d]))/(2*b))/2
```

3.788.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.788.4 Maple [A] (verified)

Time = 5.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{\left(-d^{\frac{3}{2}} a^{2}+\sqrt{d}\, a b c \right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}\, a}{x^{2} \sqrt{\left(a d -b c \right) a}}\right)+\sqrt{\left(a d -b c \right) a}\, \left(\left(a d -\frac{b c}{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}}{x^{2} \sqrt{d}}\right)-\frac{\sqrt{d\,x^{4}+c}\, b\,x^{2} \sqrt{d}}{2}\right)}{2 \sqrt{\left(a d -b c \right) a}\, \sqrt{d}\, b^{2}}$
risch	$\frac{x^{2} \sqrt{d\,x^{4}+c}}{4 b}-\frac{\frac{\left(2 a d -b c \right) \ln\left(x^{2} \sqrt{d}+\sqrt{d\,x^{4}+c}\right)}{2 b \sqrt{d}}-\frac{2 \left(a d -b c \right) a \left(\frac{\ln\left(\frac{-\frac{2 \left(a d -b c \right)}{b}-\frac{2 d \sqrt{-a b}\, \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d -b c}{b}}\, \sqrt{d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2}}}{x^{2}+\frac{\sqrt{-a b}}{b}}\right.}{4 \sqrt{-a b}\, \sqrt{-\frac{a d -b c}{b}}}\right.}{}}{}$
default	$\frac{\frac{x^{2} \sqrt{d\,x^{4}+c}}{4}+\frac{c \ln\left(x^{2} \sqrt{d}+\sqrt{d\,x^{4}+c}\right)}{4 \sqrt{d}}}{b}-\frac{a \left(-\frac{\sqrt{d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\, \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d -b c}{b}}-\frac{\sqrt{d}\, \sqrt{-a b}\, \ln\left(\frac{-\frac{d \sqrt{-a b}}{b}+d \left(x^{2}+\right.}{\sqrt{d}}\right.}{}}{}\right.}{}$
elliptic	$\frac{\frac{x^{2} \sqrt{d\,x^{4}+c}}{2}+\frac{c \ln\left(x^{2} \sqrt{d}+\sqrt{d\,x^{4}+c}\right)}{2 \sqrt{d}}}{2 b}+\frac{a \left(\sqrt{d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\, \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d -b c}{b}}-\frac{\sqrt{d}\, \sqrt{-a b}\, \ln\left(\frac{-\frac{d \sqrt{-a b}}{b}+d \left(x^{2}+\right.}{\sqrt{d}}\right.}{}\right.}{}$

input
```
int(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((a*d-b*c)*a)^(1/2)/d^(1/2)*((-d^(3/2)*a^2+d^(1/2)*a*b*c)*arctanh((d*
x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))+((a*d-b*c)*a)^(1/2)*((a*d-1/2*b*c)
*arctanh((d*x^4+c)^(1/2)/x^2/d^(1/2))-1/2*(d*x^4+c)^(1/2)*b*x^2*d^(1/2)))/
b^2
```

3.788. $\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.788.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 714, normalized size of antiderivative = 5.95

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$$

$$= \left[\frac{2\sqrt{dx^4+c}bdx^2-(bc-2\,ad)\sqrt{d}\log\left(-2\,dx^4+2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)+\sqrt{-abc+a^2d}d\log\left(\frac{(b^2c^2-8\,abcd+}{}\right.}{8\,b^2d}\right.$$

input
```
integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*s
qrt(d*x^4 + c)*sqrt(d)*x^2 - c) + sqrt(-a*b*c + a^2*d)*d*log(((b^2*c^2 - 8
*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((
b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8
 + 2*a*b*x^4 + a^2)))/(b^2*d), 1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 - 2*(b*c - 2
*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + sqrt(-a*b*c + a^2*d)
*d*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*
x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*
c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(b^2*d), 1/8*(2*sqrt(d*x^4 + c)*
b*d*x^2 - 2*sqrt(a*b*c - a^2*d)*d*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqr
t(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2
*c*d)*x^2)) - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(
d)*x^2 - c))/(b^2*d), 1/4*(sqrt(d*x^4 + c)*b*d*x^2 - (b*c - 2*a*d)*sqrt(-d
)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - sqrt(a*b*c - a^2*d)*d*arctan(1/2*
((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d -
a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)))/(b^2*d)]
```

3.788.6 Sympy [F]

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate(x**5*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(x**5*sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.788.7 Maxima [F]

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{dx^4+c}\,x^5}{bx^4+a}\,dx$$

input
```
integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)*x^5/(b*x^4 + a), x)
```

3.788.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.788.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{x^5\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input
```
int((x^5*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
int((x^5*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.789 $\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.789.1 Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{\sqrt{c+dx^4}}{2b}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

output
```
-1/2*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^
(3/2)+1/2*(d*x^4+c)^(1/2)/b
```

3.789.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{1}{2}\left(\frac{\sqrt{c+dx^4}}{b}-\frac{\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}\right)$$

input
```
Integrate[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(Sqrt[c + d*x^4]/b - (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/
Sqrt[-(b*c) + a*d]])/b^(3/2))/2
```

3.789.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {946, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{4}\int \frac{\sqrt{dx^4+c}}{bx^4+a}dx^4$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^4}{b}+\frac{2\sqrt{c+dx^4}}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{bd}+\frac{2\sqrt{c+dx^4}}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{2\sqrt{c+dx^4}}{b}-\frac{2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)$$

input
```
Int[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
((2*Sqrt[c + d*x^4])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^
4])/Sqrt[b*c - a*d]])/b^(3/2))/4
```

3.789.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.789.4 Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$\frac{\sqrt{d\,x^4+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{2b}$
risch	$\frac{\sqrt{d\,x^4+c}}{2b}-\frac{(ad-bc)\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4b\sqrt{-\frac{ad-bc}{b}}}\right.}{b}$
default	$\frac{\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{-\frac{d\sqrt{-ab}}{b}+d\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\right.}{b}}\right.}{b}}{}$
elliptic	$\frac{\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{-\frac{d\sqrt{-ab}}{b}+d\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\right.}{b}}\right.}{b}}{}$

input
```
int(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*((d*x^4+c)^(1/2)-(a*d-b*c)*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1
/2))/((a*d-b*c)*b)^(1/2))
```

3.789.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.23

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}}{4\,b},\right.$$
$$\left.-\frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx^4+c}}{2\,b}\right]$$

input
```
integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*b
*sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c))/b, -1/2*(sqrt(-(b*
c - a*d)/b)*arctan(-sqrt(d*x^4 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) -
sqrt(d*x^4 + c))/b]
```

3.789.6 Sympy [A] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \begin{cases} \frac{2\left(\frac{d\sqrt{c+dx^4}}{4b} - \frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for } d \neq 0 \\ \sqrt{c}\left(\begin{cases} \frac{x^4}{4a} & \text{for } b = 0 \\ \frac{\log\left(4a+4bx^4\right)}{4b} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Piecewise((2*(d*sqrt(c + d*x**4)/(4*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**
4)/sqrt((a*d - b*c)/b))/(4*b**2*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (sqrt(
c)*Piecewise((x**4/(4*a), Eq(b, 0)), (log(4*a + 4*b*x**4)/(4*b), True)), T
rue))
```

3.789.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.789.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}b} + \frac{\sqrt{dx^4+c}}{2b}$$

input
```
integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/2*(b*c - a*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*b) + 1/2*sqrt(d*x^4 + c)/b
```

3.789.9 Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{\sqrt{d\,x^4+c}}{2\,b} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{3/2}}$$

input
```
int((x^3*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
(c + d*x^4)^(1/2)/(2*b) - (atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1
/2))*(a*d - b*c)^(1/2))/(2*b^(3/2))
```

3.790 $\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.790.1 Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}b}+\frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

output
```
1/2*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))*d^(1/2)/b+1/2*arctan(x^2*(-a*d+b*
c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*(-a*d+b*c)^(1/2)/b/a^(1/2)
```

3.790.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.18

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{\frac{\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}}+\sqrt{d}\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{2b}$$

input
```
Integrate[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
((Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]
))/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[a] + Sqrt[d]*Log[Sqrt[d]*x^2 + Sqrt[c
+ d*x^4]])/(2*b)
```

3.790.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {965, 301, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{\sqrt{dx^4+c}}{bx^4+a}dx^2$$

$$\Big\downarrow \text{301}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}+\frac{d\int \frac{1}{\sqrt{dx^4+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}+\frac{d\int \frac{1}{1-dx^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{\sqrt{bc-ad}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{\sqrt{a}b}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b}\right)$$

input

```
Int[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
((Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])
/(Sqrt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/b)/2
```

3.790.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.790. $\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.790.4 Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$-\frac{\frac{(ad-bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}-\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{x^2\sqrt{d}}\right)}{2b}$
default	$-\frac{\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{-\frac{d\sqrt{-ab}}{b}+d\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\right.}{b}}\right.}{b}}{}$
elliptic	$-\frac{\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{-\frac{d\sqrt{-ab}}{b}+d\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\right.}{b}}\right.}{b}}{}$

input
```
int(x*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*((a*d-b*c)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))/((a*d
-b*c)*a)^(1/2)-d^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2/d^(1/2)))
```

3.790.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 612, normalized size of antiderivative = 6.73

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$$

$$= \left[\frac{2\,\sqrt{d}\log\left(-2\,dx^4-2\,\sqrt{dx^4+c}\sqrt{d}x^2-c\right)+\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((\right.}{b^2x^8+2\,abx^4+a^2}\right.}{8\,b}\right.$$

$$-\frac{4\,\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^2}{\sqrt{dx^4+c}}\right)-\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((abc-2\,a^2d)x^6-a^2cx^2)\sqrt{\right.}{b^2x^8+2\,abx^4+a^2}\right.}{8\,b}$$

$$\left.-\frac{2\,\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^2}{\sqrt{dx^4+c}}\right)-\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^6+(bc^2-acd)x^2)}\right)}{4\,b}\right]$$

input
```
integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + sqrt(-
(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 -
 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x
^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/b, -1/8*(4*sqr
t(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - sqrt(-(b*c - a*d)/a)*log(((b^
2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c
^2 + 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*
d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/b, 1/4*(sqrt((b*c - a*d)/a)*arctan(1/
2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d -
a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2)) + sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 +
 c)*sqrt(d)*x^2 - c))/b, -1/4*(2*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 +
 c)) - sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4
 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2)))/b]
```

3.790.6 Sympy [F]

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate(x*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(x*sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.790.7 Maxima [F]

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\,x}{bx^4+a}\,dx$$

input
```
integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)*x/(b*x^4 + a), x)
```

3.790.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.790.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input

```
int((x*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output

```
int((x*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.791 $\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx$

3.791.1 Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx = -\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a} + \frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}$$

output
```
-1/2*arctanh((d*x^4+c)^(1/2)/c^(1/2))*c^(1/2)/a+1/2*arctanh(b^(1/2)*(d*x^4
+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a/b^(1/2)
```

3.791.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx = \frac{\frac{\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}} - \sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

input
```
Integrate[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
```

output
```
((Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])
/Sqrt[b] - Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(2*a)
```

3.791.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{x\left(a+bx^4\right)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{4}\int \frac{\sqrt{dx^4+c}}{x^4\left(bx^4+a\right)}dx^4$$

$$\Big\downarrow \text{ 94}$$

$$\frac{1}{4}\left(\frac{c\int \frac{1}{x^4\sqrt{dx^4+c}}dx^4}{a}-\frac{(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{a}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{4}\left(\frac{2c\int \frac{1}{\frac{x^8}{d}-\frac{c}{d}}d\sqrt{dx^4+c}}{ad}-\frac{2(bc-ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{ad}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{4}\left(\frac{2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{a}\right)$$

input
```
Int[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
```

output
```
((-2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/a + (2*Sqrt[b*c - a*d]*ArcT
anh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/4
```

3.791.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.791.4 Maple [A] (verified)

Time = 4.87 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d\,x^{4}+c}}{\sqrt{(ad-bc)b}}\right)ad-\arctan\left(\frac{b\sqrt{d\,x^{4}+c}}{\sqrt{(ad-bc)b}}\right)bc-\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}}{\sqrt{c}}\right)\sqrt{(ad-bc)b}}{2a\sqrt{(ad-bc)b}}$
elliptic	$\frac{\sqrt{d\,x^{4}+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{4}+c}}{x^{2}}\right)}{2a}-$ $\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\,\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-$ $\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{-\frac{d\sqrt{-ab}}{b}+d\left(x^{2}+\sqrt{\right.}}{\sqrt{d}}\right.$
default	$\frac{\frac{\sqrt{d\,x^{4}+c}}{2}-\frac{\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{4}+c}}{x^{2}}\right)}{2}}{a}-$ $b\left(\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\,\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-\right.$ $\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{-\frac{d\sqrt{-ab}}{b}+d\left(x^{2}+\right.}{\sqrt{d}}\right.$

input
```
int((d*x^4+c)^(1/2)/x/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2))*a*d-arctan(b*(d*x^4+c)^
(1/2)/((a*d-b*c)*b)^(1/2))*b*c-c^(1/2)*arctanh((d*x^4+c)^(1/2)/c^(1/2))*((
a*d-b*c)*b)^(1/2))/a/((a*d-b*c)*b)^(1/2)
```

3.791.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.51

$$\int \frac{\sqrt{c+dx^4}}{x\,(a+bx^4)}\,dx$$

$$= \left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+\sqrt{c}\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)}{4\,a}, \frac{2\,\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}b\sqrt{}}{bc-a}\right.}{}\right.$$

input
```
integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*b
*sqrt((b*c - a*d)/b))/(b*x^4 + a)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c
)*sqrt(c) + 2*c)/x^4))/a, 1/4*(2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 +
 c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^
4 + c)*sqrt(c) + 2*c)/x^4))/a, 1/4*(2*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt
(-c)/c) + sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 +
c)*b*sqrt((b*c - a*d)/b))/(b*x^4 + a)))/a, 1/2*(sqrt(-(b*c - a*d)/b)*arcta
n(-sqrt(d*x^4 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(-c)*arctan(s
qrt(d*x^4 + c)*sqrt(-c)/c))/a]
```

3.791.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(70) = 140$.

Time = 4.43 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx = \begin{cases} \frac{2\left(\frac{cd\,\mathrm{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}+\frac{d(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4ab\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for}\ d\neq 0 \\ \sqrt{c}\left(-\frac{b\left(\begin{cases}\frac{\frac{a}{2b}+x^4}{a} & \text{for}\ b=0\\ -\frac{\log\left(a-2b\left(\frac{a}{2b}+x^4\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{2a}-\frac{b\left(\begin{cases}\frac{\frac{a}{2b}+x^4}{a} & \text{for}\ b=0\\ \frac{\log\left(a+2b\left(\frac{a}{2b}+x^4\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{2a}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((d*x**4+c)**(1/2)/x/(b*x**4+a),x)
```

output
```
Piecewise((2*(c*d*atan(sqrt(c + d*x**4)/sqrt(-c))/(4*a*sqrt(-c)) + d*(a*d
- b*c)*atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*a*b*sqrt((a*d - b*c)/
b)))/d, Ne(d, 0)), (sqrt(c)*(-b*Piecewise(((a/(2*b) + x**4)/a, Eq(b, 0)),
(-log(a - 2*b*(a/(2*b) + x**4))/(2*b), True))/(2*a) - b*Piecewise(((a/(2*b
) + x**4)/a, Eq(b, 0)), (log(a + 2*b*(a/(2*b) + x**4))/(2*b), True))/(2*a)
), True))
```

3.791.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x), x)
```

3.791.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^4}}{x\left(a+bx^4\right)}\,dx=-\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a}+\frac{c\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2\,a\sqrt{-c}}$$

input
```
integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="giac")
```

output
```
-1/2*(b*c - a*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2
*c + a*b*d)*a) + 1/2*c*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a*sqrt(-c))
```

3.791.9 Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{c+dx^4}}{x\left(a+bx^4\right)}\,dx$$
$$=\frac{\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c}\left(\sqrt{d\,x^4+c}\left(\frac{a^2\,b\,d^4}{2}-a\,b^2\,c\,d^3+b^3\,c^2\,d^2\right)+\frac{c\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\sqrt{d\,x^4+c}}{16\,a^2}\right)}{2\,a\left(\frac{b^2\,c^2\,d^3}{4}-\frac{a\,b\,c\,d^4}{4}\right)}\right)}{2\,a}+\frac{\mathrm{atanh}\left(\frac{a\,b^2\,c\,d^3\sqrt{d\,x^4+c}\sqrt{b^2\,c-a\,b\,d}}{4\left(\frac{a\,b^3\,c^2\,d^3}{4}-\frac{a^2\,b^2\,c\,d^4}{4}\right)}\right)\sqrt{b^2\,c-a\,b\,d}}{2\,a\,b}$$

input
```
int((c + d*x^4)^(1/2)/(x*(a + b*x^4)),x)
```

output
```
(c^(1/2)*atanh((c^(1/2)*((c + d*x^4)^(1/2)*((a^2*b*d^4)/2 + b^3*c^2*d^2 -
a*b^2*c*d^3) + (c*(8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2))/(1
6*a^2)))/(2*a*((b^2*c^2*d^3)/4 - (a*b*c*d^4)/4))))/(2*a) + (atanh((a*b^2*c
*d^3*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*((a*b^3*c^2*d^3)/4 - (a^2
*b^2*c*d^4)/4)))*(b^2*c - a*b*d)^(1/2))/(2*a*b)
```

3.791. $\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx$

3.792 $\int \frac{\sqrt{c+dx^4}}{x^3(a+bx^4)}\,dx$

3.792.1 Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx=-\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}}$$

output
```
-1/2*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*(-a*d+b*c)^(1/2)
/a^(3/2)-1/2*(d*x^4+c)^(1/2)/a/x^2
```

3.792.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx=-\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}}$$

input
```
Integrate[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]
```

output
```
-1/2*Sqrt[c + d*x^4]/(a*x^2) - (Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x^2*
(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2))
```

3.792.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {965, 377, 25, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{\sqrt{dx^4+c}}{x^4\left(bx^4+a\right)}dx^2$$

$$\Big\downarrow \text{377}$$

$$\frac{1}{2}\left(\frac{\int -\frac{bc-ad}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{a}-\frac{\sqrt{c+dx^4}}{ax^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{bc-ad}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{a}-\frac{\sqrt{c+dx^4}}{ax^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{a}-\frac{\sqrt{c+dx^4}}{ax^2}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(-\frac{(bc-ad)\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{a}-\frac{\sqrt{c+dx^4}}{ax^2}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(-\frac{\sqrt{bc-ad}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^4}}{ax^2}\right)$$

input

```
Int[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]
```

output
```
(-(Sqrt[c + d*x^4]/(a*x^2)) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2
)/(Sqrt[a]*Sqrt[c + d*x^4])])/a^(3/2))/2
```

3.792.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.792.4 Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{4}+c}}{x^{2}}+\frac{(ad-bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}\,a}{x^{2}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}}{2a}$
risch	$-\frac{\sqrt{d\,x^{4}+c}}{2a\,x^{2}}+\frac{(ad-bc)\left(\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\,\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\,\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^{2}+\frac{\sqrt{-ab}}{b}}\right.}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}\right.}{a}$
default	$\frac{-\frac{\left(d\,x^{4}+c\right)^{\frac{3}{2}}}{2c\,x^{2}}+\frac{d\,x^{2}\sqrt{d\,x^{4}+c}}{2c}+\frac{\sqrt{d}\,\ln\left(x^{2}\sqrt{d}+\sqrt{d\,x^{4}+c}\right)}{2}}{a}-\frac{b\left(-\frac{\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\,\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}-\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\right.}{}}}{}\right.}{}$
elliptic	$\frac{-\frac{\left(d\,x^{4}+c\right)^{\frac{3}{2}}}{c\,x^{2}}+\frac{2d\left(\frac{x^{2}\sqrt{d\,x^{4}+c}}{2}+\frac{c\ln\left(x^{2}\sqrt{d}+\sqrt{d\,x^{4}+c}\right)}{2\sqrt{d}}\right)}{c}}{2a}+\frac{b\left(\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\,\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}-\frac{\sqrt{d}\,\sqrt{-ab}\,\ln}{}}\right.}{}$

input
```
int((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(-(d*x^4+c)^(1/2)/x^2+(a*d-b*c)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-
b*c)*a)^(1/2))/((a*d-b*c)*a)^(1/2))
```

3.792.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.70

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx$$
$$= \left[\frac{x^2\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2-4\,((abc-2\,a^2d)x^6-a^2cx^2)\sqrt{dx^4+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^8+2\,abx^4+a^2}\right)-4\sqrt{dx^4}}{8\,ax^2}\right.$$
$$\left.-\frac{x^2\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^6+(bc^2-acd)x^2)}\right)+2\sqrt{dx^4+c}}{4\,ax^2}\right]$$

input
```
integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(x^2*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((a*b*c - 2*a^2*d)*x^6 - a^2
*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2))
 - 4*sqrt(d*x^4 + c))/(a*x^2), -1/4*(x^2*sqrt((b*c - a*d)/a)*arctan(1/2*((
b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^
2)*x^6 + (b*c^2 - a*c*d)*x^2)) + 2*sqrt(d*x^4 + c))/(a*x^2)]
```

3.792.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx = \int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/x**3/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/(x**3*(a + b*x**4)), x)
```

3.792.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^3}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^3), x)
```

3.792.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(60) = 120$.

Time $= 0.77$ (sec) , antiderivative size $= 121$, normalized size of antiderivative $= 1.59$

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx=\frac{\left(bc\sqrt{d}-ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a}+\frac{c\sqrt{d}}{\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2-c\right)a}$$

input
```
integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="giac")
```

output
```
1/2*(b*c*sqrt(d) - a*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^
2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a) +
c*sqrt(d)/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2 - c)*a)
```

3.792.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{d\,x^4+c}}{x^3\,\left(b\,x^4+a\right)}\,dx$$

input
```
int((c + d*x^4)^(1/2)/(x^3*(a + b*x^4)),x)
```

output
```
int((c + d*x^4)^(1/2)/(x^3*(a + b*x^4)), x)
```

3.793 $\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)}\,dx$

3.793.1 Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx=-\frac{\sqrt{c+dx^4}}{4ax^4}+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}-\frac{\sqrt{b}\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}$$

output
```
1/4*(-a*d+2*b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^2/c^(1/2)-1/2*arctanh(
b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)*(-a*d+b*c)^(1/2)/a^2-1/4
*(d*x^4+c)^(1/2)/a/x^4
```

3.793.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx=\frac{-\frac{a\sqrt{c+dx^4}}{x^4}-2\sqrt{b}\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^2}$$

input
```
Integrate[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)),x]
```

output
```
(-((a*Sqrt[c + d*x^4])/x^4) - 2*Sqrt[b]*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]
*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]] + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*
x^4]/Sqrt[c]])/Sqrt[c])/(4*a^2)
```

3.793.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{\sqrt{dx^4+c}}{x^8\left(bx^4+a\right)}dx^4$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{4}\left(\frac{\int -\frac{bdx^4+2bc-ad}{2x^4\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{a}-\frac{\sqrt{c+dx^4}}{ax^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\frac{\int \frac{bdx^4+2bc-ad}{x^4\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{2a}-\frac{\sqrt{c+dx^4}}{ax^4}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{4}\left(-\frac{\frac{(2bc-ad)\int \frac{1}{x^4\sqrt{dx^4+c}}dx^4}{a}-\frac{2b(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{a}}{2a}-\frac{\sqrt{c+dx^4}}{ax^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{\frac{2(2bc-ad)\int \frac{1}{\frac{x^8}{d}-\frac{c}{d}}d\sqrt{dx^4+c}}{ad}-\frac{4b(bc-ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{ad}}{2a}-\frac{\sqrt{c+dx^4}}{ax^4}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(-\frac{\frac{4\sqrt{b}\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{a}-\frac{2(2bc-ad)\,\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2a}-\frac{\sqrt{c+dx^4}}{ax^4}\right)$$

input
```
Int[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)),x]
```

output
```
(-(Sqrt[c + d*x^4]/(a*x^4)) - ((-2*(2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^4]/S
qrt[c]])/(a*Sqrt[c]) + (4*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^4])/Sqrt[b*c - a*d]])/a)/(2*a))/4
```

3.793.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.793.4 Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{-\frac{2(ad-bc)b\arctan\left(\frac{b\sqrt{d x^4+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{a\sqrt{d x^4+c}}{x^4}-\frac{(ad-2bc)\operatorname{arctanh}\left(\frac{\sqrt{d x^4+c}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^2}$
risch	$-\frac{\sqrt{d x^4+c}}{4a x^4}-\frac{-\frac{(-ad+2bc)\ln\left(\frac{2c+2\sqrt{c}\sqrt{d x^4+c}}{x^2}\right)}{2a\sqrt{c}}+\frac{2(ad-bc)b\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\right.}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4b\sqrt{-\frac{ad-bc}{b}}}\right.}{}}{}$
elliptic	$\frac{-\frac{\left(d x^4+c\right)^{\frac{3}{2}}}{2c x^4}+\frac{d\left(\sqrt{d x^4+c}-\sqrt{c}\ln\left(\frac{2c+2\sqrt{c}\sqrt{d x^4+c}}{x^2}\right)\right)}{2c}}{2a}-\frac{b\left(\sqrt{d x^4+c}-\sqrt{c}\ln\left(\frac{2c+2\sqrt{c}\sqrt{d x^4+c}}{x^2}\right)\right)}{2a^2}+\frac{b\left(\sqrt{d\left(x^2-\frac{\sqrt{-ab}}{b}\right)}\right.}{}$
default	$\frac{-\frac{\left(d x^4+c\right)^{\frac{3}{2}}}{4c x^4}-\frac{d\ln\left(\frac{2c+2\sqrt{c}\sqrt{d x^4+c}}{x^2}\right)}{4\sqrt{c}}+\frac{d\sqrt{d x^4+c}}{4c}}{a}-\frac{b\left(\frac{\sqrt{d x^4+c}}{2}-\frac{\sqrt{c}\ln\left(\frac{2c+2\sqrt{c}\sqrt{d x^4+c}}{x^2}\right)}{2}\right)}{a^2}+\frac{b^2\left(\frac{\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-}}{}\right.}{}$

input
```
int((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^2*(-2*(a*d-b*c)*b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^4+c)^(1/2)/((a*d
-b*c)*b)^(1/2))-a*(d*x^4+c)^(1/2)/x^4-(a*d-2*b*c)/c^(1/2)*arctanh((d*x^4+c
)^(1/2)/c^(1/2)))
```

3.793. $\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)}\,dx$

3.793.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 513, normalized size of antiderivative = 4.46

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx$$
$$= \left[\frac{2\sqrt{b^2c-abd}cx^4\log\left(\frac{bdx^4+2bc-ad-2\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)-(2bc-ad)\sqrt{c}x^4\log\left(\frac{dx^4-2\sqrt{dx^4+c}\sqrt{c}+2c}{x^4}\right)-2\sqrt{dx^4}}{8a^2cx^4}\right.$$
$$-\frac{(2bc-ad)\sqrt{-c}x^4\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-c}}{c}\right)-\sqrt{b^2c-abd}cx^4\log\left(\frac{bdx^4+2bc-ad-2\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+\sqrt{dx^4+c}}{4a^2cx^4}$$

input
```
integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(b^2*c - a*b*d)*c*x^4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^
4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) - (2*b*c - a*d)*sqrt(c)*x^4*log((
d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4 + c)*a*c)/(a^
2*c*x^4), 1/8*(4*sqrt(-b^2*c + a*b*d)*c*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^4 + b*c)) - (2*b*c - a*d)*sqrt(c)*x^4*log((d*x^4 - 2*
sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4),
-1/4*((2*b*c - a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) - sqrt
(b^2*c - a*b*d)*c*x^4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(
b^2*c - a*b*d))/(b*x^4 + a)) + sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4), 1/4*(2*sq
rt(-b^2*c + a*b*d)*c*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*
x^4 + b*c)) - (2*b*c - a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c
) - sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4)]
```

3.793.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/x**5/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/(x**5*(a + b*x**4)), x)
```

3.793.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^5}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^5), x)
```

3.793.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx = \frac{\left(b^2c-abd\right)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a^2}-\frac{\left(2\,bc-ad\right)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^2\sqrt{-c}}-\frac{\sqrt{dx^4+c}}{4\,ax^4}$$

input
```
integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="giac")
```

output
```
1/2*(b^2*c - a*b*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*a^2) - 1/4*(2*b*c - a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(
a^2*sqrt(-c)) - 1/4*sqrt(d*x^4 + c)/(a*x^4)
```

3.793.9 Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx = \frac{\mathrm{atanh}\left(\frac{b^3\,d^4\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{16\,\left(\frac{a\,b^3\,d^5}{16}-\frac{b^4\,c\,d^4}{16}\right)}\right)\,\sqrt{b^2\,c-a\,b\,d}}{2\,a^2}-\frac{\sqrt{d\,x^4+c}}{4\,a\,x^4}-\frac{\mathrm{atanh}\left(\frac{b^4\,\sqrt{c}\,d^4\,\sqrt{d\,x^4+c}}{16\,\left(\frac{b^4\,c\,d^4}{16}-\frac{3\,a\,b^3\,d^5}{32}+\frac{a^2\,b^2\,d^6}{32\,c}\right)}-\frac{3\,b^3\,d^5\,\sqrt{d\,x^4+c}}{32\,\sqrt{c}\,\left(\frac{a\,b^2\,d^6}{32\,c}-\frac{3\,b^3\,d^5}{32}+\frac{b^4\,c\,d^4}{16\,a}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^4+c}}{32\,c^{3/2}\,\left(\frac{b^2\,d^6}{32\,c}-\frac{3\,b^3\,d^5}{32\,a}+\frac{b^4\,c\,d^4}{16\,a^2}\right)}\right)\,\left(a\,d-2\,b\,c\right)}{4\,a^2\,\sqrt{c}}$$

input
```
int((c + d*x^4)^(1/2)/(x^5*(a + b*x^4)),x)
```

output
```
(atanh((b^3*d^4*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(16*((a*b^3*d^5)/
16 - (b^4*c*d^4)/16)))*(b^2*c - a*b*d)^(1/2))/(2*a^2) - (c + d*x^4)^(1/2)/
(4*a*x^4) - (atanh((b^4*c^(1/2)*d^4*(c + d*x^4)^(1/2))/(16*((b^4*c*d^4)/16
 - (3*a*b^3*d^5)/32 + (a^2*b^2*d^6)/(32*c))) - (3*b^3*d^5*(c + d*x^4)^(1/2
))/(32*c^(1/2)*((a*b^2*d^6)/(32*c) - (3*b^3*d^5)/32 + (b^4*c*d^4)/(16*a)))
 + (b^2*d^6*(c + d*x^4)^(1/2))/(32*c^(3/2)*((b^2*d^6)/(32*c) - (3*b^3*d^5)
/(32*a) + (b^4*c*d^4)/(16*a^2))))*(a*d - 2*b*c))/(4*a^2*c^(1/2))
```

3.793. $\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)}\,dx$

3.794 $\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx$

3.794.1 Optimal result

Integrand size = 24, antiderivative size = 110

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx = -\frac{\sqrt{c+dx^4}}{6ax^6}+\frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2}+\frac{b\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}}$$

output
```
1/2*b*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*(-a*d+b*c)^(1/2
)/a^(5/2)-1/6*(d*x^4+c)^(1/2)/a/x^6+1/6*(-a*d+3*b*c)*(d*x^4+c)^(1/2)/a^2/c
/x^2
```

3.794.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx = \frac{\sqrt{c+dx^4}\left(3bcx^4-a(c+dx^4)\right)}{6a^2cx^6} + \frac{b\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}}$$

input
```
Integrate[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

output
```
(Sqrt[c + d*x^4]*(3*b*c*x^4 - a*(c + d*x^4)))/(6*a^2*c*x^6) + (b*Sqrt[b*c
- a*d]*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]
*Sqrt[b*c - a*d])])/(2*a^(5/2))
```

3.794.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {965, 377, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{\sqrt{dx^4+c}}{x^8\left(bx^4+a\right)}dx^2$$

$$\Big\downarrow \text{ 377}$$

$$\frac{1}{2}\left(\frac{\int -\frac{2bdx^4+3bc-ad}{x^4\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{3a}-\frac{\sqrt{c+dx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{2bdx^4+3bc-ad}{x^4\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{3a}-\frac{\sqrt{c+dx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{ 445}$$

$$\frac{1}{2}\left(-\frac{-\frac{\int \frac{3bc(bc-ad)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{ac}-\frac{\sqrt{c+dx^4}(3bc-ad)}{acx^2}}{3a}-\frac{\sqrt{c+dx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(-\frac{-\frac{3b(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{a}-\frac{\sqrt{c+dx^4}(3bc-ad)}{acx^2}}{3a}-\frac{\sqrt{c+dx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{ 291}$$

$$\frac{1}{2}\left(-\frac{-\frac{3b(bc-ad)\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{a}-\frac{\sqrt{c+dx^4}(3bc-ad)}{acx^2}}{3a}-\frac{\sqrt{c+dx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{2}\left(-\frac{-\frac{3b\sqrt{bc-ad}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^4}(3bc-ad)}{acx^2}}{3a}-\frac{\sqrt{c+dx^4}}{3ax^6}\right)$$

input
```
Int[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

output
```
(-1/3*Sqrt[c + d*x^4]/(a*x^6) - (-(((3*b*c - a*d)*Sqrt[c + d*x^4])/(a*c*x^
2)) - (3*b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c +
d*x^4])])/a^(3/2))/(3*a))/2
```

3.794.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.794.4 Maple [A] (verified)

Time = 5.68 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^4+c}\,\left(a d\,x^4-3 b c\,x^4+a c\right)}{3 x^6}-\frac{b c (a d-b c) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2 \sqrt{(a d-b c) a}}\right)}{\sqrt{(a d-b c) a}}}{2 a^2 c}$
risch	$-\frac{\sqrt{d\,x^4+c}\,\left(a d\,x^4-3 b c\,x^4+a c\right)}{6 c\,a^2 x^6}-\frac{(a d-b c) b \left(\frac{\ln\left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\,\left(x^2+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}}\,\sqrt{d\left(x^2+\frac{\sqrt{-a b}}{b}\right)^2-\frac{2 d \sqrt{-a b}}{}}}{x^2+\frac{\sqrt{-a b}}{b}}\right.}{4 \sqrt{-a b}\,\sqrt{-\frac{a d-b c}{b}}}\right.}{}$
default	$-\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}}{6 a\,x^6 c}-\frac{b\left(-\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}}{2 c\,x^2}+\frac{d\,x^2 \sqrt{d\,x^4+c}}{2 c}+\frac{\sqrt{d}\,\ln\left(x^2 \sqrt{d}+\sqrt{d\,x^4+c}\right)}{2}\right)}{a^2}+\frac{b^2 \left(-\frac{\sqrt{d\left(x^2+\frac{\sqrt{-a b}}{b}\right)^2-\frac{2 d \sqrt{-a b}\,\left(x^2+\frac{\sqrt{-a b}}{b}\right.}{b}}}{}\right.}{}$
elliptic	$-\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}}{6 a\,x^6 c}-\frac{b\left(-\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}}{c\,x^2}+\frac{2 d\left(\frac{x^2 \sqrt{d\,x^4+c}}{2}+\frac{c \ln\left(x^2 \sqrt{d}+\sqrt{d\,x^4+c}\right)}{2 \sqrt{d}}\right)}{c}\right)}{2 a^2}-\frac{b^2 \left(\sqrt{d\left(x^2+\frac{\sqrt{-a b}}{b}\right)^2-\frac{2 d \sqrt{-a b}\,\left(x^2+\frac{\sqrt{-}}{b}\right.}{b}}\right.}{}$

input
```
int((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2*(-1/3*(d*x^4+c)^(1/2)*(a*d*x^4-3*b*c*x^4+a*c)/x^6-b*c*(a*d-b*c)/((
a*d-b*c)*a)^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2)))/c
```

3.794. $\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)}\,dx$

3.794.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.99

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx$$
$$= \left[\frac{3\,bcx^6\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\,\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\,\left(\left(abc-2\,a^2d\right)x^6-a^2cx^2\right)\sqrt{dx^4+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^8+2\,abx^4+a^2}\right)+4\,((3}{24\,a^2cx^6}\right.$$

input
```
integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/24*(3*b*c*x^6*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^
6 - a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4
+ a^2)) + 4*((3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6), 1/12*(
3*b*c*x^6*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*
x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2))
+ 2*((3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6)]
```

3.794.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx = \int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/x**7/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/(x**7*(a + b*x**4)), x)
```

3.794.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^7}\,dx$$

input

```
integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="maxima")
```

output

```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^7), x)
```

3.794.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(90) = 180$.

Time = 1.11 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx=-\frac{\left(b^2c\sqrt{d}-abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a^2}$$
$$-\frac{3\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4bc\sqrt{d}-3\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4ad^{\frac{3}{2}}-6\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc^2\sqrt{d}+3\,bc^3\sqrt{d}}{3\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2-c\right)^3a^2}$$

input

```
integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="giac")
```

output

```
-1/2*(b^2*c*sqrt(d) - a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 +
 c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*
a^2) - 1/3*(3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b*c*sqrt(d) - 3*(sqrt(d)*x
^2 - sqrt(d*x^4 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*
c^2*sqrt(d) + 3*b*c^3*sqrt(d) - a*c^2*d^(3/2))/(((sqrt(d)*x^2 - sqrt(d*x^4
 + c))^2 - c)^3*a^2)
```

3.794.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{d\,x^4+c}}{x^7\,\left(b\,x^4+a\right)}\,dx$$

input
```
int((c + d*x^4)^(1/2)/(x^7*(a + b*x^4)),x)
```

output
```
int((c + d*x^4)^(1/2)/(x^7*(a + b*x^4)), x)
```

3.795 $\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.795.1 Optimal result

Integrand size $= 24$, antiderivative size $= 857$

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{(2bc-5ad)x\sqrt{c+dx^4}}{5b^2\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^2\right)}$$

$$-\frac{a\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b^2} - \frac{a\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b^2}$$

$$-\frac{\sqrt[4]{c}(2bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{5b^2d^{3/4}\sqrt{c+dx^4}}$$

$$+\frac{\sqrt[4]{c}(b^2c^2+abcd-5a^2d^2)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{5b^2d^{3/4}(bc+ad)\sqrt{c+dx^4}}$$

$$+\frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticPi}\Bigg(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\Bigg(}{8b^{5/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

$$-\frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticPi}\Bigg(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\Bigg(}{8b^{5/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

output
```
1/5*x^3*(d*x^4+c)^(1/2)/b+1/5*(-5*a*d+2*b*c)*x*(d*x^4+c)^(1/2)/b^2/d^(1/2)
/(c^(1/2)+x^2*d^(1/2))-1/4*a*arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)
/(d*x^4+c)^(1/2))*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/b^2-1/4*a*arctan(x*
((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1
/2)/b^(1/2))^(1/2)/b^2-1/5*c^(1/4)*(-5*a*d+2*b*c)*(cos(2*arctan(d^(1/4)*x/
c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticE(sin(2*arctan
(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2
)+x^2*d^(1/2))^2)^(1/2)/b^2/d^(3/4)/(d*x^4+c)^(1/2)+1/5*c^(1/4)*(-5*a^2*d^
2+a*b*c*d+b^2*c^2)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2
))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/d^(
3/4)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*a*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^
(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(
d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/
b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2
)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(5/2)/c^
(1/4)/d^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)-1/8*a
*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4
)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^
(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1...
```

3.795.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.16

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{7ax^3(c+dx^4)-7acx^3\sqrt{1+\frac{dx^4}{c}}\,\mathrm{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+(2bc-5ad)x^7\sqrt{1+\frac{dx^4}{c}}\,\mathrm{AppellF1}\Big(}{35ab\sqrt{c+dx^4}}$$

input
```
Integrate[(x^6*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(7*a*x^3*(c + d*x^4) - 7*a*c*x^3*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4, 1/2, 1,
 7/4, -((d*x^4)/c), -((b*x^4)/a)] + (2*b*c - 5*a*d)*x^7*Sqrt[1 + (d*x^4)/c
]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(35*a*b*Sqrt[c
+ d*x^4])
```

3.795.3 Rubi [A] (warning: unable to verify)

Time = 1.78 (sec) , antiderivative size = 1069, normalized size of antiderivative = 1.25, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {978, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$$

$$\Big\downarrow \text{ 978}$$

$$\frac{x^3\sqrt{c+dx^4}}{5b}-\frac{\int \frac{x^2\left(3ac-(2bc-5ad)x^4\right)}{(bx^4+a)\sqrt{dx^4+c}}dx}{5b}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{x^3\sqrt{c+dx^4}}{5b}-\frac{\int \left(-\frac{(2bc-5ad)x^2}{b\sqrt{dx^4+c}}-\frac{5\left(a^2d-abc\right)x^2}{b(bx^4+a)\sqrt{dx^4+c}}\right)dx}{5b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^3\sqrt{dx^4+c}}{5b}-$$

$$\frac{\frac{5\sqrt{-a}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}-\frac{5(-a)^{3/4}\sqrt{bc-ad}\,\arctan(}{4b^{5/}}}{}$$

input
```
Int[(x^6*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(x^3*Sqrt[c + d*x^4])/(5*b) - (-(((2*b*c - 5*a*d)*x*Sqrt[c + d*x^4])/(b*Sq
rt[d]*(Sqrt[c] + Sqrt[d]*x^2))) - (5*(-a)^(3/4)*Sqrt[b*c - a*d]*ArcTan[(Sq
rt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(5/4)) + (5*(
-a)^(3/4)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*
Sqrt[c + d*x^4])])/(4*b^(5/4)) + (c^(1/4)*(2*b*c - 5*a*d)*(Sqrt[c] + Sqrt[
d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^
(1/4)*x)/c^(1/4)], 1/2])/(b*d^(3/4)*Sqrt[c + d*x^4]) - (c^(1/4)*(2*b*c - 5
*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*d^(3/4)*Sqrt[c + d*x^4
]) - (5*a*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt
[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^
4]) - (5*a*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[
2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x
^4]) + (5*Sqrt[-a]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi
[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d
]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(3/2)*c^(1/4)*d^(1/4)*(b*c +
 a*d)*Sqrt[c + d*x^4]) - (5*Sqrt[-a]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d...
```

3.795.3.1 Defintions of rubi rules used

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.795. $\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.795.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.13 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.39

method	result
risch	$\frac{x^3\sqrt{d\,x^4+c}}{5b}-\frac{\frac{i(5ad-2bc)\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,\sqrt{d}}-\frac{5(ad-bc)a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}-\frac{\operatorname{arctanh}\left(\frac{}{2\sqrt{}}\right.}{\sqrt{}}\right.}{}}{5b}$
elliptic	$\frac{x^3\sqrt{d\,x^4+c}}{5b}+\frac{i\left(-\frac{ad-bc}{b^2}-\frac{3c}{5b}\right)\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,\sqrt{d}}+\frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(ad-bc}{}\right.}{}$
default	$\frac{\frac{x^3\sqrt{d\,x^4+c}}{5}+\frac{2ic^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{5\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,\sqrt{d}}}{b}-\frac{a\left(\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\right.\right.}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}\right.}{}$

input
```
int(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^3*(d*x^4+c)^(1/2)/b-1/5/b*(I*(5*a*d-2*b*c)/b*c^(1/2)/(I/c^(1/2)*d^(1
/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)
/(d*x^4+c)^(1/2)/d^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-Ellipti
cE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-5/8*(a*d-b*c)*a/b^2*sum(1/_alpha*(-1/((
-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)
/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^
(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticP
i(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d
^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

3.795.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.795.6 Sympy [F]

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate(x**6*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(x**6*sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.795.7 Maxima [F]

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{dx^4+c}\,x^6}{bx^4+a}\,dx$$

input
```
integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)*x^6/(b*x^4 + a), x)
```

3.795.8 Giac [F]

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{dx^4+c}\,x^6}{bx^4+a}\,dx$$

input
```
integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)*x^6/(b*x^4 + a), x)
```

3.795.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{x^6\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input
```
int((x^6*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
int((x^6*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.796 $\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.796.1 Optimal result

Integrand size = 24, antiderivative size = 700

$$\begin{aligned}
&\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx \\
&= \frac{x\sqrt{c+dx^4}}{3b} - \frac{(bc-ad)\arctan\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}} - \frac{(bc-ad)\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}} \\
&\quad + \frac{c^{3/4}(bc-2ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{3b\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}}{\sqrt[4]{c}}\right.\right.}{8b^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)\right.}{8b^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}
\end{aligned}$$

output
```
1/3*x*(d*x^4+c)^(1/2)/b-1/4*(-a*d+b*c)*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1
/2))^(1/2)/(d*x^4+c)^(1/2))/b^2/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/4*(
-a*d+b*c)*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/
b^2/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/3*c^(3/4)*(-2*a*d+b*c)*(cos(2*
arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Ellip
ticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(
(d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/d^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/
2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(
d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/
2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^
(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+
c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a
)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^
(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(
d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)
/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/
2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/
4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

3.796.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.44 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.34

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{x\left(\frac{(2bc-3ad)x^4\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a}+5\left(c+dx^4+\frac{5a^2c^2\,\text{AppellF}}{(a+bx^4)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bc\,\text{A}\right.\right.}\right.\right.}{15b\sqrt{c+dx^4}}$$

input
```
Integrate[(x^4*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(x*(((2*b*c - 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -(
(d*x^4)/c), -((b*x^4)/a)])/a + 5*(c + d*x^4 + (5*a^2*c^2*AppellF1[1/4, 1/2
, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/((a + b*x^4)*(-5*a*c*AppellF1[1/4,
1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2,
 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d
*x^4)/c), -((b*x^4)/a)]))))))/(15*b*Sqrt[c + d*x^4])
```

3.796.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 1002, normalized size of antiderivative = 1.43, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {978, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$$

$$\Big\downarrow \ 978$$

$$\frac{x\sqrt{c+dx^4}}{3b}-\frac{\int \frac{ac-(2bc-3ad)x^4}{(bx^4+a)\sqrt{dx^4+c}}dx}{3b}$$

$$\Big\downarrow \ 1021$$

$$\frac{x\sqrt{c+dx^4}}{3b}-\frac{\frac{3a(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{b}-\frac{(2bc-3ad)\int \frac{1}{\sqrt{dx^4+c}}dx}{b}}{3b}$$

$$\Big\downarrow \ 761$$

$$\frac{x\sqrt{c+dx^4}}{3b}-\frac{\frac{3a(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{b}-\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(2bc-3ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}}{3b}$$

$$\Big\downarrow \ 925$$

$$\frac{x\sqrt{c+dx^4}}{3b}-\frac{\frac{3a(bc-ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)}{b}-\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(2bc-3ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}}{3b}$$

$$\Big\downarrow \ 1541$$

$$\frac{x\sqrt{c+dx^4}}{3b}-\frac{\frac{3a(bc-ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\right.}{2a}\right.}{b}}{3b}$$

3.796. $\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$

$$\Big\downarrow 27$$

$$\frac{x\sqrt{c+dx^4}}{3b}-$$

$$3a(bc-ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int}{ad}}{2a}\right.$$

$$b$$

$$3b$$

$$\Big\downarrow 761$$

$$\frac{x\sqrt{c+dx^4}}{3b}-$$

$$3a(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{\sqrt{b}\left(\sqrt{}\right.}{}\right.$$

$$b$$

$$\Big\downarrow 2221$$

$$\frac{x\sqrt{dx^4+c}}{3b}-$$

$$3a(bc-ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\frac{}{\sqrt[4]{-}}\right.}{2\sqrt[4]{b}\sqrt{bc-ad}}\right.}{}}{2a}\right.$$

$$\Big\downarrow 2223$$

3.796. $\int\frac{x^4\sqrt{c+dx^4}}{a+bx^4}dx$

$$\frac{x\sqrt{dx^4+c}}{3b}-$$

$$\frac{3a(bc-ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\right.}{2\sqrt[4]{b}\sqrt{bc-ad}}\right.}{2a}\right.}{}$$

input
```
Int[(x^4*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(x*Sqrt[c + d*x^4])/(3*b) - (-1/2*((2*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)
*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x
)/c^(1/4)], 1/2])/(b*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4]) + (3*a*(b*c - a*d)*(
((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)
*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x
)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt
[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] -
 Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])]
)/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(S
qrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elliptic
Pi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*S
qrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c +
d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^
(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/
4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] - (S
qrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(...
```

3.796. $\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.796.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.796. $\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.796.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.05 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.43

method	result
risch	$\frac{x\sqrt{d\,x^4+c}}{3b} - \frac{\frac{(3ad-2bc)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}} - \frac{3(ad-bc)a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+}{}\right.}{8b^2}}{3b}$
elliptic	$\frac{x\sqrt{d\,x^4+c}}{3b} + \frac{\left(-\frac{ad-bc}{b^2}-\frac{c}{3b}\right)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}} + \frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(ad-bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2}{2\sqrt{\frac{-ad+bc}{b}}}\right)}{\sqrt{\frac{-ad+bc}{b}}}\right.}{}\right.}{}$
default	$\frac{\frac{x\sqrt{d\,x^4+c}}{3}+\frac{2c\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}}{b} - \frac{a\left(\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}} - \frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(ad-bc)\left(-\right.}{}}{}\right.}{}$

input
```
int(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

3.796. $\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$

output
```
1/3*x*(d*x^4+c)^(1/2)/b-1/3/b*((3*a*d-2*b*c)/b/(I/c^(1/2)*d^(1/2))^(1/2)*(
1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(
1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-3/8*(a*d-b*c)*a/b^2*sum(1/_a
lpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+
b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1
-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1
/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,
(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*
b+a)))
```

3.796.5 Fricas [F]

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\,x^4}{bx^4+a}\,dx$$

input
```
integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*x^4/(b*x^4 + a), x)
```

3.796.6 Sympy [F]

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate(x**4*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(x**4*sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.796.7 Maxima [F]

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\,x^4}{bx^4+a}\,dx$$

input
```
integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)*x^4/(b*x^4 + a), x)
```

3.796.8 Giac [F]

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\,x^4}{bx^4+a}\,dx$$

input
```
integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)*x^4/(b*x^4 + a), x)
```

3.796.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^4\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input
```
int((x^4*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
int((x^4*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.797 $\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.797.1 Optimal result

Integrand size = 24, antiderivative size = 786

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$$

$$= \frac{\sqrt{d}x\sqrt{c+dx^4}}{b\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b} + \frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b}$$

$$- \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{b\sqrt{c+dx^4}}$$

$$+ \frac{a\sqrt[4]{c}d^{5/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{b(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8b^{3/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}}{\sqrt[4]{c}}\right.\right.}{8b^{3/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

output
```
x*d^(1/2)*(d*x^4+c)^(1/2)/b/(c^(1/2)+x^2*d^(1/2))+1/4*arctan(x*((a*d-b*c)/
(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^
(1/2)/b+1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2)
)*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/b-c^(1/4)*d^(1/4)*(cos(2*arctan(d^
(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticE(sin(
2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)
/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/(d*x^4+c)^(1/2)+a*c^(1/4)*d^(5/4)*(cos(2
*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*
((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*
(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)
*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1
/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*
(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(
1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/((-a)^(1/2)*b^(1/2)*c^(
1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)+1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(
1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d
^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2/(-a
)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2
)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)...
```

3.797.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.08

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{x^3\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{\frac{c+dx^4}{c}}}$$

input
```
Integrate[(x^2*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(x^3*Sqrt[c + d*x^4]*AppellF1[3/4, -1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a
)])/(3*a*Sqrt[(c + d*x^4)/c])
```

3.797.3 Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 1096, normalized size of antiderivative = 1.39, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {994, 834, 27, 761, 993, 1510, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \text{ 994}$$

$$\frac{(bc-ad)\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}+\frac{d\int \frac{x^2}{\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{(bc-ad)\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}+\frac{d\left(\frac{\sqrt{c}\int \frac{1}{\sqrt{dx^4+c}}dx}{\sqrt{d}}-\frac{\sqrt{c}\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{c}\sqrt{dx^4+c}}dx}{\sqrt{d}}\right)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(bc-ad)\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}+\frac{d\left(\frac{\sqrt{c}\int \frac{1}{\sqrt{dx^4+c}}dx}{\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{dx^4+c}}dx}{\sqrt{d}}\right)}{b}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{(bc-ad)\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}+\frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{dx^4+c}}dx}{\sqrt{d}}\right)}{b}$$

$$\Big\downarrow \text{ 993}$$

$$\frac{(bc-ad)\left(\frac{\int \frac{1}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}\right)}{b}+\frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{dx^4+c}}dx}{\sqrt{d}}\right)}{b}$$

$$\Big\downarrow \text{1510}$$

$$\frac{(bc-ad)\left(\frac{\int \frac{1}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}\right)}{b}+$$

$$\frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}\right)}{b}$$

$$\Big\downarrow \text{1541}$$

$$\frac{(bc-ad)\left(\frac{\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}}-\frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+}{}\right.}{b}$$

$$\frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}}-\frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+}{2\sqrt{b}}\right.}{b}$$

$$\frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}\right)}{b}$$

$$\Big\downarrow \text{761}$$

3.797. $\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$

$$\frac{(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2\sqrt{b}}-\right.}{b}$$

$$\frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}\right)}{b}$$

$$\Big\downarrow \text{2221}$$

$$\frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{dx^4+c}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{dx^4+c}}-\frac{x\sqrt{dx^4+c}}{\sqrt{d}x^2+\sqrt{c}}}{\sqrt{d}}\right)}{b}$$

$$(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\right.}{4\sqrt{-a}}\right.}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}\right.}{bc+ad}}{2\sqrt{b}}\right.$$

$$\Big\downarrow \text{2223}$$

3.797. $\int\frac{x^2\sqrt{c+dx^4}}{a+bx^4}dx$

$$\frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{dx^4+c}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{dx^4+c}}-\frac{x\sqrt{dx^4+c}}{\sqrt{d}x^2+\sqrt{c}}}{\sqrt{d}}\right)}{b}$$

$$(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\right.}{4\sqrt{-a}}\right.}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}\right.}{bc+ad}}{2\sqrt{b}}\right.$$

input
```
Int[(x^2*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(d*(-((-((x*Sqrt[c + d*x^4])/(Sqrt[c] + Sqrt[d]*x^2)) + (c^(1/4)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*Arc
Tan[(d^(1/4)*x)/c^(1/4)], 1/2])/(d^(1/4)*Sqrt[c + d*x^4]))/Sqrt[d]) + (c^(
1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*E
llipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^4]))
)/b + ((b*c - a*d)*(-1/2*(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(S
qrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elliptic
F[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x
^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((Sqrt[b]*Sqrt[c] +
Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c +
 d*x^4])])/(2*(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) - (((a*Sqrt[c])/(-a)^(3/
2) + Sqrt[d]/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[
-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1
/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d))/Sqrt[b] + (-1/2*((Sqrt[b]*Sqrt
[c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(
Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(
c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a
]*Sqrt[d])*(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*
x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*(-a)^(1/4)*b^(1/4)*Sqrt[b*...
```

3.797. $\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.797.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 994
```
Int[((x_)^2*Sqrt[(c_) + (d_.)*(x_)^4])/((a_) + (b_.)*(x_)^4), x_Symbol] :>
Simp[d/b   Int[x^2/Sqrt[c + d*x^4], x], x] + Simp[(b*c - a*d)/b   Int[x^2/(
(a + b*x^4)*Sqrt[c + d*x^4]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.797. $\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.797.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.68 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.38

method	result
default	$\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}\frac{(ad-bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2d\,x^{2}\,\_\alpha^{2}+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^{4}+}}\right.}{\sqrt{\frac{-ad+bc}{b}}}\right.}{}$
elliptic	$\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}\frac{(ad-bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2d\,x^{2}\,\_\alpha^{2}+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^{4}+}}\right.}{\sqrt{\frac{-ad+bc}{b}}}\right.}{}$

input
```
int(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
I*d^(1/2)/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1
/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2
)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8/b^2*sum(
(a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*
c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alph
a^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d
*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_al
pha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Ro
otOf(_Z^4*b+a))
```

3.797.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.797.6 Sympy [F]

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate(x**2*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(x**2*sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.797.7 Maxima [F]

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{dx^4+c}x^2}{bx^4+a}\,dx$$

input

```
integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output

```
integrate(sqrt(d*x^4 + c)*x^2/(b*x^4 + a), x)
```

3.797.8 Giac [F]

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{dx^4+c}x^2}{bx^4+a}\,dx$$

input

```
integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output

```
integrate(sqrt(d*x^4 + c)*x^2/(b*x^4 + a), x)
```

3.797.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{x^2\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input

```
int((x^2*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output

```
int((x^2*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.798 $\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.798.1 Optimal result

Integrand size = 21, antiderivative size = 679

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{(bc-ad)\arctan\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4ab\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}} + \frac{(bc-ad)\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4ab\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}$$

$$+ \frac{c^{3/4}d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{(bc+ad)\sqrt{c+dx^4}}$$

$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}}{\sqrt[4]{c}}\right.\right.}{8ab\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

$$+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8ab\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

output
```
1/4*(-a*d+b*c)*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/
2))/a/b/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/4*(-a*d+b*c)*arctan(x*((-a*
d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/a/b/((-a*d+b*c)/(-a)^(1/
2)/b^(1/2))^(1/2)+c^(3/4)*d^(3/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/
2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/
4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2
)^(1/2)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c
^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan
(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)
/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/
2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/b/c^(1/
4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)+1/8*(-a*d+
b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(
1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(
-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1
/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+
x^2*d^(1/2))^2)^(1/2)/a/b/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1
/2))/(d*x^4+c)^(1/2)
```

3.798.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$$
$$= \frac{5acx\sqrt{c+dx^4}\,\mathrm{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\left(5ac\,\mathrm{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(-2bc\,\mathrm{AppellF1}\left(\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+ad\,\mathrm{Ap}\right.\right.}$$

input
```
Integrate[Sqrt[c + d*x^4]/(a + b*x^4),x]
```

output
```
(5*a*c*x*Sqrt[c + d*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((d*x^4)/c), -((b*x^
4)/a)])/((a + b*x^4)*(5*a*c*AppellF1[1/4, -1/2, 1, 5/4, -((d*x^4)/c), -((b
*x^4)/a)] + 2*x^4*(-2*b*c*AppellF1[5/4, -1/2, 2, 9/4, -((d*x^4)/c), -((b*x
^4)/a)] + a*d*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

3.798.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 964, normalized size of antiderivative = 1.42, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {922, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \color{blue}{922}$$

$$\frac{(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}+\frac{d\int \frac{1}{\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \color{blue}{761}$$

$$\frac{(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}$$

$$\Big\downarrow \color{blue}{925}$$

$$\frac{(bc-ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)}{b}+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}$$

$$\Big\downarrow \color{blue}{1541}$$

$$\frac{(bc-ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\right.}{}}{2a}\right.}{b}$$

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{(bc-ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\right.}{}}{2a}\right.}{b}$$

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}$$

$$\Big\downarrow\ 761$$

$$\frac{(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{\sqrt{}}{}\right.}{b}$$

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}$$

$$\Big\downarrow\ 2221$$

$$(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{a}{}\right.$$

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}$$

$$\Big\downarrow\ 2223$$

3.798. $\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$

$$\frac{d^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{dx^4+c}}+$$

$$(bc-ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\right.}{2\sqrt[4]{b}\sqrt{bc-ad}}\right.}{}}{2a}\right.$$

input
```
Int[Sqrt[c + d*x^4]/(a + b*x^4),x]
```

output
```
(d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^
2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*c^(1/4)*Sqrt[c + d*
x^4]) + ((b*c - a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(
Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipti
cF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*
x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[
b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(
1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a
]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sq
rt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt
[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(
1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*S
qrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt
[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^
(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*
Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b
*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c -
a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sq
rt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sq
rt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/...
```

3.798. $\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.798.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 922
```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[b/d
  Int[1/Sqrt[a + b*x^4], x], x] - Simp[(b*c - a*d)/d   Int[1/(Sqrt[a + b*x^
4]*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

3.798. $\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.798.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.93 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.40

method	result
default	$\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4}b+a\right)}\frac{(ad-bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2d\,x^{2}\_\alpha^{2}+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^{4}+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^{3}b\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1}}{}\right)}{\_\alpha^{3}}}{8b^{2}}$
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4}b+a\right)}\frac{(ad-bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2d\,x^{2}\_\alpha^{2}+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^{4}+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^{3}b\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1}}{}\right)}{\_\alpha^{3}}}{8b^{2}}$

input
```
int((d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
d/b/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)
*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),
I)-1/8/b^2*sum((a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*
_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(
1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1
/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^
(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^
(1/2))),_alpha=RootOf(_Z^4*b+a))
```

3.798.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.798.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.798.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{dx^4+c}}{bx^4+a}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/(b*x^4 + a), x)
```

3.798.8 Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{dx^4+c}}{bx^4+a}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)/(b*x^4 + a), x)
```

3.798.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input
```
int((c + d*x^4)^(1/2)/(a + b*x^4),x)
```

output
```
int((c + d*x^4)^(1/2)/(a + b*x^4), x)
```

3.799 $\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx$

3.799.1 Optimal result

Integrand size = 24, antiderivative size = 809

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx = &-\frac{\sqrt{c+dx^4}}{ax}+\frac{\sqrt{d}x\sqrt{c+dx^4}}{a\left(\sqrt{c}+\sqrt{d}x^2\right)} \\
&-\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a}-\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a} \\
&-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{a\sqrt{c+dx^4}} \\
&+\frac{bc^{5/4}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{a(bc+ad)\sqrt{c+dx^4}} \\
&-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8\sqrt{b}\sqrt[4]{c}\left((-a)^{3/2}\sqrt{b}\sqrt{c}+a^2\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} \\
&-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}}{\sqrt[4]{c}}\right.\right.}{8a\sqrt{b}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}
\end{aligned}
$$

output
```
-(d*x^4+c)^(1/2)/a/x+x*d^(1/2)*(d*x^4+c)^(1/2)/a/(c^(1/2)+x^2*d^(1/2))-1/4
*arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b*c)
/(-a)^(1/2)/b^(1/2))^(1/2)/a-1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^
(1/2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/a-c^(1/4)*d^(
1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(
1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^
2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(d*x^4+c)^(1/2)+b*c
^(5/4)*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(
1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(
c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(a*d+b*c)
/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)
/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4
))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/
d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(
1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/c^(1/4)/d^(1/4)/b^(1/2)/((
-a)^(3/2)*b^(1/2)*c^(1/2)+a^2*d^(1/2))/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)*(cos
(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*El
lipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)
*d^(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d
^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^...
```

3.799.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx$$
$$=\frac{-21a(c+dx^4)-7(bc-2ad)x^4\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+3bdx^8\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\,\Big(}{21a^2x\sqrt{c+dx^4}}$$

input
```
Integrate[Sqrt[c + d*x^4]/(x^2*(a + b*x^4)),x]
```

output
```
(-21*a*(c + d*x^4) - 7*(b*c - 2*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4,
 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*x^8*Sqrt[1 + (d*x^4)/c]*
AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(21*a^2*x*Sqrt[c
+ d*x^4])
```

3.799. $\int \frac{\sqrt{c+dx^4}}{x^2(a+bx^4)}\,dx$

3.799.3 Rubi [A] (warning: unable to verify)

Time = 1.50 (sec) , antiderivative size = 1021, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {975, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}dx$$

$$\Big\downarrow \; 975$$

$$\frac{\int -\frac{x^2\left(-bdx^4+bc-2ad\right)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{a}-\frac{\sqrt{c+dx^4}}{ax}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{x^2\left(-bdx^4+bc-2ad\right)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{a}-\frac{\sqrt{c+dx^4}}{ax}$$

$$\Big\downarrow \; 1054$$

$$-\frac{\int \left(\frac{(bc-ad)x^2}{\left(bx^4+a\right)\sqrt{dx^4+c}}-\frac{dx^2}{\sqrt{dx^4+c}}\right)dx}{a}-\frac{\sqrt{c+dx^4}}{ax}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-\frac{(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-}}{\sqrt[4]{-a}\sqrt[4]{}}\right.}{4\sqrt[4]{-a}\sqrt[4]{b}}}{}$$

$$\frac{\sqrt{dx^4+c}}{ax}$$

input
```
Int[Sqrt[c + d*x^4]/(x^2*(a + b*x^4)),x]
```

output

```
-(Sqrt[c + d*x^4]/(a*x)) - (-((Sqrt[d]*x*Sqrt[c + d*x^4])/(Sqrt[c] + Sqrt[
d]*x^2)) + (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)
*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*b^(1/4)) - (Sqrt[b*c - a*d]*ArcTanh[(Sqr
t[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*b^(1/
4)) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
 Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/Sqrt[c + d
*x^4] - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c]
 + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*Sqrt[
c + d*x^4]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*
(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipt
icF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d
*x^4]) - ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt
[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]
) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]
*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt
[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(
d^(1/4)*x)/c^(1/4)], 1/2])/(8*Sqrt[-a]*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(S
qrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellip...
```

3.799.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 975

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

3.799. $\int \frac{\sqrt{c+dx^4}}{x^2(a+bx^4)}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.799.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.40 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.39

method	result
risch	$-\frac{\sqrt{d x^4+c}}{a x}+\frac{\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}+\frac{(ad-bc)\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\,\_\alpha}{2\sqrt{\frac{-ad+bc}{b}}}\right)}{\sqrt{\frac{-ad+bc}{b}}}}{}\right)}{}}{a}$
elliptic	$-\frac{\sqrt{d x^4+c}}{a x}+\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{(-ad+bc)\left(-\frac{\operatorname{arctanh}\left(\right.}{}\right.}{}$
default	$\frac{-\frac{\sqrt{d x^4+c}}{x}+\frac{2i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}}{a}-\frac{b\left(\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(\right.\right.}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}\right.}{}$

input
```
int((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-(d*x^4+c)^(1/2)/a/x+1/a*(I*d^(1/2)*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I
/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2
)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2)
)^(1/2),I))+1/8*(a*d-b*c)/b*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(
1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1
/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/
2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/
2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^
(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))
```

3.799.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.799.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx = \int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/x**2/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/(x**2*(a + b*x**4)), x)
```

3.799.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^2}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^2), x)
```

3.799.8 Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^2}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^2), x)
```

3.799.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{d\,x^4+c}}{x^2\,\left(b\,x^4+a\right)}\,dx$$

input
```
int((c + d*x^4)^(1/2)/(x^2*(a + b*x^4)),x)
```

output
```
int((c + d*x^4)^(1/2)/(x^2*(a + b*x^4)), x)
```

3.800 $\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)} dx$

3.800.1 Optimal result

Integrand size = 24, antiderivative size = 703

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3} - \frac{(bc-ad)\arctan\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}} - \frac{(bc-ad)\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}$$

$$- \frac{d^{3/4}(2bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{3a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}}{\sqrt[4]{c}}\right.\right.}{8a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

output
```
-1/3*(d*x^4+c)^(1/2)/a/x^3-1/4*(-a*d+b*c)*arctan(x*((b*c/a-d)*(-a)^(1/2)/b
^(1/2))^(1/2)/(d*x^4+c)^(1/2))/a^2/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/
4*(-a*d+b*c)*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2
))/a^2/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/3*d^(3/4)*(-a*d+2*b*c)*(cos
(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*El
lipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2)
)*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(a*d+b*c)/(d*x^4+c)^
(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arct
an(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^
(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2
*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x
^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+
(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x
/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arct
an(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1
/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^
(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^
(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

3.800.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.29 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx$$

$$= \frac{-bdx^8\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+\frac{a\Big(25ac\left(ac+4bcx^4-adx^4+bdx^8\right)\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-10x}{\left(a+bx^4\right)\Big(-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4}}{15a^2x^3\sqrt{c+dx^4}}$$

input
```
Integrate[Sqrt[c + d*x^4]/(x^4*(a + b*x^4)),x]
```

output
```
(-(b*d*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -(
(b*x^4)/a)]) + (a*(25*a*c*(a*c + 4*b*c*x^4 - a*d*x^4 + b*d*x^8)*AppellF1[1
/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] - 10*x^4*(a + b*x^4)*(c + d*x
^4)*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*Ap
pellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/((a + b*x^4)*(-5*a
*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*A
ppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4,
3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(15*a^2*x^3*Sqrt[c + d*x^4])
```

3.800.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 988, normalized size of antiderivative = 1.41, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {975, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}dx$$

$$\Big\downarrow \text{ 975}$$

$$\frac{\int -\frac{bdx^4+3bc-2ad}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{3a}-\frac{\sqrt{c+dx^4}}{3ax^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{bdx^4+3bc-2ad}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{3a}-\frac{\sqrt{c+dx^4}}{3ax^3}$$

$$\Big\downarrow \text{ 1021}$$

$$-\frac{3(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx+d\int \frac{1}{\sqrt{dx^4+c}}dx}{3a}-\frac{\sqrt{c+dx^4}}{3ax^3}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{3(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{3a}-\frac{\sqrt{c+dx^4}}{3ax^3}$$

$$\Big\downarrow \text{ 925}$$

$$-\frac{3(bc-ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{3a}$$

$$\frac{\sqrt{c+dx^4}}{3ax^3}$$

$$\Big\downarrow \text{ 1541}$$

$$-\frac{3(bc-ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\sqrt{b}\sqrt{c}(}{2}\right.}{3a}$$

$$\frac{\sqrt{c+dx^4}}{3ax^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{3(bc-ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\right.}{2a}\right.}{3a}$$

$$\frac{\sqrt{c+dx^4}}{3ax^3}$$

$$\Big\downarrow \; 761$$

$$-\frac{3(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\right.}{}$$

$$\frac{\sqrt{c+dx^4}}{3ax^3}$$

$$\Big\downarrow \; 2221$$

$$-\frac{3(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\right.}{}$$

$$\frac{\sqrt{c+dx^4}}{3ax^3}$$

$$\Big\downarrow \; 2223$$

3.800. $\int\frac{\sqrt{c+dx^4}}{x^4(a+bx^4)}\,dx$

$$-\frac{\frac{d^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^4+c}}+3(bc-ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(\right.}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}\right.}{}$$

$$\frac{\sqrt{dx^4+c}}{3ax^3}$$

input
```
Int[Sqrt[c + d*x^4]/(x^4*(a + b*x^4)),x]
```

output
```
-1/3*Sqrt[c + d*x^4]/(a*x^3) - ((d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)]
, 1/2])/(2*c^(1/4)*Sqrt[c + d*x^4]) + 3*(b*c - a*d)*(((a*((Sqrt[b]*Sqrt[c]
)/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sq
rt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*
c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a
]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqr
t[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c
 - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)
*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt
[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^
(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d)
)/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt
[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d
^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[
b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sq
rt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d
*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt
[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*E
llipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sq...
```

3.800. $\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)}\,dx$

3.800.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.800. $\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)} \, dx$

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.800.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.80 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.42

method	result
risch	$-\frac{\sqrt{d\,x^4+c}}{3a\,x^3}-\frac{\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}+\frac{(-3ad+3bc)\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b}{}}{}\right)}{8b}}{3a}$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{3a\,x^3}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4b+a\right)}\frac{(-ad+bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_}{}\right)}{}}{8ba}$
default	$\frac{-\frac{\sqrt{d\,x^4+c}}{3x^3}+\frac{2d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}}{a}-\frac{b\left(\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4b+a\right)}\frac{(ad-bc)\left(-\right.}{}\right)}{}$

input
```
int((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(d*x^4+c)^(1/2)/a/x^3-1/3/a*(d/(I/c^(1/2)*d^(1/2)))^(1/2)*(1-I/c^(1/2)
*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*Ellipt
icF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)+1/8*(-3*a*d+3*b*c)/b*sum(1/_alpha^3*(-1
/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1
/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)
*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*Ellipt
icPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2
)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

3.800. $\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)}\,dx$

3.800.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.800.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/x**4/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/(x**4*(a + b*x**4)), x)
```

3.800.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^4}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^4), x)
```

3.800.8 Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^4}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^4), x)
```

3.800.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{d\,x^4+c}}{x^4\,\left(b\,x^4+a\right)}\,dx$$

input
```
int((c + d*x^4)^(1/2)/(x^4*(a + b*x^4)),x)
```

output
```
int((c + d*x^4)^(1/2)/(x^4*(a + b*x^4)), x)
```

3.801 $\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx$

3.801.1 Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{2(ex)^{5/2}\sqrt{c+dx^4}\,\operatorname{AppellF1}\left(\frac{5}{8},1,-\frac{1}{2},\frac{13}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{1+\frac{dx^4}{c}}}$$

output
```
2/5*(e*x)^(5/2)*AppellF1(5/8,1,-1/2,13/8,-b*x^4/a,-d*x^4/c)*(d*x^4+c)^(1/2
)/a/e/(1+d*x^4/c)^(1/2)
```

3.801.2 Mathematica [A] (verified)

Time = 11.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{2x(ex)^{3/2}\sqrt{c+dx^4}\,\operatorname{AppellF1}\left(\frac{5}{8},-\frac{1}{2},1,\frac{13}{8},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{5a\sqrt{\frac{c+dx^4}{c}}}$$

input
```
Integrate[((e*x)^(3/2)*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(2*x*(e*x)^(3/2)*Sqrt[c + d*x^4]*AppellF1[5/8, -1/2, 1, 13/8, -((d*x^4)/c)
, -((b*x^4)/a)])/(5*a*Sqrt[(c + d*x^4)/c])
```

3.801.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {966, 27, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \textbf{966}$$

$$\frac{2\int \frac{e^6x^2\sqrt{dx^4+c}}{bx^4e^4+ae^4}d\sqrt{ex}}{e}$$

$$\Big\downarrow \textbf{27}$$

$$2e^3\int \frac{e^2x^2\sqrt{dx^4+c}}{bx^4e^4+ae^4}d\sqrt{ex}$$

$$\Big\downarrow \textbf{1013}$$

$$\frac{2e^3\sqrt{c+dx^4}\int \frac{e^2x^2\sqrt{\frac{dx^4}{c}+1}}{bx^4e^4+ae^4}d\sqrt{ex}}{\sqrt{\frac{dx^4}{c}+1}}$$

$$\Big\downarrow \textbf{1012}$$

$$\frac{2(ex)^{5/2}\sqrt{c+dx^4}\operatorname{AppellF1}\left(\frac{5}{8},1,-\frac{1}{2},\frac{13}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{\frac{dx^4}{c}+1}}$$

input
```
Int[((e*x)^(3/2)*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(2*(e*x)^(5/2)*Sqrt[c + d*x^4]*AppellF1[5/8, 1, -1/2, 13/8, -((b*x^4)/a),
-((d*x^4)/c)])/(5*a*e*Sqrt[1 + (d*x^4)/c])
```

3.801.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 966
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(
m + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(
1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.801.4 Maple [F]

$$\int \frac{(ex)^{\frac{3}{2}} \sqrt{d\,x^{4}+c}}{b\,x^{4}+a}dx$$

input
```
int((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

output
```
int((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

3.801.5 Fricas [F]

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}(ex)^{\frac{3}{2}}}{bx^4+a}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*sqrt(e*x)*e*x/(b*x^4 + a), x)
```

3.801.6 Sympy [F]

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{(ex)^{\frac{3}{2}}\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate((e*x)**(3/2)*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral((e*x)**(3/2)*sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.801.7 Maxima [F]

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}(ex)^{\frac{3}{2}}}{bx^4+a}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)*(e*x)^(3/2)/(b*x^4 + a), x)
```

3.801.8 Giac [F]

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{dx^4+c}(ex)^{\frac{3}{2}}}{bx^4+a}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)*(e*x)^(3/2)/(b*x^4 + a), x)
```

3.801.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{(e\,x)^{3/2}\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input
```
int(((e*x)^(3/2)*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
int(((e*x)^(3/2)*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.802 $\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4} \, dx$

3.802.1 Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4} \, dx = \frac{2(ex)^{3/2}\sqrt{c+dx^4}\operatorname{AppellF1}\left(\frac{3}{8},1,-\frac{1}{2},\frac{11}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{1+\frac{dx^4}{c}}}$$

output

```
2/3*(e*x)^(3/2)*AppellF1(3/8,1,-1/2,11/8,-b*x^4/a,-d*x^4/c)*(d*x^4+c)^(1/2
)/a/e/(1+d*x^4/c)^(1/2)
```

3.802.2 Mathematica [A] (verified)

Time = 11.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4} \, dx = \frac{2x\sqrt{ex}\sqrt{c+dx^4}\operatorname{AppellF1}\left(\frac{3}{8},-\frac{1}{2},1,\frac{11}{8},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{\frac{c+dx^4}{c}}}$$

input

```
Integrate[(Sqrt[e*x]*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output

```
(2*x*Sqrt[e*x]*Sqrt[c + d*x^4]*AppellF1[3/8, -1/2, 1, 11/8, -((d*x^4)/c),
-((b*x^4)/a)])/(3*a*Sqrt[(c + d*x^4)/c])
```

3.802.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {966, 27, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx$$

$$\Big\downarrow \text{ 966}$$

$$\frac{2\int \frac{e^5x\sqrt{dx^4+c}}{bx^4e^4+ae^4}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{ 27}$$

$$2e^3\int \frac{ex\sqrt{dx^4+c}}{bx^4e^4+ae^4}d\sqrt{ex}$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{2e^3\sqrt{c+dx^4}\int \frac{ex\sqrt{\frac{dx^4}{c}+1}}{bx^4e^4+ae^4}d\sqrt{ex}}{\sqrt{\frac{dx^4}{c}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{2(ex)^{3/2}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{3}{8},1,-\frac{1}{2},\frac{11}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{\frac{dx^4}{c}+1}}$$

input
```
Int[(Sqrt[e*x]*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(2*(e*x)^(3/2)*Sqrt[c + d*x^4]*AppellF1[3/8, 1, -1/2, 11/8, -((b*x^4)/a),
-((d*x^4)/c)])/(3*a*e*Sqrt[1 + (d*x^4)/c])
```

3.802.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 966
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(
m + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(
1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.802.4 Maple [F]

$$\int \frac{\sqrt{ex}\,\sqrt{d\,x^{4}+c}}{b\,x^{4}+a}dx$$

input
```
int((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

output
```
int((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

3.802.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx=\text{Timed out}$$

input
```
integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.802.6 Sympy [F]

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate((e*x)**(1/2)*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(sqrt(e*x)*sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.802.7 Maxima [F]

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{dx^4+c}\sqrt{ex}}{bx^4+a}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)*sqrt(e*x)/(b*x^4 + a), x)
```

3.802.8 Giac [F]

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{dx^4+c}\sqrt{ex}}{bx^4+a}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)*sqrt(e*x)/(b*x^4 + a), x)
```

3.802.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{e\,x}\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input
```
int(((e*x)^(1/2)*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
int(((e*x)^(1/2)*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.803 $\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\left(a+bx^4\right)}\,dx$

3.803.1 Optimal result

Integrand size = 28, antiderivative size = 69

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\left(a+bx^4\right)}\,dx=\frac{2\sqrt{ex}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{1}{8},1,-\frac{1}{2},\frac{9}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{1+\frac{dx^4}{c}}}$$

output
```
2*AppellF1(1/8,1,-1/2,9/8,-b*x^4/a,-d*x^4/c)*(e*x)^(1/2)*(d*x^4+c)^(1/2)/a
/e/(1+d*x^4/c)^(1/2)
```

3.803.2 Mathematica [A] (verified)

Time = 11.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\left(a+bx^4\right)}\,dx=\frac{2x\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{1}{8},-\frac{1}{2},1,\frac{9}{8},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a\sqrt{ex}\sqrt{\frac{c+dx^4}{c}}}$$

input
```
Integrate[Sqrt[c + d*x^4]/(Sqrt[e*x]*(a + b*x^4)),x]
```

output
```
(2*x*Sqrt[c + d*x^4]*AppellF1[1/8, -1/2, 1, 9/8, -((d*x^4)/c), -((b*x^4)/a
)])/(a*Sqrt[e*x]*Sqrt[(c + d*x^4)/c])
```

3.803.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {966, 27, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}dx$$

$$\Big\downarrow \text{966}$$

$$\frac{2\int \frac{e^4\sqrt{dx^4+c}}{bx^4e^4+ae^4}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{\sqrt{dx^4+c}}{bx^4e^4+ae^4}d\sqrt{ex}$$

$$\Big\downarrow \text{937}$$

$$\frac{2e^3\sqrt{c+dx^4}\int \frac{\sqrt{\frac{dx^4}{c}+1}}{bx^4e^4+ae^4}d\sqrt{ex}}{\sqrt{\frac{dx^4}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{2\sqrt{ex}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{1}{8},1,-\frac{1}{2},\frac{9}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{\frac{dx^4}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^4]/(Sqrt[e*x]*(a + b*x^4)),x]
```

output
```
(2*Sqrt[e*x]*Sqrt[c + d*x^4]*AppellF1[1/8, 1, -1/2, 9/8, -((b*x^4)/a), -((
d*x^4)/c)])/(a*e*Sqrt[1 + (d*x^4)/c])
```

3.803.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 966
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(
m + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(
1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && FractionQ[m] && IntegerQ[p]
```

3.803.4 Maple [F]

$$\int \frac{\sqrt{d\,x^{4}+c}}{\sqrt{ex}\,\left(b\,x^{4}+a\right)}dx$$

input
```
int((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x)
```

output
```
int((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x)
```

3.803.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx=\text{Timed out}$$

input
```
integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.803.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx=\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/(e*x)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/(sqrt(e*x)*(a + b*x**4)), x)
```

3.803.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)\sqrt{ex}}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*sqrt(e*x)), x)
```

3.803.8 Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)\sqrt{ex}}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*sqrt(e*x)), x)
```

3.803.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \int \frac{\sqrt{d\,x^4+c}}{\sqrt{e\,x}\,\left(b\,x^4+a\right)}\,dx$$

input
```
int((c + d*x^4)^(1/2)/((e*x)^(1/2)*(a + b*x^4)),x)
```

output
```
int((c + d*x^4)^(1/2)/((e*x)^(1/2)*(a + b*x^4)), x)
```

3.804 $\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}(a+bx^4)}\,dx$

3.804.1 Optimal result

Integrand size = 28, antiderivative size = 69

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx = -\frac{2\sqrt{c+dx^4}\operatorname{AppellF1}\left(-\frac{1}{8},1,-\frac{1}{2},\frac{7}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{1+\frac{dx^4}{c}}}$$

output
```
-2*AppellF1(-1/8,1,-1/2,7/8,-b*x^4/a,-d*x^4/c)*(d*x^4+c)^(1/2)/a/e/(e*x)^(
1/2)/(1+d*x^4/c)^(1/2)
```

3.804.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. 2(69) = 138.

Time = 11.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx = \frac{x\left(-70a(c+dx^4)-10(bc-4ad)x^4\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+\right.}{35a^2(ex)^{3/2}\sqrt{c+dx^4}}$$

input
```
Integrate[Sqrt[c + d*x^4]/((e*x)^(3/2)*(a + b*x^4)),x]
```

output
```
(x*(-70*a*(c + d*x^4) - 10*(b*c - 4*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[
7/8, 1/2, 1, 15/8, -((d*x^4)/c), -((b*x^4)/a)] + 14*b*d*x^8*Sqrt[1 + (d*x^
4)/c]*AppellF1[15/8, 1/2, 1, 23/8, -((d*x^4)/c), -((b*x^4)/a)]))/(35*a^2*(
e*x)^(3/2)*Sqrt[c + d*x^4])
```

3.804.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {966, 27, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\left(a+bx^4\right)}dx$$

$$\Big\downarrow \text{966}$$

$$\frac{2\int \frac{e^3\sqrt{dx^4+c}}{x(bx^4e^4+ae^4)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{\sqrt{dx^4+c}}{ex\left(bx^4e^4+ae^4\right)}d\sqrt{ex}$$

$$\Big\downarrow \text{1013}$$

$$\frac{2e^3\sqrt{c+dx^4}\int \frac{\sqrt{\frac{dx^4}{c}+1}}{ex(bx^4e^4+ae^4)}d\sqrt{ex}}{\sqrt{\frac{dx^4}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{2\sqrt{c+dx^4}\,\text{AppellF1}\left(-\frac{1}{8},1,-\frac{1}{2},\frac{7}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{\frac{dx^4}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^4]/((e*x)^(3/2)*(a + b*x^4)),x]
```

output
```
(-2*Sqrt[c + d*x^4]*AppellF1[-1/8, 1, -1/2, 7/8, -((b*x^4)/a), -((d*x^4)/c
)])/(a*e*Sqrt[e*x]*Sqrt[1 + (d*x^4)/c])
```

3.804.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 966
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(
m + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(
1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.804.4 Maple [F]

$$\int \frac{\sqrt{d\,x^{4}+c}}{\left(e x \right)^{\frac{3}{2}} \left(b\,x^{4}+a \right)}d x$$

input
```
int((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x)
```

output
```
int((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x)
```

3.804.5 Fricas [F]

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*sqrt(e*x)/(b*e^2*x^6 + a*e^2*x^2), x)
```

3.804.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx=\int \frac{\sqrt{c+dx^4}}{(ex)^{\frac{3}{2}}\,(a+bx^4)}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/(e*x)**(3/2)/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/((e*x)**(3/2)*(a + b*x**4)), x)
```

3.804.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*(e*x)^(3/2)), x)
```

3.804.8 Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*(e*x)^(3/2)), x)
```

3.804.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx=\int \frac{\sqrt{d\,x^4+c}}{{(e\,x)}^{3/2}\,\left(b\,x^4+a\right)}\,dx$$

input
```
int((c + d*x^4)^(1/2)/((e*x)^(3/2)*(a + b*x^4)),x)
```

output
```
int((c + d*x^4)^(1/2)/((e*x)^(3/2)*(a + b*x^4)), x)
```

3.805 $\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.805.1 Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx=-\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2}+\frac{(c+dx^4)^{3/2}}{6bd^2}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}$$

output
```
1/6*(d*x^4+c)^(3/2)/b/d^2-1/2*a^2*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*
c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-1/2*(a*d+b*c)*(d*x^4+c)^(1/2)/b^2/d^2
```

3.805.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\frac{\sqrt{c+dx^4}(-2bc-3ad+bdx^4)}{6b^2d^2}+\frac{a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{2b^{5/2}\sqrt{-bc+ad}}$$

input
```
Integrate[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(Sqrt[c + d*x^4]*(-2*b*c - 3*a*d + b*d*x^4))/(6*b^2*d^2) + (a^2*ArcTan[(Sq
rt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(2*b^(5/2)*Sqrt[-(b*c) + a*d])
```

3.805.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{x^8}{(bx^4+a)\sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{4}\int \left(\frac{a^2}{b^2(bx^4+a)\sqrt{dx^4+c}}+\frac{\sqrt{dx^4+c}}{bd}+\frac{-bc-ad}{b^2d\sqrt{dx^4+c}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^4}(ad+bc)}{b^2d^2}+\frac{2\left(c+dx^4\right)^{3/2}}{3bd^2}\right)$$

input
```
Int[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
((-2*(b*c + a*d)*Sqrt[c + d*x^4])/(b^2*d^2) + (2*(c + d*x^4)^(3/2))/(3*b*d
^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(b^(5/2)*
Sqrt[b*c - a*d]))/4
```

3.805.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.805.4 Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d x^4+c}}{\sqrt{(ad-bc)b}}\right)a^2d^2-\left(\left(-\frac{b x^4}{3}+a\right)d+\frac{2bc}{3}\right)\sqrt{d x^4+c}\,\sqrt{(ad-bc)b}}{2\sqrt{(ad-bc)b}\,b^2d^2}$
risch	$-\frac{(-bd x^4+3ad+2bc)\sqrt{d x^4+c}}{6d^2b^2}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\sqrt{-\right.}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4b^3\sqrt{-\frac{ad-bc}{b}}}$
default	$-\frac{\sqrt{d x^4+c}\,(-d x^4+2c)}{6b\,d^2}-\frac{a\sqrt{d x^4+c}}{2b^2d}+\frac{a^2\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-2d\sqrt{-}}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4b\sqrt{-\frac{ad-bc}{b}}}\right.}{}$
elliptic	$\frac{x^4\sqrt{d x^4+c}}{6bd}-\frac{c\sqrt{d x^4+c}}{3b\,d^2}-\frac{a\sqrt{d x^4+c}}{2b^2d}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2+2d\sqrt{-}}}{x^2-\frac{\sqrt{-ab}}{b}}\right.}{4b^3\sqrt{-\frac{ad-bc}{b}}}$

input
```
int(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2))*a^2*d^2-((-1/3*b*x^4+a)
*d+2/3*b*c)*(d*x^4+c)^(1/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b^2/d
^2
```

3.805.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.78

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$= \left[\frac{3\sqrt{b^2c-abd}a^2d^2\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)-2\,(2\,b^3c^2+ab^2cd-3\,a^2bd^2-(b^3cd-ab^2d^2)x^4)\sqrt{c}}{12\,(b^4cd^2-ab^3d^3)}\right.$$

input
```
integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d
*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) - 2*(2*b^3*c^2 + a*b^2*c*d - 3
*a^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(b^4*c*d^2 - a*b^
3*d^3), 1/6*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^4 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^4 + b*c)) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b
^3*c*d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

3.805.6 Sympy [F]

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**11/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**11/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.805.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.805.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{a^2\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}b^2} + \frac{(dx^4+c)^{\frac{3}{2}}b^2d^4 - 3\sqrt{dx^4+c}b^2cd^4 - 3\sqrt{dx^4+c}abd^5}{6\,b^3d^6}$$

input
```
integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*a^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*b^2) + 1/6*((d*x^4 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^4 + c)*b^2*c*d^4 - 3
*sqrt(d*x^4 + c)*a*b*d^5)/(b^3*d^6)
```

3.805.9 Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{(d\,x^4+c)^{3/2}}{6\,b\,d^2} - \left(\frac{c}{b\,d^2} + \frac{2\,a\,d^3 - 2\,b\,c\,d^2}{4\,b^2\,d^4}\right)\sqrt{d\,x^4+c} + \frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^11/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
(c + d*x^4)^(3/2)/(6*b*d^2) - (c/(b*d^2) + (2*a*d^3 - 2*b*c*d^2)/(4*b^2*d^
4))*(c + d*x^4)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^
(1/2)))/(2*b^(5/2)*(a*d - b*c)^(1/2))
```

3.806 $\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.806.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\sqrt{c+dx^4}}{2bd} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}$$

output
```
1/2*a*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)
^(1/2)+1/2*(d*x^4+c)^(1/2)/b/d
```

3.806.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{1}{2}\left(\frac{\sqrt{c+dx^4}}{bd} - \frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}\right)$$

input
```
Integrate[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(Sqrt[c + d*x^4]/(b*d) - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) +
 a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d]))/2
```

3.806.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{x^4}{(bx^4+a)\sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{4}\left(\frac{2\sqrt{c+dx^4}}{bd}-\frac{a\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^4}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{2\sqrt{c+dx^4}}{bd}-\frac{2a\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{bd}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^4}}{bd}\right)$$

input
```
Int[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
((2*Sqrt[c + d*x^4])/(b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b
*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/4
```

3.806.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.806.4 Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{-\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)ad+\sqrt{d\,x^4+c}\,\sqrt{(ad-bc)b}}{2bd\sqrt{(ad-bc)b}}$
risch	$\frac{\sqrt{d\,x^4+c}}{2bd}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b^2\sqrt{-\frac{ad-bc}{b}}}+$
elliptic	$\frac{\sqrt{d\,x^4+c}}{2bd}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b^2\sqrt{-\frac{ad-bc}{b}}}+$
default	$\frac{\sqrt{d\,x^4+c}}{2bd}-\frac{a\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}}-\right.}{b}$

input
```
int(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2))*a*d+(d*x^4+c)^(1/2)*((
a*d-b*c)*b)^(1/2))/b/d/((a*d-b*c)*b)^(1/2)
```

3.806.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.77

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}(b^2c-abd)}{4\,(b^3cd-ab^2d^2)}, -\frac{\sqrt{-b^2c+abd}ad\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)-\sqrt{dx^4+c}(b^2c-abd)}{2\,(b^3cd-ab^2d^2)}\right]$$

input
```
integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 +
c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2*c - a*b*d))/
(b^3*c*d - a*b^2*d^2), -1/2*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^4 +
c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) - sqrt(d*x^4 + c)*(b^2*c - a*b*d)
)/(b^3*c*d - a*b^2*d^2)]
```

3.806.6 Sympy [F]

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**7/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**7/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.806.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.806.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b} - \frac{\sqrt{dx^4+c}}{b}}{2\,d}$$

input
```
integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(a*d*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*
b*d)*b) - sqrt(d*x^4 + c)/b)/d
```

3.806.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\sqrt{d\,x^4+c}}{2\,b\,d} - \frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^7/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
(c + d*x^4)^(1/2)/(2*b*d) - (a*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c
)^(1/2)))/(2*b^(3/2)*(a*d - b*c)^(1/2))
```

3.807 $\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.807.1 Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

output
```
-1/2*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^
(1/2)
```

3.807.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{2\sqrt{b}\sqrt{-bc+ad}}$$

input
```
Integrate[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]]/(2*Sqrt[b]*Sqrt[-(b*c
) + a*d])
```

3.807.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {946, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{4}\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{2d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

input
```
Int[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
-1/2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c
- a*d])
```

3.807.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.807.4 Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)}{2\sqrt{(ad-bc)b}}$
default	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}}-$ $\ln\Big(-\frac{2(ad-bc)}{b}+$
elliptic	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}}-$ $\ln\Big(-\frac{2(ad-bc)}{b}+$

input
```
int(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

3.807.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.55

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$=\left[\frac{\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)}{4\,\sqrt{b^2c-abd}},\ \frac{\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)}{2\,(b^2c-abd)}\right]$$

input
```
integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(
b*x^4 + a))/sqrt(b^2*c - a*b*d), 1/2*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^
4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c))/(b^2*c - a*b*d)]
```

3.807.6 Sympy [A] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b\sqrt{\frac{ad-bc}{b}}} & \text{for}\ d\neq 0 \\ \begin{cases} \frac{x^4}{4a\sqrt{c}} & \text{for}\ b=0 \\ \tilde{\infty}x^4 & \text{for}\ \sqrt{c}=0 \\ \frac{\log\left(4a\sqrt{c}+4b\sqrt{c}x^4\right)}{4b\sqrt{c}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Piecewise((atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(2*b*sqrt((a*d - b*c
)/b)), Ne(d, 0)), (Piecewise((x**4/(4*a*sqrt(c)), Eq(b, 0)), (zoo*x**4, Eq
(sqrt(c), 0)), (log(4*a*sqrt(c) + 4*b*sqrt(c)*x**4)/(4*b*sqrt(c)), True)),
 True))
```

3.807.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.807.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}}$$

input
```
integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

3.807.9 Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\operatorname{atan}\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{2\sqrt{a\,b\,d-b^2\,c}}$$

input
```
int(x^3/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
atan((b*(c + d*x^4)^(1/2))/(a*b*d - b^2*c)^(1/2))/(2*(a*b*d - b^2*c)^(1/2)
)
```

3.808 $\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.808.1 Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}$$

output
```
-1/2*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a/c^(1/2)+1/2*arctanh(b^(1/2)*(d*x^4
+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

3.808.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a}$$

input
```
Integrate[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
-1/2*((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/Sqrt[
-(b*c) + a*d] + ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/Sqrt[c])/a
```

3.808.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{1}{x^4(bx^4+a)\sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{x^4\sqrt{dx^4+c}}dx^4}{a}-\frac{b\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^4}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{2\int \frac{1}{\frac{x^8}{d}-\frac{c}{d}}d\sqrt{dx^4+c}}{ad}-\frac{2b\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{ad}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{a\sqrt{c}}\right)$$

input
```
Int[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
((-2*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/4
```

3.808.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.808.4 Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$-\frac{b\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{c}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{\sqrt{c}}\right)\sqrt{(ad-bc)b}}{2a\sqrt{(ad-bc)b}\,\sqrt{c}}$
elliptic	$-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a\sqrt{c}}+\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-ad-}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4a\sqrt{-\frac{ad-bc}{b}}}$
default	$-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a\sqrt{c}}-\frac{b\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4b\sqrt{-\frac{ad-bc}{b}}}\right.}{}$

input
```
int(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2))*c^(1/2)+arctanh((d*x
^4+c)^(1/2)/c^(1/2))*((a*d-b*c)*b)^(1/2))/a/((a*d-b*c)*b)^(1/2)/c^(1/2)
```

3.808.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.07

$$\int \frac{1}{x\,(a+bx^4)\,\sqrt{c+dx^4}}\,dx = \left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+\sqrt{c}\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)}{4\,ac}, \frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\right.}{}\right.$$

input
```
integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(c*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)
*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + sqrt(c)*log((d*x^4 - 2*sq
rt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), 1/4*(2*c*sqrt(-b/(b*c - a*d))*ar
ctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) +
sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), 1/4*(c*
sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*(b*c -
a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^4 + c)
*sqrt(-c)/c))/(a*c), 1/2*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)*(
b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) + sqrt(-c)*arctan(sqrt(d*
x^4 + c)*sqrt(-c)/c))/(a*c)]
```

3.808.6 Sympy [A] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \begin{cases} \frac{2\left(-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4a\sqrt{\frac{ad-bc}{b}}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{\operatorname{atan}\left(\frac{2\left(\frac{a}{2b}+x^4\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{2b\sqrt{c}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Piecewise((2*(-d*atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*a*sqrt((a*d
 - b*c)/b)) + d*atan(sqrt(c + d*x**4)/sqrt(-c))/(4*a*sqrt(-c)))/d, Ne(d, 0
)), (atan(2*(a/(2*b) + x**4)/sqrt(-a**2/b**2))/(2*b*sqrt(c)*sqrt(-a**2/b**
2)), True))
```

3.808.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{dx^4+c}x}\,dx$$

input
```
integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x), x)
```

3.808.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = -\frac{b\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a}+\frac{\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*b*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a) + 1/2*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a*sqrt(-c))
```

3.808.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 652, normalized size of antiderivative = 7.67

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = -\frac{\mathrm{atanh}\left(\frac{\sqrt{d\,x^4+c}}{\sqrt{c}}\right)}{2\,a\,\sqrt{c}}$$

$$-\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}\right)\,1{}\mathrm{i}}{4\,\left(a^2\,d-a\,b\,c\right)}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}+\right.}{}}{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}+\right.}{}}\right.}{2\,\left(a^2\,d-a\,b\,c\right)}$$

input
```
int(1/(x*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output

```
- atanh((c + d*x^4)^(1/2)/c^(1/2))/(2*a*c^(1/2)) - (atan((((b^2*c - a*b*d)
^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) - ((b^2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3
- ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1
/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c))*1i)/(4*(a^2*d - a*b*c)) +
((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) + ((b^2*c - a*b*d)^(1/2)
*(2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2)*(b
^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c))*1i)/(4*(a^
2*d - a*b*c)))/(((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) - ((b^2*
c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c +
 d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b
*c))))/(4*(a^2*d - a*b*c)) - ((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(
1/2) + ((b^2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^
3*c*d^2)*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4
*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c))))*(b^2*c - a*b*d)^(1/2)*1i)/(2*(a^
2*d - a*b*c))
```

3.808. $\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.809 $\int \frac{1}{x^5(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.809.1 Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{1}{x^5\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=-\frac{\sqrt{c+dx^4}}{4acx^4}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}}-\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}$$

output
```
1/4*(a*d+2*b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-1/2*b^(3/2)*a
rctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/4*
(d*x^4+c)^(1/2)/a/c/x^4
```

3.809.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^5\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\frac{-\frac{a\sqrt{c+dx^4}}{cx^4}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{c^{3/2}}}{4a^2}$$

input
```
Integrate[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-((a*Sqrt[c + d*x^4])/(c*x^4)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
4])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[
c + d*x^4]/Sqrt[c]])/c^(3/2))/(4*a^2)
```

3.809.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+b x^4\right) \sqrt{c+d x^4}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4} \int \frac{1}{x^8 \left(b x^4+a\right) \sqrt{d x^4+c}}dx^4$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{4} \left(-\frac{\int \frac{b d x^4+2 b c+a d}{2 x^4 \left(b x^4+a\right) \sqrt{d x^4+c}}dx^4}{a c}-\frac{\sqrt{c+d x^4}}{a c x^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4} \left(-\frac{\int \frac{b d x^4+2 b c+a d}{x^4 \left(b x^4+a\right) \sqrt{d x^4+c}}dx^4}{2 a c}-\frac{\sqrt{c+d x^4}}{a c x^4}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{4} \left(-\frac{\frac{(a d+2 b c) \int \frac{1}{x^4 \sqrt{d x^4+c}}dx^4}{a}-\frac{2 b^2 c \int \frac{1}{\left(b x^4+a\right) \sqrt{d x^4+c}}dx^4}{a}}{2 a c}-\frac{\sqrt{c+d x^4}}{a c x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4} \left(-\frac{\frac{2 (a d+2 b c) \int \frac{1}{\frac{x^8}{d}-\frac{c}{d}}d\sqrt{d x^4+c}}{a d}-\frac{4 b^2 c \int \frac{1}{\frac{b x^8}{d}+a-\frac{b c}{d}}d\sqrt{d x^4+c}}{a d}}{2 a c}-\frac{\sqrt{c+d x^4}}{a c x^4}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(-\frac{\frac{4b^{3/2}c\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(ad+2bc)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2ac}-\frac{\sqrt{c+dx^4}}{acx^4}\right)$$

input
```
Int[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-(Sqrt[c + d*x^4]/(a*c*x^4)) - ((-2*(2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]
/Sqrt[c]])/(a*Sqrt[c]) + (4*b^(3/2)*c*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sq
rt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*c))/4
```

3.809.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.809.4 Maple [A] (verified)

Time = 4.98 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{\frac{2b^2\arctan\left(\frac{b\sqrt{dx^4+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{a\sqrt{dx^4+c}}{cx^4}+\frac{(ad+2bc)\operatorname{arctanh}\left(\frac{\sqrt{dx^4+c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{4a^2}$
risch	$-\frac{\sqrt{dx^4+c}}{4acx^4}-\frac{-\frac{(ad+2bc)\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^4+c}}{x^2}\right)}{2a\sqrt{c}}-\frac{2b^2c\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4b\sqrt{-\frac{ad-bc}{b}}}\right.}{}}{}$
elliptic	$-\frac{\sqrt{dx^4+c}}{4acx^4}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^4+c}}{x^2}\right)}{4ac^{\frac{3}{2}}}+\frac{b\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^4+c}}{x^2}\right)}{2a^2\sqrt{c}}-\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}}{x^2+}\right.}{4a^2\sqrt{-}}$
default	$\frac{-\frac{\sqrt{dx^4+c}}{4cx^4}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^4+c}}{x^2}\right)}{4c^{\frac{3}{2}}}}{a}+\frac{b\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^4+c}}{x^2}\right)}{2a^2\sqrt{c}}+\frac{b^2\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}}{x^2+}\right.}{4b\sqrt{-}}\right.}{}$

input
```
int(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^2*(2*b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^
(1/2))-a/c*(d*x^4+c)^(1/2)/x^4+(a*d+2*b*c)/c^(3/2)*arctanh((d*x^4+c)^(1/2)
/c^(1/2)))
```

3.809.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 565, normalized size of antiderivative = 4.83

$$\int \frac{1}{x^5\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

$$= \left[\frac{2\,bc^2x^4\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+(2\,bc+ad)\sqrt{c}x^4\log\left(\frac{dx^4+2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)-2\,\sqrt{}}{8\,a^2c^2x^4}\right.$$

$$-\frac{4\,bc^2x^4\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^4+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^4+bc}\right)-(2\,bc+ad)\sqrt{c}x^4\log\left(\frac{dx^4+2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)+2\,\sqrt{dx^4}}{8\,a^2c^2x^4}$$

$$-\frac{2\,bc^2x^4\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^4+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^4+bc}\right)+(2\,bc+ad)\sqrt{-c}x^4\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-c}}{c}\right)+\sqrt{dx^4+}}{4\,a^2c^2x^4}$$

input
```
integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*b*c^2*x^4*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(
d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + (2*b*c + a*d)*s
qrt(c)*x^4*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x
^4 + c)*a*c)/(a^2*c^2*x^4), -1/8*(4*b*c^2*x^4*sqrt(-b/(b*c - a*d))*arctan(
-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) - (2*b*
c + a*d)*sqrt(c)*x^4*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) +
2*sqrt(d*x^4 + c)*a*c)/(a^2*c^2*x^4), 1/4*(b*c^2*x^4*sqrt(b/(b*c - a*d))*l
og((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*
d)))/(b*x^4 + a)) - (2*b*c + a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt
(-c)/c) - sqrt(d*x^4 + c)*a*c)/(a^2*c^2*x^4), -1/4*(2*b*c^2*x^4*sqrt(-b/(b
*c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x
^4 + b*c)) + (2*b*c + a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c)
 + sqrt(d*x^4 + c)*a*c)/(a^2*c^2*x^4)]
```

3.809.6 Sympy [F]

$$\int \frac{1}{x^5\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\int \frac{1}{x^5\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**5*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.809.7 Maxima [F]

$$\int \frac{1}{x^5\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^5), x)
```

3.809.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^5\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$
$$=\frac{b^2\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a^2}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^2\sqrt{-c}c}-\frac{\sqrt{dx^4+c}}{4\,acx^4}$$

input
```
integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*b^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*a^2) - 1/4*(2*b*c + a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^2*sqrt(-c)
*c) - 1/4*sqrt(d*x^4 + c)/(a*c*x^4)
```

3.809.9 Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.38

$$
\begin{aligned}
&\int \frac{1}{x^5\,(a+bx^4)\,\sqrt{c+dx^4}}\,dx \\
&= \frac{\ln\left(\sqrt{d\,x^4+c}\,{\left(b^4\,c-a\,b^3\,d\right)}^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{4\,a^3\,d-4\,a^2\,b\,c} \\
&\quad - \frac{\ln\left(\sqrt{d\,x^4+c}\,{\left(b^4\,c-a\,b^3\,d\right)}^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{4\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{\sqrt{d\,x^4+c}}{4\,a\,c\,x^4} \\
&\quad - \frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^4+c}\,3{}\mathrm{i}}{16\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{16\,c}+\frac{5\,a\,b^3\,d^5}{32\,c^2}+\frac{a^2\,b^2\,d^6}{32\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^4+c}\,1{}\mathrm{i}}{32\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{32\,a}+\frac{b^2\,d^6}{32\,c}+\frac{3\,b^4\,c\,d^4}{16\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^4+c}\,5{}\mathrm{i}}{32\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{16\,a}+\frac{5\,b^3\,d^5}{32\,c}+\frac{a\,b^2\,d^6}{32\,c^2}\right)}\right)\,(a\,d+2\,b\,c}{4\,a^2\,\sqrt{c^3}}
\end{aligned}
$$

input
```
int(1/(x^5*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
(log((c + d*x^4)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2
*a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(4*a^3*d - 4*a^2*b*c) - (log((c + d*x
^4)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(
b^4*c - a*b^3*d)^(1/2))/(4*(a^3*d - a^2*b*c)) - (c + d*x^4)^(1/2)/(4*a*c*x
^4) - (atan((b^4*d^4*(c + d*x^4)^(1/2)*3i)/(16*(c^3)^(1/2)*((3*b^4*d^4)/(1
6*c) + (5*a*b^3*d^5)/(32*c^2) + (a^2*b^2*d^6)/(32*c^3))) + (b^2*d^6*(c + d
*x^4)^(1/2)*1i)/(32*(c^3)^(1/2)*((5*b^3*d^5)/(32*a) + (b^2*d^6)/(32*c) + (
3*b^4*c*d^4)/(16*a^2))) + (b^3*d^5*(c + d*x^4)^(1/2)*5i)/(32*(c^3)^(1/2)*(
(3*b^4*d^4)/(16*a) + (5*b^3*d^5)/(32*c) + (a*b^2*d^6)/(32*c^2))))*(a*d + 2
*b*c)*1i)/(4*a^2*(c^3)^(1/2))
```

3.810 $\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.810.1 Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x^2\sqrt{c+dx^4}}{4bd}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}$$

output
```
-1/4*(2*a*d+b*c)*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b^2/d^(3/2)+1/2*a^(3
/2)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/b^2/(-a*d+b*c)^(1
/2)+1/4*x^2*(d*x^4+c)^(1/2)/b/d
```

3.810.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\frac{bx^2\sqrt{c+dx^4}}{d}+\frac{2a^{3/2}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}-\frac{(bc+2ad)\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{d^{3/2}}}{4b^2}$$

input
```
Integrate[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
((b*x^2*Sqrt[c + d*x^4])/d + (2*a^(3/2)*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]
*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d] - ((b
*c + 2*a*d)*Log[Sqrt[d]*x^2 + Sqrt[c + d*x^4]])/d^(3/2))/(4*b^2)
```

3.810.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 381, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{x^8}{(bx^4+a)\sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \text{ 381}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2bd}-\frac{\int \frac{(bc+2ad)x^4+ac}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{2bd}\right)$$

$$\Big\downarrow \text{ 398}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2bd}-\frac{\frac{(2ad+bc)\int \frac{1}{\sqrt{dx^4+c}}dx^2}{b}-\frac{2a^2d\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}}{2bd}\right)$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2bd}-\frac{\frac{(2ad+bc)\int \frac{1}{1-dx^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}-\frac{2a^2d\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}}{2bd}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{2a^2d\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}}{2bd}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{2a^2d\int\frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}}{2bd}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{2a^{3/2}d\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{b\sqrt{bc-ad}}}{2bd}\right)$$

input

```
Int[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output

```
((x^2*Sqrt[c + d*x^4])/(2*b*d) - ((-2*a^(3/2)*d*ArcTan[(Sqrt[b*c - a*d]*x^
2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(b*Sqrt[b*c - a*d]) + ((b*c + 2*a*d)*ArcTan
h[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(b*Sqrt[d]))/(2*b*d))/2
```

3.810.3.1 Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.810.4 Maple [A] (verified)

Time = 5.62 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.24

method	result
pseudoelliptic	$\frac{\sqrt{d\,x^4+c}\,\sqrt{d}\,\sqrt{(ad-bc)a}\,b\,x^2+2d^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right)a^2-2\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{x^2\sqrt{d}}\right)\sqrt{(ad-bc)a}\,ad-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{x^2\sqrt{d}}\right)}{4b^2\sqrt{(ad-bc)a}\,d^{\frac{3}{2}}}$
risch	$\frac{x^2\sqrt{d\,x^4+c}}{4bd}-\frac{\frac{(2ad+bc)\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2b\sqrt{d}}-\frac{2a^2d\left(\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-2d}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}\right.}{}}{}$
default	$\frac{\frac{x^2\sqrt{d\,x^4+c}}{4d}-\frac{c\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{4d^{\frac{3}{2}}}}{b}-\frac{a\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2b^2\sqrt{d}}+\frac{a^2\left(\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d}}{x^2+\frac{\sqrt{-}}{b}}\right.}{4\sqrt{-ab}\,\sqrt{-a}}\right.}{}$
elliptic	$-\frac{a\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2b^2\sqrt{d}}+\frac{x^2\sqrt{d\,x^4+c}}{4bd}-\frac{c\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{4b\,d^{\frac{3}{2}}}+\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-}{b}}}{x}\right.}{4b^2\sqrt{-a}}$

input
```
int(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*((d*x^4+c)^(1/2)*d^(1/2)*((a*d-b*c)*a)^(1/2)*b*x^2+2*d^(3/2)*arctanh((
d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))*a^2-2*arctanh((d*x^4+c)^(1/2)/x^
2/d^(1/2))*((a*d-b*c)*a)^(1/2)*a*d-arctanh((d*x^4+c)^(1/2)/x^2/d^(1/2))*((
a*d-b*c)*a)^(1/2)*b*c)/b^2/((a*d-b*c)*a)^(1/2)/d^(3/2)
```

3.810. $\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.810.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 739, normalized size of antiderivative = 6.01

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$= \left[\frac{2\sqrt{dx^4+c}bdx^2+ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^6-(abc^2-}{b^2x^8+2\,abx^4+a^2}\right.}{8\,b^2d^2}\right.$$

input
```
integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 +
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d
*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + (b*c + 2*a*
d)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b^2*d^2), 1
/8*(2*sqrt(d*x^4 + c)*b*d*x^2 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 -
 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*
((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x
^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 2*(b*c + 2*a*
d)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)))/(b^2*d^2), 1/8*(2*sqrt(d
*x^4 + c)*b*d*x^2 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)
*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + (b*
c + 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b^2
*d^2), 1/4*(sqrt(d*x^4 + c)*b*d*x^2 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/
2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 +
 a*c*x^2)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)))/
(b^2*d^2)]
```

3.810.6 Sympy [F]

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**9/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**9/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.810.7 Maxima [F]

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^9}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^9/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.810.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.810.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^9}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^9/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^9/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.811 $\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.811.1 Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}$$

output
```
1/2*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b/d^(1/2)-1/2*arctan(x^2*(-a*d+b*
c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*a^(1/2)/b/(-a*d+b*c)^(1/2)
```

3.811.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{-\frac{\sqrt{a}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} + \frac{\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{d}}}{2b}$$

input
```
Integrate[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-((Sqrt[a]*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sq
rt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d]) + Log[Sqrt[d]*x^2 + Sqrt[c + d*x
^4]]/Sqrt[d])/(2*b)
```

3.811.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {965, 385, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^4+a)\sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \text{385}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\sqrt{dx^4+c}}dx^2}{b}-\frac{a\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{1-dx^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}-\frac{a\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{a\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{a\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{\sqrt{a}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{b\sqrt{bc-ad}}\right)$$

input
```
Int[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(b*Sq
rt[b*c - a*d])) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(b*Sqrt[d]))/2
```

3.811.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.811. $\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.811.4 Maple [A] (verified)

Time = 5.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{-a\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right)\sqrt{d}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{x^2\sqrt{d}}\right)\sqrt{(ad-bc)a}}{2b\sqrt{(ad-bc)a}\,\sqrt{d}}$
default	$\frac{\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2b\sqrt{d}} - \frac{a\left(\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}\right.}{}$
elliptic	$\frac{\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2b\sqrt{d}} + \frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right.}{4\sqrt{-ab}\,b\sqrt{-\frac{ad-bc}{b}}}$

input
```
int(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-a*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))*d^(1/2)+arctanh
((d*x^4+c)^(1/2)/x^2/d^(1/2))*((a*d-b*c)*a)^(1/2))/b/((a*d-b*c)*a)^(1/2)/d
^(1/2)
```

3.811.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 632, normalized size of antiderivative = 6.95

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

$$= \left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2-4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^6-(abc^2-a^2cd)x^2)\sqrt{dx^4+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^8+2\,abx^4+a^2}\right.}{8\,bd}\right.$$

input
```
integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/
(b^2*x^8 + 2*a*b*x^4 + a^2)) + 2*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*
sqrt(d)*x^2 - c))/(b*d), 1/8*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b
*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*
c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c
)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 4*sqrt(-d)*arctan(s
qrt(-d)*x^2/sqrt(d*x^4 + c)))/(b*d), 1/4*(d*sqrt(a/(b*c - a*d))*arctan(-1/
2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 +
 a*c*x^2)) + sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b
*d), 1/4*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt
(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) - 2*sqrt(-d)*arctan(s
qrt(-d)*x^2/sqrt(d*x^4 + c)))/(b*d)]
```

3.811.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.811.7 Maxima [F]

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^5}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.811.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.811.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^5}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^5/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^5/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.812 $\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.812.1 Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

output
```
1/2*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(1/2)/(-a*d+b*c
)^(1/2)
```

3.812.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\frac{\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

input
```
Integrate[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b
*c - a*d])]/(2*Sqrt[a]*Sqrt[b*c - a*d])
```

3.812.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {965, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

input
```
Int[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])]/(2*Sqrt[a]*Sqrt[b*
c - a*d])
```

3.812.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 965

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.812.4 Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}\,a}{x^{2}\sqrt{(ad-bc)a}}\right)}{2\sqrt{(ad-bc)a}}$
default	$\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^{2}+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}} -$ $\ln\Bigg(-\frac{2(ad-bc)}{b}+$
elliptic	$\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^{2}+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}} -$ $\ln\Bigg(-\frac{2(ad-bc)}{b}+$

input

```
int(x/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/2/((a*d-b*c)*a)^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))
```

3.812.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(42) = 84.

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.54

$$\int \frac{x}{\left(a+bx^{4}\right)\sqrt{c+dx^{4}}}\,dx$$
$$=\left[-\frac{\sqrt{-abc+a^{2}d}\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{8}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{4}+a^{2}c^{2}-4\left(\left(bc-2\,ad\right)x^{6}-acx^{2}\right)\sqrt{dx^{4}+c}\sqrt{-abc+a^{2}d}}{b^{2}x^{8}+2\,abx^{4}+a^{2}}\right)}{8\left(abc-a^{2}d\right)},\ \operatorname{arctan}\Bigg(\right.$$

input
```
integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*
(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sq
rt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2))/(a*b*c -
a^2*d), 1/4*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*
c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2))/sqrt(a*b*c
 - a^2*d)]
```

3.812.6 Sympy [F]

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.812.7 Maxima [F]

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.812.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}}$$

input
```
integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

3.812.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(x/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.813 $\int \frac{1}{x^3(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.813.1 Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{1}{x^3(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}}{2acx^2} - \frac{b\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}$$

output
```
-1/2*b*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(3/2)/(-a*d+
b*c)^(1/2)-1/2*(d*x^4+c)^(1/2)/a/c/x^2
```

3.813.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^3(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}}{2acx^2} - \frac{b\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}\sqrt{bc-ad}}$$

input
```
Integrate[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
-1/2*Sqrt[c + d*x^4]/(a*c*x^2) - (b*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2
 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2)*Sqrt[b*c - a*d
])
```

3.813.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {965, 382, 25, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^4) \sqrt{c+dx^4}} \, dx$$

$$\Big\downarrow \; 965$$

$$\frac{1}{2}\int \frac{1}{x^4 (bx^4+a) \sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \; 382$$

$$\frac{1}{2}\left(\frac{\int -\frac{bc}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{ac}-\frac{\sqrt{c+dx^4}}{acx^2}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(-\frac{\int \frac{bc}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{ac}-\frac{\sqrt{c+dx^4}}{acx^2}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(-\frac{b\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{a}-\frac{\sqrt{c+dx^4}}{acx^2}\right)$$

$$\Big\downarrow \; 291$$

$$\frac{1}{2}\left(-\frac{b\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{a}-\frac{\sqrt{c+dx^4}}{acx^2}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}\left(-\frac{b\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}}{acx^2}\right)$$

input

```
Int[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-(Sqrt[c + d*x^4]/(a*c*x^2)) - (b*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*S
qrt[c + d*x^4])])/(a^(3/2)*Sqrt[b*c - a*d]))/2
```

3.813.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.813. $\int \frac{1}{x^3(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.813.4 Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}\,a}{x^{2}\sqrt{\left(a d -b c \right) a}}\right) b c\,x^{2}+\sqrt{d\,x^{4}+c}\,\sqrt{\left(a d -b c \right) a}}{2 a\,x^{2} \sqrt{\left(a d -b c \right) a}\, c}$
default	$-\frac{\sqrt{d\,x^{4}+c}}{2 a c\,x^{2}}-\frac{b \left(\frac{\ln\left(\frac{-\frac{2 \left(a d -b c \right)}{b}-\frac{2 d \sqrt{-a b}\,\left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d -b c}{b}}\,\sqrt{d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\,\left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d -b c}{b}}}{x^{2}+\frac{\sqrt{-a b}}{b}}\right)}{4 \sqrt{-a b}\,\sqrt{-\frac{a d -b c}{b}}}-\ln\right.}{a}$
risch	$-\frac{\sqrt{d\,x^{4}+c}}{2 a c\,x^{2}}-\frac{b \ln\left(\frac{-\frac{2 \left(a d -b c \right)}{b}-\frac{2 d \sqrt{-a b}\,\left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d -b c}{b}}\,\sqrt{d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\,\left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d -b c}{b}}}{x^{2}+\frac{\sqrt{-a b}}{b}}\right)}{4 a \sqrt{-a b}\,\sqrt{-\frac{a d -b c}{b}}}+$
elliptic	$-\frac{\sqrt{d\,x^{4}+c}}{2 a c\,x^{2}}-\frac{b \ln\left(\frac{-\frac{2 \left(a d -b c \right)}{b}-\frac{2 d \sqrt{-a b}\,\left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d -b c}{b}}\,\sqrt{d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\,\left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d -b c}{b}}}{x^{2}+\frac{\sqrt{-a b}}{b}}\right)}{4 a \sqrt{-a b}\,\sqrt{-\frac{a d -b c}{b}}}+$

input
```
int(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))*b*c*x^2+(d*x^4+c)
^(1/2)*((a*d-b*c)*a)^(1/2))/a/x^2/((a*d-b*c)*a)^(1/2)/c
```

3.813.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(64) = 128$.

Time = 0.30 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.15

$$\int \frac{1}{x^3 (a+bx^4) \sqrt{c+dx^4}}\,dx$$
$$= \left[-\frac{\sqrt{-abc+a^2d}bcx^2 \log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((bc-2\,ad)x^6-acx^2)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)+4}{8\,(a^2bc^2-a^3cd)x^2}\right.$$
$$\left. -\frac{\sqrt{abc-a^2d}bcx^2 \arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^6+(abc^2-a^2cd)x^2)}\right)+2\,\sqrt{dx^4+c}(abc-a^2d)}{4\,(a^2bc^2-a^3cd)x^2}\right]$$

input
```
integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(-a*b*c + a^2*d)*b*c*x^2*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*
c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2))
+ 4*sqrt(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2), -1/4*(sq
rt(a*b*c - a^2*d)*b*c*x^2*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4
+ c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^
2)) + 2*sqrt(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2)]
```

3.813.6 Sympy [F]

$$\int \frac{1}{x^3 (a+bx^4) \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^3 (a+bx^4) \sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x**3/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.813.7 Maxima [F]

$$\int \frac{1}{x^3 (a+bx^4) \sqrt{c+dx^4}}\, dx = \int \frac{1}{(bx^4+a)\sqrt{dx^4+c}x^3}\, dx$$

input
```
integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^3), x)
```

3.813.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^3 (a+bx^4) \sqrt{c+dx^4}}\, dx = \frac{1}{2} d^{\frac{3}{2}} \left(\frac{b \arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}ad} + \frac{2}{\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2-c\right)ad} \right)$$

input
```
integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*d^(3/2)*(b*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a
*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*
x^2 - sqrt(d*x^4 + c))^2 - c)*a*d))
```

3.813.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^4) \sqrt{c+dx^4}}\, dx = \int \frac{1}{x^3\,\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\, dx$$

input
```
int(1/(x^3*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.814 $\int \frac{1}{x^7(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.814.1 Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=-\frac{\sqrt{c+dx^4}}{6acx^6}+\frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2}+\frac{b^2\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}$$

output
```
1/2*b^2*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(5/2)/(-a*d
+b*c)^(1/2)-1/6*(d*x^4+c)^(1/2)/a/c/x^6+1/6*(2*a*d+3*b*c)*(d*x^4+c)^(1/2)/
a^2/c^2/x^2
```

3.814.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\frac{\sqrt{c+dx^4}\left(-ac+3bcx^4+2adx^4\right)}{6a^2c^2x^6}+\frac{b^2\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}\sqrt{bc-ad}}$$

input
```
Integrate[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(Sqrt[c + d*x^4]*(-(a*c) + 3*b*c*x^4 + 2*a*d*x^4))/(6*a^2*c^2*x^6) + (b^2*
ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b
*c - a*d])])/(2*a^(5/2)*Sqrt[b*c - a*d])
```

3.814.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {965, 382, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^4) \sqrt{c+dx^4}} \, dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2} \int \frac{1}{x^8 (bx^4+a) \sqrt{dx^4+c}} dx^2$$

$$\Big\downarrow \text{ 382}$$

$$\frac{1}{2} \left(\frac{\int -\frac{2bdx^4+3bc+2ad}{x^4 (bx^4+a) \sqrt{dx^4+c}} dx^2}{3ac} - \frac{\sqrt{c+dx^4}}{3acx^6} \right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2} \left(-\frac{\int \frac{2bdx^4+3bc+2ad}{x^4 (bx^4+a) \sqrt{dx^4+c}} dx^2}{3ac} - \frac{\sqrt{c+dx^4}}{3acx^6} \right)$$

$$\Big\downarrow \text{ 445}$$

$$\frac{1}{2} \left(-\frac{-\frac{\int \frac{3b^2c^2}{\left(bx^4+a\right) \sqrt{dx^4+c}} dx^2}{ac} - \frac{\sqrt{c+dx^4} (2ad+3bc)}{acx^2}}{3ac} - \frac{\sqrt{c+dx^4}}{3acx^6} \right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2} \left(-\frac{-\frac{3b^2c \int \frac{1}{\left(bx^4+a\right) \sqrt{dx^4+c}} dx^2}{a} - \frac{\sqrt{c+dx^4} (2ad+3bc)}{acx^2}}{3ac} - \frac{\sqrt{c+dx^4}}{3acx^6} \right)$$

$$\Big\downarrow \text{ 291}$$

$$\frac{1}{2} \left(-\frac{-\frac{3b^2c \int \frac{1}{a-(ad-bc)x^4} d\frac{x^2}{\sqrt{dx^4+c}}}{a} - \frac{\sqrt{c+dx^4} (2ad+3bc)}{acx^2}}{3ac} - \frac{\sqrt{c+dx^4}}{3acx^6} \right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{2}\left(-\frac{-\frac{3b^2c\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}(2ad+3bc)}{acx^2}}{3ac}-\frac{\sqrt{c+dx^4}}{3acx^6}\right)$$

input

```
Int[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output

```
(-1/3*Sqrt[c + d*x^4]/(a*c*x^6) - (-(((3*b*c + 2*a*d)*Sqrt[c + d*x^4])/(a*
c*x^2)) - (3*b^2*c*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])]
)/(a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/2
```

3.814.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.814.4 Maple [A] (verified)

Time = 6.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{-3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right)b^2c^2x^6+\left(\left(-3b\,x^4+a\right)c-2ad\,x^4\right)\sqrt{d\,x^4+c}\,\sqrt{(ad-bc)a}}{6\sqrt{(ad-bc)a}\,a^2x^6c^2}$
risch	$-\frac{\sqrt{d\,x^4+c}\,\left(-2ad\,x^4-3bc\,x^4+ac\right)}{6c^2a^2x^6}+\frac{b^2\left(\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2\right.}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}\right.}{}$
default	$-\frac{\sqrt{d\,x^4+c}\,\left(-2d\,x^4+c\right)}{6a\,c^2x^6}+\frac{b\sqrt{d\,x^4+c}}{2a^2c\,x^2}+\frac{b^2\left(\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}}{}}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}\right.}{}$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{6ac\,x^6}+\frac{d\sqrt{d\,x^4+c}}{3a\,c^2x^2}+\frac{b\sqrt{d\,x^4+c}}{2a^2c\,x^2}+\frac{b^2\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-a}}{}}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4a^2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}$

input
```
int(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-3*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))*b^2*c^2*x^6+((
-3*b*x^4+a)*c-2*a*d*x^4)*(d*x^4+c)^(1/2)*((a*d-b*c)*a)^(1/2))/((a*d-b*c)*a
)^(1/2)/a^2/x^6/c^2
```

3.814.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.63

$$\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$
$$= \left[-\frac{3\sqrt{-abc+a^2d}b^2c^2x^6\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2-4\,((bc-2\,ad)x^6-acx^2)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right) -}{24\,(a^3bc^3-a^4c^2d)x^6}\right.$$

input
```
integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^6*log(((b^2*c^2 - 8*a*b*c*d + 8*a
^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x
^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 +
 a^2)) + 4*(a^2*b*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^
4)*sqrt(d*x^4 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^6), 1/12*(3*sqrt(a*b*c - a^
2*d)*b^2*c^2*x^6*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt
(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) - 2*(
a^2*b*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^4)*sqrt(d*x^
4 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^6)]
```

3.814.6 Sympy [F]

$$\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x**7/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**7*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.814.7 Maxima [F]

$$\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^7), x)
```

3.814.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(95) = 190$.

Time = 1.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = -\frac{1}{6}\,d^{\frac{5}{2}}\left(\frac{3\,b^2\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a^2d^2}+\frac{2\left(3\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b-6\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b\right.}{\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right.\right.}\right.$$

input
```
integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/6*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c
+ 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3
*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2
*b*c - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqr
t(d)*x^2 - sqrt(d*x^4 + c))^2 - c)^3*a^2*d^2))
```

3.814.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a+bx^4) \sqrt{c+dx^4}} \, dx = \int \frac{1}{x^7 \left(b\,x^4+a\right) \sqrt{d\,x^4+c}} \, dx$$

input

```
int(1/(x^7*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output

```
int(1/(x^7*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.815 $\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.815.1 Optimal result

Integrand size = 24, antiderivative size = 872

$$\begin{aligned}
&\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx \\
&= \frac{x\sqrt{c+dx^4}}{3bd} - \frac{(-a)^{5/4}\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}\sqrt{bc-ad}} - \frac{(-a)^{5/4}\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}\sqrt{-bc+ad}} \\
&\quad + \frac{a^2\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}} \\
&\quad + \frac{a\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}} \\
&\quad - \frac{(bc+3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{6b^2\sqrt[4]{c}d^{5/4}\sqrt{c+dx^4}} \\
&\quad + \frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}} \\
&\quad + \frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}
\end{aligned}$$

output
```
-1/4*(-a)^(5/4)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/
2))/b^(7/4)/(-a*d+b*c)^(1/2)-1/4*(-a)^(5/4)*arctan(x*(a*d-b*c)^(1/2)/(-a)^
(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(7/4)/(a*d-b*c)^(1/2)+1/3*x*(d*x^4+c)^(1/
2)/b/d-1/6*(3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*ar
ctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^
(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2
/c^(1/4)/d^(5/4)/(d*x^4+c)^(1/2)+1/4*a^2*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c
^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(
d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(
1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/(a
*d+b*c)/(d*x^4+c)^(1/2)+1/4*a*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)
^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c
^(1/4))),1/2*2^(1/2))*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+x^2*
d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/(a*d+b*c)/(
d*x^4+c)^(1/2)+1/8*a*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arct
an(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^
(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2
*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d
*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^
4+c)^(1/2)+1/8*a*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arcta...
```

3.815.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.35 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.29

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$= \frac{x\Bigg(-\frac{(bc+3ad)x^4\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{ad}+5\Bigg(\frac{c}{d}+x^4+\frac{5a^2c^2\,\text{Appell}}{d(a+bx^4)\Big(-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\Big(2bc}}{15b\sqrt{c+dx^4}}$$

input
```
Integrate[x^8/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output

```
(x*(-(((b*c + 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -(
(d*x^4)/c), -((b*x^4)/a)])/(a*d)) + 5*(c/d + x^4 + (5*a^2*c^2*AppellF1[1/4
, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/(d*(a + b*x^4)*(-5*a*c*AppellF
1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/
4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/
4, -((d*x^4)/c), -((b*x^4)/a)]))))))/(15*b*Sqrt[c + d*x^4])
```

3.815.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 1002, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {979, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{979}$$

$$\frac{x\sqrt{c+dx^4}}{3bd}-\frac{\int \frac{(bc+3ad)x^4+ac}{(bx^4+a)\sqrt{dx^4+c}}dx}{3bd}$$

$$\Big\downarrow \text{1021}$$

$$\frac{x\sqrt{c+dx^4}}{3bd}-\frac{\frac{(3ad+bc)\int \frac{1}{\sqrt{dx^4+c}}dx}{b}-\frac{3a^2d\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{b}}{3bd}$$

$$\Big\downarrow \text{761}$$

$$\frac{x\sqrt{c+dx^4}}{3bd}-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(3ad+bc)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{3a^2d\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{b}}{3bd}$$

$$\Big\downarrow \text{925}$$

$$\frac{x\sqrt{c+dx^4}}{3bd}-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(3ad+bc)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{3a^2d\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)}{b}}{3bd}$$

$$\Big\downarrow \text{1541}$$

$$\frac{x\sqrt{c+dx^4}}{3bd}-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(3ad+bc)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{3a^2d\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{}{\sqrt{c}(}}{ad+bc}}{2a}\right.}{}}{3bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt{c+dx^4}}{3bd}-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(3ad+bc)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{3a^2d\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{}}{\left(1-\frac{\sqrt{}}{\sqrt{}}\right.}}{ad+bc}}{2a}\right.}{}}{3bd}$$

$$\Big\downarrow \text{761}$$

$$\frac{x\sqrt{c+dx^4}}{3bd}-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(3ad+bc)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{3a^2d\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{}}{}}{2}\right.}{}}{}$$

$$\Big\downarrow \text{2221}$$

3.815. $\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

$$\frac{x\sqrt{dx^4+c}}{3bd}-$$

$$\frac{\frac{(bc+3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}-\frac{3a^2d\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\right.}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}\right.}{}}{}$$

$$\Big\downarrow 2223$$

$$\frac{x\sqrt{dx^4+c}}{3bd}-$$

$$\frac{\frac{(bc+3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}-\frac{3a^2d\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\right.}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}\right.}{}}{}$$

input
```
Int[x^8/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

3.815. $\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

output
```
(x*Sqrt[c + d*x^4])/(3*b*d) - (((b*c + 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt
[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(
1/4)], 1/2])/(2*b*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4]) - (3*a^2*d*(((a*((Sqrt[
b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c]
 + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*A
rcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4
)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sq
rt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sq
rt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*
ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(
b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt
[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]
) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sq
rt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*
Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqr
t[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]
*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*...
```

3.815.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

3.815. $\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.815. $\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.815.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.11 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.34

method	result
elliptic	$\frac{x\sqrt{d\,x^{4}+c}}{3bd}+\frac{\left(-\frac{a}{b^{2}}-\frac{c}{3db}\right)\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}+\frac{a^{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4}b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^{2}\_\alpha^{2}+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^{4}+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+}{}\right.}{}$
risch	$\frac{x\sqrt{d\,x^{4}+c}}{3bd}-\frac{\frac{(3ad+bc)\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}-\frac{3a^{2}d\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4}b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^{2}\_\alpha^{2}+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^{4}+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^{3}b\sqrt{}}{}}{}\right.}{8b^{2}}}{3bd}$
default	$\frac{\frac{x\sqrt{d\,x^{4}+c}}{3d}-\frac{c\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3d\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}}{b}-\frac{a\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b^{2}\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}+\frac{a^{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4}b+a\right)}\frac{-\frac{\operatorname{arct}}{}}{}\right.}{}$

input
```
int(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(d*x^4+c)^(1/2)/b/d+(-a/b^2-1/3*c/d/b)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-
I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/
2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)+1/8*a^2/b^3*sum(1/_alpha^3*(-1
/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1
/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)
*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*Ellipt
icPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2
)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

3.815.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.815.6 Sympy [F]

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**8/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**8/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.815.7 Maxima [F]

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.815.8 Giac [F]

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^8/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.815.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^8/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^8/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.816 $\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.816.1 Optimal result

Integrand size = 24, antiderivative size = 638

$$\begin{aligned}
\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx &= -\frac{\arctan\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4b\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}} - \frac{\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4b\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}} \\
&\quad + \frac{c^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}
\end{aligned}$$

output
```
-1/4*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/b/((a*
d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/
2))^(1/2)/(d*x^4+c)^(1/2))/b/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/2*c^(
3/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(
1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^
2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/d^(1/4)/(a*d+b*c)/(d*
x^4+c)^(1/2)-1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d
^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2
)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(
1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c
)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(
1/2)*d^(1/2))/(d*x^4+c)^(1/2)-1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/
2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1
/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/
2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*
d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/c^(1/4)/d^(1/4)/(b^(1
/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

3.816.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.10

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\frac{x^5\sqrt{\frac{c+dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{5a\sqrt{c+dx^4}}$$

input
```
Integrate[x^4/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(x^5*Sqrt[(c + d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4
)/a)])/(5*a*Sqrt[c + d*x^4])
```

3.816.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 958, normalized size of antiderivative = 1.50, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {983, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{983}$$

$$\frac{\int \frac{1}{\sqrt{dx^4+c}}dx}{b}-\frac{a\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{a\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \text{925}$$

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{a\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)}{b}$$

$$\Big\downarrow \text{1541}$$

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{a\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)}{a}}{2a}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-$$

$$\frac{a\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int}{ad+bc}}{2a}\right.}{b}$$

$$\Big\downarrow \textbf{761}$$

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-$$

$$\frac{a\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\right.}{}\right.}{b}$$

$$\Big\downarrow \textbf{2221}$$

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-$$

$$\frac{a\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{a\sqrt[4]{d}\left(\sqrt{c}+\sqrt{\right.}{}\right.}{}$$

$$\Big\downarrow \textbf{2223}$$

3.816. $\int\frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

$$\frac{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}-$$

$$a\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{}}\right.}{2\sqrt[4]{b}\sqrt{bc-ad}}\right.}{}}{2a}\right.$$

input

```
Int[x^4/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output

```
((Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellip
ticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*c^(1/4)*d^(1/4)*Sqrt[c + d*
x^4]) - (a*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] +
Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (S
qrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c]
)/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[
c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])
/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)
^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[
b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/
4)*Sqrt[c + d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a
*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt
[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c
 + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(
((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*
x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((
Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*
x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqr
t[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1...
```

3.816. $\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.816.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

3.816. $\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])])/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.816.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.72 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.42

method	result
default	$\frac{\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}}{\_\alpha^3}\right.}{8b^2}$
elliptic	$\frac{\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}}{\_\alpha^3}\right.}{8b^2}$

input
```
int(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)
*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),
I)-1/8*a/b^2*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha
^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^
(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^
2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/
d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))
),_alpha=RootOf(_Z^4*b+a))
```

3.816.5 Fricas [F]

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*x^4/(b*d*x^8 + (b*c + a*d)*x^4 + a*c), x)
```

3.816.6 Sympy [F]

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**4/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**4/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.816.7 Maxima [F]

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.816.8 Giac [F]

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.816.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^4/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.817 $\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.817.1 Optimal result

Integrand size = 21, antiderivative size = 638

$$\begin{aligned}\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx &= \frac{\arctan\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}+\frac{\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}\\
&+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}\\
&+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}\\
&+\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}\end{aligned}$$

output
```
1/4*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/a/((a*d
-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2
))^(1/2)/(d*x^4+c)^(1/2))/a/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/2*d^(3
/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1
/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2
*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/c^(1/4)/(a*d+b*c)/(d*x
^4+c)^(1/2)+1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^
(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)
*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1
/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)
/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1
/2)*d^(1/2))/(d*x^4+c)^(1/2)+1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2
)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/
4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2
)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d
^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/d^(1/4)/(b^(1/
2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

3.817.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.25

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx =$$

$$-\frac{5acx \operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\sqrt{c+dx^4}\left(-5ac \operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bc \operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right.\right.}$$

input
```
Integrate[1/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/((a + b*
x^4)*Sqrt[c + d*x^4]*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b
*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4
)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

3.817.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 860, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{925}$$

$$\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}$$

$$\Big\downarrow \text{1541}$$

$$\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{a\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}$$

$$\Big\downarrow \text{2221}$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+$$

$$\frac{a\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\frac{x}{\sqrt[4]{-a}}\right.}{2\sqrt[4]{b}\sqrt{bc-ad}}\right.}{2a}$$

$$\Big\downarrow \color{blue}{2223}$$

$$\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\frac{\sqrt{}}{\sqrt[4]{-a}}\right.}{2\sqrt[4]{b}\sqrt{bc-ad}}\right.}{2a}$$

$$\frac{\left(\sqrt{d}a+\sqrt{-a}\sqrt{b}\sqrt{c}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\frac{\sqrt[4]{-a}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\text{arcta}}{2\sqrt[4]{b}\sqrt{bc}}\right.}{2a}$$

input
```
Int[1/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

3.817. $\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

output

```
((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)
*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x
)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt
[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] -
 Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])]
)/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(S
qrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elliptic
Pi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*S
qrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c +
d*x^4]))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^
(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/
4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] - (S
qrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*S
qrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*
c^(1/4)*d^(1/4)*Sqrt[c + d*x^4]))/(b*c + a*d))/(2*a)
```

3.817.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1541

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.817.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.69 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.30

method	result	size
default	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^{2}\_\alpha^{2}+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^{4}+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^{3}b\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},\frac{i\sqrt{c}\,\_\alpha^{2}b}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d\,x^{4}+c}}}{\_\alpha^{3}}}{8b}$	191
elliptic	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^{2}\_\alpha^{2}+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^{4}+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^{3}b\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},\frac{i\sqrt{c}\,\_\alpha^{2}b}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d\,x^{4}+c}}}{\_\alpha^{3}}}{8b}$	191

input
```
int(1/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/b*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^
2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_
alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2
)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)
*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*b+a))
```

3.817.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.817.6 Sympy [F]

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.817.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.817.8 Giac [F]

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.817.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.818 $\int \frac{1}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.818.1 Optimal result

Integrand size = 24, antiderivative size = 677

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = {} & -\frac{\sqrt{c+dx^4}}{3acx^3}-\frac{b\arctan\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}} \\
& -\frac{b\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{d^{3/4}(4bc+ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{6ac^{5/4}(bc+ad)\sqrt{c+dx^4}} \\
& -\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} \\
& -\frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}
\end{aligned}$$

output
```
-1/3*(d*x^4+c)^(1/2)/a/c/x^3-1/4*b*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))
^(1/2)/(d*x^4+c)^(1/2))/a^2/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/4*b*arc
tan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/a^2/((-a*d+b*
c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/6*d^(3/4)*(a*d+4*b*c)*(cos(2*arctan(d^(1/4)
*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arc
tan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(
1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(5/4)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*b*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Ell
ipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d
^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(
1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2)
)^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/(d*x^4
+c)^(1/2)-1/8*b*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^
(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2
)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(
1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c
)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)
^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

3.818.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.29 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \frac{-bdx^8\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+\frac{5a\left(-5ac(ac+4bcx^4+2adx^4+bdx^8)\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2\right.}{(a+bx^4)\left(5ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-2x^4\right.}}{15a^2cx^3\sqrt{c+dx^4}}$$

input
```
Integrate[1/(x^4*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-(b*d*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -(
(b*x^4)/a)]) + (5*a*(-5*a*c*(a*c + 4*b*c*x^4 + 2*a*d*x^4 + b*d*x^8)*Appell
F1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(a + b*x^4)*(c +
d*x^4)*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d
*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/((a + b*x^4)*(5
*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] - 2*x^4*(2*b*c
*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4
, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(15*a^2*c*x^3*Sqrt[c + d*x^
4])
```

3.818.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 988, normalized size of antiderivative = 1.46, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {980, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{980}$$

$$\frac{\int -\frac{bdx^4+3bc+ad}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{3ac}-\frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{bdx^4+3bc+ad}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{3ac}-\frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{1021}$$

$$-\frac{3bc\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx+d\int \frac{1}{\sqrt{dx^4+c}}dx}{3ac}-\frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{761}$$

$$-\frac{3bc\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{3ac}-\frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{925}$$

$$-\frac{3bc\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{3ac}-\frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{1541}$$

$$-\frac{3bc\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\right.}{2a}\right.}{3ac}$$

$$-\frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3bc\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int}{ad+}}{2a}\right.}{3ac}$$

$$-\frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{761}$$

$$-\frac{3bc\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\sqrt{b}\left(\sqrt{-\right.\right.}{}$$

$$-\frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{2221}$$

3.818. $\int \frac{1}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx$

$$-\frac{3bc\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{a\sqrt[4]{d}(}{}\right.}{}$$

$$\frac{\sqrt{c+dx^4}}{3acx^3}$$

$\downarrow$ 2223

$$-\frac{\frac{d^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^4+c}}+3bc\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\right.\right.}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}\right.}{}$$

$$\frac{\sqrt{dx^4+c}}{3acx^3}$$

input
```
Int[1/(x^4*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

3.818. $\int \frac{1}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx$

output
```
-1/3*Sqrt[c + d*x^4]/(a*c*x^3) - ((d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
 + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4
)], 1/2])/(2*c^(1/4)*Sqrt[c + d*x^4]) + 3*b*c*(((a*((Sqrt[b]*Sqrt[c])/Sqrt
[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c]
+ Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4
)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt
[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c
- a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d
]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[
(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] -
Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*
x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d))/(2*a
) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^
2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sq
rt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]
*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])
])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(
Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipti
cPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*...
```

3.818.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

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rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.818. $\int \frac{1}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.818.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.32 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.43

method	result
default	$\frac{-\frac{\sqrt{dx^4+c}}{3cx^3}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3c\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{dx^4+c}}}{a}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2dx^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{dx^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1}}{}}{\_\alpha^3}}{8a}$
risch	$-\frac{\sqrt{dx^4+c}}{3acx^3}-\frac{\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{dx^4+c}}+\frac{3c\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2dx^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{dx^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x}{\sqrt{c}}}}{}}{\_\alpha^3}\right.}{8}}{3ac}$
elliptic	$-\frac{\sqrt{dx^4+c}}{3acx^3}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3ac\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{dx^4+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2dx^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{dx^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-}}{}}{\_\alpha}}{8a}$

input
```
int(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/3/c*(d*x^4+c)^(1/2)/x^3-1/3*d/c/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(
1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*El
lipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8/a*sum(1/_alpha^3*(-1/((-a*d+b*
c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4
+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x
^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/
c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))
^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

3.818.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}} \, dx = \text{Timed out}$$

input
```
integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.818.6 Sympy [F]

$$\int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}} \, dx$$

input
```
integrate(1/x**4/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.818.7 Maxima [F]

$$\int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{1}{(bx^4 + a)\sqrt{dx^4 + c}x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^4), x)
```

3.818.8 Giac [F]

$$\int \frac{1}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^4), x)
```

3.818.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\int \frac{1}{x^4\,\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/(x^4*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.819 $\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.819.1 Optimal result

Integrand size = 24, antiderivative size = 804

$$\begin{aligned}
\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx &= \frac{x\sqrt{c+dx^4}}{b\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^2\right)} \\
&- \frac{a\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b(bc-ad)} - \frac{a\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b(bc-ad)} \\
&- \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{bd^{3/4}\sqrt{c+dx^4}} \\
&+ \frac{\sqrt[4]{c}(bc+2ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2bd^{3/4}(bc+ad)\sqrt{c+dx^4}} \\
&+ \frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^{3/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} \\
&- \frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^{3/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}
\end{aligned}$$

output
```
x*(d*x^4+c)^(1/2)/b/d^(1/2)/(c^(1/2)+x^2*d^(1/2))-1/4*a*arctan(x*((a*d-b*c
)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b*c)/(-a)^(1/2)/b^(1/2)
)^(1/2)/b/(-a*d+b*c)-1/4*a*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/
(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/b/(-a*d+b*c)-c^(1/4
)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4
)))*EllipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d
^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/d^(3/4)/(d*x^4+c)^(1/2
)+1/2*c^(1/4)*(2*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2
*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2
*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/
b/d^(3/4)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*a*(cos(2*arctan(d^(1/4)*x/c^(1/4))
)^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4
)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2
)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)
^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/
d^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)-1/8*a*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Ell
ipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*
d^(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^
(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(...
```

3.819.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.08

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x^7\sqrt{\frac{c+dx^4}{c}}\,\text{AppellF1}\left(\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{7a\sqrt{c+dx^4}}$$

input
```
Integrate[x^6/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(x^7*Sqrt[(c + d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^
4)/a)])/(7*a*Sqrt[c + d*x^4])
```

3.819.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 1089, normalized size of antiderivative = 1.35, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {983, 834, 27, 761, 993, 1510, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\left(a+bx^4\right)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{983}$$

$$\frac{\int \frac{x^2}{\sqrt{dx^4+c}}dx}{b}-\frac{a\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \text{834}$$

$$\frac{\frac{\sqrt{c}\int \frac{1}{\sqrt{dx^4+c}}dx}{\sqrt{d}}-\frac{\sqrt{c}\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{c}\sqrt{dx^4+c}}dx}{\sqrt{d}}}{b}-\frac{a\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sqrt{c}\int \frac{1}{\sqrt{dx^4+c}}dx}{\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{dx^4+c}}dx}{\sqrt{d}}}{b}-\frac{a\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{dx^4+c}}dx}{\sqrt{d}}}{b}-\frac{a\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \text{993}$$

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{dx^4+c}}dx}{\sqrt{d}}}{b}-\frac{a\left(\frac{\int \frac{1}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}\right)}{b}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}}{b}-\frac{a\left(\frac{\int\frac{1}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}-\frac{\int\frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}\right)}{b}$$

$$\Big\downarrow \text{1541}$$

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}}{b}-\frac{a\Bigg(\frac{\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}}-\frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\sqrt{b}\sqrt{c}\Big(\sqrt{b}\sqrt{c}-\sqrt{-a}}{2\sqrt{b}}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}}{b}-\frac{a\Bigg(\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}}-\frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int}{ad+b}}{2\sqrt{b}}}{b}$$

$$\Big\downarrow \text{761}$$

3.819. $\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}}{b}-\frac{a\left(\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2\sqrt{b}}-\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\right.}{}\right.}{b}$$

$$\Big\downarrow \text{2221}$$

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{dx^4+c}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{dx^4+c}}-\frac{x\sqrt{dx^4+c}}{\sqrt{d}x^2+\sqrt{c}}}{\sqrt{d}}}{b}-\frac{a\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\right.}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}\right.}{bc+ad}}{2\sqrt{b}}\right.}{}$$

$$\Big\downarrow \text{2223}$$

3.819. $\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{dx^4+c}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{dx^4+c}}-\frac{x\sqrt{dx^4+c}}{\sqrt{d}x^2+\sqrt{c}}}{\sqrt{d}}}{b}-$$

$$a\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\right.}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}\right.}{bc+ad}}{2\sqrt{b}}\right.$$

input
```
Int[x^6/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-((-((x*Sqrt[c + d*x^4])/(Sqrt[c] + Sqrt[d]*x^2)) + (c^(1/4)*(Sqrt[c] + S
qrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(d^(1/4)*Sqrt[c + d*x^4]))/Sqrt[d]) + (c^(1/4
)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^4]))/b
- (a*(-1/2*(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[
d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^
(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b
]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[
d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*
(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) - (((a*Sqrt[c])/(-a)^(3/2) + Sqrt[d]/S
qrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2
]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sq
rt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sq
rt[c + d*x^4])))/(b*c + a*d))/Sqrt[b] + (-1/2*((Sqrt[b]*Sqrt[c] + Sqrt[-a]
*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt
[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(c^(1/4)*(b*c +
 a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((
Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)
*b^(1/4)*Sqrt[c + d*x^4])])/(2*(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) + ((...
```

3.819. $\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.819.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.819. $\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.819.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.76 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.36

method	result
default	$\frac{i\sqrt{c}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,\sqrt{d}}-\frac{a\left(\sum_{\_\_\alpha=\mathrm{RootOf}\left(\_\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\_}{}}{}\right.}{8b^2}$
elliptic	$\frac{i\sqrt{c}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,\sqrt{d}}-\frac{a\left(\sum_{\_\_\alpha=\mathrm{RootOf}\left(\_\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\_}{}}{}\right.}{8b^2}$

3.819. $\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

input
```
int(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I
/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)/d^(1/2)*(EllipticF(x*(I/c^(1/2
)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8*a/b^2*su
m(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-
a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/
a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c
)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/
a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_
Z^4*b+a))
```

3.819.5 Fricas [F]

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*x^6/(b*d*x^8 + (b*c + a*d)*x^4 + a*c), x)
```

3.819.6 Sympy [F]

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**6/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**6/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.819.7 Maxima [F]

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{x^6}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.819.8 Giac [F]

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{x^6}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.819.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{x^6}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^6/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^6/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.820 $\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.820.1 Optimal result

Integrand size $= 24$, antiderivative size $= 656$

$$\begin{aligned}
\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx &= \frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4(bc-ad)}+\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4(bc-ad)} \\
&\quad-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2(bc+ad)\sqrt{c+dx^4}} \\
&\quad-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt{b}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} \\
&\quad+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt{b}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}
\end{aligned}$$

output
```
1/4*arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b
*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(-a*d+b*c)+1/4*arctan(x*((-a*d+b*c)/(-a)^(1/
2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(
-a*d+b*c)-1/2*c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4)))
,1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1
/2)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/
2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1
/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2
)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d
^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/c^(1/4)/d^(1/4)/b^(1/2)/
((-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)+1/8*(cos(2*arctan(d
^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(si
n(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^
(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^
(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2
)/c^(1/4)/d^(1/4)/b^(1/2)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x^4+c)
^(1/2)
```

3.820.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.10

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x^3\sqrt{\frac{c+dx^4}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{c+dx^4}}$$

input
```
Integrate[x^2/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(x^3*Sqrt[(c + d*x^4)/c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4
)/a)])/(3*a*Sqrt[c + d*x^4])
```

3.820.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 865, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {993, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \text{ 993}$$

$$\frac{\int \frac{1}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 1541}$$

$$\frac{\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}}-\frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}}-\frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2\sqrt{b}}-\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 2221}$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\,\text{a}\right.}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}\right.}{ad+bc}}{2\sqrt{b}}$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2\sqrt{b}}$$

$$\Big\downarrow \textcolor{blue}{2223}$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\,\text{a}\right.}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}\right.}{bc+ad}}{2\sqrt{b}}$$

$$\frac{\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\text{arctanh}\left(\frac{}{\sqrt[4]{}}\right.}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-}}\right.}{}}{2\sqrt{b}}$$

input
```
Int[x^2/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

3.820. $\int\frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

output

```
-1/2*(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2
)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*
x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqr
t[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*Ar
cTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*(-a)^(
1/4)*b^(1/4)*Sqrt[b*c - a*d]) - (((a*Sqrt[c])/(-a)^(3/2) + Sqrt[d]/Sqrt[b]
)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elli
pticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*
Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c +
 d*x^4])))/(b*c + a*d))/Sqrt[b] + (-1/2*((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[
d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^
2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(c^(1/4)*(b*c + a*d)*
Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((Sqrt[b
]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/
4)*Sqrt[c + d*x^4])])/(2*(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c]/S
qrt[-a] + Sqrt[d]/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[
c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*(Sqrt[b] - (Sqrt[-a]*Sqrt[d]
)/Sqrt[c])^2)/(Sqrt[-a]*Sqrt[b]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1
/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d))/(2*Sqrt[b])
```

3.820.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 993

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

3.820. $\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.820.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.80 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.29

method	result	size
default	$\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4} b+a\right)} \frac{-\frac{\operatorname{arctanh}\left(\frac{2 d x^{2} \_\alpha^{2}+2 c}{2 \sqrt{\frac{-a d+b c}{b}} \sqrt{d x^{4}+c}}\right)}{\sqrt{\frac{-a d+b c}{b}}}+\frac{2 \_\alpha^{3} b \sqrt{1-\frac{i \sqrt{d} x^{2}}{\sqrt{c}}} \sqrt{1+\frac{i \sqrt{d} x^{2}}{\sqrt{c}}} \Pi\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}, \frac{i \sqrt{c} \_\alpha^{2} b}{\sqrt{d} a}, \frac{\sqrt{-\frac{i \sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}} a \sqrt{d x^{4}+c}}}{\_\alpha}}{8 b}$	191
elliptic	$\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4} b+a\right)} \frac{-\frac{\operatorname{arctanh}\left(\frac{2 d x^{2} \_\alpha^{2}+2 c}{2 \sqrt{\frac{-a d+b c}{b}} \sqrt{d x^{4}+c}}\right)}{\sqrt{\frac{-a d+b c}{b}}}+\frac{2 \_\alpha^{3} b \sqrt{1-\frac{i \sqrt{d} x^{2}}{\sqrt{c}}} \sqrt{1+\frac{i \sqrt{d} x^{2}}{\sqrt{c}}} \Pi\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}, \frac{i \sqrt{c} \_\alpha^{2} b}{\sqrt{d} a}, \frac{\sqrt{-\frac{i \sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}} a \sqrt{d x^{4}+c}}}{\_\alpha}}{8 b}$	191

input
```
int(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/b*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+
2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_al
pha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/
(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_
alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=
RootOf(_Z^4*b+a))
```

3.820.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.820.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**2/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.820.7 Maxima [F]

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.820.8 Giac [F]

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.820.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^2}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^2/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.821 $\int \frac{1}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.821.1 Optimal result

Integrand size = 24, antiderivative size = 833

$$
\begin{aligned}
\int \frac{1}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= -\frac{\sqrt{c+dx^4}}{acx}+\frac{\sqrt{d}x\sqrt{c+dx^4}}{ac\left(\sqrt{c}+\sqrt{d}x^2\right)} \\
&- \frac{b\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a(bc-ad)}-\frac{b\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a(bc-ad)} \\
&- \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{ac^{3/4}\sqrt{c+dx^4}} \\
&+ \frac{\sqrt[4]{d}(2bc+ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2ac^{3/4}(bc+ad)\sqrt{c+dx^4}} \\
&+ \frac{\sqrt{b}\left(\frac{\sqrt{b}\sqrt[4]{c}}{\sqrt[4]{d}}-\frac{\sqrt{-a}\sqrt[4]{d}}{\sqrt[4]{c}}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right)\sqrt{c+dx^4}} \\
&- \frac{\sqrt{b}\left(\frac{\sqrt{b}\sqrt[4]{c}}{\sqrt[4]{d}}+\frac{\sqrt{-a}\sqrt[4]{d}}{\sqrt[4]{c}}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt{c+dx^4}}
\end{aligned}
$$

output
```
-(d*x^4+c)^(1/2)/a/c/x+x*d^(1/2)*(d*x^4+c)^(1/2)/a/c/(c^(1/2)+x^2*d^(1/2))
-1/4*b*arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*
d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/a/(-a*d+b*c)-1/4*b*arctan(x*((-a*d+b*c)/(
-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^
(1/2)/a/(-a*d+b*c)-d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*x/c^(1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/
2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)
/a/c^(3/4)/(d*x^4+c)^(1/2)+1/2*d^(1/4)*(a*d+2*b*c)*(cos(2*arctan(d^(1/4)*x
/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arcta
n(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/
2)+x^2*d^(1/2))^2)^(1/2)/a/c^(3/4)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*(cos(2*ar
ctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Ellipti
cPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/
2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(-d^(1/4)*(-
a)^(1/2)/c^(1/4)+c^(1/4)*b^(1/2)/d^(1/4))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)
/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/((-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/(
d*x^4+c)^(1/2)-1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1
/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*
2^(1/2))*b^(1/2)*(d^(1/4)*(-a)^(1/2)/c^(1/4)+c^(1/4)*b^(1/2)/d^(1/4))*(...
```

3.821.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx$$

$$= \frac{-21a(c+dx^4)+7(-bc+ad)x^4\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+3bdx^8\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\,(}{21a^2cx\sqrt{c+dx^4}}$$

input
```
Integrate[1/(x^2*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-21*a*(c + d*x^4) + 7*(-(b*c) + a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4
, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*x^8*Sqrt[1 + (d*x^4)/c]
*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(21*a^2*c*x*Sqrt
[c + d*x^4])
```

3.821.3 Rubi [A] (warning: unable to verify)

Time = 1.52 (sec) , antiderivative size = 999, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {980, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^4) \sqrt{c+dx^4}} \, dx$$

$$\Big\downarrow \text{980}$$

$$\frac{\int -\frac{x^2(-bdx^4+bc-ad)}{(bx^4+a)\sqrt{dx^4+c}}dx}{ac}-\frac{\sqrt{c+dx^4}}{acx}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{x^2(-bdx^4+bc-ad)}{(bx^4+a)\sqrt{dx^4+c}}dx}{ac}-\frac{\sqrt{c+dx^4}}{acx}$$

$$\Big\downarrow \text{1054}$$

$$-\frac{\int \left(\frac{bcx^2}{(bx^4+a)\sqrt{dx^4+c}}-\frac{dx^2}{\sqrt{dx^4+c}}\right)dx}{ac}-\frac{\sqrt{c+dx^4}}{acx}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{\sqrt{b}c^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8\sqrt{-a}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}+\frac{b^{3/4}c\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{}}\right.}{4\sqrt[4]{-a}\sqrt{bc-ad}}}{}$$

$$\frac{\sqrt{dx^4+c}}{acx}$$

input
```
Int[1/(x^2*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
-(Sqrt[c + d*x^4]/(a*c*x)) - (-((Sqrt[d]*x*Sqrt[c + d*x^4])/(Sqrt[c] + Sqr
t[d]*x^2)) + (b^(3/4)*c*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqr
t[c + d*x^4])])/(4*(-a)^(1/4)*Sqrt[b*c - a*d]) - (b^(3/4)*c*ArcTanh[(Sqrt[
b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*Sqrt[b*
c - a*d]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqr
t[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/Sqrt
[c + d*x^4] - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(S
qrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2
*Sqrt[c + d*x^4]) - (b*c^(3/4)*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1
/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*El
lipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*(b*c + a*d)*Sqrt[c + d*x^4
]) - (b*c^(3/4)*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] +
Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[b]*c
^(3/4)*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt
[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt
[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)
/c^(1/4)], 1/2])/(8*Sqrt[-a]*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[
b]*c^(3/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*
Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*(S...
```

3.821.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.821.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.49 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.37

method	result
default	$\frac{-\frac{\sqrt{d\,x^4+c}}{c x}+\frac{i\sqrt{d}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{c}\,\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}}{a}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)} -\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{acx}+\frac{i\sqrt{d}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{a\sqrt{c}\,\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)} -\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d}}\right)}{\sqrt{\frac{-ad+bc}{b}}}$
risch	$-\frac{\sqrt{d\,x^4+c}}{acx}+\frac{\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-c\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)} -\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4}}\right)}{\sqrt{\frac{-ad+bc}{b}}}\right.}{ac}$

input
```
int(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/c*(d*x^4+c)^(1/2)/x+I*d^(1/2)/c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1
-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1
/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/
2))^(1/2),I)))-1/8/a*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*
_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(
1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1
/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^
(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^
(1/2))),_alpha=RootOf(_Z^4*b+a))
```

3.821.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.821.6 Sympy [F]

$$\int \frac{1}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\int \frac{1}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x**2/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.821.7 Maxima [F]

$$\int \frac{1}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}\,x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^2), x)
```

3.821.8 Giac [F]

$$\int \frac{1}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\int \frac{1}{(bx^4+a)\sqrt{dx^4+cx^2}}\,dx$$

input
```
integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^2), x)
```

3.821.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\int \frac{1}{x^2\,\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/(x^2*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.822 $\int \frac{x^{15}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

3.822.1 Optimal result

Integrand size = 24, antiderivative size = 175

$$\begin{aligned}\int \frac{x^{15}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=&\ \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)}\\&-\frac{\sqrt{c+dx^4}\left(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4\right)}{12b^3d^2(bc-ad)}\\&-\frac{a^2(6bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc-ad)^{3/2}}\end{aligned}$$

output
```
-1/4*a^2*(-5*a*d+6*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/
b^(7/2)/(-a*d+b*c)^(3/2)+1/4*a*x^8*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)-
1/12*(4*b^2*c^2+8*a*b*c*d-15*a^2*d^2-b*d*(-5*a*d+2*b*c)*x^4)*(d*x^4+c)^(1/
2)/b^3/d^2/(-a*d+b*c)
```

3.822.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int \frac{x^{15}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx\\&=-\frac{\sqrt{c+dx^4}\left(-15a^3d^2+2a^2bd\left(4c-5dx^4\right)+2b^3cx^4\left(2c-dx^4\right)+2ab^2\left(2c^2+3cdx^4+d^2x^8\right)\right)}{12b^3d^2(bc-ad)\left(a+bx^4\right)}\\&\quad+\frac{a^2(-6bc+5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{4b^{7/2}(-bc+ad)^{3/2}}\end{aligned}$$

input
```
Integrate[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
-1/12*(Sqrt[c + d*x^4]*(-15*a^3*d^2 + 2*a^2*b*d*(4*c - 5*d*x^4) + 2*b^3*c*
x^4*(2*c - d*x^4) + 2*a*b^2*(2*c^2 + 3*c*d*x^4 + d^2*x^8)))/(b^3*d^2*(b*c
- a*d)*(a + b*x^4)) + (a^2*(-6*b*c + 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4
])/Sqrt[-(b*c) + a*d]])/(4*b^(7/2)*(-(b*c) + a*d)^(3/2))
```

3.822.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 109, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{x^{12}}{\left(bx^4+a\right)^2 \sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{4}\left(\frac{ax^8\sqrt{c+dx^4}}{b\left(a+bx^4\right)(bc-ad)}-\frac{\int \frac{x^4\left(4ac-(2bc-5ad)x^4\right)}{2\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{b(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{ax^8\sqrt{c+dx^4}}{b\left(a+bx^4\right)(bc-ad)}-\frac{\int \frac{x^4\left(4ac-(2bc-5ad)x^4\right)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{164}$$

$$\frac{1}{4}\left(\frac{ax^8\sqrt{c+dx^4}}{b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{2\sqrt{c+dx^4}\left(-15a^2d^2-bdx^4(2bc-5ad)+8abcd+4b^2c^2\right)}{3b^2d^2}-\frac{a^2(6bc-5ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{b^2}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{ax^8\sqrt{c+dx^4}}{b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{2\sqrt{c+dx^4}\left(-15a^2d^2-bdx^4(2bc-5ad)+8abcd+4b^2c^2\right)}{3b^2d^2}-\frac{2a^2(6bc-5ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{b^2d}}{2b(bc-ad)}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{4}\left(\frac{ax^8\sqrt{c+dx^4}}{b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{2a^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^4}\left(-15a^2d^2-bdx^4(2bc-5ad)+8abcd+4b^2c^2\right)}{3b^2d^2}}{2b(bc-ad)}\right)$$

input
```
Int[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((a*x^8*Sqrt[c + d*x^4])/(b*(b*c - a*d)*(a + b*x^4)) - ((2*Sqrt[c + d*x^4]
*(4*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2 - b*d*(2*b*c - 5*a*d)*x^4))/(3*b^2*d^
2) + (2*a^2*(6*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a
*d]])/(b^(5/2)*Sqrt[b*c - a*d]))/(2*b*(b*c - a*d)))/4
```

3.822.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.822.4 Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$-\frac{5\left(-d^{2}\left(ad-\frac{6bc}{5}\right)\left(b\,x^{4}+a\right)a^{2}\arctan\left(\frac{b\sqrt{d\,x^{4}+c}}{\sqrt{\left(ad-bc\right)b}}\right)+\left(-\frac{4x^{4}\left(-\frac{d\,x^{4}}{2}+c\right)c\,b^{3}}{15}-\frac{4\left(d\,x^{4}+c\right)\left(\frac{d\,x^{4}}{2}+c\right)a\,b^{2}}{15}-\frac{8\left(-\frac{5d\,x^{4}}{4}+c\right)d\,a^{2}b}{15}\right.\right.}{4\sqrt{\left(ad-bc\right)b}\,d^{2}b^{3}\left(ad-bc\right)\left(b\,x^{4}+a\right)}$
risch	$-\frac{\left(-bd\,x^{4}+6ad+2bc\right)\sqrt{d\,x^{4}+c}}{6d^{2}b^{3}}-\frac{3a^{2}\ln\left(\frac{-\frac{2\left(ad-bc\right)}{b}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x^{2}+\right.}{b}}}{x^{2}+\frac{\sqrt{-ab}}{b}}\right.}{4b^{4}\sqrt{-\frac{ad-bc}{b}}}$
default	$-\frac{\sqrt{d\,x^{4}+c}\left(-d\,x^{4}+2c\right)}{6b^{2}d^{2}}-\frac{a\sqrt{d\,x^{4}+c}}{b^{3}d}+\frac{3a^{2}\left(-\frac{\ln\left(\frac{-\frac{2\left(ad-bc\right)}{b}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{}}{}}}{x^{2}+\frac{\sqrt{-ab}}{b}}\right.}{4b\sqrt{-\frac{ad-bc}{b}}}\right.}{}$
elliptic	$\frac{x^{4}\sqrt{d\,x^{4}+c}}{6b^{2}d}-\frac{c\sqrt{d\,x^{4}+c}}{3b^{2}d^{2}}-\frac{a\sqrt{d\,x^{4}+c}}{b^{3}d}-\frac{3a^{2}\ln\left(\frac{-\frac{2\left(ad-bc\right)}{b}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-}}{}}}{x^{2}+\frac{\sqrt{-ab}}{b}}\right.}{4b^{4}\sqrt{-\frac{ad-bc}{b}}}$

input
```
int(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/4*(-d^2*(a*d-6/5*b*c)*(b*x^4+a)*a^2*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)
*b)^(1/2))+(-4/15*x^4*(-1/2*d*x^4+c)*c*b^3-4/15*(d*x^4+c)*(1/2*d*x^4+c)*a*
b^2-8/15*(-5/4*d*x^4+c)*d*a^2*b+a^3*d^2)*((a*d-b*c)*b)^(1/2)*(d*x^4+c)^(1/
2))/((a*d-b*c)*b)^(1/2)/d^2/b^3/(a*d-b*c)/(b*x^4+a)
```

3.822.5 Fricas [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 622, normalized size of antiderivative = 3.55

$$\int\frac{x^{15}}{\left(a+bx^{4}\right)^{2}\sqrt{c+dx^{4}}}\,dx$$
$$=\left[\frac{3\left(6\,a^{3}bcd^{2}-5\,a^{4}d^{3}+\left(6\,a^{2}b^{2}cd^{2}-5\,a^{3}bd^{3}\right)x^{4}\right)\sqrt{b^{2}c-abd}\log\left(\frac{bdx^{4}+2\,bc-ad-2\,\sqrt{dx^{4}+c}\sqrt{b^{2}c-abd}}{bx^{4}+a}\right)+2\left(2\left(b^{5}c^{2}\right.\right.}{24\left(ab^{6}c^{2}d^{2}-2\,a^{2}b^{5}c\right.}\right.$$

input
```
integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (6*a^2*b^2*c*d^2 - 5*a^3*b*d^3)*x^4)
*sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b
^2*c - a*b*d))/(b*x^4 + a)) + 2*(2*(b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b^3*d^
3)*x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 - 15*a^4*b*d^3 -
 2*(2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*x^4)*sqrt(d
*x^4 + c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2 -
 2*a*b^6*c*d^3 + a^2*b^5*d^4)*x^4), 1/12*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (
6*a^2*b^2*c*d^2 - 5*a^3*b*d^3)*x^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4
 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + (2*(b^5*c^2*d - 2*a*b^4*c*d^
2 + a^2*b^3*d^3)*x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 -
15*a^4*b*d^3 - 2*(2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^
3)*x^4)*sqrt(d*x^4 + c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 +
(b^7*c^2*d^2 - 2*a*b^6*c*d^3 + a^2*b^5*d^4)*x^4)]
```

3.822.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\text{Timed out}$$

input
```
integrate(x**15/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Timed out
```

3.822.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{15}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.822.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx &= \frac{\sqrt{dx^4+c}a^3d}{4\,(b^4c-ab^3d)((dx^4+c)b-bc+ad)} \\ &+ \frac{(6\,a^2bc-5\,a^3d)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{4\,(b^4c-ab^3d)\sqrt{-b^2c+abd}} \\ &+ \frac{(dx^4+c)^{\frac{3}{2}}b^4d^4-3\,\sqrt{dx^4+c}b^4cd^4-6\,\sqrt{dx^4+c}ab^3d^5}{6\,b^6d^6}\end{aligned}$$

input
```
integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(d*x^4 + c)*a^3*d/((b^4*c - a*b^3*d)*((d*x^4 + c)*b - b*c + a*d))
+ 1/4*(6*a^2*b*c - 5*a^3*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))
/((b^4*c - a*b^3*d)*sqrt(-b^2*c + a*b*d)) + 1/6*((d*x^4 + c)^(3/2)*b^4*d^4
 - 3*sqrt(d*x^4 + c)*b^4*c*d^4 - 6*sqrt(d*x^4 + c)*a*b^3*d^5)/(b^6*d^6)
```

3.822.9 Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx &= \frac{(d\,x^4+c)^{3/2}}{6\,b^2\,d^2}-\left(\frac{3\,c}{2\,b^2\,d^2}+\frac{a\,d-b\,c}{b^3\,d^2}\right)\sqrt{d\,x^4+c} \\ &+ \frac{a^2\,\mathrm{atan}\left(\frac{a^2\,\sqrt{b}\,\sqrt{d\,x^4+c}\,(5\,a\,d-6\,b\,c)}{\sqrt{a\,d-b\,c}\,(5\,a^3\,d-6\,a^2\,b\,c)}\right)\,(5\,a\,d-6\,b\,c)}{4\,b^{7/2}\,(a\,d-b\,c)^{3/2}} \\ &- \frac{a^3\,d\,\sqrt{d\,x^4+c}}{2\,(a\,d-b\,c)\,(2\,b^4\,(d\,x^4+c)-2\,b^4\,c+2\,a\,b^3\,d)}\end{aligned}$$

input
```
int(x^15/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
(c + d*x^4)^(3/2)/(6*b^2*d^2) - ((3*c)/(2*b^2*d^2) + (a*d - b*c)/(b^3*d^2)
)*(c + d*x^4)^(1/2) + (a^2*atan((a^2*b^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 6*
b*c))/((a*d - b*c)^(1/2)*(5*a^3*d - 6*a^2*b*c)))*(5*a*d - 6*b*c))/(4*b^(7/
2)*(a*d - b*c)^(3/2)) - (a^3*d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b^4*(c
 + d*x^4) - 2*b^4*c + 2*a*b^3*d))
```

3.822. $\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.823 $\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.823.1 Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\sqrt{c+dx^4}}{2b^2d} - \frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\,(a+bx^4)} + \frac{a(4bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}$$

output
```
1/4*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(
5/2)/(-a*d+b*c)^(3/2)+1/2*(d*x^4+c)^(1/2)/b^2/d-1/4*a^2*(d*x^4+c)^(1/2)/b^
2/(-a*d+b*c)/(b*x^4+a)
```

3.823.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\frac{\sqrt{b}\sqrt{c+dx^4}(-3a^2d+2b^2cx^4+2ab(c-dx^4))}{d(bc-ad)(a+bx^4)} + \frac{a(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{4b^{5/2}}$$

input
```
Integrate[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((Sqrt[b]*Sqrt[c + d*x^4]*(-3*a^2*d + 2*b^2*c*x^4 + 2*a*b*(c - d*x^4)))/(d
*(b*c - a*d)*(a + b*x^4)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*
x^4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(4*b^(5/2))
```

3.823.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 100, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}dx$$

$$\Big\downarrow \textcolor{blue}{948}$$

$$\frac{1}{4}\int \frac{x^8}{\left(bx^4+a\right)^2 \sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \textcolor{blue}{100}$$

$$\frac{1}{4}\left(\frac{\int -\frac{a(2bc-ad)-2b(bc-ad)x^4}{2\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^4}}{b^2\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{1}{4}\left(-\frac{\int \frac{a(2bc-ad)-2b(bc-ad)x^4}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^4}}{b^2\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \textcolor{blue}{90}$$

$$\frac{1}{4}\left(-\frac{a(4bc-3ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4-\frac{4\sqrt{c+dx^4}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^4}}{b^2\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \textcolor{blue}{73}$$

$$\frac{1}{4}\left(-\frac{\frac{2a(4bc-3ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{d}-\frac{4\sqrt{c+dx^4}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^4}}{b^2\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$\frac{1}{4}\left(-\frac{a^2\sqrt{c+dx^4}}{b^2\left(a+bx^4\right)(bc-ad)}-\frac{-\frac{2a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}-\frac{4\sqrt{c+dx^4}(bc-ad)}{d}}{2b^2(bc-ad)}\right)$$

input
```
Int[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(-((a^2*Sqrt[c + d*x^4])/(b^2*(b*c - a*d)*(a + b*x^4))) - ((-4*(b*c - a*d)
*Sqrt[c + d*x^4])/d - (2*a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4
])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a*d]))/(2*b^2*(b*c - a*d)))/4
```

3.823.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.823.4 Maple [A] (verified)

Time = 5.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{-\frac{3d\left(b\,x^{4}+a\right)a\left(ad-\frac{4bc}{3}\right)\arctan\left(\frac{b\sqrt{d\,x^{4}+c}}{\sqrt{(ad-bc)b}}\right)}{4}+\frac{3\left(-\frac{2b^{2}c\,x^{4}}{3}-\frac{2a\left(-d\,x^{4}+c\right)b}{3}+a^{2}d\right)\sqrt{d\,x^{4}+c}\,\sqrt{(ad-bc)b}}{4}}{b^{2}(ad-bc)d(b\,x^{4}+a)\sqrt{(ad-bc)b}}$
risch	$\frac{\sqrt{d\,x^{4}+c}}{2b^{2}d}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^{2}+\frac{\sqrt{-ab}}{b}}\right)}{2b^{3}\sqrt{-\frac{ad-bc}{b}}}+$
elliptic	$\frac{\sqrt{d\,x^{4}+c}}{2b^{2}d}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^{2}+\frac{\sqrt{-ab}}{b}}\right)}{2b^{3}\sqrt{-\frac{ad-bc}{b}}}+$
default	$\frac{\sqrt{d\,x^{4}+c}}{2b^{2}d}+\frac{a^{2}\left(\frac{\sqrt{-ab}\,\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8ab(ad-bc)\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-b}{b}}}{x^{2}}\right.}{8b(ad-bc}\right.}{}$

input
```
int(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/4/((a*d-b*c)*b)^(1/2)*(-d*(b*x^4+a)*a*(a*d-4/3*b*c)*arctan(b*(d*x^4+c)^(
1/2)/((a*d-b*c)*b)^(1/2))+(-2/3*b^2*c*x^4-2/3*a*(-d*x^4+c)*b+a^2*d)*(d*x^4
+c)^(1/2)*((a*d-b*c)*b)^(1/2))/d/b^2/(a*d-b*c)/(b*x^4+a)
```

3.823.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(103) = 206$.

Time = 0.64 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.86

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$= \left[\frac{(4a^2bcd - 3a^3d^2 + (4ab^2cd - 3a^2bd^2)x^4)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2bc-ad+2\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right) + 2(2ab^3c^2 - 5}{8(ab^5c^2d - 2a^2b^4cd^2 + a^3b^3d^3 + (b^6c^2d - 2ab^5cd^2 + a}\right.$$

$$-\frac{(4a^2bcd - 3a^3d^2 + (4ab^2cd - 3a^2bd^2)x^4)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right) - (2ab^3c^2 - 5a^2b^2cd}{4(ab^5c^2d - 2a^2b^4cd^2 + a^3b^3d^3 + (b^6c^2d - 2ab^5cd^2 + a^2b^4}$$

input
```
integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(b^2
*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*
b*d))/(b*x^4 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4
*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(a*b^5*c^2*d - 2*a
^2*b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^4
), -1/4*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c
)) - (2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d
 + a^2*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3
*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^4)]
```

3.823.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x**11/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Timed out
```

3.823.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.823.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{dx^4+c}\,a^2d}{4\,(b^3c-ab^2d)((dx^4+c)b-bc+ad)} - \frac{(4\,abc-3\,a^2d)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\,(b^3c-ab^2d)\sqrt{-b^2c+abd}} + \frac{\sqrt{dx^4+c}}{2\,b^2d}$$

input
```
integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(d*x^4 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^4 + c)*b - b*c + a*d))
 - 1/4*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/
((b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/2*sqrt(d*x^4 + c)/(b^2*d)
```

3.823.9 Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\sqrt{d\,x^4+c}}{2\,b^2\,d} - \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^4+c}\,(3\,a\,d-4\,b\,c)}{(3\,a^2\,d-4\,a\,b\,c)\,\sqrt{a\,d-b\,c}}\right)\,(3\,a\,d-4\,b\,c)}{4\,b^{5/2}\,(a\,d-b\,c)^{3/2}} + \frac{a^2\,d\,\sqrt{d\,x^4+c}}{2\,(a\,d-b\,c)\,\left(2\,b^3\,\left(d\,x^4+c\right)-2\,b^3\,c+2\,a\,b^2\,d\right)}$$

input

```
int(x^11/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output

```
(c + d*x^4)^(1/2)/(2*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^4)^(1/2)*(3*a*d
- 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(4*b^(
5/2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b^3*
(c + d*x^4) - 2*b^3*c + 2*a*b^2*d))
```

3.824 $\int \frac{x^7}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

3.824.1 Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{x^7}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\frac{a\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

output
```
-1/4*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2
)/(-a*d+b*c)^(3/2)+1/4*a*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)
```

3.824.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{x^7}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\frac{\frac{a\sqrt{b}\sqrt{c+dx^4}}{(bc-ad)(a+bx^4)}-\frac{(2bc-ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{4b^{3/2}}$$

input
```
Integrate[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((a*Sqrt[b]*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) - ((2*b*c - a*d)*Ar
cTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/
(4*b^(3/2))
```

3.824.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{x^4}{(bx^4+a)^2\sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{4}\left(\frac{(2bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^4}{2b(bc-ad)}+\frac{a\sqrt{c+dx^4}}{b\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{(2bc-ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{bd(bc-ad)}+\frac{a\sqrt{c+dx^4}}{b\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{a\sqrt{c+dx^4}}{b\left(a+bx^4\right)(bc-ad)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{3/2}(bc-ad)^{3/2}}\right)$$

input
```
Int[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((a*Sqrt[c + d*x^4])/(b*(b*c - a*d)*(a + b*x^4)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(b^(3/2)*(b*c - a*d)^(3/2)))/4
```

3.824.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.824.4 Maple [A] (verified)

Time = 5.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-\frac{a\sqrt{d\,x^{4}+c}}{b\,x^{4}+a}+\frac{(ad-2bc)\arctan\left(\frac{b\sqrt{d\,x^{4}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{4(ad-bc)b}$
elliptic	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{4b^{2}\sqrt{-\frac{ad-bc}{b}}}-\ \ln\Bigg(-\frac{2(ad-bc)}{b}-$
default	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^{2}+\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}}-\ \ln\Bigg(-\frac{2(ad-bc)}{b}+2d\sqrt{}$ b

input
```
int(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a*d-b*c)/b*(-a*(d*x^4+c)^(1/2)/(b*x^4+a)+(a*d-2*b*c)/((a*d-b*c)*b)^(1
/2)*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

3.824.5 Fricas [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.52

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$= \Bigg[\frac{((2\,b^2c-abd)x^4+2\,abc-a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}(ab^2c-a^2bd)}{8\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^4)}$$

input
```
integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(((2*b^2*c - a*b*d)*x^4 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a))
 + 2*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3
*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^4), 1/4*(((2*b^2*c - a*
b*d)*x^4 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sq
rt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))
/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b
^3*d^2)*x^4)]
```

3.824.6 Sympy [F]

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**7/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.824.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.824.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\frac{\sqrt{dx^4+c}ad^2}{(b^2c-abd)((dx^4+c)b-bc+ad)} + \frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{4\,d}$$

input
```
integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(d*x^4 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^4 + c)*b - b*c + a*d)) +
 (2*b*c*d - a*d^2)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c
- a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

3.824.9 Mupad [B] (verification not implemented)

Time = 9.69 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)\,(a\,d-2\,b\,c)}{4\,b^{3/2}\,(a\,d-b\,c)^{3/2}} - \frac{a\,d\,\sqrt{d\,x^4+c}}{2\,b\,(a\,d-b\,c)\,(2\,b\,(d\,x^4+c)+2\,a\,d-2\,b\,c)}$$

input
```
int(x^7/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
(atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(4*b^(
3/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^4)^(1/2))/(2*b*(a*d - b*c)*(2*b*(c
 + d*x^4) + 2*a*d - 2*b*c))
```

3.825 $\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.825.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=-\frac{\sqrt{c+dx^4}}{4(bc-ad)\,(a+bx^4)}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}(bc-ad)^{3/2}}$$

output
```
1/4*d*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b
^(1/2)-1/4*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)
```

3.825.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\frac{1}{4}\left(-\frac{\sqrt{c+dx^4}}{(bc-ad)\,(a+bx^4)}+\frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

input
```
Integrate[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(-(Sqrt[c + d*x^4]/((b*c - a*d)*(a + b*x^4))) + (d*ArcTan[(Sqrt[b]*Sqrt[c
+ d*x^4])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/4
```

3.825.3 Rubi [A] (verified)

Time $= 0.22$ (sec) , antiderivative size $= 86$, normalized size of antiderivative $= 0.99$, number of steps used $= 5$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used $= \{946, 52, 73, 221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{4}\int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(-\frac{d\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^4}{2(bc-ad)}-\frac{\sqrt{c+dx^4}}{(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{bc-ad}-\frac{\sqrt{c+dx^4}}{(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}}{(a+bx^4)(bc-ad)}\right)$$

input
```
Int[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(-(Sqrt[c + d*x^4]/((b*c - a*d)*(a + b*x^4))) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^4])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2)))/4
```

3.825.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.825.4 Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{d(b\,x^4+a)\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{d\,x^4+c}\,\sqrt{(ad-bc)b}}{4\sqrt{(ad-bc)b}\,(ad-bc)(b\,x^4+a)}$
default	$\frac{\sqrt{-ab}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8ab(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-}}{b}\right.}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{8b(ad-bc)\sqrt{-\frac{ad-bc}{b}}}$
elliptic	$\frac{\sqrt{-ab}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8ab(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-}}{b}\right.}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{8b(ad-bc)\sqrt{-\frac{ad-bc}{b}}}$

input
```
int(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/((a*d-b*c)*b)^(1/2)*(d*(b*x^4+a)*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b
)^(1/2))+(d*x^4+c)^(1/2)*((a*d-b*c)*b)^(1/2))/(a*d-b*c)/(b*x^4+a)
```

3.825.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(71) = 142$.

Time = 0.35 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.47

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \left[-\frac{(bdx^4+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}(b^2c-abd)}{8\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^4)}, -\frac{(bdx^4+ad)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)+\sqrt{dx^4+c}(b^2c-abd)}{4\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^4)}\right]$$

input
```
integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((b*d*x^4 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*
sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2
*c - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c
*d + a^2*b^2*d^2)*x^4), -1/4*((b*d*x^4 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(
sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(b
^2*c - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3
*c*d + a^2*b^2*d^2)*x^4)]
```

3.825.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.825.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.825.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^4+c}d}{4\left((dx^4+c)b-bc+ad\right)(bc-ad)}$$

input
```
integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*d*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*(b*c - a*d)) - 1/4*sqrt(d*x^4 + c)*d/(((d*x^4 + c)*b - b*c + a*d)*(b*c -
 a*d))
```

3.825.9 Mupad [B] (verification not implemented)

Time = 9.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{d\sqrt{d\,x^4+c}}{2\,\left(a\,d-b\,c\right)\,\left(2\,b\,\left(d\,x^4+c\right)+2\,a\,d-2\,b\,c\right)} + \frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

input
```
int(x^3/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
(d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b*(c + d*x^4) + 2*a*d - 2*b*c)) +
(d*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2)))/(4*b^(1/2)*(a*d -
b*c)^(3/2))
```

3.825. $\int \frac{x^3}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

3.826 $\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

3.826.1 Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}}$$

output
```
1/4*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/
2)/a^2/(-a*d+b*c)^(3/2)-1/2*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1
/4*b*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)
```

3.826.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{-\frac{ab\sqrt{c+dx^4}}{(-bc+ad)(a+bx^4)} + \frac{\sqrt{b}(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^2}$$

input
```
Integrate[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output

```
(-((a*b*Sqrt[c + d*x^4])/((-(b*c) + a*d)*(a + b*x^4))) + (Sqrt[b]*(2*b*c -
 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*
d)^(3/2) - (2*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/Sqrt[c])/(4*a^2)
```

3.826.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \left(a+bx^4\right)^2 \sqrt{c+dx^4}}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{4}\int \frac{1}{x^4 \left(bx^4+a\right)^2 \sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \ 114$$

$$\frac{1}{4}\left(\frac{\int \frac{bdx^4+2bc-2ad}{2x^4\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{a\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}\left(\frac{\int \frac{bdx^4+2(bc-ad)}{x^4\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{a\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \ 174$$

$$\frac{1}{4}\left(\frac{\frac{2(bc-ad)\int \frac{1}{x^4\sqrt{dx^4+c}}dx^4}{a}-\frac{b(2bc-3ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{a}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{a\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{4}\left(\frac{\frac{4(bc-ad)\int \frac{1}{\frac{x^8}{d}-\frac{c}{d}}d\sqrt{dx^4+c}}{ad}-\frac{2b(2bc-3ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{ad}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{a\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{4}\left(\frac{\frac{2\sqrt{b}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{4(bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{a\left(a+bx^4\right)(bc-ad)}\right)$$

input
```
Int[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((b*Sqrt[c + d*x^4])/(a*(b*c - a*d)*(a + b*x^4)) + ((-4*(b*c - a*d)*ArcTan
h[Sqrt[c + d*x^4]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*(2*b*c - 3*a*d)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*(
b*c - a*d)))/4
```

3.826.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.826.4 Maple [A] (verified)

Time = 5.01 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$\frac{b\sqrt{c}\,(b\,x^4+a)\left(bc-\frac{3ad}{2}\right)\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)-\frac{\left(2(ad-bc)\left(b\,x^4+a\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{\sqrt{c}}\right)+\sqrt{d\,x^4+c}\,\sqrt{c}\,ab\right)\sqrt{(ad-bc)b}}{2}}{2\sqrt{c}\,\sqrt{(ad-bc)b}\,a^2(ad-bc)(b\,x^4+a)}$
elliptic	$-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a^2\sqrt{c}}+\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-ad-}}{x^2-\frac{\sqrt{-ab}}{b}}\right.}{4a^2\sqrt{-\frac{ad-bc}{b}}}$
default	$-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a^2\sqrt{c}}-\frac{b\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4b\sqrt{-\frac{ad-bc}{b}}}\right.}{}$

input
```
int(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*c^(1/2)*(b*x^4+a)*(b*c-3/2*a*d)*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)
*b)^(1/2))-1/2*(2*(a*d-b*c)*(b*x^4+a)*arctanh((d*x^4+c)^(1/2)/c^(1/2))+(d*
x^4+c)^(1/2)*c^(1/2)*a*b)*((a*d-b*c)*b)^(1/2))/c^(1/2)/((a*d-b*c)*b)^(1/2)
/a^2/(a*d-b*c)/(b*x^4+a)
```

3.826.5 Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 862, normalized size of antiderivative = 6.53

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$
$$= \left[\frac{2\sqrt{dx^4+c}abc+((2\,b^2c^2-3\,abcd)x^4+2\,abc^2-3\,a^2cd)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)-}{8\,(a^3bc^2-a^4cd+(a^2b^2c^2-a^3bcd)x^4)}\right.$$

input
```
integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(d*x^4 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 -
 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4
+ c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*((b^2*c - a*b*d)*x^
4 + a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x
^4))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sqrt(d*x^
4 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(
-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(
b*d*x^4 + b*c)) + ((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(c)*log((d*x^4
 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*
c^2 - a^3*b*c*d)*x^4), 1/8*(2*sqrt(d*x^4 + c)*a*b*c + 4*((b^2*c - a*b*d)*x
^4 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + ((2*b^2*
c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d
*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b
*x^4 + a)))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sq
rt(d*x^4 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d
)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c -
a*d))/(b*d*x^4 + b*c)) + 2*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-c)*
arctan(sqrt(d*x^4 + c)*sqrt(-c)/c))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 -
a^3*b*c*d)*x^4)]
```

3.826.6 Sympy [F]

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

input

```
integrate(1/x/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output

```
Integral(1/(x*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.826.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x}\,dx$$

input

```
integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output

```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x), x)
```

3.826.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\frac{\sqrt{dx^4+c}bd}{4\left(abc-a^2d\right)\left(\left(dx^4+c\right)b-bc+ad\right)}-\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{4\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}}$$

input

```
integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output

```
1/4*sqrt(d*x^4 + c)*b*d/((a*b*c - a^2*d)*((d*x^4 + c)*b - b*c + a*d)) - 1/
4*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2
*b*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/2*arctan(sqrt(d*x^4 + c)/sqrt(-c))
/(a^2*sqrt(-c))
```

3.826.9 Mupad [B] (verification not implemented)

Time = 10.72 (sec) , antiderivative size = 3017, normalized size of antiderivative = 22.86

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input

```
int(1/(x*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output

```
(atan((((((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d
^3))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((2*a^6*b^2*d^5 - 3*a^5*
b^3*c*d^4 + a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c +
 d*x^4)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 2
56*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^
2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d -
3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a
^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d -
 b*c)^3)^(1/2)*1i)/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c
*d^2)) + ((((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c
*d^3))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((2*a^6*b^2*d^5 - 3*a^
5*b^3*c*d^4 + a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c
 + d*x^4)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 -
 256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*
d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d
- 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 -
 a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d
 - b*c)^3)^(1/2)*1i)/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b
*c*d^2)))/(((3*a*b^3*d^4)/16 - (b^4*c*d^3)/8)/(a^5*d^2 + a^3*b^2*c^2 - 2*a
^4*b*c*d) - ((((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a...
```

3.827 $\int \frac{1}{x^5(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.827.1 Optimal result

Integrand size = 24, antiderivative size = 185

$$\begin{aligned}\int \frac{1}{x^5\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx =& -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)}-\frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)}\\ &+\frac{(4bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}}\\ &-\frac{b^{3/2}(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^3(bc-ad)^{3/2}}\end{aligned}$$

output
```
1/4*(a*d+4*b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/4*b^(3/2)*(
-5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+
b*c)^(3/2)-1/4*b*(-a*d+2*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^4+a)-1
/4*(d*x^4+c)^(1/2)/a/c/x^4/(b*x^4+a)
```

3.827.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^5\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

$$=\frac{\frac{a\sqrt{c+dx^4}\left(-a^2d+2b^2cx^4+ab\left(c-dx^4\right)\right)}{c(-bc+ad)x^4(a+bx^4)}-\frac{b^{3/2}(4bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}+\frac{(4bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{c^{3/2}}}{4a^3}$$

input
```
Integrate[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((a*Sqrt[c + d*x^4]*(-(a^2*d) + 2*b^2*c*x^4 + a*b*(c - d*x^4)))/(c*(-(b*c)
 + a*d)*x^4*(a + b*x^4)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcT
anh[Sqrt[c + d*x^4]/Sqrt[c]])/c^(3/2))/(4*a^3)
```

3.827.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {948, 114, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+b x^4\right)^2 \sqrt{c+d x^4}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4} \int \frac{1}{x^8 \left(b x^4+a\right)^2 \sqrt{d x^4+c}}dx^4$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{4} \left(-\frac{\int \frac{3 b d x^4+4 b c+a d}{2 x^4 \left(b x^4+a\right)^2 \sqrt{d x^4+c}}dx^4}{a c}-\frac{\sqrt{c+d x^4}}{a c x^4 \left(a+b x^4\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4} \left(-\frac{\int \frac{3 b d x^4+4 b c+a d}{x^4 \left(b x^4+a\right)^2 \sqrt{d x^4+c}}dx^4}{2 a c}-\frac{\sqrt{c+d x^4}}{a c x^4 \left(a+b x^4\right)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{4} \left(-\frac{\frac{\int \frac{b d (2 b c-a d) x^4+(b c-a d) (4 b c+a d)}{x^4 \left(b x^4+a\right) \sqrt{d x^4+c}}dx^4}{a (b c-a d)}+\frac{2 b \sqrt{c+d x^4} (2 b c-a d)}{a \left(a+b x^4\right) (b c-a d)}}{2 a c}-\frac{\sqrt{c+d x^4}}{a c x^4 \left(a+b x^4\right)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{4}\left(-\frac{\frac{\frac{(bc-ad)(ad+4bc)\int \frac{1}{x^4\sqrt{dx^4+c}}dx^4}{a}-\frac{b^2c(4bc-5ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{a}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^4}(2bc-ad)}{a(a+bx^4)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^4}}{acx^4\left(a+bx^4\right)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{4}\left(-\frac{\frac{\frac{2(bc-ad)(ad+4bc)\int \frac{1}{\frac{x^8}{d}-\frac{c}{d}}d\sqrt{dx^4+c}}{ad}-\frac{2b^2c(4bc-5ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{ad}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^4}(2bc-ad)}{a(a+bx^4)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^4}}{acx^4\left(a+bx^4\right)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{4}\left(-\frac{\frac{\frac{2b^{3/2}c(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)(ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{a\sqrt{c}}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^4}(2bc-ad)}{a(a+bx^4)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^4}}{acx^4\left(a+bx^4\right)}\right)$$

input

```
Int[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output

```
(-(Sqrt[c + d*x^4]/(a*c*x^4*(a + b*x^4))) - ((2*b*(2*b*c - a*d)*Sqrt[c + d
*x^4])/(a*(b*c - a*d)*(a + b*x^4)) + ((-2*(b*c - a*d)*(4*b*c + a*d)*ArcTan
h[Sqrt[c + d*x^4]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(3/2)*c*(4*b*c - 5*a*d)*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(a*(
b*c - a*d)))/(2*a*c))/4
```

3.827.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.827. $\int \frac{1}{x^5(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.827.4 Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-4x^4c^{\frac{5}{2}}b^2\left(bc-\frac{5ad}{4}\right)(b\,x^4+a)\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\,\Big(c\,x^4(b\,x^4+a)(ad+4bc)(ad-bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{\sqrt{c}}\right)+c^{\frac{3}{2}}\,\Big(}{4\sqrt{(ad-bc)b}\,a^3(ad-bc)(b\,x^4+a)x^4c^{\frac{5}{2}}}$
risch	$-\frac{\sqrt{d\,x^4+c}}{4c\,a^2x^4}-\frac{-\frac{(ad+4bc)\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a\sqrt{c}}-2b^2c\,\Bigg(\frac{\sqrt{-ab}\,\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\,\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8ab(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\Bigg(\frac{-\frac{2(ad}{}}{}}{}}{}$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{4c\,a^2x^4}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{4a^2c^{\frac{3}{2}}}+\frac{b\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{a^3\sqrt{c}}-\frac{b\ln\Bigg(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\,\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}}{x^2+}}{2a^3\sqrt{-}}$
default	$\frac{-\frac{\sqrt{d\,x^4+c}}{4c\,x^4}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{4c^{\frac{3}{2}}}}{a^2}+\frac{b\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{a^3\sqrt{c}}+\frac{b^2\Bigg(\frac{\sqrt{-ab}\,\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\,\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-}{b}}}{8ab(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)}}{}$

input
```
int(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/((a*d-b*c)*b)^(1/2)*(-4*x^4*c^(5/2)*b^2*(b*c-5/4*a*d)*(b*x^4+a)*arctan
(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(c*x^4*(b*x^4+
a)*(a*d+4*b*c)*(a*d-b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2))+c^(3/2)*(2*b^2*c
*x^4+a*(-d*x^4+c)*b-a^2*d)*a*(d*x^4+c)^(1/2)))/a^3/(a*d-b*c)/(b*x^4+a)/x^4
/c^(5/2)
```

3.827.5 Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 1236, normalized size of antiderivative = 6.68

$$\int \frac{1}{x^5 \left(a+b x^4\right)^2 \sqrt{c+d x^4}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4
)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c
- a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2
*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(c)*log((d*x^
4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*
b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^
8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -1/8*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8
 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*
x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) - ((4*b^3*c^2 -
 3*a*b^2*c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)
*sqrt(c)*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) + 2*(a^2*b*c^2
 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3
 - a^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -1/8*(2*((4*b^3*c^2 -
3*a*b^2*c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*
sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)
*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(b/(b*c - a*d))*log((b*d*x^4
 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4
 + a)) + 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^
4 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -
1/4*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x...
```

3.827.6 Sympy [F]

$$\int \frac{1}{x^5 \left(a+b x^4\right)^2 \sqrt{c+d x^4}} \, dx = \int \frac{1}{x^5 \left(a+b x^4\right)^2 \sqrt{c+d x^4}} \, dx$$

input
```
integrate(1/x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**5*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.827.7 Maxima [F]

$$\int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^5), x)
```

3.827.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.39

$$\begin{aligned}\int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx &= \frac{(4\,b^3c-5\,ab^2d)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{4\,(a^3bc-a^4d)\sqrt{-b^2c+abd}}\\ &\quad - \frac{2\,(dx^4+c)^{\frac{3}{2}}b^2cd - 2\,\sqrt{dx^4+c}b^2c^2d - (dx^4+c)^{\frac{3}{2}}abd^2 + 2\,\sqrt{dx^4+c}abcd^2 - \sqrt{dx^4+c}a^2d^3}{4\,(a^2bc^2-a^3cd)\left((dx^4+c)^2b - 2\,(dx^4+c)bc + bc^2 + (dx^4+c)ad - acd\right)}\\ &\quad - \frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^3\sqrt{-c}c}\end{aligned}$$

input
```
integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/4*(2*(d*x^4 + c)^(3/2)*b^2*c*d
 - 2*sqrt(d*x^4 + c)*b^2*c^2*d - (d*x^4 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^4
+ c)*a*b*c*d^2 - sqrt(d*x^4 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^4 +
 c)^2*b - 2*(d*x^4 + c)*b*c + b*c^2 + (d*x^4 + c)*a*d - a*c*d)) - 1/4*(4*b
*c + a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

3.827.9 Mupad [B] (verification not implemented)

Time = 11.90 (sec) , antiderivative size = 3822, normalized size of antiderivative = 20.66

$$\int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input
```
int(1/(x^5*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
(((c + d*x^4)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
 a*c*d)) + (b*(c + d*x^4)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d))
)/((c + d*x^4)*(2*a*d - 4*b*c) + 2*b*(c + d*x^4)^2 + 2*b*c^2 - 2*a*c*d) +
(atan((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2)*(a^
4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b
^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d
 - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a
^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3
*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a
^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^
2*d^5))/(64*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3
*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a
^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*
c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c
 + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b
^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^
3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^
6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^
2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*
a*d - 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c...
```

3.828 $\int \frac{x^{13}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

3.828.1 Optimal result

Integrand size = 24, antiderivative size = 191

$$\begin{aligned}\int \frac{x^{13}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=&\frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)}+\frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)}\\&+\frac{a^{3/2}(5bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}}\\&-\frac{(bc+4ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^3d^{3/2}}\end{aligned}$$

output
```
1/4*a^(3/2)*(-4*a*d+5*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(
1/2))/b^3/(-a*d+b*c)^(3/2)-1/4*(4*a*d+b*c)*arctanh(x^2*d^(1/2)/(d*x^4+c)^(
1/2))/b^3/d^(3/2)+1/4*(-2*a*d+b*c)*x^2*(d*x^4+c)^(1/2)/b^2/d/(-a*d+b*c)+1/
4*a*x^6*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)
```

3.828.2 Mathematica [A] (verified)

Time = 2.77 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.99

$$\int \frac{x^{13}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

$$=\frac{\frac{bx^2\sqrt{c+dx^4}\left(-2a^2d+b^2cx^4+ab\left(c-dx^4\right)\right)}{d(bc-ad)(a+bx^4)}+\frac{a^{3/2}(5bc-4ad)\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}-\frac{(bc+4ad)\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{d^{3/2}}}{4b^3}$$

input
```
Integrate[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((b*x^2*Sqrt[c + d*x^4]*(-2*a^2*d + b^2*c*x^4 + a*b*(c - d*x^4)))/(d*(b*c
- a*d)*(a + b*x^4)) + (a^(3/2)*(5*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*x^2*(
Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(b*c - a*d)^(3
/2) - ((b*c + 4*a*d)*Log[Sqrt[d]*x^2 + Sqrt[c + d*x^4]])/d^(3/2))/(4*b^3)
```

3.828.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {965, 372, 444, 27, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^{12}}{(bx^4+a)^2\sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \text{372}$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b(a+bx^4)(bc-ad)}-\frac{\int \frac{x^4(3ac-2(bc-2ad)x^4)}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{444}$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b(a+bx^4)(bc-ad)}-\frac{-\frac{\int -\frac{2\left((bc-ad)(bc+4ad)x^4+ac(bc-2ad)\right)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{2bd}-\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{bd}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b(a+bx^4)(bc-ad)}-\frac{\frac{\int \frac{(bc-ad)(bc+4ad)x^4+ac(bc-2ad)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{bd}-\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{bd}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{(bc-ad)(4ad+bc)\int\frac{1}{\sqrt{dx^4+c}}dx^2}{b}-\frac{a^2d(5bc-4ad)\int\frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}}{bd}-\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{bd}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \; 224$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{(bc-ad)(4ad+bc)\int\frac{1}{1-dx^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}-\frac{a^2d(5bc-4ad)\int\frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}}{bd}-\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{bd}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{(bc-ad)(4ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{a^2d(5bc-4ad)\int\frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}}{bd}-\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{bd}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \; 291$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{(bc-ad)(4ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{a^2d(5bc-4ad)\int\frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}}{bd}-\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{bd}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{(bc-ad)(4ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{a^{3/2}d(5bc-4ad)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{b\sqrt{bc-ad}}}{bd}-\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{bd}}{2b(bc-ad)}\right)$$

input

```
Int[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

3.828. $\int \frac{x^{13}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

output
```
((a*x^6*Sqrt[c + d*x^4])/(2*b*(b*c - a*d)*(a + b*x^4)) - (-(((b*c - 2*a*d)
*x^2*Sqrt[c + d*x^4])/(b*d)) + (-((a^(3/2)*d*(5*b*c - 4*a*d)*ArcTan[(Sqrt[
b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(b*Sqrt[b*c - a*d])) + ((b*c -
 a*d)*(b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(b*Sqrt[d]))/(
b*d))/(2*b*(b*c - a*d)))/2
```

3.828.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.828.4 Maple [A] (verified)

Time = 6.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$-\frac{\frac{a^{2}\left(-\frac{b\sqrt{d\,x^{4}+c}\,x^{2}}{b\,x^{4}+a}-\frac{(4ad-5bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}\,a}{x^{2}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{ad-bc}-\frac{b\sqrt{d\,x^{4}+c}\,x^{2}d^{\frac{3}{2}}-4\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}}{x^{2}\sqrt{d}}\right)a\,d^{2}-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}}{x^{2}\sqrt{d}}\right)bcd}{d^{\frac{5}{2}}}}{4b^{3}}$
risch	$\frac{x^{2}\sqrt{d\,x^{4}+c}}{4b^{2}d}-\frac{a\ln\left(x^{2}\sqrt{d}+\sqrt{d\,x^{4}+c}\right)}{b^{3}\sqrt{d}}-\frac{\ln\left(x^{2}\sqrt{d}+\sqrt{d\,x^{4}+c}\right)c}{4b^{2}d^{\frac{3}{2}}}+\frac{a^{2}\sqrt{d\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8b^{3}(ad-bc)\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}$
elliptic	$\frac{x^{2}\sqrt{d\,x^{4}+c}}{4b^{2}d}-\frac{a\ln\left(x^{2}\sqrt{d}+\sqrt{d\,x^{4}+c}\right)}{b^{3}\sqrt{d}}-\frac{\ln\left(x^{2}\sqrt{d}+\sqrt{d\,x^{4}+c}\right)c}{4b^{2}d^{\frac{3}{2}}}+\frac{a^{2}\sqrt{d\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8b^{3}(ad-bc)\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}$
default	Expression too large to display

input
```
int(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^3*(a^2/(a*d-b*c)*(-b*(d*x^4+c)^(1/2)*x^2/(b*x^4+a)-(4*a*d-5*b*c)/((
a*d-b*c)*a)^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2)))-(b*(
d*x^4+c)^(1/2)*x^2*d^(3/2)-4*arctanh((d*x^4+c)^(1/2)/x^2/d^(1/2))*a*d^2-ar
ctanh((d*x^4+c)^(1/2)/x^2/d^(1/2))*b*c*d)/d^(5/2))
```

3.828.5 Fricas [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 1386, normalized size of antiderivative = 7.26

$$\int \frac{x^{13}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input
```
integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(2*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4
*a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c)
 + (5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a
/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 -
4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b
*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*
b*x^4 + a^2)) + 4*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x
^2)*sqrt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*
x^4), 1/16*(4*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*
d - 4*a^2*b*d^2)*x^4)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + (5*a
^2*b*c*d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a/(b*c -
 a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c
*d)*x^4 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 -
a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 +
 a^2)) + 4*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*sqr
t(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4), -
1/8*((5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(
a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(
a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) - (a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2
 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 + 2*...
```

3.828.6 Sympy [F]

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**13/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**13/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.828.7 Maxima [F]

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^{13}}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^13/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.828.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(163) = 326$.

Time = 0.39 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.74

$$\begin{aligned}&\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx\\&= -\frac{\left(5\,a^2bc\sqrt{d}-4\,a^3d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\,(b^4c-ab^3d)\sqrt{abcd-a^2d^2}}+\frac{\sqrt{dx^4+c}x^2}{4\,b^2d}\\&\quad+\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2a^2bcd-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2a^3d^2-a^2bc^2d}{2\left(b^4c\sqrt{d}-ab^3d^{\frac{3}{2}}\right)\Big(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2ad+}\\&\quad+\frac{(bc+4\,ad)\log\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2\right)}{8\,b^3d^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(5*a^2*b*c*sqrt(d) - 4*a^3*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d
*x^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^4*c - a*b^3*d)*
sqrt(a*b*c*d - a^2*d^2)) + 1/4*sqrt(d*x^4 + c)*x^2/(b^2*d) + 1/2*((sqrt(d)
*x^2 - sqrt(d*x^4 + c))^2*a^2*b*c*d - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*
a^3*d^2 - a^2*b*c^2*d)/((b^4*c*sqrt(d) - a*b^3*d^(3/2))*((sqrt(d)*x^2 - sq
rt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c + 4*(sqrt(d)*
x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)) + 1/8*(b*c + 4*a*d)*log((sqrt(d)*x^
2 - sqrt(d*x^4 + c))^2)/(b^3*d^(3/2))
```

3.828.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^{13}}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^13/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^13/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.829 $\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

3.829.1 Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

output
```
-1/4*(-2*a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*a
^(1/2)/b^2/(-a*d+b*c)^(3/2)+1/2*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b^2/d
^(1/2)+1/4*a*x^2*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)
```

3.829.2 Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{\frac{abx^2\sqrt{c+dx^4}}{(bc-ad)(a+bx^4)} + \frac{\sqrt{a}(-3bc+2ad)\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{d}}}{4b^2}$$

input
```
Integrate[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((a*b*x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (Sqrt[a]*(-3*b*c +
2*a*d)*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]
*Sqrt[b*c - a*d])])/(b*c - a*d)^(3/2) + (2*Log[Sqrt[d]*x^2 + Sqrt[c + d*x^
4]])/Sqrt[d])/(4*b^2)
```

3.829.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 372, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^8}{\left(bx^4+a\right)^2 \sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \text{372}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\int \frac{ac-2(bc-ad)x^4}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}-\frac{2(bc-ad)\int \frac{1}{\sqrt{dx^4+c}}dx^2}{b}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}-\frac{2(bc-ad)\int \frac{1}{1-dx^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{b\sqrt{bc-ad}}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

input
```
Int[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((a*x^2*Sqrt[c + d*x^4])/(2*b*(b*c - a*d)*(a + b*x^4)) - ((Sqrt[a]*(3*b*c
- 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(b*Sqrt[
b*c - a*d]) - (2*(b*c - a*d)*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(b*Sq
rt[d]))/(2*b*(b*c - a*d)))/2
```

3.829.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.829. $\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.829.4 Maple [A] (verified)

Time = 5.70 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{-\frac{a\left(-\frac{b\sqrt{d\,x^{4}+c}\,x^{2}}{b\,x^{4}+a}-\frac{(2ad-3bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}\,a}{x^{2}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{ad-bc}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}}{x^{2}\sqrt{d}}\right)}{\sqrt{d}}}{4b^{2}}$
elliptic	$\frac{\ln\left(x^{2}\sqrt{d}+\sqrt{d\,x^{4}+c}\right)}{2b^{2}\sqrt{d}}-\frac{a\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8b^{2}(ad-bc)\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}-\frac{ad\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\,(x^{2}+\frac{\sqrt{-}}{b}}{b}}{}\right.}{}$
default	Expression too large to display

input
```
int(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^2*(-a/(a*d-b*c)*(-b*(d*x^4+c)^(1/2)*x^2/(b*x^4+a)-(2*a*d-3*b*c)/((a
*d-b*c)*a)^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2)))-2/d^(
1/2)*arctanh((d*x^4+c)^(1/2)/x^2/d^(1/2)))
```

3.829.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 1077, normalized size of antiderivative = 7.64

$$\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

$$= \left[\frac{4\sqrt{dx^4+c}abdx^2 + 4\left((b^2c-abd)x^4+abc-a^2d\right)\sqrt{d}\log\left(-2\,dx^4-2\,\sqrt{dx^4+c}\sqrt{d}x^2-c\right)+\left((3\,b^2cd-2\right.}{16\,(a}\right.$$

input
```
integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(4*sqrt(d*x^4 + c)*a*b*d*x^2 + 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*
d)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + ((3*b^2*c*d
 - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*
c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2
 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqr
t(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a*b^3*c*
d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/16*(4*sqrt(d*x^4 + c)*a*b*
d*x^2 - 8*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x
^2/sqrt(d*x^4 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2
)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a
*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x
^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x
^8 + 2*a*b*x^4 + a^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x
^4), 1/8*(2*sqrt(d*x^4 + c)*a*b*d*x^2 + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a
*b*c*d - 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a
*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + 2*((b^2*c -
 a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt
(d)*x^2 - c))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/8*(
2*sqrt(d*x^4 + c)*a*b*d*x^2 - 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt
(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^...
```

3.829.6 Sympy [F]

$$\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**9/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**9/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.829.7 Maxima [F]

$$\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^9}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^9/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.829.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(117) = 234$.

Time = 0.37 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.11

$$\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=-\frac{\left(3\,abc\sqrt{d}-2\,a^2d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\,(b^3c-ab^2d)\sqrt{abcd-a^2d^2}}-\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2abc\sqrt{d}-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2a^2d^{\frac{3}{2}}-abc^2\sqrt{d}}{2\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2ad+bc^2\right)(b^3c-ab^2d)}-\frac{\log\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2\right)}{4\,b^2\sqrt{d}}$$

input
```
integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*
x^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*s
qrt(a*b*c*d - a^2*d^2)) - 1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b*c*sqr
t(d) - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/
(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^
2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)
) - 1/4*log((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2)/(b^2*sqrt(d))
```

3.829.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^9}{\left(b\,x^4+a\right)^2\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^9/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^9/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.830 $\int \frac{x^5}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

3.830.1 Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^5}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=-\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{c\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4\sqrt{a}(bc-ad)^{3/2}}$$

output

```
1/4*c*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/(-a*d+b*c)^(3/2
)/a^(1/2)-1/4*x^2*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)
```

3.830.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{x^5}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\frac{1}{4}\left(-\frac{x^2\sqrt{c+dx^4}}{(bc-ad)\left(a+bx^4\right)}+\frac{c\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}(bc-ad)^{3/2}}\right)$$

input

```
Integrate[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output

```
(-((x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4))) + (c*ArcTan[(a*Sqrt[d]
 + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqr
t[a]*(b*c - a*d)^(3/2)))/4
```

3.830.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {965, 373, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^4+a)^2\sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \text{373}$$

$$\frac{1}{2}\left(\frac{\int \frac{c}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{2(bc-ad)}-\frac{x^2\sqrt{c+dx^4}}{2(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{c\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{2(bc-ad)}-\frac{x^2\sqrt{c+dx^4}}{2(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(\frac{c\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{2(bc-ad)}-\frac{x^2\sqrt{c+dx^4}}{2(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{c\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}(bc-ad)^{3/2}}-\frac{x^2\sqrt{c+dx^4}}{2(a+bx^4)(bc-ad)}\right)$$

input
```
Int[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(-1/2*(x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (c*ArcTan[(Sqrt[b*
c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*Sqrt[a]*(b*c - a*d)^(3/2)))/2
```

3.830.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 965

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.830.4 Maple [A] (verified)

Time = 5.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{c\left(-\frac{\sqrt{d\,x^{4}+c}\,x^{2}}{c\left(b\,x^{4}+a\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}\,a}{x^{2}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{4(ad-bc)}$
elliptic	$\frac{\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\,\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8b(ad-bc)\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln (\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\,\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{d (x^{2}+\sqrt{-}}}{x^{2}+\frac{\sqrt{-ab}}{b}}}{8b^{2}(ad-bc)\sqrt{-\frac{ad-bc}{b}}}$
default	Expression too large to display

input
```
int(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*c/(a*d-b*c)*(-(d*x^4+c)^(1/2)*x^2/c/(b*x^4+a)+1/((a*d-b*c)*a)^(1/2)*a
rctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2)))
```

3.830.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(77) = 154$.

Time = 0.45 (sec) , antiderivative size = 426, normalized size of antiderivative = 4.58

$$\int \frac{x^5}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

$$= \Bigg[-\frac{4\sqrt{dx^4+c}(abc-a^2d)x^2-(bcx^4+ac)\sqrt{-abc+a^2d}\log\Big(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((b}{b^2x^8+2\,abx^4+a^2}}{16\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^4)}$$

$$-\frac{2\sqrt{dx^4+c}(abc-a^2d)x^2-(bcx^4+ac)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^6+(abc^2-a^2cd)x^2)}\right)}{8\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^4)}\Bigg]$$

input
```
integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(4*sqrt(d*x^4 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(-a*b*
c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sq
rt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a^2*b^2*c^2 - 2*a^3*b*c
*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4), -1/8*(2*sqrt(
d*x^4 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(a*b*c - a^2*d)*arcta
n(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*
c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d
 + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4)]
```

3.830.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.830.7 Maxima [F]

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^5}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.830.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(77) = 154$.

Time = 0.86 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.62

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{c\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\,\sqrt{abcd-a^2d^2}(bc-ad)} + \frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{2\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2ad+bc^2\right)(b^2c-abd)}$$

input
```
integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a
*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + 1/2*(
(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x^2 - sqrt(d*x^
4 + c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b
 - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 +
 c))^2*a*d + b*c^2)*(b^2*c - a*b*d))
```

3.830.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^5}{\left(b\,x^4+a\right)^2\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^5/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^5/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.831 $\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.831.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)\,(a+bx^4)} + \frac{(bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}$$

output
```
1/4*(-2*a*d+b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(3
/2)/(-a*d+b*c)^(3/2)+1/4*b*x^2*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)
```

3.831.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.19

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{bx^2\sqrt{c+dx^4}}{4a(-bc+ad)\,(a+bx^4)} + \frac{(bc-2ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^4+bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{3/2}(bc-ad)^{3/2}}$$

input
```
Integrate[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
-1/4*(b*x^2*Sqrt[c + d*x^4])/(a*(-(b*c) + a*d)*(a + b*x^4)) + ((b*c - 2*a*
d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqr
t[b*c - a*d])])/(4*a^(3/2)*(b*c - a*d)^(3/2))
```

3.831.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {965, 296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \text{296}$$

$$\frac{1}{2}\left(\frac{(bc-2ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{2a(bc-ad)}+\frac{bx^2\sqrt{c+dx^4}}{2a(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(\frac{(bc-2ad)\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{2a(bc-ad)}+\frac{bx^2\sqrt{c+dx^4}}{2a(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{(bc-2ad)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx^2\sqrt{c+dx^4}}{2a(a+bx^4)(bc-ad)}\right)$$

input
```
Int[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((b*x^2*Sqrt[c + d*x^4])/(2*a*(b*c - a*d)*(a + b*x^4)) + ((b*c - 2*a*d)*Ar
cTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^(3/2)*(b*c - a
*d)^(3/2)))/2
```

3.831.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.831.4 Maple [A] (verified)

Time = 5.85 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{-\frac{b\sqrt{d\,x^{4}+c}\,x^{2}}{b\,x^{4}+a}+\frac{(2ad-bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}\,a}{x^{2}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}}{4(ad-bc)a}$
default	$-\frac{\sqrt{d\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8a(ad-bc)\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^{2}-\right.}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right.}{8ba(ad-bc)\sqrt{-\frac{ad-bc}{b}}}$
elliptic	$-\frac{\sqrt{d\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8a(ad-bc)\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^{2}-\right.}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right.}{8ba(ad-bc)\sqrt{-\frac{ad-bc}{b}}}$

input
```
int(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a*d-b*c)/a*(-b*(d*x^4+c)^(1/2)*x^2/(b*x^4+a)+(2*a*d-b*c)/((a*d-b*c)*a
)^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2)))
```

3.831.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(88) = 176$.

Time = 0.66 (sec) , antiderivative size = 467, normalized size of antiderivative = 4.49

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \left[\frac{4\sqrt{dx^4+c}(ab^2c-a^2bd)x^2-((b^2c-2\,abd)x^4+abc-2\,a^2d)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,a}{}\right.}{16\,(a^3b^2c^2-2\,a^4bcd+a^5d^2+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)}\right.$$

input
```
integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(4*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 - ((b^2*c - 2*a*b*d)*x^4
+ a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*
d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6
- a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^
2)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d +
 a^4*b*d^2)*x^4), 1/8*(2*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 + ((b^2*c
 - 2*a*b*d)*x^4 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c -
2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)
*x^6 + (a*b*c^2 - a^2*c*d)*x^2)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (
a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^4)]
```

3.831.6 Sympy [F]

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.831.7 Maxima [F]

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.831.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(88) = 176.

Time = 0.32 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.28

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx =$$

$$-\frac{1}{4}d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(abcd-a^2d^2)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc}{\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}x^2-\sqrt{dx^4+}\right.\right.}\right.$$

input
```
integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*
b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((
sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2
*a*d - b*c^2)/((((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqr
t(d*x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)*(a*
b*c*d - a^2*d^2)))
```

3.831.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x}{{\left(b\,x^4+a\right)}^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.832 $\int \frac{1}{x^3(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.832.1 Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{1}{x^3\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{b(3bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}}$$

output
```
-1/4*b*(-4*a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))
/a^(5/2)/(-a*d+b*c)^(3/2)-1/4*(-2*a*d+3*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b
*c)/x^2+1/4*b*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/x^2/(b*x^4+a)
```

3.832.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^3\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{\sqrt{c+dx^4}\left(2abc-2a^2d+3b^2cx^4-2abdx^4\right)}{4a^2c(-bc+ad)x^2\left(a+bx^4\right)} - \frac{b(3bc-4ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^4+bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{5/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(Sqrt[c + d*x^4]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^4 - 2*a*b*d*x^4))/(4*a^2*c
*(-(b*c) + a*d)*x^2*(a + b*x^4)) - (b*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] +
b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*a^(5
/2)*(b*c - a*d)^(3/2))
```

3.832.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 374, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+bx^4\right)^2 \sqrt{c+dx^4}} \, dx$$

$$\Big\downarrow \; 965$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(bx^4+a\right)^2 \sqrt{dx^4+c}} dx^2$$

$$\Big\downarrow \; 374$$

$$\frac{1}{2} \left(\frac{b\sqrt{c+dx^4}}{2ax^2 \left(a+bx^4\right) (bc-ad)}-\frac{\int -\frac{2bdx^4+3bc-2ad}{x^4 \left(bx^4+a\right) \sqrt{dx^4+c}}dx^2}{2a(bc-ad)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2} \left(\frac{\int \frac{2bdx^4+3bc-2ad}{x^4 \left(bx^4+a\right) \sqrt{dx^4+c}}dx^2}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^2 \left(a+bx^4\right) (bc-ad)}\right)$$

$$\Big\downarrow \; 445$$

$$\frac{1}{2} \left(\frac{-\frac{\int \frac{bc(3bc-4ad)}{\left(bx^4+a\right) \sqrt{dx^4+c}}dx^2}{ac}-\frac{\sqrt{c+dx^4}(3bc-2ad)}{acx^2}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^2 \left(a+bx^4\right) (bc-ad)}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2} \left(\frac{-\frac{b(3bc-4ad) \int \frac{1}{\left(bx^4+a\right) \sqrt{dx^4+c}}dx^2}{a}-\frac{\sqrt{c+dx^4}(3bc-2ad)}{acx^2}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^2 \left(a+bx^4\right) (bc-ad)}\right)$$

$$\Big\downarrow \; 291$$

$$\frac{1}{2}\left(\frac{-\frac{b(3bc-4ad)\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{a}-\frac{\sqrt{c+dx^4}(3bc-2ad)}{acx^2}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^2\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}\left(\frac{-\frac{b(3bc-4ad)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}(3bc-2ad)}{acx^2}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^2\left(a+bx^4\right)(bc-ad)}\right)$$

input
```
Int[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((b*Sqrt[c + d*x^4])/(2*a*(b*c - a*d)*x^2*(a + b*x^4)) + (-(((3*b*c - 2*a*
d)*Sqrt[c + d*x^4])/(a*c*x^2)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d
]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(a^(3/2)*Sqrt[b*c - a*d]))/(2*a*(b*c -
a*d)))/2
```

3.832.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.832.4 Maple [A] (verified)

Time = 6.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^4+c}}{x^2}+\frac{bc\left(\frac{b\sqrt{d\,x^4+c}\,x^2}{b\,x^4+a}-\frac{(4ad-3bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2ad-2bc}}{2a^2c}$
risch	$-\frac{\sqrt{d\,x^4+c}}{2a^2c\,x^2}-\frac{3b\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8a^2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}+$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{2a^2c\,x^2}-\frac{3b\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8a^2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}+$
default	Expression too large to display

input
```
int(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2*(-(d*x^4+c)^(1/2)/x^2+1/2*b*c/(a*d-b*c)*(b*(d*x^4+c)^(1/2)*x^2/(b*
x^4+a)-(4*a*d-3*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a
*d-b*c)*a)^(1/2))))/c
```

3.832.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(129) = 258$.

Time = 0.72 (sec) , antiderivative size = 612, normalized size of antiderivative = 4.11

$$\int \frac{1}{x^3 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx$$
$$= \left[-\frac{\left((3\,b^3c^2-4\,ab^2cd)x^6+(3\,ab^2c^2-4\,a^2bcd)x^2\right)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2}{b^2x^8+2\,ab}\right.}{16\left((a^3b^3c^3-2\,a^4b^2c^2d+a^5b\right.}\right.$$
$$-\frac{\left((3\,b^3c^2-4\,ab^2cd)x^6+(3\,ab^2c^2-4\,a^2bcd)x^2\right)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^6+(abc^2-a^2cd)x^2)}\right)+2\,(2\,a}{8\left((a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2)x^6+(a^4b^2c^3-2\,a^5\right.}$$

input
```
integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*
sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x
^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*(2*a^2*b^2*
c^2 - 4*a^3*b*c*d + 2*a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2
)*x^4)*sqrt(d*x^4 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6
 + (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^2), -1/8*(((3*b^3*c^2 - 4*a
*b^2*c*d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(a*b*c - a^2*d)*arcta
n(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*
c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) + 2*(2*a^2*b^2*c^2 - 4*a^3*
b*c*d + 2*a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4)*sqrt(
d*x^4 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6 + (a^4*b^2*
c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^2)]
```

3.832.6 Sympy [F]

$$\int \frac{1}{x^3 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx=\int \frac{1}{x^3 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.832.7 Maxima [F]

$$\int \frac{1}{x^3 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx=\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^3), x)
```

3.832.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(129) = 258$.

Time = 0.89 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.81

$$\int \frac{1}{x^3 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx$$
$$=\frac{1}{4}\,d^{\frac{5}{2}}\left(\frac{\left(3\,b^2c-4\,abd\right)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(a^2bcd^2-a^3d^3\right)\sqrt{abcd-a^2d^2}}+\frac{2\left(3\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b^2c-4\left(\sqrt{d}x^2-\sqrt{c}\right.\right.}{\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^6b-3\left(\sqrt{d}x^2-\sqrt{d}\right.\right.}\right.$$

input
```
integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c
))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sq
rt(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b^2*c - 4*
(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*a*b*d - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c)
)^2*b^2*c^2 + 14*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b*c*d - 8*(sqrt(d)*x^
2 - sqrt(d*x^4 + c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/(((sqrt(d)*x^2 -
 sqrt(d*x^4 + c))^6*b - 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b*c + 4*(sqrt(
d)*x^2 - sqrt(d*x^4 + c))^4*a*d + 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c^
2 - 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*
d^3)))
```

3.832.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^3 \left(b\,x^4+a\right)^2 \sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/(x^3*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.833 $\int \frac{1}{x^7\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

3.833.1 Optimal result

Integrand size = 24, antiderivative size = 208

$$\int \frac{1}{x^7\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6}+\frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2}+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6\left(a+bx^4\right)}+\frac{b^2(5bc-6ad)\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{7/2}(bc-ad)^{3/2}}$$

output
```
1/4*b^2*(-6*a*d+5*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2)
)/a^(7/2)/(-a*d+b*c)^(3/2)-1/12*(-2*a*d+5*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d
+b*c)/x^6+1/12*(-4*a^2*d^2-8*a*b*c*d+15*b^2*c^2)*(d*x^4+c)^(1/2)/a^3/c^2/(
-a*d+b*c)/x^2+1/4*b*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/x^6/(b*x^4+a)
```

3.833.2 Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^7\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}\left(15b^3c^2x^8+2ab^2cx^4\left(5c-4dx^4\right)+2a^3d\left(c-2dx^4\right)-2a^2b\left(c^2+3cdx^4+2d^2x^8\right)\right)}{12a^3c^2(-bc+ad)x^6\left(a+bx^4\right)}+\frac{b^2(5bc-6ad)\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{7/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
-1/12*(Sqrt[c + d*x^4]*(15*b^3*c^2*x^8 + 2*a*b^2*c*x^4*(5*c - 4*d*x^4) + 2
*a^3*d*(c - 2*d*x^4) - 2*a^2*b*(c^2 + 3*c*d*x^4 + 2*d^2*x^8)))/(a^3*c^2*(-
(b*c) + a*d)*x^6*(a + b*x^4)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(a*Sqrt[d] + b
*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*a^(7/
2)*(b*c - a*d)^(3/2))
```

3.833.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {965, 374, 25, 445, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+b x^4\right)^2 \sqrt{c+d x^4}} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2} \int \frac{1}{x^8 \left(b x^4+a\right)^2 \sqrt{d x^4+c}}dx^2$$

$$\Big\downarrow \text{374}$$

$$\frac{1}{2} \left(\frac{b \sqrt{c+d x^4}}{2 a x^6 \left(a+b x^4\right) (b c-a d)}-\frac{\int -\frac{4 b d x^4+5 b c-2 a d}{x^8 \left(b x^4+a\right) \sqrt{d x^4+c}}dx^2}{2 a (b c-a d)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(\frac{\int \frac{4 b d x^4+5 b c-2 a d}{x^8 \left(b x^4+a\right) \sqrt{d x^4+c}}dx^2}{2 a (b c-a d)}+\frac{b \sqrt{c+d x^4}}{2 a x^6 \left(a+b x^4\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{2} \left(\frac{-\frac{\int \frac{2 b d (5 b c-2 a d) x^4+15 b^2 c^2-4 a^2 d^2-8 a b c d}{x^4 \left(b x^4+a\right) \sqrt{d x^4+c}}dx^2}{3 a c}-\frac{\sqrt{c+d x^4} (5 b c-2 a d)}{3 a c x^6}}{2 a (b c-a d)}+\frac{b \sqrt{c+d x^4}}{2 a x^6 \left(a+b x^4\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{2}\left(\frac{-\frac{-\frac{\int \frac{3b^2c^2(5bc-6ad)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{ac}-\frac{\sqrt{c+dx^4}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^2}}{3ac}-\frac{\sqrt{c+dx^4}(5bc-2ad)}{3acx^6}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^6\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{2}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{a}-\frac{\sqrt{c+dx^4}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^2}}{3ac}-\frac{\sqrt{c+dx^4}(5bc-2ad)}{3acx^6}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^6\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \textbf{291}$$

$$\frac{1}{2}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{a}-\frac{\sqrt{c+dx^4}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^2}}{3ac}-\frac{\sqrt{c+dx^4}(5bc-2ad)}{3acx^6}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^6\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \textbf{218}$$

$$\frac{1}{2}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^2}}{3ac}-\frac{\sqrt{c+dx^4}(5bc-2ad)}{3acx^6}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^6\left(a+bx^4\right)(bc-ad)}\right)$$

input
```
Int[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((b*Sqrt[c + d*x^4])/(2*a*(b*c - a*d)*x^6*(a + b*x^4)) + (-1/3*((5*b*c - 2
*a*d)*Sqrt[c + d*x^4])/(a*c*x^6) - (-((((15*b^2*c)/a - 8*b*d - (4*a*d^2)/c
)*Sqrt[c + d*x^4])/x^2) - (3*b^2*c*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]
*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/(2*a
*(b*c - a*d)))/2
```

3.833. $\int \frac{1}{x^7\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

3.833.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.833. $\int \frac{1}{x^7(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.833.4 Maple [A] (verified)

Time = 6.75 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^4+c}\,\left(-2ad\,x^4-6bc\,x^4+ac\right)}{3x^6}-\frac{b^2c^2\left(\frac{b\sqrt{d\,x^4+c}\,x^2}{b\,x^4+a}-\frac{(6ad-5bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2(ad-bc)}}{2a^3c^2}$
risch	$-\frac{\sqrt{d\,x^4+c}\,(-2ad\,x^4-6bc\,x^4+ac)}{6c^2a^3x^6}-\frac{b^2\sqrt{d\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\,\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8a^3(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}+$ $bd\sqrt{-ab}\,\ln\Bigg(-\frac{2(ad-bc)}{b}+2d$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{6a^2c\,x^6}+\frac{d\sqrt{d\,x^4+c}}{3a^2c^2x^2}+\frac{b\sqrt{d\,x^4+c}}{a^3c\,x^2}-\frac{b^2\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\,\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8a^3(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)}-$ $bd\sqrt{-ab}\,\ln\Bigg(-\frac{2(ad-b}{b}$
default	Expression too large to display

input
```
int(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^3*(-1/3*(d*x^4+c)^(1/2)*(-2*a*d*x^4-6*b*c*x^4+a*c)/x^6-1/2*b^2*c^2/(
a*d-b*c)*(b*(d*x^4+c)^(1/2)*x^2/(b*x^4+a)-(6*a*d-5*b*c)/((a*d-b*c)*a)^(1/2
)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))))/c^2
```

3.833.5 Fricas [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 760, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^7\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$
$$= \Bigg[-\frac{3\left(\left(5\,b^4c^3-6\,ab^3c^2d\right)x^{10}+\left(5\,ab^3c^3-6\,a^2b^2c^2d\right)x^6\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)}{b^2x}\right.}{}$$

input
```
integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2
*d)*x^6)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)
*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 4*((
15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^8 - 2*a
^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^
2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))/((a^4*b^3*c^4 - 2*a^5
*b^2*c^3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*
d^2)*x^6), 1/24*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^
2*b^2*c^2*d)*x^6)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)
*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 -
 a^2*c*d)*x^2)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 +
4*a^4*b*d^3)*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*
b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))
/((a^4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*
a^6*b*c^3*d + a^7*c^2*d^2)*x^6)]
```

3.833.6 Sympy [F]

$$\int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**7*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.833.7 Maxima [F]

$$\int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2 \sqrt{dx^4+c}\,x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^7), x)
```

3.833.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(184) = 368$.

Time $= 0.94$ (sec) , antiderivative size $= 395$, normalized size of antiderivative $= 1.90$

$$\int \frac{1}{x^7 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx$$
$$= \frac{1}{12}\, d^{\frac{7}{2}} \left(\frac{3\left(5\,b^3c-6\,ab^2d\right)\arctan\left(-\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2 b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\left(a^3bcd^3-a^4d^4\right)\sqrt{abcd-a^2d^2}} - \frac{6\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right.\right.}{\left(a^3bcd^3-a^4d^4\right)\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4 b\right.} \right.$$

input
```
integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/12*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x
^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d
^4)*sqrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b^3*c
- 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a
^4*d^4)*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4
 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)) - 8*(3*(s
qrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*
c - 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*
x^2 - sqrt(d*x^4 + c))^2 - c)^3*a^3*d^3))
```

3.833.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^7 \left(b\,x^4+a\right)^2 \sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/(x^7*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^7*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.834 $\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.834.1 Optimal result

Integrand size = 24, antiderivative size = 996

$$
\begin{aligned}
&\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx \\
&= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\,(a+bx^4)} - \frac{\sqrt[4]{-a}(5bc-3ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16b^{7/4}(bc-ad)^{3/2}} \\
&\quad + \frac{\sqrt[4]{-a}(5bc-3ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16b^{7/4}(-bc+ad)^{3/2}} \\
&\quad + \frac{(4bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}} \\
&\quad - \frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}} \\
&\quad - \frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticPi}\biggl(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\biggl(}{32b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticPi}\biggl(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\biggl(}{32b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}
\end{aligned}
$$

output
```
-1/16*(-a)^(1/4)*(-3*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/
4)/(d*x^4+c)^(1/2))/b^(7/4)/(-a*d+b*c)^(3/2)+1/16*(-a)^(1/4)*(-3*a*d+5*b*c
)*arctan(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(7/4)/(a*
d-b*c)^(3/2)+1/4*a*x*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)+1/8*(-3*a*d+4*
b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(
1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^
2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(
-a*d+b*c)/(d*x^4+c)^(1/2)-1/16*a*d^(1/4)*(-3*a*d+5*b*c)*(cos(2*arctan(d^(1
/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*
arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2
))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(
1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/16*d^(1/4)*(-3*a*d+5*b*c)*(cos
(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*El
lipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(-a)^(1/2)*(c^(1/2)+
x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*
d^(1/2))^2)^(1/2)/b^2/c^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)-1/32*(-3*
a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)
*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1
/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*
(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)...
```

3.834.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.30 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.25

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$= \frac{x\left(\frac{(4bc-3ad)x^4\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{ab}+\frac{5a\left(c+dx^4+\frac{5ac^2\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx}{c}\right.}{-5ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bc\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\right.\right.}\right.\right.}{b(a+bx^4)}\right.}{20(bc-ad)\sqrt{c+dx^4}}$$

input
```
Integrate[x^8/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(x*(((4*b*c - 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -(
(d*x^4)/c), -((b*x^4)/a)])/(a*b) + (5*a*(c + d*x^4 + (5*a*c^2*AppellF1[1/4
, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/(-5*a*c*AppellF1[1/4, 1/2, 1,
5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4,
 -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c)
, -((b*x^4)/a)]))))/(b*(a + b*x^4))))/(20*(b*c - a*d)*Sqrt[c + d*x^4])
```

3.834.3 Rubi [A] (warning: unable to verify)

Time = 1.51 (sec) , antiderivative size = 1032, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {970, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{970}$$

$$\frac{ax\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)}-\frac{\int \frac{ac-(4bc-3ad)x^4}{(bx^4+a)\sqrt{dx^4+c}}dx}{4b(bc-ad)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{ax\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)}-\frac{\frac{a(5bc-3ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}-\frac{(4bc-3ad)\int \frac{1}{\sqrt{dx^4+c}}dx}{b}}{4b(bc-ad)}$$

$$\Big\downarrow \text{761}$$

$$\frac{ax\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)}-\frac{\frac{a(5bc-3ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}-\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(4bc-3ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}}{4b(bc-ad)}$$

$$\Big\downarrow \text{925}$$

$$\frac{ax\sqrt{c+dx^4}}{4b\left(a+bx^4\right)\left(bc-ad\right)}-\frac{\frac{a(5bc-3ad)\left(\frac{\int\frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int\frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)}{b}-\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(4bc-3ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}}{4b(bc-ad)}$$

$$\Big\downarrow \ 1541$$

$$\frac{ax\sqrt{c+dx^4}}{4b\left(a+bx^4\right)\left(bc-ad\right)}-\frac{\frac{a(5bc-3ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\right.}{}}{2a}\right.}{b}}{4b(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{ax\sqrt{c+dx^4}}{4b\left(a+bx^4\right)\left(bc-ad\right)}-\frac{\frac{a(5bc-3ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)}{a}}{2a}\right.}{b}}{4b(bc-ad)}$$

$$\Big\downarrow \ 761$$

$$\frac{ax\sqrt{c+dx^4}}{4b\left(a+bx^4\right)\left(bc-ad\right)}-\frac{a(5bc-3ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{\sqrt{b}\left(\right.}{}\right.}{b}$$

$$\Big\downarrow \ 2221$$

3.834. $\int\frac{x^8}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

$$\frac{ax\sqrt{dx^4+c}}{4b(bc-ad)\left(bx^4+a\right)}-$$

$$\frac{a(5bc-3ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\frac{}{\sqrt[4]{-}}\right.}{2\sqrt[4]{b}\sqrt{bc-ad}}\right.}{2a}\right.}{}$$

$$\Big\downarrow \text{2223}$$

$$\frac{ax\sqrt{dx^4+c}}{4b(bc-ad)\left(bx^4+a\right)}-$$

$$\frac{a(5bc-3ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\frac{}{\sqrt[4]{-}}\right.}{2\sqrt[4]{b}\sqrt{bc-ad}}\right.}{2a}\right.}{}$$

input

```
Int[x^8/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

3.834. $\int \frac{x^8}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

output

```
(a*x*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (-1/2*((4*b*c - 3*a*
d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*c^(1/4)*d^(1/4)*Sqrt[c + d*
x^4]) + (a*(5*b*c - 3*a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(
1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*E
llipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[
c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*(
(Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/
4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (S
qrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2
/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/
(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqr
t[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)
/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])
/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqr
t[-a]*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(
Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[
b*c - a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqr...
```

3.834.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

3.834. $\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.834. $\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.834.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.31 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.34

method	result
elliptic	$-\frac{ax\sqrt{d\,x^4+c}}{4(ad-bc)b(b\,x^4+a)}+\frac{\left(\frac{1}{b^2}-\frac{ad}{4(ad-bc)b^2}\right)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(3ad-5bc)\left(-\frac{\mathrm{arcta}}{}\right.}{}\right.}{}$
default	$\frac{\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b^2\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}+\frac{a^2\left(-\frac{bx\sqrt{d\,x^4+c}}{4(ad-bc)a\left(b\,x^4+a\right)}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{4(ad-bc)a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}}{}\right.}{}$

input
```
int(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/(a*d-b*c)/b*x*(d*x^4+c)^(1/2)/(b*x^4+a)+(1/b^2-1/4*a*d/(a*d-b*c)/b^
2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*
d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I
)-1/32*a/b^3*sum((3*a*d-5*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)
*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+
2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(
1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(
1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c
^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

3.834.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\text{Timed out}$$

input
```
integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.834.6 Sympy [F]

$$\int \frac{x^8}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^8}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**8/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**8/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.834.7 Maxima [F]

$$\int \frac{x^8}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^8}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.834.8 Giac [F]

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^8}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^8/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.834.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^8}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^8/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^8/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.835 $\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.835.1 Optimal result

Integrand size $=24$, antiderivative size $=908$

$$
\begin{aligned}
\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx &= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\,(a+bx^4)} \\
&\quad - \frac{(bc+ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}} + \frac{(bc+ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{3/4}b^{3/4}(-bc+ad)^{3/2}} \\
&\quad + \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}} \\
&\quad + \frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16ab\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}} \\
&\quad - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}} \\
&\quad + \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}} \\
&\quad + \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}
\end{aligned}
$$

output
```
-1/16*(a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/
2))/(-a)^(3/4)/b^(3/4)/(-a*d+b*c)^(3/2)+1/16*(a*d+b*c)*arctan(x*(a*d-b*c)^
(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(3/4)/b^(3/4)/(a*d-b*c)^(3/
2)-1/4*x*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)-1/8*d^(3/4)*(cos(2*arctan(d^
(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(
2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)
/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/16*
d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/
c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(b^(1/2)
*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2
*d^(1/2))^2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/16*d^(1/4)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Ell
ipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*((-a)^(1/2)*b^(1/2)*c
^(1/2)+a*d^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2
)^(1/2)/a/b/c^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(cos(2*arctan(d^(1/4)*
x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arc
tan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1
/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^
(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/b/
c^(1/4)/d^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(cos(2*arctan(d^(1/4)*x...
```

3.835.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.26

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{x\left(\frac{dx^4\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a}+\frac{5\left(c+dx^4+\frac{5ac^2\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{-5ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bc\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right.}\right.}{a+bx^4}\right.}{20(-bc+ad)\sqrt{c+dx^4}}$$

input
```
Integrate[x^4/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(x*((d*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -(
(b*x^4)/a)])/a + (5*(c + d*x^4 + (5*a*c^2*AppellF1[1/4, 1/2, 1, 5/4, -((d*
x^4)/c), -((b*x^4)/a)])/(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -
((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*
x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/
(a + b*x^4)))/(20*(-(b*c) + a*d)*Sqrt[c + d*x^4])
```

3.835.3 Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 1013, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {971, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{971}$$

$$\frac{\int \frac{c-dx^4}{(bx^4+a)\sqrt{dx^4+c}}dx}{4(bc-ad)}-\frac{x\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{\frac{(ad+bc)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{b}-\frac{d\int \frac{1}{\sqrt{dx^4+c}}dx}{b}}{4(bc-ad)}-\frac{x\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{(ad+bc)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{b}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}}{4(bc-ad)}-\frac{x\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{925}$$

$$\frac{\frac{(ad+bc)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)}{b}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}}{4(bc-ad)}-\frac{x\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{1541}$$

$$\frac{\frac{(ad+bc)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\right.}{2a}\right.}{b}}{4(bc-ad)}$$

$$\frac{x\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(ad+bc)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int}{ad+b}}{2a}\right.}{b}}{4(bc-ad)}$$

$$\frac{x\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{761}$$

$$\frac{(ad+bc)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{\sqrt{b}\left(\sqrt{-a}\right.}{}\right.}{b}$$

$$\frac{x\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{2221}$$

3.835. $\int \frac{x^4}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

$$\frac{(bc+ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\frac{\sqrt{b}}{\sqrt[4]{-a}}\right.}{2\sqrt[4]{b}\sqrt{bc-ad}}\right.}{}}{2a}\right.}{}$$

$$\frac{x\sqrt{dx^4+c}}{4(bc-ad)\left(bx^4+a\right)}$$

$$\Big\downarrow 2223$$

$$\frac{(bc+ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\frac{\sqrt{b}}{\sqrt[4]{-a}}\right.}{2\sqrt[4]{b}\sqrt{bc-ad}}\right.}{}}{2a}\right.}{}$$

$$\frac{x\sqrt{dx^4+c}}{4(bc-ad)\left(bx^4+a\right)}$$

input
```
Int[x^4/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

3.835. $\int \frac{x^4}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

output
```
-1/4*(x*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (-1/2*(d^(3/4)*(Sqrt[
c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*
ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*c^(1/4)*Sqrt[c + d*x^4]) + ((b*c + a
*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]
*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1
/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[
-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x
^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b
])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt
[c + d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d
])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2
)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(
1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a
)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c]
 - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(S
qrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d]...
```

3.835.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

3.835. $\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.835. $\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.835.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.20 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.36

method	result
elliptic	$\frac{x\sqrt{d\,x^{4}+c}}{4(ad-bc)(b\,x^{4}+a)}+\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{4(ad-bc)b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}\frac{(ad+bc)\left(-\frac{\mathrm{arctanh}\left(\frac{2d\,x^{2}\_\alpha^{2}+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^{4}+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}\right.}{}}{32}$
default	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^{2}\_\alpha^{2}+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^{4}+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^{3}b\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},\frac{i\sqrt{c}\,\_\alpha^{2}b}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d\,x^{4}+c}}}{\_\alpha^{3}}}{8b^{2}}-\frac{a\left(-\frac{}{4(ad}\right.}{}$

input
```
int(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a*d-b*c)*x*(d*x^4+c)^(1/2)/(b*x^4+a)+1/4*d/(a*d-b*c)/b/(I/c^(1/2)*d^(
1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2
)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/32/b^2*sum((a
*d+b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^
2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(
1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2
)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d
^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)))
,_alpha=RootOf(_Z^4*b+a))
```

3.835.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.835.6 Sympy [F]

$$\int \frac{x^4}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**4/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**4/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.835.7 Maxima [F]

$$\int \frac{x^4}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.835.8 Giac [F]

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^4}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.835.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^4}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^4/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.836 $\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.836.1 Optimal result

Integrand size = 21, antiderivative size = 983

$$\begin{aligned}
\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx &= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} \\
&+ \frac{\sqrt[4]{b}(3bc-5ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{7/4}(bc-ad)^{3/2}} - \frac{\sqrt[4]{b}(3bc-5ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{7/4}(-bc+ad)^{3/2}} \\
&+ \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}} \\
&+ \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16(-a)^{3/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\Bigg(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\Bigg(}{32a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\Bigg(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\Bigg(}{32a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}
\end{aligned}$$

output

```
1/16*b^(1/4)*(-5*a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(
d*x^4+c)^(1/2))/(-a)^(7/4)/(-a*d+b*c)^(3/2)-1/16*b^(1/4)*(-5*a*d+3*b*c)*ar
ctan(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(7/4)/(a*d
-b*c)^(3/2)+1/4*b*x*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)+1/8*d^(3/4)*(co
s(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*E
llipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2
))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(d*x^4+c
)^(1/2)+1/16*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(
1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(
1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^2*d^(1
/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(a*d+b
*c)/(d*x^4+c)^(1/2)+1/16*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^
(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d
^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a
)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(3/2)/c^(1
/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(-5*a*d+3*b*c)*(cos(2*arctan
(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(
sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^
2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b
^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2...
```

3.836.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.28 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.40

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{-5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\Bigl(5a(4bc-4ad+bdx^4)+bdx^4(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\bigl(\frac{5}{4},}{20a^2(bc-ad)\,(a+bx^4)\sqrt{c+dx^4}\,\bigl(-5ac}$$

input

```
Integrate[1/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)]*(5*a*(4*b
*c - 4*a*d + b*d*x^4) + b*d*x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[5
/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]) + 2*b*x^5*(5*a*(c + d*x^4) +
 d*x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4
)/c), -((b*x^4)/a)])*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*
x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))/(2
0*a^2*(b*c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4]*(-5*a*c*AppellF1[1/4, 1/2, 1
, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/
4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/
c), -((b*x^4)/a)]))
```

3.836.3 Rubi [A] (warning: unable to verify)

Time = 1.44 (sec) , antiderivative size = 1016, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {931, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{bx\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}-\frac{\int -\frac{bdx^4+3bc-4ad}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{4a(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{bdx^4+3bc-4ad}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{4a(bc-ad)}+\frac{bx\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 1021}$$

$$\frac{(3bc-5ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx+d\int \frac{1}{\sqrt{dx^4+c}}dx}{4a(bc-ad)}+\frac{bx\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{(3bc-5ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{4a(bc-ad)}+\frac{bx\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 925$$

$$\frac{(3bc-5ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{4a(bc-ad)}+$$

$$\frac{bx\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 1541$$

$$(3bc-5ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\right.$$

$$4a(bc-ad)$$

$$\frac{bx\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$(3bc-5ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+}{2a}\right.$$

$$4a(bc-ad)$$

$$\frac{bx\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 761$$

3.836. $\int \frac{1}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

$$(3bc-5ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\right.$$

$$\frac{bx\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 2221$$

$$\frac{b\sqrt{dx^4+c}x}{4a(bc-ad)\left(bx^4+a\right)}+$$

$$\frac{d^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^4+c}}+(3bc-5ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\right.}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}\right.$$

$$\Big\downarrow \; 2223$$

$$\frac{b\sqrt{dx^4+c}x}{4a(bc-ad)\left(bx^4+a\right)}+$$

$$\frac{d^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^4+c}}+(3bc-5ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\right.}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}\right.$$

input
```
Int[1/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

3.836. $\int \frac{1}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

output

```
(b*x*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + ((d^(3/4)*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcT
an[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*Sqrt[c + d*x^4]) + (3*b*c - 5*a*
d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*
x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/
4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(
Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-
a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^
4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b]
)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elli
pticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[
c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[
c + d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d]
)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)
^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*
Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1
/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)
^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c]
- (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sq
rt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])...
```

3.836.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.836. $\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.836.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.96 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.34

method	result
default	$-\frac{b x \sqrt{d\,x^{4}+c}}{4 (a d -b c) a \left(b\,x^{4}+a \right)}-\frac{d \sqrt{1-\frac{i \sqrt{d}\, x^{2}}{\sqrt{c}}}\, \sqrt{1+\frac{i \sqrt{d}\, x^{2}}{\sqrt{c}}}\, F\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}, i\right)}{4 (a d -b c) a \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\, \sqrt{d\,x^{4}+c}}-\frac{\sum_{\_\alpha =\operatorname{RootOf}\left(\_Z^{4} b +a \right)}\frac{(-5 a d +3 b c) \left(-\frac{\operatorname{arctanh}\left(\frac{2 d\,x^{2} \_\alpha^{2}+}{2 \sqrt{\frac{-a d +b c}{b}}\, \sqrt{d}}\right)}{\sqrt{\frac{-a d +b c}{b}}}\right.}{}}{}$
elliptic	$-\frac{b x \sqrt{d\,x^{4}+c}}{4 (a d -b c) a \left(b\,x^{4}+a \right)}-\frac{d \sqrt{1-\frac{i \sqrt{d}\, x^{2}}{\sqrt{c}}}\, \sqrt{1+\frac{i \sqrt{d}\, x^{2}}{\sqrt{c}}}\, F\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}, i\right)}{4 (a d -b c) a \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\, \sqrt{d\,x^{4}+c}}-\frac{\sum_{\_\alpha =\operatorname{RootOf}\left(\_Z^{4} b +a \right)}\frac{(-5 a d +3 b c) \left(-\frac{\operatorname{arctanh}\left(\frac{2 d\,x^{2} \_\alpha^{2}+}{2 \sqrt{\frac{-a d +b c}{b}}\, \sqrt{d}}\right)}{\sqrt{\frac{-a d +b c}{b}}}\right.}{}}{}$

input
```
int(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*b/(a*d-b*c)/a*x*(d*x^4+c)^(1/2)/(b*x^4+a)-1/4*d/(a*d-b*c)/a/(I/c^(1/2
)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)
^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/32/b/a*s
um((-5*a*d+3*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*
(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*
d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d
^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I
*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2
))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

3.836.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.836.6 Sympy [F]

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.836.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.836.8 Giac [F]

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.836.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{1}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.837 $\int \frac{1}{x^4(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.837.1 Optimal result

Integrand size = 24, antiderivative size = 1046

$$\begin{aligned}
\int \frac{1}{x^4(a+bx^4)^2\sqrt{c+dx^4}}\,dx &= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3}+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3(a+bx^4)} \\
&+\frac{b^{5/4}(7bc-9ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{11/4}(bc-ad)^{3/2}}-\frac{b^{5/4}(7bc-9ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{11/4}(-bc+ad)^{3/2}} \\
&-\frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{24a^2c^{5/4}(bc-ad)\sqrt{c+dx^4}} \\
&+\frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16(-a)^{5/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}} \\
&-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16(-a)^{5/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}} \\
&-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\Bigg(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\Bigg(}{32a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}} \\
&-\frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\Bigg(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\Bigg(}{32a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}
\end{aligned}$$

output
```
1/16*b^(5/4)*(-9*a*d+7*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(
d*x^4+c)^(1/2))/(-a)^(11/4)/(-a*d+b*c)^(3/2)-1/16*b^(5/4)*(-9*a*d+7*b*c)*a
rctan(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(11/4)/(a
*d-b*c)^(3/2)-1/12*(-4*a*d+7*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/x^3+1/4
*b*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/x^3/(b*x^4+a)-1/24*d^(3/4)*(-4*a*d+7*b*c)*
(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(
1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(5/4)/(-a*d+b*c)/(d*
x^4+c)^(1/2)+1/16*b*d^(1/4)*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)
))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4
)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/
2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/4)/(
-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/32*b*(-9*a*d+7*b*c)*(cos(2*arctan(d^
(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin
(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(
-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1
/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2
)/a^3/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)-1/16*b*d^(1/4)*(-
9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/
4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*...
```

3.837.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.55 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^4\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{bd(7bc-4ad)x^8\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+\frac{a\left(25ac\left(-7b^2cx^4\left(4c+dx^4\right)+4a^2d\left(c+2dx^4\right)+4ab\left(-c^2+5c\right.\right.\right.}{\left(a+bx^4\right)\left(\right.}}{60a^3}$$

input
```
Integrate[1/(x^4*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(b*d*(7*b*c - 4*a*d)*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -(
(d*x^4)/c), -((b*x^4)/a)] + (a*(25*a*c*(-7*b^2*c*x^4*(4*c + d*x^4) + 4*a^2
*d*(c + 2*d*x^4) + 4*a*b*(-c^2 + 5*c*d*x^4 + d^2*x^8))*AppellF1[1/4, 1/2,
1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 10*x^4*(c + d*x^4)*(-4*a^2*d + 7*b^2
*c*x^4 + 4*a*b*(c - d*x^4))*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c)
, -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a
)])))/((a + b*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x
^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/
a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(60*a
^3*c*(-(b*c) + a*d)*x^3*Sqrt[c + d*x^4])
```

3.837.3 Rubi [A] (warning: unable to verify)

Time = 1.59 (sec) , antiderivative size = 1074, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {972, 25, 1053, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+b x^4\right)^2 \sqrt{c+d x^4}} \, dx$$

$$\Big\downarrow \text{ 972}$$

$$\frac{b \sqrt{c+d x^4}}{4 a x^3 \left(a+b x^4\right) (b c-a d)}-\frac{\int -\frac{5 b d x^4+7 b c-4 a d}{x^4 \left(b x^4+a\right) \sqrt{d x^4+c}}dx}{4 a (b c-a d)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{5 b d x^4+7 b c-4 a d}{x^4 \left(b x^4+a\right) \sqrt{d x^4+c}}dx}{4 a (b c-a d)}+\frac{b \sqrt{c+d x^4}}{4 a x^3 \left(a+b x^4\right) (b c-a d)}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{-\frac{\int \frac{b d (7 b c-4 a d) x^4+21 b^2 c^2-4 a^2 d^2-20 a b c d}{\left(b x^4+a\right) \sqrt{d x^4+c}}dx}{3 a c}-\frac{\sqrt{c+d x^4} (7 b c-4 a d)}{3 a c x^3}}{4 a (b c-a d)}+\frac{b \sqrt{c+d x^4}}{4 a x^3 \left(a+b x^4\right) (b c-a d)}$$

$$\Big\downarrow \text{ 1021}$$

$$\frac{-\frac{d (7 b c-4 a d) \int \frac{1}{\sqrt{d x^4+c}}dx+3 b c (7 b c-9 a d) \int \frac{1}{\left(b x^4+a\right) \sqrt{d x^4+c}}dx}{3 a c}-\frac{\sqrt{c+d x^4} (7 b c-4 a d)}{3 a c x^3}}{4 a (b c-a d)}+\frac{b \sqrt{c+d x^4}}{4 a x^3 \left(a+b x^4\right) (b c-a d)}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{\frac{3bc(7bc-9ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(7bc-4ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{3ac}-\frac{\sqrt{c+dx^4}(7bc-4ad)}{3acx^3}}{4a(bc-ad)}+$$

$$\frac{b\sqrt{c+dx^4}}{4ax^3\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 925$$

$$-\frac{\frac{3bc(7bc-9ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(7bc-4ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{3ac}-}{4a(bc-ad)}$$

$$\frac{b\sqrt{c+dx^4}}{4ax^3\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 1541$$

$$-\frac{\frac{3bc(7bc-9ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-}\right.}{}}{2a}\right.}{3ac}}{4a(bc-}$$

$$\frac{b\sqrt{c+dx^4}}{4ax^3\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{3bc(7bc-9ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\right.}{}}{2a}\right.}{3ac}}{4a(bc-ad)}$$

$$\frac{b\sqrt{c+dx^4}}{4ax^3\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 761$$

3.837. $\int \frac{1}{x^4\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

$$-\frac{3bc(7bc-9ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\right.}{}$$

$$\frac{b\sqrt{c+dx^4}}{4ax^3\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow 2221$$

$$\frac{\sqrt{dx^4+c}\,b}{4a(bc-ad)x^3\left(bx^4+a\right)}+$$

$$-\frac{\sqrt{dx^4+c}(7bc-4ad)}{3acx^3}-\frac{\frac{d^{3/4}(7bc-4ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^4+c}}+3bc(7bc-9ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)}{2}\right.}{}$$

$$\Big\downarrow 2223$$

$$\frac{\sqrt{dx^4+c}\,b}{4a(bc-ad)x^3\left(bx^4+a\right)}+$$

$$-\frac{\sqrt{dx^4+c}(7bc-4ad)}{3acx^3}-\frac{\frac{d^{3/4}(7bc-4ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^4+c}}+3bc(7bc-9ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)}{2}\right.}{}$$

input
```
Int[1/(x^4*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

3.837. $\int \frac{1}{x^4\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

output

```
(b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x^3*(a + b*x^4)) + (-1/3*((7*b*c - 4*
a*d)*Sqrt[c + d*x^4])/(a*c*x^3) - ((d^(3/4)*(7*b*c - 4*a*d)*(Sqrt[c] + Sqr
t[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(
d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*Sqrt[c + d*x^4]) + 3*b*c*(7*b*c - 9*
a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d
]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(
1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]
*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt
[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*
x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[
b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*El
lipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqr
t[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqr
t[c + d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[
d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^
2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d
)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^
(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-
a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c
] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4...
```

3.837.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

3.837. $\int \frac{1}{x^4(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.837.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.39 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.35

method	result
elliptic	$(9ad-$ $\frac{b^{2} x \sqrt{d\,x^{4}+c}}{4 a^{2} \left(a d -b c \right) \left(b\,x^{4}+a \right)}-\frac{\sqrt{d\,x^{4}+c}}{3 c\,a^{2} x^{3}}+\frac{\left(\frac{b d}{4 \left(a d -b c \right) a^{2}}-\frac{d}{3 c\,a^{2}}\right) \sqrt{1-\frac{i \sqrt{d}\, x^{2}}{\sqrt{c}}}\, \sqrt{1+\frac{i \sqrt{d}\, x^{2}}{\sqrt{c}}}\, F\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}, i\right)}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\, \sqrt{d\,x^{4}+c}}-\sum_{\_\alpha =\operatorname{RootOf}\left(\_Z^{4} b +a \right)}$
default	$\frac{-\frac{\sqrt{d\,x^{4}+c}}{3 c\,x^{3}}-\frac{d \sqrt{1-\frac{i \sqrt{d}\, x^{2}}{\sqrt{c}}}\, \sqrt{1+\frac{i \sqrt{d}\, x^{2}}{\sqrt{c}}}\, F\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}, i\right)}{3 c \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\, \sqrt{d\,x^{4}+c}}}{a^{2}}-\frac{\sum_{\_\alpha =\operatorname{RootOf}\left(\_Z^{4} b +a \right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2 d\,x^{2} \_\alpha^{2}+2 c}{2 \sqrt{\frac{-a d +b c}{b}}\, \sqrt{d\,x^{4}+c}}\right)}{\sqrt{\frac{-a d +b c}{b}}}+\frac{2 \_\alpha^{3} b \sqrt{1-\frac{i \sqrt{d}\, x^{2}}{\sqrt{c}}}\, \sqrt{1}}{}}{\_\alpha^{3}}}{8 a^{2}}$ $\frac{d \sqrt{1-\frac{i \sqrt{d}\, x^{2}}{\sqrt{c}}}\, \sqrt{1+\frac{i \sqrt{d}\, x^{2}}{\sqrt{c}}}\, F\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}, i\right)}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\, \sqrt{d\,x^{4}+c}}+\frac{3 c \left(\sum_{\_\alpha =\operatorname{RootOf}\left(\_Z^{4} b +a \right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2 d\,x^{2} \_\alpha^{2}+2 c}{2 \sqrt{\frac{-a d +b c}{b}}\, \sqrt{d\,x^{4}+c}}\right)}{\sqrt{\frac{-a d +b c}{b}}}+\frac{2 \_\alpha^{3} b \sqrt{1-\frac{i \sqrt{d}\, x}{\sqrt{c}}}}{}}{\_\alpha^{3}}\right.}{8}$
risch	$-\frac{\sqrt{d\,x^{4}+c}}{3 c\,a^{2} x^{3}}-$

input
```
int(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

3.837. $\int \frac{1}{x^4(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

output
```
1/4*b^2/a^2/(a*d-b*c)*x*(d*x^4+c)^(1/2)/(b*x^4+a)-1/3/c/a^2*(d*x^4+c)^(1/2
)/x^3+(1/4*b*d/(a*d-b*c)/a^2-1/3*d/c/a^2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c
^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*
EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/32/a^2*sum((9*a*d-7*b*c)/(a*d-b
*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/(
(-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*
b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4
+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^
2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf
(_Z^4*b+a))
```

3.837.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.837.6 Sympy [F]

$$\int \frac{1}{x^4 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^4 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x**4/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.837.7 Maxima [F]

$$\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2 \sqrt{dx^4+c}\,x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^4), x)
```

3.837.8 Giac [F]

$$\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2 \sqrt{dx^4+c}\,x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^4), x)
```

3.837.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^4 \left(b\,x^4+a\right)^2 \sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/(x^4*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.838 $\int \frac{x^6}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

3.838.1 Optimal result

Integrand size = 24, antiderivative size = 1146

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\sqrt{d}x\sqrt{c+dx^4}}{4b(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)} - \frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\,(a+bx^4)}$$

$$+ \frac{(3bc-ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16\sqrt[4]{-a}b^{5/4}(bc-ad)^{3/2}} + \frac{(3bc-ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16\sqrt[4]{-a}b^{5/4}(-bc+ad)^{3/2}}$$

$$- \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b(bc-ad)\sqrt{c+dx^4}}$$

$$+ \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b(bc-ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

output
```
1/16*(-a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(
1/2))/(-a)^(1/4)/b^(5/4)/(-a*d+b*c)^(3/2)+1/16*(-a*d+3*b*c)*arctan(x*(a*d-
b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(1/4)/b^(5/4)/(a*d-b*c
)^(3/2)-1/4*x^3*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)+1/4*x*d^(1/2)*(d*x^4+
c)^(1/2)/b/(-a*d+b*c)/(c^(1/2)+x^2*d^(1/2))-1/4*c^(1/4)*d^(1/4)*(cos(2*arc
tan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elliptic
E(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*
x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/8*c^(
1/4)*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/
4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^
(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/(-a*d+b*c)/
(d*x^4+c)^(1/2)-1/16*d^(1/4)*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4))
)^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)
*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2
)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/(-a^2
*d^2+b^2*c^2)/(d*x^4+c)^(1/2)-1/32*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^
(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(
d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/
b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2
)-(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/...
```

3.838.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.39 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.14

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \frac{-7ax^3(c+dx^4)+7cx^3(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+dx^7(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\,\text{Ap}}{28a(bc-ad)\,(a+bx^4)\sqrt{c+dx^4}}$$

input
```
Integrate[x^6/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(-7*a*x^3*(c + d*x^4) + 7*c*x^3*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[3
/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + d*x^7*(a + b*x^4)*Sqrt[1 +
(d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(28*a*
(b*c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4])
```

3.838.3 Rubi [A] (warning: unable to verify)

Time = 1.55 (sec) , antiderivative size = 1085, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {971, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \; 971$$

$$\frac{\int \frac{x^2(dx^4+3c)}{(bx^4+a)\sqrt{dx^4+c}}dx}{4(bc-ad)}-\frac{x^3\sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \; 1054$$

$$\frac{\int \left(\frac{dx^2}{b\sqrt{dx^4+c}}+\frac{(3bc-ad)x^2}{b(bx^4+a)\sqrt{dx^4+c}}\right)dx}{4(bc-ad)}-\frac{x^3\sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$-\frac{(3bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}+\frac{(3bc-ad)\arctan\left(\frac{\sqrt{bc-}}{\sqrt[4]{-a}\sqrt[4]{}}\right.}{4\sqrt[4]{-a}b^{5/4}\sqrt{bc-}}$$

$$\frac{x^3\sqrt{dx^4+c}}{4(bc-ad)(bx^4+a)}$$

input
```
Int[x^6/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output

```
-1/4*(x^3*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + ((Sqrt[d]*x*Sqrt[c
+ d*x^4])/(b*(Sqrt[c] + Sqrt[d]*x^2)) + ((3*b*c - a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*b^(5/4)*Sqrt[
b*c - a*d]) - ((3*b*c - a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/
4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*b^(5/4)*Sqrt[b*c - a*d]) - (c^(1/4)*d^
(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*Sqrt[c + d*x^4]) + (c^(1
/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^
2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*Sqrt[c + d*x^4])
 - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*Arc
Tan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
- ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcT
an[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) -
 ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*
x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[
c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d
^(1/4)*x)/c^(1/4)], 1/2])/(8*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*
Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d...
```

3.838.3.1 Defintions of rubi rules used

rule 971

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.838. $\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.838.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.02 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.31

method	result
elliptic	$\frac{x^{3} \sqrt{d\,x^{4}+c}}{4 (a d-b c) (b\,x^{4}+a)}-\frac{i \sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\left(F\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}},i\right)-E\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}},i\right)\right)}{4 (a d-b c) b \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}-\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4} b+a\right)} (-a d+3 b c)\left(\mathrm{ar} \right.$
default	$\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4} b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2 d\,x^{2} \_\alpha^{2}+2 c}{2 \sqrt{\frac{-a d+b c}{b}}\,\sqrt{d\,x^{4}+c}}\right)}{\sqrt{\frac{-a d+b c}{b}}}+\frac{2 \_\alpha^{3} b \sqrt{1-\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\Pi\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}},\frac{i \sqrt{c}\,\_\alpha^{2} b}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i \sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\,a \sqrt{d\,x^{4}+c}}}{\_\alpha}}{8 b^{2}}-a\left(-\frac{b}{4(ad}\right.$

input

```
int(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/4/(a*d-b*c)*x^3*(d*x^4+c)^(1/2)/(b*x^4+a)-1/4*I*d^(1/2)/(a*d-b*c)/b*c^(1
/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)
*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2)
,I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/32/b^2*sum((-a*d+3*b*c)/(a
*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)
/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^
3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x
^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alph
a^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Root
Of(_Z^4*b+a))
```

3.838.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.838.6 Sympy [F]

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**6/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**6/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.838.7 Maxima [F]

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.838.8 Giac [F]

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^6}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.838.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^6}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^6/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^6/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.839 $\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.839.1 Optimal result

Integrand size = 24, antiderivative size = 1144

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{d}x\sqrt{c+dx^4}}{4a(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)}$$

$$-\frac{(bc-3ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{5/4}\sqrt[4]{b}(bc-ad)^{3/2}} - \frac{(bc-3ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{5/4}\sqrt[4]{b}(-bc+ad)^{3/2}}$$

$$+\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a(bc-ad)\sqrt{c+dx^4}}$$

$$-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a(bc-ad)\sqrt{c+dx^4}}$$

$$-\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

$$-\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

$$-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

$$+\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

output
```
-1/16*(-3*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^
(1/2))/(-a)^(5/4)/b^(1/4)/(-a*d+b*c)^(3/2)-1/16*(-3*a*d+b*c)*arctan(x*(a*d
-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(5/4)/b^(1/4)/(a*d-b*
c)^(3/2)+1/4*b*x^3*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)-1/4*x*d^(1/2)*(d
*x^4+c)^(1/2)/a/(-a*d+b*c)/(c^(1/2)+x^2*d^(1/2))+1/4*c^(1/4)*d^(1/4)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Ell
ipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))
*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(-a*d+b*c)/(d*x^4+c)^(1/2)-1/
8*c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(
d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2)
)*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(-a*d+
b*c)/(d*x^4+c)^(1/2)+1/32*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2
)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x
/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c
^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1
/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(3/2)/c^(1/4
)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/b^(1/2)/(d*x^4+c)^(1/2)-1/32*(-3*a*d+b*c)*(
cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))
*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1
/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)...
```

3.839.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.15

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \frac{21abx^3(c+dx^4)+7(bc-4ad)x^3(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-3bdx^7(a+bx^4)}{84a^2(bc-ad)(a+bx^4)\sqrt{c+dx^4}}$$

input
```
Integrate[x^2/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(21*a*b*x^3*(c + d*x^4) + 7*(b*c - 4*a*d)*x^3*(a + b*x^4)*Sqrt[1 + (d*x^4)
/c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] - 3*b*d*x^7*(a
+ b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((
b*x^4)/a)])/(84*a^2*(b*c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4])
```

3.839.3 Rubi [A] (warning: unable to verify)

Time = 1.43 (sec) , antiderivative size = 1071, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {972, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \; 972$$

$$\frac{bx^3\sqrt{c+dx^4}}{4a(a+bx^4)(bc-ad)}-\frac{\int -\frac{x^2(-bdx^4+bc-4ad)}{(bx^4+a)\sqrt{dx^4+c}}dx}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{x^2(-bdx^4+bc-4ad)}{(bx^4+a)\sqrt{dx^4+c}}dx}{4a(bc-ad)}+\frac{bx^3\sqrt{c+dx^4}}{4a(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \; 1054$$

$$\frac{\int \left(\frac{(bc-3ad)x^2}{(bx^4+a)\sqrt{dx^4+c}}-\frac{dx^2}{\sqrt{dx^4+c}}\right)dx}{4a(bc-ad)}+\frac{bx^3\sqrt{c+dx^4}}{4a(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\sqrt{dx^4+c}x^3}{4a(bc-ad)(bx^4+a)}+$$

$$-\frac{(bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}+\frac{(bc-3ad)\arctan\left(\frac{\sqrt{bc-}}{\sqrt[4]{-a}\sqrt[4]{}}\right.}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-}}$$

input
```
Int[x^2/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output

```
(b*x^3*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + (-((Sqrt[d]*x*Sqrt
[c + d*x^4])/(Sqrt[c] + Sqrt[d]*x^2)) + ((b*c - 3*a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])]])/(4*(-a)^(1/4)*b^(1/4)*Sqrt[
b*c - a*d]) - ((b*c - 3*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/
4)*Sqrt[c + d*x^4])]])/(4*(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) + (c^(1/4)*d^
(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/Sqrt[c + d*x^4] - (c^(1/4)*
d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2
]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*Sqrt[c + d*x^4]) - ((S
qrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt
[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d
^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt
[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]
*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1
/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]
*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[
(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[
-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/
c^(1/4)], 1/2])/(8*Sqrt[-a]*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d
*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c]...
```

3.839.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 972

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

3.839. $\int \frac{x^2}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.839.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.98 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.31

method	result
default	$-\frac{b\,x^{3}\sqrt{d\,x^{4}+c}}{4(ad-bc)a(b\,x^{4}+a)}+\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{4(ad-bc)a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}}{}$ $(-3ad+bc)$
elliptic	$-\frac{b\,x^{3}\sqrt{d\,x^{4}+c}}{4(ad-bc)a(b\,x^{4}+a)}+\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{4(ad-bc)a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}}{}$ $(-3ad+bc)$

input
```
int(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*b/(a*d-b*c)/a*x^3*(d*x^4+c)^(1/2)/(b*x^4+a)+1/4*I*d^(1/2)/(a*d-b*c)/a
*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^
(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^
(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/32/b/a*sum((-3*a*d+b*
c)/(a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2
+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_a
lpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)
/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*
_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha
=RootOf(_Z^4*b+a))
```

3.839.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\text{Timed out}$$

input
```
integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.839.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**2/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.839.7 Maxima [F]

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^2}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.839.8 Giac [F]

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^2}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.839.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^2}{{\left(b\,x^4+a\right)}^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^2/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.840 $\int \frac{1}{x^2(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

3.840.1 Optimal result

Integrand size = 24, antiderivative size = 1225

$$
\begin{aligned}
&\int \frac{1}{x^2\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx \\
&= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x}+\frac{\sqrt{d}(5bc-4ad)x\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)}+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} \\
&\quad-\frac{b^{3/4}(5bc-7ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{9/4}(bc-ad)^{3/2}}-\frac{b^{3/4}(5bc-7ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{9/4}(-bc+ad)^{3/2}} \\
&\quad-\frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a^2c^{3/4}(bc-ad)\sqrt{c+dx^4}} \\
&\quad+\frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a^2c^{3/4}(bc-ad)\sqrt{c+dx^4}} \\
&\quad+\frac{b\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}} \\
&\quad+\frac{b\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}} \\
&\quad-\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\Bigg(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan}{32(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}} \\
&\quad+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\Bigg(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\Bigg(}{32(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}
\end{aligned}
$$

output
```
-1/16*b^(3/4)*(-7*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/
(d*x^4+c)^(1/2))/(-a)^(9/4)/(-a*d+b*c)^(3/2)-1/16*b^(3/4)*(-7*a*d+5*b*c)*a
rctan(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(9/4)/(a*
d-b*c)^(3/2)-1/4*(-4*a*d+5*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/x+1/4*b*(
d*x^4+c)^(1/2)/a/(-a*d+b*c)/x/(b*x^4+a)+1/4*(-4*a*d+5*b*c)*x*d^(1/2)*(d*x^
4+c)^(1/2)/a^2/c/(-a*d+b*c)/(c^(1/2)+x^2*d^(1/2))-1/4*d^(1/4)*(-4*a*d+5*b*
c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/
4)))*EllipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*
d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(3/4)/(-a*d+b*c)/
(d*x^4+c)^(1/2)+1/8*d^(1/4)*(-4*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)
))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4
)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d
^(1/2))^2)^(1/2)/a^2/c^(3/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(-7*a*d+5*b*c
)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4
)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^
(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(
c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^
(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c
)/(d*x^4+c)^(1/2)-1/32*(-7*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)
^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)...
```

3.840.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^2\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

$$= \frac{21a(c+dx^4)\left(4a^2d-5b^2cx^4-4ab(c-dx^4)\right)-7(5b^2c^2-12abcd+4a^2d^2)\,x^4(a+bx^4)\,\sqrt{1+\frac{dx^4}{c}}\,\mathrm{AppellF}}{84a^3c(bc-ad)x\,(a+}$$

input
```
Integrate[1/(x^2*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(21*a*(c + d*x^4)*(4*a^2*d - 5*b^2*c*x^4 - 4*a*b*(c - d*x^4)) - 7*(5*b^2*c
^2 - 12*a*b*c*d + 4*a^2*d^2)*x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[
3/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*(5*b*c - 4*a*d)*x^8*
(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c),
-((b*x^4)/a)])/(84*a^3*c*(b*c - a*d)*x*(a + b*x^4)*Sqrt[c + d*x^4])
```

3.840.3 Rubi [A] (warning: unable to verify)

Time = 1.54 (sec) , antiderivative size = 1148, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {972, 25, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{ 972}$$

$$\frac{b\sqrt{c+dx^4}}{4ax(a+bx^4)(bc-ad)}-\frac{\int -\frac{3bdx^4+5bc-4ad}{x^2(bx^4+a)\sqrt{dx^4+c}}dx}{4a(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3bdx^4+5bc-4ad}{x^2(bx^4+a)\sqrt{dx^4+c}}dx}{4a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \text{ 1053}$$

$$\frac{-\frac{\int \frac{x^2\left((bc-2ad)(5bc-2ad)-bd(5bc-4ad)x^4\right)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{ac}-\frac{\sqrt{c+dx^4}(5bc-4ad)}{acx}}{4a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{-\frac{\int \left(\frac{\left(5b^2c^2-7abcd\right)x^2}{\left(bx^4+a\right)\sqrt{dx^4+c}}-\frac{d(5bc-4ad)x^2}{\sqrt{dx^4+c}}\right)dx}{ac}-\frac{\sqrt{c+dx^4}(5bc-4ad)}{acx}}{4a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{dx^4+c}\,b}{4a(bc-ad)x\left(bx^4+a\right)}+$$

$$-\frac{\sqrt{dx^4+c}(5bc-4ad)}{acx}-\frac{-\frac{\sqrt{b}c^{3/4}(5bc-7ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8\sqrt{-a}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}+\frac{b^{3/4}c(}{}}{}$$

input `Int[1/(x^2*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]`

3.840. $\int \frac{1}{x^2(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

output
```
(b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x*(a + b*x^4)) + (-(((5*b*c - 4*a*d)*
Sqrt[c + d*x^4])/(a*c*x)) - (-((Sqrt[d]*(5*b*c - 4*a*d)*x*Sqrt[c + d*x^4])
/(Sqrt[c] + Sqrt[d]*x^2)) + (b^(3/4)*c*(5*b*c - 7*a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*Sqrt[b*c - a*
d]) - (b^(3/4)*c*(5*b*c - 7*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b
^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*Sqrt[b*c - a*d]) + (c^(1/4)*d^(1/4
)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt
[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/Sqrt[c + d*x^4]
 - (c^(1/4)*d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^
4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2
])/(2*Sqrt[c + d*x^4]) - (b*c^(3/4)*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])
*d^(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c]
 + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*(b*c
+ a*d)*Sqrt[c + d*x^4]) - (b*c^(3/4)*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b]
)*d^(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*(b*c
 + a*d)*Sqrt[c + d*x^4]) - (Sqrt[b]*c^(3/4)*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sq
rt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c]
 + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sq
rt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(...
```

3.840.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.840.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.14 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.32

method	result
elliptic	$\frac{b^2 x^3 \sqrt{d x^4+c}}{4(ad-bc)a^2(b x^4+a)}-\frac{\sqrt{d x^4+c}}{c a^2 x}+\frac{i\left(-\frac{bd}{4(ad-bc)a^2}+\frac{d}{c a^2}\right)\sqrt{c}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d x^4+c}\sqrt{d}}-$
default	$\frac{-\frac{\sqrt{d x^4+c}}{cx}+\frac{i\sqrt{d}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{c}\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d x^4+c}}}{a^2}-$ $\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)} -\frac{\operatorname{arctanh}\left(\frac{2d x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\sqrt{d x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+$ $\frac{i\sqrt{d}\sqrt{c}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d x^4+c}}-$ $c\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)} -\frac{\operatorname{arctanh}\left(\frac{2d x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\sqrt{d x^4}}\right)}{\sqrt{\frac{-ad+bc}{b}}}\right.$
risch	$-\frac{\sqrt{d x^4+c}}{c a^2 x}+$

input
```
int(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

3.840. $\int \frac{1}{x^2(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

output
```
1/4/(a*d-b*c)/a^2*b^2*x^3*(d*x^4+c)^(1/2)/(b*x^4+a)-1/c/a^2*(d*x^4+c)^(1/2
)/x+I*(-1/4*b*d/(a*d-b*c)/a^2+d/c/a^2)*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(
1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(
1/2)/d^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1
/2)*d^(1/2))^(1/2),I))-1/32/a^2*sum((7*a*d-5*b*c)/(a*d-b*c)/_alpha*(-1/((-
a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/
(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(
1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi
(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^
(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

3.840.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.840.6 Sympy [F]

$$\int \frac{1}{x^2 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^2 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x**2/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.840.7 Maxima [F]

$$\int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^2), x)
```

3.840.8 Giac [F]

$$\int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^2), x)
```

3.840.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^2\,(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/(x^2*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.841 $\int \frac{(ex)^m\left(a+bx^4\right)^2}{\sqrt{c+dx^4}}\,dx$

3.841.1 Optimal result

Integrand size = 26, antiderivative size = 200

$$\int \frac{(ex)^m\left(a+bx^4\right)^2}{\sqrt{c+dx^4}}\,dx$$
$$= -\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)}+\frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)}$$
$$+\frac{\left(a^2d^2(3+m)(7+m)+bc(1+m)(bc(5+m)-2ad(7+m))\right)(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\big(}{d^2e(1+m)(3+m)(7+m)\sqrt{c+dx^4}}$$

output
```
-b*(b*c*(5+m)-2*a*d*(7+m))*(e*x)^(1+m)*(d*x^4+c)^(1/2)/d^2/e/(3+m)/(7+m)+b
^2*(e*x)^(5+m)*(d*x^4+c)^(1/2)/d/e^5/(7+m)+(a^2*d^2*(3+m)*(7+m)+b*c*(1+m)*
(b*c*(5+m)-2*a*d*(7+m)))*(e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m
],-d*x^4/c)*(1+d*x^4/c)^(1/2)/d^2/e/(1+m)/(3+m)/(7+m)/(d*x^4+c)^(1/2)
```

3.841.2 Mathematica [A] (verified)

Time = 11.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.82

$$\int \frac{(ex)^m\left(a+bx^4\right)^2}{\sqrt{c+dx^4}}\,dx$$
$$=\frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\Big(a^2(45+14m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)+b(1+m)x^4\Big(2a(9+m}{(1+m)(5+m)(9}$$

input
```
Integrate[((e*x)^m*(a + b*x^4)^2)/Sqrt[c + d*x^4],x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a^2*(45 + 14*m + m^2)*Hypergeometric2F1[1/
2, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*(2*a*(9 + m)*Hyperg
eometric2F1[1/2, (5 + m)/4, (9 + m)/4, -((d*x^4)/c)] + b*(5 + m)*x^4*Hyper
geometric2F1[1/2, (9 + m)/4, (13 + m)/4, -((d*x^4)/c)])))/((1 + m)*(5 + m)
*(9 + m)*Sqrt[c + d*x^4])
```

3.841.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {964, 959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^2 (ex)^m}{\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \textcolor{blue}{964}$$

$$\frac{\int \frac{(ex)^m\left(a^2d(m+7)-b(bc(m+5)-2ad(m+7))x^4\right)}{\sqrt{dx^4+c}}dx}{d(m+7)}+\frac{b^2\sqrt{c+dx^4}(ex)^{m+5}}{de^5(m+7)}$$

$$\Big\downarrow \textcolor{blue}{959}$$

$$\frac{\left(a^2d(m+7)+\frac{bc(m+1)(bc(m+5)-2ad(m+7))}{d(m+3)}\right)\int \frac{(ex)^m}{\sqrt{dx^4+c}}dx-\frac{b\sqrt{c+dx^4}(ex)^{m+1}(bc(m+5)-2ad(m+7))}{de(m+3)}}{d(m+7)}+\frac{b^2\sqrt{c+dx^4}(ex)^{m+5}}{de^5(m+7)}$$

$$\Big\downarrow \textcolor{blue}{889}$$

$$\frac{\frac{\sqrt{\frac{dx^4}{c}+1}\left(a^2d(m+7)+\frac{bc(m+1)(bc(m+5)-2ad(m+7))}{d(m+3)}\right)\int \frac{(ex)^m}{\sqrt{\frac{dx^4}{c}+1}}dx}{\sqrt{c+dx^4}}-\frac{b\sqrt{c+dx^4}(ex)^{m+1}(bc(m+5)-2ad(m+7))}{de(m+3)}}{d(m+7)}+\frac{b^2\sqrt{c+dx^4}(ex)^{m+5}}{de^5(m+7)}$$

$$\Big\downarrow \textcolor{blue}{888}$$

$$\frac{\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\left(a^2d(m+7)+\frac{bc(m+1)(bc(m+5)-2ad(m+7))}{d(m+3)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{e(m+1)\sqrt{c+dx^4}}-\frac{b\sqrt{c+dx^4}(ex)^{m+1}(bc(m+5)-2ad(m}{de(m+3)}}{d(m+7)}$$

$$\frac{b^2\sqrt{c+dx^4}(ex)^{m+5}}{de^5(m+7)}$$

input
```
Int[((e*x)^m*(a + b*x^4)^2)/Sqrt[c + d*x^4],x]
```

output
```
(b^2*(e*x)^(5 + m)*Sqrt[c + d*x^4])/(d*e^5*(7 + m)) + (-((b*(b*c*(5 + m) -
 2*a*d*(7 + m))*(e*x)^(1 + m)*Sqrt[c + d*x^4])/(d*e*(3 + m))) + ((a^2*d*(7
 + m) + (b*c*(1 + m)*(b*c*(5 + m) - 2*a*d*(7 + m)))/(d*(3 + m)))*(e*x)^(1
+ m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m)/4, -((d
*x^4)/c)])/(e*(1 + m)*Sqrt[c + d*x^4]))/(d*(7 + m))
```

3.841.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 964
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Simp[1/(b*(m + n*(p + 2) + 1))   Int[(e*x)
^m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) - d*(a*d*(m + n + 1) - 2*b*
c*(m + n*(p + 2) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

3.841.4 Maple [F]

$$\int \frac{(ex)^{m} \left(b\,x^{4}+a\right)^{2}}{\sqrt{d\,x^{4}+c}}dx$$

input
```
int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

3.841.5 Fricas [F]

$$\int \frac{(ex)^m \left(a+bx^4\right)^2}{\sqrt{c+dx^4}}\,dx=\int \frac{\left(bx^4+a\right)^2(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*x^8 + 2*a*b*x^4 + a^2)*(e*x)^m/sqrt(d*x^4 + c), x)
```

3.841.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}}\,dx = \frac{a^2e^mx^{m+1}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4\sqrt{c}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)} + \frac{abe^mx^{m+5}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{4}+\frac{5}{4}\\ \frac{m}{4}+\frac{9}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{2\sqrt{c}\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)} + \frac{b^2e^mx^{m+9}\Gamma\left(\frac{m}{4}+\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{4}+\frac{9}{4}\\ \frac{m}{4}+\frac{13}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4\sqrt{c}\Gamma\left(\frac{m}{4}+\frac{13}{4}\right)}$$

input
```
integrate((e*x)**m*(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
a**2*e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,)
, d*x**4*exp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4)) + a*b*e**m*x**(m
+ 5)*gamma(m/4 + 5/4)*hyper((1/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_pol
ar(I*pi)/c)/(2*sqrt(c)*gamma(m/4 + 9/4)) + b**2*e**m*x**(m + 9)*gamma(m/4
+ 9/4)*hyper((1/2, m/4 + 9/4), (m/4 + 13/4,), d*x**4*exp_polar(I*pi)/c)/(4
*sqrt(c)*gamma(m/4 + 13/4))
```

3.841.7 Maxima [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}}\,dx = \int \frac{(bx^4+a)^2(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^2*(e*x)^m/sqrt(d*x^4 + c), x)
```

3.841.8 Giac [F]

$$\int \frac{(ex)^m \left(a+bx^4\right)^2}{\sqrt{c+dx^4}}\,dx = \int \frac{\left(bx^4+a\right)^2 (ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^2*(e*x)^m/sqrt(d*x^4 + c), x)
```

3.841.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m \left(a+bx^4\right)^2}{\sqrt{c+dx^4}}\,dx = \int \frac{(e\,x)^m\,\left(b\,x^4+a\right)^2}{\sqrt{d\,x^4+c}}\,dx$$

input
```
int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(1/2),x)
```

output
```
int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(1/2), x)
```

3.842 $\int \frac{(ex)^m(a+bx^4)}{\sqrt{c+dx^4}}\,dx$

3.842.1 Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{(ex)^m(a+bx^4)}{\sqrt{c+dx^4}}\,dx = \frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)} - \frac{(bc(1+m)-ad(3+m))(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)}{de(1+m)(3+m)\sqrt{c+dx^4}}$$

output

```
b*(e*x)^(1+m)*(d*x^4+c)^(1/2)/d/e/(3+m)-(b*c*(1+m)-a*d*(3+m))*(e*x)^(1+m)*
hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c)^(1/2)/d/e/(1+
m)/(3+m)/(d*x^4+c)^(1/2)
```

3.842.2 Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{(ex)^m(a+bx^4)}{\sqrt{c+dx^4}}\,dx = \frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\left(a(5+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)+b(1+m)x^4\,\text{Hypergeometric2F1}\right.}{(1+m)(5+m)\sqrt{c+dx^4}}$$

input
```
Integrate[((e*x)^m*(a + b*x^4))/Sqrt[c + d*x^4],x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a*(5 + m)*Hypergeometric2F1[1/2, (1 + m)/4
, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*Hypergeometric2F1[1/2, (5 + m)/
4, (9 + m)/4, -((d*x^4)/c)]))/((1 + m)*(5 + m)*Sqrt[c + d*x^4])
```

3.842.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)(ex)^m}{\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \textit{959}$$

$$\left(a-\frac{bc(m+1)}{d(m+3)}\right)\int \frac{(ex)^m}{\sqrt{dx^4+c}}dx+\frac{b\sqrt{c+dx^4}(ex)^{m+1}}{de(m+3)}$$

$$\Big\downarrow \textit{889}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}\left(a-\frac{bc(m+1)}{d(m+3)}\right)\int \frac{(ex)^m}{\sqrt{\frac{dx^4}{c}+1}}dx}{\sqrt{c+dx^4}}+\frac{b\sqrt{c+dx^4}(ex)^{m+1}}{de(m+3)}$$

$$\Big\downarrow \textit{888}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\left(a-\frac{bc(m+1)}{d(m+3)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{e(m+1)\sqrt{c+dx^4}}+\frac{b\sqrt{c+dx^4}(ex)^{m+1}}{de(m+3)}$$

input
```
Int[((e*x)^m*(a + b*x^4))/Sqrt[c + d*x^4],x]
```

output
```
(b*(e*x)^(1 + m)*Sqrt[c + d*x^4])/(d*e*(3 + m)) + ((a - (b*c*(1 + m))/(d*(
3 + m)))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/
4, (5 + m)/4, -((d*x^4)/c)])/(e*(1 + m)*Sqrt[c + d*x^4])
```

3.842.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.842.4 Maple [F]

$$\int \frac{(ex)^m (b\,x^4+a)}{\sqrt{d\,x^4+c}}dx$$

input
```
int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

3.842.5 Fricas [F]

$$\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}}\,dx = \int \frac{(bx^4+a)(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)*(e*x)^m/sqrt(d*x^4 + c), x)
```

3.842.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

$$\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}}\,dx = \frac{ae^m x^{m+1}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\right| \frac{dx^4e^{i\pi}}{c}\right)}{4\sqrt{c}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)} + \frac{be^m x^{m+5}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{m}{4}+\frac{5}{4}\\ \frac{m}{4}+\frac{9}{4}\end{matrix}\right| \frac{dx^4e^{i\pi}}{c}\right)}{4\sqrt{c}\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)}$$

input
```
integrate((e*x)**m*(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
a*e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,), d
*x**4*exp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4)) + b*e**m*x**(m + 5)*
gamma(m/4 + 5/4)*hyper((1/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(I*
pi)/c)/(4*sqrt(c)*gamma(m/4 + 9/4))
```

3.842.7 Maxima [F]

$$\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}}\,dx = \int \frac{(bx^4+a)(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)*(e*x)^m/sqrt(d*x^4 + c), x)
```

3.842.8 Giac [F]

$$\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}}\,dx=\int \frac{(bx^4+a)(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)*(e*x)^m/sqrt(d*x^4 + c), x)
```

3.842.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}}\,dx=\int \frac{(e\,x)^m\,(b\,x^4+a)}{\sqrt{d\,x^4+c}}\,dx$$

input
```
int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(1/2),x)
```

output
```
int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(1/2), x)
```

3.843 $\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx$

3.843.1 Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx=\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)}{e(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c)^(
1/2)/e/(1+m)/(d*x^4+c)^(1/2)
```

3.843.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx=\frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},1+\frac{1+m}{4},-\frac{dx^4}{c}\right)}{(1+m)\sqrt{c+dx^4}}$$

input
```
Integrate[(e*x)^m/Sqrt[c + d*x^4],x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, 1 + (1 +
m)/4, -((d*x^4)/c)])/((1 + m)*Sqrt[c + d*x^4])
```

3.843.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}\int \frac{(ex)^m}{\sqrt{\frac{dx^4}{c}+1}}dx}{\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{e(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/Sqrt[c + d*x^4],x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, (5 +
m)/4, -((d*x^4)/c)])/(e*(1 + m)*Sqrt[c + d*x^4])
```

3.843.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

3.843.4 Maple [F]

$$\int \frac{(ex)^m}{\sqrt{d\,x^4+c}}dx$$

input
```
int((e*x)^m/(d*x^4+c)^(1/2),x)
```

output
```
int((e*x)^m/(d*x^4+c)^(1/2),x)
```

3.843.5 Fricas [F]

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((e*x)^m/sqrt(d*x^4 + c), x)
```

3.843.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx = \frac{e^m x^{m+1}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}$$

input
```
integrate((e*x)**m/(d*x**4+c)**(1/2),x)
```

output
```
e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,), d*x
**4*exp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4))
```

3.843.7 Maxima [F]

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx=\int \frac{(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/sqrt(d*x^4 + c), x)
```

3.843.8 Giac [F]

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx=\int \frac{(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/sqrt(d*x^4 + c), x)
```

3.843.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx=\int \frac{(e\,x)^m}{\sqrt{d\,x^4+c}}\,dx$$

input
```
int((e*x)^m/(c + d*x^4)^(1/2),x)
```

output
```
int((e*x)^m/(c + d*x^4)^(1/2), x)
```

3.844 $\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

3.844.1 Optimal result

Integrand size = 26, antiderivative size = 81

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1+m}{4},1,\frac{1}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*AppellF1(1/4+1/4*m,1,1/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4
/c)^(1/2)/a/e/(1+m)/(d*x^4+c)^(1/2)
```

3.844.2 Mathematica [A] (verified)

Time = 5.61 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$= \frac{x(ex)^m\sqrt{c+dx^4}\left(bc\,\text{AppellF1}\left(\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-ad\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\right.\right.}{ac(bc-ad)(1+m)\sqrt{1+\frac{dx^4}{c}}}$$

input
```
Integrate[(e*x)^m/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(x*(e*x)^m*Sqrt[c + d*x^4]*(b*c*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/4, -(
(d*x^4)/c), -((b*x^4)/a)] - a*d*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m)/
4, -((d*x^4)/c)]))/(a*c*(b*c - a*d)*(1 + m)*Sqrt[1 + (d*x^4)/c])
```

3.844.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \; 1013$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}\int \frac{(ex)^m}{(bx^4+a)\sqrt{\frac{dx^4}{c}+1}}dx}{\sqrt{c+dx^4}}$$

$$\Big\downarrow \; 1012$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\operatorname{AppellF1}\left(\frac{m+1}{4},1,\frac{1}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 1, 1/2, (5 + m)/4,
-((b*x^4)/a), -((d*x^4)/c)])/(a*e*(1 + m)*Sqrt[c + d*x^4])
```

3.844.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.844.4 Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}dx$$

input
```
int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

3.844.5 Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)\,\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*(e*x)^m/(b*d*x^8 + (b*c + a*d)*x^4 + a*c), x)
```

3.844.6 Sympy [F]

$$\int \frac{(ex)^m}{(a+bx^4)\,\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(a+bx^4)\,\sqrt{c+dx^4}}\,dx$$

input
```
integrate((e*x)**m/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral((e*x)**m/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.844.7 Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.844.8 Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.844.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{(e\,x)^m}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.845 $\int \frac{(ex)^m}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

3.845.1 Optimal result

Integrand size = 26, antiderivative size = 81

$$\int \frac{(ex)^m}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{1+m}{4},2,\frac{1}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*AppellF1(1/4+1/4*m,2,1/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4
/c)^(1/2)/a^2/e/(1+m)/(d*x^4+c)^(1/2)
```

3.845.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 179 vs. $2(81) = 162$.

Time = 11.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.21

$$\int \frac{(ex)^m}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$
$$=\frac{x(ex)^m\sqrt{c+dx^4}\left(-abcd\operatorname{AppellF1}\left(\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+bc(bc-ad)\operatorname{AppellF1}\left(\frac{1+m}{4},2,-\frac{1}{2},\right.\right.}{a^2c(bc-ad)^2(1+m)\sqrt{1+\frac{dx^4}{c}}}$$

input
```
Integrate[(e*x)^m/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(x*(e*x)^m*Sqrt[c + d*x^4]*(-(a*b*c*d*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)
/4, -((d*x^4)/c), -((b*x^4)/a)]) + b*c*(b*c - a*d)*AppellF1[(1 + m)/4, 2,
-1/2, (5 + m)/4, -((b*x^4)/a), -((d*x^4)/c)] + a^2*d^2*Hypergeometric2F1[1
/2, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)]))/(a^2*c*(b*c - a*d)^2*(1 + m)*Sqr
t[1 + (d*x^4)/c])
```

3.845.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}\int \frac{(ex)^m}{\left(bx^4+a\right)^2\sqrt{\frac{dx^4}{c}+1}}dx}{\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\operatorname{AppellF1}\left(\frac{m+1}{4},2,\frac{1}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 2, 1/2, (5 + m)/4,
-((b*x^4)/a), -((d*x^4)/c)])/(a^2*e*(1 + m)*Sqrt[c + d*x^4])
```

3.845.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.845.4 Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)^2\sqrt{d\,x^4+c}}dx$$

input
```
int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

3.845.5 Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*(e*x)^m/(b^2*d*x^12 + (b^2*c + 2*a*b*d)*x^8 + (2*
a*b*c + a^2*d)*x^4 + a^2*c), x)
```

3.845.6 Sympy [F]

$$\int \frac{(ex)^m}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx$$

input
```
integrate((e*x)**m/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral((e*x)**m/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.845.7 Maxima [F]

$$\int \frac{(ex)^m}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{\left(bx^4+a\right)^2 \sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.845.8 Giac [F]

$$\int \frac{(ex)^m}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{\left(bx^4+a\right)^2 \sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.845.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{(e\,x)^m}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input

```
int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output

```
int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.846 $\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx$

3.846.1 Optimal result

Integrand size = 26, antiderivative size = 81

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx=\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{1+m}{4},3,\frac{1}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*AppellF1(1/4+1/4*m,3,1/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4
/c)^(1/2)/a^3/e/(1+m)/(d*x^4+c)^(1/2)
```

3.846.2 Mathematica [A] (verified)

Time = 11.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx=\frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{1+m}{4},3,\frac{1}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3(1+m)\sqrt{c+dx^4}}$$

input
```
Integrate[(e*x)^m/((a + b*x^4)^3*Sqrt[c + d*x^4]),x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 1/2, (5 + m)/4, -((b
*x^4)/a), -((d*x^4)/c)])/(a^3*(1 + m)*Sqrt[c + d*x^4])
```

3.846.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}\int \frac{(ex)^m}{(bx^4+a)^3\sqrt{\frac{dx^4}{c}+1}}dx}{\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\operatorname{AppellF1}\left(\frac{m+1}{4},3,\frac{1}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/((a + b*x^4)^3*Sqrt[c + d*x^4]),x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 1/2, (5 + m)/4,
-((b*x^4)/a), -((d*x^4)/c)])/(a^3*e*(1 + m)*Sqrt[c + d*x^4])
```

3.846.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.846.4 Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)^3\sqrt{d\,x^4+c}}dx$$

input
```
int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x)
```

output
```
int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x)
```

3.846.5 Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^3\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*(e*x)^m/(b^3*d*x^16 + (b^3*c + 3*a*b^2*d)*x^12 +
3*(a*b^2*c + a^2*b*d)*x^8 + (3*a^2*b*c + a^3*d)*x^4 + a^3*c), x)
```

3.846.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m/(b*x**4+a)**3/(d*x**4+c)**(1/2),x)
```

output
```
Timed out
```

3.846.7 Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx=\int \frac{(ex)^m}{(bx^4+a)^3\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^3*sqrt(d*x^4 + c)), x)
```

3.846.8 Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx=\int \frac{(ex)^m}{(bx^4+a)^3\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^3*sqrt(d*x^4 + c)), x)
```

3.846.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx=\int \frac{(e\,x)^m}{(b\,x^4+a)^3\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(1/2)),x)
```

output
```
int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(1/2)), x)
```

3.847 $\int \frac{(ex)^m(a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx$

3.847.1 Optimal result

Integrand size = 26, antiderivative size = 198

$$\int \frac{(ex)^m\left(a+bx^4\right)^2}{\left(c+dx^4\right)^{3/2}}\,dx=\frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}}+\frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)}$$

$$-\frac{\left(2b^2c^2(1+m)-(3+m)\left(2a^2d^2-(bc-ad)^2(1+m)\right)\right)(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+}{4}\right.}{2cd^2e(1+m)(3+m)\sqrt{c+dx^4}}$$

output
```
1/2*(-a*d+b*c)^2*(e*x)^(1+m)/c/d^2/e/(d*x^4+c)^(1/2)+b^2*(e*x)^(1+m)*(d*x^
4+c)^(1/2)/d^2/e/(3+m)-1/2*(2*b^2*c^2*(1+m)-(3+m)*(2*a^2*d^2-(-a*d+b*c)^2*
(1+m)))*(e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*
x^4/c)^(1/2)/c/d^2/e/(1+m)/(3+m)/(d*x^4+c)^(1/2)
```

3.847.2 Mathematica [A] (verified)

Time = 11.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.84

$$\int \frac{(ex)^m\left(a+bx^4\right)^2}{\left(c+dx^4\right)^{3/2}}\,dx=\frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\left(a^2\left(45+14m+m^2\right)\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)+\right.}{}$$

input
```
Integrate[((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a^2*(45 + 14*m + m^2)*Hypergeometric2F1[3/
2, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*(2*a*(9 + m)*Hyperg
eometric2F1[3/2, (5 + m)/4, (9 + m)/4, -((d*x^4)/c)] + b*(5 + m)*x^4*Hyper
geometric2F1[3/2, (9 + m)/4, (13 + m)/4, -((d*x^4)/c)])))/(c*(1 + m)*(5 +
m)*(9 + m)*Sqrt[c + d*x^4])
```

3.847.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {963, 25, 959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^2 (ex)^m}{\left(c+dx^4\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{963}$$

$$\frac{(ex)^{m+1}(bc-ad)^2}{2cd^2e\sqrt{c+dx^4}}-\frac{\int -\frac{(ex)^m\left(2b^2cdx^4+2a^2d^2-(bc-ad)^2(m+1)\right)}{\sqrt{dx^4+c}}dx}{2cd^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ex)^m\left(2b^2cdx^4+2a^2d^2-(bc-ad)^2(m+1)\right)}{\sqrt{dx^4+c}}dx}{2cd^2}+\frac{(ex)^{m+1}(bc-ad)^2}{2cd^2e\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{959}$$

$$\frac{\frac{2b^2c\sqrt{c+dx^4}(ex)^{m+1}}{e(m+3)}-\frac{\left(2b^2c^2(m+1)-(m+3)\left(2a^2d^2-(m+1)(bc-ad)^2\right)\right)\int \frac{(ex)^m}{\sqrt{dx^4+c}}dx}{m+3}}{2cd^2}+\frac{(ex)^{m+1}(bc-ad)^2}{2cd^2e\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\frac{2b^2c\sqrt{c+dx^4}(ex)^{m+1}}{e(m+3)}-\frac{\sqrt{\frac{dx^4}{c}+1}\left(2b^2c^2(m+1)-(m+3)\left(2a^2d^2-(m+1)(bc-ad)^2\right)\right)\int \frac{(ex)^m}{\sqrt{\frac{dx^4}{c}+1}}dx}{(m+3)\sqrt{c+dx^4}}}{2cd^2}+\frac{(ex)^{m+1}(bc-ad)^2}{2cd^2e\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{888}$$

$$\frac{\frac{2b^2c\sqrt{c+dx^4}(ex)^{m+1}}{e(m+3)}-\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\left(2b^2c^2(m+1)-(m+3)\left(2a^2d^2-(m+1)(bc-ad)^2\right)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{e(m+1)(m+3)\sqrt{c+dx^4}}}{2cd^2}+\frac{(ex)^{m+1}(bc-ad)^2}{2cd^2e\sqrt{c+dx^4}}$$

input
```
Int[((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x]
```

output
```
((b*c - a*d)^2*(e*x)^(1 + m))/(2*c*d^2*e*Sqrt[c + d*x^4]) + ((2*b^2*c*(e*x
)^(1 + m)*Sqrt[c + d*x^4])/(e*(3 + m)) - ((2*b^2*c^2*(1 + m) - (3 + m)*(2*
a^2*d^2 - (b*c - a*d)^2*(1 + m)))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hyperg
eometric2F1[1/2, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)])/(e*(1 + m)*(3 + m)*S
qrt[c + d*x^4]))/(2*c*d^2)
```

3.847.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 963
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Simp[1/(a*b^2*n*(p + 1))   Int[(e*x)^m*(a + b*x
^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1]
```

3.847.4 Maple [F]

$$\int \frac{(ex)^{m} \left(b\,x^{4}+a\right)^{2}}{\left(d\,x^{4}+c\right)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

output
```
int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

3.847.5 Fricas [F]

$$\int \frac{(ex)^m \left(a+bx^4\right)^2}{\left(c+dx^4\right)^{3/2}}\,dx=\int \frac{\left(bx^4+a\right)^2(ex)^m}{\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*x^8 + 2*a*b*x^4 + a^2)*sqrt(d*x^4 + c)*(e*x)^m/(d^2*x^8 + 2*
c*d*x^4 + c^2), x)
```

3.847.6 Sympy [F]

$$\int \frac{(ex)^m \left(a+bx^4\right)^2}{\left(c+dx^4\right)^{3/2}}\,dx=\int \frac{(ex)^m \left(a+bx^4\right)^2}{\left(c+dx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**m*(b*x**4+a)**2/(d*x**4+c)**(3/2),x)
```

output
```
Integral((e*x)**m*(a + b*x**4)**2/(c + d*x**4)**(3/2), x)
```

3.847.7 Maxima [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx = \int \frac{(bx^4+a)^2(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^2*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

3.847.8 Giac [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx = \int \frac{(bx^4+a)^2(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^2*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

3.847.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx = \int \frac{(e\,x)^m\,(b\,x^4+a)^2}{(d\,x^4+c)^{3/2}}\,dx$$

input
```
int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x)
```

output
```
int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2), x)
```

3.848 $\int \frac{(ex)^m (a+bx^4)}{(c+dx^4)^{3/2}} \, dx$

3.848.1 Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{(ex)^m (a+bx^4)}{(c+dx^4)^{3/2}} \, dx = -\frac{(bc-ad)(ex)^{1+m}}{2cde\sqrt{c+dx^4}} + \frac{(ad(1-m)+bc(1+m))(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)}{2cde(1+m)\sqrt{c+dx^4}}$$

output
```
-1/2*(-a*d+b*c)*(e*x)^(1+m)/c/d/e/(d*x^4+c)^(1/2)+1/2*(a*d*(-m+1)+b*c*(1+m
))*(e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c
)^(1/2)/c/d/e/(1+m)/(d*x^4+c)^(1/2)
```

3.848.2 Mathematica [A] (verified)

Time = 5.58 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\int \frac{(ex)^m (a+bx^4)}{(c+dx^4)^{3/2}} \, dx = \frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\left(a(5+m)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)+b(1+m)x\right.}{c(1+m)(5+m)\sqrt{c+dx^4}}$$

input
```
Integrate[((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a*(5 + m)*Hypergeometric2F1[3/2, (1 + m)/4
, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*Hypergeometric2F1[3/2, (5 + m)/
4, (9 + m)/4, -((d*x^4)/c)]))/(c*(1 + m)*(5 + m)*Sqrt[c + d*x^4])
```

3.848.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {957, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)(ex)^m}{(c+dx^4)^{3/2}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(ad(1-m)+bc(m+1))\int \frac{(ex)^m}{\sqrt{dx^4+c}}dx}{2cd}-\frac{(ex)^{m+1}(bc-ad)}{2cde\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ad(1-m)+bc(m+1))\int \frac{(ex)^m}{\sqrt{\frac{dx^4}{c}+1}}dx}{2cd\sqrt{c+dx^4}}-\frac{(ex)^{m+1}(bc-ad)}{2cde\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}(ad(1-m)+bc(m+1))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{2cde(m+1)\sqrt{c+dx^4}}-\frac{(ex)^{m+1}(bc-ad)}{2cde\sqrt{c+dx^4}}$$

input
```
Int[((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x]
```

output
```
-1/2*((b*c - a*d)*(e*x)^(1 + m))/(c*d*e*Sqrt[c + d*x^4]) + ((a*d*(1 - m) +
 b*c*(1 + m))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1
+ m)/4, (5 + m)/4, -((d*x^4)/c)])/(2*c*d*e*(1 + m)*Sqrt[c + d*x^4])
```

3.848.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.848.4 Maple [F]

$$\int \frac{(ex)^{m} \left(b\,x^{4}+a\right)}{\left(d\,x^{4}+c\right)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

output
```
int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

3.848.5 Fricas [F]

$$\int \frac{(ex)^m \left(a+bx^4\right)}{\left(c+dx^4\right)^{3/2}}\, dx=\int \frac{{\left(bx^{4}+a\right)} (ex)^{m}}{{\left(dx^{4}+c\right)}^{\frac{3}{2}}}\, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)*sqrt(d*x^4 + c)*(e*x)^m/(d^2*x^8 + 2*c*d*x^4 + c^2),
x)
```

3.848.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.87 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$\int \frac{(ex)^m (a+bx^4)}{(c+dx^4)^{3/2}}\,dx = \frac{ae^m x^{m+1}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{m}{4}+\frac{1}{4} \\ \frac{m}{4}+\frac{5}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)} + \frac{be^m x^{m+5}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{m}{4}+\frac{5}{4} \\ \frac{m}{4}+\frac{9}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)}$$

input
```
integrate((e*x)**m*(b*x**4+a)/(d*x**4+c)**(3/2),x)
```

output
```
a*e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((3/2, m/4 + 1/4), (m/4 + 5/4,), d
*x**4*exp_polar(I*pi)/c)/(4*c**(3/2)*gamma(m/4 + 5/4)) + b*e**m*x**(m + 5)
*gamma(m/4 + 5/4)*hyper((3/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(I
*pi)/c)/(4*c**(3/2)*gamma(m/4 + 9/4))
```

3.848.7 Maxima [F]

$$\int \frac{(ex)^m (a+bx^4)}{(c+dx^4)^{3/2}}\,dx = \int \frac{(bx^4+a)(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

3.848.8 Giac [F]

$$\int \frac{(ex)^m\,(a+bx^4)}{(c+dx^4)^{3/2}}\,dx=\int \frac{(bx^4+a)(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

3.848.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m\,(a+bx^4)}{(c+dx^4)^{3/2}}\,dx=\int \frac{(e\,x)^m\,(b\,x^4+a)}{(d\,x^4+c)^{3/2}}\,dx$$

input
```
int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x)
```

output
```
int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2), x)
```

3.849 $\int \frac{(ex)^m}{\left(c+dx^4\right)^{3/2}}\,dx$

3.849.1 Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{(ex)^m}{\left(c+dx^4\right)^{3/2}}\,dx=\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)}{ce(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*hypergeom([3/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c)^(
1/2)/c/e/(1+m)/(d*x^4+c)^(1/2)
```

3.849.2 Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{(ex)^m}{\left(c+dx^4\right)^{3/2}}\,dx=\frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{4},1+\frac{1+m}{4},-\frac{dx^4}{c}\right)}{c(1+m)\sqrt{c+dx^4}}$$

input
```
Integrate[(e*x)^m/(c + d*x^4)^(3/2),x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[3/2, (1 + m)/4, 1 + (1 +
m)/4, -((d*x^4)/c)])/(c*(1 + m)*Sqrt[c + d*x^4])
```

3.849.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}\int \frac{(ex)^m}{\left(\frac{dx^4}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{ce(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/(c + d*x^4)^(3/2),x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[3/2, (1 + m)/4, (5 +
m)/4, -((d*x^4)/c)])/(c*e*(1 + m)*Sqrt[c + d*x^4])
```

3.849.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

3.849.4 Maple [F]

$$\int \frac{(ex)^m}{(d\,x^4+c)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m/(d*x^4+c)^(3/2),x)
```

output
```
int((e*x)^m/(d*x^4+c)^(3/2),x)
```

3.849.5 Fricas [F]

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*(e*x)^m/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

3.849.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \frac{e^m x^{m+1}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\right|\frac{dx^4e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}$$

input
```
integrate((e*x)**m/(d*x**4+c)**(3/2),x)
```

output
```
e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((3/2, m/4 + 1/4), (m/4 + 5/4,), d*x
**4*exp_polar(I*pi)/c)/(4*c**(3/2)*gamma(m/4 + 5/4))
```

3.849.7 Maxima [F]

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/(d*x^4 + c)^(3/2), x)
```

3.849.8 Giac [F]

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/(d*x^4 + c)^(3/2), x)
```

3.849.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \int \frac{(e\,x)^m}{(d\,x^4+c)^{3/2}}\,dx$$

input
```
int((e*x)^m/(c + d*x^4)^(3/2),x)
```

output
```
int((e*x)^m/(c + d*x^4)^(3/2), x)
```

3.850 $\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx$

3.850.1 Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx=\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1+m}{4},1,\frac{3}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*AppellF1(1/4+1/4*m,1,3/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4
/c)^(1/2)/a/c/e/(1+m)/(d*x^4+c)^(1/2)
```

3.850.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 169 vs. 2(84) = 168.

Time = 11.19 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.01

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx=\frac{x(ex)^m\sqrt{c+dx^4}\Big(b^2c^2\,\text{AppellF1}\left(\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+ad\Big(-bc\,\text{Hy}}{ac^2(b}$$

input
```
Integrate[(e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x]
```

output
```
(x*(e*x)^m*Sqrt[c + d*x^4]*(b^2*c^2*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/4
, -((d*x^4)/c), -((b*x^4)/a)] + a*d*(-(b*c*Hypergeometric2F1[1/2, (1 + m)/
4, (5 + m)/4, -((d*x^4)/c)]) + (-(b*c) + a*d)*Hypergeometric2F1[3/2, (1 +
m)/4, (5 + m)/4, -((d*x^4)/c)])))/(a*c^2*(b*c - a*d)^2*(1 + m)*Sqrt[1 + (d
*x^4)/c])
```

3.850.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}\int \frac{(ex)^m}{(bx^4+a)\left(\frac{dx^4}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\operatorname{AppellF1}\left(\frac{m+1}{4},1,\frac{3}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 1, 3/2, (5 + m)/4,
-((b*x^4)/a), -((d*x^4)/c)])/(a*c*e*(1 + m)*Sqrt[c + d*x^4])
```

3.850.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.850.4 Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)\,(d\,x^4+c)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

output
```
int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

3.850.5 Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)\,(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*(e*x)^m/(b*d^2*x^12 + (2*b*c*d + a*d^2)*x^8 + (b*
c^2 + 2*a*c*d)*x^4 + a*c^2), x)
```

3.850.6 Sympy [F]

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**m/(b*x**4+a)/(d*x**4+c)**(3/2),x)
```

output
```
Integral((e*x)**m/((a + b*x**4)*(c + d*x**4)**(3/2)), x)
```

3.850.7 Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^4 + a)*(d*x^4 + c)^(3/2)), x)
```

3.850.8 Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^4 + a)*(d*x^4 + c)^(3/2)), x)
```

3.850.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx = \int \frac{(e\,x)^m}{(b\,x^4+a)\,\left(d\,x^4+c\right)^{3/2}}\,dx$$

input
```
int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x)
```

output
```
int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)), x)
```

3.851 $\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx$

3.851.1 Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx=\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1+m}{4},2,\frac{3}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2ce(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*AppellF1(1/4+1/4*m,2,3/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4
/c)^(1/2)/a^2/c/e/(1+m)/(d*x^4+c)^(1/2)
```

3.851.2 Mathematica [A] (verified)

Time = 11.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx=\frac{x(ex)^m\left(1+\frac{dx^4}{c}\right)^{3/2}\text{AppellF1}\left(\frac{1+m}{4},2,\frac{3}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2(1+m)\,(c+dx^4)^{3/2}}$$

input
```
Integrate[(e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x]
```

output
```
(x*(e*x)^m*(1 + (d*x^4)/c)^(3/2)*AppellF1[(1 + m)/4, 2, 3/2, (5 + m)/4, -(
(b*x^4)/a), -((d*x^4)/c)])/(a^2*(1 + m)*(c + d*x^4)^(3/2))
```

3.851.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^4)^2 (c+dx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1} \int \frac{(ex)^m}{(bx^4+a)^2 \left(\frac{dx^4}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1} \operatorname{AppellF1}\left(\frac{m+1}{4},2,\frac{3}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2 ce(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 2, 3/2, (5 + m)/4,
-((b*x^4)/a), -((d*x^4)/c)])/(a^2*c*e*(1 + m)*Sqrt[c + d*x^4])
```

3.851.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.851.4 Maple [F]

$$\int \frac{(ex)^{m}}{(b\,x^{4}+a)^{2}\,(d\,x^{4}+c)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

output
```
int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

3.851.5 Fricas [F]

$$\int \frac{(ex)^{m}}{(a+bx^{4})^{2}\,(c+dx^{4})^{3/2}}\,dx=\int \frac{(ex)^{m}}{(bx^{4}+a)^{2}(dx^{4}+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*(e*x)^m/(b^2*d^2*x^16 + 2*(b^2*c*d + a*b*d^2)*x^1
2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^8 + 2*(a*b*c^2 + a^2*c*d)*x^4 + a^2*
c^2), x)
```

3.851.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{m}}{(a+bx^{4})^{2}\,(c+dx^{4})^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x)**m/(b*x**4+a)**2/(d*x**4+c)**(3/2),x)
```

output
```
Timed out
```

3.851.7 Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2 (c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^2(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^2*(d*x^4 + c)^(3/2)), x)
```

3.851.8 Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2 (c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^2(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^2*(d*x^4 + c)^(3/2)), x)
```

3.851.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^2 (c+dx^4)^{3/2}}\,dx = \int \frac{(e\,x)^m}{(b\,x^4+a)^2\,(d\,x^4+c)^{3/2}}\,dx$$

input
```
int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x)
```

output
```
int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)), x)
```

3.852 $\int \frac{(ex)^m}{(a+bx^4)^3(c+dx^4)^{3/2}}\,dx$

3.852.1 Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{(ex)^m}{(a+bx^4)^3\,(c+dx^4)^{3/2}}\,dx=\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\operatorname{AppellF1}\left(\frac{1+m}{4},3,\frac{3}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*AppellF1(1/4+1/4*m,3,3/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4
/c)^(1/2)/a^3/c/e/(1+m)/(d*x^4+c)^(1/2)
```

3.852.2 Mathematica [A] (verified)

Time = 11.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{(ex)^m}{(a+bx^4)^3\,(c+dx^4)^{3/2}}\,dx=\frac{x(ex)^m\left(1+\frac{dx^4}{c}\right)^{3/2}\operatorname{AppellF1}\left(\frac{1+m}{4},3,\frac{3}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3(1+m)\,(c+dx^4)^{3/2}}$$

input
```
Integrate[(e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x]
```

output
```
(x*(e*x)^m*(1 + (d*x^4)/c)^(3/2)*AppellF1[(1 + m)/4, 3, 3/2, (5 + m)/4, -(
(b*x^4)/a), -((d*x^4)/c)])/(a^3*(1 + m)*(c + d*x^4)^(3/2))
```

3.852.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}} \, dx$$

$$\Big\downarrow \textcolor{blue}{1013}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}\int \frac{(ex)^m}{(bx^4+a)^3 \left(\frac{dx^4}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^4}}$$

$$\Big\downarrow \textcolor{blue}{1012}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1} \operatorname{AppellF1}\left(\frac{m+1}{4},3,\frac{3}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 3/2, (5 + m)/4,
-((b*x^4)/a), -((d*x^4)/c)])/(a^3*c*e*(1 + m)*Sqrt[c + d*x^4])
```

3.852.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.852.4 Maple [F]

$$\int \frac{(ex)^{m}}{\left(b\,x^{4}+a\right)^{3} \left(d\,x^{4}+c\right)^{\frac{3}{2}}}d x$$

input
```
int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x)
```

output
```
int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x)
```

3.852.5 Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}} \, dx = \int \frac{\left(e x\right)^{m}}{{\left(b x^{4} + a\right)}^{3} {\left(d x^{4} + c\right)}^{\frac{3}{2}}} \,d x$$

input
```
integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*(e*x)^m/(b^3*d^2*x^20 + (2*b^3*c*d + 3*a*b^2*d^2)
*x^16 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^12 + (3*a*b^2*c^2 + 6*a^2*
b*c*d + a^3*d^2)*x^8 + a^3*c^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x^4), x)
```

3.852.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m/(b*x**4+a)**3/(d*x**4+c)**(3/2),x)
```

output
```
Timed out
```

3.852.7 Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^3 (dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^3*(d*x^4 + c)^(3/2)), x)
```

3.852.8 Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^3 (dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^3*(d*x^4 + c)^(3/2)), x)
```

3.852.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}}\,dx = \int \frac{(e\,x)^m}{(b\,x^4+a)^3\,(d\,x^4+c)^{3/2}}\,dx$$

input
```
int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x)
```

output
```
int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)), x)
```

3.853 $\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

3.853.1 Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx=-\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2}+\frac{(c+dx^6)^{3/2}}{9bd^2}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

output
```
1/9*(d*x^6+c)^(3/2)/b/d^2-1/3*a^2*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*
c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-1/3*(a*d+b*c)*(d*x^6+c)^(1/2)/b^2/d^2
```

3.853.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{\sqrt{c+dx^6}(-2bc-3ad+bdx^6)}{9b^2d^2}+\frac{a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}\sqrt{-bc+ad}}$$

input
```
Integrate[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(Sqrt[c + d*x^6]*(-2*b*c - 3*a*d + b*d*x^6))/(9*b^2*d^2) + (a^2*ArcTan[(Sq
rt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2)*Sqrt[-(b*c) + a*d])
```

3.853.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{6}\int \frac{x^{12}}{(bx^6+a)\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \ 99$$

$$\frac{1}{6}\int \left(\frac{a^2}{b^2(bx^6+a)\sqrt{dx^6+c}}+\frac{\sqrt{dx^6+c}}{bd}+\frac{-bc-ad}{b^2d\sqrt{dx^6+c}}\right)dx^6$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{6}\left(-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^6}(ad+bc)}{b^2d^2}+\frac{2\left(c+dx^6\right)^{3/2}}{3bd^2}\right)$$

input
```
Int[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
((-2*(b*c + a*d)*Sqrt[c + d*x^6])/(b^2*d^2) + (2*(c + d*x^6)^(3/2))/(3*b*d
^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(b^(5/2)*
Sqrt[b*c - a*d]))/6
```

3.853.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.853.4 Maple [A] (verified)

Time = 7.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$\frac{a^{2} \arctan\left(\frac{b \sqrt{d\,x^{6}+c}}{\sqrt{\left(a d -b c \right) b}}\right) d^{2}-\left(\left(-\frac{b\,x^{6}}{3}+a \right) d +\frac{2 b c}{3}\right) \sqrt{d\,x^{6}+c}\, \sqrt{\left(a d -b c \right) b}}{3 \sqrt{\left(a d -b c \right) b}\, b^{2} d^{2}}$	91

input

```
int(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/3*(a^2*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^(1/2))*d^2-((-1/3*b*x^6+a)
*d+2/3*b*c)*(d*x^6+c)^(1/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b^2/d
^2
```

3.853.5 Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.77

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \left[\frac{3\,\sqrt{b^2c-abd}a^2d^2\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,((b^3cd-ab^2d^2)x^6-2\,b^3c^2-ab^2cd+3\,a^2bd^2)}{18\,(b^4cd^2-ab^3d^3)}\right.$$

input

```
integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d
*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^6
 - 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c))/(b^4*c*d^2 - a*b^
3*d^3), 1/9*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^6 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^6 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^6 - 2*b^3*c^2 -
a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

3.853.6 Sympy [F]

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**17/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**17/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.853.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.853.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{a^2\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^2} + \frac{(dx^6+c)^{\frac{3}{2}}b^2d^4 - 3\sqrt{dx^6+c}b^2cd^4 - 3\sqrt{dx^6+c}abd^5}{9b^3d^6}$$

input
```
integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*a^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*b^2) + 1/9*((d*x^6 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^6 + c)*b^2*c*d^4 - 3
*sqrt(d*x^6 + c)*a*b*d^5)/(b^3*d^6)
```

3.853.9 Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{(d\,x^6+c)^{3/2}}{9\,b\,d^2} - \left(\frac{2\,c}{3\,b\,d^2} + \frac{3\,a\,d^3-3\,b\,c\,d^2}{9\,b^2\,d^4}\right)\sqrt{d\,x^6+c} + \frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{3\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^17/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
(c + d*x^6)^(3/2)/(9*b*d^2) - ((2*c)/(3*b*d^2) + (3*a*d^3 - 3*b*c*d^2)/(9*
b^2*d^4))*(c + d*x^6)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d -
 b*c)^(1/2)))/(3*b^(5/2)*(a*d - b*c)^(1/2))
```

3.854 $\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

3.854.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{c+dx^6}}{3bd} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

output
```
1/3*a*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)
^(1/2)+1/3*(d*x^6+c)^(1/2)/b/d
```

3.854.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{1}{3}\left(\frac{\sqrt{c+dx^6}}{bd} - \frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}\right)$$

input
```
Integrate[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(Sqrt[c + d*x^6]/(b*d) - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) +
 a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d]))/3
```

3.854.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6}\int \frac{x^6}{(bx^6+a)\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{6}\left(\frac{2\sqrt{c+dx^6}}{bd}-\frac{a\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^6}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(\frac{2\sqrt{c+dx^6}}{bd}-\frac{2a\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{bd}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}\left(\frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^6}}{bd}\right)$$

input
```
Int[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
((2*Sqrt[c + d*x^6])/(b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b
*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/6
```

3.854.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.854.4 Maple [A] (verified)

Time = 5.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

method	result	size
pseudoelliptic	$\frac{-a\arctan\left(\frac{b\sqrt{d\,x^{6}+c}}{\sqrt{(ad-bc)b}}\right)d+\sqrt{d\,x^{6}+c}\,\sqrt{(ad-bc)b}}{3bd\sqrt{(ad-bc)b}}$	72

input
```
int(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-a*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^(1/2))*d+(d*x^6+c)^(1/2)*((
a*d-b*c)*b)^(1/2))/b/d/((a*d-b*c)*b)^(1/2)
```

3.854.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.77

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \left[\frac{\sqrt{b^2c-abd}\,ad\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}(b^2c-abd)}{6\,(b^3cd-ab^2d^2)},\ -\frac{\sqrt{-b^2c+abd}\,ad\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-b^2c+abd}}{bdx^6+bc}\right)-\sqrt{dx^6+c}(b^2c-abd)}{3\,(b^3cd-ab^2d^2)}\right]$$

input

```
integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output

```
[1/6*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 +
c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^2*c - a*b*d))/
(b^3*c*d - a*b^2*d^2), -1/3*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^6 +
c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) - sqrt(d*x^6 + c)*(b^2*c - a*b*d)
)/(b^3*c*d - a*b^2*d^2)]
```

3.854.6 Sympy [F]

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input

```
integrate(x**11/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output

```
Integral(x**11/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.854.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.854.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b} - \frac{\sqrt{dx^6+c}}{b}}{3\,d}$$

input
```
integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
-1/3*(a*d*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*
b*d)*b) - sqrt(d*x^6 + c)/b)/d
```

3.854.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{d\,x^6+c}}{3\,b\,d} - \frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{3\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^11/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
(c + d*x^6)^(1/2)/(3*b*d) - (a*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c
)^(1/2)))/(3*b^(3/2)*(a*d - b*c)^(1/2))
```

3.855 $\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

3.855.1 Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

output
```
-1/3*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^
(1/2)
```

3.855.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{3\sqrt{b}\sqrt{-bc+ad}}$$

input
```
Integrate[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]]/(3*Sqrt[b]*Sqrt[-(b*c
) + a*d])
```

3.855.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {946, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{6}\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{3d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

input

```
Int[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output

```
-1/3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c
- a*d])
```

3.855.3.1 Defintions of rubi rules used

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.855.4 Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d\,x^{6}+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}$	39

input
```
int(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

3.855.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.55

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \left[\frac{\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)}{6\,\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-b^2c+abd}}{bdx^6+bc}\right)}{3\,(b^2c-abd)}\right]$$

input
```
integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(
b*x^6 + a))/sqrt(b^2*c - a*b*d), 1/3*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^
6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c))/(b^2*c - a*b*d)]
```

3.855.6 Sympy [A] (verification not implemented)

Time = 8.63 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}} & \text{for } d\neq 0 \\ \begin{cases} \frac{x^6}{6a\sqrt{c}} & \text{for } b=0 \\ \tilde{\infty}x^6 & \text{for } \sqrt{c}=0 \\ \frac{\log\left(6a\sqrt{c}+6b\sqrt{c}x^6\right)}{6b\sqrt{c}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Piecewise((atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b*c
)/b)), Ne(d, 0)), (Piecewise((x**6/(6*a*sqrt(c)), Eq(b, 0)), (zoo*x**6, Eq
(sqrt(c), 0)), (log(6*a*sqrt(c) + 6*b*sqrt(c)*x**6)/(6*b*sqrt(c)), True)),
 True))
```

3.855.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.855.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}}$$

input
```
integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

3.855.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\operatorname{atan}\left(\frac{b\sqrt{d\,x^6+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{3\sqrt{a\,b\,d-b^2\,c}}$$

input
```
int(x^5/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
atan((b*(c + d*x^6)^(1/2))/(a*b*d - b^2*c)^(1/2))/(3*(a*b*d - b^2*c)^(1/2)
)
```

3.856 $\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$

3.856.1 Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}$$

output
```
-1/3*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a/c^(1/2)+1/3*arctanh(b^(1/2)*(d*x^6
+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

3.856.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=-\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a}$$

input
```
Integrate[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
-1/3*((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/Sqrt[
-(b*c) + a*d] + ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/Sqrt[c])/a
```

3.856.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6}\int \frac{1}{x^6(bx^6+a)\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{6}\left(\frac{\int \frac{1}{x^6\sqrt{dx^6+c}}dx^6}{a}-\frac{b\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^6}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(\frac{2\int \frac{1}{\frac{x^{12}}{d}-\frac{c}{d}}d\sqrt{dx^6+c}}{ad}-\frac{2b\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{ad}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{a\sqrt{c}}\right)$$

input
```
Int[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
((-2*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/6
```

3.856.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.856.4 Maple [A] (verified)

Time = 5.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

method	result	size
pseudoelliptic	$-\frac{b \arctan\left(\frac{b \sqrt{d\,x^{6}+c}}{\sqrt{(a d-b c) b}}\right) \sqrt{c}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}}{\sqrt{c}}\right) \sqrt{(a d-b c) b}}{3 a \sqrt{(a d-b c) b}\, \sqrt{c}}$	78

input
```
int(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^(1/2))*c^(1/2)+arctanh((d*x
^6+c)^(1/2)/c^(1/2))*((a*d-b*c)*b)^(1/2))/a/((a*d-b*c)*b)^(1/2)/c^(1/2)
```

3.856.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.07

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+\sqrt{c}\log\left(\frac{dx^6-2\,\sqrt{dx^6+c}\sqrt{c}+2\,c}{x^6}\right)}{6\,ac}, \frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\right.}{}\right.$$

input
```
integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(c*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)
*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + sqrt(c)*log((d*x^6 - 2*sq
rt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), 1/6*(2*c*sqrt(-b/(b*c - a*d))*ar
ctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) +
sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), 1/6*(c*
sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*(b*c -
a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^6 + c)
*sqrt(-c)/c))/(a*c), 1/3*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(
b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) + sqrt(-c)*arctan(sqrt(d*
x^6 + c)*sqrt(-c)/c))/(a*c)]
```

3.856.6 Sympy [A] (verification not implemented)

Time = 6.99 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \begin{cases} \frac{2\left(-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{6a\sqrt{\frac{ad-bc}{b}}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{-c}}\right)}{6a\sqrt{-c}}\right)}{d} & \text{for}\ d\neq 0 \\ \frac{\operatorname{atan}\left(\frac{2\left(\frac{a}{2b}+x^6\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{3b\sqrt{c}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Piecewise((2*(-d*atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(6*a*sqrt((a*d
 - b*c)/b)) + d*atan(sqrt(c + d*x**6)/sqrt(-c))/(6*a*sqrt(-c)))/d, Ne(d, 0
)), (atan(2*(a/(2*b) + x**6)/sqrt(-a**2/b**2))/(3*b*sqrt(c)*sqrt(-a**2/b**
2)), True))
```

3.856.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x}\,dx$$

input
```
integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x), x)
```

3.856.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=-\frac{b\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a}+\frac{\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
-1/3*b*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a) + 1/3*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a*sqrt(-c))
```

3.856.9 Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 652, normalized size of antiderivative = 7.67

$$\int \frac{1}{x\,(a+bx^6)\,\sqrt{c+dx^6}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^6+c}}{\sqrt{c}}\right)}{3\,a\,\sqrt{c}}$$

$$-\frac{\operatorname{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{2\,a^2\,b^2\,d^3}{9}-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^6+c}\,\sqrt{b^2\,c-a\,b\,d}}{36\,\left(a^2\,d-a\,b\,c\right)}\right)}{6\,\left(a^2\,d-a\,b\,c\right)}\right)\,1{}\mathrm{i}}{a^2\,d-a\,b\,c}+\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}\right.}{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{2\,a^2\,b^2\,d^3}{9}-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^6+c}\,\sqrt{b^2\,c-a\,b\,d}}{36\,\left(a^2\,d-a\,b\,c\right)}\right)}{6\,\left(a^2\,d-a\,b\,c\right)}\right)}{a^2\,d-a\,b\,c}-\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}\right.}\right.}{3\,\left(a^2\,d-a\,b\,c\right)}$$

input
```
int(1/(x*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
- atanh((c + d*x^6)^(1/2)/c^(1/2))/(3*a*c^(1/2)) - (atan((((b^2*c - a*b*d)
^(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/2))/27 - ((b^2*c - a*b*d)^(1/2)*((2*a^2*
b^2*d^3)/9 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c
- a*b*d)^(1/2))/(36*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c)))*1i)/(a^2*d - a
*b*c) + ((b^2*c - a*b*d)^(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/2))/27 + ((b^2*c
 - a*b*d)^(1/2)*((2*a^2*b^2*d^3)/9 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(
c + d*x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(36*(a^2*d - a*b*c))))/(6*(a^2*d -
 a*b*c)))*1i)/(a^2*d - a*b*c))/(((b^2*c - a*b*d)^(1/2)*((2*b^3*d^2*(c + d*
x^6)^(1/2))/27 - ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^2*d^3)/9 - ((8*a^3*b^2*d
^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(36*(a^2*d
 - a*b*c))))/(6*(a^2*d - a*b*c))))/(a^2*d - a*b*c) - ((b^2*c - a*b*d)^(1/2
)*((2*b^3*d^2*(c + d*x^6)^(1/2))/27 + ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^2*d
^3)/9 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a*b
*d)^(1/2))/(36*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c))))/(a^2*d - a*b*c))*
(b^2*c - a*b*d)^(1/2)*1i)/(3*(a^2*d - a*b*c))
```

3.857 $\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$

3.857.1 Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{c+dx^6}}{6acx^6}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}}-\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}$$

output
```
1/6*(a*d+2*b*c)*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-1/3*b^(3/2)*a
rctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/6*
(d*x^6+c)^(1/2)/a/c/x^6
```

3.857.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\frac{-\frac{a\sqrt{c+dx^6}}{cx^6}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{c^{3/2}}}{6a^2}$$

input
```
Integrate[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-((a*Sqrt[c + d*x^6])/(c*x^6)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
6])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[
c + d*x^6]/Sqrt[c]])/c^(3/2))/(6*a^2)
```

3.857.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+b x^6\right) \sqrt{c+d x^6}} \, dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{6} \int \frac{1}{x^{12} \left(b x^6+a\right) \sqrt{d x^6+c}}dx^6$$

$$\Big\downarrow \ 114$$

$$\frac{1}{6} \left(-\frac{\int \frac{b d x^6+2 b c+a d}{2 x^6 \left(b x^6+a\right) \sqrt{d x^6+c}}dx^6}{a c}-\frac{\sqrt{c+d x^6}}{a c x^6}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{6} \left(-\frac{\int \frac{b d x^6+2 b c+a d}{x^6 \left(b x^6+a\right) \sqrt{d x^6+c}}dx^6}{2 a c}-\frac{\sqrt{c+d x^6}}{a c x^6}\right)$$

$$\Big\downarrow \ 174$$

$$\frac{1}{6} \left(-\frac{\frac{(a d+2 b c) \int \frac{1}{x^6 \sqrt{d x^6+c}}dx^6}{a}-\frac{2 b^2 c \int \frac{1}{\left(b x^6+a\right) \sqrt{d x^6+c}}dx^6}{a}}{2 a c}-\frac{\sqrt{c+d x^6}}{a c x^6}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{6} \left(-\frac{\frac{2 (a d+2 b c) \int \frac{1}{\frac{x^{12}}{d}-\frac{c}{d}}d\sqrt{d x^6+c}}{a d}-\frac{4 b^2 c \int \frac{1}{\frac{b x^{12}}{d}+a-\frac{b c}{d}}d\sqrt{d x^6+c}}{a d}}{2 a c}-\frac{\sqrt{c+d x^6}}{a c x^6}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{6}\left(-\frac{\frac{4b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2ac}-\frac{\sqrt{c+dx^6}}{acx^6}\right)$$

input
```
Int[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-(Sqrt[c + d*x^6]/(a*c*x^6)) - ((-2*(2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]
/Sqrt[c]])/(a*Sqrt[c]) + (4*b^(3/2)*c*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sq
rt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*c))/6
```

3.857.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.857.4 Maple [A] (verified)

Time = 5.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{\frac{2 b^{2} \arctan\left(\frac{b \sqrt{d \,x^{6}+c}}{\sqrt{\left(a d -b c \right) b}}\right)}{\sqrt{\left(a d -b c \right) b}}-\frac{a \sqrt{d \,x^{6}+c}}{c \,x^{6}}+\frac{\left(a d +2 b c \right) \operatorname{arctanh}\left(\frac{\sqrt{d \,x^{6}+c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{6 a^{2}}$	92

input
```
int(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/a^2*(2*b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^
(1/2))-a/c*(d*x^6+c)^(1/2)/x^6+(a*d+2*b*c)/c^(3/2)*arctanh((d*x^6+c)^(1/2)
/c^(1/2)))
```

3.857.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 565, normalized size of antiderivative = 4.83

$$\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

$$= \left[\frac{2\,bc^2x^6\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+(2\,bc+ad)\sqrt{c}x^6\log\left(\frac{dx^6+2\,\sqrt{dx^6+c}\sqrt{c}+2\,c}{x^6}\right)-2\,\sqrt{}}{12\,a^2c^2x^6}\right.$$

$$-\frac{4\,bc^2x^6\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^6+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^6+bc}\right)-(2\,bc+ad)\sqrt{c}x^6\log\left(\frac{dx^6+2\,\sqrt{dx^6+c}\sqrt{c}+2\,c}{x^6}\right)+2\,\sqrt{dx^6}}{12\,a^2c^2x^6}$$

$$-\frac{2\,bc^2x^6\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^6+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^6+bc}\right)+(2\,bc+ad)\sqrt{-c}x^6\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-c}}{c}\right)+\sqrt{dx^6+}}{6\,a^2c^2x^6}$$

input
```
integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(2*b*c^2*x^6*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt
(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + (2*b*c + a*d)*
sqrt(c)*x^6*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*sqrt(d*
x^6 + c)*a*c)/(a^2*c^2*x^6), -1/12*(4*b*c^2*x^6*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) - (2*
b*c + a*d)*sqrt(c)*x^6*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6)
+ 2*sqrt(d*x^6 + c)*a*c)/(a^2*c^2*x^6), 1/6*(b*c^2*x^6*sqrt(b/(b*c - a*d))
*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c -
a*d)))/(b*x^6 + a)) - (2*b*c + a*d)*sqrt(-c)*x^6*arctan(sqrt(d*x^6 + c)*sq
rt(-c)/c) - sqrt(d*x^6 + c)*a*c)/(a^2*c^2*x^6), -1/6*(2*b*c^2*x^6*sqrt(-b/
(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d
*x^6 + b*c)) + (2*b*c + a*d)*sqrt(-c)*x^6*arctan(sqrt(d*x^6 + c)*sqrt(-c)/
c) + sqrt(d*x^6 + c)*a*c)/(a^2*c^2*x^6)]
```

3.857.6 Sympy [F]

$$\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**7/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**7*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.857.7 Maxima [F]

$$\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^7), x)
```

3.857.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$
$$=\frac{b^2\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^2}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{6\,a^2\sqrt{-c}c}-\frac{\sqrt{dx^6+c}}{6\,acx^6}$$

input
```
integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*b^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*a^2) - 1/6*(2*b*c + a*d)*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a^2*sqrt(-c)
*c) - 1/6*sqrt(d*x^6 + c)/(a*c*x^6)
```

3.857.9 Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.38

$$\int \frac{1}{x^7 (a+bx^6) \sqrt{c+dx^6}}\,dx$$
$$= \frac{\ln\left(\sqrt{d\,x^6+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{6\,a^3\,d-6\,a^2\,b\,c}$$
$$-\frac{\ln\left(\sqrt{d\,x^6+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{6\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{\sqrt{d\,x^6+c}}{6\,a\,c\,x^6}$$
$$-\frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^6+c}\,1\mathrm{i}}{18\,\sqrt{c^3}\,\left(\frac{b^4\,d^4}{18\,c}+\frac{5\,a\,b^3\,d^5}{108\,c^2}+\frac{a^2\,b^2\,d^6}{108\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^6+c}\,1\mathrm{i}}{108\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{108\,a}+\frac{b^2\,d^6}{108\,c}+\frac{b^4\,c\,d^4}{18\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^6+c}\,5\mathrm{i}}{108\,\sqrt{c^3}\,\left(\frac{b^4\,d^4}{18\,a}+\frac{5\,b^3\,d^5}{108\,c}+\frac{a\,b^2\,d^6}{108\,c^2}\right)}\right)\,\left(a\,d+2\,b\,c\right)}{6\,a^2\,\sqrt{c^3}}$$

input
```
int(1/(x^7*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
(log((c + d*x^6)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2
*a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(6*a^3*d - 6*a^2*b*c) - (log((c + d*x
^6)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(
b^4*c - a*b^3*d)^(1/2))/(6*(a^3*d - a^2*b*c)) - (c + d*x^6)^(1/2)/(6*a*c*x
^6) - (atan((b^4*d^4*(c + d*x^6)^(1/2)*1i)/(18*(c^3)^(1/2)*((b^4*d^4)/(18*
c) + (5*a*b^3*d^5)/(108*c^2) + (a^2*b^2*d^6)/(108*c^3))) + (b^2*d^6*(c + d
*x^6)^(1/2)*1i)/(108*(c^3)^(1/2)*((5*b^3*d^5)/(108*a) + (b^2*d^6)/(108*c)
+ (b^4*c*d^4)/(18*a^2))) + (b^3*d^5*(c + d*x^6)^(1/2)*5i)/(108*(c^3)^(1/2)
*((b^4*d^4)/(18*a) + (5*b^3*d^5)/(108*c) + (a*b^2*d^6)/(108*c^2))))*(a*d +
 2*b*c)*1i)/(6*a^2*(c^3)^(1/2))
```

3.858 $\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

3.858.1 Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{x^3\sqrt{c+dx^6}}{6bd}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}}-\frac{(bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}}$$

output
```
-1/6*(2*a*d+b*c)*arctanh(x^3*d^(1/2)/(d*x^6+c)^(1/2))/b^2/d^(3/2)+1/3*a^(3
/2)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/b^2/(-a*d+b*c)^(1
/2)+1/6*x^3*(d*x^6+c)^(1/2)/b/d
```

3.858.2 Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{\frac{bx^3\sqrt{c+dx^6}}{d}+\frac{2a^{3/2}\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}-\frac{(bc+2ad)\log\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{d^{3/2}}}{6b^2}$$

input
```
Integrate[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
((b*x^3*Sqrt[c + d*x^6])/d + (2*a^(3/2)*ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]
*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d] - ((b
*c + 2*a*d)*Log[Sqrt[d]*x^3 + Sqrt[c + d*x^6]])/d^(3/2))/(6*b^2)
```

3.858.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 381, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{3}\int \frac{x^{12}}{(bx^6+a)\sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \text{381}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{c+dx^6}}{2bd}-\frac{\int \frac{(bc+2ad)x^6+ac}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{2bd}\right)$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{c+dx^6}}{2bd}-\frac{\frac{(2ad+bc)\int \frac{1}{\sqrt{dx^6+c}}dx^3}{b}-\frac{2a^2d\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{b}}{2bd}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{c+dx^6}}{2bd}-\frac{\frac{(2ad+bc)\int \frac{1}{1-dx^6}d\frac{x^3}{\sqrt{dx^6+c}}}{b}-\frac{2a^2d\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{b}}{2bd}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{c+dx^6}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}-\frac{2a^2d\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{b}}{2bd}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{c+dx^6}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}-\frac{2a^2d\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{b}}{2bd}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{c+dx^6}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}-\frac{2a^{3/2}d\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{b\sqrt{bc-ad}}}{2bd}\right)$$

input

```
Int[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output

```
((x^3*Sqrt[c + d*x^6])/(2*b*d) - ((-2*a^(3/2)*d*ArcTan[(Sqrt[b*c - a*d]*x^
3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(b*Sqrt[b*c - a*d]) + ((b*c + 2*a*d)*ArcTan
h[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]])/(b*Sqrt[d]))/(2*b*d))/3
```

3.858.3.1 Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.858.4 Maple [A] (verified)

Time = 9.61 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.24

method	result
pseudoelliptic	$\frac{b\sqrt{(ad-bc)a}\,\sqrt{d\,x^{6}+c}\,x^{3}\sqrt{d}+2a^{2}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}\,a}{x^{3}\sqrt{(ad-bc)a}}\right)d^{\frac{3}{2}}-2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}}{x^{3}\sqrt{d}}\right)ad\sqrt{(ad-bc)a}-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}}{x^{3}\sqrt{d}}\right)bc}{6b^{2}\sqrt{(ad-bc)a}\,d^{\frac{3}{2}}}$

input
```
int(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(b*((a*d-b*c)*a)^(1/2)*(d*x^6+c)^(1/2)*x^3*d^(1/2)+2*a^2*arctanh((d*x^
6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2))*d^(3/2)-2*arctanh((d*x^6+c)^(1/2)/x^
3/d^(1/2))*a*d*((a*d-b*c)*a)^(1/2)-arctanh((d*x^6+c)^(1/2)/x^3/d^(1/2))*b*
c*((a*d-b*c)*a)^(1/2))/b^2/((a*d-b*c)*a)^(1/2)/d^(3/2)
```

3.858.5 Fricas [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 739, normalized size of antiderivative = 6.01

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

$$= \Bigg[\frac{2\sqrt{dx^6+c}bdx^3 + ad^2\sqrt{-\frac{a}{bc-ad}}\log\Bigg(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2+4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^9-(abc^2}{b^2x^{12}+2\,abx^6+a^2}}{12\,b^2d^2}$$

input
```
integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^
2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2
+ 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt
(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + (b*c + 2
*a*d)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2)
, 1/12*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c
^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2
 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqr
t(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*(b*c
+ 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2), 1/12*(2
*sqrt(d*x^6 + c)*b*d*x^3 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c -
 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)
) + (b*c + 2*a*d)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c
))/(b^2*d^2), 1/6*(sqrt(d*x^6 + c)*b*d*x^3 - a*d^2*sqrt(a/(b*c - a*d))*arc
tan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*
d*x^9 + a*c*x^3)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6
+ c)))/(b^2*d^2)]
```

3.858.6 Sympy [F]

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**14/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**14/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.858.7 Maxima [F]

$$\int \frac{x^{14}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{x^{14}}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input

```
integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output

```
integrate(x^14/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.858.8 Giac [B] (verification not implemented)

Error detected during grading. Assigning place holder grade for now.

Time = 0.43 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.93

$$\begin{aligned}
&\int \frac{x^{14}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=Recursiveassumptionc\\
&\geq \frac{\sqrt{dx^6+c}x^3}{6\,bd}-\frac{a^2\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{3\sqrt{abc-a^2d}b^2\mathrm{sgn}\left(x\right)}\\
&\quad+\frac{\left(2\,a^2\sqrt{-d}d\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-\sqrt{abc-a^2d}bc\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)-2\sqrt{abc-a^2d}ad\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)\right)\mathrm{sgn}(x)}{6\sqrt{abc-a^2d}b^2\sqrt{-d}d}\\
&\quad+\frac{(bc+2\,ad)\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{6\,b^2\sqrt{-d}d\mathrm{sgn}\left(x\right)}-\frac{dignored}{t_{nostep}^6}
\end{aligned}$$

input

```
integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output

```
Recursive*assumption*c >= 1/6*sqrt(d*x^6 + c)*x^3/(b*d) - 1/3*a^2*arctan(a
*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)*b^2*sgn(x)) + 1
/6*(2*a^2*sqrt(-d)*d*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - sqrt(a*b*c -
a^2*d)*b*c*arctan(sqrt(d)/sqrt(-d)) - 2*sqrt(a*b*c - a^2*d)*a*d*arctan(sqr
t(d)/sqrt(-d)))*sgn(x)/(sqrt(a*b*c - a^2*d)*b^2*sqrt(-d)*d) + 1/6*(b*c + 2
*a*d)*arctan(sqrt(d + c/x^6)/sqrt(-d))/(b^2*sqrt(-d)*d*sgn(x)) - d*ignored
/t_nostep^6
```

3.858.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^{14}}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^14/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^14/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.859 $\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

3.859.1 Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}}$$

output

```
1/3*arctanh(x^3*d^(1/2)/(d*x^6+c)^(1/2))/b/d^(1/2)-1/3*arctan(x^3*(-a*d+b*
c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))*a^(1/2)/b/(-a*d+b*c)^(1/2)
```

3.859.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{-\frac{\sqrt{a}\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{\log\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{d}}}{3b}$$

input

```
Integrate[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output

```
(-((Sqrt[a]*ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sq
rt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d]) + Log[Sqrt[d]*x^3 + Sqrt[c + d*x
^6]]/Sqrt[d])/(3*b)
```

3.859.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {965, 385, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^6+a)\sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \text{ 385}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{\sqrt{dx^6+c}}dx^3}{b}-\frac{a\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{b}\right)$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{1-dx^6}d\frac{x^3}{\sqrt{dx^6+c}}}{b}-\frac{a\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{b}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}-\frac{a\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{b}\right)$$

$$\Big\downarrow \text{ 291}$$

$$\frac{1}{3}\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}-\frac{a\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{b}\right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{3}\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}-\frac{\sqrt{a}\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{b\sqrt{bc-ad}}\right)$$

input

```
Int[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(b*Sq
rt[b*c - a*d])) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(b*Sqrt[d]))/3
```

3.859.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.859. $\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

3.859.4 Maple [A] (verified)

Time = 7.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$\frac{-a\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}\,a}{x^{3}\sqrt{(ad-bc)a}}\right)\sqrt{d}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}}{x^{3}\sqrt{d}}\right)\sqrt{(ad-bc)a}}{3b\sqrt{(ad-bc)a}\,\sqrt{d}}$	85

input
```
int(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-a*arctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2))*d^(1/2)+arctanh
((d*x^6+c)^(1/2)/x^3/d^(1/2))*((a*d-b*c)*a)^(1/2))/b/((a*d-b*c)*a)^(1/2)/d
^(1/2)
```

3.859.5 Fricas [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 632, normalized size of antiderivative = 6.95

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

$$= \left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2-4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^9-(abc^2-a^2cd)x^3)\sqrt{dx^6+c}\sqrt{-\frac{a}{bc-a}}}{b^2x^{12}+2\,abx^6+a^2}\right.}{12\,bd}\right.$$

input
```
integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^
2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d))
)/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 +
c)*sqrt(d)*x^3 - c))/(b*d), 1/12*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8
*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*(
(b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^
6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) - 4*sqrt(-d)*ar
ctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b*d), 1/6*(d*sqrt(a/(b*c - a*d))*arct
an(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d
*x^9 + a*c*x^3)) + sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 -
c))/(b*d), 1/6*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c
)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) - 2*sqrt(-d)*ar
ctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b*d)]
```

3.859.6 Sympy [F]

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**8/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**8/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.859.7 Maxima [F]

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\int \frac{x^8}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.859.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(71) = 142$.

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.71

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\left(a\sqrt{-d}\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)\right)\mathrm{sgn}(x)}{3\sqrt{abc-a^2d}b\sqrt{-d}} + \frac{a\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{3\sqrt{abc-a^2d}b\mathrm{sgn}\left(x\right)} - \frac{\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{3\,b\sqrt{-d}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
-1/3*(a*sqrt(-d)*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - sqrt(a*b*c - a^2*
d)*arctan(sqrt(d)/sqrt(-d)))*sgn(x)/(sqrt(a*b*c - a^2*d)*b*sqrt(-d)) + 1/3
*a*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)*b*sg
n(x)) - 1/3*arctan(sqrt(d + c/x^6)/sqrt(-d))/(b*sqrt(-d)*sgn(x))
```

3.859.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^8}{\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^8/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^8/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.860 $\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

3.860.1 Optimal result

Integrand size = 24, antiderivative size = 54

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{\arctan\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

output
```
1/3*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(1/2)/(-a*d+b*c
)^(1/2)
```

3.860.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

input
```
Integrate[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b
*c - a*d])]/(3*Sqrt[a]*Sqrt[b*c - a*d])
```

3.860.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {965, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}dx$$
$$\Big\downarrow \text{965}$$
$$\frac{1}{3}\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3$$
$$\Big\downarrow \text{291}$$
$$\frac{1}{3}\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}$$
$$\Big\downarrow \text{218}$$
$$\frac{\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

input
```
Int[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])]/(3*Sqrt[a]*Sqrt[b*
c - a*d])
```

3.860.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.860.4 Maple [A] (verified)

Time = 6.88 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}\,a}{x^{3}\sqrt{(ad-bc)a}}\right)}{3\sqrt{(ad-bc)a}}$	42

input
```
int(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)*a)^(1/2)*arctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2))
```

3.860.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(42) = 84.

Time = 0.73 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.54

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2-4\,((bc-2\,ad)x^9-acx^3)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)}{12\,(abc-a^2d)}, \arctan\right.$$

input
```
integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*
sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2))/(a*b*c
 - a^2*d), 1/6*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a
*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3))/sqrt(a*
b*c - a^2*d)]
```

3.860.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**2/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.860.7 Maxima [F]

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\int \frac{x^2}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.860.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{3\,\sqrt{abcd-a^2d^2}}$$

input
```
integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(d)*arctan(1/2*((sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

3.860.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\int \frac{x^2}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^2/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.861 $\int \frac{1}{x^4\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$

3.861.1 Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{1}{x^4\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{c+dx^6}}{3acx^3}-\frac{b\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}$$

output
```
-1/3*b*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(3/2)/(-a*d+
b*c)^(1/2)-1/3*(d*x^6+c)^(1/2)/a/c/x^3
```

3.861.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^4\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{c+dx^6}}{3acx^3}-\frac{b\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{3a^{3/2}\sqrt{bc-ad}}$$

input
```
Integrate[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
-1/3*Sqrt[c + d*x^6]/(a*c*x^3) - (b*ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3
 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(3*a^(3/2)*Sqrt[b*c - a*d
])
```

3.861.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {965, 382, 25, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^6) \sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{3}\int \frac{1}{x^6 (bx^6+a) \sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \text{ 382}$$

$$\frac{1}{3}\left(\frac{\int -\frac{bc}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{ac}-\frac{\sqrt{c+dx^6}}{acx^3}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}\left(-\frac{\int \frac{bc}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{ac}-\frac{\sqrt{c+dx^6}}{acx^3}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(-\frac{b\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{a}-\frac{\sqrt{c+dx^6}}{acx^3}\right)$$

$$\Big\downarrow \text{ 291}$$

$$\frac{1}{3}\left(-\frac{b\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{a}-\frac{\sqrt{c+dx^6}}{acx^3}\right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{3}\left(-\frac{b\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}}{acx^3}\right)$$

input
```
Int[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-(Sqrt[c + d*x^6]/(a*c*x^3)) - (b*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*S
qrt[c + d*x^6])])/(a^(3/2)*Sqrt[b*c - a*d]))/3
```

3.861.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.861. $\int \frac{1}{x^4(a+bx^6)\sqrt{c+dx^6}}\,dx$

3.861.4 Maple [A] (verified)

Time = 8.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

method	result	size
pseudoelliptic	$-\frac{cb\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}\,a}{x^{3}\sqrt{(ad-bc)a}}\right)x^{3}+\sqrt{d\,x^{6}+c}\,\sqrt{(ad-bc)a}}{3a\,x^{3}\sqrt{(ad-bc)a}\,c}$	80

input
```
int(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(c*b*arctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2))*x^3+(d*x^6+c)
^(1/2)*((a*d-b*c)*a)^(1/2))/a/x^3/((a*d-b*c)*a)^(1/2)/c
```

3.861.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(64) = 128$.

Time = 0.37 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.15

$$\int \frac{1}{x^4\,(a+bx^6)\,\sqrt{c+dx^6}}\,dx$$

$$= \left[-\frac{\sqrt{-abc+a^2d}bcx^3\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2+4\,((bc-2\,ad)x^9-acx^3)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)+4}{12\,(a^2bc^2-a^3cd)x^3}\right.$$

$$\left.-\frac{\sqrt{abc-a^2d}bcx^3\arctan\left(\frac{((bc-2\,ad)x^6-ac)\sqrt{dx^6+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^9+(abc^2-a^2cd)x^3)}\right)+2\,\sqrt{dx^6+c}(abc-a^2d)}{6\,(a^2bc^2-a^3cd)x^3}\right]$$

input
```
integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(sqrt(-a*b*c + a^2*d)*b*c*x^3*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 -
a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2
)) + 4*sqrt(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3), -1/6*
(sqrt(a*b*c - a^2*d)*b*c*x^3*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x
^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)
*x^3)) + 2*sqrt(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3)]
```

3.861.6 Sympy [F]

$$\int \frac{1}{x^4 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int \frac{1}{x^4 (a+bx^6) \sqrt{c+dx^6}} \, dx$$

input
```
integrate(1/x**4/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.861.7 Maxima [F]

$$\int \frac{1}{x^4 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^4), x)
```

3.861.8 Giac [F]

$$\int \frac{1}{x^4 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.861.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a+b x^6\right) \sqrt{c+d x^6}} \, dx=\int \frac{1}{x^4\,\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}} \,d x$$

input
```
int(1/(x^4*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.862 $\int \frac{1}{x^{10}(a+bx^6)\sqrt{c+dx^6}}\,dx$

3.862.1 Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{1}{x^{10}(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}+\frac{b^2\arctan\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}}$$

output

```
1/3*b^2*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(5/2)/(-a*d
+b*c)^(1/2)-1/9*(d*x^6+c)^(1/2)/a/c/x^9+1/9*(2*a*d+3*b*c)*(d*x^6+c)^(1/2)/
a^2/c^2/x^3
```

3.862.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^{10}(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{c+dx^6}(-ac+3bcx^6+2adx^6)}{9a^2c^2x^9}+\frac{b^2\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{3a^{5/2}\sqrt{bc-ad}}$$

input

```
Integrate[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output

```
(Sqrt[c + d*x^6]*(-(a*c) + 3*b*c*x^6 + 2*a*d*x^6))/(9*a^2*c^2*x^9) + (b^2*
ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b
*c - a*d])])/(3*a^(5/2)*Sqrt[b*c - a*d])
```

3.862.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {965, 382, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10} (a+bx^6) \sqrt{c+dx^6}}dx$$

$$\Big\downarrow \ 965$$

$$\frac{1}{3}\int \frac{1}{x^{12} (bx^6+a) \sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \ 382$$

$$\frac{1}{3}\left(\frac{\int -\frac{2bdx^6+3bc+2ad}{x^6(bx^6+a)\sqrt{dx^6+c}}dx^3}{3ac}-\frac{\sqrt{c+dx^6}}{3acx^9}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{3}\left(-\frac{\int \frac{2bdx^6+3bc+2ad}{x^6(bx^6+a)\sqrt{dx^6+c}}dx^3}{3ac}-\frac{\sqrt{c+dx^6}}{3acx^9}\right)$$

$$\Big\downarrow \ 445$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{3b^2c^2}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{ac}-\frac{\sqrt{c+dx^6}(2ad+3bc)}{acx^3}}{3ac}-\frac{\sqrt{c+dx^6}}{3acx^9}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(-\frac{-\frac{3b^2c\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{a}-\frac{\sqrt{c+dx^6}(2ad+3bc)}{acx^3}}{3ac}-\frac{\sqrt{c+dx^6}}{3acx^9}\right)$$

$$\Big\downarrow \ 291$$

$$\frac{1}{3}\left(-\frac{-\frac{3b^2c\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{a}-\frac{\sqrt{c+dx^6}(2ad+3bc)}{acx^3}}{3ac}-\frac{\sqrt{c+dx^6}}{3acx^9}\right)$$

$$\Big\downarrow \ 218$$

$$\frac{1}{3}\left(-\frac{-\frac{3b^2c\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}(2ad+3bc)}{acx^3}}{3ac}-\frac{\sqrt{c+dx^6}}{3acx^9}\right)$$

input
```
Int[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-1/3*Sqrt[c + d*x^6]/(a*c*x^9) - (-(((3*b*c + 2*a*d)*Sqrt[c + d*x^6])/(a*
c*x^3)) - (3*b^2*c*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])]
)/(a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/3
```

3.862.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.862.4 Maple [A] (verified)

Time = 10.95 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

method	result	size
pseudoelliptic	$-\frac{-3 b^{2} c^{2} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}\,a}{x^{3} \sqrt{\left(a d -b c \right) a}}\right) x^{9}+\left(\left(-3 b\,x^{6}+a \right) c -2 a d\,x^{6}\right) \sqrt{d\,x^{6}+c}\,\sqrt{\left(a d -b c \right) a}}{9 \sqrt{\left(a d -b c \right) a}\,a^{2} x^{9} c^{2}}$	103

input
```
int(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(-3*b^2*c^2*arctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2))*x^9+((
-3*b*x^6+a)*c-2*a*d*x^6)*(d*x^6+c)^(1/2)*((a*d-b*c)*a)^(1/2))/((a*d-b*c)*a
)^(1/2)/a^2/x^9/c^2
```

3.862.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.62

$$\int \frac{1}{x^{10} \left(a+bx^6\right) \sqrt{c+dx^6}}\,dx = \Bigg[-\frac{3\,\sqrt{-abc+a^{2}d}b^{2}c^{2}x^{9}\log\left(\frac{(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2})x^{12}-2\,(3\,abc^{2}-4\,a^{2}cd)x^{6}+a^{2}c^{2}-4\,((bc-2\,ad)x^{9}-acx^{3})\sqrt{dx^{6}+c}\sqrt{-abc+a^{2}d}}{b^{2}x^{12}+2\,abx^{6}+a^{2}}\right)}{36\,(a^{3}bc^{3}-a^{4}c^{2}d)x^{9}}$$

input
```
integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/36*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^9*log(((b^2*c^2 - 8*a*b*c*d + 8*a
^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*
x^9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6
 + a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*
c*d)*sqrt(d*x^6 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^9), 1/18*(3*sqrt(a*b*c -
a^2*d)*b^2*c^2*x^9*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sq
rt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2
*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*c*d)*sqrt(d*
x^6 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^9)]
```

3.862.6 Sympy [F]

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**10/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**10*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.862.7 Maxima [F]

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^{10}}\,dx$$

input
```
integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^10), x)
```

3.862.8 Giac [F]

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^{10}}\,dx$$

input
```
integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.862.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^{10}\left(b\,x^6+a\right)\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/(x^10*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^10*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.863 $\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

3.863.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{x^5\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{5}{6},1,\frac{1}{2},\frac{11}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}$$

output
```
1/5*x^5*AppellF1(5/6,1,1/2,11/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*
x^6+c)^(1/2)
```

3.863.2 Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{x^5\sqrt{\frac{c+dx^6}{c}}\,\text{AppellF1}\left(\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{5a\sqrt{c+dx^6}}$$

input
```
Integrate[x^4/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(x^5*Sqrt[(c + d*x^6)/c]*AppellF1[5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^
6)/a)])/(5*a*Sqrt[c + d*x^6])
```

3.863.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{x^4}{(bx^6+a)\sqrt{\frac{dx^6}{c}+1}}dx}{\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^5\sqrt{\frac{dx^6}{c}+1}\operatorname{AppellF1}\left(\frac{5}{6},1,\frac{1}{2},\frac{11}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}$$

input
```
Int[x^4/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(x^5*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6, 1, 1/2, 11/6, -((b*x^6)/a), -((d*x^
6)/c)])/(5*a*Sqrt[c + d*x^6])
```

3.863.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.863.4 Maple [F]

$$\int \frac{x^4}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}}dx$$

input
```
int(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

3.863.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Not
integrable (provided residues have no relations)
```

3.863.6 Sympy [F]

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**4/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**4/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.863.7 Maxima [F]

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.863.8 Giac [F]

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.863.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^4/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.864 $\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

3.864.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{x^4\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}$$

output
```
1/4*x^4*AppellF1(2/3,1,1/2,5/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*x
^6+c)^(1/2)
```

3.864.2 Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{x^4\sqrt{\frac{c+dx^6}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{4a\sqrt{c+dx^6}}$$

input
```
Integrate[x^3/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(x^4*Sqrt[(c + d*x^6)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6
)/a)])/(4*a*Sqrt[c + d*x^6])
```

3.864.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {965, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^6+a)\sqrt{dx^6+c}}dx^2$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{x^2}{(bx^6+a)\sqrt{\frac{dx^6}{c}+1}}dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}$$

input
```
Int[x^3/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(x^4*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3, 1, 1/2, 5/3, -((b*x^6)/a), -((d*x^6
)/c)])/(4*a*Sqrt[c + d*x^6])
```

3.864.3.1 Defintions of rubi rules used

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.864.4 Maple [F]

$$\int \frac{x^{3}}{\left(b \,x^{6}+a \right) \sqrt{d \,x^{6}+c}}d x$$

input
```
int(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

3.864.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\text{Timed out}$$

input
```
integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.864.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**3/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.864.7 Maxima [F]

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^3}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.864.8 Giac [F]

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^3}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.864.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\int \frac{x^3}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^3/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.865 $\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

3.865.1 Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{x^2\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}$$

output
```
1/2*x^2*AppellF1(1/3,1,1/2,4/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*x
^6+c)^(1/2)
```

3.865.2 Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{x^2\sqrt{\frac{c+dx^6}{c}}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{2a\sqrt{c+dx^6}}$$

input
```
Integrate[x/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(x^2*Sqrt[(c + d*x^6)/c]*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6
)/a)])/(2*a*Sqrt[c + d*x^6])
```

3.865.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {965, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^2$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{1}{(bx^6+a)\sqrt{\frac{dx^6}{c}+1}}dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x^2\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}$$

input
```
Int[x/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(x^2*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 1, 1/2, 4/3, -((b*x^6)/a), -((d*x^6
)/c)])/(2*a*Sqrt[c + d*x^6])
```

3.865.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.865.4 Maple [F]

$$\int \frac{x}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}dx$$

input
```
int(x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

3.865.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^6)\,\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.865.6 Sympy [F]

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.865.7 Maxima [F]

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\int \frac{x}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.865.8 Giac [F]

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\int \frac{x}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.865.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(x/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.866 $\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

3.866.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{x\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(\frac{1}{6},1,\frac{1}{2},\frac{7}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}$$

output
```
x*AppellF1(1/6,1,1/2,7/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*x^6+c)^
(1/2)
```

3.866.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 10.19 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx=$$

$$-\frac{7acx\operatorname{AppellF1}\left(\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{(a+bx^6)\sqrt{c+dx^6}\left(-7ac\operatorname{AppellF1}\left(\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+3x^6\left(2bc\operatorname{AppellF1}\left(\frac{7}{6},\frac{1}{2},2,\frac{13}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right.\right.\right.}$$

input
```
Integrate[1/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-7*a*c*x*AppellF1[1/6, 1/2, 1, 7/6, -((d*x^6)/c), -((b*x^6)/a)])/((a + b*
x^6)*Sqrt[c + d*x^6]*(-7*a*c*AppellF1[1/6, 1/2, 1, 7/6, -((d*x^6)/c), -((b
*x^6)/a)] + 3*x^6*(2*b*c*AppellF1[7/6, 1/2, 2, 13/6, -((d*x^6)/c), -((b*x^
6)/a)] + a*d*AppellF1[7/6, 3/2, 1, 13/6, -((d*x^6)/c), -((b*x^6)/a)])))
```

3.866.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{1}{(bx^6+a)\sqrt{\frac{dx^6}{c}+1}}dx}{\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{1}{6},1,\frac{1}{2},\frac{7}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}$$

input
```
Int[1/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(x*Sqrt[1 + (d*x^6)/c]*AppellF1[1/6, 1, 1/2, 7/6, -((b*x^6)/a), -((d*x^6)/
c)])/(a*Sqrt[c + d*x^6])
```

3.866.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.866.4 Maple [F]

$$\int \frac{1}{(b\,x^{6}+a)\,\sqrt{d\,x^{6}+c}}dx$$

input
```
int(1/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(1/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

3.866.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^6)\,\sqrt{c+dx^6}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Not
integrable (provided residues have no relations)
```

3.866.6 Sympy [F]

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.866.7 Maxima [F]

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.866.8 Giac [F]

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.866.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{1}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.867 $\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$

3.867.1 Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(-\frac{1}{6},1,\frac{1}{2},\frac{5}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}$$

output
```
-AppellF1(-1/6,1,1/2,5/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/x/(d*x^6+c
)^(1/2)
```

3.867.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(62) = 124.

Time = 10.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$
$$=\frac{-55a(c+dx^6)-11(bc-2ad)x^6\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+10bdx^{12}\sqrt{1+\frac{dx^6}{c}}\,\text{Appell}}{55a^2cx\sqrt{c+dx^6}}$$

input
```
Integrate[1/(x^2*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-55*a*(c + d*x^6) - 11*(b*c - 2*a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6
, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] + 10*b*d*x^12*Sqrt[1 + (d*x^6)
/c]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c), -((b*x^6)/a)])/(55*a^2*c*x*
Sqrt[c + d*x^6])
```

3.867.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^6) \sqrt{c+dx^6}} \, dx$$

$$\Big\downarrow \; 1013$$

$$\frac{\sqrt{\frac{dx^6}{c}+1} \int \frac{1}{x^2(bx^6+a)\sqrt{\frac{dx^6}{c}+1}}dx}{\sqrt{c+dx^6}}$$

$$\Big\downarrow \; 1012$$

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(-\frac{1}{6},1,\frac{1}{2},\frac{5}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}$$

input
```
Int[1/(x^2*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
-((Sqrt[1 + (d*x^6)/c]*AppellF1[-1/6, 1, 1/2, 5/6, -((b*x^6)/a), -((d*x^6)
/c)])/(a*x*Sqrt[c + d*x^6]))
```

3.867.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.867.4 Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{6}+a \right) \sqrt{d\,x^{6}+c}}dx$$

input
```
int(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

3.867.5 Fricas [F]

$$\int \frac{1}{x^2 \left(a+bx^6\right) \sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^6 + c)/(b*d*x^14 + (b*c + a*d)*x^8 + a*c*x^2), x)
```

3.867.6 Sympy [F]

$$\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**2/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.867.7 Maxima [F]

$$\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^2), x)
```

3.867.8 Giac [F]

$$\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^2), x)
```

3.867.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^2\,\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/(x^2*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.868 $\int \frac{1}{x^3(a+bx^6)\sqrt{c+dx^6}}\,dx$

3.868.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^3\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

output
```
-1/2*AppellF1(-1/3,1,1/2,2/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/x^2/(d
*x^6+c)^(1/2)
```

3.868.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(64) = 128.

Time = 10.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^3\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$
$$=\frac{-20a(c+dx^6)+5(-2bc+ad)x^6\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+2bdx^{12}\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF}}{40a^2cx^2\sqrt{c+dx^6}}$$

input
```
Integrate[1/(x^3*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-20*a*(c + d*x^6) + 5*(-2*b*c + a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3
, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + 2*b*d*x^12*Sqrt[1 + (d*x^6)/c
]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -((b*x^6)/a)])/(40*a^2*c*x^2*Sq
rt[c + d*x^6])
```

3.868.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {965, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+bx^6\right)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{1}{x^4\left(bx^6+a\right)\sqrt{dx^6+c}}dx^2$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{1}{x^4\left(bx^6+a\right)\sqrt{\frac{dx^6}{c}+1}}dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

input
```
Int[1/(x^3*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
-1/2*(Sqrt[1 + (d*x^6)/c]*AppellF1[-1/3, 1, 1/2, 2/3, -((b*x^6)/a), -((d*x
^6)/c)])/(a*x^2*Sqrt[c + d*x^6])
```

3.868.3.1 Defintions of rubi rules used

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.868.4 Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{6}+a\right) \sqrt{d\,x^{6}+c}}dx$$

input
```
int(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

3.868.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^6) \sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.868.6 Sympy [F]

$$\int \frac{1}{x^3\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^3\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**3/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.868.7 Maxima [F]

$$\int \frac{1}{x^3\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^3), x)
```

3.868.8 Giac [F]

$$\int \frac{1}{x^3\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^3), x)
```

3.868.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^3\,\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/(x^3*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.869 $\int \frac{1}{x^5(a+bx^6)\sqrt{c+dx^6}}\,dx$

3.869.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^5\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}$$

output
```
-1/4*AppellF1(-2/3,1,1/2,1/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/x^4/(d
*x^6+c)^(1/2)
```

3.869.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(64) = 128.

Time = 10.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^5\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$
$$=\frac{-8a(c+dx^6)-4(4bc+ad)x^6\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-bdx^{12}\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\bigg(\frac{4}{3}}{32a^2cx^4\sqrt{c+dx^6}}$$

input
```
Integrate[1/(x^5*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-8*a*(c + d*x^6) - 4*(4*b*c + a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3,
1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] - b*d*x^12*Sqrt[1 + (d*x^6)/c]*Ap
pellF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b*x^6)/a)])/(32*a^2*c*x^4*Sqrt[c
 + d*x^6])
```

3.869.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {965, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^6) \sqrt{c+dx^6}} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2} \int \frac{1}{x^6 (bx^6+a) \sqrt{dx^6+c}} dx^2$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1} \int \frac{1}{x^6 (bx^6+a) \sqrt{\frac{dx^6}{c}+1}} dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^6}{c}+1} \operatorname{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}$$

input
```
Int[1/(x^5*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
-1/4*(Sqrt[1 + (d*x^6)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -((b*x^6)/a), -((d*x
^6)/c)])/(a*x^4*Sqrt[c + d*x^6])
```

3.869.3.1 Defintions of rubi rules used

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.869.4 Maple [F]

$$\int \frac{1}{x^{5} \left(b\,x^{6}+a\right) \sqrt{d\,x^{6}+c}}dx$$

input
```
int(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

3.869.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^6) \sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.869.6 Sympy [F]

$$\int \frac{1}{x^5\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^5\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**5/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**5*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.869.7 Maxima [F]

$$\int \frac{1}{x^5\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^5), x)
```

3.869.8 Giac [F]

$$\int \frac{1}{x^5\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^5), x)
```

3.869.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^5\,\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}} \,dx$$

input
```
int(1/(x^5*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^5*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.870 $\int \frac{x^{17}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

3.870.1 Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^{17}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{\sqrt{c+dx^6}}{3b^2d}-\frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)}+\frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}}$$

output
```
1/6*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(
5/2)/(-a*d+b*c)^(3/2)+1/3*(d*x^6+c)^(1/2)/b^2/d-1/6*a^2*(d*x^6+c)^(1/2)/b^
2/(-a*d+b*c)/(b*x^6+a)
```

3.870.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \frac{x^{17}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{\frac{\sqrt{b}\sqrt{c+dx^6}\left(-3a^2d+2b^2cx^6+2ab\left(c-dx^6\right)\right)}{d(bc-ad)\left(a+bx^6\right)}+\frac{a(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{6b^{5/2}}$$

input
```
Integrate[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((Sqrt[b]*Sqrt[c + d*x^6]*(-3*a^2*d + 2*b^2*c*x^6 + 2*a*b*(c - d*x^6)))/(d
*(b*c - a*d)*(a + b*x^6)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*
x^6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(6*b^(5/2))
```

3.870.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 100, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6}\int \frac{x^{12}}{(bx^6+a)^2\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{6}\left(\frac{\int -\frac{a(2bc-ad)-2b(bc-ad)x^6}{2(bx^6+a)\sqrt{dx^6+c}}dx^6}{b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^6}}{b^2\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(-\frac{\int \frac{a(2bc-ad)-2b(bc-ad)x^6}{(bx^6+a)\sqrt{dx^6+c}}dx^6}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^6}}{b^2\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{6}\left(-\frac{a(4bc-3ad)\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^6-\frac{4\sqrt{c+dx^6}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^6}}{b^2\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(-\frac{\frac{2a(4bc-3ad)\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{d}-\frac{4\sqrt{c+dx^6}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^6}}{b^2\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}\left(-\frac{a^2\sqrt{c+dx^6}}{b^2\left(a+bx^6\right)(bc-ad)}-\frac{-\frac{2a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}-\frac{4\sqrt{c+dx^6}(bc-ad)}{d}}{2b^2(bc-ad)}\right)$$

input
```
Int[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(-((a^2*Sqrt[c + d*x^6])/(b^2*(b*c - a*d)*(a + b*x^6))) - ((-4*(b*c - a*d)
*Sqrt[c + d*x^6])/d - (2*a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6
])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a*d]))/(2*b^2*(b*c - a*d)))/6
```

3.870.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.870.4 Maple [A] (verified)

Time = 5.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$\frac{-\left(b\,x^{6}+a\right)d\left(ad-\frac{4bc}{3}\right)a\arctan\left(\frac{b\sqrt{d\,x^{6}+c}}{\sqrt{\left(ad-bc\right)b}}\right)+\sqrt{\left(ad-bc\right)b}\,\sqrt{d\,x^{6}+c}\,\left(-\frac{2b^{2}c\,x^{6}}{3}-\frac{2a\left(-d\,x^{6}+c\right)b}{3}+a^{2}d\right)}{2\sqrt{\left(ad-bc\right)b}\,d\,b^{2}\left(ad-bc\right)\left(b\,x^{6}+a\right)}$	133

input
```
int(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*d-b*c)*b)^(1/2)*(-(b*x^6+a)*d*(a*d-4/3*b*c)*a*arctan(b*(d*x^6+c)^(
1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(d*x^6+c)^(1/2)*(-2/3*b^2*c*
x^6-2/3*a*(-d*x^6+c)*b+a^2*d))/d/b^2/(a*d-b*c)/(b*x^6+a)
```

3.870.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(103) = 206.

Time = 0.64 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.86

$$\int \frac{x^{17}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$
$$= \Bigg[\frac{((4\,ab^2cd-3\,a^2bd^2)x^6+4\,a^2bcd-3\,a^3d^2)\sqrt{b^2c-abd}\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,(2\,(b^4c^2-2}{12\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+}$$
$$-\frac{((4\,ab^2cd-3\,a^2bd^2)x^6+4\,a^2bcd-3\,a^3d^2)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-b^2c+abd}}{bdx^6+bc}\right)-(2\,(b^4c^2-2\,ab^3cd}{6\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a^2b^4}$$

input
```
integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^
2*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a
*b*d))/(b*x^6 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a
*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^6 + c))/(a*b^5*c^2*d - 2*
a^2*b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^
6), -1/6*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt
(-b^2*c + a*b*d)*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*
c)) - (2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a*b^3*c^2 - 5*a^2*b
^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^6 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^
3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^6)]
```

3.870.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(x**17/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Timed out
```

3.870.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.870.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{dx^6+c}a^2d}{6\,(b^3c-ab^2d)((dx^6+c)b-bc+ad)} - \frac{(4\,abc-3\,a^2d)\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{6\,(b^3c-ab^2d)\sqrt{-b^2c+abd}} + \frac{\sqrt{dx^6+c}}{3\,b^2d}$$

input

```
integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output

```
-1/6*sqrt(d*x^6 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^6 + c)*b - b*c + a*d))
 - 1/6*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/
((b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/3*sqrt(d*x^6 + c)/(b^2*d)
```

3.870.9 Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\sqrt{d\,x^6+c}}{3\,b^2\,d} - \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^6+c}\,(3\,a\,d-4\,b\,c)}{(3\,a^2\,d-4\,a\,b\,c)\,\sqrt{a\,d-b\,c}}\right)\,(3\,a\,d-4\,b\,c)}{6\,b^{5/2}\,{(a\,d-b\,c)}^{3/2}} + \frac{a^2\,d\,\sqrt{d\,x^6+c}}{2\,(a\,d-b\,c)\,\left(3\,b^3\,(d\,x^6+c)-3\,b^3\,c+3\,a\,b^2\,d\right)}$$

input

```
int(x^17/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output

```
(c + d*x^6)^(1/2)/(3*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^6)^(1/2)*(3*a*d
- 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(6*b^(
5/2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^6)^(1/2))/(2*(a*d - b*c)*(3*b^3*
(c + d*x^6) - 3*b^3*c + 3*a*b^2*d))
```

3.871 $\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

3.871.1 Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{a\sqrt{c+dx^6}}{6b(bc-ad)\,(a+bx^6)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

output
```
-1/6*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2
)/(-a*d+b*c)^(3/2)+1/6*a*(d*x^6+c)^(1/2)/b/(-a*d+b*c)/(b*x^6+a)
```

3.871.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{\frac{a\sqrt{b}\sqrt{c+dx^6}}{(bc-ad)(a+bx^6)}-\frac{(2bc-ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{6b^{3/2}}$$

input
```
Integrate[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((a*Sqrt[b]*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) - ((2*b*c - a*d)*Ar
cTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/
(6*b^(3/2))
```

3.871.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6}\int \frac{x^6}{(bx^6+a)^2\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{6}\left(\frac{(2bc-ad)\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^6}{2b(bc-ad)}+\frac{a\sqrt{c+dx^6}}{b(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(\frac{(2bc-ad)\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{bd(bc-ad)}+\frac{a\sqrt{c+dx^6}}{b(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}\left(\frac{a\sqrt{c+dx^6}}{b(a+bx^6)(bc-ad)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{b^{3/2}(bc-ad)^{3/2}}\right)$$

input
```
Int[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((a*Sqrt[c + d*x^6])/(b*(b*c - a*d)*(a + b*x^6)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(b^(3/2)*(b*c - a*d)^(3/2)))/6
```

3.871.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.871.4 Maple [A] (verified)

Time = 5.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{-\frac{a\sqrt{d\,x^{6}+c}}{b\,x^{6}+a}+\frac{(ad-2bc)\arctan\left(\frac{b\sqrt{d\,x^{6}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{6(ad-bc)b}$	83

input
```
int(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/(a*d-b*c)/b*(-a*(d*x^6+c)^(1/2)/(b*x^6+a)+(a*d-2*b*c)/((a*d-b*c)*b)^(1
/2)*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

3.871.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.52

$$\int \frac{x^{11}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$
$$=\left[\frac{((2\,b^2c-abd)x^6+2\,abc-a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}(ab^2c-a^2bd)}{12\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^6)}\right.$$

input
```
integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(((2*b^2*c - a*b*d)*x^6 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((
b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)
) + 2*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^
3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^6), 1/6*(((2*b^2*c - a
*b*d)*x^6 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^6 + c)*s
qrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)
)/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*
b^3*d^2)*x^6)]
```

3.871.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\text{Timed out}$$

input
```
integrate(x**11/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Timed out
```

3.871.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.871.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{\frac{\sqrt{dx^6+c}\,ad^2}{(b^2c-abd)((dx^6+c)b-bc+ad)} + \frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{6\,d}$$

input
```
integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
1/6*(sqrt(d*x^6 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^6 + c)*b - b*c + a*d)) +
 (2*b*c*d - a*d^2)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c
- a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

3.871.9 Mupad [B] (verification not implemented)

Time = 9.80 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{x^{11}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)\,\left(a\,d-2\,b\,c\right)}{6\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}} - \frac{a\,d\,\sqrt{d\,x^6+c}}{2\,b\,\left(a\,d-b\,c\right)\,\left(3\,b\,\left(d\,x^6+c\right)+3\,a\,d-3\,b\,c\right)}$$

input
```
int(x^11/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
(atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(6*b^(
3/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^6)^(1/2))/(2*b*(a*d - b*c)*(3*b*(c
 + d*x^6) + 3*a*d - 3*b*c))
```

3.871. $\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

3.872 $\int \frac{x^5}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

3.872.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^5}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}(bc-ad)^{3/2}}$$

output
```
1/6*d*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b
^(1/2)-1/6*(d*x^6+c)^(1/2)/(-a*d+b*c)/(b*x^6+a)
```

3.872.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{1}{6}\left(-\frac{\sqrt{c+dx^6}}{(bc-ad)\left(a+bx^6\right)}+\frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

input
```
Integrate[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(-(Sqrt[c + d*x^6]/((b*c - a*d)*(a + b*x^6))) + (d*ArcTan[(Sqrt[b]*Sqrt[c
+ d*x^6])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/6
```

3.872.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {946, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{6}\int \frac{1}{(bx^6+a)^2\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{6}\left(-\frac{d\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^6}{2(bc-ad)}-\frac{\sqrt{c+dx^6}}{(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(-\frac{\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{bc-ad}-\frac{\sqrt{c+dx^6}}{(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}}{(a+bx^6)(bc-ad)}\right)$$

input
```
Int[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(-(Sqrt[c + d*x^6]/((b*c - a*d)*(a + b*x^6))) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^6])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2)))/6
```

3.872.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.872.4 Maple [A] (verified)

Time = 5.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

method	result	size
pseudoelliptic	$\frac{d\left(b\,x^{6}+a\right)\arctan\left(\frac{b\sqrt{d\,x^{6}+c}}{\sqrt{\left(ad-bc\right)b}}\right)+\sqrt{d\,x^{6}+c}\,\sqrt{\left(ad-bc\right)b}}{6\sqrt{\left(ad-bc\right)b}\,\left(ad-bc\right)\left(b\,x^{6}+a\right)}$	90

input
```
int(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(d*(b*x^6+a)*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(d*x^6+c)^(
1/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/(a*d-b*c)/(b*x^6+a)
```

3.872.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(71) = 142.

Time = 0.41 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.47

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$= \left[-\frac{(bdx^6+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}(b^2c-abd)}{12\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^6+ab^3c^2-2\,a^2b^2cd+a^3bd^2)},\right.$$
$$\left.-\frac{(bdx^6+ad)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-b^2c+abd}}{bdx^6+bc}\right)+\sqrt{dx^6+c}(b^2c-abd)}{6\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^6+ab^3c^2-2\,a^2b^2cd+a^3bd^2)}\right]$$

input
```
integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*((b*d*x^6 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d - 2
*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^
2*c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2*a
^2*b^2*c*d + a^3*b*d^2), -1/6*((b*d*x^6 + a*d)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(
b^2*c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2
*a^2*b^2*c*d + a^3*b*d^2)]
```

3.872.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**5/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.872.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.872.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{6\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^6+c}d}{6\left((dx^6+c)b-bc+ad\right)(bc-ad)}$$

input
```
integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
-1/6*d*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*(b*c - a*d)) - 1/6*sqrt(d*x^6 + c)*d/(((d*x^6 + c)*b - b*c + a*d)*(b*c -
 a*d))
```

3.872.9 Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{d\sqrt{d\,x^6+c}}{2\,(a\,d-b\,c)\,\left(3\,b\,\left(d\,x^6+c\right)+3\,a\,d-3\,b\,c\right)} + \frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{6\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

input
```
int(x^5/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
(d*(c + d*x^6)^(1/2))/(2*(a*d - b*c)*(3*b*(c + d*x^6) + 3*a*d - 3*b*c)) +
(d*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)^(1/2)))/(6*b^(1/2)*(a*d -
b*c)^(3/2))
```

3.872. $\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

3.873 $\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

3.873.1 Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc-ad)^{3/2}}$$

output
```
1/6*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/
2)/a^2/(-a*d+b*c)^(3/2)-1/3*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1
/6*b*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/(b*x^6+a)
```

3.873.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{-\frac{ab\sqrt{c+dx^6}}{(-bc+ad)(a+bx^6)} + \frac{\sqrt{b}(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{6a^2}$$

input
```
Integrate[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(-((a*b*Sqrt[c + d*x^6])/((-(b*c) + a*d)*(a + b*x^6))) + (Sqrt[b]*(2*b*c -
 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*
d)^(3/2) - (2*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/Sqrt[c])/(6*a^2)
```

3.873.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6}\int \frac{1}{x^6\left(bx^6+a\right)^2\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{6}\left(\frac{\int \frac{bdx^6+2bc-2ad}{2x^6\left(bx^6+a\right)\sqrt{dx^6+c}}dx^6}{a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{a\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{\int \frac{bdx^6+2(bc-ad)}{x^6\left(bx^6+a\right)\sqrt{dx^6+c}}dx^6}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{a\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{6}\left(\frac{\frac{2(bc-ad)\int \frac{1}{x^6\sqrt{dx^6+c}}dx^6}{a}-\frac{b(2bc-3ad)\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^6}{a}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{a\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(\frac{\frac{4(bc-ad)\int \frac{1}{\frac{x^{12}}{d}-\frac{c}{d}}d\sqrt{dx^6+c}}{ad}-\frac{2b(2bc-3ad)\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{ad}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{a\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}\left(\frac{\frac{2\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{4(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{a\left(a+bx^6\right)\left(bc-ad\right)}\right)$$

input
```
Int[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((b*Sqrt[c + d*x^6])/(a*(b*c - a*d)*(a + b*x^6)) + ((-4*(b*c - a*d)*ArcTan
h[Sqrt[c + d*x^6]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*(2*b*c - 3*a*d)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*(
b*c - a*d)))/6
```

3.873.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.873.4 Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{-2\left(b\,x^{6}+a\right)\left(bc-\frac{3ad}{2}\right)\sqrt{c}\,b\arctan\left(\frac{b\sqrt{d\,x^{6}+c}}{\sqrt{\left(ad-bc\right)b}}\right)+\left(2\left(ad-bc\right)\left(b\,x^{6}+a\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}}{\sqrt{c}}\right)+\sqrt{d\,x^{6}+c}\,\sqrt{c}\,ab\right)\sqrt{\left(ad-bc\right)b}}{6\sqrt{c}\,\sqrt{\left(ad-bc\right)b}\,a^{2}\left(ad-bc\right)\left(b\,x^{6}+a\right)}$

input
```
int(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-2*(b*x^6+a)*(b*c-3/2*a*d)*c^(1/2)*b*arctan(b*(d*x^6+c)^(1/2)/((a*d-
b*c)*b)^(1/2))+(2*(a*d-b*c)*(b*x^6+a)*arctanh((d*x^6+c)^(1/2)/c^(1/2))+(d*
x^6+c)^(1/2)*c^(1/2)*a*b)*((a*d-b*c)*b)^(1/2))/c^(1/2)/((a*d-b*c)*b)^(1/2)
/a^2/(a*d-b*c)/(b*x^6+a)
```

3.873.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 862, normalized size of antiderivative = 6.53

$$\int \frac{1}{x\left(a+bx^{6}\right)^{2}\sqrt{c+dx^{6}}}\,dx = \left[\frac{2\,\sqrt{dx^{6}+c}abc+\left(\left(2\,b^{2}c^{2}-3\,abcd\right)x^{6}+2\,abc^{2}-3\,a^{2}cd\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^{6}+2\,bc-ad+2\,\sqrt{dx^{6}+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^{6}+a}\right)-}{12\left(\left(a^{2}b^{2}c^{2}-a^{3}bcd\right)x^{6}+a^{3}bc^{2}-a^{4}cd\right)}\right.$$

input
```
integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(2*sqrt(d*x^6 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2
- 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6
 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*((b^2*c - a*b*d)*x
^6 + a*b*c - a^2*d)*sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/
x^6))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/6*(sqrt(d*x
^6 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt
(-b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/
(b*d*x^6 + b*c)) + ((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(c)*log((d*x^
6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6
+ a^3*b*c^2 - a^4*c*d), 1/12*(2*sqrt(d*x^6 + c)*a*b*c + 4*((b^2*c - a*b*d)
*x^6 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) + ((2*b^
2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b
*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/
(b*x^6 + a)))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/6*(
sqrt(d*x^6 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c
*d)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c
- a*d))/(b*d*x^6 + b*c)) + 2*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-c
)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3
*b*c^2 - a^4*c*d)]
```

3.873.6 Sympy [F]

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.873.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x}\,dx$$

input
```
integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x), x)
```

3.873.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{\sqrt{dx^6+c}bd}{6\,(abc-a^2d)((dx^6+c)b-bc+ad)} - \frac{(2\,b^2c-3\,abd)\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{6\,(a^2bc-a^3d)\sqrt{-b^2c+abd}} + \frac{\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(d*x^6 + c)*b*d/((a*b*c - a^2*d)*((d*x^6 + c)*b - b*c + a*d)) - 1/
6*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2
*b*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/3*arctan(sqrt(d*x^6 + c)/sqrt(-c))
/(a^2*sqrt(-c))
```

3.873.9 Mupad [B] (verification not implemented)

Time = 10.31 (sec) , antiderivative size = 3025, normalized size of antiderivative = 22.92

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
(atan((((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d
^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - ((((4*a^6*b^2*d^5)/3 - 2
*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c
*d) - ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(144*a^7
*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))
/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3
*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(1
2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b
*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^
2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d
^2 - 20*a*b^4*c*d^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + ((((4*a
^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^
2*c^2 - 2*a^4*b*c*d) + ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^
3)^(1/2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*
a^5*b^4*c^2*d^3))/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^
2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d -
b*c)^3)^(1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^
2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(12*(a^5*d^3 - a^2*b^3*c
^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))/(((a*b^3*d^4)/18 - (b^4*c*d^3)/27)
/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((((c + d*x^6)^(1/2)*(13*a^2*b...
```

3.873. $\int \frac{1}{x(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

3.874 $\int \frac{1}{x^7 \left(a+bx^6\right)^2 \sqrt{c+dx^6}} \, dx$

3.874.1 Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{1}{x^7 \left(a+bx^6\right)^2 \sqrt{c+dx^6}} \, dx = -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} + \frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}} - \frac{b^{3/2}(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^3(bc-ad)^{3/2}}$$

output
```
1/6*(a*d+4*b*c)*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/6*b^(3/2)*(
-5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+
b*c)^(3/2)-1/6*b*(-a*d+2*b*c)*(d*x^6+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^6+a)-1
/6*(d*x^6+c)^(1/2)/a/c/x^6/(b*x^6+a)
```

3.874.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^7 \left(a+bx^6\right)^2 \sqrt{c+dx^6}} \, dx = \frac{\frac{a\sqrt{c+dx^6}\left(-a^2d+2b^2cx^6+ab\left(c-dx^6\right)\right)}{c(-bc+ad)x^6\left(a+bx^6\right)} - \frac{b^{3/2}(4bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{c^{3/2}}}{6a^3}$$

input
```
Integrate[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((a*Sqrt[c + d*x^6]*(-(a^2*d) + 2*b^2*c*x^6 + a*b*(c - d*x^6)))/(c*(-(b*c)
 + a*d)*x^6*(a + b*x^6)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcT
anh[Sqrt[c + d*x^6]/Sqrt[c]])/c^(3/2))/(6*a^3)
```

3.874.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {948, 114, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+b x^6\right)^2 \sqrt{c+d x^6}} \, dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{6} \int \frac{1}{x^{12} \left(b x^6+a\right)^2 \sqrt{d x^6+c}}dx^6$$

$$\Big\downarrow \ 114$$

$$\frac{1}{6} \left(-\frac{\int \frac{3 b d x^6+4 b c+a d}{2 x^6 \left(b x^6+a\right)^2 \sqrt{d x^6+c}}dx^6}{a c}-\frac{\sqrt{c+d x^6}}{a c x^6 \left(a+b x^6\right)}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{6} \left(-\frac{\int \frac{3 b d x^6+4 b c+a d}{x^6 \left(b x^6+a\right)^2 \sqrt{d x^6+c}}dx^6}{2 a c}-\frac{\sqrt{c+d x^6}}{a c x^6 \left(a+b x^6\right)}\right)$$

$$\Big\downarrow \ 168$$

$$\frac{1}{6} \left(-\frac{\frac{\int \frac{b d (2 b c-a d) x^6+(b c-a d) (4 b c+a d)}{x^6 \left(b x^6+a\right) \sqrt{d x^6+c}}dx^6}{a (b c-a d)}+\frac{2 b \sqrt{c+d x^6} (2 b c-a d)}{a \left(a+b x^6\right) (b c-a d)}}{2 a c}-\frac{\sqrt{c+d x^6}}{a c x^6 \left(a+b x^6\right)}\right)$$

$$\Big\downarrow \ 174$$

$$\frac{1}{6}\left(-\frac{\frac{\frac{(bc-ad)(ad+4bc)\int \frac{1}{x^6\sqrt{dx^6+c}}dx^6}{a}-\frac{b^2c(4bc-5ad)\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^6}{a}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^6}(2bc-ad)}{a(a+bx^6)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^6}}{acx^6\left(a+bx^6\right)}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{6}\left(-\frac{\frac{\frac{2(bc-ad)(ad+4bc)\int \frac{1}{\frac{x^{12}}{d}-\frac{c}{d}}d\sqrt{dx^6+c}}{ad}-\frac{2b^2c(4bc-5ad)\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{ad}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^6}(2bc-ad)}{a(a+bx^6)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^6}}{acx^6\left(a+bx^6\right)}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{6}\left(-\frac{\frac{\frac{2b^{3/2}c(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)(ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{a\sqrt{c}}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^6}(2bc-ad)}{a(a+bx^6)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^6}}{acx^6\left(a+bx^6\right)}\right)$$

input
```
Int[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(-(Sqrt[c + d*x^6]/(a*c*x^6*(a + b*x^6))) - ((2*b*(2*b*c - a*d)*Sqrt[c + d
*x^6])/(a*(b*c - a*d)*(a + b*x^6)) + ((-2*(b*c - a*d)*(4*b*c + a*d)*ArcTan
h[Sqrt[c + d*x^6]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(3/2)*c*(4*b*c - 5*a*d)*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(a*(
b*c - a*d)))/(2*a*c))/6
```

3.874.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.874.4 Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{-4x^{6}\left(b\,x^{6}+a\right)\left(bc-\frac{5ad}{4}\right)b^{2}c^{\frac{5}{2}}\arctan\left(\frac{b\sqrt{d\,x^{6}+c}}{\sqrt{\left(ad-bc\right)b}}\right)+\sqrt{\left(ad-bc\right)b}\,\Big(c\,x^{6}\left(b\,x^{6}+a\right)\left(ad+4bc\right)\left(ad-bc\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}}{\sqrt{c}}\right)+\Big(-}{6\sqrt{\left(ad-bc\right)b}\,c^{\frac{5}{2}}a^{3}\left(ad-bc\right)\left(b\,x^{6}+a\right)x^{6}}$

input

```
int(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/6/((a*d-b*c)*b)^(1/2)/c^(5/2)*(-4*x^6*(b*x^6+a)*(b*c-5/4*a*d)*b^2*c^(5/2
)*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(c*x^6
*(b*x^6+a)*(a*d+4*b*c)*(a*d-b*c)*arctanh((d*x^6+c)^(1/2)/c^(1/2))+(-a*d*(b
*x^6+a)*c^(3/2)+b*(2*b*x^6+a)*c^(5/2))*(d*x^6+c)^(1/2)*a))/a^3/(a*d-b*c)/(
b*x^6+a)/x^6
```

3.874.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1236, normalized size of antiderivative = 6.68

$$\int \frac{1}{x^7 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\, dx = \text{Too large to display}$$

input

```
integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output

```
[1/12*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x
^6)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*
c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a
^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(c)*log((d
*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*((2*a*b^2*c^2 - a^2*b*c*d
)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)
*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/12*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d
)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(-b/(b*c - a*d))*arctan(-s
qrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) - ((4*b^3
*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^
2)*x^6)*sqrt(c)*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) + 2*((2
*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a^3*
b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/12*(2*((4*b
^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*
d^2)*x^6)*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*
b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(b/(b*c - a*d))*l
og((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*
d)))/(b*x^6 + a)) + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d
)*sqrt(d*x^6 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^
2*d)*x^6), -1/6*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a...
```

3.874.6 Sympy [F]

$$\int \frac{1}{x^7 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx=\int \frac{1}{x^7 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**7/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**7*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.874.7 Maxima [F]

$$\int \frac{1}{x^7 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^7), x)
```

3.874.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.39

$$\begin{aligned}\int \frac{1}{x^7 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx &= \frac{\left(4\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{6\left(a^3bc-a^4d\right)\sqrt{-b^2c+abd}} \\ &\quad -\frac{2\left(dx^6+c\right)^{\frac{3}{2}}b^2cd-2\sqrt{dx^6+c}b^2c^2d-\left(dx^6+c\right)^{\frac{3}{2}}abd^2+2\sqrt{dx^6+c}abcd^2-\sqrt{dx^6+c}a^2d^3}{6\left(a^2bc^2-a^3cd\right)\left(\left(dx^6+c\right)^2b-2\left(dx^6+c\right)bc+bc^2+\left(dx^6+c\right)ad-acd\right)} \\ &\quad -\frac{\left(4\,bc+ad\right)\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{6\,a^3\sqrt{-c}c}\end{aligned}$$

input
```
integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
1/6*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/6*(2*(d*x^6 + c)^(3/2)*b^2*c*d
 - 2*sqrt(d*x^6 + c)*b^2*c^2*d - (d*x^6 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^6
+ c)*a*b*c*d^2 - sqrt(d*x^6 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^6 +
 c)^2*b - 2*(d*x^6 + c)*b*c + b*c^2 + (d*x^6 + c)*a*d - a*c*d)) - 1/6*(4*b
*c + a*d)*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

3.874.9 Mupad [B] (verification not implemented)

Time = 11.63 (sec) , antiderivative size = 3860, normalized size of antiderivative = 20.86

$$\int \frac{1}{x^7 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \text{Too large to display}$$

input
```
int(1/(x^7*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
(((c + d*x^6)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
 a*c*d)) + (b*(c + d*x^6)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d))
)/((c + d*x^6)*(3*a*d - 6*b*c) + 3*b*(c + d*x^6)^2 + 3*b*c^2 - 3*a*c*d) +
(atan((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^6)^(1/2)*(a^
4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b
^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*
d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^
3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2
*d^2 - 2*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^6)^(1/2)*(5*
a*d - 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*
d^4 - 144*a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*
d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(12*(a^6*d
^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(12*(a^6*d^3 - a
^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/
2)*(5*a*d - 4*b*c)*(((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*
a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 +
a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c
)*((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^
8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) + ((-b^3*
(a*d - b*c)^3)^(1/2)*(c + d*x^6)^(1/2)*(5*a*d - 4*b*c)*(288*a^6*b^5*c^5...
```

3.875 $\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

3.875.1 Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)} - \frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

output
```
-1/6*(-2*a*d+3*b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))*a
^(1/2)/b^2/(-a*d+b*c)^(3/2)+1/3*arctanh(x^3*d^(1/2)/(d*x^6+c)^(1/2))/b^2/d
^(1/2)+1/6*a*x^3*(d*x^6+c)^(1/2)/b/(-a*d+b*c)/(b*x^6+a)
```

3.875.2 Mathematica [A] (verified)

Time = 2.99 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\frac{abx^3\sqrt{c+dx^6}}{(bc-ad)(a+bx^6)} + \frac{\sqrt{a}(-3bc+2ad)\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{d}}}{6b^2}$$

input
```
Integrate[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((a*b*x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + (Sqrt[a]*(-3*b*c +
2*a*d)*ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]
*Sqrt[b*c - a*d])])/(b*c - a*d)^(3/2) + (2*Log[Sqrt[d]*x^3 + Sqrt[c + d*x^
6]])/Sqrt[d])/(6*b^2)
```

3.875.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 372, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{3}\int \frac{x^{12}}{\left(bx^6+a\right)^2 \sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \text{372}$$

$$\frac{1}{3}\left(\frac{ax^3\sqrt{c+dx^6}}{2b\left(a+bx^6\right)(bc-ad)}-\frac{\int \frac{ac-2(bc-ad)x^6}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{3}\left(\frac{ax^3\sqrt{c+dx^6}}{2b\left(a+bx^6\right)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{b}-\frac{2(bc-ad)\int \frac{1}{\sqrt{dx^6+c}}dx^3}{b}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{3}\left(\frac{ax^3\sqrt{c+dx^6}}{2b\left(a+bx^6\right)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{b}-\frac{2(bc-ad)\int \frac{1}{1-dx^6}d\frac{x^3}{\sqrt{dx^6+c}}}{b}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{ax^3\sqrt{c+dx^6}}{2b\left(a+bx^6\right)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{b}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

$$\Big\downarrow 291$$

$$\frac{1}{3}\left(\frac{ax^3\sqrt{c+dx^6}}{2b\left(a+bx^6\right)\left(bc-ad\right)}-\frac{\frac{a(3bc-2ad)\int\frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{b}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

$$\Big\downarrow 218$$

$$\frac{1}{3}\left(\frac{ax^3\sqrt{c+dx^6}}{2b\left(a+bx^6\right)\left(bc-ad\right)}-\frac{\frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{b\sqrt{bc-ad}}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

input
```
Int[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((a*x^3*Sqrt[c + d*x^6])/(2*b*(b*c - a*d)*(a + b*x^6)) - ((Sqrt[a]*(3*b*c
- 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(b*Sqrt[
b*c - a*d]) - (2*(b*c - a*d)*ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]])/(b*Sq
rt[d]))/(2*b*(b*c - a*d)))/3
```

3.875.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.875.4 Maple [A] (verified)

Time = 10.58 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$-\frac{-\frac{a\left(-\frac{\sqrt{d x^{6}+c}\, b\, x^{3}}{b\, x^{6}+a}-\frac{(2ad-3bc)\operatorname{arctanh}\left(\frac{\sqrt{d x^{6}+c}\, a}{x^{3}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{ad-bc}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d x^{6}+c}}{x^{3}\sqrt{d}}\right)}{\sqrt{d}}}{6b^{2}}$	117

input
```
int(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/b^2*(-a/(a*d-b*c)*(-(d*x^6+c)^(1/2)*b*x^3/(b*x^6+a)-(2*a*d-3*b*c)/((a
*d-b*c)*a)^(1/2)*arctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2)))-2/d^(
1/2)*arctanh((d*x^6+c)^(1/2)/x^3/d^(1/2)))
```

3.875.5 Fricas [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 1077, normalized size of antiderivative = 7.64

$$\int \frac{x^{14}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

$$= \left[\frac{4\sqrt{dx^6+c}abdx^3 + 4\left((b^2c-abd)x^6+abc-a^2d\right)\sqrt{d}\log\left(-2\,dx^6-2\,\sqrt{dx^6+c}\sqrt{d}x^3-c\right)+((3\,b^2cd-2}{24\,((}\right.$$

input
```
integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(4*sqrt(d*x^6 + c)*a*b*d*x^3 + 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*
d)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c) + ((3*b^2*c*d
 - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*
c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^
2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sq
rt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((b^4*c
*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/24*(4*sqrt(d*x^6 + c)*a*
b*d*x^3 - 8*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)
*x^3/sqrt(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d
^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(
3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2
)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^
2*x^12 + 2*a*b*x^6 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b
^2*d^2), 1/12*(2*sqrt(d*x^6 + c)*a*b*d*x^3 + ((3*b^2*c*d - 2*a*b*d^2)*x^6
 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^
6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) + 2*((b^
2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)
*sqrt(d)*x^3 - c))/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2),
1/12*(2*sqrt(d*x^6 + c)*a*b*d*x^3 - 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d
)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d...
```

3.875.6 Sympy [F]

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**14/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**14/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.875.7 Maxima [F]

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^{14}}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^14/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

3.875.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(117) = 234$.

Time = 0.30 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.43

$$\begin{aligned}\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx =\\ &-\frac{\left(3\,abc\sqrt{-d}\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right) - 2\,a^2\sqrt{-d}d\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right) - 2\,\sqrt{abc-a^2d}bc\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right) + 2\,\sqrt{ab}\right.}{6\left(\sqrt{abc-a^2d}b^3c\sqrt{-d} - \sqrt{abc-a^2d}ab^2\sqrt{-d}d\right.}\\ &+\frac{ac\sqrt{d+\frac{c}{x^6}}}{6\,(b^2c\,\mathrm{sgn}\,(x) - abd\,\mathrm{sgn}\,(x))\left(bc+a\left(d+\frac{c}{x^6}\right)-ad\right)}\\ &+\frac{(3\,abc-2\,a^2d)\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{6\,(b^3c\,\mathrm{sgn}\,(x) - ab^2d\,\mathrm{sgn}\,(x))\sqrt{abc-a^2d}} - \frac{\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{3\,b^2\sqrt{-d}\,\mathrm{sgn}\,(x)}\end{aligned}$$

input
```
integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
-1/6*(3*a*b*c*sqrt(-d)*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 2*a^2*sqrt(
-d)*d*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 2*sqrt(a*b*c - a^2*d)*b*c*ar
ctan(sqrt(d)/sqrt(-d)) + 2*sqrt(a*b*c - a^2*d)*a*d*arctan(sqrt(d)/sqrt(-d)
) + sqrt(a*b*c - a^2*d)*a*sqrt(-d)*sqrt(d))*sgn(x)/(sqrt(a*b*c - a^2*d)*b^
3*c*sqrt(-d) - sqrt(a*b*c - a^2*d)*a*b^2*sqrt(-d)*d) + 1/6*a*c*sqrt(d + c/
x^6)/((b^2*c*sgn(x) - a*b*d*sgn(x))*(b*c + a*(d + c/x^6) - a*d)) + 1/6*(3*
a*b*c - 2*a^2*d)*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/((b^3*c*sgn
(x) - a*b^2*d*sgn(x))*sqrt(a*b*c - a^2*d)) - 1/3*arctan(sqrt(d + c/x^6)/sq
rt(-d))/(b^2*sqrt(-d)*sgn(x))
```

3.875.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^{14}}{{\left(b\,x^6+a\right)}^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^14/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^14/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.876 $\int \frac{x^8}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

3.876.1 Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^8}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)}+\frac{c\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6\sqrt{a}(bc-ad)^{3/2}}$$

output
```
1/6*c*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/(-a*d+b*c)^(3/2
)/a^(1/2)-1/6*x^3*(d*x^6+c)^(1/2)/(-a*d+b*c)/(b*x^6+a)
```

3.876.2 Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{x^8}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{1}{6}\left(-\frac{x^3\sqrt{c+dx^6}}{(bc-ad)\left(a+bx^6\right)}+\frac{c\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}(bc-ad)^{3/2}}\right)$$

input
```
Integrate[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(-((x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6))) + (c*ArcTan[(a*Sqrt[d]
 + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqr
t[a]*(b*c - a*d)^(3/2)))/6
```

3.876.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {965, 373, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^6+a)^2\sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \text{ 373}$$

$$\frac{1}{3}\left(\frac{\int \frac{c}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{2(bc-ad)}-\frac{x^3\sqrt{c+dx^6}}{2(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{c\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{2(bc-ad)}-\frac{x^3\sqrt{c+dx^6}}{2(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 291}$$

$$\frac{1}{3}\left(\frac{c\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{2(bc-ad)}-\frac{x^3\sqrt{c+dx^6}}{2(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{3}\left(\frac{c\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{2\sqrt{a}(bc-ad)^{3/2}}-\frac{x^3\sqrt{c+dx^6}}{2(a+bx^6)(bc-ad)}\right)$$

input
```
Int[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(-1/2*(x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + (c*ArcTan[(Sqrt[b*
c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(2*Sqrt[a]*(b*c - a*d)^(3/2)))/3
```

3.876.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.876.4 Maple [A] (verified)

Time = 9.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$-\frac{c\left(-\frac{\sqrt{d\,x^{6}+c}\,x^{3}}{c\left(b\,x^{6}+a\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}\,a}{x^{3}\sqrt{\left(a d -b c \right) a}}\right)}{\sqrt{\left(a d -b c \right) a}}\right)}{6\left(a d -b c \right)}$	81

input
```
int(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*c/(a*d-b*c)*(-(d*x^6+c)^(1/2)*x^3/c/(b*x^6+a)+1/((a*d-b*c)*a)^(1/2)*a
rctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2)))
```

3.876.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(77) = 154$.

Time = 0.36 (sec) , antiderivative size = 426, normalized size of antiderivative = 4.58

$$\int \frac{x^8}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$
$$= \Bigg[-\frac{4\sqrt{dx^6+c}(abc-a^2d)x^3-(bcx^6+ac)\sqrt{-abc+a^2d}\log\Bigg(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2+4\,((}{b^2x^{12}+2\,abx^6+a^2}}{24\,((ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^6+a^2b^2c^2-2\,a^3bcd+a^4d^2)}$$
$$-\frac{2\sqrt{dx^6+c}(abc-a^2d)x^3-(bcx^6+ac)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2\,ad)x^6-ac)\sqrt{dx^6+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^9+(abc^2-a^2cd)x^3)}\right)}{12\,((ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^6+a^2b^2c^2-2\,a^3bcd+a^4d^2)}\Bigg]$$

input
```
integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/24*(4*sqrt(d*x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(-a*b*
c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*
a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6 + c)*s
qrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((a*b^3*c^2 - 2*a^2*b^
2*c*d + a^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/12*(2*sq
rt(d*x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(a*b*c - a^2*d)*ar
ctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a
*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)))/((a*b^3*c^2 - 2*a^2*b^2
*c*d + a^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]
```

3.876.6 Sympy [F]

$$\int \frac{x^8}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^8}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**8/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**8/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.876.7 Maxima [F]

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^8}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

3.876.8 Giac [F]

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^8}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.876.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^8}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^8/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^8/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.877 $\int \frac{x^2}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

3.877.1 Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^2}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)}+\frac{(bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}$$

output
```
1/6*(-2*a*d+b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(3
/2)/(-a*d+b*c)^(3/2)+1/6*b*x^3*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/(b*x^6+a)
```

3.877.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{bx^3\sqrt{c+dx^6}}{6a(-bc+ad)\left(a+bx^6\right)}+\frac{(bc-2ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^6+bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}}\right)}{6a^{3/2}(bc-ad)^{3/2}}$$

input
```
Integrate[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
-1/6*(b*x^3*Sqrt[c + d*x^6])/(a*(-(b*c) + a*d)*(a + b*x^6)) + ((b*c - 2*a*
d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqr
t[b*c - a*d])])/(6*a^(3/2)*(b*c - a*d)^(3/2))
```

3.877.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {965, 296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{3}\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \text{296}$$

$$\frac{1}{3}\left(\frac{(bc-2ad)\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{2a(bc-ad)}+\frac{bx^3\sqrt{c+dx^6}}{2a\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{3}\left(\frac{(bc-2ad)\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{2a(bc-ad)}+\frac{bx^3\sqrt{c+dx^6}}{2a\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{3}\left(\frac{(bc-2ad)\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx^3\sqrt{c+dx^6}}{2a\left(a+bx^6\right)(bc-ad)}\right)$$

input

```
Int[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output

```
((b*x^3*Sqrt[c + d*x^6])/(2*a*(b*c - a*d)*(a + b*x^6)) + ((b*c - 2*a*d)*Ar
cTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(2*a^(3/2)*(b*c - a
*d)^(3/2)))/3
```

3.877.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.877.4 Maple [A] (verified)

Time = 9.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{6}+c}\,b\,x^{3}}{b\,x^{6}+a}+\frac{(2ad-bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}\,a}{x^{3}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}}{6(ad-bc)a}$	90

input
```
int(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/(a*d-b*c)/a*(-(d*x^6+c)^(1/2)*b*x^3/(b*x^6+a)+(2*a*d-b*c)/((a*d-b*c)*a
)^(1/2)*arctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2)))
```

3.877.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(88) = 176$.

Time = 0.51 (sec) , antiderivative size = 467, normalized size of antiderivative = 4.49

$$\int \frac{x^2}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx$$
$$= \left[\frac{4\sqrt{dx^6+c}(ab^2c-a^2bd)x^3-((b^2c-2\,abd)x^6+abc-2\,a^2d)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3}{}\right.}{24\,(a^3b^2c^2-2\,a^4bcd+a^5d^2+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2}\right.$$

input
```
integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(4*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 - ((b^2*c - 2*a*b*d)*x^6
+ a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*
d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9
 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 +
a^2)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d
 + a^4*b*d^2)*x^6), 1/12*(2*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 + ((b^
2*c - 2*a*b*d)*x^6 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c
 - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d
^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2
+ (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^6)]
```

3.877.6 Sympy [F]

$$\int \frac{x^2}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \int \frac{x^2}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**2/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.877.7 Maxima [F]

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^2}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

3.877.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(88) = 176$.

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.28

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{1}{6}\,d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}} + \frac{2\left(\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^2bc}{\left(\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^4b-2\left(\sqrt{d}x^3-\sqrt{dx^6+}\right.\right.}\right.$$

input
```
integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
-1/6*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*
b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((
sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*b*c - 2*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2
*a*d - b*c^2)/((((sqrt(d)*x^3 - sqrt(d*x^6 + c))^4*b - 2*(sqrt(d)*x^3 - sqr
t(d*x^6 + c))^2*b*c + 4*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*a*d + b*c^2)*(a*
b*c*d - a^2*d^2)))
```

3.877.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^2}{{\left(b\,x^6+a\right)}^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^2/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.878 $\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

3.878.1 Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3}+\frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{b(3bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}}$$

output

```
-1/6*b*(-4*a*d+3*b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))
/a^(5/2)/(-a*d+b*c)^(3/2)-1/6*(-2*a*d+3*b*c)*(d*x^6+c)^(1/2)/a^2/c/(-a*d+b
*c)/x^3+1/6*b*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/x^3/(b*x^6+a)
```

3.878.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{\sqrt{c+dx^6}\left(2abc-2a^2d+3b^2cx^6-2abdx^6\right)}{6a^2c(-bc+ad)x^3\left(a+bx^6\right)} - \frac{b(3bc-4ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^6+bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}}\right)}{6a^{5/2}(bc-ad)^{3/2}}$$

input

```
Integrate[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(Sqrt[c + d*x^6]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^6 - 2*a*b*d*x^6))/(6*a^2*c
*(-(b*c) + a*d)*x^3*(a + b*x^6)) - (b*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] +
b*Sqrt[d]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d])])/(6*a^(5
/2)*(b*c - a*d)^(3/2))
```

3.878.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {965, 374, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+bx^6\right)^2 \sqrt{c+dx^6}} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{3}\int \frac{1}{x^6 \left(bx^6+a\right)^2 \sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \text{374}$$

$$\frac{1}{3}\left(\frac{b\sqrt{c+dx^6}}{2ax^3 \left(a+bx^6\right)(bc-ad)}-\frac{\int -\frac{2bdx^6+3bc-2ad}{x^6\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{2a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{\int \frac{2bdx^6+3bc-2ad}{x^6\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^3 \left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{3}\left(\frac{-\frac{\int \frac{bc(3bc-4ad)}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{ac}-\frac{\sqrt{c+dx^6}(3bc-2ad)}{acx^3}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^3 \left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{-\frac{b(3bc-4ad)\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{a}-\frac{\sqrt{c+dx^6}(3bc-2ad)}{acx^3}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^3 \left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow 291$$

$$\frac{1}{3}\left(\frac{-\frac{b(3bc-4ad)\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{a}-\frac{\sqrt{c+dx^6}(3bc-2ad)}{acx^3}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^3\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow 218$$

$$\frac{1}{3}\left(\frac{-\frac{b(3bc-4ad)\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}(3bc-2ad)}{acx^3}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^3\left(a+bx^6\right)(bc-ad)}\right)$$

input

```
Int[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output

```
((b*Sqrt[c + d*x^6])/(2*a*(b*c - a*d)*x^3*(a + b*x^6)) + (-(((3*b*c - 2*a*
d)*Sqrt[c + d*x^6])/(a*c*x^3)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d
]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(a^(3/2)*Sqrt[b*c - a*d]))/(2*a*(b*c -
a*d)))/3
```

3.878.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.878.4 Maple [A] (verified)

Time = 12.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{6}+c}}{x^{3}}+\frac{b c \left(\frac{\sqrt{d\,x^{6}+c}\,b\,x^{3}}{b\,x^{6}+a}-\frac{(4 a d-3 b c) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}\,a}{x^{3} \sqrt{(a d-b c) a}}\right)}{\sqrt{(a d-b c) a}}\right)}{2 a d-2 b c}}{3 a^{2} c}$	112

input
```
int(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*(-(d*x^6+c)^(1/2)/x^3+1/2*b*c/(a*d-b*c)*((d*x^6+c)^(1/2)*b*x^3/(b*
x^6+a)-(4*a*d-3*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^6+c)^(1/2)/x^3*a/((a
*d-b*c)*a)^(1/2))))/c
```

3.878.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(129) = 258$.

Time = 0.51 (sec) , antiderivative size = 612, normalized size of antiderivative = 4.11

$$\int \frac{1}{x^4 (a+bx^6)^2 \sqrt{c+dx^6}}\, dx$$
$$= \Bigg[-\frac{((3\,b^3c^2 - 4\,ab^2cd)x^9 + (3\,ab^2c^2 - 4\,a^2bcd)x^3)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a}{b^2x^{12}+2\,a}\right.}{24\,((a^3b^3c^3 - 2\,a^4b^2c^2d + a^5b}$$
$$-\frac{((3\,b^3c^2 - 4\,ab^2cd)x^9 + (3\,ab^2c^2 - 4\,a^2bcd)x^3)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2\,ad)x^6-ac)\sqrt{dx^6+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^9+(abc^2-a^2cd)x^3)}\right) + 2\,((3}{12\,((a^3b^3c^3 - 2\,a^4b^2c^2d + a^5bcd^2)x^9 + (a^4b^2c^3 - 2\,a}$$

input
```
integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/24*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*
sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*
b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*
x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 4*((3*a*b^3
*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*
a^4*d^2)*sqrt(d*x^6 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x
^9 + (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^3), -1/12*(((3*b^3*c^2 -
4*a*b^2*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*sqrt(a*b*c - a^2*d)*ar
ctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a
*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*((3*a*b^3*c^2 - 5*a^
2*b^2*c*d + 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sq
rt(d*x^6 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^9 + (a^4*b
^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^3)]
```

3.878.6 Sympy [F]

$$\int \frac{1}{x^4 (a+bx^6)^2 \sqrt{c+dx^6}}\, dx = \int \frac{1}{x^4 (a+bx^6)^2 \sqrt{c+dx^6}}\, dx$$

input
```
integrate(1/x**4/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.878.7 Maxima [F]

$$\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^4), x)
```

3.878.8 Giac [F]

$$\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.878.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^4\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/(x^4*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.879 $\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

3.879.1 Optimal result

Integrand size = 24, antiderivative size = 208

$$\begin{aligned}\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = &-\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} \\ &+\frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} \\ &+\frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9\left(a+bx^6\right)} \\ &+\frac{b^2(5bc-6ad)\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{7/2}(bc-ad)^{3/2}}\end{aligned}$$

output
```
1/6*b^2*(-6*a*d+5*b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2)
)/a^(7/2)/(-a*d+b*c)^(3/2)-1/18*(-2*a*d+5*b*c)*(d*x^6+c)^(1/2)/a^2/c/(-a*d
+b*c)/x^9+1/18*(-4*a^2*d^2-8*a*b*c*d+15*b^2*c^2)*(d*x^6+c)^(1/2)/a^3/c^2/(
-a*d+b*c)/x^3+1/6*b*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/x^9/(b*x^6+a)
```

3.879.2 Mathematica [A] (verified)

Time = 3.18 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^{10} (a+bx^6)^2 \sqrt{c+dx^6}} \, dx = -\frac{\sqrt{c+dx^6}(15b^3c^2x^{12}+2ab^2cx^6(5c-4dx^6)+2a^3d(c-2dx^6)-2a^2b(c^2+3cdx^6+2d^2x^{12}))}{18a^3c^2(-bc+ad)x^9(a+bx^6)} + \frac{b^2(5bc-6ad)\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{6a^{7/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
-1/18*(Sqrt[c + d*x^6]*(15*b^3*c^2*x^12 + 2*a*b^2*c*x^6*(5*c - 4*d*x^6) +
2*a^3*d*(c - 2*d*x^6) - 2*a^2*b*(c^2 + 3*c*d*x^6 + 2*d^2*x^12)))/(a^3*c^2*
(-(b*c) + a*d)*x^9*(a + b*x^6)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(a*Sqrt[d] +
 b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(6*a^(
7/2)*(b*c - a*d)^(3/2))
```

3.879.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {965, 374, 25, 445, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10} (a+bx^6)^2 \sqrt{c+dx^6}} dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{3} \int \frac{1}{x^{12} (bx^6+a)^2 \sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \text{374}$$

$$\frac{1}{3} \left(\frac{b\sqrt{c+dx^6}}{2ax^9 (a+bx^6) (bc-ad)}-\frac{\int -\frac{4bdx^6+5bc-2ad}{x^{12}(bx^6+a)\sqrt{dx^6+c}}dx^3}{2a(bc-ad)}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{3}\left(\frac{\int \frac{4bdx^6+5bc-2ad}{x^{12}(bx^6+a)\sqrt{dx^6+c}}dx^3}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^9\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \ 445$$

$$\frac{1}{3}\left(\frac{-\frac{\int \frac{2bd(5bc-2ad)x^6+15b^2c^2-4a^2d^2-8abcd}{x^6\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{3ac}-\frac{\sqrt{c+dx^6}(5bc-2ad)}{3acx^9}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^9\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \ 445$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{\int \frac{3b^2c^2(5bc-6ad)}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{ac}-\frac{\sqrt{c+dx^6}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^3}}{3ac}-\frac{\sqrt{c+dx^6}(5bc-2ad)}{3acx^9}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^9\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{a}-\frac{\sqrt{c+dx^6}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^3}}{3ac}-\frac{\sqrt{c+dx^6}(5bc-2ad)}{3acx^9}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^9\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \ 291$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{a}-\frac{\sqrt{c+dx^6}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^3}}{3ac}-\frac{\sqrt{c+dx^6}(5bc-2ad)}{3acx^9}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^9\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \ 218$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^3}}{3ac}-\frac{\sqrt{c+dx^6}(5bc-2ad)}{3acx^9}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^9\left(a+bx^6\right)(bc-ad)}\right)$$

input
```
Int[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

3.879. $\int \frac{1}{x^{10}(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

output
```
((b*Sqrt[c + d*x^6])/(2*a*(b*c - a*d)*x^9*(a + b*x^6)) + (-1/3*((5*b*c - 2
*a*d)*Sqrt[c + d*x^6])/(a*c*x^9) - (-((((15*b^2*c)/a - 8*b*d - (4*a*d^2)/c
)*Sqrt[c + d*x^6])/x^3) - (3*b^2*c*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]
*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/(2*a
*(b*c - a*d)))/3
```

3.879.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.879. $\int \frac{1}{x^{10}(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.879.4 Maple [A] (verified)

Time = 18.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{6}+c}\,\left(-2 a d\,x^{6}-6 b c\,x^{6}+a c \right)}{3 x^{9}}-\frac{b^{2} c^{2} \left(\frac{\sqrt{d\,x^{6}+c}\,b\,x^{3}}{b\,x^{6}+a}-\frac{\left(6 a d -5 b c \right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}\,a}{x^{3} \sqrt{\left(a d -b c \right) a}}\right)}{\sqrt{\left(a d -b c \right) a}}\right)}{2 \left(a d -b c \right)}}{3 a^{3} c^{2}}$	134

input
```
int(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^3*(-1/3*(d*x^6+c)^(1/2)*(-2*a*d*x^6-6*b*c*x^6+a*c)/x^9-1/2*b^2*c^2/(
a*d-b*c)*((d*x^6+c)^(1/2)*b*x^3/(b*x^6+a)-(6*a*d-5*b*c)/((a*d-b*c)*a)^(1/2
)*arctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2))))/c^2
```

3.879.5 Fricas [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 760, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^{10} \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx$$
$$= \Bigg[-\frac{3\,((5\,b^4c^3-6\,ab^3c^2d)x^{15}+(5\,ab^3c^3-6\,a^2b^2c^2d)x^9)\sqrt{-abc+a^2d}\log\Bigg(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2c}{b^2x}}{}$$

input
```
integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/72*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^15 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2
*d)*x^9)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3
)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) - 4*
((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^12 -
2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2
*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*sqrt(d*x^6 + c))/((a^4*b^3*c^4 - 2*
a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^15 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c
^2*d^2)*x^9), 1/36*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^15 + (5*a*b^3*c^3 - 6
*a^2*b^2*c^2*d)*x^9)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^6 - a
*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^
2 - a^2*c*d)*x^3)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2
 + 4*a^4*b*d^3)*x^12 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*
a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*sqrt(d*x^6 +
 c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^15 + (a^5*b^2*c^4
- 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^9)]
```

3.879.6 Sympy [F]

$$\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**10/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**10*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.879.7 Maxima [F]

$$\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x^{10}}\,dx$$

input
```
integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^10), x)
```

3.879.8 Giac [F]

$$\int \frac{1}{x^{10} \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\, dx = \int \frac{1}{\left(bx^6+a\right)^2 \sqrt{dx^6+c}x^{10}}\, dx$$

input
```
integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.879.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10} \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\, dx = \int \frac{1}{x^{10}\, \left(b\,x^6+a\right)^2\, \sqrt{d\,x^6+c}}\, dx$$

input
```
int(1/(x^10*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^10*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.880 $\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

3.880.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{x^5\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(\frac{5}{6},2,\frac{1}{2},\frac{11}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}$$

output
```
1/5*x^5*AppellF1(5/6,2,1/2,11/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(
d*x^6+c)^(1/2)
```

3.880.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 169 vs. $2(64) = 128$.

Time = 10.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.64

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$=\frac{x^5\Bigg(55ab(c+dx^6)+11(bc-6ad)\,(a+bx^6)\,\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-10bdx^6(a+bx}{330a^2(bc-ad)\,(a+bx^6)\,\sqrt{c+dx^6}}$$

input
```
Integrate[x^4/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(x^5*(55*a*b*(c + d*x^6) + 11*(b*c - 6*a*d)*(a + b*x^6)*Sqrt[1 + (d*x^6)/c
]*AppellF1[5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] - 10*b*d*x^6*(a
+ b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c), -(
(b*x^6)/a)]))/(330*a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

3.880.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{x^4}{\left(bx^6+a\right)^2\sqrt{\frac{dx^6}{c}+1}}dx}{\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^5\sqrt{\frac{dx^6}{c}+1}\operatorname{AppellF1}\left(\frac{5}{6},2,\frac{1}{2},\frac{11}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}$$

input
```
Int[x^4/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(x^5*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6, 2, 1/2, 11/6, -((b*x^6)/a), -((d*x^
6)/c)])/(5*a^2*Sqrt[c + d*x^6])
```

3.880.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.880.4 Maple [F]

$$\int \frac{x^4}{\left(b\,x^6+a\right)^2 \sqrt{d\,x^6+c}}dx$$

input
```
int(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

3.880.5 Fricas [F]

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^6 + c)*x^4/(b^2*d*x^18 + (b^2*c + 2*a*b*d)*x^12 + (2*a*b
*c + a^2*d)*x^6 + a^2*c), x)
```

3.880.6 Sympy [F]

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**4/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**4/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.880.7 Maxima [F]

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^4}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

3.880.8 Giac [F]

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^4}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

3.880.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^4}{{\left(b\,x^6+a\right)}^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^4/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.881 $\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

3.881.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{x^4\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{2}{3},2,\frac{1}{2},\frac{5}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

output
```
1/4*x^4*AppellF1(2/3,2,1/2,5/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(d
*x^6+c)^(1/2)
```

3.881.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 168 vs. 2(64) = 128.

Time = 10.19 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.62

$$\int \frac{x^3}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=$$
$$-\frac{x^4\Bigg(-10ab(c+dx^6)-5(bc-3ad)\left(a+bx^6\right)\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+bdx^6(a+bx}{60a^2(bc-ad)\left(a+bx^6\right)\sqrt{c+dx^6}}$$

input
```
Integrate[x^3/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
-1/60*(x^4*(-10*a*b*(c + d*x^6) - 5*(b*c - 3*a*d)*(a + b*x^6)*Sqrt[1 + (d*
x^6)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*x^6*(
a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -(
(b*x^6)/a)]))/(a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

3.881.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {965, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^6+a)^2\sqrt{dx^6+c}}dx^2$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{x^2}{(bx^6+a)^2\sqrt{\frac{dx^6}{c}+1}}dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},2,\frac{1}{2},\frac{5}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

input
```
Int[x^3/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(x^4*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3, 2, 1/2, 5/3, -((b*x^6)/a), -((d*x^6
)/c)])/(4*a^2*Sqrt[c + d*x^6])
```

3.881.3.1 Defintions of rubi rules used

rule 965

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1012

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.881.4 Maple [F]

$$\int \frac{x^3}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}dx$$

input

```
int(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output

```
int(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

3.881.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^6)^2\,\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input

```
integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output

```
Timed out
```

3.881.6 Sympy [F]

$$\int \frac{x^3}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^3}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**3/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.881.7 Maxima [F]

$$\int \frac{x^3}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^3}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

3.881.8 Giac [F]

$$\int \frac{x^3}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^3}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

3.881.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^3}{\left(b\,x^6+a\right)^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^3/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.882 $\int \frac{x}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

3.882.1 Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{x^2\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}$$

output
```
1/2*x^2*AppellF1(1/3,2,1/2,4/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(d
*x^6+c)^(1/2)
```

3.882.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. $2(64) = 128$.

Time = 10.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.69

$$\int \frac{x}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$
$$=\frac{8abx^2(c+dx^6)+8(2bc-3ad)x^2(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+bdx^8(a+bx^6)}{48a^2(bc-ad)\left(a+bx^6\right)\sqrt{c+dx^6}}$$

input
```
Integrate[x/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(8*a*b*x^2*(c + d*x^6) + 8*(2*b*c - 3*a*d)*x^2*(a + b*x^6)*Sqrt[1 + (d*x^6
)/c]*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*x^8*(a +
 b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b*
x^6)/a)])/(48*a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

3.882.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {965, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{1}{(bx^6+a)^2\sqrt{dx^6+c}}dx^2$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{1}{(bx^6+a)^2\sqrt{\frac{dx^6}{c}+1}}dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x^2\sqrt{\frac{dx^6}{c}+1}\operatorname{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}$$

input
```
Int[x/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(x^2*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 2, 1/2, 4/3, -((b*x^6)/a), -((d*x^6
)/c)])/(2*a^2*Sqrt[c + d*x^6])
```

3.882.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.882.4 Maple [F]

$$\int \frac{x}{\left(b\,x^{6}+a\right)^{2} \sqrt{d\,x^{6}+c}}d x$$

input
```
int(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

3.882.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^6)^2 \sqrt{c+dx^6}} \, dx = \text{Timed out}$$

input
```
integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.882.6 Sympy [F]

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.882.7 Maxima [F]

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

3.882.8 Giac [F]

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

3.882.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x}{{\left(b\,x^6+a\right)}^2\,\sqrt{d\,x^6+c}} \,d x$$

input

```
int(x/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output

```
int(x/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.883 $\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

3.883.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{x\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{1}{6},2,\frac{1}{2},\frac{7}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}$$

output
```
x*AppellF1(1/6,2,1/2,7/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(d*x^6+c
)^(1/2)
```

3.883.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 329 vs. $2(59) = 118$.

Time = 10.30 (sec) , antiderivative size = 329, normalized size of antiderivative = 5.58

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$=\frac{x\left(-2bdx^6\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{7}{6},\frac{1}{2},1,\frac{13}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-\frac{7a\left(7ac\left(6ad-b\left(6c+dx^6\right)\right)\text{AppellF1}\left(\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+3bx}{(a+bx^6)\left(-7ac\,\text{AppellF1}\left(\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+3x^6\right.}\right.}{42a^2(-bc+ad)\sqrt{c+dx^6}}$$

input
```
Integrate[1/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(x*(-2*b*d*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[7/6, 1/2, 1, 13/6, -((d*x^6)/c
), -((b*x^6)/a)] - (7*a*(7*a*c*(6*a*d - b*(6*c + d*x^6))*AppellF1[1/6, 1/2
, 1, 7/6, -((d*x^6)/c), -((b*x^6)/a)] + 3*b*x^6*(c + d*x^6)*(2*b*c*AppellF
1[7/6, 1/2, 2, 13/6, -((d*x^6)/c), -((b*x^6)/a)] + a*d*AppellF1[7/6, 3/2,
1, 13/6, -((d*x^6)/c), -((b*x^6)/a)])))/((a + b*x^6)*(-7*a*c*AppellF1[1/6,
 1/2, 1, 7/6, -((d*x^6)/c), -((b*x^6)/a)] + 3*x^6*(2*b*c*AppellF1[7/6, 1/2
, 2, 13/6, -((d*x^6)/c), -((b*x^6)/a)] + a*d*AppellF1[7/6, 3/2, 1, 13/6, -
((d*x^6)/c), -((b*x^6)/a)])))))/(42*a^2*(-(b*c) + a*d)*Sqrt[c + d*x^6])
```

3.883.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{1}{\left(bx^6+a\right)^2\sqrt{\frac{dx^6}{c}+1}}dx}{\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{1}{6},2,\frac{1}{2},\frac{7}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}$$

input
```
Int[1/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(x*Sqrt[1 + (d*x^6)/c]*AppellF1[1/6, 2, 1/2, 7/6, -((b*x^6)/a), -((d*x^6)/
c)])/(a^2*Sqrt[c + d*x^6])
```

3.883.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.883.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{6}+a\right)^{2} \sqrt{d\,x^{6}+c}}dx$$

input
```
int(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

3.883.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^6)^2\,\sqrt{c+dx^6}}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.883.6 Sympy [F]

$$\int \frac{1}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.883.7 Maxima [F]

$$\int \frac{1}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

3.883.8 Giac [F]

$$\int \frac{1}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

3.883.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(b\,x^6+a\right)^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.884 $\int \frac{1}{x^2\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

3.884.1 Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(-\frac{1}{6},2,\frac{1}{2},\frac{5}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2x\sqrt{c+dx^6}}$$

output
```
-AppellF1(-1/6,2,1/2,5/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/x/(d*x^6
+c)^(1/2)
```

3.884.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 226 vs. $2(62) = 124$.

Time = 10.30 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^2\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$
$$=\frac{55a(c+dx^6)\left(6a^2d-7b^2cx^6-6ab(c-dx^6)\right)-11(7b^2c^2-24abcd+12a^2d^2)\,x^6(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,\text{Appel}}{330a^3c(bc-ad)x\,(}$$

input
```
Integrate[1/(x^2*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(55*a*(c + d*x^6)*(6*a^2*d - 7*b^2*c*x^6 - 6*a*b*(c - d*x^6)) - 11*(7*b^2*
c^2 - 24*a*b*c*d + 12*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF
1[5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] + 10*b*d*(7*b*c - 6*a*d)*
x^12*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6
)/c), -((b*x^6)/a)])/(330*a^3*c*(b*c - a*d)*x*(a + b*x^6)*Sqrt[c + d*x^6])
```

3.884.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^6)^2 \sqrt{c+dx^6}}dx$$
$$\Big\downarrow \text{1013}$$
$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{1}{x^2 (bx^6+a)^2 \sqrt{\frac{dx^6}{c}+1}}dx}{\sqrt{c+dx^6}}$$
$$\Big\downarrow \text{1012}$$
$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(-\frac{1}{6},2,\frac{1}{2},\frac{5}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2 x\sqrt{c+dx^6}}$$

input
```
Int[1/(x^2*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
-((Sqrt[1 + (d*x^6)/c]*AppellF1[-1/6, 2, 1/2, 5/6, -((b*x^6)/a), -((d*x^6)
/c)])/(a^2*x*Sqrt[c + d*x^6]))
```

3.884.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.884.4 Maple [F]

$$\int \frac{1}{x^{2} \left(b \,x^{6}+a \right)^{2} \sqrt{d \,x^{6}+c}}d x$$

input
```
int(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

3.884.5 Fricas [F]

$$\int \frac{1}{x^2 \left(a+b x^6\right)^2 \sqrt{c+d x^6}} \, dx=\int { \frac{1}{{\left(b x^{6} + a\right)}^{2} \sqrt{d x^{6} + c} x^{2}} \,d x }$$

input
```
integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^6 + c)/(b^2*d*x^20 + (b^2*c + 2*a*b*d)*x^14 + (2*a*b*c +
 a^2*d)*x^8 + a^2*c*x^2), x)
```

3.884.6 Sympy [F]

$$\int \frac{1}{x^2 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx=\int \frac{1}{x^2 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**2/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.884.7 Maxima [F]

$$\int \frac{1}{x^2 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^2), x)
```

3.884.8 Giac [F]

$$\int \frac{1}{x^2 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^2), x)
```

3.884.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^2\,\left(b\,x^6+a\right)^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/(x^2*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.885 $\int \frac{1}{x^3(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

3.885.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(-\frac{1}{3},2,\frac{1}{2},\frac{2}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}$$

output
```
-1/2*AppellF1(-1/3,2,1/2,2/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/x^2/
(d*x^6+c)^(1/2)
```

3.885.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(64) = 128.

Time = 10.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.53

$$\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$
$$=\frac{20a(c+dx^6)\left(3a^2d-4b^2cx^6-3ab(c-dx^6)\right)-5\left(8b^2c^2-15abcd+3a^2d^2\right)x^6(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF}}{120a^3c(bc-ad)x^2\,(a}$$

input
```
Integrate[1/(x^3*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(20*a*(c + d*x^6)*(3*a^2*d - 4*b^2*c*x^6 - 3*a*b*(c - d*x^6)) - 5*(8*b^2*c
^2 - 15*a*b*c*d + 3*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[
2/3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + 2*b*d*(4*b*c - 3*a*d)*x^12
*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c),
-((b*x^6)/a)])/(120*a^3*c*(b*c - a*d)*x^2*(a + b*x^6)*Sqrt[c + d*x^6])
```

3.885.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {965, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{1}{x^4 \left(bx^6+a\right)^2 \sqrt{dx^6+c}}dx^2$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{1}{x^4 \left(bx^6+a\right)^2 \sqrt{\frac{dx^6}{c}+1}}dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(-\frac{1}{3},2,\frac{1}{2},\frac{2}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}$$

input
```
Int[1/(x^3*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
-1/2*(Sqrt[1 + (d*x^6)/c]*AppellF1[-1/3, 2, 1/2, 2/3, -((b*x^6)/a), -((d*x
^6)/c)])/(a^2*x^2*Sqrt[c + d*x^6])
```

3.885.3.1 Defintions of rubi rules used

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.885.4 Maple [F]

$$\int \frac{1}{x^{3} \left(b \,x^{6}+a \right)^{2} \sqrt{d \,x^{6}+c}}d x$$

input
```
int(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

3.885.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.885.6 Sympy [F]

$$\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**3/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.885.7 Maxima [F]

$$\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^3), x)
```

3.885.8 Giac [F]

$$\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^3), x)
```

3.885.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx=\int \frac{1}{x^3\,{\left(b\,x^6+a\right)}^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/(x^3*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.886 $\int \frac{1}{x^5\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

3.886.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^5\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}$$

output
```
-1/4*AppellF1(-2/3,2,1/2,1/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/x^4/
(d*x^6+c)^(1/2)
```

3.886.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 225 vs. $2(64)=128$.

Time = 10.29 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.52

$$\int \frac{1}{x^5\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

$$=\frac{8a(c+dx^6)\left(3a^2d-5b^2cx^6-3ab(c-dx^6)\right)+4(-20b^2c^2+21abcd+3a^2d^2)\,x^6(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,\text{Appell}}{96a^3c(bc-ad)x^4\,(a}$$

input
```
Integrate[1/(x^5*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(8*a*(c + d*x^6)*(3*a^2*d - 5*b^2*c*x^6 - 3*a*b*(c - d*x^6)) + 4*(-20*b^2*
c^2 + 21*a*b*c*d + 3*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1
[1/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*(-5*b*c + 3*a*d)*x^12
*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c),
-((b*x^6)/a)])/(96*a^3*c*(b*c - a*d)*x^4*(a + b*x^6)*Sqrt[c + d*x^6])
```

3.886.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {965, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\left(a+bx^6\right)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{1}{x^6\left(bx^6+a\right)^2\sqrt{dx^6+c}}dx^2$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{1}{x^6\left(bx^6+a\right)^2\sqrt{\frac{dx^6}{c}+1}}dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}$$

input
```
Int[1/(x^5*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
-1/4*(Sqrt[1 + (d*x^6)/c]*AppellF1[-2/3, 2, 1/2, 1/3, -((b*x^6)/a), -((d*x
^6)/c)])/(a^2*x^4*Sqrt[c + d*x^6])
```

3.886.3.1 Defintions of rubi rules used

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

3.886.4 Maple [F]

$$\int \frac{1}{x^{5} \left(b\,x^{6}+a\right)^{2} \sqrt{d\,x^{6}+c}}dx$$

input
```
int(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

3.886.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.886.6 Sympy [F]

$$\int \frac{1}{x^5 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx=\int \frac{1}{x^5 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**5/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**5*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.886.7 Maxima [F]

$$\int \frac{1}{x^5 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^5), x)
```

3.886.8 Giac [F]

$$\int \frac{1}{x^5 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^5), x)
```

3.886.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx=\int \frac{1}{x^5 \left(b\,x^6+a\right)^2 \sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/(x^5*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^5*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.887 $\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.887.1 Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2}+\frac{(c+dx^8)^{3/2}}{12bd^2}-\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}$$

output

```
1/12*(d*x^8+c)^(3/2)/b/d^2-1/4*a^2*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b
*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-1/4*(a*d+b*c)*(d*x^8+c)^(1/2)/b^2/d^2
```

3.887.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{c+dx^8}(-2bc-3ad+bdx^8)}{12b^2d^2}+\frac{a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{4b^{5/2}\sqrt{-bc+ad}}$$

input

```
Integrate[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output

```
(Sqrt[c + d*x^8]*(-2*b*c - 3*a*d + b*d*x^8))/(12*b^2*d^2) + (a^2*ArcTan[(S
qrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/(4*b^(5/2)*Sqrt[-(b*c) + a*d]
)
```

3.887.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{8}\int \frac{x^{16}}{(bx^8+a)\sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow \text{ 99}$$

$$\frac{1}{8}\int \left(\frac{a^2}{b^2(bx^8+a)\sqrt{dx^8+c}}+\frac{\sqrt{dx^8+c}}{bd}+\frac{-bc-ad}{b^2d\sqrt{dx^8+c}}\right)dx^8$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}\left(-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^8}(ad+bc)}{b^2d^2}+\frac{2(c+dx^8)^{3/2}}{3bd^2}\right)$$

input
```
Int[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
((-2*(b*c + a*d)*Sqrt[c + d*x^8])/(b^2*d^2) + (2*(c + d*x^8)^(3/2))/(3*b*d
^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(b^(5/2)*
Sqrt[b*c - a*d]))/8
```

3.887.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.887.4 Maple [A] (verified)

Time = 8.89 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$\frac{a^{2} \arctan\left(\frac{b \sqrt{d\,x^{8}+c}}{\sqrt{(a d-b c) b}}\right) d^{2}-\sqrt{d\,x^{8}+c}\,\left(\left(-\frac{b\,x^{8}}{3}+a\right) d+\frac{2 b c}{3}\right) \sqrt{(a d-b c) b}}{4 \sqrt{(a d-b c) b}\, b^{2} d^{2}}$	91

input
```
int(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/((a*d-b*c)*b)^(1/2)*(a^2*arctan(b*(d*x^8+c)^(1/2)/((a*d-b*c)*b)^(1/2))
*d^2-(d*x^8+c)^(1/2)*((-1/3*b*x^8+a)*d+2/3*b*c)*((a*d-b*c)*b)^(1/2))/b^2/d
^2
```

3.887.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.77

$$\int \frac{x^{23}}{(a+bx^{8})\sqrt{c+dx^{8}}}\,dx$$
$$= \left[\frac{3\sqrt{b^{2}c-abd}a^{2}d^{2}\log\left(\frac{bdx^{8}+2\,bc-ad-2\,\sqrt{dx^{8}+c}\sqrt{b^{2}c-abd}}{bx^{8}+a}\right)+2\,((b^{3}cd-ab^{2}d^{2})x^{8}-2\,b^{3}c^{2}-ab^{2}cd+3\,a^{2}bd^{2})}{24\,(b^{4}cd^{2}-ab^{3}d^{3})}\right.$$

input
```
integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d
*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^8
 - 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^
3*d^3), 1/12*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^8 + c)*sqrt(-
b^2*c + a*b*d)/(b*d*x^8 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^8 - 2*b^3*c^2 -
 a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

3.887.6 Sympy [F]

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**23/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**23/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.887.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.887.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{a^2\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}b^2} + \frac{(dx^8+c)^{\frac{3}{2}}b^2d^4 - 3\sqrt{dx^8+c}b^2cd^4 - 3\sqrt{dx^8+c}abd^5}{12\,b^3d^6}$$

input
```
integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*a^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*b^2) + 1/12*((d*x^8 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^8 + c)*b^2*c*d^4 -
3*sqrt(d*x^8 + c)*a*b*d^5)/(b^3*d^6)
```

3.887.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{(d\,x^8+c)^{3/2}}{12\,b\,d^2} - \left(\frac{c}{2\,b\,d^2} + \frac{4\,a\,d^3-4\,b\,c\,d^2}{16\,b^2\,d^4}\right)\sqrt{d\,x^8+c} + \frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^23/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
(c + d*x^8)^(3/2)/(12*b*d^2) - (c/(2*b*d^2) + (4*a*d^3 - 4*b*c*d^2)/(16*b^
2*d^4))*(c + d*x^8)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b
*c)^(1/2)))/(4*b^(5/2)*(a*d - b*c)^(1/2))
```

3.888 $\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.888.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{\sqrt{c+dx^8}}{4bd}+\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}$$

output
```
1/4*a*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)
^(1/2)+1/4*(d*x^8+c)^(1/2)/b/d
```

3.888.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{1}{4}\left(\frac{\sqrt{c+dx^8}}{bd}-\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}\right)$$

input
```
Integrate[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(Sqrt[c + d*x^8]/(b*d) - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) +
 a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d]))/4
```

3.888.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{8}\int \frac{x^8}{(bx^8+a)\sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{8}\left(\frac{2\sqrt{c+dx^8}}{bd}-\frac{a\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^8}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{8}\left(\frac{2\sqrt{c+dx^8}}{bd}-\frac{2a\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{bd}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{8}\left(\frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^8}}{bd}\right)$$

input
```
Int[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
((2*Sqrt[c + d*x^8])/(b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b
*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/8
```

3.888.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.888.4 Maple [A] (verified)

Time = 5.99 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

method	result	size
pseudoelliptic	$\frac{-a \arctan\left(\frac{b \sqrt{d\,x^{8}+c}}{\sqrt{(ad-bc)b}}\right) d+\sqrt{d\,x^{8}+c}\,\sqrt{(ad-bc)b}}{4bd\sqrt{(ad-bc)b}}$	72

input
```
int(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-a*arctan(b*(d*x^8+c)^(1/2)/((a*d-b*c)*b)^(1/2))*d+(d*x^8+c)^(1/2)*((
a*d-b*c)*b)^(1/2))/b/d/((a*d-b*c)*b)^(1/2)
```

3.888.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.77

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}(b^2c-abd)}{8\,(b^3cd-ab^2d^2)}, -\frac{\sqrt{-b^2c+abd}ad\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-b^2c+abd}}{bdx^8+bc}\right)-\sqrt{dx^8+c}(b^2c-abd)}{4\,(b^3cd-ab^2d^2)}\right]$$

input
```
integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 +
c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^2*c - a*b*d))/
(b^3*c*d - a*b^2*d^2), -1/4*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^8 +
c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) - sqrt(d*x^8 + c)*(b^2*c - a*b*d)
)/(b^3*c*d - a*b^2*d^2)]
```

3.888.6 Sympy [F]

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**15/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**15/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.888.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.888.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b} - \frac{\sqrt{dx^8+c}}{b}}{4\,d}$$

input
```
integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(a*d*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*
b*d)*b) - sqrt(d*x^8 + c)/b)/d
```

3.888.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{d\,x^8+c}}{4\,b\,d} - \frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^15/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
(c + d*x^8)^(1/2)/(4*b*d) - (a*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c
)^(1/2)))/(4*b^(3/2)*(a*d - b*c)^(1/2))
```

3.889 $\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.889.1 Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

output
```
-1/4*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^
(1/2)
```

3.889.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{4\sqrt{b}\sqrt{-bc+ad}}$$

input
```
Integrate[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]]/(4*Sqrt[b]*Sqrt[-(b*c
) + a*d])
```

3.889.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {946, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{8}\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{4d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

input

```
Int[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output

```
-1/4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c
- a*d])
```

3.889.3.1 Defintions of rubi rules used

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.889.4 Maple [A] (verified)

Time = 5.90 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d\,x^{8}+c}}{\sqrt{(ad-bc)b}}\right)}{4\sqrt{(ad-bc)b}}$	39

input
```
int(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^8+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

3.889.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.55

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \left[\frac{\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)}{8\,\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-b^2c+abd}}{bdx^8+bc}\right)}{4\,(b^2c-abd)}\right]$$

input
```
integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(
b*x^8 + a))/sqrt(b^2*c - a*b*d), 1/4*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^
8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c))/(b^2*c - a*b*d)]
```

3.889.6 Sympy [A] (verification not implemented)

Time = 10.99 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b\sqrt{\frac{ad-bc}{b}}} & \text{for } d \neq 0 \\ \begin{cases} \frac{x^8}{8a\sqrt{c}} & \text{for } b = 0 \\ \tilde{\infty}x^8 & \text{for } \sqrt{c} = 0 \\ \frac{\log\left(8a\sqrt{c}+8b\sqrt{c}x^8\right)}{8b\sqrt{c}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Piecewise((atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(4*b*sqrt((a*d - b*c
)/b)), Ne(d, 0)), (Piecewise((x**8/(8*a*sqrt(c)), Eq(b, 0)), (zoo*x**8, Eq
(sqrt(c), 0)), (log(8*a*sqrt(c) + 8*b*sqrt(c)*x**8)/(8*b*sqrt(c)), True)),
 True))
```

3.889.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.889.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}}$$

input

```
integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output

```
1/4*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

3.889.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\mathrm{atan}\left(\frac{b\sqrt{d\,x^8+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{4\sqrt{a\,b\,d-b^2\,c}}$$

input

```
int(x^7/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output

```
atan((b*(c + d*x^8)^(1/2))/(a*b*d - b^2*c)^(1/2))/(4*(a*b*d - b^2*c)^(1/2)
)
```

3.890 $\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

3.890.1 Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}$$

output
```
-1/4*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a/c^(1/2)+1/4*arctanh(b^(1/2)*(d*x^8
+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

3.890.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=-\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a}$$

input
```
Integrate[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
-1/4*((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/Sqrt[
-(b*c) + a*d] + ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/Sqrt[c])/a
```

3.890.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{8}\int \frac{1}{x^8\left(bx^8+a\right)\sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow \text{ 97}$$

$$\frac{1}{8}\left(\frac{\int \frac{1}{x^8\sqrt{dx^8+c}}dx^8}{a}-\frac{b\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^8}{a}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{8}\left(\frac{2\int \frac{1}{\frac{x^{16}}{d}-\frac{c}{d}}d\sqrt{dx^8+c}}{ad}-\frac{2b\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{ad}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{8}\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{a\sqrt{c}}\right)$$

input
```
Int[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
((-2*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/8
```

3.890.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.890.4 Maple [A] (verified)

Time = 6.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

method	result	size
pseudoelliptic	$-\frac{b \arctan\left(\frac{b\sqrt{d\,x^{8}+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{c}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{\sqrt{c}}\right)\sqrt{(ad-bc)b}}{4a\sqrt{(ad-bc)b}\,\sqrt{c}}$	78

input
```
int(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*arctan(b*(d*x^8+c)^(1/2)/((a*d-b*c)*b)^(1/2))*c^(1/2)+arctanh((d*x
^8+c)^(1/2)/c^(1/2))*((a*d-b*c)*b)^(1/2))/a/((a*d-b*c)*b)^(1/2)/c^(1/2)
```

3.890.5 Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.07

$$\int \frac{1}{x\left(a+bx^{8}\right)\sqrt{c+dx^{8}}}\,dx$$
$$=\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^{8}+2\,bc-ad+2\,\sqrt{dx^{8}+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^{8}+a}\right)+\sqrt{c}\log\left(\frac{dx^{8}-2\,\sqrt{dx^{8}+c}\sqrt{c}+2\,c}{x^{8}}\right)}{8\,ac},\ 2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\right.\right.$$

input
```
integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(c*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)
*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + sqrt(c)*log((d*x^8 - 2*sq
rt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), 1/8*(2*c*sqrt(-b/(b*c - a*d))*ar
ctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) +
sqrt(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), 1/8*(c*
sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*(b*c -
a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^8 + c)
*sqrt(-c)/c))/(a*c), 1/4*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(
b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) + sqrt(-c)*arctan(sqrt(d*
x^8 + c)*sqrt(-c)/c))/(a*c)]
```

3.890.6 Sympy [A] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(a+bx^{8}\right)\sqrt{c+dx^{8}}}\,dx=\begin{cases}\frac{2\left(-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^{8}}}{\sqrt{\frac{ad-bc}{b}}}\right)}{8a\sqrt{\frac{ad-bc}{b}}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^{8}}}{\sqrt{-c}}\right)}{8a\sqrt{-c}}\right)}{d} & \text{for}\ d\neq 0\\ \frac{\operatorname{atan}\left(\frac{2\left(\frac{a}{2b}+x^{8}\right)}{\sqrt{-\frac{a^{2}}{b^{2}}}}\right)}{4b\sqrt{c}\sqrt{-\frac{a^{2}}{b^{2}}}} & \text{otherwise}\end{cases}$$

input
```
integrate(1/x/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Piecewise((2*(-d*atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(8*a*sqrt((a*d
 - b*c)/b)) + d*atan(sqrt(c + d*x**8)/sqrt(-c))/(8*a*sqrt(-c)))/d, Ne(d, 0
)), (atan(2*(a/(2*b) + x**8)/sqrt(-a**2/b**2))/(4*b*sqrt(c)*sqrt(-a**2/b**
2)), True))
```

3.890.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x}\,dx$$

input
```
integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x), x)
```

3.890.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=-\frac{b\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}a}+\frac{\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*b*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a) + 1/4*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a*sqrt(-c))
```

3.890.9 Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 652, normalized size of antiderivative = 7.67

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^8+c}}{\sqrt{c}}\right)}{4\,a\,\sqrt{c}}$$

$$-\frac{\operatorname{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^8+c}\,\sqrt{b^2\,c-a\,b\,d}}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}\right)\,1\mathrm{i}}{8\,\left(a^2\,d-a\,b\,c\right)}+\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}+\right.}{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^8+c}\,\sqrt{b^2\,c-a\,b\,d}}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}-\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}+\right.}\right.}{4\,\left(a^2\,d-a\,b\,c\right)}$$

input
```
int(1/(x*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
- atanh((c + d*x^8)^(1/2)/c^(1/2))/(4*a*c^(1/2)) - (atan((((b^2*c - a*b*d)
^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 - ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^
3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^
(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c)))*1i)/(8*(a^2*d - a*b*c))
+ ((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 + ((b^2*c - a*b*d)
^(1/2)*(a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2
)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c)))*1i)/(8
*(a^2*d - a*b*c)))/(((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4
- ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2
)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d
 - a*b*c))))/(8*(a^2*d - a*b*c)) - ((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d
*x^8)^(1/2))/4 + ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 1
6*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*
c))))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c))))*(b^2*c - a*b*d)^(1/2)*1i
)/(4*(a^2*d - a*b*c))
```

3.891 $\int \frac{1}{x^9\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

3.891.1 Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{1}{x^9\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{c+dx^8}}{8acx^8}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}} - \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}}$$

output
```
1/8*(a*d+2*b*c)*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-1/4*b^(3/2)*a
rctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/8*
(d*x^8+c)^(1/2)/a/c/x^8
```

3.891.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^9\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{-\frac{a\sqrt{c+dx^8}}{cx^8}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{c^{3/2}}}{8a^2}$$

input
```
Integrate[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-((a*Sqrt[c + d*x^8])/(c*x^8)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
8])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[
c + d*x^8]/Sqrt[c]])/c^(3/2))/(8*a^2)
```

3.891.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 \left(a+b x^8\right) \sqrt{c+d x^8}} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{8} \int \frac{1}{x^{16} \left(b x^8+a\right) \sqrt{d x^8+c}}dx^8$$

$$\Big\downarrow \; 114$$

$$\frac{1}{8} \left(-\frac{\int \frac{b d x^8+2 b c+a d}{2 x^8 \left(b x^8+a\right) \sqrt{d x^8+c}}dx^8}{a c}-\frac{\sqrt{c+d x^8}}{a c x^8}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8} \left(-\frac{\int \frac{b d x^8+2 b c+a d}{x^8 \left(b x^8+a\right) \sqrt{d x^8+c}}dx^8}{2 a c}-\frac{\sqrt{c+d x^8}}{a c x^8}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{8} \left(-\frac{\frac{(a d+2 b c) \int \frac{1}{x^8 \sqrt{d x^8+c}}dx^8}{a}-\frac{2 b^2 c \int \frac{1}{\left(b x^8+a\right) \sqrt{d x^8+c}}dx^8}{a}}{2 a c}-\frac{\sqrt{c+d x^8}}{a c x^8}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{8} \left(-\frac{\frac{2 (a d+2 b c) \int \frac{1}{\frac{x^{16}}{d}-\frac{c}{d}}d\sqrt{d x^8+c}}{a d}-\frac{4 b^2 c \int \frac{1}{\frac{b x^{16}}{d}+a-\frac{b c}{d}}d\sqrt{d x^8+c}}{a d}}{2 a c}-\frac{\sqrt{c+d x^8}}{a c x^8}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{8}\left(-\frac{\frac{4b^{3/2}c\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(ad+2bc)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2ac}-\frac{\sqrt{c+dx^8}}{acx^8}\right)$$

input
```
Int[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-(Sqrt[c + d*x^8]/(a*c*x^8)) - ((-2*(2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]
/Sqrt[c]])/(a*Sqrt[c]) + (4*b^(3/2)*c*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sq
rt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*c))/8
```

3.891.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.891.4 Maple [A] (verified)

Time = 6.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{\frac{2b^{2}\arctan\left(\frac{b\sqrt{d\,x^{8}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{a\sqrt{d\,x^{8}+c}}{c\,x^{8}}+\frac{(ad+2bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{8a^{2}}$	92

input
```
int(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^2*(2*b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^8+c)^(1/2)/((a*d-b*c)*b)^
(1/2))-a/c*(d*x^8+c)^(1/2)/x^8+(a*d+2*b*c)/c^(3/2)*arctanh((d*x^8+c)^(1/2)
/c^(1/2)))
```

3.891.5 Fricas [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 565, normalized size of antiderivative = 4.83

$$\int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}}\,dx$$

$$= \left[\frac{2\,bc^2x^8\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)+(2\,bc+ad)\sqrt{c}x^8\log\left(\frac{dx^8+2\,\sqrt{dx^8+c}\sqrt{c}+2\,c}{x^8}\right)-2\,\sqrt{}}{16\,a^2c^2x^8} \right.$$

$$-\frac{4\,bc^2x^8\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^8+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^8+bc}\right)-(2\,bc+ad)\sqrt{c}x^8\log\left(\frac{dx^8+2\,\sqrt{dx^8+c}\sqrt{c}+2\,c}{x^8}\right)+2\,\sqrt{dx^8}}{16\,a^2c^2x^8}$$

$$-\frac{2\,bc^2x^8\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^8+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^8+bc}\right)+(2\,bc+ad)\sqrt{-c}x^8\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-c}}{c}\right)+\sqrt{dx^8+}}{8\,a^2c^2x^8}$$

input
```
integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(2*b*c^2*x^8*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt
(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + (2*b*c + a*d)*
sqrt(c)*x^8*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*sqrt(d*
x^8 + c)*a*c)/(a^2*c^2*x^8), -1/16*(4*b*c^2*x^8*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) - (2*
b*c + a*d)*sqrt(c)*x^8*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8)
+ 2*sqrt(d*x^8 + c)*a*c)/(a^2*c^2*x^8), 1/8*(b*c^2*x^8*sqrt(b/(b*c - a*d))
*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c -
a*d)))/(b*x^8 + a)) - (2*b*c + a*d)*sqrt(-c)*x^8*arctan(sqrt(d*x^8 + c)*sq
rt(-c)/c) - sqrt(d*x^8 + c)*a*c)/(a^2*c^2*x^8), -1/8*(2*b*c^2*x^8*sqrt(-b/
(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d
*x^8 + b*c)) + (2*b*c + a*d)*sqrt(-c)*x^8*arctan(sqrt(d*x^8 + c)*sqrt(-c)/
c) + sqrt(d*x^8 + c)*a*c)/(a^2*c^2*x^8)]
```

3.891.6 Sympy [F]

$$\int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**9/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**9*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.891.7 Maxima [F]

$$\int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^9}\,dx$$

input
```
integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^9), x)
```

3.891.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}}\,dx = \frac{b^2 \arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}a^2} - \frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{8\,a^2\sqrt{-c}c} - \frac{\sqrt{dx^8+c}}{8\,acx^8}$$

input
```
integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*b^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*a^2) - 1/8*(2*b*c + a*d)*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a^2*sqrt(-c)
*c) - 1/8*sqrt(d*x^8 + c)/(a*c*x^8)
```

3.891.9 Mupad [B] (verification not implemented)

Time = 10.11 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.38

$$\int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}}\,dx$$
$$= \frac{\ln\left(\sqrt{d\,x^8+c}\,(b^4\,c-a\,b^3\,d)^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{8\,a^3\,d-8\,a^2\,b\,c}$$
$$-\frac{\ln\left(\sqrt{d\,x^8+c}\,(b^4\,c-a\,b^3\,d)^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{8\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{\sqrt{d\,x^8+c}}{8\,a\,c\,x^8}$$
$$-\frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^8+c}\,3\mathrm{i}}{128\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{128\,c}+\frac{5\,a\,b^3\,d^5}{256\,c^2}+\frac{a^2\,b^2\,d^6}{256\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^8+c}\,1\mathrm{i}}{256\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{256\,a}+\frac{b^2\,d^6}{256\,c}+\frac{3\,b^4\,c\,d^4}{128\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^8+c}\,5\mathrm{i}}{256\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{128\,a}+\frac{5\,b^3\,d^5}{256\,c}+\frac{a\,b^2\,d^6}{256\,c^2}\right)}\right)\,(a\,d+2}{8\,a^2\,\sqrt{c^3}}$$

input
```
int(1/(x^9*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
(log((c + d*x^8)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2
*a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(8*a^3*d - 8*a^2*b*c) - (log((c + d*x
^8)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(
b^4*c - a*b^3*d)^(1/2))/(8*(a^3*d - a^2*b*c)) - (c + d*x^8)^(1/2)/(8*a*c*x
^8) - (atan((b^4*d^4*(c + d*x^8)^(1/2)*3i)/(128*(c^3)^(1/2)*((3*b^4*d^4)/(
128*c) + (5*a*b^3*d^5)/(256*c^2) + (a^2*b^2*d^6)/(256*c^3))) + (b^2*d^6*(c
 + d*x^8)^(1/2)*1i)/(256*(c^3)^(1/2)*((5*b^3*d^5)/(256*a) + (b^2*d^6)/(256
*c) + (3*b^4*c*d^4)/(128*a^2))) + (b^3*d^5*(c + d*x^8)^(1/2)*5i)/(256*(c^3
)^(1/2)*((3*b^4*d^4)/(128*a) + (5*b^3*d^5)/(256*c) + (a*b^2*d^6)/(256*c^2)
)))*(a*d + 2*b*c)*1i)/(8*a^2*(c^3)^(1/2))
```

3.892 $\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.892.1 Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}}$$

output
```
-1/8*(2*a*d+b*c)*arctanh(x^4*d^(1/2)/(d*x^8+c)^(1/2))/b^2/d^(3/2)+1/4*a^(3
/2)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/b^2/(-a*d+b*c)^(1
/2)+1/8*x^4*(d*x^8+c)^(1/2)/b/d
```

3.892.2 Mathematica [A] (verified)

Time = 2.36 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\frac{bx^4\sqrt{c+dx^8}}{d} + \frac{2a^{3/2}\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{(bc+2ad)\log\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{d^{3/2}}}{8b^2}$$

input
```
Integrate[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
((b*x^4*Sqrt[c + d*x^8])/d + (2*a^(3/2)*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]
*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d] - ((b
*c + 2*a*d)*Log[Sqrt[d]*x^4 + Sqrt[c + d*x^8]])/d^(3/2))/(8*b^2)
```

3.892.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 381, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19}}{\left(a+bx^8\right)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{4}\int \frac{x^{16}}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{ 381}$$

$$\frac{1}{4}\left(\frac{x^4\sqrt{c+dx^8}}{2bd}-\frac{\int \frac{(bc+2ad)x^8+ac}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{2bd}\right)$$

$$\Big\downarrow \text{ 398}$$

$$\frac{1}{4}\left(\frac{x^4\sqrt{c+dx^8}}{2bd}-\frac{\frac{(2ad+bc)\int \frac{1}{\sqrt{dx^8+c}}dx^4}{b}-\frac{2a^2d\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{b}}{2bd}\right)$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{4}\left(\frac{x^4\sqrt{c+dx^8}}{2bd}-\frac{\frac{(2ad+bc)\int \frac{1}{1-dx^8}d\frac{x^4}{\sqrt{dx^8+c}}}{b}-\frac{2a^2d\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{b}}{2bd}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{4}\left(\frac{x^4\sqrt{c+dx^8}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}-\frac{2a^2d\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{b}}{2bd}\right)$$

$$\Big\downarrow 291$$

$$\frac{1}{4}\left(\frac{x^4\sqrt{c+dx^8}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}-\frac{2a^2d\int\frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{b}}{2bd}\right)$$

$$\Big\downarrow 218$$

$$\frac{1}{4}\left(\frac{x^4\sqrt{c+dx^8}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}-\frac{2a^{3/2}d\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{b\sqrt{bc-ad}}}{2bd}\right)$$

input
```
Int[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
((x^4*Sqrt[c + d*x^8])/(2*b*d) - ((-2*a^(3/2)*d*ArcTan[(Sqrt[b*c - a*d]*x^
4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(b*Sqrt[b*c - a*d]) + ((b*c + 2*a*d)*ArcTan
h[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]])/(b*Sqrt[d]))/(2*b*d))/4
```

3.892.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.892.4 Maple [A] (verified)

Time = 22.37 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.24

method	result
pseudoelliptic	$\frac{b\sqrt{(ad-bc)a}\,\sqrt{d\,x^{8}+c}\,\sqrt{d}\,x^{4}+2a^{2}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}\,a}{x^{4}\sqrt{(ad-bc)a}}\right)d^{\frac{3}{2}}-2\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{x^{4}\sqrt{d}}\right)ad\sqrt{(ad-bc)a}-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{x^{4}\sqrt{d}}\right)b}{8b^{2}\sqrt{(ad-bc)a}\,d^{\frac{3}{2}}}$

input
```
int(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(b*((a*d-b*c)*a)^(1/2)*(d*x^8+c)^(1/2)*d^(1/2)*x^4+2*a^2*arctanh((d*x^
8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2))*d^(3/2)-2*arctanh((d*x^8+c)^(1/2)/x^
4/d^(1/2))*a*d*((a*d-b*c)*a)^(1/2)-arctanh((d*x^8+c)^(1/2)/x^4/d^(1/2))*b*
c*((a*d-b*c)*a)^(1/2))/b^2/((a*d-b*c)*a)^(1/2)/d^(3/2)
```

3.892.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 739, normalized size of antiderivative = 6.01

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \left[\frac{2\sqrt{dx^8+c}bdx^4 + ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2+4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^{12}-(abc}{b^2x^{16}+2\,abx^8+a^2}\right.}{16\,b^2d^2}\right.$$

input
```
integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^
2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2
+ 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqr
t(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + (b*c +
2*a*d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b^2*d^2
), 1/16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*
c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^
2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*s
qrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 2*(b*
c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)))/(b^2*d^2), 1/16*
(2*sqrt(d*x^8 + c)*b*d*x^4 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c
 - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x
^4)) + (b*c + 2*a*d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*x^4
- c))/(b^2*d^2), 1/8*(sqrt(d*x^8 + c)*b*d*x^4 - a*d^2*sqrt(a/(b*c - a*d))*
arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/
(a*d*x^12 + a*c*x^4)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*
x^8 + c)))/(b^2*d^2)]
```

3.892.6 Sympy [F]

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**19/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**19/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.892.7 Maxima [F]

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^{19}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^19/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.892.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.892.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^{19}}{\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^19/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^19/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.893 $\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.893.1 Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}$$

output
```
1/4*arctanh(x^4*d^(1/2)/(d*x^8+c)^(1/2))/b/d^(1/2)-1/4*arctan(x^4*(-a*d+b*
c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))*a^(1/2)/b/(-a*d+b*c)^(1/2)
```

3.893.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{-\frac{\sqrt{a}\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{\log\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{d}}}{4b}$$

input
```
Integrate[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-((Sqrt[a]*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sq
rt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d]) + Log[Sqrt[d]*x^4 + Sqrt[c + d*x
^8]]/Sqrt[d])/(4*b)
```

3.893.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {965, 385, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{4}\int \frac{x^8}{(bx^8+a)\sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{385}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{\sqrt{dx^8+c}}dx^4}{b}-\frac{a\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{b}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{1-dx^8}d\frac{x^4}{\sqrt{dx^8+c}}}{b}-\frac{a\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{b}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}-\frac{a\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{b}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{4}\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}-\frac{a\int \frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{b}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{4}\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}-\frac{\sqrt{a}\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{b\sqrt{bc-ad}}\right)$$

input

```
Int[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(b*Sq
rt[b*c - a*d])) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(b*Sqrt[d]))/4
```

3.893.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.893. $\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.893.4 Maple [A] (verified)

Time = 12.91 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$\frac{-a\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}\,a}{x^{4}\sqrt{\left(a d -b c \right) a}}\right)\sqrt{d}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{x^{4}\sqrt{d}}\right)\sqrt{\left(a d -b c \right) a}}{4 b \sqrt{\left(a d -b c \right) a}\,\sqrt{d}}$	85

input
```
int(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-a*arctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2))*d^(1/2)+arctanh
((d*x^8+c)^(1/2)/x^4/d^(1/2))*((a*d-b*c)*a)^(1/2))/b/((a*d-b*c)*a)^(1/2)/d
^(1/2)
```

3.893.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 632, normalized size of antiderivative = 6.95

$$\int \frac{x^{11}}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2-4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^{12}-\left(abc^2-a^2cd\right)x^4\right)\sqrt{dx^8+c}\sqrt{-\frac{a}{bc-}}}{b^2x^{16}+2\,abx^8+a^2}\right.}{16\,bd}\right.$$

input
```
integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^
2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)
))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 2*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 +
 c)*sqrt(d)*x^4 - c))/(b*d), 1/16*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 -
8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*
((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*
x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) - 4*sqrt(-d)*
arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)))/(b*d), 1/8*(d*sqrt(a/(b*c - a*d))*ar
ctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a
*d*x^12 + a*c*x^4)) + sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4
 - c))/(b*d), 1/8*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 -
a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) - 2*sqrt(-d
)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)))/(b*d)]
```

3.893.6 Sympy [F]

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**11/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**11/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.893.7 Maxima [F]

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^{11}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^11/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.893.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.893.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{11}}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^11/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^11/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.894 $\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.894.1 Optimal result

Integrand size = 24, antiderivative size = 54

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{\arctan\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

output
```
1/4*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(1/2)/(-a*d+b*c
)^(1/2)
```

3.894.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

input
```
Integrate[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b
*c - a*d])]/(4*Sqrt[a]*Sqrt[b*c - a*d])
```

3.894.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {965, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{4}\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{4}\int \frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

input
```
Int[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])]/(4*Sqrt[a]*Sqrt[b*
c - a*d])
```

3.894.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.894.4 Maple [A] (verified)

Time = 10.89 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}\,a}{x^{4}\sqrt{(ad-bc)a}}\right)}{4\sqrt{(ad-bc)a}}$	42

input
```
int(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/((a*d-b*c)*a)^(1/2)*arctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2))
```

3.894.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(42) = 84.

Time = 0.45 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.54

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2-4\,((bc-2\,ad)x^{12}-acx^4)\sqrt{dx^8+c}\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)}{16\,(abc-a^2d)},\ \text{arctan}\right.$$

input
```
integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x^4)
*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2))/(a*b*
c - a^2*d), 1/8*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(
a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4))/sqrt(
a*b*c - a^2*d)]
```

3.894.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**3/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.894.7 Maxima [F]

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^3}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.894.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx=-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\,\sqrt{abcd-a^2d^2}}$$

input
```
integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

3.894.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^3}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^3/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.895 $\int \frac{1}{x^5\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

3.895.1 Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{1}{x^5\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=-\frac{\sqrt{c+dx^8}}{4acx^4}-\frac{b\arctan\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}$$

output
```
-1/4*b*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(3/2)/(-a*d+
b*c)^(1/2)-1/4*(d*x^8+c)^(1/2)/a/c/x^4
```

3.895.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^5\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=-\frac{\sqrt{c+dx^8}}{4acx^4}-\frac{b\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{3/2}\sqrt{bc-ad}}$$

input
```
Integrate[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
-1/4*Sqrt[c + d*x^8]/(a*c*x^4) - (b*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4
 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*a^(3/2)*Sqrt[b*c - a*d
])
```

3.895.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {965, 382, 25, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^8) \sqrt{c+dx^8}} \, dx$$
$$\Big\downarrow \text{965}$$
$$\frac{1}{4} \int \frac{1}{x^8 (bx^8+a) \sqrt{dx^8+c}} dx^4$$
$$\Big\downarrow \text{382}$$
$$\frac{1}{4} \left(\frac{\int -\frac{bc}{(bx^8+a)\sqrt{dx^8+c}} dx^4}{ac} - \frac{\sqrt{c+dx^8}}{acx^4} \right)$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{4} \left(-\frac{\int \frac{bc}{(bx^8+a)\sqrt{dx^8+c}} dx^4}{ac} - \frac{\sqrt{c+dx^8}}{acx^4} \right)$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{4} \left(-\frac{b \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}} dx^4}{a} - \frac{\sqrt{c+dx^8}}{acx^4} \right)$$
$$\Big\downarrow \text{291}$$
$$\frac{1}{4} \left(-\frac{b \int \frac{1}{a-(ad-bc)x^8} d\frac{x^4}{\sqrt{dx^8+c}}}{a} - \frac{\sqrt{c+dx^8}}{acx^4} \right)$$
$$\Big\downarrow \text{218}$$
$$\frac{1}{4} \left(-\frac{b \arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{a^{3/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^8}}{acx^4} \right)$$

input
```
Int[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-(Sqrt[c + d*x^8]/(a*c*x^4)) - (b*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*S
qrt[c + d*x^8])])/(a^(3/2)*Sqrt[b*c - a*d]))/4
```

3.895.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.895. $\int \frac{1}{x^5(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.895.4 Maple [A] (verified)

Time = 14.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

method	result	size
pseudoelliptic	$-\frac{cb\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}\,a}{x^{4}\sqrt{(ad-bc)a}}\right)x^{4}+\sqrt{d\,x^{8}+c}\,\sqrt{(ad-bc)a}}{4a\,x^{4}\sqrt{(ad-bc)a}\,c}$	80

input
```
int(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*b*arctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2))*x^4+(d*x^8+c)
^(1/2)*((a*d-b*c)*a)^(1/2))/a/x^4/((a*d-b*c)*a)^(1/2)/c
```

3.895.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(64) = 128.

Time = 0.50 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.15

$$\int \frac{1}{x^{5}\left(a+bx^{8}\right)\sqrt{c+dx^{8}}}\,dx$$

$$= \left[-\frac{\sqrt{-abc+a^{2}d}bcx^{4}\log\left(\frac{(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2})x^{16}-2\,(3\,abc^{2}-4\,a^{2}cd)x^{8}+a^{2}c^{2}+4\,((bc-2\,ad)x^{12}-acx^{4})\sqrt{dx^{8}+c}\sqrt{-abc+a^{2}d}}{b^{2}x^{16}+2\,abx^{8}+a^{2}}\right)+}{16\,(a^{2}bc^{2}-a^{3}cd)x^{4}}\right.$$

$$\left.-\frac{\sqrt{abc-a^{2}d}bcx^{4}\arctan\left(\frac{((bc-2\,ad)x^{8}-ac)\sqrt{dx^{8}+c}\sqrt{abc-a^{2}d}}{2\,((abcd-a^{2}d^{2})x^{12}+(abc^{2}-a^{2}cd)x^{4})}\right)+2\,\sqrt{dx^{8}+c}(abc-a^{2}d)}{8\,(a^{2}bc^{2}-a^{3}cd)x^{4}}\right]$$

input
```
integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(sqrt(-a*b*c + a^2*d)*b*c*x^4*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 -
 a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^
2)) + 4*sqrt(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4), -1/8
*(sqrt(a*b*c - a^2*d)*b*c*x^4*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*
x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*
d)*x^4)) + 2*sqrt(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4)]
```

3.895.6 Sympy [F]

$$\int \frac{1}{x^5 \left(a+bx^8\right) \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^5 \left(a+bx^8\right) \sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**5/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**5*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.895.7 Maxima [F]

$$\int \frac{1}{x^5 \left(a+bx^8\right) \sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^5), x)
```

3.895.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^5 \left(a+bx^8\right) \sqrt{c+dx^8}}\,dx$$
$$= \frac{1}{4} d^{\frac{3}{2}} \left(\frac{b \arctan\left(\frac{\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 b - bc + 2\,ad}{2\sqrt{abcd - a^2d^2}}\right)}{\sqrt{abcd - a^2d^2}ad} + \frac{2}{\left(\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 - c\right)ad} \right)$$

input
```
integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*d^(3/2)*(b*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a
*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*
x^4 - sqrt(d*x^8 + c))^2 - c)*a*d))
```

3.895.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{x^5\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/(x^5*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^5*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.896 $\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.896.1 Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=-\frac{\sqrt{c+dx^8}}{12acx^{12}}+\frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4}+\frac{b^2\arctan\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}$$

output

```
1/4*b^2*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(5/2)/(-a*d
+b*c)^(1/2)-1/12*(d*x^8+c)^(1/2)/a/c/x^12+1/12*(2*a*d+3*b*c)*(d*x^8+c)^(1/
2)/a^2/c^2/x^4
```

3.896.2 Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\frac{\sqrt{c+dx^8}\left(-ac+3bcx^8+2adx^8\right)}{12a^2c^2x^{12}}+\frac{b^2\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{5/2}\sqrt{bc-ad}}$$

input

```
Integrate[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output

```
(Sqrt[c + d*x^8]*(-(a*c) + 3*b*c*x^8 + 2*a*d*x^8))/(12*a^2*c^2*x^12) + (b^
2*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt
[b*c - a*d])])/(4*a^(5/2)*Sqrt[b*c - a*d])
```

3.896.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {965, 382, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{13} \left(a+b x^8\right) \sqrt{c+d x^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{4} \int \frac{1}{x^{16} \left(b x^8+a\right) \sqrt{d x^8+c}}dx^4$$

$$\Big\downarrow \text{382}$$

$$\frac{1}{4} \left(\frac{\int -\frac{2 b d x^8+3 b c+2 a d}{x^8 \left(b x^8+a\right) \sqrt{d x^8+c}}dx^4}{3 a c}-\frac{\sqrt{c+d x^8}}{3 a c x^{12}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4} \left(-\frac{\int \frac{2 b d x^8+3 b c+2 a d}{x^8 \left(b x^8+a\right) \sqrt{d x^8+c}}dx^4}{3 a c}-\frac{\sqrt{c+d x^8}}{3 a c x^{12}}\right)$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{4} \left(-\frac{-\frac{\int \frac{3 b^2 c^2}{\left(b x^8+a\right) \sqrt{d x^8+c}}dx^4}{a c}-\frac{\sqrt{c+d x^8} (2 a d+3 b c)}{a c x^4}}{3 a c}-\frac{\sqrt{c+d x^8}}{3 a c x^{12}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4} \left(-\frac{-\frac{3 b^2 c \int \frac{1}{\left(b x^8+a\right) \sqrt{d x^8+c}}dx^4}{a}-\frac{\sqrt{c+d x^8} (2 a d+3 b c)}{a c x^4}}{3 a c}-\frac{\sqrt{c+d x^8}}{3 a c x^{12}}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{4} \left(-\frac{-\frac{3 b^2 c \int \frac{1}{a-(a d-b c) x^8}d\frac{x^4}{\sqrt{d x^8+c}}}{a}-\frac{\sqrt{c+d x^8} (2 a d+3 b c)}{a c x^4}}{3 a c}-\frac{\sqrt{c+d x^8}}{3 a c x^{12}}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{4}\left(-\frac{-\frac{3b^2c\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}(2ad+3bc)}{acx^4}}{3ac}-\frac{\sqrt{c+dx^8}}{3acx^{12}}\right)$$

input
```
Int[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-1/3*Sqrt[c + d*x^8]/(a*c*x^12) - (-(((3*b*c + 2*a*d)*Sqrt[c + d*x^8])/(a
*c*x^4)) - (3*b^2*c*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])
])/(a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/4
```

3.896.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.896.4 Maple [A] (verified)

Time = 25.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

method	result	size
pseudoelliptic	$-\frac{-3 b^{2} c^{2} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}\,a}{x^{4} \sqrt{\left(a d -b c \right) a}}\right) x^{12}+\sqrt{d\,x^{8}+c}\,\left(\left(-3 b\,x^{8}+a \right) c -2 a d\,x^{8}\right) \sqrt{\left(a d -b c \right) a}}{12 \sqrt{\left(a d -b c \right) a}\,x^{12} a^{2} c^{2}}$	103

input
```
int(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-3*b^2*c^2*arctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2))*x^12+
(d*x^8+c)^(1/2)*((-3*b*x^8+a)*c-2*a*d*x^8)*((a*d-b*c)*a)^(1/2))/((a*d-b*c)
*a)^(1/2)/x^12/a^2/c^2
```

3.896.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.62

$$\int \frac{1}{x^{13} \left(a+b x^8\right) \sqrt{c+d x^8}} \, dx$$
$$= \Bigg[-\frac{3 \sqrt{-abc + a^2 d} b^2 c^2 x^{12} \log \Bigg(\frac{(b^2 c^2 - 8\,abcd + 8\,a^2 d^2) x^{16} - 2\,(3\,abc^2 - 4\,a^2 cd) x^8 + a^2 c^2 - 4\,((bc - 2\,ad) x^{12} - acx^4) \sqrt{dx^8 + c} \sqrt{-abc + a^2 d}}{b^2 x^{16} + 2\,abx^8 + a^2}}{48\,(a^3 bc^3 - a^4 c^2 d) x^{12}}$$

input
```
integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^12*log(((b^2*c^2 - 8*a*b*c*d + 8*
a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)
*x^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x
^8 + a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^
3*c*d)*sqrt(d*x^8 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^12), 1/24*(3*sqrt(a*b*c
 - a^2*d)*b^2*c^2*x^12*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c
)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)
) + 2*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^3*c*d)*sq
rt(d*x^8 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^12)]
```

3.896.6 Sympy [F]

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**13/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**13*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.896.7 Maxima [F]

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^{13}}\,dx$$

input
```
integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^13), x)
```

3.896.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(95) = 190$.

Time = 1.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}}\,dx =$$

$$-\frac{1}{12}d^{\frac{5}{2}}\Bigg(\frac{3\,b^2\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a^2d^2}+\frac{2\left(3\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b-6\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2}{\Big(\Big(\sqrt{d}x^4-\sqrt{dx^8+}}$$

input
```
integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/12*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c
 + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(
3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^
2*b*c - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sq
rt(d)*x^4 - sqrt(d*x^8 + c))^2 - c)^3*a^2*d^2))
```

3.896.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{x^{13}\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/(x^13*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^13*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.897 $\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.897.1 Optimal result

Integrand size = 24, antiderivative size = 851

$$\begin{aligned}
\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx
&= -\frac{\sqrt[4]{-a}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{-a}\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{-bc+ad}} \\
&\quad + \frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}} \\
&\quad - \frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}} \\
&\quad - \frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}
\end{aligned}$$

output
```
-1/8*(-a)^(1/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(
1/2))/b^(3/4)/(-a*d+b*c)^(1/2)-1/8*(-a)^(1/4)*arctan(x^2*(a*d-b*c)^(1/2)/(
-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/b^(3/4)/(a*d-b*c)^(1/2)+1/4*(cos(2*arct
an(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Ellip
ticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))
*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/d^(1/4)/(d*x^8+c)^(1/
2)-1/8*a*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^
(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+
c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/8*
d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*
x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*((
-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(
1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/16*(cos(2
*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*
EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1
/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^
4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*
d^(1/2))^2)^(1/2)/b/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/16*(cos(2*
arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4))...
```

3.897.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.08

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{x^{10}\sqrt{\frac{c+dx^8}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{10a\sqrt{c+dx^8}}$$

input
```
Integrate[x^9/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x^10*Sqrt[(c + d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^
8)/a)])/(10*a*Sqrt[c + d*x^8])
```

3.897.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {965, 983, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{x^8}{(bx^8+a)\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{ 983}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\sqrt{dx^8+c}}dx^2}{b}-\frac{a\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{2}\left(\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{a\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{ 925}$$

$$\frac{1}{2}\left(\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{a\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^4}{\sqrt{-a}}+1\right)\sqrt{dx^8+c}}}{2a}\right.}{b}\right.$$

$$\Big\downarrow \text{ 1541}$$

$$\frac{1}{2}\left(\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{a\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\surd\right.}{}}{2a}\right.}{}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{a\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\right.}{}}{2a}\right.}{}\right.$$

$$\Big\downarrow \; 761$$

$$\frac{1}{2}\left(\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{a\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{d}\left(\right.}{}}{}\right.}{}\right.$$

$$\Big\downarrow \; 2221$$

3.897. $\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

$$\frac{1}{2}\left(\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{a\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{d}(}{}}{}\right.}{}\right.$$

$$\Big\downarrow \text{2223}$$

$$\frac{1}{2}\left(\frac{\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^8+c}}-\frac{a\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{Ellipti}}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}}{}\right.}{}\right.$$

input
```
Int[x^9/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

3.897. $\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

output

```
(((Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*b*c^(1/4)*d^(1/4)*Sqrt[c +
 d*x^8]) - (a*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c]
 + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*Ar
cTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
 + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sq
rt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4
)*Sqrt[c + d*x^8])]])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*S
qrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[
d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a
]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1
/4)*d^(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sq
rt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[
c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c
^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]
*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[
b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])]])/(2*b^(1/4)*Sqrt[b*c
 - a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)
*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] +
 Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^...
```

3.897.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.897. $\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.897.4 Maple [F]

$$\int \frac{x^{9}}{\left(b \,x^{8}+a \right) \sqrt{d \,x^{8}+c}}d x$$

input
```
int(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

3.897.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.897.6 Sympy [F]

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{9}}{\left(a + b x^{8}\right) \sqrt{c + d x^{8}}}\,dx$$

input
```
integrate(x**9/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**9/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.897.7 Maxima [F]

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^9/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.897.8 Giac [F]

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^9/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.897.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^9/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^9/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.898 $\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.898.1 Optimal result

Integrand size = 22, antiderivative size = 754

$$\begin{aligned}
\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = &-\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{-bc+ad}}\\
&+\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}\\
&+\frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}\\
&+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}\\
&+\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}
\end{aligned}$$

output
```
-1/8*b^(1/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2
))/(-a)^(3/4)/(-a*d+b*c)^(1/2)-1/8*b^(1/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a)
^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(3/4)/(a*d-b*c)^(1/2)+1/8*d^(1/4)*(co
s(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)
))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1
/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1
/2))^2)^(1/2)/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/8*d^(1/4)*(cos(2*arctan(
d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Elliptic
F(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*((-a)^(1/2)*b^(1/2)*c^(1
/2)+a*d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(
1/2)/a/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/16*(cos(2*arctan(d^(1/4)*x^2/c^
(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arcta
n(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1
/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^
(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^
(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/16*(cos(2*arctan(d^(1/4)*x^2/c^(
1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan
(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1
/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^
(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a...
```

3.898.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.09

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^2\sqrt{\frac{c+dx^8}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{2a\sqrt{c+dx^8}}$$

input
```
Integrate[x/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x^2*Sqrt[(c + d*x^8)/c]*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8
)/a)])/(2*a*Sqrt[c + d*x^8])
```

3.898.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 876, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {965, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{925}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^4}{\sqrt{-a}}+1\right)\sqrt{dx^8+c}}dx^2}{2a}\right)$$

$$\Big\downarrow \text{1541}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+}{2a}\right.$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}}{2a}\right.$$

$$\Big\downarrow \text{2221}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}}{2a}\right.$$

$$\Big\downarrow \text{2223}$$

$$\frac{1}{2}\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\right.}{2\sqrt[4]{b}\sqrt{bc-}}\right.}{}}{2a}\right.$$

input `Int[x/((a + b*x^8)*Sqrt[c + d*x^8]),x]`

3.898. $\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

output
```
(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4
)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*
x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(S
qrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a
] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x
^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b
])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ell
ipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sq
rt[c + d*x^8])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt
[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x
^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c +
a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-
a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x^2
)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((S
qrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x
^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt
[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4
)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/(2*a))/2
```

3.898.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.898.4 Maple [F]

$$\int \frac{x}{(b\,x^{8}+a)\,\sqrt{d\,x^{8}+c}}dx$$

input
```
int(x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

3.898.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.898.6 Sympy [F]

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.898.7 Maxima [F]

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.898.8 Giac [F]

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.898.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.899 $\int \frac{1}{x^7(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.899.1 Optimal result

Integrand size = 24, antiderivative size = 878

$$\begin{aligned}
&\int \frac{1}{x^7\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx\\
&=-\frac{\sqrt{c+dx^8}}{6acx^6}-\frac{b^{5/4}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{7/4}\sqrt{bc-ad}}-\frac{b^{5/4}\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{7/4}\sqrt{-bc+ad}}\\
&\quad-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{12ac^{5/4}\sqrt{c+dx^8}}\\
&\quad-\frac{b\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}\\
&\quad-\frac{b\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a^2\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}\\
&\quad-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}\\
&\quad-\frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}
\end{aligned}$$

output
```
-1/8*b^(5/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2
))/(-a)^(7/4)/(-a*d+b*c)^(1/2)-1/8*b^(5/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a)
^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(7/4)/(a*d-b*c)^(1/2)-1/6*(d*x^8+c)^(
1/2)/a/c/x^6-1/12*d^(3/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos
(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4)
)),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^
(1/2)/a/c^(5/4)/(d*x^8+c)^(1/2)-1/8*b*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^
(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan
(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(
c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/(
a*d+b*c)/(d*x^8+c)^(1/2)-1/8*b*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))
^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4
)*x^2/c^(1/4))),1/2*2^(1/2))*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/
2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/(a*d
+b*c)/(d*x^8+c)^(1/2)-1/16*b*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/
cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(
1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/
2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*
d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(
a*d+b*c)/(d*x^8+c)^(1/2)-1/16*b*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^...
```

3.899.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^7(a+bx^8)\sqrt{c+dx^8}}\,dx$$
$$=\frac{-5a(c+dx^8)-5(3bc+ad)x^8\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-bdx^{16}\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}\left(\frac{5}{4}\right.}{30a^2cx^6\sqrt{c+dx^8}}$$

input
```
Integrate[1/(x^7*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-5*a*(c + d*x^8) - 5*(3*b*c + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[1/4,
1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] - b*d*x^16*Sqrt[1 + (d*x^8)/c]*Ap
pellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a)])/(30*a^2*c*x^6*Sqrt[c
 + d*x^8])
```

3.899.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 1006, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {965, 980, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+bx^8\right) \sqrt{c+dx^8}}dx$$

$$\Big\downarrow \; 965$$

$$\frac{1}{2}\int \frac{1}{x^8 \left(bx^8+a\right) \sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \; 980$$

$$\frac{1}{2}\left(\frac{\int -\frac{bdx^8+3bc+ad}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2}{3ac}-\frac{\sqrt{c+dx^8}}{3acx^6}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(-\frac{\int \frac{bdx^8+3bc+ad}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2}{3ac}-\frac{\sqrt{c+dx^8}}{3acx^6}\right)$$

$$\Big\downarrow \; 1021$$

$$\frac{1}{2}\left(-\frac{3bc\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2+d\int \frac{1}{\sqrt{dx^8+c}}dx^2}{3ac}-\frac{\sqrt{c+dx^8}}{3acx^6}\right)$$

$$\Big\downarrow \; 761$$

$$\frac{1}{2}\left(-\frac{3bc\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}}}{3ac}-\frac{\sqrt{c+dx^8}}{3acx^6}\right)$$

$$\Big\downarrow \; 925$$

$$\frac{1}{2}\left(-\frac{3bc\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^4}{\sqrt{-a}}+1\right)\sqrt{dx^8+c}}dx^2}{2a}\right)+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}}}{3ac}-\right.$$

$$\Big\downarrow \ 1541$$

$$\frac{1}{2}\left(-\frac{3bc\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-}\right.}{}}{2a}\right.}{3ac}\right.$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(-\frac{3bc\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\right.}{}}{2a}\right.}{3ac}\right.$$

$$\Big\downarrow \ 761$$

$$\frac{1}{2}\left(-\frac{3bc\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}}{2a}+\right.}{}\right.$$

3.899. $\int \frac{1}{x^7(a+bx^8)\sqrt{c+dx^8}}\,dx$

$$\Big\downarrow \; 2221$$

$$\frac{1}{2}\Bigg(-\frac{\frac{d^{3/4}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^8+c}}+3bc\Bigg(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{EllipticF}\Big(2\,\mathrm{arc}}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}}{}$$

$$\Big\downarrow \; 2223$$

$$\frac{1}{2}\Bigg(-\frac{\frac{d^{3/4}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^8+c}}+3bc\Bigg(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{EllipticF}\Big(2\,\mathrm{arc}}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}}{}$$

3.899. $\int \frac{1}{x^7(a+bx^8)\sqrt{c+dx^8}}\,dx$

input

```
Int[1/(x^7*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output

```
(-1/3*Sqrt[c + d*x^8]/(a*c*x^6) - ((d^(3/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(
c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(
1/4)], 1/2])/(2*c^(1/4)*Sqrt[c + d*x^8]) + 3*b*c*(((a*((Sqrt[b]*Sqrt[c])/S
qrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[
c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c
^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]
*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt
[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*
c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4
)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqr
t[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d
^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^8])))/(b*c + a
*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + S
qrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan
[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (
Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c]
 + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqr
t[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d
])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^
4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqr...
```

3.899.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

3.899. $\int \frac{1}{x^7(a+bx^8)\sqrt{c+dx^8}}\,dx$

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.899.4 Maple [F]

$$\int \frac{1}{x^{7} \left(b\,x^{8}+a \right) \sqrt{d\,x^{8}+c}}d x$$

input
```
int(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

3.899.5 Fricas [F]

$$\int \frac{1}{x^7 (a+bx^8) \sqrt{c+dx^8}} \, dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^7} \, dx$$

input
```
integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^8 + c)/(b*d*x^23 + (b*c + a*d)*x^15 + a*c*x^7), x)
```

3.899.6 Sympy [F]

$$\int \frac{1}{x^7 (a+bx^8) \sqrt{c+dx^8}} \, dx = \int \frac{1}{x^7 \left(a+bx^8\right) \sqrt{c+dx^8}} \, dx$$

input
```
integrate(1/x**7/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**7*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.899.7 Maxima [F]

$$\int \frac{1}{x^7\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}\,x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^7), x)
```

3.899.8 Giac [F]

$$\int \frac{1}{x^7\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}\,x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^7), x)
```

3.899.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{x^7\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/(x^7*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^7*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.900 $\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.900.1 Optimal result

Integrand size = 24, antiderivative size = 1005

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^2\sqrt{c+dx^8}}{2b\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^4\right)}$$

$$+\frac{(-a)^{3/4}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{5/4}\sqrt{bc-ad}}-\frac{(-a)^{3/4}\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{5/4}\sqrt{-bc+ad}}$$

$$-\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2bd^{3/4}\sqrt{c+dx^8}}$$

$$+\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4bd^{3/4}\sqrt{c+dx^8}}$$

$$+\frac{a\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$

$$+\frac{a\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$

$$+\frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)\right.}{16b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$

$$-\frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\right.}{16b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$

output
```
1/8*(-a)^(3/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1
/2))/b^(5/4)/(-a*d+b*c)^(1/2)-1/8*(-a)^(3/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-
a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/b^(5/4)/(a*d-b*c)^(1/2)+1/2*x^2*(d*x^8+c
)^(1/2)/b/d^(1/2)/(c^(1/2)+x^4*d^(1/2))-1/2*c^(1/4)*(cos(2*arctan(d^(1/4)*
x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticE(sin(2*
arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)
/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/d^(3/4)/(d*x^8+c)^(1/2)+1/4*c^(1/4)*(cos
(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4))
)*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d
^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/d^(3/4)/(d*x^8+c)^(1/2
)-1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x
^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^
(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2)
)*(-a)^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*
((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/(a*d+b*c
)/(d*x^8+c)^(1/2)+1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*
arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4)))
,-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^
(1/2),1/2*2^(1/2))*(-a)^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^
(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1...
```

3.900.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.06

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^{14}\sqrt{\frac{c+dx^8}{c}}\,\text{AppellF1}\left(\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{14a\sqrt{c+dx^8}}$$

input
```
Integrate[x^13/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x^14*Sqrt[(c + d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x
^8)/a)])/(14*a*Sqrt[c + d*x^8])
```

3.900.3 Rubi [A] (warning: unable to verify)

Time = 1.57 (sec) , antiderivative size = 1111, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {965, 983, 834, 27, 761, 993, 1510, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{x^{12}}{(bx^8+a)\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{ 983}$$

$$\frac{1}{2}\left(\frac{\int \frac{x^4}{\sqrt{dx^8+c}}dx^2}{b}-\frac{a\int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{ 834}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{c}\int \frac{1}{\sqrt{dx^8+c}}dx^2}{\sqrt{d}}-\frac{\sqrt{c}\int \frac{\sqrt{c}-\sqrt{d}x^4}{\sqrt{c}\sqrt{dx^8+c}}dx^2}{\sqrt{d}}}{b}-\frac{a\int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{c}\int \frac{1}{\sqrt{dx^8+c}}dx^2}{\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^4}{\sqrt{dx^8+c}}dx^2}{\sqrt{d}}}{b}-\frac{a\int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^8}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^4}{\sqrt{dx^8+c}}dx^2}{\sqrt{d}}}{b}-\frac{a\int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{ 993}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^8}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x^4}{\sqrt{dx^8+c}}dx^2}{\sqrt{d}}}{b}-\frac{a\left(\frac{\int\frac{1}{\left(\sqrt{b}x^4+\sqrt{-a}\right)\sqrt{dx^8+c}}dx^2}{2\sqrt{b}}-\frac{\int\frac{}{\left(\sqrt{-a}-\sqrt{b}\right.}}{2}\right.}{b}\right.$$

$$\Big\downarrow \text{1510}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^8}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{x^2\sqrt{c+dx^8}}{\sqrt{c}+\sqrt{d}x^4}}{\sqrt{d}}}{b}\right.$$

$$\Big\downarrow \text{1541}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^8}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{x^2\sqrt{c+dx^8}}{\sqrt{c}+\sqrt{d}x^4}}{\sqrt{d}}}{b}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^8}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{x^2\sqrt{c+dx^8}}{\sqrt{c}+\sqrt{d}x^4}}{\sqrt{d}}}{b}\right.$$

$$\Big\downarrow \text{761}$$

3.900. $\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^8}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{x^2\sqrt{c+dx^8}}{\sqrt{c}+\sqrt{d}x^4}}{\sqrt{d}}}{b}\right.$$

$$\Big\downarrow \text{2221}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{dx^8+c}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{dx^8+c}}-\frac{x^2\sqrt{dx^8+c}}{\sqrt{d}x^4+\sqrt{c}}}{\sqrt{d}}}{b}\right.$$

$$\Big\downarrow \text{2223}$$

3.900. $\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{dx^8+c}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{dx^8+c}}-\frac{x^2\sqrt{dx^8+c}}{\sqrt{d}x^4+\sqrt{c}}}{\sqrt{d}}}{b}\right.$$

input
```
Int[x^13/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

3.900. $\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

output

```
((-((-((x^2*Sqrt[c + d*x^8])/(Sqrt[c] + Sqrt[d]*x^4)) + (c^(1/4)*(Sqrt[c]
+ Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*Arc
Tan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(d^(1/4)*Sqrt[c + d*x^8]))/Sqrt[d]) + (c
^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]
*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^
8]))/b - (a*(-1/2*(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(Sqrt[c]
+ Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*Arc
Tan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
+ (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((Sqrt[b]*Sqrt[c] + Sqrt[
-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*
x^8])])/(2*(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) - (((a*Sqrt[c])/(-a)^(3/2)
+ Sqrt[d]/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqr
t[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]
*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/
4)*d^(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/Sqrt[b] + (-1/2*((Sqrt[b]*Sqrt[
c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(S
qrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/
(c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-
a]*Sqrt[d])*(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]
*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*(-a)^(1/4)*b^(1/4)*Sqrt...
```

3.900.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 965

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

3.900. $\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.900.4 Maple [F]

$$\int \frac{x^{13}}{(b\,x^{8}+a)\,\sqrt{d\,x^{8}+c}}dx$$

input
```
int(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

3.900.5 Fricas [F]

$$\int \frac{x^{13}}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx=\int \frac{x^{13}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^8 + c)*x^13/(b*d*x^16 + (b*c + a*d)*x^8 + a*c), x)
```

3.900.6 Sympy [F]

$$\int \frac{x^{13}}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx=\int \frac{x^{13}}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**13/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**13/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.900.7 Maxima [F]

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^{13}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^13/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.900.8 Giac [F]

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^{13}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^13/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.900.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^{13}}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^13/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^13/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.901 $\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.901.1 Optimal result

Integrand size $=24$, antiderivative size $=768$

$$
\begin{aligned}
\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx &= \frac{\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}-\frac{\arctan\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{-bc+ad}}\\
&-\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}\\
&-\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}\\
&+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}\\
&-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\mathrm{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}
\end{aligned}
$$

output

```
1/8*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(
1/4)/b^(1/4)/(-a*d+b*c)^(1/2)-1/8*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^
(1/4)/(d*x^8+c)^(1/2))/(-a)^(1/4)/b^(1/4)/(a*d-b*c)^(1/2)-1/16*(cos(2*arct
an(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Ellip
ticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d
^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(
1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/
2))^2)^(1/2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(-a)^(1/2)/b^(1/2)/(d*x^8+c)^(1/2)+
1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2
/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(
1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))
*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(
c^(1/2)+x^4*d^(1/2))^2)^(1/2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(-a)^(1/2)/b^(1/2)
/(d*x^8+c)^(1/2)-1/8*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/
cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1
/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)-(-a)^(1/2)*d^(1/2)/b^(1/
2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/c^(1/4)/(a*d+b*c)/(d*x^8+c)^
(1/2)-1/8*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arcta
n(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2
^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*...
```

3.901.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.08

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^6\sqrt{\frac{c+dx^8}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{6a\sqrt{c+dx^8}}$$

input

```
Integrate[x^5/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output

```
(x^6*Sqrt[(c + d*x^8)/c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8
)/a)])/(6*a*Sqrt[c + d*x^8])
```

3.901.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 881, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {965, 993, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(a+bx^8\right)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^4}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{993}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\left(\sqrt{b}x^4+\sqrt{-a}\right)\sqrt{dx^8+c}}dx^2}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^4\right)\sqrt{dx^8+c}}dx^2}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{1541}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\sqrt{c}\left(\sqrt{b}x^4+\sqrt{-a}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}}{2\sqrt{b}}-\frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(\sqrt{b}x^4+\sqrt{-a}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}}{2\sqrt{b}}-\frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}(}{}}{2\sqrt{}}\right.$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(\sqrt{b}x^4+\sqrt{-a}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}}{2\sqrt{b}}\right.$$

$$\Big\downarrow \text{2221}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)}{4\sqrt{-a}\sqrt{b}\sqrt{d}}\right.}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}\right.}{ad+bc}}{2\sqrt{b}}\right.$$

$$\Big\downarrow \text{2223}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)}{4\sqrt{-a}\sqrt{b}\sqrt{d}}\right.}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^8+c}}\right.}{bc+ad}}{2\sqrt{b}}\right.$$

input

```
Int[x^5/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

3.901. $\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

output
```
(-1/2*(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^
4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(
Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])
*ArcTanh[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*(
-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) - (((a*Sqrt[c])/(-a)^(3/2) + Sqrt[d]/Sq
rt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]
*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqr
t[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*S
qrt[c + d*x^8])))/(b*c + a*d))/Sqrt[b] + (-1/2*((Sqrt[b]*Sqrt[c] + Sqrt[-a
]*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqr
t[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(c^(1/4)*(b*
c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*
(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) +
((Sqrt[c]/Sqrt[-a] + Sqrt[d]/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*
x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[c]*(Sqrt[b] - (Sqrt[
-a]*Sqrt[d])/Sqrt[c])^2)/(Sqrt[-a]*Sqrt[b]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2
)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/(2*Sq
rt[b]))/2
```

3.901.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.901. $\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.901.4 Maple [F]

$$\int \frac{x^{5}}{\left(b\,x^{8}+a\right)\sqrt{d\,x^{8}+c}}d x$$

input
```
int(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

3.901.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.901.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**5/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.901.7 Maxima [F]

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^5}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.901.8 Giac [F]

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^5}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^5/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.901.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^5}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^5/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^5/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.902 $\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}} \, dx$

3.902.1 Optimal result

Integrand size $= 24$, antiderivative size $= 1032$

$$\int \frac{1}{x^3(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{c+dx^8}}{2acx^2}+\frac{\sqrt{d}x^2\sqrt{c+dx^8}}{2ac\left(\sqrt{c}+\sqrt{d}x^4\right)}$$

$$+\frac{b^{3/4}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{5/4}\sqrt{bc-ad}}-\frac{b^{3/4}\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{5/4}\sqrt{-bc+ad}}$$

$$-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2ac^{3/4}\sqrt{c+dx^8}}$$

$$+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4ac^{3/4}\sqrt{c+dx^8}}$$

$$+\frac{b\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$

$$+\frac{b\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$

$$+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\right.}{16(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$

$$-\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right.}{16(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$

output
```
1/8*b^(3/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2)
)/(-a)^(5/4)/(-a*d+b*c)^(1/2)-1/8*b^(3/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^
(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(5/4)/(a*d-b*c)^(1/2)-1/2*(d*x^8+c)^(1
/2)/a/c/x^2+1/2*x^2*d^(1/2)*(d*x^8+c)^(1/2)/a/c/(c^(1/2)+x^4*d^(1/2))-1/2*
d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*
x^2/c^(1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c
^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(3/4)/(d
*x^8+c)^(1/2)+1/4*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos
(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4)
)),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^
(1/2)/a/c^(3/4)/(d*x^8+c)^(1/2)-1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2
)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)
*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/
2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(
1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^
(3/2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/16*(cos(2*arctan(d^(1/4)
*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(
2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2
/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(c^(1/2)+x^4*d^(1
/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(...
```

3.902.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}}\,dx = \frac{-21a(c+dx^8)+7(-bc+ad)x^8\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+3bdx^{16}\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}}{42a^2cx^2\sqrt{c+dx^8}}$$

input
```
Integrate[1/(x^3*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-21*a*(c + d*x^8) + 7*(-(b*c) + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[3/4
, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*x^16*Sqrt[1 + (d*x^8)/c
]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)/a)])/(42*a^2*c*x^2*S
qrt[c + d*x^8])
```

3.902.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 1021, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {965, 980, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^8+a) \sqrt{dx^8+c}} dx^2$$

$$\Big\downarrow \text{980}$$

$$\frac{1}{2} \left(\frac{\int -\frac{x^4(-bdx^8+bc-ad)}{(bx^8+a)\sqrt{dx^8+c}} dx^2}{ac} - \frac{\sqrt{c+dx^8}}{acx^2} \right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(-\frac{\int \frac{x^4(-bdx^8+bc-ad)}{(bx^8+a)\sqrt{dx^8+c}} dx^2}{ac} - \frac{\sqrt{c+dx^8}}{acx^2} \right)$$

$$\Big\downarrow \text{1054}$$

$$\frac{1}{2} \left(-\frac{\int \left(\frac{bcx^4}{(bx^8+a)\sqrt{dx^8+c}} - \frac{dx^4}{\sqrt{dx^8+c}} \right) dx^2}{ac} - \frac{\sqrt{c+dx^8}}{acx^2} \right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{-\frac{\sqrt{b}c^{3/4}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8\sqrt{-a}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}} + \frac{b^{3/4}c\arctan\left(\frac{\sqrt{b}}{\sqrt[4]{-a}}\right.}{4\sqrt[4]{-a}\sqrt{bc}}}{} \right.$$

input
```
Int[1/(x^3*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output

```
(-(Sqrt[c + d*x^8]/(a*c*x^2)) - (-((Sqrt[d]*x^2*Sqrt[c + d*x^8])/(Sqrt[c]
+ Sqrt[d]*x^4)) + (b^(3/4)*c*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1
/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4)*Sqrt[b*c - a*d]) - (b^(3/4)*c*ArcTanh
[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4
)*Sqrt[b*c - a*d]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*
x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
 1/2])/Sqrt[c + d*x^8] - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
+ d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/
4)], 1/2])/(2*Sqrt[c + d*x^8]) - (b*c^(3/4)*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/
Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[
d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*(b*c + a*d)
*Sqrt[c + d*x^8]) - (b*c^(3/4)*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1
/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*El
lipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*(b*c + a*d)*Sqrt[c + d*x
^8]) - (Sqrt[b]*c^(3/4)*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] +
Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[
b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*A
rcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*Sqrt[-a]*d^(1/4)*(b*c + a*d)*Sqrt[c
 + d*x^8]) + (Sqrt[b]*c^(3/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt
[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Elliptic...
```

3.902.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 965

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 980

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.902.4 Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{8}+a\right) \sqrt{d\,x^{8}+c}}dx$$

input
```
int(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

3.902.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.902.6 Sympy [F]

$$\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**3/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.902.7 Maxima [F]

$$\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^3), x)
```

3.902.8 Giac [F]

$$\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^3), x)
```

3.902.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^3\,(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/(x^3*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.903 $\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.903.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{x^5\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}\left(\frac{5}{8},1,\frac{1}{2},\frac{13}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}$$

output
```
1/5*x^5*AppellF1(5/8,1,1/2,13/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/(d*
x^8+c)^(1/2)
```

3.903.2 Mathematica [A] (verified)

Time = 10.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{x^5\sqrt{\frac{c+dx^8}{c}}\operatorname{AppellF1}\left(\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{5a\sqrt{c+dx^8}}$$

input
```
Integrate[x^4/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x^5*Sqrt[(c + d*x^8)/c]*AppellF1[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^
8)/a)])/(5*a*Sqrt[c + d*x^8])
```

3.903.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^8}{c}+1}\int \frac{x^4}{(bx^8+a)\sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^5\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(\frac{5}{8},1,\frac{1}{2},\frac{13}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}$$

input
```
Int[x^4/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x^5*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8, 1, 1/2, 13/8, -((b*x^8)/a), -((d*x^
8)/c)])/(5*a*Sqrt[c + d*x^8])
```

3.903.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.903.4 Maple [F]

$$\int \frac{x^{4}}{\left(b\,x^{8}+a\right)\sqrt{d\,x^{8}+c}}dx$$

input
```
int(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

3.903.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\text{Timed out}$$

input
```
integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.903.6 Sympy [F]

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**4/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**4/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.903.7 Maxima [F]

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.903.8 Giac [F]

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.903.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^4}{\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^4/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.904 $\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.904.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{x^3\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}\left(\frac{3}{8},1,\frac{1}{2},\frac{11}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}$$

output
```
1/3*x^3*AppellF1(3/8,1,1/2,11/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/(d*
x^8+c)^(1/2)
```

3.904.2 Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{x^3\sqrt{\frac{c+dx^8}{c}}\operatorname{AppellF1}\left(\frac{3}{8},\frac{1}{2},1,\frac{11}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{3a\sqrt{c+dx^8}}$$

input
```
Integrate[x^2/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x^3*Sqrt[(c + d*x^8)/c]*AppellF1[3/8, 1/2, 1, 11/8, -((d*x^8)/c), -((b*x^
8)/a)])/(3*a*Sqrt[c + d*x^8])
```

3.904.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}dx$$
$$\Big\downarrow \text{1013}$$
$$\frac{\sqrt{\frac{dx^8}{c}+1}\int \frac{x^2}{(bx^8+a)\sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$
$$\Big\downarrow \text{1012}$$
$$\frac{x^3\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(\frac{3}{8},1,\frac{1}{2},\frac{11}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}$$

input
```
Int[x^2/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x^3*Sqrt[1 + (d*x^8)/c]*AppellF1[3/8, 1, 1/2, 11/8, -((b*x^8)/a), -((d*x^
8)/c)])/(3*a*Sqrt[c + d*x^8])
```

3.904.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.904.4 Maple [F]

$$\int \frac{x^2}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}dx$$

input
```
int(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

3.904.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Not
integrable (provided residues have no relations)
```

3.904.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**2/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.904.7 Maxima [F]

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.904.8 Giac [F]

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.904.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^2/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.905 $\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.905.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{x\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}\left(\frac{1}{8},1,\frac{1}{2},\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}$$

output
```
x*AppellF1(1/8,1,1/2,9/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/(d*x^8+c)^
(1/2)
```

3.905.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. $2(59) = 118$.

Time = 10.20 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx =$$

$$-\frac{9acx\operatorname{AppellF1}\left(\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{(a+bx^8)\sqrt{c+dx^8}\,\big(-9ac\operatorname{AppellF1}\big(\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\big)+4x^8\,\big(2bc\operatorname{AppellF1}\big(\frac{9}{8},\frac{1}{2},2,\frac{17}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}}$$

input
```
Integrate[1/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-9*a*c*x*AppellF1[1/8, 1/2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)])/((a + b*
x^8)*Sqrt[c + d*x^8]*(-9*a*c*AppellF1[1/8, 1/2, 1, 9/8, -((d*x^8)/c), -((b
*x^8)/a)] + 4*x^8*(2*b*c*AppellF1[9/8, 1/2, 2, 17/8, -((d*x^8)/c), -((b*x^
8)/a)] + a*d*AppellF1[9/8, 3/2, 1, 17/8, -((d*x^8)/c), -((b*x^8)/a)])))
```

3.905.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}dx$$
$$\Big\downarrow \text{937}$$
$$\frac{\sqrt{\frac{dx^8}{c}+1}\int \frac{1}{(bx^8+a)\sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$
$$\Big\downarrow \text{936}$$
$$\frac{x\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(\frac{1}{8},1,\frac{1}{2},\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}$$

input
```
Int[1/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x*Sqrt[1 + (d*x^8)/c]*AppellF1[1/8, 1, 1/2, 9/8, -((b*x^8)/a), -((d*x^8)/
c)])/(a*Sqrt[c + d*x^8])
```

3.905.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.905.4 Maple [F]

$$\int \frac{1}{(b\,x^{8}+a)\,\sqrt{d\,x^{8}+c}}dx$$

input
```
int(1/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(1/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

3.905.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.905.6 Sympy [F]

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.905.7 Maxima [F]

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.905.8 Giac [F]

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.905.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.906 $\int \frac{1}{x^2(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.906.1 Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=-\frac{\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(-\frac{1}{8},1,\frac{1}{2},\frac{7}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}$$

output
```
-AppellF1(-1/8,1,1/2,7/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/x/(d*x^8+c
)^(1/2)
```

3.906.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(62) = 124.

Time = 10.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^2\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$
$$=\frac{-35a(c+dx^8)-5(bc-3ad)x^8\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+7bdx^{16}\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}}{35a^2cx\sqrt{c+dx^8}}$$

input
```
Integrate[1/(x^2*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-35*a*(c + d*x^8) - 5*(b*c - 3*a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[7/8,
 1/2, 1, 15/8, -((d*x^8)/c), -((b*x^8)/a)] + 7*b*d*x^16*Sqrt[1 + (d*x^8)/c
]*AppellF1[15/8, 1/2, 1, 23/8, -((d*x^8)/c), -((b*x^8)/a)])/(35*a^2*c*x*Sq
rt[c + d*x^8])
```

3.906.3 Rubi [A] (verified)

Time $=0.21$ (sec) , antiderivative size $=62$, normalized size of antiderivative $=1.00$, number of steps used $=2$, number of rules used $=2$, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used $=\{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+bx^8\right)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^8}{c}+1}\int \frac{1}{x^2(bx^8+a)\sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\operatorname{AppellF1}\left(-\frac{1}{8},1,\frac{1}{2},\frac{7}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}$$

input
```
Int[1/(x^2*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
-((Sqrt[1 + (d*x^8)/c]*AppellF1[-1/8, 1, 1/2, 7/8, -((b*x^8)/a), -((d*x^8)
/c)])/(a*x*Sqrt[c + d*x^8]))
```

3.906.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.906.4 Maple [F]

$$\int \frac{1}{x^2 \left(b\,x^8+a\right) \sqrt{d\,x^8+c}}dx$$

input
```
int(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

3.906.5 Fricas [F]

$$\int \frac{1}{x^2 (a+bx^8) \sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^8 + c)/(b*d*x^18 + (b*c + a*d)*x^10 + a*c*x^2), x)
```

3.906.6 Sympy [F]

$$\int \frac{1}{x^2\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{x^2\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**2/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.906.7 Maxima [F]

$$\int \frac{1}{x^2\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^2), x)
```

3.906.8 Giac [F]

$$\int \frac{1}{x^2\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^2), x)
```

3.906.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^8) \sqrt{c+dx^8}}\,dx=\int \frac{1}{x^2\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/(x^2*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.907 $\int \frac{1}{x^4(a+bx^8)\sqrt{c+dx^8}}\,dx$

3.907.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=-\frac{\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(-\frac{3}{8},1,\frac{1}{2},\frac{5}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}$$

output
```
-1/3*AppellF1(-3/8,1,1/2,5/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/x^3/(d
*x^8+c)^(1/2)
```

3.907.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(64) = 128.

Time = 10.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$
$$=\frac{-65a(c+dx^8)+13(-3bc+ad)x^8\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+5bdx^{16}\sqrt{1+\frac{dx^8}{c}}\,\text{Appell}}{195a^2cx^3\sqrt{c+dx^8}}$$

input
```
Integrate[1/(x^4*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-65*a*(c + d*x^8) + 13*(-3*b*c + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[5/
8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] + 5*b*d*x^16*Sqrt[1 + (d*x^8)
/c]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c), -((b*x^8)/a)])/(195*a^2*c*x
^3*Sqrt[c + d*x^8])
```

3.907.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^8) \sqrt{c+dx^8}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^8}{c}+1} \int \frac{1}{x^4(bx^8+a)\sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(-\frac{3}{8},1,\frac{1}{2},\frac{5}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}$$

input
```
Int[1/(x^4*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
-1/3*(Sqrt[1 + (d*x^8)/c]*AppellF1[-3/8, 1, 1/2, 5/8, -((b*x^8)/a), -((d*x
^8)/c)])/(a*x^3*Sqrt[c + d*x^8])
```

3.907.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.907.4 Maple [F]

$$\int \frac{1}{x^{4} \left(b \,x^{8}+a \right) \sqrt{d \,x^{8}+c}}d x$$

input
```
int(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

3.907.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^8) \sqrt{c+dx^8}}\, dx = \text{Timed out}$$

input
```
integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.907.6 Sympy [F]

$$\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**4/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.907.7 Maxima [F]

$$\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^4), x)
```

3.907.8 Giac [F]

$$\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^4), x)
```

3.907.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^8) \sqrt{c+dx^8}}\, dx = \int \frac{1}{x^4 \left(b\,x^8+a\right) \sqrt{d\,x^8+c}}\, dx$$

input

```
int(1/(x^4*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output

```
int(1/(x^4*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.908 $\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

3.908.1 Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{c+dx^8}}{4b^2d} - \frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)(a+bx^8)} + \frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}$$

output
```
1/8*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(
5/2)/(-a*d+b*c)^(3/2)+1/4*(d*x^8+c)^(1/2)/b^2/d-1/8*a^2*(d*x^8+c)^(1/2)/b^
2/(-a*d+b*c)/(b*x^8+a)
```

3.908.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\frac{\sqrt{b}\sqrt{c+dx^8}(-3a^2d+2b^2cx^8+2ab(c-dx^8))}{d(bc-ad)(a+bx^8)} + \frac{a(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{8b^{5/2}}$$

input
```
Integrate[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((Sqrt[b]*Sqrt[c + d*x^8]*(-3*a^2*d + 2*b^2*c*x^8 + 2*a*b*(c - d*x^8)))/(d
*(b*c - a*d)*(a + b*x^8)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*
x^8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(8*b^(5/2))
```

3.908.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 100, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{23}}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{8} \int \frac{x^{16}}{\left(bx^8+a\right)^2 \sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{8} \left(\frac{\int -\frac{a(2bc-ad)-2b(bc-ad)x^8}{2(bx^8+a)\sqrt{dx^8+c}}dx^8}{b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^8}}{b^2\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8} \left(-\frac{\int \frac{a(2bc-ad)-2b(bc-ad)x^8}{(bx^8+a)\sqrt{dx^8+c}}dx^8}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^8}}{b^2\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{8} \left(-\frac{a(4bc-3ad)\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^8-\frac{4\sqrt{c+dx^8}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^8}}{b^2\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{8} \left(-\frac{\frac{2a(4bc-3ad)\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{d}-\frac{4\sqrt{c+dx^8}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^8}}{b^2\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{8}\left(-\frac{a^2\sqrt{c+dx^8}}{b^2\left(a+bx^8\right)(bc-ad)}-\frac{-\frac{2a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}-\frac{4\sqrt{c+dx^8}(bc-ad)}{d}}{2b^2(bc-ad)}\right)$$

input
```
Int[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-((a^2*Sqrt[c + d*x^8])/(b^2*(b*c - a*d)*(a + b*x^8))) - ((-4*(b*c - a*d)
*Sqrt[c + d*x^8])/d - (2*a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8
])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a*d]))/(2*b^2*(b*c - a*d)))/8
```

3.908.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.908.4 Maple [A] (verified)

Time = 5.95 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$\frac{-\frac{3d\left(ad-\frac{4bc}{3}\right)a\left(b\,x^{8}+a\right)\arctan\left(\frac{b\sqrt{d\,x^{8}+c}}{\sqrt{(ad-bc)b}}\right)}{8}+\frac{3\left(-\frac{2b^{2}c\,x^{8}}{3}-\frac{2a\left(-d\,x^{8}+c\right)b}{3}+a^{2}d\right)\sqrt{(ad-bc)b}\,\sqrt{d\,x^{8}+c}}{8}}{b^{2}(ad-bc)d(b\,x^{8}+a)\sqrt{(ad-bc)b}}$	133

input
```
int(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/8/((a*d-b*c)*b)^(1/2)*(-d*(a*d-4/3*b*c)*a*(b*x^8+a)*arctan(b*(d*x^8+c)^(
1/2)/((a*d-b*c)*b)^(1/2))+(-2/3*b^2*c*x^8-2/3*a*(-d*x^8+c)*b+a^2*d)*((a*d-
b*c)*b)^(1/2)*(d*x^8+c)^(1/2))/d/b^2/(a*d-b*c)/(b*x^8+a)
```

3.908.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(103) = 206.

Time = 0.28 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.86

$$\int \frac{x^{23}}{\left(a+bx^{8}\right)^{2}\sqrt{c+dx^{8}}}\,dx$$

$$= \Bigg[\frac{((4\,ab^{2}cd-3\,a^{2}bd^{2})x^{8}+4\,a^{2}bcd-3\,a^{3}d^{2})\sqrt{b^{2}c-abd}\log\left(\frac{bdx^{8}+2\,bc-ad+2\,\sqrt{dx^{8}+c}\sqrt{b^{2}c-abd}}{bx^{8}+a}\right)+2\,(2\,(b^{4}c^{2}-2}{16\,(ab^{5}c^{2}d-2\,a^{2}b^{4}cd^{2}+a^{3}b^{3}d^{3}+(b^{6}c^{2}d-2\,ab^{5}cd^{2}+a}$$

$$-\frac{((4\,ab^{2}cd-3\,a^{2}bd^{2})x^{8}+4\,a^{2}bcd-3\,a^{3}d^{2})\sqrt{-b^{2}c+abd}\arctan\left(\frac{\sqrt{dx^{8}+c}\sqrt{-b^{2}c+abd}}{bdx^{8}+bc}\right)-(2\,(b^{4}c^{2}-2\,ab^{3}cd-}{8\,(ab^{5}c^{2}d-2\,a^{2}b^{4}cd^{2}+a^{3}b^{3}d^{3}+(b^{6}c^{2}d-2\,ab^{5}cd^{2}+a^{2}b^{4}}$$

input
```
integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^
2*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a
*b*d))/(b*x^8 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a
*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^8 + c))/(a*b^5*c^2*d - 2*
a^2*b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^
8), -1/8*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt
(-b^2*c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*
c)) - (2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a*b^3*c^2 - 5*a^2*b
^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^8 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^
3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^8)]
```

3.908.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\text{Timed out}$$

input
```
integrate(x**23/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Timed out
```

3.908.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.908.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{dx^8+c}a^2d}{8\,(b^3c-ab^2d)((dx^8+c)b-bc+ad)} - \frac{(4\,abc-3\,a^2d)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{8\,(b^3c-ab^2d)\sqrt{-b^2c+abd}} + \frac{\sqrt{dx^8+c}}{4\,b^2d}$$

input
```
integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/8*sqrt(d*x^8 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^8 + c)*b - b*c + a*d))
 - 1/8*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/
((b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/4*sqrt(d*x^8 + c)/(b^2*d)
```

3.908.9 Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{d\,x^8+c}}{4\,b^2\,d} - \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^8+c}\,(3\,a\,d-4\,b\,c)}{(3\,a^2\,d-4\,a\,b\,c)\,\sqrt{a\,d-b\,c}}\right)\,(3\,a\,d-4\,b\,c)}{8\,b^{5/2}\,{(a\,d-b\,c)}^{3/2}} + \frac{a^2\,d\,\sqrt{d\,x^8+c}}{2\,(a\,d-b\,c)\,\left(4\,b^3\,\left(d\,x^8+c\right)-4\,b^3\,c+4\,a\,b^2\,d\right)}$$

input
```
int(x^23/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
(c + d*x^8)^(1/2)/(4*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^8)^(1/2)*(3*a*d
- 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(8*b^(
5/2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^8)^(1/2))/(2*(a*d - b*c)*(4*b^3*
(c + d*x^8) - 4*b^3*c + 4*a*b^2*d))
```

3.909 $\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

3.909.1 Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{a\sqrt{c+dx^8}}{8b(bc-ad)\,(a+bx^8)} - \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

output
```
-1/8*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2
)/(-a*d+b*c)^(3/2)+1/8*a*(d*x^8+c)^(1/2)/b/(-a*d+b*c)/(b*x^8+a)
```

3.909.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\frac{a\sqrt{b}\sqrt{c+dx^8}}{(bc-ad)(a+bx^8)} - \frac{(2bc-ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{8b^{3/2}}$$

input
```
Integrate[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((a*Sqrt[b]*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) - ((2*b*c - a*d)*Ar
cTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/
(8*b^(3/2))
```

3.909.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow 948$$

$$\frac{1}{8}\int \frac{x^8}{(bx^8+a)^2\sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow 87$$

$$\frac{1}{8}\left(\frac{(2bc-ad)\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^8}{2b(bc-ad)}+\frac{a\sqrt{c+dx^8}}{b(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{8}\left(\frac{(2bc-ad)\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{bd(bc-ad)}+\frac{a\sqrt{c+dx^8}}{b(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{8}\left(\frac{a\sqrt{c+dx^8}}{b(a+bx^8)(bc-ad)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{b^{3/2}(bc-ad)^{3/2}}\right)$$

input
```
Int[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((a*Sqrt[c + d*x^8])/(b*(b*c - a*d)*(a + b*x^8)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(b^(3/2)*(b*c - a*d)^(3/2)))/8
```

3.909.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.909.4 Maple [A] (verified)

Time = 5.84 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{8}+c}\,a}{b\,x^{8}+a}+\frac{(a d-2 b c)\,\arctan\left(\frac{b\sqrt{d\,x^{8}+c}}{\sqrt{(a d-b c) b}}\right)}{\sqrt{(a d-b c) b}}}{8(a d-b c) b}$	83

input
```
int(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/(a*d-b*c)/b*(-(d*x^8+c)^(1/2)*a/(b*x^8+a)+(a*d-2*b*c)/((a*d-b*c)*b)^(1
/2)*arctan(b*(d*x^8+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

3.909.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.52

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$
$$= \left[\frac{((2\,b^2c-abd)x^8+2\,abc-a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}(ab^2c-a^2bd)}{16\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)x^8+ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)}\right.$$

input
```
integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(((2*b^2*c - a*b*d)*x^8 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((
b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)
) + 2*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b
^3*d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2), 1/8*(((2*b^2*c - a
*b*d)*x^8 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8 + c)*s
qrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)
)/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d +
 a^3*b^2*d^2)]
```

3.909.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x**15/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Timed out
```

3.909.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{15}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.909.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^{15}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{\frac{\sqrt{dx^8+c}ad^2}{(b^2c-abd)((dx^8+c)b-bc+ad)} + \frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{8\,d}$$

input
```
integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*(sqrt(d*x^8 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^8 + c)*b - b*c + a*d)) +
 (2*b*c*d - a*d^2)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c
- a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

3.909.9 Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{x^{15}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)\,\left(a\,d-2\,b\,c\right)}{8\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}} - \frac{a\,d\,\sqrt{d\,x^8+c}}{2\,b\,\left(a\,d-b\,c\right)\,\left(4\,b\,\left(d\,x^8+c\right)+4\,a\,d-4\,b\,c\right)}$$

input
```
int(x^15/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
(atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(8*b^(
3/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^8)^(1/2))/(2*b*(a*d - b*c)*(4*b*(c
 + d*x^8) + 4*a*d - 4*b*c))
```

3.909. $\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

3.910 $\int \frac{x^7}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

3.910.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^7}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=-\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}(bc-ad)^{3/2}}$$

output
```
1/8*d*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b
^(1/2)-1/8*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)
```

3.910.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{x^7}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{1}{8}\left(-\frac{\sqrt{c+dx^8}}{(bc-ad)\left(a+bx^8\right)}+\frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

input
```
Integrate[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-(Sqrt[c + d*x^8]/((b*c - a*d)*(a + b*x^8))) + (d*ArcTan[(Sqrt[b]*Sqrt[c
+ d*x^8])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/8
```

3.910.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {946, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{8}\int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{8}\left(-\frac{d\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^8}{2(bc-ad)}-\frac{\sqrt{c+dx^8}}{(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{8}\left(-\frac{\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{bc-ad}-\frac{\sqrt{c+dx^8}}{(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{8}\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}}{(a+bx^8)(bc-ad)}\right)$$

input

```
Int[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output

```
(-(Sqrt[c + d*x^8]/((b*c - a*d)*(a + b*x^8))) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^8])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2)))/8
```

3.910.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.910.4 Maple [A] (verified)

Time = 5.82 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

method	result	size
pseudoelliptic	$\frac{d\left(b\,x^{8}+a\right)\arctan\left(\frac{b\sqrt{d\,x^{8}+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{d\,x^{8}+c}\,\sqrt{(ad-bc)b}}{8\sqrt{(ad-bc)b}\,(ad-bc)\left(b\,x^{8}+a\right)}$	90

input
```
int(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(d*(b*x^8+a)*arctan(b*(d*x^8+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(d*x^8+c)^(
1/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/(a*d-b*c)/(b*x^8+a)
```

3.910.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(71) = 142$.

Time = 0.41 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.47

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \left[-\frac{(bdx^8+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}(b^2c-abd)}{16\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^8+ab^3c^2-2\,a^2b^2cd+a^3bd^2)},\; -\frac{(bdx^8+ad)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-b^2c+abd}}{bdx^8+bc}\right)+\sqrt{dx^8+c}(b^2c-abd)}{8\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^8+ab^3c^2-2\,a^2b^2cd+a^3bd^2)}\right]$$

input
```
integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((b*d*x^8 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d - 2
*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^
2*c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2*a
^2*b^2*c*d + a^3*b*d^2), -1/8*((b*d*x^8 + a*d)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(
b^2*c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2
*a^2*b^2*c*d + a^3*b*d^2)]
```

3.910.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x**7/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Timed out
```

3.910.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.910.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{8\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^8+c}d}{8\left((dx^8+c)b-bc+ad\right)(bc-ad)}$$

input
```
integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/8*d*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*(b*c - a*d)) - 1/8*sqrt(d*x^8 + c)*d/(((d*x^8 + c)*b - b*c + a*d)*(b*c -
 a*d))
```

3.910.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{d\sqrt{d\,x^8+c}}{2\,\left(a\,d-b\,c\right)\,\left(4\,b\,\left(d\,x^8+c\right)+4\,a\,d-4\,b\,c\right)} + \frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{8\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

input
```
int(x^7/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
(d*(c + d*x^8)^(1/2))/(2*(a*d - b*c)*(4*b*(c + d*x^8) + 4*a*d - 4*b*c)) +
(d*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(1/2)*(a*d -
b*c)^(3/2))
```

3.910. $\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

3.911 $\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

3.911.1 Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}+\frac{\sqrt{b}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}}$$

output
```
1/8*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/
2)/a^2/(-a*d+b*c)^(3/2)-1/4*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1
/8*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)
```

3.911.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{-\frac{ab\sqrt{c+dx^8}}{(-bc+ad)(a+bx^8)}+\frac{\sqrt{b}(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{8a^2}$$

input
```
Integrate[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-((a*b*Sqrt[c + d*x^8])/((-(b*c) + a*d)*(a + b*x^8))) + (Sqrt[b]*(2*b*c -
 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*
d)^(3/2) - (2*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/Sqrt[c])/(8*a^2)
```

3.911.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{8} \int \frac{1}{x^8 \left(b x^8+a\right)^2 \sqrt{d x^8+c}}dx^8$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{8} \left(\frac{\int \frac{b d x^8+2 b c-2 a d}{2 x^8 \left(b x^8+a\right) \sqrt{d x^8+c}}dx^8}{a (b c-a d)}+\frac{b \sqrt{c+d x^8}}{a \left(a+b x^8\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8} \left(\frac{\int \frac{b d x^8+2 (b c-a d)}{x^8 \left(b x^8+a\right) \sqrt{d x^8+c}}dx^8}{2 a (b c-a d)}+\frac{b \sqrt{c+d x^8}}{a \left(a+b x^8\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{8} \left(\frac{\frac{2 (b c-a d) \int \frac{1}{x^8 \sqrt{d x^8+c}}dx^8}{a}-\frac{b (2 b c-3 a d) \int \frac{1}{\left(b x^8+a\right) \sqrt{d x^8+c}}dx^8}{a}}{2 a (b c-a d)}+\frac{b \sqrt{c+d x^8}}{a \left(a+b x^8\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{8} \left(\frac{\frac{4 (b c-a d) \int \frac{1}{\frac{x^{16}}{d}-\frac{c}{d}}d\sqrt{d x^8+c}}{a d}-\frac{2 b (2 b c-3 a d) \int \frac{1}{\frac{b x^{16}}{d}+a-\frac{b c}{d}}d\sqrt{d x^8+c}}{a d}}{2 a (b c-a d)}+\frac{b \sqrt{c+d x^8}}{a \left(a+b x^8\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{8}\left(\frac{\frac{2\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{4(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{a\left(a+bx^8\right)(bc-ad)}\right)$$

input
```
Int[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((b*Sqrt[c + d*x^8])/(a*(b*c - a*d)*(a + b*x^8)) + ((-4*(b*c - a*d)*ArcTan
h[Sqrt[c + d*x^8]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*(2*b*c - 3*a*d)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*(
b*c - a*d)))/8
```

3.911.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.911.4 Maple [A] (verified)

Time = 5.92 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{-2b\left(bc-\frac{3ad}{2}\right)\sqrt{c}\,(b\,x^{8}+a)\,\arctan\left(\frac{b\sqrt{d\,x^{8}+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\,\left(2(ad-bc)\,(b\,x^{8}+a)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{\sqrt{c}}\right)+\sqrt{c}\,\sqrt{d\,x^{8}+c}\,ab\right)}{8\sqrt{c}\,\sqrt{(ad-bc)b}\,a^{2}(ad-bc)(b\,x^{8}+a)}$

input
```
int(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-2*b*(b*c-3/2*a*d)*c^(1/2)*(b*x^8+a)*arctan(b*(d*x^8+c)^(1/2)/((a*d-
b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(2*(a*d-b*c)*(b*x^8+a)*arctanh((d*x^8+c
)^(1/2)/c^(1/2))+c^(1/2)*(d*x^8+c)^(1/2)*a*b))/c^(1/2)/((a*d-b*c)*b)^(1/2)
/a^2/(a*d-b*c)/(b*x^8+a)
```

3.911.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 862, normalized size of antiderivative = 6.53

$$\int \frac{1}{x\left(a+bx^{8}\right)^{2}\sqrt{c+dx^{8}}}\,dx$$
$$=\left[\frac{2\sqrt{dx^{8}+c}abc+((2\,b^{2}c^{2}-3\,abcd)x^{8}+2\,abc^{2}-3\,a^{2}cd)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^{8}+2\,bc-ad+2\,\sqrt{dx^{8}+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^{8}+a}\right)-}{16\left((a^{2}b^{2}c^{2}-a^{3}bcd)x^{8}+a^{3}bc^{2}-a^{4}cd\right)}\right.$$

input
```
integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(2*sqrt(d*x^8 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2
- 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8
 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*((b^2*c - a*b*d)*x
^8 + a*b*c - a^2*d)*sqrt(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/
x^8))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/8*(sqrt(d*x
^8 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt
(-b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/
(b*d*x^8 + b*c)) + ((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(c)*log((d*x^
8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8
+ a^3*b*c^2 - a^4*c*d), 1/16*(2*sqrt(d*x^8 + c)*a*b*c + 4*((b^2*c - a*b*d)
*x^8 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) + ((2*b^
2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b
*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/
(b*x^8 + a)))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/8*(
sqrt(d*x^8 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c
*d)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c
- a*d))/(b*d*x^8 + b*c)) + 2*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-c
)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3
*b*c^2 - a^4*c*d)]
```

3.911.6 Sympy [F]

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.911.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}x}\,dx$$

input
```
integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x), x)
```

3.911.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{dx^8+c}bd}{8\,(abc-a^2d)((dx^8+c)b-bc+ad)} - \frac{(2\,b^2c-3\,abd)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{8\,(a^2bc-a^3d)\sqrt{-b^2c+abd}} + \frac{\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{4\,a^2\sqrt{-c}}$$

input

```
integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output

```
1/8*sqrt(d*x^8 + c)*b*d/((a*b*c - a^2*d)*((d*x^8 + c)*b - b*c + a*d)) - 1/
8*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2
*b*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/4*arctan(sqrt(d*x^8 + c)/sqrt(-c))
/(a^2*sqrt(-c))
```

3.911.9 Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 3017, normalized size of antiderivative = 22.86

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

input

```
int(1/(x*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output

```
(atan((((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d
^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((a^6*b^2*d^5 - (3*a^5*
b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)
- ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(256*a^7*b^2
*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(
512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b
^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*
(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c
)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*
d - 3*a^4*b*c*d^2)) + ((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2
 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((a^6*b^
2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2
- 2*a^4*b*c*d) + ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/
2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*
b^4*c^2*d^3))/(512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^
3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)
^3)^(1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))
*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(16*(a^5*d^3 - a^2*b^3*c^3 +
 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))/(((3*a*b^3*d^4)/128 - (b^4*c*d^3)/64)/
(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((((c + d*x^8)^(1/2)*(13*a^2*b^...
```

3.911. $\int \frac{1}{x(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

3.912 $\int \frac{1}{x^9\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

3.912.1 Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{1}{x^9\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=-\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)}-\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)}+\frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}}-\frac{b^{3/2}(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^3(bc-ad)^{3/2}}$$

output
```
1/8*(a*d+4*b*c)*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/8*b^(3/2)*(
-5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+
b*c)^(3/2)-1/8*b*(-a*d+2*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^8+a)-1
/8*(d*x^8+c)^(1/2)/a/c/x^8/(b*x^8+a)
```

3.912.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^9\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{\frac{a\sqrt{c+dx^8}\left(-a^2d+2b^2cx^8+ab\left(c-dx^8\right)\right)}{c(-bc+ad)x^8\left(a+bx^8\right)}-\frac{b^{3/2}(4bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}+\frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{c^{3/2}}}{8a^3}$$

input
```
Integrate[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((a*Sqrt[c + d*x^8]*(-(a^2*d) + 2*b^2*c*x^8 + a*b*(c - d*x^8)))/(c*(-(b*c)
 + a*d)*x^8*(a + b*x^8)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcT
anh[Sqrt[c + d*x^8]/Sqrt[c]])/c^(3/2))/(8*a^3)
```

3.912.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {948, 114, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 \left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{8} \int \frac{1}{x^{16} \left(b x^8+a\right)^2 \sqrt{d x^8+c}}dx^8$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{8} \left(-\frac{\int \frac{3 b d x^8+4 b c+a d}{2 x^8 \left(b x^8+a\right)^2 \sqrt{d x^8+c}}dx^8}{a c}-\frac{\sqrt{c+d x^8}}{a c x^8 \left(a+b x^8\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8} \left(-\frac{\int \frac{3 b d x^8+4 b c+a d}{x^8 \left(b x^8+a\right)^2 \sqrt{d x^8+c}}dx^8}{2 a c}-\frac{\sqrt{c+d x^8}}{a c x^8 \left(a+b x^8\right)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{8} \left(-\frac{\frac{\int \frac{b d (2 b c-a d) x^8+(b c-a d) (4 b c+a d)}{x^8 \left(b x^8+a\right) \sqrt{d x^8+c}}dx^8}{a (b c-a d)}+\frac{2 b \sqrt{c+d x^8} (2 b c-a d)}{a \left(a+b x^8\right) (b c-a d)}}{2 a c}-\frac{\sqrt{c+d x^8}}{a c x^8 \left(a+b x^8\right)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{8}\left(-\frac{\frac{\frac{(bc-ad)(ad+4bc)\int \frac{1}{x^8\sqrt{dx^8+c}}dx^8}{a}-\frac{b^2c(4bc-5ad)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^8}{a}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^8}(2bc-ad)}{a(a+bx^8)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^8}}{acx^8\left(a+bx^8\right)}\right)$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{8}\left(-\frac{\frac{\frac{2(bc-ad)(ad+4bc)\int \frac{1}{\frac{x^{16}}{d}-\frac{c}{d}}d\sqrt{dx^8+c}}{ad}-\frac{2b^2c(4bc-5ad)\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{ad}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^8}(2bc-ad)}{a(a+bx^8)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^8}}{acx^8\left(a+bx^8\right)}\right)$$

$$\Big\downarrow \textbf{221}$$

$$\frac{1}{8}\left(-\frac{\frac{\frac{2b^{3/2}c(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)(ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{a\sqrt{c}}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^8}(2bc-ad)}{a(a+bx^8)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^8}}{acx^8\left(a+bx^8\right)}\right)$$

input
```
Int[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-(Sqrt[c + d*x^8]/(a*c*x^8*(a + b*x^8))) - ((2*b*(2*b*c - a*d)*Sqrt[c + d
*x^8])/(a*(b*c - a*d)*(a + b*x^8)) + ((-2*(b*c - a*d)*(4*b*c + a*d)*ArcTan
h[Sqrt[c + d*x^8]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(3/2)*c*(4*b*c - 5*a*d)*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(a*(
b*c - a*d)))/(2*a*c))/8
```

3.912.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.912.4 Maple [A] (verified)

Time = 6.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{x^{8} b^{2} \left(b\,x^{8}+a\right) \left(bc-\frac{5ad}{4}\right) c^{\frac{5}{2}} \arctan\left(\frac{b\sqrt{d\,x^{8}+c}}{\sqrt{\left(ad-bc\right)b}}\right)-\frac{\left(c\,x^{8} \left(b\,x^{8}+a\right) \left(ad+4bc\right) \left(ad-bc\right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{\sqrt{c}}\right)+\left(2b^{2} c\,x^{8}+a\left(-d\,x^{8}+c\right)\right)\right)}{4}}{2\sqrt{\left(ad-bc\right)b}\,a^{3}\left(ad-bc\right)\left(b\,x^{8}+a\right)c^{\frac{5}{2}}\,x^{8}}$

input
```
int(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((a*d-b*c)*b)^(1/2)*(x^8*b^2*(b*x^8+a)*(b*c-5/4*a*d)*c^(5/2)*arctan(b
*(d*x^8+c)^(1/2)/((a*d-b*c)*b)^(1/2))-1/4*(c*x^8*(b*x^8+a)*(a*d+4*b*c)*(a*
d-b*c)*arctanh((d*x^8+c)^(1/2)/c^(1/2))+(2*b^2*c*x^8+a*(-d*x^8+c)*b-a^2*d)
*a*c^(3/2)*(d*x^8+c)^(1/2))*((a*d-b*c)*b)^(1/2))/a^3/(a*d-b*c)/(b*x^8+a)/c
^(5/2)/x^8
```

3.912.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 1236, normalized size of antiderivative = 6.68

$$\int \frac{1}{x^9 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x
^8)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*
c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a
^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(c)*log((d
*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*((2*a*b^2*c^2 - a^2*b*c*d
)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)
*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/16*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d
)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(-b/(b*c - a*d))*arctan(-s
qrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) - ((4*b^3
*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^
2)*x^8)*sqrt(c)*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) + 2*((2
*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*
b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/16*(2*((4*b
^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*
d^2)*x^8)*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*
b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(b/(b*c - a*d))*l
og((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*
d)))/(b*x^8 + a)) + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d
)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^
2*d)*x^8), -1/8*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a...
```

3.912.6 Sympy [F]

$$\int \frac{1}{x^9\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{1}{x^9\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**9/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**9*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.912.7 Maxima [F]

$$\int \frac{1}{x^9\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}\,x^9}\,dx$$

input
```
integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^9), x)
```

3.912.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^9\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{\left(4\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{8\,\left(a^3bc-a^4d\right)\sqrt{-b^2c+abd}}$$
$$-\frac{2\,\left(dx^8+c\right)^{\frac{3}{2}}b^2cd-2\,\sqrt{dx^8+c}b^2c^2d-\left(dx^8+c\right)^{\frac{3}{2}}abd^2+2\,\sqrt{dx^8+c}abcd^2-\sqrt{dx^8+c}a^2d^3}{8\,\left(a^2bc^2-a^3cd\right)\left(\left(dx^8+c\right)^2b-2\,\left(dx^8+c\right)bc+bc^2+\left(dx^8+c\right)ad-acd\right)}$$
$$-\frac{\left(4\,bc+ad\right)\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{8\,a^3\sqrt{-c}c}$$

input
```
integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/8*(2*(d*x^8 + c)^(3/2)*b^2*c*d
 - 2*sqrt(d*x^8 + c)*b^2*c^2*d - (d*x^8 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^8
+ c)*a*b*c*d^2 - sqrt(d*x^8 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^8 +
 c)^2*b - 2*(d*x^8 + c)*b*c + b*c^2 + (d*x^8 + c)*a*d - a*c*d)) - 1/8*(4*b
*c + a*d)*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

3.912.9 Mupad [B] (verification not implemented)

Time = 12.25 (sec) , antiderivative size = 3832, normalized size of antiderivative = 20.71

$$\int \frac{1}{x^9 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

input
```
int(1/(x^9*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
(((c + d*x^8)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
 a*c*d)) + (b*(c + d*x^8)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d))
)/((c + d*x^8)*(4*a*d - 8*b*c) + 4*b*(c + d*x^8)^2 + 4*b*c^2 - 4*a*c*d) +
(atan((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^8)^(1/2)*(a^
4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b
^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*
d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 -
2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^
7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^8)^(1/2)*(5*a*d - 4*b*c)
*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*
a^9*b^2*c^2*d^5))/(512*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^
3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d^3 - a^3*b
^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(16*(a^6*d^3 - a^3*b^3*c^3
 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d
- 4*b*c)*(((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*
d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^
2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b
^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(
a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(
c + d*x^8)^(1/2)*(5*a*d - 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^...
```

3.913 $\int \frac{x^{19}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

3.913.1 Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{x^{19}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

output
```
-1/8*(-2*a*d+3*b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))*a
^(1/2)/b^2/(-a*d+b*c)^(3/2)+1/4*arctanh(x^4*d^(1/2)/(d*x^8+c)^(1/2))/b^2/d
^(1/2)+1/8*a*x^4*(d*x^8+c)^(1/2)/b/(-a*d+b*c)/(b*x^8+a)
```

3.913.2 Mathematica [A] (verified)

Time = 4.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int \frac{x^{19}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{\frac{abx^4\sqrt{c+dx^8}}{(bc-ad)(a+bx^8)} + \frac{\sqrt{a}(-3bc+2ad)\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{d}}}{8b^2}$$

input
```
Integrate[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output

```
((a*b*x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (Sqrt[a]*(-3*b*c +
2*a*d)*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]
*Sqrt[b*c - a*d])])/(b*c - a*d)^(3/2) + (2*Log[Sqrt[d]*x^4 + Sqrt[c + d*x^
8]])/Sqrt[d])/(8*b^2)
```

3.913.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {965, 372, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19}}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{4}\int \frac{x^{16}}{\left(bx^8+a\right)^2 \sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{ 372}$$

$$\frac{1}{4}\left(\frac{ax^4\sqrt{c+dx^8}}{2b\left(a+bx^8\right)(bc-ad)}-\frac{\int \frac{ac-2(bc-ad)x^8}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{ 398}$$

$$\frac{1}{4}\left(\frac{ax^4\sqrt{c+dx^8}}{2b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{b}-\frac{2(bc-ad)\int \frac{1}{\sqrt{dx^8+c}}dx^4}{b}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{4}\left(\frac{ax^4\sqrt{c+dx^8}}{2b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{b}-\frac{2(bc-ad)\int \frac{1}{1-dx^8}d\frac{x^4}{\sqrt{dx^8+c}}}{b}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{4}\left(\frac{ax^4\sqrt{c+dx^8}}{2b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{b}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{4}\left(\frac{ax^4\sqrt{c+dx^8}}{2b\left(a+bx^8\right)\left(bc-ad\right)}-\frac{\frac{a(3bc-2ad)\int\frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{b}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{4}\left(\frac{ax^4\sqrt{c+dx^8}}{2b\left(a+bx^8\right)\left(bc-ad\right)}-\frac{\frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{b\sqrt{bc-ad}}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

input
```
Int[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((a*x^4*Sqrt[c + d*x^8])/(2*b*(b*c - a*d)*(a + b*x^8)) - ((Sqrt[a]*(3*b*c
- 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(b*Sqrt[
b*c - a*d]) - (2*(b*c - a*d)*ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]])/(b*Sq
rt[d]))/(2*b*(b*c - a*d)))/4
```

3.913.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.913. $\int \frac{x^{19}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.913.4 Maple [A] (verified)

Time = 24.90 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$-\frac{-\frac{a\left(-\frac{b\sqrt{d\,x^{8}+c}\,x^{4}}{b\,x^{8}+a}-\frac{(2ad-3bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}\,a}{x^{4}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{ad-bc}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{x^{4}\sqrt{d}}\right)}{\sqrt{d}}}{8b^{2}}$	117

input
```
int(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/b^2*(-a/(a*d-b*c)*(-b*(d*x^8+c)^(1/2)*x^4/(b*x^8+a)-(2*a*d-3*b*c)/((a
*d-b*c)*a)^(1/2)*arctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2)))-2/d^(
1/2)*arctanh((d*x^8+c)^(1/2)/x^4/d^(1/2)))
```

3.913.5 Fricas [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 1077, normalized size of antiderivative = 7.64

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$= \left[\frac{4\sqrt{dx^8+c}abdx^4+4\left((b^2c-abd)x^8+abc-a^2d\right)\sqrt{d}\log\left(-2\,dx^8-2\,\sqrt{dx^8+c}\sqrt{d}x^4-c\right)+((3\,b^2cd-}{32\,((}\right.$$

input

```
integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output

```
[1/32*(4*sqrt(d*x^8 + c)*a*b*d*x^4 + 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*
d)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c) + ((3*b^2*c*d
 - 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*
c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^
2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*s
qrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((b^4*
c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/32*(4*sqrt(d*x^8 + c)*a
*b*d*x^4 - 8*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d
)*x^4/sqrt(d*x^8 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*
d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*
(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^
2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(
b^2*x^16 + 2*a*b*x^8 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2
*b^2*d^2), 1/16*(2*sqrt(d*x^8 + c)*a*b*d*x^4 + ((3*b^2*c*d - 2*a*b*d^2)*x^
8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*
x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) + 2*(
(b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 +
 c)*sqrt(d)*x^4 - c))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2
), 1/16*(2*sqrt(d*x^8 + c)*a*b*d*x^4 - 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^
2*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)) + ((3*b^2*c*d - 2*a*...
```

3.913.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{19}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\text{Timed out}$$

input
```
integrate(x**19/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Timed out
```

3.913.7 Maxima [F]

$$\int \frac{x^{19}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^{19}}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^19/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.913.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(117) = 234$.

Time $= 0.37$ (sec) , antiderivative size $= 298$, normalized size of antiderivative $= 2.11$

$$\int \frac{x^{19}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=-\frac{\left(3\,abc\sqrt{d}-2\,a^2d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{8\,(b^3c-ab^2d)\sqrt{abcd-a^2d^2}}-\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2abc\sqrt{d}-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2a^2d^{\frac{3}{2}}-abc^2\sqrt{d}}{4\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2bc+4\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2ad+bc^2\right)(b^3c-ab^2d)}-\frac{\log\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2\right)}{8\,b^2\sqrt{d}}$$

input
```
integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*
x^8 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*s
qrt(a*b*c*d - a^2*d^2)) - 1/4*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b*c*sqr
t(d) - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/
(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^
2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)
) - 1/8*log((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2)/(b^2*sqrt(d))
```

3.913.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^{19}}{{\left(b\,x^8+a\right)}^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^19/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^19/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.914 $\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

3.914.1 Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=-\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)\,(a+bx^8)}+\frac{c\arctan\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8\sqrt{a}(bc-ad)^{3/2}}$$

output
```
1/8*c*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/(-a*d+b*c)^(3/2
)/a^(1/2)-1/8*x^4*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)
```

3.914.2 Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\frac{1}{8}\left(-\frac{x^4\sqrt{c+dx^8}}{(bc-ad)\,(a+bx^8)}+\frac{c\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}(bc-ad)^{3/2}}\right)$$

input
```
Integrate[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-((x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8))) + (c*ArcTan[(a*Sqrt[d]
 + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqr
t[a]*(b*c - a*d)^(3/2)))/8
```

3.914.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {965, 373, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{4}\int \frac{x^8}{(bx^8+a)^2\sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{373}$$

$$\frac{1}{4}\left(\frac{\int \frac{c}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{2(bc-ad)}-\frac{x^4\sqrt{c+dx^8}}{2\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{c\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{2(bc-ad)}-\frac{x^4\sqrt{c+dx^8}}{2\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{4}\left(\frac{c\int \frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{2(bc-ad)}-\frac{x^4\sqrt{c+dx^8}}{2\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{4}\left(\frac{c\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{2\sqrt{a}(bc-ad)^{3/2}}-\frac{x^4\sqrt{c+dx^8}}{2\left(a+bx^8\right)(bc-ad)}\right)$$

input
```
Int[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-1/2*(x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (c*ArcTan[(Sqrt[b*
c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(2*Sqrt[a]*(b*c - a*d)^(3/2)))/4
```

3.914.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.914.4 Maple [A] (verified)

Time = 20.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$-\frac{c\left(-\frac{\sqrt{d x^{8}+c}\,x^{4}}{c\left(b x^{8}+a\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^{8}+c}\,a}{x^{4}\sqrt{\left(a d-b c\right) a}}\right)}{\sqrt{\left(a d-b c\right) a}}\right)}{8\left(a d-b c\right)}$	81

input
```
int(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*c/(a*d-b*c)*(-(d*x^8+c)^(1/2)*x^4/c/(b*x^8+a)+1/((a*d-b*c)*a)^(1/2)*a
rctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2)))
```

3.914.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(77) = 154$.

Time = 0.35 (sec) , antiderivative size = 426, normalized size of antiderivative = 4.58

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$
$$= \left[-\frac{4\sqrt{dx^8+c}(abc-a^2d)x^4-(bcx^8+ac)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2+4\,((}{b^2x^{16}+2\,abx^8+a^2}\right.}{32\,((ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^8+a^2b^2c^2-2\,a^3bcd+a^4d^2)}\right.$$
$$\left.-\frac{2\sqrt{dx^8+c}(abc-a^2d)x^4-(bcx^8+ac)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2\,ad)x^8-ac)\sqrt{dx^8+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^{12}+(abc^2-a^2cd)x^4)}\right)}{16\,((ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^8+a^2b^2c^2-2\,a^3bcd+a^4d^2)}\right]$$

input
```
integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*(4*sqrt(d*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(-a*b*
c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*
a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x^8 + c)*
sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((a*b^3*c^2 - 2*a^2*b
^2*c*d + a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/16*(2*s
qrt(d*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(a*b*c - a^2*d)*a
rctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((
a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)))/((a*b^3*c^2 - 2*a^2*b
^2*c*d + a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]
```

3.914.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x**11/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Timed out
```

3.914.7 Maxima [F]

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^{11}}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^11/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.914.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(77) = 154$.

Time = 0.99 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.62

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{c\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{8\sqrt{abcd-a^2d^2}(bc-ad)} + \frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 bc\sqrt{d}-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 ad^{\frac{3}{2}}-bc^2\sqrt{d}}{4\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4 b-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 bc+4\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 ad+bc^2\right)(b^2c-abd)}$$

input
```
integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a
*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + 1/4*(
(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x^4 - sqrt(d*x^
8 + c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b
 - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 +
 c))^2*a*d + b*c^2)*(b^2*c - a*b*d))
```

3.914.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^{11}}{{\left(b\,x^8+a\right)}^2\,\sqrt{d\,x^8+c}} \,d x$$

input
```
int(x^11/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^11/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.915 $\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

3.915.1 Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)}+\frac{(bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}$$

output
```
1/8*(-2*a*d+b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(3
/2)/(-a*d+b*c)^(3/2)+1/8*b*x^4*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)
```

3.915.2 Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=-\frac{bx^4\sqrt{c+dx^8}}{8a(-bc+ad)(a+bx^8)}+\frac{(bc-2ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^8+bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{3/2}(bc-ad)^{3/2}}$$

input
```
Integrate[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
-1/8*(b*x^4*Sqrt[c + d*x^8])/(a*(-(b*c) + a*d)*(a + b*x^8)) + ((b*c - 2*a*
d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqr
t[b*c - a*d])])/(8*a^(3/2)*(b*c - a*d)^(3/2))
```

3.915.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {965, 296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \; 965$$

$$\frac{1}{4}\int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \; 296$$

$$\frac{1}{4}\left(\frac{(bc-2ad)\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{2a(bc-ad)}+\frac{bx^4\sqrt{c+dx^8}}{2a\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 291$$

$$\frac{1}{4}\left(\frac{(bc-2ad)\int \frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{2a(bc-ad)}+\frac{bx^4\sqrt{c+dx^8}}{2a\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{1}{4}\left(\frac{(bc-2ad)\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx^4\sqrt{c+dx^8}}{2a\left(a+bx^8\right)(bc-ad)}\right)$$

input
```
Int[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((b*x^4*Sqrt[c + d*x^8])/(2*a*(b*c - a*d)*(a + b*x^8)) + ((b*c - 2*a*d)*Ar
cTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(2*a^(3/2)*(b*c - a
*d)^(3/2)))/4
```

3.915.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.915.4 Maple [A] (verified)

Time = 20.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$\frac{-\frac{b\sqrt{d\,x^{8}+c}\,x^{4}}{b\,x^{8}+a}+\frac{(2ad-bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}\,a}{x^{4}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}}{8a(ad-bc)}$	90

input
```
int(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/a/(a*d-b*c)*(-b*(d*x^8+c)^(1/2)*x^4/(b*x^8+a)+(2*a*d-b*c)/((a*d-b*c)*a
)^(1/2)*arctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2)))
```

3.915.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(88) = 176$.

Time = 0.39 (sec) , antiderivative size = 467, normalized size of antiderivative = 4.49

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$= \left[\frac{4\sqrt{dx^8+c}(ab^2c-a^2bd)x^4-((b^2c-2\,abd)x^8+abc-2\,a^2d)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3}{}\right.}{32\,((a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)x^8+a^3b^2c^2-2\,a^4bcd+}\right.$$

input
```
integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*(4*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 - ((b^2*c - 2*a*b*d)*x^8
+ a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*
d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^1
2 - a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 +
 a^2)))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^8 + a^3*b^2*c^2 - 2*a
^4*b*c*d + a^5*d^2), 1/16*(2*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 + ((b
^2*c - 2*a*b*d)*x^8 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*
c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*
d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4
*b*d^2)*x^8 + a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)]
```

3.915.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**3/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.915.7 Maxima [F]

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^3}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.915.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(88) = 176$.

Time = 0.37 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.28

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{1}{8}d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{(abcd-a^2d^2)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2bc\right.}{\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b-2\left(\sqrt{d}x^4-\sqrt{dx^8+\right.\right.}\right.$$

input
```
integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/8*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*
b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((
sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2
*a*d - b*c^2)/(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqr
t(d*x^8 + c))^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)*(a*
b*c*d - a^2*d^2)))
```

3.915.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^3}{{\left(b\,x^8+a\right)}^2\,\sqrt{d\,x^8+c}} \,d x$$

input
```
int(x^3/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.916 $\int \frac{1}{x^5\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

3.916.1 Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{1}{x^5\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = -\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)} - \frac{b(3bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

output
```
-1/8*b*(-4*a*d+3*b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))
/a^(5/2)/(-a*d+b*c)^(3/2)-1/8*(-2*a*d+3*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b
*c)/x^4+1/8*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^4/(b*x^8+a)
```

3.916.2 Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^5\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{c+dx^8}\left(2abc-2a^2d+3b^2cx^8-2abdx^8\right)}{8a^2c(-bc+ad)x^4\left(a+bx^8\right)} - \frac{b(3bc-4ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^8+bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(Sqrt[c + d*x^8]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^8 - 2*a*b*d*x^8))/(8*a^2*c
*(-(b*c) + a*d)*x^4*(a + b*x^8)) - (b*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] +
b*Sqrt[d]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*a^(5
/2)*(b*c - a*d)^(3/2))
```

3.916.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 374, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{4}\int \frac{1}{x^8 \left(bx^8+a\right)^2 \sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{374}$$

$$\frac{1}{4}\left(\frac{b\sqrt{c+dx^8}}{2ax^4 \left(a+bx^8\right) (bc-ad)}-\frac{\int -\frac{2bdx^8+3bc-2ad}{x^8 \left(bx^8+a\right) \sqrt{dx^8+c}}dx^4}{2a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{\int \frac{2bdx^8+3bc-2ad}{x^8 \left(bx^8+a\right) \sqrt{dx^8+c}}dx^4}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^4 \left(a+bx^8\right) (bc-ad)}\right)$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{4}\left(\frac{-\frac{\int \frac{bc(3bc-4ad)}{\left(bx^8+a\right) \sqrt{dx^8+c}}dx^4}{ac}-\frac{\sqrt{c+dx^8}(3bc-2ad)}{acx^4}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^4 \left(a+bx^8\right) (bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{-\frac{b(3bc-4ad) \int \frac{1}{\left(bx^8+a\right) \sqrt{dx^8+c}}dx^4}{a}-\frac{\sqrt{c+dx^8}(3bc-2ad)}{acx^4}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^4 \left(a+bx^8\right) (bc-ad)}\right)$$

$$\Big\downarrow \ 291$$

$$\frac{1}{4}\left(\frac{-\frac{b(3bc-4ad)\int \frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{a}-\frac{\sqrt{c+dx^8}(3bc-2ad)}{acx^4}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^4\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \ 218$$

$$\frac{1}{4}\left(\frac{-\frac{b(3bc-4ad)\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}(3bc-2ad)}{acx^4}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^4\left(a+bx^8\right)(bc-ad)}\right)$$

input
```
Int[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((b*Sqrt[c + d*x^8])/(2*a*(b*c - a*d)*x^4*(a + b*x^8)) + (-(((3*b*c - 2*a*
d)*Sqrt[c + d*x^8])/(a*c*x^4)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d
]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(a^(3/2)*Sqrt[b*c - a*d]))/(2*a*(b*c -
a*d)))/4
```

3.916.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.916.4 Maple [A] (verified)

Time = 31.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{8}+c}}{x^{4}}+\frac{b c \left(\frac{b\sqrt{d\,x^{8}+c}\,x^{4}}{b\,x^{8}+a}-\frac{(4 a d-3 b c)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}\,a}{x^{4}\sqrt{(a d-b c) a}}\right)}{\sqrt{(a d-b c) a}}\right)}{2 a d-2 b c}}{4 a^{2} c}$	112

input
```
int(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^2*(-(d*x^8+c)^(1/2)/x^4+1/2*b*c/(a*d-b*c)*(b*(d*x^8+c)^(1/2)*x^4/(b*
x^8+a)-(4*a*d-3*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^8+c)^(1/2)/x^4*a/((a
*d-b*c)*a)^(1/2))))/c
```

3.916.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(129) = 258$.

Time = 0.55 (sec) , antiderivative size = 612, normalized size of antiderivative = 4.11

$$\int \frac{1}{x^5 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx$$
$$= \left[-\frac{((3\,b^3c^2-4\,ab^2cd)x^{12}+(3\,ab^2c^2-4\,a^2bcd)x^4)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+}{b^2x^{16}+2}\right.}{32\,((a^3b^3c^3-2\,a^4b^2c^2d+a^5}\right.$$
$$-\frac{((3\,b^3c^2-4\,ab^2cd)x^{12}+(3\,ab^2c^2-4\,a^2bcd)x^4)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2\,ad)x^8-ac)\sqrt{dx^8+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^{12}+(abc^2-a^2cd)x^4)}\right)+2\,((}{16\,((a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2)x^{12}+(a^4b^2c^3-2}$$

input
```
integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)
*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a
*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(
d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 4*((3*a*b
^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d +
2*a^4*d^2)*sqrt(d*x^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)
*x^12 + (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4), -1/16*(((3*b^3*c^2
 - 4*a*b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)*sqrt(a*b*c - a^2*d
)*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)
/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)) + 2*((3*a*b^3*c^2 -
 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^
2)*sqrt(d*x^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^12 +
(a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4)]
```

3.916.6 Sympy [F]

$$\int \frac{1}{x^5 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^5 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**5/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**5*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.916.7 Maxima [F]

$$\int \frac{1}{x^5 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)^2 \sqrt{dx^8+c}\,x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^5), x)
```

3.916.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(129) = 258.

Time = 0.93 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.81

$$\int \frac{1}{x^5 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx$$
$$= \frac{1}{8} d^{\frac{5}{2}} \left(\frac{(3\,b^2c - 4\,abd)\arctan\left(\frac{\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^2 b - bc + 2\,ad}{2\sqrt{abcd - a^2d^2}}\right)}{(a^2bcd^2 - a^3d^3)\sqrt{abcd - a^2d^2}} + \frac{2\left(3\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^4 b^2c - 4\left(\sqrt{d}x^4 - \sqrt{\right.\right.}{\left(\left(\sqrt{d}x^4 - \sqrt{dx^8+c}\right)^6 b - 3\left(\sqrt{d}x^4 - \sqrt{d\right.\right.} \right.$$

input
```
integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c
))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sq
rt(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b^2*c - 4*
(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*a*b*d - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c)
)^2*b^2*c^2 + 14*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b*c*d - 8*(sqrt(d)*x^
4 - sqrt(d*x^8 + c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/(((sqrt(d)*x^4 -
 sqrt(d*x^8 + c))^6*b - 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b*c + 4*(sqrt(
d)*x^4 - sqrt(d*x^8 + c))^4*a*d + 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c^
2 - 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*
d^3)))
```

3.916.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^5 \left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}}\,dx$$

input

```
int(1/(x^5*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output

```
int(1/(x^5*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.917 $\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

3.917.1 Optimal result

Integrand size = 24, antiderivative size = 208

$$\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)} + \frac{b^2(5bc-6ad)\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{7/2}(bc-ad)^{3/2}}$$

output
```
1/8*b^2*(-6*a*d+5*b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2)
)/a^(7/2)/(-a*d+b*c)^(3/2)-1/24*(-2*a*d+5*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d
+b*c)/x^12+1/24*(-4*a^2*d^2-8*a*b*c*d+15*b^2*c^2)*(d*x^8+c)^(1/2)/a^3/c^2/
(-a*d+b*c)/x^4+1/8*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^12/(b*x^8+a)
```

3.917.2 Mathematica [A] (verified)

Time = 5.34 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^{13} \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx = -\frac{\sqrt{c+dx^8}(15b^3c^2x^{16}+2ab^2cx^8(5c-4dx^8)+2a^3d(c-2dx^8)-2a^2b(c^2+3cdx^8+2d^2x^{16}))}{24a^3c^2(-bc+ad)x^{12}\left(a+bx^8\right)} + \frac{b^2(5bc-6ad)\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{7/2}(bc-ad)^{3/2}}$$

input

```
Integrate[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output

```
-1/24*(Sqrt[c + d*x^8]*(15*b^3*c^2*x^16 + 2*a*b^2*c*x^8*(5*c - 4*d*x^8) +
2*a^3*d*(c - 2*d*x^8) - 2*a^2*b*(c^2 + 3*c*d*x^8 + 2*d^2*x^16)))/(a^3*c^2*
(-(b*c) + a*d)*x^12*(a + b*x^8)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(a*Sqrt[d]
+ b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*a^
(7/2)*(b*c - a*d)^(3/2))
```

3.917.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {965, 374, 25, 445, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{13} \left(a+bx^8\right)^2 \sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{4}\int \frac{1}{x^{16} \left(bx^8+a\right)^2 \sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{374}$$

$$\frac{1}{4}\left(\frac{b\sqrt{c+dx^8}}{2ax^{12}\left(a+bx^8\right)(bc-ad)}-\frac{\int -\frac{4bdx^8+5bc-2ad}{x^{16}\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{2a(bc-ad)}\right)$$

$$\Big\downarrow 25$$

$$\frac{1}{4}\left(\frac{\int \frac{4bdx^8+5bc-2ad}{x^{16}(bx^8+a)\sqrt{dx^8+c}}dx^4}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^{12}\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow 445$$

$$\frac{1}{4}\left(\frac{-\frac{\int \frac{2bd(5bc-2ad)x^8+15b^2c^2-4a^2d^2-8abcd}{x^8\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{3ac}-\frac{\sqrt{c+dx^8}(5bc-2ad)}{3acx^{12}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^{12}\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow 445$$

$$\frac{1}{4}\left(\frac{-\frac{-\frac{\int \frac{3b^2c^2(5bc-6ad)}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{ac}-\frac{\sqrt{c+dx^8}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^4}}{3ac}-\frac{\sqrt{c+dx^8}(5bc-2ad)}{3acx^{12}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^{12}\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow 27$$

$$\frac{1}{4}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{a}-\frac{\sqrt{c+dx^8}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^4}}{3ac}-\frac{\sqrt{c+dx^8}(5bc-2ad)}{3acx^{12}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^{12}\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow 291$$

$$\frac{1}{4}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\int \frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{a}-\frac{\sqrt{c+dx^8}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^4}}{3ac}-\frac{\sqrt{c+dx^8}(5bc-2ad)}{3acx^{12}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^{12}\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow 218$$

$$\frac{1}{4}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^4}}{3ac}-\frac{\sqrt{c+dx^8}(5bc-2ad)}{3acx^{12}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^{12}\left(a+bx^8\right)(bc-ad)}\right)$$

input
```
Int[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

3.917. $\int \frac{1}{x^{13}(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

output
```
((b*Sqrt[c + d*x^8])/(2*a*(b*c - a*d)*x^12*(a + b*x^8)) + (-1/3*((5*b*c -
2*a*d)*Sqrt[c + d*x^8])/(a*c*x^12) - (-((((15*b^2*c)/a - 8*b*d - (4*a*d^2)
/c)*Sqrt[c + d*x^8])/x^4) - (3*b^2*c*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*
d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/(2
*a*(b*c - a*d)))/4
```

3.917.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.917. $\int \frac{1}{x^{13}(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

rule 965

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.917.4 Maple [A] (verified)

Time = 47.79 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{8}+c}\,\left(-2ad\,x^{8}-6bc\,x^{8}+ac\right)}{3x^{12}}-\frac{b^{2}c^{2}\left(\frac{b\sqrt{d\,x^{8}+c}\,x^{4}}{b\,x^{8}+a}-\frac{(6ad-5bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}\,a}{x^{4}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2(ad-bc)}}{4a^{3}c^{2}}$	134

input

```
int(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/4/a^3*(-1/3*(d*x^8+c)^(1/2)*(-2*a*d*x^8-6*b*c*x^8+a*c)/x^12-1/2*b^2*c^2/
(a*d-b*c)*(b*(d*x^8+c)^(1/2)*x^4/(b*x^8+a)-(6*a*d-5*b*c)/((a*d-b*c)*a)^(1/
2)*arctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2))))/c^2
```

3.917.5 Fricas [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 760, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^{13}\left(a+bx^{8}\right)^{2}\sqrt{c+dx^{8}}}\,dx$$
$$=\left[-\frac{3\left(\left(5\,b^{4}c^{3}-6\,ab^{3}c^{2}d\right)x^{20}+\left(5\,ab^{3}c^{3}-6\,a^{2}b^{2}c^{2}d\right)x^{12}\right)\sqrt{-abc+a^{2}d}\,\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{16}-2\,\left(3\,abc^{2}-4\,a^{2}\right.}{b}\right.}{}\right.$$

input

```
integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^20 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2
*d)*x^12)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16
 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x
^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) -
4*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^16
+ 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^8 - 2*a^
3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*sqrt(d*x^8 + c))/((a^4*b^3*c^4 -
2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^20 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7
*c^2*d^2)*x^12), 1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^20 + (5*a*b^3*c^3
- 6*a^2*b^2*c^2*d)*x^12)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^8
 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a
*b*c^2 - a^2*c*d)*x^4)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*
c*d^2 + 4*a^4*b*d^3)*x^16 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d
^2 + 2*a^5*d^3)*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*sqrt(d*
x^8 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^20 + (a^5*b^2
*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^12)]
```

3.917.6 Sympy [F]

$$\int \frac{1}{x^{13} (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^{13} (a+bx^8)^2 \sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**13/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**13*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.917.7 Maxima [F]

$$\int \frac{1}{x^{13} (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)^2 \sqrt{dx^8+c}\,x^{13}}\,dx$$

input
```
integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^13), x)
```

3.917.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(184) = 368$.

Time = 1.05 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.90

$$\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

$$= \frac{1}{24}\,d^{\frac{7}{2}}\left(\frac{3\left(5\,b^3c-6\,ab^2d\right)\arctan\left(-\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(a^3bcd^3-a^4d^4\right)\sqrt{abcd-a^2d^2}}-\frac{6\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right.\right.}{\left(a^3bcd^3-a^4d^4\right)\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b\right.}\right.$$

input
```
integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/24*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x
^8 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d
^4)*sqrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b^3*c
- 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a
^4*d^4)*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*x^8
 + c))^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)) - 8*(3*(s
qrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*
c - 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*
x^4 - sqrt(d*x^8 + c))^2 - c)^3*a^3*d^3))
```

3.917.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \int \frac{1}{x^{13}\,\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/(x^13*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^13*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.918 $\int \frac{x^9}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

3.918.1 Optimal result

Integrand size = 24, antiderivative size = 924

$$
\begin{aligned}
\int \frac{x^9}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx &= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} \\
&\quad -\frac{(bc+ad)\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}}+\frac{(bc+ad)\arctan\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{3/4}b^{3/4}(-bc+ad)^{3/2}} \\
&\quad +\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}} \\
&\quad +\frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32ab\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}} \\
&\quad -\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}} \\
&\quad +\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{64ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}} \\
&\quad +\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{64ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}}
\end{aligned}
$$

output
```
-1/32*(a*d+b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(
1/2))/(-a)^(3/4)/b^(3/4)/(-a*d+b*c)^(3/2)+1/32*(a*d+b*c)*arctan(x^2*(a*d-b
*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(3/4)/b^(3/4)/(a*d-b*c)
^(3/2)-1/8*x^2*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)-1/16*d^(3/4)*(cos(2*ar
ctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Ell
ipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2
))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^8+c
)^(1/2)+1/32*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*ar
ctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/
2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*
x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)
+1/32*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^
(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/
2))*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c
)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/b/c^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/
64*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c
^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2
)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c
^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(
1/2)+x^4*d^(1/2))^2)^(1/2)/a/b/c^(1/4)/d^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/...
```

3.918.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.17

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx =$$

$$-\frac{x^2\left(5a(c+dx^8)-5c(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+dx^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\mathrm{Ap}\right.}{40a(bc-ad)\left(a+bx^8\right)\sqrt{c+dx^8}}$$

input
```
Integrate[x^9/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
-1/40*(x^2*(5*a*(c + d*x^8) - 5*c*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1
[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + d*x^8*(a + b*x^8)*Sqrt[1
+ (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a)]))/(a*(
b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

3.918.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 1033, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {965, 971, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^8}{(bx^8+a)^2\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{971}$$

$$\frac{1}{2}\left(\frac{\int \frac{c-dx^8}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{4(bc-ad)}-\frac{x^2\sqrt{c+dx^8}}{4(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{1021}$$

$$\frac{1}{2}\left(\frac{\frac{(ad+bc)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2}{b}-\frac{d\int \frac{1}{\sqrt{dx^8+c}}dx^2}{b}}{4(bc-ad)}-\frac{x^2\sqrt{c+dx^8}}{4(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\left(\frac{\frac{(ad+bc)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2}{b}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^8}}}{4(bc-ad)}-\frac{x^2\sqrt{c+dx^8}}{4(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{925}$$

$$\frac{1}{2}\left(\frac{\frac{(ad+bc)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^4}{\sqrt{-a}}+1\right)\sqrt{dx^8+c}}dx^2}{2a}\right)}{b}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^8}}}{4(bc-ad)}-\right.$$

$$\Big\downarrow \; 1541$$

$$\frac{1}{2}\left(\frac{\frac{(ad+bc)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^4+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\sqrt{b}\sqrt{c}\left(\sqrt{-}\right.}{2a}\right.}{b}}{4(bc-ad)}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{\frac{(ad+bc)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\right.}{2a}\right.}{b}}{4(bc-ad)}\right.$$

$$\Big\downarrow \; 761$$

$$\frac{1}{2}\left(\frac{(ad+bc)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}}{2a}+\right.}{b}\right.$$

$$\Big\downarrow \; 2221$$

3.918. $\int \frac{x^9}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

$$\frac{1}{2}\Bigg(\frac{(bc+ad)\Bigg(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\Bigg(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\Big(}{2\sqrt[4]{b}\sqrt{bc-a}}}{2a}}{}$$

$$\Big\downarrow \text{2223}$$

$$\frac{1}{2}\Bigg(\frac{(bc+ad)\Bigg(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\Bigg(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\Big(}{2\sqrt[4]{b}\sqrt{bc-a}}}{2a}}{}$$

3.918. $\int \frac{x^9}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

input
```
Int[x^9/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-1/4*(x^2*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (-1/2*(d^(3/4)*(Sq
rt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF
[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(b*c^(1/4)*Sqrt[c + d*x^8]) + ((b*
c + a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sq
rt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[
(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (S
qrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c]
)/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqr
t[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d
])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^
4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqr
t[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*d
^(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c]
 + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] +
Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4
)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt
[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c -
 a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*
d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt
[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + S...
```

3.918.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

3.918. $\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.918.4 Maple [F]

$$\int \frac{x^{9}}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}d x$$

input
```
int(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.918.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\text{Timed out}$$

input
```
integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.918.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\text{Timed out}$$

input
```
integrate(x**9/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Timed out
```

3.918.7 Maxima [F]

$$\int \frac{x^9}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^9}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^9/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.918.8 Giac [F]

$$\int \frac{x^9}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^9}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^9/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.918.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^9}{\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^9/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^9/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.919 $\int \frac{x}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

3.919.1 Optimal result

Integrand size = 22, antiderivative size = 999

$$
\begin{aligned}
\int \frac{x}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx &= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} \\
&+ \frac{\sqrt[4]{b}(3bc-5ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{7/4}(bc-ad)^{3/2}} - \frac{\sqrt[4]{b}(3bc-5ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{7/4}(-bc+ad)^{3/2}} \\
&+ \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}} \\
&+ \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}} \\
&+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32(-a)^{3/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}} \\
&+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}} \\
&+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}
\end{aligned}
$$

output
```
1/32*b^(1/4)*(-5*a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)
/(d*x^8+c)^(1/2))/(-a)^(7/4)/(-a*d+b*c)^(3/2)-1/32*b^(1/4)*(-5*a*d+3*b*c)*
arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(7/4)/
(a*d-b*c)^(3/2)+1/8*b*x^2*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)+1/16*d^(3
/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/
c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/
2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+
b*c)/(d*x^8+c)^(1/2)+1/32*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2
/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arc
tan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2)
)*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4
)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/32*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*
arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*E
llipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1
/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))
^2)^(1/2)/(-a)^(3/2)/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(-5
*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1
/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/
2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^
(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d...
```

3.919.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.17

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$= \frac{x^2\Bigg(5ab(c+dx^8)+5(3bc-4ad)\,(a+bx^8)\,\sqrt{1+\frac{dx^8}{c}}\,\mathrm{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+bdx^8(a+bx^8)\,\sqrt{}}{40a^2(bc-ad)\,(a+bx^8)\,\sqrt{c+dx^8}}$$

input
```
Integrate[x/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^2*(5*a*b*(c + d*x^8) + 5*(3*b*c - 4*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/c
]*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + b*d*x^8*(a + b*
x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8
)/a)]))/(40*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

3.919.3 Rubi [A] (warning: unable to verify)

Time = 1.43 (sec) , antiderivative size = 1036, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {965, 931, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{ 931}$$

$$\frac{1}{2}\left(\frac{bx^2\sqrt{c+dx^8}}{4a\left(a+bx^8\right)(bc-ad)}-\frac{\int -\frac{bdx^8+3bc-4ad}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2}{4a(bc-ad)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\int \frac{bdx^8+3bc-4ad}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2}{4a(bc-ad)}+\frac{bx^2\sqrt{c+dx^8}}{4a\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 1021}$$

$$\frac{1}{2}\left(\frac{(3bc-5ad)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2+d\int \frac{1}{\sqrt{dx^8+c}}dx^2}{4a(bc-ad)}+\frac{bx^2\sqrt{c+dx^8}}{4a\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{2}\left(\frac{(3bc-5ad)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}}}{4a(bc-ad)}+\frac{bx^2\sqrt{c+dx^8}}{4a\left(a+bx^8\right)(bc-\right.$$

$$\Big\downarrow \text{ 925}$$

$$\frac{1}{2}\Bigg(\frac{(3bc-5ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^4}{\sqrt{-a}}+1\right)\sqrt{dx^8+c}}dx^2}{2a}\right)+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right.\right.}{2\sqrt[4]{c}\sqrt{c+dx^8}}}{4a(bc-ad)}$$

$$\Big\downarrow \color{blue}{1541}$$

$$\frac{1}{2}\Bigg(\frac{(3bc-5ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+}{}\right.}{4a(bc-ad)}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{1}{2}\Bigg(\frac{(3bc-5ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\big(\sqrt{}}{}}{2}\right.}{4a(bc-ad)}$$

$$\Big\downarrow \color{blue}{761}$$

$$\frac{1}{2}\Bigg(\frac{(3bc-5ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\right.}{2\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}}{2a}\right.}{}$$

3.919. $\int \frac{x}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

$$\Big\downarrow \text{2221}$$

$$\frac{1}{2}\left(\frac{b\sqrt{dx^8+c}x^2}{4a(bc-ad)\left(bx^8+a\right)}+\frac{\frac{d^{3/4}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^8+c}}+(3bc-5ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}}{}}{}\right.}{}\right.$$

$$\Big\downarrow \text{2223}$$

$$\frac{1}{2}\left(\frac{b\sqrt{dx^8+c}x^2}{4a(bc-ad)\left(bx^8+a\right)}+\frac{\frac{d^{3/4}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^8+c}}+(3bc-5ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}}{}}{}\right.}{}\right.$$

3.919. $\int \frac{x}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

input
```
Int[x/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((b*x^2*Sqrt[c + d*x^8])/(4*a*(b*c - a*d)*(a + b*x^8)) + ((d^(3/4)*(Sqrt[c
] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*A
rcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*Sqrt[c + d*x^8]) + (3*b*c -
 5*a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqr
t[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(
d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sq
rt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])
/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt
[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d]
)/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4
)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt
[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*d^
(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c]
+ a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + S
qrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)
*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[
d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c -
a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d
]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[
(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sq...
```

3.919.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

3.919. $\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.919.4 Maple [F]

$$\int \frac{x}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}dx$$

input
```
int(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.919.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.919.6 Sympy [F]

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.919.7 Maxima [F]

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.919.8 Giac [F]

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.919.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.920 $\int \frac{1}{x^7(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

3.920.1 Optimal result

Integrand size = 24, antiderivative size = 1060

$$\int \frac{1}{x^7\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=-\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)}$$

$$+\frac{b^{5/4}(7bc-9ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{11/4}(bc-ad)^{3/2}}-\frac{b^{5/4}(7bc-9ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{11/4}(-bc+ad)^{3/2}}$$

$$-\frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{48a^2c^{5/4}(bc-ad)\sqrt{c+dx^8}}$$

$$+\frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32(-a)^{5/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32(-a)^{5/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\Bigg(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\Bigg(}{64a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$-\frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\Bigg(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\Bigg(}{64a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

output
```
1/32*b^(5/4)*(-9*a*d+7*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)
/(d*x^8+c)^(1/2))/(-a)^(11/4)/(-a*d+b*c)^(3/2)-1/32*b^(5/4)*(-9*a*d+7*b*c)
*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(11/4
)/(a*d-b*c)^(3/2)-1/24*(-4*a*d+7*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/x^6
+1/8*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^6/(b*x^8+a)-1/48*d^(3/4)*(-4*a*d+7*b
*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/
c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/
2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(5/4)/(-a*
d+b*c)/(d*x^8+c)^(1/2)+1/32*b*d^(1/4)*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)
*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2
*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*
c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)
^(5/2)/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/64*b*(-9*a*d+7*b*c)*
(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1
/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(
-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1
/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2
)+x^4*d^(1/2))^2)^(1/2)/a^3/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^8+c)^(
1/2)-1/32*b*d^(1/4)*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^
(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)...
```

3.920.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.32 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^7\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

$$= \frac{5a(c+dx^8)\left(4a^2d-7b^2cx^8-4ab(c-dx^8)\right)+5\left(-21b^2c^2+20abcd+4a^2d^2\right)x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\text{Appell}}{120a^3c(bc-ad)x^6\,(a}$$

input
```
Integrate[1/(x^7*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(5*a*(c + d*x^8)*(4*a^2*d - 7*b^2*c*x^8 - 4*a*b*(c - d*x^8)) + 5*(-21*b^2*
c^2 + 20*a*b*c*d + 4*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1
[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + b*d*(-7*b*c + 4*a*d)*x^16
*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c),
-((b*x^8)/a)])/(120*a^3*c*(b*c - a*d)*x^6*(a + b*x^8)*Sqrt[c + d*x^8])
```

3.920.3 Rubi [A] (warning: unable to verify)

Time = 1.57 (sec) , antiderivative size = 1092, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {965, 972, 25, 1053, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{1}{x^8 \left(bx^8+a\right)^2 \sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{972}$$

$$\frac{1}{2}\left(\frac{b\sqrt{c+dx^8}}{4ax^6 \left(a+bx^8\right) (bc-ad)}-\frac{\int -\frac{5bdx^8+7bc-4ad}{x^8 \left(bx^8+a\right) \sqrt{dx^8+c}}dx^2}{4a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{5bdx^8+7bc-4ad}{x^8 \left(bx^8+a\right) \sqrt{dx^8+c}}dx^2}{4a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{4ax^6 \left(a+bx^8\right) (bc-ad)}\right)$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{2}\left(\frac{-\frac{\int \frac{bd(7bc-4ad)x^8+21b^2c^2-4a^2d^2-20abcd}{\left(bx^8+a\right) \sqrt{dx^8+c}}dx^2}{3ac}-\frac{\sqrt{c+dx^8}(7bc-4ad)}{3acx^6}}{4a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{4ax^6 \left(a+bx^8\right) (bc-ad)}\right)$$

$$\Big\downarrow \text{1021}$$

$$\frac{1}{2}\left(\frac{-\frac{d(7bc-4ad)\int \frac{1}{\sqrt{dx^8+c}}dx^2+3bc(7bc-9ad)\int \frac{1}{\left(bx^8+a\right) \sqrt{dx^8+c}}dx^2}{3ac}-\frac{\sqrt{c+dx^8}(7bc-4ad)}{3acx^6}}{4a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{4ax^6 \left(a+bx^8\right) (bc-ad)}\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\Bigg(\frac{-\frac{3bc(7bc-9ad)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}(7bc-4ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}}}{3ac}-\frac{\sqrt{c+dx^8}(7bc-4ad)}{3acx^6}}{4a(bc-ad)}+$$

$$\Big\downarrow 925$$

$$\frac{1}{2}\Bigg(\frac{-\frac{3bc(7bc-9ad)\Bigg(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^4}{\sqrt{-a}}+1\right)\sqrt{dx^8+c}}dx^2}{2a}\Bigg)+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}(7bc-4ad)\,\text{EllipticF}\Bigg(2\arctan\Bigg(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}}{2\sqrt[4]{c}\sqrt{c+dx^8}}}{3ac}}{4a(bc-ad)}$$

$$\Big\downarrow 1541$$

$$\frac{1}{2}\Bigg(\frac{-\frac{3bc(7bc-9ad)\Bigg(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{}}{}}{}}{3ac}}{4a(}$$

$$\Big\downarrow 27$$

$$\frac{1}{2}\Bigg(\frac{-\frac{3bc(7bc-9ad)\Bigg(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\Big(\sqrt{-}}{}}{2a}}{3ac}}{4a(bc-}$$

$$\Big\downarrow 761$$

3.920. $\int \frac{1}{x^7\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

$$\frac{1}{2}\left(-\frac{3bc(7bc-9ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)\right.}{2\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}}{2a}\right.}{}\right.$$

$$\Big\downarrow \text{ 2221}$$

$$\frac{1}{2}\left(\frac{\sqrt{dx^8+c}\,b}{4a(bc-ad)x^6\left(bx^8+a\right)}+\frac{-\frac{\sqrt{dx^8+c}(7bc-4ad)}{3acx^6}-\frac{\frac{d^{3/4}(7bc-4ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^8+c}}+3bc(7b}{}}{}\right.$$

$$\Big\downarrow \text{ 2223}$$

3.920. $\int \frac{1}{x^7\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

$$\frac{1}{2}\left(\frac{\sqrt{dx^8+c}\,b}{4a(bc-ad)x^6\left(bx^8+a\right)}+\frac{-\frac{\sqrt{dx^8+c}(7bc-4ad)}{3acx^6}-\frac{\frac{d^{3/4}(7bc-4ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^8+c}}+3bc(7}{}}{}\right.$$

input
```
Int[1/(x^7*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

3.920. $\int \frac{1}{x^7\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

output
```
((b*Sqrt[c + d*x^8])/(4*a*(b*c - a*d)*x^6*(a + b*x^8)) + (-1/3*((7*b*c - 4
*a*d)*Sqrt[c + d*x^8])/(a*c*x^6) - ((d^(3/4)*(7*b*c - 4*a*d)*(Sqrt[c] + Sq
rt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[
(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*Sqrt[c + d*x^8]) + 3*b*c*(7*b*c -
 9*a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqr
t[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(
d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sq
rt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])
/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt
[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d]
)/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4
)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt
[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*d^
(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c]
+ a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + S
qrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)
*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[
d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c -
a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d
]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sq...
```

3.920.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.920. $\int \frac{1}{x^7(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.920.4 Maple [F]

$$\int \frac{1}{x^{7} \left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}dx$$

input
```
int(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.920.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.920.6 Sympy [F]

$$\int \frac{1}{x^7 \left(a+b x^8\right)^2 \sqrt{c+d x^8}}\,dx=\int \frac{1}{x^7 \left(a+b x^8\right)^2 \sqrt{c+d x^8}}\,dx$$

input
```
integrate(1/x**7/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**7*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.920.7 Maxima [F]

$$\int \frac{1}{x^7 \left(a+b x^8\right)^2 \sqrt{c+d x^8}}\,dx=\int \frac{1}{{\left(b x^{8}+a\right)}^{2} \sqrt{d x^{8}+c} x^{7}}\,dx$$

input
```
integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^7), x)
```

3.920.8 Giac [F]

$$\int \frac{1}{x^7 \left(a+b x^8\right)^2 \sqrt{c+d x^8}}\,dx=\int \frac{1}{{\left(b x^{8}+a\right)}^{2} \sqrt{d x^{8}+c} x^{7}}\,dx$$

input
```
integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^7), x)
```

3.920.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^7 \left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/(x^7*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^7*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.921 $\int \frac{x^{13}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

3.921.1 Optimal result

Integrand size = 24, antiderivative size = 1164

$$\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\frac{\sqrt{d}x^2\sqrt{c+dx^8}}{8b(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)}-\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\,(a+bx^8)}$$

$$+\frac{(3bc-ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32\sqrt[4]{-a}b^{5/4}(bc-ad)^{3/2}}+\frac{(3bc-ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32\sqrt[4]{-a}b^{5/4}(-bc+ad)^{3/2}}$$

$$-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b(bc-ad)\sqrt{c+dx^8}}$$

$$+\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b(bc-ad)\sqrt{c+dx^8}}$$

$$-\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$-\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

output
```
1/32*(-a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)
^(1/2))/(-a)^(1/4)/b^(5/4)/(-a*d+b*c)^(3/2)+1/32*(-a*d+3*b*c)*arctan(x^2*(
a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(1/4)/b^(5/4)/(a*d
-b*c)^(3/2)-1/8*x^6*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)+1/8*x^2*d^(1/2)*(
d*x^8+c)^(1/2)/b/(-a*d+b*c)/(c^(1/2)+x^4*d^(1/2))-1/8*c^(1/4)*d^(1/4)*(cos
(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4))
)*EllipticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d
^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/(-a*d+b*c)/(d*x^8+c)^(
1/2)+1/16*c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos
(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4)
)),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^
(1/2)/b/(-a*d+b*c)/(d*x^8+c)^(1/2)-1/32*d^(1/4)*(-a*d+3*b*c)*(cos(2*arctan
(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Ellipti
cF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(
b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1
/2)/b^(3/2)/c^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^8+c)^(1/2)-1/64*(-a*d+3*b*c)*(
cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/
4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-
a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/
2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1...
```

3.921.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.14

$$\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{x^6\left(-7a(c+dx^8)+7c(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+dx^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\mathrm{Ap}\right.}{56a(bc-ad)\,(a+bx^8)\sqrt{c+dx^8}}$$

input
```
Integrate[x^13/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^6*(-7*a*(c + d*x^8) + 7*c*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[3/4,
 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + d*x^8*(a + b*x^8)*Sqrt[1 + (d*
x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)/a)]))/(56*a*(b
*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

3.921.3 Rubi [A] (warning: unable to verify)

Time = 1.55 (sec) , antiderivative size = 1107, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {965, 971, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^{12}}{(bx^8+a)^2\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{971}$$

$$\frac{1}{2}\left(\frac{\int \frac{x^4(dx^8+3c)}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{4(bc-ad)}-\frac{x^6\sqrt{c+dx^8}}{4(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{1054}$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{dx^4}{b\sqrt{dx^8+c}}+\frac{(3bc-ad)x^4}{b(bx^8+a)\sqrt{dx^8+c}}\right)dx^2}{4(bc-ad)}-\frac{x^6\sqrt{c+dx^8}}{4(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{(3bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}+\frac{(3bc-ad)\arctan\left(\right.}{4\sqrt[4]{-a}b^{5/4}}\right.$$

input
```
Int[x^13/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output

```
(-1/4*(x^6*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + ((Sqrt[d]*x^2*Sqrt
[c + d*x^8])/(b*(Sqrt[c] + Sqrt[d]*x^4)) + ((3*b*c - a*d)*ArcTan[(Sqrt[b*c
 - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4)*b^(5/4)*
Sqrt[b*c - a*d]) - ((3*b*c - a*d)*ArcTanh[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4
)*b^(1/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4)*b^(5/4)*Sqrt[b*c - a*d]) - (c^(
1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x
^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(b*Sqrt[c + d*x^8]
) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + S
qrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*b*Sqrt[
c + d*x^8]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d
)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^8]) - ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*
d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ell
ipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqr
t[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqr
t[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi
[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d
]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d...
```

3.921.3.1 Defintions of rubi rules used

rule 965

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 971

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))    Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.921.4 Maple [F]

$$\int \frac{x^{13}}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}dx$$

input
```
int(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.921.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\text{Timed out}$$

input
```
integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.921.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\text{Timed out}$$

input
```
integrate(x**13/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Timed out
```

3.921.7 Maxima [F]

$$\int \frac{x^{13}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^{13}}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^13/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.921.8 Giac [F]

$$\int \frac{x^{13}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^{13}}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^13/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.921.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^{13}}{\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^13/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^13/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.922 $\int \frac{x^5}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

3.922.1 Optimal result

Integrand size = 24, antiderivative size = 1162

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{d}x^2\sqrt{c+dx^8}}{8a(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)}+\frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)}$$

$$-\frac{(bc-3ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{5/4}\sqrt[4]{b}(bc-ad)^{3/2}}-\frac{(bc-3ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{5/4}\sqrt[4]{b}(-bc+ad)^{3/2}}$$

$$+\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a(bc-ad)\sqrt{c+dx^8}}$$

$$-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a(bc-ad)\sqrt{c+dx^8}}$$

$$-\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$-\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\Bigg(}{64(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$+\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\Bigg(}{64(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

output
```
-1/32*(-3*a*d+b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c
)^(1/2))/(-a)^(5/4)/b^(1/4)/(-a*d+b*c)^(3/2)-1/32*(-3*a*d+b*c)*arctan(x^2*
(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(5/4)/b^(1/4)/(a*
d-b*c)^(3/2)+1/8*b*x^6*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)-1/8*x^2*d^(1
/2)*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(c^(1/2)+x^4*d^(1/2))+1/8*c^(1/4)*d^(1/4)
*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(
1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+
x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/(-a*d+b*c)/(d*x^8
+c)^(1/2)-1/16*c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2
)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^
(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2)
)^2)^(1/2)/a/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(-3*a*d+b*c)*(cos(2*arctan(d^
(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi
(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2
))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))
*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2
)^(1/2)/(-a)^(3/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/b^(1/2)/(d*x^8+c)^
(1/2)-1/64*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2
*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))
),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2...
```

3.922.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.15

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$
$$= \frac{x^6\left(21ab(c+dx^8)+7(bc-4ad)\,(a+bx^8)\,\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-3bdx^8(a+bx^8)\right.}{168a^2(bc-ad)\,(a+bx^8)\,\sqrt{c+dx^8}}$$

input
```
Integrate[x^5/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^6*(21*a*b*(c + d*x^8) + 7*(b*c - 4*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]
*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] - 3*b*d*x^8*(a + b
*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x
^8)/a)]))/(168*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

3.922.3 Rubi [A] (warning: unable to verify)

Time = 1.42 (sec) , antiderivative size = 1093, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {965, 972, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^8+a)^2\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{972}$$

$$\frac{1}{2}\left(\frac{bx^6\sqrt{c+dx^8}}{4a(a+bx^8)(bc-ad)}-\frac{\int -\frac{x^4(-bdx^8+bc-4ad)}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{4a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{x^4(-bdx^8+bc-4ad)}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{4a(bc-ad)}+\frac{bx^6\sqrt{c+dx^8}}{4a(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{1054}$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{(bc-3ad)x^4}{(bx^8+a)\sqrt{dx^8+c}}-\frac{dx^4}{\sqrt{dx^8+c}}\right)dx^2}{4a(bc-ad)}+\frac{bx^6\sqrt{c+dx^8}}{4a(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{b\sqrt{dx^8+c}x^6}{4a(bc-ad)(bx^8+a)}+\frac{-\frac{(bc-3ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}}{8\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}}{}\right.$$

input
```
Int[x^5/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

---

output
```
((b*x^6*Sqrt[c + d*x^8])/(4*a*(b*c - a*d)*(a + b*x^8)) + (-((Sqrt[d]*x^2*S
qrt[c + d*x^8])/(Sqrt[c] + Sqrt[d]*x^4)) + ((b*c - 3*a*d)*ArcTan[(Sqrt[b*c
 - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4)*b^(1/4)*
Sqrt[b*c - a*d]) - ((b*c - 3*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4
)*b^(1/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) + (c^(
1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x
^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/Sqrt[c + d*x^8] -
(c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[
d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*Sqrt[c + d*
x^8]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqr
t[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[
2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x
^8]) - ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt
[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2
*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^
8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqr
t[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*
Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcT
an[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*Sqrt[-a]*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c
+ a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c ...
```

3.922.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.922.4 Maple [F]

$$\int \frac{x^{5}}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}dx$$

input
```
int(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.922.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^8)^2 \sqrt{c+dx^8}}\, dx = \text{Timed out}$$

input
```
integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.922.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^8)^2 \sqrt{c+dx^8}}\, dx = \int \frac{x^{5}}{\left(a+bx^{8}\right)^{2} \sqrt{c+dx^{8}}}\, dx$$

input
```
integrate(x**5/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.922.7 Maxima [F]

$$\int \frac{x^5}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^5}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.922.8 Giac [F]

$$\int \frac{x^5}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^5}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^5/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.922.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^5}{\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^5/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^5/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.923 $\int \frac{1}{x^3\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

3.923.1 Optimal result

Integrand size = 24, antiderivative size = 1243

$$
\begin{aligned}
&\int \frac{1}{x^3\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx\\
&= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2}+\frac{\sqrt{d}(5bc-4ad)x^2\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)}\\
&\quad-\frac{b^{3/4}(5bc-7ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{9/4}(bc-ad)^{3/2}}-\frac{b^{3/4}(5bc-7ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{9/4}(-bc+ad)^{3/2}}\\
&\quad-\frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a^2c^{3/4}(bc-ad)\sqrt{c+dx^8}}\\
&\quad+\frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a^2c^{3/4}(bc-ad)\sqrt{c+dx^8}}\\
&\quad+\frac{b\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}\\
&\quad+\frac{b\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}\\
&\quad-\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\right.}{64(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}\\
&\quad+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}
\end{aligned}
$$

output

```
-1/32*b^(3/4)*(-7*a*d+5*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4
)/(d*x^8+c)^(1/2))/(-a)^(9/4)/(-a*d+b*c)^(3/2)-1/32*b^(3/4)*(-7*a*d+5*b*c)
*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(9/4)
/(a*d-b*c)^(3/2)-1/8*(-4*a*d+5*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/x^2+1
/8*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^2/(b*x^8+a)+1/8*(-4*a*d+5*b*c)*x^2*d^(
1/2)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/(c^(1/2)+x^4*d^(1/2))-1/8*d^(1/4)*(-
4*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(
1/4)*x^2/c^(1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2
))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(
3/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/16*d^(1/4)*(-4*a*d+5*b*c)*(cos(2*arctan(
d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Elliptic
F(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((
d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(3/4)/(-a*d+b*c)/(d*x^8+c)^(
1/2)+1/64*(-7*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4)
)),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d
^(1/2),1/2*2^(1/2))*b^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1
/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/4
)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/64*(-7*a*d+5*b*c)*(cos(2*
arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4))...
```

3.923.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.31 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^3\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

$$=\frac{21a(c+dx^8)\left(4a^2d-5b^2cx^8-4ab(c-dx^8)\right)-7\left(5b^2c^2-12abcd+4a^2d^2\right)x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\mathrm{AppellF}}{168a^3c(bc-ad)x^2\,(a}$$

input

```
Integrate[1/(x^3*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output

```
(21*a*(c + d*x^8)*(4*a^2*d - 5*b^2*c*x^8 - 4*a*b*(c - d*x^8)) - 7*(5*b^2*c
^2 - 12*a*b*c*d + 4*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[
3/4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*(5*b*c - 4*a*d)*x^16
*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c),
 -((b*x^8)/a)])/(168*a^3*c*(b*c - a*d)*x^2*(a + b*x^8)*Sqrt[c + d*x^8])
```

3.923.3 Rubi [A] (warning: unable to verify)

Time = 1.53 (sec) , antiderivative size = 1170, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {965, 972, 25, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(bx^8+a\right)^2 \sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{972}$$

$$\frac{1}{2} \left(\frac{b\sqrt{c+dx^8}}{4ax^2 \left(a+bx^8\right) (bc-ad)}-\frac{\int -\frac{3bdx^8+5bc-4ad}{x^4 \left(bx^8+a\right) \sqrt{dx^8+c}}dx^2}{4a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(\frac{\int \frac{3bdx^8+5bc-4ad}{x^4 \left(bx^8+a\right) \sqrt{dx^8+c}}dx^2}{4a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{4ax^2 \left(a+bx^8\right) (bc-ad)}\right)$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{2} \left(\frac{-\frac{\int \frac{x^4 \left((bc-2ad)(5bc-2ad)-bd(5bc-4ad)x^8\right)}{\left(bx^8+a\right) \sqrt{dx^8+c}}dx^2}{ac}-\frac{\sqrt{c+dx^8}(5bc-4ad)}{acx^2}}{4a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{4ax^2 \left(a+bx^8\right) (bc-ad)}\right)$$

$$\Big\downarrow \text{1054}$$

$$\frac{1}{2} \left(\frac{-\frac{\int \left(\frac{\left(5b^2c^2-7abcd\right)x^4}{\left(bx^8+a\right) \sqrt{dx^8+c}}-\frac{d(5bc-4ad)x^4}{\sqrt{dx^8+c}}\right)dx^2}{ac}-\frac{\sqrt{c+dx^8}(5bc-4ad)}{acx^2}}{4a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{4ax^2 \left(a+bx^8\right) (bc-ad)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\sqrt{dx^8+c}\,b}{4a(bc-ad)x^2\left(bx^8+a\right)}+\frac{-\frac{\sqrt{dx^8+c}(5bc-4ad)}{acx^2}-\frac{-\frac{\sqrt{b}c^{3/4}(5bc-7ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{arcta}\right.}{8\sqrt{-a}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}}{}}{}\right.$$

input
```
Int[1/(x^3*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((b*Sqrt[c + d*x^8])/(4*a*(b*c - a*d)*x^2*(a + b*x^8)) + (-(((5*b*c - 4*a*
d)*Sqrt[c + d*x^8])/(a*c*x^2)) - (-((Sqrt[d]*(5*b*c - 4*a*d)*x^2*Sqrt[c +
d*x^8])/(Sqrt[c] + Sqrt[d]*x^4)) + (b^(3/4)*c*(5*b*c - 7*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4)*Sqrt
[b*c - a*d]) - (b^(3/4)*c*(5*b*c - 7*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x^2)/((
-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4)*Sqrt[b*c - a*d]) + (c^(
1/4)*d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqr
t[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/Sq
rt[c + d*x^8] - (c^(1/4)*d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*S
qrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2
)/c^(1/4)], 1/2])/(2*Sqrt[c + d*x^8]) - (b*c^(3/4)*(Sqrt[c] - (Sqrt[-a]*Sq
rt[d])/Sqrt[b])*d^(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)
], 1/2])/(4*(b*c + a*d)*Sqrt[c + d*x^8]) - (b*c^(3/4)*(Sqrt[c] + (Sqrt[-a]
*Sqrt[d])/Sqrt[b])*d^(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
 + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1
/4)], 1/2])/(4*(b*c + a*d)*Sqrt[c + d*x^8]) - (Sqrt[b]*c^(3/4)*(Sqrt[b]*Sq
rt[c] - Sqrt[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(
c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-
a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x...
```

3.923. $\int \frac{1}{x^3(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

3.923.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.923. $\int \frac{1}{x^3(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

3.923.4 Maple [F]

$$\int \frac{1}{x^{3} \left(b \,x^{8}+a \right)^{2} \sqrt{d \,x^{8}+c}}d x$$

input
```
int(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.923.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}}\, dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.923.6 Sympy [F]

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}}\, dx = \int \frac{1}{x^{3} \left(a + b x^{8}\right)^{2} \sqrt{c + d x^{8}}}\, dx$$

input
```
integrate(1/x**3/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.923.7 Maxima [F]

$$\int \frac{1}{x^3 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx=\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^3), x)
```

3.923.8 Giac [F]

$$\int \frac{1}{x^3 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx=\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^3), x)
```

3.923.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx=\int \frac{1}{x^3\,{\left(b\,x^8+a\right)}^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/(x^3*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.924 $\int \frac{x^4}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

3.924.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{x^5\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{5}{8},2,\frac{1}{2},\frac{13}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}$$

output
```
1/5*x^5*AppellF1(5/8,2,1/2,13/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/(
d*x^8+c)^(1/2)
```

3.924.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 170 vs. 2(64) = 128.

Time = 10.22 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.66

$$\int \frac{x^4}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

$$=\frac{x^5\Bigg(65ab(c+dx^8)+13(3bc-8ad)\left(a+bx^8\right)\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-5bdx^8(a+bx}{520a^2(bc-ad)\left(a+bx^8\right)\sqrt{c+dx^8}}$$

input
```
Integrate[x^4/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^5*(65*a*b*(c + d*x^8) + 13*(3*b*c - 8*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)
/c]*AppellF1[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] - 5*b*d*x^8*(a
 + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c), -
((b*x^8)/a)]))/(520*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

3.924.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^8}{c}+1}\int \frac{x^4}{(bx^8+a)^2\sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^5\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(\frac{5}{8},2,\frac{1}{2},\frac{13}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}$$

input
```
Int[x^4/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^5*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8, 2, 1/2, 13/8, -((b*x^8)/a), -((d*x^
8)/c)])/(5*a^2*Sqrt[c + d*x^8])
```

3.924.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.924.4 Maple [F]

$$\int \frac{x^{4}}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}dx$$

input
```
int(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.924.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.924.6 Sympy [F]

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**4/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**4/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.924.7 Maxima [F]

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^4}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.924.8 Giac [F]

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^4}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.924.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^4}{{\left(b\,x^8+a\right)}^2\,\sqrt{d\,x^8+c}} \,d x$$

input

```
int(x^4/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output

```
int(x^4/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.925 $\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

3.925.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\frac{x^3\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{3}{8},2,\frac{1}{2},\frac{11}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}$$

output
```
1/3*x^3*AppellF1(3/8,2,1/2,11/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/(
d*x^8+c)^(1/2)
```

3.925.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 170 vs. 2(64) = 128.

Time = 10.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.66

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$
$$=\frac{x^3\Bigg(33ab(c+dx^8)+11(5bc-8ad)\,(a+bx^8)\,\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{3}{8},\frac{1}{2},1,\frac{11}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+3bdx^8(a+bx}{264a^2(bc-ad)\,(a+bx^8)\,\sqrt{c+dx^8}}$$

input
```
Integrate[x^2/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^3*(33*a*b*(c + d*x^8) + 11*(5*b*c - 8*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)
/c]*AppellF1[3/8, 1/2, 1, 11/8, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*x^8*(a
 + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[11/8, 1/2, 1, 19/8, -((d*x^8)/c), -
((b*x^8)/a)]))/(264*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

3.925.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{dx^8}{c}+1}\int \frac{x^2}{(bx^8+a)^2\sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^3\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(\frac{3}{8},2,\frac{1}{2},\frac{11}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}$$

input
```
Int[x^2/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^3*Sqrt[1 + (d*x^8)/c]*AppellF1[3/8, 2, 1/2, 11/8, -((b*x^8)/a), -((d*x^
8)/c)])/(3*a^2*Sqrt[c + d*x^8])
```

3.925.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.925.4 Maple [F]

$$\int \frac{x^2}{\left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}}dx$$

input
```
int(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.925.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.925.6 Sympy [F]

$$\int \frac{x^2}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^2}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**2/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.925.7 Maxima [F]

$$\int \frac{x^2}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^2}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.925.8 Giac [F]

$$\int \frac{x^2}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^2}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.925.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^2/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.926 $\int \frac{1}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

3.926.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{x\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{1}{8},2,\frac{1}{2},\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}$$

output

```
x*AppellF1(1/8,2,1/2,9/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/(d*x^8+c
)^(1/2)
```

3.926.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 328 vs. $2(59) = 118$.

Time = 10.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 5.56

$$\int \frac{1}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=-\frac{x\left(bdx^8\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{9}{8},\frac{1}{2},1,\frac{17}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+\frac{3a\left(9ac\left(8ad-b\left(8c+dx^8\right)\right)\text{AppellF1}\left(\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+4bx^8\right.}{\left(a+bx^8\right)\left(-9ac\,\text{AppellF1}\left(\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+4x^8\left(\right.\right.}\right.}{24a^2(-bc+ad)\sqrt{c+dx^8}}$$

input

```
Integrate[1/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
-1/24*(x*(b*d*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[9/8, 1/2, 1, 17/8, -((d*x^8
)/c), -((b*x^8)/a)] + (3*a*(9*a*c*(8*a*d - b*(8*c + d*x^8))*AppellF1[1/8,
1/2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)] + 4*b*x^8*(c + d*x^8)*(2*b*c*Appe
llF1[9/8, 1/2, 2, 17/8, -((d*x^8)/c), -((b*x^8)/a)] + a*d*AppellF1[9/8, 3/
2, 1, 17/8, -((d*x^8)/c), -((b*x^8)/a)])))/((a + b*x^8)*(-9*a*c*AppellF1[1
/8, 1/2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)] + 4*x^8*(2*b*c*AppellF1[9/8,
1/2, 2, 17/8, -((d*x^8)/c), -((b*x^8)/a)] + a*d*AppellF1[9/8, 3/2, 1, 17/8
, -((d*x^8)/c), -((b*x^8)/a)])))))/(a^2*(-(b*c) + a*d)*Sqrt[c + d*x^8])
```

3.926.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^8}{c}+1}\int \frac{1}{\left(bx^8+a\right)^2\sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^8}{c}+1}\operatorname{AppellF1}\left(\frac{1}{8},2,\frac{1}{2},\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}$$

input
```
Int[1/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x*Sqrt[1 + (d*x^8)/c]*AppellF1[1/8, 2, 1/2, 9/8, -((b*x^8)/a), -((d*x^8)/
c)])/(a^2*Sqrt[c + d*x^8])
```

3.926.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.926.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}dx$$

input
```
int(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.926.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.926.6 Sympy [F]

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.926.7 Maxima [F]

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.926.8 Giac [F]

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.926.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{1}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

input

```
int(1/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output

```
int(1/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.927 $\int \frac{1}{x^2(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

3.927.1 Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=-\frac{\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(-\frac{1}{8},2,\frac{1}{2},\frac{7}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2x\sqrt{c+dx^8}}$$

output
```
-AppellF1(-1/8,2,1/2,7/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/x/(d*x^8
+c)^(1/2)
```

3.927.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 226 vs. $2(62) = 124$.

Time = 10.33 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^2\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

$$=\frac{35a(c+dx^8)\left(8a^2d-9b^2cx^8-8ab(c-dx^8)\right)-5(9b^2c^2-40abcd+24a^2d^2)\,x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\text{Appell}}{280a^3c(bc-ad)x\,(a}$$

input
```
Integrate[1/(x^2*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(35*a*(c + d*x^8)*(8*a^2*d - 9*b^2*c*x^8 - 8*a*b*(c - d*x^8)) - 5*(9*b^2*c
^2 - 40*a*b*c*d + 24*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1
[7/8, 1/2, 1, 15/8, -((d*x^8)/c), -((b*x^8)/a)] + 7*b*d*(9*b*c - 8*a*d)*x^
16*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[15/8, 1/2, 1, 23/8, -((d*x^8)/
c), -((b*x^8)/a)])/(280*a^3*c*(b*c - a*d)*x*(a + b*x^8)*Sqrt[c + d*x^8])
```

3.927.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^8}{c}+1}\int \frac{1}{x^2 \left(bx^8+a\right)^2 \sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\text{AppellF1}\left(-\frac{1}{8},2,\frac{1}{2},\frac{7}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2 x\sqrt{c+dx^8}}$$

input
```
Int[1/(x^2*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
-((Sqrt[1 + (d*x^8)/c]*AppellF1[-1/8, 2, 1/2, 7/8, -((b*x^8)/a), -((d*x^8)
/c)])/(a^2*x*Sqrt[c + d*x^8]))
```

3.927.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.927.4 Maple [F]

$$\int \frac{1}{x^{2} \left(b \,x^{8}+a \right)^{2} \sqrt{d \,x^{8}+c}}dx$$

input
```
int(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.927.5 Fricas [F]

$$\int \frac{1}{x^2 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx = \int \frac{1}{{\left(bx^{8} + a\right)}^{2} \sqrt{dx^{8} + c} x^{2}}\, dx$$

input
```
integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^8 + c)/(b^2*d*x^26 + (b^2*c + 2*a*b*d)*x^18 + (2*a*b*c +
 a^2*d)*x^10 + a^2*c*x^2), x)
```

3.927.6 Sympy [F]

$$\int \frac{1}{x^2 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^2 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**2/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.927.7 Maxima [F]

$$\int \frac{1}{x^2 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)^2 \sqrt{dx^8+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^2), x)
```

3.927.8 Giac [F]

$$\int \frac{1}{x^2 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)^2 \sqrt{dx^8+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^2), x)
```

3.927.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^2 \left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/(x^2*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.928 $\int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

3.928.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=-\frac{\sqrt{1+\frac{dx^8}{c}}\,\operatorname{AppellF1}\left(-\frac{3}{8},2,\frac{1}{2},\frac{5}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}$$

output
```
-1/3*AppellF1(-3/8,2,1/2,5/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/x^3/
(d*x^8+c)^(1/2)
```

3.928.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(64) = 128.

Time = 10.32 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.53

$$\int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$
$$=\frac{65a(c+dx^8)\left(8a^2d-11b^2cx^8-8ab(c-dx^8)\right)-13\left(33b^2c^2-56abcd+8a^2d^2\right)x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\text{Appe}}{1560a^3c(bc-ad)x^3}$$

input
```
Integrate[1/(x^4*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(65*a*(c + d*x^8)*(8*a^2*d - 11*b^2*c*x^8 - 8*a*b*(c - d*x^8)) - 13*(33*b^
2*c^2 - 56*a*b*c*d + 8*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*Appell
F1[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] + 5*b*d*(11*b*c - 8*a*d)
*x^16*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^
8)/c), -((b*x^8)/a)])/(1560*a^3*c*(b*c - a*d)*x^3*(a + b*x^8)*Sqrt[c + d*x
^8])
```

3.928.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^8}{c}+1} \int \frac{1}{x^4 (bx^8+a)^2 \sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^8}{c}+1} \text{AppellF1}\left(-\frac{3}{8},2,\frac{1}{2},\frac{5}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}$$

input
```
Int[1/(x^4*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
-1/3*(Sqrt[1 + (d*x^8)/c]*AppellF1[-3/8, 2, 1/2, 5/8, -((b*x^8)/a), -((d*x
^8)/c)])/(a^2*x^3*Sqrt[c + d*x^8])
```

3.928.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.928.4 Maple [F]

$$\int \frac{1}{x^{4} \left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}dx$$

input
```
int(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.928.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.928.6 Sympy [F]

$$\int \frac{1}{x^4 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx = \int \frac{1}{x^4 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx$$

input
```
integrate(1/x**4/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.928.7 Maxima [F]

$$\int \frac{1}{x^4 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx = \int \frac{1}{\left(bx^8+a\right)^2 \sqrt{dx^8+c}x^4}\, dx$$

input
```
integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^4), x)
```

3.928.8 Giac [F]

$$\int \frac{1}{x^4 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx = \int \frac{1}{\left(bx^8+a\right)^2 \sqrt{dx^8+c}x^4}\, dx$$

input
```
integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^4), x)
```

3.928.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^4 \left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/(x^4*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.929 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^5 \, dx$

3.929.1 Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^5 \, dx = \frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c^2} + \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} - \frac{d^2(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}}$$

output
```
1/6*a*(c+d/x^2)^(3/2)*x^6/c-1/16*d^2*(-a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/
c^(1/2))/c^(5/2)+1/16*d*(-a*d+2*b*c)*x^2*(c+d/x^2)^(1/2)/c^2+1/8*(-a*d+2*b
*c)*x^4*(c+d/x^2)^(1/2)/c
```

3.929.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.96

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^5 \, dx = \frac{\sqrt{c+\frac{d}{x^2}}x\left(\sqrt{c}x(6bc(d+2cx^2)+a(-3d^2+2cdx^2+8c^2x^4)) + \frac{6d^2(-2bc+ad)\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{d}+\sqrt{d+cx^2}}\right)}{\sqrt{d+cx^2}}\right)}{48c^{5/2}}$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]
```

output
```
(Sqrt[c + d/x^2]*x*(Sqrt[c]*x*(6*b*c*(d + 2*c*x^2) + a*(-3*d^2 + 2*c*d*x^2
 + 8*c^2*x^4)) + (6*d^2*(-2*b*c + a*d)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[d] + Sqr
t[d + c*x^2])])/Sqrt[d + c*x^2]))/(48*c^(5/2))
```

3.929.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {948, 87, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8d\frac{1}{x^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-\frac{(2bc-ad)\int \sqrt{c+\frac{d}{x^2}}x^6d\frac{1}{x^2}}{2c}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-\frac{(2bc-ad)\left(\frac{1}{4}d\int \frac{x^4}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}-\frac{1}{2}x^4\sqrt{c+\frac{d}{x^2}}\right)}{2c}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-\frac{(2bc-ad)\left(\frac{1}{4}d\left(-\frac{d\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}}{2c}-\frac{x^2\sqrt{c+\frac{d}{x^2}}}{c}\right)-\frac{1}{2}x^4\sqrt{c+\frac{d}{x^2}}\right)}{2c}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-\frac{(2bc-ad)\left(\frac{1}{4}d\left(-\frac{\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{c}-\frac{x^2\sqrt{c+\frac{d}{x^2}}}{c}\right)-\frac{1}{2}x^4\sqrt{c+\frac{d}{x^2}}\right)}{2c}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-\frac{(2bc-ad)\left(\frac{1}{4}d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{x^2\sqrt{c+\frac{d}{x^2}}}{c}\right)-\frac{1}{2}x^4\sqrt{c+\frac{d}{x^2}}\right)}{2c}\right)$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]
```

output
```
((a*(c + d/x^2)^(3/2)*x^6)/(3*c) - ((2*b*c - a*d)*(-1/2*(Sqrt[c + d/x^2]*x
^4) + (d*(-((Sqrt[c + d/x^2]*x^2)/c) + (d*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]
)/c^(3/2)))/4))/(2*c))/2
```

3.929.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

3.929. $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.929.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

method	result
risch	$\frac{x^{2} \left(8 a \,x^{4} c^{2}+2 a c d \,x^{2}+12 b \,c^{2} x^{2}-3 a \,d^{2}+6 b c d \right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}{48 c^{2}}+\frac{d^{2} \left(a d -2 b c \right) \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+d}\right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}{16 c^{\frac{5}{2}} \sqrt{c \,x^{2}+d}}$
default	$\frac{\sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x \Big(8 \left(c \,x^{2}+d \right)^{\frac{3}{2}} c^{\frac{3}{2}} a \,x^{3}-6 \left(c \,x^{2}+d \right)^{\frac{3}{2}} \sqrt{c}\, a d x +12 \left(c \,x^{2}+d \right)^{\frac{3}{2}} c^{\frac{3}{2}} b x +3 \sqrt{c \,x^{2}+d}\, \sqrt{c}\, a \,d^{2} x -6 \sqrt{c \,x^{2}+d}\, c^{\frac{3}{2}} b d x +3 \ln \Big(\sqrt{c}\, x +\sqrt{c \,x^{2}}}{48 \sqrt{c \,x^{2}+d}\, c^{\frac{5}{2}}}$

input
```
int((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*x^2*(8*a*c^2*x^4+2*a*c*d*x^2+12*b*c^2*x^2-3*a*d^2+6*b*c*d)/c^2*((c*x^
2+d)/x^2)^(1/2)+1/16*d^2*(a*d-2*b*c)/c^(5/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))
*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

3.929.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.97

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx$$

$$= \left[-\frac{3\,(2\,bcd^2-ad^3)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,(8\,ac^3x^6+2\,(6\,bc^3+ac^2d)x^4+3\,(2\,bc^2d-}{96\,c^3}\right.$$

input

```
integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output

```
[-1/96*(3*(2*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c
*x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*
c^2*d - a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^3, 1/48*(3*(2*b*c*d^2 - a*d
^3)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (8*a
*c^3*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*c^2*d - a*c*d^2)*x^2)*sqrt((
c*x^2 + d)/x^2))/c^3]
```

3.929.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(107) = 214$.

Time = 24.90 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.84

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx &= \frac{acx^7}{6\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{5a\sqrt{d}x^5}{24\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{3}{2}}x^3}{48c\sqrt{\frac{cx^2}{d}+1}}\\ &\quad-\frac{ad^{\frac{5}{2}}x}{16c^2\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{16c^{\frac{5}{2}}}+\frac{bcx^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}\\ &\quad+\frac{3b\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{bd^{\frac{3}{2}}x}{8c\sqrt{\frac{cx^2}{d}+1}}-\frac{bd^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{3}{2}}}\end{aligned}$$

input

```
integrate((a+b/x**2)*x**5*(c+d/x**2)**(1/2),x)
```

output
```
a*c*x**7/(6*sqrt(d)*sqrt(c*x**2/d + 1)) + 5*a*sqrt(d)*x**5/(24*sqrt(c*x**2
/d + 1)) - a*d**(3/2)*x**3/(48*c*sqrt(c*x**2/d + 1)) - a*d**(5/2)*x/(16*c*
*2*sqrt(c*x**2/d + 1)) + a*d**3*asinh(sqrt(c)*x/sqrt(d))/(16*c**(5/2)) + b
*c*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*b*sqrt(d)*x**3/(8*sqrt(c*x**2/d
 + 1)) + b*d**(3/2)*x/(8*c*sqrt(c*x**2/d + 1)) - b*d**2*asinh(sqrt(c)*x/sq
rt(d))/(8*c**(3/2))
```

3.929.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(103) = 206$.

Time = 0.29 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.98

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx$$

$$= -\frac{1}{96}\left(\frac{3\,d^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3-8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}cd^3-3\sqrt{c+\frac{d}{x^2}}c^2d^3\right)}{\left(c+\frac{d}{x^2}\right)^3c^2-3\left(c+\frac{d}{x^2}\right)^2c^3+3\left(c+\frac{d}{x^2}\right)c^4-c^5}\right)a$$
$$+\frac{1}{16}\left(\frac{d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2+\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c-2\left(c+\frac{d}{x^2}\right)c^2+c^3}\right)b$$

input
```
integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/96*(3*d^3*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/
c^(5/2) + 2*(3*(c + d/x^2)^(5/2)*d^3 - 8*(c + d/x^2)^(3/2)*c*d^3 - 3*sqrt(
c + d/x^2)*c^2*d^3)/((c + d/x^2)^3*c^2 - 3*(c + d/x^2)^2*c^3 + 3*(c + d/x^
2)*c^4 - c^5))*a + 1/16*(d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x
^2) + sqrt(c)))/c^(3/2) + 2*((c + d/x^2)^(3/2)*d^2 + sqrt(c + d/x^2)*c*d^2
)/((c + d/x^2)^2*c - 2*(c + d/x^2)*c^2 + c^3))*b
```

3.929.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.16

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx$$
$$=\frac{1}{48}\left(2\left(4\,ax^2\mathrm{sgn}(x)+\frac{6\,bc^4\mathrm{sgn}(x)+ac^3d\mathrm{sgn}(x)}{c^4}\right)x^2+\frac{3\,(2\,bc^3d\mathrm{sgn}(x)-ac^2d^2\mathrm{sgn}(x))}{c^4}\right)\sqrt{cx^2+d}x$$
$$+\frac{(2\,bcd^2\mathrm{sgn}(x)-ad^3\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{16\,c^{\frac{5}{2}}}$$
$$-\frac{(2\,bcd^2\log\left(|d|\right)-ad^3\log\left(|d|\right))\mathrm{sgn}(x)}{32\,c^{\frac{5}{2}}}$$

input

```
integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="giac")
```

output

```
1/48*(2*(4*a*x^2*sgn(x) + (6*b*c^4*sgn(x) + a*c^3*d*sgn(x))/c^4)*x^2 + 3*(
2*b*c^3*d*sgn(x) - a*c^2*d^2*sgn(x))/c^4)*sqrt(c*x^2 + d)*x + 1/16*(2*b*c*
d^2*sgn(x) - a*d^3*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(5/2)
- 1/32*(2*b*c*d^2*log(abs(d)) - a*d^3*log(abs(d)))*sgn(x)/c^(5/2)
```

3.929.9 Mupad [B] (verification not implemented)

Time = 10.01 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx=\frac{a\,x^6\,\sqrt{c+\frac{d}{x^2}}}{16}+\frac{b\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{a\,x^6\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{6\,c}$$
$$-\frac{a\,x^6\,{\left(c+\frac{d}{x^2}\right)}^{5/2}}{16\,c^2}+\frac{b\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c}$$
$$-\frac{b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{3/2}}-\frac{a\,d^3\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,1{}\mathrm{i}}{16\,c^{5/2}}$$

input

```
int(x^5*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output

```
(a*x^6*(c + d/x^2)^(1/2))/16 + (b*x^4*(c + d/x^2)^(1/2))/8 + (a*x^6*(c + d
/x^2)^(3/2))/(6*c) - (a*x^6*(c + d/x^2)^(5/2))/(16*c^2) + (b*x^4*(c + d/x^
2)^(3/2))/(8*c) - (a*d^3*atan(((c + d/x^2)^(1/2)*1i)/c^(1/2))*1i)/(16*c^(5
/2)) - (b*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(3/2))
```

3.930 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^3\,dx$

3.930.1 Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^3\,dx=\frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c}+\frac{d(4bc-ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}$$

output
```
1/4*a*(c+d/x^2)^(3/2)*x^4/c+1/8*d*(-a*d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(
1/2))/c^(3/2)+1/8*(-a*d+4*b*c)*x^2*(c+d/x^2)^(1/2)/c
```

3.930.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^3\,dx=\frac{\sqrt{c+\frac{d}{x^2}}x\left(\sqrt{c}x\sqrt{d+cx^2}(4bc+a(d+2cx^2))+d(-4bc+ad)\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)\right)}{8c^{3/2}\sqrt{d+cx^2}}$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]
```

output
```
(Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[d + c*x^2]*(4*b*c + a*(d + 2*c*x^2)) +
d*(-4*b*c + a*d)*Log[-(Sqrt[c]*x) + Sqrt[d + c*x^2]]))/(8*c^(3/2)*Sqrt[d +
 c*x^2])
```

3.930.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {948, 87, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\,dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^6d\frac{1}{x^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{ax^4\left(c+\frac{d}{x^2}\right)^{3/2}}{2c}-\frac{(4bc-ad)\int \sqrt{c+\frac{d}{x^2}}x^4d\frac{1}{x^2}}{4c}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{ax^4\left(c+\frac{d}{x^2}\right)^{3/2}}{2c}-\frac{(4bc-ad)\left(\frac{1}{2}d\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}-x^2\sqrt{c+\frac{d}{x^2}}\right)}{4c}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{ax^4\left(c+\frac{d}{x^2}\right)^{3/2}}{2c}-\frac{(4bc-ad)\left(\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}-x^2\sqrt{c+\frac{d}{x^2}}\right)}{4c}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{ax^4\left(c+\frac{d}{x^2}\right)^{3/2}}{2c}-\frac{(4bc-ad)\left(x^2\left(-\sqrt{c+\frac{d}{x^2}}\right)-\frac{d\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{4c}\right)$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]
```

output
```
((a*(c + d/x^2)^(3/2)*x^4)/(2*c) - ((4*b*c - a*d)*(-(Sqrt[c + d/x^2]*x^2)
- (d*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/Sqrt[c]))/(4*c))/2
```

3.930.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.930. $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx$

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.930.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

method	result	size
risch	$\frac{x^{2}\left(2 a c\,x^{2}+a d+4 b c\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{8 c}-\frac{d \left(a d-4 b c\right)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{8 c^{\frac{3}{2}}\sqrt{c\,x^{2}+d}}$	91
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(2\left(c\,x^{2}+d\right)^{\frac{3}{2}}\sqrt{c}\,a x-\sqrt{c\,x^{2}+d}\,\sqrt{c}\,a d x+4\sqrt{c\,x^{2}+d}\,c^{\frac{3}{2}} b x-\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right) a\,d^{2}+4\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right) b c d\right)}{8\sqrt{c\,x^{2}+d}\,c^{\frac{3}{2}}}$	122

input
```
int((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x^2*(2*a*c*x^2+a*d+4*b*c)/c*((c*x^2+d)/x^2)^(1/2)-1/8*d*(a*d-4*b*c)/c^
(3/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2
)
```

3.930.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.12

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx$$

$$=\left[-\frac{(4\,bcd-ad^2)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,(2\,ac^2x^4+(4\,bc^2+acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16\,c^2},\right.$$
$$\left.-\frac{(4\,bcd-ad^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(2\,ac^2x^4+(4\,bc^2+acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8\,c^2}\right]$$

input
```
integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((4*b*c*d - a*d^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*x^2
 + d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*x^2 + d)
/x^2))/c^2, -1/8*((4*b*c*d - a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x
^2 + d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*
x^2 + d)/x^2))/c^2]
```

3.930.6 Sympy [A] (verification not implemented)

Time = 17.85 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx = \frac{acx^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + \frac{3a\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}} + \frac{ad^{\frac{3}{2}}x}{8c\sqrt{\frac{cx^2}{d}+1}} - \frac{ad^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{3}{2}}} + \frac{b\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2} + \frac{bd\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2\sqrt{c}}$$

input
```
integrate((a+b/x**2)*x**3*(c+d/x**2)**(1/2),x)
```

output
```
a*c*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*a*sqrt(d)*x**3/(8*sqrt(c*x**2/
d + 1)) + a*d**(3/2)*x/(8*c*sqrt(c*x**2/d + 1)) - a*d**2*asinh(sqrt(c)*x/s
qrt(d))/(8*c**(3/2)) + b*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 + b*d*asinh(sqrt(c
)*x/sqrt(d))/(2*sqrt(c))
```

3.930.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(74) = 148.

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.77

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx = \frac{1}{16}\left(\frac{d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2+\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c-2\left(c+\frac{d}{x^2}\right)c^2+c^3}\right)a + \frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}x^2-\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}\right)b$$

input
```
integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/16*(d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(
3/2) + 2*((c + d/x^2)^(3/2)*d^2 + sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)^2*c
- 2*(c + d/x^2)*c^2 + c^3))*a + 1/4*(2*sqrt(c + d/x^2)*x^2 - d*log((sqrt(c
 + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c))*b
```

3.930.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx = \frac{1}{8}\left(2\,ax^2\mathrm{sgn}(x)+\frac{4\,bc^2\mathrm{sgn}(x)+acd\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+d}x - \frac{(4\,bcd\mathrm{sgn}(x)-ad^2\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{8\,c^{\frac{3}{2}}} + \frac{(4\,bcd\log(|d|)-ad^2\log(|d|))\mathrm{sgn}(x)}{16\,c^{\frac{3}{2}}}$$

input
```
integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(2*a*x^2*sgn(x) + (4*b*c^2*sgn(x) + a*c*d*sgn(x))/c^2)*sqrt(c*x^2 + d)
*x - 1/8*(4*b*c*d*sgn(x) - a*d^2*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 d)))/c^(3/2) + 1/16*(4*b*c*d*log(abs(d)) - a*d^2*log(abs(d)))*sgn(x)/c^(3
/2)
```

3.930.9 Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx = \frac{a\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{b\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}+\frac{a\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c} + \frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,\sqrt{c}} - \frac{a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{3/2}}$$

input
```
int(x^3*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
(a*x^4*(c + d/x^2)^(1/2))/8 + (b*x^2*(c + d/x^2)^(1/2))/2 + (a*x^4*(c + d/
x^2)^(3/2))/(8*c) + (b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(1/2)) - (
a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(3/2))
```

3.930. $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx$

3.931 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx$

3.931.1 Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx=-\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

output
```
1/2*a*(c+d/x^2)^(3/2)*x^2/c+1/2*(a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2
))/c^(1/2)-1/2*(a*d+2*b*c)*(c+d/x^2)^(1/2)/c
```

3.931.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx=\frac{1}{2}\sqrt{c+\frac{d}{x^2}}\left(-2b+ax^2+\frac{2(2bc+ad)x\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{d}+\sqrt{d+cx^2}}\right)}{\sqrt{c}\sqrt{d+cx^2}}\right)$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]
```

output
```
(Sqrt[c + d/x^2]*(-2*b + a*x^2 + (2*(2*b*c + a*d)*x*ArcTanh[(Sqrt[c]*x)/(-
Sqrt[d] + Sqrt[d + c*x^2])])/(Sqrt[c]*Sqrt[d + c*x^2])))/2
```

3.931.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \ 948$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4d\frac{1}{x^2}$$

$$\Big\downarrow \ 87$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(ad+2bc)\int \sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x^2}}{2c}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(ad+2bc)\left(c\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}+2\sqrt{c+\frac{d}{x^2}}\right)}{2c}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(ad+2bc)\left(\frac{2c\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{d}+2\sqrt{c+\frac{d}{x^2}}\right)}{2c}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(ad+2bc)\left(2\sqrt{c+\frac{d}{x^2}}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)\right)}{2c}\right)$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]
```

output
```
((a*(c + d/x^2)^(3/2)*x^2)/c - ((2*b*c + a*d)*(2*Sqrt[c + d/x^2] - 2*Sqrt[
c]*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]))/(2*c))/2
```

3.931.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.931. $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x \, dx$

3.931.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

method	result
risch	$\frac{\left(a\,x^{2}-2 b\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{2}+\frac{\left(\frac{a d}{2}+b c\right)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{\sqrt{c}\,\sqrt{c\,x^{2}+d}}$
default	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(-\sqrt{c\,x^{2}+d}\,\sqrt{c}\,a d\,x^{2}-2\sqrt{c\,x^{2}+d}\,c^{\frac{3}{2}}\,b\,x^{2}+2\left(c\,x^{2}+d\right)^{\frac{3}{2}}\sqrt{c}\,b-\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)a\,d^{2}x-2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)b c d x\right)}{2\sqrt{c\,x^{2}+d}\,d\sqrt{c}}$

input
```
int((a+b/x^2)*x*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^2-2*b)*((c*x^2+d)/x^2)^(1/2)+(1/2*a*d+b*c)*ln(c^(1/2)*x+(c*x^2+d)
^(1/2))/c^(1/2)*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

3.931.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.85

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx = \left[\frac{(2\,bc+ad)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\,(acx^2-2\,bc)\sqrt{\frac{cx^2+d}{x^2}}}{4\,c},\right.$$
$$\left.-\frac{(2\,bc+ad)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(acx^2-2\,bc)\sqrt{\frac{cx^2+d}{x^2}}}{2\,c}\right]$$

input
```
integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((2*b*c + a*d)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/
x^2) - d) + 2*(a*c*x^2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c, -1/2*((2*b*c + a
*d)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (a*c
*x^2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c]
```

3.931.6 Sympy [A] (verification not implemented)

Time = 17.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx=\frac{a\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{ad\,\mathrm{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2\sqrt{c}}+b\sqrt{c}\,\mathrm{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)-\frac{bcx}{\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{b\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}}$$

input
```
integrate((a+b/x**2)*x*(c+d/x**2)**(1/2),x)
```

output
```
a*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 + a*d*asinh(sqrt(c)*x/sqrt(d))/(2*sqrt(c)
) + b*sqrt(c)*asinh(sqrt(c)*x/sqrt(d)) - b*c*x/(sqrt(d)*sqrt(c*x**2/d + 1)
) - b*sqrt(d)/(x*sqrt(c*x**2/d + 1))
```

3.931.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx=\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}x^2-\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}\right)a-\frac{1}{2}\left(\sqrt{c}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\sqrt{c+\frac{d}{x^2}}\right)b$$

input
```
integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(c + d/x^2)*x^2 - d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d
/x^2) + sqrt(c)))/sqrt(c))*a - 1/2*(sqrt(c)*log((sqrt(c + d/x^2) - sqrt(c)
)/(sqrt(c + d/x^2) + sqrt(c))) + 2*sqrt(c + d/x^2))*b
```

3.931.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx = \frac{1}{2}\sqrt{cx^2+d}ax\mathrm{sgn}(x)+\frac{2b\sqrt{c}d\mathrm{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d} - \frac{\left(2bc\mathrm{sgn}(x)+ad\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)}{4\sqrt{c}}$$

input
```
integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^2 + d)*a*x*sgn(x) + 2*b*sqrt(c)*d*sgn(x)/((sqrt(c)*x - sqrt(c
*x^2 + d))^2 - d) - 1/4*(2*b*c*sgn(x) + a*d*sgn(x))*log((sqrt(c)*x - sqrt(
c*x^2 + d))^2)/sqrt(c)
```

3.931.9 Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx = \frac{a\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}-b\,\sqrt{c+\frac{d}{x^2}} + b\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)+\frac{a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,\sqrt{c}}$$

input
```
int(x*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
(a*x^2*(c + d/x^2)^(1/2))/2 - b*(c + d/x^2)^(1/2) + b*c^(1/2)*atanh((c + d
/x^2)^(1/2)/c^(1/2)) + (a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(1/2))
```

3.932 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx$

3.932.1 Optimal result

Integrand size = 22, antiderivative size = 59

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = -a\sqrt{c+\frac{d}{x^2}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}+a\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

output
```
-1/3*b*(c+d/x^2)^(3/2)/d+a*arctanh((c+d/x^2)^(1/2)/c^(1/2))*c^(1/2)-a*(c+d
/x^2)^(1/2)
```

3.932.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}\left(3adx^2+b(d+cx^2)+\frac{3a\sqrt{c}dx^3\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)}{\sqrt{d+cx^2}}\right)}{3dx^2}$$

input
```
Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]
```

output
```
-1/3*(Sqrt[c + d/x^2]*(3*a*d*x^2 + b*(d + c*x^2) + (3*a*Sqrt[c]*d*x^3*Log[
-(Sqrt[c]*x) + Sqrt[d + c*x^2]])/Sqrt[d + c*x^2]))/(d*x^2)
```

3.932.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {948, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(-a\int \sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x^2}-\frac{2b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-a\left(c\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}+2\sqrt{c+\frac{d}{x^2}}\right)-\frac{2b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-a\left(\frac{2c\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{d}+2\sqrt{c+\frac{d}{x^2}}\right)-\frac{2b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-a\left(2\sqrt{c+\frac{d}{x^2}}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)\right)-\frac{2b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}\right)$$

input
```
Int[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]
```

output
```
((-2*b*(c + d/x^2)^(3/2))/(3*d) - a*(2*Sqrt[c + d/x^2] - 2*Sqrt[c]*ArcTanh
[Sqrt[c + d/x^2]/Sqrt[c]]))/2
```

3.932. $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}dx$

3.932.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.932. $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx$

3.932.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.42

method	result	size
risch	$-\frac{\left(3 a d \,x^{2}+c b \,x^{2}+b d \right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}{3 x^{2} d}+\frac{a \sqrt{c}\, \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+d}\right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}{\sqrt{c \,x^{2}+d}}$	84
default	$-\frac{\sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, \left(-3 \sqrt{c \,x^{2}+d}\, c^{\frac{3}{2}} a \,x^{4}+3 \left(c \,x^{2}+d \right)^{\frac{3}{2}} \sqrt{c}\, a \,x^{2}-3 \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+d}\right) a c d \,x^{3}+\left(c \,x^{2}+d \right)^{\frac{3}{2}} \sqrt{c}\, b \right)}{3 x^{2} \sqrt{c \,x^{2}+d}\, d \sqrt{c}}$	109

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*a*d*x^2+b*c*x^2+b*d)/x^2/d*((c*x^2+d)/x^2)^(1/2)+a*c^(1/2)*ln(c^(1
/2)*x+(c*x^2+d)^(1/2))*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

3.932.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.81

$$\int \frac{\left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}}{x} \, dx = \left[\frac{3 \, a \sqrt{c} d x^{2} \log \left(-2 \, c x^{2} - 2 \, \sqrt{c} x^{2} \sqrt{\frac{c x^{2} + d}{x^{2}}} - d\right) - 2 \, {\left({\left(b c + 3 \, a d\right)} x^{2} + b d\right)} \sqrt{\frac{c x^{2} + d}{x^{2}}}}{6 \, d x^{2}}, -\frac{3 \, a \sqrt{-c} d x^{2} \arctan \left(\frac{\sqrt{-c} x^{2} \sqrt{\frac{c x^{2} + d}{x^{2}}}}{c x^{2} + d}\right) + {\left({\left(b c + 3 \, a d\right)} x^{2} + b d\right)} \sqrt{\frac{c x^{2} + d}{x^{2}}}}{3 \, d x^{2}}\right]$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/6*(3*a*sqrt(c)*d*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2)
 - d) - 2*((b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2), -1/3*(
3*a*sqrt(-c)*d*x^2*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d))
+ ((b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2)]
```

3.932.6 Sympy [A] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.44

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = -\frac{a\left(\begin{cases}\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{c+\frac{d}{x^2}} & \text{for } d\neq 0\\ -\sqrt{c}\log\left(x^2\right) & \text{otherwise}\end{cases}\right)}{2} + \frac{b\left(\begin{cases}-\frac{\sqrt{c}}{x^2} & \text{for } d=0\\ -\frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x,x)
```

output
```
-a*Piecewise((2*c*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c) + 2*sqrt(c + d/
x**2), Ne(d, 0)), (-sqrt(c)*log(x**2), True))/2 + b*Piecewise((-sqrt(c)/x*
*2, Eq(d, 0)), (-2*(c + d/x**2)**(3/2)/(3*d), True))/2
```

3.932.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = -\frac{1}{2}\left(\sqrt{c}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\sqrt{c+\frac{d}{x^2}}\right)a-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3\,d}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
-1/2*(sqrt(c)*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))
 + 2*sqrt(c + d/x^2))*a - 1/3*b*(c + d/x^2)^(3/2)/d
```

3.932.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(47) = 94$.

Time = 0.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.76

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = -\frac{1}{2}a\sqrt{c}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)\operatorname{sgn}(x) + \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{3}{2}}\operatorname{sgn}(x)+3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4a\sqrt{c}d\operatorname{sgn}(x)-6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2a\sqrt{c}d^2s}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^3}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
-1/2*a*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)*sgn(x) + 2/3*(3*(sqrt(
c)*x - sqrt(c*x^2 + d))^4*b*c^(3/2)*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + d
))^4*a*sqrt(c)*d*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d^2*
sgn(x) + b*c^(3/2)*d^2*sgn(x) + 3*a*sqrt(c)*d^3*sgn(x))/((sqrt(c)*x - sqrt
(c*x^2 + d))^2 - d)^3
```

3.932.9 Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = a\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right) - a\sqrt{c+\frac{d}{x^2}} - \frac{b\sqrt{c+\frac{d}{x^2}}\,(c\,x^2+d)}{3\,d\,x^2}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x,x)
```

output
```
a*c^(1/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - a*(c + d/x^2)^(1/2) - (b*(c +
 d/x^2)^(1/2)*(d + c*x^2))/(3*d*x^2)
```

3.933 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx$

3.933.1 Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx=\frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}$$

output
```
1/3*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^2-1/5*b*(c+d/x^2)^(5/2)/d^2
```

3.933.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)\left(-3bd+2bcx^2-5adx^2\right)}{15d^2x^4}$$

input
```
Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]
```

output
```
(Sqrt[c + d/x^2]*(d + c*x^2)*(-3*b*d + 2*b*c*x^2 - 5*a*d*x^2))/(15*d^2*x^4
)
```

3.933.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}dx$$

$$\Big\downarrow \ 946$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}d\frac{1}{x^2}$$

$$\Big\downarrow \ 53$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{d}+\frac{(ad-bc)\sqrt{c+\frac{d}{x^2}}}{d}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{2\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^2}-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}\right)$$

input

```
Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]
```

output

```
((2*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^2) - (2*b*(c + d/x^2)^(5/2))/(5*d^
2))/2
```

3.933.3.1 Defintions of rubi rules used

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.933.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result	size
gosper	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(5ad\,x^{2}-2cb\,x^{2}+3bd\right)\left(c\,x^{2}+d\right)}{15d^{2}x^{4}}$	48
default	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(5ad\,x^{2}-2cb\,x^{2}+3bd\right)\left(c\,x^{2}+d\right)}{15d^{2}x^{4}}$	48
risch	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(5acd\,x^{4}-2b\,c^{2}x^{4}+5a\,d^{2}x^{2}+bcd\,x^{2}+3b\,d^{2}\right)}{15x^{4}d^{2}}$	62
trager	$-\frac{\left(5acd\,x^{4}-2b\,c^{2}x^{4}+5a\,d^{2}x^{2}+bcd\,x^{2}+3b\,d^{2}\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{15x^{4}d^{2}}$	66

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/15*((c*x^2+d)/x^2)^(1/2)*(5*a*d*x^2-2*b*c*x^2+3*b*d)*(c*x^2+d)/d^2/x^4
```

3.933.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx=\frac{\left((2\,bc^2-5\,acd)x^4-3\,bd^2-(bcd+5\,ad^2)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,d^2x^4}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/15*((2*b*c^2 - 5*a*c*d)*x^4 - 3*b*d^2 - (b*c*d + 5*a*d^2)*x^2)*sqrt((c*x
^2 + d)/x^2)/(d^2*x^4)
```

3.933.6 Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = -\frac{a\left(\begin{cases}\frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{x^2} & \text{otherwise}\end{cases}\right)}{2} - \frac{b\left(\begin{cases}\frac{2\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{2x^4} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**3,x)
```

output
```
-a*Piecewise((2*(c + d/x**2)**(3/2)/(3*d), Ne(d, 0)), (sqrt(c)/x**2, True)
)/2 - b*Piecewise((2*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d*
*2, Ne(d, 0)), (sqrt(c)/(2*x**4), True))/2
```

3.933.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = -\frac{1}{15}\,b\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^2} - \frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^2}\right) - \frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3\,d}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-1/15*b*(3*(c + d/x^2)^(5/2)/d^2 - 5*(c + d/x^2)^(3/2)*c/d^2) - 1/3*a*(c +
 d/x^2)^(3/2)/d
```

3.933.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(38) = 76$.

Time = 0.59 (sec) , antiderivative size = 250, normalized size of antiderivative = 5.43

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx$$
$$= \frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8 ac^{\frac{3}{2}}\operatorname{sgn}(x)+30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6 bc^{\frac{5}{2}}\operatorname{sgn}(x)-30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6 ac^{\frac{3}{2}}d\,\mathrm{sgn}}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
2/15*(15*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(3/2)*sgn(x) + 30*(sqrt(c)*x
- sqrt(c*x^2 + d))^6*b*c^(5/2)*sgn(x) - 30*(sqrt(c)*x - sqrt(c*x^2 + d))^6
*a*c^(3/2)*d*sgn(x) + 10*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*d*sgn(x
) + 20*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^2*sgn(x) + 10*(sqrt(c)*
x - sqrt(c*x^2 + d))^2*b*c^(5/2)*d^2*sgn(x) - 10*(sqrt(c)*x - sqrt(c*x^2 +
 d))^2*a*c^(3/2)*d^3*sgn(x) - 2*b*c^(5/2)*d^3*sgn(x) + 5*a*c^(3/2)*d^4*sgn
(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^5
```

3.933.9 Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.98

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx &= \frac{\sqrt{c+\frac{d}{x^2}}\,\left(b\,c^2+a\,d\,c\right)}{5\,d^2}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4}\\ &\quad-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(5\,a\,d^2+b\,c\,d\right)}{15\,d^2\,x^2}-\frac{c\,\sqrt{c+\frac{d}{x^2}}\,\left(8\,a\,d+b\,c\right)}{15\,d^2}\end{aligned}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^3,x)
```

output
```
((c + d/x^2)^(1/2)*(b*c^2 + a*c*d))/(5*d^2) - (b*(c + d/x^2)^(1/2))/(5*x^4
) - ((c + d/x^2)^(1/2)*(5*a*d^2 + b*c*d))/(15*d^2*x^2) - (c*(c + d/x^2)^(1
/2)*(8*a*d + b*c))/(15*d^2)
```

3.934 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx$

3.934.1 Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx=-\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}$$

output
```
-1/3*c*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^3+1/5*(-a*d+2*b*c)*(c+d/x^2)^(5/2)/d^3
-1/7*b*(c+d/x^2)^(7/2)/d^3
```

3.934.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx =\frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)\left(-15bd^2+12bcdx^2-21ad^2x^2-8bc^2x^4+14acdx^4\right)}{105d^3x^6}$$

input
```
Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]
```

output
```
(Sqrt[c + d/x^2]*(d + c*x^2)*(-15*b*d^2 + 12*b*c*d*x^2 - 21*a*d^2*x^2 - 8*
b*c^2*x^4 + 14*a*c*d*x^4))/(105*d^3*x^6)
```

3.934.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}dx$$

$$\Big\downarrow \ 948$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}d\frac{1}{x^2}$$

$$\Big\downarrow \ 86$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{d^2}+\frac{(ad-2bc)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^2}+\frac{c(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^2}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{2\left(c+\frac{d}{x^2}\right)^{5/2}(2bc-ad)}{5d^3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^3}-\frac{2b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}\right)$$

input
```
Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]
```

output
```
((-2*c*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^3) + (2*(2*b*c - a*d)*(c + d/x^
2)^(5/2))/(5*d^3) - (2*b*(c + d/x^2)^(7/2))/(7*d^3))/2
```

3.934.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

---

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.934.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(14acd\,x^{4}-8b\,c^{2}x^{4}-21a\,d^{2}x^{2}+12bcd\,x^{2}-15b\,d^{2}\right)\left(c\,x^{2}+d\right)}{105d^{3}x^{6}}$	70
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(14acd\,x^{4}-8b\,c^{2}x^{4}-21a\,d^{2}x^{2}+12bcd\,x^{2}-15b\,d^{2}\right)\left(c\,x^{2}+d\right)}{105d^{3}x^{6}}$	70
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(14a\,c^{2}d\,x^{6}-8b\,c^{3}x^{6}-7ac\,d^{2}x^{4}+4b\,c^{2}d\,x^{4}-21a\,d^{3}x^{2}-3bc\,d^{2}x^{2}-15b\,d^{3}\right)}{105x^{6}d^{3}}$	87
trager	$\frac{\left(14a\,c^{2}d\,x^{6}-8b\,c^{3}x^{6}-7ac\,d^{2}x^{4}+4b\,c^{2}d\,x^{4}-21a\,d^{3}x^{2}-3bc\,d^{2}x^{2}-15b\,d^{3}\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{105x^{6}d^{3}}$	91

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/105*((c*x^2+d)/x^2)^(1/2)*(14*a*c*d*x^4-8*b*c^2*x^4-21*a*d^2*x^2+12*b*c*
d*x^2-15*b*d^2)*(c*x^2+d)/d^3/x^6
```

3.934.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx$$
$$= -\frac{\left(2\,(4\,bc^3-7\,ac^2d)x^6-(4\,bc^2d-7\,acd^2)x^4+15\,bd^3+3\,(bcd^2+7\,ad^3)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,d^3x^6}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/105*(2*(4*b*c^3 - 7*a*c^2*d)*x^6 - (4*b*c^2*d - 7*a*c*d^2)*x^4 + 15*b*d
^3 + 3*(b*c*d^2 + 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^3*x^6)
```

3.934.6 Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.47

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = -\frac{a\left(\begin{cases} \frac{2\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} & \text{for}\ d\neq 0 \\ \frac{\sqrt{c}}{2x^4} & \text{otherwise} \end{cases}\right)}{2} - \frac{b\left(\begin{cases} \frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for}\ d\neq 0 \\ \frac{\sqrt{c}}{3x^6} & \text{otherwise} \end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**5,x)
```

output
```
-a*Piecewise((2*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2, N
e(d, 0)), (sqrt(c)/(2*x**4), True))/2 - b*Piecewise((2*(c**2*(c + d/x**2)*
*(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3, Ne(d,
0)), (sqrt(c)/(3*x**6), True))/2
```

3.934.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = -\frac{1}{105}\,b\left(\frac{15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^3}-\frac{42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^3}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^3}\right) - \frac{1}{15}\,a\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^2}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^2}\right)$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/105*b*(15*(c + d/x^2)^(7/2)/d^3 - 42*(c + d/x^2)^(5/2)*c/d^3 + 35*(c +
d/x^2)^(3/2)*c^2/d^3) - 1/15*a*(3*(c + d/x^2)^(5/2)/d^2 - 5*(c + d/x^2)^(3
/2)*c/d^2)
```

3.934.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(62) = 124$.

Time = 0.81 (sec) , antiderivative size = 310, normalized size of antiderivative = 4.19

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx$$
$$= \frac{4\left(105\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{5}{2}}\operatorname{sgn}(x)+280\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{7}{2}}\operatorname{sgn}(x)-175\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{5}{2}}}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
4/105*(105*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(5/2)*sgn(x) + 280*(sqrt(c
)*x - sqrt(c*x^2 + d))^8*b*c^(7/2)*sgn(x) - 175*(sqrt(c)*x - sqrt(c*x^2 +
d))^8*a*c^(5/2)*d*sgn(x) + 140*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d
*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(5/2)*d^2*sgn(x) + 84*(sq
rt(c)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)*d^2*sgn(x) - 42*(sqrt(c)*x - sqrt(c
*x^2 + d))^4*a*c^(5/2)*d^3*sgn(x) - 28*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c
^(7/2)*d^3*sgn(x) + 49*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(5/2)*d^4*sgn(x
) + 4*b*c^(7/2)*d^4*sgn(x) - 7*a*c^(5/2)*d^5*sgn(x))/((sqrt(c)*x - sqrt(c*
x^2 + d))^2 - d)^7
```

3.934.9 Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.70

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = \frac{2\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{15\,d^2}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{a\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4}-\frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{105\,d^3}-\frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{15\,d\,x^2}-\frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^4}+\frac{4\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^2}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^5,x)
```

output
```
(2*a*c^2*(c + d/x^2)^(1/2))/(15*d^2) - (b*(c + d/x^2)^(1/2))/(7*x^6) - (a*
(c + d/x^2)^(1/2))/(5*x^4) - (8*b*c^3*(c + d/x^2)^(1/2))/(105*d^3) - (a*c*
(c + d/x^2)^(1/2))/(15*d*x^2) - (b*c*(c + d/x^2)^(1/2))/(35*d*x^4) + (4*b*
c^2*(c + d/x^2)^(1/2))/(105*d^2*x^2)
```

3.935 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx$

3.935.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx=\frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}$$

output
```
1/3*c^2*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^4-1/5*c*(-2*a*d+3*b*c)*(c+d/x^2)^(5/2
)/d^4+1/7*(-a*d+3*b*c)*(c+d/x^2)^(7/2)/d^4-1/9*b*(c+d/x^2)^(9/2)/d^4
```

3.935.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)\left(-35bd^3+30bcd^2x^2-45ad^3x^2-24bc^2dx^4+36acd^2x^4+16bc^3x^6-24ac^2dx^6\right)}{315d^4x^8}$$

input
```
Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]
```

output
```
(Sqrt[c + d/x^2]*(d + c*x^2)*(-35*b*d^3 + 30*b*c*d^2*x^2 - 45*a*d^3*x^2 -
24*b*c^2*d*x^4 + 36*a*c*d^2*x^4 + 16*b*c^3*x^6 - 24*a*c^2*d*x^6))/(315*d^4
*x^8)
```

3.935.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}dx$$

$$\Big\downarrow \; 948$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}d\frac{1}{x^2}$$

$$\Big\downarrow \; 86$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{d^3}+\frac{(ad-3bc)\left(c+\frac{d}{x^2}\right)^{5/2}}{d^3}+\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^3}-\frac{c^2(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{2c^2\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^4}+\frac{2\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-ad)}{7d^4}-\frac{2c\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-2ad)}{5d^4}-\frac{2b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}\right)$$

input
```
Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]
```

output
```
((2*c^2*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (2*c*(3*b*c - 2*a*d)*(c +
 d/x^2)^(5/2))/(5*d^4) + (2*(3*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^4) - (2*
b*(c + d/x^2)^(9/2))/(9*d^4))/2
```

3.935. $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}dx$

3.935.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.935.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

method	result	size
gosper	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(24 a\,c^{2} d\,x^{6}-16 b\,c^{3} x^{6}-36 a c\,d^{2} x^{4}+24 b\,c^{2} d\,x^{4}+45 a\,d^{3} x^{2}-30 b c\,d^{2} x^{2}+35 b\,d^{3}\right) \left(c\,x^{2}+d\right)}{315 d^{4} x^{8}}$	94
default	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(24 a\,c^{2} d\,x^{6}-16 b\,c^{3} x^{6}-36 a c\,d^{2} x^{4}+24 b\,c^{2} d\,x^{4}+45 a\,d^{3} x^{2}-30 b c\,d^{2} x^{2}+35 b\,d^{3}\right) \left(c\,x^{2}+d\right)}{315 d^{4} x^{8}}$	94
risch	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(24 a\,c^{3} d\,x^{8}-16 b\,c^{4} x^{8}-12 a\,c^{2} d^{2} x^{6}+8 b\,c^{3} d\,x^{6}+9 a c\,d^{3} x^{4}-6 b\,c^{2} d^{2} x^{4}+45 a\,d^{4} x^{2}+5 b c\,d^{3} x^{2}+35 b\,d^{4}\right)}{315 x^{8} d^{4}}$	111
trager	$-\frac{\left(24 a\,c^{3} d\,x^{8}-16 b\,c^{4} x^{8}-12 a\,c^{2} d^{2} x^{6}+8 b\,c^{3} d\,x^{6}+9 a c\,d^{3} x^{4}-6 b\,c^{2} d^{2} x^{4}+45 a\,d^{4} x^{2}+5 b c\,d^{3} x^{2}+35 b\,d^{4}\right) \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{315 x^{8} d^{4}}$	115

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/315*((c*x^2+d)/x^2)^(1/2)*(24*a*c^2*d*x^6-16*b*c^3*x^6-36*a*c*d^2*x^4+2
4*b*c^2*d*x^4+45*a*d^3*x^2-30*b*c*d^2*x^2+35*b*d^3)*(c*x^2+d)/d^4/x^8
```

3.935.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx$$
$$= \frac{(8\,(2\,bc^4-3\,ac^3d)x^8-4\,(2\,bc^3d-3\,ac^2d^2)x^6-35\,bd^4+3\,(2\,bc^2d^2-3\,acd^3)x^4-5\,(bcd^3+9\,ad^4)x^2)\sqrt{\frac{cx^2}{x^2}}}{315\,d^4x^8}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
1/315*(8*(2*b*c^4 - 3*a*c^3*d)*x^8 - 4*(2*b*c^3*d - 3*a*c^2*d^2)*x^6 - 35*
b*d^4 + 3*(2*b*c^2*d^2 - 3*a*c*d^3)*x^4 - 5*(b*c*d^3 + 9*a*d^4)*x^2)*sqrt(
(c*x^2 + d)/x^2)/(d^4*x^8)
```

3.935.6 Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.38

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx$$
$$= -\frac{a\left(\begin{cases}\frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{3x^6} & \text{otherwise}\end{cases}\right)}{2}$$
$$-\frac{b\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**7,x)
```

output
```
-a*Piecewise((2*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 +
(c + d/x**2)**(7/2)/7)/d**3, Ne(d, 0)), (sqrt(c)/(3*x**6), True))/2 - b*Pi
ecewise((2*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3
*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4, Ne(d, 0)), (sqrt(c
)/(4*x**8), True))/2
```

3.935.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13

$$
\begin{aligned}
&\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx \\
&= -\frac{1}{315}\,b\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^4}-\frac{135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^4}+\frac{189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^4}-\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3}{d^4}\right) \\
&\quad -\frac{1}{105}\,a\left(\frac{15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^3}-\frac{42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^3}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^3}\right)
\end{aligned}
$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/315*b*(35*(c + d/x^2)^(9/2)/d^4 - 135*(c + d/x^2)^(7/2)*c/d^4 + 189*(c
+ d/x^2)^(5/2)*c^2/d^4 - 105*(c + d/x^2)^(3/2)*c^3/d^4) - 1/105*a*(15*(c +
 d/x^2)^(7/2)/d^3 - 42*(c + d/x^2)^(5/2)*c/d^3 + 35*(c + d/x^2)^(3/2)*c^2/
d^3)
```

3.935.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(88) = 176.

Time = 0.99 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.56

$$
\begin{aligned}
&\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx \\
&= \frac{16\left(210\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{7}{2}}\mathrm{sgn}(x)+630\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{9}{2}}\mathrm{sgn}(x)-315\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}\right.}{}
\end{aligned}
$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="giac")
```

output
```
16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(7/2)*sgn(x) + 630*(sqrt(
c)*x - sqrt(c*x^2 + d))^10*b*c^(9/2)*sgn(x) - 315*(sqrt(c)*x - sqrt(c*x^2
+ d))^10*a*c^(7/2)*d*sgn(x) + 378*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(9/2
)*d*sgn(x) + 63*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(7/2)*d^2*sgn(x) + 168
*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(9/2)*d^2*sgn(x) - 42*(sqrt(c)*x - sq
rt(c*x^2 + d))^6*a*c^(7/2)*d^3*sgn(x) - 72*(sqrt(c)*x - sqrt(c*x^2 + d))^4
*b*c^(9/2)*d^3*sgn(x) + 108*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(7/2)*d^4*
sgn(x) + 18*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(9/2)*d^4*sgn(x) - 27*(sqr
t(c)*x - sqrt(c*x^2 + d))^2*a*c^(7/2)*d^5*sgn(x) - 2*b*c^(9/2)*d^5*sgn(x)
+ 3*a*c^(7/2)*d^6*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^9
```

3.935.9 Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = \frac{16\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^4}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{8\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{105\,d^3}-\frac{a\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^4}-\frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,d\,x^6}+\frac{4\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^2}+\frac{2\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^4}-\frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3\,x^2}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^7,x)
```

output
```
(16*b*c^4*(c + d/x^2)^(1/2))/(315*d^4) - (b*(c + d/x^2)^(1/2))/(9*x^8) - (
8*a*c^3*(c + d/x^2)^(1/2))/(105*d^3) - (a*(c + d/x^2)^(1/2))/(7*x^6) - (a*
c*(c + d/x^2)^(1/2))/(35*d*x^4) - (b*c*(c + d/x^2)^(1/2))/(63*d*x^6) + (4*
a*c^2*(c + d/x^2)^(1/2))/(105*d^2*x^2) + (2*b*c^2*(c + d/x^2)^(1/2))/(105*
d^2*x^4) - (8*b*c^3*(c + d/x^2)^(1/2))/(315*d^3*x^2)
```

3.936 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$

3.936.1 Optimal result

Integrand size = 22, antiderivative size = 134

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx = &-\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^5}+\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5}\\ &-\frac{3c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}\\ &+\frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}\end{aligned}$$

output
```
-1/3*c^3*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^5+1/5*c^2*(-3*a*d+4*b*c)*(c+d/x^2)^(
5/2)/d^5-3/7*c*(-a*d+2*b*c)*(c+d/x^2)^(7/2)/d^5+1/9*(-a*d+4*b*c)*(c+d/x^2)
^(9/2)/d^5-1/11*b*(c+d/x^2)^(11/2)/d^5
```

3.936.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$$

$$=\frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)(11adx^2(-35d^3+30cd^2x^2-24c^2dx^4+16c^3x^6)+b(-315d^4+280cd^3x^2-240c^2d^2x^4+}{3465d^5x^{10}}$$

input
```
Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]
```

output
```
(Sqrt[c + d/x^2]*(d + c*x^2)*(11*a*d*x^2*(-35*d^3 + 30*c*d^2*x^2 - 24*c^2*
d*x^4 + 16*c^3*x^6) + b*(-315*d^4 + 280*c*d^3*x^2 - 240*c^2*d^2*x^4 + 192*
c^3*d*x^6 - 128*c^4*x^8)))/(3465*d^5*x^10)
```

3.936.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^6}d\frac{1}{x^2}$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{2}\int \Bigg(\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{d^4}+\frac{(ad-4bc)\left(c+\frac{d}{x^2}\right)^{7/2}}{d^4}+\frac{3c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{d^4}-\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^4}+\frac{c^3(bc-$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\Bigg(-\frac{2c^3\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^5}+\frac{2c^2\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-3ad)}{5d^5}+\frac{2\left(c+\frac{d}{x^2}\right)^{9/2}(4bc-ad)}{9d^5}-\frac{6c\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad$$

input
```
Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]
```

output
```
((-2*c^3*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^5) + (2*c^2*(4*b*c - 3*a*d)*(
c + d/x^2)^(5/2))/(5*d^5) - (6*c*(2*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^5)
+ (2*(4*b*c - a*d)*(c + d/x^2)^(9/2))/(9*d^5) - (2*b*(c + d/x^2)^(11/2))/(
11*d^5))/2
```

3.936.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.936.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88

method	result
gosper	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(176a\,c^{3}d\,x^{8}-128b\,c^{4}x^{8}-264a\,c^{2}d^{2}x^{6}+192b\,c^{3}d\,x^{6}+330ac\,d^{3}x^{4}-240b\,c^{2}d^{2}x^{4}-385a\,d^{4}x^{2}+280bc\,d^{3}x^{2}-315b\,d^{4}\right)\left(c\,x^{2}+d\right)}{3465d^{5}x^{10}}$
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(176a\,c^{3}d\,x^{8}-128b\,c^{4}x^{8}-264a\,c^{2}d^{2}x^{6}+192b\,c^{3}d\,x^{6}+330ac\,d^{3}x^{4}-240b\,c^{2}d^{2}x^{4}-385a\,d^{4}x^{2}+280bc\,d^{3}x^{2}-315b\,d^{4}\right)\left(c\,x^{2}+d\right)}{3465d^{5}x^{10}}$
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(176a\,c^{4}d\,x^{10}-128b\,c^{5}x^{10}-88a\,c^{3}d^{2}x^{8}+64b\,c^{4}d\,x^{8}+66a\,c^{2}d^{3}x^{6}-48b\,c^{3}d^{2}x^{6}-55ac\,d^{4}x^{4}+40b\,c^{2}d^{3}x^{4}-385a\,d^{5}x^{2}-35bc\,d^{4}x\right.}{3465x^{10}d^{5}}$
trager	$\frac{\left(176a\,c^{4}d\,x^{10}-128b\,c^{5}x^{10}-88a\,c^{3}d^{2}x^{8}+64b\,c^{4}d\,x^{8}+66a\,c^{2}d^{3}x^{6}-48b\,c^{3}d^{2}x^{6}-55ac\,d^{4}x^{4}+40b\,c^{2}d^{3}x^{4}-385a\,d^{5}x^{2}-35bc\,d^{4}x^{2}-315b\,d\right.}{3465x^{10}d^{5}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/3465*((c*x^2+d)/x^2)^(1/2)*(176*a*c^3*d*x^8-128*b*c^4*x^8-264*a*c^2*d^2*
x^6+192*b*c^3*d*x^6+330*a*c*d^3*x^4-240*b*c^2*d^2*x^4-385*a*d^4*x^2+280*b*
c*d^3*x^2-315*b*d^4)*(c*x^2+d)/d^5/x^10
```

3.936.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx =$$

$$-\frac{(16\,(8\,bc^5-11\,ac^4d)x^{10}-8\,(8\,bc^4d-11\,ac^3d^2)x^8+6\,(8\,bc^3d^2-11\,ac^2d^3)x^6+315\,bd^5-5\,(8\,bc^2d^3-11}{3465\,d^5x^{10}}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
-1/3465*(16*(8*b*c^5 - 11*a*c^4*d)*x^10 - 8*(8*b*c^4*d - 11*a*c^3*d^2)*x^8
 + 6*(8*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 + 315*b*d^5 - 5*(8*b*c^2*d^3 - 11*a*
c*d^4)*x^4 + 35*(b*c*d^4 + 11*a*d^5)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^5*x^10)
```

3.936.6 Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.32

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$$

$$= -\frac{a\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2}$$

$$-\frac{b\left(\begin{cases}\frac{2\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{5x^{10}} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**9,x)
```

output
```
-a*Piecewise((2*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/
5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4, Ne(d, 0)), (s
qrt(c)/(4*x**8), True))/2 - b*Piecewise((2*(c**4*(c + d/x**2)**(3/2)/3 - 4
*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x*
*2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5, Ne(d, 0)), (sqrt(c)/(5*x**10
), True))/2
```

3.936.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx =$$

$$-\frac{1}{3465}\,b\left(\frac{315\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^5}-\frac{1540\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^5}+\frac{2970\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^5}-\frac{2772\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^5}+\frac{1155\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^5}\right.$$

$$\left.-\frac{1}{315}\,a\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^4}-\frac{135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^4}+\frac{189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^4}-\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3}{d^4}\right)\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
-1/3465*b*(315*(c + d/x^2)^(11/2)/d^5 - 1540*(c + d/x^2)^(9/2)*c/d^5 + 297
0*(c + d/x^2)^(7/2)*c^2/d^5 - 2772*(c + d/x^2)^(5/2)*c^3/d^5 + 1155*(c + d
/x^2)^(3/2)*c^4/d^5) - 1/315*a*(35*(c + d/x^2)^(9/2)/d^4 - 135*(c + d/x^2)
^(7/2)*c/d^4 + 189*(c + d/x^2)^(5/2)*c^2/d^4 - 105*(c + d/x^2)^(3/2)*c^3/d
^4)
```

3.936.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(114) = 228$.

Time = 1.49 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.21

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$$

$$= \frac{32\left(3465\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{9}{2}}\operatorname{sgn}(x)+11088\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{11}{2}}\operatorname{sgn}(x)-4851\left(\sqrt{c}x-\sqrt{cx^2+\right.\right.}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="giac")
```

output
```
32/3465*(3465*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(9/2)*sgn(x) + 11088*(s
qrt(c)*x - sqrt(c*x^2 + d))^12*b*c^(11/2)*sgn(x) - 4851*(sqrt(c)*x - sqrt(
c*x^2 + d))^12*a*c^(9/2)*d*sgn(x) + 7392*(sqrt(c)*x - sqrt(c*x^2 + d))^10*
b*c^(11/2)*d*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(9/2)*d^2*s
gn(x) + 2640*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(11/2)*d^2*sgn(x) - 165*(
sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(9/2)*d^3*sgn(x) - 1320*(sqrt(c)*x - sq
rt(c*x^2 + d))^6*b*c^(11/2)*d^3*sgn(x) + 1815*(sqrt(c)*x - sqrt(c*x^2 + d)
)^6*a*c^(9/2)*d^4*sgn(x) + 440*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(11/2)*
d^4*sgn(x) - 605*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(9/2)*d^5*sgn(x) - 88
*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(11/2)*d^5*sgn(x) + 121*(sqrt(c)*x -
sqrt(c*x^2 + d))^2*a*c^(9/2)*d^6*sgn(x) + 8*b*c^(11/2)*d^6*sgn(x) - 11*a*c
^(9/2)*d^7*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^11
```

3.936.9 Mupad [B] (verification not implemented)

Time = 10.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.57

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx &= \frac{16\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^4}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{a\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{128\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{3465\,d^5}\\ &\quad-\frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,d\,x^6}-\frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{99\,d\,x^8}+\frac{2\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^4}-\frac{8\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3\,x^2}\\ &\quad+\frac{8\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{693\,d^2\,x^6}-\frac{16\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^3\,x^4}+\frac{64\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{3465\,d^4\,x^2}\end{aligned}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^9,x)
```

output
```
(16*a*c^4*(c + d/x^2)^(1/2))/(315*d^4) - (b*(c + d/x^2)^(1/2))/(11*x^10) -
 (a*(c + d/x^2)^(1/2))/(9*x^8) - (128*b*c^5*(c + d/x^2)^(1/2))/(3465*d^5)
- (a*c*(c + d/x^2)^(1/2))/(63*d*x^6) - (b*c*(c + d/x^2)^(1/2))/(99*d*x^8)
+ (2*a*c^2*(c + d/x^2)^(1/2))/(105*d^2*x^4) - (8*a*c^3*(c + d/x^2)^(1/2))/
(315*d^3*x^2) + (8*b*c^2*(c + d/x^2)^(1/2))/(693*d^2*x^6) - (16*b*c^3*(c +
 d/x^2)^(1/2))/(1155*d^3*x^4) + (64*b*c^4*(c + d/x^2)^(1/2))/(3465*d^4*x^2
)
```

3.937 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^{10}\,dx$

3.937.1 Optimal result

Integrand size = 22, antiderivative size = 150

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^{10}\,dx = &-\frac{16d^3(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3465c^5} \\ &+\frac{8d^2(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{1155c^4} \\ &-\frac{2d(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{231c^3} \\ &+\frac{(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{99c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c}\end{aligned}$$

output
```
-16/3465*d^3*(-8*a*d+11*b*c)*(c+d/x^2)^(3/2)*x^3/c^5+8/1155*d^2*(-8*a*d+11
*b*c)*(c+d/x^2)^(3/2)*x^5/c^4-2/231*d*(-8*a*d+11*b*c)*(c+d/x^2)^(3/2)*x^7/
c^3+1/99*(-8*a*d+11*b*c)*(c+d/x^2)^(3/2)*x^9/c^2+1/11*a*(c+d/x^2)^(3/2)*x^
11/c
```

3.937.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.72

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx$$
$$=\frac{\sqrt{c+\frac{d}{x^2}}x(d+cx^2)\left(11bc(-16d^3+24cd^2x^2-30c^2dx^4+35c^3x^6)+a(128d^4-192cd^3x^2+240c^2d^2x^4-28\right.}{3465c^5}$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]
```

output
```
(Sqrt[c + d/x^2]*x*(d + c*x^2)*(11*b*c*(-16*d^3 + 24*c*d^2*x^2 - 30*c^2*d*
x^4 + 35*c^3*x^6) + a*(128*d^4 - 192*c*d^3*x^2 + 240*c^2*d^2*x^4 - 280*c^3
*d*x^6 + 315*c^4*x^8)))/(3465*c^5)
```

3.937.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {955, 803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{10}\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}dx$$
$$\Big\downarrow \text{955}$$
$$\frac{(11bc-8ad)\int\sqrt{c+\frac{d}{x^2}}x^8dx}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{3/2}}{11c}$$
$$\Big\downarrow \text{803}$$
$$\frac{(11bc-8ad)\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}-\frac{2d\int\sqrt{c+\frac{d}{x^2}}x^6dx}{3c}\right)}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{3/2}}{11c}$$
$$\Big\downarrow \text{803}$$

$$\frac{(11bc-8ad)\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}-\frac{2d\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}-\frac{4d\int\sqrt{c+\frac{d}{x^2}}x^4dx}{7c}\right)}{3c}\right)}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{3/2}}{11c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(11bc-8ad)\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}-\frac{2d\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}-\frac{4d\left(\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}-\frac{2d\int\sqrt{c+\frac{d}{x^2}}x^2dx}{5c}\right)}{7c}\right)}{3c}\right)}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{3/2}}{11c}$$

$$\Big\downarrow \text{796}$$

$$\frac{\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}-\frac{2d\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}-\frac{4d\left(\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}-\frac{2dx^3\left(c+\frac{d}{x^2}\right)^{3/2}}{15c^2}\right)}{7c}\right)}{3c}\right)(11bc-8ad)}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{3/2}}{11c}$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]
```

output
```
(a*(c + d/x^2)^(3/2)*x^11)/(11*c) + ((11*b*c - 8*a*d)*(((c + d/x^2)^(3/2)*
x^9)/(9*c) - (2*d*(((c + d/x^2)^(3/2)*x^7)/(7*c) - (4*d*((-2*d*(c + d/x^2)
^(3/2)*x^3)/(15*c^2) + ((c + d/x^2)^(3/2)*x^5)/(5*c)))/(7*c)))/(3*c)))/(11
*c)
```

3.937. $\int\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx$

3.937.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.937.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

method	result
gosper	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(315a\,x^{8}c^{4}-280a\,c^{3}d\,x^{6}+385b\,c^{4}x^{6}+240a\,c^{2}d^{2}x^{4}-330b\,c^{3}d\,x^{4}-192ac\,d^{3}x^{2}+264b\,c^{2}d^{2}x^{2}+128a\,d^{4}-176bc\,d^{3}\right)\left(c\,x^{2}+d\right)}{3465c^{5}}$
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(315a\,x^{8}c^{4}-280a\,c^{3}d\,x^{6}+385b\,c^{4}x^{6}+240a\,c^{2}d^{2}x^{4}-330b\,c^{3}d\,x^{4}-192ac\,d^{3}x^{2}+264b\,c^{2}d^{2}x^{2}+128a\,d^{4}-176bc\,d^{3}\right)\left(c\,x^{2}+d\right)}{3465c^{5}}$
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x(315a\,c^{5}x^{10}+35a\,c^{4}d\,x^{8}+385b\,c^{5}x^{8}-40a\,c^{3}d^{2}x^{6}+55b\,c^{4}d\,x^{6}+48a\,c^{2}d^{3}x^{4}-66b\,c^{3}d^{2}x^{4}-64ac\,d^{4}x^{2}+88b\,c^{2}d^{3}x^{2}+128a\,d^{5}-}{3465c^{5}}$
trager	$\frac{(315a\,c^{5}x^{10}+35a\,c^{4}d\,x^{8}+385b\,c^{5}x^{8}-40a\,c^{3}d^{2}x^{6}+55b\,c^{4}d\,x^{6}+48a\,c^{2}d^{3}x^{4}-66b\,c^{3}d^{2}x^{4}-64ac\,d^{4}x^{2}+88b\,c^{2}d^{3}x^{2}+128a\,d^{5}-176bc\,d^{4})x}{3465c^{5}}$

input
```
int((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3465*((c*x^2+d)/x^2)^(1/2)*x*(315*a*c^4*x^8-280*a*c^3*d*x^6+385*b*c^4*x^
6+240*a*c^2*d^2*x^4-330*b*c^3*d*x^4-192*a*c*d^3*x^2+264*b*c^2*d^2*x^2+128*
a*d^4-176*b*c*d^3)*(c*x^2+d)/c^5
```

3.937.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx$$
$$=\frac{(315\,ac^5x^{11}+35\,(11\,bc^5+ac^4d)x^9+5\,(11\,bc^4d-8\,ac^3d^2)x^7-6\,(11\,bc^3d^2-8\,ac^2d^3)x^5+8\,(11\,bc^2d^3-8\,a}{3465\,c^5}$$

input
```
integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3465*(315*a*c^5*x^11 + 35*(11*b*c^5 + a*c^4*d)*x^9 + 5*(11*b*c^4*d - 8*a
*c^3*d^2)*x^7 - 6*(11*b*c^3*d^2 - 8*a*c^2*d^3)*x^5 + 8*(11*b*c^2*d^3 - 8*a
*c*d^4)*x^3 - 16*(11*b*c*d^4 - 8*a*d^5)*x)*sqrt((c*x^2 + d)/x^2)/c^5
```

3.937.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. 2(146) = 292.

Time = 2.76 (sec) , antiderivative size = 1386, normalized size of antiderivative = 9.24

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x**2)*x**10*(c+d/x**2)**(1/2),x)
```

output
```
315*a*c**9*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c
**5*d**20) + 1295*a*c**8*d**(35/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**
16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19
*x**2 + 3465*c**5*d**20) + 1990*a*c**7*d**(37/2)*x**14*sqrt(c*x**2/d + 1)/
(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13
860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**6*d**(39/2)*x**12*sqrt(
c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d
**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 343*a*c**5*d**(41/2
)*x**10*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c
**4*d**(43/2)*x**8*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d
**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**2
0) + 280*a*c**3*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 +
13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 34
65*c**5*d**20) + 560*a*c**2*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d
**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**
19*x**2 + 3465*c**5*d**20) + 448*a*c*d**(49/2)*x**2*sqrt(c*x**2/d + 1)/(34
65*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860
*c**6*d**19*x**2 + 3465*c**5*d**20) + 128*a*d**(51/2)*sqrt(c*x**2/d + 1...
```

3.937.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx$$
$$=\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5-105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^3x^3\right)b}{315\,c^4}$$
$$+\frac{\big(315\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-1540\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+2970\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-2772\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5+1155\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3465\,c^5}$$

input
```
integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/315*(35*(c + d/x^2)^(9/2)*x^9 - 135*(c + d/x^2)^(7/2)*d*x^7 + 189*(c + d
/x^2)^(5/2)*d^2*x^5 - 105*(c + d/x^2)^(3/2)*d^3*x^3)*b/c^4 + 1/3465*(315*(
c + d/x^2)^(11/2)*x^11 - 1540*(c + d/x^2)^(9/2)*d*x^9 + 2970*(c + d/x^2)^(
7/2)*d^2*x^7 - 2772*(c + d/x^2)^(5/2)*d^3*x^5 + 1155*(c + d/x^2)^(3/2)*d^4
*x^3)*a/c^5
```

3.937.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.17

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx = \frac{16\left(11\,bcd^{\frac{9}{2}}-8\,ad^{\frac{11}{2}}\right)\mathrm{sgn}(x)}{3465\,c^5}$$

$$+\frac{315\,(cx^2+d)^{\frac{11}{2}}a\mathrm{sgn}(x)+385\,(cx^2+d)^{\frac{9}{2}}bc\mathrm{sgn}(x)-1540\,(cx^2+d)^{\frac{9}{2}}ad\mathrm{sgn}(x)-1485\,(cx^2+d)^{\frac{7}{2}}bcd\mathrm{sgn}(}{}$$

input
```
integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
16/3465*(11*b*c*d^(9/2) - 8*a*d^(11/2))*sgn(x)/c^5 + 1/3465*(315*(c*x^2 +
d)^(11/2)*a*sgn(x) + 385*(c*x^2 + d)^(9/2)*b*c*sgn(x) - 1540*(c*x^2 + d)^(
9/2)*a*d*sgn(x) - 1485*(c*x^2 + d)^(7/2)*b*c*d*sgn(x) + 2970*(c*x^2 + d)^(
7/2)*a*d^2*sgn(x) + 2079*(c*x^2 + d)^(5/2)*b*c*d^2*sgn(x) - 2772*(c*x^2 +
d)^(5/2)*a*d^3*sgn(x) - 1155*(c*x^2 + d)^(3/2)*b*c*d^3*sgn(x) + 1155*(c*x^
2 + d)^(3/2)*a*d^4*sgn(x))/c^5
```

3.937.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.78

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx = \sqrt{c+\frac{d}{x^2}}\left(\frac{a\,x^{11}}{11}+\frac{x\,(128\,a\,d^5-176\,b\,c\,d^4)}{3465\,c^5}\right.\\ +\frac{x^9\,(385\,b\,c^5+35\,a\,d\,c^4)}{3465\,c^5}-\frac{d\,x^7\,(8\,a\,d-11\,b\,c)}{693\,c^2}\\ \left.+\frac{2\,d^2\,x^5\,(8\,a\,d-11\,b\,c)}{1155\,c^3}-\frac{8\,d^3\,x^3\,(8\,a\,d-11\,b\,c)}{3465\,c^4}\right)\end{aligned}$$

input
```
int(x^10*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
(c + d/x^2)^(1/2)*((a*x^11)/11 + (x*(128*a*d^5 - 176*b*c*d^4))/(3465*c^5)
+ (x^9*(385*b*c^5 + 35*a*c^4*d))/(3465*c^5) - (d*x^7*(8*a*d - 11*b*c))/(69
3*c^2) + (2*d^2*x^5*(8*a*d - 11*b*c))/(1155*c^3) - (8*d^3*x^3*(8*a*d - 11*
b*c))/(3465*c^4))
```

3.938 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\, x^8 \, dx$

3.938.1 Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\, x^8 \, dx = \frac{8d^2(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{315c^4} - \frac{4d(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{105c^3} + \frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^7}{21c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^9}{9c}$$

output
```
8/315*d^2*(-2*a*d+3*b*c)*(c+d/x^2)^(3/2)*x^3/c^4-4/105*d*(-2*a*d+3*b*c)*(c
+d/x^2)^(3/2)*x^5/c^3+1/21*(-2*a*d+3*b*c)*(c+d/x^2)^(3/2)*x^7/c^2+1/9*a*(c
+d/x^2)^(3/2)*x^9/c
```

3.938.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.75

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\, x^8 \, dx = \frac{\sqrt{c+\frac{d}{x^2}}\, x(d+cx^2)\left(24bcd^2-16ad^3-36bc^2dx^2+24acd^2x^2+45bc^3x^4-30ac^2dx^4+35ac^3x^6\right)}{315c^4}$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]
```

output
```
(Sqrt[c + d/x^2]*x*(d + c*x^2)*(24*b*c*d^2 - 16*a*d^3 - 36*b*c^2*d*x^2 + 2
4*a*c*d^2*x^2 + 45*b*c^3*x^4 - 30*a*c^2*d*x^4 + 35*a*c^3*x^6))/(315*c^4)
```

3.938.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(3bc-2ad)\int \sqrt{c+\frac{d}{x^2}}x^6dx}{3c}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(3bc-2ad)\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}-\frac{4d\int \sqrt{c+\frac{d}{x^2}}x^4dx}{7c}\right)}{3c}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(3bc-2ad)\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}-\frac{4d\left(\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}-\frac{2d\int \sqrt{c+\frac{d}{x^2}}x^2dx}{5c}\right)}{7c}\right)}{3c}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$

$$\Big\downarrow \text{796}$$

$$\frac{\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}-\frac{4d\left(\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}-\frac{2dx^3\left(c+\frac{d}{x^2}\right)^{3/2}}{15c^2}\right)}{7c}\right)(3bc-2ad)}{3c}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]
```

3.938. $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx$

output
```
(a*(c + d/x^2)^(3/2)*x^9)/(9*c) + ((3*b*c - 2*a*d)*(((c + d/x^2)^(3/2)*x^7
)/(7*c) - (4*d*((-2*d*(c + d/x^2)^(3/2)*x^3)/(15*c^2) + ((c + d/x^2)^(3/2)
*x^5)/(5*c)))/(7*c)))/(3*c)
```

3.938.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.938.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(35x^{6}a\,c^{3}-30a\,c^{2}d\,x^{4}+45b\,c^{3}x^{4}+24ac\,d^{2}x^{2}-36b\,c^{2}d\,x^{2}-16a\,d^{3}+24bc\,d^{2}\right)\left(c\,x^{2}+d\right)}{315c^{4}}$	89
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(35x^{6}a\,c^{3}-30a\,c^{2}d\,x^{4}+45b\,c^{3}x^{4}+24ac\,d^{2}x^{2}-36b\,c^{2}d\,x^{2}-16a\,d^{3}+24bc\,d^{2}\right)\left(c\,x^{2}+d\right)}{315c^{4}}$	89
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(35a\,x^{8}c^{4}+5a\,c^{3}d\,x^{6}+45b\,c^{4}x^{6}-6a\,c^{2}d^{2}x^{4}+9b\,c^{3}d\,x^{4}+8ac\,d^{3}x^{2}-12b\,c^{2}d^{2}x^{2}-16a\,d^{4}+24bc\,d^{3}\right)}{315c^{4}}$	106
trager	$\frac{\left(35a\,x^{8}c^{4}+5a\,c^{3}d\,x^{6}+45b\,c^{4}x^{6}-6a\,c^{2}d^{2}x^{4}+9b\,c^{3}d\,x^{4}+8ac\,d^{3}x^{2}-12b\,c^{2}d^{2}x^{2}-16a\,d^{4}+24bc\,d^{3}\right)x\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{315c^{4}}$	110

input
```
int((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*((c*x^2+d)/x^2)^(1/2)*x*(35*a*c^3*x^6-30*a*c^2*d*x^4+45*b*c^3*x^4+24
*a*c*d^2*x^2-36*b*c^2*d*x^2-16*a*d^3+24*b*c*d^2)*(c*x^2+d)/c^4
```

3.938.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx$$
$$=\frac{\left(35\,ac^4x^9+5\,(9\,bc^4+ac^3d)x^7+3\,(3\,bc^3d-2\,ac^2d^2)x^5-4\,(3\,bc^2d^2-2\,acd^3)x^3+8\,(3\,bcd^3-2\,ad^4)x\right)\sqrt{\frac{cx}{}}}{315\,c^4}$$

input
```
integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + a*c^3*d)*x^7 + 3*(3*b*c^3*d - 2*a*c^2*d
^2)*x^5 - 4*(3*b*c^2*d^2 - 2*a*c*d^3)*x^3 + 8*(3*b*c*d^3 - 2*a*d^4)*x)*sqr
t((c*x^2 + d)/x^2)/c^4
```

3.938.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(112) = 224$.

Time = 2.25 (sec) , antiderivative size = 910, normalized size of antiderivative = 7.78

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx = \frac{35ac^7d^{\frac{19}{2}}x^{14}\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} + \frac{110ac^6d^{\frac{21}{2}}x^{12}\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} + \frac{114ac^5d^{\frac{23}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} + \frac{40ac^4d^{\frac{25}{2}}x^{8}\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} - \frac{5ac^3d^{\frac{27}{2}}x^{6}\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} - \frac{30ac^2d^{\frac{29}{2}}x^{4}\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} - \frac{40acd^{\frac{31}{2}}x^{2}\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} - \frac{16ad^{\frac{33}{2}}\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} + \frac{15bc^5d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{33bc^4d^{\frac{11}{2}}x^{8}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{17bc^3d^{\frac{13}{2}}x^{6}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{3bc^2d^{\frac{15}{2}}x^{4}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{12bcd^{\frac{17}{2}}x^{2}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{8bd^{\frac{19}{2}}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}$$

input

```
integrate((a+b/x**2)*x**8*(c+d/x**2)**(1/2),x)
```

3.938. $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx$

output
```
35*a*c**7*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**
6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**6*d**(21/2
)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945
*c**5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**5*d**(23/2)*x**10*sqrt(c*x**
2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 +
 315*c**4*d**12) + 40*a*c**4*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d
**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 5
*a*c**3*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d
**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 30*a*c**2*d**(29/2)*x*
*4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5
*d**11*x**2 + 315*c**4*d**12) - 40*a*c*d**(31/2)*x**2*sqrt(c*x**2/d + 1)/(
315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*
d**12) - 16*a*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*
d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*b*c**5*d**(9/2)*x*
*10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3
*d**6) + 33*b*c**4*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 +
 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*b*c**3*d**(13/2)*x**6*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*b*c
**2*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*
x**2 + 105*c**3*d**6) + 12*b*c*d**(17/2)*x**2*sqrt(c*x**2/d + 1)/(105*c...
```

3.938.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx &= \frac{\left(15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5+35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3\right)b}{105\,c^3}\\&\quad+\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5-105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^3x^3\right)a}{315\,c^4}\end{aligned}$$

input
```
integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/105*(15*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x
^2)^(3/2)*d^2*x^3)*b/c^3 + 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 135*(c + d/x^
2)^(7/2)*d*x^7 + 189*(c + d/x^2)^(5/2)*d^2*x^5 - 105*(c + d/x^2)^(3/2)*d^3
*x^3)*a/c^4
```

3.938.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx=-\frac{8\left(3\,bcd^{\frac{7}{2}}-2\,ad^{\frac{9}{2}}\right)\operatorname{sgn}(x)}{315\,c^4}$$
$$+\frac{35\,(cx^2+d)^{\frac{9}{2}}a\operatorname{sgn}(x)+45\,(cx^2+d)^{\frac{7}{2}}bc\operatorname{sgn}(x)-135\,(cx^2+d)^{\frac{7}{2}}ad\operatorname{sgn}(x)-126\,(cx^2+d)^{\frac{5}{2}}bcd\operatorname{sgn}(x)+}{315\,c^4}$$

input

```
integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="giac")
```

output

```
-8/315*(3*b*c*d^(7/2) - 2*a*d^(9/2))*sgn(x)/c^4 + 1/315*(35*(c*x^2 + d)^(9
/2)*a*sgn(x) + 45*(c*x^2 + d)^(7/2)*b*c*sgn(x) - 135*(c*x^2 + d)^(7/2)*a*d
*sgn(x) - 126*(c*x^2 + d)^(5/2)*b*c*d*sgn(x) + 189*(c*x^2 + d)^(5/2)*a*d^2
*sgn(x) + 105*(c*x^2 + d)^(3/2)*b*c*d^2*sgn(x) - 105*(c*x^2 + d)^(3/2)*a*d
^3*sgn(x))/c^4
```

3.938.9 Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.83

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx=\sqrt{c+\frac{d}{x^2}}\left(\frac{a\,x^9}{9}-\frac{x\,(16\,a\,d^4-24\,b\,c\,d^3)}{315\,c^4}+\frac{x^7\,(45\,b\,c^4+5\,a\,d\,c^3)}{315\,c^4}-\frac{d\,x^5\,(2\,a\,d-3\,b\,c)}{105\,c^2}+\frac{4\,d^2\,x^3\,(2\,a\,d-3\,b\,c)}{315\,c^3}\right)$$

input

```
int(x^8*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output

```
(c + d/x^2)^(1/2)*((a*x^9)/9 - (x*(16*a*d^4 - 24*b*c*d^3))/(315*c^4) + (x^
7*(45*b*c^4 + 5*a*c^3*d))/(315*c^4) - (d*x^5*(2*a*d - 3*b*c))/(105*c^2) +
(4*d^2*x^3*(2*a*d - 3*b*c))/(315*c^3))
```

3.939 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^6 \, dx$

3.939.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^6 \, dx = -\frac{2d(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{105c^3} + \frac{(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{35c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^7}{7c}$$

output
```
-2/105*d*(-4*a*d+7*b*c)*(c+d/x^2)^(3/2)*x^3/c^3+1/35*(-4*a*d+7*b*c)*(c+d/x
^2)^(3/2)*x^5/c^2+1/7*a*(c+d/x^2)^(3/2)*x^7/c
```

3.939.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^6 \, dx = \frac{\sqrt{c+\frac{d}{x^2}} x (d+cx^2) (-14bcd+8ad^2+21bc^2x^2-12acdx^2+15ac^2x^4)}{105c^3}$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]
```

output
```
(Sqrt[c + d/x^2]*x*(d + c*x^2)*(-14*b*c*d + 8*a*d^2 + 21*b*c^2*x^2 - 12*a*
c*d*x^2 + 15*a*c^2*x^4))/(105*c^3)
```

3.939.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {955, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(7bc-4ad)\int \sqrt{c+\frac{d}{x^2}}x^4dx}{7c}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(7bc-4ad)\left(\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}-\frac{2d\int \sqrt{c+\frac{d}{x^2}}x^2dx}{5c}\right)}{7c}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{796}$$

$$\frac{\left(\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}-\frac{2dx^3\left(c+\frac{d}{x^2}\right)^{3/2}}{15c^2}\right)(7bc-4ad)}{7c}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]
```

output
```
(a*(c + d/x^2)^(3/2)*x^7)/(7*c) + ((7*b*c - 4*a*d)*((-2*d*(c + d/x^2)^(3/2
)*x^3)/(15*c^2) + ((c + d/x^2)^(3/2)*x^5)/(5*c)))/(7*c)
```

3.939.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.939.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(15a\,x^{4}c^{2}-12acd\,x^{2}+21b\,c^{2}x^{2}+8a\,d^{2}-14bcd\right)\left(c\,x^{2}+d\right)}{105c^{3}}$	65
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(15a\,x^{4}c^{2}-12acd\,x^{2}+21b\,c^{2}x^{2}+8a\,d^{2}-14bcd\right)\left(c\,x^{2}+d\right)}{105c^{3}}$	65
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(15x^{6}a\,c^{3}+3a\,c^{2}d\,x^{4}+21b\,c^{3}x^{4}-4ac\,d^{2}x^{2}+7b\,c^{2}d\,x^{2}+8a\,d^{3}-14bc\,d^{2}\right)}{105c^{3}}$	82
trager	$\frac{\left(15x^{6}a\,c^{3}+3a\,c^{2}d\,x^{4}+21b\,c^{3}x^{4}-4ac\,d^{2}x^{2}+7b\,c^{2}d\,x^{2}+8a\,d^{3}-14bc\,d^{2}\right)x\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{105c^{3}}$	86

input
```
int((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*((c*x^2+d)/x^2)^(1/2)*x*(15*a*c^2*x^4-12*a*c*d*x^2+21*b*c^2*x^2+8*a*
d^2-14*b*c*d)*(c*x^2+d)/c^3
```

3.939.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx$$
$$=\frac{\left(15\,ac^3x^7+3\,(7\,bc^3+ac^2d)x^5+(7\,bc^2d-4\,acd^2)x^3-2\,(7\,bcd^2-4\,ad^3)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,c^3}$$

input

```
integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output

```
1/105*(15*a*c^3*x^7 + 3*(7*b*c^3 + a*c^2*d)*x^5 + (7*b*c^2*d - 4*a*c*d^2)*
x^3 - 2*(7*b*c*d^2 - 4*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^3
```

3.939.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(78) = 156.

Time = 1.65 (sec) , antiderivative size = 422, normalized size of antiderivative = 5.02

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx=\frac{15ac^5d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}$$
$$+\frac{33ac^4d^{\frac{11}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}$$
$$+\frac{17ac^3d^{\frac{13}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}$$
$$+\frac{3ac^2d^{\frac{15}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}$$
$$+\frac{12acd^{\frac{17}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}$$
$$+\frac{8ad^{\frac{19}{2}}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}$$
$$+\frac{b\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{bd^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c}-\frac{2bd^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2}$$

input
```
integrate((a+b/x**2)*x**6*(c+d/x**2)**(1/2),x)
```

output
```
15*a*c**5*d**(9/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4
*d**5*x**2 + 105*c**3*d**6) + 33*a*c**4*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/
(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**3*d**(
13/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 1
05*c**3*d**6) + 3*a*c**2*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*
x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c*d**(17/2)*x**2*sqrt(c*
x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*
a*d**(19/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 +
105*c**3*d**6) + b*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + b*d**(3/2)*x**2*sqr
t(c*x**2/d + 1)/(15*c) - 2*b*d**(5/2)*sqrt(c*x**2/d + 1)/(15*c**2)
```

3.939.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx=\frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3\right)b}{15\,c^2}+\frac{\left(15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5+35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3\right)a}{105\,c^3}$$

input
```
integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3)*b/c^2 + 1/105*(
15*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x^2)^(3/
2)*d^2*x^3)*a/c^3
```

3.939.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx=\frac{2\left(7\,bcd^{\frac{5}{2}}-4\,ad^{\frac{7}{2}}\right)\mathrm{sgn}(x)}{105\,c^3}+\frac{15\left(cx^2+d\right)^{\frac{7}{2}}a\mathrm{sgn}(x)+21\left(cx^2+d\right)^{\frac{5}{2}}bc\mathrm{sgn}(x)-42\left(cx^2+d\right)^{\frac{5}{2}}ad\mathrm{sgn}(x)-35\left(cx^2+d\right)^{\frac{3}{2}}bcd\mathrm{sgn}(x)+35}{105\,c^3}$$

input
```
integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
2/105*(7*b*c*d^(5/2) - 4*a*d^(7/2))*sgn(x)/c^3 + 1/105*(15*(c*x^2 + d)^(7/
2)*a*sgn(x) + 21*(c*x^2 + d)^(5/2)*b*c*sgn(x) - 42*(c*x^2 + d)^(5/2)*a*d*s
gn(x) - 35*(c*x^2 + d)^(3/2)*b*c*d*sgn(x) + 35*(c*x^2 + d)^(3/2)*a*d^2*sgn
(x))/c^3
```

3.939.9 Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx=\sqrt{c+\frac{d}{x^2}}\left(\frac{a\,x^7}{7}+\frac{x\,(8\,a\,d^3-14\,b\,c\,d^2)}{105\,c^3}+\frac{x^5\,(21\,b\,c^3+3\,a\,d\,c^2)}{105\,c^3}-\frac{d\,x^3\,(4\,a\,d-7\,b\,c)}{105\,c^2}\right)$$

input
```
int(x^6*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
(c + d/x^2)^(1/2)*((a*x^7)/7 + (x*(8*a*d^3 - 14*b*c*d^2))/(105*c^3) + (x^5
*(21*b*c^3 + 3*a*c^2*d))/(105*c^3) - (d*x^3*(4*a*d - 7*b*c))/(105*c^2))
```

3.940 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^4 \, dx$

3.940.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^4 \, dx = \frac{(5bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{15c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{5c}$$

output

```
1/15*(-2*a*d+5*b*c)*(c+d/x^2)^(3/2)*x^3/c^2+1/5*a*(c+d/x^2)^(3/2)*x^5/c
```

3.940.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^4 \, dx = \frac{\sqrt{c+\frac{d}{x^2}} x(d+cx^2)\left(5bc-2ad+3acx^2\right)}{15c^2}$$

input

```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]
```

output

```
(Sqrt[c + d/x^2]*x*(d + c*x^2)*(5*b*c - 2*a*d + 3*a*c*x^2))/(15*c^2)
```

3.940.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(5bc-2ad)\int \sqrt{c+\frac{d}{x^2}}x^2dx}{5c}+\frac{ax^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}$$

$$\Big\downarrow \text{796}$$

$$\frac{x^3\left(c+\frac{d}{x^2}\right)^{3/2}(5bc-2ad)}{15c^2}+\frac{ax^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]
```

output
```
((5*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(15*c^2) + (a*(c + d/x^2)^(3/2)*x^
5)/(5*c)
```

3.940.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.940.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(3ac\,x^{2}-2ad+5bc\right)\left(c\,x^{2}+d\right)}{15c^{2}}$	43
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(3ac\,x^{2}-2ad+5bc\right)\left(c\,x^{2}+d\right)}{15c^{2}}$	43
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(3a\,x^{4}c^{2}+acd\,x^{2}+5b\,c^{2}x^{2}-2a\,d^{2}+5bcd\right)}{15c^{2}}$	57
trager	$\frac{\left(3a\,x^{4}c^{2}+acd\,x^{2}+5b\,c^{2}x^{2}-2a\,d^{2}+5bcd\right)x\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{15c^{2}}$	61

input
```
int((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*((c*x^2+d)/x^2)^(1/2)*x*(3*a*c*x^2-2*a*d+5*b*c)*(c*x^2+d)/c^2
```

3.940.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx = \frac{\left(3\,ac^{2}x^{5}+\left(5\,bc^{2}+acd\right)x^{3}+\left(5\,bcd-2\,ad^{2}\right)x\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{15\,c^{2}}$$

input
```
integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*a*c^2*x^5 + (5*b*c^2 + a*c*d)*x^3 + (5*b*c*d - 2*a*d^2)*x)*sqrt((c
*x^2 + d)/x^2)/c^2
```

3.940.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(46) = 92.

Time = 1.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.25

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx = \frac{a\sqrt{d}x^{4}\sqrt{\frac{cx^{2}}{d}+1}}{5}+\frac{ad^{\frac{3}{2}}x^{2}\sqrt{\frac{cx^{2}}{d}+1}}{15c} - \frac{2ad^{\frac{5}{2}}\sqrt{\frac{cx^{2}}{d}+1}}{15c^{2}}+\frac{b\sqrt{d}x^{2}\sqrt{\frac{cx^{2}}{d}+1}}{3}+\frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^{2}}{d}+1}}{3c}$$

input
```
integrate((a+b/x**2)*x**4*(c+d/x**2)**(1/2),x)
```

output
```
a*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + a*d**(3/2)*x**2*sqrt(c*x**2/d + 1)/(
15*c) - 2*a*d**(5/2)*sqrt(c*x**2/d + 1)/(15*c**2) + b*sqrt(d)*x**2*sqrt(c*
x**2/d + 1)/3 + b*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c)
```

3.940.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx = \frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3}{3\,c}+\frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3\right)a}{15\,c^2}$$

input
```
integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*b*(c + d/x^2)^(3/2)*x^3/c + 1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x
^2)^(3/2)*d*x^3)*a/c^2
```

3.940.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\begin{aligned}&\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx\\&=-\frac{\left(5\,bcd^{\frac{3}{2}}-2\,ad^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{15\,c^2}\\&\quad+\frac{3\left(cx^2+d\right)^{\frac{5}{2}}a\mathrm{sgn}(x)+5\left(cx^2+d\right)^{\frac{3}{2}}bc\mathrm{sgn}(x)-5\left(cx^2+d\right)^{\frac{3}{2}}ad\mathrm{sgn}(x)}{15\,c^2}\end{aligned}$$

input
```
integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/15*(5*b*c*d^(3/2) - 2*a*d^(5/2))*sgn(x)/c^2 + 1/15*(3*(c*x^2 + d)^(5/2)
*a*sgn(x) + 5*(c*x^2 + d)^(3/2)*b*c*sgn(x) - 5*(c*x^2 + d)^(3/2)*a*d*sgn(x
))/c^2
```

3.940.9 Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx=\sqrt{c+\frac{d}{x^2}}\left(\frac{a\,x^5}{5}-\frac{x\,\left(2\,a\,d^2-5\,b\,c\,d\right)}{15\,c^2}+\frac{x^3\,\left(5\,b\,c^2+a\,d\,c\right)}{15\,c^2}\right)$$

input
```
int(x^4*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
(c + d/x^2)^(1/2)*((a*x^5)/5 - (x*(2*a*d^2 - 5*b*c*d))/(15*c^2) + (x^3*(5*
b*c^2 + a*c*d))/(15*c^2))
```

3.941 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^2 \, dx$

3.941.1 Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^2 \, dx = b\sqrt{c+\frac{d}{x^2}}x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-b\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)$$

output
```
1/3*a*(c+d/x^2)^(3/2)*x^3/c-b*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))*d^(1/2)+b
*x*(c+d/x^2)^(1/2)
```

3.941.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^2 \, dx = \frac{1}{3}\sqrt{c+\frac{d}{x^2}}x\left(3b+a\left(\frac{d}{c}+x^2\right)-\frac{3b\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d+cx^2}}\right)$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]
```

output
```
(Sqrt[c + d/x^2]*x*(3*b + a*(d/c + x^2) - (3*b*Sqrt[d]*ArcTanh[Sqrt[d + c*
x^2]/Sqrt[d]])/Sqrt[d + c*x^2]))/3
```

3.941.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {953, 773, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{953}$$

$$b\int \sqrt{c+\frac{d}{x^2}}dx+\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}$$

$$\Big\downarrow \text{773}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-b\int \sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{247}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-b\left(d\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}-x\sqrt{c+\frac{d}{x^2}}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-b\left(d\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}-x\sqrt{c+\frac{d}{x^2}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-b\left(\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)-x\sqrt{c+\frac{d}{x^2}}\right)$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]
```

output
```
(a*(c + d/x^2)^(3/2)*x^3)/(3*c) - b*(-(Sqrt[c + d/x^2]*x) + Sqrt[d]*ArcTan
h[Sqrt[d]/(Sqrt[c + d/x^2]*x)])
```

3.941. $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx$

3.941.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 953
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[d/e^n   Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] &&
(IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && G
tQ[m + n, -1]))
```

3.941.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(3\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)bc-a\left(c\,x^{2}+d\right)^{\frac{3}{2}}-3\sqrt{c\,x^{2}+d}\,bc\right)}{3\sqrt{c\,x^{2}+d}\,c}$	83

input
```
int((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((c*x^2+d)/x^2)^(1/2)*x*(3*d^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x
)*b*c-a*(c*x^2+d)^(3/2)-3*(c*x^2+d)^(1/2)*b*c)/(c*x^2+d)^(1/2)/c
```

3.941.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.36

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx$$
$$= \Bigg[\frac{3\,bc\sqrt{d}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(acx^3+(3\,bc+ad)x)\sqrt{\frac{cx^2+d}{x^2}}}{6\,c}, \frac{3\,bc\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+}{3\,c}$$

input
```
integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)
/x^2) + 2*(a*c*x^3 + (3*b*c + a*d)*x)*sqrt((c*x^2 + d)/x^2))/c, 1/3*(3*b*c
*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (a*c*x^3
+ (3*b*c + a*d)*x)*sqrt((c*x^2 + d)/x^2))/c]
```

3.941.6 Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.62

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx = \frac{a\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c}+\frac{b\sqrt{c}x}{\sqrt{1+\frac{d}{cx^2}}} - b\sqrt{d}\,\mathrm{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)+\frac{bd}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}$$

input
```
integrate((a+b/x**2)*x**2*(c+d/x**2)**(1/2),x)
```

output
```
a*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + a*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c)
+ b*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - b*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x))
+ b*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2)))
```

3.941.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx=\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3}{3\,c}+\frac{1}{2}\left(2\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)b$$

input
```
integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*a*(c + d/x^2)^(3/2)*x^3/c + 1/2*(2*sqrt(c + d/x^2)*x + sqrt(d)*log((sq
rt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*b
```

3.941.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(54) = 108$.

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.76

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx=\frac{bd\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)\operatorname{sgn}(x)}{\sqrt{-d}}-\frac{\left(3\,bcd\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+3\,bc\sqrt{-d}\sqrt{d}+a\sqrt{-d}d^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{3\,c\sqrt{-d}}+\frac{(cx^2+d)^{\frac{3}{2}}ac^2\operatorname{sgn}(x)+3\sqrt{cx^2+d}bc^3\operatorname{sgn}(x)}{3\,c^3}$$

input
```
integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
b*d*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/3*(3*b*c*d*arctan
(sqrt(d)/sqrt(-d)) + 3*b*c*sqrt(-d)*sqrt(d) + a*sqrt(-d)*d^(3/2))*sgn(x)/(
c*sqrt(-d)) + 1/3*((c*x^2 + d)^(3/2)*a*c^2*sgn(x) + 3*sqrt(c*x^2 + d)*b*c^
3*sgn(x))/c^3
```

3.941.9 Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx = b\,x\,\sqrt{c+\frac{d}{x^2}}+\frac{a\,x\,\sqrt{c+\frac{d}{x^2}}\,\left(c\,x^2+d\right)}{3\,c}+\frac{b\,\sqrt{d}\,\mathrm{asin}\left(\frac{\sqrt{d}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c+\frac{d}{x^2}}\,1\mathrm{i}}{\sqrt{c}\,\sqrt{\frac{d}{c\,x^2}+1}}$$

input
```
int(x^2*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
b*x*(c + d/x^2)^(1/2) + (a*x*(c + d/x^2)^(1/2)*(d + c*x^2))/(3*c) + (b*d^(
1/2)*asin((d^(1/2)*1i)/(c^(1/2)*x))*(c + d/x^2)^(1/2)*1i)/(c^(1/2)*(d/(c*x
^2) + 1)^(1/2))
```

3.942 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx$

3.942.1 Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx=-\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c}-\frac{(bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2\sqrt{d}}$$

output
```
a*(c+d/x^2)^(3/2)*x/c-1/2*(2*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d
^(1/2)-1/2*(2*a*d+b*c)*(c+d/x^2)^(1/2)/c/x
```

3.942.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}\left(-b+2ax^2-\frac{(bc+2ad)x^2\operatorname{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{d+cx^2}}\right)}{2x}$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2],x]
```

output
```
(Sqrt[c + d/x^2]*(-b + 2*a*x^2 - ((b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2
]/Sqrt[d]])/(Sqrt[d]*Sqrt[d + c*x^2])))/(2*x)
```

3.942.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {899, 359, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{359}$$

$$\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(2ad+bc)\int \sqrt{c+\frac{d}{x^2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{211}$$

$$\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(2ad+bc)\left(\frac{1}{2}c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)}{c}$$

$$\Big\downarrow \text{224}$$

$$\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(2ad+bc)\left(\frac{1}{2}c\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(2ad+bc)\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)}{c}$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2],x]
```

output
```
(a*(c + d/x^2)^(3/2)*x)/c - ((b*c + 2*a*d)*(Sqrt[c + d/x^2]/(2*x) + (c*Arc
Tanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*Sqrt[d])))/c
```

3.942.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.942.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

method	result	si
risch	$-\frac{b\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{2x}+\frac{\left(a\sqrt{c\,x^{2}+d}-\frac{(2ad+bc)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)}{2\sqrt{d}}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{\sqrt{c\,x^{2}+d}}$	9
default	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(2d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)a\,x^{2}+\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)bc\,x^{2}-2\sqrt{c\,x^{2}+d}\,ad\,x^{2}-\sqrt{c\,x^{2}+d}\,bc\,x^{2}+\left(c\,x^{2}+d\right)^{\frac{3}{2}}b\right)}{2x\sqrt{c\,x^{2}+d}\,d}$	1

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/x*((c*x^2+d)/x^2)^(1/2)+(a*(c*x^2+d)^(1/2)-1/2*(2*a*d+b*c)/d^(1/2)*
ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x))*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(
1/2)
```

3.942.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.93

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx$$
$$=\left[\frac{(bc+2\,ad)\sqrt{d}x\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(2\,adx^2-bd)\sqrt{\frac{cx^2+d}{x^2}}}{4\,dx}, \frac{(bc+2\,ad)\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x\sqrt{}}{cx^2+}\right.}{2\,d}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x
^2) + 2*d)/x^2) + 2*(2*a*d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x), 1/2*((
b*c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d
)) + (2*a*d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x)]
```

3.942.6 Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx=\frac{a\sqrt{c}x}{\sqrt{1+\frac{d}{cx^2}}}-a\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)+\frac{ad}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}-\frac{b\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{bc\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2\sqrt{d}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2),x)
```

output
```
a*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - a*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x)) +
a*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2))) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x
) - b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*sqrt(d))
```

3.942.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx=\frac{1}{2}\left(2\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)a-\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}cx}{\left(c+\frac{d}{x^2}\right)x^2-d}-\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{\sqrt{d}}\right)b$$

input

```
integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output

```
1/2*(2*sqrt(c + d/x^2)*x + sqrt(d)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt
(c + d/x^2)*x + sqrt(d))))*a - 1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*x^2
 - d) - c*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))
/sqrt(d))*b
```

3.942.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx=\frac{2\sqrt{cx^2+d}ac\operatorname{sgn}(x)+\frac{\left(bc^2\operatorname{sgn}(x)+2\,acd\operatorname{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\frac{\sqrt{cx^2+d}bc\operatorname{sgn}(x)}{x^2}}{2\,c}$$

input

```
integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="giac")
```

output

```
1/2*(2*sqrt(c*x^2 + d)*a*c*sgn(x) + (b*c^2*sgn(x) + 2*a*c*d*sgn(x))*arctan
(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) - sqrt(c*x^2 + d)*b*c*sgn(x)/x^2)/c
```

3.942.9 Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx = a\,x\,\sqrt{c+\frac{d}{x^2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{2\,x}-\frac{b\,c\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{2\,\sqrt{d}} + \frac{a\,\sqrt{d}\,\mathrm{asin}\left(\frac{\sqrt{d}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c+\frac{d}{x^2}}\,1\mathrm{i}}{\sqrt{c}\,\sqrt{\frac{d}{c\,x^2}+1}}$$

input
```
int((a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
a*x*(c + d/x^2)^(1/2) - (b*(c + d/x^2)^(1/2))/(2*x) - (b*c*log((c + d/x^2)
^(1/2) + d^(1/2)/x))/(2*d^(1/2)) + (a*d^(1/2)*asin((d^(1/2)*1i)/(c^(1/2)*x
))*(c + d/x^2)^(1/2)*1i)/(c^(1/2)*(d/(c*x^2) + 1)^(1/2))
```

3.943 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$

3.943.1 Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx=\frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{c(bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{3/2}}$$

output
```
-1/4*b*(c+d/x^2)^(3/2)/d/x+1/8*c*(-4*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^
(1/2))/d^(3/2)+1/8*(-4*a*d+b*c)*(c+d/x^2)^(1/2)/d/x
```

3.943.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$$
$$=\frac{\sqrt{c+\frac{d}{x^2}}\left(-\sqrt{d}(2bd+bcx^2+4adx^2)+\frac{c(bc-4ad)x^4\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d+cx^2}}\right)}{8d^{3/2}x^3}$$

input
```
Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]
```

output
```
(Sqrt[c + d/x^2]*(-(Sqrt[d]*(2*b*d + b*c*x^2 + 4*a*d*x^2)) + (c*(b*c - 4*a
*d)*x^4*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/Sqrt[d + c*x^2]))/(8*d^(3/2)*x^3
)
```

3.943.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 858, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}dx$$

$$\Big\downarrow \text{959}$$

$$-\frac{(bc-4ad)\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2}dx}{4d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

$$\Big\downarrow \text{858}$$

$$\frac{(bc-4ad)\int \sqrt{c+\frac{d}{x^2}}d\frac{1}{x}}{4d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

$$\Big\downarrow \text{211}$$

$$\frac{(bc-4ad)\left(\frac{1}{2}c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)}{4d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

$$\Big\downarrow \text{224}$$

$$\frac{(bc-4ad)\left(\frac{1}{2}c\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)}{4d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

$$\Big\downarrow \text{219}$$

$$\frac{(bc-4ad)\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)}{4d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

---

input
```
Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]
```

output
```
-1/4*(b*(c + d/x^2)^(3/2))/(d*x) + ((b*c - 4*a*d)*(Sqrt[c + d/x^2]/(2*x) +
 (c*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*Sqrt[d])))/(4*d)
```

3.943.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.943. $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$

3.943.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\left(4 a d \,x^{2}+c b \,x^{2}+2 b d \right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}{8 x^{3} d}-\frac{c \left(4 a d -b c \right) \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{c \,x^{2}+d}}{x}\right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}{8 d^{\frac{3}{2}} \sqrt{c \,x^{2}+d}}$
default	$-\frac{\sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, \big(4 d^{\frac{3}{2}} \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{c \,x^{2}+d}}{x}\right) a c \,x^{4}-\sqrt{d}\, \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{c \,x^{2}+d}}{x}\right) b \,c^{2} x^{4}-4 \sqrt{c \,x^{2}+d}\, a c d \,x^{4}+\sqrt{c \,x^{2}+d}\, b \,c^{2} x^{4}+4 \left(c \,x^{2}+d \right)^{\frac{3}{2}} a d}{8 x^{3} \sqrt{c \,x^{2}+d}\, d^{2}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(4*a*d*x^2+b*c*x^2+2*b*d)/x^3/d*((c*x^2+d)/x^2)^(1/2)-1/8*c*(4*a*d-b*
c)/d^(3/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)*((c*x^2+d)/x^2)^(1/2)*x/(
c*x^2+d)^(1/2)
```

3.943.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx = \left[-\frac{(bc^2-4\,acd)\sqrt{d}x^3\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(2\,bd^2+(bcd+4\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16\,d^2x^3},\right.$$
$$\left.-\frac{(bc^2-4\,acd)\sqrt{-d}x^3\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+(2\,bd^2+(bcd+4\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8\,d^2x^3}\right]$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[-1/16*((b*c^2 - 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^
2 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2
+ d)/x^2))/(d^2*x^3), -1/8*((b*c^2 - 4*a*c*d)*sqrt(-d)*x^3*arctan(sqrt(-d)
*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*
sqrt((c*x^2 + d)/x^2))/(d^2*x^3)]
```

3.943.6 Sympy [A] (verification not implemented)

Time = 3.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.58

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx = -\frac{a\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2x} - \frac{ac\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2\sqrt{d}} - \frac{bc^{\frac{3}{2}}}{8dx\sqrt{1+\frac{d}{cx^2}}} - \frac{3b\sqrt{c}}{8x^3\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{3}{2}}} - \frac{bd}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**2,x)
```

output
```
-a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x) - a*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*
sqrt(d)) - b*c**(3/2)/(8*d*x*sqrt(1 + d/(c*x**2))) - 3*b*sqrt(c)/(8*x**3*s
qrt(1 + d/(c*x**2))) + b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(3/2)) - b*
d/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))
```

3.943.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(75) = 150$.

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.12

$$\begin{aligned}&\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx \\ &= -\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}cx}{\left(c+\frac{d}{x^2}\right)x^2-d} - \frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{\sqrt{d}}\right)a \\ &\quad - \frac{1}{16}\left(\frac{c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3+\sqrt{c+\frac{d}{x^2}}c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2dx^4-2\left(c+\frac{d}{x^2}\right)d^2x^2+d^3}\right)b\end{aligned}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*x^2 - d) - c*log((sqrt(c + d/x^2)
*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sqrt(d))*a - 1/16*(c^2*log((s
qrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2) + 2*((c
 + d/x^2)^(3/2)*c^2*x^3 + sqrt(c + d/x^2)*c^2*d*x)/((c + d/x^2)^2*d*x^4 -
2*(c + d/x^2)*d^2*x^2 + d^3))*b
```

3.943.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.43

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx = -\frac{\frac{\left(bc^3\operatorname{sgn}(x)-4\,ac^2d\operatorname{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d}+\frac{\left(cx^2+d\right)^{\frac{3}{2}}bc^3\operatorname{sgn}(x)+4\left(cx^2+d\right)^{\frac{3}{2}}ac^2d\operatorname{sgn}(x)+\sqrt{cx^2+d}bc^3d\operatorname{sgn}(x)-4\sqrt{cx^2+d}ac^2d^2\operatorname{sgn}(x)}{c^2dx^4}}{8\,c}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-1/8*((b*c^3*sgn(x) - 4*a*c^2*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(
sqrt(-d)*d) + ((c*x^2 + d)^(3/2)*b*c^3*sgn(x) + 4*(c*x^2 + d)^(3/2)*a*c^2*
d*sgn(x) + sqrt(c*x^2 + d)*b*c^3*d*sgn(x) - 4*sqrt(c*x^2 + d)*a*c^2*d^2*sg
n(x))/(c^2*d*x^4))/c
```

3.943.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^2, x)
```

3.944 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$

3.944.1 Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx = \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x} - \frac{c^2(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{16d^{5/2}}$$

output
```
-1/6*b*(c+d/x^2)^(3/2)/d/x^3-1/16*c^2*(-2*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/
x^2)^(1/2))/d^(5/2)+1/8*(-2*a*d+b*c)*(c+d/x^2)^(1/2)/d/x^3+1/16*c*(-2*a*d+
b*c)*(c+d/x^2)^(1/2)/d^2/x
```

3.944.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\,(-8bd^2-2bcdx^2-12ad^2x^2+3bc^2x^4-6acdx^4)}{48d^2x^5} - \frac{c^2(bc-2ad)\sqrt{c+\frac{d}{x^2}}\,x\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{16d^{5/2}\sqrt{d+cx^2}}$$

input
```
Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]
```

output
```
(Sqrt[c + d/x^2]*(-8*b*d^2 - 2*b*c*d*x^2 - 12*a*d^2*x^2 + 3*b*c^2*x^4 - 6*
a*c*d*x^4))/(48*d^2*x^5) - (c^2*(b*c - 2*a*d)*Sqrt[c + d/x^2]*x*ArcTanh[Sq
rt[d + c*x^2]/Sqrt[d]])/(16*d^(5/2)*Sqrt[d + c*x^2])
```

3.944.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {959, 858, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}dx$$

$$\Big\downarrow \text{ 959}$$

$$-\frac{(bc-2ad)\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^4}dx}{2d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{(bc-2ad)\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2}d\frac{1}{x}}{2d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{(bc-2ad)\left(\frac{1}{4}c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)}{2d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{(bc-2ad)\left(\frac{1}{4}c\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{2d}\right)+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)}{2d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{(bc-2ad)\left(\frac{1}{4}c\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{2d}\right)+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)}{2d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

$$\Big\downarrow 219$$

$$\frac{(bc-2ad)\left(\frac{1}{4}c\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}\right)+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)}{2d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

input

```
Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]
```

output

```
-1/6*(b*(c + d/x^2)^(3/2))/(d*x^3) + ((b*c - 2*a*d)*(Sqrt[c + d/x^2]/(4*x^
3) + (c*(Sqrt[c + d/x^2]/(2*d*x) - (c*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]
)/(2*d^(3/2))))/4))/(2*d)
```

3.944.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

3.944. $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.944.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\left(6acd\,x^{4}-3b\,c^{2}x^{4}+12a\,d^{2}x^{2}+2bcd\,x^{2}+8b\,d^{2}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{48x^{5}d^{2}}+\frac{c^{2}\left(2ad-bc\right)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{16d^{\frac{5}{2}}\sqrt{c\,x^{2}+d}}$
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(6d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)a\,c^{2}x^{6}-3\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)b\,c^{3}x^{6}-6\sqrt{c\,x^{2}+d}\,a\,c^{2}d\,x^{6}+3\sqrt{c\,x^{2}+d}\,b\,c^{3}x^{6}+6\left(c\,x^{2}+d\right)^{\frac{3}{2}}}{48x^{5}\sqrt{c\,x^{2}+d}\,d^{3}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(6*a*c*d*x^4-3*b*c^2*x^4+12*a*d^2*x^2+2*b*c*d*x^2+8*b*d^2)/x^5/d^2*(
(c*x^2+d)/x^2)^(1/2)+1/16*c^2*(2*a*d-b*c)/d^(5/2)*ln((2*d+2*d^(1/2)*(c*x^2
+d)^(1/2))/x)*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

3.944.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.98

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$$

$$= \left[-\frac{3\,(bc^3-2\,ac^2d)\sqrt{d}x^5\log\left(-\frac{cx^2+2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\,(3\,(bc^2d-2\,acd^2)x^4-8\,bd^3-2\,(bcd^2+6\,ad^3)x^2}{96\,d^3x^5}\right.$$

---

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[-1/96*(3*(b*c^3 - 2*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((
c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2
*(b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^5), 1/48*(3*(b*c^3
 - 2*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2
+ d)) + (3*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2*(b*c*d^2 + 6*a*d^3)*x^2
)*sqrt((c*x^2 + d)/x^2))/(d^3*x^5)]
```

3.944.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(107) = 214$.

Time = 7.14 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.84

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx = -\frac{ac^{\frac{3}{2}}}{8dx\sqrt{1+\frac{d}{cx^2}}} - \frac{3a\sqrt{c}}{8x^3\sqrt{1+\frac{d}{cx^2}}} + \frac{ac^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{3}{2}}} - \frac{ad}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^{\frac{5}{2}}}{16d^2x\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^{\frac{3}{2}}}{48dx^3\sqrt{1+\frac{d}{cx^2}}} - \frac{5b\sqrt{c}}{24x^5\sqrt{1+\frac{d}{cx^2}}} - \frac{bc^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{16d^{\frac{5}{2}}} - \frac{bd}{6\sqrt{c}x^7\sqrt{1+\frac{d}{cx^2}}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**4,x)
```

output
```
-a*c**(3/2)/(8*d*x*sqrt(1 + d/(c*x**2))) - 3*a*sqrt(c)/(8*x**3*sqrt(1 + d/
(c*x**2))) + a*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(3/2)) - a*d/(4*sqrt(
c)*x**5*sqrt(1 + d/(c*x**2))) + b*c**(5/2)/(16*d**2*x*sqrt(1 + d/(c*x**2))
) + b*c**(3/2)/(48*d*x**3*sqrt(1 + d/(c*x**2))) - 5*b*sqrt(c)/(24*x**5*sqr
t(1 + d/(c*x**2))) - b*c**3*asinh(sqrt(d)/(sqrt(c)*x))/(16*d**(5/2)) - b*d
/(6*sqrt(c)*x**7*sqrt(1 + d/(c*x**2)))
```

3.944.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(103) = 206$.

Time = 0.28 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.25

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$$
$$= -\frac{1}{16}\left(\frac{c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3+\sqrt{c+\frac{d}{x^2}}c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2dx^4-2\left(c+\frac{d}{x^2}\right)d^2x^2+d^3}\right)a$$
$$+\frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5-8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3-3\sqrt{c+\frac{d}{x^2}}c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3d^2x^6-3\left(c+\frac{d}{x^2}\right)^2d^3x^4+3\left(c+\frac{d}{x^2}\right)d^4x^2-d^5}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-1/16*(c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))
)/d^(3/2) + 2*((c + d/x^2)^(3/2)*c^2*x^3 + sqrt(c + d/x^2)*c^2*d*x)/((c +
d/x^2)^2*d*x^4 - 2*(c + d/x^2)*d^2*x^2 + d^3))*a + 1/96*(3*c^3*log((sqrt(c
 + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) + 2*(3*(c +
d/x^2)^(5/2)*c^3*x^5 - 8*(c + d/x^2)^(3/2)*c^3*d*x^3 - 3*sqrt(c + d/x^2)*c
^3*d^2*x)/((c + d/x^2)^3*d^2*x^6 - 3*(c + d/x^2)^2*d^3*x^4 + 3*(c + d/x^2)
*d^4*x^2 - d^5))*b
```

3.944.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.24

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$$
$$= \frac{\frac{3\left(bc^4\operatorname{sgn}(x)-2\,ac^3d\operatorname{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d^2}+\frac{3\left(cx^2+d\right)^{\frac{5}{2}}bc^4\operatorname{sgn}(x)-6\left(cx^2+d\right)^{\frac{5}{2}}ac^3d\operatorname{sgn}(x)-8\left(cx^2+d\right)^{\frac{3}{2}}bc^4d\operatorname{sgn}(x)-3\sqrt{cx^2+d}bc^4d^2\operatorname{sgn}}{c^3d^2x^6}}{48\,c}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/48*(3*(b*c^4*sgn(x) - 2*a*c^3*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))
/(sqrt(-d)*d^2) + (3*(c*x^2 + d)^(5/2)*b*c^4*sgn(x) - 6*(c*x^2 + d)^(5/2)*
a*c^3*d*sgn(x) - 8*(c*x^2 + d)^(3/2)*b*c^4*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*
c^4*d^2*sgn(x) + 6*sqrt(c*x^2 + d)*a*c^3*d^3*sgn(x))/(c^3*d^2*x^6))/c
```

3.944.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^4,x)
```

output
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^4, x)
```

3.945 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^5 \, dx$

3.945.1 Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^5 \, dx = \frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c} + \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^6}{6c} + \frac{d^2(6bc-ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}}$$

output
```
1/24*(-a*d+6*b*c)*(c+d/x^2)^(3/2)*x^4/c+1/6*a*(c+d/x^2)^(5/2)*x^6/c+1/16*d
^2*(-a*d+6*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(3/2)+1/16*d*(-a*d+6*b*
c)*x^2*(c+d/x^2)^(1/2)/c
```

3.945.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^5 \, dx = \frac{\sqrt{c+\frac{d}{x^2}}x\left(\sqrt{c}x\sqrt{d+cx^2}(6bc(5d+2cx^2)+a(3d^2+14cdx^2+8c^2x^4))+3d^2(-6bc+ad)\log\right.}{48c^{3/2}\sqrt{d+cx^2}}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]
```

output
```
(Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[d + c*x^2]*(6*b*c*(5*d + 2*c*x^2) + a*(
3*d^2 + 14*c*d*x^2 + 8*c^2*x^4)) + 3*d^2*(-6*b*c + a*d)*Log[-(Sqrt[c]*x) +
 Sqrt[d + c*x^2]]))/(48*c^(3/2)*Sqrt[d + c*x^2])
```

3.945.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^8d\frac{1}{x^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(6bc-ad)\int\left(c+\frac{d}{x^2}\right)^{3/2}x^6d\frac{1}{x^2}}{6c}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(6bc-ad)\left(\frac{3}{4}d\int\sqrt{c+\frac{d}{x^2}}x^4d\frac{1}{x^2}-\frac{1}{2}x^4\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{6c}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(6bc-ad)\left(\frac{3}{4}d\left(\frac{1}{2}d\int\frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}-x^2\sqrt{c+\frac{d}{x^2}}\right)-\frac{1}{2}x^4\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{6c}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(6bc-ad)\left(\frac{3}{4}d\left(\int\frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}-x^2\sqrt{c+\frac{d}{x^2}}\right)-\frac{1}{2}x^4\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{6c}\right)$$

$$\Big\downarrow {\scriptstyle 221}$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(6bc-ad)\left(\frac{3}{4}d\left(x^2\left(-\sqrt{c+\frac{d}{x^2}}\right)-\frac{d\,\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{1}{2}x^4\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{6c}\right)$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]
```

output
```
((a*(c + d/x^2)^(5/2)*x^6)/(3*c) - ((6*b*c - a*d)*(-1/2*((c + d/x^2)^(3/2)
*x^4) + (3*d*(-(Sqrt[c + d/x^2]*x^2) - (d*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]
)/Sqrt[c]))/4))/(6*c))/2
```

3.945.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.945.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

method	result
risch	$\frac{x^{2} \left(8 a \,x^{4} c^{2}+14 a c d \,x^{2}+12 b \,c^{2} x^{2}+3 a \,d^{2}+30 b c d \right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}{48 c}-\frac{d^{2} \left(a d -6 b c \right) \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+d}\right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}{16 c^{\frac{3}{2}} \sqrt{c \,x^{2}+d}}$
default	$\frac{\left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \Big(8 \left(c \,x^{2}+d \right)^{\frac{5}{2}} \sqrt{c}\, a x -2 \left(c \,x^{2}+d \right)^{\frac{3}{2}} \sqrt{c}\, a d x +12 \left(c \,x^{2}+d \right)^{\frac{3}{2}} c^{\frac{3}{2}} b x -3 \sqrt{c \,x^{2}+d}\, \sqrt{c}\, a \,d^{2} x +18 \sqrt{c \,x^{2}+d}\, c^{\frac{3}{2}} b d x -3 \ln \Big(\sqrt{c}\, x +\sqrt{c}}{48 \left(c \,x^{2}+d \right)^{\frac{3}{2}} c^{\frac{3}{2}}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x,method=_RETURNVERBOSE)
```

output
```
1/48/c*x^2*(8*a*c^2*x^4+14*a*c*d*x^2+12*b*c^2*x^2+3*a*d^2+30*b*c*d)*((c*x^
2+d)/x^2)^(1/2)-1/16*d^2*(a*d-6*b*c)/c^(3/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))
*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

3.945.5 Fricas [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.98

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^5\,dx = \left[-\frac{3\,(6\,bcd^2-ad^3)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,(8\,ac^3x^6+2\,(6\,bc^3+7\,ac^2d)}{96\,c^2}\right.$$

$$-\frac{3\,(6\,bcd^2-ad^3)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(8\,ac^3x^6+2\,(6\,bc^3+7\,ac^2d)x^4+3\,(10\,bc^2d+acd^2)x^2)\sqrt{\frac{cx^2}{x}}}{48\,c^2}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="fricas")
```

output
```
[-1/96*(3*(6*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c
*x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10
*b*c^2*d + a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^2, -1/48*(3*(6*b*c*d^2 -
 a*d^3)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) -
(8*a*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10*b*c^2*d + a*c*d^2)*x^2)
*sqrt((c*x^2 + d)/x^2))/c^2]
```

3.945.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(105) = 210.

Time = 33.53 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.06

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^5\,dx &= \frac{ac^2x^7}{6\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{11ac\sqrt{d}x^5}{24\sqrt{\frac{cx^2}{d}+1}}\\ &+\frac{17ad^{\frac{3}{2}}x^3}{48\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{5}{2}}x}{16c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{16c^{\frac{3}{2}}}+\frac{bc^2x^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}\\ &+\frac{3bc\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{bd^{\frac{3}{2}}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{bd^{\frac{3}{2}}x}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{3bd^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8\sqrt{c}}\end{aligned}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**5,x)
```

output
```
a*c**2*x**7/(6*sqrt(d)*sqrt(c*x**2/d + 1)) + 11*a*c*sqrt(d)*x**5/(24*sqrt(
c*x**2/d + 1)) + 17*a*d**(3/2)*x**3/(48*sqrt(c*x**2/d + 1)) + a*d**(5/2)*x
/(16*c*sqrt(c*x**2/d + 1)) - a*d**3*asinh(sqrt(c)*x/sqrt(d))/(16*c**(3/2))
 + b*c**2*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*b*c*sqrt(d)*x**3/(8*sqrt
(c*x**2/d + 1)) + b*d**(3/2)*x*sqrt(c*x**2/d + 1)/2 + b*d**(3/2)*x/(8*sqrt
(c*x**2/d + 1)) + 3*b*d**2*asinh(sqrt(c)*x/sqrt(d))/(8*sqrt(c))
```

3.945.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(103) = 206$.

Time = 0.30 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.95

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^5\,dx = \frac{1}{96}\left(\frac{3\,d^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3+8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}cd^3-3\sqrt{c+\frac{d}{x^2}}c^2d^3\right)}{\left(c+\frac{d}{x^2}\right)^3c-3\left(c+\frac{d}{x^2}\right)^2c^2+3\left(c+\frac{d}{x^2}\right)c^3-c^4}\right)a - \frac{1}{16}\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2-3\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2-2\left(c+\frac{d}{x^2}\right)c+c^2}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="maxima")
```

output
```
1/96*(3*d^3*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c
^(3/2) + 2*(3*(c + d/x^2)^(5/2)*d^3 + 8*(c + d/x^2)^(3/2)*c*d^3 - 3*sqrt(c
 + d/x^2)*c^2*d^3)/((c + d/x^2)^3*c - 3*(c + d/x^2)^2*c^2 + 3*(c + d/x^2)*
c^3 - c^4))*a - 1/16*(3*d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^
2) + sqrt(c)))/sqrt(c) - 2*(5*(c + d/x^2)^(3/2)*d^2 - 3*sqrt(c + d/x^2)*c*
d^2)/((c + d/x^2)^2 - 2*(c + d/x^2)*c + c^2))*b
```

3.945.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^5\,dx = \frac{1}{48}\Bigg(2\left(4\,acx^2\mathrm{sgn}(x)+\frac{6\,bc^5\mathrm{sgn}(x)+7\,ac^4d\mathrm{sgn}(x)}{c^4}\right)x^2+\frac{3\,(10\,bc^4d\mathrm{sgn}(x)+ac^3d^2\mathrm{sgn}(x))}{c^4} - \frac{(6\,bcd^2\mathrm{sgn}(x)-ad^3\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{16\,c^{\frac{3}{2}}} + \frac{(6\,bcd^2\log(|d|)-ad^3\log(|d|))\mathrm{sgn}(x)}{32\,c^{\frac{3}{2}}}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="giac")
```

output
```
1/48*(2*(4*a*c*x^2*sgn(x) + (6*b*c^5*sgn(x) + 7*a*c^4*d*sgn(x))/c^4)*x^2 +
 3*(10*b*c^4*d*sgn(x) + a*c^3*d^2*sgn(x))/c^4)*sqrt(c*x^2 + d)*x - 1/16*(6
*b*c*d^2*sgn(x) - a*d^3*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(
3/2) + 1/32*(6*b*c*d^2*log(abs(d)) - a*d^3*log(abs(d)))*sgn(x)/c^(3/2)
```

3.945.9 Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^5\,dx = \frac{a\,x^6\,\left(c+\frac{d}{x^2}\right)^{3/2}}{6}+\frac{5\,b\,x^4\,\left(c+\frac{d}{x^2}\right)^{3/2}}{8}+\frac{a\,x^6\,\left(c+\frac{d}{x^2}\right)^{5/2}}{16\,c} + \frac{3\,b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,\sqrt{c}}-\frac{a\,c\,x^6\,\sqrt{c+\frac{d}{x^2}}}{16}-\frac{3\,b\,c\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{a\,d^3\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}}{16\,c^{3/2}}$$

input
```
int(x^5*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
(a*x^6*(c + d/x^2)^(3/2))/6 + (5*b*x^4*(c + d/x^2)^(3/2))/8 + (a*x^6*(c +
d/x^2)^(5/2))/(16*c) + (a*d^3*atan(((c + d/x^2)^(1/2)*1i)/c^(1/2))*1i)/(16
*c^(3/2)) + (3*b*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(1/2)) - (a*c*
x^6*(c + d/x^2)^(1/2))/16 - (3*b*c*x^4*(c + d/x^2)^(1/2))/8
```

3.946 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^3 \, dx$

3.946.1 Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^3 \, dx = -\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c} + \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^4}{4c} + \frac{3d(4bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}}$$

output
```
1/8*(a*d+4*b*c)*(c+d/x^2)^(3/2)*x^2/c+1/4*a*(c+d/x^2)^(5/2)*x^4/c+3/8*d*(a
*d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(1/2)-3/8*d*(a*d+4*b*c)*(c+d/
x^2)^(1/2)/c
```

3.946.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^3 \, dx = \frac{1}{8}\sqrt{c+\frac{d}{x^2}}\left(-8bd+4bcx^2 + 5adx^2 + 2acx^4 + \frac{6d(4bc+ad)x\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{d}+\sqrt{d+cx^2}}\right)}{\sqrt{c}\sqrt{d+cx^2}}\right)$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]
```

output
```
(Sqrt[c + d/x^2]*(-8*b*d + 4*b*c*x^2 + 5*a*d*x^2 + 2*a*c*x^4 + (6*d*(4*b*c
 + a*d)*x*ArcTanh[(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[d + c*x^2])])/(Sqrt[c]*Sqrt
[d + c*x^2])))/8
```

3.946.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2} \int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 d\frac{1}{x^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2} \left(\frac{a x^4 \left(c+\frac{d}{x^2}\right)^{5/2}}{2 c}-\frac{(a d+4 b c) \int \left(c+\frac{d}{x^2}\right)^{3/2} x^4 d\frac{1}{x^2}}{4 c}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2} \left(\frac{a x^4 \left(c+\frac{d}{x^2}\right)^{5/2}}{2 c}-\frac{(a d+4 b c) \left(\frac{3}{2} d \int \sqrt{c+\frac{d}{x^2}} x^2 d\frac{1}{x^2}-x^2 \left(c+\frac{d}{x^2}\right)^{3/2}\right)}{4 c}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2} \left(\frac{a x^4 \left(c+\frac{d}{x^2}\right)^{5/2}}{2 c}-\frac{(a d+4 b c) \left(\frac{3}{2} d \left(c \int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}} d\frac{1}{x^2}+2 \sqrt{c+\frac{d}{x^2}}\right)-x^2 \left(c+\frac{d}{x^2}\right)^{3/2}\right)}{4 c}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{ax^4\left(c+\frac{d}{x^2}\right)^{5/2}}{2c}-\frac{(ad+4bc)\left(\frac{3}{2}d\left(\frac{2c\int\frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{d}+2\sqrt{c+\frac{d}{x^2}}\right)-x^2\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{4c}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{ax^4\left(c+\frac{d}{x^2}\right)^{5/2}}{2c}-\frac{(ad+4bc)\left(\frac{3}{2}d\left(2\sqrt{c+\frac{d}{x^2}}-2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)\right)-x^2\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{4c}\right)$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]
```

output
```
((a*(c + d/x^2)^(5/2)*x^4)/(2*c) - ((4*b*c + a*d)*(-((c + d/x^2)^(3/2)*x^2
) + (3*d*(2*Sqrt[c + d/x^2] - 2*Sqrt[c]*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]))
/2))/(4*c))/2
```

3.946.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

---

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.946.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\left(2a\,x^{4}c+5ad\,x^{2}+4cb\,x^{2}-8bd\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{8}+\frac{3d\left(ad+4bc\right)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{8\sqrt{c}\,\sqrt{c\,x^{2}+d}}$
default	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}x^{2}\Big(-2\left(c\,x^{2}+d\right)^{\frac{3}{2}}\sqrt{c}\,ad\,x^{2}-8\left(c\,x^{2}+d\right)^{\frac{3}{2}}c^{\frac{3}{2}}b\,x^{2}+8\left(c\,x^{2}+d\right)^{\frac{5}{2}}\sqrt{c}\,b-3\sqrt{c\,x^{2}+d}\,\sqrt{c}\,a\,d^{2}x^{2}-12\sqrt{c\,x^{2}+d}\,c^{\frac{3}{2}}bd\,x^{2}-3\ln\Big(\sqrt{}}{8\left(c\,x^{2}+d\right)^{\frac{3}{2}}d\sqrt{c}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*a*c*x^4+5*a*d*x^2+4*b*c*x^2-8*b*d)*((c*x^2+d)/x^2)^(1/2)+3/8*d*(a*d
+4*b*c)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))/c^(1/2)*((c*x^2+d)/x^2)^(1/2)*x/(c*x
^2+d)^(1/2)
```

3.946.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.77

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx = \left[\frac{3\,(4\,bcd+ad^2)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\,(2\,ac^2x^4-8\,bcd+(4\,bc^2+5\,ac}{16\,c}\right.$$

$$\left.-\frac{3\,(4\,bcd+ad^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(2\,ac^2x^4-8\,bcd+(4\,bc^2+5\,acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8\,c}\right]$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="fricas")
```

output
```
[1/16*(3*(4*b*c*d + a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^
2 + d)/x^2) - d) + 2*(2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2 + 5*a*c*d)*x^2)*sqr
t((c*x^2 + d)/x^2))/c, -1/8*(3*(4*b*c*d + a*d^2)*sqrt(-c)*arctan(sqrt(-c)*
x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2
 + 5*a*c*d)*x^2)*sqrt((c*x^2 + d)/x^2))/c]
```

3.946.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(104) = 208$.

Time = 55.22 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.88

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx = \frac{ac^2x^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3ac\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{3}{2}}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{ad^{\frac{3}{2}}x}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8\sqrt{c}}+\frac{3b\sqrt{c}d\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2}+\frac{bc\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2}-\frac{bc\sqrt{d}x}{\sqrt{\frac{cx^2}{d}+1}}-\frac{bd^{\frac{3}{2}}}{x\sqrt{\frac{cx^2}{d}+1}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**3,x)
```

output
```
a*c**2*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*a*c*sqrt(d)*x**3/(8*sqrt(c*
x**2/d + 1)) + a*d**(3/2)*x*sqrt(c*x**2/d + 1)/2 + a*d**(3/2)*x/(8*sqrt(c*
x**2/d + 1)) + 3*a*d**2*asinh(sqrt(c)*x/sqrt(d))/(8*sqrt(c)) + 3*b*sqrt(c)
*d*asinh(sqrt(c)*x/sqrt(d))/2 + b*c*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 - b*c*s
qrt(d)*x/sqrt(c*x**2/d + 1) - b*d**(3/2)/(x*sqrt(c*x**2/d + 1))
```

3.946.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.49

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx = -\frac{1}{16}\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2-3\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2-2\left(c+\frac{d}{x^2}\right)c+c^2}\right)a + \frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}cx^2-3\sqrt{c}d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)-4\sqrt{c+\frac{d}{x^2}}d\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="maxima")
```

output
```
-1/16*(3*d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/
sqrt(c) - 2*(5*(c + d/x^2)^(3/2)*d^2 - 3*sqrt(c + d/x^2)*c*d^2)/((c + d/x^
2)^2 - 2*(c + d/x^2)*c + c^2))*a + 1/4*(2*sqrt(c + d/x^2)*c*x^2 - 3*sqrt(c
)*d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))) - 4*sqrt(
c + d/x^2)*d)*b
```

3.946.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx=\frac{2\,b\sqrt{c}d^2\mathrm{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d}+\frac{1}{8}\left(2\,acx^2\mathrm{sgn}(x)+\frac{4\,bc^3\mathrm{sgn}(x)+5\,ac^2d\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+d}x-\frac{3\,(4\,bcd\mathrm{sgn}(x)+ad^2\mathrm{sgn}(x))\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)}{16\,\sqrt{c}}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="giac")
```

output
```
2*b*sqrt(c)*d^2*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d) + 1/8*(2*a*c*
x^2*sgn(x) + (4*b*c^3*sgn(x) + 5*a*c^2*d*sgn(x))/c^2)*sqrt(c*x^2 + d)*x -
3/16*(4*b*c*d*sgn(x) + a*d^2*sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)/
sqrt(c)
```

3.946.9 Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx=\frac{5\,a\,x^4\,\left(c+\frac{d}{x^2}\right)^{3/2}}{8}-b\,d\,\sqrt{c+\frac{d}{x^2}}+\frac{3\,b\,\sqrt{c}\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2}+\frac{3\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,\sqrt{c}}-\frac{3\,a\,c\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{b\,c\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}$$

input
```
int(x^3*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
(5*a*x^4*(c + d/x^2)^(3/2))/8 - b*d*(c + d/x^2)^(1/2) + (3*b*c^(1/2)*d*ata
nh((c + d/x^2)^(1/2)/c^(1/2)))/2 + (3*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2
)))/(8*c^(1/2)) - (3*a*c*x^4*(c + d/x^2)^(1/2))/8 + (b*c*x^2*(c + d/x^2)^(
1/2))/2
```

3.947 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x \, dx$

3.947.1 Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x \, dx = -\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}} - \frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c} + \frac{1}{2}\sqrt{c}(2bc+3ad)\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

output
```
-1/6*(3*a*d+2*b*c)*(c+d/x^2)^(3/2)/c+1/2*a*(c+d/x^2)^(5/2)*x^2/c+1/2*(3*a*
d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))*c^(1/2)-1/2*(3*a*d+2*b*c)*(c+d/x
^2)^(1/2)
```

3.947.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x \, dx = \frac{\sqrt{c+\frac{d}{x^2}}\left(-6adx^2+3acx^4-2b(d+4cx^2)+\frac{6\sqrt{c}(2bc+3ad)x^3\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{d}+\sqrt{d+cx^2}}\right)}{\sqrt{d+cx^2}}\right)}{6x^2}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]
```

output
```
(Sqrt[c + d/x^2]*(-6*a*d*x^2 + 3*a*c*x^4 - 2*b*(d + 4*c*x^2) + (6*Sqrt[c]*
(2*b*c + 3*a*d)*x^3*ArcTanh[(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[d + c*x^2])])/Sqr
t[d + c*x^2]))/(6*x^2)
```

3.947.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {948, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^4d\frac{1}{x^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(3ad+2bc)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^2d\frac{1}{x^2}}{2c}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(3ad+2bc)\left(c\int \sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x^2}+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{2c}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(3ad+2bc)\left(c\left(c\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}+2\sqrt{c+\frac{d}{x^2}}\right)+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{2c}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(3ad+2bc)\left(c\left(\frac{2c\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{d}+2\sqrt{c+\frac{d}{x^2}}\right)+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{2c}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(3ad+2bc)\left(c\left(2\sqrt{c+\frac{d}{x^2}}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)\right)+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{2c}\right)$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]
```

output
```
((a*(c + d/x^2)^(5/2)*x^2)/c - ((2*b*c + 3*a*d)*((2*(c + d/x^2)^(3/2))/3 +
 c*(2*Sqrt[c + d/x^2] - 2*Sqrt[c]*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])))/(2*c
))/2
```

3.947.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.947. $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x\,dx$

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.947.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90

method	result
risch	$\frac{\left(3 a\,x^{4} c-6 a d\,x^{2}-8 c b\,x^{2}-2 b d\right) \sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{6 x^{2}}+\frac{\left(3 a d+2 b c\right) \sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right) \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{2 \sqrt{c\,x^{2}+d}}$
default	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(-6 \left(c\,x^{2}+d\right)^{\frac{3}{2}} c^{\frac{3}{2}} a d\,x^{4}-4 \left(c\,x^{2}+d\right)^{\frac{3}{2}} c^{\frac{5}{2}} b\,x^{4}+6 \left(c\,x^{2}+d\right)^{\frac{5}{2}} \sqrt{c}\,a d\,x^{2}+4 \left(c\,x^{2}+d\right)^{\frac{5}{2}} c^{\frac{3}{2}} b\,x^{2}-9 \sqrt{c\,x^{2}+d}\,c^{\frac{3}{2}} a\,d^{2} x^{4}-6 \sqrt{c\,x^{2}}}{6 \left(c\,x^{2}+d\right)^{\frac{3}{2}} d^{2} \sqrt{c}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/6*(3*a*c*x^4-6*a*d*x^2-8*b*c*x^2-2*b*d)/x^2*((c*x^2+d)/x^2)^(1/2)+1/2*(3
*a*d+2*b*c)*c^(1/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*((c*x^2+d)/x^2)^(1/2)*x/
(c*x^2+d)^(1/2)
```

3.947.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.77

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = \left[\frac{3\,(2\,bc+3\,ad)\sqrt{c}x^2\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\,(3\,acx^4-2\,(4\,bc+3\,ad)x^2-2}{12\,x^2}\right.$$

$$\left.-\frac{3\,(2\,bc+3\,ad)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(3\,acx^4-2\,(4\,bc+3\,ad)x^2-2\,bd)\sqrt{\frac{cx^2+d}{x^2}}}{6\,x^2}\right]$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="fricas")
```

output
```
[1/12*(3*(2*b*c + 3*a*d)*sqrt(c)*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*
x^2 + d)/x^2) - d) + 2*(3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^2 - 2*b*d)*sqrt((c
*x^2 + d)/x^2))/x^2, -1/6*(3*(2*b*c + 3*a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)*
x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^
2 - 2*b*d)*sqrt((c*x^2 + d)/x^2))/x^2]
```

3.947.6 Sympy [A] (verification not implemented)

Time = 16.51 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.70

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = \frac{3a\sqrt{c}d\,\mathrm{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2} + \frac{ac\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2} - \frac{ac\sqrt{d}x}{\sqrt{\frac{cx^2}{d}+1}} - \frac{ad^{\frac{3}{2}}}{x\sqrt{\frac{cx^2}{d}+1}} + bc^{\frac{3}{2}}\,\mathrm{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) - \frac{bc^2x}{\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} - \frac{bc\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}} + bd\left(\begin{cases} -\frac{\sqrt{c}}{2x^2} & \text{for } d=0 \\ -\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x,x)
```

output
```
3*a*sqrt(c)*d*asinh(sqrt(c)*x/sqrt(d))/2 + a*c*sqrt(d)*x*sqrt(c*x**2/d + 1
)/2 - a*c*sqrt(d)*x/sqrt(c*x**2/d + 1) - a*d**(3/2)/(x*sqrt(c*x**2/d + 1))
 + b*c**(3/2)*asinh(sqrt(c)*x/sqrt(d)) - b*c**2*x/(sqrt(d)*sqrt(c*x**2/d +
 1)) - b*c*sqrt(d)/(x*sqrt(c*x**2/d + 1)) + b*d*Piecewise((-sqrt(c)/(2*x**
2), Eq(d, 0)), (-(c + d/x**2)**(3/2)/(3*d), True))
```

3.947.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = \frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}cx^2 - 3\sqrt{c}d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right) - 4\sqrt{c+\frac{d}{x^2}}d\right)a - \frac{1}{6}\left(3\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right) + 2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}} + 6\sqrt{c+\frac{d}{x^2}}c\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(c + d/x^2)*c*x^2 - 3*sqrt(c)*d*log((sqrt(c + d/x^2) - sqrt(c))
/(sqrt(c + d/x^2) + sqrt(c))) - 4*sqrt(c + d/x^2)*d)*a - 1/6*(3*c^(3/2)*lo
g((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))) + 2*(c + d/x^2)
^(3/2) + 6*sqrt(c + d/x^2)*c)*b
```

3.947.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(90) = 180$.

Time = 0.48 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.05

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = \frac{1}{2}\sqrt{cx^2+d}acx\,\mathrm{sgn}(x) - \frac{1}{4}\left(2\,bc^{\frac{3}{2}}\mathrm{sgn}(x) + 3\,a\sqrt{c}d\,\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right) + \frac{2\left(6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4 bc^{\frac{3}{2}}d\,\mathrm{sgn}(x) + 3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4 a\sqrt{c}d^2\mathrm{sgn}(x) - 6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2 bc^{\frac{3}{2}}d^2\mathrm{sg}\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^2 + d)*a*c*x*sgn(x) - 1/4*(2*b*c^(3/2)*sgn(x) + 3*a*sqrt(c)*d
*sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + d))^2) + 2/3*(6*(sqrt(c)*x - sqrt(c
*x^2 + d))^4*b*c^(3/2)*d*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*sqrt
(c)*d^2*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(3/2)*d^2*sgn(x) -
6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d^3*sgn(x) + 4*b*c^(3/2)*d^3*s
gn(x) + 3*a*sqrt(c)*d^4*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^3
```

3.947.9 Mupad [B] (verification not implemented)

Time = 10.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = b\,c^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3} - a\,d\sqrt{c+\frac{d}{x^2}}-b\,c\sqrt{c+\frac{d}{x^2}}+\frac{3\,a\,\sqrt{c}\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2}+\frac{a\,c\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}$$

input
```
int(x*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
b*c^(3/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - (b*(c + d/x^2)^(3/2))/3 - a*d
*(c + d/x^2)^(1/2) - b*c*(c + d/x^2)^(1/2) + (3*a*c^(1/2)*d*atanh((c + d/x
^2)^(1/2)/c^(1/2)))/2 + (a*c*x^2*(c + d/x^2)^(1/2))/2
```

3.948 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx$

3.948.1 Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = -ac\sqrt{c+\frac{d}{x^2}} - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} + ac^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

output
```
-1/3*a*(c+d/x^2)^(3/2)-1/5*b*(c+d/x^2)^(5/2)/d+a*c^(3/2)*arctanh((c+d/x^2)
^(1/2)/c^(1/2))-a*c*(c+d/x^2)^(1/2)
```

3.948.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\left(-\frac{3b(d+cx^2)^2}{d} - 5ax^2(d+4cx^2) - \frac{15ac^{3/2}x^5\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)}{\sqrt{d+cx^2}}\right)}{15x^4}$$

input
```
Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]
```

output
```
(Sqrt[c + d/x^2]*((-3*b*(d + c*x^2)^2)/d - 5*a*x^2*(d + 4*c*x^2) - (15*a*c
^(3/2)*x^5*Log[-(Sqrt[c]*x) + Sqrt[d + c*x^2]])/Sqrt[d + c*x^2]))/(15*x^4)
```

3.948.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {948, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(-a\int \left(c+\frac{d}{x^2}\right)^{3/2}x^2d\frac{1}{x^2}-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-a\left(c\int \sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x^2}+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-a\left(c\left(c\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}+2\sqrt{c+\frac{d}{x^2}}\right)+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-a\left(c\left(\frac{2c\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{d}+2\sqrt{c+\frac{d}{x^2}}\right)+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-a\left(c\left(2\sqrt{c+\frac{d}{x^2}}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)\right)+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}\right)$$

input

```
Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]
```

output
```
((-2*b*(c + d/x^2)^(5/2))/(5*d) - a*((2*(c + d/x^2)^(3/2))/3 + c*(2*Sqrt[c
 + d/x^2] - 2*Sqrt[c]*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])))/2
```

3.948.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.948. $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx$

3.948.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.42

method	result
risch	$-\frac{(20acd\,x^4+3b\,c^2x^4+5a\,d^2x^2+6bcd\,x^2+3b\,d^2)\sqrt{\frac{c\,x^2+d}{x^2}}}{15x^4d}+\frac{c^{\frac{3}{2}}a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{\frac{c\,x^2+d}{x^2}}\,x}{\sqrt{c\,x^2+d}}$
default	$-\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\big(-10(c\,x^2+d)^{\frac{3}{2}}c^{\frac{5}{2}}a\,x^6+10(c\,x^2+d)^{\frac{5}{2}}c^{\frac{3}{2}}a\,x^4-15\sqrt{c\,x^2+d}\,c^{\frac{5}{2}}ad\,x^6-15\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)a\,c^2d^2x^5+5(c\,x^2+d)^{\frac{5}{2}}\sqrt{c}}{15x^2(c\,x^2+d)^{\frac{3}{2}}d^2\sqrt{c}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(20*a*c*d*x^4+3*b*c^2*x^4+5*a*d^2*x^2+6*b*c*d*x^2+3*b*d^2)/x^4/d*((c
*x^2+d)/x^2)^(1/2)+c^(3/2)*a*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*((c*x^2+d)/x^2)
^(1/2)*x/(c*x^2+d)^(1/2)
```

3.948.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.80

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = \left[\frac{15\,ac^{\frac{3}{2}}dx^4\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,((3\,bc^2+20\,acd)x^4+3\,bd^2}{30\,dx^4}\right.$$

$$\left.-\frac{15\,a\sqrt{-c}cdx^4\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+((3\,bc^2+20\,acd)x^4+3\,bd^2+(6\,bcd+5\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{15\,dx^4}\right]$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/30*(15*a*c^(3/2)*d*x^4*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^
2) - d) - 2*((3*b*c^2 + 20*a*c*d)*x^4 + 3*b*d^2 + (6*b*c*d + 5*a*d^2)*x^2)
*sqrt((c*x^2 + d)/x^2))/(d*x^4), -1/15*(15*a*sqrt(-c)*c*d*x^4*arctan(sqrt(
-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + ((3*b*c^2 + 20*a*c*d)*x^4 + 3
*b*d^2 + (6*b*c*d + 5*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^4)]
```

3.948.6 Sympy [A] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.50

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx=\frac{\begin{cases} -\frac{2ac^2\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}}-2ac\sqrt{c+\frac{d}{x^2}}-\frac{2a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5d} & \text{for } d\neq 0\\ -ac^{\frac{3}{2}}\log\left(-\frac{bc^{\frac{3}{2}}}{x^2}\right)-\frac{bc^{\frac{3}{2}}}{x^2} & \text{otherwise}\end{cases}}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x,x)
```

output
```
Piecewise((-2*a*c**2*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c) - 2*a*c*sqrt
(c + d/x**2) - 2*a*(c + d/x**2)**(3/2)/3 - 2*b*(c + d/x**2)**(5/2)/(5*d),
Ne(d, 0)), (-a*c**(3/2)*log(-b*c**(3/2)/x**2) - b*c**(3/2)/x**2, True))/2
```

3.948.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx=-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5\,d}-\frac{1}{6}\left(3\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}+6\sqrt{c+\frac{d}{x^2}}c\right)a$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
-1/5*b*(c + d/x^2)^(5/2)/d - 1/6*(3*c^(3/2)*log((sqrt(c + d/x^2) - sqrt(c)
)/(sqrt(c + d/x^2) + sqrt(c))) + 2*(c + d/x^2)^(3/2) + 6*sqrt(c + d/x^2)*c
)*a
```

3.948.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(60) = 120$.

Time = 0.84 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.34

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = -\frac{1}{2}\,ac^{\frac{3}{2}}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)\operatorname{sgn}(x)$$

$$+\frac{2\Big(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8 bc^{\frac{5}{2}}\operatorname{sgn}(x)+30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8 ac^{\frac{3}{2}}d\operatorname{sgn}(x)-90\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6 ac^{\frac{3}{2}}d^2\mathrm{sg}}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="giac")
```

output
```
-1/2*a*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)*sgn(x) + 2/15*(15*(sqr
t(c)*x - sqrt(c*x^2 + d))^8*b*c^(5/2)*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2
+ d))^8*a*c^(3/2)*d*sgn(x) - 90*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(3/2)*
d^2*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*d^2*sgn(x) + 110
*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^3*sgn(x) - 70*(sqrt(c)*x - sq
rt(c*x^2 + d))^2*a*c^(3/2)*d^4*sgn(x) + 3*b*c^(5/2)*d^4*sgn(x) + 20*a*c^(3
/2)*d^5*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^5
```

3.948.9 Mupad [B] (verification not implemented)

Time = 10.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = a\,c^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

$$-\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}}{3}-a\,c\,\sqrt{c+\frac{d}{x^2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}\,\left(c\,x^2+d\right)^2}{5\,d\,x^4}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x,x)
```

output
```
a*c^(3/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - (a*(c + d/x^2)^(3/2))/3 - a*c
*(c + d/x^2)^(1/2) - (b*(c + d/x^2)^(1/2)*(d + c*x^2)^2)/(5*d*x^4)
```

3.949 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx$

3.949.1 Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx=\frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}$$

output
```
1/5*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^2-1/7*b*(c+d/x^2)^(7/2)/d^2
```

3.949.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx=-\frac{\sqrt{c+\frac{d}{x^2}}\,(d+cx^2)^2\,(5bd-2bcx^2+7adx^2)}{35d^2x^6}$$

input
```
Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]
```

output
```
-1/35*(Sqrt[c + d/x^2]*(d + c*x^2)^2*(5*b*d - 2*b*c*x^2 + 7*a*d*x^2))/(d^2
*x^6)
```

3.949.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{946}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{d}+\frac{(ad-bc)\left(c+\frac{d}{x^2}\right)^{3/2}}{d}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^2}-\frac{2b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}\right)$$

input
```
Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]
```

output
```
((2*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^2) - (2*b*(c + d/x^2)^(7/2))/(7*d^
2))/2
```

3.949.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.949.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result	size
gosper	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(7ad\,x^{2}-2cb\,x^{2}+5bd\right) \left(c\,x^{2}+d\right)}{35d^{2} x^{4}}$	48
default	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(7ad\,x^{2}-2cb\,x^{2}+5bd\right) \left(c\,x^{2}+d\right)}{35d^{2} x^{4}}$	48
risch	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, \left(7a\,c^{2} d\,x^{6}-2b\,c^{3} x^{6}+14ac\,d^{2} x^{4}+b\,c^{2} d\,x^{4}+7a\,d^{3} x^{2}+8bc\,d^{2} x^{2}+5b\,d^{3}\right)}{35x^{6} d^{2}}$	86
trager	$-\frac{\left(7a\,c^{2} d\,x^{6}-2b\,c^{3} x^{6}+14ac\,d^{2} x^{4}+b\,c^{2} d\,x^{4}+7a\,d^{3} x^{2}+8bc\,d^{2} x^{2}+5b\,d^{3}\right) \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{35x^{6} d^{2}}$	90

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/35*((c*x^2+d)/x^2)^(3/2)*(7*a*d*x^2-2*b*c*x^2+5*b*d)*(c*x^2+d)/d^2/x^4
```

3.949.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(38) = 76$.

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx = \frac{\left((2\,bc^3-7\,ac^2d)x^6-(bc^2d+14\,acd^2)x^4-5\,bd^3-(8\,bcd^2+7\,ad^3)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{35\,d^2x^6}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")
```

---

output
```
1/35*((2*b*c^3 - 7*a*c^2*d)*x^6 - (b*c^2*d + 14*a*c*d^2)*x^4 - 5*b*d^3 - (
8*b*c*d^2 + 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^2*x^6)
```

3.949.6 Sympy [A] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.11

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx = -\frac{ac\left(\begin{cases}\frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{x^2} & \text{otherwise}\end{cases}\right)}{2} - \frac{ad\left(\begin{cases}\frac{2\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{2x^4} & \text{otherwise}\end{cases}\right)}{2} - \frac{bc\left(\begin{cases}\frac{2\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{2x^4} & \text{otherwise}\end{cases}\right)}{2} - \frac{bd\left(\begin{cases}\frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{3x^6} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**3,x)
```

output
```
-a*c*Piecewise((2*(c + d/x**2)**(3/2)/(3*d), Ne(d, 0)), (sqrt(c)/x**2, Tru
e))/2 - a*d*Piecewise((2*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5
)/d**2, Ne(d, 0)), (sqrt(c)/(2*x**4), True))/2 - b*c*Piecewise((2*(-c*(c +
 d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2, Ne(d, 0)), (sqrt(c)/(2*x*
*4), True))/2 - b*d*Piecewise((2*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/
x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3, Ne(d, 0)), (sqrt(c)/(3*x**6)
, True))/2
```

3.949.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx = -\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5\,d} - \frac{1}{35}\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^2} - \frac{7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^2}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-1/5*a*(c + d/x^2)^(5/2)/d - 1/35*(5*(c + d/x^2)^(7/2)/d^2 - 7*(c + d/x^2)
^(5/2)*c/d^2)*b
```

3.949.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(38) = 76.

Time = 0.94 (sec) , antiderivative size = 370, normalized size of antiderivative = 8.04

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx = \frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{5}{2}}\operatorname{sgn}(x)+70\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{7}{2}}\operatorname{sgn}(x)-70}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(5/2)*sgn(x) + 70*(sqrt(c)*x
 - sqrt(c*x^2 + d))^10*b*c^(7/2)*sgn(x) - 70*(sqrt(c)*x - sqrt(c*x^2 + d))
^10*a*c^(5/2)*d*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(7/2)*d*sg
n(x) + 105*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(5/2)*d^2*sgn(x) + 140*(sqr
t(c)*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d^2*sgn(x) - 140*(sqrt(c)*x - sqrt(c
*x^2 + d))^6*a*c^(5/2)*d^3*sgn(x) + 28*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c
^(7/2)*d^3*sgn(x) + 77*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(5/2)*d^4*sgn(x
) + 14*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7/2)*d^4*sgn(x) - 14*(sqrt(c)*
x - sqrt(c*x^2 + d))^2*a*c^(5/2)*d^5*sgn(x) - 2*b*c^(7/2)*d^5*sgn(x) + 7*a
*c^(5/2)*d^6*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^7
```

3.949.9 Mupad [B] (verification not implemented)

Time = 10.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.65

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx = \frac{2\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{35\,d^2}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{5\,d}-\frac{2\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{5\,x^2}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4}-\frac{8\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,x^4}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^2}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x)
```

output
```
(2*b*c^3*(c + d/x^2)^(1/2))/(35*d^2) - (a*c^2*(c + d/x^2)^(1/2))/(5*d) - (
2*a*c*(c + d/x^2)^(1/2))/(5*x^2) - (a*d*(c + d/x^2)^(1/2))/(5*x^4) - (8*b*
c*(c + d/x^2)^(1/2))/(35*x^4) - (b*d*(c + d/x^2)^(1/2))/(7*x^6) - (b*c^2*(
c + d/x^2)^(1/2))/(35*d*x^2)
```

3.949. $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx$

3.950 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx$

3.950.1 Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx=-\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}$$

output
```
-1/5*c*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^3+1/7*(-a*d+2*b*c)*(c+d/x^2)^(7/2)/d^3
-1/9*b*(c+d/x^2)^(9/2)/d^3
```

3.950.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)^2\left(9adx^2(-5d+2cx^2)+b(-35d^2+20cdx^2-8c^2x^4)\right)}{315d^3x^8}$$

input
```
Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]
```

output
```
(Sqrt[c + d/x^2]*(d + c*x^2)^2*(9*a*d*x^2*(-5*d + 2*c*x^2) + b*(-35*d^2 +
20*c*d*x^2 - 8*c^2*x^4)))/(315*d^3*x^8)
```

3.950.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \ 948$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}d\frac{1}{x^2}$$

$$\Big\downarrow \ 86$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{d^2}+\frac{(ad-2bc)\left(c+\frac{d}{x^2}\right)^{5/2}}{d^2}+\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^2}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{2\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^3}-\frac{2b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}\right)$$

input
```
Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]
```

output
```
((-2*c*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^3) + (2*(2*b*c - a*d)*(c + d/x^
2)^(7/2))/(7*d^3) - (2*b*(c + d/x^2)^(9/2))/(9*d^3))/2
```

3.950.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.950.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(18acd\,x^{4}-8b\,c^{2} x^{4}-45a\,d^{2} x^{2}+20bcd\,x^{2}-35b\,d^{2}\right) \left(c\,x^{2}+d\right)}{315d^{3} x^{6}}$	70
default	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(18acd\,x^{4}-8b\,c^{2} x^{4}-45a\,d^{2} x^{2}+20bcd\,x^{2}-35b\,d^{2}\right) \left(c\,x^{2}+d\right)}{315d^{3} x^{6}}$	70
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, \left(18a\,c^{3} d\,x^{8}-8b\,c^{4} x^{8}-9a\,c^{2} d^{2} x^{6}+4b\,c^{3} d\,x^{6}-72ac\,d^{3} x^{4}-3b\,c^{2} d^{2} x^{4}-45a\,d^{4} x^{2}-50bc\,d^{3} x^{2}-35b\,d^{4}\right)}{315x^{8} d^{3}}$	111
trager	$\frac{\left(18a\,c^{3} d\,x^{8}-8b\,c^{4} x^{8}-9a\,c^{2} d^{2} x^{6}+4b\,c^{3} d\,x^{6}-72ac\,d^{3} x^{4}-3b\,c^{2} d^{2} x^{4}-45a\,d^{4} x^{2}-50bc\,d^{3} x^{2}-35b\,d^{4}\right) \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{315x^{8} d^{3}}$	115

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/315*((c*x^2+d)/x^2)^(3/2)*(18*a*c*d*x^4-8*b*c^2*x^4-45*a*d^2*x^2+20*b*c*
d*x^2-35*b*d^2)*(c*x^2+d)/d^3/x^6
```

3.950.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.47

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx =$$
$$-\frac{\left(2\left(4\,bc^4-9\,ac^3d\right)x^8-\left(4\,bc^3d-9\,ac^2d^2\right)x^6+35\,bd^4+3\left(bc^2d^2+24\,acd^3\right)x^4+5\left(10\,bcd^3+9\,ad^4\right)x^2\right)\sqrt{\frac{cx^2}{x}}}{315\,d^3x^8}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
-1/315*(2*(4*b*c^4 - 9*a*c^3*d)*x^8 - (4*b*c^3*d - 9*a*c^2*d^2)*x^6 + 35*b
*d^4 + 3*(b*c^2*d^2 + 24*a*c*d^3)*x^4 + 5*(10*b*c*d^3 + 9*a*d^4)*x^2)*sqrt
((c*x^2 + d)/x^2)/(d^3*x^8)
```

3.950.6 Sympy [A] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 258, normalized size of antiderivative = 3.49

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = -\frac{ac\left(\begin{cases}\frac{2\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{2x^4} & \text{otherwise}\end{cases}\right)}{2} - \frac{ad\left(\begin{cases}\frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{3x^6} & \text{otherwise}\end{cases}\right)}{2} - \frac{bc\left(\begin{cases}\frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{3x^6} & \text{otherwise}\end{cases}\right)}{2} - \frac{bd\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**5,x)
```

output
```
-a*c*Piecewise((2*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2,
 Ne(d, 0)), (sqrt(c)/(2*x**4), True))/2 - a*d*Piecewise((2*(c**2*(c + d/x*
*2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3, Ne
(d, 0)), (sqrt(c)/(3*x**6), True))/2 - b*c*Piecewise((2*(c**2*(c + d/x**2)
**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3, Ne(d,
 0)), (sqrt(c)/(3*x**6), True))/2 - b*d*Piecewise((2*(-c**3*(c + d/x**2)**
(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c +
d/x**2)**(9/2)/9)/d**4, Ne(d, 0)), (sqrt(c)/(4*x**8), True))/2
```

3.950.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = -\frac{1}{35}\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^2}-\frac{7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^2}\right)a - \frac{1}{315}\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^3}-\frac{90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^3}+\frac{63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^3}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
-1/35*(5*(c + d/x^2)^(7/2)/d^2 - 7*(c + d/x^2)^(5/2)*c/d^2)*a - 1/315*(35*
(c + d/x^2)^(9/2)/d^3 - 90*(c + d/x^2)^(7/2)*c/d^3 + 63*(c + d/x^2)^(5/2)*
c^2/d^3)*b
```

3.950.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(62) = 124.

Time = 1.31 (sec) , antiderivative size = 430, normalized size of antiderivative = 5.81

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = \frac{4\left(315\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{7}{2}}\mathrm{sgn}(x)+840\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{9}{2}}\mathrm{sgn}(x)-\right.}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
4/315*(315*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(7/2)*sgn(x) + 840*(sqrt(c
)*x - sqrt(c*x^2 + d))^12*b*c^(9/2)*sgn(x) - 315*(sqrt(c)*x - sqrt(c*x^2 +
 d))^12*a*c^(7/2)*d*sgn(x) + 1260*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(9/
2)*d*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(7/2)*d^2*sgn(x) +
1764*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(9/2)*d^2*sgn(x) - 819*(sqrt(c)*x
 - sqrt(c*x^2 + d))^8*a*c^(7/2)*d^3*sgn(x) + 504*(sqrt(c)*x - sqrt(c*x^2 +
 d))^6*b*c^(9/2)*d^3*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(7/2
)*d^4*sgn(x) + 144*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(9/2)*d^4*sgn(x) -
9*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(7/2)*d^5*sgn(x) - 36*(sqrt(c)*x - s
qrt(c*x^2 + d))^2*b*c^(9/2)*d^5*sgn(x) + 81*(sqrt(c)*x - sqrt(c*x^2 + d))^
2*a*c^(7/2)*d^6*sgn(x) + 4*b*c^(9/2)*d^6*sgn(x) - 9*a*c^(7/2)*d^7*sgn(x))/
((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^9
```

3.950.9 Mupad [B] (verification not implemented)

Time = 10.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.22

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = \frac{2\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{35\,d^2} - \frac{8\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3} - \frac{8\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,x^4} - \frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6} - \frac{10\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,x^6} - \frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8} - \frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^2} - \frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d\,x^4} + \frac{4\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^2\,x^2}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x)
```

output
```
(2*a*c^3*(c + d/x^2)^(1/2))/(35*d^2) - (8*b*c^4*(c + d/x^2)^(1/2))/(315*d^
3) - (8*a*c*(c + d/x^2)^(1/2))/(35*x^4) - (a*d*(c + d/x^2)^(1/2))/(7*x^6)
- (10*b*c*(c + d/x^2)^(1/2))/(63*x^6) - (b*d*(c + d/x^2)^(1/2))/(9*x^8) -
(a*c^2*(c + d/x^2)^(1/2))/(35*d*x^2) - (b*c^2*(c + d/x^2)^(1/2))/(105*d*x^
4) + (4*b*c^3*(c + d/x^2)^(1/2))/(315*d^2*x^2)
```

3.951 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx$

3.951.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = \frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4} - \frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4} + \frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4} - \frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}$$

output
```
1/5*c^2*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^4-1/7*c*(-2*a*d+3*b*c)*(c+d/x^2)^(7/2
)/d^4+1/9*(-a*d+3*b*c)*(c+d/x^2)^(9/2)/d^4-1/11*b*(c+d/x^2)^(11/2)/d^4
```

3.951.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)^2\left(-11adx^2(35d^2-20cdx^2+8c^2x^4)-3b(105d^3-70cd^2x^2+\right.}{3465d^4x^{10}}$$

input
```
Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x]
```

output
```
(Sqrt[c + d/x^2]*(d + c*x^2)^2*(-11*a*d*x^2*(35*d^2 - 20*c*d*x^2 + 8*c^2*x
^4) - 3*b*(105*d^3 - 70*c*d^2*x^2 + 40*c^2*d*x^4 - 16*c^3*x^6)))/(3465*d^4
*x^10)
```

3.951.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}d\frac{1}{x^2}$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{d^3}+\frac{(ad-3bc)\left(c+\frac{d}{x^2}\right)^{7/2}}{d^3}+\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{d^3}-\frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^3}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2c^2\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^4}+\frac{2\left(c+\frac{d}{x^2}\right)^{9/2}(3bc-ad)}{9d^4}-\frac{2c\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-2ad)}{7d^4}-\frac{2b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}\right)$$

input
```
Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x]
```

output
```
((2*c^2*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (2*c*(3*b*c - 2*a*d)*(c +
 d/x^2)^(7/2))/(7*d^4) + (2*(3*b*c - a*d)*(c + d/x^2)^(9/2))/(9*d^4) - (2*
b*(c + d/x^2)^(11/2))/(11*d^4))/2
```

3.951.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.951.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

method	result
gosper	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(88a\,c^{2} d\,x^{6}-48b\,c^{3} x^{6}-220ac\,d^{2} x^{4}+120b\,c^{2} d\,x^{4}+385a\,d^{3} x^{2}-210bc\,d^{2} x^{2}+315b\,d^{3}\right) \left(c\,x^{2}+d\right)}{3465d^{4} x^{8}}$
default	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(88a\,c^{2} d\,x^{6}-48b\,c^{3} x^{6}-220ac\,d^{2} x^{4}+120b\,c^{2} d\,x^{4}+385a\,d^{3} x^{2}-210bc\,d^{2} x^{2}+315b\,d^{3}\right) \left(c\,x^{2}+d\right)}{3465d^{4} x^{8}}$
risch	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, (88a\,c^{4} d\,x^{10}-48b\,c^{5} x^{10}-44a\,c^{3} d^{2} x^{8}+24b\,c^{4} d\,x^{8}+33a\,c^{2} d^{3} x^{6}-18b\,c^{3} d^{2} x^{6}+550ac\,d^{4} x^{4}+15b\,c^{2} d^{3} x^{4}+385a\,d^{5} x^{2}+420bc}{3465x^{10} d^{4}}$
trager	$-\frac{(88a\,c^{4} d\,x^{10}-48b\,c^{5} x^{10}-44a\,c^{3} d^{2} x^{8}+24b\,c^{4} d\,x^{8}+33a\,c^{2} d^{3} x^{6}-18b\,c^{3} d^{2} x^{6}+550ac\,d^{4} x^{4}+15b\,c^{2} d^{3} x^{4}+385a\,d^{5} x^{2}+420bc\,d^{4} x^{2}+315}{3465x^{10} d^{4}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/3465*((c*x^2+d)/x^2)^(3/2)*(88*a*c^2*d*x^6-48*b*c^3*x^6-220*a*c*d^2*x^4
+120*b*c^2*d*x^4+385*a*d^3*x^2-210*b*c*d^2*x^2+315*b*d^3)*(c*x^2+d)/d^4/x^
8
```

3.951.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = \frac{(8\,(6\,bc^5-11\,ac^4d)x^{10}-4\,(6\,bc^4d-11\,ac^3d^2)x^8+3\,(6\,bc^3d^2-11\,ac^2d^3)x^6-3}{3465\,d^4x^{10}}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
1/3465*(8*(6*b*c^5 - 11*a*c^4*d)*x^10 - 4*(6*b*c^4*d - 11*a*c^3*d^2)*x^8 +
 3*(6*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 - 315*b*d^5 - 5*(3*b*c^2*d^3 + 110*a*c
*d^4)*x^4 - 35*(12*b*c*d^4 + 11*a*d^5)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^4*x^1
0)
```

3.951.6 Sympy [A] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = -\frac{ac\left(\begin{cases}\frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{3x^6} & \text{otherwise}\end{cases}\right)}{2} - \frac{ad\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2} - \frac{bc\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2} - \frac{bd\left(\begin{cases}\frac{2\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{5x^{10}} & \text{otherwise}\end{cases}\right)}{2}$$

input

```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**7,x)
```

output
```
-a*c*Piecewise((2*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5
+ (c + d/x**2)**(7/2)/7)/d**3, Ne(d, 0)), (sqrt(c)/(3*x**6), True))/2 - a*
d*Piecewise((2*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5
 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4, Ne(d, 0)), (sq
rt(c)/(4*x**8), True))/2 - b*c*Piecewise((2*(-c**3*(c + d/x**2)**(3/2)/3 +
 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**
(9/2)/9)/d**4, Ne(d, 0)), (sqrt(c)/(4*x**8), True))/2 - b*d*Piecewise((2*(
c**4*(c + d/x**2)**(3/2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/
x**2)**(7/2)/7 - 4*c*(c + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5
, Ne(d, 0)), (sqrt(c)/(5*x**10), True))/2
```

3.951.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = -\frac{1}{315}\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^3} - \frac{90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^3} + \frac{63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^3}\right)a - \frac{1}{1155}\left(\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4} - \frac{385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^4} + \frac{495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^4} - \frac{231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^4}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-1/315*(35*(c + d/x^2)^(9/2)/d^3 - 90*(c + d/x^2)^(7/2)*c/d^3 + 63*(c + d/
x^2)^(5/2)*c^2/d^3)*a - 1/1155*(105*(c + d/x^2)^(11/2)/d^4 - 385*(c + d/x^
2)^(9/2)*c/d^4 + 495*(c + d/x^2)^(7/2)*c^2/d^4 - 231*(c + d/x^2)^(5/2)*c^3
/d^4)*b
```

3.951.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(88) = 176$.

Time = 2.03 (sec) , antiderivative size = 490, normalized size of antiderivative = 4.71

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = \frac{16\left(2310\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{16}ac^{\frac{9}{2}}\operatorname{sgn}(x) + 6930\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}bc^{\frac{11}{2}}\operatorname{sgn}(\right.}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
16/3465*(2310*(sqrt(c)*x - sqrt(c*x^2 + d))^16*a*c^(9/2)*sgn(x) + 6930*(sq
rt(c)*x - sqrt(c*x^2 + d))^14*b*c^(11/2)*sgn(x) - 1155*(sqrt(c)*x - sqrt(c
*x^2 + d))^14*a*c^(9/2)*d*sgn(x) + 12474*(sqrt(c)*x - sqrt(c*x^2 + d))^12*
b*c^(11/2)*d*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(9/2)*d^2*s
gn(x) + 15246*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(11/2)*d^2*sgn(x) - 485
1*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(9/2)*d^3*sgn(x) + 4950*(sqrt(c)*x
- sqrt(c*x^2 + d))^8*b*c^(11/2)*d^3*sgn(x) + 2475*(sqrt(c)*x - sqrt(c*x^2
+ d))^8*a*c^(9/2)*d^4*sgn(x) + 990*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(11
/2)*d^4*sgn(x) + 495*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(9/2)*d^5*sgn(x)
- 330*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(11/2)*d^5*sgn(x) + 605*(sqrt(c)
*x - sqrt(c*x^2 + d))^4*a*c^(9/2)*d^6*sgn(x) + 66*(sqrt(c)*x - sqrt(c*x^2
+ d))^2*b*c^(11/2)*d^6*sgn(x) - 121*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(9
/2)*d^7*sgn(x) - 6*b*c^(11/2)*d^7*sgn(x) + 11*a*c^(9/2)*d^8*sgn(x))/((sqrt
(c)*x - sqrt(c*x^2 + d))^2 - d)^11
```

3.951.9 Mupad [B] (verification not implemented)

Time = 10.68 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.98

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = \frac{16\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^4}-\frac{8\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3}-\frac{10\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,x^6} - \frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{4\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{33\,x^8}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d\,x^4} + \frac{4\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^2\,x^2}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{231\,d\,x^6}+\frac{2\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{385\,d^2\,x^4}-\frac{8\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^3\,x^2}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x)
```

output
```
(16*b*c^5*(c + d/x^2)^(1/2))/(1155*d^4) - (8*a*c^4*(c + d/x^2)^(1/2))/(315
*d^3) - (10*a*c*(c + d/x^2)^(1/2))/(63*x^6) - (a*d*(c + d/x^2)^(1/2))/(9*x
^8) - (4*b*c*(c + d/x^2)^(1/2))/(33*x^8) - (b*d*(c + d/x^2)^(1/2))/(11*x^1
0) - (a*c^2*(c + d/x^2)^(1/2))/(105*d*x^4) + (4*a*c^3*(c + d/x^2)^(1/2))/(
315*d^2*x^2) - (b*c^2*(c + d/x^2)^(1/2))/(231*d*x^6) + (2*b*c^3*(c + d/x^2
)^(1/2))/(385*d^2*x^4) - (8*b*c^4*(c + d/x^2)^(1/2))/(1155*d^3*x^2)
```

3.951. $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx$

3.952 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx$

3.952.1 Optimal result

Integrand size = 22, antiderivative size = 134

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx=-\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5}+\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}-\frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{3d^5}+\frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{13/2}}{13d^5}$$

output
```
-1/5*c^3*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^5+1/7*c^2*(-3*a*d+4*b*c)*(c+d/x^2)^(
7/2)/d^5-1/3*c*(-a*d+2*b*c)*(c+d/x^2)^(9/2)/d^5+1/11*(-a*d+4*b*c)*(c+d/x^2
)^(11/2)/d^5-1/13*b*(c+d/x^2)^(13/2)/d^5
```

3.952.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)^2\left(13adx^2(-105d^3+70cd^2x^2-40c^2dx^4+16c^3x^6)+b(-115\right.}{15015d^5x^{12}}$$

input
```
Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]
```

output
```
(Sqrt[c + d/x^2]*(d + c*x^2)^2*(13*a*d*x^2*(-105*d^3 + 70*c*d^2*x^2 - 40*c
^2*d*x^4 + 16*c^3*x^6) + b*(-1155*d^4 + 840*c*d^3*x^2 - 560*c^2*d^2*x^4 +
320*c^3*d*x^6 - 128*c^4*x^8)))/(15015*d^5*x^12)
```

3.952.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}dx$$

$$\Big\downarrow \; 948$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^6}d\frac{1}{x^2}$$

$$\Big\downarrow \; 86$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{d^4}+\frac{(ad-4bc)\left(c+\frac{d}{x^2}\right)^{9/2}}{d^4}+\frac{3c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{d^4}-\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{d^4}+c^3(bc-\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{2c^3\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^5}+\frac{2c^2\left(c+\frac{d}{x^2}\right)^{7/2}(4bc-3ad)}{7d^5}+\frac{2\left(c+\frac{d}{x^2}\right)^{11/2}(4bc-ad)}{11d^5}-\frac{2c\left(c+\frac{d}{x^2}\right)^{9/2}(2bc-a}{3d^5}\right.$$

input
```
Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]
```

output
```
((-2*c^3*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^5) + (2*c^2*(4*b*c - 3*a*d)*(
c + d/x^2)^(7/2))/(7*d^5) - (2*c*(2*b*c - a*d)*(c + d/x^2)^(9/2))/(3*d^5)
+ (2*(4*b*c - a*d)*(c + d/x^2)^(11/2))/(11*d^5) - (2*b*(c + d/x^2)^(13/2))
/(13*d^5))/2
```

3.952.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.952.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88

method	result
gosper	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} (208a\,c^3d\,x^8-128b\,c^4x^8-520a\,c^2d^2x^6+320b\,c^3d\,x^6+910ac\,d^3x^4-560b\,c^2d^2x^4-1365a\,d^4x^2+840bc\,d^3x^2-1155b\,d^4) (c\,x^2-}{15015d^5x^{10}}$
default	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} (208a\,c^3d\,x^8-128b\,c^4x^8-520a\,c^2d^2x^6+320b\,c^3d\,x^6+910ac\,d^3x^4-560b\,c^2d^2x^4-1365a\,d^4x^2+840bc\,d^3x^2-1155b\,d^4) (c\,x^2-}{15015d^5x^{10}}$
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\, (208a\,c^5d\,x^{12}-128b\,c^6x^{12}-104a\,c^4d^2x^{10}+64b\,c^5d\,x^{10}+78a\,c^3d^3x^8-48b\,c^4d^2x^8-65a\,c^2d^4x^6+40b\,c^3d^3x^6-1820ac\,d^5x^4-3}{15015x^{12}d^5}$
trager	$\frac{(208a\,c^5d\,x^{12}-128b\,c^6x^{12}-104a\,c^4d^2x^{10}+64b\,c^5d\,x^{10}+78a\,c^3d^3x^8-48b\,c^4d^2x^8-65a\,c^2d^4x^6+40b\,c^3d^3x^6-1820ac\,d^5x^4-35b\,c^2d^4x^4-}{15015x^{12}d^5}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/15015*((c*x^2+d)/x^2)^(3/2)*(208*a*c^3*d*x^8-128*b*c^4*x^8-520*a*c^2*d^2
*x^6+320*b*c^3*d*x^6+910*a*c*d^3*x^4-560*b*c^2*d^2*x^4-1365*a*d^4*x^2+840*
b*c*d^3*x^2-1155*b*d^4)*(c*x^2+d)/d^5/x^10
```

3.952.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.17

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx =$$

$$-\frac{(16\,(8\,bc^6-13\,ac^5d)x^{12}-8\,(8\,bc^5d-13\,ac^4d^2)x^{10}+6\,(8\,bc^4d^2-13\,ac^3d^3)x^8+1155\,bd^6-5\,(8\,bc^3d^3-13}{15015\,d^5x^{12}}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
-1/15015*(16*(8*b*c^6 - 13*a*c^5*d)*x^12 - 8*(8*b*c^5*d - 13*a*c^4*d^2)*x^
10 + 6*(8*b*c^4*d^2 - 13*a*c^3*d^3)*x^8 + 1155*b*d^6 - 5*(8*b*c^3*d^3 - 13
*a*c^2*d^4)*x^6 + 35*(b*c^2*d^4 + 52*a*c*d^5)*x^4 + 105*(14*b*c*d^5 + 13*a
*d^6)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^5*x^12)
```

3.952.6 Sympy [A] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.93

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx =$$

$$-\frac{ac\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2}$$

$$-\frac{ad\left(\begin{cases}\frac{2\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{5x^{10}} & \text{otherwise}\end{cases}\right)}{2}$$

$$-\frac{bc\left(\begin{cases}\frac{2\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{5x^{10}} & \text{otherwise}\end{cases}\right)}{2}$$

$$-\frac{bd\left(\begin{cases}\frac{2\left(-\frac{c^5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+c^4\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}-\frac{10c^3\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{10c^2\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}-\frac{5c\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}}{13}\right)}{d^6} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{6x^{12}} & \text{otherwise}\end{cases}\right)}{2}$$

input

```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**9,x)
```

output
```
-a*c*Piecewise((2*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2
)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4, Ne(d, 0)),
(sqrt(c)/(4*x**8), True))/2 - a*d*Piecewise((2*(c**4*(c + d/x**2)**(3/2)/3
 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c +
d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5, Ne(d, 0)), (sqrt(c)/(5*x
**10), True))/2 - b*c*Piecewise((2*(c**4*(c + d/x**2)**(3/2)/3 - 4*c**3*(c
 + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x**2)**(9/
2)/9 + (c + d/x**2)**(11/2)/11)/d**5, Ne(d, 0)), (sqrt(c)/(5*x**10), True)
)/2 - b*d*Piecewise((2*(-c**5*(c + d/x**2)**(3/2)/3 + c**4*(c + d/x**2)**(
5/2) - 10*c**3*(c + d/x**2)**(7/2)/7 + 10*c**2*(c + d/x**2)**(9/2)/9 - 5*c
*(c + d/x**2)**(11/2)/11 + (c + d/x**2)**(13/2)/13)/d**6, Ne(d, 0)), (sqrt
(c)/(6*x**12), True))/2
```

3.952.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx =$$

$$-\frac{1}{1155}\left(\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4}-\frac{385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^4}+\frac{495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^4}-\frac{231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^4}\right)a$$

$$-\frac{1}{15015}\left(\frac{1155\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}}{d^5}-\frac{5460\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}c}{d^5}+\frac{10010\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c^2}{d^5}-\frac{8580\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^3}{d^5}+\frac{3003\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^5}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
-1/1155*(105*(c + d/x^2)^(11/2)/d^4 - 385*(c + d/x^2)^(9/2)*c/d^4 + 495*(c
 + d/x^2)^(7/2)*c^2/d^4 - 231*(c + d/x^2)^(5/2)*c^3/d^4)*a - 1/15015*(1155
*(c + d/x^2)^(13/2)/d^5 - 5460*(c + d/x^2)^(11/2)*c/d^5 + 10010*(c + d/x^2
)^(9/2)*c^2/d^5 - 8580*(c + d/x^2)^(7/2)*c^3/d^5 + 3003*(c + d/x^2)^(5/2)*
c^4/d^5)*b
```

3.952.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(114) = 228$.

Time = 2.05 (sec) , antiderivative size = 550, normalized size of antiderivative = 4.10

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx = \frac{32\left(15015\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{18}ac^{\frac{11}{2}}\operatorname{sgn}(x)+48048\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{16}bc^{\frac{13}{2}}\mathrm{sg}}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
32/15015*(15015*(sqrt(c)*x - sqrt(c*x^2 + d))^18*a*c^(11/2)*sgn(x) + 48048
*(sqrt(c)*x - sqrt(c*x^2 + d))^16*b*c^(13/2)*sgn(x) - 3003*(sqrt(c)*x - sq
rt(c*x^2 + d))^16*a*c^(11/2)*d*sgn(x) + 96096*(sqrt(c)*x - sqrt(c*x^2 + d)
)^14*b*c^(13/2)*d*sgn(x) - 6006*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(11/2
)*d^2*sgn(x) + 109824*(sqrt(c)*x - sqrt(c*x^2 + d))^12*b*c^(13/2)*d^2*sgn(
x) - 28314*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(11/2)*d^3*sgn(x) + 37752*
(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(13/2)*d^3*sgn(x) + 13728*(sqrt(c)*x
- sqrt(c*x^2 + d))^10*a*c^(11/2)*d^4*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2
 + d))^8*b*c^(13/2)*d^4*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^
(11/2)*d^5*sgn(x) - 2288*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(13/2)*d^5*sg
n(x) + 3718*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(11/2)*d^6*sgn(x) + 624*(s
qrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(13/2)*d^6*sgn(x) - 1014*(sqrt(c)*x - sq
rt(c*x^2 + d))^4*a*c^(11/2)*d^7*sgn(x) - 104*(sqrt(c)*x - sqrt(c*x^2 + d))
^2*b*c^(13/2)*d^7*sgn(x) + 169*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(11/2)*
d^8*sgn(x) + 8*b*c^(13/2)*d^8*sgn(x) - 13*a*c^(11/2)*d^9*sgn(x))/((sqrt(c)
*x - sqrt(c*x^2 + d))^2 - d)^13
```

3.952.9 Mupad [B] (verification not implemented)

Time = 11.33 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.85

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx &= \frac{16\,a\,c^5\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^4}-\frac{128\,b\,c^6\,\sqrt{c+\frac{d}{x^2}}}{15015\,d^5}-\frac{4\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{33\,x^8}\\ &-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{14\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{143\,x^{10}}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{13\,x^{12}}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{231\,d\,x^6}+\frac{2\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{385\,d^2\,x^4}\\ &-\frac{8\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^3\,x^2}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{429\,d\,x^8}+\frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{3003\,d^2\,x^6}-\frac{16\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{5005\,d^3\,x^4}+\frac{64\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{15015\,d^4\,x^2}\end{aligned}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x)
```

output
```
(16*a*c^5*(c + d/x^2)^(1/2))/(1155*d^4) - (128*b*c^6*(c + d/x^2)^(1/2))/(1
5015*d^5) - (4*a*c*(c + d/x^2)^(1/2))/(33*x^8) - (a*d*(c + d/x^2)^(1/2))/(
11*x^10) - (14*b*c*(c + d/x^2)^(1/2))/(143*x^10) - (b*d*(c + d/x^2)^(1/2))
/(13*x^12) - (a*c^2*(c + d/x^2)^(1/2))/(231*d*x^6) + (2*a*c^3*(c + d/x^2)^
(1/2))/(385*d^2*x^4) - (8*a*c^4*(c + d/x^2)^(1/2))/(1155*d^3*x^2) - (b*c^2
*(c + d/x^2)^(1/2))/(429*d*x^8) + (8*b*c^3*(c + d/x^2)^(1/2))/(3003*d^2*x^
6) - (16*b*c^4*(c + d/x^2)^(1/2))/(5005*d^3*x^4) + (64*b*c^5*(c + d/x^2)^(
1/2))/(15015*d^4*x^2)
```

3.952. $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx$

3.953 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{12} \, dx$

3.953.1 Optimal result

Integrand size = 22, antiderivative size = 150

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx &= -\frac{16d^3(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{15015c^5}\\ &+\frac{8d^2(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{3003c^4}-\frac{2d(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{429c^3}\\ &+\frac{(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^{11}}{143c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^{13}}{13c}\end{aligned}$$

output
```
-16/15015*d^3*(-8*a*d+13*b*c)*(c+d/x^2)^(5/2)*x^5/c^5+8/3003*d^2*(-8*a*d+1
3*b*c)*(c+d/x^2)^(5/2)*x^7/c^4-2/429*d*(-8*a*d+13*b*c)*(c+d/x^2)^(5/2)*x^9
/c^3+1/143*(-8*a*d+13*b*c)*(c+d/x^2)^(5/2)*x^11/c^2+1/13*a*(c+d/x^2)^(5/2)
*x^13/c
```

3.953.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.73

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\,x(d+cx^2)^2\,(13bc(-16d^3+40cd^2x^2-70c^2dx^4+105c^3x^6)+a(128d^4-320cd^3x^2}{15015c^5}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]
```

output
```
(Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(13*b*c*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*
d*x^4 + 105*c^3*x^6) + a*(128*d^4 - 320*c*d^3*x^2 + 560*c^2*d^2*x^4 - 840*
c^3*d*x^6 + 1155*c^4*x^8)))/(15015*c^5)
```

3.953.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(13bc-8ad)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^{10}dx}{13c}+\frac{ax^{13}\left(c+\frac{d}{x^2}\right)^{5/2}}{13c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(13bc-8ad)\left(\frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}-\frac{6d\int \left(c+\frac{d}{x^2}\right)^{3/2}x^8dx}{11c}\right)}{13c}+\frac{ax^{13}\left(c+\frac{d}{x^2}\right)^{5/2}}{13c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(13bc-8ad)\left(\frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}-\frac{6d\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}-\frac{4d\int \left(c+\frac{d}{x^2}\right)^{3/2}x^6dx}{9c}\right)}{11c}\right)}{13c}+\frac{ax^{13}\left(c+\frac{d}{x^2}\right)^{5/2}}{13c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(13bc-8ad)\left(\frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}-\frac{6d\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}-\frac{4d\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}-\frac{2d\int\left(c+\frac{d}{x^2}\right)^{3/2}x^4dx}{7c}\right)}{9c}\right)}{11c}\right)}{13c}+\frac{ax^{13}\left(c+\frac{d}{x^2}\right)^{5/2}}{13c}$$

$$\Big\downarrow \; 796$$

$$\frac{\left(\frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}-\frac{6d\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}-\frac{4d\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}-\frac{2dx^5\left(c+\frac{d}{x^2}\right)^{5/2}}{35c^2}\right)}{9c}\right)}{11c}\right)(13bc-8ad)}{13c}+\frac{ax^{13}\left(c+\frac{d}{x^2}\right)^{5/2}}{13c}$$

input

```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]
```

output

```
(a*(c + d/x^2)^(5/2)*x^13)/(13*c) + ((13*b*c - 8*a*d)*(((c + d/x^2)^(5/2)*
x^11)/(11*c) - (6*d*(((c + d/x^2)^(5/2)*x^9)/(9*c) - (4*d*((-2*d*(c + d/x^
2)^(5/2)*x^5)/(35*c^2) + ((c + d/x^2)^(5/2)*x^7)/(7*c)))/(9*c)))/(11*c)))/
(13*c)
```

3.953. $\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{12}\,dx$

3.953.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.953.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77

method	result
gosper	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(1155 a\,x^{8} c^{4}-840 a\,c^{3} d\,x^{6}+1365 b\,c^{4} x^{6}+560 a\,c^{2} d^{2} x^{4}-910 b\,c^{3} d\,x^{4}-320 a c\,d^{3} x^{2}+520 b\,c^{2} d^{2} x^{2}+128 a\,d^{4}-208 b c\,d^{3}\right) \left(c\,x^{2}\right.}{15015 c^{5}}$
default	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(1155 a\,x^{8} c^{4}-840 a\,c^{3} d\,x^{6}+1365 b\,c^{4} x^{6}+560 a\,c^{2} d^{2} x^{4}-910 b\,c^{3} d\,x^{4}-320 a c\,d^{3} x^{2}+520 b\,c^{2} d^{2} x^{2}+128 a\,d^{4}-208 b c\,d^{3}\right) \left(c\,x^{2}\right.}{15015 c^{5}}$
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x \left(1155 a\,c^{6} x^{12}+1470 a\,c^{5} d\,x^{10}+1365 b\,c^{6} x^{10}+35 a\,c^{4} d^{2} x^{8}+1820 b\,c^{5} d\,x^{8}-40 a\,c^{3} d^{3} x^{6}+65 b\,c^{4} d^{2} x^{6}+48 a\,c^{2} d^{4} x^{4}-78 b\,c^{3} d^{3} x^{4}-\right.}{15015 c^{5}}$
trager	$\frac{\left(1155 a\,c^{6} x^{12}+1470 a\,c^{5} d\,x^{10}+1365 b\,c^{6} x^{10}+35 a\,c^{4} d^{2} x^{8}+1820 b\,c^{5} d\,x^{8}-40 a\,c^{3} d^{3} x^{6}+65 b\,c^{4} d^{2} x^{6}+48 a\,c^{2} d^{4} x^{4}-78 b\,c^{3} d^{3} x^{4}-64 a c\,d^{5} x^{2}\right.}{15015 c^{5}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x,method=_RETURNVERBOSE)
```

output
```
1/15015*((c*x^2+d)/x^2)^(3/2)*x^3*(1155*a*c^4*x^8-840*a*c^3*d*x^6+1365*b*c
^4*x^6+560*a*c^2*d^2*x^4-910*b*c^3*d*x^4-320*a*c*d^3*x^2+520*b*c^2*d^2*x^2
+128*a*d^4-208*b*c*d^3)*(c*x^2+d)/c^5
```

3.953.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{12}\,dx = \frac{(1155\,ac^6x^{13}+105\,(13\,bc^6+14\,ac^5d)x^{11}+35\,(52\,bc^5d+ac^4d^2)x^9+5\,(13\,bc^4d^2-8\,ac^3d^3)}{15015}$$

input

```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="fricas")
```

output

```
1/15015*(1155*a*c^6*x^13 + 105*(13*b*c^6 + 14*a*c^5*d)*x^11 + 35*(52*b*c^5
*d + a*c^4*d^2)*x^9 + 5*(13*b*c^4*d^2 - 8*a*c^3*d^3)*x^7 - 6*(13*b*c^3*d^3
 - 8*a*c^2*d^4)*x^5 + 8*(13*b*c^2*d^4 - 8*a*c*d^5)*x^3 - 16*(13*b*c*d^5 -
8*a*d^6)*x)*sqrt((c*x^2 + d)/x^2)/c^5
```

3.953.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3351 vs. 2(146) = 292.

Time = 5.61 (sec) , antiderivative size = 3351, normalized size of antiderivative = 22.34

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{12}\,dx = \text{Too large to display}$$

input

```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**12,x)
```

output
```
693*a*c**12*d**(51/2)*x**22*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 4
5045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45
045*c**7*d**29*x**2 + 9009*c**6*d**30) + 3528*a*c**11*d**(53/2)*x**20*sqrt
(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c*
*9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*
d**30) + 7175*a*c**10*d**(55/2)*x**18*sqrt(c*x**2/d + 1)/(9009*c**11*d**25
*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28
*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 7290*a*c**9*d**(57/2)*x
**16*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 +
 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9
009*c**6*d**30) + 315*a*c**9*d**(35/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9
*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d
**19*x**2 + 3465*c**5*d**20) + 3699*a*c**8*d**(59/2)*x**14*sqrt(c*x**2/d +
 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x*
*6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 12
95*a*c**8*d**(37/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860
*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) + 756*a*c**7*d**(61/2)*x**12*sqrt(c*x**2/d + 1)/(9009*c**11*d**2
5*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**2
8*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 1990*a*c**7*d**(39/...
```

3.953.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx = \frac{\left(105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5\right)b}{1155\,c^4} + \frac{\left(1155\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}x^{13}-5460\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}dx^{11}+10010\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}d^2x^9-8580\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^3x^7+3003\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{15015\,c^5}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="maxima")
```

output
```
1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c + d/x^2)^(9/2)*d*x^9 + 495*(c
 + d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)*d^3*x^5)*b/c^4 + 1/15015*(
1155*(c + d/x^2)^(13/2)*x^13 - 5460*(c + d/x^2)^(11/2)*d*x^11 + 10010*(c +
 d/x^2)^(9/2)*d^2*x^9 - 8580*(c + d/x^2)^(7/2)*d^3*x^7 + 3003*(c + d/x^2)^
(5/2)*d^4*x^5)*a/c^5
```

3.953.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.17

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx = \frac{16\left(13\,bcd^{\frac{11}{2}}-8\,ad^{\frac{13}{2}}\right)\mathrm{sgn}(x)}{15015\,c^5}$$

$$+\frac{1155\,(cx^2+d)^{\frac{13}{2}}a\mathrm{sgn}(x)+1365\,(cx^2+d)^{\frac{11}{2}}bc\mathrm{sgn}(x)-5460\,(cx^2+d)^{\frac{11}{2}}ad\mathrm{sgn}(x)-5005\,(cx^2+d)^{\frac{9}{2}}bcd\mathrm{sgn}}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="giac")
```

output
```
16/15015*(13*b*c*d^(11/2) - 8*a*d^(13/2))*sgn(x)/c^5 + 1/15015*(1155*(c*x^
2 + d)^(13/2)*a*sgn(x) + 1365*(c*x^2 + d)^(11/2)*b*c*sgn(x) - 5460*(c*x^2
+ d)^(11/2)*a*d*sgn(x) - 5005*(c*x^2 + d)^(9/2)*b*c*d*sgn(x) + 10010*(c*x^
2 + d)^(9/2)*a*d^2*sgn(x) + 6435*(c*x^2 + d)^(7/2)*b*c*d^2*sgn(x) - 8580*(
c*x^2 + d)^(7/2)*a*d^3*sgn(x) - 3003*(c*x^2 + d)^(5/2)*b*c*d^3*sgn(x) + 30
03*(c*x^2 + d)^(5/2)*a*d^4*sgn(x))/c^5
```

3.953.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx = \sqrt{c+\frac{d}{x^2}}\left(\frac{x\,(128\,a\,d^6-208\,b\,c\,d^5)}{15015\,c^5} + \frac{x^{11}\,(1365\,b\,c^6+1470\,a\,d\,c^5)}{15015\,c^5}+\frac{a\,c\,x^{13}}{13}+\frac{d\,x^9\,(a\,d+52\,b\,c)}{429\,c} - \frac{d^2\,x^7\,(8\,a\,d-13\,b\,c)}{3003\,c^2}+\frac{2\,d^3\,x^5\,(8\,a\,d-13\,b\,c)}{5005\,c^3}-\frac{8\,d^4\,x^3\,(8\,a\,d-13\,b\,c)}{15015\,c^4}\right)$$

input
```
int(x^12*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
(c + d/x^2)^(1/2)*((x*(128*a*d^6 - 208*b*c*d^5))/(15015*c^5) + (x^11*(1365
*b*c^6 + 1470*a*c^5*d))/(15015*c^5) + (a*c*x^13)/13 + (d*x^9*(a*d + 52*b*c
))/(429*c) - (d^2*x^7*(8*a*d - 13*b*c))/(3003*c^2) + (2*d^3*x^5*(8*a*d - 1
3*b*c))/(5005*c^3) - (8*d^4*x^3*(8*a*d - 13*b*c))/(15015*c^4))
```

3.954 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{10} \, dx$

3.954.1 Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{10} \, dx = \frac{8d^2(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{3465c^4} - \frac{4d(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{693c^3} + \frac{(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{99c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{11c}$$

output
```
8/3465*d^2*(-6*a*d+11*b*c)*(c+d/x^2)^(5/2)*x^5/c^4-4/693*d*(-6*a*d+11*b*c)
*(c+d/x^2)^(5/2)*x^7/c^3+1/99*(-6*a*d+11*b*c)*(c+d/x^2)^(5/2)*x^9/c^2+1/11
*a*(c+d/x^2)^(5/2)*x^11/c
```

3.954.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right) \left(c +\frac{d}{x^2}\right)^{3/2} x^{10} \, dx = \frac{\sqrt{c+\frac{d}{x^2}}x(d+cx^2)^2 \left(11bc(8d^2-20cdx^2+35c^2x^4)+3a(-16d^3+40cd^2x^2-70c^2dx^4+10\right.}{3465c^4}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]
```

output
```
(Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(11*b*c*(8*d^2 - 20*c*d*x^2 + 35*c^2*x^4)
 + 3*a*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*d*x^4 + 105*c^3*x^6)))/(3465*c^4)
```

3.954.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{10}\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(11bc-6ad)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^8dx}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(11bc-6ad)\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}-\frac{4d\int \left(c+\frac{d}{x^2}\right)^{3/2}x^6dx}{9c}\right)}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(11bc-6ad)\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}-\frac{4d\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}-\frac{2d\int \left(c+\frac{d}{x^2}\right)^{3/2}x^4dx}{7c}\right)}{9c}\right)}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{796}$$

$$\frac{\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}-\frac{4d\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}-\frac{2dx^5\left(c+\frac{d}{x^2}\right)^{5/2}}{35c^2}\right)}{9c}\right)(11bc-6ad)}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]
```

output
```
(a*(c + d/x^2)^(5/2)*x^11)/(11*c) + ((11*b*c - 6*a*d)*(((c + d/x^2)^(5/2)*
x^9)/(9*c) - (4*d*((-2*d*(c + d/x^2)^(5/2)*x^5)/(35*c^2) + ((c + d/x^2)^(5
/2)*x^7)/(7*c)))/(9*c)))/(11*c)
```

3.954. $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{10}\,dx$

3.954.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.954.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

method	result
gosper	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(315 x^{6} a\,c^{3}-210 a\,c^{2} d\,x^{4}+385 b\,c^{3} x^{4}+120 a c\,d^{2} x^{2}-220 b\,c^{2} d\,x^{2}-48 a\,d^{3}+88 b c\,d^{2}\right) \left(c\,x^{2}+d\right)}{3465 c^{4}}$
default	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(315 x^{6} a\,c^{3}-210 a\,c^{2} d\,x^{4}+385 b\,c^{3} x^{4}+120 a c\,d^{2} x^{2}-220 b\,c^{2} d\,x^{2}-48 a\,d^{3}+88 b c\,d^{2}\right) \left(c\,x^{2}+d\right)}{3465 c^{4}}$
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x \left(315 a\,c^{5} x^{10}+420 a\,c^{4} d\,x^{8}+385 b\,c^{5} x^{8}+15 a\,c^{3} d^{2} x^{6}+550 b\,c^{4} d\,x^{6}-18 a\,c^{2} d^{3} x^{4}+33 b\,c^{3} d^{2} x^{4}+24 a c\,d^{4} x^{2}-44 b\,c^{2} d^{3} x^{2}-48 a\,d^{5}\right.}{3465 c^{4}}$
trager	$\frac{\left(315 a\,c^{5} x^{10}+420 a\,c^{4} d\,x^{8}+385 b\,c^{5} x^{8}+15 a\,c^{3} d^{2} x^{6}+550 b\,c^{4} d\,x^{6}-18 a\,c^{2} d^{3} x^{4}+33 b\,c^{3} d^{2} x^{4}+24 a c\,d^{4} x^{2}-44 b\,c^{2} d^{3} x^{2}-48 a\,d^{5}+88 b c\,d^{4}\right) x}{3465 c^{4}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x,method=_RETURNVERBOSE)
```

output
```
1/3465*((c*x^2+d)/x^2)^(3/2)*x^3*(315*a*c^3*x^6-210*a*c^2*d*x^4+385*b*c^3*
x^4+120*a*c*d^2*x^2-220*b*c^2*d*x^2-48*a*d^3+88*b*c*d^2)*(c*x^2+d)/c^4
```

3.954.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{10}\,dx=\frac{(315\,ac^5x^{11}+35\,(11\,bc^5+12\,ac^4d)x^9+5\,(110\,bc^4d+3\,ac^3d^2)x^7+3\,(11\,bc^3d^2-6\,ac^2d^3)x^5}{3465\,c^4}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="fricas")
```

output
```
1/3465*(315*a*c^5*x^11 + 35*(11*b*c^5 + 12*a*c^4*d)*x^9 + 5*(110*b*c^4*d +
 3*a*c^3*d^2)*x^7 + 3*(11*b*c^3*d^2 - 6*a*c^2*d^3)*x^5 - 4*(11*b*c^2*d^3 -
 6*a*c*d^4)*x^3 + 8*(11*b*c*d^4 - 6*a*d^5)*x)*sqrt((c*x^2 + d)/x^2)/c^4
```

3.954.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2304 vs. 2(112) = 224.

Time = 4.20 (sec) , antiderivative size = 2304, normalized size of antiderivative = 19.69

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{10}\,dx=\text{Too large to display}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**10,x)
```

output

```
315*a*c**10*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 138
60*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*
c**5*d**20) + 1295*a*c**9*d**(35/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d*
*16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**1
9*x**2 + 3465*c**5*d**20) + 1990*a*c**8*d**(37/2)*x**14*sqrt(c*x**2/d + 1)
/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 1
3860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**7*d**(39/2)*x**12*sqrt
(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*
d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c**7*d**(21/2
)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945
*c**5*d**11*x**2 + 315*c**4*d**12) + 343*a*c**6*d**(41/2)*x**10*sqrt(c*x**
2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*
x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 110*a*c**6*d**(23/2)*x**
12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5
*d**11*x**2 + 315*c**4*d**12) + 35*a*c**5*d**(43/2)*x**8*sqrt(c*x**2/d + 1
)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 +
13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 114*a*c**5*d**(25/2)*x**10*sqrt
(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*
x**2 + 315*c**4*d**12) + 280*a*c**4*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(346
5*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 138...
```

3.954.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{10}\,dx=\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)b}{315\,c^3}+\frac{\left(105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5\right)a}{1155\,c^4}$$

input

```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="maxima")
```

output

```
1/315*(35*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x
^2)^(5/2)*d^2*x^5)*b/c^3 + 1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c +
d/x^2)^(9/2)*d*x^9 + 495*(c + d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)
*d^3*x^5)*a/c^4
```

3.954.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx = -\frac{8\left(11\,bcd^{\frac{9}{2}}-6\,ad^{\frac{11}{2}}\right)\mathrm{sgn}(x)}{3465\,c^4}$$

$$+\frac{315\left(cx^2+d\right)^{\frac{11}{2}}a\mathrm{sgn}(x)+385\left(cx^2+d\right)^{\frac{9}{2}}bc\mathrm{sgn}(x)-1155\left(cx^2+d\right)^{\frac{9}{2}}ad\mathrm{sgn}(x)-990\left(cx^2+d\right)^{\frac{7}{2}}bcd\mathrm{sgn}(x)+}{3465\,c^4}$$

input

```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="giac")
```

output

```
-8/3465*(11*b*c*d^(9/2) - 6*a*d^(11/2))*sgn(x)/c^4 + 1/3465*(315*(c*x^2 +
d)^(11/2)*a*sgn(x) + 385*(c*x^2 + d)^(9/2)*b*c*sgn(x) - 1155*(c*x^2 + d)^(
9/2)*a*d*sgn(x) - 990*(c*x^2 + d)^(7/2)*b*c*d*sgn(x) + 1485*(c*x^2 + d)^(7
/2)*a*d^2*sgn(x) + 693*(c*x^2 + d)^(5/2)*b*c*d^2*sgn(x) - 693*(c*x^2 + d)^
(5/2)*a*d^3*sgn(x))/c^4
```

3.954.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx = \sqrt{c+\frac{d}{x^2}}\left(\frac{x^9\,(385\,b\,c^5+420\,a\,d\,c^4)}{3465\,c^4}-\frac{x\,(48\,a\,d^5-88\,b\,c\,d^4)}{3465\,c^4}+\frac{a\,c\,x^{11}}{11}+\frac{d\,x^7\,(3\,a\,d+110\,b\,c)}{693\,c}-\frac{d^2\,x^5\,(6\,a\,d-11\,b\,c)}{1155\,c^2}+\frac{4\,d^3\,x^3\,(6\,a\,d-11\,b\,c)}{3465\,c^3}\right)$$

input

```
int(x^10*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output

```
(c + d/x^2)^(1/2)*((x^9*(385*b*c^5 + 420*a*c^4*d))/(3465*c^4) - (x*(48*a*d
^5 - 88*b*c*d^4))/(3465*c^4) + (a*c*x^11)/11 + (d*x^7*(3*a*d + 110*b*c))/(
693*c) - (d^2*x^5*(6*a*d - 11*b*c))/(1155*c^2) + (4*d^3*x^3*(6*a*d - 11*b*
c))/(3465*c^3))
```

3.955 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^8 \, dx$

3.955.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^8 \, dx = -\frac{2d(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{315c^3} + \frac{(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{63c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{9c}$$

output
```
-2/315*d*(-4*a*d+9*b*c)*(c+d/x^2)^(5/2)*x^5/c^3+1/63*(-4*a*d+9*b*c)*(c+d/x
^2)^(5/2)*x^7/c^2+1/9*a*(c+d/x^2)^(5/2)*x^9/c
```

3.955.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^8 \, dx = \frac{\sqrt{c+\frac{d}{x^2}}x(d+cx^2)^2 \left(9bc(-2d+5cx^2)+a(8d^2-20cdx^2+35c^2x^4)\right)}{315c^3}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]
```

output
```
(Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(9*b*c*(-2*d + 5*c*x^2) + a*(8*d^2 - 20*c
*d*x^2 + 35*c^2*x^4)))/(315*c^3)
```

3.955.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \textcolor{blue}{955}$$

$$\frac{(9bc-4ad)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^6dx}{9c}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}$$

$$\Big\downarrow \textcolor{blue}{803}$$

$$\frac{(9bc-4ad)\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}-\frac{2d\int \left(c+\frac{d}{x^2}\right)^{3/2}x^4dx}{7c}\right)}{9c}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}$$

$$\Big\downarrow \textcolor{blue}{796}$$

$$\frac{\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}-\frac{2dx^5\left(c+\frac{d}{x^2}\right)^{5/2}}{35c^2}\right)(9bc-4ad)}{9c}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]
```

output
```
(a*(c + d/x^2)^(5/2)*x^9)/(9*c) + ((9*b*c - 4*a*d)*((-2*d*(c + d/x^2)^(5/2
)*x^5)/(35*c^2) + ((c + d/x^2)^(5/2)*x^7)/(7*c)))/(9*c)
```

3.955.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

---

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.955.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(35 a\,x^{4} c^{2}-20 a c d\,x^{2}+45 b\,c^{2} x^{2}+8 a\,d^{2}-18 b c d \right) \left(c\,x^{2}+d \right)}{315 c^{3}}$	67
default	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(35 a\,x^{4} c^{2}-20 a c d\,x^{2}+45 b\,c^{2} x^{2}+8 a\,d^{2}-18 b c d \right) \left(c\,x^{2}+d \right)}{315 c^{3}}$	67
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x \left(35 a\,x^{8} c^{4}+50 a\,c^{3} d\,x^{6}+45 b\,c^{4} x^{6}+3 a\,c^{2} d^{2} x^{4}+72 b\,c^{3} d\,x^{4}-4 a c\,d^{3} x^{2}+9 b\,c^{2} d^{2} x^{2}+8 a\,d^{4}-18 b c\,d^{3}\right)}{315 c^{3}}$	106
trager	$\frac{\left(35 a\,x^{8} c^{4}+50 a\,c^{3} d\,x^{6}+45 b\,c^{4} x^{6}+3 a\,c^{2} d^{2} x^{4}+72 b\,c^{3} d\,x^{4}-4 a c\,d^{3} x^{2}+9 b\,c^{2} d^{2} x^{2}+8 a\,d^{4}-18 b c\,d^{3}\right) x \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{315 c^{3}}$	110

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x,method=_RETURNVERBOSE)
```

output
```
1/315*((c*x^2+d)/x^2)^(3/2)*x^3*(35*a*c^2*x^4-20*a*c*d*x^2+45*b*c^2*x^2+8*
a*d^2-18*b*c*d)*(c*x^2+d)/c^3
```

3.955.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^8 \, dx = \frac{(35\, ac^4x^9 + 5\,(9\, bc^4 + 10\, ac^3d)x^7 + 3\,(24\, bc^3d + ac^2d^2)x^5 + (9\, bc^2d^2 - 4\, acd^3)x^3 - 2\,(9\, bcd^3}{315\, c^3}$$

input

```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="fricas")
```

output

```
1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + 10*a*c^3*d)*x^7 + 3*(24*b*c^3*d + a*c^2
*d^2)*x^5 + (9*b*c^2*d^2 - 4*a*c*d^3)*x^3 - 2*(9*b*c*d^3 - 4*a*d^4)*x)*sqr
t((c*x^2 + d)/x^2)/c^3
```

3.955.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1340 vs. 2(78) = 156.

Time = 3.15 (sec) , antiderivative size = 1340, normalized size of antiderivative = 15.95

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^8 \, dx = \text{Too large to display}$$

input

```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**8,x)
```

output
```
35*a*c**8*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**
6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**7*d**(21/2
)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945
*c**5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**6*d**(23/2)*x**10*sqrt(c*x**
2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 +
 315*c**4*d**12) + 40*a*c**5*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d
**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 1
5*a*c**5*d**(11/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4
*d**5*x**2 + 105*c**3*d**6) - 5*a*c**4*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(
315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*
d**12) + 33*a*c**4*d**(13/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 +
 210*c**4*d**5*x**2 + 105*c**3*d**6) - 30*a*c**3*d**(29/2)*x**4*sqrt(c*x**
2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 +
 315*c**4*d**12) + 17*a*c**3*d**(15/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d
**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) - 40*a*c**2*d**(31/2)*x**2*
sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d*
*11*x**2 + 315*c**4*d**12) + 3*a*c**2*d**(17/2)*x**4*sqrt(c*x**2/d + 1)/(1
05*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) - 16*a*c*d**(33/2)
*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d
**11*x**2 + 315*c**4*d**12) + 12*a*c*d**(19/2)*x**2*sqrt(c*x**2/d + 1)/...
```

3.955.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^8\,dx=\frac{\left(5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5\right)b}{35\,c^2}+\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)a}{315\,c^3}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="maxima")
```

output
```
1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x^2)^(5/2)*d*x^5)*b/c^2 + 1/315*(
35*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x^2)^(5/
2)*d^2*x^5)*a/c^3
```

3.955.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx = \frac{2\left(9\,bcd^{\frac{7}{2}}-4\,ad^{\frac{9}{2}}\right)\mathrm{sgn}(x)}{315\,c^3}$$
$$+\frac{35\,(cx^2+d)^{\frac{9}{2}}a\mathrm{sgn}(x)+45\,(cx^2+d)^{\frac{7}{2}}bc\mathrm{sgn}(x)-90\,(cx^2+d)^{\frac{7}{2}}ad\mathrm{sgn}(x)-63\,(cx^2+d)^{\frac{5}{2}}bcd\mathrm{sgn}(x)+63\,(c}{315\,c^3}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="giac")
```

output
```
2/315*(9*b*c*d^(7/2) - 4*a*d^(9/2))*sgn(x)/c^3 + 1/315*(35*(c*x^2 + d)^(9/
2)*a*sgn(x) + 45*(c*x^2 + d)^(7/2)*b*c*sgn(x) - 90*(c*x^2 + d)^(7/2)*a*d*s
gn(x) - 63*(c*x^2 + d)^(5/2)*b*c*d*sgn(x) + 63*(c*x^2 + d)^(5/2)*a*d^2*sgn
(x))/c^3
```

3.955.9 Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx = \sqrt{c+\frac{d}{x^2}}\,\left(\frac{x\,(8\,a\,d^4-18\,b\,c\,d^3)}{315\,c^3}\right.$$
$$\left.+\frac{x^7\,(45\,b\,c^4+50\,a\,d\,c^3)}{315\,c^3}+\frac{a\,c\,x^9}{9}+\frac{d\,x^5\,(a\,d+24\,b\,c)}{105\,c}-\frac{d^2\,x^3\,(4\,a\,d-9\,b\,c)}{315\,c^2}\right)$$

input
```
int(x^8*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
(c + d/x^2)^(1/2)*((x*(8*a*d^4 - 18*b*c*d^3))/(315*c^3) + (x^7*(45*b*c^4 +
 50*a*c^3*d))/(315*c^3) + (a*c*x^9)/9 + (d*x^5*(a*d + 24*b*c))/(105*c) - (
d^2*x^3*(4*a*d - 9*b*c))/(315*c^2))
```

3.956 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx$

3.956.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx=\frac{(7bc-2ad) \left(c+\frac{d}{x^2}\right)^{5/2} x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{7c}$$

output
```
1/35*(-2*a*d+7*b*c)*(c+d/x^2)^(5/2)*x^5/c^2+1/7*a*(c+d/x^2)^(5/2)*x^7/c
```

3.956.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx=\frac{\sqrt{c+\frac{d}{x^2}}x(d+cx^2)^2 \left(7bc-2ad+5acx^2\right)}{35c^2}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]
```

output
```
(Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(7*b*c - 2*a*d + 5*a*c*x^2))/(35*c^2)
```

3.956.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(7bc-2ad)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^4dx}{7c}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}$$

$$\Big\downarrow \text{796}$$

$$\frac{x^5\left(c+\frac{d}{x^2}\right)^{5/2}(7bc-2ad)}{35c^2}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]
```

output
```
((7*b*c - 2*a*d)*(c + d/x^2)^(5/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(5/2)*x^
7)/(7*c)
```

3.956.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.956.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(5ac\,x^{2}-2ad+7bc\right) \left(c\,x^{2}+d\right)}{35c^{2}}$	45
default	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(5ac\,x^{2}-2ad+7bc\right) \left(c\,x^{2}+d\right)}{35c^{2}}$	45
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x \left(5x^{6}a\,c^{3}+8a\,c^{2}d\,x^{4}+7b\,c^{3}x^{4}+ac\,d^{2}x^{2}+14b\,c^{2}d\,x^{2}-2a\,d^{3}+7bc\,d^{2}\right)}{35c^{2}}$	81
trager	$\frac{\left(5x^{6}a\,c^{3}+8a\,c^{2}d\,x^{4}+7b\,c^{3}x^{4}+ac\,d^{2}x^{2}+14b\,c^{2}d\,x^{2}-2a\,d^{3}+7bc\,d^{2}\right) x \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{35c^{2}}$	85

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x,method=_RETURNVERBOSE)
```

output
```
1/35*((c*x^2+d)/x^2)^(3/2)*x^3*(5*a*c*x^2-2*a*d+7*b*c)*(c*x^2+d)/c^2
```

3.956.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^6\,dx = \frac{\left(5\,ac^{3}x^{7}+\left(7\,bc^{3}+8\,ac^{2}d\right)x^{5}+\left(14\,bc^{2}d+acd^{2}\right)x^{3}+\left(7\,bcd^{2}-2\,ad^{3}\right)x\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{35\,c^{2}}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="fricas")
```

output
```
1/35*(5*a*c^3*x^7 + (7*b*c^3 + 8*a*c^2*d)*x^5 + (14*b*c^2*d + a*c*d^2)*x^3
 + (7*b*c*d^2 - 2*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^2
```

3.956.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(46) = 92.

Time = 2.37 (sec) , antiderivative size = 498, normalized size of antiderivative = 9.40

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx &= \frac{15ac^6d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} \\ &+ \frac{33ac^5d^{\frac{11}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{17ac^4d^{\frac{13}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} \\ &+ \frac{3ac^3d^{\frac{15}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{12ac^2d^{\frac{17}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} \\ &+ \frac{8acd^{\frac{19}{2}}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{ad^{\frac{3}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{ad^{\frac{5}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c} \\ &- \frac{2ad^{\frac{7}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2}+\frac{bc\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{2bd^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{bd^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c}\end{aligned}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**6,x)
```

output
```
15*a*c**6*d**(9/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4
*d**5*x**2 + 105*c**3*d**6) + 33*a*c**5*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/
(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**4*d**(
13/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 1
05*c**3*d**6) + 3*a*c**3*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*
x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c**2*d**(17/2)*x**2*sqrt
(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) +
 8*a*c*d**(19/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x*
*2 + 105*c**3*d**6) + a*d**(3/2)*x**4*sqrt(c*x**2/d + 1)/5 + a*d**(5/2)*x*
*2*sqrt(c*x**2/d + 1)/(15*c) - 2*a*d**(7/2)*sqrt(c*x**2/d + 1)/(15*c**2) +
 b*c*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + 2*b*d**(3/2)*x**2*sqrt(c*x**2/d +
 1)/5 + b*d**(5/2)*sqrt(c*x**2/d + 1)/(5*c)
```

3.956.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx=\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5}{5\,c}+\frac{\left(5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5\right)a}{35\,c^2}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="maxima")
```

output
```
1/5*b*(c + d/x^2)^(5/2)*x^5/c + 1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x
^2)^(5/2)*d*x^5)*a/c^2
```

3.956.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx=-\frac{\left(7\,bcd^{\frac{5}{2}}-2\,ad^{\frac{7}{2}}\right)\mathrm{sgn}(x)}{35\,c^2}+\frac{5\left(cx^2+d\right)^{\frac{7}{2}}a\mathrm{sgn}(x)+7\left(cx^2+d\right)^{\frac{5}{2}}bc\mathrm{sgn}(x)-7\left(cx^2+d\right)^{\frac{5}{2}}ad\mathrm{sgn}(x)}{35\,c^2}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="giac")
```

output
```
-1/35*(7*b*c*d^(5/2) - 2*a*d^(7/2))*sgn(x)/c^2 + 1/35*(5*(c*x^2 + d)^(7/2)
*a*sgn(x) + 7*(c*x^2 + d)^(5/2)*b*c*sgn(x) - 7*(c*x^2 + d)^(5/2)*a*d*sgn(x
))/c^2
```

3.956.9 Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx=\sqrt{c+\frac{d}{x^2}}\left(\frac{x^5\left(7\,b\,c^3+8\,a\,d\,c^2\right)}{35\,c^2}-\frac{x\left(2\,a\,d^3-7\,b\,c\,d^2\right)}{35\,c^2}+\frac{a\,c\,x^7}{7}+\frac{d\,x^3\left(a\,d+14\,b\,c\right)}{35\,c}\right)$$

input
```
int(x^6*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
(c + d/x^2)^(1/2)*((x^5*(7*b*c^3 + 8*a*c^2*d))/(35*c^2) - (x*(2*a*d^3 - 7*
b*c*d^2))/(35*c^2) + (a*c*x^7)/7 + (d*x^3*(a*d + 14*b*c))/(35*c))
```

3.956. $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx$

3.957 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^4 \, dx$

3.957.1 Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^4 \, dx = bd\sqrt{c+\frac{d}{x^2}}x + \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - bd^{3/2}\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)$$

output
```
1/3*b*(c+d/x^2)^(3/2)*x^3+1/5*a*(c+d/x^2)^(5/2)*x^5/c-b*d^(3/2)*arctanh(d^
(1/2)/x/(c+d/x^2)^(1/2))+b*d*x*(c+d/x^2)^(1/2)
```

3.957.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^4 \, dx = \frac{1}{15}\sqrt{c+\frac{d}{x^2}}x\left(\frac{3a(d+cx^2)^2}{c} + 5b(4d+cx^2) - \frac{15bd^{3/2}\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d+cx^2}}\right)$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]
```

output
```
(Sqrt[c + d/x^2]*x*((3*a*(d + c*x^2)^2)/c + 5*b*(4*d + c*x^2) - (15*b*d^(3
/2)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/Sqrt[d + c*x^2]))/15
```

3.957.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = $\{953, 858, 247, 247, 224, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{953}$$

$$b\int\left(c+\frac{d}{x^2}\right)^{3/2}x^2dx+\frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c}$$

$$\Big\downarrow \text{858}$$

$$\frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-b\int\left(c+\frac{d}{x^2}\right)^{3/2}x^4d\frac{1}{x}$$

$$\Big\downarrow \text{247}$$

$$\frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-b\left(d\int\sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x}-\frac{1}{3}x^3\left(c+\frac{d}{x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{247}$$

$$\frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-b\left(d\left(d\int\frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}-x\sqrt{c+\frac{d}{x^2}}\right)-\frac{1}{3}x^3\left(c+\frac{d}{x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-b\left(d\left(d\int\frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}-x\sqrt{c+\frac{d}{x^2}}\right)-\frac{1}{3}x^3\left(c+\frac{d}{x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-b\left(d\left(\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)-x\sqrt{c+\frac{d}{x^2}}\right)-\frac{1}{3}x^3\left(c+\frac{d}{x^2}\right)^{3/2}\right)$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]
```

output
```
(a*(c + d/x^2)^(5/2)*x^5)/(5*c) - b*(-1/3*((c + d/x^2)^(3/2)*x^3) + d*(-(S
qrt[c + d/x^2]*x) + Sqrt[d]*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]))
```

3.957.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 953
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[d/e^n   Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] &&
(IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && G
tQ[m + n, -1]))
```

3.957.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(3a\left(c\,x^{2}+d\right)^{\frac{5}{2}}+5\left(c\,x^{2}+d\right)^{\frac{3}{2}} bc-15d^{\frac{3}{2}} \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right) bc+15\sqrt{c\,x^{2}+d}\,bcd\right)}{15\left(c\,x^{2}+d\right)^{\frac{3}{2}} c}$	99

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/15*((c*x^2+d)/x^2)^(3/2)*x^3*(3*a*(c*x^2+d)^(5/2)+5*(c*x^2+d)^(3/2)*b*c-
15*d^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c+15*(c*x^2+d)^(1/2)*b*c*
d)/(c*x^2+d)^(3/2)/c
```

3.957.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.36

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^4\,dx = \left[\frac{15\,bcd^{\frac{3}{2}}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(3\,ac^2x^5+(5\,bc^2+6\,acd)x^3+(20\,bcd+3\,ad^2)x)}{30\,c}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="fricas")
```

output
```
[1/30*(15*b*c*d^(3/2)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*
d)/x^2) + 2*(3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 + (20*b*c*d + 3*a*d^2)*
x)*sqrt((c*x^2 + d)/x^2))/c, 1/15*(15*b*c*sqrt(-d)*d*arctan(sqrt(-d)*x*sqr
t((c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 +
 (20*b*c*d + 3*a*d^2)*x)*sqrt((c*x^2 + d)/x^2))/c]
```

3.957.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(75) = 150$.

Time = 2.18 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.14

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx = \frac{ac\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{2ad^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{ad^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c}+\frac{b\sqrt{c}dx}{\sqrt{1+\frac{d}{cx^2}}}+\frac{bc\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3}-bd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)+\frac{bd^2}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**4,x)
```

output
```
a*c*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + 2*a*d**(3/2)*x**2*sqrt(c*x**2/d +
1)/5 + a*d**(5/2)*sqrt(c*x**2/d + 1)/(5*c) + b*sqrt(c)*d*x/sqrt(1 + d/(c*x
**2)) + b*c*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + b*d**(3/2)*sqrt(c*x**2/d +
 1)/3 - b*d**(3/2)*asinh(sqrt(d)/(sqrt(c)*x)) + b*d**2/(sqrt(c)*x*sqrt(1 +
 d/(c*x**2)))
```

3.957.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5}{5\,c}+\frac{1}{6}\left(2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3+6\sqrt{c+\frac{d}{x^2}}dx+3\,d^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="maxima")
```

output
```
1/5*a*(c + d/x^2)^(5/2)*x^5/c + 1/6*(2*(c + d/x^2)^(3/2)*x^3 + 6*sqrt(c +
d/x^2)*d*x + 3*d^(3/2)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*
x + sqrt(d))))*b
```

3.957.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.63

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx = \frac{bd^2\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)\mathrm{sgn}(x)}{\sqrt{-d}} - \frac{\left(15\,bcd^2\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+20\,bc\sqrt{-d}d^{\frac{3}{2}}+3\,a\sqrt{-d}d^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{15\,c\sqrt{-d}} + \frac{3\,(cx^2+d)^{\frac{5}{2}}ac^4\mathrm{sgn}(x)+5\,(cx^2+d)^{\frac{3}{2}}bc^5\mathrm{sgn}(x)+15\,\sqrt{cx^2+d}bc^5d\mathrm{sgn}(x)}{15\,c^5}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="giac")
```

output
```
b*d^2*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/15*(15*b*c*d^2*
arctan(sqrt(d)/sqrt(-d)) + 20*b*c*sqrt(-d)*d^(3/2) + 3*a*sqrt(-d)*d^(5/2))
*sgn(x)/(c*sqrt(-d)) + 1/15*(3*(c*x^2 + d)^(5/2)*a*c^4*sgn(x) + 5*(c*x^2 +
 d)^(3/2)*b*c^5*sgn(x) + 15*sqrt(c*x^2 + d)*b*c^5*d*sgn(x))/c^5
```

3.957.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx = \int x^4\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}\,dx$$

input
```
int(x^4*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
int(x^4*(a + b/x^2)*(c + d/x^2)^(3/2), x)
```

3.958 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^2 \, dx$

3.958.1 Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^2 \, dx = -\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x}{3c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^3}{3c}-\frac{1}{2}\sqrt{d}(3bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)$$

output
```
1/3*(2*a*d+3*b*c)*(c+d/x^2)^(3/2)*x/c+1/3*a*(c+d/x^2)^(5/2)*x^3/c-1/2*(2*a
*d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))*d^(1/2)-1/2*d*(2*a*d+3*b*c)*(
c+d/x^2)^(1/2)/c/x
```

3.958.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^2 \, dx = \frac{\sqrt{c+\frac{d}{x^2}}\left(\sqrt{d+cx^2}(-3bd+6bcx^2+8adx^2+2acx^4)-3\sqrt{d}(3bc+2ad)x^2\operatorname{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{6x\sqrt{d+cx^2}}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]
```

output
```
(Sqrt[c + d/x^2]*(Sqrt[d + c*x^2]*(-3*b*d + 6*b*c*x^2 + 8*a*d*x^2 + 2*a*c*
x^4) - 3*Sqrt[d]*(3*b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(6
*x*Sqrt[d + c*x^2])
```

3.958.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 773, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(2ad+3bc)\int \left(c+\frac{d}{x^2}\right)^{3/2}dx}{3c}+\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}$$

$$\Big\downarrow \text{773}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(2ad+3bc)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^2d\frac{1}{x}}{3c}$$

$$\Big\downarrow \text{247}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(2ad+3bc)\left(3d\int \sqrt{c+\frac{d}{x^2}}d\frac{1}{x}-x\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{3c}$$

$$\Big\downarrow \text{211}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(2ad+3bc)\left(3d\left(\frac{1}{2}c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)-x\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{3c}$$

$$\Big\downarrow \text{224}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(2ad+3bc)\left(3d\left(\frac{1}{2}c\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)-x\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{3c}$$

$$\Big\downarrow \text{219}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(2ad+3bc)\left(3d\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)-x\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{3c}$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]
```

output
```
(a*(c + d/x^2)^(5/2)*x^3)/(3*c) - ((3*b*c + 2*a*d)*(-((c + d/x^2)^(3/2)*x)
 + 3*d*(Sqrt[c + d/x^2]/(2*x) + (c*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(
2*Sqrt[d]))))/(3*c)
```

3.958.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.958.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{bd\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{2x}+\frac{\left(c^{2}a\left(\frac{x^{2}\sqrt{c\,x^{2}+d}}{3c}-\frac{2d\sqrt{c\,x^{2}+d}}{3c^{2}}\right)+\sqrt{c\,x^{2}+d}\,bc+2ad\sqrt{c\,x^{2}+d}-\frac{\sqrt{d}\,(2ad+3bc)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)}{2}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{\sqrt{c\,x^{2}+d}}$
default	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}x\left(6d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)a\,x^{2}+9d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)bc\,x^{2}-2(c\,x^{2}+d)^{\frac{3}{2}}ad\,x^{2}-3(c\,x^{2}+d)^{\frac{3}{2}}bc\,x^{2}+3(c\,x^{2}+d)\right.}{6(c\,x^{2}+d)^{\frac{3}{2}}d}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*d/x*((c*x^2+d)/x^2)^(1/2)+(c^2*a*(1/3*x^2/c*(c*x^2+d)^(1/2)-2/3*d/c
^2*(c*x^2+d)^(1/2))+(c*x^2+d)^(1/2)*b*c+2*a*d*(c*x^2+d)^(1/2)-1/2*d^(1/2)*
(2*a*d+3*b*c)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x))*((c*x^2+d)/x^2)^(1/2)
*x/(c*x^2+d)^(1/2)
```

3.958.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.57

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx=\left[\frac{3\,(3\,bc+2\,ad)\sqrt{d}x\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(2\,acx^4+2\,(3\,bc+4\,ad)x^2-3\,bd)}{12\,x}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="fricas")
```

output
```
[1/12*(3*(3*b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 +
 d)/x^2) + 2*d)/x^2) + 2*(2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3*b*d)*sqrt(
(c*x^2 + d)/x^2))/x, 1/6*(3*(3*b*c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*s
qrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3
*b*d)*sqrt((c*x^2 + d)/x^2))/x]
```

3.958.6 Sympy [A] (verification not implemented)

Time = 3.24 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.67

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx = \frac{a\sqrt{c}dx}{\sqrt{1+\frac{d}{cx^2}}}+\frac{ac\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3} + \frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3} - ad^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)+\frac{ad^2}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{3}{2}}x}{\sqrt{1+\frac{d}{cx^2}}} - \frac{b\sqrt{c}d\sqrt{1+\frac{d}{cx^2}}}{2x}+\frac{b\sqrt{c}d}{x\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**2,x)
```

output
```
a*sqrt(c)*d*x/sqrt(1 + d/(c*x**2)) + a*c*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3
 + a*d**(3/2)*sqrt(c*x**2/d + 1)/3 - a*d**(3/2)*asinh(sqrt(d)/(sqrt(c)*x))
 + a*d**2/(sqrt(c)*x*sqrt(1 + d/(c*x**2))) + b*c**(3/2)*x/sqrt(1 + d/(c*x*
*2)) - b*sqrt(c)*d*sqrt(1 + d/(c*x**2))/(2*x) + b*sqrt(c)*d/(x*sqrt(1 + d/
(c*x**2))) - 3*b*c*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x))/2
```

3.958.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.35

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx=\frac{1}{6}\left(2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3+6\sqrt{c+\frac{d}{x^2}}dx+3\,d^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)a+\frac{1}{4}\left(4\sqrt{c+\frac{d}{x^2}}cx-\frac{2\sqrt{c+\frac{d}{x^2}}cdx}{(c+\frac{d}{x^2})x^2-d}+3\,c\sqrt{d}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="maxima")
```

output
```
1/6*(2*(c + d/x^2)^(3/2)*x^3 + 6*sqrt(c + d/x^2)*d*x + 3*d^(3/2)*log((sqrt
(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*a + 1/4*(4*sqrt(c
 + d/x^2)*c*x - 2*sqrt(c + d/x^2)*c*d*x/((c + d/x^2)*x^2 - d) + 3*c*sqrt(d
)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*b
```

3.958.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx=\frac{2\left(cx^2+d\right)^{\frac{3}{2}}ac\mathrm{sgn}(x)+6\sqrt{cx^2+d}bc^2\mathrm{sgn}(x)+6\sqrt{cx^2+d}acd\mathrm{sgn}(x)-\frac{3\sqrt{cx^2+d}bcd\mathrm{sgn}(x)}{x^2}+}{6\,c}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="giac")
```

output
```
1/6*(2*(c*x^2 + d)^(3/2)*a*c*sgn(x) + 6*sqrt(c*x^2 + d)*b*c^2*sgn(x) + 6*s
qrt(c*x^2 + d)*a*c*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c*d*sgn(x)/x^2 + 3*(3*b*
c^2*d*sgn(x) + 2*a*c*d^2*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d)
)/c
```

3.958.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx=\int x^2\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

input
```
int(x^2*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
int(x^2*(a + b/x^2)*(c + d/x^2)^(3/2), x)
```

3.959 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} \, dx$

3.959.1 Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} \, dx = -\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x} - \frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c} - \frac{3c(bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8\sqrt{d}}$$

output
```
-1/4*(4*a*d+b*c)*(c+d/x^2)^(3/2)/c/x+a*(c+d/x^2)^(5/2)*x/c-3/8*c*(4*a*d+b*
c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(1/2)-3/8*(4*a*d+b*c)*(c+d/x^2)^(1
/2)/x
```

3.959.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} \, dx = \frac{\sqrt{c+\frac{d}{x^2}}\left(-2bd-5bcx^2-4adx^2+8acx^4-\frac{3c(bc+4ad)x^4\operatorname{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{d+cx^2}}\right)}{8x^3}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2),x]
```

output
```
(Sqrt[c + d/x^2]*(-2*b*d - 5*b*c*x^2 - 4*a*d*x^2 + 8*a*c*x^4 - (3*c*(b*c +
 4*a*d)*x^4*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(Sqrt[d]*Sqrt[d + c*x^2])))/
(8*x^3)
```

3.959.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {899, 359, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{359}$$

$$\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(4ad+bc)\int \left(c+\frac{d}{x^2}\right)^{3/2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{211}$$

$$\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(4ad+bc)\left(\frac{3}{4}c\int \sqrt{c+\frac{d}{x^2}}d\frac{1}{x}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{c}$$

$$\Big\downarrow \text{211}$$

$$\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(4ad+bc)\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{c}$$

$$\Big\downarrow \text{224}$$

$$\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(4ad+bc)\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{c}$$

$$\Big\downarrow 219$$

$$\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(4ad+bc)\left(\frac{3}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{c}$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2),x]
```

output
```
(a*(c + d/x^2)^(5/2)*x)/c - ((b*c + 4*a*d)*((c + d/x^2)^(3/2)/(4*x) + (3*c
*(Sqrt[c + d/x^2]/(2*x) + (c*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*Sqrt
[d])))/4))/c
```

3.959.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.959.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\left(4 a d \,x^{2}+5 c b \,x^{2}+2 b d \right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}{8 x^{3}}+\frac{c \left(8 a \sqrt{c \,x^{2}+d}-\frac{\left(12 a d +3 b c \right) \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{c \,x^{2}+d}}{x}\right)}{\sqrt{d}}\right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}{8 \sqrt{c \,x^{2}+d}}$
default	$-\frac{\left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(-4 \left(c \,x^{2}+d \right)^{\frac{3}{2}} a c d \,x^{4}-\left(c \,x^{2}+d \right)^{\frac{3}{2}} b \,c^{2} x^{4}+12 d^{\frac{5}{2}} \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{c \,x^{2}+d}}{x}\right) a c \,x^{4}+3 d^{\frac{3}{2}} \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{c \,x^{2}+d}}{x}\right) b \,c^{2} x^{4}+4 (c \,x^{2}}{8 x \left(c \,x^{2}+d \right)^{\frac{3}{2}} d^{2}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(4*a*d*x^2+5*b*c*x^2+2*b*d)/x^3*((c*x^2+d)/x^2)^(1/2)+1/8*c*(8*a*(c*x
^2+d)^(1/2)-(12*a*d+3*b*c)/d^(1/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x))*
((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

3.959.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.93

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} \, dx = \left[\frac{3 \, {\left(b c^{2} + 4 \, a c d\right)} \sqrt{d} x^{3} \log\left(-\frac{c x^{2} - 2 \, \sqrt{d} x \sqrt{\frac{c x^{2} + d}{x^{2}}} + 2 \, d}{x^{2}}\right) + 2 \, {\left(8 \, a c d x^{4} - 2 \, b d^{2} - {\left(5 \, b c d + 4 \, a d^{2}\right)} x^{2}\right)} \sqrt{}}{16 \, d x^{3}}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*(b*c^2 + 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x
^2 + d)/x^2) + 2*d)/x^2) + 2*(8*a*c*d*x^4 - 2*b*d^2 - (5*b*c*d + 4*a*d^2)*
x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^3), 1/8*(3*(b*c^2 + 4*a*c*d)*sqrt(-d)*x^3
*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (8*a*c*d*x^4 - 2*b
*d^2 - (5*b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^3)]
```

3.959.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(100) = 200$.

Time = 5.38 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.93

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}\,dx = \frac{ac^{\frac{3}{2}}x}{\sqrt{1+\frac{d}{cx^2}}} - \frac{a\sqrt{c}d\sqrt{1+\frac{d}{cx^2}}}{2x} + \frac{a\sqrt{c}d}{x\sqrt{1+\frac{d}{cx^2}}} - \frac{3ac\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2} - \frac{bc^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x} - \frac{bc^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}} - \frac{3b\sqrt{c}d}{8x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{3bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8\sqrt{d}} - \frac{bd^2}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2),x)
```

output
```
a*c**(3/2)*x/sqrt(1 + d/(c*x**2)) - a*sqrt(c)*d*sqrt(1 + d/(c*x**2))/(2*x)
 + a*sqrt(c)*d/(x*sqrt(1 + d/(c*x**2))) - 3*a*c*sqrt(d)*asinh(sqrt(d)/(sqr
t(c)*x))/2 - b*c**(3/2)*sqrt(1 + d/(c*x**2))/(2*x) - b*c**(3/2)/(8*x*sqrt(
1 + d/(c*x**2))) - 3*b*sqrt(c)*d/(8*x**3*sqrt(1 + d/(c*x**2))) - 3*b*c**2*
asinh(sqrt(d)/(sqrt(c)*x))/(8*sqrt(d)) - b*d**2/(4*sqrt(c)*x**5*sqrt(1 + d
/(c*x**2)))
```

3.959.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(94) = 188$.

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.85

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx = \frac{1}{4}\left(4\sqrt{c+\frac{d}{x^2}}cx - \frac{2\sqrt{c+\frac{d}{x^2}}cdx}{\left(c+\frac{d}{x^2}\right)x^2-d} + 3c\sqrt{d}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)a + \frac{1}{16}\left(\frac{3c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{\sqrt{d}} - \frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3 - 3\sqrt{c+\frac{d}{x^2}}c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2x^4 - 2\left(c+\frac{d}{x^2}\right)dx^2 + d^2}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*sqrt(c + d/x^2)*c*x - 2*sqrt(c + d/x^2)*c*d*x/((c + d/x^2)*x^2 - d)
 + 3*c*sqrt(d)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt
(d))))*a + 1/16*(3*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*
x + sqrt(d)))/sqrt(d) - 2*(5*(c + d/x^2)^(3/2)*c^2*x^3 - 3*sqrt(c + d/x^2)
*c^2*d*x)/((c + d/x^2)^2*x^4 - 2*(c + d/x^2)*d*x^2 + d^2))*b
```

3.959.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.29

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}\,dx=\frac{8\sqrt{cx^2+d}ac^2\mathrm{sgn}(x)+\frac{3\left(bc^3\mathrm{sgn}(x)+4\,ac^2d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\frac{5\left(cx^2+d\right)^{\frac{3}{2}}bc^3\mathrm{sgn}(x)+4\left(cx^2+d\right)^{\frac{3}{2}}ac^2d}{}}{8\,c}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(8*sqrt(c*x^2 + d)*a*c^2*sgn(x) + 3*(b*c^3*sgn(x) + 4*a*c^2*d*sgn(x))*
arctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) - (5*(c*x^2 + d)^(3/2)*b*c^3*sgn
(x) + 4*(c*x^2 + d)^(3/2)*a*c^2*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^3*d*sgn(x
) - 4*sqrt(c*x^2 + d)*a*c^2*d^2*sgn(x))/(c^2*x^4))/c
```

3.959.9 Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}\,dx=\frac{a\,x\,{\left(c\,x^2+d\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{d}{c\,x^2}\right)}{{\left(\frac{d}{c}+x^2\right)}^{3/2}}-\frac{b\,{\left(c\,x^2+d\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{d}{c\,x^2}\right)}{x\,{\left(\frac{d}{c}+x^2\right)}^{3/2}}$$

input
```
int((a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
(a*x*(d + c*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -d/(c*x^2)))/(d/c + x^
2)^(3/2) - (b*(d + c*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -d/(c*x^2)))/(
x*(d/c + x^2)^(3/2))
```

3.959. $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} dx$

3.960 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx$

3.960.1 Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx=\frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx}+\frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}+\frac{c^2(bc-6ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{16d^{3/2}}$$

output

```
1/24*(-6*a*d+b*c)*(c+d/x^2)^(3/2)/d/x-1/6*b*(c+d/x^2)^(5/2)/d/x+1/16*c^2*(
-6*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(3/2)+1/16*c*(-6*a*d+b*c)
*(c+d/x^2)^(1/2)/d/x
```

3.960.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}\left(-\sqrt{d}(6adx^2(2d+5cx^2))+b(8d^2+14cdx^2+3c^2x^4))+\frac{3c^2(bc-6ad)x^6\mathrm{ar}}{\sqrt{d}}\right.}{48d^{3/2}x^5}$$

input

```
Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[c + d/x^2]*(-(Sqrt[d]*(6*a*d*x^2*(2*d + 5*c*x^2) + b*(8*d^2 + 14*c*d
*x^2 + 3*c^2*x^4))) + (3*c^2*(b*c - 6*a*d)*x^6*ArcTanh[Sqrt[d + c*x^2]/Sqr
t[d]])/Sqrt[d + c*x^2]))/(48*d^(3/2)*x^5)
```

3.960.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {959, 858, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{ 959}$$

$$-\frac{(bc-6ad)\int \frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}dx}{6d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{(bc-6ad)\int \left(c+\frac{d}{x^2}\right)^{3/2}d\frac{1}{x}}{6d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{(bc-6ad)\left(\frac{3}{4}c\int \sqrt{c+\frac{d}{x^2}}d\frac{1}{x}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{6d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{(bc-6ad)\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{6d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{(bc-6ad)\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{6d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(bc-6ad)\left(\frac{3}{4}c\left(\frac{c\operatorname{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{6d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

input
```
Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]
```

output
```
-1/6*(b*(c + d/x^2)^(5/2))/(d*x) + ((b*c - 6*a*d)*((c + d/x^2)^(3/2)/(4*x)
 + (3*c*(Sqrt[c + d/x^2]/(2*x) + (c*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/
(2*Sqrt[d])))/4))/(6*d)
```

3.960.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.960. $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx$

3.960.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\left(30acd\,x^{4}+3b\,c^{2}x^{4}+12a\,d^{2}x^{2}+14bcd\,x^{2}+8b\,d^{2}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{48x^{5}d}-\frac{c^{2}\left(6ad-bc\right)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{16d^{\frac{3}{2}}\sqrt{c\,x^{2}+d}}$
default	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}\left(18d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)a\,c^{2}x^{6}-3d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)b\,c^{3}x^{6}-6\left(c\,x^{2}+d\right)^{\frac{3}{2}}a\,c^{2}d\,x^{6}+\left(c\,x^{2}+d\right)^{\frac{3}{2}}b\,c^{3}x^{6}+6(c\,x\right.}{48x^{3}(}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(30*a*c*d*x^4+3*b*c^2*x^4+12*a*d^2*x^2+14*b*c*d*x^2+8*b*d^2)/x^5/d*(
(c*x^2+d)/x^2)^(1/2)-1/16*c^2*(6*a*d-b*c)/d^(3/2)*ln((2*d+2*d^(1/2)*(c*x^2
+d)^(1/2))/x)*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

3.960.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.00

$$\int\frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx=\left[-\frac{3\left(bc^3-6\,ac^2d\right)\sqrt{d}x^5\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(3\left(bc^2d+10\,acd^2\right)x^4\right.}{96\,d^2x^5}\right.$$

$$-\frac{3\left(bc^3-6\,ac^2d\right)\sqrt{-d}x^5\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(3\left(bc^2d+10\,acd^2\right)x^4+8\,bd^3+2\left(7\,bcd^2+6\,ad^3\right)x^2\right)\sqrt{\frac{cx^2}{x^2}}}{48\,d^2x^5}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[-1/96*(3*(b*c^3 - 6*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((
c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 +
2*(7*b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^2*x^5), -1/48*(3*(b
*c^3 - 6*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*
x^2 + d)) + (3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 + 2*(7*b*c*d^2 + 6*a*d
^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^2*x^5)]
```

3.960.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(102) = 204$.

Time = 10.45 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.06

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx = -\frac{ac^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x} - \frac{ac^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}} - \frac{3a\sqrt{c}d}{8x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{3ac^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8\sqrt{d}} - \frac{ad^2}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}} - \frac{bc^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{cx^2}}} - \frac{17bc^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{11b\sqrt{c}d}{24x^5\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{16d^{\frac{3}{2}}} - \frac{bd^2}{6\sqrt{c}x^7\sqrt{1+\frac{d}{cx^2}}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**2,x)
```

output
```
-a*c**(3/2)*sqrt(1 + d/(c*x**2))/(2*x) - a*c**(3/2)/(8*x*sqrt(1 + d/(c*x**
2))) - 3*a*sqrt(c)*d/(8*x**3*sqrt(1 + d/(c*x**2))) - 3*a*c**2*asinh(sqrt(d
)/(sqrt(c)*x))/(8*sqrt(d)) - a*d**2/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))
- b*c**(5/2)/(16*d*x*sqrt(1 + d/(c*x**2))) - 17*b*c**(3/2)/(48*x**3*sqrt(1
 + d/(c*x**2))) - 11*b*sqrt(c)*d/(24*x**5*sqrt(1 + d/(c*x**2))) + b*c**3*a
sinh(sqrt(d)/(sqrt(c)*x))/(16*d**(3/2)) - b*d**2/(6*sqrt(c)*x**7*sqrt(1 +
d/(c*x**2)))
```

3.960.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(103) = 206$.

Time = 0.30 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.24

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx = \frac{1}{16}\left(\frac{3\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{\sqrt{d}} - \frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3 - 3\sqrt{c+\frac{d}{x^2}}c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2x^4 - 2\left(c+\frac{d}{x^2}\right)dx^2 + d^2}\right)a - \frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5 + 8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3 - 3\sqrt{c+\frac{d}{x^2}}c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3dx^6 - 3\left(c+\frac{d}{x^2}\right)^2d^2x^4 + 3\left(c+\frac{d}{x^2}\right)d^3x^2 - d^4}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
1/16*(3*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)
))/sqrt(d) - 2*(5*(c + d/x^2)^(3/2)*c^2*x^3 - 3*sqrt(c + d/x^2)*c^2*d*x)/(
(c + d/x^2)^2*x^4 - 2*(c + d/x^2)*d*x^2 + d^2))*a - 1/96*(3*c^3*log((sqrt(
c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2) + 2*(3*(c +
 d/x^2)^(5/2)*c^3*x^5 + 8*(c + d/x^2)^(3/2)*c^3*d*x^3 - 3*sqrt(c + d/x^2)*
c^3*d^2*x)/((c + d/x^2)^3*d*x^6 - 3*(c + d/x^2)^2*d^2*x^4 + 3*(c + d/x^2)*
d^3*x^2 - d^4))*b
```

3.960.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.41

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx =$$
$$-\frac{\frac{3\left(bc^4\operatorname{sgn}(x)-6\,ac^3d\operatorname{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d}+\frac{3\left(cx^2+d\right)^{\frac{5}{2}}bc^4\operatorname{sgn}(x)+30\left(cx^2+d\right)^{\frac{5}{2}}ac^3d\operatorname{sgn}(x)+8\left(cx^2+d\right)^{\frac{3}{2}}bc^4d\operatorname{sgn}(x)-48\left(cx^2+d\right)^{\frac{3}{2}}ac^3d}{c^3dx^6}}{48\,c}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
-1/48*(3*(b*c^4*sgn(x) - 6*a*c^3*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d)
)/(sqrt(-d)*d) + (3*(c*x^2 + d)^(5/2)*b*c^4*sgn(x) + 30*(c*x^2 + d)^(5/2)*
a*c^3*d*sgn(x) + 8*(c*x^2 + d)^(3/2)*b*c^4*d*sgn(x) - 48*(c*x^2 + d)^(3/2)
*a*c^3*d^2*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^4*d^2*sgn(x) + 18*sqrt(c*x^2 + d
)*a*c^3*d^3*sgn(x))/(c^3*d*x^6))/c
```

3.960.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)\,\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^2, x)
```

3.961 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$

3.961.1 Optimal result

Integrand size = 22, antiderivative size = 159

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{c^2(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{128d^2x}-\frac{c^3(3bc-8ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{128d^{5/2}}$$

output
```
1/48*(-8*a*d+3*b*c)*(c+d/x^2)^(3/2)/d/x^3-1/8*b*(c+d/x^2)^(5/2)/d/x^3-1/12
8*c^3*(-8*a*d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(5/2)+1/64*c*(-8
*a*d+3*b*c)*(c+d/x^2)^(1/2)/d/x^3+1/128*c^2*(-8*a*d+3*b*c)*(c+d/x^2)^(1/2)
/d^2/x
```

3.961.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx =$$

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,\Big(\sqrt{d}\sqrt{d+cx^2}(8adx^2(8d^2+14cdx^2+3c^2x^4)+b(48d^3+72cd^2x^2+6c^2dx^4-9c^3x^6))+3c^3(3bc-}{384d^{5/2}x^7\sqrt{d+cx^2}}$$

input
```
Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]
```

output
```
-1/384*(Sqrt[c + d/x^2]*(Sqrt[d]*Sqrt[d + c*x^2]*(8*a*d*x^2*(8*d^2 + 14*c*
d*x^2 + 3*c^2*x^4) + b*(48*d^3 + 72*c*d^2*x^2 + 6*c^2*d*x^4 - 9*c^3*x^6))
+ 3*c^3*(3*b*c - 8*a*d)*x^8*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(d^(5/2)*x^
7*Sqrt[d + c*x^2])
```

3.961.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {959, 858, 248, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{959}$$

$$-\frac{(3bc-8ad)\int \frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}dx}{8d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

$$\Big\downarrow \text{858}$$

$$\frac{(3bc-8ad)\int \frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}d\frac{1}{x}}{8d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

$$\Big\downarrow \text{248}$$

$$\frac{(3bc-8ad)\left(\frac{1}{2}c\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2}d\frac{1}{x}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{6x^3}\right)}{8d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

$$\Big\downarrow \text{248}$$

$$\frac{(3bc-8ad)\left(\frac{1}{2}c\left(\frac{1}{4}c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{6x^3}\right)}{8d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

$$\Big\downarrow \text{262}$$

$$\frac{(3bc-8ad)\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int\frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{2d}\right)+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{6x^3}\right)}{8d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

$$\Big\downarrow \text{224}$$

$$\frac{(3bc-8ad)\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int\frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{2d}\right)+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{6x^3}\right)}{8d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3bc-8ad)\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\operatorname{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}\right)+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{6x^3}\right)}{8d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

input
```
Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]
```

output
```
-1/8*(b*(c + d/x^2)^(5/2))/(d*x^3) + ((3*b*c - 8*a*d)*((c + d/x^2)^(3/2)/(
6*x^3) + (c*(Sqrt[c + d/x^2]/(4*x^3) + (c*(Sqrt[c + d/x^2]/(2*d*x) - (c*Ar
cTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*d^(3/2))))/4))/2))/(8*d)
```

3.961.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.961. $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.961.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{\left(24a\,c^{2}d\,x^{6}-9b\,c^{3}x^{6}+112ac\,d^{2}x^{4}+6b\,c^{2}d\,x^{4}+64a\,d^{3}x^{2}+72bc\,d^{2}x^{2}+48b\,d^{3}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{384x^{7}d^{2}}+\frac{c^{3}(8ad-3bc)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)}{128d^{\frac{5}{2}}\sqrt{c\,x^{2}+d}}$
default	$\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}\left(-8\left(c\,x^{2}+d\right)^{\frac{3}{2}}a\,c^{3}d\,x^{8}+3\left(c\,x^{2}+d\right)^{\frac{3}{2}}b\,c^{4}x^{8}+24d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)a\,c^{3}x^{8}-9d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)b\,c^{4}x^{8}+8(c\,x$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/384*(24*a*c^2*d*x^6-9*b*c^3*x^6+112*a*c*d^2*x^4+6*b*c^2*d*x^4+64*a*d^3*
x^2+72*b*c*d^2*x^2+48*b*d^3)/x^7/d^2*((c*x^2+d)/x^2)^(1/2)+1/128*c^3*(8*a*
d-3*b*c)/d^(5/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)*((c*x^2+d)/x^2)^(1/
2)*x/(c*x^2+d)^(1/2)
```

3.961.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = \left[-\frac{3\,(3\,bc^4-8\,ac^3d)\sqrt{d}x^7\log\left(-\frac{cx^2+2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\,(3\,(3\,bc^3d-8\,ac^2d^2)}{768\,d^3x}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[-1/768*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(d)*x^7*log(-(c*x^2 + 2*sqrt(d)*x*sqr
t((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(3*b*c^3*d - 8*a*c^2*d^2)*x^6 - 48*b
*d^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^3 + 8*a*d^4)*x^2)*sqr
t((c*x^2 + d)/x^2))/(d^3*x^7), 1/384*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(-d)*x^7
*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*(3*b*c^3*d - 8*
a*c^2*d^2)*x^6 - 48*b*d^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^
3 + 8*a*d^4)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^7)]
```

3.961.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(141) = 282.

Time = 31.78 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.81

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx &= -\frac{ac^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{cx^2}}}-\frac{17ac^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{11a\sqrt{c}d}{24x^5\sqrt{1+\frac{d}{cx^2}}}\\&\quad+\frac{ac^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{16d^{\frac{3}{2}}}-\frac{ad^2}{6\sqrt{c}x^7\sqrt{1+\frac{d}{cx^2}}}+\frac{3bc^{\frac{7}{2}}}{128d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{5}{2}}}{128dx^3\sqrt{1+\frac{d}{cx^2}}}\\&\quad-\frac{13bc^{\frac{3}{2}}}{64x^5\sqrt{1+\frac{d}{cx^2}}}-\frac{5b\sqrt{c}d}{16x^7\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc^4\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{128d^{\frac{5}{2}}}-\frac{bd^2}{8\sqrt{c}x^9\sqrt{1+\frac{d}{cx^2}}}\end{aligned}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**4,x)
```

output
```
-a*c**(5/2)/(16*d*x*sqrt(1 + d/(c*x**2))) - 17*a*c**(3/2)/(48*x**3*sqrt(1
+ d/(c*x**2))) - 11*a*sqrt(c)*d/(24*x**5*sqrt(1 + d/(c*x**2))) + a*c**3*as
inh(sqrt(d)/(sqrt(c)*x))/(16*d**(3/2)) - a*d**2/(6*sqrt(c)*x**7*sqrt(1 + d
/(c*x**2))) + 3*b*c**(7/2)/(128*d**2*x*sqrt(1 + d/(c*x**2))) + b*c**(5/2)/
(128*d*x**3*sqrt(1 + d/(c*x**2))) - 13*b*c**(3/2)/(64*x**5*sqrt(1 + d/(c*x
**2))) - 5*b*sqrt(c)*d/(16*x**7*sqrt(1 + d/(c*x**2))) - 3*b*c**4*asinh(sqr
t(d)/(sqrt(c)*x))/(128*d**(5/2)) - b*d**2/(8*sqrt(c)*x**9*sqrt(1 + d/(c*x*
*2)))
```

3.961.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(135) = 270$.

Time = 0.28 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.23

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx =$$

$$-\frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5+8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3-3\sqrt{c+\frac{d}{x^2}}c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3dx^6-3\left(c+\frac{d}{x^2}\right)^2d^2x^4+3\left(c+\frac{d}{x^2}\right)d^3x^2-d^4}\right)a$$

$$+\frac{1}{256}\left(\frac{3\,c^4\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^4x^7-11\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^4dx^5-11\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^4d^2x^3+3\sqrt{c+\frac{d}{x^2}}\right.}{\left(c+\frac{d}{x^2}\right)^4d^2x^8-4\left(c+\frac{d}{x^2}\right)^3d^3x^6+6\left(c+\frac{d}{x^2}\right)^2d^4x^4-4\left(c+\frac{d}{x^2}\right)d^5x^2+}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
-1/96*(3*c^3*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d
)))/d^(3/2) + 2*(3*(c + d/x^2)^(5/2)*c^3*x^5 + 8*(c + d/x^2)^(3/2)*c^3*d*x
^3 - 3*sqrt(c + d/x^2)*c^3*d^2*x)/((c + d/x^2)^3*d*x^6 - 3*(c + d/x^2)^2*d
^2*x^4 + 3*(c + d/x^2)*d^3*x^2 - d^4))*a + 1/256*(3*c^4*log((sqrt(c + d/x^
2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) + 2*(3*(c + d/x^2)^
(7/2)*c^4*x^7 - 11*(c + d/x^2)^(5/2)*c^4*d*x^5 - 11*(c + d/x^2)^(3/2)*c^4*
d^2*x^3 + 3*sqrt(c + d/x^2)*c^4*d^3*x)/((c + d/x^2)^4*d^2*x^8 - 4*(c + d/x
^2)^3*d^3*x^6 + 6*(c + d/x^2)^2*d^4*x^4 - 4*(c + d/x^2)*d^5*x^2 + d^6))*b
```

3.961.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = \frac{\frac{3\left(3\,bc^5\operatorname{sgn}(x)-8\,ac^4d\operatorname{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d^2} + \frac{9\left(cx^2+d\right)^{\frac{7}{2}}bc^5\operatorname{sgn}(x)-24\left(cx^2+d\right)^{\frac{7}{2}}ac^4d\operatorname{sgn}(x)-33\,(}{}}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
1/384*(3*(3*b*c^5*sgn(x) - 8*a*c^4*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-
d))/(sqrt(-d)*d^2) + (9*(c*x^2 + d)^(7/2)*b*c^5*sgn(x) - 24*(c*x^2 + d)^(7
/2)*a*c^4*d*sgn(x) - 33*(c*x^2 + d)^(5/2)*b*c^5*d*sgn(x) - 40*(c*x^2 + d)^
(5/2)*a*c^4*d^2*sgn(x) - 33*(c*x^2 + d)^(3/2)*b*c^5*d^2*sgn(x) + 88*(c*x^2
 + d)^(3/2)*a*c^4*d^3*sgn(x) + 9*sqrt(c*x^2 + d)*b*c^5*d^3*sgn(x) - 24*sqr
t(c*x^2 + d)*a*c^4*d^4*sgn(x))/(c^4*d^2*x^8))/c
```

3.961.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)\,\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x)
```

output
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^4, x)
```

3.962 $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx$

3.962.1 Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}\,x^4}{4c}-\frac{d(4bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}$$

output
```
-1/8*d*(-3*a*d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(5/2)+1/8*(-3*a*d
+4*b*c)*x^2*(c+d/x^2)^(1/2)/c^2+1/4*a*x^4*(c+d/x^2)^(1/2)/c
```

3.962.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

$$=\frac{\sqrt{c}(d+cx^2)\,(4bc-3ad+2acx^2)+\frac{2d(-4bc+3ad)\sqrt{d+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{d}+\sqrt{d+cx^2}}\right)}{x}}{8c^{5/2}\sqrt{c+\frac{d}{x^2}}}$$

input
```
Integrate[((a + b/x^2)*x^3)/Sqrt[c + d/x^2],x]
```

output
```
(Sqrt[c]*(d + c*x^2)*(4*b*c - 3*a*d + 2*a*c*x^2) + (2*d*(-4*b*c + 3*a*d)*S
qrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[d + c*x^2])])/x)/(8*c^
(5/2)*Sqrt[c + d/x^2])
```

3.962.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {948, 87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+\frac{b}{x^2}\right)}{\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)x^6}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{2c}-\frac{(4bc-3ad)\int \frac{x^4}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}}{4c}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{2c}-\frac{(4bc-3ad)\left(-\frac{d\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}}{2c}-\frac{x^2\sqrt{c+\frac{d}{x^2}}}{c}\right)}{4c}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{2c}-\frac{(4bc-3ad)\left(-\frac{\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{c}-\frac{x^2\sqrt{c+\frac{d}{x^2}}}{c}\right)}{4c}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{2c}-\frac{(4bc-3ad)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{x^2\sqrt{c+\frac{d}{x^2}}}{c}\right)}{4c}\right)$$

input
```
Int[((a + b/x^2)*x^3)/Sqrt[c + d/x^2],x]
```

output
```
((a*Sqrt[c + d/x^2]*x^4)/(2*c) - ((4*b*c - 3*a*d)*(-((Sqrt[c + d/x^2]*x^2)
/c) + (d*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/c^(3/2)))/(4*c))/2
```

3.962.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.962.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

method	result	size
risch	$\frac{\left(2 a c \,x^{2}-3 a d +4 b c \right) \left(c \,x^{2}+d \right)}{8 c^{2} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}+\frac{d \left(3 a d -4 b c \right) \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+d}\right) \sqrt{c \,x^{2}+d}}{8 c^{\frac{5}{2}} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}$	99
default	$\frac{\sqrt{c \,x^{2}+d}\, \left(2 \sqrt{c \,x^{2}+d}\, c^{\frac{5}{2}} a \,x^{3}-3 \sqrt{c \,x^{2}+d}\, c^{\frac{3}{2}} a d x +4 \sqrt{c \,x^{2}+d}\, c^{\frac{5}{2}} b x +3 \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+d}\right) a c \,d^{2}-4 \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+d}\right) b \,c^{2} d \right)}{8 \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x \,c^{\frac{7}{2}}}$	129

input
```
int((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*a*c*x^2-3*a*d+4*b*c)*(c*x^2+d)/c^2/((c*x^2+d)/x^2)^(1/2)+1/8*d*(3*a
*d-4*b*c)/c^(5/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))/((c*x^2+d)/x^2)^(1/2)/x*(c
*x^2+d)^(1/2)
```

3.962.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = \left[-\frac{(4\,bcd - 3\,ad^2)\sqrt{c}\log\left(-2\,cx^2 - 2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}} - d\right) - 2\,(2\,ac^2x^4 + (4\,bc^2 - 3\,acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16\,c^3}, \quad (4\,bcd\right.$$

input
```
integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((4*b*c*d - 3*a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x
^2 + d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2
+ d)/x^2))/c^3, 1/8*((4*b*c*d - 3*a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt
((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*s
qrt((c*x^2 + d)/x^2))/c^3]
```

3.962.6 Sympy [A] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.67

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{ax^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} - \frac{a\sqrt{d}x^3}{8c\sqrt{\frac{cx^2}{d}+1}} - \frac{3ad^{\frac{3}{2}}x}{8c^2\sqrt{\frac{cx^2}{d}+1}} + \frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{5}{2}}} + \frac{b\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2c} - \frac{bd\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}}$$

input
```
integrate((a+b/x**2)*x**3/(c+d/x**2)**(1/2),x)
```

output
```
a*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) - a*sqrt(d)*x**3/(8*c*sqrt(c*x**2/d
+ 1)) - 3*a*d**(3/2)*x/(8*c**2*sqrt(c*x**2/d + 1)) + 3*a*d**2*asinh(sqrt(c
)*x/sqrt(d))/(8*c**(5/2)) + b*sqrt(d)*x*sqrt(c*x**2/d + 1)/(2*c) - b*d*asi
nh(sqrt(c)*x/sqrt(d))/(2*c**(3/2))
```

3.962.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(74) = 148.

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.98

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{1}{4}\,b\left(\frac{2\sqrt{c+\frac{d}{x^2}}d}{\left(c+\frac{d}{x^2}\right)c-c^2} + \frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right) - \frac{1}{16}\,a\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}} + \frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2-5\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c^2-2\left(c+\frac{d}{x^2}\right)c^3+c^4}\right)$$

input
```
integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*b*(2*sqrt(c + d/x^2)*d/((c + d/x^2)*c - c^2) + d*log((sqrt(c + d/x^2)
- sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2)) - 1/16*a*(3*d^2*log((sqrt
(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(5/2) + 2*(3*(c + d/
x^2)^(3/2)*d^2 - 5*sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)^2*c^2 - 2*(c + d/x^
2)*c^3 + c^4))
```

3.962.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.23

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{1}{8}\sqrt{cx^2+d}x\left(\frac{2\,ax^2}{c\mathrm{sgn}\left(x\right)}+\frac{4\,bc^2\mathrm{sgn}(x)-3\,acd\mathrm{sgn}(x)}{c^3}\right) - \frac{\left(4\,bcd\log\left(|d|\right)-3\,ad^2\log\left(|d|\right)\right)\mathrm{sgn}(x)}{16\,c^{\frac{5}{2}}} + \frac{\left(4\,bcd-3\,ad^2\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{8\,c^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(c*x^2 + d)*x*(2*a*x^2/(c*sgn(x)) + (4*b*c^2*sgn(x) - 3*a*c*d*sgn(
x))/c^3) - 1/16*(4*b*c*d*log(abs(d)) - 3*a*d^2*log(abs(d)))*sgn(x)/c^(5/2)
 + 1/8*(4*b*c*d - 3*a*d^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(5/2)
*sgn(x))
```

3.962.9 Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{5\,a\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8\,c} - \frac{3\,a\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c^2} + \frac{b\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2\,c} - \frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{3/2}} + \frac{3\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{5/2}}$$

input
```
int((x^3*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

output
```
(5*a*x^4*(c + d/x^2)^(1/2))/(8*c) - (3*a*x^4*(c + d/x^2)^(3/2))/(8*c^2) +
(b*x^2*(c + d/x^2)^(1/2))/(2*c) - (b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(
2*c^(3/2)) + (3*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(5/2))
```

3.962. $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx$

3.963 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx$

3.963.1 Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\sqrt{c+\frac{d}{x^2}}x^2}{2c}+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}$$

output
```
1/2*(-a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(3/2)+1/2*a*x^2*(c+d/x
^2)^(1/2)/c
```

3.963.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\sqrt{c}x(d+cx^2)+(-2bc+ad)\sqrt{d+cx^2}\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)}{2c^{3/2}\sqrt{c+\frac{d}{x^2}}x}$$

input
```
Integrate[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]
```

output
```
(a*Sqrt[c]*x*(d + c*x^2) + (-2*b*c + a*d)*Sqrt[d + c*x^2]*Log[-(Sqrt[c]*x)
 + Sqrt[d + c*x^2]])/(2*c^(3/2)*Sqrt[c + d/x^2]*x)
```

3.963.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+\frac{b}{x^2}\right)}{\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \ 948$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \ 87$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+\frac{d}{x^2}}}{c}-\frac{(2bc-ad)\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}}{2c}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+\frac{d}{x^2}}}{c}-\frac{(2bc-ad)\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{cd}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2}\left(\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{ax^2\sqrt{c+\frac{d}{x^2}}}{c}\right)$$

input
```
Int[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]
```

output
```
((a*Sqrt[c + d/x^2]*x^2)/c + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c
]])/c^(3/2))/2
```

3.963. $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}dx$

3.963.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.963.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

method	result	size
risch	$\frac{a\left(c\,x^{2}+d\right)}{2c\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}-\frac{\left(a d -2 b c \right) \ln\left(\sqrt{c}\,x +\sqrt{c\,x^{2}+d}\right) \sqrt{c\,x^{2}+d}}{2 c^{\frac{3}{2}} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x}$	82
default	$\frac{\sqrt{c\,x^{2}+d}\,\left(\sqrt{c\,x^{2}+d}\,c^{\frac{3}{2}} a x +2 b \ln\left(\sqrt{c}\,x +\sqrt{c\,x^{2}+d}\right) c^{2}-\ln\left(\sqrt{c}\,x +\sqrt{c\,x^{2}+d}\right) a c d \right)}{2 \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x\, c^{\frac{5}{2}}}$	90

input
```
int((a+b/x^2)*x/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/c*a*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)-1/2*(a*d-2*b*c)/c^(3/2)*ln(c^(1/2)
*x+(c*x^2+d)^(1/2))/((c*x^2+d)/x^2)^(1/2)/x*(c*x^2+d)^(1/2)
```

3.963.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.47

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

$$= \left[\frac{2\,acx^2\sqrt{\frac{cx^2+d}{x^2}} - (2\,bc-ad)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)}{4\,c^2}, \frac{acx^2\sqrt{\frac{cx^2+d}{x^2}} - (2\,bc-ad)\sqrt{-c}\,\mathrm{arct}}{2\,c^2}\right.$$

input
```
integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*a*c*x^2*sqrt((c*x^2 + d)/x^2) - (2*b*c - a*d)*sqrt(c)*log(-2*c*x^2
 + 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d))/c^2, 1/2*(a*c*x^2*sqrt((c*x^2
 + d)/x^2) - (2*b*c - a*d)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x
^2)/(c*x^2 + d)))/c^2]
```

3.963.6 Sympy [A] (verification not implemented)

Time = 14.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2c} - \frac{ad\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}} + \frac{b\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{\sqrt{c}}$$

input
```
integrate((a+b/x**2)*x/(c+d/x**2)**(1/2),x)
```

output
```
a*sqrt(d)*x*sqrt(c*x**2/d + 1)/(2*c) - a*d*asinh(sqrt(c)*x/sqrt(d))/(2*c**
(3/2)) + b*asinh(sqrt(c)*x/sqrt(d))/sqrt(c)
```

3.963.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(47) = 94$.

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.85

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{1}{4}\,a\left(\frac{2\,\sqrt{c+\frac{d}{x^2}}d}{\left(c+\frac{d}{x^2}\right)c-c^2}+\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)-\frac{b\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{2\,\sqrt{c}}$$

input
```
integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*a*(2*sqrt(c + d/x^2)*d/((c + d/x^2)*c - c^2) + d*log((sqrt(c + d/x^2)
- sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2)) - 1/2*b*log((sqrt(c + d/x
^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c)
```

3.963.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{\sqrt{cx^2+d}ax}{2\,c\mathrm{sgn}\left(x\right)}+\frac{\left(2\,bc\log\left(|d|\right)-ad\log\left(|d|\right)\right)\mathrm{sgn}(x)}{4\,c^{\frac{3}{2}}}-\frac{\left(2\,bc-ad\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{2\,c^{\frac{3}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^2 + d)*a*x/(c*sgn(x)) + 1/4*(2*b*c*log(abs(d)) - a*d*log(abs(
d)))*sgn(x)/c^(3/2) - 1/2*(2*b*c - a*d)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
d)))/(c^(3/2)*sgn(x))
```

3.963.9 Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{a\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2\,c}-\frac{a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{3/2}}$$

input

```
int((x*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

output

```
(b*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(1/2) + (a*x^2*(c + d/x^2)^(1/2))/(
2*c) - (a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(3/2))
```

3.964 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x}\,dx$

3.964.1 Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x}\,dx=-\frac{b\sqrt{c+\frac{d}{x^2}}}{d}+\frac{a\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}$$

output

```
a*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(1/2)-b*(c+d/x^2)^(1/2)/d
```

3.964.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x}\,dx=\frac{-b\sqrt{c}(d+cx^2)-adx\sqrt{d+cx^2}\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)}{\sqrt{c}d\sqrt{c+\frac{d}{x^2}}x^2}$$

input

```
Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x),x]
```

output

```
(-(b*Sqrt[c]*(d + c*x^2)) - a*d*x*Sqrt[d + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[
d + c*x^2]])/(Sqrt[c]*d*Sqrt[c + d/x^2]*x^2)
```

3.964.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{ 948}$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{2}\left(-a\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}-\frac{2b\sqrt{c+\frac{d}{x^2}}}{d}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(-\frac{2a\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{d}-\frac{2b\sqrt{c+\frac{d}{x^2}}}{d}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(\frac{2a\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{2b\sqrt{c+\frac{d}{x^2}}}{d}\right)$$

input
```
Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x),x]
```

output
```
((-2*b*Sqrt[c + d/x^2])/d + (2*a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/Sqrt[c]
)/2
```

3.964.3.1 Defintions of rubi rules used

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.964.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

method	result	size
default	$\frac{\sqrt{c\,x^{2}+d}\,\left(a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)d x-b\sqrt{c\,x^{2}+d}\,\sqrt{c}\right)}{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x^{2}\sqrt{c}\,d}$	69
risch	$-\frac{b\left(c\,x^{2}+d\right)}{d\,x^{2}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}+\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\sqrt{c\,x^{2}+d}}{\sqrt{c}\,\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}$	77

input

```
int((a+b/x^2)/x/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
(c*x^2+d)^(1/2)*(a*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*d*x-b*(c*x^2+d)^(1/2)*c^(
1/2))/((c*x^2+d)/x^2)^(1/2)/x^2/c^(1/2)/d
```

3.964.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.02

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx = \left[\frac{a\sqrt{c}d\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,bc\sqrt{\frac{cx^2+d}{x^2}}}{2\,cd},\right.$$
$$\left.-\frac{a\sqrt{-c}d\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+bc\sqrt{\frac{cx^2+d}{x^2}}}{cd}\right]$$

input
```
integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(a*sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d)
- 2*b*c*sqrt((c*x^2 + d)/x^2))/(c*d), -(a*sqrt(-c)*d*arctan(sqrt(-c)*x^2*s
qrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + b*c*sqrt((c*x^2 + d)/x^2))/(c*d)]
```

3.964.6 Sympy [A] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx = -\frac{a\left(\begin{cases}\frac{2\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}} & \text{for } d\neq 0\\ -\frac{\log(x^2)}{\sqrt{c}} & \text{otherwise}\end{cases}\right)}{2}+\frac{b\left(\begin{cases}-\frac{1}{\sqrt{c}x^2} & \text{for } d=0\\ -\frac{2\sqrt{c+\frac{d}{x^2}}}{d} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)/x/(c+d/x**2)**(1/2),x)
```

output
```
-a*Piecewise((2*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c), Ne(d, 0)), (-log
(x**2)/sqrt(c), True))/2 + b*Piecewise((-1/(sqrt(c)*x**2), Eq(d, 0)), (-2*
sqrt(c + d/x**2)/d, True))/2
```

3.964.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x}\,dx = -\frac{a\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{2\sqrt{c}} - \frac{b\sqrt{c+\frac{d}{x^2}}}{d}$$

input
```
integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*a*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c
) - b*sqrt(c + d/x^2)/d
```

3.964.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x}\,dx = -\frac{a\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)}{2\sqrt{c}\operatorname{sgn}(x)} + \frac{2b\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)\operatorname{sgn}(x)}$$

input
```
integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*a*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)/(sqrt(c)*sgn(x)) + 2*b*sqrt(c)
/(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)*sgn(x))
```

3.964.9 Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x}\,dx = \frac{a\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{b\sqrt{c+\frac{d}{x^2}}}{d}$$

input
```
int((a + b/x^2)/(x*(c + d/x^2)^(1/2)),x)
```

output
```
(a*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(1/2) - (b*(c + d/x^2)^(1/2))/d
```

3.965 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx$

3.965.1 Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx = \frac{(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^2} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}$$

output
```
-1/3*b*(c+d/x^2)^(3/2)/d^2+(-a*d+b*c)*(c+d/x^2)^(1/2)/d^2
```

3.965.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}(3adx^2+b(d-2cx^2))}{3d^2x^2}$$

input
```
Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]
```

output
```
-1/3*(Sqrt[c + d/x^2]*(3*a*d*x^2 + b*(d - 2*c*x^2)))/(d^2*x^2)
```

3.965.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^3\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{946}$$

$$-\frac{1}{2}\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2}\int \left(\frac{\sqrt{c+\frac{d}{x^2}}b}{d}+\frac{ad-bc}{d\sqrt{c+\frac{d}{x^2}}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\sqrt{c+\frac{d}{x^2}}(bc-ad)}{d^2}-\frac{2b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}\right)$$

input
```
Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]
```

output
```
((2*(b*c - a*d)*Sqrt[c + d/x^2])/d^2 - (2*b*(c + d/x^2)^(3/2))/(3*d^2))/2
```

3.965.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.965.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

method	result	size
trager	$-\frac{\left(3ad\,x^{2}-2cb\,x^{2}+bd\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{3x^{2}d^{2}}$	44
gosper	$-\frac{\left(3ad\,x^{2}-2cb\,x^{2}+bd\right)\left(c\,x^{2}+d\right)}{3\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,d^{2}x^{4}}$	47
default	$-\frac{\left(3ad\,x^{2}-2cb\,x^{2}+bd\right)\left(c\,x^{2}+d\right)}{3\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,d^{2}x^{4}}$	47
risch	$-\frac{\left(3ad\,x^{2}-2cb\,x^{2}+bd\right)\left(c\,x^{2}+d\right)}{3\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,d^{2}x^{4}}$	47

input
```
int((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^2*(3*a*d*x^2-2*b*c*x^2+b*d)/d^2*(-(-c*x^2-d)/x^2)^(1/2)
```

3.965.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx=\frac{\left((2\,bc-3\,ad)x^2-bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\,d^2x^2}$$

input
```
integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*((2*b*c - 3*a*d)*x^2 - b*d)*sqrt((c*x^2 + d)/x^2)/(d^2*x^2)
```

3.965.6 Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx = \frac{\begin{cases} \frac{-2a\sqrt{c+\frac{d}{x^2}}-\frac{2b\left(-c\sqrt{c+\frac{d}{x^2}}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d}}{d} & \text{for } d\neq 0 \\ \frac{-\frac{a}{x^2}-\frac{b}{2x^4}}{\sqrt{c}} & \text{otherwise} \end{cases}}{2}$$

input
```
integrate((a+b/x**2)/x**3/(c+d/x**2)**(1/2),x)
```

output
```
Piecewise(((-2*a*sqrt(c + d/x**2) - 2*b*(-c*sqrt(c + d/x**2) + (c + d/x**2
)**(3/2)/3)/d)/d, Ne(d, 0)), ((-a/x**2 - b/(2*x**4))/sqrt(c), True))/2
```

3.965.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx = -\frac{1}{3}b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2}-\frac{3\sqrt{c+\frac{d}{x^2}}c}{d^2}\right)-\frac{a\sqrt{c+\frac{d}{x^2}}}{d}$$

input
```
integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*b*((c + d/x^2)^(3/2)/d^2 - 3*sqrt(c + d/x^2)*c/d^2) - a*sqrt(c + d/x^
2)/d
```

3.965.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(37) = 74$.

Time = 0.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.88

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx$$
$$= \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4 a\sqrt{c}+6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2 bc^{\frac{3}{2}}-6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2 a\sqrt{c}d-2\,bc^{\frac{3}{2}}d+3\,a\sqrt{c}}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^3\operatorname{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*sqrt(c) + 6*(sqrt(c)*x - sqrt(c*x
^2 + d))^2*b*c^(3/2) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d - 2*b
*c^(3/2)*d + 3*a*sqrt(c)*d^2)/(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^3*sgn
(x))
```

3.965.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}\,\left(b\,d+3\,a\,d\,x^2-2\,b\,c\,x^2\right)}{3\,d^2\,x^2}$$

input
```
int((a + b/x^2)/(x^3*(c + d/x^2)^(1/2)),x)
```

output
```
-((c + d/x^2)^(1/2)*(b*d + 3*a*d*x^2 - 2*b*c*x^2))/(3*d^2*x^2)
```

3.966 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx$

3.966.1 Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx=-\frac{c(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}$$

output
```
1/3*(-a*d+2*b*c)*(c+d/x^2)^(3/2)/d^3-1/5*b*(c+d/x^2)^(5/2)/d^3-c*(-a*d+b*c
)*(c+d/x^2)^(1/2)/d^3
```

3.966.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}(-5adx^2(d-2cx^2)+b(-3d^2+4cdx^2-8c^2x^4))}{15d^3x^4}$$

input
```
Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5),x]
```

output
```
(Sqrt[c + d/x^2]*(-5*a*d*x^2*(d - 2*c*x^2) + b*(-3*d^2 + 4*c*d*x^2 - 8*c^2
*x^4)))/(15*d^3*x^4)
```

3.966.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^5\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{d^2}+\frac{(ad-2bc)\sqrt{c+\frac{d}{x^2}}}{d^2}+\frac{c(bc-ad)}{d^2\sqrt{c+\frac{d}{x^2}}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{3d^3}-\frac{2c\sqrt{c+\frac{d}{x^2}}(bc-ad)}{d^3}-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}\right)$$

input
```
Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5),x]
```

output
```
((-2*c*(b*c - a*d)*Sqrt[c + d/x^2])/d^3 + (2*(2*b*c - a*d)*(c + d/x^2)^(3/
2))/(3*d^3) - (2*b*(c + d/x^2)^(5/2))/(5*d^3))/2
```

3.966.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.966.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

method	result	size
trager	$\frac{\left(10acd\,x^4-8b\,c^2x^4-5a\,d^2x^2+4bcd\,x^2-3b\,d^2\right)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{15x^4d^3}$	67
gosper	$\frac{\left(10acd\,x^4-8b\,c^2x^4-5a\,d^2x^2+4bcd\,x^2-3b\,d^2\right)\left(c\,x^2+d\right)}{15\sqrt{\frac{c\,x^2+d}{x^2}}\,d^3x^6}$	70
default	$\frac{\left(10acd\,x^4-8b\,c^2x^4-5a\,d^2x^2+4bcd\,x^2-3b\,d^2\right)\left(c\,x^2+d\right)}{15\sqrt{\frac{c\,x^2+d}{x^2}}\,d^3x^6}$	70
risch	$\frac{\left(10acd\,x^4-8b\,c^2x^4-5a\,d^2x^2+4bcd\,x^2-3b\,d^2\right)\left(c\,x^2+d\right)}{15\sqrt{\frac{c\,x^2+d}{x^2}}\,d^3x^6}$	70

input

```
int((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/15/x^4*(10*a*c*d*x^4-8*b*c^2*x^4-5*a*d^2*x^2+4*b*c*d*x^2-3*b*d^2)/d^3*(-
(-c*x^2-d)/x^2)^(1/2)
```

3.966.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx = -\frac{\left(2\,(4\,bc^2-5\,acd)x^4+3\,bd^2-(4\,bcd-5\,ad^2)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,d^3x^4}$$

input

```
integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(2*(4*b*c^2 - 5*a*c*d)*x^4 + 3*b*d^2 - (4*b*c*d - 5*a*d^2)*x^2)*sqrt
((c*x^2 + d)/x^2)/(d^3*x^4)
```

3.966.6 Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^5}\,dx = \frac{\begin{cases} \frac{-\frac{2a\left(-c\sqrt{c+\frac{d}{x^2}}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d}-\frac{2b\left(c^2\sqrt{c+\frac{d}{x^2}}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}}{d} & \text{for } d\neq 0 \\ \frac{-\frac{a}{2x^4}-\frac{b}{3x^6}}{\sqrt{c}} & \text{otherwise} \end{cases}}{2}$$

input
```
integrate((a+b/x**2)/x**5/(c+d/x**2)**(1/2),x)
```

output
```
Piecewise(((-2*a*(-c*sqrt(c + d/x**2) + (c + d/x**2)**(3/2)/3)/d - 2*b*(c*
*2*sqrt(c + d/x**2) - 2*c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d
**2)/d, Ne(d, 0)), ((-a/(2*x**4) - b/(3*x**6))/sqrt(c), True))/2
```

3.966.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^5}\,dx = -\frac{1}{15}\,b\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3}-\frac{10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^3}+\frac{15\sqrt{c+\frac{d}{x^2}}\,c^2}{d^3}\right) - \frac{1}{3}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2}-\frac{3\sqrt{c+\frac{d}{x^2}}\,c}{d^2}\right)$$

input
```
integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/15*b*(3*(c + d/x^2)^(5/2)/d^3 - 10*(c + d/x^2)^(3/2)*c/d^3 + 15*sqrt(c
+ d/x^2)*c^2/d^3) - 1/3*a*((c + d/x^2)^(3/2)/d^2 - 3*sqrt(c + d/x^2)*c/d^2
)
```

3.966.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(62) = 124$.

Time = 0.66 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.50

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx$$
$$= \frac{4\left(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6ac^{\frac{3}{2}}+40\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{5}{2}}-35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4ac^{\frac{3}{2}}d-20\left(\sqrt{c}x-\right.\right.}{15\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^5\operatorname{sgn}(x)}$$

input
```
integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
4/15*(15*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(3/2) + 40*(sqrt(c)*x - sqrt(
c*x^2 + d))^4*b*c^(5/2) - 35*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d -
 20*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(5/2)*d + 25*(sqrt(c)*x - sqrt(c*x
^2 + d))^2*a*c^(3/2)*d^2 + 4*b*c^(5/2)*d^2 - 5*a*c^(3/2)*d^3)/(((sqrt(c)*x
 - sqrt(c*x^2 + d))^2 - d)^5*sgn(x))
```

3.966.9 Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}\,\left(8\,b\,c^2\,x^4-10\,a\,c\,d\,x^4-4\,b\,c\,d\,x^2+5\,a\,d^2\,x^2+3\,b\,d^2\right)}{15\,d^3\,x^4}$$

input
```
int((a + b/x^2)/(x^5*(c + d/x^2)^(1/2)),x)
```

output
```
-((c + d/x^2)^(1/2)*(3*b*d^2 + 5*a*d^2*x^2 + 8*b*c^2*x^4 - 10*a*c*d*x^4 -
4*b*c*d*x^2))/(15*d^3*x^4)
```

3.967 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx$

3.967.1 Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx = \frac{c^2(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^4} - \frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4} + \frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4} - \frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}$$

output
```
-1/3*c*(-2*a*d+3*b*c)*(c+d/x^2)^(3/2)/d^4+1/5*(-a*d+3*b*c)*(c+d/x^2)^(5/2)
/d^4-1/7*b*(c+d/x^2)^(7/2)/d^4+c^2*(-a*d+b*c)*(c+d/x^2)^(1/2)/d^4
```

3.967.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx = \frac{(d+cx^2)\left(-15bd^3+18bcd^2x^2-21ad^3x^2-24bc^2dx^4+28acd^2x^4+48bc^3x^6-56ac^2dx^6\right)}{105d^4\sqrt{c+\frac{d}{x^2}}x^8}$$

input
```
Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7),x]
```

output
```
((d + c*x^2)*(-15*b*d^3 + 18*b*c*d^2*x^2 - 21*a*d^3*x^2 - 24*b*c^2*d*x^4 +
 28*a*c*d^2*x^4 + 48*b*c^3*x^6 - 56*a*c^2*d*x^6))/(105*d^4*Sqrt[c + d/x^2]
*x^8)
```

3.967.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^7\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \textcolor{blue}{948}$$

$$-\frac{1}{2}\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}d\frac{1}{x^2}$$

$$\Big\downarrow \textcolor{blue}{86}$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{d^3}+\frac{(ad-3bc)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^3}+\frac{c(3bc-2ad)\sqrt{c+\frac{d}{x^2}}}{d^3}-\frac{c^2(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{1}{2}\left(\frac{2c^2\sqrt{c+\frac{d}{x^2}}(bc-ad)}{d^4}+\frac{2\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-ad)}{5d^4}-\frac{2c\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{3d^4}-\frac{2b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}\right)$$

input
```
Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7),x]
```

output
```
((2*c^2*(b*c - a*d)*Sqrt[c + d/x^2])/d^4 - (2*c*(3*b*c - 2*a*d)*(c + d/x^2
)^(3/2))/(3*d^4) + (2*(3*b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (2*b*(c +
 d/x^2)^(7/2))/(7*d^4))/2
```

3.967. $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}dx$

3.967.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.967.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

method	result	size
trager	$-\frac{\left(56a\,c^{2}d\,x^{6}-48b\,c^{3}x^{6}-28ac\,d^{2}x^{4}+24b\,c^{2}d\,x^{4}+21a\,d^{3}x^{2}-18bc\,d^{2}x^{2}+15b\,d^{3}\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{105x^{6}d^{4}}$	91
gosper	$-\frac{\left(56a\,c^{2}d\,x^{6}-48b\,c^{3}x^{6}-28ac\,d^{2}x^{4}+24b\,c^{2}d\,x^{4}+21a\,d^{3}x^{2}-18bc\,d^{2}x^{2}+15b\,d^{3}\right)\left(c\,x^{2}+d\right)}{105\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,d^{4}x^{8}}$	94
default	$-\frac{\left(56a\,c^{2}d\,x^{6}-48b\,c^{3}x^{6}-28ac\,d^{2}x^{4}+24b\,c^{2}d\,x^{4}+21a\,d^{3}x^{2}-18bc\,d^{2}x^{2}+15b\,d^{3}\right)\left(c\,x^{2}+d\right)}{105\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,d^{4}x^{8}}$	94
risch	$-\frac{\left(56a\,c^{2}d\,x^{6}-48b\,c^{3}x^{6}-28ac\,d^{2}x^{4}+24b\,c^{2}d\,x^{4}+21a\,d^{3}x^{2}-18bc\,d^{2}x^{2}+15b\,d^{3}\right)\left(c\,x^{2}+d\right)}{105\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,d^{4}x^{8}}$	94

input
```
int((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105/x^6*(56*a*c^2*d*x^6-48*b*c^3*x^6-28*a*c*d^2*x^4+24*b*c^2*d*x^4+21*a
*d^3*x^2-18*b*c*d^2*x^2+15*b*d^3)/d^4*(-(-c*x^2-d)/x^2)^(1/2)
```

3.967. $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^7}\,dx$

3.967.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx$$
$$=\frac{\left(8\left(6\,bc^3-7\,ac^2d\right)x^6-4\left(6\,bc^2d-7\,acd^2\right)x^4-15\,bd^3+3\left(6\,bcd^2-7\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,d^4x^6}$$

input
```
integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(8*(6*b*c^3 - 7*a*c^2*d)*x^6 - 4*(6*b*c^2*d - 7*a*c*d^2)*x^4 - 15*b*
d^3 + 3*(6*b*c*d^2 - 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^4*x^6)
```

3.967.6 Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.32

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx$$
$$=\frac{\begin{cases}\frac{-\frac{2a\left(c^2\sqrt{c+\frac{d}{x^2}}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{2b\left(-c^3\sqrt{c+\frac{d}{x^2}}+c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}}{d} & \text{for } d\neq 0\\ \frac{-\frac{a}{3x^6}-\frac{b}{4x^8}}{\sqrt{c}} & \text{otherwise}\end{cases}}{2}$$

input
```
integrate((a+b/x**2)/x**7/(c+d/x**2)**(1/2),x)
```

output
```
Piecewise(((-2*a*(c**2*sqrt(c + d/x**2) - 2*c*(c + d/x**2)**(3/2)/3 + (c +
 d/x**2)**(5/2)/5)/d**2 - 2*b*(-c**3*sqrt(c + d/x**2) + c**2*(c + d/x**2)*
*(3/2) - 3*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3)/d, Ne(d,
 0)), ((-a/(3*x**6) - b/(4*x**8))/sqrt(c), True))/2
```

3.967.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx = -\frac{1}{35}\,b\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^4}-\frac{21\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^4}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^4}-\frac{35\sqrt{c+\frac{d}{x^2}}c^3}{d^4}\right) - \frac{1}{15}\,a\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3}-\frac{10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^3}+\frac{15\sqrt{c+\frac{d}{x^2}}c^2}{d^3}\right)$$

input
```
integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/35*b*(5*(c + d/x^2)^(7/2)/d^4 - 21*(c + d/x^2)^(5/2)*c/d^4 + 35*(c + d/
x^2)^(3/2)*c^2/d^4 - 35*sqrt(c + d/x^2)*c^3/d^4) - 1/15*a*(3*(c + d/x^2)^(
5/2)/d^3 - 10*(c + d/x^2)^(3/2)*c/d^3 + 15*sqrt(c + d/x^2)*c^2/d^3)
```

3.967.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(87) = 174.

Time = 0.87 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.34

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx$$

$$= \frac{16\left(70\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8ac^{\frac{5}{2}}+210\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6bc^{\frac{7}{2}}-175\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6ac^{\frac{5}{2}}d-126\left(\sqrt{c}x\right.}{105}$$

input
```
integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
16/105*(70*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(5/2) + 210*(sqrt(c)*x - sq
rt(c*x^2 + d))^6*b*c^(7/2) - 175*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(5/2)
*d - 126*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)*d + 147*(sqrt(c)*x - sq
rt(c*x^2 + d))^4*a*c^(5/2)*d^2 + 42*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7
/2)*d^2 - 49*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(5/2)*d^3 - 6*b*c^(7/2)*d
^3 + 7*a*c^(5/2)*d^4)/(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^7*sgn(x))
```

3.967.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^7}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\,(48\,b\,c^3-56\,a\,c^2\,d)}{105\,d^4}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{7\,d\,x^6}-\frac{\sqrt{c+\frac{d}{x^2}}\,(24\,b\,c^2-28\,a\,c\,d)}{105\,d^3\,x^2}-\frac{\sqrt{c+\frac{d}{x^2}}\,(7\,a\,d-6\,b\,c)}{35\,d^2\,x^4}$$

input
```
int((a + b/x^2)/(x^7*(c + d/x^2)^(1/2)),x)
```

output
```
((c + d/x^2)^(1/2)*(48*b*c^3 - 56*a*c^2*d))/(105*d^4) - (b*(c + d/x^2)^(1/
2))/(7*d*x^6) - ((c + d/x^2)^(1/2)*(24*b*c^2 - 28*a*c*d))/(105*d^3*x^2) -
((c + d/x^2)^(1/2)*(7*a*d - 6*b*c))/(35*d^2*x^4)
```

3.968 $\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx$

3.968.1 Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\frac{2d(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x}{15c^3}+\frac{(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x^3}{15c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}\,x^5}{5c}$$

output
```
-2/15*d*(-4*a*d+5*b*c)*x*(c+d/x^2)^(1/2)/c^3+1/15*(-4*a*d+5*b*c)*x^3*(c+d/
x^2)^(1/2)/c^2+1/5*a*x^5*(c+d/x^2)^(1/2)/c
```

3.968.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.68

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\,x(5bc(-2d+cx^2)+a(8d^2-4cdx^2+3c^2x^4))}{15c^3}$$

input
```
Integrate[((a + b/x^2)*x^4)/Sqrt[c + d/x^2],x]
```

output
```
(Sqrt[c + d/x^2]*x*(5*b*c*(-2*d + c*x^2) + a*(8*d^2 - 4*c*d*x^2 + 3*c^2*x^
4)))/(15*c^3)
```

3.968.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(a+\frac{b}{x^2}\right)}{\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(5bc-4ad)\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}dx}{5c}+\frac{ax^5\sqrt{c+\frac{d}{x^2}}}{5c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(5bc-4ad)\left(\frac{x^3\sqrt{c+\frac{d}{x^2}}}{3c}-\frac{2d\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}dx}{3c}\right)}{5c}+\frac{ax^5\sqrt{c+\frac{d}{x^2}}}{5c}$$

$$\Big\downarrow \text{746}$$

$$\frac{\left(\frac{x^3\sqrt{c+\frac{d}{x^2}}}{3c}-\frac{2dx\sqrt{c+\frac{d}{x^2}}}{3c^2}\right)(5bc-4ad)}{5c}+\frac{ax^5\sqrt{c+\frac{d}{x^2}}}{5c}$$

input
```
Int[((a + b/x^2)*x^4)/Sqrt[c + d/x^2],x]
```

output
```
(a*Sqrt[c + d/x^2]*x^5)/(5*c) + ((5*b*c - 4*a*d)*((-2*d*Sqrt[c + d/x^2]*x)
/(3*c^2) + (Sqrt[c + d/x^2]*x^3)/(3*c)))/(5*c)
```

3.968.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.968.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

method	result	size
trager	$\frac{\left(3a\,x^{4}c^{2}-4acd\,x^{2}+5b\,c^{2}x^{2}+8a\,d^{2}-10bcd\right)x\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{15c^{3}}$	62
gosper	$\frac{\left(3a\,x^{4}c^{2}-4acd\,x^{2}+5b\,c^{2}x^{2}+8a\,d^{2}-10bcd\right)\left(c\,x^{2}+d\right)}{15x\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,c^{3}}$	67
default	$\frac{\left(3a\,x^{4}c^{2}-4acd\,x^{2}+5b\,c^{2}x^{2}+8a\,d^{2}-10bcd\right)\left(c\,x^{2}+d\right)}{15x\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,c^{3}}$	67
risch	$\frac{\left(3a\,x^{4}c^{2}-4acd\,x^{2}+5b\,c^{2}x^{2}+8a\,d^{2}-10bcd\right)\left(c\,x^{2}+d\right)}{15x\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,c^{3}}$	67

input
```
int((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(3*a*c^2*x^4-4*a*c*d*x^2+5*b*c^2*x^2+8*a*d^2-10*b*c*d)*x/c^3*(-(-c*x^
2-d)/x^2)^(1/2)
```

3.968.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.72

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\left(3\,ac^2x^5+(5\,bc^2-4\,acd)x^3-2\,(5\,bcd-4\,ad^2)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,c^3}$$

input

```
integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output

```
1/15*(3*a*c^2*x^5 + (5*b*c^2 - 4*a*c*d)*x^3 - 2*(5*b*c*d - 4*a*d^2)*x)*sqr
t((c*x^2 + d)/x^2)/c^3
```

3.968.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(76) = 152.

Time = 1.46 (sec) , antiderivative size = 338, normalized size of antiderivative = 4.12

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx &= \frac{3ac^4d^{\frac{9}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}+\frac{2ac^3d^{\frac{11}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}\\ &\quad+\frac{3ac^2d^{\frac{13}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}+\frac{12acd^{\frac{15}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}\\ &\quad+\frac{8ad^{\frac{17}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}+\frac{b\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3c}-\frac{2bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^2}\end{aligned}$$

input

```
integrate((a+b/x**2)*x**4/(c+d/x**2)**(1/2),x)
```

output

```
3*a*c**4*d**(9/2)*x**8*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**
5*x**2 + 15*c**3*d**6) + 2*a*c**3*d**(11/2)*x**6*sqrt(c*x**2/d + 1)/(15*c*
*5*d**4*x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6) + 3*a*c**2*d**(13/2)*x**4
*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6)
 + 12*a*c*d**(15/2)*x**2*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d
**5*x**2 + 15*c**3*d**6) + 8*a*d**(17/2)*sqrt(c*x**2/d + 1)/(15*c**5*d**4*
x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6) + b*sqrt(d)*x**2*sqrt(c*x**2/d +
1)/(3*c) - 2*b*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c**2)
```

3.968.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3 - 3\sqrt{c+\frac{d}{x^2}}dx\right)b}{3\,c^2} + \frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5 - 10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3 + 15\sqrt{c+\frac{d}{x^2}}d^2x\right)a}{15\,c^3}$$

input
```
integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*((c + d/x^2)^(3/2)*x^3 - 3*sqrt(c + d/x^2)*d*x)*b/c^2 + 1/15*(3*(c + d
/x^2)^(5/2)*x^5 - 10*(c + d/x^2)^(3/2)*d*x^3 + 15*sqrt(c + d/x^2)*d^2*x)*a
/c^3
```

3.968.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.21

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{2\left(5\,bcd^{\frac{3}{2}} - 4\,ad^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{15\,c^3} - \frac{(bcd - ad^2)\sqrt{cx^2+d}}{c^3\mathrm{sgn}\,(x)} + \frac{3\,(cx^2+d)^{\frac{5}{2}}a + 5\,(cx^2+d)^{\frac{3}{2}}bc - 10\,(cx^2+d)^{\frac{3}{2}}ad}{15\,c^3\mathrm{sgn}\,(x)}$$

input
```
integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
2/15*(5*b*c*d^(3/2) - 4*a*d^(5/2))*sgn(x)/c^3 - (b*c*d - a*d^2)*sqrt(c*x^2
 + d)/(c^3*sgn(x)) + 1/15*(3*(c*x^2 + d)^(5/2)*a + 5*(c*x^2 + d)^(3/2)*b*c
 - 10*(c*x^2 + d)^(3/2)*a*d)/(c^3*sgn(x))
```

3.968.9 Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{x\,\sqrt{c+\frac{d}{x^2}}\,\left(3\,a\,c^2\,x^4+5\,b\,c^2\,x^2-4\,a\,c\,d\,x^2-10\,b\,c\,d+8\,a\,d^2\right)}{15\,c^3}$$

input
```
int((x^4*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

output
```
(x*(c + d/x^2)^(1/2)*(8*a*d^2 + 3*a*c^2*x^4 + 5*b*c^2*x^2 - 10*b*c*d - 4*a
*c*d*x^2))/(15*c^3)
```

3.969 $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx$

3.969.1 Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{(3bc-2ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}\,x^3}{3c}$$

output
```
1/3*(-2*a*d+3*b*c)*x*(c+d/x^2)^(1/2)/c^2+1/3*a*x^3*(c+d/x^2)^(1/2)/c
```

3.969.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\,x(3bc-2ad+acx^2)}{3c^2}$$

input
```
Integrate[((a + b/x^2)*x^2)/Sqrt[c + d/x^2],x]
```

output
```
(Sqrt[c + d/x^2]*x*(3*b*c - 2*a*d + a*c*x^2))/(3*c^2)
```

3.969.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {955, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+\frac{b}{x^2}\right)}{\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(3bc-2ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}dx}{3c}+\frac{ax^3\sqrt{c+\frac{d}{x^2}}}{3c}$$

$$\Big\downarrow \text{746}$$

$$\frac{x\sqrt{c+\frac{d}{x^2}}(3bc-2ad)}{3c^2}+\frac{ax^3\sqrt{c+\frac{d}{x^2}}}{3c}$$

input
```
Int[((a + b/x^2)*x^2)/Sqrt[c + d/x^2],x]
```

output
```
((3*b*c - 2*a*d)*Sqrt[c + d/x^2]*x)/(3*c^2) + (a*Sqrt[c + d/x^2]*x^3)/(3*c
)
```

3.969.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.969.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result	size
trager	$\frac{\left(a c \,x^{2}-2 a d +3 b c \right) x \sqrt{-\frac{-c \,x^{2}-d}{x^{2}}}}{3 c^{2}}$	39
gosper	$\frac{\left(a c \,x^{2}-2 a d +3 b c \right) \left(c \,x^{2}+d \right)}{3 x \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, c^{2}}$	44
default	$\frac{\left(a c \,x^{2}-2 a d +3 b c \right) \left(c \,x^{2}+d \right)}{3 x \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, c^{2}}$	44
risch	$\frac{\left(a c \,x^{2}-2 a d +3 b c \right) \left(c \,x^{2}+d \right)}{3 x \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, c^{2}}$	44

input
```
int((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*c*x^2-2*a*d+3*b*c)*x/c^2*(-(-c*x^2-d)/x^2)^(1/2)
```

3.969.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{(acx^3+(3\,bc-2\,ad)x)\sqrt{\frac{cx^2+d}{x^2}}}{3\,c^2}$$

input
```
integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(a*c*x^3 + (3*b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/c^2
```

3.969.6 Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3c} - \frac{2ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^2} + \frac{b\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}{c}$$

input
```
integrate((a+b/x**2)*x**2/(c+d/x**2)**(1/2),x)
```

output
```
a*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/(3*c) - 2*a*d**(3/2)*sqrt(c*x**2/d + 1)/
(3*c**2) + b*sqrt(d)*sqrt(c*x**2/d + 1)/c
```

3.969.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{b\sqrt{c+\frac{d}{x^2}}x}{c} + \frac{\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3 - 3\sqrt{c+\frac{d}{x^2}}dx\right)a}{3\,c^2}$$

input
```
integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
b*sqrt(c + d/x^2)*x/c + 1/3*((c + d/x^2)^(3/2)*x^3 - 3*sqrt(c + d/x^2)*d*x
)*a/c^2
```

3.969.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\frac{\left(3\,bc\sqrt{d}-2\,ad^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{3\,c^2} + \frac{\left(cx^2+d\right)^{\frac{3}{2}}a}{3\,c^2\mathrm{sgn}\left(x\right)} + \frac{\sqrt{cx^2+d}(bc-ad)}{c^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/3*(3*b*c*sqrt(d) - 2*a*d^(3/2))*sgn(x)/c^2 + 1/3*(c*x^2 + d)^(3/2)*a/(c
^2*sgn(x)) + sqrt(c*x^2 + d)*(b*c - a*d)/(c^2*sgn(x))
```

3.969. $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx$

3.969.9 Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\,x^3\,\sqrt{c+\frac{d}{x^2}}\,\left(c-\frac{2\,d}{x^2}\right)}{3\,c^2}+\frac{b\,x\,\sqrt{\frac{c\,x^2}{d}+1}}{\sqrt{c+\frac{d}{x^2}}\,\left(\sqrt{\frac{c\,x^2}{d}+1}+1\right)}$$

input

```
int((x^2*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

output

```
(a*x^3*(c + d/x^2)^(1/2)*(c - (2*d)/x^2))/(3*c^2) + (b*x*((c*x^2)/d + 1)^(
1/2))/((c + d/x^2)^(1/2)*(((c*x^2)/d + 1)^(1/2) + 1))
```

3.970 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx$

3.970.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\sqrt{c+\frac{d}{x^2}}x}{c}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{\sqrt{d}}$$

output
```
-b*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(1/2)+a*x*(c+d/x^2)^(1/2)/c
```

3.970.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.51

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\sqrt{d}(d+cx^2)-bc\sqrt{d+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{c\sqrt{d}\sqrt{c+\frac{d}{x^2}}x}$$

input
```
Integrate[(a + b/x^2)/Sqrt[c + d/x^2],x]
```

output
```
(a*Sqrt[d]*(d + c*x^2) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[
d]])/(c*Sqrt[d]*Sqrt[c + d/x^2]*x)
```

3.970.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {899, 358, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{358}$$

$$\frac{ax\sqrt{c+\frac{d}{x^2}}}{c}-b\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{224}$$

$$\frac{ax\sqrt{c+\frac{d}{x^2}}}{c}-b\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}$$

$$\Big\downarrow \text{219}$$

$$\frac{ax\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{\sqrt{d}}$$

input
```
Int[(a + b/x^2)/Sqrt[c + d/x^2],x]
```

output
```
(a*Sqrt[c + d/x^2]*x)/c - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/Sqrt[d]
```

3.970.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 899

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.970.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{\sqrt{c\,x^{2}+d}\,\left(a\sqrt{c\,x^{2}+d}\,\sqrt{d}-b\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)c\right)}{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x c\sqrt{d}}$	73

input

```
int((a+b/x^2)/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
(c*x^2+d)^(1/2)*(a*(c*x^2+d)^(1/2)*d^(1/2)-b*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)
+d)/x)*c)/((c*x^2+d)/x^2)^(1/2)/x/c/d^(1/2)
```

3.970.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.79

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \left[\frac{2\,adx\sqrt{\frac{cx^2+d}{x^2}}+bc\sqrt{d}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)}{2\,cd}, \frac{adx\sqrt{\frac{cx^2+d}{x^2}}+bc\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{cd}\right]$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*d*x*sqrt((c*x^2 + d)/x^2) + b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*
x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c*d), (a*d*x*sqrt((c*x^2 + d)/x^2) +
 b*c*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c*d)]
```

3.970.6 Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}{c} - \frac{b\,\mathrm{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{\sqrt{d}}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(1/2),x)
```

output
```
a*sqrt(d)*sqrt(c*x**2/d + 1)/c - b*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d)
```

3.970.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\sqrt{c+\frac{d}{x^2}}x}{c} + \frac{b\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{2\,\sqrt{d}}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
a*sqrt(c + d/x^2)*x/c + 1/2*b*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c +
d/x^2)*x + sqrt(d)))/sqrt(d)
```

3.970.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(39) = 78$.

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\frac{\left(bc\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+a\sqrt{-d}\sqrt{d}\right)\mathrm{sgn}(x)}{c\sqrt{-d}}+\frac{\frac{b\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}+\frac{\sqrt{cx^2+d}a}{c}}{\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
-(b*c*arctan(sqrt(d)/sqrt(-d)) + a*sqrt(-d)*sqrt(d))*sgn(x)/(c*sqrt(-d)) +
 (b*arctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) + sqrt(c*x^2 + d)*a/c)/sgn(x
)
```

3.970.9 Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\,x\,\sqrt{\frac{c\,x^2}{d}+1}}{\sqrt{c+\frac{d}{x^2}}\,\left(\sqrt{\frac{c\,x^2}{d}+1}+1\right)}-\frac{b\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{\sqrt{d}}$$

input
```
int((a + b/x^2)/(c + d/x^2)^(1/2),x)
```

output
```
(a*x*((c*x^2)/d + 1)^(1/2))/((c + d/x^2)^(1/2)*(((c*x^2)/d + 1)^(1/2) + 1)
) - (b*log((c + d/x^2)^(1/2) + d^(1/2)/x))/d^(1/2)
```

3.971 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx$

3.971.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx=-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}+\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{2d^{3/2}}$$

output
```
1/2*(-2*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(3/2)-1/2*b*(c+d/x^2
)^(1/2)/d/x
```

3.971.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx=\frac{-b\sqrt{d}(d+cx^2)+(bc-2ad)x^2\sqrt{d+cx^2}\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{c+\frac{d}{x^2}}x^3}$$

input
```
Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]
```

output
```
(-(b*Sqrt[d]*(d + c*x^2)) + (b*c - 2*a*d)*x^2*Sqrt[d + c*x^2]*ArcTanh[Sqrt
[d + c*x^2]/Sqrt[d]])/(2*d^(3/2)*Sqrt[c + d/x^2]*x^3)
```

3.971.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {959, 858, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^2\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{959}$$

$$-\frac{(bc-2ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}x^2}dx}{2d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

$$\Big\downarrow \text{858}$$

$$\frac{(bc-2ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{2d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

$$\Big\downarrow \text{224}$$

$$\frac{(bc-2ad)\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{2d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

$$\Big\downarrow \text{219}$$

$$\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

input
```
Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]
```

output
```
-1/2*(b*Sqrt[c + d/x^2])/(d*x) + ((b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c +
d/x^2]*x)])/(2*d^(3/2))
```

3.971.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.971.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52

method	result	size
risch	$-\frac{b\left(c\,x^{2}+d\right)}{2 d\,x^{3} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}}-\frac{\left(2 a d -b c \right) \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{c\,x^{2}+d}}{x}\right) \sqrt{c\,x^{2}+d}}{2 d^{\frac{3}{2}} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x}$	93
default	$-\frac{\sqrt{c\,x^{2}+d}\, \left(2 a \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{c\,x^{2}+d}}{x}\right) d^{2} x^{2}-\ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{c\,x^{2}+d}}{x}\right) b c d\,x^{2}+\sqrt{c\,x^{2}+d}\, d^{\frac{3}{2}} b \right)}{2 \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x^{3} d^{\frac{5}{2}}}$	105

input
```
int((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*b*(c*x^2+d)/x^3/((c*x^2+d)/x^2)^(1/2)-1/2*(2*a*d-b*c)/d^(3/2)*ln((2
*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)/((c*x^2+d)/x^2)^(1/2)/x*(c*x^2+d)^(1/2)
```

3.971.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.36

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = \left[-\frac{(bc-2\,ad)\sqrt{d}x\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,bd\sqrt{\frac{cx^2+d}{x^2}}}{4\,d^2x},\right.$$
$$\left.-\frac{(bc-2\,ad)\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+bd\sqrt{\frac{cx^2+d}{x^2}}}{2\,d^2x}\right]$$

input
```
integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((b*c - 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/
x^2) + 2*d)/x^2) + 2*b*d*sqrt((c*x^2 + d)/x^2))/(d^2*x), -1/2*((b*c - 2*a*
d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + b*d*s
qrt((c*x^2 + d)/x^2))/(d^2*x)]
```

3.971.6 Sympy [A] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = -\frac{a\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{\sqrt{d}}-\frac{b\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2dx}+\frac{bc\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{3}{2}}}$$

input
```
integrate((a+b/x**2)/x**2/(c+d/x**2)**(1/2),x)
```

output
```
-a*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*
d*x) + b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(3/2))
```

3.971.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.98

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = -\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}cx}{\left(c+\frac{d}{x^2}\right)dx^2-d^2}+\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right)b+\frac{a\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{2\sqrt{d}}$$

input
```
integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*d*x^2 - d^2) + c*log((sqrt(c + d/
x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2))*b + 1/2*a*log((s
qrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sqrt(d)
```

3.971.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = -\frac{\frac{\left(bc^2-2\,acd\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d}+\frac{\sqrt{cx^2+d}bc}{dx^2}}{2\,c\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*((b*c^2 - 2*a*c*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d) + sq
rt(c*x^2 + d)*b*c/(d*x^2))/(c*sgn(x))
```

3.971.9 Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.54

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = \begin{cases} -\frac{3\,a\,x^2+b}{3\,\sqrt{c}\,x^3} & \text{if } d=0 \\ \frac{b\,c\,\ln\left(2\,\sqrt{c+\frac{d}{x^2}}+\frac{2\,\sqrt{d}}{x}\right)}{2\,d^{3/2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{2\,d\,x}-\frac{a\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{\sqrt{d}} & \text{if } d\neq 0 \end{cases}$$

input
```
int((a + b/x^2)/(x^2*(c + d/x^2)^(1/2)),x)
```

output
```
piecewise(d == 0, -(b + 3*a*x^2)/(3*c^(1/2)*x^3), d ~= 0, - (a*log((c + d/
x^2)^(1/2) + d^(1/2)/x))/d^(1/2) - (b*(c + d/x^2)^(1/2))/(2*d*x) + (b*c*lo
g(2*(c + d/x^2)^(1/2) + (2*d^(1/2))/x))/(2*d^(3/2)))
```

3.971. $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx$

3.972 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx$

3.972.1 Optimal result

Integrand size = 22, antiderivative size = 93

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx = -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{c(3bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{5/2}}$$

output

```
-1/8*c*(-4*a*d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(5/2)-1/4*b*(c+
d/x^2)^(1/2)/d/x^3+1/8*(-4*a*d+3*b*c)*(c+d/x^2)^(1/2)/d^2/x
```

3.972.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.09

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx = \frac{-\sqrt{d}(d+cx^2)(2bd-3bcx^2+4adx^2)-c(3bc-4ad)x^4\sqrt{d+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{c+\frac{d}{x^2}}x^5}$$

input

```
Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4),x]
```

output
```
(-(Sqrt[d]*(d + c*x^2)*(2*b*d - 3*b*c*x^2 + 4*a*d*x^2)) - c*(3*b*c - 4*a*d
)*x^4*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(8*d^(5/2)*Sqrt[c
+ d/x^2]*x^5)
```

3.972.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 858, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^4\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{959}$$

$$-\frac{(3bc-4ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}x^4}dx}{4d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

$$\Big\downarrow \text{858}$$

$$\frac{(3bc-4ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}}{4d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

$$\Big\downarrow \text{262}$$

$$\frac{(3bc-4ad)\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{2d}\right)}{4d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

$$\Big\downarrow \text{224}$$

$$\frac{(3bc-4ad)\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{2d}\right)}{4d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3bc-4ad)\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}\right)}{4d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

input
```
Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4),x]
```

output
```
-1/4*(b*Sqrt[c + d/x^2])/(d*x^3) + ((3*b*c - 4*a*d)*(Sqrt[c + d/x^2]/(2*d*
x) - (c*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*d^(3/2))))/(4*d)
```

3.972.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.972.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{\left(c\,x^{2}+d\right)\left(4ad\,x^{2}-3cb\,x^{2}+2bd\right)}{8d^{2}x^{5}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}+\frac{c(4ad-3bc)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)\sqrt{c\,x^{2}+d}}{8d^{\frac{5}{2}}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}$
default	$-\frac{\sqrt{c\,x^{2}+d}\,\left(-4\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)ac\,d^{2}x^{4}+3\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)b\,c^{2}d\,x^{4}+4d^{\frac{5}{2}}\sqrt{c\,x^{2}+d}\,a\,x^{2}-3d^{\frac{3}{2}}\sqrt{c\,x^{2}+d}\,bc\,x^{2}+2d^{\frac{5}{2}}\sqrt{c\,x^{2}+}\right.}{8\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x^{5}d^{\frac{7}{2}}}$

input
```
int((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(c*x^2+d)*(4*a*d*x^2-3*b*c*x^2+2*b*d)/d^2/x^5/((c*x^2+d)/x^2)^(1/2)+1
/8*c*(4*a*d-3*b*c)/d^(5/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)/((c*x^2+d
)/x^2)^(1/2)/x*(c*x^2+d)^(1/2)
```

3.972.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.16

$$\int\frac{a+\frac{b}{x^{2}}}{\sqrt{c+\frac{d}{x^{2}}}\,x^{4}}\,dx$$

$$=\left[-\frac{(3\,bc^{2}-4\,acd)\sqrt{d}x^{3}\log\left(-\frac{cx^{2}+2\,\sqrt{d}x\sqrt{\frac{cx^{2}+d}{x^{2}}}+2\,d}{x^{2}}\right)+2\,(2\,bd^{2}-(3\,bcd-4\,ad^{2})x^{2})\sqrt{\frac{cx^{2}+d}{x^{2}}}}{16\,d^{3}x^{3}},\ (3\,bc^{2}-4\,acd\right.$$

input
```
integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((3*b*c^2 - 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*
x^2 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 - (3*b*c*d - 4*a*d^2)*x^2)*sqrt((c*
x^2 + d)/x^2))/(d^3*x^3), 1/8*((3*b*c^2 - 4*a*c*d)*sqrt(-d)*x^3*arctan(sqr
t(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*b*d^2 - (3*b*c*d - 4*a*d^2
)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^3)]
```

3.972.6 Sympy [A] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.61

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^4}\,dx = -\frac{a\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2dx} + \frac{ac\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{3}{2}}} + \frac{3bc^{\frac{3}{2}}}{8d^2x\sqrt{1+\frac{d}{cx^2}}} + \frac{b\sqrt{c}}{8dx^3\sqrt{1+\frac{d}{cx^2}}} - \frac{3bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{5}{2}}} - \frac{b}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}$$

input
```
integrate((a+b/x**2)/x**4/(c+d/x**2)**(1/2),x)
```

output
```
-a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*d*x) + a*c*asinh(sqrt(d)/(sqrt(c)*x))/(
2*d**(3/2)) + 3*b*c**(3/2)/(8*d**2*x*sqrt(1 + d/(c*x**2))) + b*sqrt(c)/(8*
d*x**3*sqrt(1 + d/(c*x**2))) - 3*b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(
5/2)) - b/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))
```

3.972.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(77) = 154.

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.15

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^4}\,dx = -\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}\,cx}{\left(c+\frac{d}{x^2}\right)dx^2-d^2} + \frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right)a + \frac{1}{16}\,b\left(\frac{3\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}} + \frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3 - 5\sqrt{c+\frac{d}{x^2}}\,c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2d^2x^4 - 2\left(c+\frac{d}{x^2}\right)d^3x^2 + d^4}\right)$$

input
```
integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*d*x^2 - d^2) + c*log((sqrt(c + d/
x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2))*a + 1/16*b*(3*c^
2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2)
 + 2*(3*(c + d/x^2)^(3/2)*c^2*x^3 - 5*sqrt(c + d/x^2)*c^2*d*x)/((c + d/x^2
)^2*d^2*x^4 - 2*(c + d/x^2)*d^3*x^2 + d^4))
```

3.972.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.34

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^4}\,dx$$
$$= \frac{\frac{\left(3\,bc^3-4\,ac^2d\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d^2}+\frac{3\,\left(cx^2+d\right)^{\frac{3}{2}}bc^3-4\,\left(cx^2+d\right)^{\frac{3}{2}}ac^2d-5\,\sqrt{cx^2+d}bc^3d+4\,\sqrt{cx^2+d}ac^2d^2}{c^2d^2x^4}}{8\,c\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*((3*b*c^3 - 4*a*c^2*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d^2)
 + (3*(c*x^2 + d)^(3/2)*b*c^3 - 4*(c*x^2 + d)^(3/2)*a*c^2*d - 5*sqrt(c*x^2
 + d)*b*c^3*d + 4*sqrt(c*x^2 + d)*a*c^2*d^2)/(c^2*d^2*x^4))/(c*sgn(x))
```

3.972.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^4}\,dx = \int \frac{a+\frac{b}{x^2}}{x^4\,\sqrt{c+\frac{d}{x^2}}}\,dx$$

input
```
int((a + b/x^2)/(x^4*(c + d/x^2)^(1/2)),x)
```

output
```
int((a + b/x^2)/(x^4*(c + d/x^2)^(1/2)), x)
```

3.973 $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

3.973.1 Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{3d(4bc-5ad)}{8c^3\sqrt{c+\frac{d}{x^2}}}+\frac{(4bc-5ad)x^2}{8c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}}-\frac{3d(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}$$

output
```
-3/8*d*(-5*a*d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(7/2)+3/8*d*(-5*a
*d+4*b*c)/c^3/(c+d/x^2)^(1/2)+1/8*(-5*a*d+4*b*c)*x^2/c^2/(c+d/x^2)^(1/2)+1
/4*a*x^4/c/(c+d/x^2)^(1/2)
```

3.973.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{\sqrt{c}x(4bc(3d+cx^2)+a(-15d^2-5cdx^2+2c^2x^4))+24bcd\sqrt{d+cx^2}\operatorname{arctanh}\Bigg(\frac{\sqrt{c}x}{\sqrt{d}-\sqrt{d+cx^2}}}{8c^{7/2}\sqrt{c+\frac{d}{x^2}}x}$$

input
```
Integrate[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2),x]
```

output
```
(Sqrt[c]*x*(4*b*c*(3*d + c*x^2) + a*(-15*d^2 - 5*c*d*x^2 + 2*c^2*x^4)) + 2
4*b*c*d*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/(Sqrt[d] - Sqrt[d + c*x^2])] +
 30*a*d^2*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[d + c*x^2])
])/(8*c^(7/2)*Sqrt[c + d/x^2]*x)
```

3.973.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {948, 87, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+\frac{b}{x^2}\right)}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)x^6}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{ax^4}{2c\sqrt{c+\frac{d}{x^2}}}-\frac{(4bc-5ad)\int \frac{x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x^2}}{4c}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{ax^4}{2c\sqrt{c+\frac{d}{x^2}}}-\frac{(4bc-5ad)\left(-\frac{3d\int \frac{x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x^2}}{2c}-\frac{x^2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{4c}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{ax^4}{2c\sqrt{c+\frac{d}{x^2}}}-\frac{(4bc-5ad)\left(-\frac{3d\left(\frac{\int\frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}}{c}+\frac{2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{2c}-\frac{x^2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{4c}\right)$$

$$\Big\downarrow \color{blue}{73}$$

$$\frac{1}{2}\left(\frac{ax^4}{2c\sqrt{c+\frac{d}{x^2}}}-\frac{(4bc-5ad)\left(-\frac{3d\left(\frac{2\int\frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{cd}+\frac{2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{2c}-\frac{x^2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{4c}\right)$$

$$\Big\downarrow \color{blue}{221}$$

$$\frac{1}{2}\left(\frac{ax^4}{2c\sqrt{c+\frac{d}{x^2}}}-\frac{(4bc-5ad)\left(-\frac{3d\left(\frac{2}{c\sqrt{c+\frac{d}{x^2}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}\right)}{2c}-\frac{x^2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{4c}\right)$$

3.973. $\int\frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

input
```
Int[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2),x]
```

output
```
((a*x^4)/(2*c*Sqrt[c + d/x^2]) - ((4*b*c - 5*a*d)*(-(x^2/(c*Sqrt[c + d/x^2
])) - (3*d*(2/(c*Sqrt[c + d/x^2]) - (2*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/c
^(3/2)))/(2*c)))/(4*c))/2
```

3.973.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.973. $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.973.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.19

method	result
default	$\frac{\left(c x^{2}+d\right)\left(2c^{\frac{7}{2}} a x^{5}-5c^{\frac{5}{2}} a d x^{3}+4c^{\frac{7}{2}} b x^{3}-15c^{\frac{3}{2}} a d^{2} x+12c^{\frac{5}{2}} b d x+15\ln\left(\sqrt{c}\, x+\sqrt{c x^{2}+d}\right)\sqrt{c x^{2}+d}\, a c d^{2}-12\ln\left(\sqrt{c}\, x+\sqrt{c x^{2}+d}\right)\right.}{8\left(\frac{c x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} c^{\frac{9}{2}}}$
risch	$\frac{\left(2ac x^{2}-7ad+4bc\right)\left(c x^{2}+d\right)}{8c^{3}\sqrt{\frac{c x^{2}+d}{x^{2}}}}+\frac{d\left(\frac{7adx}{\sqrt{c x^{2}+d}}-\frac{4bcx}{\sqrt{c x^{2}+d}}+\left(15acd-12b c^{2}\right)\left(-\frac{x}{c\sqrt{c x^{2}+d}}+\frac{\ln\left(\sqrt{c}\, x+\sqrt{c x^{2}+d}\right)}{c^{\frac{3}{2}}}\right)\right)\sqrt{c x^{2}+d}}{8c^{3}\sqrt{\frac{c x^{2}+d}{x^{2}}}\, x}$

input
```
int((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(c*x^2+d)*(2*c^(7/2)*a*x^5-5*c^(5/2)*a*d*x^3+4*c^(7/2)*b*x^3-15*c^(3/2
)*a*d^2*x+12*c^(5/2)*b*d*x+15*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*(c*x^2+d)^(1/2
)*a*c*d^2-12*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*(c*x^2+d)^(1/2)*b*c^2*d)/((c*x^
2+d)/x^2)^(3/2)/x^3/c^(9/2)
```

3.973.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.58

$$\int \frac{\left(a+\frac{b}{x^2}\right) x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\, dx = \left[-\frac{3\,(4\,bcd^2-5\,ad^3+(4\,bc^2d-5\,acd^2)x^2)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2}{16\,(c^5x^2+c^4d)}\right.$$

input
```
integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

output

```
[-1/16*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt(c)*log(
-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(2*a*c^3*x^6 + (4*
b*c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x
^2))/(c^5*x^2 + c^4*d), 1/8*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d
^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) +
 (2*a*c^3*x^6 + (4*b*c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)
*sqrt((c*x^2 + d)/x^2))/(c^5*x^2 + c^4*d)]
```

3.973.6 Sympy [A] (verification not implemented)

Time = 34.41 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.50

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = a\left(\frac{x^5}{4c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} - \frac{5\sqrt{d}x^3}{8c^2\sqrt{\frac{cx^2}{d}+1}} - \frac{15d^{\frac{3}{2}}x}{8c^3\sqrt{\frac{cx^2}{d}+1}} + \frac{15d^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{7}{2}}}\right) + b\left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + \frac{3\sqrt{d}x}{2c^2\sqrt{\frac{cx^2}{d}+1}} - \frac{3d\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{5}{2}}}\right)$$

input

```
integrate((a+b/x**2)*x**3/(c+d/x**2)**(3/2),x)
```

output

```
a*(x**5/(4*c*sqrt(d)*sqrt(c*x**2/d + 1)) - 5*sqrt(d)*x**3/(8*c**2*sqrt(c*x
**2/d + 1)) - 15*d**(3/2)*x/(8*c**3*sqrt(c*x**2/d + 1)) + 15*d**2*asinh(sq
rt(c)*x/sqrt(d))/(8*c**(7/2))) + b*(x**3/(2*c*sqrt(d)*sqrt(c*x**2/d + 1))
+ 3*sqrt(d)*x/(2*c**2*sqrt(c*x**2/d + 1)) - 3*d*asinh(sqrt(c)*x/sqrt(d))/(
2*c**(5/2)))
```

3.973.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(98) = 196$.

Time = 0.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.82

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\frac{1}{16}\,a\left(\frac{2\left(15\left(c+\frac{d}{x^2}\right)^2d^2-25\left(c+\frac{d}{x^2}\right)cd^2+8\,c^2d^2\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3-2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^4+\sqrt{c+\frac{d}{x^2}}c^5}+\frac{15\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{7}{2}}}\right) + \frac{1}{4}\,b\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)d-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2-\sqrt{c+\frac{d}{x^2}}c^3}+\frac{3\,d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)$$

input
```
integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/16*a*(2*(15*(c + d/x^2)^2*d^2 - 25*(c + d/x^2)*c*d^2 + 8*c^2*d^2)/((c +
 d/x^2)^(5/2)*c^3 - 2*(c + d/x^2)^(3/2)*c^4 + sqrt(c + d/x^2)*c^5) + 15*d^
2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(7/2)) +
1/4*b*(2*(3*(c + d/x^2)*d - 2*c*d)/((c + d/x^2)^(3/2)*c^2 - sqrt(c + d/x^2
)*c^3) + 3*d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/
c^(5/2))
```

3.973.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.22

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{\left(x^2\left(\frac{2\,ax^2}{c\mathrm{sgn}(x)}+\frac{4\,bc^4\mathrm{sgn}(x)-5\,ac^3d\mathrm{sgn}(x)}{c^5}\right)+\frac{3\left(4\,bc^3d\mathrm{sgn}(x)-5\,ac^2d^2\mathrm{sgn}(x)\right)}{c^5}\right)x}{8\sqrt{cx^2+d}} - \frac{3\left(4\,bcd\log\left(|d|\right)-5\,ad^2\log\left(|d|\right)\right)\mathrm{sgn}(x)}{16\,c^{\frac{7}{2}}} + \frac{3\left(4\,bcd-5\,ad^2\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{8\,c^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(x^2*(2*a*x^2/(c*sgn(x)) + (4*b*c^4*sgn(x) - 5*a*c^3*d*sgn(x))/c^5) +
3*(4*b*c^3*d*sgn(x) - 5*a*c^2*d^2*sgn(x))/c^5)*x/sqrt(c*x^2 + d) - 3/16*(4
*b*c*d*log(abs(d)) - 5*a*d^2*log(abs(d)))*sgn(x)/c^(7/2) + 3/8*(4*b*c*d -
5*a*d^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(7/2)*sgn(x))
```

3.973.9 Mupad [B] (verification not implemented)

Time = 10.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{a\,x^4}{4\,c\,\sqrt{c+\frac{d}{x^2}}}-\frac{15\,a\,d^2}{8\,c^3\,\sqrt{c+\frac{d}{x^2}}}+\frac{b\,x^2}{2\,c\,\sqrt{c+\frac{d}{x^2}}}-\frac{3\,b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{5/2}}+\frac{15\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{7/2}}+\frac{3\,b\,d}{2\,c^2\,\sqrt{c+\frac{d}{x^2}}}-\frac{5\,a\,d\,x^2}{8\,c^2\,\sqrt{c+\frac{d}{x^2}}}$$

input
```
int((x^3*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

output
```
(a*x^4)/(4*c*(c + d/x^2)^(1/2)) - (15*a*d^2)/(8*c^3*(c + d/x^2)^(1/2)) + (
b*x^2)/(2*c*(c + d/x^2)^(1/2)) - (3*b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/
(2*c^(5/2)) + (15*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(7/2)) + (3
*b*d)/(2*c^2*(c + d/x^2)^(1/2)) - (5*a*d*x^2)/(8*c^2*(c + d/x^2)^(1/2))
```

3.974 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

3.974.1 Optimal result

Integrand size = 20, antiderivative size = 86

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=-\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}}+\frac{(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}$$

output
```
1/2*(-3*a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(5/2)+1/2*(3*a*d-2*b
*c)/c^2/(c+d/x^2)^(1/2)+1/2*a*x^2/c/(c+d/x^2)^(1/2)
```

3.974.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{6ad\sqrt{d+cx^2}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{d}-\sqrt{d+cx^2}}\right)+\sqrt{c}\left(-2bcx+3adx+acx^3+4b\sqrt{c}\sqrt{d+cx^2}\text{arctan}\right.}{2c^{5/2}\sqrt{c+\frac{d}{x^2}}x}$$

input
```
Integrate[((a + b/x^2)*x)/(c + d/x^2)^(3/2),x]
```

output
```
(6*a*d*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/(Sqrt[d] - Sqrt[d + c*x^2])] +
Sqrt[c]*(-2*b*c*x + 3*a*d*x + a*c*x^3 + 4*b*Sqrt[c]*Sqrt[d + c*x^2]*ArcTan
h[(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[d + c*x^2])]))/(2*c^(5/2)*Sqrt[c + d/x^2]*x
)
```

3.974.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+\frac{b}{x^2}\right)}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{ax^2}{c\sqrt{c+\frac{d}{x^2}}}-\frac{(2bc-3ad)\int \frac{x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x^2}}{2c}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{ax^2}{c\sqrt{c+\frac{d}{x^2}}}-\frac{(2bc-3ad)\left(\frac{\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}}{c}+\frac{2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{2c}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{ax^2}{c\sqrt{c+\frac{d}{x^2}}}-\frac{(2bc-3ad)\left(\frac{2\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{cd}+\frac{2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{2c}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{2}\left(\frac{ax^2}{c\sqrt{c+\frac{d}{x^2}}}-\frac{(2bc-3ad)\left(\frac{2}{c\sqrt{c+\frac{d}{x^2}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}\right)}{2c}\right)$$

input
```
Int[((a + b/x^2)*x)/(c + d/x^2)^(3/2),x]
```

output
```
((a*x^2)/(c*Sqrt[c + d/x^2]) - ((2*b*c - 3*a*d)*(2/(c*Sqrt[c + d/x^2]) - (
2*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/c^(3/2)))/(2*c))/2
```

3.974.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.974. $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.974.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{\left(c\,x^{2}+d\right)\left(-c^{\frac{5}{2}} a\,x^{3}-3 c^{\frac{3}{2}} a d x+2 c^{\frac{5}{2}} b x+3 \sqrt{c\,x^{2}+d}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right) a c d-2 \sqrt{c\,x^{2}+d}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right) b\,c^{2}\right)}{2 \left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} c^{\frac{7}{2}}}$	115
risch	$\frac{a\left(c\,x^{2}+d\right)}{2 c^{2} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}}-\frac{\left(\frac{a d x}{\sqrt{c\,x^{2}+d}}+\left(3 a c d-2 b\,c^{2}\right)\left(-\frac{x}{c \sqrt{c\,x^{2}+d}}+\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)}{c^{\frac{3}{2}}}\right)\right)\sqrt{c\,x^{2}+d}}{2 c^{2} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x}$	121

input
```
int((a+b/x^2)*x/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*x^2+d)*(-c^(5/2)*a*x^3-3*c^(3/2)*a*d*x+2*c^(5/2)*b*x+3*(c*x^2+d)^(
1/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*c*d-2*(c*x^2+d)^(1/2)*ln(c^(1/2)*x+(c
*x^2+d)^(1/2))*b*c^2)/((c*x^2+d)/x^2)^(3/2)/x^3/c^(7/2)
```

3.974.5 Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.90

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \left[-\frac{\left(2\,bcd-3\,ad^2+\left(2\,bc^2-3\,acd\right)x^2\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,(ac^2x^4}{4\,(c^4x^2+c^3d)}\right.$$

$$\left.-\frac{\left(2\,bcd-3\,ad^2+\left(2\,bc^2-3\,acd\right)x^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-\left(ac^2x^4-\left(2\,bc^2-3\,acd\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{2\,(c^4x^2+c^3d)}\right]$$

input
```
integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*b*c*d - 3*a*d^2 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(c)*log(-2*c*x^2
+ 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(a*c^2*x^4 - (2*b*c^2 - 3*a
*c*d)*x^2)*sqrt((c*x^2 + d)/x^2))/(c^4*x^2 + c^3*d), -1/2*((2*b*c*d - 3*a*
d^2 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 +
d)/x^2)/(c*x^2 + d)) - (a*c^2*x^4 - (2*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 +
 d)/x^2))/(c^4*x^2 + c^3*d)]
```

3.974.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(73) = 146.

Time = 16.94 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.07

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = a\left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3\sqrt{d}x}{2c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{3d\,\mathrm{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{5}{2}}}\right)$$
$$+b\left(-\frac{2c^3x^2\sqrt{1+\frac{d}{cx^2}}}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}-\frac{c^3x^2\log\left(\frac{d}{cx^2}\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}+\frac{2c^3x^2\log\left(\sqrt{1+\frac{d}{cx^2}}+1\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}\right.$$
$$\left.-\frac{c^2d\log\left(\frac{d}{cx^2}\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}+\frac{2c^2d\log\left(\sqrt{1+\frac{d}{cx^2}}+1\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}\right)$$

input
```
integrate((a+b/x**2)*x/(c+d/x**2)**(3/2),x)
```

output
```
a*(x**3/(2*c*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*sqrt(d)*x/(2*c**2*sqrt(c*x**2
/d + 1)) - 3*d*asinh(sqrt(c)*x/sqrt(d))/(2*c**(5/2))) + b*(-2*c**3*x**2*sq
rt(1 + d/(c*x**2))/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) - c**3*x**2*log(d/(c*x
**2))/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) + 2*c**3*x**2*log(sqrt(1 + d/(c*x**
2)) + 1)/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) - c**2*d*log(d/(c*x**2))/(2*c**(
9/2)*x**2 + 2*c**(7/2)*d) + 2*c**2*d*log(sqrt(1 + d/(c*x**2)) + 1)/(2*c**(
9/2)*x**2 + 2*c**(7/2)*d))
```

3.974.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(70) = 140.

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.67

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{1}{4}\,a\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)d-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2-\sqrt{c+\frac{d}{x^2}}c^3}+\frac{3\,d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right) - \frac{1}{2}\,b\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^2}}c}\right)$$

input
```
integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*a*(2*(3*(c + d/x^2)*d - 2*c*d)/((c + d/x^2)^(3/2)*c^2 - sqrt(c + d/x^2
)*c^3) + 3*d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/
c^(5/2)) - 1/2*b*(log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(
c)))/c^(3/2) + 2/(sqrt(c + d/x^2)*c))
```

3.974.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.22

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{x\left(\frac{ax^2}{c\mathrm{sgn}(x)} - \frac{2\,bc^2\mathrm{sgn}(x)-3\,acd\mathrm{sgn}(x)}{c^3}\right)}{2\sqrt{cx^2+d}} + \frac{(2\,bc\log\left(|d|\right) - 3\,ad\log\left(|d|\right))\mathrm{sgn}(x)}{4\,c^{\frac{5}{2}}} - \frac{(2\,bc-3\,ad)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{2\,c^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*x*(a*x^2/(c*sgn(x)) - (2*b*c^2*sgn(x) - 3*a*c*d*sgn(x))/c^3)/sqrt(c*x^
2 + d) + 1/4*(2*b*c*log(abs(d)) - 3*a*d*log(abs(d)))*sgn(x)/c^(5/2) - 1/2*
(2*b*c - 3*a*d)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(5/2)*sgn(x))
```

3.974.9 Mupad [B] (verification not implemented)

Time = 9.82 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}} - \frac{b}{c\sqrt{c+\frac{d}{x^2}}} + \frac{a\,x^2}{2\,c\sqrt{c+\frac{d}{x^2}}} - \frac{3\,a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{5/2}} + \frac{3\,a\,d}{2\,c^2\sqrt{c+\frac{d}{x^2}}}$$

input
```
int((x*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

output
```
(b*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(3/2) - b/(c*(c + d/x^2)^(1/2)) + (
a*x^2)/(2*c*(c + d/x^2)^(1/2)) - (3*a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/
(2*c^(5/2)) + (3*a*d)/(2*c^2*(c + d/x^2)^(1/2))
```

3.975 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x} \, dx$

3.975.1 Optimal result

Integrand size = 22, antiderivative size = 52

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x} \, dx=\frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}}+\frac{a\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}$$

output
```
a*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(3/2)+(-a*d+b*c)/c/d/(c+d/x^2)^(1/2)
```

3.975.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.44

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x} \, dx=\frac{\sqrt{c}(bc-ad)x-ad\sqrt{d+cx^2}\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)}{c^{3/2}d\sqrt{c+\frac{d}{x^2}}x}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]
```

output
```
(Sqrt[c]*(b*c - a*d)*x - a*d*Sqrt[d + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[d + c
*x^2]])/(c^(3/2)*d*Sqrt[c + d/x^2]*x)
```

3.975.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 948$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x^2}$$

$$\Big\downarrow \; 87$$

$$\frac{1}{2}\left(\frac{2(bc-ad)}{cd\sqrt{c+\frac{d}{x^2}}}-\frac{a\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}}{c}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{2(bc-ad)}{cd\sqrt{c+\frac{d}{x^2}}}-\frac{2a\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{cd}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{2a\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{2(bc-ad)}{cd\sqrt{c+\frac{d}{x^2}}}\right)$$

input
```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]
```

output
```
((2*(b*c - a*d))/(c*d*Sqrt[c + d/x^2]) + (2*a*ArcTanh[Sqrt[c + d/x^2]/Sqrt
[c]])/c^(3/2))/2
```

3.975.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.975.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.44

method	result	size
default	$\frac{\left(c\,x^{2}+d\right)\left(c^{\frac{5}{2}} b x-c^{\frac{3}{2}} a d x+\sqrt{c\,x^{2}+d}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right) a c d\right)}{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} d\,c^{\frac{5}{2}}}$	75

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+d)*(c^(5/2)*b*x-c^(3/2)*a*d*x+(c*x^2+d)^(1/2)*ln(c^(1/2)*x+(c*x^2+d
)^(1/2))*a*c*d)/((c*x^2+d)/x^2)^(3/2)/x^3/d/c^(5/2)
```

3.975.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.

Time = 0.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 3.85

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x}\,dx = \left[\frac{2\,(bc^2-acd)x^2\sqrt{\frac{cx^2+d}{x^2}}+(acdx^2+ad^2)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)}{2\,(c^3dx^2+c^2d^2)},\ (bc^2\right.$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/2*(2*(b*c^2 - a*c*d)*x^2*sqrt((c*x^2 + d)/x^2) + (a*c*d*x^2 + a*d^2)*sq
rt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d))/(c^3*d*x^2
+ c^2*d^2), ((b*c^2 - a*c*d)*x^2*sqrt((c*x^2 + d)/x^2) - (a*c*d*x^2 + a*d^
2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c^3*d
*x^2 + c^2*d^2)]
```

3.975.6 Sympy [A] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.44

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x}\,dx = \begin{cases} \frac{2\left(-\frac{ad\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{2c\sqrt{-c}}-\frac{ad-bc}{2c\sqrt{c+\frac{d}{x^2}}}\right)}{d} & \text{for } d\neq 0 \\ \frac{-a\log\left(-\frac{b}{x^2}\right)-\frac{b}{x^2}}{2c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x,x)
```

output
```
Piecewise((2*(-a*d*atan(sqrt(c + d/x**2)/sqrt(-c))/(2*c*sqrt(-c)) - (a*d -
 b*c)/(2*c*sqrt(c + d/x**2)))/d, Ne(d, 0)), ((-a*log(-b/x**2) - b/x**2)/(2
*c**(3/2)), True))
```

3.975.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x} \, dx = -\frac{1}{2} a \left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2}{\sqrt{c+\frac{d}{x^2}}c}\right) + \frac{b}{\sqrt{c+\frac{d}{x^2}}d}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
-1/2*a*(log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/
2) + 2/(sqrt(c + d/x^2)*c)) + b/(sqrt(c + d/x^2)*d)
```

3.975.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x} \, dx = \frac{a\log\left(|d|\right)\operatorname{sgn}(x)}{2\,c^{\frac{3}{2}}} + \frac{(bc\operatorname{sgn}(x) - ad\operatorname{sgn}(x))x}{\sqrt{cx^2+d}cd} - \frac{a\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{c^{\frac{3}{2}}\operatorname{sgn}(x)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="giac")
```

output
```
1/2*a*log(abs(d))*sgn(x)/c^(3/2) + (b*c*sgn(x) - a*d*sgn(x))*x/(sqrt(c*x^2
 + d)*c*d) - a*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(3/2)*sgn(x))
```

3.975.9 Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x} \, dx = \frac{a\,\operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}} - \frac{a}{c\sqrt{c+\frac{d}{x^2}}} + \frac{b\sqrt{x^2}}{d\sqrt{c\,x^2+d}}$$

input
```
int((a + b/x^2)/(x*(c + d/x^2)^(3/2)),x)
```

output
```
(a*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(3/2) - a/(c*(c + d/x^2)^(1/2)) + (
b*(x^2)^(1/2))/(d*(d + c*x^2)^(1/2))
```

3.976 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^3} \, dx$

3.976.1 Optimal result

Integrand size = 22, antiderivative size = 42

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^3} \, dx = -\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}$$

output
```
(a*d-b*c)/d^2/(c+d/x^2)^(1/2)-b*(c+d/x^2)^(1/2)/d^2
```

3.976.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^3} \, dx = \frac{adx^2-b(d+2cx^2)}{d^2\sqrt{c+\frac{d}{x^2}}x^2}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3),x]
```

output
```
(a*d*x^2 - b*(d + 2*c*x^2))/(d^2*Sqrt[c + d/x^2]*x^2)
```

3.976.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^3\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{946}$$

$$-\frac{1}{2}\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2}\int \left(\frac{b}{d\sqrt{c+\frac{d}{x^2}}}+\frac{ad-bc}{d\left(c+\frac{d}{x^2}\right)^{3/2}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2(bc-ad)}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{2b\sqrt{c+\frac{d}{x^2}}}{d^2}\right)$$

input
```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3),x]
```

output
```
((-2*(b*c - a*d))/(d^2*Sqrt[c + d/x^2]) - (2*b*Sqrt[c + d/x^2])/d^2)/2
```

3.976.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.976.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result	size
gosper	$\frac{\left(a d \,x^{2}-2 c b \,x^{2}-b d \right) \left(c \,x^{2}+d \right)}{\left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} d^{2} x^{4}}$	46
default	$\frac{\left(a d \,x^{2}-2 c b \,x^{2}-b d \right) \left(c \,x^{2}+d \right)}{\left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} d^{2} x^{4}}$	46
trager	$\frac{\left(a d \,x^{2}-2 c b \,x^{2}-b d \right) \sqrt{-\frac{-c \,x^{2}-d}{x^{2}}}}{d^{2} \left(c \,x^{2}+d \right)}$	49
risch	$-\frac{b \left(c \,x^{2}+d \right)}{d^{2} x^{2} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}+\frac{a d -b c}{d^{2} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}$	56

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
(a*d*x^2-2*b*c*x^2-b*d)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/d^2/x^4
```

3.976.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^3}\,dx = -\frac{\left((2\,bc-ad)x^2+bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{cd^2x^2+d^3}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
-((2*b*c - a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2)/(c*d^2*x^2 + d^3)
```

3.976.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.62

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx = \begin{cases} \frac{a}{d\sqrt{c+\frac{d}{x^2}}} - \frac{2bc}{d^2\sqrt{c+\frac{d}{x^2}}} - \frac{b}{dx^2\sqrt{c+\frac{d}{x^2}}} & \text{for } d \neq 0 \\ \frac{-\frac{a}{2x^2}-\frac{b}{4x^4}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**3,x)
```

output
```
Piecewise((a/(d*sqrt(c + d/x**2)) - 2*b*c/(d**2*sqrt(c + d/x**2)) - b/(d*x
**2*sqrt(c + d/x**2)), Ne(d, 0)), ((-a/(2*x**2) - b/(4*x**4))/c**(3/2), Tr
ue))
```

3.976.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx = -b\left(\frac{\sqrt{c+\frac{d}{x^2}}}{d^2} + \frac{c}{\sqrt{c+\frac{d}{x^2}}d^2}\right) + \frac{a}{\sqrt{c+\frac{d}{x^2}}d}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-b*(sqrt(c + d/x^2)/d^2 + c/(sqrt(c + d/x^2)*d^2)) + a/(sqrt(c + d/x^2)*d)
```

3.976.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.57

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx = \frac{2\,b\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)d\operatorname{sgn}(x)} - \frac{(bc-ad)x}{\sqrt{cx^2+d}d^2\operatorname{sgn}(x)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
2*b*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)*d*sgn(x)) - (b*c - a*d)
*x/(sqrt(c*x^2 + d)*d^2*sgn(x))
```

3.976.9 Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^3}\,dx = \frac{x\sqrt{c+\frac{d}{x^2}}\left(x^2\left(\frac{a}{d}-\frac{2\,b\,c}{d^2}\right)-\frac{b}{d}\right)}{c\,x^3+d\,x}$$

input
```
int((a + b/x^2)/(x^3*(c + d/x^2)^(3/2)),x)
```

output
```
(x*(c + d/x^2)^(1/2)*(x^2*(a/d - (2*b*c)/d^2) - b/d))/(d*x + c*x^3)
```

3.977 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5} \, dx$

3.977.1 Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5} \, dx = \frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}+\frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}$$

output
```
-1/3*b*(c+d/x^2)^(3/2)/d^3+c*(-a*d+b*c)/d^3/(c+d/x^2)^(1/2)+(-a*d+2*b*c)*(
c+d/x^2)^(1/2)/d^3
```

3.977.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5} \, dx = \frac{-3adx^2(d+2cx^2)+b(-d^2+4cdx^2+8c^2x^4)}{3d^3\sqrt{c+\frac{d}{x^2}}x^4}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5),x]
```

output
```
(-3*a*d*x^2*(d + 2*c*x^2) + b*(-d^2 + 4*c*d*x^2 + 8*c^2*x^4))/(3*d^3*Sqrt[
c + d/x^2]*x^4)
```

3.977.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^5\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{2}\int \left(\frac{\sqrt{c+\frac{d}{x^2}}b}{d^2}+\frac{ad-2bc}{d^2\sqrt{c+\frac{d}{x^2}}}+\frac{c(bc-ad)}{d^2\left(c+\frac{d}{x^2}\right)^{3/2}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\sqrt{c+\frac{d}{x^2}}(2bc-ad)}{d^3}+\frac{2c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}-\frac{2b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}\right)$$

input
```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5),x]
```

output
```
((2*c*(b*c - a*d))/(d^3*Sqrt[c + d/x^2]) + (2*(2*b*c - a*d)*Sqrt[c + d/x^2
])/d^3 - (2*b*(c + d/x^2)^(3/2))/(3*d^3))/2
```

3.977.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.977.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

method	result	size
gosper	$-\frac{\left(6acd\,x^{4}-8b\,c^{2}x^{4}+3a\,d^{2}x^{2}-4bcd\,x^{2}+b\,d^{2}\right)\left(c\,x^{2}+d\right)}{3\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}d^{3}x^{6}}$	69
default	$-\frac{\left(6acd\,x^{4}-8b\,c^{2}x^{4}+3a\,d^{2}x^{2}-4bcd\,x^{2}+b\,d^{2}\right)\left(c\,x^{2}+d\right)}{3\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}d^{3}x^{6}}$	69
trager	$-\frac{\left(6acd\,x^{4}-8b\,c^{2}x^{4}+3a\,d^{2}x^{2}-4bcd\,x^{2}+b\,d^{2}\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{3x^{2}d^{3}\left(c\,x^{2}+d\right)}$	75
risch	$-\frac{\left(c\,x^{2}+d\right)\left(3ad\,x^{2}-5cb\,x^{2}+bd\right)}{3d^{3}x^{4}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}-\frac{\left(ad-bc\right)c}{d^{3}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}$	75

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(6*a*c*d*x^4-8*b*c^2*x^4+3*a*d^2*x^2-4*b*c*d*x^2+b*d^2)*(c*x^2+d)/((c
*x^2+d)/x^2)^(3/2)/d^3/x^6
```

3.977.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx=\frac{\left(2\left(4\,bc^2-3\,acd\right)x^4-bd^2+\left(4\,bcd-3\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\left(cd^3x^4+d^4x^2\right)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
1/3*(2*(4*b*c^2 - 3*a*c*d)*x^4 - b*d^2 + (4*b*c*d - 3*a*d^2)*x^2)*sqrt((c*
x^2 + d)/x^2)/(c*d^3*x^4 + d^4*x^2)
```

3.977.6 Sympy [A] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5}\,dx = \begin{cases} \frac{2\left(-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{6d^2} - \frac{c(ad-bc)}{2d^2\sqrt{c+\frac{d}{x^2}}} - \frac{\sqrt{c+\frac{d}{x^2}}\,(ad-2bc)}{2d^2}\right)}{d} & \text{for } d \neq 0 \\ \frac{-\frac{a}{2x^4}-\frac{b}{3x^6}}{2c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**5,x)
```

output
```
Piecewise((2*(-b*(c + d/x**2)**(3/2)/(6*d**2) - c*(a*d - b*c)/(2*d**2*sqrt
(c + d/x**2)) - sqrt(c + d/x**2)*(a*d - 2*b*c)/(2*d**2))/d, Ne(d, 0)), ((-
a/(2*x**4) - b/(3*x**6))/(2*c**(3/2)), True))
```

3.977.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5}\,dx = -\frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3} - \frac{6\sqrt{c+\frac{d}{x^2}}\,c}{d^3} - \frac{3\,c^2}{\sqrt{c+\frac{d}{x^2}}\,d^3}\right) - a\left(\frac{\sqrt{c+\frac{d}{x^2}}}{d^2} + \frac{c}{\sqrt{c+\frac{d}{x^2}}\,d^2}\right)$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
-1/3*b*((c + d/x^2)^(3/2)/d^3 - 6*sqrt(c + d/x^2)*c/d^3 - 3*c^2/(sqrt(c +
d/x^2)*d^3)) - a*(sqrt(c + d/x^2)/d^2 + c/(sqrt(c + d/x^2)*d^2))
```

3.977.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(60) = 120$.

Time = 0.53 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.76

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx = \frac{(bc^2-acd)x}{\sqrt{cx^2+d}d^3\mathrm{sgn}\,(x)}$$

$$-\frac{2\,(3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{3}{2}}-3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4a\sqrt{c}d-12\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2bc^{\frac{3}{2}}d+6\,(\sqrt{c}x-\sqrt{cx}}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^3d^2\mathrm{sgn}\,(x)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
(b*c^2 - a*c*d)*x/(sqrt(c*x^2 + d)*d^3*sgn(x)) - 2/3*(3*(sqrt(c)*x - sqrt(
c*x^2 + d))^4*b*c^(3/2) - 3*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*sqrt(c)*d -
12*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(3/2)*d + 6*(sqrt(c)*x - sqrt(c*x^2
 + d))^2*a*sqrt(c)*d^2 + 5*b*c^(3/2)*d^2 - 3*a*sqrt(c)*d^3)/(((sqrt(c)*x -
 sqrt(c*x^2 + d))^2 - d)^3*d^2*sgn(x))
```

3.977.9 Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}\,\left(-8\,b\,c^2\,x^4+6\,a\,c\,d\,x^4-4\,b\,c\,d\,x^2+3\,a\,d^2\,x^2+b\,d^2\right)}{3\,d^3\,x^2\,\left(c\,x^2+d\right)}$$

input
```
int((a + b/x^2)/(x^5*(c + d/x^2)^(3/2)),x)
```

output
```
-((c + d/x^2)^(1/2)*(b*d^2 + 3*a*d^2*x^2 - 8*b*c^2*x^4 + 6*a*c*d*x^4 - 4*b
*c*d*x^2))/(3*d^3*x^2*(d + c*x^2))
```

3.978 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7} \, dx$

3.978.1 Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7} \, dx = -\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}} - \frac{c(3bc-2ad)\sqrt{c+\frac{d}{x^2}}}{d^4} + \frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}$$

output
```
1/3*(-a*d+3*b*c)*(c+d/x^2)^(3/2)/d^4-1/5*b*(c+d/x^2)^(5/2)/d^4-c^2*(-a*d+b
*c)/d^4/(c+d/x^2)^(1/2)-c*(-2*a*d+3*b*c)*(c+d/x^2)^(1/2)/d^4
```

3.978.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7} \, dx = \frac{-5adx^2(d^2-4cdx^2-8c^2x^4)-3b(d^3-2cd^2x^2+8c^2dx^4+16c^3x^6)}{15d^4\sqrt{c+\frac{d}{x^2}}x^6}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7),x]
```

output
```
(-5*a*d*x^2*(d^2 - 4*c*d*x^2 - 8*c^2*x^4) - 3*b*(d^3 - 2*c*d^2*x^2 + 8*c^2
*d*x^4 + 16*c^3*x^6))/(15*d^4*Sqrt[c + d/x^2]*x^6)
```

3.978.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^7\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}d\frac{1}{x^2}$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{2}\int \left(-\frac{(bc-ad)c^2}{d^3\left(c+\frac{d}{x^2}\right)^{3/2}}+\frac{(3bc-2ad)c}{d^3\sqrt{c+\frac{d}{x^2}}}+\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{d^3}+\frac{(ad-3bc)\sqrt{c+\frac{d}{x^2}}}{d^3}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}+\frac{2\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-ad)}{3d^4}-\frac{2c\sqrt{c+\frac{d}{x^2}}(3bc-2ad)}{d^4}-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}\right)$$

input
```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7),x]
```

output
```
((-2*c^2*(b*c - a*d))/(d^4*Sqrt[c + d/x^2]) - (2*c*(3*b*c - 2*a*d)*Sqrt[c
+ d/x^2])/d^4 + (2*(3*b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (2*b*(c + d/
x^2)^(5/2))/(5*d^4))/2
```

3.978.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.978.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{\left(40a\,c^{2}d\,x^{6}-48b\,c^{3}x^{6}+20ac\,d^{2}x^{4}-24b\,c^{2}d\,x^{4}-5a\,d^{3}x^{2}+6bc\,d^{2}x^{2}-3b\,d^{3}\right)\left(c\,x^{2}+d\right)}{15\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}d^{4}x^{8}}$	94
default	$\frac{\left(40a\,c^{2}d\,x^{6}-48b\,c^{3}x^{6}+20ac\,d^{2}x^{4}-24b\,c^{2}d\,x^{4}-5a\,d^{3}x^{2}+6bc\,d^{2}x^{2}-3b\,d^{3}\right)\left(c\,x^{2}+d\right)}{15\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}d^{4}x^{8}}$	94
risch	$\frac{\left(c\,x^{2}+d\right)\left(25acd\,x^{4}-33b\,c^{2}x^{4}-5a\,d^{2}x^{2}+9bcd\,x^{2}-3b\,d^{2}\right)}{15d^{4}x^{6}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}+\frac{(ad-bc)c^{2}}{d^{4}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}$	99
trager	$\frac{\left(40a\,c^{2}d\,x^{6}-48b\,c^{3}x^{6}+20ac\,d^{2}x^{4}-24b\,c^{2}d\,x^{4}-5a\,d^{3}x^{2}+6bc\,d^{2}x^{2}-3b\,d^{3}\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{15x^{4}d^{4}\left(c\,x^{2}+d\right)}$	100

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/15*(40*a*c^2*d*x^6-48*b*c^3*x^6+20*a*c*d^2*x^4-24*b*c^2*d*x^4-5*a*d^3*x^
2+6*b*c*d^2*x^2-3*b*d^3)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/d^4/x^8
```

3.978.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^7}\,dx = -\frac{\left(8\,(6\,bc^3-5\,ac^2d)x^6+4\,(6\,bc^2d-5\,acd^2)x^4+3\,bd^3-(6\,bcd^2-5\,ad^3)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,(cd^4x^6+d^5x^4)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
-1/15*(8*(6*b*c^3 - 5*a*c^2*d)*x^6 + 4*(6*b*c^2*d - 5*a*c*d^2)*x^4 + 3*b*d
^3 - (6*b*c*d^2 - 5*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(c*d^4*x^6 + d^5*x^4
)
```

3.978.6 Sympy [A] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^7}\,dx = \begin{cases} \frac{2\left(-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{10d^3}+\frac{c^2(ad-bc)}{2d^3\sqrt{c+\frac{d}{x^2}}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}(ad-3bc)}{6d^3}-\frac{\sqrt{c+\frac{d}{x^2}}\left(-2acd+3bc^2\right)}{2d^3}\right)}{d} & \text{for } d\neq 0 \\ \frac{-\frac{a}{3x^6}-\frac{b}{4x^8}}{2c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**7,x)
```

output
```
Piecewise((2*(-b*(c + d/x**2)**(5/2)/(10*d**3) + c**2*(a*d - b*c)/(2*d**3*
sqrt(c + d/x**2)) - (c + d/x**2)**(3/2)*(a*d - 3*b*c)/(6*d**3) - sqrt(c +
d/x**2)*(-2*a*c*d + 3*b*c**2)/(2*d**3))/d, Ne(d, 0)), ((-a/(3*x**6) - b/(4
*x**8))/(2*c**(3/2)), True))
```

3.978.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^7}\,dx =$$

$$-\frac{1}{5}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^4}+\frac{15\,\sqrt{c+\frac{d}{x^2}}c^2}{d^4}+\frac{5\,c^3}{\sqrt{c+\frac{d}{x^2}}d^4}\right)$$
$$-\frac{1}{3}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3}-\frac{6\,\sqrt{c+\frac{d}{x^2}}c}{d^3}-\frac{3\,c^2}{\sqrt{c+\frac{d}{x^2}}d^3}\right)$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-1/5*b*((c + d/x^2)^(5/2)/d^4 - 5*(c + d/x^2)^(3/2)*c/d^4 + 15*sqrt(c + d/
x^2)*c^2/d^4 + 5*c^3/(sqrt(c + d/x^2)*d^4)) - 1/3*a*((c + d/x^2)^(3/2)/d^3
 - 6*sqrt(c + d/x^2)*c/d^3 - 3*c^2/(sqrt(c + d/x^2)*d^3))
```

3.978.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(88) = 176.

Time = 0.70 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.03

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^7}\,dx = -\frac{(bc^3-ac^2d)x}{\sqrt{cx^2+d}d^4\mathrm{sgn}\left(x\right)}$$
$$+\frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8bc^{\frac{5}{2}}-15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8ac^{\frac{3}{2}}d-90\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6bc^{\frac{5}{2}}d+90\left(\sqrt{c}x-\right.\right.}{}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
-(b*c^3 - a*c^2*d)*x/(sqrt(c*x^2 + d)*d^4*sgn(x)) + 2/15*(15*(sqrt(c)*x -
sqrt(c*x^2 + d))^8*b*c^(5/2) - 15*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(3/2
)*d - 90*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(5/2)*d + 90*(sqrt(c)*x - sqr
t(c*x^2 + d))^6*a*c^(3/2)*d^2 + 240*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5
/2)*d^2 - 160*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^3 - 150*(sqrt(c)
*x - sqrt(c*x^2 + d))^2*b*c^(5/2)*d^3 + 110*(sqrt(c)*x - sqrt(c*x^2 + d))^
2*a*c^(3/2)*d^4 + 33*b*c^(5/2)*d^4 - 25*a*c^(3/2)*d^5)/(((sqrt(c)*x - sqrt
(c*x^2 + d))^2 - d)^5*d^3*sgn(x))
```

3.978.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^7}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}\,(48\,b\,c^3\,x^6-40\,a\,c^2\,d\,x^6+24\,b\,c^2\,d\,x^4-20\,a\,c\,d^2\,x^4-6\,b\,c\,d^2\,x^2+5\,a\,d^3\,x^2+3\,b\,d^3)}{15\,d^4\,x^4\,\left(c\,x^2+d\right)}$$

input
```
int((a + b/x^2)/(x^7*(c + d/x^2)^(3/2)),x)
```

output
```
-((c + d/x^2)^(1/2)*(3*b*d^3 + 5*a*d^3*x^2 + 48*b*c^3*x^6 - 20*a*c*d^2*x^4
 - 40*a*c^2*d*x^6 - 6*b*c*d^2*x^2 + 24*b*c^2*d*x^4))/(15*d^4*x^4*(d + c*x^
2))
```

3.979 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9} \, dx$

3.979.1 Optimal result

Integrand size = 22, antiderivative size = 126

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9} \, dx = \frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}} + \frac{c^2(4bc-3ad)\sqrt{c+\frac{d}{x^2}}}{d^5} - \frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^5} + \frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5} - \frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}$$

output
```
-c*(-a*d+2*b*c)*(c+d/x^2)^(3/2)/d^5+1/5*(-a*d+4*b*c)*(c+d/x^2)^(5/2)/d^5-1
/7*b*(c+d/x^2)^(7/2)/d^5+c^3*(-a*d+b*c)/d^5/(c+d/x^2)^(1/2)+c^2*(-3*a*d+4*
b*c)*(c+d/x^2)^(1/2)/d^5
```

3.979.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9} \, dx = \frac{-7adx^2(d^3-2cd^2x^2+8c^2dx^4+16c^3x^6)+b(-5d^4+8cd^3x^2-16c^2d^2x^4+64c^3dx^6+}{35d^5\sqrt{c+\frac{d}{x^2}}x^8}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9),x]
```

output
```
(-7*a*d*x^2*(d^3 - 2*c*d^2*x^2 + 8*c^2*d*x^4 + 16*c^3*x^6) + b*(-5*d^4 + 8
*c*d^3*x^2 - 16*c^2*d^2*x^4 + 64*c^3*d*x^6 + 128*c^4*x^8))/(35*d^5*Sqrt[c
+ d/x^2]*x^8)
```

3.979.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^9\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}d\frac{1}{x^2}$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{2}\int \left(\frac{(bc-ad)c^3}{d^4\left(c+\frac{d}{x^2}\right)^{3/2}}-\frac{(4bc-3ad)c^2}{d^4\sqrt{c+\frac{d}{x^2}}}+\frac{3(2bc-ad)\sqrt{c+\frac{d}{x^2}}c}{d^4}+\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{d^4}+\frac{(ad-4bc)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^4}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}}+\frac{2c^2\sqrt{c+\frac{d}{x^2}}(4bc-3ad)}{d^5}+\frac{2\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-ad)}{5d^5}-\frac{2c\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{d^5}-\frac{2b\left(c+\frac{d}{x^2}\right)^{7/}}{7d^5}\right.$$

input
```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9),x]
```

output
```
((2*c^3*(b*c - a*d))/(d^5*Sqrt[c + d/x^2]) + (2*c^2*(4*b*c - 3*a*d)*Sqrt[c
 + d/x^2])/d^5 - (2*c*(2*b*c - a*d)*(c + d/x^2)^(3/2))/d^5 + (2*(4*b*c - a
*d)*(c + d/x^2)^(5/2))/(5*d^5) - (2*b*(c + d/x^2)^(7/2))/(7*d^5))/2
```

3.979.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.979.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{\left(112 a \,c^{3} d \,x^{8}-128 b \,c^{4} x^{8}+56 a \,c^{2} d^{2} x^{6}-64 b \,c^{3} d \,x^{6}-14 a c \,d^{3} x^{4}+16 b \,c^{2} d^{2} x^{4}+7 a \,d^{4} x^{2}-8 b c \,d^{3} x^{2}+5 b \,d^{4}\right) \left(c \,x^{2}+d \right)}{35 \left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} d^{5} x^{10}}$	118
default	$-\frac{\left(112 a \,c^{3} d \,x^{8}-128 b \,c^{4} x^{8}+56 a \,c^{2} d^{2} x^{6}-64 b \,c^{3} d \,x^{6}-14 a c \,d^{3} x^{4}+16 b \,c^{2} d^{2} x^{4}+7 a \,d^{4} x^{2}-8 b c \,d^{3} x^{2}+5 b \,d^{4}\right) \left(c \,x^{2}+d \right)}{35 \left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} d^{5} x^{10}}$	118
trager	$-\frac{\left(112 a \,c^{3} d \,x^{8}-128 b \,c^{4} x^{8}+56 a \,c^{2} d^{2} x^{6}-64 b \,c^{3} d \,x^{6}-14 a c \,d^{3} x^{4}+16 b \,c^{2} d^{2} x^{4}+7 a \,d^{4} x^{2}-8 b c \,d^{3} x^{2}+5 b \,d^{4}\right) \sqrt{-\frac{-c \,x^{2}-d}{x^{2}}}}{35 x^{6} d^{5} \left(c \,x^{2}+d \right)}$	124
risch	$-\frac{\left(c \,x^{2}+d \right) \left(77 a \,c^{2} d \,x^{6}-93 b \,c^{3} x^{6}-21 a c \,d^{2} x^{4}+29 b \,c^{2} d \,x^{4}+7 a \,d^{3} x^{2}-13 b c \,d^{2} x^{2}+5 b \,d^{3}\right)}{35 d^{5} x^{8} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}-\frac{c^{3} \left(a d -b c \right)}{d^{5} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}$	124

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/35*(112*a*c^3*d*x^8-128*b*c^4*x^8+56*a*c^2*d^2*x^6-64*b*c^3*d*x^6-14*a*
c*d^3*x^4+16*b*c^2*d^2*x^4+7*a*d^4*x^2-8*b*c*d^3*x^2+5*b*d^4)*(c*x^2+d)/((
c*x^2+d)/x^2)^(3/2)/d^5/x^10
```

3.979.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.96

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9}\,dx = \frac{\left(16\,(8\,bc^4-7\,ac^3d)x^8+8\,(8\,bc^3d-7\,ac^2d^2)x^6-5\,bd^4-2\,(8\,bc^2d^2-7\,acd^3)x^4+(8\,b\right.}{35\,(cd^5x^8+d^6x^6)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
1/35*(16*(8*b*c^4 - 7*a*c^3*d)*x^8 + 8*(8*b*c^3*d - 7*a*c^2*d^2)*x^6 - 5*b
*d^4 - 2*(8*b*c^2*d^2 - 7*a*c*d^3)*x^4 + (8*b*c*d^3 - 7*a*d^4)*x^2)*sqrt((
c*x^2 + d)/x^2)/(c*d^5*x^8 + d^6*x^6)
```

3.979.6 Sympy [A] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9}\,dx = \begin{cases} \frac{2\left(-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{14d^4}-\frac{c^3(ad-bc)}{2d^4\sqrt{c+\frac{d}{x^2}}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}(ad-4bc)}{10d^4}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}\left(-3acd+6bc^2\right)}{6d^4}-\frac{\sqrt{c+\frac{d}{x^2}}\cdot\left(3ac^2d-4bc^3\right)}{2d^4}\right)}{d} & \text{for } d \\ \frac{-\frac{a}{4x^8}-\frac{b}{5x^{10}}}{2c^{\frac{3}{2}}} & \text{other} \end{cases}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**9,x)
```

output
```
Piecewise((2*(-b*(c + d/x**2)**(7/2)/(14*d**4) - c**3*(a*d - b*c)/(2*d**4*
sqrt(c + d/x**2)) - (c + d/x**2)**(5/2)*(a*d - 4*b*c)/(10*d**4) - (c + d/x
**2)**(3/2)*(-3*a*c*d + 6*b*c**2)/(6*d**4) - sqrt(c + d/x**2)*(3*a*c**2*d
- 4*b*c**3)/(2*d**4))/d, Ne(d, 0)), ((-a/(4*x**8) - b/(5*x**10))/(2*c**(3/
2)), True))
```

3.979.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.20

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^9}\,dx =$$

$$-\frac{1}{35}\,b\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^5}-\frac{28\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^5}+\frac{70\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^5}-\frac{140\sqrt{c+\frac{d}{x^2}}\,c^3}{d^5}-\frac{35\,c^4}{\sqrt{c+\frac{d}{x^2}}\,d^5}\right)$$
$$-\frac{1}{5}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^4}+\frac{15\sqrt{c+\frac{d}{x^2}}\,c^2}{d^4}+\frac{5\,c^3}{\sqrt{c+\frac{d}{x^2}}\,d^4}\right)$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
-1/35*b*(5*(c + d/x^2)^(7/2)/d^5 - 28*(c + d/x^2)^(5/2)*c/d^5 + 70*(c + d/
x^2)^(3/2)*c^2/d^5 - 140*sqrt(c + d/x^2)*c^3/d^5 - 35*c^4/(sqrt(c + d/x^2)
*d^5)) - 1/5*a*((c + d/x^2)^(5/2)/d^4 - 5*(c + d/x^2)^(3/2)*c/d^4 + 15*sqr
t(c + d/x^2)*c^2/d^4 + 5*c^3/(sqrt(c + d/x^2)*d^4))
```

3.979.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(112) = 224.

Time = 1.19 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.29

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^9}\,dx = \frac{(bc^4-ac^3d)x}{\sqrt{cx^2+d}\,d^5\operatorname{sgn}(x)}$$
$$-\frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{7}{2}}-35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{5}{2}}d-280\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{7}{2}}d+280\left(\sqrt{c}x\right.\right.}{}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
(b*c^4 - a*c^3*d)*x/(sqrt(c*x^2 + d)*d^5*sgn(x)) - 2/35*(35*(sqrt(c)*x - s
qrt(c*x^2 + d))^12*b*c^(7/2) - 35*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(5/
2)*d - 280*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(7/2)*d + 280*(sqrt(c)*x -
 sqrt(c*x^2 + d))^10*a*c^(5/2)*d^2 + 1015*(sqrt(c)*x - sqrt(c*x^2 + d))^8*
b*c^(7/2)*d^2 - 1015*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(5/2)*d^3 - 2240*
(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d^3 + 1680*(sqrt(c)*x - sqrt(c*x
^2 + d))^6*a*c^(5/2)*d^4 + 1673*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)*
d^4 - 1337*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(5/2)*d^5 - 616*(sqrt(c)*x
- sqrt(c*x^2 + d))^2*b*c^(7/2)*d^5 + 504*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a
*c^(5/2)*d^6 + 93*b*c^(7/2)*d^6 - 77*a*c^(5/2)*d^7)/(((sqrt(c)*x - sqrt(c*
x^2 + d))^2 - d)^7*d^4*sgn(x))
```

3.979.9 Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.22

$$
\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9}\,dx &= \frac{c\sqrt{c+\frac{d}{x^2}}\,(21\,a\,d-29\,b\,c)}{35\,d^4\,x^2} \\
&\quad - \frac{b\sqrt{c+\frac{d}{x^2}}}{7\,d^2\,x^6} - \frac{\sqrt{c+\frac{d}{x^2}}\,(7\,a\,d^2-13\,b\,c\,d)}{35\,d^4\,x^4} \\
&\quad - \frac{\sqrt{c+\frac{d}{x^2}}\left(x^2\left(\frac{58\,b\,c^4-42\,a\,c^3\,d}{35\,d^5}+\frac{2\,c^3\,(77\,a\,d-93\,b\,c)}{35\,d^5}\right)+\frac{c^2\,(77\,a\,d-93\,b\,c)}{35\,d^4}\right)}{c\,x^2+d}
\end{aligned}
$$

input
```
int((a + b/x^2)/(x^9*(c + d/x^2)^(3/2)),x)
```

output
```
(c*(c + d/x^2)^(1/2)*(21*a*d - 29*b*c))/(35*d^4*x^2) - (b*(c + d/x^2)^(1/2
))/(7*d^2*x^6) - ((c + d/x^2)^(1/2)*(7*a*d^2 - 13*b*c*d))/(35*d^4*x^4) - (
(c + d/x^2)^(1/2)*(x^2*((58*b*c^4 - 42*a*c^3*d)/(35*d^5) + (2*c^3*(77*a*d
- 93*b*c))/(35*d^5)) + (c^2*(77*a*d - 93*b*c))/(35*d^4)))/(d + c*x^2)
```

3.980 $\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

3.980.1 Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{4d(5bc-6ad)x}{15c^3\sqrt{c+\frac{d}{x^2}}}-\frac{8d(5bc-6ad)\sqrt{c+\frac{d}{x^2}}\,x}{15c^4}+\frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

output
```
4/15*d*(-6*a*d+5*b*c)*x/c^3/(c+d/x^2)^(1/2)+1/15*(-6*a*d+5*b*c)*x^3/c^2/(c
+d/x^2)^(1/2)+1/5*a*x^5/c/(c+d/x^2)^(1/2)-8/15*d*(-6*a*d+5*b*c)*x*(c+d/x^2
)^(1/2)/c^4
```

3.980.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.72

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{5bc(-8d^2-4cdx^2+c^2x^4)+3a(16d^3+8cd^2x^2-2c^2dx^4+c^3x^6)}{15c^4\sqrt{c+\frac{d}{x^2}}\,x}$$

input
```
Integrate[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]
```

output
```
(5*b*c*(-8*d^2 - 4*c*d*x^2 + c^2*x^4) + 3*a*(16*d^3 + 8*c*d^2*x^2 - 2*c^2*
d*x^4 + c^3*x^6))/(15*c^4*Sqrt[c + d/x^2]*x)
```

3.980.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {955, 803, 773, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(a+\frac{b}{x^2}\right)}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(5bc-6ad)\int \frac{x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx}{5c}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{803}$$

$$\frac{(5bc-6ad)\left(\frac{x^3}{3c\sqrt{c+\frac{d}{x^2}}}-\frac{4d\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx}{3c}\right)}{5c}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{773}$$

$$\frac{(5bc-6ad)\left(\frac{4d\int \frac{x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{3c}+\frac{x^3}{3c\sqrt{c+\frac{d}{x^2}}}\right)}{5c}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{245}$$

$$\frac{(5bc-6ad)\left(\frac{4d\left(-\frac{2d\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{c}-\frac{x}{c\sqrt{c+\frac{d}{x^2}}}\right)}{3c}+\frac{x^3}{3c\sqrt{c+\frac{d}{x^2}}}\right)}{5c}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\left(\frac{4d\left(-\frac{2d}{c^2x\sqrt{c+\frac{d}{x^2}}}-\frac{x}{c\sqrt{c+\frac{d}{x^2}}}\right)}{3c}+\frac{x^3}{3c\sqrt{c+\frac{d}{x^2}}}\right)(5bc-6ad)}{5c}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

input
```
Int[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]
```

output
```
(a*x^5)/(5*c*Sqrt[c + d/x^2]) + ((5*b*c - 6*a*d)*(x^3/(3*c*Sqrt[c + d/x^2]
) + (4*d*((-2*d)/(c^2*Sqrt[c + d/x^2]*x) - x/(c*Sqrt[c + d/x^2])))/(3*c)))
/(5*c)
```

3.980.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.980.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{\left(3x^{6} a \,c^{3}-6a \,c^{2} d \,x^{4}+5b \,c^{3} x^{4}+24ac \,d^{2} x^{2}-20b \,c^{2} d \,x^{2}+48a \,d^{3}-40bc \,d^{2}\right) \left(c \,x^{2}+d \right)}{15 \left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} c^{4}}$	91
default	$\frac{\left(3x^{6} a \,c^{3}-6a \,c^{2} d \,x^{4}+5b \,c^{3} x^{4}+24ac \,d^{2} x^{2}-20b \,c^{2} d \,x^{2}+48a \,d^{3}-40bc \,d^{2}\right) \left(c \,x^{2}+d \right)}{15 \left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} c^{4}}$	91
trager	$\frac{\left(3x^{6} a \,c^{3}-6a \,c^{2} d \,x^{4}+5b \,c^{3} x^{4}+24ac \,d^{2} x^{2}-20b \,c^{2} d \,x^{2}+48a \,d^{3}-40bc \,d^{2}\right) x \sqrt{-\frac{-c \,x^{2}-d}{x^{2}}}}{15(c \,x^{2}+d) c^{4}}$	95
risch	$\frac{\left(3a \,x^{4} c^{2}-9acd \,x^{2}+5b \,c^{2} x^{2}+33a \,d^{2}-25bcd \right) \left(c \,x^{2}+d \right)}{15c^{4} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}+\frac{(ad-bc) d^{2}}{c^{4} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}$	99

input
```
int((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(3*a*c^3*x^6-6*a*c^2*d*x^4+5*b*c^3*x^4+24*a*c*d^2*x^2-20*b*c^2*d*x^2+
48*a*d^3-40*b*c*d^2)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^4
```

3.980.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{\left(3\,ac^{3}x^{7} + \left(5\,bc^{3} - 6\,ac^{2}d\right)x^{5} - 4\,\left(5\,bc^{2}d - 6\,acd^{2}\right)x^{3} - 8\,\left(5\,bcd^{2} - 6\,ad^{3}\right)x\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{15\,\left(c^{5}x^{2} + c^{4}d\right)}$$

input
```
integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(3*a*c^3*x^7 + (5*b*c^3 - 6*a*c^2*d)*x^5 - 4*(5*b*c^2*d - 6*a*c*d^2)*
x^3 - 8*(5*b*c*d^2 - 6*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/(c^5*x^2 + c^4*d)
```

3.980.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(107) = 214$.

Time = 2.92 (sec) , antiderivative size = 561, normalized size of antiderivative = 5.05

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = a\left(\frac{c^5d^{\frac{19}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}} + \frac{5c^3d^{\frac{23}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}} + \frac{30c^2d^{\frac{25}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}} + \frac{40cd^{\frac{27}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}} + \frac{16d^{\frac{29}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}}\right) + b\left(\frac{c^3d^{\frac{9}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6} - \frac{3c^2d^{\frac{11}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6} - \frac{12cd^{\frac{13}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6} - \frac{8d^{\frac{15}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}\right)$$

input
```
integrate((a+b/x**2)*x**4/(c+d/x**2)**(3/2),x)
```

output
```
a*(c**5*d**(19/2)*x**10*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**
10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 5*c**3*d**(23/2)*x**6*sqrt(
c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2
+ 5*c**4*d**12) + 30*c**2*d**(25/2)*x**4*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x
**6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 40*c*d**(2
7/2)*x**2*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c
**5*d**11*x**2 + 5*c**4*d**12) + 16*d**(29/2)*sqrt(c*x**2/d + 1)/(5*c**7*d
**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12)) + b*(c
**3*d**(9/2)*x**6*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2
+ 3*c**3*d**6) - 3*c**2*d**(11/2)*x**4*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**
4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 12*c*d**(13/2)*x**2*sqrt(c*x**2/d +
1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 8*d**(15/2)*sqrt(
c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6))
```

3.980.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{1}{3}b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-6\sqrt{c+\frac{d}{x^2}}dx}{c^3}-\frac{3\,d^2}{\sqrt{c+\frac{d}{x^2}}c^3x}\right) + \frac{1}{5}a\left(\frac{5\,d^3}{\sqrt{c+\frac{d}{x^2}}c^4x}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3+15\sqrt{c+\frac{d}{x^2}}d^2x}{c^4}\right)$$

input
```
integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/3*b*(((c + d/x^2)^(3/2)*x^3 - 6*sqrt(c + d/x^2)*d*x)/c^3 - 3*d^2/(sqrt(c
 + d/x^2)*c^3*x)) + 1/5*a*(5*d^3/(sqrt(c + d/x^2)*c^4*x) + ((c + d/x^2)^(5
/2)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3 + 15*sqrt(c + d/x^2)*d^2*x)/c^4)
```

3.980.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.32

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{8\,(5\,bcd^2-6\,ad^3)\mathrm{sgn}(x)}{15\,c^4\sqrt{d}}-\frac{bcd^2-ad^3}{\sqrt{cx^2+d}c^4\mathrm{sgn}\,(x)} + \frac{3\,(cx^2+d)^{\frac{5}{2}}ac^{16}+5\,(cx^2+d)^{\frac{3}{2}}bc^{17}-15\,(cx^2+d)^{\frac{3}{2}}ac^{16}d-30\,\sqrt{cx^2+d}bc^{17}d+45\,\sqrt{cx^2+d}ac^{16}d^2}{15\,c^{20}\mathrm{sgn}\,(x)}$$

input
```
integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="giac")
```

output
```
8/15*(5*b*c*d^2 - 6*a*d^3)*sgn(x)/(c^4*sqrt(d)) - (b*c*d^2 - a*d^3)/(sqrt(
c*x^2 + d)*c^4*sgn(x)) + 1/15*(3*(c*x^2 + d)^(5/2)*a*c^16 + 5*(c*x^2 + d)^
(3/2)*b*c^17 - 15*(c*x^2 + d)^(3/2)*a*c^16*d - 30*sqrt(c*x^2 + d)*b*c^17*d
 + 45*sqrt(c*x^2 + d)*a*c^16*d^2)/(c^20*sgn(x))
```

3.980.9 Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{3\,a\,c^3\,x^6+5\,b\,c^3\,x^4-6\,a\,c^2\,d\,x^4-20\,b\,c^2\,d\,x^2+24\,a\,c\,d^2\,x^2-40\,b\,c\,d^2+48\,a\,d^3}{15\,c^4\,x\,\sqrt{c+\frac{d}{x^2}}}$$

input
```
int((x^4*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

output
```
(48*a*d^3 + 3*a*c^3*x^6 + 5*b*c^3*x^4 - 40*b*c*d^2 + 24*a*c*d^2*x^2 - 6*a*
c^2*d*x^4 - 20*b*c^2*d*x^2)/(15*c^4*x*(c + d/x^2)^(1/2))
```

3.981 $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

3.981.1 Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=-\frac{(3bc-4ad)x}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{2(3bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^3}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

output
```
-1/3*(-4*a*d+3*b*c)*x/c^2/(c+d/x^2)^(1/2)+1/3*a*x^3/c/(c+d/x^2)^(1/2)+2/3*
(-4*a*d+3*b*c)*x*(c+d/x^2)^(1/2)/c^3
```

3.981.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{3bc(2d+cx^2)+a(-8d^2-4cdx^2+c^2x^4)}{3c^3\sqrt{c+\frac{d}{x^2}}\,x}$$

input
```
Integrate[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2),x]
```

output
```
(3*b*c*(2*d + c*x^2) + a*(-8*d^2 - 4*c*d*x^2 + c^2*x^4))/(3*c^3*Sqrt[c + d
/x^2]*x)
```

3.981.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 773, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+\frac{b}{x^2}\right)}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(3bc-4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx}{3c}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{773}$$

$$\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}-\frac{(3bc-4ad)\int \frac{x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{3c}$$

$$\Big\downarrow \text{245}$$

$$\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}-\frac{(3bc-4ad)\left(-\frac{2d\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{c}-\frac{x}{c\sqrt{c+\frac{d}{x^2}}}\right)}{3c}$$

$$\Big\downarrow \text{208}$$

$$\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}-\frac{\left(-\frac{2d}{c^2x\sqrt{c+\frac{d}{x^2}}}-\frac{x}{c\sqrt{c+\frac{d}{x^2}}}\right)(3bc-4ad)}{3c}$$

input
```
Int[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2),x]
```

output
```
(a*x^3)/(3*c*Sqrt[c + d/x^2]) - ((3*b*c - 4*a*d)*((-2*d)/(c^2*Sqrt[c + d/x
^2]*x) - x/(c*Sqrt[c + d/x^2])))/(3*c)
```

3.981. $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx$

3.981.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.981.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{\left(a\,x^{4} c^{2}-4 a c d\,x^{2}+3 b\,c^{2} x^{2}-8 a\,d^{2}+6 b c d \right) \left(c\,x^{2}+d \right)}{3 \left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} c^{3} x^{3}}$	66
default	$\frac{\left(a\,x^{4} c^{2}-4 a c d\,x^{2}+3 b\,c^{2} x^{2}-8 a\,d^{2}+6 b c d \right) \left(c\,x^{2}+d \right)}{3 \left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} c^{3} x^{3}}$	66
trager	$\frac{\left(a\,x^{4} c^{2}-4 a c d\,x^{2}+3 b\,c^{2} x^{2}-8 a\,d^{2}+6 b c d \right) x \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{3 \left(c\,x^{2}+d \right) c^{3}}$	70
risch	$\frac{\left(a c\,x^{2}-5 a d +3 b c \right) \left(c\,x^{2}+d \right)}{3 c^{3} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x}-\frac{\left(a d -b c \right) d}{c^{3} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x}$	75

input
```
int((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

3.981. $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

output
```
1/3*(a*c^2*x^4-4*a*c*d*x^2+3*b*c^2*x^2-8*a*d^2+6*b*c*d)*(c*x^2+d)/((c*x^2+
d)/x^2)^(3/2)/c^3/x^3
```

3.981.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{\left(ac^2x^5+\left(3\,bc^2-4\,acd\right)x^3+2\left(3\,bcd-4\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\left(c^4x^2+c^3d\right)}$$

input
```
integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(a*c^2*x^5 + (3*b*c^2 - 4*a*c*d)*x^3 + 2*(3*b*c*d - 4*a*d^2)*x)*sqrt((
c*x^2 + d)/x^2)/(c^4*x^2 + c^3*d)
```

3.981.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(70)=140$.

Time = 2.61 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.38

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx &= a\left(\frac{c^3d^{\frac{9}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{3c^2d^{\frac{11}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}\right.\\ &\left.-\frac{12cd^{\frac{13}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{8d^{\frac{15}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}\right)\\ &+b\left(\frac{x^2}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{2\sqrt{d}}{c^2\sqrt{\frac{cx^2}{d}+1}}\right)\end{aligned}$$

input
```
integrate((a+b/x**2)*x**2/(c+d/x**2)**(3/2),x)
```

output
```
a*(c**3*d**(9/2)*x**6*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x
**2 + 3*c**3*d**6) - 3*c**2*d**(11/2)*x**4*sqrt(c*x**2/d + 1)/(3*c**5*d**4
*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 12*c*d**(13/2)*x**2*sqrt(c*x**2/
d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 8*d**(15/2)*s
qrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6)) + b
*(x**2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1)
))
```

3.981.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = b\left(\frac{\sqrt{c+\frac{d}{x^2}}x}{c^2}+\frac{d}{\sqrt{c+\frac{d}{x^2}}c^2x}\right) + \frac{1}{3}a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-6\sqrt{c+\frac{d}{x^2}}dx}{c^3}-\frac{3d^2}{\sqrt{c+\frac{d}{x^2}}c^3x}\right)$$

input
```
integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

output
```
b*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) + 1/3*a*(((c + d/x^2
)^(3/2)*x^3 - 6*sqrt(c + d/x^2)*d*x)/c^3 - 3*d^2/(sqrt(c + d/x^2)*c^3*x))
```

3.981.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.34

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\frac{2\,(3\,bcd-4\,ad^2)\mathrm{sgn}(x)}{3\,c^3\sqrt{d}}+\frac{bcd-ad^2}{\sqrt{cx^2+d}c^3\mathrm{sgn}\,(x)} + \frac{(cx^2+d)^{\frac{3}{2}}ac^6+3\,\sqrt{cx^2+d}bc^7-6\,\sqrt{cx^2+d}ac^6d}{3\,c^9\mathrm{sgn}\,(x)}$$

input
```
integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="giac")
```

output
```
-2/3*(3*b*c*d - 4*a*d^2)*sgn(x)/(c^3*sqrt(d)) + (b*c*d - a*d^2)/(sqrt(c*x^
2 + d)*c^3*sgn(x)) + 1/3*((c*x^2 + d)^(3/2)*a*c^6 + 3*sqrt(c*x^2 + d)*b*c^
7 - 6*sqrt(c*x^2 + d)*a*c^6*d)/(c^9*sgn(x))
```

3.981.9 Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{b\,c^2\,x^4+3\,b\,c\,d\,x^2+2\,b\,d^2}{c^2\,x^3\,\left(c+\frac{d}{x^2}\right)^{3/2}}-\frac{-a\,c^2\,x^4+4\,a\,c\,d\,x^2+8\,a\,d^2}{3\,c^3\,x\,\sqrt{c+\frac{d}{x^2}}}$$

input

```
int((x^2*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

output

```
(2*b*d^2 + b*c^2*x^4 + 3*b*c*d*x^2)/(c^2*x^3*(c + d/x^2)^(3/2)) - (8*a*d^2
 - a*c^2*x^4 + 4*a*c*d*x^2)/(3*c^3*x*(c + d/x^2)^(1/2))
```

3.982 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

3.982.1 Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\frac{bc-2ad}{c^2\sqrt{c+\frac{d}{x^2}}\,x}+\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}$$

output
```
(2*a*d-b*c)/c^2/x/(c+d/x^2)^(1/2)+a*x/c/(c+d/x^2)^(1/2)
```

3.982.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{-bc+2ad+acx^2}{c^2\sqrt{c+\frac{d}{x^2}}\,x}$$

input
```
Integrate[(a + b/x^2)/(c + d/x^2)^(3/2),x]
```

output
```
(-(b*c) + 2*a*d + a*c*x^2)/(c^2*Sqrt[c + d/x^2]*x)
```

3.982.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {899, 359, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{359}$$

$$\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}-\frac{(bc-2ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{208}$$

$$\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}-\frac{bc-2ad}{c^2x\sqrt{c+\frac{d}{x^2}}}$$

input
```
Int[(a + b/x^2)/(c + d/x^2)^(3/2),x]
```

output
```
-((b*c - 2*a*d)/(c^2*Sqrt[c + d/x^2]*x)) + (a*x)/(c*Sqrt[c + d/x^2])
```

3.982.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.982.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{(a c x^2+2ad-bc)(c x^2+d)}{\left(\frac{c x^2+d}{x^2}\right)^{\frac{3}{2}} c^2 x^3}$	43
default	$\frac{(a c x^2+2ad-bc)(c x^2+d)}{\left(\frac{c x^2+d}{x^2}\right)^{\frac{3}{2}} c^2 x^3}$	43
trager	$\frac{(a c x^2+2ad-bc) x \sqrt{-\frac{-c x^2-d}{x^2}}}{(c x^2+d) c^2}$	47
risch	$\frac{a(c x^2+d)}{c^2\sqrt{\frac{c x^2+d}{x^2}}\, x}+\frac{ad-bc}{c^2\sqrt{\frac{c x^2+d}{x^2}}\, x}$	58

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*c*x^2+2*a*d-b*c)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/c^2/x^3
```

3.982.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{(acx^3-(bc-2\,ad)x)\sqrt{\frac{cx^2+d}{x^2}}}{c^3x^2+c^2d}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

output
```
(a*c*x^3 - (b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/(c^3*x^2 + c^2*d)
```

3.982.6 Sympy [A] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=a\left(\frac{x^2}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{2\sqrt{d}}{c^2\sqrt{\frac{cx^2}{d}+1}}\right)-\frac{b}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2),x)
```

output
```
a*(x**2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1
))) - b/(c*sqrt(d)*sqrt(c*x**2/d + 1))
```

3.982.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=a\left(\frac{\sqrt{c+\frac{d}{x^2}}x}{c^2}+\frac{d}{\sqrt{c+\frac{d}{x^2}}c^2x}\right)-\frac{b}{\sqrt{c+\frac{d}{x^2}}cx}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

output
```
a*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) - b/(sqrt(c + d/x^2)
*c*x)
```

3.982.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{(bc-2\,ad)\mathrm{sgn}(x)}{c^2\sqrt{d}} + \frac{\sqrt{cx^2+d}a}{c^2\mathrm{sgn}\left(x\right)} - \frac{bc-ad}{\sqrt{cx^2+d}c^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="giac")
```

output
```
(b*c - 2*a*d)*sgn(x)/(c^2*sqrt(d)) + sqrt(c*x^2 + d)*a/(c^2*sgn(x)) - (b*c
 - a*d)/(sqrt(c*x^2 + d)*c^2*sgn(x))
```

3.982.9 Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{\left(c\,x^2+d\right)\,\left(a\,c\,x^2+2\,a\,d-b\,c\right)}{c^2\,x^3\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}$$

input
```
int((a + b/x^2)/(c + d/x^2)^(3/2),x)
```

output
```
((d + c*x^2)*(2*a*d - b*c + a*c*x^2))/(c^2*x^3*(c + d/x^2)^(3/2))
```

3.983 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2} \, dx$

3.983.1 Optimal result

Integrand size = 22, antiderivative size = 59

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2} \, dx = \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{d^{3/2}}$$

output
```
-b*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(3/2)+(-a*d+b*c)/c/d/x/(c+d/x^2)^(
1/2)
```

3.983.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2} \, dx = \frac{\sqrt{d}(bc-ad) - bc\sqrt{d+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{cd^{3/2}\sqrt{c+\frac{d}{x^2}}\,x}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2),x]
```

output
```
(Sqrt[d]*(b*c - a*d) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]
])/(c*d^(3/2)*Sqrt[c + d/x^2]*x)
```

3.983.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {954, 858, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^2\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{954}$$

$$\frac{b\int \frac{1}{\sqrt{c+\frac{d}{x^2}}x^2}dx}{d}+\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{858}$$

$$\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}-\frac{b\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{d}$$

$$\Big\downarrow \text{224}$$

$$\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}-\frac{b\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}-\frac{b\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{d^{3/2}}$$

input
```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2),x]
```

output
```
(b*c - a*d)/(c*d*Sqrt[c + d/x^2]*x) - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*
x)])/d^(3/2)
```

3.983. $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}dx$

3.983.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

3.983.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{\left(c\,x^{2}+d\right)\left(\sqrt{c\,x^{2}+d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)bcd+d^{\frac{5}{2}}a-b\,d^{\frac{3}{2}}c\right)}{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}x^{3}c\,d^{\frac{5}{2}}}$	79

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+d)*((c*x^2+d)^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c*d+d^(5
/2)*a-b*d^(3/2)*c)/((c*x^2+d)/x^2)^(3/2)/x^3/c/d^(5/2)
```

3.983.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.31

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2}\,dx = \left[\frac{2\,(bcd-ad^2)x\sqrt{\frac{cx^2+d}{x^2}}+(bc^2x^2+bcd)\sqrt{d}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)}{2\,(c^2d^2x^2+cd^3)}, (bcd-ad^2)x\right.$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*d)*sqrt
(d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c^2*d^2*
x^2 + c*d^3), ((b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*
d)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c^2*d^2
*x^2 + c*d^3)]
```

3.983.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(46) = 92.

Time = 3.84 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.49

$$\begin{aligned}\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2}\,dx = &-\frac{a}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} \\ &+ b\left(\frac{cd^2x^2\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} - \frac{2cd^2x^2\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}\right. \\ &\left.+ \frac{2d^3\sqrt{\frac{cx^2}{d}+1}}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} + \frac{d^3\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} - \frac{2d^3\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}\right)\end{aligned}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**2,x)
```

output
```
-a/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + b*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(
7/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x**2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d
**(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*sqrt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2
 + 2*d**(9/2)) + d**3*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d*
*3*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)))
```

3.983.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx = \frac{1}{2}b\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^2}}dx}\right)-\frac{a}{\sqrt{c+\frac{d}{x^2}}cx}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*(log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^
(3/2) + 2/(sqrt(c + d/x^2)*d*x)) - a/(sqrt(c + d/x^2)*c*x)
```

3.983.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(51) = 102$.

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.83

$$\begin{aligned}\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx &= \frac{b\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d\operatorname{sgn}(x)} \\ &\quad - \frac{\left(bc\sqrt{d}\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+bc\sqrt{-d}-a\sqrt{-d}d\right)\operatorname{sgn}(x)}{c\sqrt{-d}d^{\frac{3}{2}}}+\frac{bc-ad}{\sqrt{cx^2+d}cd\operatorname{sgn}(x)}\end{aligned}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
b*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d*sgn(x)) - (b*c*sqrt(d)*arct
an(sqrt(d)/sqrt(-d)) + b*c*sqrt(-d) - a*sqrt(-d)*d)*sgn(x)/(c*sqrt(-d)*d^(
3/2)) + (b*c - a*d)/(sqrt(c*x^2 + d)*c*d*sgn(x))
```

3.983.9 Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx=\frac{b}{d\,x\,\sqrt{c+\frac{d}{x^2}}}-\frac{a}{c\,x\,\sqrt{c+\frac{d}{x^2}}}-\frac{b\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{d^{3/2}}$$

input

```
int((a + b/x^2)/(x^2*(c + d/x^2)^(3/2)),x)
```

output

```
b/(d*x*(c + d/x^2)^(1/2)) - a/(c*x*(c + d/x^2)^(1/2)) - (b*log((c + d/x^2)
^(1/2) + d^(1/2)/x))/d^(3/2)
```

3.984 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^4} \, dx$

3.984.1 Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^4} \, dx = -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}x^3} - \frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}x} + \frac{(3bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{2d^{5/2}}$$

output
```
1/2*(-2*a*d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(5/2)-1/2*b/d/x^3/
(c+d/x^2)^(1/2)+1/2*(2*a*d-3*b*c)/d^2/x/(c+d/x^2)^(1/2)
```

3.984.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^4} \, dx = \frac{\sqrt{d}(2adx^2-b(d+3cx^2)) + (3bc-2ad)x^2\sqrt{d+cx^2}\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{c+\frac{d}{x^2}}x^3}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4),x]
```

output
```
(Sqrt[d]*(2*a*d*x^2 - b*(d + 3*c*x^2)) + (3*b*c - 2*a*d)*x^2*Sqrt[d + c*x^
2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(2*d^(5/2)*Sqrt[c + d/x^2]*x^3)
```

3.984.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {959, 858, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^4\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{959}$$

$$-\frac{(3bc-2ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}dx}{2d}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{858}$$

$$\frac{(3bc-2ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}d\frac{1}{x}}{2d}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(3bc-2ad)\left(\frac{\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{d}-\frac{1}{dx\sqrt{c+\frac{d}{x^2}}}\right)}{2d}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{224}$$

$$\frac{(3bc-2ad)\left(\frac{\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{d}-\frac{1}{dx\sqrt{c+\frac{d}{x^2}}}\right)}{2d}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3bc-2ad)\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{d^{3/2}}-\frac{1}{dx\sqrt{c+\frac{d}{x^2}}}\right)}{2d}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

3.984. $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}dx$

input
```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4),x]
```

output
```
-1/2*b/(d*Sqrt[c + d/x^2]*x^3) + ((3*b*c - 2*a*d)*(-(1/(d*Sqrt[c + d/x^2]*
x)) + ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]/d^(3/2)))/(2*d)
```

3.984.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.984. $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^4} \, dx$

3.984.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.38

method	result	size
risch	$-\frac{b\left(c\,x^{2}+d\right)}{2 d^{2} x^{3} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}}+\frac{\left(\frac{b c}{\sqrt{c\,x^{2}+d}}+d(2 a d-3 b c)\left(\frac{1}{d\sqrt{c\,x^{2}+d}}-\frac{\ln\left(\frac{2 d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)}{d^{\frac{3}{2}}}\right)\right)\sqrt{c\,x^{2}+d}}{2 d^{2}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}$	127
default	$-\frac{\left(c\,x^{2}+d\right)\left(2\sqrt{c\,x^{2}+d}\,\ln\left(\frac{2 d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right) a\,d^{2} x^{2}-3\sqrt{c\,x^{2}+d}\,\ln\left(\frac{2 d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right) b c d\,x^{2}-2 d^{\frac{5}{2}} a\,x^{2}+3 d^{\frac{3}{2}} b c\,x^{2}+d^{\frac{5}{2}} b\right)}{2\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{5} d^{\frac{7}{2}}}$	131

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d^2*b*(c*x^2+d)/x^3/((c*x^2+d)/x^2)^(1/2)+1/2/d^2*(b*c/(c*x^2+d)^(1/2
)+d*(2*a*d-3*b*c)*(1/d/(c*x^2+d)^(1/2)-1/d^(3/2)*ln((2*d+2*d^(1/2)*(c*x^2+
d)^(1/2))/x)))/((c*x^2+d)/x^2)^(1/2)/x*(c*x^2+d)^(1/2)
```

3.984.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.70

$$\int \frac{a+\frac{b}{x^{2}}}{\left(c+\frac{d}{x^{2}}\right)^{3/2} x^{4}}\,dx=\left[-\frac{\left(\left(3\,bc^{2}-2\,acd\right)x^{3}+\left(3\,bcd-2\,ad^{2}\right)x\right)\sqrt{d}\log\left(-\frac{cx^{2}-2\,\sqrt{d}x\sqrt{\frac{cx^{2}+d}{x^{2}}}+2\,d}{x^{2}}\right)+2\,(bd^{2}+}{4\,(cd^{3}x^{3}+d^{4}x)}\right.$$

$$\left.-\frac{\left(\left(3\,bc^{2}-2\,acd\right)x^{3}+\left(3\,bcd-2\,ad^{2}\right)x\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^{2}+d}{x^{2}}}}{cx^{2}+d}\right)+\left(bd^{2}+\left(3\,bcd-2\,ad^{2}\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{2\,(cd^{3}x^{3}+d^{4}x)}\right]$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[-1/4*(((3*b*c^2 - 2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(d)*log(-(c*x
^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(b*d^2 + (3*b*c*d -
 2*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^3*x^3 + d^4*x), -1/2*(((3*b*c^2
 - 2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((
c*x^2 + d)/x^2)/(c*x^2 + d)) + (b*d^2 + (3*b*c*d - 2*a*d^2)*x^2)*sqrt((c*x
^2 + d)/x^2))/(c*d^3*x^3 + d^4*x)]
```

3.984.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(76) = 152$.

Time = 6.59 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.85

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx = a\left(\frac{cd^2x^2\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} - \frac{2cd^2x^2\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} + \frac{2d^3\sqrt{\frac{cx^2}{d}+1}}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} + \frac{d^3\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} - \frac{2d^3\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}\right) + b\left(-\frac{3\sqrt{c}}{2d^2x\sqrt{1+\frac{d}{cx^2}}} + \frac{3c\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{5}{2}}} - \frac{1}{2\sqrt{c}dx^3\sqrt{1+\frac{d}{cx^2}}}\right)$$

input

```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**4,x)
```

output

```
a*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x
**2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*
sqrt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + d**3*log(c*x**2/d)/(
2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d**3*log(sqrt(c*x**2/d + 1) + 1)/(2*c*
d**(7/2)*x**2 + 2*d**(9/2))) + b*(-3*sqrt(c)/(2*d**2*x*sqrt(1 + d/(c*x**2)
)) + 3*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(5/2)) - 1/(2*sqrt(c)*d*x**3*sqr
t(1 + d/(c*x**2))))
```

3.984.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(76) = 152$.

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.76

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx = -\frac{1}{4}b\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)cx^2-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3-\sqrt{c+\frac{d}{x^2}}\,d^3x} + \frac{3\,c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}\right) + \frac{1}{2}a\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{2}{\sqrt{c+\frac{d}{x^2}}\,dx}\right)$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
-1/4*b*(2*(3*(c + d/x^2)*c*x^2 - 2*c*d)/((c + d/x^2)^(3/2)*d^2*x^3 - sqrt(
c + d/x^2)*d^3*x) + 3*c*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)
*x + sqrt(d)))/d^(5/2)) + 1/2*a*(log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c
 + d/x^2)*x + sqrt(d)))/d^(3/2) + 2/(sqrt(c + d/x^2)*d*x))
```

3.984.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx = -\frac{(3\,bc-2\,ad)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{2\,\sqrt{-d}d^2\mathrm{sgn}\left(x\right)} - \frac{3\,(cx^2+d)bc-2\,(cx^2+d)ad-2\,bcd+2\,ad^2}{2\left((cx^2+d)^{\frac{3}{2}}-\sqrt{cx^2+d}d\right)d^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
-1/2*(3*b*c - 2*a*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d^2*sgn(x)
) - 1/2*(3*(c*x^2 + d)*b*c - 2*(c*x^2 + d)*a*d - 2*b*c*d + 2*a*d^2)/(((c*x
^2 + d)^(3/2) - sqrt(c*x^2 + d)*d)*d^2*sgn(x))
```

3.984.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx = \int \frac{a+\frac{b}{x^2}}{x^4\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$$

input
```
int((a + b/x^2)/(x^4*(c + d/x^2)^(3/2)),x)
```

output
```
int((a + b/x^2)/(x^4*(c + d/x^2)^(3/2)), x)
```

3.985 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx$

3.985.1 Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx=-\frac{b}{4d\sqrt{c+\frac{d}{x^2}}x^5}-\frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}x^3}+\frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x}-\frac{3c(5bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{8d^{7/2}}$$

output
```
-3/8*c*(-4*a*d+5*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(7/2)-1/4*b/d/x
^5/(c+d/x^2)^(1/2)+1/4*(4*a*d-5*b*c)/d^2/x^3/(c+d/x^2)^(1/2)+3/8*(-4*a*d+5
*b*c)*(c+d/x^2)^(1/2)/d^3/x
```

3.985.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx=\frac{\sqrt{d}(-4adx^2(d+3cx^2)+b(-2d^2+5cdx^2+15c^2x^4))-3c(5bc-4ad)x^4\sqrt{d+cx^2}\text{arct}}{8d^{7/2}\sqrt{c+\frac{d}{x^2}}x^5}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6),x]
```

output
```
(Sqrt[d]*(-4*a*d*x^2*(d + 3*c*x^2) + b*(-2*d^2 + 5*c*d*x^2 + 15*c^2*x^4))
- 3*c*(5*b*c - 4*a*d)*x^4*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]
)/(8*d^(7/2)*Sqrt[c + d/x^2]*x^5)
```

3.985.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {959, 858, 252, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^6\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{959}$$

$$-\frac{(5bc-4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}dx}{4d}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{858}$$

$$\frac{(5bc-4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}d\frac{1}{x}}{4d}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(5bc-4ad)\left(\frac{3\int \frac{1}{\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}}{d}-\frac{1}{dx^3\sqrt{c+\frac{d}{x^2}}}\right)}{4d}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{262}$$

$$\frac{(5bc-4ad)\left(\frac{3\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{2d}\right)}{d}-\frac{1}{dx^3\sqrt{c+\frac{d}{x^2}}}\right)}{4d}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \color{blue}{224}$$

$$\frac{(5bc-4ad)\left(\frac{3\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{2d}\right)}{d}-\frac{1}{dx^3\sqrt{c+\frac{d}{x^2}}}\right)}{4d}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \color{blue}{219}$$

$$\frac{(5bc-4ad)\left(\frac{3\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}\right)}{d}-\frac{1}{dx^3\sqrt{c+\frac{d}{x^2}}}\right)}{4d}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

input

```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6),x]
```

output

```
-1/4*b/(d*Sqrt[c + d/x^2]*x^5) + ((5*b*c - 4*a*d)*(-(1/(d*Sqrt[c + d/x^2]*
x^3)) + (3*(Sqrt[c + d/x^2]/(2*d*x) - (c*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*
x)])/(2*d^(3/2))))/d))/(4*d)
```

3.985.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.985.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{\left(c\,x^{2}+d\right)\left(4 a d\,x^{2}-7 c b\,x^{2}+2 b d\right)}{8 d^{3} x^{5} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}}-\frac{c \left(-\frac{4 a d -7 b c}{\sqrt{c\,x^{2}+d}}+3 d \left(4 a d -5 b c\right) \left(\frac{1}{d \sqrt{c\,x^{2}+d}}-\frac{\ln\left(\frac{2 d +2 \sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)}{d^{\frac{3}{2}}}\right)\right) \sqrt{c\,x^{2}+d}}{8 d^{3} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}$
default	$-\frac{\left(c\,x^{2}+d\right)\left(12 d^{\frac{5}{2}} a c\,x^{4}-15 d^{\frac{3}{2}} b\,c^{2} x^{4}-12 \ln\left(\frac{2 d +2 \sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right) \sqrt{c\,x^{2}+d}\,a c\,d^{2} x^{4}+15 \ln\left(\frac{2 d +2 \sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right) \sqrt{c\,x^{2}+d}\,b\,c^{2} d\,x^{4}+4 d^{\frac{7}{2}} a\right.}{8 \left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{7} d^{\frac{9}{2}}}$

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

3.985. $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^6}\,dx$

output
```
-1/8*(c*x^2+d)*(4*a*d*x^2-7*b*c*x^2+2*b*d)/d^3/x^5/((c*x^2+d)/x^2)^(1/2)-1
/8*c/d^3*(-(4*a*d-7*b*c)/(c*x^2+d)^(1/2)+3*d*(4*a*d-5*b*c)*(1/d/(c*x^2+d)^
(1/2)-1/d^(3/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)))/((c*x^2+d)/x^2)^(1
/2)/x*(c*x^2+d)^(1/2)
```

3.985.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.55

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx = \left[-\frac{3\left(\left(5\,bc^3-4\,ac^2d\right)x^5+\left(5\,bc^2d-4\,acd^2\right)x^3\right)\sqrt{d}\log\left(-\frac{cx^2+2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\,(3}{16\,\left(cd^4x^5+d^5x^3\right)}\right.$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
[-1/16*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b*c^2*d - 4*a*c*d^2)*x^3)*sqrt(d
)*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(5*b*
c^2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a*d^3)*x^2)*sqrt((c*x^2
+ d)/x^2))/(c*d^4*x^5 + d^5*x^3), 1/8*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b
*c^2*d - 4*a*c*d^2)*x^3)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/
(c*x^2 + d)) + (3*(5*b*c^2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a
*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^4*x^5 + d^5*x^3)]
```

3.985.6 Sympy [A] (verification not implemented)

Time = 11.33 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.46

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx = a\left(-\frac{3\sqrt{c}}{2d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{3c\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{5}{2}}}-\frac{1}{2\sqrt{c}dx^3\sqrt{1+\frac{d}{cx^2}}}\right) + b\left(\frac{15c^{\frac{3}{2}}}{8d^3x\sqrt{1+\frac{d}{cx^2}}}+\frac{5\sqrt{c}}{8d^2x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{15c^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{7}{2}}}-\frac{1}{4\sqrt{c}dx^5\sqrt{1+\frac{d}{cx^2}}}\right)$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**6,x)
```

---

output
```
a*(-3*sqrt(c)/(2*d**2*x*sqrt(1 + d/(c*x**2))) + 3*c*asinh(sqrt(d)/(sqrt(c)
*x))/(2*d**(5/2)) - 1/(2*sqrt(c)*d*x**3*sqrt(1 + d/(c*x**2)))) + b*(15*c**
(3/2)/(8*d**3*x*sqrt(1 + d/(c*x**2))) + 5*sqrt(c)/(8*d**2*x**3*sqrt(1 + d/
(c*x**2))) - 15*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(7/2)) - 1/(4*sqrt(c
)*d*x**5*sqrt(1 + d/(c*x**2))))
```

3.985.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(103) = 206$.

Time = 0.27 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.98

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx = \frac{1}{16}\,b\left(\frac{2\left(15\left(c+\frac{d}{x^2}\right)^2c^2x^4-25\left(c+\frac{d}{x^2}\right)c^2dx^2+8\,c^2d^2\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5-2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^4x^3+\sqrt{c+\frac{d}{x^2}}\,d^5x}+\frac{15\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{7}{2}}}\right) - \frac{1}{4}\,a\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)cx^2-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3-\sqrt{c+\frac{d}{x^2}}\,d^3x}+\frac{3\,c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}\right)$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
1/16*b*(2*(15*(c + d/x^2)^2*c^2*x^4 - 25*(c + d/x^2)*c^2*d*x^2 + 8*c^2*d^2
)/((c + d/x^2)^(5/2)*d^3*x^5 - 2*(c + d/x^2)^(3/2)*d^4*x^3 + sqrt(c + d/x^
2)*d^5*x) + 15*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x +
sqrt(d)))/d^(7/2)) - 1/4*a*(2*(3*(c + d/x^2)*c*x^2 - 2*c*d)/((c + d/x^2)^(
3/2)*d^2*x^3 - sqrt(c + d/x^2)*d^3*x) + 3*c*log((sqrt(c + d/x^2)*x - sqrt(
d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2))
```

3.985.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.20

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx = \frac{3\left(5\,bc^2-4\,acd\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{8\sqrt{-d}\,d^3\mathrm{sgn}\left(x\right)}+\frac{bc^2-acd}{\sqrt{cx^2+d}\,d^3\mathrm{sgn}\left(x\right)} + \frac{7\left(cx^2+d\right)^{\frac{3}{2}}bc^2-4\left(cx^2+d\right)^{\frac{3}{2}}acd-9\sqrt{cx^2+d}\,bc^2d+4\sqrt{cx^2+d}\,acd^2}{8\,c^2d^3x^4\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
3/8*(5*b*c^2 - 4*a*c*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d^3*sgn
(x)) + (b*c^2 - a*c*d)/(sqrt(c*x^2 + d)*d^3*sgn(x)) + 1/8*(7*(c*x^2 + d)^(
3/2)*b*c^2 - 4*(c*x^2 + d)^(3/2)*a*c*d - 9*sqrt(c*x^2 + d)*b*c^2*d + 4*sqr
t(c*x^2 + d)*a*c*d^2)/(c^2*d^3*x^4*sgn(x))
```

3.985.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx = \int \frac{a+\frac{b}{x^2}}{x^6\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$$

input
```
int((a + b/x^2)/(x^6*(c + d/x^2)^(3/2)),x)
```

output
```
int((a + b/x^2)/(x^6*(c + d/x^2)^(3/2)), x)
```

3.986 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx$

3.986.1 Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx = \frac{\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} (ex)^{1+m} \operatorname{AppellF1}\left(\frac{1}{2}(-1-m),-p,-q,\frac{1-m}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e(1+m)}$$

output
```
(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1+m)*AppellF1(-1/2-1/2*m,-p,-q,-1/2*m+1/2,-
b/a/x^2,-d/c/x^2)/e/(1+m)/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

3.986.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx = \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x(ex)^m \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{2}(1+m-2p-2q),-p,-q,\frac{1}{2}(3+m-}{1+m-2p-2q}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^m,x]
```

output
```
((a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^m*AppellF1[(1 + m - 2*p - 2*q)/2, -p,
 -q, (3 + m - 2*p - 2*q)/2, -((a*x^2)/b), -((c*x^2)/d)])/((1 + m - 2*p - 2
*q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q)
```

3.986.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {999, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

$$\Big\downarrow 999$$

$$-\left(\frac{1}{x}\right)^m (ex)^m \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \left(\frac{1}{x}\right)^{-m-2} d\frac{1}{x}$$

$$\Big\downarrow 395$$

$$\left(\frac{1}{x}\right)^m (ex)^m \left(-\left(a+\frac{b}{x^2}\right)^p\right) \left(\frac{b}{ax^2}+1\right)^{-p} \int \left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q \left(\frac{1}{x}\right)^{-m-2} d\frac{1}{x}$$

$$\Big\downarrow 395$$

$$\left(\frac{1}{x}\right)^m (ex)^m \left(-\left(a+\frac{b}{x^2}\right)^p\right) \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \int \left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q \left(\frac{1}{x}\right)^{-m-2} d\frac{1}{x}$$

$$\Big\downarrow 394$$

$$\frac{x(ex)^m \left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{2}(-m-1),-p,-q,\frac{1-m}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{m+1}$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^m,x]
```

output
```
((a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^m*AppellF1[(-1 - m)/2, -p, -q, (1 - m
)/2, -(b/(a*x^2)), -(d/(c*x^2))])/((1 + m)*(1 + b/(a*x^2))^p*(1 + d/(c*x^2
))^q)
```

3.986.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 999
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(-(e*x)^m)*(x^(-1))^m   Subst[Int[(a + b/x^n)^p*(
(c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q}
, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

3.986.4 Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x)
```

3.986.5 Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx=\int (ex)^m \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="fricas")
```

output
```
integral((e*x)^m*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

3.986.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx=\text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**m,x)
```

output
```
Timed out
```

3.986.7 Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx=\int (ex)^m \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.986.8 Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx=\int (ex)^m \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="giac")
```

output
```
integrate((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.986.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q(ex)^m\,dx=\int (e\,x)^m\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
int((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.987 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4 \, dx$

3.987.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4 \, dx = \frac{1}{5}\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x^5 \operatorname{AppellF1}\left(-\frac{5}{2},-p,-q,-\frac{3}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

output
```
1/5*(a+b/x^2)^p*(c+d/x^2)^q*x^5*AppellF1(-5/2,-p,-q,-3/2,-b/a/x^2,-d/c/x^2
)/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

3.987.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4 \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^5 \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \operatorname{AppellF1}\left(\frac{5}{2}-p-q,-p,-q,\frac{7}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{-5+2p+2q}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^4,x]
```

output
```
-(((a + b/x^2)^p*(c + d/x^2)^q*x^5*AppellF1[5/2 - p - q, -p, -q, 7/2 - p -
 q, -((a*x^2)/b), -((c*x^2)/d)])/((-5 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 +
(c*x^2)/d)^q))
```

3.987.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {997, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$$

$$\Big\downarrow \ 997$$

$$-\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^6 d\frac{1}{x}$$

$$\Big\downarrow \ 395$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \int \left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q x^6 d\frac{1}{x}$$

$$\Big\downarrow \ 395$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \int \left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q x^6 d\frac{1}{x}$$

$$\Big\downarrow \ 394$$

$$\frac{1}{5}x^5 \left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \text{AppellF1}\left(-\frac{5}{2},-p,-q,-\frac{3}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q*x^4,x]
```

output
```
((a + b/x^2)^p*(c + d/x^2)^q*x^5*AppellF1[-5/2, -p, -q, -3/2, -(b/(a*x^2))
, -(d/(c*x^2))])/(5*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

3.987.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 997
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

3.987.4 Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*x^4,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q*x^4,x)
```

3.987.5 Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="fricas")
```

output
```
integral(x^4*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

3.987.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q*x**4,x)
```

output
```
Timed out
```

3.987.7 Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x^4, x)
```

3.987.8 Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x^4, x)
```

3.987.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx=\int x^4 \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input

```
int(x^4*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

output

```
int(x^4*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.988 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 \, dx$

3.988.1 Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 \, dx = \frac{b^2 \left(a+\frac{b}{x^2}\right)^{1+p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} \operatorname{AppellF1}\left(1+p,-q,3,2+p,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^3(1+p)}$$

output
```
1/2*b^2*(a+b/x^2)^(p+1)*(c+d/x^2)^q*AppellF1(p+1,3,-q,2+p,(a+b/x^2)/a,-d*(
a+b/x^2)/(-a*d+b*c))/a^3/(p+1)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

3.988.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4 \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \operatorname{AppellF1}\left(2-p-q,-p,-q,3-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(-2+p+q)}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^3,x]
```

output
```
-1/2*((a + b/x^2)^p*(c + d/x^2)^q*x^4*AppellF1[2 - p - q, -p, -q, 3 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((-2 + p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^
2)/d)^q)
```

3.988.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^6d\frac{1}{x^2}$$

$$\Big\downarrow \text{154}$$

$$-\frac{1}{2}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\int\left(a+\frac{b}{x^2}\right)^p\left(\frac{bc}{bc-ad}+\frac{bd}{(bc-ad)x^2}\right)^q x^6d\frac{1}{x^2}$$

$$\Big\downarrow \text{153}$$

$$\frac{b^2\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\text{AppellF1}\left(p+1,-q,3,p+2,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^3(p+1)}$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q*x^3,x]
```

output
```
(b^2*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 3, 2 + p, -((d*
(a + b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a^3*(1 + p)*((b*(c + d/x^2))
/(b*c - a*d))^q)
```

3.988.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.988.4 Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*x^3,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q*x^3,x)
```

3.988.5 Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="fricas")
```

output
```
integral(x^3*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

3.988.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx=\text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q*x**3,x)
```

output
```
Timed out
```

3.988.7 Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x^3, x)
```

3.988.8 Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x^3, x)
```

3.988.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx=\int x^3 \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
int(x^3*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int(x^3*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.989 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$

3.989.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx=\frac{1}{3}\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x^3 \operatorname{AppellF1}\left(-\frac{3}{2},-p,-q,-\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

output
```
1/3*(a+b/x^2)^p*(c+d/x^2)^q*x^3*AppellF1(-3/2,-p,-q,-1/2,-b/a/x^2,-d/c/x^2
)/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

3.989.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \operatorname{AppellF1}\left(\frac{3}{2}-p-q,-p,-q,\frac{5}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{-3+2p+2q}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^2,x]
```

output
```
-(((a + b/x^2)^p*(c + d/x^2)^q*x^3*AppellF1[3/2 - p - q, -p, -q, 5/2 - p -
 q, -((a*x^2)/b), -((c*x^2)/d)])/((-3 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 +
(c*x^2)/d)^q))
```

3.989.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {997, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

$$\Big\downarrow \text{997}$$

$$-\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \int \left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \int \left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{394}$$

$$\frac{1}{3}x^3 \left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \text{AppellF1}\left(-\frac{3}{2},-p,-q,-\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q*x^2,x]
```

output
```
((a + b/x^2)^p*(c + d/x^2)^q*x^3*AppellF1[-3/2, -p, -q, -1/2, -(b/(a*x^2))
, -(d/(c*x^2))])/(3*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

3.989.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 997
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

3.989.4 Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*x^2,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q*x^2,x)
```

3.989.5 Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="fricas")
```

output
```
integral(x^2*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

3.989.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx=\text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q*x**2,x)
```

output
```
Timed out
```

3.989.7 Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x^2, x)
```

3.989.8 Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x^2, x)
```

3.989.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx=\int x^2\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q \,dx$$

input
```
int(x^2*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int(x^2*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.990 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$

3.990.1 Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = -\frac{b\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\text{AppellF1}\left(1+p,-q,2,2+p,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^2(1+p)}$$

output
```
-1/2*b*(a+b/x^2)^(p+1)*(c+d/x^2)^q*AppellF1(p+1,2,-q,2+p,(a+b/x^2)/a,-d*(a
+b/x^2)/(-a*d+b*c))/a^2/(p+1)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

3.990.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^2\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\text{AppellF1}\left(1-p-q,-p,-q,2-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(-1+p+q)}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x,x]
```

output
```
-1/2*((a + b/x^2)^p*(c + d/x^2)^q*x^2*AppellF1[1 - p - q, -p, -q, 2 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((-1 + p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^
2)/d)^q)
```

3.990.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^4 d\frac{1}{x^2}$$

$$\Big\downarrow \text{154}$$

$$-\frac{1}{2}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\int \left(a+\frac{b}{x^2}\right)^p\left(\frac{bc}{bc-ad}+\frac{bd}{(bc-ad)x^2}\right)^q x^4 d\frac{1}{x^2}$$

$$\Big\downarrow \text{153}$$

$$-\frac{b\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\text{AppellF1}\left(p+1,-q,2,p+2,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^2(p+1)}$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q*x,x]
```

output
```
-1/2*(b*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 2, 2 + p, -(
(d*(a + b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(a^2*(1 + p)*((b*(c + d/x^2)
)/(b*c - a*d))^q)
```

3.990.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.990.4 Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*x,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q*x,x)
```

3.990.5 Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="fricas")
```

output
```
integral(x*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

3.990.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q*x,x)
```

output
```
Timed out
```

3.990.7 Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x, x)
```

3.990.8 Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$$

input

```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="giac")
```

output

```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x, x)
```

3.990.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx=\int x\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input

```
int(x*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

output

```
int(x*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.991 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$

3.991.1 Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x \operatorname{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

output
```
(a+b/x^2)^p*(c+d/x^2)^q*x*AppellF1(-1/2,-p,-q,1/2,-b/a/x^2,-d/c/x^2)/((1+b
/a/x^2)^p)/((1+d/c/x^2)^q)
```

3.991.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.32

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{2}-p-q,-p,-q,\frac{3}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{-1+2p+2q}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

output
```
-(((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/2 - p - q, -p, -q, 3/2 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c
*x^2)/d)^q))
```

3.991.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {899, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

$$\Big\downarrow \text{899}$$

$$-\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2d\frac{1}{x}$$

$$\Big\downarrow \text{395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \int \left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q x^2d\frac{1}{x}$$

$$\Big\downarrow \text{395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \int \left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q x^2d\frac{1}{x}$$

$$\Big\downarrow \text{394}$$

$$x\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \text{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

output
```
((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a*x^2)), -
(d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

3.991.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.991.4 Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

3.991.5 Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="fricas")
```

output
```
integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

3.991.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q,x)
```

output
```
Timed out
```

3.991.7 Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.991.8 Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.991.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$$

input
```
int((a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int((a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.992 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$

3.992.1 Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx = \frac{\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\operatorname{AppellF1}\left(1+p,-q,1,2+p,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a(1+p)}$$

output
```
1/2*(a+b/x^2)^(p+1)*(c+d/x^2)^q*AppellF1(p+1,1,-q,2+p,(a+b/x^2)/a,-d*(a+b/
x^2)/(-a*d+b*c))/a/(p+1)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

3.992.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\operatorname{AppellF1}\left(-p-q,-p,-q,1-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(p+q)}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x,x]
```

output
```
-1/2*((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-p - q, -p, -q, 1 - p - q, -((a
*x^2)/b), -((c*x^2)/d)])/((p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q)
```

3.992.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{154}$$

$$-\frac{1}{2}\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\int \left(a+\frac{b}{x^2}\right)^p \left(\frac{bc}{bc-ad}+\frac{bd}{(bc-ad)x^2}\right)^q x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{153}$$

$$\frac{\left(a+\frac{b}{x^2}\right)^{p+1} \left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} \text{AppellF1}\left(p+1,-q,1,p+2,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a(p+1)}$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/x,x]
```

output
```
((a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 1, 2 + p, -((d*(a +
 b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a*(1 + p)*((b*(c + d/x^2))/(b*c
- a*d))^q)
```

3.992.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.992.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/x,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/x,x)
```

3.992.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="fricas")
```

output
```
integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x, x)
```

3.992.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/x,x)
```

output
```
Timed out
```

3.992.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x, x)
```

3.992.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x, x)
```

3.992.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

input
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x,x)
```

output
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x, x)
```

3.993 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$

3.993.1 Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}$$

output
```
-(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(1/2,-p,-q,3/2,-b/a/x^2,-d/c/x^2)/((1+b/a
/x^2)^p)/((1+d/c/x^2)^q)/x
```

3.993.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\text{AppellF1}\left(-\frac{1}{2}-p-q,-p,-q,\frac{1}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(1+2p+2q)x}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x]
```

output
```
-(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-1/2 - p - q, -p, -q, 1/2 - p - q,
 -((a*x^2)/b), -((c*x^2)/d)])/((1 + 2*p + 2*q)*x*(1 + (a*x^2)/b)^p*(1 + (c
*x^2)/d)^q))
```

3.993.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {997, 334, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}dx$$

$$\Big\downarrow \text{997}$$

$$-\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q d\frac{1}{x}$$

$$\Big\downarrow \text{334}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \int \left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q d\frac{1}{x}$$

$$\Big\downarrow \text{334}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \int \left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q d\frac{1}{x}$$

$$\Big\downarrow \text{333}$$

$$-\frac{\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x]
```

output
```
-(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[1/2, -p, -q, 3/2, -(b/(a*x^2)), -(
d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*x))
```

3.993.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 997
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

3.993.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^2,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^2,x)
```

3.993.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="fricas")
```

output
```
integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^2, x)
```

3.993.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/x**2,x)
```

output
```
Timed out
```

3.993.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^2, x)
```

3.993.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^2, x)
```

3.993.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

input
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x)
```

output
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^2, x)
```

3.994 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$

3.994.1 Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(1+p,-q,2+p,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2b(1+p)}$$

output
```
-1/2*(a+b/x^2)^(p+1)*(c+d/x^2)^q*hypergeom([-q, p+1],[2+p],-d*(a+b/x^2)/(-
a*d+b*c))/b/(p+1)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

3.994.2 Mathematica [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{ax^2}{b}\right)^{-p}(d+cx^2)\left(1+\frac{cx^2}{d}\right)^p\text{Hypergeometric2F1}\Big(-p,-1-p-q,-p-q,\frac{(bc}{b(}}{2d(1+p+q)x^2}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x]
```

output
```
-1/2*((a + b/x^2)^p*(c + d/x^2)^q*(d + c*x^2)*(1 + (c*x^2)/d)^p*Hypergeome
tric2F1[-p, -1 - p - q, -p - q, ((b*c - a*d)*x^2)/(b*(d + c*x^2))])/(d*(1
+ p + q)*x^2*(1 + (a*x^2)/b)^p)
```

3.994.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}dx$$

$$\Big\downarrow \text{946}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q d\frac{1}{x^2}$$

$$\Big\downarrow \text{80}$$

$$-\frac{1}{2}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\int \left(a+\frac{b}{x^2}\right)^p\left(\frac{bc}{bc-ad}+\frac{bd}{(bc-ad)x^2}\right)^q d\frac{1}{x^2}$$

$$\Big\downarrow \text{79}$$

$$-\frac{\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2b(p+1)}$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x]
```

output
```
-1/2*((a + b/x^2)^(1 + p)*(c + d/x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p
, -((d*(a + b/x^2))/(b*c - a*d))])/(b*(1 + p)*((b*(c + d/x^2))/(b*c - a*d)
)^q)
```

3.994.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.994.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^3,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^3,x)
```

3.994.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="fricas")
```

output
```
integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^3, x)
```

3.994.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/x**3,x)
```

output
```
Timed out
```

3.994.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^3, x)
```

3.994.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^3, x)
```

3.994.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

input
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x)
```

output
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^3, x)
```

3.995 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4} \, dx$

3.995.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} \operatorname{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3x^3}$$

output
```
-1/3*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(3/2,-p,-q,5/2,-b/a/x^2,-d/c/x^2)/((1
+b/a/x^2)^p)/((1+d/c/x^2)^q)/x^3
```

3.995.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \operatorname{AppellF1}\left(-\frac{3}{2}-p-q,-p,-q,-\frac{1}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(3+2p+2q)x^3}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x]
```

output
```
-(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-3/2 - p - q, -p, -q, -1/2 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((3 + 2*p + 2*q)*x^3*(1 + (a*x^2)/b)^p*(1 +
 (c*x^2)/d)^q))
```

3.995.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {997, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4} \, dx$$

$$\Big\downarrow \text{997}$$

$$-\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{a x^2}+1\right)^{-p} \int \frac{\left(\frac{b}{a x^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{a x^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{c x^2}+1\right)^{-q} \int \frac{\left(\frac{b}{a x^2}+1\right)^p \left(\frac{d}{c x^2}+1\right)^q}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{394}$$

$$-\frac{\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{a x^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{c x^2}+1\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{b}{a x^2},-\frac{d}{c x^2}\right)}{3 x^3}$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x]
```

output
```
-1/3*((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[3/2, -p, -q, 5/2, -(b/(a*x^2)),
 -(d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*x^3)
```

3.995.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 997
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

3.995.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^4,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^4,x)
```

3.995.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="fricas")
```

output
```
integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^4, x)
```

3.995.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\, dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/x**4,x)
```

output
```
Timed out
```

3.995.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\, dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\, dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^4, x)
```

3.995.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\, dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\, dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^4, x)
```

3.995.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

input
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x)
```

output
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^4, x)
```

3.996 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx$

3.996.1 Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c +\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx=\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{7/2}\operatorname{AppellF1}\left(-\frac{7}{4},-p,-q,-\frac{3}{4},-\frac{b}{ax^2},-\right.}{7e}$$

output
```
2/7*(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(7/2)*AppellF1(-7/4,-p,-q,-3/4,-b/a/x^2,
-d/c/x^2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

3.996.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x(ex)^{5/2}\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{7}{4}-p-q,-p,-q,\frac{11}{4}-p-q,-\frac{ax^2}{b},-\right.}{-7+4p+4q}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2),x]
```

output
```
(-2*(a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^(5/2)*AppellF1[7/4 - p - q, -p, -q
, 11/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-7 + 4*p + 4*q)*(1 + (a*x^2
)/b)^p*(1 + (c*x^2)/d)^q)
```

3.996.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {998, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{5/2} \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

$$\Big\downarrow \text{998}$$

$$-\frac{2\int e^4\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4 d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \int e^4\left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q x^4 d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \int e^4\left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q x^4 d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1012}$$

$$\frac{2(ex)^{7/2}\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \operatorname{AppellF1}\left(-\frac{7}{4},-p,-q,-\frac{3}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{7e}$$

input

```
Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2),x]
```

output

```
(2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(7/2)*AppellF1[-7/4, -p, -q, -3/4, -(
b/(a*x^2)), -(d/(c*x^2))])/(7*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

3.996.3.1 Defintions of rubi rules used

rule 998
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Simp[-g/e   Subst[Int[(a +
b/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e
*x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fractio
nQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.996.4 Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{\frac{5}{2}}\,dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x)
```

3.996.5 Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx = \int (ex)^{\frac{5}{2}}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*e^2*x^2*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

3.996.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(5/2),x)
```

output
```
Timed out
```

3.996.7 Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx = \int (ex)^{\frac{5}{2}} \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(5/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.996.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,6,1,1,0]%%%} / %%%{1,[0,0,0,0,3]%%%} Error: Bad Argum
ent Value
```

3.996.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx=\int (e\,x)^{5/2}\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q \,dx$$

input
```
int((e*x)^(5/2)*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int((e*x)^(5/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.997 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx$

3.997.1 Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{5/2}\operatorname{AppellF1}\left(-\frac{5}{4},-p,-q,-\frac{1}{4},-\frac{b}{ax^2},-\right.}{5e}$$

output
```
2/5*(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2)*AppellF1(-5/4,-p,-q,-1/4,-b/a/x^2,
-d/c/x^2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

3.997.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x(ex)^{3/2}\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{5}{4}-p-q,-p,-q,\frac{9}{4}-p-q,-\frac{ax^2}{b},-\right.}{-5+4p+4q}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2),x]
```

output
```
(-2*(a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^(3/2)*AppellF1[5/4 - p - q, -p, -q
, 9/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-5 + 4*p + 4*q)*(1 + (a*x^2)
/b)^p*(1 + (c*x^2)/d)^q)
```

3.997.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {998, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2} \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$$

$$\Big\downarrow \text{998}$$

$$-\frac{2\int e^3\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^3 d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\int e^3\left(\frac{b}{ax^2}+1\right)^p\left(c+\frac{d}{x^2}\right)^q x^3 d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\int e^3\left(\frac{b}{ax^2}+1\right)^p\left(\frac{d}{cx^2}+1\right)^q x^3 d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1012}$$

$$\frac{2(ex)^{5/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\operatorname{AppellF1}\left(-\frac{5}{4},-p,-q,-\frac{1}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{5e}$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2),x]
```

output
```
(2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2)*AppellF1[-5/4, -p, -q, -1/4, -(
b/(a*x^2)), -(d/(c*x^2))])/(5*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

3.997.3.1 Defintions of rubi rules used

rule 998
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Simp[-g/e   Subst[Int[(a +
b/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e
*x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fractio
nQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.997.4 Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{\frac{3}{2}}\,dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x)
```

3.997.5 Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=\int (ex)^{\frac{3}{2}}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*e*x*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

3.997.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(3/2),x)
```

output
```
Timed out
```

3.997.7 Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx = \int (ex)^{\frac{3}{2}} \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.997.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,4,1,1,0]%%%} / %%%{1,[0,0,0,0,2]%%%} Error: Bad Argum
ent Value
```

3.997.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=\int (e\,x)^{3/2}\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
int((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.998 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx$

3.998.1 Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx = \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{3/2}\operatorname{AppellF1}\left(-\frac{3}{4},-p,-q,\frac{1}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e}$$

output
```
2/3*(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2)*AppellF1(-3/4,-p,-q,1/4,-b/a/x^2,-
d/c/x^2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

3.998.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\sqrt{ex}\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\operatorname{AppellF1}\bigl(\frac{3}{4}-p-q,-p,-q,\frac{7}{4}-p-q,-\frac{ax^2}{b},-}{-3+4p+4q}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x],x]
```

output
```
(-2*(a + b/x^2)^p*(c + d/x^2)^q*x*Sqrt[e*x]*AppellF1[3/4 - p - q, -p, -q,
7/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-3 + 4*p + 4*q)*(1 + (a*x^2)/b
)^p*(1 + (c*x^2)/d)^q)
```

3.998.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {998, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

$$\Big\downarrow \text{998}$$

$$-\frac{2\int e^2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^2 d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\int e^2\left(\frac{b}{ax^2}+1\right)^p\left(c+\frac{d}{x^2}\right)^q x^2 d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\int e^2\left(\frac{b}{ax^2}+1\right)^p\left(\frac{d}{cx^2}+1\right)^q x^2 d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1012}$$

$$\frac{2(ex)^{3/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\operatorname{AppellF1}\left(-\frac{3}{4},-p,-q,\frac{1}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e}$$

input

```
Int[(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x],x]
```

output

```
(2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2)*AppellF1[-3/4, -p, -q, 1/4, -(b
/(a*x^2)), -(d/(c*x^2))])/(3*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

3.998.3.1 Defintions of rubi rules used

rule 998
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Simp[-g/e   Subst[Int[(a +
b/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e
*x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fractio
nQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.998.4 Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x)
```

3.998.5 Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx = \int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

3.998.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\, dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(1/2),x)
```

output
```
Timed out
```

3.998.7 Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\, dx = \int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.998.8 Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\, dx = \int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.998.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\sqrt{ex}\,dx=\int \sqrt{e\,x}\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
int((e*x)^(1/2)*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int((e*x)^(1/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.999 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$

3.999.1 Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = \frac{2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} \sqrt{ex}\,\text{AppellF1}\left(-\frac{1}{4},-p,-q,\frac{3}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e}$$

output
```
2*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(-1/4,-p,-q,3/4,-b/a/x^2,-d/c/x^2)*(e*x)
^(1/2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

3.999.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \text{AppellF1}\left(\frac{1}{4}-p-q,-p,-q,\frac{5}{4}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(-1+4p+4q)\sqrt{ex}}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/Sqrt[e*x],x]
```

output
```
(-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/4 - p - q, -p, -q, 5/4 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 4*p + 4*q)*Sqrt[e*x]*(1 + (a*x^2)/b
)^p*(1 + (c*x^2)/d)^q)
```

3.999.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {998, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}} \, dx$$

$$\Big\downarrow \text{998}$$

$$-\frac{2\int e\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q xd\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p}\int e\left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q xd\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q}\int e\left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q xd\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1012}$$

$$\frac{2\sqrt{ex}\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q}\text{AppellF1}\left(-\frac{1}{4},-p,-q,\frac{3}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e}$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/Sqrt[e*x],x]
```

output
```
(2*(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x]*AppellF1[-1/4, -p, -q, 3/4, -(b/(
a*x^2)), -(d/(c*x^2))])/(e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

3.999. $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}} \, dx$

3.999.3.1 Defintions of rubi rules used

rule 998
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Simp[-g/e   Subst[Int[(a +
b/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e
*x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fractio
nQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.999.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x)
```

3.999.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/(e*x), x)
```

3.999.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx=\text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(1/2),x)
```

output
```
Timed out
```

3.999.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/sqrt(e*x), x)
```

3.999.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/sqrt(e*x), x)
```

3.999.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{e\,x}}\,dx$$

input
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(1/2),x)
```

output
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(1/2), x)
```

3.1000 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx$

3.1000.1 Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}$$

output
```
-2*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(1/4,-p,-q,5/4,-b/a/x^2,-d/c/x^2)/e/((1
+b/a/x^2)^p)/((1+d/c/x^2)^q)/(e*x)^(1/2)
```

3.1000.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\operatorname{AppellF1}\left(-\frac{1}{4}-p-q,-p,-q,\frac{3}{4}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(1+4p+4q)(ex)^{3/2}}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x]
```

output
```
(-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/4 - p - q, -p, -q, 3/4 - p -
 q, -((a*x^2)/b), -((c*x^2)/d)])/((1 + 4*p + 4*q)*(e*x)^(3/2)*(1 + (a*x^2)
/b)^p*(1 + (c*x^2)/d)^q)
```

3.1000.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {998, 937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}} \, dx$$

$$\Big\downarrow \; 998$$

$$-\frac{2\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \; 937$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p}\int \left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \; 937$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q}\int \left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \; 936$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x]
```

output
```
(-2*(a + b/x^2)^p*(c + d/x^2)^q*AppellF1[1/4, -p, -q, 5/4, -(b/(a*x^2)), -
(d/(c*x^2))])/(e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*Sqrt[e*x])
```

3.1000.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 998
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Simp[-g/e   Subst[Int[(a +
b/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e
*x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fractio
nQ[m]
```

3.1000.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{3}{2}}}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x)
```

3.1000.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/(e^2*x^2), x)
```

3.1000.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(3/2),x)
```

output
```
Timed out
```

3.1000.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(3/2), x)
```

3.1000.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(3/2), x)
```

3.1000.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(e\,x)^{3/2}}\,dx$$

input

```
int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x)
```

output

```
int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2), x)
```

3.1001 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx$

3.1001.1 Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\operatorname{AppellF1}\left(\frac{3}{4},-p,-q,\frac{7}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

output
```
-2/3*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(3/4,-p,-q,7/4,-b/a/x^2,-d/c/x^2)/e/(
(1+b/a/x^2)^p)/((1+d/c/x^2)^q)/(e*x)^(3/2)
```

3.1001.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\operatorname{AppellF1}\left(-\frac{3}{4}-p-q,-p,-q,\frac{1}{4}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(3+4p+4q)(ex)^{5/2}}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x]
```

output
```
(-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-3/4 - p - q, -p, -q, 1/4 - p -
 q, -((a*x^2)/b), -((c*x^2)/d)])/((3 + 4*p + 4*q)*(e*x)^(5/2)*(1 + (a*x^2)
/b)^p*(1 + (c*x^2)/d)^q)
```

3.1001.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {998, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}dx$$

$$\Big\downarrow \text{998}$$

$$-\frac{2\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{ex}d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\int \frac{\left(\frac{b}{ax^2}+1\right)^p\left(c+\frac{d}{x^2}\right)^q}{ex}d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\int \frac{\left(\frac{b}{ax^2}+1\right)^p\left(\frac{d}{cx^2}+1\right)^q}{ex}d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{4},-p,-q,\frac{7}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x]
```

output
```
(-2*(a + b/x^2)^p*(c + d/x^2)^q*AppellF1[3/4, -p, -q, 7/4, -(b/(a*x^2)), -
(d/(c*x^2))])/(3*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*(e*x)^(3/2))
```

3.1001. $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}dx$

3.1001.3.1 Defintions of rubi rules used

rule 998
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Simp[-g/e   Subst[Int[(a +
b/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e
*x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fractio
nQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.1001.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x^{2}}\right)^{p} \left(c+\frac{d}{x^{2}}\right)^{q}}{\left(e x \right)^{\frac{5}{2}}}d x$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x)
```

3.1001.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = \int \frac{{\left(a + \frac{b}{x^{2}}\right)}^{p} {\left(c + \frac{d}{x^{2}}\right)}^{q}}{\left(e x\right)^{\frac{5}{2}}}\,{d x}$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/(e^3*x^3), x)
```

3.1001.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(5/2),x)
```

output
```
Timed out
```

3.1001.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(5/2), x)
```

3.1001.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(5/2), x)
```

3.1001.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(e\,x)^{5/2}}\,dx$$

input

```
int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x)
```

output

```
int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2), x)
```

3.1002 $\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx$

3.1002.1 Optimal result

Integrand size = 28, antiderivative size = 135

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx=-\frac{5}{64}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{5}{96}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}-\frac{1}{24}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}+\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}-\frac{5\operatorname{arccosh}\left(\sqrt{x}\right)}{64}$$

output
```
-5/64*arccosh(x^(1/2))-5/96*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-1
/24*x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+1/4*x^(7/2)*(-1+x^(1/2))^
(1/2)*(1+x^(1/2))^(1/2)-5/64*x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

3.1002.2 Mathematica [A] (warning: unable to verify)

Time = 7.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.73

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx=\frac{1}{192}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}\left(-15-15\sqrt{x}-10x-10x^{3/2}-8x^2-8x^{5/2}+48x^3+48\right.\right.$$

input
```
Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2),x]
```

output
```
(Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]*(-15 - 15*Sqrt[x] - 10*x - 10*
x^(3/2) - 8*x^2 - 8*x^(5/2) + 48*x^3 + 48*x^(7/2)) - 30*ArcTanh[Sqrt[(-1 +
 Sqrt[x])/(1 + Sqrt[x])]])/192
```

3.1002.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {812, 845, 845, 845, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}\,dx$$

$$\Big\downarrow \text{812}$$

$$\frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}-\frac{1}{8}\int \frac{x^{5/2}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{845}$$

$$\frac{1}{8}\left(-\frac{5}{6}\int \frac{x^{3/2}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx-\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}\right)+\frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}$$

$$\Big\downarrow \text{845}$$

$$\frac{1}{8}\left(-\frac{5}{6}\left(\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)-\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}\right)+ \\ \frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}$$

$$\Big\downarrow \text{845}$$

$$\frac{1}{8}\left(-\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)-\frac{1}{3}\sqrt{\sqrt{x}-}\right. \\ \frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}$$

$$\Big\downarrow \text{852}$$

$$\frac{1}{8}\left(-\frac{5}{6}\left(\frac{3}{4}\left(\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)-\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{}\right. \\ \frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}$$

$$\Big\downarrow \text{43}$$

$$\frac{1}{8}\left(-\frac{5}{6}\left(\frac{3}{4}\left(\operatorname{arccosh}(\sqrt{x})+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)-\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}\right)+$$
$$\frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}$$

input

```
Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2),x]
```

output

```
(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2))/4 + (-1/3*(Sqrt[-1 + Sqrt[x
]]*Sqrt[1 + Sqrt[x]]*x^(5/2)) - (5*((Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*
x^(3/2))/2 + (3*(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + ArcCosh[Sq
rt[x]]))/4))/6)/8
```

3.1002.3.1 Defintions of rubi rules used

rule 43

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 812

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Simp[2*a1*a2*n*(p/(m + 2*n*p + 1))   Int[(c*x
)^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a
2, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && N
eQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 845

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

3.1002.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.56

method	result	size
derivativedivides	$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-48\sqrt{-1+x}\,x^{\frac{7}{2}}+8x^{\frac{5}{2}}\sqrt{-1+x}+10x^{\frac{3}{2}}\sqrt{-1+x}+15\sqrt{x}\,\sqrt{-1+x}+15\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{192\sqrt{-1+x}}$	75
default	$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-48\sqrt{-1+x}\,x^{\frac{7}{2}}+8x^{\frac{5}{2}}\sqrt{-1+x}+10x^{\frac{3}{2}}\sqrt{-1+x}+15\sqrt{x}\,\sqrt{-1+x}+15\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{192\sqrt{-1+x}}$	75

input
```
int(x^(5/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-48*(-1+x)^(1/2)*x^(7/2)+8*x^(
5/2)*(-1+x)^(1/2)+10*x^(3/2)*(-1+x)^(1/2)+15*x^(1/2)*(-1+x)^(1/2)+15*ln(x^
(1/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

3.1002.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.46

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \frac{1}{192}\left(48\,x^{3}-8\,x^{2}-10\,x-15\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{5}{128}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

input
```
integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="frica
s")
```

output
```
1/192*(48*x^3 - 8*x^2 - 10*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x)
- 1) + 5/128*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

3.1002.6 Sympy [F]

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx=\int x^{\frac{5}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,dx$$

input
```
integrate(x**(5/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

output
```
Integral(x**(5/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

3.1002.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.42

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx=\frac{1}{4}\,(x-1)^{\frac{3}{2}}x^{\frac{5}{2}}+\frac{5}{24}\,(x-1)^{\frac{3}{2}}x^{\frac{3}{2}}+\frac{5}{32}\,(x-1)^{\frac{3}{2}}\sqrt{x}+\frac{5}{64}\,\sqrt{x-1}\sqrt{x}-\frac{5}{64}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

input
```
integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxim
a")
```

output
```
1/4*(x - 1)^(3/2)*x^(5/2) + 5/24*(x - 1)^(3/2)*x^(3/2) + 5/32*(x - 1)^(3/2
)*sqrt(x) + 5/64*sqrt(x - 1)*sqrt(x) - 5/64*log(2*sqrt(x - 1) + 2*sqrt(x))
```

3.1002.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.20

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx=\frac{1}{6720}\,((2\,((4\,(5\,(6\,(7\,\sqrt{x}-50)\,(\sqrt{x}+1)+1219)\,(\sqrt{x}+1)-12463)\,(\sqrt{x}$$
$$+\frac{1}{840}\,((2\,((4\,(5\,(6\,\sqrt{x}-37)\,(\sqrt{x}+1)+661)\,(\sqrt{x}+1)-4551)\,(\sqrt{x}+1)+4781)\,(\sqrt{x}+1)-6335)\,(\sqrt{x}+$$
$$+\frac{5}{32}\,\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

input

```
integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac"
)
```

output

```
1/6720*((2*((4*(5*(6*(7*sqrt(x) - 50)*(sqrt(x) + 1) + 1219)*(sqrt(x) + 1)
- 12463)*(sqrt(x) + 1) + 64233)*(sqrt(x) + 1) - 53963)*(sqrt(x) + 1) + 594
65)*(sqrt(x) + 1) - 23205)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/840*((2
*((4*(5*(6*sqrt(x) - 37)*(sqrt(x) + 1) + 661)*(sqrt(x) + 1) - 4551)*(sqrt(
x) + 1) + 4781)*(sqrt(x) + 1) - 6335)*(sqrt(x) + 1) + 2835)*sqrt(sqrt(x) +
 1)*sqrt(sqrt(x) - 1) + 5/32*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

3.1002.9 Mupad [B] (verification not implemented)

Time = 75.08 (sec) , antiderivative size = 831, normalized size of antiderivative = 6.16

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \text{Too large to display}$$

input

```
int(x^(5/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

output

```
((1723*((x^(1/2) - 1)^(1/2) - 1i)^5)/(48*((x^(1/2) + 1)^(1/2) - 1)^5) - (2
35*((x^(1/2) - 1)^(1/2) - 1i)^3)/(48*((x^(1/2) + 1)^(1/2) - 1)^3) + (72283
*((x^(1/2) - 1)^(1/2) - 1i)^7)/(16*((x^(1/2) + 1)^(1/2) - 1)^7) + (848801*
((x^(1/2) - 1)^(1/2) - 1i)^9)/(16*((x^(1/2) + 1)^(1/2) - 1)^9) + (4181067*
((x^(1/2) - 1)^(1/2) - 1i)^11)/(16*((x^(1/2) + 1)^(1/2) - 1)^11) + (109941
81*((x^(1/2) - 1)^(1/2) - 1i)^13)/(16*((x^(1/2) + 1)^(1/2) - 1)^13) + (174
57599*((x^(1/2) - 1)^(1/2) - 1i)^15)/(16*((x^(1/2) + 1)^(1/2) - 1)^15) + (
17457599*((x^(1/2) - 1)^(1/2) - 1i)^17)/(16*((x^(1/2) + 1)^(1/2) - 1)^17)
+ (10994181*((x^(1/2) - 1)^(1/2) - 1i)^19)/(16*((x^(1/2) + 1)^(1/2) - 1)^1
9) + (4181067*((x^(1/2) - 1)^(1/2) - 1i)^21)/(16*((x^(1/2) + 1)^(1/2) - 1)
^21) + (848801*((x^(1/2) - 1)^(1/2) - 1i)^23)/(16*((x^(1/2) + 1)^(1/2) - 1
)^23) + (72283*((x^(1/2) - 1)^(1/2) - 1i)^25)/(16*((x^(1/2) + 1)^(1/2) - 1
)^25) + (1723*((x^(1/2) - 1)^(1/2) - 1i)^27)/(48*((x^(1/2) + 1)^(1/2) - 1)
^27) - (235*((x^(1/2) - 1)^(1/2) - 1i)^29)/(48*((x^(1/2) + 1)^(1/2) - 1)^2
9) + (5*((x^(1/2) - 1)^(1/2) - 1i)^31)/(16*((x^(1/2) + 1)^(1/2) - 1)^31) +
 (5*((x^(1/2) - 1)^(1/2) - 1i))/(16*((x^(1/2) + 1)^(1/2) - 1)))/((120*((x^
(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (16*((x^(1/2) - 1)
^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (560*((x^(1/2) - 1)^(1/2) -
1i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (1820*((x^(1/2) - 1)^(1/2) - 1i)^8)/(
(x^(1/2) + 1)^(1/2) - 1)^8 - (4368*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(...
```

3.1003 $\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx$

3.1003.1 Optimal result

Integrand size = 28, antiderivative size = 104

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx=-\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{1}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}+\frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{8}$$

output
```
-1/8*arccosh(x^(1/2))-1/12*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+1/
3*x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-1/8*x^(1/2)*(-1+x^(1/2))^(1
/2)*(1+x^(1/2))^(1/2)
```

3.1003.2 Mathematica [A] (warning: unable to verify)

Time = 1.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx=\frac{1}{24}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}\left(-3-3\sqrt{x}-2x-2x^{3/2}+8x^2+8x^{5/2}\right)-6\operatorname{arctanh}\right.$$

input
```
Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2),x]
```

output
```
(Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]*(-3 - 3*Sqrt[x] - 2*x - 2*x^(3
/2) + 8*x^2 + 8*x^(5/2)) - 6*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/
24
```

3.1003.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {812, 845, 845, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\,dx$$

$$\Big\downarrow \text{ 812}$$

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}-\frac{1}{6}\int \frac{x^{3/2}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{ 845}$$

$$\frac{1}{6}\left(-\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

$$\Big\downarrow \text{ 845}$$

$$\frac{1}{6}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

$$\Big\downarrow \text{ 852}$$

$$\frac{1}{6}\left(-\frac{3}{4}\left(\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

$$\Big\downarrow \text{ 43}$$

$$\frac{1}{6}\left(-\frac{3}{4}\left(\text{arccosh}\left(\sqrt{x}\right)+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

input

```
Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2),x]
```

output
```
(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2))/3 + (-1/2*(Sqrt[-1 + Sqrt[x
]]*Sqrt[1 + Sqrt[x]]*x^(3/2)) - (3*(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*S
qrt[x] + ArcCosh[Sqrt[x]]))/4)/6
```

3.1003.3.1 Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 812
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Simp[2*a1*a2*n*(p/(m + 2*n*p + 1))   Int[(c*x
)^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a
2, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && N
eQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

3.1003.4 Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-8 x^{\frac{5}{2}} \sqrt{-1+x}+2 x^{\frac{3}{2}} \sqrt{-1+x}+3 \sqrt{x}\, \sqrt{-1+x}+3 \ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{24 \sqrt{-1+x}}$	65
default	$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-8 x^{\frac{5}{2}} \sqrt{-1+x}+2 x^{\frac{3}{2}} \sqrt{-1+x}+3 \sqrt{x}\, \sqrt{-1+x}+3 \ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{24 \sqrt{-1+x}}$	65

input

```
int(x^(3/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/24*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-8*x^(5/2)*(-1+x)^(1/2)+2*x^(3/
2)*(-1+x)^(1/2)+3*x^(1/2)*(-1+x)^(1/2)+3*ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^
(1/2)
```

3.1003.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.55

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = \frac{1}{24}\left(8\,x^{2}-2\,x-3\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{16}\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

input

```
integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="frica
s")
```

output

```
1/24*(8*x^2 - 2*x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/16*
log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

3.1003.6 Sympy [F]

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx=\int x^{\frac{3}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,dx$$

input
```
integrate(x**(3/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

output
```
Integral(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

3.1003.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.45

$$\begin{aligned}\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx&=\frac{1}{3}\,(x-1)^{\frac{3}{2}}x^{\frac{3}{2}}\\&+\frac{1}{4}\,(x-1)^{\frac{3}{2}}\sqrt{x}+\frac{1}{8}\,\sqrt{x-1}\sqrt{x}-\frac{1}{8}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)\end{aligned}$$

input
```
integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxim
a")
```

output
```
1/3*(x - 1)^(3/2)*x^(3/2) + 1/4*(x - 1)^(3/2)*sqrt(x) + 1/8*sqrt(x - 1)*sq
rt(x) - 1/8*log(2*sqrt(x - 1) + 2*sqrt(x))
```

3.1003.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx=&\frac{1}{120}\left(\left(2\left(\left(4\left(5\sqrt{x}-26\right)\left(\sqrt{x}+1\right)+321\right)\left(\sqrt{x}+1\right)-451\right)\left(\sqrt{x}+1\right)+74\right.\right.\\&+\frac{1}{60}\left(\left(2\left(3\left(4\sqrt{x}-17\right)\left(\sqrt{x}+1\right)+133\right)\left(\sqrt{x}+1\right)-295\right)\left(\sqrt{x}+1\right)+195\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\\&+\frac{1}{4}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)\end{aligned}$$

input
```
integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac"
)
```

output
```
1/120*((2*((4*(5*sqrt(x) - 26)*(sqrt(x) + 1) + 321)*(sqrt(x) + 1) - 451)*(
sqrt(x) + 1) + 745)*(sqrt(x) + 1) - 405)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) -
1) + 1/60*((2*(3*(4*sqrt(x) - 17)*(sqrt(x) + 1) + 133)*(sqrt(x) + 1) - 295
)*(sqrt(x) + 1) + 195)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/4*log(sqrt(
sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

3.1003.9 Mupad [B] (verification not implemented)

Time = 45.13 (sec) , antiderivative size = 632, normalized size of antiderivative = 6.08

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right)}{2} - \frac{\frac{35\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{6\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{757\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^5}{2\left(\sqrt{\sqrt{x}+1}-1\right)^5}+\frac{7339\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^7}{2\left(\sqrt{\sqrt{x}+1}-1\right)^7}+\frac{41929\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^9}{3\left(\sqrt{\sqrt{x}+1}-1\right)^9}+\frac{25661\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}}+\frac{25661\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)}{\left(\sqrt{\sqrt{x}+1}-1\right)^{13}}}{1+\frac{66\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4}-\frac{220\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6}+\frac{495\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8}-\frac{792\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}}+\frac{924\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}}-\frac{792\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right.}{\left(\sqrt{\sqrt{x}+1}-1\right)}}$$

input
```
int(x^(3/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

output
```
- atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1))/2 - ((35*((x
^(1/2) - 1)^(1/2) - 1i)^3)/(6*((x^(1/2) + 1)^(1/2) - 1)^3) + (757*((x^(1/2
) - 1)^(1/2) - 1i)^5)/(2*((x^(1/2) + 1)^(1/2) - 1)^5) + (7339*((x^(1/2) -
1)^(1/2) - 1i)^7)/(2*((x^(1/2) + 1)^(1/2) - 1)^7) + (41929*((x^(1/2) - 1)^
(1/2) - 1i)^9)/(3*((x^(1/2) + 1)^(1/2) - 1)^9) + (25661*((x^(1/2) - 1)^(1/
2) - 1i)^11)/((x^(1/2) + 1)^(1/2) - 1)^11 + (25661*((x^(1/2) - 1)^(1/2) -
1i)^13)/((x^(1/2) + 1)^(1/2) - 1)^13 + (41929*((x^(1/2) - 1)^(1/2) - 1i)^1
5)/(3*((x^(1/2) + 1)^(1/2) - 1)^15) + (7339*((x^(1/2) - 1)^(1/2) - 1i)^17)
/(2*((x^(1/2) + 1)^(1/2) - 1)^17) + (757*((x^(1/2) - 1)^(1/2) - 1i)^19)/(2
*((x^(1/2) + 1)^(1/2) - 1)^19) + (35*((x^(1/2) - 1)^(1/2) - 1i)^21)/(6*((x
^(1/2) + 1)^(1/2) - 1)^21) - ((x^(1/2) - 1)^(1/2) - 1i)^23/(2*((x^(1/2) +
1)^(1/2) - 1)^23) - ((x^(1/2) - 1)^(1/2) - 1i)/(2*((x^(1/2) + 1)^(1/2) - 1
)))/((66*((x^(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (12*(
(x^(1/2) - 1)^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (220*((x^(1/2)
- 1)^(1/2) - 1i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (495*((x^(1/2) - 1)^(1/2
) - 1i)^8)/((x^(1/2) + 1)^(1/2) - 1)^8 - (792*((x^(1/2) - 1)^(1/2) - 1i)^1
0)/((x^(1/2) + 1)^(1/2) - 1)^10 + (924*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^
(1/2) + 1)^(1/2) - 1)^12 - (792*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) +
 1)^(1/2) - 1)^14 + (495*((x^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/
2) - 1)^16 - (220*((x^(1/2) - 1)^(1/2) - 1i)^18)/((x^(1/2) + 1)^(1/2) -...
```

3.1003. $\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx$

3.1004 $\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx$

3.1004.1 Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = -\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} - \frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{4}$$

output
```
-1/4*arccosh(x^(1/2))+1/2*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-1/4
*x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

3.1004.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 404 vs. 2(73) = 146.

Time = 1.93 (sec) , antiderivative size = 404, normalized size of antiderivative = 5.53

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{-4\sqrt{1+\sqrt{x}}\left(-18816+28224\sqrt{x}+55360x+17296x^{3/2}+7240x^2-1096x^{5/2}-4752x^3-1136x^{7/2}\right)-4}{-12416} + \operatorname{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

input
```
Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

output
```
(-4*Sqrt[1 + Sqrt[x]]*(-18816 + 28224*Sqrt[x] + 55360*x + 17296*x^(3/2) +
7240*x^2 - 1096*x^(5/2) - 4752*x^3 - 1136*x^(7/2)) - 4*Sqrt[-1 + Sqrt[x]]*
Sqrt[1 + Sqrt[x]]*(32592 + 74488*Sqrt[x] + 38632*x + 6992*x^(3/2) - 104*x^
2 - 6079*x^(5/2) - 3120*x^3 - 194*x^(7/2)) + Sqrt[3]*(-4*Sqrt[-1 + Sqrt[x]
]*(-18816 - 52416*Sqrt[x] - 41472*x - 10928*x^(3/2) - 1192*x^2 + 3832*x^(5
/2) + 3408*x^3 + 656*x^(7/2)) - 4*(10864 - 10872*Sqrt[x] - 41440*x - 23268
*x^(3/2) - 6678*x^2 - 1148*x^(5/2) + 3416*x^3 + 1800*x^(7/2) + 112*x^4)))/
(-12416 + 13312*Sqrt[x] + 49408*x + 24960*x^(3/2) + 1552*x^2 + Sqrt[3]*Sqr
t[1 + Sqrt[x]]*(7168 - 11264*Sqrt[x] - 22016*x - 5248*x^(3/2)) + Sqrt[-1 +
 Sqrt[x]]*(21504 + 60416*Sqrt[x] + 47104*x + 9088*x^(3/2) + Sqrt[3]*Sqrt[1
 + Sqrt[x]]*(-12416 - 28672*Sqrt[x] - 14400*x - 896*x^(3/2)))) + ArcTanh[(
-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

3.1004.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {812, 845, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\,dx$$

$$\Big\downarrow \text{812}$$

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}-\frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{845}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx-\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}$$

$$\Big\downarrow \text{852}$$

$$\frac{1}{4}\left(-\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}-\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}$$

$$\Big\downarrow \text{43}$$

$$\frac{1}{4}\left(-\operatorname{arccosh}(\sqrt{x})-\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}$$

input
```
Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

output
```
(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2))/2 + (-(Sqrt[-1 + Sqrt[x]]*S
qrt[1 + Sqrt[x]]*Sqrt[x]) - ArcCosh[Sqrt[x]])/4
```

3.1004.3.1 Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 812
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Simp[2*a1*a2*n*(p/(m + 2*n*p + 1))   Int[(c*x
)^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a
2, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && N
eQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

3.1004.4 Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-2x^{\frac{3}{2}}\sqrt{-1+x}+\sqrt{x}\,\sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{4\sqrt{-1+x}}$	52
default	$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-2x^{\frac{3}{2}}\sqrt{-1+x}+\sqrt{x}\,\sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{4\sqrt{-1+x}}$	52

input

```
int(x^(1/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/4*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-2*x^(3/2)*(-1+x)^(1/2)+x^(1/2)*
(-1+x)^(1/2)+ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

3.1004.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{1}{4}\,(2\,x-1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{8}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

input

```
integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="frica
s")
```

output

```
1/4*(2*x - 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/8*log(2*sqrt
(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

3.1004.6 Sympy [F]

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \int \sqrt{x}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,dx$$

input

```
integrate(x**(1/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

output

```
Integral(sqrt(x)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

3.1004.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{1}{2}\,(x-1)^{\frac{3}{2}}\sqrt{x} + \frac{1}{4}\,\sqrt{x-1}\sqrt{x} - \frac{1}{4}\,\log\left(2\,\sqrt{x-1} + 2\,\sqrt{x}\right)$$

input
```
integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxim
a")
```

output
```
1/2*(x - 1)^(3/2)*sqrt(x) + 1/4*sqrt(x - 1)*sqrt(x) - 1/4*log(2*sqrt(x - 1
) + 2*sqrt(x))
```

3.1004.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(45) = 90.

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\begin{aligned}&\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx\\&= \frac{1}{12}\left(\left(2\left(3\sqrt{x}-10\right)\left(\sqrt{x}+1\right)+43\right)\left(\sqrt{x}+1\right)-39\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\\&\quad+\frac{1}{3}\left(\left(2\sqrt{x}-5\right)\left(\sqrt{x}+1\right)+9\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{2}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)\end{aligned}$$

input
```
integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac"
)
```

output
```
1/12*((2*(3*sqrt(x) - 10)*(sqrt(x) + 1) + 43)*(sqrt(x) + 1) - 39)*sqrt(sqr
t(x) + 1)*sqrt(sqrt(x) - 1) + 1/3*((2*sqrt(x) - 5)*(sqrt(x) + 1) + 9)*sqrt
(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x)
 - 1))
```

3.1004.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx=\int \sqrt{x}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,dx$$

input

```
int(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

output

```
int(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2), x)
```

3.1005 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$

3.1005.1 Optimal result

Integrand size = 28, antiderivative size = 37

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \operatorname{arccosh}\left(\sqrt{x}\right)$$

output
```
-arccosh(x^(1/2))+x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

3.1005.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 264 vs. $2(37) = 74$.

Time = 1.49 (sec) , antiderivative size = 264, normalized size of antiderivative = 7.14

$$\begin{aligned}&\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx\\ &=4\left(\frac{4\sqrt{1+\sqrt{x}}\left(-12-24\sqrt{x}+x+5x^{3/2}\right)+\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(-84-10\sqrt{x}+28x+7x^{3/2}\right)+\sqrt{3}\left(2\right.}{56-16\sqrt{3}\sqrt{1+\sqrt{x}}\left(2+3\sqrt{x}\right)+\sqrt{-1+\sqrt{x}}\left(96-8\sqrt{3}\sqrt{1+\right.}\right.\\ &\qquad\left.+\operatorname{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)\right)\end{aligned}$$

input
```
Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x],x]
```

output
```
4*((4*Sqrt[1 + Sqrt[x]]*(-12 - 24*Sqrt[x] + x + 5*x^(3/2)) + Sqrt[-1 + Sqr
t[x]]*Sqrt[1 + Sqrt[x]]*(-84 - 10*Sqrt[x] + 28*x + 7*x^(3/2)) + Sqrt[3]*(2
8 + 70*Sqrt[x] + 18*x - 14*x^(3/2) - 4*x^2 - 4*Sqrt[-1 + Sqrt[x]]*(-12 - 8
*Sqrt[x] + 5*x + 3*x^(3/2))))/(56 - 16*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(2 + 3*Sq
rt[x]) + Sqrt[-1 + Sqrt[x]]*(96 - 8*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(7 + 2*Sqrt[
x]) + 80*Sqrt[x]) + 112*Sqrt[x] + 28*x) + ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]]
)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])])
```

3.1005.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = $\{812, 852, 43\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{812}$$

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx$$

$$\Big\downarrow \text{852}$$

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}$$

$$\Big\downarrow \text{43}$$

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\operatorname{arccosh}\left(\sqrt{x}\right)$$

input
```
Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x],x]
```

output
```
Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - ArcCosh[Sqrt[x]]
```

3.1005.3.1 Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 812
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Simp[2*a1*a2*n*(p/(m + 2*n*p + 1))   Int[(c*x
)^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a
2, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && N
eQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

3.1005.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(25) = 50$.

Time = 4.59 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.95

method	result	size
derivativedivides	$\sqrt{\sqrt{x}-1}\,\left(\sqrt{x}+1\right)^{\frac{3}{2}}-\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}-\frac{\sqrt{\left(\sqrt{x}+1\right)\left(\sqrt{x}-1\right)}\,\ln\left(\sqrt{x}+\sqrt{-1+x}\right)}{\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}$	72
default	$\sqrt{\sqrt{x}-1}\,\left(\sqrt{x}+1\right)^{\frac{3}{2}}-\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}-\frac{\sqrt{\left(\sqrt{x}+1\right)\left(\sqrt{x}-1\right)}\,\ln\left(\sqrt{x}+\sqrt{-1+x}\right)}{\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}$	72

input
```
int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(3/2)-(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)-((
x^(1/2)+1)*(x^(1/2)-1))^(1/2)/(x^(1/2)+1)^(1/2)/(x^(1/2)-1)^(1/2)*ln(x^(1/
2)+(-1+x)^(1/2))
```

3.1005.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{2}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - 2x + 1\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="frica
s")
```

output
```
sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/2*log(2*sqrt(x)*sqrt(sqrt(
x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

3.1005.6 Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = 4\sqrt{\sqrt{x}-1}\left(\frac{(\sqrt{x}+1)^{\frac{3}{2}}}{4} - \frac{\sqrt{\sqrt{x}+1}}{4}\right) - 2\log\left(2\sqrt{\sqrt{x}-1} + 2\sqrt{\sqrt{x}+1}\right)$$

input
```
integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(1/2),x)
```

output
```
4*sqrt(sqrt(x) - 1)*((sqrt(x) + 1)**(3/2)/4 - sqrt(sqrt(x) + 1)/4) - 2*log
(2*sqrt(sqrt(x) - 1) + 2*sqrt(sqrt(x) + 1))
```

3.1005.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{x-1}\sqrt{x} - \log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="maxim
a")
```

output
```
sqrt(x - 1)*sqrt(x) - log(2*sqrt(x - 1) + 2*sqrt(x))
```

3.1005.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}(\sqrt{x}-2) + 2\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + 2\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="giac"
)
```

output
```
sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*(sqrt(x) - 2) + 2*sqrt(sqrt(x) + 1)*sq
rt(sqrt(x) - 1) + 2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

3.1005.9 Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1} - \ln\left(\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}+\sqrt{x}\right)$$

input
```
int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(1/2),x)
```

output
```
x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2) - log((x^(1/2) - 1)^(1/2)*
(x^(1/2) + 1)^(1/2) + x^(1/2))
```

3.1006 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx$

3.1006.1 Optimal result

Integrand size = 28, antiderivative size = 67

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx=\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}}-2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+2\text{arccosh}\left(\sqrt{x}\right)$$

output
```
2*arccosh(x^(1/2))+2*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(1/2)-2*x^(1/2
)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

3.1006.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 184 vs. 2(67) = 134.

Time = 1.19 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.75

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx=\frac{\Big(-1+\sqrt{-1+\sqrt{x}}\Big)\Big(\sqrt{3}-\sqrt{1+\sqrt{x}}\Big)\Big(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+\sqrt{x}}-}{\Big(-3-2\sqrt{-1+\sqrt{x}}+2\sqrt{3}\sqrt{1+\sqrt{x}}+\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}-2\sqrt{x}\Big)}-8\text{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

input
```
Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]
```

output
```
((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + S
qrt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x]))/((-3 - 2*Sqrt[-1 + Sqrt[x]
] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt
[x]] - 2*Sqrt[x])*Sqrt[x]) - 8*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3]
- Sqrt[1 + Sqrt[x]])]
```

3.1006.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {849, 812, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{3/2}}dx$$

$$\Big\downarrow \text{849}$$

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}}-2\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{812}$$

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}}-2\left(\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx\right)$$

$$\Big\downarrow \text{852}$$

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}}-2\left(\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{43}$$

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}}-2\left(\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\operatorname{arccosh}\left(\sqrt{x}\right)\right)$$

input
```
Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]
```

output
```
(2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/Sqrt[x] - 2*(Sqrt[-1 + Sqrt[x
]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - ArcCosh[Sqrt[x]])
```

3.1006.3.1 Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 812
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Simp[2*a1*a2*n*(p/(m + 2*n*p + 1))   Int[(c*x
)^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a
2, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && N
eQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 849
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2
*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a
1*a2*c^(2*n)*(m + 1)))   Int[(c*x)^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^
p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && I
GtQ[2*n, 0] && LtQ[m, -1] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

3.1006.4 Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\sqrt{x}-\sqrt{-1+x}\right)}{\sqrt{x}\,\sqrt{-1+x}}$	47
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\sqrt{x}-\sqrt{-1+x}\right)}{\sqrt{x}\,\sqrt{-1+x}}$	47

input
```
int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(ln(x^(1/2)+(-1+x)^(1/2))*x^(1/2)-(-
1+x)^(1/2))/x^(1/2)/(-1+x)^(1/2)
```

3.1006.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = -\frac{x\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)+2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2x}{x}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="frica
s")
```

output
```
-(x*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1) + 2*sqrt(
x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*x)/x
```

3.1006.6 Sympy [F]

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = \int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(3/2),x)
```

output
```
Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(3/2), x)
```

3.1006.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = -\frac{2\sqrt{x-1}}{\sqrt{x}} + 2\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="maxim
a")
```

output
```
-2*sqrt(x - 1)/sqrt(x) + 2*log(2*sqrt(x - 1) + 2*sqrt(x))
```

3.1006.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="giac"
)
```

output
```
Timed out
```

3.1006.9 Mupad [B] (verification not implemented)

Time = 10.55 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.93

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = 8\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right) - \frac{\frac{5\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^2}{2\left(\sqrt{\sqrt{x}+1}-1\right)^2}+\frac{1}{2}}{\frac{\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}} - \frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{2\left(\sqrt{\sqrt{x}+1}-1\right)}$$

input
```
int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(3/2),x)
```

output
```
8*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)) - ((5*((x^(1
/2) - 1)^(1/2) - 1i)^2)/(2*((x^(1/2) + 1)^(1/2) - 1)^2) + 1/2)/((((x^(1/2)
- 1)^(1/2) - 1i)^3/((x^(1/2) + 1)^(1/2) - 1)^3 + ((x^(1/2) - 1)^(1/2) - 1i
)/((x^(1/2) + 1)^(1/2) - 1)) - ((x^(1/2) - 1)^(1/2) - 1i)/(2*((x^(1/2) + 1
)^(1/2) - 1))
```

3.1006. $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx$

3.1007 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx$

3.1007.1 Optimal result

Integrand size = 28, antiderivative size = 31

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{3x^{3/2}}$$

output
```
2/3*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(3/2)
```

3.1007.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 421 vs. 2(31) = 62.

Time = 2.51 (sec) , antiderivative size = 421, normalized size of antiderivative = 13.58

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+\sqrt{x}}-\right.}{}$$

input
```
Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]
```

output
```
((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + S
qrt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x])*(8*(7 + 12*Sqrt[-1 + Sqrt[x
]] - 4*Sqrt[3]*Sqrt[1 + Sqrt[x]] - 7*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + S
qrt[x]]) + 4*(49 + 8*Sqrt[-1 + Sqrt[x]] - 24*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 3
*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*Sqrt[x] + 2*(-23 - 144*Sqrt
[-1 + Sqrt[x]] + 32*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 77*Sqrt[3]*Sqrt[-1 + Sqrt[
x]]*Sqrt[1 + Sqrt[x]])*x + 2*(-140 - 106*Sqrt[-1 + Sqrt[x]] + 62*Sqrt[3]*S
qrt[1 + Sqrt[x]] + 21*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x^(3/2
) - 73*x^2))/(12*(-3 - 2*Sqrt[-1 + Sqrt[x]] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]]
+ Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]] - 2*Sqrt[x])^3*x^(3/2))
```

3.1007.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{797\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{797}$$

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{3x^{3/2}}$$

input
```
Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]
```

output
```
(2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(3*x^(3/2))
```

3.1007.3.1 Defintions of rubi rules used

rule 797
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

3.1007.4 Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)}{3x^{\frac{3}{2}}}$	23
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)}{3x^{\frac{3}{2}}}$	23

input
```
int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-1+x)/x^(3/2)
```

3.1007.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{2\left((x-1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+x^2\right)}{3\,x^2}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="frica
s")
```

output
```
2/3*((x - 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + x^2)/x^2
```

3.1007.6 Sympy [F]

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(5/2),x)
```

output
```
Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(5/2), x)
```

3.1007.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{2\,(x-1)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="maxim
a")
```

output
```
2/3*(x - 1)^(3/2)/x^(3/2)
```

3.1007.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(19) = 38.

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{16\left(3\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+16\right)}{3\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{3}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="giac"
)
```

output
```
16/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 16)/((sqrt(sqrt(x) + 1
) - sqrt(sqrt(x) - 1))^4 + 4)^3
```

3.1007.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{3}-\frac{2\,\sqrt{\sqrt{x}+1}}{3}\right)}{x^{3/2}}$$

input
```
int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(5/2),x)
```

output
```
((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/3 - (2*(x^(1/2) + 1)^(1/2)
)/3))/x^(3/2)
```

3.1007. $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx$

3.1008 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}} dx$

3.1008.1 Optimal result

Integrand size = 28, antiderivative size = 63

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}} dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{5x^{5/2}} + \frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{15x^{3/2}}$$

output
```
2/5*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(5/2)+4/15*(-1+x^(1/2))^(3/2)*(
1+x^(1/2))^(3/2)/x^(3/2)
```

3.1008.2 Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}} dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}(3+2x)}{15x^{5/2}}$$

input
```
Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2),x]
```

output
```
(2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(3 + 2*x))/(15*x^(5/2))
```

3.1008.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {804, 797}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{7/2}}dx$$

$$\Big\downarrow \text{804}$$

$$\frac{2}{5}\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{5/2}}dx+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}$$

$$\Big\downarrow \text{797}$$

$$\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{15x^{3/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}$$

input
```
Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2),x]
```

output
```
(2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(5*x^(5/2)) + (4*(-1 + Sqrt[x
])^(3/2)*(1 + Sqrt[x])^(3/2))/(15*x^(3/2))
```

3.1008.3.1 Defintions of rubi rules used

rule 797
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

rule 804
```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
   Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1
, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2
*n) + p + 1], 0] && NeQ[m, -1]
```

3.1008.4 Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.44

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)(2x+3)}{15x^{\frac{5}{2}}}$	28
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)(2x+3)}{15x^{\frac{5}{2}}}$	28

input
```
int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-1+x)*(2*x+3)/x^(5/2)
```

3.1008.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \frac{2\left(2\,x^3+(2\,x^2+x-3)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{15\,x^3}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="frica
s")
```

output
```
2/15*(2*x^3 + (2*x^2 + x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1))
/x^3
```

3.1008.6 Sympy [F]

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(7/2),x)
```

output
```
Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(7/2), x)
```

3.1008.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \frac{4\,(x-1)^{\frac{3}{2}}}{15\,x^{\frac{3}{2}}} + \frac{2\,(x-1)^{\frac{3}{2}}}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="maxim
a")
```

output
```
4/15*(x - 1)^(3/2)/x^(3/2) + 2/5*(x - 1)^(3/2)/x^(5/2)
```

3.1008.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(39) = 78.

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \frac{128\,\Big(15\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12} - 20\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8} + 80\,\big(\sqrt{}}{15\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{5}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="giac"
)
```

output
```
128/15*(15*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^12 - 20*(sqrt(sqrt(x) +
 1) - sqrt(sqrt(x) - 1))^8 + 80*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4
+ 64)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)^5
```

3.1008.9 Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\,\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{15}-\frac{2\,\sqrt{\sqrt{x}+1}}{5}+\frac{4\,x^2\,\sqrt{\sqrt{x}+1}}{15}\right)}{x^{5/2}}$$

input
```
int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(7/2),x)
```

output
```
((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/15 - (2*(x^(1/2) + 1)^(1/2
))/5 + (4*x^2*(x^(1/2) + 1)^(1/2))/15))/x^(5/2)
```

3.1008. $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx$

3.1009 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}} \, dx$

3.1009.1 Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}} \, dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}} + \frac{8\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{35x^{5/2}} + \frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{105x^{3/2}}$$

output
```
2/7*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(7/2)+8/35*(-1+x^(1/2))^(3/2)*(
1+x^(1/2))^(3/2)/x^(5/2)+16/105*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(3/
2)
```

3.1009.2 Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}} \, dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}(15+12x+8x^2)}{105x^{7/2}}$$

input
```
Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2),x]
```

output
```
(2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(15 + 12*x + 8*x^2))/(105*x^(7
/2))
```

3.1009.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {804, 804, 797}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{9/2}}dx$$

$$\Big\downarrow 804$$

$$\frac{4}{7}\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{7/2}}dx+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}$$

$$\Big\downarrow 804$$

$$\frac{4}{7}\left(\frac{2}{5}\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{5/2}}dx+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}\right)+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}$$

$$\Big\downarrow 797$$

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}+\frac{4}{7}\left(\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{15x^{3/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}\right)$$

input
```
Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2),x]
```

output
```
(4*((2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(5*x^(5/2)) + (4*(-1 + Sq
rt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(15*x^(3/2))))/7 + (2*(-1 + Sqrt[x])^(3/
2)*(1 + Sqrt[x])^(3/2))/(7*x^(7/2))
```

3.1009.3.1 Defintions of rubi rules used

rule 797
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

rule 804
```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
   Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1
, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2
*n) + p + 1], 0] && NeQ[m, -1]
```

3.1009.4 Maple [A] (verified)

Time = 4.61 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.35

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)\left(8x^{2}+12x+15\right)}{105x^{\frac{7}{2}}}$	33
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)\left(8x^{2}+12x+15\right)}{105x^{\frac{7}{2}}}$	33

input
```
int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-1+x)*(8*x^2+12*x+15)/x^(7/2)
```

3.1009.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{2\left(8\,x^{4}+\left(8\,x^{3}+4\,x^{2}+3\,x-15\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{105\,x^{4}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="frica
s")
```

output
```
2/105*(8*x^4 + (8*x^3 + 4*x^2 + 3*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(s
qrt(x) - 1))/x^4
```

3.1009.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(9/2),x)
```

output
```
Timed out
```

3.1009.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{16\,(x-1)^{\frac{3}{2}}}{105\,x^{\frac{3}{2}}} + \frac{8\,(x-1)^{\frac{3}{2}}}{35\,x^{\frac{5}{2}}} + \frac{2\,(x-1)^{\frac{3}{2}}}{7\,x^{\frac{7}{2}}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="maxim
a")
```

output
```
16/105*(x - 1)^(3/2)/x^(3/2) + 8/35*(x - 1)^(3/2)/x^(5/2) + 2/7*(x - 1)^(3
/2)/x^(7/2)
```

3.1009.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{4096\left(35\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{16}-70\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}+168\right.}{105\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\right.\right.}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="giac"
)
```

output
```
4096/105*(35*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^16 - 70*(sqrt(sqrt(x)
 + 1) - sqrt(sqrt(x) - 1))^12 + 168*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1)
)^8 + 224*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 128)/((sqrt(sqrt(x)
+ 1) - sqrt(sqrt(x) - 1))^4 + 4)^7
```

3.1009.9 Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{35}-\frac{2\,\sqrt{\sqrt{x}+1}}{7}+\frac{8\,x^2\,\sqrt{\sqrt{x}+1}}{105}+\frac{16\,x^3\,\sqrt{\sqrt{x}+1}}{105}\right)}{x^{7/2}}$$

input

```
int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(9/2),x)
```

output

```
((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/35 - (2*(x^(1/2) + 1)^(1/2
))/7 + (8*x^2*(x^(1/2) + 1)^(1/2))/105 + (16*x^3*(x^(1/2) + 1)^(1/2))/105)
)/x^(7/2)
```

3.1010 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx$

3.1010.1 Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}} + \frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}} + \frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{105x^{5/2}} + \frac{32\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{315x^{3/2}}$$

output
```
2/9*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(9/2)+4/21*(-1+x^(1/2))^(3/2)*(
1+x^(1/2))^(3/2)/x^(7/2)+16/105*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(5/
2)+32/315*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(3/2)
```

3.1010.2 Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}\left(35+30x+24x^2+16x^3\right)}{315x^{9/2}}$$

input
```
Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2),x]
```

output
```
(2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(35 + 30*x + 24*x^2 + 16*x^3))
/(315*x^(9/2))
```

3.1010.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {804, 804, 804, 797}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{11/2}}dx$$

$$\Big\downarrow \text{804}$$

$$\frac{2}{3}\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{9/2}}dx+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}$$

$$\Big\downarrow \text{804}$$

$$\frac{2}{3}\left(\frac{4}{7}\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{7/2}}dx+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}\right)+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}$$

$$\Big\downarrow \text{804}$$

$$\frac{2}{3}\left(\frac{4}{7}\left(\frac{2}{5}\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{5/2}}dx+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}\right)+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}\right)+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}$$

$$\Big\downarrow \text{797}$$

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}+\frac{2}{3}\left(\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}+\frac{4}{7}\left(\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{15x^{3/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}\right)\right)$$

input
```
Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2),x]
```

output
```
(2*((4*((2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(5*x^(5/2)) + (4*(-1
+ Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(15*x^(3/2))))/7 + (2*(-1 + Sqrt[x])
^(3/2)*(1 + Sqrt[x])^(3/2))/(7*x^(7/2))))/3 + (2*(-1 + Sqrt[x])^(3/2)*(1 +
 Sqrt[x])^(3/2))/(9*x^(9/2))
```

3.1010.3.1 Defintions of rubi rules used

rule 797
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

rule 804
```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
   Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1
, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2
*n) + p + 1], 0] && NeQ[m, -1]
```

3.1010.4 Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.30

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)\left(16x^{3}+24x^{2}+30x+35\right)}{315x^{\frac{9}{2}}}$	38
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)\left(16x^{3}+24x^{2}+30x+35\right)}{315x^{\frac{9}{2}}}$	38

input
```
int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-1+x)*(16*x^3+24*x^2+30*x+35)/x
^(9/2)
```

3.1010.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{2\left(16\,x^{5} + (16\,x^{4} + 8\,x^{3} + 6\,x^{2} + 5\,x - 35)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{315\,x^{5}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="fric
as")
```

output
```
2/315*(16*x^5 + (16*x^4 + 8*x^3 + 6*x^2 + 5*x - 35)*sqrt(x)*sqrt(sqrt(x) +
 1)*sqrt(sqrt(x) - 1))/x^5
```

3.1010.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx=\text{Timed out}$$

input
```
integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(11/2),x)
```

output
```
Timed out
```

3.1010.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx=\frac{32\,(x-1)^{\frac{3}{2}}}{315\,x^{\frac{3}{2}}}+\frac{16\,(x-1)^{\frac{3}{2}}}{105\,x^{\frac{5}{2}}}+\frac{4\,(x-1)^{\frac{3}{2}}}{21\,x^{\frac{7}{2}}}+\frac{2\,(x-1)^{\frac{3}{2}}}{9\,x^{\frac{9}{2}}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="maxi
ma")
```

output
```
32/315*(x - 1)^(3/2)/x^(3/2) + 16/105*(x - 1)^(3/2)/x^(5/2) + 4/21*(x - 1)
^(3/2)/x^(7/2) + 2/9*(x - 1)^(3/2)/x^(9/2)
```

3.1010.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{16384\left(315\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{20}-756\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{16}+\right.}{31}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="giac
")
```

output
```
16384/315*(315*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^20 - 756*(sqrt(sqrt
(x) + 1) - sqrt(sqrt(x) - 1))^16 + 1344*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x)
- 1))^12 + 2304*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 2304*(sqrt(sqr
t(x) + 1) - sqrt(sqrt(x) - 1))^4 + 1024)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x
) - 1))^4 + 4)^9
```

3.1010.9 Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{63}-\frac{2\,\sqrt{\sqrt{x}+1}}{9}+\frac{4\,x^2\,\sqrt{\sqrt{x}+1}}{105}+\frac{16\,x^3\,\sqrt{\sqrt{x}+1}}{315}+\frac{32\,x^4\,\sqrt{\sqrt{x}+1}}{315}\right)}{x^{9/2}}$$

input
```
int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(11/2),x)
```

output
```
((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/63 - (2*(x^(1/2) + 1)^(1/2
))/9 + (4*x^2*(x^(1/2) + 1)^(1/2))/105 + (16*x^3*(x^(1/2) + 1)^(1/2))/315
+ (32*x^4*(x^(1/2) + 1)^(1/2))/315))/x^(9/2)
```

3.1011 $\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$

3.1011.1 Optimal result

Integrand size = 28, antiderivative size = 104

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{5}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{5}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{5\text{arccosh}\left(\sqrt{x}\right)}{8}$$

output
```
5/8*arccosh(x^(1/2))+5/12*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+1/3
*x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+5/8*x^(1/2)*(-1+x^(1/2))^(1/
2)*(1+x^(1/2))^(1/2)
```

3.1011.2 Mathematica [A] (warning: unable to verify)

Time = 1.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{24}\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}\left(15+15\sqrt{x} + 10x+10x^{3/2}+8x^2+8x^{5/2}\right) + \frac{5}{4}\text{arctanh}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\right)$$

input
```
Integrate[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

output
```
(Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]*(15 + 15*Sqrt[x] + 10*x + 10*x
^(3/2) + 8*x^2 + 8*x^(5/2)))/24 + (5*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt
[x])]])/4
```

3.1011.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {845, 845, 845, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{845}$$

$$\frac{5}{6}\int \frac{x^{3/2}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

$$\Big\downarrow \text{845}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

$$\Big\downarrow \text{845}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

$$\Big\downarrow \text{852}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

$$\Big\downarrow \text{43}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\text{arccosh}\left(\sqrt{x}\right)+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

input
```
Int[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

output
```
(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2))/3 + (5*((Sqrt[-1 + Sqrt[x]]
*Sqrt[1 + Sqrt[x]]*x^(3/2))/2 + (3*(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*S
qrt[x] + ArcCosh[Sqrt[x]]))/4))/6
```

3.1011.3.1 Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

3.1011.4 Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(8x^{\frac{5}{2}}\sqrt{-1+x}+10x^{\frac{3}{2}}\sqrt{-1+x}+15\sqrt{x}\,\sqrt{-1+x}+15\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{24\sqrt{-1+x}}$	65
default	$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(8x^{\frac{5}{2}}\sqrt{-1+x}+10x^{\frac{3}{2}}\sqrt{-1+x}+15\sqrt{x}\,\sqrt{-1+x}+15\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{24\sqrt{-1+x}}$	65

input
```
int(x^(5/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(8*x^(5/2)*(-1+x)^(1/2)+10*x^(3/2
)*(-1+x)^(1/2)+15*x^(1/2)*(-1+x)^(1/2)+15*ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)
^(1/2)
```

3.1011.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.55

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{24}\left(8\,x^2+10\,x+15\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - \frac{5}{16}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

input
```
integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="frica
s")
```

output
```
1/24*(8*x^2 + 10*x + 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 5/1
6*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

3.1011.6 Sympy [F]

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

input
```
integrate(x**(5/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

output
```
Integral(x**(5/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

3.1011.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.45

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{3}\,\sqrt{x-1}x^{\frac{5}{2}} + \frac{5}{12}\,\sqrt{x-1}x^{\frac{3}{2}} + \frac{5}{8}\,\sqrt{x-1}\sqrt{x} + \frac{5}{8}\,\log\left(2\,\sqrt{x-1} + 2\,\sqrt{x}\right)$$

input
```
integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxim
a")
```

output
```
1/3*sqrt(x - 1)*x^(5/2) + 5/12*sqrt(x - 1)*x^(3/2) + 5/8*sqrt(x - 1)*sqrt(
x) + 5/8*log(2*sqrt(x - 1) + 2*sqrt(x))
```

3.1011.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.73

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{24}\,((2\,((4\,(\sqrt{x}+1)\,(\sqrt{x}-4)+45)\,(\sqrt{x}+1)-55)\,(\sqrt{x}+1)+85)\,(\sqrt{x}+1$$

$$-\frac{5}{4}\,\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

input
```
integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac"
)
```

output
```
1/24*((2*((4*(sqrt(x) + 1)*(sqrt(x) - 4) + 45)*(sqrt(x) + 1) - 55)*(sqrt(x
) + 1) + 85)*(sqrt(x) + 1) - 33)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 5/4
*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

3.1011.9 Mupad [B] (verification not implemented)

Time = 38.44 (sec) , antiderivative size = 632, normalized size of antiderivative = 6.08

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{5\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right)}{2} - \frac{-\frac{175\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{6\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{311\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^5}{2\left(\sqrt{\sqrt{x}+1}-1\right)^5}+\frac{8361\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^7}{2\left(\sqrt{\sqrt{x}+1}-1\right)^7}+\frac{42259\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^9}{3\left(\sqrt{\sqrt{x}+1}-1\right)^9}+\frac{25295\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}}+\frac{25295\left(\sqrt{\sqrt{x}-1}\right.}{\left(\sqrt{\sqrt{x}+1}-1\right)}}{1+\frac{66\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4}-\frac{220\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6}+\frac{495\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8}-\frac{792\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}}+\frac{924\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}}-\frac{792\left(\sqrt{\sqrt{x}-1}\right.}{\left(\sqrt{\sqrt{x}+1}-1\right.}}$$

input
```
int(x^(5/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

output
```
(5*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)))/2 - ((311*
((x^(1/2) - 1)^(1/2) - 1i)^5)/(2*((x^(1/2) + 1)^(1/2) - 1)^5) - (175*((x^(
1/2) - 1)^(1/2) - 1i)^3)/(6*((x^(1/2) + 1)^(1/2) - 1)^3) + (8361*((x^(1/2)
 - 1)^(1/2) - 1i)^7)/(2*((x^(1/2) + 1)^(1/2) - 1)^7) + (42259*((x^(1/2) -
1)^(1/2) - 1i)^9)/(3*((x^(1/2) + 1)^(1/2) - 1)^9) + (25295*((x^(1/2) - 1)^
(1/2) - 1i)^11)/((x^(1/2) + 1)^(1/2) - 1)^11 + (25295*((x^(1/2) - 1)^(1/2)
 - 1i)^13)/((x^(1/2) + 1)^(1/2) - 1)^13 + (42259*((x^(1/2) - 1)^(1/2) - 1i
)^15)/(3*((x^(1/2) + 1)^(1/2) - 1)^15) + (8361*((x^(1/2) - 1)^(1/2) - 1i)^
17)/(2*((x^(1/2) + 1)^(1/2) - 1)^17) + (311*((x^(1/2) - 1)^(1/2) - 1i)^19)
/(2*((x^(1/2) + 1)^(1/2) - 1)^19) - (175*((x^(1/2) - 1)^(1/2) - 1i)^21)/(6
*((x^(1/2) + 1)^(1/2) - 1)^21) + (5*((x^(1/2) - 1)^(1/2) - 1i)^23)/(2*((x^
(1/2) + 1)^(1/2) - 1)^23) + (5*((x^(1/2) - 1)^(1/2) - 1i))/(2*((x^(1/2) +
1)^(1/2) - 1)))/((66*((x^(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) -
1)^4 - (12*((x^(1/2) - 1)^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (22
0*((x^(1/2) - 1)^(1/2) - 1i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (495*((x^(1/
2) - 1)^(1/2) - 1i)^8)/((x^(1/2) + 1)^(1/2) - 1)^8 - (792*((x^(1/2) - 1)^(
1/2) - 1i)^10)/((x^(1/2) + 1)^(1/2) - 1)^10 + (924*((x^(1/2) - 1)^(1/2) -
1i)^12)/((x^(1/2) + 1)^(1/2) - 1)^12 - (792*((x^(1/2) - 1)^(1/2) - 1i)^14)
/((x^(1/2) + 1)^(1/2) - 1)^14 + (495*((x^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1
/2) + 1)^(1/2) - 1)^16 - (220*((x^(1/2) - 1)^(1/2) - 1i)^18)/((x^(1/2) ...
```

3.1012 $\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$

3.1012.1 Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{3}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{3\operatorname{arccosh}\left(\sqrt{x}\right)}{4}$$

output
```
3/4*arccosh(x^(1/2))+1/2*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+3/4*
x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

3.1012.2 Mathematica [A] (warning: unable to verify)

Time = 1.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{4}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}\left(3+3\sqrt{x}+2x+2x^{3/2}\right)+6\operatorname{arctanh}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\right)\right)$$

input
```
Integrate[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

output
```
(Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]*(3 + 3*Sqrt[x] + 2*x + 2*x^(3/
2)) + 6*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/4
```

3.1012.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {845, 845, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{ 845}$$

$$\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}$$

$$\Big\downarrow \text{ 845}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}$$

$$\Big\downarrow \text{ 852}$$

$$\frac{3}{4}\left(\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}$$

$$\Big\downarrow \text{ 43}$$

$$\frac{3}{4}\left(\text{arccosh}\left(\sqrt{x}\right)+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}$$

input
```
Int[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

output
```
(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2))/2 + (3*(Sqrt[-1 + Sqrt[x]]*
Sqrt[1 + Sqrt[x]]*Sqrt[x] + ArcCosh[Sqrt[x]]))/4
```

3.1012.3.1 Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

3.1012.4 Maple [A] (verified)

Time = 4.75 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(2x^{\frac{3}{2}}\sqrt{-1+x}+3\sqrt{x}\,\sqrt{-1+x}+3\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{4\sqrt{-1+x}}$	55
default	$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(2x^{\frac{3}{2}}\sqrt{-1+x}+3\sqrt{x}\,\sqrt{-1+x}+3\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{4\sqrt{-1+x}}$	55

input
```
int(x^(3/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(2*x^(3/2)*(-1+x)^(1/2)+3*x^(1/2)*
(-1+x)^(1/2)+3*ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

3.1012.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{4}\,(2\,x+3)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - \frac{3}{8}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

input
```
integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="frica
s")
```

output
```
1/4*(2*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 3/8*log(2*sqrt
(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

3.1012.6 Sympy [F]

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

input
```
integrate(x**(3/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

output
```
Integral(x**(3/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

3.1012.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{2}\,\sqrt{x-1}x^{\frac{3}{2}} + \frac{3}{4}\,\sqrt{x-1}\sqrt{x} + \frac{3}{4}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

input
```
integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxim
a")
```

output
```
1/2*sqrt(x - 1)*x^(3/2) + 3/4*sqrt(x - 1)*sqrt(x) + 3/4*log(2*sqrt(x - 1)
+ 2*sqrt(x))
```

3.1012.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{4}\left(\left(2\left(\sqrt{x}+1\right)\left(\sqrt{x}-2\right)+9\right)\left(\sqrt{x}+1\right)-5\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - \frac{3}{2}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

input
```
integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac"
)
```

output
```
1/4*((2*(sqrt(x) + 1)*(sqrt(x) - 2) + 9)*(sqrt(x) + 1) - 5)*sqrt(sqrt(x) +
 1)*sqrt(sqrt(x) - 1) - 3/2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

3.1012.9 Mupad [B] (verification not implemented)

Time = 26.04 (sec) , antiderivative size = 429, normalized size of antiderivative = 5.88

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = 3\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right) + \frac{\frac{23\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{333\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\sqrt{x}+1}-1\right)^5}+\frac{671\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\sqrt{x}+1}-1\right)^7}+\frac{671\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^9}{\left(\sqrt{\sqrt{x}+1}-1\right)^9}+\frac{333\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}}+\frac{23\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{13}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{13}}-3}{1+\frac{28\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4}-\frac{56\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6}+\frac{70\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8}-\frac{56\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}}+\frac{28\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}}-\frac{8\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{14}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{14}}+}$$

input
```
int(x^(3/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

output

```
3*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)) + ((23*((x^(
1/2) - 1)^(1/2) - 1i)^3)/((x^(1/2) + 1)^(1/2) - 1)^3 + (333*((x^(1/2) - 1)
^(1/2) - 1i)^5)/((x^(1/2) + 1)^(1/2) - 1)^5 + (671*((x^(1/2) - 1)^(1/2) -
1i)^7)/((x^(1/2) + 1)^(1/2) - 1)^7 + (671*((x^(1/2) - 1)^(1/2) - 1i)^9)/((
x^(1/2) + 1)^(1/2) - 1)^9 + (333*((x^(1/2) - 1)^(1/2) - 1i)^11)/((x^(1/2)
+ 1)^(1/2) - 1)^11 + (23*((x^(1/2) - 1)^(1/2) - 1i)^13)/((x^(1/2) + 1)^(1/
2) - 1)^13 - (3*((x^(1/2) - 1)^(1/2) - 1i)^15)/((x^(1/2) + 1)^(1/2) - 1)^1
5 - (3*((x^(1/2) - 1)^(1/2) - 1i))/((x^(1/2) + 1)^(1/2) - 1))/((28*((x^(1/
2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (8*((x^(1/2) - 1)^(1/
2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (56*((x^(1/2) - 1)^(1/2) - 1i)^6
)/((x^(1/2) + 1)^(1/2) - 1)^6 + (70*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^(1/2
) + 1)^(1/2) - 1)^8 - (56*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1)^(1
/2) - 1)^10 + (28*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2) - 1)
^12 - (8*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14 + ((x
^(1/2) - 1)^(1/2) - 1i)^16/((x^(1/2) + 1)^(1/2) - 1)^16 + 1)
```

3.1012. $\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$

3.1013 $\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$

3.1013.1 Optimal result

Integrand size = 28, antiderivative size = 35

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx=\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\operatorname{arccosh}\left(\sqrt{x}\right)$$

output
```
arccosh(x^(1/2))+x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

3.1013.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 265 vs. 2(35) = 70.

Time = 1.52 (sec) , antiderivative size = 265, normalized size of antiderivative = 7.57

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$$

$$=\frac{4\Bigl(4\sqrt{1+\sqrt{x}}\left(-12-24\sqrt{x}+x+5x^{3/2}\right)+\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(-84-10\sqrt{x}+28x+7x^{3/2}\right)+\sqrt{3}\Bigl(28}{56-16\sqrt{3}\sqrt{1+\sqrt{x}}\left(2+3\sqrt{x}\right)+\sqrt{-1+\sqrt{x}}\Bigl(96-8\sqrt{3}\sqrt{1+}$$

$$-4\operatorname{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

input
```
Integrate[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

output
```
(4*(4*Sqrt[1 + Sqrt[x]]*(-12 - 24*Sqrt[x] + x + 5*x^(3/2)) + Sqrt[-1 + Sqr
t[x]]*Sqrt[1 + Sqrt[x]]*(-84 - 10*Sqrt[x] + 28*x + 7*x^(3/2)) + Sqrt[3]*(2
8 + 70*Sqrt[x] + 18*x - 14*x^(3/2) - 4*x^2 - 4*Sqrt[-1 + Sqrt[x]]*(-12 - 8
*Sqrt[x] + 5*x + 3*x^(3/2)))))/(56 - 16*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(2 + 3*S
qrt[x]) + Sqrt[-1 + Sqrt[x]]*(96 - 8*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(7 + 2*Sqrt
[x]) + 80*Sqrt[x]) + 112*Sqrt[x] + 28*x) - 4*ArcTanh[(-1 + Sqrt[-1 + Sqrt[
x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

3.1013.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {845, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{845}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}$$

$$\Big\downarrow \text{852}$$

$$\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}$$

$$\Big\downarrow \text{43}$$

$$\text{arccosh}\left(\sqrt{x}\right)+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}$$

input
```
Int[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

output
```
Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + ArcCosh[Sqrt[x]]
```

3.1013.3.1 Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

3.1013.4 Maple [A] (verified)

Time = 4.61 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(\sqrt{x}\,\sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{\sqrt{-1+x}}$	41
default	$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(\sqrt{x}\,\sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{\sqrt{-1+x}}$	41

input
```
int(x^(1/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(x^(1/2)*(-1+x)^(1/2)+ln(x^(1/2)+(-1+x
)^(1/2)))/(-1+x)^(1/2)
```

3.1013.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - \frac{1}{2}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

input
```
integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="frica
s")
```

output
```
sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 1/2*log(2*sqrt(x)*sqrt(sqrt(
x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

3.1013.6 Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

input
```
integrate(x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(x)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

3.1013.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \sqrt{x-1}\sqrt{x} + \log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

input
```
integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxim
a")
```

output
```
sqrt(x - 1)*sqrt(x) + log(2*sqrt(x - 1) + 2*sqrt(x))
```

3.1013.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

input
```
integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac"
)
```

output
```
sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*log(sqrt(sqrt(x) + 1) - sq
rt(sqrt(x) - 1))
```

3.1013.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

input
```
int(x^(1/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

output
```
int(x^(1/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)), x)
```

3.1014 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx$

3.1014.1 Optimal result

Integrand size = 28, antiderivative size = 8

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = 2\operatorname{arccosh}\left(\sqrt{x}\right)$$

output
```
2*arccosh(x^(1/2))
```

3.1014.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(8) = 16.

Time = 1.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 4.75

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = -8\operatorname{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

input
```
Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]
```

output
```
-8*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

3.1014.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx$$

$$\Big\downarrow \textcolor{blue}{852}$$

$$2\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}$$

$$\Big\downarrow \textcolor{blue}{43}$$

$$2\text{arccosh}\left(\sqrt{x}\right)$$

input

```
Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]
```

output

```
2*ArcCosh[Sqrt[x]]
```

3.1014.3.1 Defintions of rubi rules used

rule 43

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 852

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

3.1014.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(6) = 12$.

Time = 4.59 (sec) , antiderivative size = 40, normalized size of antiderivative = 5.00

method	result	size
derivativedivides	$\frac{2\sqrt{\left(\sqrt{x}+1\right)\left(\sqrt{x}-1\right)}\,\ln\left(\sqrt{x}+\sqrt{-1+x}\right)}{\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}$	40
default	$\frac{2\sqrt{\left(\sqrt{x}+1\right)\left(\sqrt{x}-1\right)}\,\ln\left(\sqrt{x}+\sqrt{-1+x}\right)}{\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}$	40

input
```
int(1/x^(1/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((x^(1/2)+1)*(x^(1/2)-1))^(1/2)/(x^(1/2)+1)^(1/2)/(x^(1/2)-1)^(1/2)*ln(x
^(1/2)+(-1+x)^(1/2))
```

3.1014.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = -\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

input
```
integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fri
cas")
```

output
```
-log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

3.1014.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx=\int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

input
```
integrate(1/x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

output
```
Integral(1/(sqrt(x)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

3.1014.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx=2\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

input
```
integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="max
ima")
```

output
```
2*log(2*sqrt(x - 1) + 2*sqrt(x))
```

3.1014.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx=-4\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

input
```
integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="gia
c")
```

output
```
-4*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

3.1014.9 Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = 2\,\mathrm{acosh}\left(\sqrt{x}\right)$$

input
```
int(1/(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

output
```
2*acosh(x^(1/2))
```

3.1015 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx$

3.1015.1 Optimal result

Integrand size = 28, antiderivative size = 29

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx=\frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}$$

output
```
2*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

3.1015.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 146 vs. 2(29) = 58.

Time = 1.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 5.03

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx=\frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+\right.}{\left(3+2\sqrt{-1+\sqrt{x}}-2\sqrt{3}\sqrt{1+\sqrt{x}}-\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}+2\right.}$$

input
```
Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]
```

output
```
((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + S
qrt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x]))/((3 + 2*Sqrt[-1 + Sqrt[x]]
 - 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[
x]] + 2*Sqrt[x])*Sqrt[x])
```

3.1015.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {797}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}}dx$$

$$\Big\downarrow 797$$

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

input
```
Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]
```

output
```
(2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x]
```

3.1015.3.1 Defintions of rubi rules used

rule 797
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

3.1015.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$	20
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$	20

input
```
int(1/x^(3/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(1/2)
```

3.1015.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{2\left(\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+x\right)}{x}$$

input
```
integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fri
cas")
```

output
```
2*(sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + x)/x
```

3.1015.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

input
```
integrate(1/x**(3/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

output
```
Integral(1/(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

3.1015.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{2\sqrt{x-1}}{\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="max
ima")
```

output
```
2*sqrt(x - 1)/sqrt(x)
```

3.1015.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{16}{\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4}$$

input
```
integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="gia
c")
```

output
```
16/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)
```

3.1015.9 Mupad [B] (verification not implemented)

Time = 9.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

input
```
int(1/(x^(3/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

output
```
(2*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(1/2)
```

3.1016 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx$

3.1016.1 Optimal result

Integrand size = 28, antiderivative size = 63

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3x^{3/2}} + \frac{4\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3\sqrt{x}}$$

output
```
2/3*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2)+4/3*(-1+x^(1/2))^(1/2)*(1
+x^(1/2))^(1/2)/x^(1/2)
```

3.1016.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 407 vs. 2(63) = 126.

Time = 2.56 (sec) , antiderivative size = 407, normalized size of antiderivative = 6.46

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+}\right.}{}$$

input
```
Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]
```

output

```
((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + S
qrt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x])*(8*(-7 - 12*Sqrt[-1 + Sqrt[
x]] + 4*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 7*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 +
Sqrt[x]]) - 4*(49 + 8*Sqrt[-1 + Sqrt[x]] - 24*Sqrt[3]*Sqrt[1 + Sqrt[x]] +
3*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*Sqrt[x] + 2*(-61 + 16*Sqrt
[3]*Sqrt[1 + Sqrt[x]] + 7*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x
+ (-56 - 28*Sqrt[-1 + Sqrt[x]] + 20*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 6*Sqrt[3]*
Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x^(3/2) - 11*x^2))/(12*(-3 - 2*Sqrt[
-1 + Sqrt[x]] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*S
qrt[1 + Sqrt[x]] - 2*Sqrt[x])^3*x^(3/2))
```

3.1016.3 Rubi [A] (verified)

Time $= 0.17$ (sec) , antiderivative size $= 63$, normalized size of antiderivative $= 1.00$, number of steps used $= 2$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used $= \{804, 797\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}}dx$$

$$\Big\downarrow \text{ 804}$$

$$\frac{2}{3}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}}dx+\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}}$$

$$\Big\downarrow \text{ 797}$$

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}}+\frac{4\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

input

```
Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]
```

output

```
(2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(3*x^(3/2)) + (4*Sqrt[-1 + Sqrt[x
]]*Sqrt[1 + Sqrt[x]])/(3*Sqrt[x])
```

3.1016.3.1 Defintions of rubi rules used

rule 797

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

rule 804

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
   Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1
, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2
*n) + p + 1], 0] && NeQ[m, -1]
```

3.1016.4 Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.40

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(1+2x)}{3x^{\frac{3}{2}}}$	25
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(1+2x)}{3x^{\frac{3}{2}}}$	25

input

```
int(1/x^(5/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/3*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(1+2*x)/x^(3/2)
```

3.1016.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{2\left((2\,x+1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2\,x^2\right)}{3\,x^2}$$

input

```
integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fri
cas")
```

output
```
2/3*((2*x + 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*x^2)/x^2
```

3.1016.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \int \frac{1}{x^{\frac{5}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

input
```
integrate(1/x**(5/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

output
```
Integral(1/(x**(5/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

3.1016.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{4\sqrt{x-1}}{3\sqrt{x}} + \frac{2\sqrt{x-1}}{3x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="max
ima")
```

output
```
4/3*sqrt(x - 1)/sqrt(x) + 2/3*sqrt(x - 1)/x^(3/2)
```

3.1016.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{128\left(3\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)}{3\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)^3}$$

input
```
integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="gia
c")
```

output
```
128/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)/((sqrt(sqrt(x) + 1
) - sqrt(sqrt(x) - 1))^4 + 4)^3
```

3.1016.9 Mupad [B] (verification not implemented)

Time = 9.68 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\left(\frac{4\,x}{3}+\frac{2\,\sqrt{x}}{3}+\frac{4\,x^{3/2}}{3}+\frac{2}{3}\right)}{x^{3/2}\,\sqrt{\sqrt{x}+1}}$$

input
```
int(1/(x^(5/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

output
```
((x^(1/2) - 1)^(1/2)*((4*x)/3 + (2*x^(1/2))/3 + (4*x^(3/2))/3 + 2/3))/(x^(
3/2)*(x^(1/2) + 1)^(1/2))
```

3.1017 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx$

3.1017.1 Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{5x^{5/2}} + \frac{8\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{15x^{3/2}} + \frac{16\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{15\sqrt{x}}$$

output
```
2/5*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2)+8/15*(-1+x^(1/2))^(1/2)*(
1+x^(1/2))^(1/2)/x^(3/2)+16/15*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2
)
```

3.1017.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}(3+4x+8x^2)}{15x^{5/2}}$$

input
```
Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]
```

output
```
(2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*(3 + 4*x + 8*x^2))/(15*x^(5/2))
```

3.1017.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {804, 804, 797}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}}dx$$

$$\Big\downarrow \text{804}$$

$$\frac{4}{5}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}}dx+\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{5x^{5/2}}$$

$$\Big\downarrow \text{804}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}}dx+\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}}\right)+\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{5x^{5/2}}$$

$$\Big\downarrow \text{797}$$

$$\frac{4}{5}\left(\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}}+\frac{4\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3\sqrt{x}}\right)+\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{5x^{5/2}}$$

input
```
Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]
```

output
```
(4*((2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(3*x^(3/2)) + (4*Sqrt[-1 + Sq
rt[x]]*Sqrt[1 + Sqrt[x]])/(3*Sqrt[x])))/5 + (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 +
 Sqrt[x]])/(5*x^(5/2))
```

3.1017.3.1 Defintions of rubi rules used

rule 797
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

rule 804
```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
   Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1
, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2
*n) + p + 1], 0] && NeQ[m, -1]
```

3.1017.4 Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.32

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(8x^2+4x+3)}{15x^{\frac{5}{2}}}$	30
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(8x^2+4x+3)}{15x^{\frac{5}{2}}}$	30

input
```
int(1/x^(7/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(8*x^2+4*x+3)/x^(5/2)
```

3.1017.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{2\left(8\,x^3+(8\,x^2+4\,x+3)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{15\,x^3}$$

input
```
integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fri
cas")
```

output
```
2/15*(8*x^3 + (8*x^2 + 4*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1
))/x^3
```

3.1017.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \int \frac{1}{x^{\frac{7}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

input
```
integrate(1/x**(7/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

output
```
Integral(1/(x**(7/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

3.1017.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{16\sqrt{x-1}}{15\sqrt{x}} + \frac{8\sqrt{x-1}}{15\,x^{\frac{3}{2}}} + \frac{2\sqrt{x-1}}{5\,x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="max
ima")
```

output
```
16/15*sqrt(x - 1)/sqrt(x) + 8/15*sqrt(x - 1)/x^(3/2) + 2/5*sqrt(x - 1)/x^(
5/2)
```

3.1017.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{4096\left(5\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+10\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+8\right)}{15\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{5}}$$

input
```
integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="gia
c")
```

output
```
4096/15*(5*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 10*(sqrt(sqrt(x) +
1) - sqrt(sqrt(x) - 1))^4 + 8)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4
+ 4)^5
```

3.1017.9 Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\,\left(\frac{8\,x}{15}+\frac{16\,x^2}{15}+\frac{2\,\sqrt{x}}{5}+\frac{8\,x^{3/2}}{15}+\frac{16\,x^{5/2}}{15}+\frac{2}{5}\right)}{x^{5/2}\,\sqrt{\sqrt{x}+1}}$$

input
```
int(1/(x^(7/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

output
```
((x^(1/2) - 1)^(1/2)*((8*x)/15 + (16*x^2)/15 + (2*x^(1/2))/5 + (8*x^(3/2))
/15 + (16*x^(5/2))/15 + 2/5))/(x^(5/2)*(x^(1/2) + 1)^(1/2))
```

3.1018 $\int x^2(-a+bx^n)^p(a+bx^n)^p\,dx$

3.1018.1 Optimal result

Integrand size = 24, antiderivative size = 78

$$\int x^2(-a+bx^n)^p(a+bx^n)^p\,dx=\frac{1}{3}x^3(-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2n},-p,1+\frac{3}{2n},\frac{b^2x^{2n}}{a^2}\right)$$

output
```
1/3*x^3*(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 3/2/n],[1+3/2/n],b^2*x^(2*
n)/a^2)/((1-b^2*x^(2*n)/a^2)^p)
```

3.1018.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

$$\int x^2(-a+bx^n)^p(a+bx^n)^p\,dx=\frac{1}{3}x^3(-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{3}{2n},-p;1+\frac{3}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

input
```
Integrate[x^2*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x^3*(-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{3/(2*n), -p}, {1 + 3/
(2*n)}, (b^2*x^(2*n))/a^2])/(3*(1 - (b^2*x^(2*n))/a^2)^p)
```

3.1018.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {890, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (bx^n-a)^p (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{890}$$

$$(bx^n-a)^p (a+bx^n)^p \left(b^2x^{2n}-a^2\right)^{-p} \int x^2 \left(b^2x^{2n}-a^2\right)^p dx$$

$$\Big\downarrow \text{889}$$

$$(bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \int x^2 \left(1-\frac{b^2x^{2n}}{a^2}\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{3}x^3 (bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2n},-p,1+\frac{3}{2n},\frac{b^2x^{2n}}{a^2}\right)$$

input
```
Int[x^2*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x^3*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[3/(2*n), -p, 1 + 3/(2*
n), (b^2*x^(2*n))/a^2])/(3*(1 - (b^2*x^(2*n))/a^2)^p)
```

3.1018.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 890
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p])   Int[(c*x)^m*(a1*a2 + b1*b2*
x^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
 a1*b2, 0] &&  !IntegerQ[p]
```

3.1018.4 Maple [F]

$$\int x^{2}(b\,x^{n}-a)^{p}\left(a+b\,x^{n}\right)^{p}dx$$

input
```
int(x^2*(b*x^n-a)^p*(a+b*x^n)^p,x)
```

output
```
int(x^2*(b*x^n-a)^p*(a+b*x^n)^p,x)
```

3.1018.5 Fricas [F]

$$\int x^{2}(-a+bx^{n})^{p}\left(a+bx^{n}\right)^{p}\,dx=\int (bx^{n}+a)^{p}(bx^{n}-a)^{p}x^{2}\,dx$$

input
```
integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

3.1018.6 Sympy [F]

$$\int x^2(-a+bx^n)^p\,(a+bx^n)^p\,dx=\int x^2(-a+bx^n)^p\,(a+bx^n)^p\,dx$$

input
```
integrate(x**2*(-a+b*x**n)**p*(a+b*x**n)**p,x)
```

output
```
Integral(x**2*(-a + b*x**n)**p*(a + b*x**n)**p, x)
```

3.1018.7 Maxima [F]

$$\int x^2(-a+bx^n)^p\,(a+bx^n)^p\,dx=\int (bx^n+a)^p(bx^n-a)^px^2\,dx$$

input
```
integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

3.1018.8 Giac [F]

$$\int x^2(-a+bx^n)^p\,(a+bx^n)^p\,dx=\int (bx^n+a)^p(bx^n-a)^px^2\,dx$$

input
```
integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

3.1018.9 Mupad [F(-1)]

Timed out.

$$\int x^2(-a+bx^n)^p\,(a+bx^n)^p\;dx=\int x^2\,(a+b\,x^n)^p\,(b\,x^n-a)^p\,dx$$

input
```
int(x^2*(a + b*x^n)^p*(b*x^n - a)^p,x)
```

output
```
int(x^2*(a + b*x^n)^p*(b*x^n - a)^p, x)
```

3.1019 $\int x(-a+bx^n)^p (a+bx^n)^p \, dx$

3.1019.1 Optimal result

Integrand size = 22, antiderivative size = 70

$$\int x(-a+bx^n)^p (a+bx^n)^p \, dx = \frac{1}{2}x^2(-a+bx^n)^p (a+bx^n)^p \left(1 - \frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n}, -p, 1+\frac{1}{n}, \frac{b^2x^{2n}}{a^2}\right)$$

output
```
1/2*x^2*(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 1/n],[1+1/n],b^2*x^(2*n)/a
^2)/((1-b^2*x^(2*n)/a^2)^p)
```

3.1019.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int x(-a+bx^n)^p (a+bx^n)^p \, dx = \frac{1}{2}x^2(-a+bx^n)^p (a+bx^n)^p \left(1 - \frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{n}, -p; 1+\frac{1}{n}; \frac{b^2x^{2n}}{a^2}\right)$$

input
```
Integrate[x*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x^2*(-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{n^(-1), -p}, {1 + n^(
-1)}, (b^2*x^(2*n))/a^2])/(2*(1 - (b^2*x^(2*n))/a^2)^p)
```

3.1019.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {890, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(bx^n-a)^p(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{890}$$

$$(bx^n-a)^p(a+bx^n)^p\left(b^2x^{2n}-a^2\right)^{-p}\int x\left(b^2x^{2n}-a^2\right)^p\,dx$$

$$\Big\downarrow \text{889}$$

$$(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\int x\left(1-\frac{b^2x^{2n}}{a^2}\right)^p\,dx$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{2}x^2(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},\frac{b^2x^{2n}}{a^2}\right)$$

input
```
Int[x*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x^2*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1)
, (b^2*x^(2*n))/a^2])/(2*(1 - (b^2*x^(2*n))/a^2)^p)
```

3.1019.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 890
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p])   Int[(c*x)^m*(a1*a2 + b1*b2*
x^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
 a1*b2, 0] &&  !IntegerQ[p]
```

3.1019.4 Maple [F]

$$\int x(b\,x^{n}-a)^{p}\,(a+b\,x^{n})^{p}\,dx$$

input
```
int(x*(b*x^n-a)^p*(a+b*x^n)^p,x)
```

output
```
int(x*(b*x^n-a)^p*(a+b*x^n)^p,x)
```

3.1019.5 Fricas [F]

$$\int x(-a+bx^n)^p\,(a+bx^n)^p\,\,dx=\int(bx^n+a)^p(bx^n-a)^px\,dx$$

input
```
integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

3.1019.6 Sympy [F]

$$\int x(-a+bx^n)^p\,(a+bx^n)^p\,\,dx=\int x(-a+bx^n)^p\,(a+bx^n)^p\,\,dx$$

input
```
integrate(x*(-a+b*x**n)**p*(a+b*x**n)**p,x)
```

output
```
Integral(x*(-a + b*x**n)**p*(a + b*x**n)**p, x)
```

3.1019.7 Maxima [F]

$$\int x(-a+bx^n)^p\,(a+bx^n)^p\,dx=\int (bx^n+a)^p(bx^n-a)^px\,dx$$

input
```
integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

3.1019.8 Giac [F]

$$\int x(-a+bx^n)^p\,(a+bx^n)^p\,dx=\int (bx^n+a)^p(bx^n-a)^px\,dx$$

input
```
integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

3.1019.9 Mupad [F(-1)]

Timed out.

$$\int x(-a+bx^n)^p\,(a+bx^n)^p\,dx=\int x\,(a+b\,x^n)^p\,(b\,x^n-a)^p\,dx$$

input
```
int(x*(a + b*x^n)^p*(b*x^n - a)^p,x)
```

output
```
int(x*(a + b*x^n)^p*(b*x^n - a)^p, x)
```

3.1020 $\int (-a+bx^n)^p (a+bx^n)^p \, dx$

3.1020.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = x(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)$$

output
```
x*(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2
)/((1-b^2*x^(2*n)/a^2)^p)
```

3.1020.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = x(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,1+\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)$$

input
```
Integrate[(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n)
, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

3.1020.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {785, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx^n-a)^p (a+bx^n)^p \, dx$$

$$\Big\downarrow 785$$

$$(bx^n-a)^p (a+bx^n)^p \left(b^2x^{2n}-a^2\right)^{-p} \int \left(b^2x^{2n}-a^2\right)^p dx$$

$$\Big\downarrow 779$$

$$(bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p dx$$

$$\Big\downarrow 778$$

$$x(bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)$$

input
```
Int[(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1)
)/2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

3.1020.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 785
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p])   Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Fre
eQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

3.1020.4 Maple [F]

$$\int (b\,x^{n}-a)^{p} (a+b\,x^{n})^{p} \, dx$$

input
```
int((b*x^n-a)^p*(a+b*x^n)^p,x)
```

output
```
int((b*x^n-a)^p*(a+b*x^n)^p,x)
```

3.1020.5 Fricas [F]

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = \int (bx^n+a)^p(bx^n-a)^p \, dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(b*x^n - a)^p, x)
```

3.1020.6 Sympy [F]

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = \int (-a+bx^n)^p (a+bx^n)^p \, dx$$

input
```
integrate((-a+b*x**n)**p*(a+b*x**n)**p,x)
```

output
```
Integral((-a + b*x**n)**p*(a + b*x**n)**p, x)
```

3.1020.7 Maxima [F]

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = \int (bx^n+a)^p (bx^n-a)^p \, dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p, x)
```

3.1020.8 Giac [F]

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = \int (bx^n+a)^p (bx^n-a)^p \, dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p, x)
```

3.1020.9 Mupad [F(-1)]

Timed out.

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = \int (a+b\,x^n)^p\,(b\,x^n-a)^p\,dx$$

input
```
int((a + b*x^n)^p*(b*x^n - a)^p,x)
```

output
```
int((a + b*x^n)^p*(b*x^n - a)^p, x)
```

3.1021 $\int \frac{(-a+bx^n)^p(a+bx^n)^p}{x}\,dx$

3.1021.1 Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x}\,dx = -\frac{(-a+bx^n)^p\,(a+bx^n)^p\,(a^2-b^2x^{2n})\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(1+p)}$$

output
```
-1/2*(-a+b*x^n)^p*(a+b*x^n)^p*(a^2-b^2*x^(2*n))*hypergeom([1, p+1],[2+p],1
-b^2*x^(2*n)/a^2)/a^2/n/(p+1)
```

3.1021.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x}\,dx = \frac{(-a+bx^n)^p\,(a+bx^n)^p\,(-a^2+b^2x^{2n})\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(1+p)}$$

input
```
Integrate[((-a + b*x^n)^p*(a + b*x^n)^p)/x,x]
```

output
```
((-a + b*x^n)^p*(a + b*x^n)^p*(-a^2 + b^2*x^(2*n))*Hypergeometric2F1[1, 1
+ p, 2 + p, 1 - (b^2*x^(2*n))/a^2])/(2*a^2*n*(1 + p))
```

3.1021.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {799, 136, 243, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^n-a)^p (a+bx^n)^p}{x} \, dx$$

$$\Big\downarrow \text{799}$$

$$\frac{\int x^{-n}(bx^n-a)^p (bx^n+a)^p dx^n}{n}$$

$$\Big\downarrow \text{136}$$

$$\frac{(bx^n-a)^p (a+bx^n)^p \left(b^2x^{2n}-a^2\right)^{-p} \int x^{-n}\left(b^2x^{2n}-a^2\right)^p dx^n}{n}$$

$$\Big\downarrow \text{243}$$

$$\frac{(bx^n-a)^p (a+bx^n)^p \left(b^2x^{2n}-a^2\right)^{-p} \int x^{-n}\left(b^2x^{2n}-a^2\right)^p dx^{2n}}{2n}$$

$$\Big\downarrow \text{75}$$

$$\frac{\left(b^2x^{2n}-a^2\right) (bx^n-a)^p (a+bx^n)^p \text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(p+1)}$$

input
```
Int[((-a + b*x^n)^p*(a + b*x^n)^p)/x,x]
```

output
```
((-a + b*x^n)^p*(a + b*x^n)^p*(-a^2 + b^2*x^(2*n))*Hypergeometric2F1[1, 1
+ p, 2 + p, 1 - (b^2*x^(2*n))/a^2])/(2*a^2*n*(1 + p))
```

3.1021.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 136
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^Fr
acPart[m])   Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b, c, d, f,
 m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 799
```
Int[(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(
p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a1 + b1
*x)^p*(a2 + b2*x)^p, x], x, x^n], x] /; FreeQ[{a1, b1, a2, b2, m, n, p}, x]
 && EqQ[a2*b1 + a1*b2, 0] && IntegerQ[Simplify[(m + 1)/(2*n)]]
```

3.1021.4 Maple [F]

$$\int \frac{(b\,x^{n}-a)^{p}\,(a+b\,x^{n})^{p}}{x}dx$$

input
```
int((b*x^n-a)^p*(a+b*x^n)^p/x,x)
```

output
```
int((b*x^n-a)^p*(a+b*x^n)^p/x,x)
```

3.1021.5 Fricas [F]

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x} \, dx = \int \frac{(bx^n+a)^p (bx^n-a)^p}{x} \, dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

3.1021.6 Sympy [F]

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x} \, dx = \int \frac{(-a+bx^n)^p (a+bx^n)^p}{x} \, dx$$

input
```
integrate((-a+b*x**n)**p*(a+b*x**n)**p/x,x)
```

output
```
Integral((-a + b*x**n)**p*(a + b*x**n)**p/x, x)
```

3.1021.7 Maxima [F]

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x} \, dx = \int \frac{(bx^n+a)^p (bx^n-a)^p}{x} \, dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

3.1021.8 Giac [F]

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x}\,dx=\int \frac{(bx^n+a)^p(bx^n-a)^p}{x}\,dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

3.1021.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x}\,dx=\int \frac{(a+b\,x^n)^p\,(b\,x^n-a)^p}{x}\,dx$$

input
```
int(((a + b*x^n)^p*(b*x^n - a)^p)/x,x)
```

output
```
int(((a + b*x^n)^p*(b*x^n - a)^p)/x, x)
```

3.1022 $\int \frac{(-a+bx^n)^p(a+bx^n)^p}{x^2}\,dx$

3.1022.1 Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x^2}\,dx = -\frac{(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2n},-p,1-\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)}{x}$$

output
```
-(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, -1/2/n],[1-1/2/n],b^2*x^(2*n)/a^2
)/x/((1-b^2*x^(2*n)/a^2)^p)
```

3.1022.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x^2}\,dx = -\frac{(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2n},-p;1-\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{x}$$

input
```
Integrate[((-a + b*x^n)^p*(a + b*x^n)^p)/x^2,x]
```

output
```
-(((-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{-1/2*1/n, -p}, {1 - 1/(
2*n)}, (b^2*x^(2*n))/a^2])/(x*(1 - (b^2*x^(2*n))/a^2)^p))
```

3.1022.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {890, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^n-a)^p (a+bx^n)^p}{x^2} \, dx$$

$$\Big\downarrow \text{890}$$

$$(bx^n-a)^p (a+bx^n)^p \left(b^2x^{2n}-a^2\right)^{-p} \int \frac{\left(b^2x^{2n}-a^2\right)^p}{x^2}dx$$

$$\Big\downarrow \text{889}$$

$$(bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \int \frac{\left(1-\frac{b^2x^{2n}}{a^2}\right)^p}{x^2}dx$$

$$\Big\downarrow \text{888}$$

$$-\frac{(bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2n},-p,1-\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)}{x}$$

input

```
Int[((-a + b*x^n)^p*(a + b*x^n)^p)/x^2,x]
```

output

```
-(((-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[-1/2*1/n, -p, 1 - 1/(2*n
), (b^2*x^(2*n))/a^2])/(x*(1 - (b^2*x^(2*n))/a^2)^p))
```

3.1022.3.1 Defintions of rubi rules used

rule 888

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 890
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p])   Int[(c*x)^m*(a1*a2 + b1*b2*
x^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
 a1*b2, 0] &&  !IntegerQ[p]
```

3.1022.4 Maple [F]

$$\int \frac{(b\,x^{n}-a)^{p}\,(a+b\,x^{n})^{p}}{x^{2}}dx$$

input
```
int((b*x^n-a)^p*(a+b*x^n)^p/x^2,x)
```

output
```
int((b*x^n-a)^p*(a+b*x^n)^p/x^2,x)
```

3.1022.5 Fricas [F]

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x^2}\,dx = \int \frac{(bx^n+a)^p(bx^n-a)^p}{x^2}\,dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

3.1022.6 Sympy [F]

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x^2} \, dx = \int \frac{(-a+bx^n)^p (a+bx^n)^p}{x^2} \, dx$$

input
```
integrate((-a+b*x**n)**p*(a+b*x**n)**p/x**2,x)
```

output
```
Integral((-a + b*x**n)**p*(a + b*x**n)**p/x**2, x)
```

3.1022.7 Maxima [F]

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x^2} \, dx = \int \frac{(bx^n+a)^p (bx^n-a)^p}{x^2} \, dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

3.1022.8 Giac [F]

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x^2} \, dx = \int \frac{(bx^n+a)^p (bx^n-a)^p}{x^2} \, dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

3.1022.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+bx^n)^p(a+bx^n)^p}{x^2}\,dx=\int \frac{(a+b\,x^n)^p\,(b\,x^n-a)^p}{x^2}\,dx$$

input
```
int(((a + b*x^n)^p*(b*x^n - a)^p)/x^2,x)
```

output
```
int(((a + b*x^n)^p*(b*x^n - a)^p)/x^2, x)
```

3.1023 $\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx$

3.1023.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx=\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

output
```
ln(x)-1/3*ln(-x^6+1)
```

3.1023.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx=\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

input
```
Integrate[(1 + x^6)/(x*(1 - x^6)),x]
```

output
```
Log[x] - Log[1 - x^6]/3
```

3.1023.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6+1}{x\left(1-x^6\right)}dx$$
$$\Big\downarrow \text{948}$$
$$\frac{1}{6}\int \frac{x^6+1}{x^6\left(1-x^6\right)}dx^6$$
$$\Big\downarrow \text{86}$$
$$\frac{1}{6}\int \left(\frac{1}{x^6}-\frac{2}{x^6-1}\right)dx^6$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{6}\left(\log\left(x^6\right)-2\log\left(1-x^6\right)\right)$$

input
```
Int[(1 + x^6)/(x*(1 - x^6)),x]
```

output
```
(Log[x^6] - 2*Log[1 - x^6])/6
```

3.1023.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1023.4 Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln\left(x\right)-\frac{\ln\left(x^{6}-1\right)}{3}$	12
meijerg	$-\frac{\ln\left(-x^{6}+1\right)}{3}+\ln\left(x\right)+\frac{i \pi}{6}$	18
default	$\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{3}-\frac{\ln\left(1+x\right)}{3}-\frac{\ln\left(x^{2}+x+1\right)}{3}-\frac{\ln\left(x^{2}-x+1\right)}{3}$	36
norman	$\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{3}-\frac{\ln\left(1+x\right)}{3}-\frac{\ln\left(x^{2}+x+1\right)}{3}-\frac{\ln\left(x^{2}-x+1\right)}{3}$	36
parallelrisch	$\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{3}-\frac{\ln\left(1+x\right)}{3}-\frac{\ln\left(x^{2}+x+1\right)}{3}-\frac{\ln\left(x^{2}-x+1\right)}{3}$	36

input
```
int((x^6+1)/x/(-x^6+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/3*ln(x^6-1)
```

3.1023.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx=-\frac{1}{3}\log\left(x^6-1\right)+\log\left(x\right)$$

input
```
integrate((x^6+1)/x/(-x^6+1),x, algorithm="fricas")
```

output
```
-1/3*log(x^6 - 1) + log(x)
```

3.1023.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx = \log\left(x\right) - \frac{\log\left(x^6-1\right)}{3}$$

input
```
integrate((x**6+1)/x/(-x**6+1),x)
```

output
```
log(x) - log(x**6 - 1)/3
```

3.1023.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx = -\frac{1}{3}\log\left(x^6-1\right) + \frac{1}{6}\log\left(x^6\right)$$

input
```
integrate((x^6+1)/x/(-x^6+1),x, algorithm="maxima")
```

output
```
-1/3*log(x^6 - 1) + 1/6*log(x^6)
```

3.1023.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx = \frac{1}{6}\log\left(x^6\right) - \frac{1}{3}\log\left(\left|x^6-1\right|\right)$$

input
```
integrate((x^6+1)/x/(-x^6+1),x, algorithm="giac")
```

output
```
1/6*log(x^6) - 1/3*log(abs(x^6 - 1))
```

3.1023.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx=\ln\left(x\right)-\frac{\ln\left(x^6-1\right)}{3}$$

input
```
int(-(x^6 + 1)/(x*(x^6 - 1)),x)
```

output
```
log(x) - log(x^6 - 1)/3
```

3.1024 $\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)$

3.1024.1 Optimal result

Integrand size = 33, antiderivative size = 22

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n)\,dx=\frac{(ex)^{1+m}(a+bx^n)^{1+p}}{e}$$

output

```
(e*x)^(1+m)*(a+b*x^n)^(p+1)/e
```

3.1024.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 5.00

$$\begin{aligned}&\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n)\,dx\\&=x(ex)^m(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(a\,\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)\right.\\&\left.\quad+\frac{b(1+m+n+np)x^n\,\text{Hypergeometric2F1}\left(\frac{1+m+n}{n},-p,\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{1+m+n}\right)\end{aligned}$$

input

```
Integrate[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n),x]
```

output

```
(x*(e*x)^m*(a + b*x^n)^p*(a*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n
, -((b*x^n)/a)] + (b*(1 + m + n + n*p)*x^n*Hypergeometric2F1[(1 + m + n)/n
, -p, (1 + m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n)))/(1 + (b*x^n)/a)^p
```

3.1024.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a+bx^n)^p (a(m+1)+bx^n(m+np+n+1)) \, dx$$

$$\Big\downarrow \text{951}$$

$$\frac{(ex)^{m+1} (a+bx^n)^{p+1}}{e}$$

input
```
Int[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n),x]
```

output
```
((e*x)^(1 + m)*(a + b*x^n)^(1 + p))/e
```

3.1024.3.1 Defintions of rubi rules used

rule 951
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

3.1024.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 12.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

method	result	size
parallelrisch	$\frac{x\,x^{n} (ex)^{m} (a+b\,x^{n})^{p} b^{2}+x (ex)^{m} (a+b\,x^{n})^{p} a b}{b}$	46

input
```
int((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x,method=_RETURNVERBOS
E)
```

output
```
(x*x^n*(e*x)^m*(a+b*x^n)^p*b^2+x*(e*x)^m*(a+b*x^n)^p*a*b)/b
```

3.1024.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n)\,dx = \left(bxx^n e^{(m\log(e)+m\log(x))}+axe^{(m\log(e)+m\log(x))}\right)(bx^n+a)^p$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="fr
icas")
```

output
```
(b*x*x^n*e^(m*log(e) + m*log(x)) + a*x*e^(m*log(e) + m*log(x)))*(b*x^n + a
)^p
```

3.1024.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 2.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n)\,dx = ax(ex)^m (a+bx^n)^p+bxx^n(ex)^m (a+bx^n)^p$$

input
```
integrate((e*x)**m*(a+b*x**n)**p*(a*(1+m)+b*(n*p+m+n+1)*x**n),x)
```

output
```
a*x*(e*x)**m*(a + b*x**n)**p + b*x*x**n*(e*x)**m*(a + b*x**n)**p
```

3.1024.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n) \, dx$$
$$= \left(ae^m x x^m + be^m x e^{(m\log(x)+n\log(x))}\right)(bx^n+a)^p$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="ma
xima")
```

output
```
(a*e^m*x*x^m + b*e^m*x*e^(m*log(x) + n*log(x)))*(b*x^n + a)^p
```

3.1024.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(22) = 44.

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n) \, dx$$
$$= (bx^n+a)^p bxx^n e^{(m\log(e)+m\log(x))} + (bx^n+a)^p axe^{(m\log(e)+m\log(x))}$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="gi
ac")
```

output
```
(b*x^n + a)^p*b*x*x^n*e^(m*log(e) + m*log(x)) + (b*x^n + a)^p*a*x*e^(m*log
(e) + m*log(x))
```

3.1024.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n) \, dx$$
$$= \left(a\,x\,(e\,x)^m + b\,x^{n+1}\,(e\,x)^m\right)\,(a+b\,x^n)^p$$

input
```
int((e*x)^m*(a*(m + 1) + b*x^n*(m + n + n*p + 1))*(a + b*x^n)^p,x)
```

output
```
(a*x*(e*x)^m + b*x^(n + 1)*(e*x)^m)*(a + b*x^n)^p
```

3.1025 $\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx$

3.1025.1 Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx=\frac{b(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)e(1+m)}-\frac{d(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)e(1+m)}$$

output
```
b*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/(-a*d+b*c)/e/
(1+m)-d*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c/(-a*d+b
*c)/e/(1+m)
```

3.1025.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx =\frac{x(ex)^m\left(-bc\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+ad\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)\right.}{ac(-bc+ad)(1+m)}$$

input
```
Integrate[(e*x)^m/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(x*(e*x)^m*(-(b*c*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)
/a)]) + a*d*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)]))
/(a*c*(-(b*c) + a*d)*(1 + m))
```

3.1025.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1010, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}dx$$

$$\Big\downarrow \text{1010}$$

$$\frac{b\int \frac{(ex)^m}{bx^n+a}dx}{bc-ad}-\frac{d\int \frac{(ex)^m}{dx^n+c}dx}{bc-ad}$$

$$\Big\downarrow \text{888}$$

$$\frac{b(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{ae(m+1)(bc-ad)}-\frac{d(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)}$$

input
```
Int[(e*x)^m/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/
a)])/(a*(b*c - a*d)*e*(1 + m)) - (d*(e*x)^(1 + m)*Hypergeometric2F1[1, (1
+ m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)*e*(1 + m))
```

3.1025.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1010
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n,
m}, x] && NeQ[b*c - a*d, 0]
```

3.1025.4 Maple [F]

$$\int \frac{(ex)^m}{(a+b\,x^n)\,(c+d\,x^n)}dx$$

input
```
int((e*x)^m/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int((e*x)^m/(a+b*x^n)/(c+d*x^n),x)
```

3.1025.5 Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((e*x)^m/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

3.1025.6 Sympy [F]

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx$$

input
```
integrate((e*x)**m/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Integral((e*x)**m/((a + b*x**n)*(c + d*x**n)), x)
```

3.1025.7 Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^n + a)*(d*x^n + c)), x)
```

3.1025.8 Giac [F]

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^n + a)*(d*x^n + c)), x)
```

3.1025.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{(e\,x)^m}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

input

```
int((e*x)^m/((a + b*x^n)*(c + d*x^n)),x)
```

output

```
int((e*x)^m/((a + b*x^n)*(c + d*x^n)), x)
```

3.1026 $\int \frac{x^2}{(a+bx^n)(c+dx^n)} dx$

3.1026.1 Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx = \frac{bx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a(bc-ad)} - \frac{dx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{dx^n}{c}\right)}{3c(bc-ad)}$$

output
```
1/3*b*x^3*hypergeom([1, 3/n],[(3+n)/n],-b*x^n/a)/a/(-a*d+b*c)-1/3*d*x^3*hy
pergeom([1, 3/n],[(3+n)/n],-d*x^n/c)/c/(-a*d+b*c)
```

3.1026.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx = \frac{bcx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right) - adx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{dx^n}{c}\right)}{3abc^2-3a^2cd}$$

input
```
Integrate[x^2/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*c*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)] - a*d*x^3*Hype
rgeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*a*b*c^2 - 3*a^2*c*d)
```

3.1026.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1010, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)} \, dx$$

$$\Big\downarrow \text{1010}$$

$$\frac{b\int \frac{x^2}{bx^n+a}dx}{bc-ad}-\frac{d\int \frac{x^2}{dx^n+c}dx}{bc-ad}$$

$$\Big\downarrow \text{888}$$

$$\frac{bx^3 \text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3a(bc-ad)}-\frac{dx^3 \text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{dx^n}{c}\right)}{3c(bc-ad)}$$

input
```
Int[x^2/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a*(b*c - a*d
)) - (d*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*c*(b*c
- a*d))
```

3.1026.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1010
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n,
m}, x] && NeQ[b*c - a*d, 0]
```

3.1026.4 Maple [F]

$$\int \frac{x^{2}}{\left(a+b\,x^{n}\right)\left(c+d\,x^{n}\right)}dx$$

input
```
int(x^2/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(x^2/(a+b*x^n)/(c+d*x^n),x)
```

3.1026.5 Fricas [F]

$$\int \frac{x^2}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{x^2}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(x^2/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

3.1026.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{x^2}{(a+bx^n)\,(c+dx^n)}\,dx$$

input
```
integrate(x**2/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Integral(x**2/((a + b*x**n)*(c + d*x**n)), x)
```

3.1026.7 Maxima [F]

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^2}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^n + a)*(d*x^n + c)), x)
```

3.1026.8 Giac [F]

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^2}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^n + a)*(d*x^n + c)), x)
```

3.1026.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^2}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

input
```
int(x^2/((a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(x^2/((a + b*x^n)*(c + d*x^n)), x)
```

3.1027 $\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx$

3.1027.1 Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \frac{bx^2 \,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a(bc-ad)} - \frac{dx^2 \,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)}$$

output
```
1/2*b*x^2*hypergeom([1, 2/n],[(2+n)/n],-b*x^n/a)/a/(-a*d+b*c)-1/2*d*x^2*hy
pergeom([1, 2/n],[(2+n)/n],-d*x^n/c)/c/(-a*d+b*c)
```

3.1027.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \frac{bcx^2 \,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right) - adx^2 \,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{dx^n}{c}\right)}{2abc^2-2a^2cd}$$

input
```
Integrate[x/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*c*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)] - a*d*x^2*Hype
rgeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*a*b*c^2 - 2*a^2*c*d)
```

3.1027.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1010, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^n)(c+dx^n)}dx$$

$$\Big\downarrow \text{1010}$$

$$\frac{b\int \frac{x}{bx^n+a}dx}{bc-ad}-\frac{d\int \frac{x}{dx^n+c}dx}{bc-ad}$$

$$\Big\downarrow \text{888}$$

$$\frac{bx^2 \text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a(bc-ad)}-\frac{dx^2 \text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)}$$

input
```
Int[x/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a*(b*c - a*d
)) - (d*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*c*(b*c
- a*d))
```

3.1027.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1010
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n,
m}, x] && NeQ[b*c - a*d, 0]
```

3.1027.4 Maple [F]

$$\int \frac{x}{(a+b\,x^n)\,(c+d\,x^n)}dx$$

input
```
int(x/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(x/(a+b*x^n)/(c+d*x^n),x)
```

3.1027.5 Fricas [F]

$$\int \frac{x}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{x}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(x/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

3.1027.6 Sympy [F]

$$\int \frac{x}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{x}{(a+bx^n)\,(c+dx^n)}\,dx$$

input
```
integrate(x/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Integral(x/((a + b*x**n)*(c + d*x**n)), x)
```

3.1027.7 Maxima [F]

$$\int \frac{x}{(a+bx^n)\,(c+dx^n)}\,dx=\int \frac{x}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^n + a)*(d*x^n + c)), x)
```

3.1027.8 Giac [F]

$$\int \frac{x}{(a+bx^n)\,(c+dx^n)}\,dx=\int \frac{x}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x/((b*x^n + a)*(d*x^n + c)), x)
```

3.1027.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^n)\,(c+dx^n)}\,dx=\int \frac{x}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

input
```
int(x/((a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(x/((a + b*x^n)*(c + d*x^n)), x)
```

3.1028 $\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$

3.1028.1 Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx=\frac{bx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

output
```
b*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(-a*d+b*c)-d*x*hypergeom([1, 1/
n],[1+1/n],-d*x^n/c)/c/(-a*d+b*c)
```

3.1028.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx =\frac{x\left(-bc\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)+ad\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)\right)}{ac(-bc+ad)}$$

input
```
Integrate[1/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(x*(-(b*c*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]) + a*d*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(a*c*(-(b*c) + a*d)
)
```

3.1028.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {917, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)(c+dx^n)}dx$$

$$\Big\downarrow \ 917$$

$$\frac{b\int \frac{1}{bx^n+a}dx}{bc-ad}-\frac{d\int \frac{1}{dx^n+c}dx}{bc-ad}$$

$$\Big\downarrow \ 778$$

$$\frac{bx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

input

```
Int[1/((a + b*x^n)*(c + d*x^n)),x]
```

output

```
(b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d
)) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c
- a*d))
```

3.1028.3.1 Defintions of rubi rules used

rule 778

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 917

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^n), x], x] - Simp[d/(b*c - a*d)   Int[1/(c
 + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

3.1028.4 Maple [F]

$$\int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)}dx$$

input
```
int(1/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(1/(a+b*x^n)/(c+d*x^n),x)
```

3.1028.5 Fricas [F]

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

3.1028.6 Sympy [F]

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{(a+bx^n)\,(c+dx^n)}\,dx$$

input
```
integrate(1/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Integral(1/((a + b*x**n)*(c + d*x**n)), x)
```

3.1028.7 Maxima [F]

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

3.1028.8 Giac [F]

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

3.1028.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{(a+b\,x^n)\,\left(c+d\,x^n\right)}\,dx$$

input
```
int(1/((a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(1/((a + b*x^n)*(c + d*x^n)), x)
```

3.1029 $\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx$

3.1029.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx=\frac{\log(x)}{ac}-\frac{b\log(a+bx^n)}{a(bc-ad)n}+\frac{d\log(c+dx^n)}{c(bc-ad)n}$$

output
```
ln(x)/a/c-b*ln(a+b*x^n)/a/(-a*d+b*c)/n+d*ln(c+d*x^n)/c/(-a*d+b*c)/n
```

3.1029.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx=\frac{bc\log(x^n)-ad\log(x^n)-bc\log(a+bx^n)+ad\log(c+dx^n)}{abc^2n-a^2cdn}$$

input
```
Integrate[1/(x*(a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*c*Log[x^n] - a*d*Log[x^n] - b*c*Log[a + b*x^n] + a*d*Log[c + d*x^n])/(a
*b*c^2*n - a^2*c*d*n)
```

3.1029.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{-n}}{(bx^n+a)(dx^n+c)}dx^n}{n}$$

$$\Big\downarrow \text{93}$$

$$\frac{\int \left(\frac{x^{-n}}{ac}+\frac{b^2}{a(ad-bc)(bx^n+a)}+\frac{d^2}{c(bc-ad)(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b\log(a+bx^n)}{a(bc-ad)}+\frac{d\log(c+dx^n)}{c(bc-ad)}+\frac{\log(x^n)}{ac}}{n}$$

input
```
Int[1/(x*(a + b*x^n)*(c + d*x^n)),x]
```

output
```
(Log[x^n]/(a*c) - (b*Log[a + b*x^n])/(a*(b*c - a*d)) + (d*Log[c + d*x^n])/
(c*(b*c - a*d)))/n
```

3.1029.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1029.4 Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{\ln(x)adn-\ln(x)bcn+b\ln(a+b\,x^{n})c-d\ln(c+d\,x^{n})a}{(ad-bc)acn}$	58
derivativedivides	$\frac{\frac{\ln(x^{n})}{ac}-\frac{d\ln(c+d\,x^{n})}{c(ad-bc)}+\frac{b\ln(a+b\,x^{n})}{(ad-bc)a}}{n}$	64
default	$\frac{\frac{\ln(x^{n})}{ac}-\frac{d\ln(c+d\,x^{n})}{c(ad-bc)}+\frac{b\ln(a+b\,x^{n})}{(ad-bc)a}}{n}$	64
norman	$\frac{\ln(x)}{ac}+\frac{b\ln(a+b\,\mathrm{e}^{n\ln(x)})}{(ad-bc)an}-\frac{d\ln(c+d\,\mathrm{e}^{n\ln(x)})}{cn(ad-bc)}$	68
risch	$-\frac{\ln(x)b}{(ad-bc)a}+\frac{\ln(x)d}{c(ad-bc)}+\frac{b\ln(x^{n}+\frac{a}{b})}{(ad-bc)an}-\frac{d\ln(x^{n}+\frac{c}{d})}{cn(ad-bc)}$	94

input
```
int(1/x/(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
(ln(x)*a*d*n-ln(x)*b*c*n+b*ln(a+b*x^n)*c-d*ln(c+d*x^n)*a)/(a*d-b*c)/a/c/n
```

3.1029.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a+bx^{n}\right)\left(c+dx^{n}\right)}\,dx=-\frac{bc\log\left(bx^{n}+a\right)-ad\log\left(dx^{n}+c\right)-(bc-ad)n\log\left(x\right)}{(abc^{2}-a^{2}cd)n}$$

input
```
integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
-(b*c*log(b*x^n + a) - a*d*log(d*x^n + c) - (b*c - a*d)*n*log(x))/((a*b*c^
2 - a^2*c*d)*n)
```

3.1029.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(46) = 92$.

Time = 1.34 (sec) , antiderivative size = 348, normalized size of antiderivative = 5.52

$$\int \frac{1}{x\left(a+bx^{n}\right)\left(c+dx^{n}\right)}\,dx = \begin{cases} \frac{\frac{\log\left(x\right)}{c}-\frac{\log\left(\frac{c}{d}+x^{n}\right)}{cn}}{a} & \text{for}\ b=0 \\ \frac{\frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{a}{b}+x^{n}\right)}{an}}{c} & \text{for}\ d=0 \\ \frac{-\frac{x^{-n}}{cn}+\frac{d\log\left(x^{-n}+\frac{d}{c}\right)}{c^{2}n}}{b} & \text{for}\ a=0 \\ \frac{cdn\log\left(x\right)}{bc^{3}n+bc^{2}dnx^{n}}-\frac{cd\log\left(\frac{c}{d}+x^{n}\right)}{bc^{3}n+bc^{2}dnx^{n}}+\frac{cd}{bc^{3}n+bc^{2}dnx^{n}}+\frac{d^{2}nx^{n}\log\left(x\right)}{bc^{3}n+bc^{2}dnx^{n}}-\frac{d^{2}x^{n}\log\left(\frac{c}{d}+x^{n}\right)}{bc^{3}n+bc^{2}dnx^{n}} & \text{for}\ a=\frac{bc}{d} \\ \frac{-\frac{x^{-n}}{an}+\frac{b\log\left(x^{-n}+\frac{b}{a}\right)}{a^{2}n}}{d} & \text{for}\ c=0 \\ \frac{\log\left(x\right)}{(a+b)(c+d)} & \text{for}\ n=0 \\ \frac{adn\log\left(x\right)}{a^{2}cdn-abc^{2}n}-\frac{ad\log\left(\frac{c}{d}+x^{n}\right)}{a^{2}cdn-abc^{2}n}-\frac{bcn\log\left(x\right)}{a^{2}cdn-abc^{2}n}+\frac{bc\log\left(\frac{a}{b}+x^{n}\right)}{a^{2}cdn-abc^{2}n} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Piecewise(((log(x)/c - log(c/d + x**n)/(c*n))/a, Eq(b, 0)), ((log(x)/a - l
og(a/b + x**n)/(a*n))/c, Eq(d, 0)), ((-1/(c*n*x**n) + d*log(x**(-n) + d/c)
/(c**2*n))/b, Eq(a, 0)), (c*d*n*log(x)/(b*c**3*n + b*c**2*d*n*x**n) - c*d*
log(c/d + x**n)/(b*c**3*n + b*c**2*d*n*x**n) + c*d/(b*c**3*n + b*c**2*d*n*
x**n) + d**2*n*x**n*log(x)/(b*c**3*n + b*c**2*d*n*x**n) - d**2*x**n*log(c/
d + x**n)/(b*c**3*n + b*c**2*d*n*x**n), Eq(a, b*c/d)), ((-1/(a*n*x**n) + b
*log(x**(-n) + b/a)/(a**2*n))/d, Eq(c, 0)), (log(x)/((a + b)*(c + d)), Eq(
n, 0)), (a*d*n*log(x)/(a**2*c*d*n - a*b*c**2*n) - a*d*log(c/d + x**n)/(a**
2*c*d*n - a*b*c**2*n) - b*c*n*log(x)/(a**2*c*d*n - a*b*c**2*n) + b*c*log(a
/b + x**n)/(a**2*c*d*n - a*b*c**2*n), True))
```

3.1029.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+bx^n\right)\left(c+dx^n\right)}\,dx = -\frac{b\log\left(\frac{bx^n+a}{b}\right)}{abcn-a^2dn}+\frac{d\log\left(\frac{dx^n+c}{d}\right)}{bc^2n-acdn}+\frac{\log\left(x\right)}{ac}$$

input
```
integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
-b*log((b*x^n + a)/b)/(a*b*c*n - a^2*d*n) + d*log((d*x^n + c)/d)/(b*c^2*n
- a*c*d*n) + log(x)/(a*c)
```

3.1029.8 Giac [F]

$$\int \frac{1}{x\left(a+bx^n\right)\left(c+dx^n\right)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)x}\,dx$$

input
```
integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)*x), x)
```

3.1029.9 Mupad [B] (verification not implemented)

Time = 9.98 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.57

$$\int \frac{1}{x\left(a+bx^n\right)\left(c+dx^n\right)}\,dx = \frac{b\,\ln\left(-\frac{1}{b\,d\,x}-\frac{2\,a\,c\,n+a\,d\,n\,x^n+b\,c\,n\,x^n}{d\,x\,\left(a^2\,d\,n-a\,b\,c\,n\right)}\right)}{a^2\,d\,n-a\,b\,c\,n} + \frac{d\,\ln\left(-\frac{1}{b\,d\,x}-\frac{2\,a\,c\,n+a\,d\,n\,x^n+b\,c\,n\,x^n}{b\,x\,\left(b\,c^2\,n-a\,c\,d\,n\right)}\right)}{b\,c^2\,n-a\,c\,d\,n}+\frac{\ln\left(x\right)\,\left(n-1\right)}{a\,c\,n}$$

input
```
int(1/(x*(a + b*x^n)*(c + d*x^n)),x)
```

output
```
(b*log(- 1/(b*d*x) - (2*a*c*n + a*d*n*x^n + b*c*n*x^n)/(d*x*(a^2*d*n - a*b
*c*n))))/(a^2*d*n - a*b*c*n) + (d*log(- 1/(b*d*x) - (2*a*c*n + a*d*n*x^n +
 b*c*n*x^n)/(b*x*(b*c^2*n - a*c*d*n))))/(b*c^2*n - a*c*d*n) + (log(x)*(n -
 1))/(a*c*n)
```

3.1030 $\int \frac{1}{x^2(a+bx^n)(c+dx^n)}\,dx$

3.1030.1 Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{1}{x^2\left(a+bx^n\right)\left(c+dx^n\right)}\,dx=-\frac{b\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)x}+\frac{d\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)x}$$

output
```
-b*hypergeom([1, -1/n],[(-1+n)/n],-b*x^n/a)/a/(-a*d+b*c)/x+d*hypergeom([1,
 -1/n],[(-1+n)/n],-d*x^n/c)/c/(-a*d+b*c)/x
```

3.1030.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\left(a+bx^n\right)\left(c+dx^n\right)}\,dx=\frac{bc\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},\frac{-1+n}{n},-\frac{bx^n}{a}\right)-ad\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},\frac{-1+n}{n},-\frac{dx^n}{c}\right)}{ac(-bc+ad)x}$$

input
```
Integrate[1/(x^2*(a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*c*Hypergeometric2F1[1, -n^(-1), (-1 + n)/n, -((b*x^n)/a)] - a*d*Hyperge
ometric2F1[1, -n^(-1), (-1 + n)/n, -((d*x^n)/c)])/(a*c*(-(b*c) + a*d)*x)
```

3.1030.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1010, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^n) (c+dx^n)} \, dx$$

$$\Big\downarrow \text{1010}$$

$$\frac{b \int \frac{1}{x^2 (bx^n+a)}dx}{bc-ad}-\frac{d \int \frac{1}{x^2 (dx^n+c)}dx}{bc-ad}$$

$$\Big\downarrow \text{888}$$

$$\frac{d \,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{dx^n}{c}\right)}{cx(bc-ad)}-\frac{b \,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax(bc-ad)}$$

input
```
Int[1/(x^2*(a + b*x^n)*(c + d*x^n)),x]
```

output
```
-((b*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(a*(b*c -
a*d)*x)) + (d*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((d*x^n)/c)])/(
c*(b*c - a*d)*x)
```

3.1030.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1010
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n,
m}, x] && NeQ[b*c - a*d, 0]
```

3.1030.4 Maple [F]

$$\int \frac{1}{x^2\,(a+b\,x^n)\,(c+d\,x^n)}dx$$

input
```
int(1/x^2/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(1/x^2/(a+b*x^n)/(c+d*x^n),x)
```

3.1030.5 Fricas [F]

$$\int \frac{1}{x^2\,(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b*d*x^2*x^(2*n) + (b*c + a*d)*x^2*x^n + a*c*x^2), x)
```

3.1030.6 Sympy [F]

$$\int \frac{1}{x^2\,(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{x^2\,(a+bx^n)\,(c+dx^n)}\,dx$$

input
```
integrate(1/x**2/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Integral(1/(x**2*(a + b*x**n)*(c + d*x**n)), x)
```

3.1030.7 Maxima [F]

$$\int \frac{1}{x^2 (a+bx^n) (c+dx^n)} \, dx = \int \frac{1}{(bx^n+a)(dx^n+c)x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)*x^2), x)
```

3.1030.8 Giac [F]

$$\int \frac{1}{x^2 (a+bx^n) (c+dx^n)} \, dx = \int \frac{1}{(bx^n+a)(dx^n+c)x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)*x^2), x)
```

3.1030.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^n) (c+dx^n)} \, dx = \int \frac{1}{x^2 \, (a+b\,x^n) \, (c+d\,x^n)} \, dx$$

input
```
int(1/(x^2*(a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(1/(x^2*(a + b*x^n)*(c + d*x^n)), x)
```

3.1031 $\int \frac{1}{x^3(a+bx^n)(c+dx^n)}\,dx$

3.1031.1 Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{1}{x^3(a+bx^n)(c+dx^n)}\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a(bc-ad)x^2} + \frac{d\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)x^2}$$

output
```
-1/2*b*hypergeom([1, -2/n],[(-2+n)/n],-b*x^n/a)/a/(-a*d+b*c)/x^2+1/2*d*hyp
ergeom([1, -2/n],[(-2+n)/n],-d*x^n/c)/c/(-a*d+b*c)/x^2
```

3.1031.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3(a+bx^n)(c+dx^n)}\,dx = \frac{bc\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},\frac{-2+n}{n},-\frac{bx^n}{a}\right) - ad\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},\frac{-2+n}{n},-\frac{dx^n}{c}\right)}{2ac(-bc+ad)x^2}$$

input
```
Integrate[1/(x^3*(a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*c*Hypergeometric2F1[1, -2/n, (-2 + n)/n, -((b*x^n)/a)] - a*d*Hypergeome
tric2F1[1, -2/n, (-2 + n)/n, -((d*x^n)/c)])/(2*a*c*(-(b*c) + a*d)*x^2)
```

3.1031.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1010, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^n)(c+dx^n)} \, dx$$
$$\Big\downarrow \; 1010$$
$$\frac{b\int \frac{1}{x^3(bx^n+a)}dx}{bc-ad} - \frac{d\int \frac{1}{x^3(dx^n+c)}dx}{bc-ad}$$
$$\Big\downarrow \; 888$$
$$\frac{d \operatorname{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)} - \frac{b \operatorname{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2(bc-ad)}$$

input
```
Int[1/(x^3*(a + b*x^n)*(c + d*x^n)),x]
```

output
```
-1/2*(b*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(a*(b*c -
a*d)*x^2) + (d*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((d*x^n)/c)])/(2*
c*(b*c - a*d)*x^2)
```

3.1031.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1010
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n,
m}, x] && NeQ[b*c - a*d, 0]
```

3.1031.4 Maple [F]

$$\int \frac{1}{x^{3} \left(a+b \,x^{n}\right) \left(c+d \,x^{n}\right)}d x$$

input
```
int(1/x^3/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(1/x^3/(a+b*x^n)/(c+d*x^n),x)
```

3.1031.5 Fricas [F]

$$\int \frac{1}{x^3 (a+bx^n) (c+dx^n)} \, dx = \int \frac{1}{(bx^n+a)(dx^n+c)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b*d*x^3*x^(2*n) + (b*c + a*d)*x^3*x^n + a*c*x^3), x)
```

3.1031.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a+bx^n) (c+dx^n)} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/x**3/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.1031.7 Maxima [F]

$$\int \frac{1}{x^3 (a+bx^n) (c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)*x^3), x)
```

3.1031.8 Giac [F]

$$\int \frac{1}{x^3 (a+bx^n) (c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)*x^3), x)
```

3.1031.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^n) (c+dx^n)}\,dx = \int \frac{1}{x^3\,(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

input
```
int(1/(x^3*(a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(1/(x^3*(a + b*x^n)*(c + d*x^n)), x)
```

3.1032 $\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx$

3.1032.1 Optimal result

Integrand size = 24, antiderivative size = 175

$$\begin{aligned}\int \frac{(ex)^m}{(a+bx^n)^2\,(c+dx^n)}\,dx &= \frac{b(ex)^{1+m}}{a(bc-ad)en\,(a+bx^n)}\\ &\quad+\frac{b(ad(1+m-2n)-bc(1+m-n))(ex)^{1+m}\operatorname{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2e(1+m)n}\\ &\quad+\frac{d^2(ex)^{1+m}\operatorname{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^2e(1+m)}\end{aligned}$$

output
```
b*(e*x)^(1+m)/a/(-a*d+b*c)/e/n/(a+b*x^n)+b*(a*d*(1+m-2*n)-b*c*(1+m-n))*(e*
x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/e/(
1+m)/n+d^2*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c/(-a*
d+b*c)^2/e/(1+m)
```

3.1032.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int \frac{(ex)^m}{(a+bx^n)^2\,(c+dx^n)}\,dx = \frac{x(ex)^m\left(\frac{b^2c-abd}{a^2n+abnx^n}+\frac{b(ad(1+m-2n)-bc(1+m-n))\operatorname{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a^2(1+m)n}+\frac{d^2\operatorname{Hypergeometric2F1}\left(1,\frac{1+m}{n},1-\right.}{c+cm}\right.}{(bc-ad)^2}$$

input
```
Integrate[(e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(x*(e*x)^m*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 + m - 2*n) -
b*c*(1 + m - n))*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/
a)])/(a^2*(1 + m)*n) + (d^2*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
 -((d*x^n)/c)])/(c + c*m)))/(b*c - a*d)^2
```

3.1032.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1006, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}dx$$

$$\Big\downarrow \text{1006}$$

$$\frac{b(ex)^{m+1}}{aen(bc-ad)(a+bx^n)}-\frac{\int \frac{(ex)^m(bd(m-n+1)x^n+bc(m-n+1)+adn)}{(bx^n+a)(dx^n+c)}dx}{an(bc-ad)}$$

$$\Big\downarrow \text{1067}$$

$$\frac{b(ex)^{m+1}}{aen(bc-ad)(a+bx^n)}-\frac{\int \left(\frac{b(bc(m-n+1)-ad(m-2n+1))(ex)^m}{(bc-ad)(bx^n+a)}+\frac{ad^2n(ex)^m}{(ad-bc)(dx^n+c)}\right)dx}{an(bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b(ex)^{m+1}}{aen(bc-ad)(a+bx^n)}-$$

$$\frac{-\frac{ad^2n(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)}-\frac{b(ex)^{m+1}(ad(m-2n+1)-bc(m-n+1))\,\text{Hypergeometric2F1}\Bigl(1,\frac{m+1}{n},\frac{m+n+1}{n},-}{ae(m+1)(bc-ad)}}{an(bc-ad)}$$

input
```
Int[(e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b*(e*x)^(1 + m))/(a*(b*c - a*d)*e*n*(a + b*x^n)) - (-((b*(a*d*(1 + m - 2*
n) - b*c*(1 + m - n))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m
 + n)/n, -((b*x^n)/a)])/(a*(b*c - a*d)*e*(1 + m))) - (a*d^2*n*(e*x)^(1 + m
)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c -
a*d)*e*(1 + m)))/(a*(b*c - a*d)*n)
```

3.1032.3.1 Defintions of rubi rules used

rule 1006
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1032.4 Maple [F]

$$\int \frac{(ex)^m}{(a+b\,x^n)^2\,(c+d\,x^n)}dx$$

input
```
int((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x)
```

3.1032.5 Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((e*x)^m/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a
*b*c + a^2*d)*x^n), x)
```

3.1032.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(ex)^m}{(a+bx^n)^2 (c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.1032.7 Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
d^2*e^m*integrate(x^m/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*x^n), x) + b*e^m*x*x^m/(a^2*b*c*n - a^3*d*n + (a*b^2*
c*n - a^2*b*d*n)*x^n) - (b^2*c*e^m*(m - n + 1) - a*b*d*e^m*(m - 2*n + 1))*
integrate(x^m/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n -
2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^n), x)
```

3.1032.8 Giac [F]

$$\int \frac{(ex)^m}{(a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^n + a)^2*(d*x^n + c)), x)
```

3.1032.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{(e\,x)^m}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

input
```
int((e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int((e*x)^m/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1033 $\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx$

3.1033.1 Optimal result

Integrand size = 22, antiderivative size = 142

$$\begin{aligned}\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx &= \frac{bx^3}{a(bc-ad)n(a+bx^n)} \\ &\quad + \frac{b(ad(3-2n)-bc(3-n))x^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a^2(bc-ad)^2n} \\ &\quad + \frac{d^2x^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{dx^n}{c}\right)}{3c(bc-ad)^2}\end{aligned}$$

output
```
b*x^3/a/(-a*d+b*c)/n/(a+b*x^n)+1/3*b*(a*d*(3-2*n)-b*c*(3-n))*x^3*hypergeom
([1, 3/n],[(3+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+1/3*d^2*x^3*hypergeom([1,
 3/n],[(3+n)/n],-d*x^n/c)/c/(-a*d+b*c)^2
```

3.1033.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{x^3\left(bc(ad(3-2n)+bc(-3+n))(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)+a\big(3bc(bc-ad)+ad^2}{3a^2c(bc-ad)^2n(a+bx^n)}$$

input
```
Integrate[x^2/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(x^3*(b*c*(a*d*(3 - 2*n) + b*c*(-3 + n))*(a + b*x^n)*Hypergeometric2F1[1,
3/n, (3 + n)/n, -((b*x^n)/a)] + a*(3*b*c*(b*c - a*d) + a*d^2*n*(a + b*x^n)
*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])))/(3*a^2*c*(b*c - a*d
)^2*n*(a + b*x^n))
```

3.1033.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1006, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}dx$$

$$\Big\downarrow \text{1006}$$

$$\frac{bx^3}{an(bc-ad)(a+bx^n)}-\frac{\int \frac{x^2(bd(3-n)x^n+bc(3-n)+adn)}{(bx^n+a)(dx^n+c)}dx}{an(bc-ad)}$$

$$\Big\downarrow \text{1067}$$

$$\frac{bx^3}{an(bc-ad)(a+bx^n)}-\frac{\int \left(\frac{b(bc(3-n)-ad(3-2n))x^2}{(bc-ad)(bx^n+a)}+\frac{ad^2nx^2}{(ad-bc)(dx^n+c)}\right)dx}{an(bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx^3}{an(bc-ad)(a+bx^n)}-\frac{-\frac{ad^2nx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{dx^n}{c}\right)}{3c(bc-ad)}-\frac{bx^3(ad(3-2n)-bc(3-n))\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3a(bc-ad)}}{an(bc-ad)}$$

input
```
Int[x^2/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b*x^3)/(a*(b*c - a*d)*n*(a + b*x^n)) - (-1/3*(b*(a*d*(3 - 2*n) - b*c*(3 -
 n))*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(a*(b*c - a*d
)) - (a*d^2*n*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*c
*(b*c - a*d)))/(a*(b*c - a*d)*n)
```

3.1033.3.1 Defintions of rubi rules used

rule 1006
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1033.4 Maple [F]

$$\int \frac{x^2}{(a+b\,x^n)^2\,(c+d\,x^n)}dx$$

input
```
int(x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

3.1033.5 Fricas [F]

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx=\int \frac{x^2}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(x^2/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c
 + a^2*d)*x^n), x)
```

3.1033.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**2/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.1033.7 Maxima [F]

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx=\int \frac{x^2}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
b*x^3/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) + d^2*integrate(
x^2/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)*x^n), x) - (a*b*d*(2*n - 3) - b^2*c*(n - 3))*integrate(x^2/(a^2*b^2*c^2
*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^
2*n)*x^n), x)
```

3.1033.8 Giac [F]

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx=\int \frac{x^2}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^n + a)^2*(d*x^n + c)), x)
```

3.1033.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx=\int \frac{x^2}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

input
```
int(x^2/((a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(x^2/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1034 $\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx$

3.1034.1 Optimal result

Integrand size = 20, antiderivative size = 143

$$\begin{aligned}\int \frac{x}{(a+bx^n)^2\,(c+dx^n)}\,dx &= \frac{bx^2}{a(bc-ad)n\,(a+bx^n)} \\ &\quad + \frac{b(2ad(1-n)-bc(2-n))x^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a^2(bc-ad)^2n} \\ &\quad + \frac{d^2x^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)^2}\end{aligned}$$

output
```
b*x^2/a/(-a*d+b*c)/n/(a+b*x^n)+1/2*b*(2*a*d*(1-n)-b*c*(2-n))*x^2*hypergeom
([1, 2/n],[(2+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+1/2*d^2*x^2*hypergeom([1,
 2/n],[(2+n)/n],-d*x^n/c)/c/(-a*d+b*c)^2
```

3.1034.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a+bx^n)^2\,(c+dx^n)}\,dx = \frac{x^2\big(bc(bc(-2+n)-2ad(-1+n))\,(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)+a\big(2bc(bc-ad)+ad}{2a^2c(bc-ad)^2n\,(a+bx^n)}$$

input
```
Integrate[x/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(x^2*(b*c*(b*c*(-2 + n) - 2*a*d*(-1 + n))*(a + b*x^n)*Hypergeometric2F1[1,
 2/n, (2 + n)/n, -((b*x^n)/a)] + a*(2*b*c*(b*c - a*d) + a*d^2*n*(a + b*x^n
)*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])))/(2*a^2*c*(b*c - a*
d)^2*n*(a + b*x^n))
```

3.1034.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1006, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^n)^2(c+dx^n)}dx$$

$$\Big\downarrow \text{1006}$$

$$\frac{bx^2}{an(bc-ad)(a+bx^n)}-\frac{\int \frac{x(bd(2-n)x^n+bc(2-n)+adn)}{(bx^n+a)(dx^n+c)}dx}{an(bc-ad)}$$

$$\Big\downarrow \text{1067}$$

$$\frac{bx^2}{an(bc-ad)(a+bx^n)}-\frac{\int \left(\frac{anxd^2}{(ad-bc)(dx^n+c)}+\frac{b(bc(2-n)-2ad(1-n))x}{(bc-ad)(bx^n+a)}\right)dx}{an(bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx^2}{an(bc-ad)(a+bx^n)}-\frac{-\frac{ad^2nx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)}-\frac{bx^2(2ad(1-n)-bc(2-n))\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a(bc-ad)}}{an(bc-ad)}$$

input
```
Int[x/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b*x^2)/(a*(b*c - a*d)*n*(a + b*x^n)) - (-1/2*(b*(2*a*d*(1 - n) - b*c*(2 -
 n))*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(a*(b*c - a*d
)) - (a*d^2*n*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*c
*(b*c - a*d)))/(a*(b*c - a*d)*n)
```

3.1034.3.1 Defintions of rubi rules used

rule 1006
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1034.4 Maple [F]

$$\int \frac{x}{\left(a+b\,x^{n}\right)^{2}\left(c+d\,x^{n}\right)}dx$$

input
```
int(x/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(x/(a+b*x^n)^2/(c+d*x^n),x)
```

3.1034.5 Fricas [F]

$$\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{x}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(x/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
 a^2*d)*x^n), x)
```

3.1034.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.1034.7 Maxima [F]

$$\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{x}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
d^2*integrate(x/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)*x^n), x) + b*x^2/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*
d*n)*x^n) - (2*a*b*d*(n - 1) - b^2*c*(n - 2))*integrate(x/(a^2*b^2*c^2*n -
 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)
*x^n), x)
```

3.1034.8 Giac [F]

$$\int \frac{x}{(a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{x}{(bx^n+a)^2(dx^n+c)} \, dx$$

input
```
integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x/((b*x^n + a)^2*(d*x^n + c)), x)
```

3.1034.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{x}{(a+b\,x^n)^2\,(c+d\,x^n)} \, dx$$

input
```
int(x/((a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(x/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1035 $\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx$

3.1035.1 Optimal result

Integrand size = 19, antiderivative size = 122

$$\begin{aligned}\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx &= \frac{bx}{a(bc-ad)n(a+bx^n)} \\ &\quad + \frac{b(ad(1-2n)-bc(1-n))x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n} \\ &\quad + \frac{d^2x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^2}\end{aligned}$$

output
```
b*x/a/(-a*d+b*c)/n/(a+b*x^n)+b*(a*d*(1-2*n)-b*c*(1-n))*x*hypergeom([1, 1/n
],[1+1/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+d^2*x*hypergeom([1, 1/n],[1+1/n],-d
*x^n/c)/c/(-a*d+b*c)^2
```

3.1035.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{x\left(\frac{b^2c-abd}{a^2n+abnx^n}+\frac{b(ad(1-2n)+bc(-1+n))\operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2n}+\frac{d^2\operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c}\right)}{(bc-ad)^2}$$

input
```
Integrate[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(x*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 - 2*n) + b*c*(-1 + n)
)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n) + (d^2*H
ypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c))/(b*c - a*d)^2
```

3.1035.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {931, 1020, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{an(bc-ad)(a+bx^n)}-\frac{\int \frac{bd(1-n)x^n+adn+b(c-cn)}{(bx^n+a)(dx^n+c)}dx}{an(bc-ad)}$$

$$\Big\downarrow \text{1020}$$

$$\frac{bx}{an(bc-ad)(a+bx^n)}-\frac{-\frac{ad^2n\int \frac{1}{dx^n+c}dx}{bc-ad}-\frac{b(ad(1-2n)-bc(1-n))\int \frac{1}{bx^n+a}dx}{bc-ad}}{an(bc-ad)}$$

$$\Big\downarrow \text{778}$$

$$\frac{bx}{an(bc-ad)(a+bx^n)}-\frac{-\frac{ad^2nx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}-\frac{bx(ad(1-2n)-bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}}{an(bc-ad)}$$

input
```
Int[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) - (-((b*(a*d*(1 - 2*n) - b*c*(1 - n))*
x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)))
 - (a*d^2*n*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(
b*c - a*d)))/(a*(b*c - a*d)*n)
```

3.1035.3.1 Defintions of rubi rules used

rule 778

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 931

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

3.1035.4 Maple [F]

$$\int \frac{1}{\left(a+b\,x^{n}\right)^{2}\left(c+d\,x^{n}\right)}dx$$

input

```
int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

output

```
int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

3.1035.5 Fricas [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx=\int \frac{1}{(bx^n+a)^2(dx^n+c)}\,dx$$

input

```
integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
 a^2*d)*x^n), x)
```

3.1035.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.1035.7 Maxima [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
d^2*integrate(1/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)*x^n), x) - (a*b*d*(2*n - 1) - b^2*c*(n - 1))*integrate(1/(a
^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n
+ a^3*b*d^2*n)*x^n), x) + b*x/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*
n)*x^n)
```

3.1035.8 Giac [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)), x)
```

3.1035.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx=\int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

input
```
int(1/((a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(1/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1036 $\int \frac{1}{x(a+bx^n)^2(c+dx^n)}\,dx$

3.1036.1 Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = \frac{b}{a(bc-ad)n\left(a+bx^n\right)}+\frac{\log(x)}{a^2c} - \frac{b(bc-2ad)\log\left(a+bx^n\right)}{a^2(bc-ad)^2n}-\frac{d^2\log\left(c+dx^n\right)}{c(bc-ad)^2n}$$

output

```
b/a/(-a*d+b*c)/n/(a+b*x^n)+ln(x)/a^2/c-b*(-2*a*d+b*c)*ln(a+b*x^n)/a^2/(-a*
d+b*c)^2/n-d^2*ln(c+d*x^n)/c/(-a*d+b*c)^2/n
```

3.1036.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = -\frac{b}{a(-bc+ad)n\left(a+bx^n\right)}+\frac{\log\left(x^n\right)}{a^2cn} + \frac{b(-bc+2ad)\log\left(a+bx^n\right)}{a^2(-bc+ad)^2n}-\frac{d^2\log\left(c+dx^n\right)}{c(bc-ad)^2n}$$

input

```
Integrate[1/(x*(a + b*x^n)^2*(c + d*x^n)),x]
```

output

```
-(b/(a*(-(b*c) + a*d)*n*(a + b*x^n))) + Log[x^n]/(a^2*c*n) + (b*(-(b*c) +
2*a*d)*Log[a + b*x^n])/(a^2*(-(b*c) + a*d)^2*n) - (d^2*Log[c + d*x^n])/(c*
(b*c - a*d)^2*n)
```

3.1036.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^n)^2(c+dx^n)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{\int \frac{x^{-n}}{(bx^n+a)^2(dx^n+c)}dx^n}{n}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{\int \left(\frac{x^{-n}}{a^2c}+\frac{b^2(2ad-bc)}{a^2(ad-bc)^2(bx^n+a)}-\frac{d^3}{c(bc-ad)^2(dx^n+c)}+\frac{b^2}{a(ad-bc)(bx^n+a)^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{b(bc-2ad)\log(a+bx^n)}{a^2(bc-ad)^2}+\frac{\log(x^n)}{a^2c}-\frac{d^2\log(c+dx^n)}{c(bc-ad)^2}+\frac{b}{a(bc-ad)(a+bx^n)}}{n}$$

input
```
Int[1/(x*(a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b/(a*(b*c - a*d)*(a + b*x^n)) + Log[x^n]/(a^2*c) - (b*(b*c - 2*a*d)*Log[a
 + b*x^n])/(a^2*(b*c - a*d)^2) - (d^2*Log[c + d*x^n])/(c*(b*c - a*d)^2))/n
```

3.1036.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1036.4 Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{\ln\left(x^{n}\right)}{a^{2} c}-\frac{d^{2} \ln\left(c+d\,x^{n}\right)}{\left(a d-b c\right)^{2} c}-\frac{b}{\left(a d-b c\right) a \left(a+b\,x^{n}\right)}+\frac{b \left(2 a d-b c\right) \ln\left(a+b\,x^{n}\right)}{a^{2} \left(a d-b c\right)^{2}}}{n}$
default	$\frac{\frac{\ln\left(x^{n}\right)}{a^{2} c}-\frac{d^{2} \ln\left(c+d\,x^{n}\right)}{\left(a d-b c\right)^{2} c}-\frac{b}{\left(a d-b c\right) a \left(a+b\,x^{n}\right)}+\frac{b \left(2 a d-b c\right) \ln\left(a+b\,x^{n}\right)}{a^{2} \left(a d-b c\right)^{2}}}{n}$
norman	$\frac{\frac{b^{2} {\mathrm e}^{n \ln\left(x\right)}}{n\,a^{2} \left(a d-b c\right)}+\frac{\ln\left(x\right)}{a c}+\frac{b \ln\left(x\right) {\mathrm e}^{n \ln\left(x\right)}}{a^{2} c}}{a+b\,{\mathrm e}^{n \ln\left(x\right)}}+\frac{b \left(2 a d-b c\right) \ln\left(a+b\,{\mathrm e}^{n \ln\left(x\right)}\right)}{\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) a^{2} n}-\frac{d^{2} \ln\left(c+d\,{\mathrm e}^{n \ln\left(x\right)}\right)}{c n \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}$
parallelrisch	$\frac{\ln\left(x\right) x^{n} a^{2} b\,d^{2} n-2 \ln\left(x\right) x^{n} a\,b^{2} c d n+\ln\left(x\right) x^{n} b^{3} c^{2} n+\ln\left(x\right) a^{3} d^{2} n-2 \ln\left(x\right) a^{2} b c d n+\ln\left(x\right) a\,b^{2} c^{2} n+2 \ln\left(a+b\,x^{n}\right) x^{n} a\,b^{2} c d-\mathrm{l}}{\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right.}$
risch	$-\frac{2 \ln\left(x\right) b d}{\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) a}+\frac{\ln\left(x\right) b^{2} c}{\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) a^{2}}+\frac{\ln\left(x\right) d^{2}}{c \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}-\frac{b}{\left(a d-b c\right) a n \left(a+b\,x^{n}\right)}+\frac{2 b \ln\left(x^{n}\right.}{\left(a^{2} d^{2}-2 a b c\right.}$

input
```
int(1/x/(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/a^2/c*ln(x^n)-d^2/(a*d-b*c)^2/c*ln(c+d*x^n)-b/(a*d-b*c)/a/(a+b*x^n)
+b*(2*a*d-b*c)/a^2/(a*d-b*c)^2*ln(a+b*x^n))
```

3.1036.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(101) = 202.

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.22

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx$$
$$=\frac{ab^2c^2-a^2bcd+(b^3c^2-2\,ab^2cd+a^2bd^2)nx^n\log\left(x\right)+(ab^2c^2-2\,a^2bcd+a^3d^2)n\log\left(x\right)-(ab^2c^2-2\,a^2bc}{(a^2b^3c^3-2\,a^3b^2c^2d+a^4bcd^2)nx^n+(a^3b^2c^3-2\,a^4}$$

input
```
integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
(a*b^2*c^2 - a^2*b*c*d + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*n*x^n*log(x)
+ (a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*n*log(x) - (a*b^2*c^2 - 2*a^2*b*c*d
+ (b^3*c^2 - 2*a*b^2*c*d)*x^n)*log(b*x^n + a) - (a^2*b*d^2*x^n + a^3*d^2)*
log(d*x^n + c))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*n*x^n + (a^
3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*n)
```

3.1036.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/x/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.1036.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.50

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx=-\frac{d^2\log\left(\frac{dx^n+c}{d}\right)}{b^2c^3n-2\,abc^2dn+a^2cd^2n}-\frac{\left(b^2c-2\,abd\right)\log\left(\frac{bx^n+a}{b}\right)}{a^2b^2c^2n-2\,a^3bcdn+a^4d^2n}+\frac{b}{a^2bcn-a^3dn+\left(ab^2cn-a^2bdn\right)x^n}+\frac{\log\left(x\right)}{a^2c}$$

input
```
integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
-d^2*log((d*x^n + c)/d)/(b^2*c^3*n - 2*a*b*c^2*d*n + a^2*c*d^2*n) - (b^2*c
 - 2*a*b*d)*log((b*x^n + a)/b)/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n)
 + b/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) + log(x)/(a^2*c)
```

3.1036.8 Giac [F]

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx=\int \frac{1}{(bx^n+a)^2(dx^n+c)x}\,dx$$

input
```
integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)*x), x)
```

3.1036.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx=\int \frac{1}{x\left(a+b\,x^n\right)^2\,\left(c+d\,x^n\right)}\,dx$$

input
```
int(1/(x*(a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(1/(x*(a + b*x^n)^2*(c + d*x^n)), x)
```

3.1037 $\int \frac{1}{x^2(a+bx^n)^2(c+dx^n)}\,dx$

3.1037.1 Optimal result

Integrand size = 22, antiderivative size = 142

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx &= \frac{b}{a(bc-ad)nx\left(a+bx^n\right)} \\ &\quad-\frac{b(bc(1+n)-ad(1+2n))\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2nx} \\ &\quad-\frac{d^2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^2x}\end{aligned}$$

output
```
b/a/(-a*d+b*c)/n/x/(a+b*x^n)-b*(b*c*(1+n)-a*d*(1+2*n))*hypergeom([1, -1/n]
,[(-1+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n/x-d^2*hypergeom([1, -1/n],[(-1+n)
/n],-d*x^n/c)/c/(-a*d+b*c)^2/x
```

3.1037.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = \frac{bc(-bc(1+n)+ad(1+2n))\left(a+bx^n\right)\text{Hypergeometric2F1}\left(1,-\frac{1}{n},\frac{-1+n}{n},-\frac{bx^n}{a}\right)-a\big(bc(-bc+ad)+ad^2}{a^2c(bc-ad)^2nx\left(a+bx^n\right)}$$

input
```
Integrate[1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b*c*(-(b*c*(1 + n)) + a*d*(1 + 2*n))*(a + b*x^n)*Hypergeometric2F1[1, -n^
(-1), (-1 + n)/n, -((b*x^n)/a)] - a*(b*c*(-(b*c) + a*d) + a*d^2*n*(a + b*x
^n)*Hypergeometric2F1[1, -n^(-1), (-1 + n)/n, -((d*x^n)/c)]))/(a^2*c*(b*c
- a*d)^2*n*x*(a + b*x^n))
```

3.1037.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1006, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)}dx$$

$$\Big\downarrow \text{1006}$$

$$\frac{b}{anx(bc-ad)(a+bx^n)}-\frac{\int \frac{-bd(n+1)x^n+adn-bc(n+1)}{x^2(bx^n+a)(dx^n+c)}dx}{an(bc-ad)}$$

$$\Big\downarrow \text{1067}$$

$$\frac{b}{anx(bc-ad)(a+bx^n)}-\frac{\int \left(\frac{and^2}{(ad-bc)x^2(dx^n+c)}+\frac{b(ad(2n+1)-bc(n+1))}{(bc-ad)x^2(bx^n+a)}\right)dx}{an(bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b}{anx(bc-ad)(a+bx^n)}-\frac{\frac{ad^2n\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{dx^n}{c}\right)}{cx(bc-ad)}+\frac{b(bc(n+1)-ad(2n+1))\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax(bc-ad)}}{an(bc-ad)}$$

input
```
Int[1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
b/(a*(b*c - a*d)*n*x*(a + b*x^n)) - ((b*(b*c*(1 + n) - a*d*(1 + 2*n))*Hype
rgeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(a*(b*c - a*d)*x) +
 (a*d^2*n*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((d*x^n)/c)])/(c*(b
*c - a*d)*x))/(a*(b*c - a*d)*n)
```

3.1037.3.1 Defintions of rubi rules used

rule 1006
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1037.4 Maple [F]

$$\int \frac{1}{x^{2} \left(a+b\,x^{n}\right)^{2} \left(c+d\,x^{n}\right)}dx$$

input
```
int(1/x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(1/x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

3.1037.5 Fricas [F]

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b^2*d*x^2*x^(3*n) + a^2*c*x^2 + (b^2*c + 2*a*b*d)*x^2*x^(2*n)
+ (2*a*b*c + a^2*d)*x^2*x^n), x)
```

3.1037.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/x**2/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.1037.7 Maxima [F]

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
d^2*integrate(1/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^2*x^n + (b^2*c^3 -
2*a*b*c^2*d + a^2*c*d^2)*x^2), x) - (a*b*d*(2*n + 1) - b^2*c*(n + 1))*inte
grate(1/((a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^2*x^n + (a^2*b^2*
c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n)*x^2), x) + b/((a*b^2*c*n - a^2*b*d*n)*x
*x^n + (a^2*b*c*n - a^3*d*n)*x)
```

3.1037.8 Giac [F]

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{1}{(bx^n+a)^2 (dx^n+c) x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)*x^2), x)
```

3.1037.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{1}{x^2 \left(a+b\,x^n\right)^2 \left(c+d\,x^n\right)} \, dx$$

input
```
int(1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(1/(x^2*(a + b*x^n)^2*(c + d*x^n)), x)
```

3.1038 $\int \frac{1}{x^3(a+bx^n)^2(c+dx^n)}\,dx$

3.1038.1 Optimal result

Integrand size = 22, antiderivative size = 145

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx &= \frac{b}{a(bc-ad)nx^2\left(a+bx^n\right)}\\ &\quad+\frac{b(2ad(1+n)-bc(2+n))\operatorname{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^2(bc-ad)^2nx^2}\\ &\quad-\frac{d^2\operatorname{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)^2x^2}\end{aligned}$$

output
```
b/a/(-a*d+b*c)/n/x^2/(a+b*x^n)+1/2*b*(2*a*d*(1+n)-b*c*(2+n))*hypergeom([1,
 -2/n],[(-2+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n/x^2-1/2*d^2*hypergeom([1, -
2/n],[(-2+n)/n],-d*x^n/c)/c/(-a*d+b*c)^2/x^2
```

3.1038.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = \frac{bc(2ad(1+n)-bc(2+n))\left(a+bx^n\right)\operatorname{Hypergeometric2F1}\left(1,-\frac{2}{n},\frac{-2+n}{n},-\frac{bx^n}{a}\right)-a\big(2bc(-bc+ad)+ad^2}{2a^2c(bc-ad)^2nx^2\left(a+bx^n\right)}$$

input
```
Integrate[1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b*c*(2*a*d*(1 + n) - b*c*(2 + n))*(a + b*x^n)*Hypergeometric2F1[1, -2/n,
(-2 + n)/n, -((b*x^n)/a)] - a*(2*b*c*(-(b*c) + a*d) + a*d^2*n*(a + b*x^n)*
Hypergeometric2F1[1, -2/n, (-2 + n)/n, -((d*x^n)/c)]))/(2*a^2*c*(b*c - a*d
)^2*n*x^2*(a + b*x^n))
```

3.1038.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1006, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)} \, dx$$

$$\Big\downarrow \; 1006$$

$$\frac{b}{anx^2(bc-ad)(a+bx^n)}-\frac{\int \frac{-bd(n+2)x^n+adn-bc(n+2)}{x^3(bx^n+a)(dx^n+c)}dx}{an(bc-ad)}$$

$$\Big\downarrow \; 1067$$

$$\frac{b}{anx^2(bc-ad)(a+bx^n)}-\frac{\int \left(\frac{and^2}{(ad-bc)x^3(dx^n+c)}+\frac{b(2ad(n+1)-bc(n+2))}{(bc-ad)x^3(bx^n+a)}\right)dx}{an(bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$\frac{b}{anx^2(bc-ad)(a+bx^n)}-\frac{\frac{ad^2n\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)}-\frac{b(2ad(n+1)-bc(n+2))\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2(bc-ad)}}{an(bc-ad)}$$

input
```
Int[1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
b/(a*(b*c - a*d)*n*x^2*(a + b*x^n)) - (-1/2*(b*(2*a*d*(1 + n) - b*c*(2 + n
))*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(a*(b*c - a*d)*
x^2) + (a*d^2*n*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((d*x^n)/c)])/(2
*c*(b*c - a*d)*x^2))/(a*(b*c - a*d)*n)
```

3.1038.3.1 Defintions of rubi rules used

rule 1006
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1038.4 Maple [F]

$$\int \frac{1}{x^{3} \left(a+b\,x^{n}\right)^{2} \left(c+d\,x^{n}\right)}dx$$

input
```
int(1/x^3/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(1/x^3/(a+b*x^n)^2/(c+d*x^n),x)
```

3.1038.5 Fricas [F]

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{1}{(bx^n+a)^2 (dx^n+c)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b^2*d*x^3*x^(3*n) + a^2*c*x^3 + (b^2*c + 2*a*b*d)*x^3*x^(2*n)
+ (2*a*b*c + a^2*d)*x^3*x^n), x)
```

3.1038.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/x**3/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.1038.7 Maxima [F]

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{1}{(bx^n+a)^2 (dx^n+c)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
d^2*integrate(1/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^3*x^n + (b^2*c^3 -
2*a*b*c^2*d + a^2*c*d^2)*x^3), x) + (b^2*c*(n + 2) - 2*a*b*d*(n + 1))*inte
grate(1/((a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^3*x^n + (a^2*b^2*
c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n)*x^3), x) + b/((a*b^2*c*n - a^2*b*d*n)*x
^2*x^n + (a^2*b*c*n - a^3*d*n)*x^2)
```

3.1038.8 Giac [F]

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)*x^3), x)
```

3.1038.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{1}{x^3\,(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

input
```
int(1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(1/(x^3*(a + b*x^n)^2*(c + d*x^n)), x)
```

3.1039 $\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx$

3.1039.1 Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx = -\frac{(bc-ad)^3x^n}{d^4n} + \frac{b(b^2c^2-3abcd+3a^2d^2)\,x^{2n}}{2d^3n} - \frac{b^2(bc-3ad)x^{3n}}{3d^2n} + \frac{b^3x^{4n}}{4dn} + \frac{c(bc-ad)^3\log\left(c+dx^n\right)}{d^5n}$$

output
```
-(-a*d+b*c)^3*x^n/d^4/n+1/2*b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)*x^(2*n)/d^3/n-
1/3*b^2*(-3*a*d+b*c)*x^(3*n)/d^2/n+1/4*b^3*x^(4*n)/d/n+c*(-a*d+b*c)^3*ln(c
+d*x^n)/d^5/n
```

3.1039.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx = \frac{dx^n(12a^3d^3+18a^2bd^2(-2c+dx^n)+6ab^2d(6c^2-3cdx^n+2d^2x^{2n})+b^3(-12c^3+6c^2dx^n-4cd^2x^{2n}+3d^3}{12d^5n}$$

input
```
Integrate[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

output
```
(d*x^n*(12*a^3*d^3 + 18*a^2*b*d^2*(-2*c + d*x^n) + 6*a*b^2*d*(6*c^2 - 3*c*
d*x^n + 2*d^2*x^(2*n)) + b^3*(-12*c^3 + 6*c^2*d*x^n - 4*c*d^2*x^(2*n) + 3*
d^3*x^(3*n))) + 12*c*(b*c - a*d)^3*Log[c + d*x^n])/(12*d^5*n)
```

3.1039.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}(a+bx^n)^3}{c+dx^n}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{\int \frac{x^n(bx^n+a)^3}{dx^n+c}dx^n}{n}$$

$$\Big\downarrow \; 86$$

$$\frac{\int \left(\frac{b(b^2c^2-3abdc+3a^2d^2)x^n}{d^3}-\frac{b^2(bc-3ad)x^{2n}}{d^2}+\frac{b^3x^{3n}}{d}+\frac{(ad-bc)^3}{d^4}+\frac{c(bc-ad)^3}{d^4(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{bx^{2n}(3a^2d^2-3abcd+b^2c^2)}{2d^3}-\frac{b^2x^{3n}(bc-3ad)}{3d^2}+\frac{c(bc-ad)^3\log(c+dx^n)}{d^5}-\frac{x^n(bc-ad)^3}{d^4}+\frac{b^3x^{4n}}{4d}}{n}$$

input
```
Int[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

output
```
(-(((b*c - a*d)^3*x^n)/d^4) + (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(2*n)
)/(2*d^3) - (b^2*(b*c - 3*a*d)*x^(3*n))/(3*d^2) + (b^3*x^(4*n))/(4*d) + (c
*(b*c - a*d)^3*Log[c + d*x^n])/d^5)/n
```

3.1039.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1039.4 Maple [A] (verified)

Time = 5.41 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.45

method	result
norman	$\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\mathrm{e}^{n\ln(x)}}{d^4n}+\frac{b^3\mathrm{e}^{4n\ln(x)}}{4dn}+\frac{b\left(3a^2d^2-3abcd+b^2c^2\right)\mathrm{e}^{2n\ln(x)}}{2d^3n}+\frac{b^2(3ad-bc)\mathrm{e}^{3n\ln(x)}}{3d^2n}-\frac{c(a^3d^3-3a}{}$
risch	$\frac{b^3x^{4n}}{4dn}+\frac{b^2x^{3n}a}{dn}-\frac{b^3x^{3n}c}{3d^2n}+\frac{3b\,x^{2n}a^2}{2dn}-\frac{3b^2x^{2n}ac}{2d^2n}+\frac{b^3x^{2n}c^2}{2d^3n}+\frac{x^na^3}{dn}-\frac{3x^na^2bc}{d^2n}+\frac{3x^na\,b^2c^2}{d^3n}-\frac{x^nb^3c^3}{d^4n}-\frac{c\ln(x^n+}{d^2}$

input

```
int(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output

```
1/d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/n*exp(n*ln(x))+1/4*b^3
/d/n*exp(n*ln(x))^4+1/2*b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/d^3/n*exp(n*ln(x))
^2+1/3*b^2*(3*a*d-b*c)/d^2/n*exp(n*ln(x))^3-c*(a^3*d^3-3*a^2*b*c*d^2+3*a*b
^2*c^2*d-b^3*c^3)/d^5/n*ln(c+d*exp(n*ln(x)))
```

3.1039.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.36

$$\int\frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx$$
$$=\frac{3\,b^3d^4x^{4\,n}-4\,(b^3cd^3-3\,ab^2d^4)x^{3\,n}+6\,(b^3c^2d^2-3\,ab^2cd^3+3\,a^2bd^4)x^{2\,n}-12\,(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd}{12\,d^5n}$$

input

```
integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

output
```
1/12*(3*b^3*d^4*x^(4*n) - 4*(b^3*c*d^3 - 3*a*b^2*d^4)*x^(3*n) + 6*(b^3*c^2
*d^2 - 3*a*b^2*c*d^3 + 3*a^2*b*d^4)*x^(2*n) - 12*(b^3*c^3*d - 3*a*b^2*c^2*
d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*x^n + 12*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b
*c^2*d^2 - a^3*c*d^3)*log(d*x^n + c))/(d^5*n)
```

3.1039.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(114) = 228$.

Time = 3.83 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.67

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx$$

$$= \begin{cases} \frac{(a+b)^3\log(x)}{c} \\ \frac{\frac{a^3xx^{2n-1}}{2n}+\frac{a^2bxx^nx^{2n-1}}{n}+\frac{3ab^2xx^{2n}x^{2n-1}}{4n}+\frac{b^3xx^{3n}x^{2n-1}}{5n}}{c} \\ \frac{(a+b)^3\log(x)}{c+d} \\ -\frac{a^3c\log\left(\frac{c}{d}+x^n\right)}{d^2n}+\frac{a^3x^n}{dn}+\frac{3a^2bc^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{3a^2bcx^n}{d^2n}+\frac{3a^2bx^{2n}}{2dn}-\frac{3ab^2c^3\log\left(\frac{c}{d}+x^n\right)}{d^4n}+\frac{3ab^2c^2x^n}{d^3n}-\frac{3ab^2cx^{2n}}{2d^2n}+\frac{ab^2x^{3n}}{dn} \end{cases}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**3/(c+d*x**n),x)
```

output
```
Piecewise(((a + b)**3*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**3*x*x**(2*n - 1
)/(2*n) + a**2*b*x*x**n*x**(2*n - 1)/n + 3*a*b**2*x*x**(2*n)*x**(2*n - 1)/
(4*n) + b**3*x*x**(3*n)*x**(2*n - 1)/(5*n))/c, Eq(d, 0)), ((a + b)**3*log(
x)/(c + d), Eq(n, 0)), (-a**3*c*log(c/d + x**n)/(d**2*n) + a**3*x**n/(d*n)
 + 3*a**2*b*c**2*log(c/d + x**n)/(d**3*n) - 3*a**2*b*c*x**n/(d**2*n) + 3*a
**2*b*x**(2*n)/(2*d*n) - 3*a*b**2*c**3*log(c/d + x**n)/(d**4*n) + 3*a*b**2
*c**2*x**n/(d**3*n) - 3*a*b**2*c*x**(2*n)/(2*d**2*n) + a*b**2*x**(3*n)/(d*
n) + b**3*c**4*log(c/d + x**n)/(d**5*n) - b**3*c**3*x**n/(d**4*n) + b**3*c
**2*x**(2*n)/(2*d**3*n) - b**3*c*x**(3*n)/(3*d**2*n) + b**3*x**(4*n)/(4*d*
n), True))
```

3.1039.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.78

$$\begin{aligned}\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx &= a^3\left(\frac{x^n}{dn}-\frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right)\\ &\quad+\frac{1}{12}\,b^3\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n}+\frac{3\,d^3x^{4\,n}-4\,cd^2x^{3\,n}+6\,c^2dx^{2\,n}-12\,c^3x^n}{d^4n}\right)\\ &\quad-\frac{1}{2}\,ab^2\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+6\,c^2x^n}{d^3n}\right)\\ &\quad+\frac{3}{2}\,a^2b\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)\end{aligned}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

output
```
a^3*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) + 1/12*b^3*(12*c^4*log((d*x
^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3*n) + 6*c^2*d*x^(2*n) -
12*c^3*x^n)/(d^4*n)) - 1/2*a*b^2*(6*c^3*log((d*x^n + c)/d)/(d^4*n) - (2*d^
2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 3/2*a^2*b*(2*c^2*log((d*
x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

3.1039.8 Giac [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx=\int \frac{(bx^n+a)^3x^{2\,n-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^3*x^(2*n - 1)/(d*x^n + c), x)
```

3.1039.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx = \int \frac{x^{2\,n-1}\,(a+b\,x^n)^3}{c+d\,x^n}\,dx$$

input
```
int((x^(2*n - 1)*(a + b*x^n)^3)/(c + d*x^n),x)
```

output
```
int((x^(2*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)
```

3.1040 $\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx$

3.1040.1 Optimal result

Integrand size = 26, antiderivative size = 90

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx=\frac{(bc-ad)^2x^n}{d^3n}-\frac{b(bc-2ad)x^{2n}}{2d^2n}+\frac{b^2x^{3n}}{3dn}-\frac{c(bc-ad)^2\log(c+dx^n)}{d^4n}$$

output
```
(-a*d+b*c)^2*x^n/d^3/n-1/2*b*(-2*a*d+b*c)*x^(2*n)/d^2/n+1/3*b^2*x^(3*n)/d/
n-c*(-a*d+b*c)^2*ln(c+d*x^n)/d^4/n
```

3.1040.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx =\frac{dx^n(6a^2d^2+6abd(-2c+dx^n)+b^2(6c^2-3cdx^n+2d^2x^{2n}))-6c(bc-ad)^2\log(c+dx^n)}{6d^4n}$$

input
```
Integrate[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

output
```
(d*x^n*(6*a^2*d^2 + 6*a*b*d*(-2*c + d*x^n) + b^2*(6*c^2 - 3*c*d*x^n + 2*d^
2*x^(2*n))) - 6*c*(b*c - a*d)^2*Log[c + d*x^n])/(6*d^4*n)
```

3.1040.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}(a+bx^n)^2}{c+dx^n} \, dx$$
$$\Big\downarrow \ 948$$
$$\frac{\int \frac{x^n(bx^n+a)^2}{dx^n+c}dx^n}{n}$$
$$\Big\downarrow \ 86$$
$$\frac{\int \left(-\frac{b(bc-2ad)x^n}{d^2}+\frac{b^2x^{2n}}{d}+\frac{(ad-bc)^2}{d^3}-\frac{c(bc-ad)^2}{d^3(dx^n+c)}\right)dx^n}{n}$$
$$\Big\downarrow \ 2009$$
$$\frac{-\frac{c(bc-ad)^2\log(c+dx^n)}{d^4}+\frac{x^n(bc-ad)^2}{d^3}-\frac{bx^{2n}(bc-2ad)}{2d^2}+\frac{b^2x^{3n}}{3d}}{n}$$

input
```
Int[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

output
```
(((b*c - a*d)^2*x^n)/d^3 - (b*(b*c - 2*a*d)*x^(2*n))/(2*d^2) + (b^2*x^(3*n
))/(3*d) - (c*(b*c - a*d)^2*Log[c + d*x^n])/d^4)/n
```

3.1040.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1040.4 Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.31

method	result	size
norman	$\frac{\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) {\mathrm e}^{n \ln \left(x \right)}}{d^{3} n}+\frac{b^{2} {\mathrm e}^{3 n \ln \left(x \right)}}{3 d n}+\frac{b \left(2 a d -b c \right) {\mathrm e}^{2 n \ln \left(x \right)}}{2 d^{2} n}-\frac{c \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) \ln \left(c +d \,{\mathrm e}^{n \ln \left(x \right)}\right)}{d^{4} n}$	118
risch	$\frac{b^{2} x^{3 n}}{3 d n}+\frac{b \,x^{2 n} a}{d n}-\frac{b^{2} x^{2 n} c}{2 d^{2} n}+\frac{x^{n} a^{2}}{d n}-\frac{2 x^{n} a b c}{d^{2} n}+\frac{x^{n} b^{2} c^{2}}{d^{3} n}-\frac{c \ln \left(x^{n}+\frac{c}{d}\right) a^{2}}{d^{2} n}+\frac{2 c^{2} \ln \left(x^{n}+\frac{c}{d}\right) a b}{d^{3} n}-\frac{c^{3} \ln \left(x^{n}+\frac{c}{d}\right) b^{2}}{d^{4} n}$	161

input
```
int(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*exp(n*ln(x))+1/3*b^2/d/n*exp(n*ln(x))^
3+1/2*b*(2*a*d-b*c)/d^2/n*exp(n*ln(x))^2-c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^4
/n*ln(c+d*exp(n*ln(x)))
```

3.1040.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx$$
$$= \frac{2\,b^2d^3x^{3\,n} - 3\,(b^2cd^2 - 2\,abd^3)x^{2\,n} + 6\,(b^2c^2d - 2\,abcd^2 + a^2d^3)x^n - 6\,(b^2c^3 - 2\,abc^2d + a^2cd^2)\log\left(dx^n +\right.}{6\,d^4n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
1/6*(2*b^2*d^3*x^(3*n) - 3*(b^2*c*d^2 - 2*a*b*d^3)*x^(2*n) + 6*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)*x^n - 6*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d
*x^n + c))/(d^4*n)
```

3.1040.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(75) = 150$.

Time = 2.13 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.46

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx$$
$$= \begin{cases} \frac{(a+b)^2\log(x)}{c} & \text{for } d = \\ \frac{\frac{a^2xx^{2n-1}}{2n}+\frac{2abxx^nx^{2n-1}}{3n}+\frac{b^2xx^{2n}x^{2n-1}}{4n}}{c} & \text{for } d = \\ \frac{(a+b)^2\log(x)}{c+d} & \text{for } n = \\ -\frac{a^2c\log\left(\frac{c}{d}+x^n\right)}{d^2n}+\frac{a^2x^n}{dn}+\frac{2abc^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{2abcx^n}{d^2n}+\frac{abx^{2n}}{dn}-\frac{b^2c^3\log\left(\frac{c}{d}+x^n\right)}{d^4n}+\frac{b^2c^2x^n}{d^3n}-\frac{b^2cx^{2n}}{2d^2n}+\frac{b^2x^{3n}}{3dn} & \text{otherw} \end{cases}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Piecewise(((a + b)**2*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**2*x*x**(2*n - 1
)/(2*n) + 2*a*b*x*x**n*x**(2*n - 1)/(3*n) + b**2*x*x**(2*n)*x**(2*n - 1)/(
4*n))/c, Eq(d, 0)), ((a + b)**2*log(x)/(c + d), Eq(n, 0)), (-a**2*c*log(c/
d + x**n)/(d**2*n) + a**2*x**n/(d*n) + 2*a*b*c**2*log(c/d + x**n)/(d**3*n)
 - 2*a*b*c*x**n/(d**2*n) + a*b*x**(2*n)/(d*n) - b**2*c**3*log(c/d + x**n)/
(d**4*n) + b**2*c**2*x**n/(d**3*n) - b**2*c*x**(2*n)/(2*d**2*n) + b**2*x**
(3*n)/(3*d*n), True))
```

3.1040.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.67

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = a^2\left(\frac{x^n}{dn}-\frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right) - \frac{1}{6}b^2\left(\frac{6c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2d^2x^{3n}-3cdx^{2n}+6c^2x^n}{d^3n}\right) + ab\left(\frac{2c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2n}-2cx^n}{d^2n}\right)$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
a^2*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) - 1/6*b^2*(6*c^3*log((d*x^n
 + c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) +
a*b*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

3.1040.8 Giac [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{(bx^n+a)^2x^{2\,n-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2*x^(2*n - 1)/(d*x^n + c), x)
```

3.1040.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{x^{2\,n-1}\,(a+b\,x^n)^2}{c+d\,x^n}\,dx$$

input
```
int((x^(2*n - 1)*(a + b*x^n)^2)/(c + d*x^n),x)
```

output
```
int((x^(2*n - 1)*(a + b*x^n)^2)/(c + d*x^n), x)
```

3.1041 $\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx$

3.1041.1 Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx=-\frac{(bc-ad)x^n}{d^2n}+\frac{bx^{2n}}{2dn}+\frac{c(bc-ad)\log(c+dx^n)}{d^3n}$$

output
```
-(-a*d+b*c)*x^n/d^2/n+1/2*b*x^(2*n)/d/n+c*(-a*d+b*c)*ln(c+d*x^n)/d^3/n
```

3.1041.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx=\frac{dx^n(-2bc+2ad+bdx^n)+2c(bc-ad)\log(c+dx^n)}{2d^3n}$$

input
```
Integrate[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n),x]
```

output
```
(d*x^n*(-2*b*c + 2*a*d + b*d*x^n) + 2*c*(b*c - a*d)*Log[c + d*x^n])/(2*d^3
*n)
```

3.1041.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}(a+bx^n)}{c+dx^n} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^n(bx^n+a)}{dx^n+c}dx^n}{n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{bx^n}{d}+\frac{ad-bc}{d^2}+\frac{c(bc-ad)}{d^2(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{c(bc-ad)\log(c+dx^n)}{d^3}-\frac{x^n(bc-ad)}{d^2}+\frac{bx^{2n}}{2d}}{n}$$

input
```
Int[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n),x]
```

output
```
(-(((b*c - a*d)*x^n)/d^2) + (b*x^(2*n))/(2*d) + (c*(b*c - a*d)*Log[c + d*x
^n])/d^3)/n
```

3.1041.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1041.4 Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

method	result	size
norman	$\frac{(ad-bc)\,\mathrm{e}^{n\ln(x)}}{d^2 n}+\frac{b\,\mathrm{e}^{2n\ln(x)}}{2dn}-\frac{(ad-bc)\,c\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}{d^3 n}$	65
risch	$\frac{b\,x^{2n}}{2dn}+\frac{x^n a}{dn}-\frac{x^n bc}{d^2 n}-\frac{c\ln\left(x^n+\frac{c}{d}\right)a}{d^2 n}+\frac{c^2\ln\left(x^n+\frac{c}{d}\right)b}{d^3 n}$	81

input
```
int(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(a*d-b*c)/n*exp(n*ln(x))+1/2*b/d/n*exp(n*ln(x))^2-(a*d-b*c)*c/d^3/n*
ln(c+d*exp(n*ln(x)))
```

3.1041.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx=\frac{bd^2x^{2\,n}-2\,(bcd-ad^2)x^n+2\,(bc^2-acd)\log\left(dx^n+c\right)}{2\,d^3n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
1/2*(b*d^2*x^(2*n) - 2*(b*c*d - a*d^2)*x^n + 2*(b*c^2 - a*c*d)*log(d*x^n +
 c))/(d^3*n)
```

3.1041.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(48) = 96$.

Time = 1.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.93

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx = \begin{cases} \frac{(a+b)\log(x)}{c} & \text{for } d=0 \wedge n=0 \\ \frac{\frac{axx^{2n-1}}{2n}+\frac{bxx^nx^{2n-1}}{3n}}{c} & \text{for } d=0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n=0 \\ -\frac{ac\log\left(\frac{c}{d}+x^n\right)}{d^2n}+\frac{ax^n}{dn}+\frac{bc^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{bcx^n}{d^2n}+\frac{bx^{2n}}{2dn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)/(c+d*x**n),x)
```

output
```
Piecewise(((a + b)*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a*x*x**(2*n - 1)/(2*n
) + b*x*x**n*x**(2*n - 1)/(3*n))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq
(n, 0)), (-a*c*log(c/d + x**n)/(d**2*n) + a*x**n/(d*n) + b*c**2*log(c/d +
x**n)/(d**3*n) - b*c*x**n/(d**2*n) + b*x**(2*n)/(2*d*n), True))
```

3.1041.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx = a\left(\frac{x^n}{dn}-\frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right)+\frac{1}{2}b\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
a*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) + 1/2*b*(2*c^2*log((d*x^n + c
)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

3.1041.8 Giac [F]

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx=\int \frac{(bx^n+a)x^{2\,n-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)*x^(2*n - 1)/(d*x^n + c), x)
```

3.1041.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx=\int \frac{x^{2\,n-1}\,(a+b\,x^n)}{c+d\,x^n}\,dx$$

input
```
int((x^(2*n - 1)*(a + b*x^n))/(c + d*x^n),x)
```

output
```
int((x^(2*n - 1)*(a + b*x^n))/(c + d*x^n), x)
```

3.1042 $\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx$

3.1042.1 Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = -\frac{a\log(a+bx^n)}{b(bc-ad)n}+\frac{c\log(c+dx^n)}{d(bc-ad)n}$$

output
```
-a*ln(a+b*x^n)/b/(-a*d+b*c)/n+c*ln(c+d*x^n)/d/(-a*d+b*c)/n
```

3.1042.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = -\frac{ad\log(a+bx^n)-bc\log(c+dx^n)}{b^2cdn-abd^2n}$$

input
```
Integrate[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
-((a*d*Log[a + b*x^n] - b*c*Log[c + d*x^n])/(b^2*c*d*n - a*b*d^2*n))
```

3.1042.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a+bx^n)(c+dx^n)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{\int \frac{x^n}{(bx^n+a)(dx^n+c)}dx^n}{n}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{\int \left(\frac{c}{(bc-ad)(dx^n+c)}-\frac{a}{(bc-ad)(bx^n+a)}\right)dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{c\log(c+dx^n)}{d(bc-ad)}-\frac{a\log(a+bx^n)}{b(bc-ad)}}{n}$$

input
```
Int[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(-((a*Log[a + b*x^n])/(b*(b*c - a*d))) + (c*Log[c + d*x^n])/(d*(b*c - a*d)
))/n
```

3.1042.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1042.4 Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

method	result	size
norman	$\frac{a\ln\left(a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)}{\left(a d-b c\right) b n}-\frac{c\ln\left(c+d\,{\mathrm e}^{n\ln\left(x\right)}\right)}{d n\left(a d-b c\right)}$	59
risch	$\frac{\ln\left(x\right)}{b d}-\frac{\ln\left(x\right) a}{\left(a d-b c\right) b}+\frac{\ln\left(x\right) c}{d\left(a d-b c\right)}+\frac{a\ln\left(x^{n}+\frac{a}{b}\right)}{\left(a d-b c\right) b n}-\frac{c\ln\left(x^{n}+\frac{c}{d}\right)}{d n\left(a d-b c\right)}$	103

input
```
int(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
a/(a*d-b*c)/b/n*ln(a+b*exp(n*ln(x)))-c/d/n/(a*d-b*c)*ln(c+d*exp(n*ln(x)))
```

3.1042.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+2n}}{(a+bx^n)\,(c+dx^n)}\,dx=-\frac{ad\log\left(bx^n+a\right)-bc\log\left(dx^n+c\right)}{(b^2cd-abd^2)n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
-(a*d*log(b*x^n + a) - b*c*log(d*x^n + c))/((b^2*c*d - a*b*d^2)*n)
```

3.1042.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Exception raised: NotImplementedError >> no valid subset found
```

3.1042.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = -\frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2cn-abdn}+\frac{c\log\left(\frac{dx^n+c}{d}\right)}{bcdn-ad^2n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
-a*log((b*x^n + a)/b)/(b^2*c*n - a*b*d*n) + c*log((d*x^n + c)/d)/(b*c*d*n
- a*d^2*n)
```

3.1042.8 Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^{2\,n-1}}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/((b*x^n + a)*(d*x^n + c)), x)
```

3.1042.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^{2\,n-1}}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

input
```
int(x^(2*n - 1)/((a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(x^(2*n - 1)/((a + b*x^n)*(c + d*x^n)), x)
```

3.1043 $\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx$

3.1043.1 Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{a}{b(bc-ad)n(a+bx^n)}+\frac{c\log(a+bx^n)}{(bc-ad)^2n}-\frac{c\log(c+dx^n)}{(bc-ad)^2n}$$

output
```
a/b/(-a*d+b*c)/n/(a+b*x^n)+c*ln(a+b*x^n)/(-a*d+b*c)^2/n-c*ln(c+d*x^n)/(-a*
d+b*c)^2/n
```

3.1043.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{a}{b(bc-ad)n(a+bx^n)}+\frac{c\log(a+bx^n)}{(bc-ad)^2n}-\frac{c\log(c+dx^n)}{(bc-ad)^2n}$$

input
```
Integrate[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
a/(b*(b*c - a*d)*n*(a + b*x^n)) + (c*Log[a + b*x^n])/((b*c - a*d)^2*n) - (
c*Log[c + d*x^n])/((b*c - a*d)^2*n)
```

3.1043.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a+bx^n)^2(c+dx^n)}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^n}{(bx^n+a)^2(dx^n+c)}dx^n}{n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(-\frac{a}{(bc-ad)(bx^n+a)^2}+\frac{bc}{(bc-ad)^2(bx^n+a)}-\frac{cd}{(bc-ad)^2(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a}{b(bc-ad)(a+bx^n)}+\frac{c\log(a+bx^n)}{(bc-ad)^2}-\frac{c\log(c+dx^n)}{(bc-ad)^2}}{n}$$

input
```
Int[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(a/(b*(b*c - a*d)*(a + b*x^n)) + (c*Log[a + b*x^n])/(b*c - a*d)^2 - (c*Log
[c + d*x^n])/(b*c - a*d)^2)/n
```

3.1043.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1043.4 Maple [A] (verified)

Time = 5.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

method	result	size
risch	$-\frac{a}{(ad-bc)bn(a+b\,x^{n})}-\frac{c\ln\left(x^{n}+\frac{c}{d}\right)}{n(a^{2}d^{2}-2abcd+b^{2}c^{2})}+\frac{c\ln\left(x^{n}+\frac{a}{b}\right)}{n(a^{2}d^{2}-2abcd+b^{2}c^{2})}$	107
norman	$\frac{\mathrm{e}^{n\ln(x)}}{(ad-bc)n\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}+\frac{c\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{n(a^{2}d^{2}-2abcd+b^{2}c^{2})}-\frac{c\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}{n(a^{2}d^{2}-2abcd+b^{2}c^{2})}$	109

input
```
int(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
-a/(a*d-b*c)/b/n/(a+b*x^n)-c/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(x^n+c/d)+c/n
/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(x^n+a/b)
```

3.1043.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.60

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{abc-a^2d+(b^2cx^n+abc)\log(bx^n+a)-(b^2cx^n+abc)\log(dx^n+c)}{(b^4c^2-2\,ab^3cd+a^2b^2d^2)nx^n+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
(a*b*c - a^2*d + (b^2*c*x^n + a*b*c)*log(b*x^n + a) - (b^2*c*x^n + a*b*c)*
log(d*x^n + c))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^n + (a*b^3*c^2
- 2*a^2*b^2*c*d + a^3*b*d^2)*n)
```

3.1043.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.1043.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.61

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{c\log\left(\frac{bx^n+a}{b}\right)}{b^2c^2n-2\,abcdn+a^2d^2n} - \frac{c\log\left(\frac{dx^n+c}{d}\right)}{b^2c^2n-2\,abcdn+a^2d^2n} + \frac{a}{ab^2cn-a^2bdn+(b^3cn-ab^2dn)x^n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
c*log((b*x^n + a)/b)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) - c*log((d*x^n
+ c)/d)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) + a/(a*b^2*c*n - a^2*b*d*n +
 (b^3*c*n - a*b^2*d*n)*x^n)
```

3.1043.8 Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{x^{2\,n-1}}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)
```

3.1043.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx=\int \frac{x^{2\,n-1}}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

input

```
int(x^(2*n - 1)/((a + b*x^n)^2*(c + d*x^n)),x)
```

output

```
int(x^(2*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1044 $\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx$

3.1044.1 Optimal result

Integrand size = 26, antiderivative size = 105

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = \frac{a}{2b(bc-ad)n(a+bx^n)^2}-\frac{c}{(bc-ad)^2n(a+bx^n)}-\frac{cd\log(a+bx^n)}{(bc-ad)^3n}+\frac{cd\log(c+dx^n)}{(bc-ad)^3n}$$

output
```
1/2*a/b/(-a*d+b*c)/n/(a+b*x^n)^2-c/(-a*d+b*c)^2/n/(a+b*x^n)-c*d*ln(a+b*x^n
)/(-a*d+b*c)^3/n+c*d*ln(c+d*x^n)/(-a*d+b*c)^3/n
```

3.1044.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = \frac{-abc-a^2d-2b^2cx^n}{2b(bc-ad)^2n(a+bx^n)^2}-\frac{cd\log(a+bx^n)}{(bc-ad)^3n}+\frac{cd\log(c+dx^n)}{(bc-ad)^3n}$$

input
```
Integrate[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

output
```
(-(a*b*c) - a^2*d - 2*b^2*c*x^n)/(2*b*(b*c - a*d)^2*n*(a + b*x^n)^2) - (c*
d*Log[a + b*x^n])/((b*c - a*d)^3*n) + (c*d*Log[c + d*x^n])/((b*c - a*d)^3*
n)
```

3.1044.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a+bx^n)^3 (c+dx^n)} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{\int \frac{x^n}{(bx^n+a)^3 (dx^n+c)}dx^n}{n}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{\int \left(\frac{cd^2}{(bc-ad)^3 (dx^n+c)}-\frac{bcd}{(bc-ad)^3 (bx^n+a)}+\frac{bc}{(bc-ad)^2 (bx^n+a)^2}-\frac{a}{(bc-ad) (bx^n+a)^3}\right)dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a}{2b(bc-ad)(a+bx^n)^2}-\frac{c}{(bc-ad)^2 (a+bx^n)}-\frac{cd \log (a+bx^n)}{(bc-ad)^3}+\frac{cd \log (c+dx^n)}{(bc-ad)^3}}{n}$$

input

```
Int[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

output

```
(a/(2*b*(b*c - a*d)*(a + b*x^n)^2) - c/((b*c - a*d)^2*(a + b*x^n)) - (c*d*
Log[a + b*x^n])/(b*c - a*d)^3 + (c*d*Log[c + d*x^n])/(b*c - a*d)^3)/n
```

3.1044.3.1 Defintions of rubi rules used

rule 86

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1044.4 Maple [A] (verified)

Time = 6.24 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.50

method	result
risch	$-\frac{2b^2c\,x^n+a^2d+abc}{2n(ad-bc)^2b(a+b\,x^n)^2}-\frac{cd\ln\left(x^n+\frac{c}{d}\right)}{n(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}+\frac{cd\ln\left(x^n+\frac{a}{b}\right)}{n(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$
norman	$\frac{-\frac{bc\,\mathrm{e}^{n\ln(x)}}{\left(a^2d^2-2abcd+b^2c^2\right)n}+\frac{a\left(-abd-b^2c\right)}{2\left(a^2d^2-2abcd+b^2c^2\right)b^2n}}{\left(a+b\,\mathrm{e}^{n\ln(x)}\right)^2}+\frac{cd\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{n(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}-\frac{cd\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}{n(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$

input
```
int(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b^2*c*x^n+a^2*d+a*b*c)/n/(a*d-b*c)^2/b/(a+b*x^n)^2-c*d/n/(a^3*d^3-
3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+c/d)+c*d/n/(a^3*d^3-3*a^2*b*c*
d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+a/b)
```

3.1044.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(103) = 206$.

Time = 0.31 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.54

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3\,(c+dx^n)}\,dx =$$
$$-\frac{ab^2c^2-a^3d^2+2\,(b^3c^2-ab^2cd)x^n+2\,(b^3cdx^{2\,n}+2\,ab^2cdx^n+a^2bcd)\log\left(bx^n+a\right)-2\,(b^3cdx^{2\,n}+}{2\,((b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)nx^{2\,n}+2\,(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3)nx^n+(a^2b^4c}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

output
```
-1/2*(a*b^2*c^2 - a^3*d^2 + 2*(b^3*c^2 - a*b^2*c*d)*x^n + 2*(b^3*c*d*x^(2*
n) + 2*a*b^2*c*d*x^n + a^2*b*c*d)*log(b*x^n + a) - 2*(b^3*c*d*x^(2*n) + 2*
a*b^2*c*d*x^n + a^2*b*c*d)*log(d*x^n + c))/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a
^2*b^4*c*d^2 - a^3*b^3*d^3)*n*x^(2*n) + 2*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3
*a^3*b^3*c*d^2 - a^4*b^2*d^3)*n*x^n + (a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a
^4*b^2*c*d^2 - a^5*b*d^3)*n)
```

3.1044.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n)**3/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.1044.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(103) = 206$.

Time = 0.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.31

$$\begin{aligned}\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx \\ = -\frac{cd\log\left(\frac{bx^n+a}{b}\right)}{b^3c^3n-3\,ab^2c^2dn+3\,a^2bcd^2n-a^3d^3n}+\frac{cd\log\left(\frac{dx^n+c}{d}\right)}{b^3c^3n-3\,ab^2c^2dn+3\,a^2bcd^2n-a^3d^3n} \\ -\frac{2\,b^2cx^n+abc+a^2d}{2\,(a^2b^3c^2n-2\,a^3b^2cdn+a^4bd^2n+(b^5c^2n-2\,ab^4cdn+a^2b^3d^2n)x^{2\,n}+2\,(ab^4c^2n-2\,a^2b^3cdn+a^3b^2d^2n}\end{aligned}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

output
```
-c*d*log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a
^3*d^3*n) + c*d*log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*
c*d^2*n - a^3*d^3*n) - 1/2*(2*b^2*c*x^n + a*b*c + a^2*d)/(a^2*b^3*c^2*n -
2*a^3*b^2*c*d*n + a^4*b*d^2*n + (b^5*c^2*n - 2*a*b^4*c*d*n + a^2*b^3*d^2*n
)*x^(2*n) + 2*(a*b^4*c^2*n - 2*a^2*b^3*c*d*n + a^3*b^2*d^2*n)*x^n)
```

3.1044.8 Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = \int \frac{x^{2\,n-1}}{(bx^n+a)^3(dx^n+c)}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)
```

3.1044.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = \int \frac{x^{2\,n-1}}{(a+b\,x^n)^3\,(c+d\,x^n)}\,dx$$

input
```
int(x^(2*n - 1)/((a + b*x^n)^3*(c + d*x^n)),x)
```

output
```
int(x^(2*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)
```

3.1045 $\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx$

3.1045.1 Optimal result

Integrand size = 26, antiderivative size = 158

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx = \frac{c(bc-ad)^3x^n}{d^5n} - \frac{(bc-ad)^3x^{2n}}{2d^4n} + \frac{b(b^2c^2-3abcd+3a^2d^2)\,x^{3n}}{3d^3n} - \frac{b^2(bc-3ad)x^{4n}}{4d^2n} + \frac{b^3x^{5n}}{5dn} - \frac{c^2(bc-ad)^3\log\left(c+dx^n\right)}{d^6n}$$

output
```
c*(-a*d+b*c)^3*x^n/d^5/n-1/2*(-a*d+b*c)^3*x^(2*n)/d^4/n+1/3*b*(3*a^2*d^2-3
*a*b*c*d+b^2*c^2)*x^(3*n)/d^3/n-1/4*b^2*(-3*a*d+b*c)*x^(4*n)/d^2/n+1/5*b^3
*x^(5*n)/d/n-c^2*(-a*d+b*c)^3*ln(c+d*x^n)/d^6/n
```

3.1045.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.17

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx = \frac{dx^n(30a^3d^3(-2c+dx^n)+30a^2bd^2(6c^2-3cdx^n+2d^2x^{2n})+15ab^2d(-12c^3+6c^2dx^n-4cd^2x^{2n}+3d^3x^{3n}}{60d^6n}$$

input
```
Integrate[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

output
```
(d*x^n*(30*a^3*d^3*(-2*c + d*x^n) + 30*a^2*b*d^2*(6*c^2 - 3*c*d*x^n + 2*d^
2*x^(2*n)) + 15*a*b^2*d*(-12*c^3 + 6*c^2*d*x^n - 4*c*d^2*x^(2*n) + 3*d^3*x
^(3*n)) + b^3*(60*c^4 - 30*c^3*d*x^n + 20*c^2*d^2*x^(2*n) - 15*c*d^3*x^(3*
n) + 12*d^4*x^(4*n))) - 60*c^2*(b*c - a*d)^3*Log[c + d*x^n])/(60*d^6*n)
```

3.1045.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}(a+bx^n)^3}{c+dx^n}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{2n}(bx^n+a)^3}{dx^n+c}dx^n}{n}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{(ad-bc)^3x^n}{d^4}+\frac{b(b^2c^2-3abdc+3a^2d^2)x^{2n}}{d^3}-\frac{b^2(bc-3ad)x^{3n}}{d^2}+\frac{b^3x^{4n}}{d}+\frac{c(bc-ad)^3}{d^5}-\frac{c^2(bc-ad)^3}{d^5(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{bx^{3n}(3a^2d^2-3abcd+b^2c^2)}{3d^3}-\frac{b^2x^{4n}(bc-3ad)}{4d^2}-\frac{c^2(bc-ad)^3\log(c+dx^n)}{d^6}+\frac{cx^n(bc-ad)^3}{d^5}-\frac{x^{2n}(bc-ad)^3}{2d^4}+\frac{b^3x^{5n}}{5d}}{n}$$

input
```
Int[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

output
```
((c*(b*c - a*d)^3*x^n)/d^5 - ((b*c - a*d)^3*x^(2*n))/(2*d^4) + (b*(b^2*c^2
 - 3*a*b*c*d + 3*a^2*d^2)*x^(3*n))/(3*d^3) - (b^2*(b*c - 3*a*d)*x^(4*n))/(
4*d^2) + (b^3*x^(5*n))/(5*d) - (c^2*(b*c - a*d)^3*Log[c + d*x^n])/d^6)/n
```

3.1045.3.1 Defintions of rubi rules used

rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1045.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(150) = 300$.

Time = 5.72 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.16

method	result
risch	$\frac{b^3 x^{5n}}{5dn} + \frac{3b^2 x^{4n} a}{4dn} - \frac{b^3 x^{4n} c}{4d^2 n} + \frac{b\,x^{3n} a^2}{dn} - \frac{b^2 x^{3n} ac}{d^2 n} + \frac{b^3 x^{3n} c^2}{3d^3 n} + \frac{x^{2n} a^3}{2dn} - \frac{3x^{2n} a^2 bc}{2d^2 n} + \frac{3x^{2n} a\,b^2 c^2}{2d^3 n} - \frac{x^{2n} b^3 c^3}{2d^4 n} - \frac{c\,x^n a}{d^2 n}$

input

```
int(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output

```
1/5*b^3/d/n*(x^n)^5+3/4*b^2/d/n*(x^n)^4*a-1/4*b^3/d^2/n*(x^n)^4*c+b/d/n*(x
^n)^3*a^2-b^2/d^2/n*(x^n)^3*a*c+1/3*b^3/d^3/n*(x^n)^3*c^2+1/2/d/n*(x^n)^2*
a^3-3/2/d^2/n*(x^n)^2*a^2*b*c+3/2/d^3/n*(x^n)^2*a*b^2*c^2-1/2/d^4/n*(x^n)^
2*b^3*c^3-c/d^2/n*x^n*a^3+3*c^2/d^3/n*x^n*a^2*b-3*c^3/d^4/n*x^n*a*b^2+c^4/
d^5/n*x^n*b^3+c^2/d^3/n*ln(x^n+c/d)*a^3-3*c^3/d^4/n*ln(x^n+c/d)*a^2*b+3*c^
4/d^5/n*ln(x^n+c/d)*a*b^2-c^5/d^6/n*ln(x^n+c/d)*b^3
```

3.1045.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.46

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx$$
$$=\frac{12\,b^3d^5x^{5\,n}-15\,(b^3cd^4-3\,ab^2d^5)x^{4\,n}+20\,(b^3c^2d^3-3\,ab^2cd^4+3\,a^2bd^5)x^{3\,n}-30\,(b^3c^3d^2-3\,ab^2c^2d^3+3\,a}{}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

output
```
1/60*(12*b^3*d^5*x^(5*n) - 15*(b^3*c*d^4 - 3*a*b^2*d^5)*x^(4*n) + 20*(b^3*
c^2*d^3 - 3*a*b^2*c*d^4 + 3*a^2*b*d^5)*x^(3*n) - 30*(b^3*c^3*d^2 - 3*a*b^2
*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^(2*n) + 60*(b^3*c^4*d - 3*a*b^2*c^3*
d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x^n - 60*(b^3*c^5 - 3*a*b^2*c^4*d + 3*a
^2*b*c^3*d^2 - a^3*c^2*d^3)*log(d*x^n + c))/(d^6*n)
```

3.1045.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(138) = 276.

Time = 5.48 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.71

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx$$
$$=\begin{cases}\frac{(a+b)^3\log(x)}{c} \\ \frac{\frac{a^3xx^{3n-1}}{3n}+\frac{3a^2bxx^nx^{3n-1}}{4n}+\frac{3ab^2xx^{2n}x^{3n-1}}{5n}+\frac{b^3xx^{3n}x^{3n-1}}{6n}}{c} \\ \frac{(a+b)^3\log(x)}{c+d} \\ \frac{a^3c^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{a^3cx^n}{d^2n}+\frac{a^3x^{2n}}{2dn}-\frac{3a^2bc^3\log\left(\frac{c}{d}+x^n\right)}{d^4n}+\frac{3a^2bc^2x^n}{d^3n}-\frac{3a^2bcx^{2n}}{2d^2n}+\frac{a^2bx^{3n}}{dn}+\frac{3ab^2c^4\log\left(\frac{c}{d}+x^n\right)}{d^5n}-\frac{3ab^2c^3x^n}{d^4n}\end{cases}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**3/(c+d*x**n),x)
```

output
```
Piecewise(((a + b)**3*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**3*x*x**(3*n - 1
)/(3*n) + 3*a**2*b*x*x**n*x**(3*n - 1)/(4*n) + 3*a*b**2*x*x**(2*n)*x**(3*n
 - 1)/(5*n) + b**3*x*x**(3*n)*x**(3*n - 1)/(6*n))/c, Eq(d, 0)), ((a + b)**
3*log(x)/(c + d), Eq(n, 0)), (a**3*c**2*log(c/d + x**n)/(d**3*n) - a**3*c*
x**n/(d**2*n) + a**3*x**(2*n)/(2*d*n) - 3*a**2*b*c**3*log(c/d + x**n)/(d**
4*n) + 3*a**2*b*c**2*x**n/(d**3*n) - 3*a**2*b*c*x**(2*n)/(2*d**2*n) + a**2
*b*x**(3*n)/(d*n) + 3*a*b**2*c**4*log(c/d + x**n)/(d**5*n) - 3*a*b**2*c**3
*x**n/(d**4*n) + 3*a*b**2*c**2*x**(2*n)/(2*d**3*n) - a*b**2*c*x**(3*n)/(d*
*2*n) + 3*a*b**2*x**(4*n)/(4*d*n) - b**3*c**5*log(c/d + x**n)/(d**6*n) + b
**3*c**4*x**n/(d**5*n) - b**3*c**3*x**(2*n)/(2*d**4*n) + b**3*c**2*x**(3*n
)/(3*d**3*n) - b**3*c*x**(4*n)/(4*d**2*n) + b**3*x**(5*n)/(5*d*n), True))
```

3.1045.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.81

$$\begin{aligned}\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx &= -\frac{1}{60}\,b^3\left(\frac{60\,c^5\log\left(\frac{dx^n+c}{d}\right)}{d^6n}-\frac{12\,d^4x^{5\,n}-15\,cd^3x^{4\,n}+20\,c^2d^2x^{3\,n}-30\,c^3dx^{2\,n}+60\,c^4x^n}{d^5n}\right)\\&\quad+\frac{1}{4}\,ab^2\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n}+\frac{3\,d^3x^{4\,n}-4\,cd^2x^{3\,n}+6\,c^2dx^{2\,n}-12\,c^3x^n}{d^4n}\right)\\&\quad-\frac{1}{2}\,a^2b\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+6\,c^2x^n}{d^3n}\right)\\&\quad+\frac{1}{2}\,a^3\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)\end{aligned}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

output
```
-1/60*b^3*(60*c^5*log((d*x^n + c)/d)/(d^6*n) - (12*d^4*x^(5*n) - 15*c*d^3*
x^(4*n) + 20*c^2*d^2*x^(3*n) - 30*c^3*d*x^(2*n) + 60*c^4*x^n)/(d^5*n)) + 1
/4*a*b^2*(12*c^4*log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(
3*n) + 6*c^2*d*x^(2*n) - 12*c^3*x^n)/(d^4*n)) - 1/2*a^2*b*(6*c^3*log((d*x^
n + c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) +
 1/2*a^3*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n)
)
```

3.1045.8 Giac [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx=\int \frac{(bx^n+a)^3x^{3\,n-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^3*x^(3*n - 1)/(d*x^n + c), x)
```

3.1045.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx=\int \frac{x^{3\,n-1}\,(a+b\,x^n)^3}{c+d\,x^n}\,dx$$

input
```
int((x^(3*n - 1)*(a + b*x^n)^3)/(c + d*x^n),x)
```

output
```
int((x^(3*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)
```

3.1046 $\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx$

3.1046.1 Optimal result

Integrand size = 26, antiderivative size = 118

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = -\frac{c(bc-ad)^2x^n}{d^4n}+\frac{(bc-ad)^2x^{2n}}{2d^3n}-\frac{b(bc-2ad)x^{3n}}{3d^2n}+\frac{b^2x^{4n}}{4dn}+\frac{c^2(bc-ad)^2\log(c+dx^n)}{d^5n}$$

output
```
-c*(-a*d+b*c)^2*x^n/d^4/n+1/2*(-a*d+b*c)^2*x^(2*n)/d^3/n-1/3*b*(-2*a*d+b*c
)*x^(3*n)/d^2/n+1/4*b^2*x^(4*n)/d/n+c^2*(-a*d+b*c)^2*ln(c+d*x^n)/d^5/n
```

3.1046.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{dx^n(6a^2d^2(-2c+dx^n)+4abd(6c^2-3cdx^n+2d^2x^{2n})+b^2(-12c^3+6c^2dx^n-4cd^2x^{2n}+3d^3x^{3n}))+12c^2}{12d^5n}$$

input
```
Integrate[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

output
```
(d*x^n*(6*a^2*d^2*(-2*c + d*x^n) + 4*a*b*d*(6*c^2 - 3*c*d*x^n + 2*d^2*x^(2
*n)) + b^2*(-12*c^3 + 6*c^2*d*x^n - 4*c*d^2*x^(2*n) + 3*d^3*x^(3*n))) + 12
*c^2*(b*c - a*d)^2*Log[c + d*x^n])/(12*d^5*n)
```

3.1046.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}(a+bx^n)^2}{c+dx^n} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{2n}(bx^n+a)^2}{dx^n+c}dx^n}{n}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{(ad-bc)^2x^n}{d^3}-\frac{b(bc-2ad)x^{2n}}{d^2}+\frac{b^2x^{3n}}{d}-\frac{c(bc-ad)^2}{d^4}+\frac{c^2(bc-ad)^2}{d^4(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{c^2(bc-ad)^2\log(c+dx^n)}{d^5}-\frac{cx^n(bc-ad)^2}{d^4}+\frac{x^{2n}(bc-ad)^2}{2d^3}-\frac{bx^{3n}(bc-2ad)}{3d^2}+\frac{b^2x^{4n}}{4d}}{n}$$

input
```
Int[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

output
```
(-((c*(b*c - a*d)^2*x^n)/d^4) + ((b*c - a*d)^2*x^(2*n))/(2*d^3) - (b*(b*c
- 2*a*d)*x^(3*n))/(3*d^2) + (b^2*x^(4*n))/(4*d) + (c^2*(b*c - a*d)^2*Log[c
 + d*x^n])/d^5)/n
```

3.1046.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1046.4 Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.33

method	result
norman	$\frac{b^2 e^{4n\ln(x)}}{4dn} + \frac{(a^2d^2-2abcd+b^2c^2)e^{2n\ln(x)}}{2d^3n} + \frac{b(2ad-bc)e^{3n\ln(x)}}{3d^2n} - \frac{c(a^2d^2-2abcd+b^2c^2)e^{n\ln(x)}}{d^4n} + \frac{c^2(a^2d^2-2abcd+b^2c^2)\ln(}{d^5n}$
risch	$\frac{b^2x^{4n}}{4dn} + \frac{2b\,x^{3n}a}{3dn} - \frac{b^2x^{3n}c}{3d^2n} + \frac{x^{2n}a^2}{2dn} - \frac{x^{2n}abc}{d^2n} + \frac{x^{2n}b^2c^2}{2d^3n} - \frac{c\,x^na^2}{d^2n} + \frac{2c^2x^nab}{d^3n} - \frac{c^3x^nb^2}{d^4n} + \frac{c^2\ln\left(x^n+\frac{c}{d}\right)a^2}{d^3n} - 2c^3\ln$

input
```
int(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2/d/n*exp(n*ln(x))^4+1/2/d^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*exp(n*ln(
x))^2+1/3*b*(2*a*d-b*c)/d^2/n*exp(n*ln(x))^3-c*(a^2*d^2-2*a*b*c*d+b^2*c^2)
/d^4/n*exp(n*ln(x))+c^2/d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*ln(c+d*exp(n*ln(
x)))
```

3.1046.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.24

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx$$
$$= \frac{3\,b^2d^4x^{4\,n} - 4\,(b^2cd^3 - 2\,abd^4)x^{3\,n} + 6\,(b^2c^2d^2 - 2\,abcd^3 + a^2d^4)x^{2\,n} - 12\,(b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x^n + 1}{12\,d^5n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
1/12*(3*b^2*d^4*x^(4*n) - 4*(b^2*c*d^3 - 2*a*b*d^4)*x^(3*n) + 6*(b^2*c^2*d
^2 - 2*a*b*c*d^3 + a^2*d^4)*x^(2*n) - 12*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*
c*d^3)*x^n + 12*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*log(d*x^n + c))/(d^5
*n)
```

3.1046.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(99) = 198$.

Time = 3.20 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.35

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx$$
$$= \begin{cases} \frac{(a+b)^2 \log(x)}{c} \\ \frac{\frac{a^2xx^{3n-1}}{3n}+\frac{abxx^nx^{3n-1}}{2n}+\frac{b^2xx^{2n}x^{3n-1}}{5n}}{c} \\ \frac{(a+b)^2 \log(x)}{c+d} \\ \frac{a^2c^2\log\left(\frac{c}{d}+x^n\right)}{d^3n} - \frac{a^2cx^n}{d^2n} + \frac{a^2x^{2n}}{2dn} - \frac{2abc^3\log\left(\frac{c}{d}+x^n\right)}{d^4n} + \frac{2abc^2x^n}{d^3n} - \frac{abcx^{2n}}{d^2n} + \frac{2abx^{3n}}{3dn} + \frac{b^2c^4\log\left(\frac{c}{d}+x^n\right)}{d^5n} - \frac{b^2c^3x^n}{d^4n} + \end{cases}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Piecewise(((a + b)**2*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**2*x*x**(3*n - 1
)/(3*n) + a*b*x*x**n*x**(3*n - 1)/(2*n) + b**2*x*x**(2*n)*x**(3*n - 1)/(5*
n))/c, Eq(d, 0)), ((a + b)**2*log(x)/(c + d), Eq(n, 0)), (a**2*c**2*log(c/
d + x**n)/(d**3*n) - a**2*c*x**n/(d**2*n) + a**2*x**(2*n)/(2*d*n) - 2*a*b*
c**3*log(c/d + x**n)/(d**4*n) + 2*a*b*c**2*x**n/(d**3*n) - a*b*c*x**(2*n)/
(d**2*n) + 2*a*b*x**(3*n)/(3*d*n) + b**2*c**4*log(c/d + x**n)/(d**5*n) - b
**2*c**3*x**n/(d**4*n) + b**2*c**2*x**(2*n)/(2*d**3*n) - b**2*c*x**(3*n)/(
3*d**2*n) + b**2*x**(4*n)/(4*d*n), True))
```

3.1046.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.63

$$
\begin{aligned}
&\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx \\
&= \frac{1}{12}b^2\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n}+\frac{3\,d^3x^{4\,n}-4\,cd^2x^{3\,n}+6\,c^2dx^{2\,n}-12\,c^3x^n}{d^4n}\right) \\
&\quad -\frac{1}{3}ab\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+6\,c^2x^n}{d^3n}\right) \\
&\quad +\frac{1}{2}a^2\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)
\end{aligned}
$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
1/12*b^2*(12*c^4*log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(
3*n) + 6*c^2*d*x^(2*n) - 12*c^3*x^n)/(d^4*n)) - 1/3*a*b*(6*c^3*log((d*x^n
+ c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 1
/2*a^2*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

3.1046.8 Giac [F]

$$
\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{(bx^n+a)^2x^{3\,n-1}}{dx^n+c}\,dx
$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2*x^(3*n - 1)/(d*x^n + c), x)
```

3.1046.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx=\int \frac{x^{3\,n-1}\,{\left(a+b\,x^n\right)}^2}{c+d\,x^n} \,d x$$

input

```
int((x^(3*n - 1)*(a + b*x^n)^2)/(c + d*x^n),x)
```

output

```
int((x^(3*n - 1)*(a + b*x^n)^2)/(c + d*x^n), x)
```

3.1047 $\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx$

3.1047.1 Optimal result

Integrand size = 24, antiderivative size = 86

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx=\frac{c(bc-ad)x^n}{d^3n}-\frac{(bc-ad)x^{2n}}{2d^2n}+\frac{bx^{3n}}{3dn}-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4n}$$

output
```
c*(-a*d+b*c)*x^n/d^3/n-1/2*(-a*d+b*c)*x^(2*n)/d^2/n+1/3*b*x^(3*n)/d/n-c^2*
(-a*d+b*c)*ln(c+d*x^n)/d^4/n
```

3.1047.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx =\frac{dx^n(3ad(-2c+dx^n)+b(6c^2-3cdx^n+2d^2x^{2n}))+6c^2(-bc+ad)\log(c+dx^n)}{6d^4n}$$

input
```
Integrate[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n),x]
```

output
```
(d*x^n*(3*a*d*(-2*c + d*x^n) + b*(6*c^2 - 3*c*d*x^n + 2*d^2*x^(2*n))) + 6*
c^2*(-(b*c) + a*d)*Log[c + d*x^n])/(6*d^4*n)
```

3.1047.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}(a+bx^n)}{c+dx^n} \, dx$$
$$\Big\downarrow \ 948$$
$$\frac{\int \frac{x^{2n}(bx^n+a)}{dx^n+c}dx^n}{n}$$
$$\Big\downarrow \ 86$$
$$\frac{\int \left(\frac{(ad-bc)x^n}{d^2}+\frac{bx^{2n}}{d}+\frac{c(bc-ad)}{d^3}-\frac{c^2(bc-ad)}{d^3(dx^n+c)}\right)dx^n}{n}$$
$$\Big\downarrow \ 2009$$
$$\frac{-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4}+\frac{cx^n(bc-ad)}{d^3}-\frac{x^{2n}(bc-ad)}{2d^2}+\frac{bx^{3n}}{3d}}{n}$$

input
```
Int[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n),x]
```

output
```
((c*(b*c - a*d)*x^n)/d^3 - ((b*c - a*d)*x^(2*n))/(2*d^2) + (b*x^(3*n))/(3*
d) - (c^2*(b*c - a*d)*Log[c + d*x^n])/d^4)/n
```

3.1047.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1047.4 Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06

method	result	size
norman	$\frac{b\,\mathrm{e}^{3n\ln(x)}}{3dn}+\frac{(ad-bc)\mathrm{e}^{2n\ln(x)}}{2d^2n}-\frac{(ad-bc)c\,\mathrm{e}^{n\ln(x)}}{d^3n}+\frac{c^2(ad-bc)\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}{d^4n}$	91
risch	$\frac{b\,x^{3n}}{3dn}+\frac{x^{2n}a}{2dn}-\frac{x^{2n}bc}{2d^2n}-\frac{c\,x^{n}a}{d^2n}+\frac{c^2x^{n}b}{d^3n}+\frac{c^2\ln\left(x^{n}+\frac{c}{d}\right)a}{d^3n}-\frac{c^3\ln\left(x^{n}+\frac{c}{d}\right)b}{d^4n}$	115

input
```
int(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/3*b/d/n*exp(n*ln(x))^3+1/2/d^2*(a*d-b*c)/n*exp(n*ln(x))^2-(a*d-b*c)*c/d^
3/n*exp(n*ln(x))+c^2/d^4*(a*d-b*c)/n*ln(c+d*exp(n*ln(x)))
```

3.1047.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = \frac{2\,bd^3x^{3\,n}-3\,(bcd^2-ad^3)x^{2\,n}+6\,(bc^2d-acd^2)x^n-6\,(bc^3-ac^2d)\log\left(dx^n+c\right)}{6\,d^4n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
1/6*(2*b*d^3*x^(3*n) - 3*(b*c*d^2 - a*d^3)*x^(2*n) + 6*(b*c^2*d - a*c*d^2)
*x^n - 6*(b*c^3 - a*c^2*d)*log(d*x^n + c))/(d^4*n)
```

3.1047.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(71) = 142$.

Time = 1.86 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.74

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = \begin{cases} \frac{(a+b)\log(x)}{c} & \text{for } d = 0 \wedge n = 0 \\ \frac{\frac{axx^{3n-1}}{3n} + \frac{bxx^n x^{3n-1}}{4n}}{c} & \text{for } d = 0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n = 0 \\ \frac{ac^2\log\left(\frac{c}{d}+x^n\right)}{d^3n} - \frac{acx^n}{d^2n} + \frac{ax^{2n}}{2dn} - \frac{bc^3\log\left(\frac{c}{d}+x^n\right)}{d^4n} + \frac{bc^2x^n}{d^3n} - \frac{bcx^{2n}}{2d^2n} + \frac{bx^{3n}}{3dn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)/(c+d*x**n),x)
```

output
```
Piecewise(((a + b)*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a*x*x**(3*n - 1)/(3*n
) + b*x*x**n*x**(3*n - 1)/(4*n))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq
(n, 0)), (a*c**2*log(c/d + x**n)/(d**3*n) - a*c*x**n/(d**2*n) + a*x**(2*n)
/(2*d*n) - b*c**3*log(c/d + x**n)/(d**4*n) + b*c**2*x**n/(d**3*n) - b*c*x*
*(2*n)/(2*d**2*n) + b*x**(3*n)/(3*d*n), True))
```

3.1047.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = -\frac{1}{6}\,b\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n} - \frac{2\,d^2x^{3\,n} - 3\,cdx^{2\,n} + 6\,c^2x^n}{d^3n}\right) + \frac{1}{2}\,a\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n} + \frac{dx^{2\,n} - 2\,cx^n}{d^2n}\right)$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
-1/6*b*(6*c^3*log((d*x^n + c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n)
+ 6*c^2*x^n)/(d^3*n)) + 1/2*a*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*
n) - 2*c*x^n)/(d^2*n))
```

3.1047.8 Giac [F]

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx=\int \frac{(bx^n+a)x^{3\,n-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)*x^(3*n - 1)/(d*x^n + c), x)
```

3.1047.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx=\int \frac{x^{3\,n-1}\,(a+b\,x^n)}{c+d\,x^n}\,dx$$

input
```
int((x^(3*n - 1)*(a + b*x^n))/(c + d*x^n),x)
```

output
```
int((x^(3*n - 1)*(a + b*x^n))/(c + d*x^n), x)
```

3.1048 $\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx$

3.1048.1 Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx=\frac{x^n}{bdn}+\frac{a^2\log(a+bx^n)}{b^2(bc-ad)n}-\frac{c^2\log(c+dx^n)}{d^2(bc-ad)n}$$

output
```
x^n/b/d/n+a^2*ln(a+b*x^n)/b^2/(-a*d+b*c)/n-c^2*ln(c+d*x^n)/d^2/(-a*d+b*c)/
n
```

3.1048.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx=\frac{a^2d^2\log(a+bx^n)+b(d(bc-ad)x^n-bc^2\log(c+dx^n))}{b^2d^2(bc-ad)n}$$

input
```
Integrate[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(a^2*d^2*Log[a + b*x^n] + b*(d*(b*c - a*d)*x^n - b*c^2*Log[c + d*x^n]))/(b
^2*d^2*(b*c - a*d)*n)
```

3.1048.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{(a+bx^n)(c+dx^n)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{\int \frac{x^{2n}}{(bx^n+a)(dx^n+c)}dx^n}{n}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{\int \left(\frac{a^2}{b(bc-ad)(bx^n+a)}+\frac{1}{bd}+\frac{c^2}{d(ad-bc)(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^2\log(a+bx^n)}{b^2(bc-ad)}-\frac{c^2\log(c+dx^n)}{d^2(bc-ad)}+\frac{x^n}{bd}}{n}$$

input
```
Int[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(x^n/(b*d) + (a^2*Log[a + b*x^n])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x^n])
/(d^2*(b*c - a*d)))/n
```

3.1048.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1048.4 Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

method	result	size
norman	$\frac{{\mathrm e}^{n \ln\left(x \right)}}{b d n}+\frac{c^{2} \ln\left(c +d \,{\mathrm e}^{n \ln\left(x \right)}\right)}{d^{2} n \left(a d -b c \right)}-\frac{a^{2} \ln\left(a +b \,{\mathrm e}^{n \ln\left(x \right)}\right)}{\left(a d -b c \right) b^{2} n}$	78
risch	$-\frac{\ln\left(x \right) a}{b^{2} d}-\frac{\ln\left(x \right) c}{b \,d^{2}}+\frac{x^{n}}{b d n}+\frac{\ln\left(x \right) a^{2}}{\left(a d -b c \right) b^{2}}-\frac{\ln\left(x \right) c^{2}}{d^{2} \left(a d -b c \right)}-\frac{a^{2} \ln\left(x^{n}+\frac{a}{b}\right)}{\left(a d -b c \right) b^{2} n}+\frac{c^{2} \ln\left(x^{n}+\frac{c}{d}\right)}{d^{2} n \left(a d -b c \right)}$	137

input
```
int(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/b/d/n*exp(n*ln(x))+c^2/d^2/n/(a*d-b*c)*ln(c+d*exp(n*ln(x)))-a^2/(a*d-b*c
)/b^2/n*ln(a+b*exp(n*ln(x)))
```

3.1048.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \frac{a^2d^2\log\left(bx^n+a\right)-b^2c^2\log\left(dx^n+c\right)+(b^2cd-abd^2)x^n}{(b^3cd^2-ab^2d^3)n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
(a^2*d^2*log(b*x^n + a) - b^2*c^2*log(d*x^n + c) + (b^2*c*d - a*b*d^2)*x^n
)/((b^3*c*d^2 - a*b^2*d^3)*n)
```

3.1048.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.1048.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \frac{a^2\log\left(\frac{bx^n+a}{b}\right)}{b^3cn-ab^2dn}-\frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{bcd^2n-ad^3n}+\frac{x^n}{bdn}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
a^2*log((b*x^n + a)/b)/(b^3*c*n - a*b^2*d*n) - c^2*log((d*x^n + c)/d)/(b*c
*d^2*n - a*d^3*n) + x^n/(b*d*n)
```

3.1048.8 Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^{3n-1}}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/((b*x^n + a)*(d*x^n + c)), x)
```

3.1048.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx=\int \frac{x^{3\,n-1}}{\left(a+b\,x^n\right)\,\left(c+d\,x^n\right)}\,dx$$

input
```
int(x^(3*n - 1)/((a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(x^(3*n - 1)/((a + b*x^n)*(c + d*x^n)), x)
```

3.1049 $\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx$

3.1049.1 Optimal result

Integrand size = 26, antiderivative size = 95

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = -\frac{a^2}{b^2(bc-ad)n(a+bx^n)} - \frac{a(2bc-ad)\log(a+bx^n)}{b^2(bc-ad)^2n} + \frac{c^2\log(c+dx^n)}{d(bc-ad)^2n}$$

output
```
-a^2/b^2/(-a*d+b*c)/n/(a+b*x^n)-a*(-a*d+2*b*c)*ln(a+b*x^n)/b^2/(-a*d+b*c)^
2/n+c^2*ln(c+d*x^n)/d/(-a*d+b*c)^2/n
```

3.1049.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = -\frac{a^2}{b^2(bc-ad)n(a+bx^n)} + \frac{a(-2bc+ad)\log(a+bx^n)}{b^2(bc-ad)^2n} + \frac{c^2\log(c+dx^n)}{d(-bc+ad)^2n}$$

input
```
Integrate[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
-(a^2/(b^2*(b*c - a*d)*n*(a + b*x^n))) + (a*(-2*b*c + a*d)*Log[a + b*x^n])
/(b^2*(b*c - a*d)^2*n) + (c^2*Log[c + d*x^n])/(d*(-(b*c) + a*d)^2*n)
```

3.1049.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{(a+bx^n)^2(c+dx^n)}dx$$

$$\Big\downarrow \; 948$$

$$\frac{\int \frac{x^{2n}}{(bx^n+a)^2(dx^n+c)}dx^n}{n}$$

$$\Big\downarrow \; 99$$

$$\frac{\int \left(\frac{a^2}{b(bc-ad)(bx^n+a)^2}+\frac{(ad-2bc)a}{b(bc-ad)^2(bx^n+a)}+\frac{c^2}{(bc-ad)^2(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{a^2}{b^2(bc-ad)(a+bx^n)}-\frac{a(2bc-ad)\log(a+bx^n)}{b^2(bc-ad)^2}+\frac{c^2\log(c+dx^n)}{d(bc-ad)^2}}{n}$$

input
```
Int[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(-(a^2/(b^2*(b*c - a*d)*(a + b*x^n))) - (a*(2*b*c - a*d)*Log[a + b*x^n])/(
b^2*(b*c - a*d)^2) + (c^2*Log[c + d*x^n])/(d*(b*c - a*d)^2))/n
```

3.1049.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1049.4 Maple [A] (verified)

Time = 5.40 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.32

method	result
norman	$\frac{a^{2}}{\left(a d -b c \right) b^{2} n \left(a +b \,{\mathrm e}^{n \ln \left(x \right)}\right)}+\frac{c^{2} \ln \left(c +d \,{\mathrm e}^{n \ln \left(x \right)}\right)}{d n \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right)}+\frac{a \left(a d -2 b c \right) \ln \left(a +b \,{\mathrm e}^{n \ln \left(x \right)}\right)}{\left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) b^{2} n}$
risch	$\frac{\ln \left(x \right)}{b^{2} d}-\frac{\ln \left(x \right) a^{2} d}{\left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) b^{2}}+\frac{2 \ln \left(x \right) a c}{\left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) b}-\frac{\ln \left(x \right) c^{2}}{d \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right)}+\frac{a^{2}}{\left(a d -b c \right) b^{2} n \left(a +b \,x^{n}\right)}+\frac{a^{2} \ln \left(x^{n}+\frac{a}{b}\right.}{\left(a^{2} d^{2}-2 a b c d +b^{2}\right.}$

input
```
int(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
a^2/(a*d-b*c)/b^2/n/(a+b*exp(n*ln(x)))+c^2/d/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)
*ln(c+d*exp(n*ln(x)))+a*(a*d-2*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/n*ln(a
+b*exp(n*ln(x)))
```

3.1049.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.75

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2\,(c+dx^n)}\,dx = -\frac{a^{2} b c d - a^{3} d^{2} + (2\,a^{2} b c d - a^{3} d^{2} + (2\,a b^{2} c d - a^{2} b d^{2})x^{n})\log\left(b x^{n} + a\right) - (b^{3} c^{2} x^{n} + a b^{2} c^{2})\log\left(d x^{n} + c\right)}{(b^{5} c^{2} d - 2\,a b^{4} c d^{2} + a^{2} b^{3} d^{3}) n x^{n} + (a b^{4} c^{2} d - 2\,a^{2} b^{3} c d^{2} + a^{3} b^{2} d^{3}) n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
-(a^2*b*c*d - a^3*d^2 + (2*a^2*b*c*d - a^3*d^2 + (2*a*b^2*c*d - a^2*b*d^2)
*x^n)*log(b*x^n + a) - (b^3*c^2*x^n + a*b^2*c^2)*log(d*x^n + c))/((b^5*c^2
*d - 2*a*b^4*c*d^2 + a^2*b^3*d^3)*n*x^n + (a*b^4*c^2*d - 2*a^2*b^3*c*d^2 +
 a^3*b^2*d^3)*n)
```

3.1049.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+3n}}{\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.1049.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.55

$$\begin{aligned}\int \frac{x^{-1+3n}}{\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx &= \frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{b^2c^2dn-2\,abcd^2n+a^2d^3n} \\ &\quad - \frac{a^2}{ab^3cn-a^2b^2dn+\left(b^4cn-ab^3dn\right)x^n} \\ &\quad - \frac{\left(2\,abc-a^2d\right)\log\left(\frac{bx^n+a}{b}\right)}{b^4c^2n-2\,ab^3cdn+a^2b^2d^2n}\end{aligned}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
c^2*log((d*x^n + c)/d)/(b^2*c^2*d*n - 2*a*b*c*d^2*n + a^2*d^3*n) - a^2/(a*
b^3*c*n - a^2*b^2*d*n + (b^4*c*n - a*b^3*d*n)*x^n) - (2*a*b*c - a^2*d)*log
((b*x^n + a)/b)/(b^4*c^2*n - 2*a*b^3*c*d*n + a^2*b^2*d^2*n)
```

3.1049.8 Giac [F]

$$\int \frac{x^{-1+3n}}{\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = \int \frac{x^{3\,n-1}}{\left(bx^n+a\right)^2\left(dx^n+c\right)}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)
```

3.1049.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2\,(c+dx^n)}\,dx=\int \frac{x^{3\,n-1}}{{\left(a+b\,x^n\right)}^2\,\left(c+d\,x^n\right)}\,dx$$

input

```
int(x^(3*n - 1)/((a + b*x^n)^2*(c + d*x^n)),x)
```

output

```
int(x^(3*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1050 $\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx$

3.1050.1 Optimal result

Integrand size = 26, antiderivative size = 120

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3\,(c+dx^n)}\,dx = -\frac{a^2}{2b^2(bc-ad)n\,(a+bx^n)^2} + \frac{a(2bc-ad)}{b^2(bc-ad)^2n\,(a+bx^n)} + \frac{c^2\log\left(a+bx^n\right)}{(bc-ad)^3n} - \frac{c^2\log\left(c+dx^n\right)}{(bc-ad)^3n}$$

output
```
-1/2*a^2/b^2/(-a*d+b*c)/n/(a+b*x^n)^2+a*(-a*d+2*b*c)/b^2/(-a*d+b*c)^2/n/(a
+b*x^n)+c^2*ln(a+b*x^n)/(-a*d+b*c)^3/n-c^2*ln(c+d*x^n)/(-a*d+b*c)^3/n
```

3.1050.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3\,(c+dx^n)}\,dx = \frac{\frac{a(-bc+ad)\left(-3abc+a^2d-4b^2cx^n+2abdx^n\right)}{b^2(a+bx^n)^2} + 2c^2\log\left(a+bx^n\right) - 2c^2\log\left(c+dx^n\right)}{2(bc-ad)^3n}$$

input
```
Integrate[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

output
```
((a*(-(b*c) + a*d)*(-3*a*b*c + a^2*d - 4*b^2*c*x^n + 2*a*b*d*x^n))/(b^2*(a
 + b*x^n)^2) + 2*c^2*Log[a + b*x^n] - 2*c^2*Log[c + d*x^n])/(2*(b*c - a*d)
^3*n)
```

3.1050.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{(a+bx^n)^3 (c+dx^n)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{2n}}{(bx^n+a)^3(dx^n+c)}dx^n}{n}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{a^2}{b(bc-ad)(bx^n+a)^3}+\frac{(ad-2bc)a}{b(bc-ad)^2(bx^n+a)^2}+\frac{bc^2}{(bc-ad)^3(bx^n+a)}-\frac{c^2d}{(bc-ad)^3(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2}{2b^2(bc-ad)(a+bx^n)^2}+\frac{a(2bc-ad)}{b^2(bc-ad)^2(a+bx^n)}+\frac{c^2\log(a+bx^n)}{(bc-ad)^3}-\frac{c^2\log(c+dx^n)}{(bc-ad)^3}}{n}$$

input
```
Int[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

output
```
(-1/2*a^2/(b^2*(b*c - a*d)*(a + b*x^n)^2) + (a*(2*b*c - a*d))/(b^2*(b*c -
a*d)^2*(a + b*x^n)) + (c^2*Log[a + b*x^n])/(b*c - a*d)^3 - (c^2*Log[c + d*
x^n])/(b*c - a*d)^3)/n
```

3.1050.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1050.4 Maple [A] (verified)

Time = 6.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.41

method	result	s
risch	$-\frac{a\left(2abd\,x^{n}-4b^{2}c\,x^{n}+a^{2}d-3abc\right)}{2n\,b^{2}\left(ad-bc\right)^{2}\left(a+b\,x^{n}\right)^{2}}+\frac{c^{2}\ln\left(x^{n}+\frac{c}{d}\right)}{n\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}-\frac{c^{2}\ln\left(x^{n}+\frac{a}{b}\right)}{n\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$	1
norman	$\frac{\frac{\left(-ad+2bc\right)a\,\mathrm{e}^{n\ln\left(x\right)}}{nb\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}+\frac{a^{2}\left(-ad+3bc\right)}{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)b^{2}n}}{\left(a+b\,\mathrm{e}^{n\ln\left(x\right)}\right)^{2}}+\frac{c^{2}\ln\left(c+d\,\mathrm{e}^{n\ln\left(x\right)}\right)}{n\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}-\frac{c^{2}\ln\left(a+b\,\mathrm{e}^{n\ln\left(x\right)}\right)}{n\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$	2

input
```
int(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(2*a*b*d*x^n-4*b^2*c*x^n+a^2*d-3*a*b*c)/n/b^2/(a*d-b*c)^2/(a+b*x^n)
^2+c^2/n/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+c/d)-c^2/n/(
a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+a/b)
```

3.1050.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(118) = 236.

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.51

$$\int \frac{x^{-1+3n}}{\left(a+bx^n\right)^3\left(c+dx^n\right)}\,dx = \frac{3\,a^2b^2c^2-4\,a^3bcd+a^4d^2+2\,(2\,ab^3c^2-3\,a^2b^2cd+a^3bd^2)x^n+2\,(b^4c^2x^{2\,n}+2\,ab^3c^2x^n+a^2b^2c^2)\log{(bx^n+}}{2\,((b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)nx^{2\,n}+2\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3)nx^n+}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

output
```
1/2*(3*a^2*b^2*c^2 - 4*a^3*b*c*d + a^4*d^2 + 2*(2*a*b^3*c^2 - 3*a^2*b^2*c*
d + a^3*b*d^2)*x^n + 2*(b^4*c^2*x^(2*n) + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*l
og(b*x^n + a) - 2*(b^4*c^2*x^(2*n) + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*log(d*
x^n + c))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*n*x^(
2*n) + 2*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*n*x
^n + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*n)
```

3.1050.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3\,(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n)**3/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.1050.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(118) = 236$.

Time = 0.21 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.18

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3\,(c+dx^n)}\,dx$$
$$= \frac{c^2\log\left(\frac{bx^n+a}{b}\right)}{b^3c^3n-3\,ab^2c^2dn+3\,a^2bcd^2n-a^3d^3n} - \frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{b^3c^3n-3\,ab^2c^2dn+3\,a^2bcd^2n-a^3d^3n}$$
$$+ \frac{3\,a^2bc-a^3d+2\,(2\,ab^2c-a^2bd)x^n}{2\,(a^2b^4c^2n-2\,a^3b^3cdn+a^4b^2d^2n+(b^6c^2n-2\,ab^5cdn+a^2b^4d^2n)x^{2\,n}+2\,(ab^5c^2n-2\,a^2b^4cdn+a^3b^3d^2}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

output
```
c^2*log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^
3*d^3*n) - c^2*log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c
*d^2*n - a^3*d^3*n) + 1/2*(3*a^2*b*c - a^3*d + 2*(2*a*b^2*c - a^2*b*d)*x^n
)/(a^2*b^4*c^2*n - 2*a^3*b^3*c*d*n + a^4*b^2*d^2*n + (b^6*c^2*n - 2*a*b^5*
c*d*n + a^2*b^4*d^2*n)*x^(2*n) + 2*(a*b^5*c^2*n - 2*a^2*b^4*c*d*n + a^3*b^
3*d^2*n)*x^n)
```

3.1050.8 Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3\,(c+dx^n)}\,dx=\int \frac{x^{3\,n-1}}{(bx^n+a)^3(dx^n+c)}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)
```

3.1050.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3\,(c+dx^n)}\,dx=\int \frac{x^{3\,n-1}}{(a+b\,x^n)^3\,\,(c+d\,x^n)}\,dx$$

input
```
int(x^(3*n - 1)/((a + b*x^n)^3*(c + d*x^n)),x)
```

output
```
int(x^(3*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)
```

3.1051 $\int x^{13}(b+cx)^{13}(b+2cx)\,dx$

3.1051.1 Optimal result

Integrand size = 17, antiderivative size = 14

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx=\frac{1}{14}x^{14}(b+cx)^{14}$$

output
```
1/14*x^14*(c*x+b)^14
```

3.1051.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(14) = 28.

Time = 0.01 (sec) , antiderivative size = 172, normalized size of antiderivative = 12.29

$$\begin{aligned}\int x^{13}(b+cx)^{13}(b+2cx)\,dx=\;&\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}\\&+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}\\&+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}\\&+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}\end{aligned}$$

input
```
Integrate[x^13*(b + c*x)^13*(b + 2*c*x),x]
```

output
```
(b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (
143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7
*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^2
4)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)
/14
```

3.1051.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{83\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx$$

$$\Big\downarrow \, 83$$

$$\frac{1}{14}x^{14}(b+cx)^{14}$$

input
```
Int[x^13*(b + c*x)^13*(b + 2*c*x),x]
```

output
```
(x^14*(b + c*x)^14)/14
```

3.1051.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

3.1051.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(12) = 24.

Time = 4.54 (sec) , antiderivative size = 155, normalized size of antiderivative = 11.07

method	result
gosper	$143b^5c^9x^{23} + \frac{143}{2}b^4c^{10}x^{24} + 26b^3c^{11}x^{25} + \frac{13}{2}b^2c^{12}x^{26} + b\,c^{13}x^{27} + \frac{1}{14}c^{14}x^{28} + \frac{1}{14}b^{14}x^{14} + b^{13}c\,x$
default	$143b^5c^9x^{23} + \frac{143}{2}b^4c^{10}x^{24} + 26b^3c^{11}x^{25} + \frac{13}{2}b^2c^{12}x^{26} + b\,c^{13}x^{27} + \frac{1}{14}c^{14}x^{28} + \frac{1}{14}b^{14}x^{14} + b^{13}c\,x$
norman	$143b^5c^9x^{23} + \frac{143}{2}b^4c^{10}x^{24} + 26b^3c^{11}x^{25} + \frac{13}{2}b^2c^{12}x^{26} + b\,c^{13}x^{27} + \frac{1}{14}c^{14}x^{28} + \frac{1}{14}b^{14}x^{14} + b^{13}c\,x$
risch	$143b^5c^9x^{23} + \frac{143}{2}b^4c^{10}x^{24} + 26b^3c^{11}x^{25} + \frac{13}{2}b^2c^{12}x^{26} + b\,c^{13}x^{27} + \frac{1}{14}c^{14}x^{28} + \frac{1}{14}b^{14}x^{14} + b^{13}c\,x$
parallelrisch	$143b^5c^9x^{23} + \frac{143}{2}b^4c^{10}x^{24} + 26b^3c^{11}x^{25} + \frac{13}{2}b^2c^{12}x^{26} + b\,c^{13}x^{27} + \frac{1}{14}c^{14}x^{28} + \frac{1}{14}b^{14}x^{14} + b^{13}c\,x$

input
```
int(x^13*(c*x+b)^13*(2*c*x+b),x,method=_RETURNVERBOSE)
```

output
```
143*b^5*c^9*x^23+143/2*b^4*c^10*x^24+26*b^3*c^11*x^25+13/2*b^2*c^12*x^26+b
*c^13*x^27+1/14*c^14*x^28+1/14*b^14*x^14+b^13*c*x^15+13/2*b^12*c^2*x^16+26
*b^11*c^3*x^17+143/2*b^10*c^4*x^18+143*b^9*c^5*x^19+429/2*b^8*c^6*x^20+171
6/7*b^7*c^7*x^21+429/2*b^6*c^8*x^22
```

3.1051.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 11.00

$$\begin{aligned}\int x^{13}(b+cx)^{13}(b+2cx)\,dx &= \frac{1}{14}\,c^{14}x^{28} + bc^{13}x^{27} + \frac{13}{2}\,b^2c^{12}x^{26} + 26\,b^3c^{11}x^{25} \\ &\quad + \frac{143}{2}\,b^4c^{10}x^{24} + 143\,b^5c^9x^{23} + \frac{429}{2}\,b^6c^8x^{22} \\ &\quad + \frac{1716}{7}\,b^7c^7x^{21} + \frac{429}{2}\,b^8c^6x^{20} + 143\,b^9c^5x^{19} + \frac{143}{2}\,b^{10}c^4x^{18} \\ &\quad + 26\,b^{11}c^3x^{17} + \frac{13}{2}\,b^{12}c^2x^{16} + b^{13}cx^{15} + \frac{1}{14}\,b^{14}x^{14}\end{aligned}$$

input
```
integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="fricas")
```

output
```
1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143
/2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*
x^21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^
11*c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

3.1051.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 175, normalized size of antiderivative = 12.50

$$\begin{aligned}\int x^{13}(b+cx)^{13}(b+2cx)\,dx=&\ \frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13b^{12}c^2x^{16}}{2}+26b^{11}c^3x^{17}\\&+\frac{143b^{10}c^4x^{18}}{2}+143b^9c^5x^{19}+\frac{429b^8c^6x^{20}}{2}\\&+\frac{1716b^7c^7x^{21}}{7}+\frac{429b^6c^8x^{22}}{2}+143b^5c^9x^{23}+\frac{143b^4c^{10}x^{24}}{2}\\&+26b^3c^{11}x^{25}+\frac{13b^2c^{12}x^{26}}{2}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}\end{aligned}$$

input
```
integrate(x**13*(c*x+b)**13*(2*c*x+b),x)
```

output
```
b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**
17 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2
+ 1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 1
43*b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c*
*13*x**27 + c**14*x**28/14
```

3.1051.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(12) = 24$.

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 11.00

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx=\frac{1}{14}\,c^{14}x^{28}+bc^{13}x^{27}+\frac{13}{2}\,b^2c^{12}x^{26}+26\,b^3c^{11}x^{25}+\frac{143}{2}\,b^4c^{10}x^{24}+143\,b^5c^9x^{23}+\frac{429}{2}\,b^6c^8x^{22}+\frac{1716}{7}\,b^7c^7x^{21}+\frac{429}{2}\,b^8c^6x^{20}+143\,b^9c^5x^{19}+\frac{143}{2}\,b^{10}c^4x^{18}+26\,b^{11}c^3x^{17}+\frac{13}{2}\,b^{12}c^2x^{16}+b^{13}cx^{15}+\frac{1}{14}\,b^{14}x^{14}$$

input
```
integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="maxima")
```

output
```
1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143
/2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*
x^21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^
11*c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

3.1051.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx=\frac{1}{14}\,{\left(cx^2+bx\right)}^{14}$$

input
```
integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="giac")
```

output
```
1/14*(c*x^2 + b*x)^14
```

3.1051.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 11.00

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx=\frac{b^{14}\,x^{14}}{14}+b^{13}\,c\,x^{15}+\frac{13\,b^{12}\,c^2\,x^{16}}{2}+26\,b^{11}\,c^3\,x^{17}+\frac{143\,b^{10}\,c^4\,x^{18}}{2}+143\,b^9\,c^5\,x^{19}+\frac{429\,b^8\,c^6\,x^{20}}{2}+\frac{1716\,b^7\,c^7\,x^{21}}{7}+\frac{429\,b^6\,c^8\,x^{22}}{2}+143\,b^5\,c^9\,x^{23}+\frac{143\,b^4\,c^{10}\,x^{24}}{2}+26\,b^3\,c^{11}\,x^{25}+\frac{13\,b^2\,c^{12}\,x^{26}}{2}+b\,c^{13}\,x^{27}+\frac{c^{14}\,x^{28}}{14}$$

input

```
int(x^13*(b + c*x)^13*(b + 2*c*x),x)
```

output

```
(b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2
*x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (
429*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b
^5*c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26
)/2
```

3.1051. $\int x^{13}(b+cx)^{13}(b+2cx)\,dx$

3.1052 $\int x^{27}(b+cx^2)^{13}\left(b+2cx^2\right)\,dx$

3.1052.1 Optimal result

Integrand size = 21, antiderivative size = 16

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx=\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

output
```
1/28*x^28*(c*x^2+b)^14
```

3.1052.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\begin{aligned}\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx &= \frac{b^{14}x^{28}}{28}+\frac{1}{2}b^{13}cx^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}\\&\quad+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}\\&\quad+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{143}{2}b^5c^9x^{46}+\frac{143}{4}b^4c^{10}x^{48}\\&\quad+13b^3c^{11}x^{50}+\frac{13}{4}b^2c^{12}x^{52}+\frac{1}{2}bc^{13}x^{54}+\frac{c^{14}x^{56}}{28}\end{aligned}$$

input
```
Integrate[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]
```

output
```
(b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (
858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b
^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/
2 + (c^14*x^56)/28
```

3.1052.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {354, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^{26}\left(cx^2+b\right)^{13}\left(2cx^2+b\right)dx^2$$

$$\Big\downarrow \text{83}$$

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

input
```
Int[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]
```

output
```
(x^28*(b + c*x^2)^14)/28
```

3.1052.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 354

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.1052.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 4.74 (sec) , antiderivative size = 157, normalized size of antiderivative = 9.81

method	result
gosper	$\frac{1}{28}b^{14}x^{28}+\frac{1}{2}b^{13}c\,x^{30}+\frac{13}{4}b^{12}c^{2}x^{32}+13b^{11}c^{3}x^{34}+\frac{143}{4}b^{10}c^{4}x^{36}+\frac{143}{2}b^{9}c^{5}x^{38}+\frac{429}{4}b^{8}c^{6}x^{40}+\frac{858}{7}$
default	$\frac{1}{28}b^{14}x^{28}+\frac{1}{2}b^{13}c\,x^{30}+\frac{13}{4}b^{12}c^{2}x^{32}+13b^{11}c^{3}x^{34}+\frac{143}{4}b^{10}c^{4}x^{36}+\frac{143}{2}b^{9}c^{5}x^{38}+\frac{429}{4}b^{8}c^{6}x^{40}+\frac{858}{7}$
risch	$\frac{1}{28}b^{14}x^{28}+\frac{1}{2}b^{13}c\,x^{30}+\frac{13}{4}b^{12}c^{2}x^{32}+13b^{11}c^{3}x^{34}+\frac{143}{4}b^{10}c^{4}x^{36}+\frac{143}{2}b^{9}c^{5}x^{38}+\frac{429}{4}b^{8}c^{6}x^{40}+\frac{858}{7}$
parallelrisch	$\frac{1}{28}b^{14}x^{28}+\frac{1}{2}b^{13}c\,x^{30}+\frac{13}{4}b^{12}c^{2}x^{32}+13b^{11}c^{3}x^{34}+\frac{143}{4}b^{10}c^{4}x^{36}+\frac{143}{2}b^{9}c^{5}x^{38}+\frac{429}{4}b^{8}c^{6}x^{40}+\frac{858}{7}$

input

```
int(x^27*(c*x^2+b)^13*(2*c*x^2+b),x,method=_RETURNVERBOSE)
```

output

```
1/28*b^14*x^28+1/2*b^13*c*x^30+13/4*b^12*c^2*x^32+13*b^11*c^3*x^34+143/4*b
^10*c^4*x^36+143/2*b^9*c^5*x^38+429/4*b^8*c^6*x^40+858/7*b^7*c^7*x^42+429/
4*b^6*c^8*x^44+143/2*b^5*c^9*x^46+143/4*b^4*c^10*x^48+13*b^3*c^11*x^50+13/
4*b^2*c^12*x^52+1/2*b*c^13*x^54+1/28*c^14*x^56
```

3.1052.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\begin{aligned}\int x^{27}(b+cx^2)^{13}\left(b+2cx^2\right)\,dx &= \frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^{2}c^{12}x^{52}+13\,b^{3}c^{11}x^{50}\\ &\quad+\frac{143}{4}\,b^{4}c^{10}x^{48}+\frac{143}{2}\,b^{5}c^{9}x^{46}+\frac{429}{4}\,b^{6}c^{8}x^{44}+\frac{858}{7}\,b^{7}c^{7}x^{42}\\ &\quad+\frac{429}{4}\,b^{8}c^{6}x^{40}+\frac{143}{2}\,b^{9}c^{5}x^{38}+\frac{143}{4}\,b^{10}c^{4}x^{36}\\ &\quad+13\,b^{11}c^{3}x^{34}+\frac{13}{4}\,b^{12}c^{2}x^{32}+\frac{1}{2}\,b^{13}cx^{30}+\frac{1}{28}\,b^{14}x^{28}\end{aligned}$$

input
```
integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="fricas")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7
*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36
+ 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

3.1052.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(12) = 24$.

Time = 0.04 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\begin{aligned}\int x^{27}\left(b+cx^{2}\right)^{13}\left(b+2cx^{2}\right)\,dx=&\frac{b^{14}x^{28}}{28}+\frac{b^{13}cx^{30}}{2}+\frac{13b^{12}c^{2}x^{32}}{4}+13b^{11}c^{3}x^{34}\\&+\frac{143b^{10}c^{4}x^{36}}{4}+\frac{143b^{9}c^{5}x^{38}}{2}+\frac{429b^{8}c^{6}x^{40}}{4}\\&+\frac{858b^{7}c^{7}x^{42}}{7}+\frac{429b^{6}c^{8}x^{44}}{4}+\frac{143b^{5}c^{9}x^{46}}{2}+\frac{143b^{4}c^{10}x^{48}}{4}\\&+13b^{3}c^{11}x^{50}+\frac{13b^{2}c^{12}x^{52}}{4}+\frac{bc^{13}x^{54}}{2}+\frac{c^{14}x^{56}}{28}\end{aligned}$$

input
```
integrate(x**27*(c*x**2+b)**13*(2*c*x**2+b),x)
```

output
```
b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x
**34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**4
0/4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/
2 + 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 +
 b*c**13*x**54/2 + c**14*x**56/28
```

3.1052.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{27}(b+cx^2)^{13}(b+2cx^2)\,dx = \frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13\,b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

input
```
integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="maxima")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7
*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36
+ 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

3.1052.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{27}(b+cx^2)^{13}(b+2cx^2)\,dx = \frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13\,b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

input
```
integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="giac")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7
*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36
+ 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

3.1052.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\begin{aligned}\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx=&\frac{b^{14}\,x^{28}}{28}+\frac{b^{13}\,c\,x^{30}}{2}+\frac{13\,b^{12}\,c^2\,x^{32}}{4}+13\,b^{11}\,c^3\,x^{34}+\frac{143\,b^{10}\,c^4\,x^{36}}{4}\\&+\frac{143\,b^9\,c^5\,x^{38}}{2}+\frac{429\,b^8\,c^6\,x^{40}}{4}+\frac{858\,b^7\,c^7\,x^{42}}{7}\\&+\frac{429\,b^6\,c^8\,x^{44}}{4}+\frac{143\,b^5\,c^9\,x^{46}}{2}+\frac{143\,b^4\,c^{10}\,x^{48}}{4}\\&+13\,b^3\,c^{11}\,x^{50}+\frac{13\,b^2\,c^{12}\,x^{52}}{4}+\frac{b\,c^{13}\,x^{54}}{2}+\frac{c^{14}\,x^{56}}{28}\end{aligned}$$

input
```
int(x^27*(b + c*x^2)^13*(b + 2*c*x^2),x)
```

output
```
(b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*
b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5
*x^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44
)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (1
3*b^2*c^12*x^52)/4
```

3.1053 $\int x^{41}(b+cx^3)^{13}(b+2cx^3)\,dx$

3.1053.1 Optimal result

Integrand size = 21, antiderivative size = 16

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx=\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

output
```
1/42*x^42*(c*x^3+b)^14
```

3.1053.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 186 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 186, normalized size of antiderivative = 11.62

$$\begin{aligned}\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx&=\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}\\&+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}\\&+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}\\&+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}\end{aligned}$$

input
```
Integrate[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]
```

output
```
(b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^5
1)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2
 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (1
43*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^1
3*x^81)/3 + (c^14*x^84)/42
```

3.1053.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int x^{39}\left(cx^3+b\right)^{13}\left(2cx^3+b\right)dx^3$$

$$\Big\downarrow \text{83}$$

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

input
```
Int[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]
```

output
```
(x^42*(b + c*x^3)^14)/42
```

3.1053.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1053.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 4.66 (sec) , antiderivative size = 157, normalized size of antiderivative = 9.81

method	result
gosper	$\frac{1}{42}b^{14}x^{42}+\frac{1}{3}b^{13}c\,x^{45}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{1}{3}b$
default	$\frac{1}{42}b^{14}x^{42}+\frac{1}{3}b^{13}c\,x^{45}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{1}{3}b$
risch	$\frac{1}{42}b^{14}x^{42}+\frac{1}{3}b^{13}c\,x^{45}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{1}{3}b$
parallelrisch	$\frac{1}{42}b^{14}x^{42}+\frac{1}{3}b^{13}c\,x^{45}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{1}{3}b$

input
```
int(x^41*(c*x^3+b)^13*(2*c*x^3+b),x,method=_RETURNVERBOSE)
```

output
```
1/42*b^14*x^42+1/3*b^13*c*x^45+13/6*b^12*c^2*x^48+26/3*b^11*c^3*x^51+143/6
*b^10*c^4*x^54+143/3*b^9*c^5*x^57+13/6*b^2*c^12*x^78+1/3*b*c^13*x^81+1/42*
c^14*x^84+143/2*b^8*c^6*x^60+572/7*b^7*c^7*x^63+143/2*b^6*c^8*x^66+143/3*b
^5*c^9*x^69+143/6*b^4*c^10*x^72+26/3*b^3*c^11*x^75
```

3.1053.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\begin{aligned}\int x^{41}\left(b+cx^{3}\right)^{13}\left(b+2cx^{3}\right)\,dx &= \frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^{2}c^{12}x^{78}+\frac{26}{3}\,b^{3}c^{11}x^{75}\\ &\quad+\frac{143}{6}\,b^{4}c^{10}x^{72}+\frac{143}{3}\,b^{5}c^{9}x^{69}+\frac{143}{2}\,b^{6}c^{8}x^{66}\\ &\quad+\frac{572}{7}\,b^{7}c^{7}x^{63}+\frac{143}{2}\,b^{8}c^{6}x^{60}+\frac{143}{3}\,b^{9}c^{5}x^{57}+\frac{143}{6}\,b^{10}c^{4}x^{54}\\ &\quad+\frac{26}{3}\,b^{11}c^{3}x^{51}+\frac{13}{6}\,b^{12}c^{2}x^{48}+\frac{1}{3}\,b^{13}cx^{45}+\frac{1}{42}\,b^{14}x^{42}\end{aligned}$$

input
```
integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="fricas")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b
^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^5
4 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*
x^42
```

3.1053.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(12) = 24$.

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 11.56

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx=\frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^2x^{48}}{6}+\frac{26b^{11}c^3x^{51}}{3}+\frac{143b^{10}c^4x^{54}}{6}+\frac{143b^9c^5x^{57}}{3}+\frac{143b^8c^6x^{60}}{2}+\frac{572b^7c^7x^{63}}{7}+\frac{143b^6c^8x^{66}}{2}+\frac{143b^5c^9x^{69}}{3}+\frac{143b^4c^{10}x^{72}}{6}+\frac{26b^3c^{11}x^{75}}{3}+\frac{13b^2c^{12}x^{78}}{6}+\frac{bc^{13}x^{81}}{3}+\frac{c^{14}x^{84}}{42}$$

input
```
integrate(x**41*(c*x**3+b)**13*(2*c*x**3+b),x)
```

output
```
b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x
**51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x*
*60/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**6
9/3 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78
/6 + b*c**13*x**81/3 + c**14*x**84/42
```

3.1053.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx = \frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^2c^{12}x^{78}+\frac{26}{3}b^3c^{11}x^{75}+\frac{143}{6}b^4c^{10}x^{72}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{2}b^6c^8x^{66}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^8c^6x^{60}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{6}b^{10}c^4x^{54}+\frac{26}{3}b^{11}c^3x^{51}+\frac{13}{6}b^{12}c^2x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

input
```
integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="maxima")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b
^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^5
4 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*
x^42
```

3.1053.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx = \frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^2c^{12}x^{78}+\frac{26}{3}b^3c^{11}x^{75}+\frac{143}{6}b^4c^{10}x^{72}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{2}b^6c^8x^{66}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^8c^6x^{60}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{6}b^{10}c^4x^{54}+\frac{26}{3}b^{11}c^3x^{51}+\frac{13}{6}b^{12}c^2x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

input
```
integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="giac")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b
^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^5
4 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*
x^42
```

3.1053.9 Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx = \frac{b^{14}\,x^{42}}{42}+\frac{b^{13}\,c\,x^{45}}{3}+\frac{13\,b^{12}\,c^2\,x^{48}}{6}+\frac{26\,b^{11}\,c^3\,x^{51}}{3}+\frac{143\,b^{10}\,c^4\,x^{54}}{6}+\frac{143\,b^9\,c^5\,x^{57}}{3}+\frac{143\,b^8\,c^6\,x^{60}}{2}+\frac{572\,b^7\,c^7\,x^{63}}{7}+\frac{143\,b^6\,c^8\,x^{66}}{2}+\frac{143\,b^5\,c^9\,x^{69}}{3}+\frac{143\,b^4\,c^{10}\,x^{72}}{6}+\frac{26\,b^3\,c^{11}\,x^{75}}{3}+\frac{13\,b^2\,c^{12}\,x^{78}}{6}+\frac{b\,c^{13}\,x^{81}}{3}+\frac{c^{14}\,x^{84}}{42}$$

input
```
int(x^41*(b + c*x^3)^13*(b + 2*c*x^3),x)
```

output
```
(b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*
b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9
*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*
x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75
)/3 + (13*b^2*c^12*x^78)/6
```

3.1054 $\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx$

3.1054.1 Optimal result

Integrand size = 25, antiderivative size = 21

$$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

output
```
1/14*x^(14*n)*(b+c*x^n)^14/n
```

3.1054.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

input
```
Integrate[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n),x]
```

output
```
(x^(14*n)*(b + c*x^n)^14)/(14*n)
```

3.1054.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14n-1}(b+cx^n)^{13}\,(b+2cx^n)\;dx$$

$$\Big\downarrow \; 948$$

$$\frac{\int x^{13n}(cx^n+b)^{13}\,(2cx^n+b)\,dx^n}{n}$$

$$\Big\downarrow \; 83$$

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

input
```
Int[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n),x]
```

output
```
(x^(14*n)*(b + c*x^n)^14)/(14*n)
```

3.1054.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1054.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(19) = 38.

Time = 183.42 (sec) , antiderivative size = 230, normalized size of antiderivative = 10.95

method	result
risch	$\frac{c^{14}x^{28n}}{14n}+\frac{b\,c^{13}x^{27n}}{n}+\frac{13b^2c^{12}x^{26n}}{2n}+\frac{26b^3c^{11}x^{25n}}{n}+\frac{143b^4c^{10}x^{24n}}{2n}+\frac{143b^5c^9x^{23n}}{n}+\frac{429b^6c^8x^{22n}}{2n}+\frac{1716b^7c^7x^2}{7n}$
parallelrisch	$c^{14}x^{-1+14n}x^{14n}x+14b\,c^{13}x^{-1+14n}x^{13n}x+91b^2c^{12}x^{-1+14n}x^{12n}x+364b^3c^{11}x^{-1+14n}x^{11n}x+1001b^4c^{10}x^{-1+14n}x^{10n}x+2002$

input
```
int(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*b^2*c^12/n*(x^n)^26+26*b^3*c^1
1/n*(x^n)^25+143/2*b^4*c^10/n*(x^n)^24+143*b^5*c^9/n*(x^n)^23+429/2*b^6*c^
8/n*(x^n)^22+1716/7*b^7*c^7/n*(x^n)^21+429/2*b^8*c^6/n*(x^n)^20+143*b^9*c^
5/n*(x^n)^19+143/2*b^10*c^4/n*(x^n)^18+26*b^11*c^3/n*(x^n)^17+13/2*b^12*c^
2/n*(x^n)^16+b^13*c/n*(x^n)^15+1/14*b^14/n*(x^n)^14
```

3.1054.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(19) = 38.

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 9.00

$$\int x^{-1+14n}(b+cx^n)^{13}\left(b+2cx^n\right)\,dx$$
$$=\frac{c^{14}x^{28\,n}+14\,bc^{13}x^{27\,n}+91\,b^2c^{12}x^{26\,n}+364\,b^3c^{11}x^{25\,n}+1001\,b^4c^{10}x^{24\,n}+2002\,b^5c^9x^{23\,n}+3003\,b^6c^8x^{22\,n}+}{}$$

input
```
integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="fricas")
```

output
```
1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*
c^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*
c^8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^
5*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*
x^(16*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

3.1054.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(15) = 30$.

Time = 10.80 (sec) , antiderivative size = 360, normalized size of antiderivative = 17.14

$$\int x^{-1+14n}(b+cx^n)^{13}\,(b+2cx^n)\;dx$$
$$=\begin{cases} \frac{b^{14}xx^{14n-1}}{14n} + \frac{b^{13}cxx^{n}x^{14n-1}}{n} + \frac{13b^{12}c^{2}xx^{2n}x^{14n-1}}{2n} + \frac{26b^{11}c^{3}xx^{3n}x^{14n-1}}{n} + \frac{143b^{10}c^{4}xx^{4n}x^{14n-1}}{2n} + \frac{143b^{9}c^{5}xx^{5n}x^{14n-1}}{n} + 429b \\ (b+c)^{13}\,(b+2c)\log{(x)} \end{cases}$$

input
```
integrate(x**(-1+14*n)*(b+c*x**n)**13*(b+2*c*x**n),x)
```

output
```
Piecewise((b**14*x*x**(14*n - 1)/(14*n) + b**13*c*x*x**n*x**(14*n - 1)/n +
 13*b**12*c**2*x*x**(2*n)*x**(14*n - 1)/(2*n) + 26*b**11*c**3*x*x**(3*n)*x
**(14*n - 1)/n + 143*b**10*c**4*x*x**(4*n)*x**(14*n - 1)/(2*n) + 143*b**9*
c**5*x*x**(5*n)*x**(14*n - 1)/n + 429*b**8*c**6*x*x**(6*n)*x**(14*n - 1)/(
2*n) + 1716*b**7*c**7*x*x**(7*n)*x**(14*n - 1)/(7*n) + 429*b**6*c**8*x*x**
(8*n)*x**(14*n - 1)/(2*n) + 143*b**5*c**9*x*x**(9*n)*x**(14*n - 1)/n + 143
*b**4*c**10*x*x**(10*n)*x**(14*n - 1)/(2*n) + 26*b**3*c**11*x*x**(11*n)*x*
*(14*n - 1)/n + 13*b**2*c**12*x*x**(12*n)*x**(14*n - 1)/(2*n) + b*c**13*x*
x**(13*n)*x**(14*n - 1)/n + c**14*x*x**(14*n)*x**(14*n - 1)/(14*n), Ne(n,
0)), ((b + c)**13*(b + 2*c)*log(x), True))
```

3.1054.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(19) = 38$.

Time = 0.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\begin{aligned}\int x^{-1+14n}(b+cx^n)^{13}\,(b+2cx^n)\;dx &= \frac{c^{14}x^{28\,n}}{14\,n} + \frac{bc^{13}x^{27\,n}}{n} + \frac{13\,b^{2}c^{12}x^{26\,n}}{2\,n} \\ &+ \frac{26\,b^{3}c^{11}x^{25\,n}}{n} + \frac{143\,b^{4}c^{10}x^{24\,n}}{2\,n} + \frac{143\,b^{5}c^{9}x^{23\,n}}{n} \\ &+ \frac{429\,b^{6}c^{8}x^{22\,n}}{2\,n} + \frac{1716\,b^{7}c^{7}x^{21\,n}}{7\,n} + \frac{429\,b^{8}c^{6}x^{20\,n}}{2\,n} \\ &+ \frac{143\,b^{9}c^{5}x^{19\,n}}{n} + \frac{143\,b^{10}c^{4}x^{18\,n}}{2\,n} + \frac{26\,b^{11}c^{3}x^{17\,n}}{n} \\ &+ \frac{13\,b^{12}c^{2}x^{16\,n}}{2\,n} + \frac{b^{13}cx^{15\,n}}{n} + \frac{b^{14}x^{14\,n}}{14\,n}\end{aligned}$$

input
```
integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="maxima")
```

output
```
1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b
^3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n +
429/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20
*n)/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x
^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14
*n)/n
```

3.1054.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(19) = 38$.

Time = 0.36 (sec) , antiderivative size = 189, normalized size of antiderivative = 9.00

$$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx$$
$$= \frac{c^{14}x^{28\,n} + 14\,bc^{13}x^{27\,n} + 91\,b^2c^{12}x^{26\,n} + 364\,b^3c^{11}x^{25\,n} + 1001\,b^4c^{10}x^{24\,n} + 2002\,b^5c^9x^{23\,n} + 3003\,b^6c^8x^{22\,n} +}{}$$

input
```
integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="giac")
```

output
```
1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*
c^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*
c^8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^
5*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*
x^(16*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

3.1054.9 Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\begin{aligned}\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx &= \frac{b^{14}\,x^{14\,n}}{14\,n} + \frac{c^{14}\,x^{28\,n}}{14\,n} + \frac{13\,b^{12}\,c^2\,x^{16\,n}}{2\,n} \\ &+ \frac{26\,b^{11}\,c^3\,x^{17\,n}}{n} + \frac{143\,b^{10}\,c^4\,x^{18\,n}}{2\,n} + \frac{143\,b^9\,c^5\,x^{19\,n}}{n} \\ &+ \frac{429\,b^8\,c^6\,x^{20\,n}}{2\,n} + \frac{1716\,b^7\,c^7\,x^{21\,n}}{7\,n} + \frac{429\,b^6\,c^8\,x^{22\,n}}{2\,n} \\ &+ \frac{143\,b^5\,c^9\,x^{23\,n}}{n} + \frac{143\,b^4\,c^{10}\,x^{24\,n}}{2\,n} + \frac{26\,b^3\,c^{11}\,x^{25\,n}}{n} \\ &+ \frac{13\,b^2\,c^{12}\,x^{26\,n}}{2\,n} + \frac{b^{13}\,c\,x^{15\,n}}{n} + \frac{b\,c^{13}\,x^{27\,n}}{n}\end{aligned}$$

input

```
int(x^(14*n - 1)*(b + c*x^n)^13*(b + 2*c*x^n),x)
```

output

```
(b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(
2*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9
*c^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/
(7*n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4
*c^10*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/
(2*n) + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

3.1054. $\int x^{-1+14n}(b+cx^n)^{13}\left(b+2cx^n\right)\,dx$

3.1055 $\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx$

3.1055.1 Optimal result

Integrand size = 31, antiderivative size = 13

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx=x^m(a+bx^n)^p$$

output
```
x^m*(a+b*x^n)^p
```

3.1055.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 8.23

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx$$
$$=\frac{x^m(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(a(m+n)\,\text{Hypergeometric2F1}\left(\frac{m}{n},1-p,\frac{m+n}{n},-\frac{bx^n}{a}\right)+b(m+np)x^n\,\text{Hypergeo}\right.}{a(m+n)}$$

input
```
Integrate[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n),x]
```

output
```
(x^m*(a + b*x^n)^p*(a*(m + n)*Hypergeometric2F1[m/n, 1 - p, (m + n)/n, -((
b*x^n)/a)] + b*(m + n*p)*x^n*Hypergeometric2F1[(m + n)/n, 1 - p, 2 + m/n,
-((b*x^n)/a)]))/(a*(m + n)*(1 + (b*x^n)/a)^p)
```

3.1055.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-1}(a+bx^n)^{p-1}\left(am+bx^n(m+np)\right)\,dx$$

$$\Big\downarrow \; 951$$

$$x^m(a+bx^n)^p$$

input
```
Int[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n),x]
```

output
```
x^m*(a + b*x^n)^p
```

3.1055.3.1 Defintions of rubi rules used

rule 951
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

3.1055.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(13) = 26.

Time = 12.57 (sec) , antiderivative size = 50, normalized size of antiderivative = 3.85

method	result	size
parallelrisch	$\frac{x\,x^{n} x^{-1+m} \left(a+b\,x^{n}\right)^{p-1} b^{2}+x\,x^{-1+m} \left(a+b\,x^{n}\right)^{p-1} a b}{b}$	50

input
```
int(x^(-1+m)*(a+b*x^n)^(p-1)*(a*m+b*(n*p+m)*x^n),x,method=_RETURNVERBOSE)
```

output
```
(x*x^n*x^(-1+m)*(a+b*x^n)^(p-1)*b^2+x*x^(-1+m)*(a+b*x^n)^(p-1)*a*b)/b
```

3.1055.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.46

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx = \left(bxx^{m-1}x^n+axx^{m-1}\right)(bx^n+a)^{p-1}$$

input
```
integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="fric
as")
```

output
```
(b*x*x^(m - 1)*x^n + a*x*x^(m - 1))*(b*x^n + a)^(p - 1)
```

3.1055.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(10) = 20$.

Time = 2.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.00

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx = axx^{m-1}(a+bx^n)^{p-1}+bxx^nx^{m-1}(a+bx^n)^{p-1}$$

input
```
integrate(x**(-1+m)*(a+b*x**n)**(-1+p)*(a*m+b*(n*p+m)*x**n),x)
```

output
```
a*x*x**(m - 1)*(a + b*x**n)**(p - 1) + b*x*x**n*x**(m - 1)*(a + b*x**n)**(
p - 1)
```

3.1055.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx = e^{(p\log(bx^n+a)+m\log(x))}$$

input
```
integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="maxi
ma")
```

output
```
e^(p*log(b*x^n + a) + m*log(x))
```

3.1055.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(13) = 26$.

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 5.38

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx$$
$$= bxx^ne^{(p\log(bx^n+a)+m\log(x)-\log(bx^n+a)-\log(x))}+axe^{(p\log(bx^n+a)+m\log(x)-\log(bx^n+a)-\log(x))}$$

input
```
integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="giac
")
```

output
```
b*x*x^n*e^(p*log(b*x^n + a) + m*log(x) - log(b*x^n + a) - log(x)) + a*x*e^
(p*log(b*x^n + a) + m*log(x) - log(b*x^n + a) - log(x))
```

3.1055.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx = \left(a\,x^m+b\,x^{m+n}\right)\,\left(a+b\,x^n\right)^{p-1}$$

input
```
int(x^(m - 1)*(a*m + b*x^n*(m + n*p))*(a + b*x^n)^(p - 1),x)
```

output
```
(a*x^m + b*x^(m + n))*(a + b*x^n)^(p - 1)
```

3.1056 $\int \frac{b+2cx}{x(b+cx)}\,dx$

3.1056.1 Optimal result

Integrand size = 17, antiderivative size = 8

$$\int \frac{b+2cx}{x(b+cx)}\,dx=\log(x(b+cx))$$

output
```
ln(x*(c*x+b))
```

3.1056.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \frac{b+2cx}{x(b+cx)}\,dx=\log(x)+\log(b+cx)$$

input
```
Integrate[(b + 2*c*x)/(x*(b + c*x)),x]
```

output
```
Log[x] + Log[b + c*x]
```

3.1056.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx}{x(b+cx)}\,dx$$

$$\Big\downarrow \ 86$$

$$\int \left(\frac{c}{b+cx}+\frac{1}{x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\log(b+cx)+\log(x)$$

input
```
Int[(b + 2*c*x)/(x*(b + c*x)),x]
```

output
```
Log[x] + Log[b + c*x]
```

3.1056.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1056.4 Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\ln\left(x(cx+b)\right)$	9
norman	$\ln\left(x\right)+\ln\left(cx+b\right)$	10
parallelrisch	$\ln\left(x\right)+\ln\left(cx+b\right)$	10
risch	$\ln\left(c\,x^{2}+bx\right)$	11

input
```
int((2*c*x+b)/x/(c*x+b),x,method=_RETURNVERBOSE)
```

output
```
ln(x*(c*x+b))
```

3.1056.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{b+2cx}{x(b+cx)}\,dx = \log\left(cx^2+bx\right)$$

input
```
integrate((2*c*x+b)/x/(c*x+b),x, algorithm="fricas")
```

output
```
log(c*x^2 + b*x)
```

3.1056.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{x(b+cx)}\,dx = \log\left(bx+cx^2\right)$$

input
```
integrate((2*c*x+b)/x/(c*x+b),x)
```

output
```
log(b*x + c*x**2)
```

3.1056.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \frac{b+2cx}{x(b+cx)}\,dx=\log\left(cx+b\right)+\log\left(x\right)$$

input
```
integrate((2*c*x+b)/x/(c*x+b),x, algorithm="maxima")
```

output
```
log(c*x + b) + log(x)
```

3.1056.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{b+2cx}{x(b+cx)}\,dx=\log\left(|cx+b|\right)+\log\left(|x|\right)$$

input
```
integrate((2*c*x+b)/x/(c*x+b),x, algorithm="giac")
```

output
```
log(abs(c*x + b)) + log(abs(x))
```

3.1056.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{x(b+cx)}\,dx=\ln\left(x\,\left(b+c\,x\right)\right)$$

input
```
int((b + 2*c*x)/(x*(b + c*x)),x)
```

output
```
log(x*(b + c*x))
```

3.1057 $\int \frac{b+2cx^2}{x(b+cx^2)}\,dx$

3.1057.1 Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx=\log(x)+\frac{1}{2}\log\left(b+cx^2\right)$$

output

```
ln(x)+1/2*ln(c*x^2+b)
```

3.1057.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx=\log(x)+\frac{1}{2}\log\left(b+cx^2\right)$$

input

```
Integrate[(b + 2*c*x^2)/(x*(b + c*x^2)),x]
```

output

```
Log[x] + Log[b + c*x^2]/2
```

3.1057.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{2cx^2+b}{x^2\left(cx^2+b\right)}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{c}{cx^2+b}+\frac{1}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\log\left(b+cx^2\right)+\log\left(x^2\right)\right)$$

input
```
Int[(b + 2*c*x^2)/(x*(b + c*x^2)),x]
```

output
```
(Log[x^2] + Log[b + c*x^2])/2
```

3.1057.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1057.4 Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln\left(x\right)+\frac{\ln\left(c\,x^{2}+b\right)}{2}$	14
norman	$\ln\left(x\right)+\frac{\ln\left(c\,x^{2}+b\right)}{2}$	14
risch	$\ln\left(x\right)+\frac{\ln\left(c\,x^{2}+b\right)}{2}$	14
parallelrisch	$\ln\left(x\right)+\frac{\ln\left(c\,x^{2}+b\right)}{2}$	14

input
```
int((2*c*x^2+b)/x/(c*x^2+b),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+1/2*ln(c*x^2+b)
```

3.1057.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx=\frac{1}{2}\log\left(cx^2+b\right)+\log\left(x\right)$$

input
```
integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="fricas")
```

output
```
1/2*log(c*x^2 + b) + log(x)
```

3.1057.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx=\log{\left(x\right)}+\frac{\log{\left(\frac{b}{c}+x^2\right)}}{2}$$

input
```
integrate((2*c*x**2+b)/x/(c*x**2+b),x)
```

output
```
log(x) + log(b/c + x**2)/2
```

3.1057.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx=\frac{1}{2}\,\log\left(cx^2+b\right)+\frac{1}{2}\,\log\left(x^2\right)$$

input
```
integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="maxima")
```

output
```
1/2*log(c*x^2 + b) + 1/2*log(x^2)
```

3.1057.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx=\frac{1}{2}\,\log\left(x^2\right)+\frac{1}{2}\,\log\left(\left|cx^2+b\right|\right)$$

input
```
integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="giac")
```

output
```
1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))
```

3.1057.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx=\frac{\ln\left(c\,x^2+b\right)}{2}+\ln\left(x\right)$$

input
```
int((b + 2*c*x^2)/(x*(b + c*x^2)),x)
```

output
```
log(b + c*x^2)/2 + log(x)
```

3.1058 $\int \frac{b+2cx^3}{x(b+cx^3)}\,dx$

3.1058.1 Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \frac{b+2cx^3}{x(b+cx^3)}\,dx = \log(x)+\frac{1}{3}\log\left(b+cx^3\right)$$

output
```
ln(x)+1/3*ln(c*x^3+b)
```

3.1058.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{x(b+cx^3)}\,dx = \log(x)+\frac{1}{3}\log\left(b+cx^3\right)$$

input
```
Integrate[(b + 2*c*x^3)/(x*(b + c*x^3)),x]
```

output
```
Log[x] + Log[b + c*x^3]/3
```

3.1058.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^3}{x\left(b+cx^3\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{2cx^3+b}{x^3\left(cx^3+b\right)}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{c}{cx^3+b}+\frac{1}{x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\log\left(b+cx^3\right)+\log\left(x^3\right)\right)$$

input
```
Int[(b + 2*c*x^3)/(x*(b + c*x^3)),x]
```

output
```
(Log[x^3] + Log[b + c*x^3])/3
```

3.1058.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1058.4 Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln\left(x\right)+\frac{\ln\left(c\,x^{3}+b\right)}{3}$	14
norman	$\ln\left(x\right)+\frac{\ln\left(c\,x^{3}+b\right)}{3}$	14
risch	$\ln\left(x\right)+\frac{\ln\left(c\,x^{3}+b\right)}{3}$	14
parallelrisch	$\ln\left(x\right)+\frac{\ln\left(c\,x^{3}+b\right)}{3}$	14

input
```
int((2*c*x^3+b)/x/(c*x^3+b),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+1/3*ln(c*x^3+b)
```

3.1058.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx=\frac{1}{3}\log\left(cx^3+b\right)+\log\left(x\right)$$

input
```
integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="fricas")
```

output
```
1/3*log(c*x^3 + b) + log(x)
```

3.1058.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx=\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

input
```
integrate((2*c*x**3+b)/x/(c*x**3+b),x)
```

output
```
log(x) + log(b/c + x**3)/3
```

3.1058.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx=\frac{1}{3}\,\log\left(cx^3+b\right)+\frac{1}{3}\,\log\left(x^3\right)$$

input
```
integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="maxima")
```

output
```
1/3*log(c*x^3 + b) + 1/3*log(x^3)
```

3.1058.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx=\frac{1}{3}\,\log\left(\left|cx^3+b\right|\right)+\log\left(\left|x\right|\right)$$

input
```
integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="giac")
```

output
```
1/3*log(abs(c*x^3 + b)) + log(abs(x))
```

3.1058.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^3}{x(b+cx^3)}\,dx = \frac{\ln\left(c\,x^3+b\right)}{3}+\ln\left(x\right)$$

input
```
int((b + 2*c*x^3)/(x*(b + c*x^3)),x)
```

output
```
log(b + c*x^3)/3 + log(x)
```

3.1059 $\int \frac{b+2cx^n}{x(b+cx^n)}\,dx$

3.1059.1 Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \frac{b+2cx^n}{x(b+cx^n)}\,dx = \log(x)+\frac{\log(b+cx^n)}{n}$$

output
```
ln(x)+ln(b+c*x^n)/n
```

3.1059.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{b+2cx^n}{x(b+cx^n)}\,dx = \frac{\log(x^n)+\log(n(b+cx^n))}{n}$$

input
```
Integrate[(b + 2*c*x^n)/(x*(b + c*x^n)),x]
```

output
```
(Log[x^n] + Log[n*(b + c*x^n)])/n
```

3.1059.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^n}{x\left(b+cx^n\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{-n}\left(2cx^n+b\right)}{cx^n+b}dx^n}{n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(x^{-n}+\frac{c}{cx^n+b}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log\left(b+cx^n\right)+\log\left(x^n\right)}{n}$$

input
```
Int[(b + 2*c*x^n)/(x*(b + c*x^n)),x]
```

output
```
(Log[x^n] + Log[b + c*x^n])/n
```

3.1059.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1059.4 Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$\frac{\ln(x^n(b+c\,x^n))}{n}$	17
default	$\frac{\ln(x^n(b+c\,x^n))}{n}$	17
norman	$\ln(x)+\frac{\ln\left(b+c\,\mathrm{e}^{n\ln(x)}\right)}{n}$	18
risch	$\ln(x)+\frac{\ln\left(x^n+\frac{b}{c}\right)}{n}$	18
parallelrisch	$\frac{n\ln(x)+\ln(b+c\,x^n)}{n}$	18

input
```
int((b+2*c*x^n)/x/(b+c*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/n*ln(x^n*(b+c*x^n))
```

3.1059.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{b+2cx^n}{x(b+cx^n)}\,dx=\frac{n\log(x)+\log(cx^n+b)}{n}$$

input
```
integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="fricas")
```

output
```
(n*log(x) + log(c*x^n + b))/n
```

3.1059.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{b+2cx^n}{x\left(b+cx^n\right)}\,dx=\begin{cases}\log\left(x\right) & \text{for } c=0\wedge\left(c=0\vee n=0\right)\\ \frac{(b+2c)\log\left(x\right)}{b+c} & \text{for } n=0\\ \log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise}\end{cases}$$

input
```
integrate((b+2*c*x**n)/x/(b+c*x**n),x)
```

output
```
Piecewise((log(x), Eq(c, 0) & (Eq(c, 0) | Eq(n, 0))), ((b + 2*c)*log(x)/(b
 + c), Eq(n, 0)), (log(x) + log(b/c + x**n)/n, True))
```

3.1059.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int \frac{b+2cx^n}{x\left(b+cx^n\right)}\,dx=b\left(\frac{\log\left(x\right)}{b}-\frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right)+\frac{2\,\log\left(\frac{cx^n+b}{c}\right)}{n}$$

input
```
integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="maxima")
```

output
```
b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

3.1059.8 Giac [F]

$$\int \frac{b+2cx^n}{x\left(b+cx^n\right)}\,dx=\int \frac{2\,cx^n+b}{(cx^n+b)x}\,dx$$

input
```
integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="giac")
```

output
```
integrate((2*c*x^n + b)/((c*x^n + b)*x), x)
```

3.1059.9 Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^n}{x\left(b+cx^n\right)}\,dx=\ln\left(x\right)+\frac{\ln\left(b+c\,x^n\right)}{n}$$

input
```
int((b + 2*c*x^n)/(x*(b + c*x^n)),x)
```

output
```
log(x) + log(b + c*x^n)/n
```

3.1060 $\int \frac{b+2cx}{x^8(b+cx)^8}\,dx$

3.1060.1 Optimal result

Integrand size = 17, antiderivative size = 14

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx = -\frac{1}{7x^7(b+cx)^7}$$

output
```
-1/7/x^7/(c*x+b)^7
```

3.1060.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx = -\frac{1}{7x^7(b+cx)^7}$$

input
```
Integrate[(b + 2*c*x)/(x^8*(b + c*x)^8),x]
```

output
```
-1/7*1/(x^7*(b + c*x)^7)
```

3.1060.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx$$

$$\Big\downarrow \, 83$$

$$-\frac{1}{7x^7(b+cx)^7}$$

input
```
Int[(b + 2*c*x)/(x^8*(b + c*x)^8),x]
```

output
```
-1/7*1/(x^7*(b + c*x)^7)
```

3.1060.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

3.1060.4 Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result
gosper	$-\frac{1}{7x^7(cx+b)^7}$
norman	$-\frac{1}{7x^7(cx+b)^7}$
risch	$-\frac{1}{7x^7(cx+b)^7}$
parallelrisch	$-\frac{1}{7x^7(cx+b)^7}$
default	$-\frac{1}{7b^7x^7}-\frac{132c^6}{b^{13}x}+\frac{66c^5}{b^{12}x^2}-\frac{30c^4}{b^{11}x^3}+\frac{12c^3}{b^{10}x^4}-\frac{4c^2}{b^9x^5}+\frac{c}{b^8x^6}+\frac{132c^7}{b^{13}(cx+b)}+\frac{66c^7}{b^{12}(cx+b)^2}+\frac{30c^7}{b^{11}(cx+b)^3}+$

input
```
int((2*c*x+b)/x^8/(c*x+b)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7/x^7/(c*x+b)^7
```

3.1060.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(12) = 24$.

Time = 0.49 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.79

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx = -\frac{1}{7\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

input
```
integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="fricas")
```

output
```
-1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4
*c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

3.1060.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(14) = 28$.

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 6.21

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx = -\frac{1}{7b^7x^7+49b^6cx^8+147b^5c^2x^9+245b^4c^3x^{10}+245b^3c^4x^{11}+147b^2c^5x^{12}+49bc^6x^{13}+7c^7x^{14}}$$

input
```
integrate((2*c*x+b)/x**8/(c*x+b)**8,x)
```

output
```
-1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**1
0 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x
**14)
```

3.1060.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(12) = 24$.

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.79

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx$$
$$= -\frac{1}{7\,(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7)}$$

input
```
integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="maxima")
```

output
```
-1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4
*c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

3.1060.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx = -\frac{1}{7\,(cx^2+bx)^7}$$

input
```
integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="giac")
```

output
```
-1/7/(c*x^2 + b*x)^7
```

3.1060.9 Mupad [B] (verification not implemented)

Time = 11.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx = -\frac{1}{7\,x^7\,(b+c\,x)^7}$$

input
```
int((b + 2*c*x)/(x^8*(b + c*x)^8),x)
```

output
```
-1/(7*x^7*(b + c*x)^7)
```

3.1061 $\int \frac{b+2cx^2}{x^{15}(b+cx^2)^8}\,dx$

3.1061.1 Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx=-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

output
```
-1/14/x^14/(c*x^2+b)^7
```

3.1061.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx=-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

input
```
Integrate[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x]
```

output
```
-1/14*1/(x^14*(b + c*x^2)^7)
```

3.1061.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {354, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{2cx^2+b}{x^{16}\left(cx^2+b\right)^8}dx^2$$

$$\Big\downarrow \text{83}$$

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

input
```
Int[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x]
```

output
```
-1/14*1/(x^14*(b + c*x^2)^7)
```

3.1061.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.1061.4 Maple [A] (verified)

Time = 4.65 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
norman	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
risch	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
parallelrisch	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
default	$-\frac{1}{14b^7x^{14}}-\frac{66c^6}{b^{13}x^2}+\frac{33c^5}{b^{12}x^4}-\frac{15c^4}{b^{11}x^6}+\frac{6c^3}{b^{10}x^8}-\frac{2c^2}{b^9x^{10}}+\frac{c}{2b^8x^{12}}-\frac{c^8\left(-\frac{132}{c\left(c\,x^2+b\right)}-\frac{b^5}{c\left(c\,x^2+b\right)^6}-\frac{b^6}{7c\left(c\,x^2+b\right)^7}-\right.}{2}$

input
```
int((2*c*x^2+b)/x^15/(c*x^2+b)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/14/x^14/(c*x^2+b)^7
```

3.1061.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(14) = 28.

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx =$$
$$-\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

input
```
integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="fricas")
```

output
```
-1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^
4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

3.1061.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 0.66 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx = -\frac{1}{14b^7x^{14}+98b^6cx^{16}+294b^5c^2x^{18}+490b^4c^3x^{20}+490b^3c^4x^{22}+294b^2c^5x^{24}+98bc^6x^{26}+14c^7x^{28}}$$

input

```
integrate((2*c*x**2+b)/x**15/(c*x**2+b)**8,x)
```

output

```
-1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*
x**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c
**7*x**28)
```

3.1061.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx = -\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

input

```
integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="maxima")
```

output

```
-1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^
4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

3.1061.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx=-\frac{1}{14\left(cx^4+bx^2\right)^7}$$

input
```
integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="giac")
```

output
```
-1/14/(c*x^4 + b*x^2)^7
```

3.1061.9 Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx=-\frac{1}{14\,x^{14}\left(c\,x^2+b\right)^7}$$

input
```
int((b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x)
```

output
```
-1/(14*x^14*(b + c*x^2)^7)
```

3.1062 $\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx$

3.1062.1 Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx=-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

output
```
-1/21/x^21/(c*x^3+b)^7
```

3.1062.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx=-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

input
```
Integrate[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]
```

output
```
-1/21*1/(x^21*(b + c*x^3)^7)
```

3.1062.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3}\int \frac{2cx^3+b}{x^{24}\left(cx^3+b\right)^8}dx^3$$

$$\Big\downarrow \ 83$$

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

input
```
Int[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]
```

output
```
-1/21*1/(x^21*(b + c*x^3)^7)
```

3.1062.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1062.4 Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{21x^{21}(cx^3+b)^7}$
risch	$-\frac{1}{21x^{21}(cx^3+b)^7}$
parallelrisch	$-\frac{1}{21x^{21}(cx^3+b)^7}$
default	$-\frac{1}{21b^7x^{21}}-\frac{44c^6}{b^{13}x^3}+\frac{22c^5}{b^{12}x^6}-\frac{10c^4}{b^{11}x^9}+\frac{4c^3}{b^{10}x^{12}}-\frac{4c^2}{3b^9x^{15}}+\frac{c}{3b^8x^{18}}-\frac{c^8\left(-\frac{132}{c(cx^3+b)}-\frac{b^5}{c(cx^3+b)^6}-\frac{b^6}{7c(cx^3+b)^7}-\right.}{}$

input
```
int((2*c*x^3+b)/x^22/(c*x^3+b)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/21/x^21/(c*x^3+b)^7
```

3.1062.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.69 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^3}{x^{22}(b+cx^3)^8}\,dx = -\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

input
```
integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="fricas")
```

output
```
-1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^
4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

3.1062.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 0.89 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx = -\frac{1}{21b^7x^{21}+147b^6cx^{24}+441b^5c^2x^{27}+735b^4c^3x^{30}+735b^3c^4x^{33}+441b^2c^5x^{36}+147bc^6x^{39}+21c^7x^{42}}$$

input
```
integrate((2*c*x**3+b)/x**22/(c*x**3+b)**8,x)
```

output
```
-1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3
*x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21
*c**7*x**42)
```

3.1062.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx = -\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

input
```
integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="maxima")
```

output
```
-1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^
4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

3.1062.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx = -\frac{1}{21\left(cx^6+bx^3\right)^7}$$

input
```
integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="giac")
```

output
```
-1/21/(c*x^6 + b*x^3)^7
```

3.1062.9 Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx = -\frac{1}{21\,x^{21}\left(c\,x^3+b\right)^7}$$

input
```
int((b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x)
```

output
```
-1/(21*x^21*(b + c*x^3)^7)
```

3.1063 $\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx$

3.1063.1 Optimal result

Integrand size = 25, antiderivative size = 21

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx = -\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

output
```
-1/7/n/(x^(7*n))/(b+c*x^n)^7
```

3.1063.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx = -\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

input
```
Integrate[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]
```

output
```
-1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

3.1063.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-7n-1}(b+2cx^n)}{(b+cx^n)^8} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{-8n}(2cx^n+b)}{(cx^n+b)^8}dx^n}{n}$$

$$\Big\downarrow \text{83}$$

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

input

```
Int[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]
```

output

```
-1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

3.1063.3.1 Defintions of rubi rules used

rule 83

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1063.4 Maple [A] (verified)

Time = 14.92 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result
parallelrisch	$-\frac{x\,x^{-1-7n}}{7n\left(b+c\,x^{n}\right)^{7}}$
risch	$-\frac{132c^{6}x^{-n}}{b^{13}n}+\frac{66c^{5}x^{-2n}}{b^{12}n}-\frac{30c^{4}x^{-3n}}{b^{11}n}+\frac{12c^{3}x^{-4n}}{b^{10}n}-\frac{4c^{2}x^{-5n}}{b^{9}n}+\frac{c\,x^{-6n}}{b^{8}n}-\frac{x^{-7n}}{7b^{7}n}+\frac{c^{7}\left(924x^{6n}c^{6}+6006b\,c^{5}x^{5n}+1\right.}{}$

input
```
int(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*x*x^(-1-7*n)/n/(b+c*x^n)^7
```

3.1063.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(21) = 42.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.00

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx =$$

$$-\frac{1}{7\left(c^{7}nx^{14\,n}+7\,bc^{6}nx^{13\,n}+21\,b^{2}c^{5}nx^{12\,n}+35\,b^{3}c^{4}nx^{11\,n}+35\,b^{4}c^{3}nx^{10\,n}+21\,b^{5}c^{2}nx^{9\,n}+7\,b^{6}cnx^{8\,n}+b^{7}\right.}$$

input
```
integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="fricas")
```

output
```
-1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n) + 35*b^3
*c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x^(9*n) + 7*b^6*c*n
*x^(8*n) + b^7*n*x^(7*n))
```

3.1063.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(19) = 38.

Time = 41.77 (sec) , antiderivative size = 129, normalized size of antiderivative = 6.14

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx$$

$$= \begin{cases} -\frac{xx^{-7n-1}}{7b^{7}n+49b^{6}cnx^{n}+147b^{5}c^{2}nx^{2n}+245b^{4}c^{3}nx^{3n}+245b^{3}c^{4}nx^{4n}+147b^{2}c^{5}nx^{5n}+49bc^{6}nx^{6n}+7c^{7}nx^{7n}} & \text{for}\ n\neq 0 \\ \frac{(b+2c)\log{(x)}}{(b+c)^{8}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-7*n)*(b+2*c*x**n)/(b+c*x**n)**8,x)
```

output
```
Piecewise((-x*x**(-7*n - 1)/(7*b**7*n + 49*b**6*c*n*x**n + 147*b**5*c**2*n
*x**(2*n) + 245*b**4*c**3*n*x**(3*n) + 245*b**3*c**4*n*x**(4*n) + 147*b**2
*c**5*n*x**(5*n) + 49*b*c**6*n*x**(6*n) + 7*c**7*n*x**(7*n)), Ne(n, 0)), (
(b + 2*c)*log(x)/(b + c)**8, True))
```

3.1063.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(21) = 42$.

Time = 0.25 (sec) , antiderivative size = 612, normalized size of antiderivative = 29.14

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx =$$

$$-\frac{1}{105}\,b\left(\frac{360360\,c^{13}x^{13\,n}+2342340\,bc^{12}x^{12\,n}+6426420\,b^2c^{11}x^{11\,n}+9579570\,b^3c^{10}x^{10\,n}+8270262\,b^4c^9x^{9\,n}}{b^{14}c^7nx^{14\,n}+7\,b^{15}c^6nx^{13\,n}+21\,b^{16}c^5}\right.$$

$$+\frac{1}{105}\,c\left(\frac{360360\,c^{12}x^{12\,n}+2342340\,bc^{11}x^{11\,n}+6426420\,b^2c^{10}x^{10\,n}+9579570\,b^3c^9x^{9\,n}+8270262\,b^4c^8x^{8\,n}+}{b^{13}c^7nx^{13\,n}+7\,b^{14}c^6nx^{12\,n}+21\,b^{15}c^5nx^{11\,n}+}\right.$$

input
```
integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="maxima")
```

output
```
-1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^
11*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 401801
4*b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*
b^8*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^
2*x^(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x
^(13*n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*
x^(10*n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) +
360360*c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c
*((360360*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10
*n) + 9579570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*
x^(7*n) + 934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^
(4*n) + 1001*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b
^12)/(b^13*c^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n)
 + 35*b^16*c^4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n)
+ 7*b^19*c*n*x^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c
^6*log((c*x^n + b)/c)/(b^14*n))
```

3.1063.8 Giac [F]

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx=\int \frac{(2\,cx^n+b)x^{-7\,n-1}}{(cx^n+b)^8}\,dx$$

input
```
integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="giac")
```

output
```
integrate((2*c*x^n + b)*x^(-7*n - 1)/(c*x^n + b)^8, x)
```

3.1063.9 Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.00

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx=$$

$$-\frac{1}{7\,x^{7\,n}\,\left(b^7\,n+c^7\,n\,x^{7\,n}+7\,b^6\,c\,n\,x^n+7\,b\,c^6\,n\,x^{6\,n}+21\,b^5\,c^2\,n\,x^{2\,n}+35\,b^4\,c^3\,n\,x^{3\,n}+35\,b^3\,c^4\,n\,x^{4\,n}+21\,b^2\right.}$$

input
```
int((b + 2*c*x^n)/(x^(7*n + 1)*(b + c*x^n)^8),x)
```

output
```
-1/(7*x^(7*n)*(b^7*n + c^7*n*x^(7*n) + 7*b^6*c*n*x^n + 7*b*c^6*n*x^(6*n) +
 21*b^5*c^2*n*x^(2*n) + 35*b^4*c^3*n*x^(3*n) + 35*b^3*c^4*n*x^(4*n) + 21*b
^2*c^5*n*x^(5*n)))
```

3.1064 $\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx$

3.1064.1 Optimal result

Integrand size = 22, antiderivative size = 52

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx=-\frac{1}{8}\sqrt{1+x^{16}}-\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+x^{16}}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

output
```
-1/24*(x^16+1)^(3/2)+1/8*arctanh(1/2*(x^16+1)^(1/2)*2^(1/2))*2^(1/2)-1/8*(
x^16+1)^(1/2)
```

3.1064.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx=\frac{1}{24}\left(-4-x^{16}\right)\sqrt{1+x^{16}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+x^{16}}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

input
```
Integrate[(x^31*Sqrt[1 + x^16])/(1 - x^16),x]
```

output
```
((-4 - x^16)*Sqrt[1 + x^16])/24 + ArcTanh[Sqrt[1 + x^16]/Sqrt[2]]/(4*Sqrt[
2])
```

3.1064.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {948, 90, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{31}\sqrt{x^{16}+1}}{1-x^{16}}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{16}\int \frac{x^{16}\sqrt{x^{16}+1}}{1-x^{16}}dx^{16}$$

$$\Big\downarrow \; 90$$

$$\frac{1}{16}\left(\int \frac{\sqrt{x^{16}+1}}{1-x^{16}}dx^{16}-\frac{2}{3}\left(x^{16}+1\right)^{3/2}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{16}\left(2\int \frac{1}{\left(1-x^{16}\right)\sqrt{x^{16}+1}}dx^{16}-\frac{2}{3}\left(x^{16}+1\right)^{3/2}-2\sqrt{x^{16}+1}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{16}\left(4\int \frac{1}{2-x^{32}}d\sqrt{x^{16}+1}-\frac{2}{3}\left(x^{16}+1\right)^{3/2}-2\sqrt{x^{16}+1}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{16}\left(2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)-\frac{2}{3}\left(x^{16}+1\right)^{3/2}-2\sqrt{x^{16}+1}\right)$$

input

```
Int[(x^31*Sqrt[1 + x^16])/(1 - x^16),x]
```

output

```
(-2*Sqrt[1 + x^16] - (2*(1 + x^16)^(3/2))/3 + 2*Sqrt[2]*ArcTanh[Sqrt[1 + x
^16]/Sqrt[2]])/16
```

3.1064.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1064.4 Maple [A] (verified)

Time = 5.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

method	result	size
pseudoelliptic	$-\frac{x^{16}\sqrt{x^{16}+1}}{24}-\frac{\sqrt{x^{16}+1}}{6}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(x^{8}+1\right)\sqrt{2}}{2\sqrt{x^{16}+1}}\right)}{16}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(x^{8}-1\right)\sqrt{2}}{2\sqrt{x^{16}+1}}\right)}{16}$	69
risch	$-\frac{\left(x^{16}+4\right)\sqrt{x^{16}+1}}{24}-\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)x^{16}+3\operatorname{RootOf}\left(\_Z^{2}-2\right)-4\sqrt{x^{16}+1}}{(-1+x)(1+x)\left(x^{2}+1\right)\left(x^{4}+1\right)\left(x^{8}+1\right)}\right)}{16}$	85
trager	$\left(-\frac{x^{16}}{24}-\frac{1}{6}\right)\sqrt{x^{16}+1}+\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)x^{16}+3\operatorname{RootOf}\left(\_Z^{2}-2\right)+4\sqrt{x^{16}+1}}{(-1+x)(1+x)\left(x^{2}+1\right)\left(x^{4}+1\right)\left(x^{8}+1\right)}\right)}{16}$	87

input
```
int(x^31*(x^16+1)^(1/2)/(-x^16+1),x,method=_RETURNVERBOSE)
```

output
```
-1/24*x^16*(x^16+1)^(1/2)-1/6*(x^16+1)^(1/2)+1/16*2^(1/2)*arctanh(1/2*(x^8
+1)*2^(1/2)/(x^16+1)^(1/2))-1/16*2^(1/2)*arctanh(1/2*(x^8-1)*2^(1/2)/(x^16
+1)^(1/2))
```

3.1064.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx=-\frac{1}{24}\left(x^{16}+4\right)\sqrt{x^{16}+1}+\frac{1}{16}\sqrt{2}\log\left(\frac{x^{16}+2\sqrt{2}\sqrt{x^{16}+1}+3}{x^{16}-1}\right)$$

input
```
integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="fricas")
```

output
```
-1/24*(x^16 + 4)*sqrt(x^16 + 1) + 1/16*sqrt(2)*log((x^16 + 2*sqrt(2)*sqrt(
x^16 + 1) + 3)/(x^16 - 1))
```

3.1064.6 Sympy [A] (verification not implemented)

Time = 27.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx = -\frac{(x^{16}+1)^{\frac{3}{2}}}{24} - \frac{\sqrt{x^{16}+1}}{8} - \frac{\sqrt{2}\left(\log\left(\sqrt{x^{16}+1}-\sqrt{2}\right)-\log\left(\sqrt{x^{16}+1}+\sqrt{2}\right)\right)}{16}$$

input
```
integrate(x**31*(x**16+1)**(1/2)/(-x**16+1),x)
```

output
```
-(x**16 + 1)**(3/2)/24 - sqrt(x**16 + 1)/8 - sqrt(2)*(log(sqrt(x**16 + 1)
- sqrt(2)) - log(sqrt(x**16 + 1) + sqrt(2)))/16
```

3.1064.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx = -\frac{1}{24}\left(x^{16}+1\right)^{\frac{3}{2}} - \frac{1}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^{16}+1}}{\sqrt{2}+\sqrt{x^{16}+1}}\right) - \frac{1}{8}\sqrt{x^{16}+1}$$

input
```
integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="maxima")
```

output
```
-1/24*(x^16 + 1)^(3/2) - 1/16*sqrt(2)*log(-(sqrt(2) - sqrt(x^16 + 1))/(sqr
t(2) + sqrt(x^16 + 1))) - 1/8*sqrt(x^16 + 1)
```

3.1064.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx = -\frac{1}{24}\left(x^{16}+1\right)^{\frac{3}{2}} - \frac{1}{16}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{x^{16}+1}\right|}{2\left(\sqrt{2}+\sqrt{x^{16}+1}\right)}\right) - \frac{1}{8}\sqrt{x^{16}+1}$$

input
```
integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="giac")
```

output
```
-1/24*(x^16 + 1)^(3/2) - 1/16*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(x^16
 + 1))/(sqrt(2) + sqrt(x^16 + 1))) - 1/8*sqrt(x^16 + 1)
```

3.1064.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.71

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx=\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{x^{16}+1}}{2}\right)}{8}-\frac{\sqrt{x^{16}+1}}{8}-\frac{\left(x^{16}+1\right)^{3/2}}{24}$$

input
```
int(-(x^31*(x^16 + 1)^(1/2))/(x^16 - 1),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(x^16 + 1)^(1/2))/2))/8 - (x^16 + 1)^(1/2)/8 - (x^
16 + 1)^(3/2)/24
```

3.1065 $\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx$

3.1065.1 Optimal result

Integrand size = 26, antiderivative size = 93

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx=\frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}}-\frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

output
```
2*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))*c^(1/2)/a^(1/2)-2*a
rctanh(d^(1/2)*(a+b/x)^(1/2)/b^(1/2)/(c+d/x)^(1/2))*d^(1/2)/b^(1/2)
```

3.1065.2 Mathematica [A] (verified)

Time = 10.50 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx=-\frac{2\sqrt{d}\sqrt{bc-ad}\sqrt{c+\frac{d}{x}}\,x\sqrt{\frac{b(d+cx)}{(bc-ad)x}}\operatorname{arcsinh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{bd+bcx}+\frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]
```

output
```
(-2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[c + d/x]*x*Sqrt[(b*(d + c*x))/((b*c - a*d
)*x)]*ArcSinh[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*d + b*c*x) + (2
*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/Sqrt[a]
```

3.1065.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {948, 140, 27, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+\frac{d}{x}}}{x\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{948}$$

$$-\int \frac{\sqrt{c+\frac{d}{x}}x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{140}$$

$$-d\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}-\int \frac{cx}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-d\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}-c\int \frac{x}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{66}$$

$$-2d\int \frac{1}{b-\frac{d}{x^2}}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}-c\int \frac{x}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{104}$$

$$-2c\int \frac{1}{\frac{c}{x^2}-a}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}-2d\int \frac{1}{b-\frac{d}{x^2}}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}}-\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

input

```
Int[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]
```

output

```
(2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/Sqrt[
a] - (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])])/
Sqrt[b]
```

3.1065.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

3.1065. $\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1065.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(69) = 138$.

Time = 0.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.54

method	result	size
default	$-\frac{\sqrt{\frac{ax+b}{x}}\, x \sqrt{\frac{cx+d}{x}}\, \left(\sqrt{ac}\, \ln\left(\frac{adx+bcx+2\sqrt{bd}\, \sqrt{(ax+b)(cx+d)}+2bd}{x}\right) d-\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\, \sqrt{ac}+ad+bc}{2\sqrt{ac}}\right) \sqrt{bd}\, c\right)}{\sqrt{(ax+b)(cx+d)}\, \sqrt{ac}\, \sqrt{bd}}$	143

input
```
int((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*((a*c)^(1/2)*ln((a*d*x+b*c*x+2*(b*d
)^(1/2)*((a*x+b)*(c*x+d))^(1/2)+2*b*d)/x)*d-ln(1/2*(2*a*c*x+2*((a*x+b)*(c*
x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*(b*d)^(1/2)*c)/((a*x+b)*(c*x
+d))^(1/2)/(a*c)^(1/2)/(b*d)^(1/2)
```

3.1065.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(69) = 138$.

Time = 0.42 (sec) , antiderivative size = 757, normalized size of antiderivative = 8.14

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}\,x}\,dx = \left[\frac{1}{2}\sqrt{\frac{c}{a}}\log\left(-8\,a^2c^2x^2 - b^2c^2 - 6\,abcd - a^2d^2\right.\right.$$

$$\left. - 4\left(2\,a^2cx^2 + \left(abc + a^2d\right)x\right)\sqrt{\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} - 8\left(abc^2 + a^2cd\right)x\right)$$

$$+\frac{1}{2}\sqrt{\frac{d}{b}}\log\left(-\frac{8\,b^2d^2 + (b^2c^2 + 6\,abcd + a^2d^2)x^2 - 4\,(2\,b^2dx + (b^2c + abd)x^2)\sqrt{\frac{d}{b}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} + 8\,(b^2ca}{x^2}\right.$$

$$-\sqrt{-\frac{c}{a}}\arctan\left(\frac{2\,ax\sqrt{-\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,acx + bc + ad}\right)$$

$$+\frac{1}{2}\sqrt{\frac{d}{b}}\log\left(-\frac{8\,b^2d^2 + (b^2c^2 + 6\,abcd + a^2d^2)x^2 - 4\,(2\,b^2dx + (b^2c + abd)x^2)\sqrt{\frac{d}{b}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} + 8\,(b^2ca}{x^2}\right.$$

$$+\frac{1}{2}\sqrt{\frac{c}{a}}\log\left(-8\,a^2c^2x^2 - b^2c^2 - 6\,abcd - a^2d^2\right.$$

$$\left. - 4\left(2\,a^2cx^2 + \left(abc + a^2d\right)x\right)\sqrt{\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} - 8\left(abc^2 + a^2cd\right)x\right),$$

$$-\sqrt{-\frac{c}{a}}\arctan\left(\frac{2\,ax\sqrt{-\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,acx + bc + ad}\right)$$

$$\left.+\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2\,bdx + (bc + ad)x^2)\sqrt{-\frac{d}{b}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,(acdx^2 + bd^2 + (bcd + ad^2)x)}\right)\right]$$

input
```
integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(c/a)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a
^2*c*x^2 + (a*b*c + a^2*d)*x)*sqrt(c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x
) - 8*(a*b*c^2 + a^2*c*d)*x) + 1/2*sqrt(d/b)*log(-(8*b^2*d^2 + (b^2*c^2 +
6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*s
qrt((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2), -sqrt(
-c/a)*arctan(2*a*x*sqrt(-c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x
 + b*c + a*d)) + 1/2*sqrt(d/b)*log(-(8*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*sqrt((a*x + b)/
x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2), sqrt(-d/b)*arctan(1/
2*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(-d/b)*sqrt((a*x + b)/x)*sqrt((c*x + d)/
x)/(a*c*d*x^2 + b*d^2 + (b*c*d + a*d^2)*x)) + 1/2*sqrt(c/a)*log(-8*a^2*c^2
*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a^2*c*x^2 + (a*b*c + a^2*d)*x)
*sqrt(c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x),
 -sqrt(-c/a)*arctan(2*a*x*sqrt(-c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(
2*a*c*x + b*c + a*d)) + sqrt(-d/b)*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2)*
sqrt(-d/b)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(a*c*d*x^2 + b*d^2 + (b*c*d
 + a*d^2)*x))]
```

3.1065.6 Sympy [F]

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx = \int \frac{\sqrt{c+\frac{d}{x}}}{x\sqrt{a+\frac{b}{x}}}\,dx$$

input
```
integrate((c+d/x)**(1/2)/x/(a+b/x)**(1/2),x)
```

output
```
Integral(sqrt(c + d/x)/(x*sqrt(a + b/x)), x)
```

3.1065.7 Maxima [F]

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx = \int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx$$

input
```
integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)
```

3.1065.8 Giac [F]

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx=\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx$$

input
```
integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)
```

3.1065.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx=\int \frac{\sqrt{c+\frac{d}{x}}}{x\,\sqrt{a+\frac{b}{x}}}\,dx$$

input
```
int((c + d/x)^(1/2)/(x*(a + b/x)^(1/2)),x)
```

output
```
int((c + d/x)^(1/2)/(x*(a + b/x)^(1/2)), x)
```

3.1066 $\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

3.1066.1 Optimal result

Integrand size = 30, antiderivative size = 252

$$\begin{aligned}\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx &= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n}\\ &+\frac{5(bc-ad)(7bc+ad)\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{96bd^3n}-\frac{(7bc+ad)\,(a+bx^n)^{5/2}\,\sqrt{c+dx^n}}{24bd^2n}\\ &+\frac{(a+bx^n)^{7/2}\,\sqrt{c+dx^n}}{4bdn}+\frac{5(bc-ad)^3(7bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{3/2}d^{9/2}n}\end{aligned}$$

output
```
5/64*(-a*d+b*c)^3*(a*d+7*b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d
*x^n)^(1/2))/b^(3/2)/d^(9/2)/n+5/96*(-a*d+b*c)*(a*d+7*b*c)*(a+b*x^n)^(3/2)
*(c+d*x^n)^(1/2)/b/d^3/n-1/24*(a*d+7*b*c)*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/
b/d^2/n+1/4*(a+b*x^n)^(7/2)*(c+d*x^n)^(1/2)/b/d/n-5/64*(-a*d+b*c)^2*(a*d+7
*b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d^4/n
```

3.1066.2 Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.88

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(15a^3d^3+a^2bd^2(-191c+118dx^n)+ab^2d(265c^2-172c\right.}{}$$

input
```
Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

output
```
(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(15*a^3*d^3 + a^2*b*d^2*(-191*c + 1
18*d*x^n) + a*b^2*d*(265*c^2 - 172*c*d*x^n + 136*d^2*x^(2*n)) + b^3*(-105*
c^3 + 70*c^2*d*x^n - 56*c*d^2*x^(2*n) + 48*d^3*x^(3*n))) + 15*(b*c - a*d)^
(7/2)*(7*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqr
t[a + b*x^n])/Sqrt[b*c - a*d]])/(192*b^2*d^(9/2)*n*Sqrt[c + d*x^n])
```

3.1066.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {948, 90, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}\left(a+bx^n\right)^{5/2}}{\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \; 948$$

$$\frac{\int \frac{x^n\left(bx^n+a\right)^{5/2}}{\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \; 90$$

$$\frac{\frac{\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{(ad+7bc)\int \frac{\left(bx^n+a\right)^{5/2}}{\sqrt{dx^n+c}}dx^n}{8bd}}{n}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{(ad+7bc)\left(\frac{\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\int \frac{\left(bx^n+a\right)^{3/2}}{\sqrt{dx^n+c}}dx^n}{6d}\right)}{8bd}}{n}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{(ad+7bc)\left(\frac{\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\int \frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}dx^n}{4d}\right)}{6d}\right)}{8bd}}{n}$$

$$\Big\downarrow \color{blue}{60}$$

$$\frac{\frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{(ad+7bc)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx}{2d}\right.}{4d}\right.}{6d}\right.}{8bd}}{n}$$

$$\Big\downarrow \color{blue}{66}$$

$$\frac{\frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{(ad+7bc)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int\frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{d}\right)}{4d}\right.}{6d}\right.}{8bd}}{n}$$

$$\Big\downarrow \color{blue}{221}$$

$$\frac{\frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{(ad+7bc)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right.}{\sqrt{b}d^{3/2}}\right.}{4d}\right.}{6d}\right.}{8bd}}{n}$$

input
```
Int[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

3.1066. $\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

output

```
(((a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(4*b*d) - ((7*b*c + a*d)*(((a + b*x^n
)^(5/2)*Sqrt[c + d*x^n])/(3*d) - (5*(b*c - a*d)*(((a + b*x^n)^(3/2)*Sqrt[c
 + d*x^n])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/d - (
(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/
(Sqrt[b]*d^(3/2))))/(4*d)))/(6*d)))/(8*b*d))/n
```

3.1066.3.1 Defintions of rubi rules used

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1066.4 Maple [F]

$$\int \frac{x^{-1+2n}(a+b\,x^{n})^{\frac{5}{2}}}{\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

3.1066.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.41

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \left[-\frac{15\,(7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+\right.}{}\right.$$

$$-\frac{15\,(7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4)\sqrt{-bd}\arctan\left(\frac{(2\sqrt{-bd}bdx^n+(bc+ad)\sqrt{-bd})\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\,(b^2d^2x^{2\,n}+abcd+(b^2cd+abd^2)x^n)}\right.}{}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[-1/768*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d
^3 - a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*
d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt
(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(48*b^4*d^4*x^(3*n) - 105*b^4
*c^3*d + 265*a*b^3*c^2*d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c
*d^3 - 17*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2
*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^5*n), -1/384*(15*(7
*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*
sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt
(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^
2)*x^n)) - 2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 265*a*b^3*c^2*d^2 - 191
*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3*d^4)*x^(2*n) + 2
*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*s
qrt(d*x^n + c))/(b^2*d^5*n)]
```

3.1066.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

output
```
Timed out
```

3.1066.7 Maxima [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}x^{2\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

3.1066.8 Giac [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}x^{2\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

3.1066.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx=\int \frac{x^{2\,n-1}\,{\left(a+b\,x^n\right)}^{5/2}}{\sqrt{c+d\,x^n}}\,dx$$

input
```
int((x^(2*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2),x)
```

output
```
int((x^(2*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2), x)
```

3.1067 $\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$

3.1067.1 Optimal result

Integrand size = 30, antiderivative size = 199

$$\begin{aligned}\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx &= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n}\\ &\quad-\frac{(5bc+ad)\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{12bd^2n}+\frac{(a+bx^n)^{5/2}\,\sqrt{c+dx^n}}{3bdn}\\ &\quad-\frac{(bc-ad)^2(5bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{3/2}d^{7/2}n}\end{aligned}$$

output
```
-1/8*(-a*d+b*c)^2*(a*d+5*b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d
*x^n)^(1/2))/b^(3/2)/d^(7/2)/n-1/12*(a*d+5*b*c)*(a+b*x^n)^(3/2)*(c+d*x^n)^
(1/2)/b/d^2/n+1/3*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b/d/n+1/8*(-a*d+b*c)*(a*
d+5*b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d^3/n
```

3.1067.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(3a^2d^2+2abd(-11c+7dx^n)+b^2(15c^2-10cdx^n+8d^2}{24b^2d^{7/2}n\sqrt{c+dx^n}}$$

input
```
Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

output
```
(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(3*a^2*d^2 + 2*a*b*d*(-11*c + 7*d*x
^n) + b^2*(15*c^2 - 10*c*d*x^n + 8*d^2*x^(2*n))) - 3*(b*c - a*d)^(5/2)*(5*
b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x
^n])/Sqrt[b*c - a*d]])/(24*b^2*d^(7/2)*n*Sqrt[c + d*x^n])
```

3.1067.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {948, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \textbf{948}$$

$$\frac{\int \frac{x^n(bx^n+a)^{3/2}}{\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \textbf{90}$$

$$\frac{\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{(ad+5bc)\int \frac{(bx^n+a)^{3/2}}{\sqrt{dx^n+c}}dx^n}{6bd}}{n}$$

$$\Big\downarrow \textbf{60}$$

$$\frac{\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{(ad+5bc)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\int \frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}dx^n}{4d}\right)}{6bd}}{n}$$

$$\Big\downarrow \textbf{60}$$

$$\frac{\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{(ad+5bc)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{2d}\right)}{4d}\right)}{6bd}}{n}$$

$$\Big\downarrow \textbf{66}$$

$$\frac{\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{(ad+5bc)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int \frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{d}\right)}{4d}\right)}{6bd}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{(ad+5bc)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{\sqrt{b}d^{3/2}}\right)}{4d}\right)}{6bd}}{n}$$

input
```
Int[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

output
```
(((a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(3*b*d) - ((5*b*c + a*d)*(((a + b*x^n
)^(3/2)*Sqrt[c + d*x^n])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x^n]*Sqrt[c +
 d*x^n])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[
c + d*x^n])])/(Sqrt[b]*d^(3/2))))/(4*d)))/(6*b*d))/n
```

3.1067.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1067.4 Maple [F]

$$\int \frac{x^{-1+2n}(a+b\,x^{n})^{\frac{3}{2}}}{\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

3.1067.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.36

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \left[\frac{3\left(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3\right)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2\right.}{}\right.$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/96*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(b*d)*l
og(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*
x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d
+ a*b*d^2)*x^n) + 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3
*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n
 + c))/(b^2*d^4*n), 1/48*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a
^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*
d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d
+ a*b*d^2)*x^n)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 +
3*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^
n + c))/(b^2*d^4*n)]
```

3.1067.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

output
```
Timed out
```

3.1067.7 Maxima [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}x^{2n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

3.1067.8 Giac [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx=\int \frac{(bx^n+a)^{\frac{3}{2}}x^{2\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

3.1067.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx=\int \frac{x^{2\,n-1}\,(a+b\,x^n)^{3/2}}{\sqrt{c+d\,x^n}}\,dx$$

input
```
int((x^(2*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2),x)
```

output
```
int((x^(2*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2), x)
```

3.1068 $\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$

3.1068.1 Optimal result

Integrand size = 30, antiderivative size = 146

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n}+\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}+\frac{(bc-ad)(3bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{3/2}d^{5/2}n}$$

output
```
1/4*(-a*d+b*c)*(a*d+3*b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^
n)^(1/2))/b^(3/2)/d^(5/2)/n+1/2*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b/d/n-1/4*
(a*d+3*b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d^2/n
```

3.1068.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(-3bc+ad+2bdx^n\right)+(bc-ad)^{3/2}(3bc+ad)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\operatorname{arcsinh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{4b^2d^{5/2}n\sqrt{c+dx^n}}$$

input
```
Integrate[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

output

```
(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c + a*d + 2*b*d*x^n) + (b*c -
 a*d)^(3/2)*(3*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[
d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^2*d^(5/2)*n*Sqrt[c + d*x^n])
```

3.1068.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^n\sqrt{bx^n+a}}{\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{(ad+3bc)\int \frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}dx^n}{4bd}}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{(ad+3bc)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{2d}\right)}{4bd}}{n}$$

$$\Big\downarrow \text{66}$$

$$\frac{\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{(ad+3bc)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int \frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{d}\right)}{4bd}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{(ad+3bc)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{\sqrt{b}d^{3/2}}\right)}{4bd}}{n}$$

input

```
Int[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

output
```
(((a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(2*b*d) - ((3*b*c + a*d)*((Sqrt[a + b
*x^n]*Sqrt[c + d*x^n])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/
(Sqrt[b]*Sqrt[c + d*x^n])])/(Sqrt[b]*d^(3/2))))/(4*b*d))/n
```

3.1068.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1068.4 Maple [F]

$$\int \frac{x^{-1+2n}\sqrt{a+b\,x^{n}}}{\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

3.1068.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.46

$$\int \frac{x^{-1+2n}\sqrt{a+bx^{n}}}{\sqrt{c+dx^{n}}}\,dx$$
$$= \Bigg[-\frac{(3\,b^{2}c^{2}-2\,abcd-a^{2}d^{2})\sqrt{bd}\log\Big(8\,b^{2}d^{2}x^{2\,n}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}-4\left(2\,\sqrt{bd}bdx^{n}+(bc+ad)\sqrt{bd}\right)\sqrt{}}{16\,b^{2}d^{3}n}$$
$$-\frac{(3\,b^{2}c^{2}-2\,abcd-a^{2}d^{2})\sqrt{-bd}\arctan\left(\frac{(2\,\sqrt{-bd}bdx^{n}+(bc+ad)\sqrt{-bd})\sqrt{bx^{n}+a}\sqrt{dx^{n}+c}}{2\,(b^{2}d^{2}x^{2\,n}+abcd+(b^{2}cd+abd^{2})x^{n})}\right)-2\,(2\,b^{2}d^{2}x^{n}-3\,b^{2}cd+a}{8\,b^{2}d^{3}n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[-1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n)
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqr
t(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(
2*b^2*d^2*x^n - 3*b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2
*d^3*n), -1/8*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*
sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n +
c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(2*b^2*d^2*x
^n - 3*b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^3*n)]
```

3.1068.6 Sympy [F]

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx=\int \frac{x^{2n-1}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

output
```
Integral(x**(2*n - 1)*sqrt(a + b*x**n)/sqrt(c + d*x**n), x)
```

3.1068.7 Maxima [F]

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx=\int \frac{\sqrt{bx^n+a}x^{2\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(b*x^n + a)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

3.1068.8 Giac [F]

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx=\int \frac{\sqrt{bx^n+a}x^{2\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

3.1068.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx=\int \frac{x^{2\,n-1}\,\sqrt{a+b\,x^n}}{\sqrt{c+d\,x^n}}\,dx$$

input
```
int((x^(2*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2),x)
```

output
```
int((x^(2*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2), x)
```

3.1069 $\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$

3.1069.1 Optimal result

Integrand size = 30, antiderivative size = 89

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

output

```
-(a*d+b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(3/2
)/d^(3/2)/n+(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d/n
```

3.1069.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.38

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)-\sqrt{bc-ad}(bc+ad)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\text{arcsinh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{b^2d^{3/2}n\sqrt{c+dx^n}}$$

input

```
Integrate[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

output

```
(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n) - Sqrt[b*c - a*d]*(b*c + a*d)*Sqrt[
(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c -
a*d]])/(b^2*d^(3/2)*n*Sqrt[c + d*x^n])
```

3.1069.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {948, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \textcolor{blue}{948}$$

$$\frac{\int \frac{x^n}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \textcolor{blue}{90}$$

$$\frac{\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bd}-\frac{(ad+bc)\int \frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{2bd}}{n}$$

$$\Big\downarrow \textcolor{blue}{66}$$

$$\frac{\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bd}-\frac{(ad+bc)\int \frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{bd}}{n}$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$\frac{\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bd}-\frac{(ad+bc)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}}}{n}$$

input
```
Int[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

output
```
((Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*d^(3/2)))/n
```

3.1069.3.1 Defintions of rubi rules used

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1069.4 Maple [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a+b\,x^{n}}\,\sqrt{c+d\,x^{n}}}dx$$

input

```
int(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

output

```
int(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

3.1069.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.16

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$$

$$= \left[\frac{4\sqrt{bx^n+a}\sqrt{dx^n+c}bd+(bc+ad)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2-4\left(2\,\sqrt{bd}bdx^n+(bc+a\right.\right.}{4\,b^2d^2n}\right.$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/4*(4*sqrt(b*x^n + a)*sqrt(d*x^n + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*
b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n +
 (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b
*d^2)*x^n))/(b^2*d^2*n), 1/2*(2*sqrt(b*x^n + a)*sqrt(d*x^n + c)*b*d + (b*c
 + a*d)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*
d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d
+ a*b*d^2)*x^n)))/(b^2*d^2*n)]
```

3.1069.6 Sympy [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n-1}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

output
```
Integral(x**(2*n - 1)/(sqrt(a + b*x**n)*sqrt(c + d*x**n)), x)
```

3.1069.7 Maxima [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx=\int \frac{x^{2\,n-1}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^(2*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

3.1069.8 Giac [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx=\int \frac{x^{2\,n-1}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

3.1069.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx=\int \frac{x^{2\,n-1}}{\sqrt{a+b\,x^n}\,\sqrt{c+d\,x^n}}\,dx$$

input
```
int(x^(2*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)),x)
```

output
```
int(x^(2*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)), x)
```

3.1070 $\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$

3.1070.1 Optimal result

Integrand size = 30, antiderivative size = 91

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}n}$$

output
```
2*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(3/2)/n/d^(1/
2)+2*a*(c+d*x^n)^(1/2)/b/(-a*d+b*c)/n/(a+b*x^n)^(1/2)
```

3.1070.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.34

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \frac{2\left(\frac{ab(c+dx^n)}{(bc-ad)\sqrt{a+bx^n}} + \frac{\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}\operatorname{arcsinh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{\sqrt{d}}\right)}{b^2n\sqrt{c+dx^n}}$$

input
```
Integrate[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

output
```
(2*((a*b*(c + d*x^n))/((b*c - a*d)*Sqrt[a + b*x^n]) + (Sqrt[b*c - a*d]*Sqr
t[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c
- a*d]])/Sqrt[d]))/(b^2*n*Sqrt[c + d*x^n])
```

3.1070.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {948, 87, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^n}{(bx^n+a)^{3/2}\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{\int \frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{b}+\frac{2a\sqrt{c+dx^n}}{b(bc-ad)\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{66}$$

$$\frac{\frac{2\int \frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{b}+\frac{2a\sqrt{c+dx^n}}{b(bc-ad)\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}}+\frac{2a\sqrt{c+dx^n}}{b(bc-ad)\sqrt{a+bx^n}}}{n}$$

input
```
Int[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

output
```
((2*a*Sqrt[c + d*x^n])/(b*(b*c - a*d)*Sqrt[a + b*x^n]) + (2*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*Sqrt[d]))/n
```

3.1070.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1070.4 Maple [F]

$$\int \frac{x^{-1+2n}}{\left(a+b\,x^{n}\right)^{\frac{3}{2}}\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

3.1070.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(75) = 150$.

Time = 0.35 (sec) , antiderivative size = 408, normalized size of antiderivative = 4.48

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \Bigg[\frac{4\sqrt{bx^n+a}\sqrt{dx^n+c}abd + \Big((b^2c-abd)\sqrt{bd}x^n + (abc-a^2d)\sqrt{bd}\Big)\log\Big(8\,b}{2\,((b^4c}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/2*(4*sqrt(b*x^n + a)*sqrt(d*x^n + c)*a*b*d + ((b^2*c - a*b*d)*sqrt(b*d)
*x^n + (a*b*c - a^2*d)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*
c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n
 + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^4*c*d - a*b^3*d^2)
*n*x^n + (a*b^3*c*d - a^2*b^2*d^2)*n), (2*sqrt(b*x^n + a)*sqrt(d*x^n + c)*
a*b*d - ((b^2*c - a*b*d)*sqrt(-b*d)*x^n + (a*b*c - a^2*d)*sqrt(-b*d))*arct
an(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqr
t(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^4
*c*d - a*b^3*d^2)*n*x^n + (a*b^3*c*d - a^2*b^2*d^2)*n)]
```

3.1070.6 Sympy [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n-1}}{(a+bx^n)^{\frac{3}{2}}\sqrt{c+dx^n}}\,dx$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

output
```
Integral(x**(2*n - 1)/((a + b*x**n)**(3/2)*sqrt(c + d*x**n)), x)
```

3.1070.7 Maxima [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx=\int \frac{x^{2n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

3.1070.8 Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx=\int \frac{x^{2n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

3.1070.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx=\int \frac{x^{2n-1}}{(a+b\,x^n)^{3/2}\sqrt{c+d\,x^n}}\,dx$$

input
```
int(x^(2*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)),x)
```

output
```
int(x^(2*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)), x)
```

3.1071 $\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$

3.1071.1 Optimal result

Integrand size = 30, antiderivative size = 95

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx=\frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n\,(a+bx^n)^{3/2}}-\frac{2(3bc-ad)\sqrt{c+dx^n}}{3b(bc-ad)^2n\sqrt{a+bx^n}}$$

output
```
2/3*a*(c+d*x^n)^(1/2)/b/(-a*d+b*c)/n/(a+b*x^n)^(3/2)-2/3*(-a*d+3*b*c)*(c+d
*x^n)^(1/2)/b/(-a*d+b*c)^2/n/(a+b*x^n)^(1/2)
```

3.1071.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.60

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx=\frac{2\sqrt{c+dx^n}(-2ac-3bcx^n+adx^n)}{3(bc-ad)^2n\,(a+bx^n)^{3/2}}$$

input
```
Integrate[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

output
```
(2*Sqrt[c + d*x^n]*(-2*a*c - 3*b*c*x^n + a*d*x^n))/(3*(b*c - a*d)^2*n*(a +
 b*x^n)^(3/2))
```

3.1071.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {948, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^n}{(bx^n+a)^{5/2}\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{(3bc-ad)\int \frac{1}{(bx^n+a)^{3/2}\sqrt{dx^n+c}}dx^n}{3b(bc-ad)}+\frac{2a\sqrt{c+dx^n}}{3b(bc-ad)(a+bx^n)^{3/2}}}{n}$$

$$\Big\downarrow \text{48}$$

$$\frac{\frac{2a\sqrt{c+dx^n}}{3b(bc-ad)(a+bx^n)^{3/2}}-\frac{2(3bc-ad)\sqrt{c+dx^n}}{3b(bc-ad)^2\sqrt{a+bx^n}}}{n}$$

input
```
Int[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

output
```
((2*a*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)*(a + b*x^n)^(3/2)) - (2*(3*b*c - a
*d)*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)^2*Sqrt[a + b*x^n]))/n
```

3.1071.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1071.4 Maple [F]

$$\int \frac{x^{-1+2n}}{(a+b\,x^n)^{\frac{5}{2}}\sqrt{c+d\,x^n}}dx$$

input

```
int(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

output

```
int(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

3.1071.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.42

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = -\frac{2\,(2\,ac+(3\,bc-ad)x^n)\sqrt{bx^n+a}\sqrt{dx^n+c}}{3\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)nx^{2\,n}+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)nx^n+(a^2b^2c^2-2\,a^3bcd+a^4d^2)n)}$$

input

```
integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output

```
-2/3*(2*a*c + (3*b*c - a*d)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c)/((b^4*c^2
 - 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a
^3*b*d^2)*n*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*n)
```

3.1071.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

output
```
Timed out
```

3.1071.7 Maxima [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^(2*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

3.1071.8 Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

3.1071.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2\,n-1}}{(a+b\,x^n)^{5/2}\,\sqrt{c+d\,x^n}}\,dx$$

input
```
int(x^(2*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)),x)
```

output
```
int(x^(2*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)), x)
```

3.1072 $\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

3.1072.1 Optimal result

Integrand size = 30, antiderivative size = 358

$$\begin{aligned}\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx &= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n}\\
&- \frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n}\\
&+ \frac{\left(63b^2c^2+14abcd+3a^2d^2\right)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n}\\
&- \frac{3(3bc+ad)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n} + \frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bdn}\\
&- \frac{(bc-ad)^3\left(63b^2c^2+14abcd+3a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{128b^{5/2}d^{11/2}n}\end{aligned}$$

output
```
-1/128*(-a*d+b*c)^3*(3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*arctanh(d^(1/2)*(a+b
*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(11/2)/n-1/192*(-a*d+b*c)*(
3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b^2/d^4/n
+1/240*(3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b
^2/d^3/n-3/40*(a*d+3*b*c)*(a+b*x^n)^(7/2)*(c+d*x^n)^(1/2)/b^2/d^2/n+1/5*x^
n*(a+b*x^n)^(7/2)*(c+d*x^n)^(1/2)/b/d/n+1/128*(-a*d+b*c)^2*(3*a^2*d^2+14*a
*b*c*d+63*b^2*c^2)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b^2/d^5/n
```

3.1072.2 Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.77

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx=\frac{\sqrt{c+dx^n}\Bigg(-\frac{24(3bc+ad)(a+bx^n)^4}{bd}+64x^n(a+bx^n)^4+\frac{5(bc-ad)^3(63b^2c^2+14abcd+3a^2d^2)\Big(-}{}}{320bdn\sqrt{a+bx^n}}$$

input
```
Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

output
```
(Sqrt[c + d*x^n]*((-24*(3*b*c + a*d)*(a + b*x^n)^4)/(b*d) + 64*x^n*(a + b*
x^n)^4 + (5*(b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*((-2*d*(a
+ b*x^n))/(-(b*c) + a*d) - (4*d^2*(a + b*x^n)^2)/(3*(b*c - a*d)^2) - (16*d
^3*(a + b*x^n)^3)/(15*(-(b*c) + a*d)^3) - (2*Sqrt[d]*Sqrt[a + b*x^n]*ArcSi
nh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(Sqrt[b*c - a*d]*Sqrt[(b*(c
 + d*x^n))/(b*c - a*d)])))/(4*b*d^5)))/(320*b*d*n*Sqrt[a + b*x^n])
```

3.1072.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.82, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {948, 101, 27, 90, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{2n}(bx^n+a)^{5/2}}{\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{101}$$

$$\frac{\frac{\int -\frac{(bx^n+a)^{5/2}(3(3bc+ad)x^n+2ac)}{2\sqrt{dx^n+c}}dx^n}{5bd}+\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}-\frac{\int\frac{(bx^n+a)^{5/2}(3(3bc+ad)x^n+2ac)}{\sqrt{dx^n+c}}dx^n}{10bd}}{n}$$

$$\Big\downarrow 90$$

$$\frac{\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}-\frac{\frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\int\frac{(bx^n+a)^{5/2}}{\sqrt{dx^n+c}}dx^n}{8bd}}{10bd}}{n}$$

$$\Big\downarrow 60$$

$$\frac{\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}-\frac{\frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\int\frac{(bx^n+a)^{3/2}}{\sqrt{dx^n+c}}dx^n}{6d}\right)}{8bd}}{10bd}}{n}$$

$$\Big\downarrow 60$$

$$\frac{\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}-\frac{\frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\int}{}\right.}{6d}\right.}{8bd}}{10bd}}{n}$$

$$\Big\downarrow 60$$

$$\frac{\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}-\frac{\frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\right.}{}\right.}{}\right.}{8bd}}{10bd}}{n}$$

$$\Big\downarrow 66$$

3.1072. $\int\frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

$$\frac{\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}-\frac{\frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)}{}\right)}{}\right)}{8bd}}{10bd}}{n}$$

$$\Big\downarrow 221$$

$$\frac{\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}-\frac{\frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)}{}\right)}{}\right)}{8bd}}{10bd}}{n}$$

input
```
Int[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

output
```
((x^n*(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(5*b*d) - ((3*(3*b*c + a*d)*(a +
b*x^n)^(7/2)*Sqrt[c + d*x^n])/(4*b*d) - ((63*b^2*c^2 + 14*a*b*c*d + 3*a^2*
d^2)*(((a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(3*d) - (5*(b*c - a*d)*(((a + b*
x^n)^(3/2)*Sqrt[c + d*x^n])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x^n]*Sqrt[
c + d*x^n])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sq
rt[c + d*x^n])])/(Sqrt[b]*d^(3/2))))/(4*d)))/(6*d)))/(8*b*d))/(10*b*d))/n
```

3.1072. $\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

3.1072.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1072.4 Maple [F]

$$\int \frac{x^{-1+3n}(a+b\,x^n)^{\frac{5}{2}}}{\sqrt{c+d\,x^n}}dx$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

3.1072.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 771, normalized size of antiderivative = 2.15

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \left[-\frac{15\,(63\,b^5c^5-175\,ab^4c^4d+150\,a^2b^3c^3d^2-30\,a^3b^2c^2d^3-5\,a^4bcd^4-3\,a^5d^5)\sqrt{}}{}\right.$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[-1/7680*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*
b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) +
 b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt
(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(3
84*b^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2
*d^3 - 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x
^(3*n) + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^3*d^5)*x^(2*n) - 2
*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5
)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^6*n), 1/3840*(15*(63*b^5*c^
5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c
*d^4 - 3*a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d
)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d +
 (b^2*c*d + a*b*d^2)*x^n)) + 2*(384*b^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310
*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 -
144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^(3*n) + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*
d^4 + 93*a^2*b^3*d^5)*x^(2*n) - 2*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 4
81*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(
b^3*d^6*n)]
```

3.1072.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

output
```
Timed out
```

3.1072.7 Maxima [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((b*x^n + a)^(5/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

3.1072.8 Giac [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(5/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

3.1072.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx=\int \frac{x^{3\,n-1}\,{\left(a+b\,x^n\right)}^{5/2}}{\sqrt{c+d\,x^n}}\,dx$$

input
```
int((x^(3*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2),x)
```

output
```
int((x^(3*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2), x)
```

3.1073 $\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$

3.1073.1 Optimal result

Integrand size $= 30$, antiderivative size $= 291$

$$\begin{aligned}\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx &= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n}\\ &+\frac{\left(35b^2c^2+10abcd+3a^2d^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n}\\ &-\frac{(7bc+3ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n}+\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn}\\ &+\frac{(bc-ad)^2\left(35b^2c^2+10abcd+3a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{5/2}d^{9/2}n}\end{aligned}$$

output
```
1/64*(-a*d+b*c)^2*(3*a^2*d^2+10*a*b*c*d+35*b^2*c^2)*arctanh(d^(1/2)*(a+b*x
^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(9/2)/n+1/96*(3*a^2*d^2+10*a*
b*c*d+35*b^2*c^2)*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b^2/d^3/n-1/24*(3*a*d+7*
b*c)*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b^2/d^2/n+1/4*x^n*(a+b*x^n)^(5/2)*(c+
d*x^n)^(1/2)/b/d/n-1/64*(-a*d+b*c)*(3*a^2*d^2+10*a*b*c*d+35*b^2*c^2)*(a+b*
x^n)^(1/2)*(c+d*x^n)^(1/2)/b^2/d^4/n
```

3.1073.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{-b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(9a^3d^3+3a^2bd^2(5c-2dx^n)-ab^2d(145c^2-92cdx^n+\right.}{}$$

input
```
Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

output
```
(-(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(9*a^3*d^3 + 3*a^2*b*d^2*(5*c - 2
*d*x^n) - a*b^2*d*(145*c^2 - 92*c*d*x^n + 72*d^2*x^(2*n)) + b^3*(105*c^3 -
 70*c^2*d*x^n + 56*c*d^2*x^(2*n) - 48*d^3*x^(3*n)))) + 3*(b*c - a*d)^(5/2)
*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*A
rcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(192*b^3*d^(9/2)*n*Sqrt
[c + d*x^n])
```

3.1073.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {948, 101, 27, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{2n}(bx^n+a)^{3/2}}{\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{101}$$

$$\frac{\frac{\int -\frac{(bx^n+a)^{3/2}((7bc+3ad)x^n+2ac)}{2\sqrt{dx^n+c}}dx^n}{4bd}+\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bd}}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bd}-\frac{\int \frac{(bx^n+a)^{3/2}((7bc+3ad)x^n+2ac)}{\sqrt{dx^n+c}}dx^n}{8bd}}{n}$$

$$\Big\downarrow 90$$

$$\frac{\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bd}-\frac{\frac{(3ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)\int\frac{(bx^n+a)^{3/2}}{\sqrt{dx^n+c}}dx^n}{6bd}}{8bd}}{n}$$

$$\Big\downarrow 60$$

$$\frac{\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bd}-\frac{\frac{(3ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\int\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}dx^n}{4d}\right)}{6bd}}{8bd}}{n}$$

$$\Big\downarrow 60$$

$$\frac{\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bd}-\frac{\frac{(3ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int}{}\right.}{4d}\right.}{6bd}}{8bd}}{n}$$

$$\Big\downarrow 66$$

$$\frac{\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bd}-\frac{\frac{(3ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int}{}\right.}{4d}\right.}{6bd}}{8bd}}{n}$$

$$\Big\downarrow 221$$

$$\frac{\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bd}-\frac{\frac{(3ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\text{arct}}{}\right.}{4d}\right.}{6bd}}{8bd}}{n}$$

input
```
Int[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

3.1073. $\int\frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$

output
```
((x^n*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(4*b*d) - (((7*b*c + 3*a*d)*(a +
b*x^n)^(5/2)*Sqrt[c + d*x^n])/(3*b*d) - ((35*b^2*c^2 + 10*a*b*c*d + 3*a^2*
d^2)*(((a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(2*d) - (3*(b*c - a*d)*((Sqrt[a
+ b*x^n]*Sqrt[c + d*x^n])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n
])/(Sqrt[b]*Sqrt[c + d*x^n])])/(Sqrt[b]*d^(3/2))))/(4*d)))/(6*b*d))/(8*b*d
))/n
```

3.1073.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1073.4 Maple [F]

$$\int \frac{x^{-1+3n}(a+b\,x^{n})^{\frac{3}{2}}}{\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

3.1073.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.09

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \Bigg[\frac{3\,(35\,b^4c^4-60\,ab^3c^3d+18\,a^2b^2c^2d^2+4\,a^3bcd^3+3\,a^4d^4)\sqrt{bd}\log\Big(8\,b^2d^2x^{2\,n}+}{}$$

$$-\frac{3\,(35\,b^4c^4-60\,ab^3c^3d+18\,a^2b^2c^2d^2+4\,a^3bcd^3+3\,a^4d^4)\sqrt{-bd}\arctan\Big(\frac{(2\,\sqrt{-bd}bdx^n+(bc+ad)\sqrt{-bd})\sqrt{bx^n+a}\sqrt{dx^n+}}{2\,(b^2d^2x^{2\,n}+abcd+(b^2cd+abd^2)x^n)}}{}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/768*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^
3 + 3*a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2
*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqr
t(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*(48*b^4*d^4*x^(3*n) - 105*b^
4*c^3*d + 145*a*b^3*c^2*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*
d^3 - 9*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^
2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^5*n), -1/384*(3*(35*b^
4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*s
qrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(
b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2
)*x^n)) - 2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 145*a*b^3*c^2*d^2 - 15*a
^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4)*x^(2*n) + 2*(35
*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d
*x^n + c))/(b^3*d^5*n)]
```

3.1073.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

output
```
Timed out
```

3.1073.7 Maxima [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

3.1073.8 Giac [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

3.1073.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}\,(a+b\,x^n)^{3/2}}{\sqrt{c+d\,x^n}}\,dx$$

input
```
int((x^(3*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2),x)
```

output
```
int((x^(3*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2), x)
```

3.1074 $\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$

3.1074.1 Optimal result

Integrand size = 30, antiderivative size = 221

$$\begin{aligned}\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx &= \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} \\ &\quad - \frac{(5bc+3ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn} \\ &\quad - \frac{(bc-ad)(5b^2c^2+2abcd+a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{5/2}d^{7/2}n}\end{aligned}$$

output
```
-1/8*(-a*d+b*c)*(a^2*d^2+2*a*b*c*d+5*b^2*c^2)*arctanh(d^(1/2)*(a+b*x^n)^(1
/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(7/2)/n-1/12*(3*a*d+5*b*c)*(a+b*x^n
)^(3/2)*(c+d*x^n)^(1/2)/b^2/d^2/n+1/3*x^n*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/
b/d/n+1/8*(a^2*d^2+2*a*b*c*d+5*b^2*c^2)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b^
2/d^3/n
```

3.1074.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$

$$= \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(-3a^2d^2+2abd(-2c+dx^n)+b^2(15c^2-10cdx^n+8d^2x^{2n})\right)-3(bc-ad)^{3/2}\left(5b^2c\right.}{24b^3d^{7/2}n\sqrt{c+dx^n}}$$

input
```
Integrate[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

output
```
(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*a^2*d^2 + 2*a*b*d*(-2*c + d*x^n
) + b^2*(15*c^2 - 10*c*d*x^n + 8*d^2*x^(2*n))) - 3*(b*c - a*d)^(3/2)*(5*b^
2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sq
rt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(24*b^3*d^(7/2)*n*Sqrt[c + d*x^n]
)
```

3.1074.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {948, 101, 27, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \; 948$$

$$\frac{\int \frac{x^{2n}\sqrt{bx^n+a}}{\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \; 101$$

$$\frac{\frac{\int -\frac{\sqrt{bx^n+a}((5bc+3ad)x^n+2ac)}{2\sqrt{dx^n+c}}dx^n}{3bd}+\frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bd}}{n}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bd}-\frac{\int \frac{\sqrt{bx^n+a}((5bc+3ad)x^n+2ac)}{\sqrt{dx^n+c}}dx^n}{6bd}}{n}$$

$$\Big\downarrow \; 90$$

$$\frac{\frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bd}-\frac{\frac{(3ad+5bc)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{3\left(a^2d^2+2abcd+5b^2c^2\right)\int \frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}dx^n}{4bd}}{6bd}}{n}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bd}-\frac{\frac{(3ad+5bc)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{3\left(a^2d^2+2abcd+5b^2c^2\right)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{2d}\right)}{4bd}}{6bd}}{n}$$

$$\Big\downarrow \color{blue}{66}$$

$$\frac{\frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bd}-\frac{\frac{(3ad+5bc)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{3\left(a^2d^2+2abcd+5b^2c^2\right)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int\frac{1}{b-\frac{dx^{2n}}{d}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{d}\right)}{4bd}}{6bd}}{n}$$

$$\Big\downarrow \color{blue}{221}$$

$$\frac{\frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bd}-\frac{\frac{(3ad+5bc)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{3\left(a^2d^2+2abcd+5b^2c^2\right)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{\sqrt{b}d^{3/2}}\right)}{4bd}}{6bd}}{n}$$

input
```
Int[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

output
```
((x^n*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(3*b*d) - (((5*b*c + 3*a*d)*(a +
b*x^n)^(3/2)*Sqrt[c + d*x^n])/(2*b*d) - (3*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^
2)*((Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sq
rt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(Sqrt[b]*d^(3/2))))/(4*b*d))/(6
*b*d))/n
```

3.1074.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

3.1074. $\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1074.4 Maple [F]

$$\int \frac{x^{-1+3n}\sqrt{a+b\,x^{n}}}{\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

3.1074.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.13

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$
$$= \left[-\frac{3\,(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2+4\left(2\,\sqrt{bd}bdx^n+(bc\right.\right.}{}\right.$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[-1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x
^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d +
 a*b*d^2)*x^n) - 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a
^2*b*d^3 - 2*(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c
))/(b^3*d^4*n), 1/48*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3
)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sq
rt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*
d^2)*x^n)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b
*d^3 - 2*(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(
b^3*d^4*n)]
```

3.1074.6 Sympy [F]

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n-1}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

output
```
Integral(x**(3*n - 1)*sqrt(a + b*x**n)/sqrt(c + d*x**n), x)
```

3.1074.7 Maxima [F]

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx=\int \frac{\sqrt{bx^n+a}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(b*x^n + a)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

3.1074.8 Giac [F]

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx=\int \frac{\sqrt{bx^n+a}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

3.1074.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx=\int \frac{x^{3\,n-1}\,\sqrt{a+b\,x^n}}{\sqrt{c+d\,x^n}}\,dx$$

input
```
int((x^(3*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2),x)
```

output
```
int((x^(3*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2), x)
```

3.1075 $\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$

3.1075.1 Optimal result

Integrand size = 30, antiderivative size = 150

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n}+\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{\left(4abcd-3(bc+ad)^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{5/2}d^{5/2}n}$$

output
```
-1/4*(4*a*b*c*d-3*(a*d+b*c)^2)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+
d*x^n)^(1/2))/b^(5/2)/d^(5/2)/n-3/4*(a*d+b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1
/2)/b^2/d^2/n+1/2*x^n*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d/n
```

3.1075.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$$

$$= \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(-3bc-3ad+2bdx^n\right)+\sqrt{bc-ad}(3b^2c^2+2abcd+3a^2d^2)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\operatorname{arcsinh}\Bigg(}{4b^3d^{5/2}n\sqrt{c+dx^n}}$$

input
```
Integrate[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

output
```
(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c - 3*a*d + 2*b*d*x^n) + Sqrt
[b*c - a*d]*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c
- a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^3*d^(5/2)
*n*Sqrt[c + d*x^n])
```

3.1075.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {948, 101, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{2n}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{101}$$

$$\frac{\frac{\int -\frac{3(bc+ad)x^n+2ac}{2\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{2bd}+\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bd}}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bd}-\frac{\int \frac{3(bc+ad)x^n+2ac}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{4bd}}{n}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bd}-\frac{\frac{\left(4abcd-3(ad+bc)^2\right)\int \frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{2bd}+\frac{3(ad+bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{bd}}{4bd}}{n}$$

$$\Big\downarrow \text{66}$$

$$\frac{\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bd}-\frac{\frac{\left(4abcd-3(ad+bc)^2\right)\int \frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{bd}+\frac{3(ad+bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{bd}}{4bd}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bd}-\frac{\frac{\left(4abcd-3(ad+bc)^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}}+\frac{3(ad+bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{bd}}{4bd}}{n}$$

input
```
Int[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

output
```
((x^n*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(2*b*d) - ((3*(b*c + a*d)*Sqrt[a +
b*x^n]*Sqrt[c + d*x^n])/(b*d) + ((4*a*b*c*d - 3*(b*c + a*d)^2)*ArcTanh[(Sq
rt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*d^(3/2)))/(4*b
*d))/n
```

3.1075.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1075.4 Maple [F]

$$\int \frac{x^{-1+3n}}{\sqrt{a+b\,x^{n}}\,\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

3.1075.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.41

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^{n}}\sqrt{c+dx^{n}}}\,dx$$
$$= \Biggl[\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\sqrt{bd}\log\Bigl(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2+4\left(2\,\sqrt{bd}bdx^n+(bc+ad)\sqrt{bd}\right)\surd}{16\,b^3d^3n}$$
$$-\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\sqrt{-bd}\arctan\left(\frac{\left(2\,\sqrt{-bd}bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\,(b^2d^2x^{2\,n}+abcd+(b^2cd+abd^2)x^n)}\right)-2\,(2\,b^2d^2x^n-3\,b^2cd-}{8\,b^3d^3n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/16*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n)
 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sq
rt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*
(2*b^2*d^2*x^n - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(
b^3*d^3*n), -1/8*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-b*d)*arctan(1/
2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x
^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(2*b^2*
d^2*x^n - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^3
*n)]
```

3.1075.6 Sympy [F]

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx=\int \frac{x^{3n-1}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

output
```
Integral(x**(3*n - 1)/(sqrt(a + b*x**n)*sqrt(c + d*x**n)), x)
```

3.1075.7 Maxima [F]

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx=\int \frac{x^{3\,n-1}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^(3*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

3.1075.8 Giac [F]

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx=\int \frac{x^{3\,n-1}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

3.1075.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx=\int \frac{x^{3\,n-1}}{\sqrt{a+b\,x^n}\,\sqrt{c+d\,x^n}}\,dx$$

input
```
int(x^(3*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)),x)
```

output
```
int(x^(3*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)), x)
```

3.1076 $\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$

3.1076.1 Optimal result

Integrand size = 30, antiderivative size = 133

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}$$

output
```
-(3*a*d+b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5
/2)/d^(3/2)/n-2*a^2*(c+d*x^n)^(1/2)/b^2/(-a*d+b*c)/n/(a+b*x^n)^(1/2)+(a+b*
x^n)^(1/2)*(c+d*x^n)^(1/2)/b^2/d/n
```

3.1076.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.39

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \frac{-b\sqrt{d}(c+dx^n)\left(-3a^2d+b^2cx^n+ab(c-dx^n)\right)+\sqrt{bc-ad}(b^2c^2+2abcd-}{b^3d^{3/2}(-bc+ad)n\sqrt{a+bx^n}\sqrt{c+}$$

input
```
Integrate[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

output
```
(-(b*Sqrt[d]*(c + d*x^n)*(-3*a^2*d + b^2*c*x^n + a*b*(c - d*x^n))) + Sqrt[
b*c - a*d]*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[(b*(c +
d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(
b^3*d^(3/2)*(-(b*c) + a*d)*n*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])
```

3.1076.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {948, 100, 27, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{\int \frac{x^{2n}}{(bx^n+a)^{3/2}\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{\frac{2\int -\frac{a(bc-ad)-b(bc-ad)x^n}{2\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{\int \frac{(bc-ad)(a-bx^n)}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{\int \frac{a-bx^n}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{b^2}-\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{-\frac{\frac{(3ad+bc)\int \frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{2d}-\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}}{b^2}-\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{ 66}$$

$$\frac{-\frac{\frac{(3ad+bc)\int \frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{d}-\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}}{b^2}-\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)\sqrt{a+bx^n}}-\frac{\frac{(3ad+bc)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{\sqrt{b}d^{3/2}}-\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}}{b^2}}{n}$$

input
```
Int[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

output
```
((-2*a^2*Sqrt[c + d*x^n])/(b^2*(b*c - a*d)*Sqrt[a + b*x^n]) - (-((Sqrt[a +
 b*x^n]*Sqrt[c + d*x^n])/d) + ((b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x
^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(Sqrt[b]*d^(3/2)))/b^2)/n
```

3.1076.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1076.4 Maple [F]

$$\int \frac{x^{-1+3n}}{(a+b\,x^{n})^{\frac{3}{2}}\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

3.1076.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(113) = 226$.

Time = 0.39 (sec) , antiderivative size = 540, normalized size of antiderivative = 4.06

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \left[\frac{4\left(ab^2cd-3\,a^2bd^2+(b^3cd-ab^2d^2)x^n\right)\sqrt{bx^n+a}\sqrt{dx^n+c}+\left(\left(b^3c^2+2\,ab^2\right.\right.}{}\right.$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output

```
[1/4*(4*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n +
 a)*sqrt(d*x^n + c) + ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(b*d)*x^n
 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n)
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqr
t(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^
5*c*d^2 - a*b^4*d^3)*n*x^n + (a*b^4*c*d^2 - a^2*b^3*d^3)*n), 1/2*(2*(a*b^2
*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n
 + c) + ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(-b*d)*x^n + (a*b^2*c^2
 + 2*a^2*b*c*d - 3*a^3*d^2)*sqrt(-b*d))*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n +
 (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n)
+ a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^5*c*d^2 - a*b^4*d^3)*n*x^n + (a
*b^4*c*d^2 - a^2*b^3*d^3)*n)]
```

3.1076.6 Sympy [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n-1}}{(a+bx^n)^{\frac{3}{2}}\sqrt{c+dx^n}}\,dx$$

input

```
integrate(x**(-1+3*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

output

```
Integral(x**(3*n - 1)/((a + b*x**n)**(3/2)*sqrt(c + d*x**n)), x)
```

3.1076.7 Maxima [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

input

```
integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output

```
integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

3.1076.8 Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx=\int \frac{x^{3n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

3.1076.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx=\int \frac{x^{3n-1}}{(a+b\,x^n)^{3/2}\sqrt{c+d\,x^n}}\,dx$$

input
```
int(x^(3*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)),x)
```

output
```
int(x^(3*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)), x)
```

3.1077 $\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$

3.1077.1 Optimal result

Integrand size = 30, antiderivative size = 147

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}n}$$

output
```
2*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/n/d^(1/
2)-2/3*a^2*(c+d*x^n)^(1/2)/b^2/(-a*d+b*c)/n/(a+b*x^n)^(3/2)+4/3*a*(-2*a*d+
3*b*c)*(c+d*x^n)^(1/2)/b^2/(-a*d+b*c)^2/n/(a+b*x^n)^(1/2)
```

3.1077.2 Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.48

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \frac{2\sqrt{c+dx^n}\Bigg(\frac{a^2}{-bc+ad}+\frac{(3b^2c^2-a^2d^2)(a+bx^n)}{d(bc-ad)^2}-\frac{3(a+bx^n)\bigg(\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}-\sqrt{d}\sqrt{a+bx^n}\mathrm{a}}{d\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}}}{3b^2n\,(a+bx^n)^{3/2}}$$

input
```
Integrate[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

output
```
(2*Sqrt[c + d*x^n]*(a^2/(-(b*c) + a*d) + ((3*b^2*c^2 - a^2*d^2)*(a + b*x^n
))/(d*(b*c - a*d)^2) - (3*(a + b*x^n)*(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n)
)/(b*c - a*d)] - Sqrt[d]*Sqrt[a + b*x^n]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])
/Sqrt[b*c - a*d]]))/(d*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]))
)/(3*b^2*n*(a + b*x^n)^(3/2))
```

3.1077.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {948, 100, 27, 87, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{2n}}{(bx^n+a)^{5/2}\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{100}$$

$$\frac{\frac{2\int -\frac{a(3bc-ad)-3b(bc-ad)x^n}{2(bx^n+a)^{3/2}\sqrt{dx^n+c}}dx^n}{3b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)(a+bx^n)^{3/2}}}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{a(3bc-ad)-3b(bc-ad)x^n}{(bx^n+a)^{3/2}\sqrt{dx^n+c}}dx^n}{3b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)(a+bx^n)^{3/2}}}{n}$$

$$\Big\downarrow \text{87}$$

$$\frac{-\frac{-3(bc-ad)\int \frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n-\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{(bc-ad)\sqrt{a+bx^n}}}{3b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)(a+bx^n)^{3/2}}}{n}$$

$$\Big\downarrow \text{66}$$

$$\frac{-\frac{-6(bc-ad)\int \frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}-\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{(bc-ad)\sqrt{a+bx^n}}}{3b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)(a+bx^n)^{3/2}}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)(a+bx^n)^{3/2}}-\frac{-\frac{6(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{\sqrt{b}\sqrt{d}}-\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{(bc-ad)\sqrt{a+bx^n}}}{3b^2(bc-ad)}}{n}$$

input
```
Int[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

output
```
((-2*a^2*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)*(a + b*x^n)^(3/2)) - ((-4*a*(
3*b*c - 2*a*d)*Sqrt[c + d*x^n])/((b*c - a*d)*Sqrt[a + b*x^n]) - (6*(b*c -
a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(Sqrt[b
]*Sqrt[d]))/(3*b^2*(b*c - a*d)))/n
```

3.1077.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.1077.4 Maple [F]

$$\int \frac{x^{-1+3n}}{(a+b\,x^{n})^{\frac{5}{2}}\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

3.1077.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(123) = 246$.

Time = 0.49 (sec) , antiderivative size = 769, normalized size of antiderivative = 5.23

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \left[\frac{4\left(5\,a^2b^2cd-3\,a^3bd^2+2\left(3\,ab^3cd-2\,a^2b^2d^2\right)x^n\right)\sqrt{bx^n+a}\sqrt{dx^n+c}+3\left(\right(}{}\right.$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/6*(4*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^n
)*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^
2)*sqrt(b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(b*d)
*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2
*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)
*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/
((b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*
a^2*b^5*c*d^2 + a^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^
4*b^3*d^3)*n), 1/3*(2*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^
2*b^2*d^2)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) - 3*((b^4*c^2 - 2*a*b^3*c*
d + a^2*b^2*d^2)*sqrt(-b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b
*d^2)*sqrt(-b*d)*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b*d))*a
rctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*
sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((
b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^
2*b^5*c*d^2 + a^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*
b^3*d^3)*n)]
```

3.1077.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

output
```
Timed out
```

3.1077.7 Maxima [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^(3*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

3.1077.8 Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx=\int \frac{x^{3\,n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

3.1077.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx=\int \frac{x^{3\,n-1}}{(a+b\,x^n)^{5/2}\sqrt{c+d\,x^n}}\,dx$$

input
```
int(x^(3*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)),x)
```

output
```
int(x^(3*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)), x)
```

3.1078 $\int x^p(b+cx)^p(b+2cx)\,dx$

3.1078.1 Optimal result

Integrand size = 17, antiderivative size = 20

$$\int x^p(b+cx)^p(b+2cx)\,dx=\frac{x^{1+p}(b+cx)^{1+p}}{1+p}$$

output
```
x^(p+1)*(c*x+b)^(p+1)/(p+1)
```

3.1078.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^p(b+cx)^p(b+2cx)\,dx=\frac{x^{1+p}(b+cx)^{1+p}}{1+p}$$

input
```
Integrate[x^p*(b + c*x)^p*(b + 2*c*x),x]
```

output
```
(x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)
```

3.1078.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^p(b+2cx)(b+cx)^p\,dx$$

$$\Big\downarrow \ 83$$

$$\frac{x^{p+1}(b+cx)^{p+1}}{p+1}$$

input
```
Int[x^p*(b + c*x)^p*(b + 2*c*x),x]
```

output
```
(x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)
```

3.1078.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

3.1078.4 Maple [A] (verified)

Time = 4.81 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{x^{1+p}(cx+b)^{1+p}}{1+p}$	21
risch	$\frac{x(cx+b)x^{p}(cx+b)^{p}}{1+p}$	23
parallelrisch	$\frac{x^{2}x^{p}(cx+b)^{p}bc+x\,x^{p}(cx+b)^{p}b^{2}}{b(1+p)}$	42

input
```
int(x^p*(c*x+b)^p*(2*c*x+b),x,method=_RETURNVERBOSE)
```

output
```
x^(1+p)*(c*x+b)^(1+p)/(1+p)
```

3.1078.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int x^p(b+cx)^p(b+2cx)\,dx=\frac{(cx^2+bx)(cx+b)^px^p}{p+1}$$

input
```
integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="fricas")
```

output
```
(c*x^2 + b*x)*(c*x + b)^p*x^p/(p + 1)
```

3.1078.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(15) = 30.

Time = 0.79 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int x^p(b+cx)^p(b+2cx)\,dx=\begin{cases}\frac{bxx^p(b+cx)^p}{p+1}+\frac{cx^2x^p(b+cx)^p}{p+1} & \text{for } p\neq -1\\ \log{(x)}+\log{\left(\frac{b}{c}+x\right)} & \text{otherwise}\end{cases}$$

input
```
integrate(x**p*(c*x+b)**p*(2*c*x+b),x)
```

output
```
Piecewise((b*x*x**p*(b + c*x)**p/(p + 1) + c*x**2*x**p*(b + c*x)**p/(p + 1
), Ne(p, -1)), (log(x) + log(b/c + x), True))
```

3.1078.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int x^p(b+cx)^p(b+2cx)\,dx=\frac{(cx^2+bx)e^{(p\log(cx+b)+p\log(x))}}{p+1}$$

input
```
integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="maxima")
```

output
```
(c*x^2 + b*x)*e^(p*log(c*x + b) + p*log(x))/(p + 1)
```

3.1078.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int x^p(b+cx)^p(b+2cx)\,dx=\frac{(cx+b)^p cx^2x^p+(cx+b)^p bxx^p}{p+1}$$

input
```
integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="giac")
```

output
```
((c*x + b)^p*c*x^2*x^p + (c*x + b)^p*b*x*x^p)/(p + 1)
```

3.1078.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^p(b+cx)^p(b+2cx)\,dx=\frac{x\,x^p\,(b+c\,x)^p\,(b+c\,x)}{p+1}$$

input
```
int(x^p*(b + c*x)^p*(b + 2*c*x),x)
```

output
```
(x*x^p*(b + c*x)^p*(b + c*x))/(p + 1)
```

3.1079 $\int x^{-1+2(1+p)}(b+cx^2)^p\,(b+2cx^2)\;dx$

3.1079.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx=\frac{x^{2(1+p)}\left(b+cx^2\right)^{1+p}}{2(1+p)}$$

output

```
1/2*x^(2+2*p)*(c*x^2+b)^(p+1)/(p+1)
```

3.1079.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx$$
$$=\frac{x^{2+2p}(b+cx^2)^p\left(1+\frac{cx^2}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^2}{b}\right)+2c(1+p)x^2\,\text{Hyper}\right.}{2(1+p)(2+p)}$$

input

```
Integrate[x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2),x]
```

output

```
(x^(2 + 2*p)*(b + c*x^2)^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p,
-((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x
^2)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

3.1079.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {356}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2(p+1)-1}\left(b+2cx^2\right)\left(b+cx^2\right)^p\,dx$$

$$\Big\downarrow 356$$

$$\frac{x^{2(p+1)}\left(b+cx^2\right)^{p+1}}{2(p+1)}$$

input
```
Int[x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2),x]
```

output
```
(x^(2*(1 + p))*(b + c*x^2)^(1 + p))/(2*(1 + p))
```

3.1079.3.1 Defintions of rubi rules used

rule 356
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] /;
 FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) -
b*c*(m + 2*p + 3), 0] && NeQ[m, -1]
```

3.1079.4 Maple [A] (verified)

Time = 5.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{x^{2+2p}\left(c\,x^{2}+b\right)^{1+p}}{2+2p}$	26
risch	$\frac{x\left(c\,x^{2}+b\right)x^{1+2p}\left(c\,x^{2}+b\right)^{p}}{2+2p}$	32
parallelrisch	$\frac{x^{3}x^{1+2p}\left(c\,x^{2}+b\right)^{p}bc+x\,x^{1+2p}\left(c\,x^{2}+b\right)^{p}b^{2}}{2b(1+p)}$	55

input
```
int(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(2+2*p)*(c*x^2+b)^(1+p)/(1+p)
```

3.1079.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx=\frac{(cx^3+bx)(cx^2+b)^px^{2\,p+1}}{2\,(p+1)}$$

input
```
integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="fricas")
```

output
```
1/2*(c*x^3 + b*x)*(c*x^2 + b)^p*x^(2*p + 1)/(p + 1)
```

3.1079.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(20) = 40.

Time = 26.65 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx=\begin{cases}\frac{bxx^{2p+1}(b+cx^2)^p}{2p+2}+\frac{cx^3x^{2p+1}(b+cx^2)^p}{2p+2} & \text{for}\ p\neq -1\\ \log{(x)}+\frac{\log{\left(x-\sqrt{-\frac{b}{c}}\right)}}{2}+\frac{\log{\left(x+\sqrt{-\frac{b}{c}}\right)}}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(1+2*p)*(c*x**2+b)**p*(2*c*x**2+b),x)
```

output
```
Piecewise((b*x*x**(2*p + 1)*(b + c*x**2)**p/(2*p + 2) + c*x**3*x**(2*p + 1
)*(b + c*x**2)**p/(2*p + 2), Ne(p, -1)), (log(x) + log(x - sqrt(-b/c))/2 +
 log(x + sqrt(-b/c))/2, True))
```

3.1079.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx=\frac{(cx^4+bx^2)e^{(p\log(cx^2+b)+2\,p\log(x))}}{2\,(p+1)}$$

input
```
integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="maxima")
```

output
```
1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

3.1079.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(25) = 50.

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.93

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx =\frac{(cx^2+b)^p cx^3e^{(2\,p\log(x)+\log(x))}+(cx^2+b)^p bxe^{(2\,p\log(x)+\log(x))}}{2\,(p+1)}$$

input
```
integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="giac")
```

output
```
1/2*((c*x^2 + b)^p*c*x^3*e^(2*p*log(x) + log(x)) + (c*x^2 + b)^p*b*x*e^(2*
p*log(x) + log(x)))/(p + 1)
```

3.1079.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx=\left(c\,x^2+b\right)^p\left(\frac{c\,x^{2\,p+1}\,x^3}{2\,p+2}+\frac{b\,x\,x^{2\,p+1}}{2\,p+2}\right)$$

input
```
int(x^(2*p + 1)*(b + c*x^2)^p*(b + 2*c*x^2),x)
```

output
```
(b + c*x^2)^p*((c*x^(2*p + 1)*x^3)/(2*p + 2) + (b*x*x^(2*p + 1))/(2*p + 2)
)
```

3.1080 $\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx$

3.1080.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{x^{3(1+p)}\left(b+cx^3\right)^{1+p}}{3(1+p)}$$

output
```
1/3*x^(3+3*p)*(c*x^3+b)^(p+1)/(p+1)
```

3.1080.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx$$
$$=\frac{x^{3+3p}(b+cx^3)^p\left(1+\frac{cx^3}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^3}{b}\right)+2c(1+p)x^3\,\text{Hyper}\right.}{3(1+p)(2+p)}$$

input
```
Integrate[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3),x]
```

output
```
(x^(3 + 3*p)*(b + c*x^3)^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p,
-((c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x
^3)/b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

3.1080.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3(p+1)-1}\left(b+2cx^3\right)\left(b+cx^3\right)^p\,dx$$

$$\Big\downarrow \textcolor{blue}{951}$$

$$\frac{x^{3(p+1)}\left(b+cx^3\right)^{p+1}}{3(p+1)}$$

input

```
Int[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3),x]
```

output

```
(x^(3*(1 + p))*(b + c*x^3)^(1 + p))/(3*(1 + p))
```

3.1080.3.1 Defintions of rubi rules used

rule 951

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

3.1080.4 Maple [A] (verified)

Time = 6.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{x^{3+3p}\left(c\,x^{3}+b\right)^{1+p}}{3+3p}$	26
risch	$\frac{x\left(c\,x^{3}+b\right)x^{2+3p}\left(c\,x^{3}+b\right)^{p}}{3+3p}$	32
parallelrisch	$\frac{x^{4}x^{2+3p}\left(c\,x^{3}+b\right)^{p}c^{2}+x\,x^{2+3p}\left(c\,x^{3}+b\right)^{p}b c}{3c(1+p)}$	55

input

```
int(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(3+3*p)*(c*x^3+b)^(1+p)/(1+p)
```

3.1080.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{(cx^4+bx)(cx^3+b)^p x^{3p+2}}{3(p+1)}$$

input
```
integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="fricas")
```

output
```
1/3*(c*x^4 + b*x)*(c*x^3 + b)^p*x^(3*p + 2)/(p + 1)
```

3.1080.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\text{Timed out}$$

input
```
integrate(x**(2+3*p)*(c*x**3+b)**p*(2*c*x**3+b),x)
```

output
```
Timed out
```

3.1080.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{(cx^6+bx^3)e^{(p\log(cx^3+b)+3p\log(x))}}{3(p+1)}$$

input
```
integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="maxima")
```

output
```
1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

3.1080.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx = \frac{(cx^3+b)^p cx^4 e^{(3\,p\log(x)+2\,\log(x))} + (cx^3+b)^p bxe^{(3\,p\log(x)+2\,\log(x))}}{3\,(p+1)}$$

input
```
integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="giac")
```

output
```
1/3*((c*x^3 + b)^p*c*x^4*e^(3*p*log(x) + 2*log(x)) + (c*x^3 + b)^p*b*x*e^(
3*p*log(x) + 2*log(x)))/(p + 1)
```

3.1080.9 Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx = \left(c\,x^3+b\right)^p\left(\frac{c\,x^{3\,p+2}\,x^4}{3\,p+3}+\frac{b\,x\,x^{3\,p+2}}{3\,p+3}\right)$$

input
```
int(x^(3*p + 2)*(b + c*x^3)^p*(b + 2*c*x^3),x)
```

output
```
(b + c*x^3)^p*((c*x^(3*p + 2)*x^4)/(3*p + 3) + (b*x*x^(3*p + 2))/(3*p + 3)
)
```

3.1081 $\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\;dx$

3.1081.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\;dx=\frac{x^{n(1+p)}(b+cx^n)^{1+p}}{n(1+p)}$$

output

```
x^(n*(p+1))*(b+c*x^n)^(p+1)/n/(p+1)
```

3.1081.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.74

$$\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\;dx$$
$$=\frac{(b+cx^n)^p\,(1+\frac{cx^n}{b})^{-p}\,(b(2+p)x^{n(1+p)}\text{ Hypergeometric2F1}\,(-p,1+p,2+p,-\frac{cx^n}{b})+2c(1+p)x^{n(2+p)}\text{ H}}{n(1+p)(2+p)}$$

input

```
Integrate[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n),x]
```

output

```
((b + c*x^n)^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p, 2 + p
, -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 + p, 3
 + p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*(1 + (c*x^n)/b)^p)
```

3.1081.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n(p+1)-1}(b+2cx^n)(b+cx^n)^p\,dx$$

$$\Big\downarrow \text{951}$$

$$\frac{x^{n(p+1)}(b+cx^n)^{p+1}}{n(p+1)}$$

input
```
Int[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n),x]
```

output
```
(x^(n*(1 + p))*(b + c*x^n)^(1 + p))/(n*(1 + p))
```

3.1081.3.1 Defintions of rubi rules used

rule 951
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

3.1081.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(27) = 54.

Time = 6.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

method	result	size
parallelrisch	$\frac{x\,x^{n}x^{np+n-1}(b+c\,x^{n})^{p}c^{2}+x\,x^{np+n-1}(b+c\,x^{n})^{p}bc}{n(1+p)c}$	60

input
```
int(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x,method=_RETURNVERBOSE)
```

output
```
(x*x^n*x^(n*p+n-1)*(b+c*x^n)^p*c^2+x*x^(n*p+n-1)*(b+c*x^n)^p*b*c)/n/(1+p)/
c
```

3.1081.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\;dx=\frac{(cxx^n+bx)(cx^n+b)^px^{np+n-1}}{np+n}$$

input
```
integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="fricas")
```

output
```
(c*x*x^n + b*x)*(c*x^n + b)^p*x^(n*p + n - 1)/(n*p + n)
```

3.1081.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.65 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.30

$$\begin{aligned}&\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\;dx\\&=\frac{b^{p+1}c^{-p-1}c^{p+1}x^{np+n}\Gamma(p+1)\,{}_2F_1\left(\begin{matrix}-p,p+1\\p+2\end{matrix}\middle|\frac{cx^ne^{i\pi}}{b}\right)}{n\Gamma\left(p+2\right)}\\&\quad+\frac{2b^{p+2}cc^{-p-2}c^{p+2}x^{np+2n}\Gamma(p+2)\,{}_2F_1\left(\begin{matrix}-p,p+2\\p+3\end{matrix}\middle|\frac{cx^ne^{i\pi}}{b}\right)}{b^2n\Gamma\left(p+3\right)}\end{aligned}$$

input
```
integrate(x**(-1+n*(1+p))*(b+c*x**n)**p*(b+2*c*x**n),x)
```

output
```
b**(p + 1)*c**(-p - 1)*c**(p + 1)*x**(n*p + n)*gamma(p + 1)*hyper((-p, p +
 1), (p + 2,), c*x**n*exp_polar(I*pi)/b)/(n*gamma(p + 2)) + 2*b**(p + 2)*c
*c**(-p - 2)*c**(p + 2)*x**(n*p + 2*n)*gamma(p + 2)*hyper((-p, p + 2), (p
+ 3,), c*x**n*exp_polar(I*pi)/b)/(b**2*n*gamma(p + 3))
```

3.1081.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int x^{-1+n(1+p)}(b+cx^n)^p\left(b+2cx^n\right)\,dx=\frac{(cx^{2\,n}+bx^n)e^{(np\log(x)+p\log(cx^n+b))}}{n(p+1)}$$

input
```
integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="maxima")
```

output
```
(c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))
```

3.1081.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int x^{-1+n(1+p)}(b+cx^n)^p\left(b+2cx^n\right)\,dx$$
$$=\frac{(cx^n+b)^pcxx^ne^{(np\log(x)+n\log(x)-\log(x))}+(cx^n+b)^pbxe^{(np\log(x)+n\log(x)-\log(x))}}{np+n}$$

input
```
integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="giac")
```

output
```
((c*x^n + b)^p*c*x*x^n*e^(n*p*log(x) + n*log(x) - log(x)) + (c*x^n + b)^p*
b*x*e^(n*p*log(x) + n*log(x) - log(x)))/(n*p + n)
```

3.1081.9 Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int x^{-1+n(1+p)}(b+cx^n)^p\left(b+2cx^n\right)\,dx=\left(\frac{b\,x\,x^{n\,(p+1)-1}}{n\,\left(p+1\right)}+\frac{c\,x\,x^n\,x^{n\,(p+1)-1}}{n\,\left(p+1\right)}\right)\,{\left(b+c\,x^n\right)}^p$$

input
```
int(x^(n*(p + 1) - 1)*(b + c*x^n)^p*(b + 2*c*x^n),x)
```

output
```
((b*x*x^(n*(p + 1) - 1))/(n*(p + 1)) + (c*x*x^n*x^(n*(p + 1) - 1))/(n*(p +
 1)))*(b + c*x^n)^p
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != −1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```